Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.2-Quartic-trinomial/115-1.2.2.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**1109**]. This is test number [115].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	99.82 (1107)	0.18 (2)
Rubi	99.55 (1104)	0.45 (5)
Maple	93.87 (1041)	6.13 (68)
Fricas	93.51 (1037)	6.49 (72)
Giac	81.15 (900)	18.85 (209)
Reduce	79.44 (881)	20.56 (228)
Mupad	68.44 (759)	31.56 (350)
Maxima	66.37 (736)	33.63 (373)
Sympy	46.08 (511)	53.92 (598)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

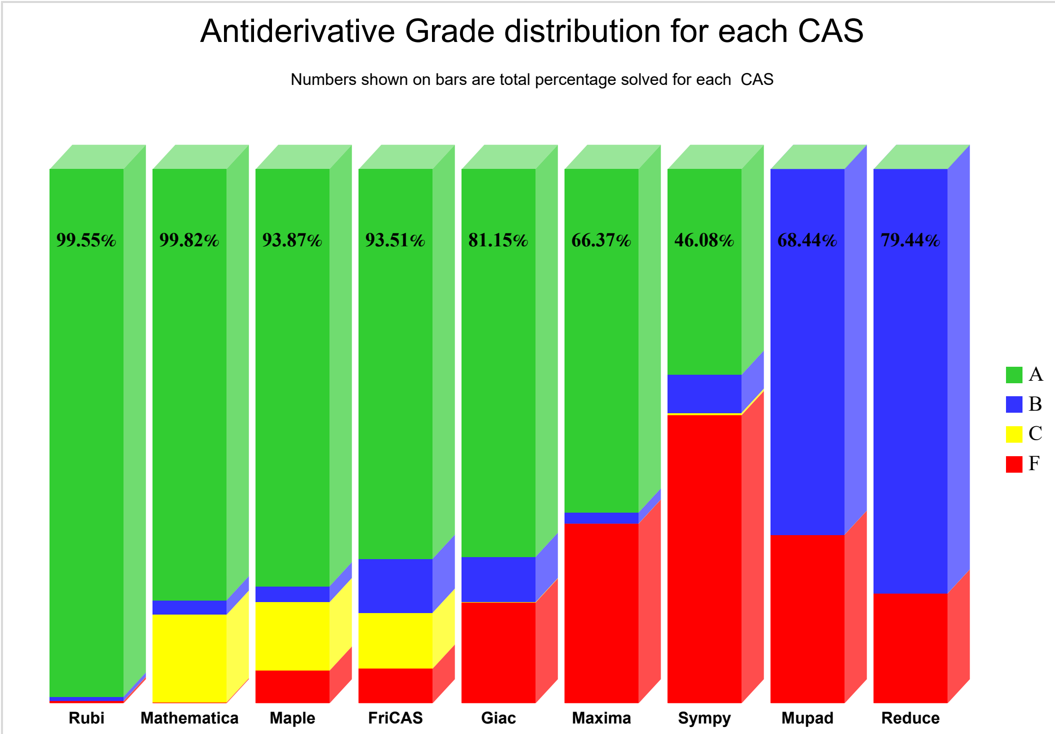
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

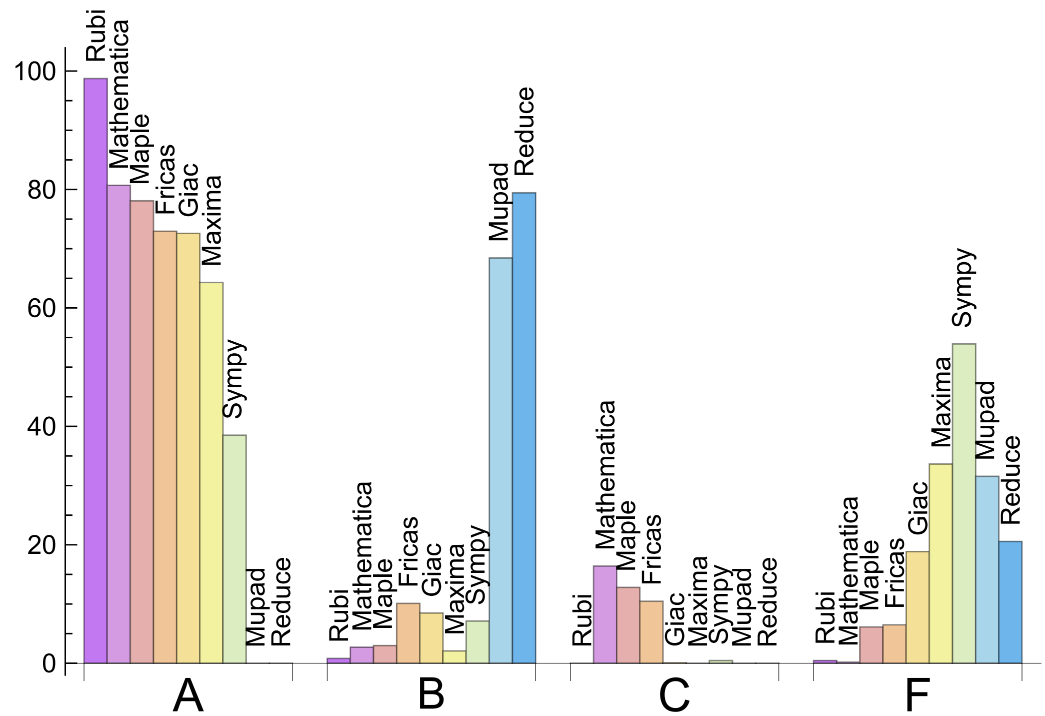
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.738	0.812	0.000	0.451
Mathematica	80.703	2.705	16.411	0.180
Maple	78.088	2.976	12.804	6.132
Fricas	72.949	10.099	10.460	6.492
Giac	72.588	8.476	0.090	18.846
Maxima	64.292	2.074	0.000	33.634
Sympy	38.503	7.124	0.451	53.922
Mupad	0.000	68.440	0.000	31.560
Reduce	0.000	79.441	0.000	20.559

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	2	100.00	0.00	0.00
Rubi	5	100.00	0.00	0.00
Maple	68	100.00	0.00	0.00
Fricas	72	100.00	0.00	0.00
Giac	209	99.04	0.00	0.96
Reduce	228	100.00	0.00	0.00
Mupad	350	0.00	100.00	0.00
Maxima	373	81.23	0.00	18.77
Sympy	598	82.44	17.56	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Reduce	0.21
Giac	0.24
Rubi	0.50
Fricas	0.59
Maple	0.84
Mathematica	1.21
Sympy	2.43
Mupad	10.09

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	87.42	0.87	54.00	0.88
Mathematica	108.99	0.92	78.00	0.90
Sympy	110.65	1.46	63.00	0.99
Maple	113.23	0.83	66.00	0.82
Rubi	147.17	1.04	91.00	1.00
Giac	192.25	1.31	69.00	0.90
Reduce	295.01	1.94	63.00	1.00
Fricas	502.66	2.05	75.00	1.00
Mupad	1382.05	4.19	64.00	0.88

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

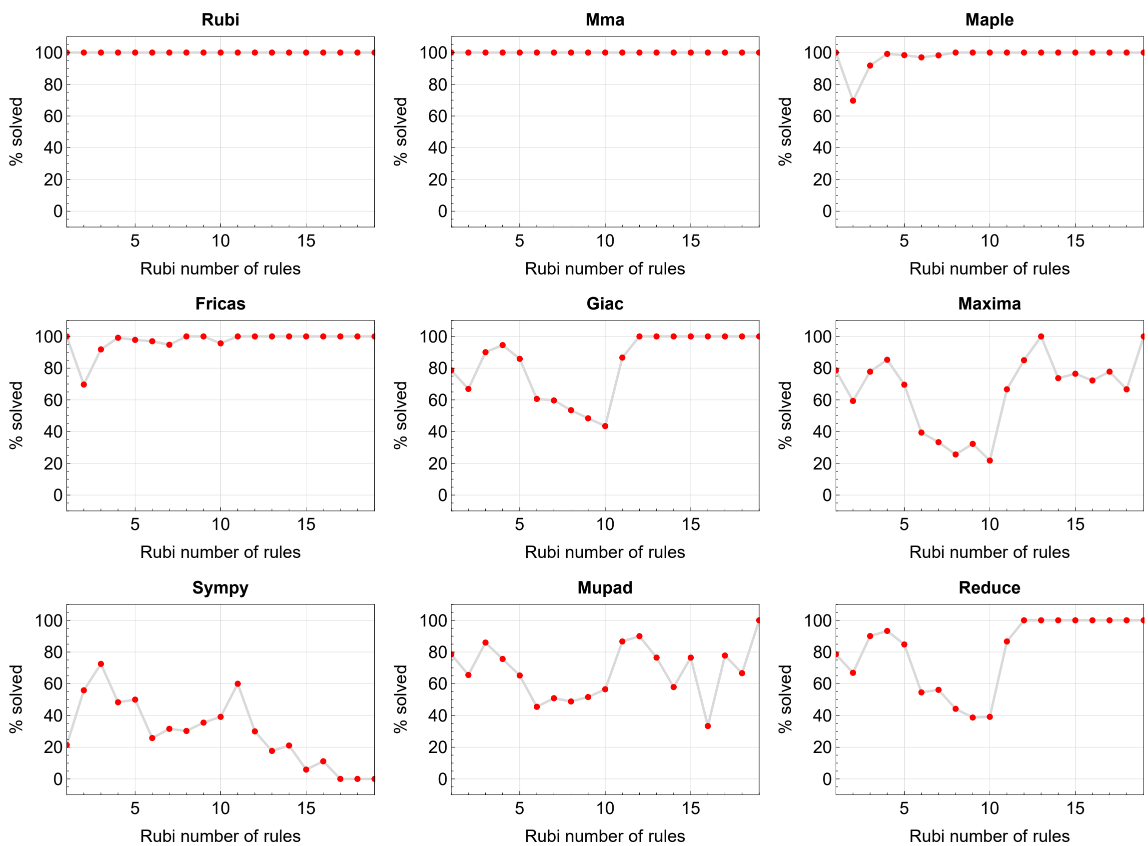


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

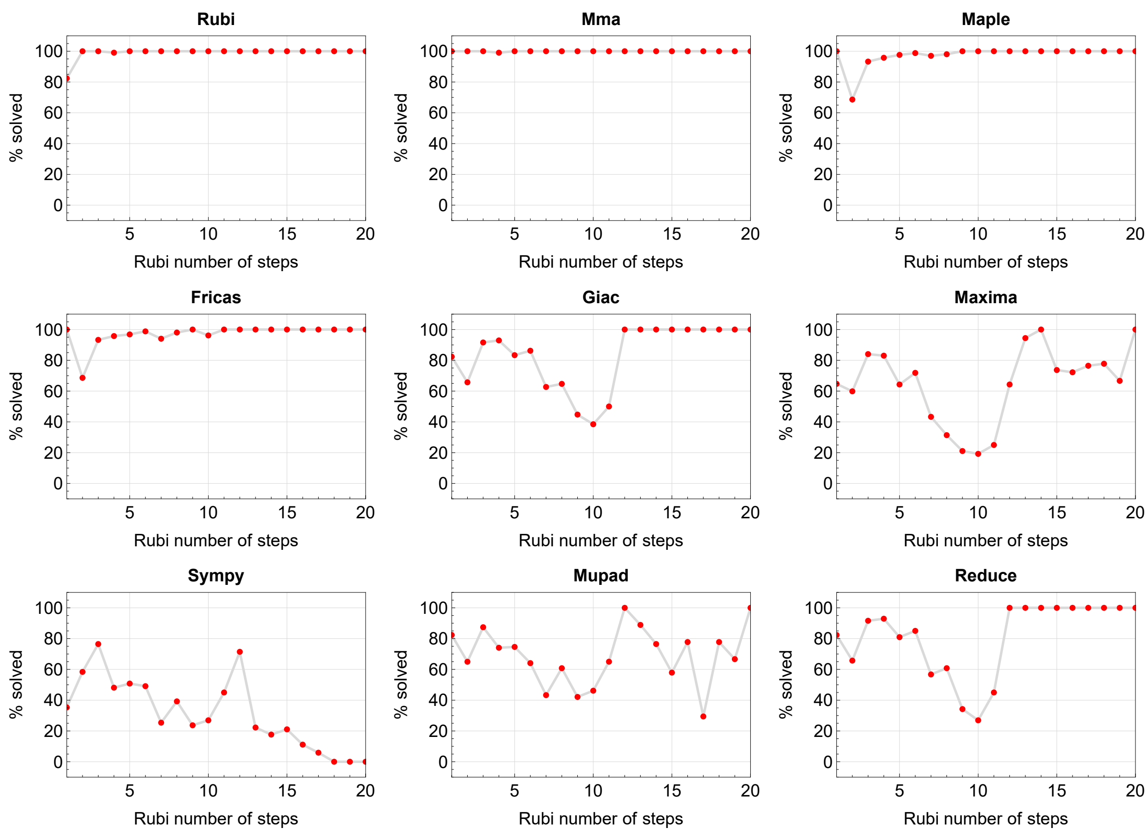


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

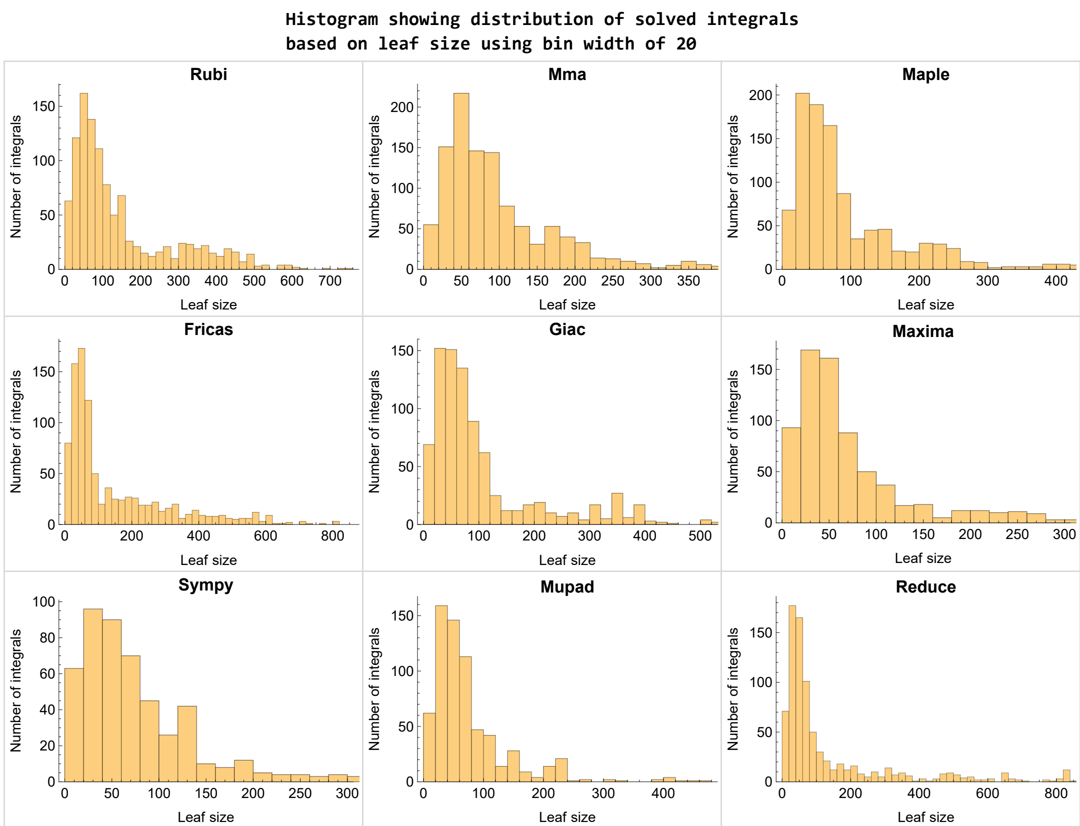


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

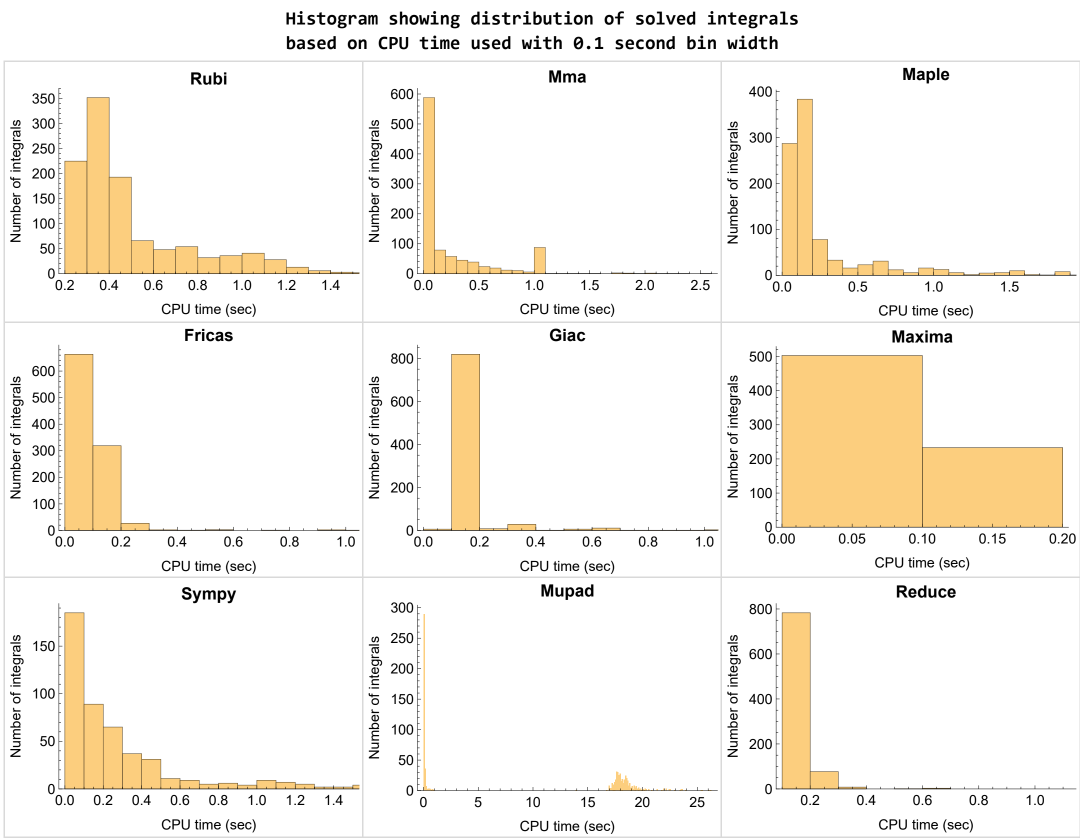


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

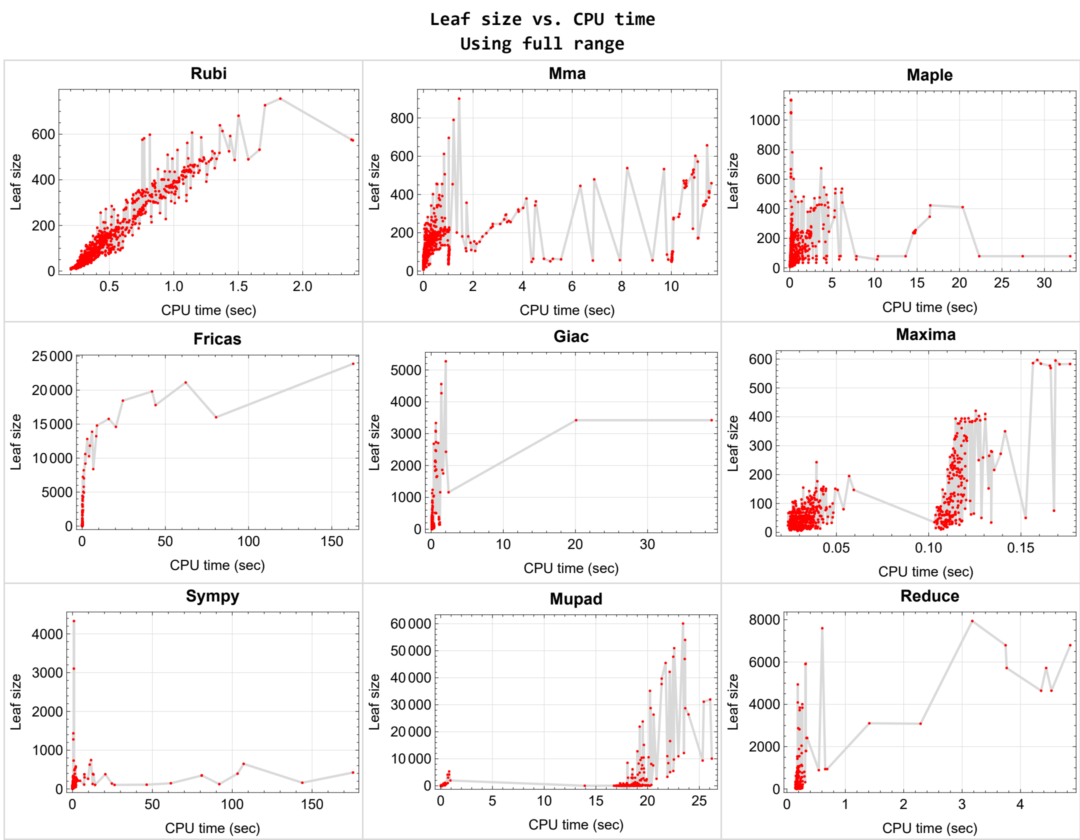


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {669, 670, 671, 1107}

Mathematica {1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1078, 1079, 1080, 1081, 1082, 1086, 1088, 1089, 1091, 1104, 1107, 1109}

Maple $\{4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 288, 289, 290, 291, 292, 293, 297, 298, 299, 300, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 320, 321, 322, 323, 324, 325, 326, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 351, 352, 353, 354, 355, 356, 357, 358, 488, 489, 490, 491, 492, 493, 494, 495, 496, 505, 506, 507, 508, 510, 511, 512, 513, 514, 515, 516, 517, 518, 532, 533, 534, 535, 536, 537, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 571, 572, 573, 574, 575, 581, 582, 583, 584, 585, 586, 593, 594, 595, 596, 597, 598, 599, 600, 733, 735\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
      leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

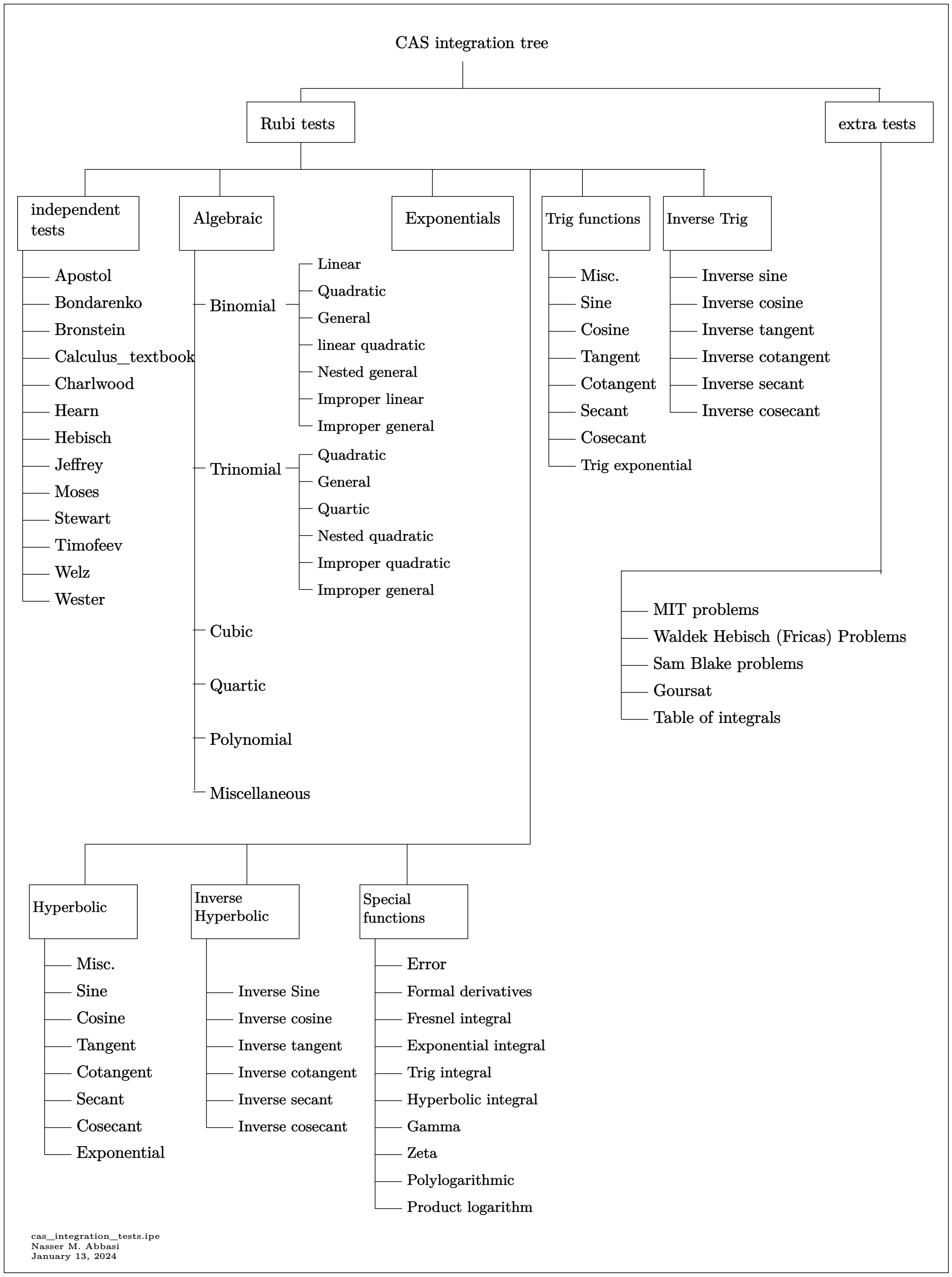
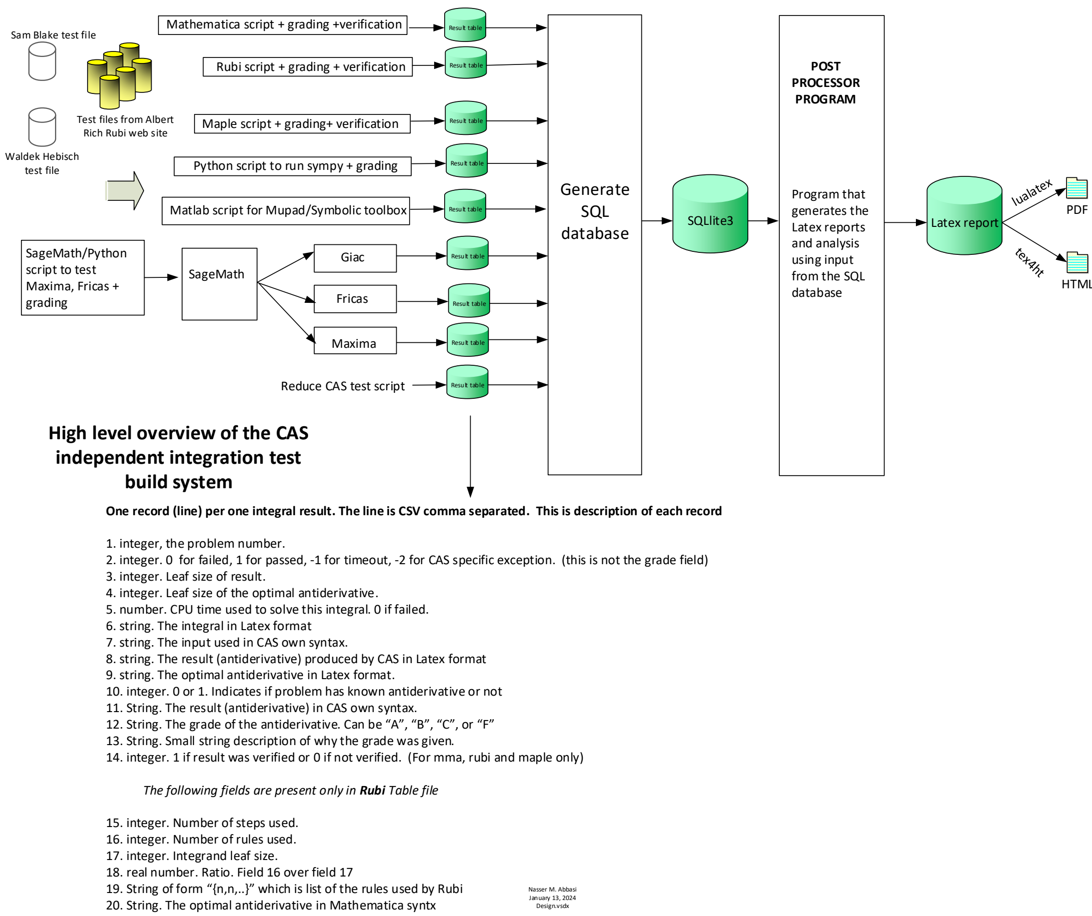


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462,

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871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889,
890, 891, 892, 893, 894, 896, 898, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911,
912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930,
931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949,
950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968,
969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987,
988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005,
1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020,
1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035,
1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050,
1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065,
1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080,
1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095,
1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1107, 1108, 1109 }

B grade { 703, 704, 705, 710, 712, 717, 718, 719, 720 }

C grade { }

F normal fail { 895, 897, 899, 1105, 1106 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 584, 587, 588, 589, 590, 591, 592, 593, 594, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 704, 711, 718, 725, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 844, 845, 846, 847, 848, 868, 869, 870, 871, 874, 875, 876, 877, 878, 879, 880, 889, 890, 891, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 982, 983, 984, 985, 986, 987, 988, 989, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1031, 1033, 1035, 1037, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1075, 1076, 1077, 1083, 1084, 1085, 1089, 1090, 1091, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1107, 1108, 1109 }

B grade { 34, 311, 331, 340, 399, 490, 491, 492, 512, 543, 583, 585, 586, 595, 596, 597, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1078, 1079, 1080, 1081, 1082, 1088 }

C grade { 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 666, 667, 668, 669, 670, 671, 701, 702, 703, 705, 706, 707, 708, 709, 710, 712, 713, 714, 715, 716, 717, 719, 720, 721, 722, 723, 724, 726, 727, 838, 839, 840, 841, 842, 843, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 872, 873, 881, 882, 883, 884, 885, 886, 887, 888, 892, 893, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 958, 959, 960, 961, 962, 963, 975, 976, 977, 978, 979, 980, 981, 990, 991, 992, 993, 994, 1003, 1004, 1005, 1006, 1007, 1016, 1017, 1018, 1019, 1020, 1030, 1032, 1034, 1036, 1038, 1086, 1087, 1092, 1093, 1094 }

F normal fail { 1105, 1106 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 497, 498, 499, 500, 501, 502, 503, 504, 509, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 538, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 576, 577, 578, 579, 580, 587, 588, 589, 590, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 647, 648, 649, 650, 651, 652, 663, 664, 665, 674, 678, 679, 680, 685, 686, 687, 688, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 774, 775, 776, 777, 778, 779, 780, 781, 782, 788, 789, 790, 791, 792, 793, 794, 795, 801, 802, 805, 806, 807, 808, 809, 816, 817, 818, 819, 820, 821, 825, 826, 827, 828, 830, 834, 835, 836, 837, 839, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 889, 890, 891, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1026, 1027, 1028, 1029, 1031, 1033, 1035, 1037, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1085 }

B grade { 34, 204, 279, 305, 311, 331, 332, 340, 380, 399, 641, 642, 643, 644, 645, 646, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 672, 673, 803, 804, 1025, 1083, 1084 }

C grade { 488, 489, 490, 491, 492, 493, 494, 495, 496, 505, 506, 507, 508, 510, 511, 512, 513, 514, 515, 516, 517, 518, 532, 533, 534, 535, 536, 537, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 571, 572, 573, 574, 575, 581, 582, 583, 584, 585, 586, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 770, 771, 772, 773, 783, 784, 785, 786, 787, 796, 797, 798, 799, 800, 810, 811, 812, 813, 814, 815, 822, 823, 824, 829, 831, 832, 833, 838, 840, 841, 842, 843, 881, 882, 883, 884, 885, 886, 887, 888, 892, 893, 894, 895, 896, 897, 898, 899, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 1030, 1032, 1034, 1036, 1038 }

F normal fail { 282, 283, 284, 666, 667, 668, 669, 670, 671, 675, 676, 677, 681, 682, 683, 684, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 381, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 401, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 539, 540, 541, 542, 543, 544, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 674, 678, 679, 680, 685, 686, 687, 688, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 776, 777, 778, 779, 780, 781, 782, 817, 818, 819, 820, 821, 826, 827, 828, 829, 830, 835, 836, 837, 838, 839, 841, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1085 }

B grade { 29, 34, 78, 305, 311, 331, 332, 340, 380, 382, 383, 384, 399, 400, 402, 403, 404, 405, 406, 538, 545, 595, 672, 673, 770, 771, 772, 773, 774, 775, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 822, 823, 824, 825, 831, 832, 833, 834, 840, 842, 843, 877, 894, 895, 896, 897, 898, 899, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 1013, 1083, 1084 }

C grade { 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 223, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 728 }

F normal fail { 282, 283, 284, 666, 667, 668, 669, 670, 671, 675, 676, 677, 681, 682, 683, 684, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 961, 978, 979, 980, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 170, 171, 172, 173, 174, 175, 178, 179, 180, 181, 182, 183, 184, 185, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 222, 223, 224, 225, 226, 227, 279, 280, 281, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 401, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 539, 540, 541, 542, 543, 544, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 642, 643, 644, 645, 655, 657, 659, 660, 661, 672, 673, 674, 678, 679, 680, 685, 686, 687, 688, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 826, 827, 828, 829, 830, 835, 836, 837, 838, 839, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 889, 890, 891, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1021, 1022, 1023, 1024, 1026, 1028, 1031, 1035, 1037, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1083, 1084, 1085 }

B grade { 29, 34, 78, 176, 177, 305, 311, 331, 332, 340, 380, 382, 399, 400, 402, 403, 538, 545, 595, 654, 656, 658, 1033 }

C grade { }

F normal fail { 166, 167, 168, 169, 186, 187, 188, 189, 190, 191, 204, 205, 206, 219, 220, 221, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 282, 283, 284, 638, 639, 640, 641, 646, 647, 648, 649, 650, 651, 652, 653, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 675, 676, 677, 681, 682, 683, 684, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 770, 771, 772, 773, 774, 775, 783, 784, 785, 786, 787, 788, 789, 796, 797, 798, 799, 800, 801, 810, 811, 812, 813, 814, 815, 816, 822, 823, 824, 825, 831, 832, 833, 834, 840, 841, 842, 843, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 892, 893, 894, 895, 896, 897, 898, 899, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 958, 959, 960, 961, 962, 963, 975, 976, 977, 978, 979, 980, 981, 990, 991, 992, 993, 994, 1003, 1004, 1005, 1006, 1007, 1016, 1017, 1018, 1019, 1020, 1025, 1027, 1029, 1030, 1032, 1034, 1036, 1038, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109 }

F(-1) timedout fail { }

F(-2) exception fail { 763, 764, 765, 766, 767, 768, 769, 776, 777, 778, 779, 780, 781, 782, 790, 791, 792, 793, 794, 795, 802, 803, 804, 805, 806, 807, 808, 809, 817, 818, 819, 820, 821, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 982, 983, 984, 985, 986, 987, 988, 989, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 281, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 539, 540, 541, 542, 543, 544, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 680, 686, 687, 688, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 776, 777, 778, 779, 780, 781, 782, 790, 791, 792, 793, 794, 795, 802, 803, 804, 805, 806, 807, 808, 809, 817, 818, 819, 820, 821, 826, 827, 828, 829, 830, 835, 836, 837, 838, 839, 841, 842, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 948, 949, 950, 951, 953, 969, 982, 983, 984, 985, 986, 987, 995, 996, 997, 998, 999, 1000, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1031, 1033, 1035, 1037, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066 }

B grade { 29, 34, 158, 159, 160, 176, 177, 178, 179, 180, 211, 212, 213, 214, 279, 280, 305, 311, 331, 332, 340, 399, 449, 450, 514, 538, 545, 672, 673, 674, 678, 679, 685, 770, 771, 772, 773, 774, 775, 783, 784, 785, 786, 787, 788, 789, 796, 797, 798, 799, 800, 801, 810, 811, 812, 813, 814, 815, 816, 822, 823, 824, 825, 831, 832, 833, 834, 840, 843, 879, 896, 897, 898, 899, 954, 955, 956, 957, 964, 965, 966, 967, 970, 971, 972, 973, 974, 988, 989, 1001, 1002, 1083, 1084, 1085 }

C grade { 223 }

F normal fail { 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 282, 283, 284, 666, 667, 668, 669, 670, 671, 675, 676, 677, 681, 682, 683, 684, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 958, 959, 960, 961, 962, 963, 975, 976, 977, 978, 979, 980, 981, 990, 991, 992, 993, 994, 1003, 1004, 1005, 1006, 1007, 1016, 1017, 1018, 1019, 1020, 1030, 1032, 1034, 1036, 1038, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109 }

F(-1) timedout fail { }

F(-2) exception fail { 952, 968 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 170, 171, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 279, 280, 281, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 501, 502, 503, 504, 508, 509, 514, 515, 516, 517, 518, 527, 528, 529, 530, 531, 538, 545, 546, 547, 548, 549, 550, 551, 564, 565, 566, 567, 568, 569, 570, 572, 573, 574, 575, 583, 584, 595, 596, 597, 598, 612, 613, 614, 615, 619, 620, 621, 622, 626, 627, 628, 629, 672, 673, 674, 680, 685, 686, 687, 688, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 966, 967, 984, 985, 986, 987, 997, 998, 999, 1000, 1010, 1011, 1012, 1021, 1022, 1023, 1024, 1026, 1027, 1028, 1031, 1034, 1035, 1036, 1037, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1083, 1084, 1085 }

C grade { }

F normal fail { }

F(-1) timedout fail { 157, 167, 168, 169, 172, 173, 174, 175, 186, 187, 188, 189, 190, 191, 192, 193, 204, 206, 207, 208, 219, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 282, 283, 284, 497, 498, 499, 500, 505, 506, 507, 510, 511, 512, 513, 519, 520, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 536, 537, 539, 540, 541, 542, 543, 544, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 571, 576, 577, 578, 579, 580, 581, 582, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 616, 617, 618, 623, 624, 625, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 675, 676, 677, 678, 679, 681, 682, 683, 684, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 988, 989, 990, 991, 992, 993, 994, 995, 996, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1025, 1029, 1030, 1032, 1033, 1038, 1039, 1051, 1052, 1053, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 172, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 333, 334, 335, 336, 337, 338, 339, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 376, 377, 378, 379, 381, 383, 384, 385, 386, 387, 388, 389, 392, 393, 394, 395, 396, 397, 398, 401, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 770, 771, 772, 773, 774, 775, 783, 784, 785, 786, 787, 788, 789, 797, 798, 799, 800, 822, 823, 824, 825, 828, 831, 832, 833, 834, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 1031, 1033, 1035, 1037, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066 }

B grade { 14, 29, 34, 54, 55, 56, 57, 58, 59, 70, 71, 78, 87, 279, 280, 281, 287, 305, 311, 331, 332, 340, 341, 372, 373, 380, 382, 390, 391, 399, 400, 402, 403, 672, 673, 674, 763, 764, 765, 766, 767, 768, 769, 776, 777, 778, 779, 780, 790, 791, 792, 793, 803, 804, 805, 806, 807, 812, 813, 817, 818, 819, 820, 821, 826, 827, 829, 830, 835, 836, 837, 838, 839, 877, 894, 895, 1083, 1084, 1085 }

C grade { 1030, 1032, 1034, 1036, 1038 }

F normal fail { 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 282, 283, 284, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 611, 612, 613, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 632, 633, 634, 635, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1108 }

F(-1) timedout fail { 117, 118, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 250, 251, 252, 253, 267, 268, 470, 471, 472, 473, 474, 475, 476, 496, 504, 609, 610, 614, 615, 630, 631, 636, 637, 638, 639, 650, 651, 652, 653, 654, 655, 781, 782, 794, 795, 796, 801, 802, 808, 809, 810, 811, 814, 815, 816, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 1103, 1104, 1105, 1106, 1107, 1109 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 279, 280, 281, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 672, 673, 674, 678, 679, 680, 685, 686, 687, 688, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 948, 949, 950, 951, 964, 965, 966, 967, 982, 983, 984, 985, 986, 995, 996, 997, 998, 999, 1008, 1009, 1010, 1011, 1012, 1013, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1031, 1033, 1035, 1037, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1083, 1084, 1085 }

C grade { }

F normal fail { 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 282, 283, 284, 666, 667, 668, 669, 670, 671, 675, 676, 677, 681, 682, 683, 684, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 987, 988, 989, 990, 991, 992, 993, 994, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1030, 1032, 1034, 1036, 1038, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.231	0.002	0.066	0.025	0.079	0.018	0.133	0.163	0.024

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.236	0.002	0.051	0.031	0.083	0.017	0.108	0.163	0.021

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.246	0.000	0.058	0.030	0.101	0.016	0.110	0.165	0.020

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	17	17	14	13	13	12	13	14	13
N.S.	1	1.06	1.06	0.88	0.81	0.81	0.75	0.81	0.88	0.81
time (sec)	N/A	0.253	0.002	0.065	0.029	0.099	0.017	0.106	0.165	0.022

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	1.00	0.83
time (sec)	N/A	0.241	0.000	0.030	0.025	0.095	0.033	0.106	0.178	0.018

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	14	11	11
N.S.	1	1.00	1.00	0.92	1.08	0.85	0.77	1.08	0.85	0.85
time (sec)	N/A	0.249	0.001	0.035	0.036	0.092	0.031	0.126	0.163	0.024

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	13	5	10	13	10
N.S.	1	1.00	1.00	1.10	1.00	1.30	0.50	1.00	1.30	1.00
time (sec)	N/A	0.243	0.001	0.036	0.025	0.083	0.032	0.138	0.161	0.025

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	20	17	11
N.S.	1	1.00	1.00	0.92	1.08	1.31	0.77	1.54	1.31	0.85
time (sec)	N/A	0.257	0.003	0.040	0.025	0.062	0.045	0.118	0.170	17.220

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	15	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	1.00	0.87
time (sec)	N/A	0.255	0.002	0.041	0.029	0.095	0.053	0.112	0.162	0.024

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	19	17	17	14	13	13	14	13	15	13
N.S.	1	0.89	0.89	0.74	0.68	0.68	0.74	0.68	0.79	0.68
time (sec)	N/A	0.256	0.002	0.042	0.025	0.145	0.051	0.103	0.171	0.026

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.245	0.002	0.042	0.036	0.070	0.061	0.102	0.163	0.027

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	24	24	24	24	26	24
N.S.	1	1.13	1.00	0.83	0.80	0.80	0.80	0.80	0.87	0.80
time (sec)	N/A	0.287	0.001	0.094	0.032	0.088	0.017	0.140	0.165	17.405

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	24	24	24	24	26	24
N.S.	1	1.13	1.00	0.83	0.80	0.80	0.80	0.80	0.87	0.80
time (sec)	N/A	0.289	0.001	0.082	0.030	0.085	0.018	0.112	0.169	0.034

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	24	24	24	24	25	24
N.S.	1	1.00	1.00	0.94	1.50	1.50	1.50	1.50	1.56	1.50
time (sec)	N/A	0.233	0.002	0.076	0.027	0.067	0.020	0.109	0.166	0.034

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	30	23	22	24	21	20	24	21	21
N.S.	1	1.30	1.00	0.96	1.04	0.91	0.87	1.04	0.91	0.91
time (sec)	N/A	0.277	0.001	0.071	0.033	0.113	0.048	0.134	0.167	17.233

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	27	24	24	27	24	32	27	23
N.S.	1	1.04	1.00	0.89	0.89	1.00	0.89	1.19	1.00	0.85
time (sec)	N/A	0.280	0.001	0.051	0.024	0.136	0.053	0.119	0.169	0.033

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	24	30	24	23	26	28	24	34	28	24
N.S.	1	1.25	1.00	0.96	1.08	1.17	1.00	1.42	1.17	1.00
time (sec)	N/A	0.277	0.001	0.054	0.032	0.107	0.078	0.110	0.160	17.311

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	24	24	26	24	26	26
N.S.	1	1.00	1.58	1.32	1.26	1.26	1.37	1.26	1.37	1.37
time (sec)	N/A	0.230	0.001	0.053	0.032	0.102	0.085	0.128	0.172	0.039

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	26	26	27	26	26	26
N.S.	1	1.13	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.281	0.001	0.056	0.026	0.097	0.095	0.125	0.169	0.040

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.283	0.001	0.078	0.031	0.098	0.020	0.111	0.164	0.033

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.270	0.001	0.095	0.024	0.110	0.019	0.148	0.178	0.039

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	24	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.96	0.84
time (sec)	N/A	0.268	0.001	0.046	0.025	0.083	0.034	0.134	0.167	0.033

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	25	22
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	1.04	0.92
time (sec)	N/A	0.267	0.001	0.050	0.027	0.092	0.039	0.151	0.162	0.037

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	22	26	22	22	26	24
N.S.	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.13	1.04
time (sec)	N/A	0.266	0.001	0.050	0.031	0.091	0.072	0.147	0.169	0.030

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	26	25
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.93	0.89
time (sec)	N/A	0.266	0.001	0.052	0.026	0.090	0.095	0.120	0.163	0.040

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.275	0.001	0.055	0.029	0.100	0.106	0.124	0.170	0.039

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	35	35	39	35	37	35
N.S.	1	1.09	1.00	0.84	0.81	0.81	0.91	0.81	0.86	0.81
time (sec)	N/A	0.303	0.002	0.099	0.032	0.070	0.022	0.175	0.164	0.048

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	43	36	35	35	37	35	37	35
N.S.	1	1.12	1.26	1.06	1.03	1.03	1.09	1.03	1.09	1.03
time (sec)	N/A	0.285	0.002	0.080	0.030	0.086	0.025	0.171	0.166	0.044

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	35	35	37	35	36	35
N.S.	1	1.00	1.00	0.94	2.19	2.19	2.31	2.19	2.25	2.19
time (sec)	N/A	0.223	0.002	0.081	0.029	0.082	0.024	0.123	0.179	0.044

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	39	34	36	33	37	36	33	33
N.S.	1	1.10	1.00	0.87	0.92	0.85	0.95	0.92	0.85	0.85
time (sec)	N/A	0.289	0.004	0.054	0.031	0.163	0.051	0.105	0.167	0.040

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	40	35	36	38	37	46	38	34
N.S.	1	1.05	1.00	0.88	0.90	0.95	0.92	1.15	0.95	0.85
time (sec)	N/A	0.287	0.007	0.074	0.025	0.086	0.061	0.106	0.170	0.038

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	41	40	35	37	39	37	46	39	37
N.S.	1	1.02	1.00	0.88	0.92	0.98	0.92	1.15	0.98	0.92
time (sec)	N/A	0.296	0.004	0.056	0.030	0.066	0.090	0.118	0.196	17.435

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	39	34	39	39	37	47	39	36
N.S.	1	1.10	1.00	0.87	1.00	1.00	0.95	1.21	1.00	0.92
time (sec)	N/A	0.293	0.004	0.056	0.026	0.100	0.114	0.128	0.163	0.050

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	35	35	37	35	37	37
N.S.	1	1.00	2.26	1.89	1.84	1.84	1.95	1.84	1.95	1.95
time (sec)	N/A	0.231	0.006	0.059	0.025	0.083	0.127	0.108	0.169	16.957

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	43	36	37	37	39	37	37	37
N.S.	1	1.10	1.08	0.90	0.92	0.92	0.98	0.92	0.92	0.92
time (sec)	N/A	0.255	0.004	0.062	0.032	0.098	0.140	0.160	0.164	16.983

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	37	37	39	37	37	37
N.S.	1	1.09	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.297	0.004	0.063	0.024	0.091	0.167	0.139	0.165	17.019

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	37	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.86	0.81
time (sec)	N/A	0.288	0.002	0.099	0.040	0.084	0.021	0.150	0.170	0.045

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	37	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.86	0.81
time (sec)	N/A	0.292	0.002	0.098	0.028	0.131	0.020	0.133	0.161	0.044

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	35	31
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	1.00	0.89
time (sec)	N/A	0.287	0.001	0.048	0.025	0.094	0.021	0.147	0.169	0.041

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	29	32	36	32
N.S.	1	1.00	1.00	0.97	0.94	1.06	0.85	0.94	1.06	0.94
time (sec)	N/A	0.286	0.004	0.057	0.025	0.112	0.041	0.129	0.169	0.044

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	36	36	34	36	36
N.S.	1	1.00	1.00	0.92	0.92	0.97	0.97	0.92	0.97	0.97
time (sec)	N/A	0.287	0.003	0.054	0.032	0.115	0.067	0.165	0.164	0.040

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	33	37	34	33	37	34
N.S.	1	1.00	1.00	0.97	0.97	1.09	1.00	0.97	1.09	1.00
time (sec)	N/A	0.304	0.005	0.054	0.047	0.105	0.098	0.128	0.167	0.032

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	37	37	39	37	37	35
N.S.	1	1.00	1.00	0.92	0.95	0.95	1.00	0.95	0.95	0.90
time (sec)	N/A	0.301	0.004	0.056	0.025	0.077	0.118	0.140	0.168	0.031

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37	37
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.291	0.004	0.058	0.027	0.087	0.131	0.130	0.168	16.956

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	52	53	46	46	45	44	47	45	45
N.S.	1	0.98	1.00	0.87	0.87	0.85	0.83	0.89	0.85	0.85
time (sec)	N/A	0.330	0.005	0.095	0.025	0.122	0.086	0.129	0.174	0.050

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	40	34	34	33	32	35	33	33
N.S.	1	0.98	1.00	0.85	0.85	0.82	0.80	0.88	0.82	0.82
time (sec)	N/A	0.306	0.005	0.096	0.028	0.092	0.065	0.112	0.166	0.047

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	23	22	20	24	22	22
N.S.	1	0.96	1.00	0.85	0.85	0.81	0.74	0.89	0.81	0.81
time (sec)	N/A	0.287	0.004	0.076	0.026	0.105	0.065	0.130	0.164	0.037

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87	0.87
time (sec)	N/A	0.235	0.002	0.073	0.026	0.095	0.045	0.128	0.170	0.032

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	22	21	23	18	15	22	20	18
N.S.	1	1.18	1.00	0.95	1.05	0.82	0.68	1.00	0.91	0.82
time (sec)	N/A	0.255	0.005	0.082	0.025	0.085	0.094	0.121	0.165	17.132

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	36	35	32	33	33	31	43	33	31
N.S.	1	1.03	1.00	0.91	0.94	0.94	0.89	1.23	0.94	0.89
time (sec)	N/A	0.295	0.006	0.090	0.024	0.084	0.121	0.147	0.164	16.993

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	50	49	44	47	45	42	57	47	46
N.S.	1	1.02	1.00	0.90	0.96	0.92	0.86	1.16	0.96	0.94
time (sec)	N/A	0.318	0.006	0.093	0.032	0.111	0.144	0.114	0.169	16.970

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	60	148	107	65	62	54
N.S.	1	1.00	1.00	0.88	0.88	2.18	1.57	0.96	0.91	0.79
time (sec)	N/A	0.332	0.021	0.161	0.104	0.168	0.088	0.110	0.163	0.033

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	50	126	95	55	51	43
N.S.	1	1.00	1.00	0.89	0.91	2.29	1.73	1.00	0.93	0.78
time (sec)	N/A	0.320	0.019	0.096	0.106	0.105	0.084	0.126	0.167	0.052

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	37	99	80	40	37	32
N.S.	1	1.00	1.00	0.90	0.88	2.36	1.90	0.95	0.88	0.76
time (sec)	N/A	0.298	0.014	0.093	0.104	0.155	0.073	0.134	0.170	0.048

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	26	23
N.S.	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.84	0.74
time (sec)	N/A	0.255	0.007	0.085	0.109	0.180	0.063	0.137	0.165	0.038

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	23	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.96	0.67
time (sec)	N/A	0.238	0.004	0.087	0.107	0.107	0.058	0.120	0.167	0.055

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	30	26
N.S.	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.88	0.76
time (sec)	N/A	0.261	0.010	0.102	0.110	0.123	0.074	0.128	0.165	16.979

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	51	43	39	40	106	87	40	43	37
N.S.	1	1.19	1.00	0.91	0.93	2.47	2.02	0.93	1.00	0.86
time (sec)	N/A	0.274	0.016	0.108	0.118	0.141	0.099	0.134	0.166	18.235

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	68	58	52	52	132	100	52	56	48
N.S.	1	1.17	1.00	0.90	0.90	2.28	1.72	0.90	0.97	0.83
time (sec)	N/A	0.292	0.019	0.106	0.112	0.119	0.122	0.119	0.168	18.164

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	56	49	55	54	70	53	67	70	57
N.S.	1	0.98	0.86	0.96	0.95	1.23	0.93	1.18	1.23	1.00
time (sec)	N/A	0.330	0.014	0.134	0.037	0.093	0.119	0.125	0.172	17.133

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	43	38	41	43	56	39	49	57	45
N.S.	1	0.98	0.86	0.93	0.98	1.27	0.89	1.11	1.30	1.02
time (sec)	N/A	0.310	0.012	0.092	0.036	0.109	0.127	0.105	0.163	17.022

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	31	27	30	32	35	29	32	44	29
N.S.	1	0.94	0.82	0.91	0.97	1.06	0.88	0.97	1.33	0.88
time (sec)	N/A	0.288	0.007	0.088	0.025	0.097	0.085	0.102	0.163	0.044

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	15	15	14	17	14
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.94	0.88	1.06	0.88
time (sec)	N/A	0.233	0.002	0.080	0.026	0.113	0.068	0.145	0.160	0.026

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	39	33	35	37	47	34	36	59	34
N.S.	1	1.03	0.87	0.92	0.97	1.24	0.89	0.95	1.55	0.89
time (sec)	N/A	0.304	0.014	0.096	0.032	0.173	0.138	0.120	0.179	0.050

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	52	41	53	52	73	51	50	81	51
N.S.	1	1.06	0.84	1.08	1.06	1.49	1.04	1.02	1.65	1.04
time (sec)	N/A	0.323	0.030	0.106	0.031	0.086	0.172	0.130	0.167	17.345

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	68	57	65	70	90	68	86	94	67
N.S.	1	1.03	0.86	0.98	1.06	1.36	1.03	1.30	1.42	1.02
time (sec)	N/A	0.348	0.038	0.115	0.027	0.173	0.206	0.112	0.162	17.651

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	82	71	65	71	190	124	73	96	66
N.S.	1	1.05	0.91	0.83	0.91	2.44	1.59	0.94	1.23	0.85
time (sec)	N/A	0.338	0.034	0.108	0.107	0.186	0.170	0.129	0.164	0.040

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	69	60	54	59	164	107	61	83	56
N.S.	1	1.06	0.92	0.83	0.91	2.52	1.65	0.94	1.28	0.86
time (sec)	N/A	0.320	0.032	0.099	0.107	0.118	0.135	0.130	0.160	0.059

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	58	51	42	45	136	83	42	69	43
N.S.	1	1.14	1.00	0.82	0.88	2.67	1.63	0.82	1.35	0.84
time (sec)	N/A	0.279	0.026	0.096	0.112	0.093	0.119	0.178	0.162	18.604

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	36	120	78	35	62	33
N.S.	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	1.38	0.73
time (sec)	N/A	0.261	0.015	0.100	0.108	0.095	0.093	0.115	0.158	0.042

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	61	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	1.36	0.73
time (sec)	N/A	0.254	0.018	0.097	0.109	0.114	0.095	0.121	0.166	13.925

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	54	45	49	136	92	47	72	44
N.S.	1	1.13	1.00	0.83	0.91	2.52	1.70	0.87	1.33	0.81
time (sec)	N/A	0.272	0.027	0.105	0.106	0.105	0.136	0.107	0.163	16.748

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	78	67	55	64	172	114	59	88	58
N.S.	1	1.16	1.00	0.82	0.96	2.57	1.70	0.88	1.31	0.87
time (sec)	N/A	0.308	0.028	0.113	0.106	0.125	0.190	0.111	0.165	18.478

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	95	80	67	75	198	126	70	101	70
N.S.	1	1.19	1.00	0.84	0.94	2.48	1.58	0.88	1.26	0.88
time (sec)	N/A	0.327	0.036	0.133	0.110	0.110	0.187	0.173	0.158	17.493

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	76	63	71	77	103	78	80	107	78
N.S.	1	1.03	0.85	0.96	1.04	1.39	1.05	1.08	1.45	1.05
time (sec)	N/A	0.367	0.018	0.144	0.029	0.156	0.205	0.131	0.165	0.061

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	62	48	54	66	91	68	62	95	68
N.S.	1	0.95	0.74	0.83	1.02	1.40	1.05	0.95	1.46	1.05
time (sec)	N/A	0.343	0.042	0.103	0.029	0.176	0.180	0.138	0.156	18.141

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	51	39	42	55	69	53	42	81	52
N.S.	1	1.04	0.80	0.86	1.12	1.41	1.08	0.86	1.65	1.06
time (sec)	N/A	0.316	0.013	0.099	0.026	0.082	0.147	0.146	0.165	0.061

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	28	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.47	1.95
time (sec)	N/A	0.234	0.006	0.095	0.036	0.123	0.128	0.137	0.166	18.284

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	26	26	27	14	25	28
N.S.	1	1.00	1.00	0.94	1.62	1.62	1.69	0.88	1.56	1.75
time (sec)	N/A	0.230	0.002	0.088	0.028	0.081	0.108	0.172	0.169	18.355

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	43	46	60	90	56	59	109	56
N.S.	1	1.02	0.80	0.85	1.11	1.67	1.04	1.09	2.02	1.04
time (sec)	N/A	0.314	0.023	0.109	0.035	0.181	0.196	0.137	0.172	17.814

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	69	59	66	77	119	80	66	133	75
N.S.	1	1.03	0.88	0.99	1.15	1.78	1.19	0.99	1.99	1.12
time (sec)	N/A	0.342	0.044	0.121	0.030	0.084	0.233	0.151	0.169	17.641

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	74	77	92	134	90	79	148	88
N.S.	1	1.02	0.86	0.90	1.07	1.56	1.05	0.92	1.72	1.02
time (sec)	N/A	0.386	0.041	0.127	0.028	0.183	0.260	0.129	0.167	0.081

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	101	85	89	103	145	104	110	159	101
N.S.	1	1.06	0.89	0.94	1.08	1.53	1.09	1.16	1.67	1.06
time (sec)	N/A	0.419	0.052	0.131	0.033	0.181	0.279	0.143	0.168	17.479

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	96	77	63	82	230	133	73	132	77
N.S.	1	1.13	0.91	0.74	0.96	2.71	1.56	0.86	1.55	0.91
time (sec)	N/A	0.375	0.032	0.114	0.120	0.109	0.207	0.133	0.166	0.068

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	85	66	51	68	202	107	54	118	64
N.S.	1	1.20	0.93	0.72	0.96	2.85	1.51	0.76	1.66	0.90
time (sec)	N/A	0.312	0.034	0.109	0.114	0.108	0.189	0.144	0.168	17.538

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	72	55	47	59	188	110	45	113	56
N.S.	1	1.12	0.86	0.73	0.92	2.94	1.72	0.70	1.77	0.88
time (sec)	N/A	0.288	0.030	0.114	0.107	0.215	0.178	0.122	0.174	17.737

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	70	58	49	62	190	110	50	110	55
N.S.	1	1.08	0.89	0.75	0.95	2.92	1.69	0.77	1.69	0.85
time (sec)	N/A	0.286	0.021	0.112	0.111	0.174	0.149	0.157	0.164	0.075

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	70	55	57	58	188	105	45	113	55
N.S.	1	1.13	0.89	0.92	0.94	3.03	1.69	0.73	1.82	0.89
time (sec)	N/A	0.280	0.026	0.105	0.113	0.178	0.144	0.106	0.168	17.665

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	88	68	54	71	202	116	57	121	66
N.S.	1	1.22	0.94	0.75	0.99	2.81	1.61	0.79	1.68	0.92
time (sec)	N/A	0.310	0.028	0.122	0.110	0.179	0.191	0.111	0.167	17.567

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	105	79	64	86	238	138	71	137	80
N.S.	1	1.21	0.91	0.74	0.99	2.74	1.59	0.82	1.57	0.92
time (sec)	N/A	0.333	0.030	0.133	0.113	0.121	0.219	0.135	0.169	17.786

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	122	90	75	97	264	150	80	150	92
N.S.	1	1.26	0.93	0.77	1.00	2.72	1.55	0.82	1.55	0.95
time (sec)	N/A	0.366	0.040	0.135	0.111	0.114	0.264	0.111	0.178	17.687

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	139	101	86	108	286	162	93	161	102
N.S.	1	1.25	0.91	0.77	0.97	2.58	1.46	0.84	1.45	0.92
time (sec)	N/A	0.389	0.041	0.145	0.116	0.214	0.277	0.126	0.180	17.873

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	17	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.81	0.71
time (sec)	N/A	0.249	0.012	0.123	0.034	0.086	0.547	0.122	0.171	0.032

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	17	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.81	0.71
time (sec)	N/A	0.252	0.012	0.115	0.032	0.131	0.344	0.156	0.177	0.029

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	17	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.81	0.71
time (sec)	N/A	0.250	0.013	0.113	0.026	0.090	0.215	0.105	0.180	0.027

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	14	13	18	19	13	17	15
N.S.	1	1.00	0.90	0.67	0.62	0.86	0.90	0.62	0.81	0.71
time (sec)	N/A	0.248	0.011	0.110	0.031	0.126	0.380	0.126	0.167	0.027

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	17	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.81	0.71
time (sec)	N/A	0.247	0.013	0.108	0.026	0.138	0.131	0.112	0.164	0.026

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	16	19	13	15	15
N.S.	1	1.00	1.00	0.67	0.62	0.76	0.90	0.62	0.71	0.71
time (sec)	N/A	0.268	0.011	0.063	0.030	0.132	0.135	0.116	0.168	0.027

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	20	14	13	14	17	13	13	14
N.S.	1	1.00	1.05	0.74	0.68	0.74	0.89	0.68	0.68	0.74
time (sec)	N/A	0.274	0.011	0.060	0.026	0.116	0.166	0.144	0.169	0.028

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	14	17	13	15	15
N.S.	1	1.00	1.00	0.74	0.68	0.74	0.89	0.68	0.79	0.79
time (sec)	N/A	0.249	0.013	0.070	0.031	0.098	0.263	0.130	0.170	0.031

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	28	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.78	0.69
time (sec)	N/A	0.271	0.017	0.150	0.026	0.120	1.136	0.125	0.167	17.775

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	28	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.78	0.69
time (sec)	N/A	0.272	0.016	0.144	0.032	0.140	0.805	0.124	0.168	17.557

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	28	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.78	0.69
time (sec)	N/A	0.274	0.016	0.143	0.032	0.101	0.592	0.141	0.166	0.040

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	28	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.78	0.69
time (sec)	N/A	0.268	0.017	0.140	0.027	0.101	0.626	0.125	0.166	17.535

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	28	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.78	0.69
time (sec)	N/A	0.264	0.018	0.138	0.028	0.140	0.347	0.104	0.163	0.039

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	28	26
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.78	0.72
time (sec)	N/A	0.261	0.016	0.135	0.028	0.076	0.364	0.137	0.163	0.041

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	28	26
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.78	0.72
time (sec)	N/A	0.261	0.016	0.125	0.026	0.107	0.402	0.142	0.174	0.041

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	27	34	24	26	26
N.S.	1	1.00	0.83	0.69	0.67	0.75	0.94	0.67	0.72	0.72
time (sec)	N/A	0.260	0.015	0.121	0.027	0.106	0.497	0.115	0.161	17.388

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	36	35	40	49	35	39	35
N.S.	1	1.00	1.00	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.278	0.027	0.185	0.032	0.113	2.077	0.108	0.162	0.048

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	36	35	40	49	35	39	35
N.S.	1	1.00	1.00	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.284	0.032	0.178	0.027	0.092	1.588	0.105	0.169	0.047

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	36	35	40	49	35	39	35
N.S.	1	1.00	0.92	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.288	0.019	0.176	0.028	0.143	1.151	0.108	0.164	0.047

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	39	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.291	0.019	0.171	0.032	0.087	1.055	0.100	0.174	0.047

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	39	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.291	0.018	0.173	0.032	0.115	0.845	0.129	0.162	0.047

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	39	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.282	0.018	0.169	0.034	0.127	0.853	0.120	0.161	0.046

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	39	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.279	0.020	0.167	0.028	0.090	0.925	0.104	0.170	0.050

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	39	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.283	0.018	0.168	0.028	0.157	1.125	0.115	0.163	0.047

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	250	129	128	198	177	0	197	158	66
N.S.	1	1.54	0.80	0.79	1.22	1.09	0.00	1.22	0.98	0.41
time (sec)	N/A	0.757	0.167	0.143	0.114	0.119	0.000	0.165	0.160	0.105

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	248	128	125	194	167	0	196	157	67
N.S.	1	1.56	0.81	0.79	1.22	1.05	0.00	1.23	0.99	0.42
time (sec)	N/A	0.726	0.151	0.132	0.111	0.130	0.000	0.131	0.173	0.104

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	231	119	116	186	158	124	178	147	54
N.S.	1	1.56	0.80	0.78	1.26	1.07	0.84	1.20	0.99	0.36
time (sec)	N/A	0.697	0.132	0.134	0.117	0.149	91.921	0.159	0.165	17.649

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	229	118	115	185	117	110	178	145	55
N.S.	1	1.56	0.80	0.78	1.26	0.80	0.75	1.21	0.99	0.37
time (sec)	N/A	0.684	0.134	0.118	0.111	0.160	46.436	0.106	0.163	18.017

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	214	91	106	172	132	104	182	112	38
N.S.	1	1.56	0.66	0.77	1.26	0.96	0.76	1.33	0.82	0.28
time (sec)	N/A	0.649	0.107	0.097	0.118	0.141	26.321	0.126	0.169	0.076

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	214	92	106	172	120	104	182	112	37
N.S.	1	1.57	0.68	0.78	1.26	0.88	0.76	1.34	0.82	0.27
time (sec)	N/A	0.635	0.112	0.096	0.111	0.128	14.133	0.129	0.168	0.087

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	229	117	115	186	140	114	190	155	54
N.S.	1	1.57	0.80	0.79	1.27	0.96	0.78	1.30	1.06	0.37
time (sec)	N/A	0.701	0.147	0.125	0.119	0.147	7.260	0.172	0.170	17.877

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	231	119	116	187	165	128	178	163	53
N.S.	1	1.55	0.80	0.78	1.26	1.11	0.86	1.19	1.09	0.36
time (sec)	N/A	0.695	0.147	0.125	0.113	0.104	12.578	0.133	0.166	17.724

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	248	127	124	198	191	139	200	180	66
N.S.	1	1.55	0.79	0.78	1.24	1.19	0.87	1.25	1.12	0.41
time (sec)	N/A	0.737	0.177	0.135	0.115	0.148	24.623	0.121	0.165	0.089

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	250	129	127	201	191	146	192	180	65
N.S.	1	1.55	0.80	0.79	1.25	1.19	0.91	1.19	1.12	0.40
time (sec)	N/A	0.723	0.178	0.136	0.113	0.106	61.568	0.131	0.177	17.721

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	267	140	138	209	202	160	199	191	77
N.S.	1	1.53	0.80	0.79	1.20	1.16	0.92	1.14	1.10	0.44
time (sec)	N/A	0.762	0.195	0.142	0.117	0.151	143.903	0.135	0.163	0.094

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	277	149	147	217	240	0	216	326	92
N.S.	1	1.46	0.78	0.77	1.14	1.26	0.00	1.14	1.72	0.48
time (sec)	N/A	0.762	0.294	0.204	0.118	0.190	0.000	0.112	0.171	0.096

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	260	138	136	207	238	0	196	314	80
N.S.	1	1.47	0.78	0.77	1.17	1.34	0.00	1.11	1.77	0.45
time (sec)	N/A	0.748	0.293	0.195	0.117	0.159	0.000	0.164	0.166	18.518

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	258	138	136	206	209	0	196	313	80
N.S.	1	1.46	0.78	0.77	1.16	1.18	0.00	1.11	1.77	0.45
time (sec)	N/A	0.736	0.277	0.188	0.112	0.171	0.000	0.117	0.170	18.583

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	241	128	124	195	200	0	199	307	64
N.S.	1	1.46	0.78	0.75	1.18	1.21	0.00	1.21	1.86	0.39
time (sec)	N/A	0.699	0.268	0.161	0.110	0.150	0.000	0.140	0.176	0.085

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	241	127	127	195	191	0	199	304	64
N.S.	1	1.46	0.77	0.77	1.18	1.16	0.00	1.21	1.84	0.39
time (sec)	N/A	0.689	0.264	0.132	0.118	0.159	0.000	0.108	0.173	0.093

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	241	128	127	194	204	0	199	305	64
N.S.	1	1.46	0.78	0.77	1.18	1.24	0.00	1.21	1.85	0.39
time (sec)	N/A	0.693	0.257	0.123	0.118	0.120	0.000	0.154	0.168	0.093

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	241	128	124	194	193	0	199	306	64
N.S.	1	1.46	0.78	0.75	1.18	1.17	0.00	1.21	1.85	0.39
time (sec)	N/A	0.716	0.259	0.122	0.117	0.135	0.000	0.154	0.171	17.573

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	258	138	136	208	218	0	210	329	77
N.S.	1	1.46	0.78	0.77	1.18	1.23	0.00	1.19	1.86	0.44
time (sec)	N/A	0.736	0.307	0.175	0.117	0.155	0.000	0.178	0.166	0.089

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	260	138	136	209	244	0	196	336	77
N.S.	1	1.47	0.78	0.77	1.18	1.38	0.00	1.11	1.90	0.44
time (sec)	N/A	0.724	0.313	0.170	0.115	0.132	0.000	0.150	0.167	17.578

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	277	149	145	221	263	0	220	355	87
N.S.	1	1.46	0.78	0.76	1.16	1.38	0.00	1.16	1.87	0.46
time (sec)	N/A	0.760	0.325	0.183	0.115	0.112	0.000	0.151	0.174	0.095

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	279	149	147	224	263	0	212	355	87
N.S.	1	1.47	0.78	0.77	1.18	1.38	0.00	1.12	1.87	0.46
time (sec)	N/A	0.755	0.330	0.181	0.112	0.149	0.000	0.126	0.165	17.631

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	296	160	158	232	274	0	219	366	99
N.S.	1	1.44	0.78	0.77	1.13	1.34	0.00	1.07	1.79	0.48
time (sec)	N/A	0.786	0.361	0.188	0.113	0.146	0.000	0.158	0.170	0.109

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	287	149	145	229	275	0	208	484	101
N.S.	1	1.45	0.75	0.73	1.16	1.39	0.00	1.05	2.44	0.51
time (sec)	N/A	0.770	0.380	0.281	0.120	0.124	0.000	0.144	0.169	17.669

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	270	138	136	218	276	0	209	478	87
N.S.	1	1.45	0.74	0.73	1.17	1.48	0.00	1.12	2.57	0.47
time (sec)	N/A	0.736	0.367	0.231	0.118	0.192	0.000	0.122	0.175	17.744

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	270	138	139	218	275	0	209	477	87
N.S.	1	1.45	0.74	0.75	1.17	1.48	0.00	1.12	2.56	0.47
time (sec)	N/A	0.742	0.374	0.229	0.110	0.160	0.000	0.116	0.171	0.096

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	270	136	138	222	289	0	212	478	85
N.S.	1	1.43	0.72	0.73	1.17	1.53	0.00	1.12	2.53	0.45
time (sec)	N/A	0.761	0.367	0.230	0.114	0.192	0.000	0.117	0.171	17.521

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	270	137	138	221	279	0	211	477	85
N.S.	1	1.43	0.72	0.73	1.17	1.48	0.00	1.12	2.52	0.45
time (sec)	N/A	0.760	0.338	0.149	0.116	0.138	0.000	0.121	0.171	0.095

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	270	138	150	217	281	0	209	478	86
N.S.	1	1.45	0.74	0.81	1.17	1.51	0.00	1.12	2.57	0.46
time (sec)	N/A	0.765	0.245	0.135	0.116	0.196	0.000	0.113	0.167	17.573

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	270	138	147	217	268	0	209	477	86
N.S.	1	1.45	0.74	0.79	1.17	1.44	0.00	1.12	2.56	0.46
time (sec)	N/A	0.756	0.236	0.134	0.113	0.129	0.000	0.114	0.173	0.097

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	287	149	145	230	284	0	220	506	99
N.S.	1	1.45	0.75	0.73	1.16	1.43	0.00	1.11	2.56	0.50
time (sec)	N/A	0.778	0.387	0.218	0.114	0.134	0.000	0.129	0.176	17.561

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	289	149	145	231	310	0	208	513	99
N.S.	1	1.46	0.75	0.73	1.17	1.57	0.00	1.05	2.59	0.50
time (sec)	N/A	0.765	0.386	0.217	0.112	0.129	0.000	0.132	0.171	0.119

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	306	160	155	243	329	0	232	532	109
N.S.	1	1.45	0.76	0.73	1.15	1.56	0.00	1.10	2.52	0.52
time (sec)	N/A	0.818	0.422	0.224	0.117	0.126	0.000	0.116	0.169	17.525

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	308	160	155	246	329	0	224	532	109
N.S.	1	1.46	0.76	0.73	1.17	1.56	0.00	1.06	2.52	0.52
time (sec)	N/A	0.803	0.406	0.222	0.114	0.243	0.000	0.145	0.175	0.134

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	325	171	167	254	340	0	231	543	121
N.S.	1	1.44	0.76	0.74	1.12	1.50	0.00	1.02	2.40	0.54
time (sec)	N/A	0.832	0.441	0.232	0.118	0.164	0.000	0.149	0.174	0.140

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	327	171	167	257	340	0	243	543	121
N.S.	1	1.45	0.76	0.74	1.14	1.50	0.00	1.08	2.40	0.54
time (sec)	N/A	0.823	0.441	0.231	0.117	0.269	0.000	0.146	0.173	18.412

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	137	121	100	121	188	0	101	99	105
N.S.	1	1.15	1.02	0.84	1.02	1.58	0.00	0.85	0.83	0.88
time (sec)	N/A	0.463	0.343	0.271	0.037	0.192	0.000	0.169	0.167	18.192

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	103	110	90	97	167	0	85	80	77
N.S.	1	1.13	1.21	0.99	1.07	1.84	0.00	0.93	0.88	0.85
time (sec)	N/A	0.420	0.255	0.172	0.036	0.221	0.000	0.136	0.165	17.970

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	72	97	77	73	140	0	69	60	64
N.S.	1	1.06	1.43	1.13	1.07	2.06	0.00	1.01	0.88	0.94
time (sec)	N/A	0.345	0.218	0.150	0.034	0.117	0.000	0.150	0.249	17.996

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	52	64	63	49	115	0	52	40	50
N.S.	1	0.95	1.16	1.15	0.89	2.09	0.00	0.95	0.73	0.91
time (sec)	N/A	0.348	0.053	0.138	0.034	0.224	0.000	0.135	0.228	18.422

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	57	68	65	51	115	0	61	43	0
N.S.	1	1.10	1.31	1.25	0.98	2.21	0.00	1.17	0.83	0.00
time (sec)	N/A	0.350	0.060	0.150	0.034	0.122	0.000	0.151	0.232	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	41	28	0	63	42	28
N.S.	1	1.00	1.00	1.16	1.64	1.12	0.00	2.52	1.68	1.12
time (sec)	N/A	0.261	0.048	0.133	0.036	0.098	0.000	0.144	0.262	18.323

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	37	65	42	0	120	63	41
N.S.	1	1.00	0.88	0.71	1.25	0.81	0.00	2.31	1.21	0.79
time (sec)	N/A	0.309	0.059	0.147	0.033	0.174	0.000	0.183	0.245	18.566

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	57	48	89	53	0	148	82	89
N.S.	1	1.08	0.71	0.60	1.11	0.66	0.00	1.85	1.02	1.11
time (sec)	N/A	0.367	0.068	0.170	0.046	0.239	0.000	0.170	0.241	18.669

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	68	61	113	64	0	178	101	113
N.S.	1	1.11	0.63	0.56	1.05	0.59	0.00	1.65	0.94	1.05
time (sec)	N/A	0.430	0.071	0.200	0.038	0.197	0.000	0.189	0.265	18.822

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	154	79	72	137	75	0	206	120	137
N.S.	1	1.13	0.58	0.53	1.01	0.55	0.00	1.51	0.88	1.01
time (sec)	N/A	0.507	0.082	0.230	0.038	0.253	0.000	0.133	0.252	19.057

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	84	57	50	46	53	0	60	45	53
N.S.	1	1.08	0.73	0.64	0.59	0.68	0.00	0.77	0.58	0.68
time (sec)	N/A	0.352	0.030	0.670	0.042	0.110	0.000	0.151	0.214	17.945

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	39	34	41	0	44	33	41
N.S.	1	1.00	0.87	0.75	0.65	0.79	0.00	0.85	0.63	0.79
time (sec)	N/A	0.298	0.025	0.552	0.037	0.139	0.000	0.132	0.177	17.921

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	20	29
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	0.80	1.16
time (sec)	N/A	0.239	0.005	0.474	0.039	0.145	0.000	0.137	0.168	18.031

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	60	65	0	112	0	69	60	68
N.S.	1	1.00	1.20	1.30	0.00	2.24	0.00	1.38	1.20	1.36
time (sec)	N/A	0.308	0.048	0.538	0.000	0.208	0.000	0.116	0.170	18.164

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	63	74	0	129	0	47	79	0
N.S.	1	1.00	1.12	1.32	0.00	2.30	0.00	0.84	1.41	0.00
time (sec)	N/A	0.293	0.088	0.651	0.000	0.216	0.000	0.115	0.175	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	87	91	88	0	154	0	78	100	0
N.S.	1	1.04	1.08	1.05	0.00	1.83	0.00	0.93	1.19	0.00
time (sec)	N/A	0.371	0.109	0.794	0.000	0.098	0.000	0.133	0.175	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	121	103	100	0	180	0	88	120	0
N.S.	1	1.08	0.92	0.89	0.00	1.61	0.00	0.79	1.07	0.00
time (sec)	N/A	0.440	0.122	0.973	0.000	0.114	0.000	0.144	0.186	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	144	132	111	142	210	0	115	118	134
N.S.	1	1.16	1.06	0.90	1.15	1.69	0.00	0.93	0.95	1.08
time (sec)	N/A	0.448	0.452	0.214	0.039	0.124	0.000	0.138	0.170	18.330

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	113	121	100	118	189	0	99	99	99
N.S.	1	1.12	1.20	0.99	1.17	1.87	0.00	0.98	0.98	0.98
time (sec)	N/A	0.393	0.337	0.186	0.036	0.143	0.000	0.121	0.175	18.904

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	97	110	90	91	166	255	84	80	0
N.S.	1	1.10	1.25	1.02	1.03	1.89	2.90	0.95	0.91	0.00
time (sec)	N/A	0.386	0.291	0.168	0.037	0.201	1.888	0.141	0.174	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	74	76	70	145	0	68	61	0
N.S.	1	1.00	0.92	0.95	0.88	1.81	0.00	0.85	0.76	0.00
time (sec)	N/A	0.378	0.073	0.152	0.038	0.193	0.000	0.131	0.166	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	88	77	71	139	0	79	62	0
N.S.	1	1.00	1.14	1.00	0.92	1.81	0.00	1.03	0.81	0.00
time (sec)	N/A	0.384	0.140	0.173	0.038	0.116	0.000	0.166	0.172	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	82	85	73	89	135	0	122	58	0
N.S.	1	1.09	1.13	0.97	1.19	1.80	0.00	1.63	0.77	0.00
time (sec)	N/A	0.383	0.082	0.174	0.040	0.096	0.000	0.263	0.168	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	81	39	0	92	63	30
N.S.	1	1.00	1.00	1.16	3.24	1.56	0.00	3.68	2.52	1.20
time (sec)	N/A	0.252	0.063	0.148	0.045	0.096	0.000	0.163	0.170	20.172

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	105	53	0	178	82	87
N.S.	1	1.00	0.67	0.75	2.02	1.02	0.00	3.42	1.58	1.67
time (sec)	N/A	0.302	0.074	0.167	0.040	0.113	0.000	0.196	0.179	20.259

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	46	50	129	64	0	206	101	111
N.S.	1	1.08	0.58	0.62	1.61	0.80	0.00	2.58	1.26	1.39
time (sec)	N/A	0.372	0.080	0.197	0.038	0.166	0.000	0.185	0.173	19.308

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	57	61	153	75	0	236	120	135
N.S.	1	1.11	0.53	0.56	1.42	0.69	0.00	2.19	1.11	1.25
time (sec)	N/A	0.441	0.090	0.246	0.039	0.245	0.000	0.196	0.173	19.780

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	154	68	72	177	86	0	264	139	159
N.S.	1	1.13	0.50	0.53	1.30	0.63	0.00	1.94	1.02	1.17
time (sec)	N/A	0.510	0.099	0.346	0.040	0.123	0.000	0.150	0.186	19.865

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	152	68	72	79	86	0	92	78	73
N.S.	1	1.13	0.51	0.54	0.59	0.64	0.00	0.69	0.58	0.54
time (sec)	N/A	0.508	0.037	0.819	0.038	0.120	0.000	0.127	0.167	18.540

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	118	57	61	68	75	0	76	67	62
N.S.	1	1.11	0.54	0.58	0.64	0.71	0.00	0.72	0.63	0.58
time (sec)	N/A	0.433	0.038	0.671	0.040	0.233	0.000	0.153	0.176	17.804

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	46	50	57	64	0	60	56	51
N.S.	1	1.08	0.58	0.62	0.71	0.80	0.00	0.75	0.70	0.64
time (sec)	N/A	0.363	0.032	0.569	0.041	0.114	0.000	0.141	0.170	18.714

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	45	52	0	44	44	40
N.S.	1	1.00	0.67	0.75	0.87	1.00	0.00	0.85	0.85	0.77
time (sec)	N/A	0.292	0.029	0.493	0.036	0.095	0.000	0.120	0.171	19.480

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	32	39	0	27	31	30
N.S.	1	1.00	1.00	1.16	1.28	1.56	0.00	1.08	1.24	1.20
time (sec)	N/A	0.249	0.006	0.553	0.039	0.088	0.000	0.132	0.174	18.927

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	75	76	78	0	135	0	89	79	0
N.S.	1	1.03	1.04	1.07	0.00	1.85	0.00	1.22	1.08	0.00
time (sec)	N/A	0.361	0.058	0.646	0.000	0.100	0.000	0.162	0.168	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	78	83	88	0	142	0	63	91	0
N.S.	1	0.99	1.05	1.11	0.00	1.80	0.00	0.80	1.15	0.00
time (sec)	N/A	0.362	0.080	0.794	0.000	0.102	0.000	0.151	0.175	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	84	92	86	0	159	0	76	101	0
N.S.	1	1.04	1.14	1.06	0.00	1.96	0.00	0.94	1.25	0.00
time (sec)	N/A	0.367	0.101	0.964	0.000	0.151	0.000	0.148	0.175	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	115	104	100	0	180	0	88	120	0
N.S.	1	1.06	0.95	0.92	0.00	1.65	0.00	0.81	1.10	0.00
time (sec)	N/A	0.429	0.110	1.189	0.000	0.110	0.000	0.177	0.184	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	149	114	111	0	202	0	119	139	0
N.S.	1	1.09	0.83	0.81	0.00	1.47	0.00	0.87	1.01	0.00
time (sec)	N/A	0.505	0.141	1.484	0.000	0.112	0.000	0.200	0.201	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	183	126	122	0	224	0	120	158	0
N.S.	1	1.11	0.76	0.74	0.00	1.36	0.00	0.73	0.96	0.00
time (sec)	N/A	0.573	0.160	1.826	0.000	0.108	0.000	0.191	0.218	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	125	110	89	100	166	0	97	80	0
N.S.	1	1.10	0.96	0.78	0.88	1.46	0.00	0.85	0.70	0.00
time (sec)	N/A	0.449	0.222	0.195	0.036	0.094	0.000	0.157	0.185	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	91	99	85	76	145	0	79	61	0
N.S.	1	1.06	1.15	0.99	0.88	1.69	0.00	0.92	0.71	0.00
time (sec)	N/A	0.377	0.159	0.161	0.039	0.088	0.000	0.169	0.172	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	83	63	52	114	0	59	41	53
N.S.	1	0.98	1.43	1.09	0.90	1.97	0.00	1.02	0.71	0.91
time (sec)	N/A	0.321	0.121	0.148	0.037	0.077	0.000	0.120	0.169	18.363

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	52	42	32	74	0	38	25	33
N.S.	1	1.00	1.68	1.35	1.03	2.39	0.00	1.23	0.81	1.06
time (sec)	N/A	0.272	0.010	0.132	0.036	0.098	0.000	0.120	0.177	18.377

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	21	0	34	23	21
N.S.	1	1.00	1.00	0.96	0.91	0.91	0.00	1.48	1.00	0.91
time (sec)	N/A	0.243	0.032	0.129	0.036	0.077	0.000	0.133	0.172	17.277

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	30	44	31	0	59	42	29
N.S.	1	1.00	0.67	0.58	0.85	0.60	0.00	1.13	0.81	0.56
time (sec)	N/A	0.297	0.047	0.138	0.040	0.071	0.000	0.138	0.173	17.597

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	46	43	68	42	0	81	63	42
N.S.	1	1.08	0.58	0.54	0.85	0.52	0.00	1.01	0.79	0.52
time (sec)	N/A	0.358	0.054	0.154	0.048	0.080	0.000	0.128	0.172	17.969

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	57	54	92	53	0	107	82	92
N.S.	1	1.11	0.53	0.50	0.85	0.49	0.00	0.99	0.76	0.85
time (sec)	N/A	0.432	0.060	0.181	0.041	0.078	0.000	0.161	0.177	18.059

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	118	57	54	57	53	0	79	45	53
N.S.	1	1.11	0.54	0.51	0.54	0.50	0.00	0.75	0.42	0.50
time (sec)	N/A	0.428	0.031	0.955	0.038	0.074	0.000	0.128	0.180	17.686

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	84	46	43	46	42	0	64	34	42
N.S.	1	1.08	0.59	0.55	0.59	0.54	0.00	0.82	0.44	0.54
time (sec)	N/A	0.375	0.029	0.790	0.037	0.071	0.000	0.112	0.171	16.986

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	32	34	30	0	48	22	33
N.S.	1	1.00	0.68	0.64	0.68	0.60	0.00	0.96	0.44	0.66
time (sec)	N/A	0.326	0.022	0.649	0.037	0.078	0.000	0.114	0.172	17.629

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	13	20	0	28	12	20
N.S.	1	1.00	1.00	0.95	0.59	0.91	0.00	1.27	0.55	0.91
time (sec)	N/A	0.262	0.004	0.546	0.035	0.069	0.000	0.153	0.170	18.640

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	52	50	0	75	0	46	53	0
N.S.	1	1.00	1.73	1.67	0.00	2.50	0.00	1.53	1.77	0.00
time (sec)	N/A	0.252	0.022	0.480	0.000	0.078	0.000	0.154	0.172	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	76	73	0	128	0	52	79	76
N.S.	1	1.00	1.29	1.24	0.00	2.17	0.00	0.88	1.34	1.29
time (sec)	N/A	0.305	0.060	0.561	0.000	0.090	0.000	0.137	0.181	18.351

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	93	91	94	0	158	0	79	101	0
N.S.	1	1.07	1.05	1.08	0.00	1.82	0.00	0.91	1.16	0.00
time (sec)	N/A	0.362	0.080	0.670	0.000	0.085	0.000	0.175	0.177	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	115	99	87	103	209	0	96	137	0
N.S.	1	1.03	0.88	0.78	0.92	1.87	0.00	0.86	1.22	0.00
time (sec)	N/A	0.518	0.222	0.196	0.037	0.085	0.000	0.158	0.176	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	85	85	73	77	180	0	74	114	0
N.S.	1	1.05	1.05	0.90	0.95	2.22	0.00	0.91	1.41	0.00
time (sec)	N/A	0.404	0.172	0.182	0.044	0.081	0.000	0.119	0.179	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	73	63	54	150	0	57	88	55
N.S.	1	1.09	1.33	1.15	0.98	2.73	0.00	1.04	1.60	1.00
time (sec)	N/A	0.329	0.086	0.160	0.045	0.079	0.000	0.127	0.176	18.339

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	26	0	18	39	26
N.S.	1	1.00	1.00	0.95	0.91	1.18	0.00	0.82	1.77	1.18
time (sec)	N/A	0.251	0.032	0.128	0.037	0.067	0.000	0.138	0.185	17.546

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	29	27	41	41	0	58	56	26
N.S.	1	1.00	1.04	0.96	1.46	1.46	0.00	2.07	2.00	0.93
time (sec)	N/A	0.272	0.045	0.156	0.038	0.070	0.000	0.121	0.172	16.913

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	41	65	54	0	114	81	51
N.S.	1	1.00	0.81	0.72	1.14	0.95	0.00	2.00	1.42	0.89
time (sec)	N/A	0.331	0.063	0.173	0.038	0.074	0.000	0.145	0.179	17.293

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	91	57	52	89	63	0	169	102	60
N.S.	1	1.07	0.67	0.61	1.05	0.74	0.00	1.99	1.20	0.71
time (sec)	N/A	0.397	0.078	0.182	0.041	0.078	0.000	0.128	0.180	17.397

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	125	68	65	113	76	0	222	121	114
N.S.	1	1.11	0.60	0.58	1.00	0.67	0.00	1.96	1.07	1.01
time (sec)	N/A	0.469	0.086	0.211	0.039	0.084	0.000	0.137	0.180	18.547

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	113	54	60	45	62	0	86	53	61
N.S.	1	1.07	0.51	0.57	0.42	0.58	0.00	0.81	0.50	0.58
time (sec)	N/A	0.429	0.033	1.196	0.043	0.071	0.000	0.104	0.178	18.612

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	79	43	50	34	51	0	76	42	51
N.S.	1	1.01	0.55	0.64	0.44	0.65	0.00	0.97	0.54	0.65
time (sec)	N/A	0.362	0.030	0.970	0.039	0.076	0.000	0.110	0.171	18.168

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	29	37	22	39	0	51	30	38
N.S.	1	1.00	0.64	0.82	0.49	0.87	0.00	1.13	0.67	0.84
time (sec)	N/A	0.295	0.025	0.789	0.039	0.083	0.000	0.122	0.173	18.441

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	29	14	29	0	28	22	30
N.S.	1	1.00	1.00	1.38	0.67	1.38	0.00	1.33	1.05	1.43
time (sec)	N/A	0.245	0.004	0.648	0.040	0.078	0.000	0.129	0.184	18.154

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	59	65	0	157	0	79	136	0
N.S.	1	1.00	1.16	1.27	0.00	3.08	0.00	1.55	2.67	0.00
time (sec)	N/A	0.302	0.040	0.560	0.000	0.082	0.000	0.118	0.179	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	87	78	77	0	194	0	80	172	42
N.S.	1	1.07	0.96	0.95	0.00	2.40	0.00	0.99	2.12	0.52
time (sec)	N/A	0.375	0.074	0.522	0.000	0.083	0.000	0.153	0.179	17.838

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	121	92	94	0	224	0	99	195	44
N.S.	1	1.11	0.84	0.86	0.00	2.06	0.00	0.91	1.79	0.40
time (sec)	N/A	0.439	0.096	0.602	0.000	0.095	0.000	0.131	0.182	17.498

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	58	27	26	43	0	30	22	42
N.S.	1	1.11	1.61	0.75	0.72	1.19	0.00	0.83	0.61	1.17
time (sec)	N/A	0.301	0.037	0.351	0.117	0.070	0.000	0.133	0.168	17.240

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	59	27	26	59	0	43	23	41
N.S.	1	1.11	1.64	0.75	0.72	1.64	0.00	1.19	0.64	1.14
time (sec)	N/A	0.297	0.034	0.333	0.108	0.065	0.000	0.117	0.178	17.487

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	49	58	37	41	45	0	43	32	40
N.S.	1	1.09	1.29	0.82	0.91	1.00	0.00	0.96	0.71	0.89
time (sec)	N/A	0.306	0.006	0.246	0.115	0.070	0.000	0.114	0.175	18.456

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	49	58	42	41	45	0	44	32	40
N.S.	1	1.09	1.29	0.93	0.91	1.00	0.00	0.98	0.71	0.89
time (sec)	N/A	0.323	0.006	0.263	0.109	0.070	0.000	0.125	0.181	18.501

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	83	63	52	114	0	59	41	53
N.S.	1	0.98	1.43	1.09	0.90	1.97	0.00	1.02	0.71	0.91
time (sec)	N/A	0.340	0.127	0.216	0.036	0.085	0.000	0.116	0.170	18.460

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	59	88	56	42	120	0	73	32	60
N.S.	1	0.98	1.47	0.93	0.70	2.00	0.00	1.22	0.53	1.00
time (sec)	N/A	0.337	0.117	0.612	0.115	0.092	0.000	0.176	0.172	17.930

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	344	102	237	0	73	0	0	84	0
N.S.	1	1.07	0.32	0.73	0.00	0.23	0.00	0.00	0.26	0.00
time (sec)	N/A	0.799	10.047	0.666	0.000	0.088	0.000	0.000	0.203	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	187	102	157	0	68	0	0	85	0
N.S.	1	1.06	0.58	0.89	0.00	0.39	0.00	0.00	0.48	0.00
time (sec)	N/A	0.554	10.038	0.358	0.000	0.090	0.000	0.000	0.206	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	308	86	226	0	61	0	0	63	0
N.S.	1	1.05	0.29	0.77	0.00	0.21	0.00	0.00	0.22	0.00
time (sec)	N/A	0.685	10.032	0.352	0.000	0.094	0.000	0.000	0.201	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	151	86	145	0	58	0	0	64	0
N.S.	1	1.03	0.59	0.99	0.00	0.40	0.00	0.00	0.44	0.00
time (sec)	N/A	0.467	9.824	0.287	0.000	0.104	0.000	0.000	0.199	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	272	57	213	0	48	0	0	39	0
N.S.	1	1.03	0.22	0.81	0.00	0.18	0.00	0.00	0.15	0.00
time (sec)	N/A	0.629	7.925	0.279	0.000	0.089	0.000	0.000	0.201	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	55	130	0	43	0	0	40	0
N.S.	1	1.00	0.47	1.10	0.00	0.36	0.00	0.00	0.34	0.00
time (sec)	N/A	0.398	6.835	0.276	0.000	0.091	0.000	0.000	0.192	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	268	55	202	0	48	0	0	45	0
N.S.	1	1.06	0.22	0.80	0.00	0.19	0.00	0.00	0.18	0.00
time (sec)	N/A	0.612	10.013	0.356	0.000	0.090	0.000	0.000	0.207	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	57	125	0	41	0	0	51	0
N.S.	1	1.00	0.48	1.06	0.00	0.35	0.00	0.00	0.43	0.00
time (sec)	N/A	0.394	10.013	0.373	0.000	0.093	0.000	0.000	0.219	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	304	57	224	0	59	0	0	54	0
N.S.	1	1.04	0.19	0.76	0.00	0.20	0.00	0.00	0.18	0.00
time (sec)	N/A	0.696	10.013	0.460	0.000	0.086	0.000	0.000	0.226	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	151	57	142	0	53	0	0	54	0
N.S.	1	1.03	0.39	0.97	0.00	0.36	0.00	0.00	0.37	0.00
time (sec)	N/A	0.467	10.014	0.494	0.000	0.092	0.000	0.000	0.254	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	340	57	239	0	72	0	0	54	0
N.S.	1	1.05	0.18	0.74	0.00	0.22	0.00	0.00	0.17	0.00
time (sec)	N/A	0.751	10.014	0.600	0.000	0.093	0.000	0.000	0.282	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	187	57	156	0	64	0	0	54	0
N.S.	1	1.06	0.32	0.89	0.00	0.36	0.00	0.00	0.31	0.00
time (sec)	N/A	0.596	10.012	0.625	0.000	0.089	0.000	0.000	0.304	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	374	101	248	0	84	0	0	105	0
N.S.	1	1.07	0.29	0.71	0.00	0.24	0.00	0.00	0.30	0.00
time (sec)	N/A	0.838	10.066	1.457	0.000	0.098	0.000	0.000	0.207	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	217	101	168	0	79	0	0	106	0
N.S.	1	1.07	0.50	0.83	0.00	0.39	0.00	0.00	0.52	0.00
time (sec)	N/A	0.609	10.042	1.138	0.000	0.098	0.000	0.000	0.212	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	338	90	237	0	72	0	0	84	0
N.S.	1	1.06	0.28	0.74	0.00	0.22	0.00	0.00	0.26	0.00
time (sec)	N/A	0.755	10.032	0.950	0.000	0.095	0.000	0.000	0.210	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	181	90	157	0	69	0	0	85	0
N.S.	1	1.05	0.52	0.91	0.00	0.40	0.00	0.00	0.49	0.00
time (sec)	N/A	0.518	10.035	0.740	0.000	0.091	0.000	0.000	0.198	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	302	58	226	0	62	0	0	58	0
N.S.	1	1.04	0.20	0.78	0.00	0.21	0.00	0.00	0.20	0.00
time (sec)	N/A	0.698	10.013	0.636	0.000	0.085	0.000	0.000	0.194	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	148	56	145	0	56	0	0	59	0
N.S.	1	1.03	0.39	1.01	0.00	0.39	0.00	0.00	0.41	0.00
time (sec)	N/A	0.474	9.242	0.586	0.000	0.089	0.000	0.000	0.201	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	298	56	216	0	59	0	0	64	0
N.S.	1	1.04	0.20	0.76	0.00	0.21	0.00	0.00	0.22	0.00
time (sec)	N/A	0.680	10.014	0.678	0.000	0.078	0.000	0.000	0.211	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	146	58	130	0	50	0	0	68	0
N.S.	1	1.02	0.41	0.91	0.00	0.35	0.00	0.00	0.48	0.00
time (sec)	N/A	0.452	10.014	0.798	0.000	0.093	0.000	0.000	0.223	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	298	58	220	0	56	0	0	70	0
N.S.	1	1.04	0.20	0.77	0.00	0.20	0.00	0.00	0.24	0.00
time (sec)	N/A	0.680	10.014	1.036	0.000	0.086	0.000	0.000	0.230	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	148	58	140	0	49	0	0	70	0
N.S.	1	1.03	0.41	0.98	0.00	0.34	0.00	0.00	0.49	0.00
time (sec)	N/A	0.451	10.016	1.241	0.000	0.070	0.000	0.000	0.256	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	334	58	239	0	72	0	0	71	0
N.S.	1	1.04	0.18	0.75	0.00	0.22	0.00	0.00	0.22	0.00
time (sec)	N/A	0.748	10.014	1.527	0.000	0.070	0.000	0.000	0.287	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	181	58	156	0	64	0	0	71	0
N.S.	1	1.05	0.34	0.90	0.00	0.37	0.00	0.00	0.41	0.00
time (sec)	N/A	0.544	10.018	1.806	0.000	0.070	0.000	0.000	0.317	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	370	58	250	0	83	0	0	71	0
N.S.	1	1.06	0.17	0.71	0.00	0.24	0.00	0.00	0.20	0.00
time (sec)	N/A	0.853	10.016	2.265	0.000	0.077	0.000	0.000	0.389	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	217	58	167	0	75	0	0	71	0
N.S.	1	1.07	0.29	0.82	0.00	0.37	0.00	0.00	0.35	0.00
time (sec)	N/A	0.603	10.031	2.651	0.000	0.079	0.000	0.000	0.430	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	193	97	148	0	69	0	0	85	0
N.S.	1	1.08	0.54	0.83	0.00	0.39	0.00	0.00	0.47	0.00
time (sec)	N/A	0.570	10.042	1.073	0.000	0.082	0.000	0.000	0.215	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	314	86	217	0	62	0	0	63	0
N.S.	1	1.06	0.29	0.73	0.00	0.21	0.00	0.00	0.21	0.00
time (sec)	N/A	0.714	10.039	0.947	0.000	0.077	0.000	0.000	0.207	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	157	86	137	0	57	0	0	64	0
N.S.	1	1.05	0.58	0.92	0.00	0.38	0.00	0.00	0.43	0.00
time (sec)	N/A	0.485	10.023	0.713	0.000	0.082	0.000	0.000	0.200	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	278	70	206	0	47	0	0	44	0
N.S.	1	1.05	0.26	0.77	0.00	0.18	0.00	0.00	0.17	0.00
time (sec)	N/A	0.631	10.022	0.626	0.000	0.080	0.000	0.000	0.209	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	70	123	0	43	0	0	45	0
N.S.	1	1.00	0.58	1.02	0.00	0.36	0.00	0.00	0.37	0.00
time (sec)	N/A	0.428	10.029	0.290	0.000	0.086	0.000	0.000	0.195	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	244	57	131	0	22	0	0	22	0
N.S.	1	1.06	0.25	0.57	0.00	0.10	0.00	0.00	0.10	0.00
time (sec)	N/A	0.567	10.017	0.237	0.000	0.075	0.000	0.000	0.179	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	55	106	0	14	0	0	24	0
N.S.	1	1.00	0.61	1.18	0.00	0.16	0.00	0.00	0.27	0.00
time (sec)	N/A	0.358	10.014	0.629	0.000	0.074	0.000	0.000	0.177	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	274	55	195	0	50	0	0	26	0
N.S.	1	1.06	0.21	0.75	0.00	0.19	0.00	0.00	0.10	0.00
time (sec)	N/A	0.624	10.013	0.856	0.000	0.080	0.000	0.000	0.181	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	57	119	0	42	0	0	26	0
N.S.	1	1.00	0.47	0.98	0.00	0.35	0.00	0.00	0.21	0.00
time (sec)	N/A	0.428	10.016	1.137	0.000	0.071	0.000	0.000	0.195	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	310	57	215	0	61	0	0	26	0
N.S.	1	1.05	0.19	0.73	0.00	0.21	0.00	0.00	0.09	0.00
time (sec)	N/A	0.712	10.013	1.396	0.000	0.078	0.000	0.000	0.185	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	157	57	134	0	53	0	0	26	0
N.S.	1	1.05	0.38	0.90	0.00	0.36	0.00	0.00	0.17	0.00
time (sec)	N/A	0.475	10.013	1.802	0.000	0.087	0.000	0.000	0.193	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	346	57	230	0	72	0	0	26	0
N.S.	1	1.06	0.17	0.71	0.00	0.22	0.00	0.00	0.08	0.00
time (sec)	N/A	0.782	10.025	2.252	0.000	0.070	0.000	0.000	0.206	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	193	57	147	0	64	0	0	26	0
N.S.	1	1.08	0.32	0.82	0.00	0.36	0.00	0.00	0.15	0.00
time (sec)	N/A	0.554	10.028	2.726	0.000	0.077	0.000	0.000	0.212	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	190	86	144	0	88	0	0	149	0
N.S.	1	1.09	0.49	0.83	0.00	0.51	0.00	0.00	0.86	0.00
time (sec)	N/A	0.575	10.035	1.879	0.000	0.085	0.000	0.000	0.240	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	311	72	213	0	80	0	0	125	0
N.S.	1	1.07	0.25	0.73	0.00	0.27	0.00	0.00	0.43	0.00
time (sec)	N/A	0.712	10.025	1.879	0.000	0.088	0.000	0.000	0.229	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	154	73	131	0	76	0	0	128	0
N.S.	1	1.05	0.50	0.90	0.00	0.52	0.00	0.00	0.88	0.00
time (sec)	N/A	0.496	10.021	1.575	0.000	0.089	0.000	0.000	0.236	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	273	61	200	0	65	0	0	105	0
N.S.	1	1.05	0.24	0.77	0.00	0.25	0.00	0.00	0.41	0.00
time (sec)	N/A	0.622	10.023	0.976	0.000	0.080	0.000	0.000	0.226	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	60	120	0	59	0	0	106	0
N.S.	1	1.00	0.50	1.01	0.00	0.50	0.00	0.00	0.89	0.00
time (sec)	N/A	0.394	10.016	0.348	0.000	0.078	0.000	0.000	0.224	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	272	60	203	0	64	0	0	33	0
N.S.	1	1.05	0.23	0.78	0.00	0.25	0.00	0.00	0.13	0.00
time (sec)	N/A	0.625	10.014	0.351	0.000	0.074	0.000	0.000	0.180	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	60	123	0	59	0	0	35	0
N.S.	1	1.00	0.51	1.04	0.00	0.50	0.00	0.00	0.30	0.00
time (sec)	N/A	0.400	9.874	1.161	0.000	0.072	0.000	0.000	0.186	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	306	58	203	0	81	0	0	37	0
N.S.	1	1.07	0.20	0.71	0.00	0.28	0.00	0.00	0.13	0.00
time (sec)	N/A	0.698	10.015	2.036	0.000	0.073	0.000	0.000	0.190	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	153	60	127	0	73	0	0	37	0
N.S.	1	1.06	0.41	0.88	0.00	0.50	0.00	0.00	0.26	0.00
time (sec)	N/A	0.459	10.014	2.310	0.000	0.075	0.000	0.000	0.199	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	342	60	222	0	95	0	0	37	0
N.S.	1	1.07	0.19	0.69	0.00	0.30	0.00	0.00	0.12	0.00
time (sec)	N/A	0.761	10.016	2.296	0.000	0.089	0.000	0.000	0.211	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	189	60	141	0	87	0	0	37	0
N.S.	1	1.09	0.35	0.82	0.00	0.50	0.00	0.00	0.21	0.00
time (sec)	N/A	0.562	10.024	1.752	0.000	0.087	0.000	0.000	0.222	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	378	60	237	0	108	0	0	37	0
N.S.	1	1.08	0.17	0.68	0.00	0.31	0.00	0.00	0.11	0.00
time (sec)	N/A	0.850	10.024	2.002	0.000	0.084	0.000	0.000	0.258	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	85	59	181	77	161	731	264	181	171
N.S.	1	1.05	0.73	2.23	0.95	1.99	9.02	3.26	2.23	2.11
time (sec)	N/A	0.397	0.063	0.198	0.039	0.085	0.673	0.119	0.183	17.803

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	62	43	96	55	89	337	141	96	97
N.S.	1	1.07	0.74	1.66	0.95	1.53	5.81	2.43	1.66	1.67
time (sec)	N/A	0.364	0.050	0.086	0.037	0.233	0.424	0.111	0.180	17.925

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	39	27	36	33	39	112	56	39	38
N.S.	1	1.11	0.77	1.03	0.94	1.11	3.20	1.60	1.11	1.09
time (sec)	N/A	0.318	0.027	0.070	0.035	0.086	0.255	0.111	0.191	18.061

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	0	0	0	0	0	23	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.292	0.029	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	0	0	0	0	0	34	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.288	0.030	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	0	0	0	0	0	45	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.96	0.00
time (sec)	N/A	0.288	0.032	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	24	24	24	24	26	24
N.S.	1	1.13	1.00	0.83	0.80	0.80	0.80	0.80	0.87	0.80
time (sec)	N/A	0.315	0.002	0.109	0.031	0.070	0.017	0.109	0.167	0.042

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	24	24	24	24	26	24
N.S.	1	1.13	1.00	0.83	0.80	0.80	0.80	0.80	0.87	0.80
time (sec)	N/A	0.314	0.001	0.107	0.031	0.068	0.017	0.127	0.160	0.033

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	24	24	24	24	25	24
N.S.	1	1.00	1.00	1.56	1.50	1.50	1.50	1.50	1.56	1.50
time (sec)	N/A	0.253	0.002	0.040	0.033	0.068	0.018	0.133	0.168	0.034

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	30	23	22	24	21	20	24	21	21
N.S.	1	1.30	1.00	0.96	1.04	0.91	0.87	1.04	0.91	0.91
time (sec)	N/A	0.296	0.001	0.063	0.031	0.073	0.034	0.123	0.163	0.031

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	27	24	24	27	24	32	27	23
N.S.	1	1.04	1.00	0.89	0.89	1.00	0.89	1.19	1.00	0.85
time (sec)	N/A	0.306	0.001	0.050	0.031	0.073	0.050	0.132	0.166	17.717

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	24	30	24	23	26	28	24	34	28	24
N.S.	1	1.25	1.00	0.96	1.08	1.17	1.00	1.42	1.17	1.00
time (sec)	N/A	0.312	0.001	0.049	0.031	0.070	0.076	0.144	0.168	0.047

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	24	24	26	24	26	26
N.S.	1	1.00	1.58	1.32	1.26	1.26	1.37	1.26	1.37	1.37
time (sec)	N/A	0.258	0.001	0.050	0.025	0.071	0.084	0.174	0.169	0.038

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	26	26	27	26	26	26
N.S.	1	1.13	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.308	0.001	0.053	0.026	0.072	0.094	0.172	0.172	0.039

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	26	26	27	26	26	26
N.S.	1	1.13	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.305	0.001	0.056	0.026	0.131	0.102	0.103	0.166	0.039

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.287	0.001	0.109	0.025	0.157	0.017	0.133	0.169	0.034

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.286	0.001	0.042	0.029	0.105	0.017	0.105	0.169	0.034

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	24	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.96	0.84
time (sec)	N/A	0.250	0.000	0.036	0.025	0.188	0.017	0.138	0.159	0.030

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	25	22
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	1.04	0.92
time (sec)	N/A	0.302	0.001	0.046	0.029	0.157	0.034	0.133	0.169	0.036

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	22	26	22	22	26	24
N.S.	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.13	1.04
time (sec)	N/A	0.302	0.001	0.044	0.026	0.105	0.058	0.152	0.174	0.030

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	26	25
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.93	0.89
time (sec)	N/A	0.294	0.001	0.049	0.025	0.061	0.078	0.109	0.162	0.037

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.288	0.001	0.049	0.026	0.135	0.087	0.100	0.169	0.038

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	60	56	47	46	46	49	46	48	46
N.S.	1	1.07	1.00	0.84	0.82	0.82	0.88	0.82	0.86	0.82
time (sec)	N/A	0.360	0.002	0.125	0.030	0.131	0.020	0.105	0.169	0.027

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	57	56	47	46	46	51	46	48	46
N.S.	1	1.02	1.00	0.84	0.82	0.82	0.91	0.82	0.86	0.82
time (sec)	N/A	0.357	0.002	0.125	0.026	0.067	0.019	0.120	0.170	0.024

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	56	47	46	46	49	46	48	46
N.S.	1	1.08	1.06	0.89	0.87	0.87	0.92	0.87	0.91	0.87
time (sec)	N/A	0.346	0.002	0.115	0.026	0.066	0.021	0.122	0.181	0.025

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	56	47	46	46	53	46	48	46
N.S.	1	1.12	1.65	1.38	1.35	1.35	1.56	1.35	1.41	1.35
time (sec)	N/A	0.312	0.002	0.113	0.031	0.144	0.021	0.109	0.165	0.025

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	45	44	44	44	44	47	44
N.S.	1	1.00	1.00	2.81	2.75	2.75	2.75	2.75	2.94	2.75
time (sec)	N/A	0.247	0.002	0.099	0.025	0.134	0.021	0.116	0.169	0.024

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	54	50	45	47	44	49	47	44	44
N.S.	1	1.08	1.00	0.90	0.94	0.88	0.98	0.94	0.88	0.88
time (sec)	N/A	0.333	0.003	0.102	0.031	0.142	0.047	0.102	0.168	0.028

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	48	53	48	45	46	49	46	56	49	44
N.S.	1	1.10	1.00	0.94	0.96	1.02	0.96	1.17	1.02	0.92
time (sec)	N/A	0.344	0.004	0.060	0.026	0.084	0.064	0.135	0.167	0.030

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	49	55	49	46	48	49	49	59	49	48
N.S.	1	1.12	1.00	0.94	0.98	1.00	1.00	1.20	1.00	0.98
time (sec)	N/A	0.348	0.004	0.076	0.030	0.077	0.096	0.171	0.168	0.040

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	49	52	49	46	48	50	49	57	50	47
N.S.	1	1.06	1.00	0.94	0.98	1.02	1.00	1.16	1.02	0.96
time (sec)	N/A	0.340	0.004	0.058	0.031	0.088	0.124	0.139	0.167	0.046

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	54	50	45	50	50	49	58	50	47
N.S.	1	1.08	1.00	0.90	1.00	1.00	0.98	1.16	1.00	0.94
time (sec)	N/A	0.346	0.004	0.056	0.028	0.165	0.155	0.138	0.170	0.054

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	52	47	46	46	49	46	48	46
N.S.	1	1.00	2.74	2.47	2.42	2.42	2.58	2.42	2.53	2.42
time (sec)	N/A	0.261	0.004	0.056	0.031	0.113	0.168	0.124	0.174	17.652

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	56	47	48	48	51	48	48	48
N.S.	1	1.10	1.40	1.18	1.20	1.20	1.28	1.20	1.20	1.20
time (sec)	N/A	0.288	0.003	0.060	0.033	0.158	0.186	0.125	0.166	18.646

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	56	58	56	47	48	48	51	48	48	48
N.S.	1	1.04	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.344	0.004	0.062	0.026	0.100	0.197	0.134	0.167	0.039

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	56	58	56	47	48	48	51	48	48	48
N.S.	1	1.04	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.341	0.004	0.066	0.031	0.078	0.213	0.108	0.177	0.040

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	56	60	56	47	48	48	51	48	48	48
N.S.	1	1.07	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.342	0.004	0.065	0.027	0.141	0.224	0.144	0.165	18.800

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	53	46	48	46
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.95	0.82	0.86	0.82
time (sec)	N/A	0.333	0.003	0.121	0.025	0.136	0.019	0.103	0.168	0.025

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	53	46	48	46
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.95	0.82	0.86	0.82
time (sec)	N/A	0.338	0.002	0.113	0.029	0.064	0.020	0.124	0.167	0.024

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	53	46	48	46
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.95	0.82	0.86	0.82
time (sec)	N/A	0.333	0.002	0.111	0.031	0.153	0.019	0.101	0.165	0.025

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	60	51	44	55	43	49	43	46	43
N.S.	1	1.18	1.00	0.86	1.08	0.84	0.96	0.84	0.90	0.84
time (sec)	N/A	0.347	0.001	0.040	0.032	0.147	0.019	0.142	0.172	0.023

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	44	48	44	44	48	44
N.S.	1	1.00	1.00	0.94	0.92	1.00	0.92	0.92	1.00	0.92
time (sec)	N/A	0.338	0.006	0.088	0.026	0.089	0.041	0.126	0.164	0.026

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	45	48	49	45	48	47
N.S.	1	1.00	1.00	0.90	0.90	0.96	0.98	0.90	0.96	0.94
time (sec)	N/A	0.328	0.005	0.057	0.031	0.119	0.065	0.120	0.161	0.049

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	47	48	49	47	48	47
N.S.	1	1.00	1.00	0.90	0.94	0.96	0.98	0.94	0.96	0.94
time (sec)	N/A	0.323	0.006	0.056	0.026	0.141	0.094	0.171	0.171	0.044

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	46	48	48	46	48	46
N.S.	1	1.00	1.00	0.94	0.98	1.02	1.02	0.98	1.02	0.98
time (sec)	N/A	0.320	0.005	0.056	0.027	0.090	0.128	0.111	0.166	0.039

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	47	48	48	51	48	48	47
N.S.	1	1.00	1.00	0.87	0.89	0.89	0.94	0.89	0.89	0.87
time (sec)	N/A	0.330	0.008	0.056	0.028	0.099	0.155	0.106	0.175	0.037

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	48	48	51	48	48	48
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.327	0.006	0.056	0.031	0.081	0.170	0.108	0.161	0.038

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	48	48	51	48	48	48
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.339	0.008	0.060	0.035	0.163	0.180	0.103	0.163	18.651

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	86	82	69	68	68	80	68	70	68
N.S.	1	1.05	1.00	0.84	0.83	0.83	0.98	0.83	0.85	0.83
time (sec)	N/A	0.405	0.003	0.139	0.026	0.091	0.022	0.129	0.167	0.032

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	95	82	69	68	68	78	68	70	68
N.S.	1	1.04	0.90	0.76	0.75	0.75	0.86	0.75	0.77	0.75
time (sec)	N/A	0.414	0.002	0.131	0.031	0.127	0.024	0.130	0.159	0.031

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	82	69	68	68	78	68	70	68
N.S.	1	1.06	1.14	0.96	0.94	0.94	1.08	0.94	0.97	0.94
time (sec)	N/A	0.386	0.002	0.127	0.025	0.163	0.024	0.130	0.161	0.031

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	82	69	68	68	80	68	70	68
N.S.	1	1.08	1.55	1.30	1.28	1.28	1.51	1.28	1.32	1.28
time (sec)	N/A	0.351	0.002	0.124	0.035	0.133	0.024	0.166	0.179	0.031

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	77	68	67	67	75	67	70	67
N.S.	1	1.12	2.26	2.00	1.97	1.97	2.21	1.97	2.06	1.97
time (sec)	N/A	0.317	0.002	0.118	0.030	0.082	0.025	0.105	0.163	0.031

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	69	68	68	78	68	69	68
N.S.	1	1.00	1.00	4.31	4.25	4.25	4.88	4.25	4.31	4.25
time (sec)	N/A	0.254	0.002	0.109	0.027	0.085	0.024	0.127	0.158	0.032

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	76	82	76	67	69	66	76	69	66	66
N.S.	1	1.08	1.00	0.88	0.91	0.87	1.00	0.91	0.87	0.87
time (sec)	N/A	0.366	0.004	0.128	0.030	0.125	0.049	0.103	0.167	0.035

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	68	69	72	76	79	72	67
N.S.	1	1.00	1.00	0.88	0.90	0.94	0.99	1.03	0.94	0.87
time (sec)	N/A	0.383	0.006	0.063	0.027	0.097	0.067	0.128	0.166	0.037

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	72	79	72	67	69	71	73	80	71	69
N.S.	1	1.10	1.00	0.93	0.96	0.99	1.01	1.11	0.99	0.96
time (sec)	N/A	0.380	0.004	0.064	0.034	0.083	0.098	0.141	0.174	0.036

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	79	68	70	71	76	81	71	70
N.S.	1	0.97	1.00	0.86	0.89	0.90	0.96	1.03	0.90	0.89
time (sec)	N/A	0.381	0.004	0.065	0.027	0.081	0.134	0.137	0.166	18.698

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	73	79	73	68	70	72	73	81	72	69
N.S.	1	1.08	1.00	0.93	0.96	0.99	1.00	1.11	0.99	0.95
time (sec)	N/A	0.380	0.006	0.083	0.028	0.109	0.175	0.152	0.162	17.659

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	77	76	77	68	72	72	75	81	72	70
N.S.	1	0.99	1.00	0.88	0.94	0.94	0.97	1.05	0.94	0.91
time (sec)	N/A	0.372	0.004	0.063	0.029	0.124	0.220	0.113	0.168	0.052

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	76	82	76	67	72	72	73	80	72	69
N.S.	1	1.08	1.00	0.88	0.95	0.95	0.96	1.05	0.95	0.91
time (sec)	N/A	0.383	0.004	0.062	0.040	0.149	0.260	0.124	0.164	17.893

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	82	69	68	68	73	68	70	70
N.S.	1	1.00	4.32	3.63	3.58	3.58	3.84	3.58	3.68	3.68
time (sec)	N/A	0.259	0.006	0.064	0.028	0.088	0.283	0.133	0.163	0.053

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	78	69	70	70	75	70	70	69
N.S.	1	1.10	1.95	1.72	1.75	1.75	1.88	1.75	1.75	1.72
time (sec)	N/A	0.296	0.004	0.069	0.026	0.110	0.300	0.108	0.175	17.909

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	72	82	69	70	70	75	70	70	70
N.S.	1	1.16	1.32	1.11	1.13	1.13	1.21	1.13	1.13	1.13
time (sec)	N/A	0.309	0.004	0.071	0.030	0.085	0.309	0.105	0.169	0.050

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	100	82	69	70	70	75	70	70	70
N.S.	1	1.19	0.98	0.82	0.83	0.83	0.89	0.83	0.83	0.83
time (sec)	N/A	0.339	0.006	0.075	0.025	0.093	0.327	0.103	0.165	18.890

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	82	84	82	69	70	70	75	70	70	70
N.S.	1	1.02	1.00	0.84	0.85	0.85	0.91	0.85	0.85	0.85
time (sec)	N/A	0.375	0.004	0.079	0.026	0.083	0.344	0.127	0.160	0.054

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	82	86	82	69	70	70	75	70	70	70
N.S.	1	1.05	1.00	0.84	0.85	0.85	0.91	0.85	0.85	0.85
time (sec)	N/A	0.389	0.004	0.081	0.028	0.115	0.376	0.125	0.165	19.286

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	68	68	80	68	70	68
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.85	0.83
time (sec)	N/A	0.381	0.002	0.131	0.029	0.132	0.022	0.113	0.168	0.032

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	67	67	76	67	70	67
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.96	0.85	0.89	0.85
time (sec)	N/A	0.384	0.002	0.128	0.026	0.078	0.021	0.141	0.161	0.031

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	68	68	80	68	70	68
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.85	0.83
time (sec)	N/A	0.367	0.002	0.122	0.030	0.142	0.021	0.120	0.158	0.031

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	68	68	80	68	70	68
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.85	0.83
time (sec)	N/A	0.384	0.002	0.116	0.027	0.104	0.021	0.108	0.167	0.032

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	82	73	66	100	65	73	65	68	65
N.S.	1	1.12	1.00	0.90	1.37	0.89	1.00	0.89	0.93	0.89
time (sec)	N/A	0.390	0.001	0.048	0.033	0.150	0.021	0.112	0.167	0.030

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	66	70	70	66	70	66
N.S.	1	1.00	1.00	0.93	0.92	0.97	0.97	0.92	0.97	0.92
time (sec)	N/A	0.375	0.007	0.200	0.026	0.123	0.049	0.105	0.160	0.034

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	67	67	70	75	67	70	69
N.S.	1	1.00	1.00	0.91	0.91	0.95	1.01	0.91	0.95	0.93
time (sec)	N/A	0.371	0.007	0.069	0.024	0.156	0.073	0.108	0.167	0.032

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	67	70	73	67	70	69
N.S.	1	1.00	1.00	0.93	0.93	0.97	1.01	0.93	0.97	0.96
time (sec)	N/A	0.376	0.005	0.065	0.025	0.082	0.103	0.124	0.168	0.033

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	69	70	73	69	70	69
N.S.	1	1.00	1.00	0.93	0.96	0.97	1.01	0.96	0.97	0.96
time (sec)	N/A	0.371	0.008	0.066	0.027	0.083	0.138	0.105	0.167	0.054

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	67	69	70	73	69	70	70
N.S.	1	1.00	1.00	0.91	0.93	0.95	0.99	0.93	0.95	0.95
time (sec)	N/A	0.370	0.007	0.067	0.030	0.081	0.179	0.125	0.164	0.054

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	66	68	70	71	68	70	68
N.S.	1	1.00	1.00	0.93	0.96	0.99	1.00	0.96	0.99	0.96
time (sec)	N/A	0.367	0.005	0.062	0.036	0.135	0.219	0.106	0.164	0.048

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	69	70	70	75	70	70	69
N.S.	1	1.00	1.00	0.91	0.92	0.92	0.99	0.92	0.92	0.91
time (sec)	N/A	0.376	0.007	0.066	0.026	0.075	0.249	0.149	0.167	18.443

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	70	70	75	70	70	70
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.91	0.85	0.85	0.85
time (sec)	N/A	0.372	0.008	0.067	0.027	0.172	0.267	0.107	0.170	0.049

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	82	72	74	77	93	80	92	93	79
N.S.	1	0.99	0.87	0.89	0.93	1.12	0.96	1.11	1.12	0.95
time (sec)	N/A	0.420	0.015	0.084	0.034	0.154	0.131	0.103	0.166	19.390

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	68	60	63	65	81	66	80	81	68
N.S.	1	0.97	0.86	0.90	0.93	1.16	0.94	1.14	1.16	0.97
time (sec)	N/A	0.396	0.017	0.076	0.028	0.186	0.124	0.110	0.173	0.043

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	56	49	53	54	70	53	67	70	57
N.S.	1	0.98	0.86	0.93	0.95	1.23	0.93	1.18	1.23	1.00
time (sec)	N/A	0.364	0.012	0.096	0.027	0.120	0.118	0.108	0.166	0.054

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	43	38	41	43	56	39	49	57	45
N.S.	1	0.98	0.86	0.93	0.98	1.27	0.89	1.11	1.30	1.02
time (sec)	N/A	0.333	0.012	0.062	0.034	0.119	0.103	0.126	0.166	19.210

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	31	27	30	32	35	29	30	44	29
N.S.	1	0.94	0.82	0.91	0.97	1.06	0.88	0.91	1.33	0.88
time (sec)	N/A	0.317	0.009	0.056	0.030	0.104	0.083	0.154	0.167	0.050

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	15	15	14	17	14
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.94	0.88	1.06	0.88
time (sec)	N/A	0.258	0.002	0.050	0.026	0.085	0.064	0.164	0.164	0.031

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	39	33	35	37	47	34	47	59	34
N.S.	1	1.03	0.87	0.92	0.97	1.24	0.89	1.24	1.55	0.89
time (sec)	N/A	0.332	0.010	0.076	0.028	0.103	0.136	0.110	0.171	0.058

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	52	41	51	52	73	51	51	81	51
N.S.	1	1.06	0.84	1.04	1.06	1.49	1.04	1.04	1.65	1.04
time (sec)	N/A	0.359	0.024	0.089	0.030	0.153	0.169	0.156	0.161	0.075

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	68	57	65	70	90	68	86	94	67
N.S.	1	1.03	0.86	0.98	1.06	1.36	1.03	1.30	1.42	1.02
time (sec)	N/A	0.376	0.039	0.097	0.034	0.117	0.203	0.129	0.160	0.074

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	95	82	76	82	212	134	84	107	77
N.S.	1	1.08	0.93	0.86	0.93	2.41	1.52	0.95	1.22	0.88
time (sec)	N/A	0.399	0.039	0.099	0.106	0.097	0.146	0.152	0.174	19.469

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	82	71	65	71	190	124	73	96	66
N.S.	1	1.05	0.91	0.83	0.91	2.44	1.59	0.94	1.23	0.85
time (sec)	N/A	0.379	0.034	0.086	0.111	0.273	0.137	0.113	0.167	0.039

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	69	60	54	59	164	107	61	83	56
N.S.	1	1.06	0.92	0.83	0.91	2.52	1.65	0.94	1.28	0.86
time (sec)	N/A	0.361	0.032	0.078	0.109	0.100	0.128	0.135	0.167	0.059

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	58	51	42	45	136	83	42	69	43
N.S.	1	1.14	1.00	0.82	0.88	2.67	1.63	0.82	1.35	0.84
time (sec)	N/A	0.315	0.024	0.072	0.117	0.159	0.114	0.152	0.167	18.857

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	36	120	78	35	62	33
N.S.	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	1.38	0.73
time (sec)	N/A	0.287	0.015	0.075	0.110	0.104	0.089	0.106	0.163	0.043

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	52	45	36	35	120	78	35	61	33
N.S.	1	1.16	1.00	0.80	0.78	2.67	1.73	0.78	1.36	0.73
time (sec)	N/A	0.282	0.016	0.064	0.103	0.207	0.095	0.118	0.172	18.397

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	54	45	49	136	92	47	72	44
N.S.	1	1.13	1.00	0.83	0.91	2.52	1.70	0.87	1.33	0.81
time (sec)	N/A	0.302	0.027	0.099	0.107	0.203	0.135	0.163	0.166	0.075

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	78	67	55	64	172	114	59	88	58
N.S.	1	1.16	1.00	0.82	0.96	2.57	1.70	0.88	1.31	0.87
time (sec)	N/A	0.326	0.028	0.125	0.106	0.083	0.161	0.119	0.165	18.401

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	95	80	67	75	198	126	70	101	70
N.S.	1	1.19	1.00	0.84	0.94	2.48	1.58	0.88	1.26	0.88
time (sec)	N/A	0.350	0.032	0.119	0.105	0.092	0.186	0.168	0.173	18.227

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	95	78	76	99	137	100	91	147	98
N.S.	1	1.04	0.86	0.84	1.09	1.51	1.10	1.00	1.62	1.08
time (sec)	N/A	0.424	0.023	0.142	0.031	0.079	0.284	0.103	0.167	17.762

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	59	65	88	124	90	73	134	88
N.S.	1	1.03	0.77	0.84	1.14	1.61	1.17	0.95	1.74	1.14
time (sec)	N/A	0.406	0.040	0.099	0.027	0.080	0.250	0.106	0.168	17.657

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	70	50	54	77	102	76	53	122	75
N.S.	1	0.99	0.70	0.76	1.08	1.44	1.07	0.75	1.72	1.06
time (sec)	N/A	0.379	0.014	0.091	0.027	0.067	0.209	0.123	0.169	0.072

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	35	37	58	58	60	33	39	60
N.S.	1	1.00	1.84	1.95	3.05	3.05	3.16	1.74	2.05	3.16
time (sec)	N/A	0.259	0.011	0.087	0.031	0.071	0.176	0.155	0.165	0.040

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	24	26	47	47	48	22	46	48
N.S.	1	1.12	0.71	0.76	1.38	1.38	1.41	0.65	1.35	1.41
time (sec)	N/A	0.318	0.007	0.088	0.033	0.062	0.155	0.143	0.171	0.035

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	37	37	39	14	36	39
N.S.	1	1.00	1.00	0.94	2.31	2.31	2.44	0.88	2.25	2.44
time (sec)	N/A	0.259	0.002	0.082	0.027	0.063	0.141	0.135	0.164	0.038

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	71	54	63	82	134	80	70	161	78
N.S.	1	1.01	0.77	0.90	1.17	1.91	1.14	1.00	2.30	1.11
time (sec)	N/A	0.382	0.028	0.113	0.027	0.069	0.242	0.119	0.164	0.073

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	76	99	163	102	93	187	97
N.S.	1	1.00	0.83	0.90	1.18	1.94	1.21	1.11	2.23	1.15
time (sec)	N/A	0.412	0.044	0.130	0.028	0.067	0.288	0.131	0.171	17.482

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	107	85	89	114	178	116	108	202	111
N.S.	1	1.06	0.84	0.88	1.13	1.76	1.15	1.07	2.00	1.10
time (sec)	N/A	0.452	0.044	0.139	0.035	0.069	0.313	0.161	0.168	17.425

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	136	99	85	116	322	172	96	194	109
N.S.	1	1.15	0.84	0.72	0.98	2.73	1.46	0.81	1.64	0.92
time (sec)	N/A	0.456	0.037	0.125	0.112	0.072	0.291	0.127	0.161	0.059

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	123	89	74	104	296	156	84	181	99
N.S.	1	1.17	0.85	0.70	0.99	2.82	1.49	0.80	1.72	0.94
time (sec)	N/A	0.427	0.032	0.109	0.110	0.071	0.279	0.136	0.163	0.076

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	112	77	62	90	268	131	65	167	86
N.S.	1	1.23	0.85	0.68	0.99	2.95	1.44	0.71	1.84	0.95
time (sec)	N/A	0.387	0.032	0.101	0.107	0.073	0.252	0.115	0.162	17.423

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	99	66	58	81	254	134	56	162	78
N.S.	1	1.19	0.80	0.70	0.98	3.06	1.61	0.67	1.95	0.94
time (sec)	N/A	0.361	0.027	0.108	0.106	0.071	0.209	0.116	0.160	17.205

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	97	69	58	87	258	143	62	162	75
N.S.	1	1.15	0.82	0.69	1.04	3.07	1.70	0.74	1.93	0.89
time (sec)	N/A	0.351	0.033	0.109	0.105	0.104	0.188	0.132	0.159	17.191

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	69	58	87	258	139	62	162	74
N.S.	1	1.12	0.81	0.68	1.02	3.04	1.64	0.73	1.91	0.87
time (sec)	N/A	0.335	0.028	0.111	0.104	0.083	0.184	0.168	0.160	17.070

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	114	66	78	80	254	129	56	162	77
N.S.	1	1.44	0.84	0.99	1.01	3.22	1.63	0.71	2.05	0.97
time (sec)	N/A	0.348	0.024	0.095	0.106	0.086	0.186	0.132	0.154	17.288

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	115	79	65	93	268	139	68	170	88
N.S.	1	1.28	0.88	0.72	1.03	2.98	1.54	0.76	1.89	0.98
time (sec)	N/A	0.372	0.029	0.135	0.116	0.080	0.250	0.127	0.154	17.433

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	132	91	75	108	304	162	82	186	102
N.S.	1	1.23	0.85	0.70	1.01	2.84	1.51	0.77	1.74	0.95
time (sec)	N/A	0.403	0.035	0.152	0.113	0.085	0.276	0.123	0.147	17.663

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	149	101	87	119	330	173	93	199	114
N.S.	1	1.24	0.84	0.72	0.99	2.75	1.44	0.78	1.66	0.95
time (sec)	N/A	0.436	0.037	0.164	0.111	0.084	0.305	0.135	0.154	0.158

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	131	114	98	143	203	150	113	229	142
N.S.	1	0.98	0.86	0.74	1.08	1.53	1.13	0.85	1.72	1.07
time (sec)	N/A	0.512	0.017	0.190	0.043	0.063	0.450	0.106	0.149	17.749

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	115	101	87	132	190	138	95	216	132
N.S.	1	0.97	0.86	0.74	1.12	1.61	1.17	0.81	1.83	1.12
time (sec)	N/A	0.508	0.019	0.142	0.043	0.070	0.428	0.167	0.154	17.402

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	106	72	76	121	168	124	75	204	119
N.S.	1	0.97	0.66	0.70	1.11	1.54	1.14	0.69	1.87	1.09
time (sec)	N/A	0.449	0.019	0.129	0.034	0.065	0.365	0.152	0.149	0.115

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	57	59	102	102	107	55	61	104
N.S.	1	1.00	3.00	3.11	5.37	5.37	5.63	2.89	3.21	5.47
time (sec)	N/A	0.265	0.014	0.125	0.027	0.075	0.315	0.110	0.152	0.049

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	46	48	91	91	95	44	90	93
N.S.	1	1.10	1.18	1.23	2.33	2.33	2.44	1.13	2.31	2.38
time (sec)	N/A	0.289	0.011	0.125	0.037	0.061	0.285	0.107	0.159	17.861

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	35	37	80	80	83	33	79	81
N.S.	1	1.08	0.66	0.70	1.51	1.51	1.57	0.62	1.49	1.53
time (sec)	N/A	0.364	0.011	0.125	0.029	0.061	0.262	0.113	0.162	18.391

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	24	26	69	69	71	22	68	70
N.S.	1	1.12	0.71	0.76	2.03	2.03	2.09	0.65	2.00	2.06
time (sec)	N/A	0.326	0.007	0.125	0.032	0.065	0.238	0.110	0.159	0.044

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	59	59	63	14	58	61
N.S.	1	1.00	1.00	0.94	3.69	3.69	3.94	0.88	3.62	3.81
time (sec)	N/A	0.263	0.002	0.120	0.034	0.068	0.218	0.120	0.165	0.047

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	103	76	85	126	222	128	92	269	122
N.S.	1	1.01	0.75	0.83	1.24	2.18	1.25	0.90	2.64	1.20
time (sec)	N/A	0.438	0.033	0.164	0.036	0.067	0.346	0.137	0.168	18.654

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	92	98	143	251	150	115	295	141
N.S.	1	1.00	0.79	0.84	1.23	2.16	1.29	0.99	2.54	1.22
time (sec)	N/A	0.483	0.055	0.187	0.035	0.070	0.403	0.107	0.160	0.181

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	141	107	111	158	266	165	130	310	155
N.S.	1	1.01	0.76	0.79	1.13	1.90	1.18	0.93	2.21	1.11
time (sec)	N/A	0.520	0.040	0.198	0.043	0.077	0.438	0.106	0.166	18.167

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	190	122	107	159	454	218	117	292	153
N.S.	1	1.22	0.78	0.69	1.02	2.91	1.40	0.75	1.87	0.98
time (sec)	N/A	0.553	0.043	0.163	0.116	0.078	0.468	0.111	0.161	17.530

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	177	111	96	148	428	204	106	279	143
N.S.	1	1.22	0.77	0.66	1.02	2.95	1.41	0.73	1.92	0.99
time (sec)	N/A	0.530	0.040	0.145	0.108	0.074	0.447	0.137	0.165	0.112

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	166	100	84	134	400	178	87	265	130
N.S.	1	1.27	0.76	0.64	1.02	3.05	1.36	0.66	2.02	0.99
time (sec)	N/A	0.446	0.035	0.136	0.109	0.074	0.420	0.121	0.176	17.908

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	153	88	80	125	386	182	78	260	122
N.S.	1	1.26	0.73	0.66	1.03	3.19	1.50	0.64	2.15	1.01
time (sec)	N/A	0.415	0.037	0.148	0.118	0.086	0.363	0.106	0.162	18.357

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	151	91	80	131	390	194	84	260	119
N.S.	1	1.24	0.75	0.66	1.07	3.20	1.59	0.69	2.13	0.98
time (sec)	N/A	0.407	0.040	0.144	0.116	0.080	0.324	0.114	0.161	18.812

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	149	91	78	133	390	196	84	260	117
N.S.	1	1.21	0.74	0.63	1.08	3.17	1.59	0.68	2.11	0.95
time (sec)	N/A	0.407	0.039	0.144	0.112	0.082	0.299	0.135	0.173	0.122

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	147	91	78	133	390	196	84	260	116
N.S.	1	1.19	0.73	0.63	1.07	3.15	1.58	0.68	2.10	0.94
time (sec)	N/A	0.395	0.034	0.142	0.113	0.085	0.277	0.110	0.168	18.428

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	145	91	80	131	390	190	84	260	118
N.S.	1	1.16	0.73	0.64	1.05	3.12	1.52	0.67	2.08	0.94
time (sec)	N/A	0.390	0.038	0.148	0.118	0.093	0.270	0.107	0.164	18.171

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	176	89	120	124	386	177	78	260	121
N.S.	1	1.56	0.79	1.06	1.10	3.42	1.57	0.69	2.30	1.07
time (sec)	N/A	0.424	0.030	0.130	0.112	0.101	0.280	0.108	0.168	17.971

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	169	101	87	137	400	187	90	268	132
N.S.	1	1.34	0.80	0.69	1.09	3.17	1.48	0.71	2.13	1.05
time (sec)	N/A	0.446	0.037	0.170	0.114	0.087	0.367	0.137	0.163	0.170

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	186	113	97	152	436	209	104	284	146
N.S.	1	1.27	0.77	0.66	1.03	2.97	1.42	0.71	1.93	0.99
time (sec)	N/A	0.464	0.043	0.194	0.132	0.096	0.393	0.133	0.170	18.642

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	203	123	109	163	462	221	115	297	158
N.S.	1	1.28	0.78	0.69	1.03	2.92	1.40	0.73	1.88	1.00
time (sec)	N/A	0.515	0.042	0.220	0.118	0.082	0.423	0.110	0.166	18.318

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	15	19	12	15	20	16
N.S.	1	1.00	0.84	0.84	0.79	1.00	0.63	0.79	1.05	0.84
time (sec)	N/A	0.242	0.005	0.056	0.104	0.066	0.043	0.129	0.167	0.031

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	13	11
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	1.18	1.00
time (sec)	N/A	0.239	0.001	0.036	0.029	0.068	0.028	0.110	0.174	0.018

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	22	17
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	1.16	0.89
time (sec)	N/A	0.251	0.006	0.062	0.107	0.073	0.037	0.107	0.163	0.028

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	18	18	19	18	23	15	18	32	18
N.S.	1	0.82	0.82	0.86	0.82	1.05	0.68	0.82	1.45	0.82
time (sec)	N/A	0.279	0.005	0.042	0.028	0.065	0.032	0.102	0.165	17.606

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	9	8	9	14	11
N.S.	1	1.00	0.85	0.77	0.69	0.69	0.62	0.69	1.08	0.85
time (sec)	N/A	0.239	0.001	0.043	0.029	0.061	0.028	0.149	0.166	0.044

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	19	18	23	14	19	44	18
N.S.	1	1.00	0.83	0.79	0.75	0.96	0.58	0.79	1.83	0.75
time (sec)	N/A	0.293	0.006	0.049	0.030	0.066	0.031	0.108	0.166	0.062

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	33	30	41	40	48	48	33	40
N.S.	1	1.00	0.65	0.59	0.80	0.78	0.94	0.94	0.65	0.78
time (sec)	N/A	0.323	0.022	0.127	0.032	0.101	0.320	0.106	0.168	0.072

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	33	30	41	34	48	42	31	41
N.S.	1	1.00	0.65	0.59	0.80	0.67	0.94	0.82	0.61	0.80
time (sec)	N/A	0.329	0.018	0.100	0.034	0.077	0.197	0.106	0.164	17.625

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	33	30	41	29	48	37	28	41
N.S.	1	1.00	0.65	0.59	0.80	0.57	0.94	0.73	0.55	0.80
time (sec)	N/A	0.329	0.015	0.103	0.029	0.096	0.118	0.123	0.164	0.051

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	41	31	46	41	30	41
N.S.	1	1.00	0.67	0.61	0.84	0.63	0.94	0.84	0.61	0.84
time (sec)	N/A	0.320	0.017	0.108	0.028	0.073	0.211	0.136	0.171	0.047

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	44	34	46	51	32	31
N.S.	1	1.00	0.67	0.61	0.90	0.69	0.94	1.04	0.65	0.63
time (sec)	N/A	0.319	0.021	0.117	0.027	0.111	0.216	0.101	0.163	0.053

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	43	34	46	55	35	34
N.S.	1	1.00	0.67	0.61	0.88	0.69	0.94	1.12	0.71	0.69
time (sec)	N/A	0.318	0.020	0.120	0.031	0.155	0.250	0.110	0.166	0.052

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	38	30	47	34	46	48	35	34
N.S.	1	1.00	0.78	0.61	0.96	0.69	0.94	0.98	0.71	0.69
time (sec)	N/A	0.312	0.023	0.126	0.031	0.159	0.341	0.140	0.172	17.757

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	52	73	68	88	86	55	71
N.S.	1	1.00	0.60	0.57	0.80	0.75	0.97	0.95	0.60	0.78
time (sec)	N/A	0.383	0.028	0.336	0.026	0.087	0.515	0.127	0.163	0.040

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	52	73	58	88	74	53	71
N.S.	1	1.00	0.60	0.57	0.80	0.64	0.97	0.81	0.58	0.78
time (sec)	N/A	0.380	0.028	0.221	0.027	0.093	0.334	0.123	0.166	0.031

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	50	73	51	88	69	50	71
N.S.	1	1.00	0.60	0.55	0.80	0.56	0.97	0.76	0.55	0.78
time (sec)	N/A	0.384	0.027	0.245	0.030	0.076	0.206	0.127	0.167	0.031

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	55	52	90	53	87	73	52	71
N.S.	1	1.00	0.62	0.58	1.01	0.60	0.98	0.82	0.58	0.80
time (sec)	N/A	0.366	0.027	0.153	0.032	0.126	0.289	0.110	0.164	0.031

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	55	52	76	56	87	89	54	71
N.S.	1	1.00	0.62	0.58	0.85	0.63	0.98	1.00	0.61	0.80
time (sec)	N/A	0.374	0.026	0.164	0.027	0.115	0.291	0.170	0.168	0.033

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	55	52	76	56	87	94	57	71
N.S.	1	1.00	0.62	0.58	0.85	0.63	0.98	1.06	0.64	0.80
time (sec)	N/A	0.369	0.024	0.167	0.027	0.148	0.319	0.146	0.163	0.033

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	60	52	82	56	85	95	57	75
N.S.	1	1.00	0.69	0.60	0.94	0.64	0.98	1.09	0.66	0.86
time (sec)	N/A	0.366	0.027	0.179	0.033	0.094	0.372	0.148	0.162	0.060

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	77	74	105	96	128	124	77	103
N.S.	1	1.00	0.60	0.57	0.81	0.74	0.99	0.96	0.60	0.80
time (sec)	N/A	0.435	0.030	0.819	0.027	0.098	0.792	0.167	0.229	0.041

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	77	74	105	82	129	106	75	103
N.S.	1	1.00	0.59	0.56	0.80	0.63	0.98	0.81	0.57	0.79
time (sec)	N/A	0.432	0.027	0.316	0.035	0.140	0.524	0.149	0.217	0.038

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	77	72	105	73	129	101	72	103
N.S.	1	1.00	0.59	0.55	0.80	0.56	0.98	0.77	0.55	0.79
time (sec)	N/A	0.452	0.023	0.342	0.026	0.070	0.341	0.152	0.232	0.039

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	77	74	155	75	128	105	74	103
N.S.	1	1.00	0.60	0.57	1.20	0.58	0.99	0.81	0.57	0.80
time (sec)	N/A	0.420	0.023	0.256	0.032	0.130	0.402	0.136	0.220	0.038

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	77	74	108	78	124	127	76	103
N.S.	1	1.00	0.62	0.59	0.86	0.62	0.99	1.02	0.61	0.82
time (sec)	N/A	0.410	0.028	0.273	0.028	0.086	0.422	0.155	0.227	0.041

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	77	74	108	78	126	132	79	103
N.S.	1	1.00	0.61	0.58	0.85	0.61	0.99	1.04	0.62	0.81
time (sec)	N/A	0.414	0.028	0.274	0.028	0.079	0.453	0.117	0.222	0.041

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	82	74	114	78	126	133	79	107
N.S.	1	1.00	0.65	0.58	0.90	0.61	0.99	1.05	0.62	0.84
time (sec)	N/A	0.422	0.029	0.285	0.031	0.121	0.535	0.131	0.224	0.041

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	349	178	202	300	295	0	306	331	129
N.S.	1	1.46	0.74	0.85	1.26	1.23	0.00	1.28	1.38	0.54
time (sec)	N/A	1.013	0.308	1.925	0.118	0.128	0.000	0.126	0.233	17.922

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	326	167	188	273	290	0	277	319	112
N.S.	1	1.48	0.76	0.85	1.24	1.31	0.00	1.25	1.44	0.51
time (sec)	N/A	0.919	0.297	0.963	0.120	0.120	0.000	0.129	0.313	0.129

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	324	167	190	282	261	0	278	318	112
N.S.	1	1.47	0.76	0.86	1.28	1.18	0.00	1.26	1.44	0.51
time (sec)	N/A	0.897	0.278	0.487	0.114	0.169	0.000	0.116	0.178	0.126

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	301	154	171	256	259	0	277	312	92
N.S.	1	1.48	0.75	0.84	1.25	1.27	0.00	1.36	1.53	0.45
time (sec)	N/A	0.860	0.276	0.433	0.114	0.107	0.000	0.116	0.177	18.236

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	301	153	177	265	234	0	275	307	92
N.S.	1	1.48	0.75	0.87	1.30	1.15	0.00	1.35	1.50	0.45
time (sec)	N/A	0.843	0.268	0.351	0.113	0.150	0.000	0.121	0.172	17.797

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	301	152	180	255	247	0	264	307	90
N.S.	1	1.46	0.74	0.87	1.24	1.20	0.00	1.28	1.49	0.44
time (sec)	N/A	0.848	0.264	0.355	0.116	0.185	0.000	0.120	0.174	0.112

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	305	154	177	261	244	0	269	311	90
N.S.	1	1.48	0.75	0.86	1.27	1.18	0.00	1.31	1.51	0.44
time (sec)	N/A	0.840	0.255	0.342	0.112	0.107	0.000	0.162	0.168	17.906

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	325	165	186	268	294	0	294	334	102
N.S.	1	1.46	0.74	0.83	1.20	1.32	0.00	1.32	1.50	0.46
time (sec)	N/A	0.901	0.287	0.426	0.113	0.123	0.000	0.141	0.170	0.130

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	329	165	192	275	320	0	276	341	102
N.S.	1	1.48	0.74	0.86	1.23	1.43	0.00	1.24	1.53	0.46
time (sec)	N/A	0.900	0.290	0.455	0.114	0.130	0.000	0.109	0.178	17.755

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	352	181	198	290	339	0	307	360	113
N.S.	1	1.46	0.75	0.82	1.20	1.41	0.00	1.27	1.49	0.47
time (sec)	N/A	0.936	0.305	0.445	0.113	0.115	0.000	0.163	0.178	17.844

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	419	205	232	361	433	0	345	673	188
N.S.	1	1.44	0.70	0.80	1.24	1.49	0.00	1.19	2.31	0.65
time (sec)	N/A	1.070	0.610	2.516	0.118	0.168	0.000	0.161	0.184	0.136

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	396	194	218	334	428	0	316	661	171
N.S.	1	1.45	0.71	0.80	1.22	1.57	0.00	1.16	2.42	0.63
time (sec)	N/A	1.002	0.589	2.501	0.115	0.140	0.000	0.154	0.178	18.368

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	394	175	220	343	399	0	317	660	171
N.S.	1	1.44	0.64	0.81	1.26	1.46	0.00	1.16	2.42	0.63
time (sec)	N/A	1.008	0.507	2.382	0.115	0.172	0.000	0.130	0.178	17.806

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	371	164	203	317	406	0	314	654	153
N.S.	1	1.45	0.64	0.79	1.24	1.59	0.00	1.23	2.55	0.60
time (sec)	N/A	0.948	0.521	2.353	0.121	0.139	0.000	0.122	0.183	0.115

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	371	164	209	326	405	0	313	653	153
N.S.	1	1.45	0.64	0.82	1.27	1.58	0.00	1.22	2.55	0.60
time (sec)	N/A	0.981	0.496	2.388	0.114	0.126	0.000	0.148	0.182	18.417

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	371	164	206	323	429	0	317	654	150
N.S.	1	1.43	0.63	0.80	1.25	1.66	0.00	1.22	2.53	0.58
time (sec)	N/A	1.010	0.477	2.398	0.120	0.162	0.000	0.188	0.185	18.407

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	375	164	206	332	428	0	316	653	150
N.S.	1	1.45	0.63	0.80	1.28	1.65	0.00	1.22	2.52	0.58
time (sec)	N/A	0.986	0.464	2.352	0.120	0.149	0.000	0.118	0.174	18.132

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	370	161	206	323	429	0	317	654	149
N.S.	1	1.43	0.62	0.80	1.25	1.66	0.00	1.23	2.53	0.58
time (sec)	N/A	0.962	0.424	2.300	0.114	0.190	0.000	0.144	0.178	18.732

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	374	161	206	332	406	0	316	651	149
N.S.	1	1.45	0.62	0.80	1.29	1.57	0.00	1.22	2.52	0.58
time (sec)	N/A	0.969	0.411	0.477	0.115	0.183	0.000	0.122	0.182	0.120

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	369	161	212	317	394	0	302	651	150
N.S.	1	1.43	0.62	0.82	1.23	1.53	0.00	1.17	2.52	0.58
time (sec)	N/A	0.994	0.305	0.510	0.119	0.135	0.000	0.133	0.178	0.108

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	373	161	209	322	395	0	308	653	150
N.S.	1	1.45	0.62	0.81	1.25	1.53	0.00	1.19	2.53	0.58
time (sec)	N/A	0.961	0.287	0.440	0.114	0.148	0.000	0.176	0.185	18.607

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	393	173	217	328	456	0	327	688	166
N.S.	1	1.43	0.63	0.79	1.19	1.66	0.00	1.19	2.50	0.60
time (sec)	N/A	0.997	0.517	0.599	0.121	0.157	0.000	0.155	0.176	0.144

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	397	173	222	335	482	0	308	695	166
N.S.	1	1.44	0.63	0.81	1.22	1.75	0.00	1.12	2.53	0.60
time (sec)	N/A	1.041	0.513	0.565	0.119	0.187	0.000	0.158	0.181	0.180

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	420	189	229	350	501	0	349	714	179
N.S.	1	1.43	0.65	0.78	1.19	1.71	0.00	1.19	2.44	0.61
time (sec)	N/A	1.060	0.563	0.652	0.141	0.188	0.000	0.186	0.184	18.672

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	489	227	256	421	571	0	383	1015	248
N.S.	1	1.43	0.66	0.75	1.23	1.66	0.00	1.12	2.96	0.72
time (sec)	N/A	1.199	1.034	14.829	0.126	0.167	0.000	0.151	0.187	18.674

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	466	216	250	394	566	0	354	1003	231
N.S.	1	1.43	0.66	0.77	1.21	1.74	0.00	1.09	3.09	0.71
time (sec)	N/A	1.162	1.040	14.726	0.120	0.124	0.000	0.146	0.190	18.628

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	464	216	245	403	537	0	355	1002	231
N.S.	1	1.43	0.66	0.75	1.24	1.65	0.00	1.09	3.08	0.71
time (sec)	N/A	1.150	0.961	14.768	0.127	0.207	0.000	0.136	0.188	18.491

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	441	205	235	377	544	0	352	996	213
N.S.	1	1.43	0.67	0.76	1.22	1.77	0.00	1.14	3.23	0.69
time (sec)	N/A	1.120	1.012	14.742	0.116	0.157	0.000	0.165	0.190	0.231

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	441	205	240	386	543	0	351	995	213
N.S.	1	1.43	0.67	0.78	1.25	1.76	0.00	1.14	3.23	0.69
time (sec)	N/A	1.105	0.957	14.626	0.125	0.136	0.000	0.150	0.183	0.131

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	441	205	238	383	567	0	355	996	210
N.S.	1	1.42	0.66	0.77	1.23	1.82	0.00	1.14	3.20	0.68
time (sec)	N/A	1.082	0.978	14.700	0.120	0.181	0.000	0.141	0.186	17.733

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	445	186	238	392	566	0	354	995	210
N.S.	1	1.43	0.60	0.77	1.26	1.82	0.00	1.14	3.20	0.68
time (sec)	N/A	1.122	0.806	14.635	0.131	0.170	0.000	0.118	0.190	0.131

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	440	186	236	385	577	0	355	996	208
N.S.	1	1.40	0.59	0.75	1.23	1.84	0.00	1.13	3.17	0.66
time (sec)	N/A	1.092	0.782	14.659	0.117	0.130	0.000	0.154	0.183	17.808

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	444	186	236	394	576	0	354	995	208
N.S.	1	1.41	0.59	0.75	1.25	1.83	0.00	1.13	3.17	0.66
time (sec)	N/A	1.126	0.711	14.665	0.115	0.161	0.000	0.119	0.188	17.758

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	439	186	236	385	577	0	355	996	207
N.S.	1	1.38	0.59	0.74	1.21	1.82	0.00	1.12	3.14	0.65
time (sec)	N/A	1.093	0.666	14.628	0.124	0.121	0.000	0.125	0.186	0.121

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	443	186	236	394	576	0	354	995	207
N.S.	1	1.40	0.59	0.74	1.24	1.82	0.00	1.12	3.14	0.65
time (sec)	N/A	1.098	0.705	14.567	0.118	0.187	0.000	0.118	0.187	0.133

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	438	183	238	383	567	0	355	996	209
N.S.	1	1.40	0.59	0.76	1.23	1.82	0.00	1.14	3.19	0.67
time (sec)	N/A	1.134	0.580	14.545	0.122	0.110	0.000	0.125	0.190	17.755

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	442	183	238	392	540	0	354	993	209
N.S.	1	1.42	0.59	0.76	1.26	1.73	0.00	1.13	3.18	0.67
time (sec)	N/A	1.098	0.523	0.586	0.126	0.119	0.000	0.160	0.189	0.140

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	437	183	244	377	526	0	340	993	210
N.S.	1	1.41	0.59	0.79	1.22	1.70	0.00	1.10	3.20	0.68
time (sec)	N/A	1.093	0.386	0.668	0.117	0.124	0.000	0.145	0.190	0.132

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	441	183	237	382	537	0	346	995	210
N.S.	1	1.42	0.59	0.76	1.23	1.73	0.00	1.12	3.21	0.68
time (sec)	N/A	1.076	0.389	0.668	0.123	0.134	0.000	0.138	0.182	17.757

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	461	195	249	388	618	0	365	1042	226
N.S.	1	1.41	0.60	0.76	1.19	1.89	0.00	1.12	3.19	0.69
time (sec)	N/A	1.158	0.873	0.752	0.128	0.228	0.000	0.133	0.193	0.218

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	465	195	254	395	644	0	356	1049	226
N.S.	1	1.42	0.60	0.78	1.21	1.97	0.00	1.09	3.21	0.69
time (sec)	N/A	1.129	0.843	0.840	0.122	0.246	0.000	0.160	0.190	17.828

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	488	211	261	410	663	0	362	1068	239
N.S.	1	1.41	0.61	0.76	1.19	1.92	0.00	1.05	3.10	0.69
time (sec)	N/A	1.287	0.861	0.918	0.131	0.220	0.000	0.149	0.190	17.902

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	91	24	13	13	0	29	15	71
N.S.	1	0.62	1.15	0.30	0.16	0.16	0.00	0.37	0.19	0.90
time (sec)	N/A	0.329	0.327	0.174	0.026	0.090	0.000	0.114	0.169	18.034

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	39	24	13	13	0	23	15	59
N.S.	1	1.06	0.58	0.36	0.19	0.19	0.00	0.34	0.22	0.88
time (sec)	N/A	0.342	1.009	0.131	0.031	0.097	0.000	0.155	0.172	18.716

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	88	23	14	13	0	22	14	33
N.S.	1	1.00	2.44	0.64	0.39	0.36	0.00	0.61	0.39	0.92
time (sec)	N/A	0.284	0.269	0.127	0.025	0.120	0.000	0.141	0.168	18.328

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	45	454	26	14	11	0	30	11	109
N.S.	1	0.60	6.05	0.35	0.19	0.15	0.00	0.40	0.15	1.45
time (sec)	N/A	0.314	0.557	0.142	0.031	0.091	0.000	0.148	0.168	17.863

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	45	178	28	14	17	0	45	17	112
N.S.	1	0.60	2.37	0.37	0.19	0.23	0.00	0.60	0.23	1.49
time (sec)	N/A	0.321	0.197	0.123	0.025	0.074	0.000	0.142	0.171	17.881

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	22	13	13	0	30	15	33
N.S.	1	1.00	0.95	0.56	0.33	0.33	0.00	0.77	0.38	0.85
time (sec)	N/A	0.304	1.011	0.104	0.031	0.131	0.000	0.119	0.169	17.488

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	24	15	15	0	31	15	35
N.S.	1	0.62	0.49	0.30	0.19	0.19	0.00	0.39	0.19	0.44
time (sec)	N/A	0.320	1.007	0.105	0.030	0.107	0.000	0.145	0.165	17.508

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	24	15	15	0	31	15	35
N.S.	1	0.62	0.49	0.30	0.19	0.19	0.00	0.39	0.19	0.44
time (sec)	N/A	0.325	1.013	0.107	0.032	0.151	0.000	0.149	0.168	17.547

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	24	15	15	0	31	15	35
N.S.	1	0.62	0.49	0.30	0.19	0.19	0.00	0.39	0.19	0.44
time (sec)	N/A	0.323	1.009	0.111	0.027	0.122	0.000	0.102	0.175	17.817

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	36	13	13	0	29	15	0
N.S.	1	0.62	0.49	0.46	0.16	0.16	0.00	0.37	0.19	0.00
time (sec)	N/A	0.325	1.008	1.546	0.030	0.123	0.000	0.156	0.165	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	36	13	13	0	29	15	0
N.S.	1	0.62	0.49	0.46	0.16	0.16	0.00	0.37	0.19	0.00
time (sec)	N/A	0.328	1.007	1.043	0.030	0.075	0.000	0.111	0.163	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	50	36	33	10	10	0	20	12	0
N.S.	1	0.68	0.49	0.45	0.14	0.14	0.00	0.27	0.16	0.00
time (sec)	N/A	0.306	0.005	0.648	0.029	0.144	0.000	0.118	0.171	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	42	35	34	10	13	0	26	13	0
N.S.	1	0.58	0.49	0.47	0.14	0.18	0.00	0.36	0.18	0.00
time (sec)	N/A	0.325	1.007	0.998	0.027	0.083	0.000	0.140	0.166	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	47	37	34	13	13	0	30	15	33
N.S.	1	0.61	0.48	0.44	0.17	0.17	0.00	0.39	0.19	0.43
time (sec)	N/A	0.317	1.008	1.538	0.026	0.082	0.000	0.116	0.176	17.709

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	35	15	15	0	31	15	35
N.S.	1	0.62	0.49	0.44	0.19	0.19	0.00	0.39	0.19	0.44
time (sec)	N/A	0.326	1.008	2.191	0.025	0.102	0.000	0.128	0.173	18.299

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	35	15	15	0	31	15	35
N.S.	1	0.62	0.49	0.44	0.19	0.19	0.00	0.39	0.19	0.44
time (sec)	N/A	0.316	1.007	3.117	0.024	0.094	0.000	0.111	0.173	18.001

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	49	39	35	15	15	0	31	15	35
N.S.	1	0.62	0.49	0.44	0.19	0.19	0.00	0.39	0.19	0.44
time (sec)	N/A	0.327	1.009	4.326	0.031	0.140	0.000	0.127	0.175	18.085

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	167	78	113	46	35	35	0	67	37	0
N.S.	1	0.47	0.68	0.28	0.21	0.21	0.00	0.40	0.22	0.00
time (sec)	N/A	0.384	0.620	0.174	0.031	0.172	0.000	0.112	0.168	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	167	78	113	46	35	35	0	67	37	0
N.S.	1	0.47	0.68	0.28	0.21	0.21	0.00	0.40	0.22	0.00
time (sec)	N/A	0.381	0.552	0.165	0.033	0.158	0.000	0.119	0.170	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	106	78	113	46	35	35	0	67	37	0
N.S.	1	0.74	1.07	0.43	0.33	0.33	0.00	0.63	0.35	0.00
time (sec)	N/A	0.380	0.523	0.170	0.027	0.196	0.000	0.139	0.170	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	113	46	35	35	0	45	37	46
N.S.	1	1.06	1.64	0.67	0.51	0.51	0.00	0.65	0.54	0.67
time (sec)	N/A	0.348	0.460	0.149	0.026	0.126	0.000	0.121	0.173	17.684

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	35	35	0	44	36	36
N.S.	1	1.00	0.71	0.63	0.92	0.92	0.00	1.16	0.95	0.95
time (sec)	N/A	0.290	0.011	0.148	0.027	0.091	0.000	0.132	0.177	17.929

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	163	74	60	47	33	33	0	68	33	0
N.S.	1	0.45	0.37	0.29	0.20	0.20	0.00	0.42	0.20	0.00
time (sec)	N/A	0.346	1.018	0.125	0.025	0.078	0.000	0.136	0.169	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	164	73	62	48	34	38	0	87	38	0
N.S.	1	0.45	0.38	0.29	0.21	0.23	0.00	0.53	0.23	0.00
time (sec)	N/A	0.374	1.019	0.150	0.034	0.091	0.000	0.111	0.170	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	164	72	612	48	34	39	0	87	39	0
N.S.	1	0.44	3.73	0.29	0.21	0.24	0.00	0.53	0.24	0.00
time (sec)	N/A	0.367	0.828	0.130	0.026	0.082	0.000	0.108	0.176	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	163	74	266	50	33	39	0	87	39	0
N.S.	1	0.45	1.63	0.31	0.20	0.24	0.00	0.53	0.24	0.00
time (sec)	N/A	0.371	0.324	0.134	0.026	0.089	0.000	0.124	0.165	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	59	41	35	35	0	68	37	151
N.S.	1	1.00	1.44	1.00	0.85	0.85	0.00	1.66	0.90	3.68
time (sec)	N/A	0.290	1.016	0.129	0.028	0.181	0.000	0.123	0.172	17.649

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	75	61	44	35	37	0	69	37	151
N.S.	1	0.89	0.73	0.52	0.42	0.44	0.00	0.82	0.44	1.80
time (sec)	N/A	0.314	1.015	0.125	0.028	0.177	0.000	0.122	0.165	17.814

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	167	78	61	44	35	37	0	69	37	151
N.S.	1	0.47	0.37	0.26	0.21	0.22	0.00	0.41	0.22	0.90
time (sec)	N/A	0.363	1.016	0.132	0.036	0.137	0.000	0.110	0.168	17.737

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	167	78	61	46	35	37	0	69	37	151
N.S.	1	0.47	0.37	0.28	0.21	0.22	0.00	0.41	0.22	0.90
time (sec)	N/A	0.377	1.016	0.141	0.030	0.083	0.000	0.117	0.172	17.785

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	167	78	61	44	35	37	0	69	37	151
N.S.	1	0.47	0.37	0.26	0.21	0.22	0.00	0.41	0.22	0.90
time (sec)	N/A	0.375	1.016	0.137	0.028	0.102	0.000	0.112	0.173	17.618

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	37	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.22	0.00
time (sec)	N/A	0.365	1.015	3.154	0.027	0.102	0.000	0.104	0.167	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	37	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.22	0.00
time (sec)	N/A	0.362	1.011	2.258	0.027	0.122	0.000	0.134	0.174	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	37	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.22	0.00
time (sec)	N/A	0.361	1.013	1.641	0.025	0.084	0.000	0.150	0.168	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	75	61	58	35	35	0	67	37	0
N.S.	1	0.45	0.37	0.35	0.21	0.21	0.00	0.40	0.22	0.00
time (sec)	N/A	0.349	1.011	1.075	0.026	0.087	0.000	0.110	0.167	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	75	59	56	31	31	0	63	35	0
N.S.	1	0.47	0.37	0.35	0.19	0.19	0.00	0.40	0.22	0.00
time (sec)	N/A	0.358	1.009	0.639	0.027	0.087	0.000	0.111	0.167	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	66	60	58	32	36	0	64	36	0
N.S.	1	0.42	0.38	0.37	0.20	0.23	0.00	0.41	0.23	0.00
time (sec)	N/A	0.361	1.013	0.979	0.030	0.130	0.000	0.104	0.167	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	69	59	56	33	36	0	67	36	0
N.S.	1	0.43	0.37	0.35	0.20	0.22	0.00	0.42	0.22	0.00
time (sec)	N/A	0.357	1.012	1.515	0.031	0.137	0.000	0.104	0.179	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	66	59	56	32	37	0	66	37	0
N.S.	1	0.42	0.37	0.35	0.20	0.23	0.00	0.42	0.23	0.00
time (sec)	N/A	0.358	1.015	2.214	0.025	0.090	0.000	0.114	0.173	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	71	61	57	35	37	0	69	37	151
N.S.	1	0.44	0.37	0.35	0.21	0.23	0.00	0.42	0.23	0.93
time (sec)	N/A	0.353	1.012	3.113	0.033	0.129	0.000	0.142	0.169	18.872

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	75	61	57	35	37	0	69	37	151
N.S.	1	0.45	0.37	0.34	0.21	0.22	0.00	0.41	0.22	0.90
time (sec)	N/A	0.350	1.011	4.263	0.032	0.147	0.000	0.108	0.165	18.423

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	75	61	57	35	37	0	69	37	151
N.S.	1	0.45	0.37	0.34	0.21	0.22	0.00	0.41	0.22	0.90
time (sec)	N/A	0.362	1.015	5.825	0.026	0.115	0.000	0.115	0.171	17.741

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	75	61	57	35	37	0	69	37	151
N.S.	1	0.45	0.37	0.34	0.21	0.22	0.00	0.41	0.22	0.90
time (sec)	N/A	0.365	1.011	7.834	0.026	0.150	0.000	0.114	0.175	17.522

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	75	61	57	35	37	0	69	37	151
N.S.	1	0.45	0.37	0.34	0.21	0.22	0.00	0.41	0.22	0.90
time (sec)	N/A	0.372	1.014	10.311	0.031	0.122	0.000	0.105	0.167	17.779

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	255	101	83	66	57	57	0	105	59	0
N.S.	1	0.40	0.33	0.26	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.430	1.020	0.432	0.033	0.100	0.000	0.127	0.177	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	255	104	135	66	57	57	0	105	59	0
N.S.	1	0.41	0.53	0.26	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.433	0.981	0.311	0.027	0.090	0.000	0.146	0.185	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	255	104	135	68	57	57	0	105	59	0
N.S.	1	0.41	0.53	0.27	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.438	0.995	0.265	0.026	0.087	0.000	0.113	0.180	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	160	107	135	66	57	57	0	105	59	0
N.S.	1	0.67	0.84	0.41	0.36	0.36	0.00	0.66	0.37	0.00
time (sec)	N/A	0.420	0.865	0.208	0.026	0.139	0.000	0.143	0.172	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	119	88	135	66	56	56	0	104	59	0
N.S.	1	0.74	1.13	0.55	0.47	0.47	0.00	0.87	0.50	0.00
time (sec)	N/A	0.400	0.777	0.191	0.026	0.101	0.000	0.111	0.169	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	135	66	56	56	0	67	59	0
N.S.	1	1.06	1.96	0.96	0.81	0.81	0.00	0.97	0.86	0.00
time (sec)	N/A	0.348	0.721	0.161	0.033	0.123	0.000	0.104	0.172	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	57	57	0	66	58	36
N.S.	1	1.00	0.71	0.63	1.50	1.50	0.00	1.74	1.53	0.95
time (sec)	N/A	0.287	0.014	0.167	0.033	0.117	0.000	0.125	0.177	17.511

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	251	98	82	69	55	55	0	106	55	0
N.S.	1	0.39	0.33	0.27	0.22	0.22	0.00	0.42	0.22	0.00
time (sec)	N/A	0.389	1.022	0.137	0.028	0.093	0.000	0.139	0.172	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	250	97	85	70	56	61	0	125	61	0
N.S.	1	0.39	0.34	0.28	0.22	0.24	0.00	0.50	0.24	0.00
time (sec)	N/A	0.393	1.021	0.153	0.030	0.157	0.000	0.131	0.173	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	250	99	276	70	56	61	0	127	61	0
N.S.	1	0.40	1.10	0.28	0.22	0.24	0.00	0.51	0.24	0.00
time (sec)	N/A	0.411	0.864	0.155	0.035	0.179	0.000	0.134	0.175	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	250	99	279	70	56	61	0	128	61	0
N.S.	1	0.40	1.12	0.28	0.22	0.24	0.00	0.51	0.24	0.00
time (sec)	N/A	0.413	0.741	0.194	0.033	0.198	0.000	0.132	0.166	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	250	96	696	70	56	61	0	126	61	0
N.S.	1	0.38	2.78	0.28	0.22	0.24	0.00	0.50	0.24	0.00
time (sec)	N/A	0.403	1.022	0.206	0.032	0.102	0.000	0.112	0.174	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	251	98	268	70	55	61	0	125	61	0
N.S.	1	0.39	1.07	0.28	0.22	0.24	0.00	0.50	0.24	0.00
time (sec)	N/A	0.404	0.521	0.256	0.026	0.153	0.000	0.110	0.169	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	81	58	57	57	0	106	59	231
N.S.	1	1.00	1.98	1.41	1.39	1.39	0.00	2.59	1.44	5.63
time (sec)	N/A	0.286	1.016	0.349	0.026	0.176	0.000	0.126	0.169	18.489

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	75	83	68	57	59	0	107	59	231
N.S.	1	0.89	0.99	0.81	0.68	0.70	0.00	1.27	0.70	2.75
time (sec)	N/A	0.316	1.019	0.522	0.034	0.201	0.000	0.103	0.175	17.602

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	128	103	83	66	57	59	0	107	59	231
N.S.	1	0.80	0.65	0.52	0.45	0.46	0.00	0.84	0.46	1.80
time (sec)	N/A	0.352	1.016	0.769	0.032	0.081	0.000	0.104	0.170	17.908

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	255	102	83	66	57	59	0	107	59	231
N.S.	1	0.40	0.33	0.26	0.22	0.23	0.00	0.42	0.23	0.91
time (sec)	N/A	0.409	1.020	1.182	0.030	0.086	0.000	0.120	0.169	18.174

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	255	104	83	66	57	59	0	107	59	231
N.S.	1	0.41	0.33	0.26	0.22	0.23	0.00	0.42	0.23	0.91
time (sec)	N/A	0.400	1.022	1.843	0.030	0.090	0.000	0.150	0.169	17.890

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	255	104	83	66	57	59	0	107	59	231
N.S.	1	0.41	0.33	0.26	0.22	0.23	0.00	0.42	0.23	0.91
time (sec)	N/A	0.410	1.025	2.712	0.032	0.080	0.000	0.129	0.172	18.056

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	255	102	83	66	57	59	0	107	59	231
N.S.	1	0.40	0.33	0.26	0.22	0.23	0.00	0.42	0.23	0.91
time (sec)	N/A	0.408	1.026	4.033	0.030	0.089	0.000	0.107	0.171	18.166

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	59	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.406	1.022	5.989	0.035	0.078	0.000	0.106	0.170	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	59	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.407	1.015	4.385	0.027	0.071	0.000	0.125	0.165	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	59	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.404	1.017	3.164	0.027	0.072	0.000	0.110	0.174	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	59	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.389	1.017	2.211	0.027	0.079	0.000	0.131	0.168	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	101	83	80	57	57	0	105	59	0
N.S.	1	0.40	0.33	0.31	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.390	1.015	1.522	0.035	0.074	0.000	0.153	0.169	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	98	83	80	56	56	0	104	59	0
N.S.	1	0.39	0.33	0.32	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.392	1.016	1.054	0.031	0.077	0.000	0.150	0.170	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	102	81	78	54	54	0	102	57	0
N.S.	1	0.41	0.33	0.31	0.22	0.22	0.00	0.41	0.23	0.00
time (sec)	N/A	0.405	1.014	0.638	0.029	0.067	0.000	0.134	0.165	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	93	83	80	55	59	0	103	59	0
N.S.	1	0.38	0.34	0.32	0.22	0.24	0.00	0.42	0.24	0.00
time (sec)	N/A	0.402	1.021	1.003	0.031	0.065	0.000	0.118	0.170	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	92	83	80	54	59	0	104	59	0
N.S.	1	0.37	0.34	0.33	0.22	0.24	0.00	0.42	0.24	0.00
time (sec)	N/A	0.404	1.017	1.461	0.026	0.075	0.000	0.125	0.166	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	95	83	80	55	59	0	106	59	0
N.S.	1	0.38	0.33	0.32	0.22	0.24	0.00	0.43	0.24	0.00
time (sec)	N/A	0.393	1.017	2.148	0.031	0.084	0.000	0.136	0.170	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	93	83	80	55	59	0	106	59	0
N.S.	1	0.38	0.34	0.32	0.22	0.24	0.00	0.43	0.24	0.00
time (sec)	N/A	0.402	1.015	3.112	0.025	0.075	0.000	0.134	0.170	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	92	83	80	54	59	0	105	59	0
N.S.	1	0.37	0.34	0.33	0.22	0.24	0.00	0.43	0.24	0.00
time (sec)	N/A	0.400	1.016	4.307	0.033	0.069	0.000	0.138	0.168	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	97	83	79	57	59	0	107	59	231
N.S.	1	0.39	0.33	0.31	0.23	0.24	0.00	0.43	0.24	0.92
time (sec)	N/A	0.399	1.016	5.860	0.035	0.076	0.000	0.126	0.172	19.300

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	99	83	79	57	59	0	107	59	231
N.S.	1	0.39	0.33	0.31	0.23	0.23	0.00	0.42	0.23	0.91
time (sec)	N/A	0.396	1.014	7.861	0.027	0.084	0.000	0.110	0.172	18.961

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	101	83	79	57	59	0	107	59	231
N.S.	1	0.40	0.33	0.31	0.22	0.23	0.00	0.42	0.23	0.91
time (sec)	N/A	0.404	1.014	10.395	0.031	0.074	0.000	0.110	0.165	20.039

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	101	83	79	57	59	0	107	59	231
N.S.	1	0.40	0.33	0.31	0.22	0.23	0.00	0.42	0.23	0.91
time (sec)	N/A	0.397	1.018	13.645	0.032	0.064	0.000	0.112	0.170	18.576

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	101	83	79	57	59	0	107	59	231
N.S.	1	0.40	0.33	0.31	0.22	0.23	0.00	0.42	0.23	0.91
time (sec)	N/A	0.401	1.017	22.323	0.028	0.064	0.000	0.119	0.163	17.691

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	101	83	79	57	59	0	107	59	231
N.S.	1	0.40	0.33	0.31	0.22	0.23	0.00	0.42	0.23	0.91
time (sec)	N/A	0.401	1.015	27.442	0.027	0.063	0.000	0.153	0.164	17.352

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	101	83	79	57	59	0	107	59	231
N.S.	1	0.40	0.33	0.31	0.22	0.23	0.00	0.42	0.23	0.91
time (sec)	N/A	0.399	1.014	33.045	0.041	0.071	0.000	0.108	0.170	18.188

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	127	68	167	42	34	33	0	59	33	0
N.S.	1	0.54	1.31	0.33	0.27	0.26	0.00	0.46	0.26	0.00
time (sec)	N/A	0.376	0.372	0.204	0.031	0.073	0.000	0.151	0.169	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	74	113	31	23	22	0	33	22	64
N.S.	1	0.99	1.51	0.41	0.31	0.29	0.00	0.44	0.29	0.85
time (sec)	N/A	0.351	0.302	0.158	0.035	0.067	0.000	0.107	0.167	19.233

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	49	22	13	13	0	22	13	33
N.S.	1	1.00	1.11	0.50	0.30	0.30	0.00	0.50	0.30	0.75
time (sec)	N/A	0.288	0.109	0.148	0.026	0.069	0.000	0.105	0.164	18.153

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	80	55	134	29	23	18	0	33	20	40
N.S.	1	0.69	1.68	0.36	0.29	0.22	0.00	0.41	0.25	0.50
time (sec)	N/A	0.304	0.181	0.167	0.032	0.070	0.000	0.136	0.161	18.692

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	125	65	175	42	33	33	0	52	33	75
N.S.	1	0.52	1.40	0.34	0.26	0.26	0.00	0.42	0.26	0.60
time (sec)	N/A	0.365	0.190	0.177	0.026	0.071	0.000	0.113	0.172	18.593

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	72	66	64	37	99	0	64	37	0
N.S.	1	0.56	0.51	0.50	0.29	0.77	0.00	0.50	0.29	0.00
time (sec)	N/A	0.343	1.023	1.511	0.111	0.073	0.000	0.111	0.165	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	61	54	48	26	82	0	42	26	0
N.S.	1	0.69	0.61	0.54	0.29	0.92	0.00	0.47	0.29	0.00
time (sec)	N/A	0.323	1.012	1.023	0.109	0.079	0.000	0.140	0.162	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	34	15	67	0	23	23	0
N.S.	1	1.00	0.83	0.64	0.28	1.26	0.00	0.43	0.43	0.00
time (sec)	N/A	0.293	1.011	0.717	0.106	0.078	0.000	0.139	0.169	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	64	56	50	29	82	0	37	30	0
N.S.	1	0.70	0.61	0.54	0.32	0.89	0.00	0.40	0.33	0.00
time (sec)	N/A	0.310	1.013	1.067	0.104	0.070	0.000	0.141	0.166	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	81	70	68	40	106	0	50	43	0
N.S.	1	0.61	0.53	0.51	0.30	0.80	0.00	0.38	0.32	0.00
time (sec)	N/A	0.326	1.019	1.535	0.104	0.077	0.000	0.116	0.167	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	158	91	81	74	66	91	0	92	95	0
N.S.	1	0.58	0.51	0.47	0.42	0.58	0.00	0.58	0.60	0.00
time (sec)	N/A	0.410	1.027	0.148	0.033	0.065	0.000	0.112	0.167	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	113	80	192	54	55	69	0	62	81	0
N.S.	1	0.71	1.70	0.48	0.49	0.61	0.00	0.55	0.72	0.00
time (sec)	N/A	0.394	0.468	0.130	0.036	0.071	0.000	0.125	0.166	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	41	71	119	31	36	36	0	32	28	42
N.S.	1	1.73	2.90	0.76	0.88	0.88	0.00	0.78	0.68	1.02
time (sec)	N/A	0.345	0.298	0.116	0.027	0.072	0.000	0.106	0.171	18.079

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	23	26	26	0	24	25	34
N.S.	1	1.00	0.71	0.61	0.68	0.68	0.00	0.63	0.66	0.89
time (sec)	N/A	0.285	0.006	0.102	0.033	0.064	0.000	0.108	0.161	18.016

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	147	84	790	68	57	90	0	89	109	0
N.S.	1	0.57	5.37	0.46	0.39	0.61	0.00	0.61	0.74	0.00
time (sec)	N/A	0.390	1.213	0.150	0.026	0.075	0.000	0.107	0.167	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	189	98	901	90	75	119	0	122	133	0
N.S.	1	0.52	4.77	0.48	0.40	0.63	0.00	0.65	0.70	0.00
time (sec)	N/A	0.424	1.439	0.161	0.036	0.065	0.000	0.146	0.167	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	115	89	116	68	202	0	84	118	0
N.S.	1	0.70	0.54	0.71	0.41	1.23	0.00	0.51	0.72	0.00
time (sec)	N/A	0.386	1.036	2.102	0.116	0.078	0.000	0.142	0.162	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	102	84	97	59	188	0	65	113	0
N.S.	1	0.80	0.66	0.76	0.46	1.47	0.00	0.51	0.88	0.00
time (sec)	N/A	0.356	1.023	1.454	0.113	0.075	0.000	0.138	0.164	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	100	81	99	62	190	0	70	110	0
N.S.	1	0.78	0.63	0.77	0.48	1.47	0.00	0.54	0.85	0.00
time (sec)	N/A	0.360	1.025	1.005	0.115	0.080	0.000	0.110	0.171	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	112	83	97	58	188	0	65	113	0
N.S.	1	0.89	0.66	0.77	0.46	1.49	0.00	0.52	0.90	0.00
time (sec)	N/A	0.358	1.021	0.668	0.113	0.074	0.000	0.132	0.166	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	118	93	110	71	202	0	87	121	0
N.S.	1	0.72	0.56	0.67	0.43	1.22	0.00	0.53	0.73	0.00
time (sec)	N/A	0.382	1.026	1.006	0.114	0.099	0.000	0.130	0.171	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	135	105	127	86	238	0	101	137	0
N.S.	1	0.65	0.50	0.61	0.41	1.14	0.00	0.48	0.66	0.00
time (sec)	N/A	0.423	1.030	1.481	0.116	0.091	0.000	0.150	0.165	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	238	127	103	96	110	157	0	114	175	0
N.S.	1	0.53	0.43	0.40	0.46	0.66	0.00	0.48	0.74	0.00
time (sec)	N/A	0.475	1.033	0.164	0.036	0.076	0.000	0.123	0.165	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	196	116	261	76	99	135	0	84	163	0
N.S.	1	0.59	1.33	0.39	0.51	0.69	0.00	0.43	0.83	0.00
time (sec)	N/A	0.461	0.782	0.148	0.036	0.077	0.000	0.110	0.171	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	B	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	161	50	80	80	0	54	50	144
N.S.	1	1.00	3.93	1.22	1.95	1.95	0.00	1.32	1.22	3.51
time (sec)	N/A	0.289	0.525	0.135	0.054	0.079	0.000	0.117	0.164	19.848

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	74	86	206	42	69	69	0	43	68	53
N.S.	1	1.16	2.78	0.57	0.93	0.93	0.00	0.58	0.92	0.72
time (sec)	N/A	0.392	0.466	0.128	0.037	0.078	0.000	0.106	0.167	18.540

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	232	31	58	58	0	32	57	42
N.S.	1	1.06	3.36	0.45	0.84	0.84	0.00	0.46	0.83	0.61
time (sec)	N/A	0.343	0.479	0.121	0.028	0.075	0.000	0.174	0.235	17.772

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	23	48	48	0	24	47	34
N.S.	1	1.00	0.71	0.61	1.26	1.26	0.00	0.63	1.24	0.89
time (sec)	N/A	0.283	0.008	0.110	0.033	0.067	0.000	0.107	0.223	18.241

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	223	116	96	90	101	178	0	111	215	0
N.S.	1	0.52	0.43	0.40	0.45	0.80	0.00	0.50	0.96	0.00
time (sec)	N/A	0.448	1.035	0.168	0.037	0.080	0.000	0.143	0.234	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	267	132	119	112	119	207	0	144	241	0
N.S.	1	0.49	0.45	0.42	0.45	0.78	0.00	0.54	0.90	0.00
time (sec)	N/A	0.475	1.037	0.180	0.038	0.075	0.000	0.139	0.226	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	169	111	176	112	334	0	106	216	0
N.S.	1	0.68	0.45	0.71	0.45	1.35	0.00	0.43	0.87	0.00
time (sec)	N/A	0.447	1.056	4.197	0.120	0.087	0.000	0.162	0.223	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	156	106	146	103	320	0	87	211	0
N.S.	1	0.74	0.50	0.70	0.49	1.52	0.00	0.41	1.00	0.00
time (sec)	N/A	0.434	1.033	3.040	0.117	0.093	0.000	0.110	0.224	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	154	105	149	109	324	0	93	211	0
N.S.	1	0.73	0.50	0.71	0.52	1.54	0.00	0.44	1.00	0.00
time (sec)	N/A	0.425	1.036	2.137	0.118	0.085	0.000	0.145	0.223	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	152	105	147	111	324	0	93	211	0
N.S.	1	0.72	0.50	0.69	0.52	1.53	0.00	0.44	1.00	0.00
time (sec)	N/A	0.421	1.036	1.497	0.112	0.087	0.000	0.140	0.225	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	150	105	149	109	324	0	93	211	0
N.S.	1	0.70	0.49	0.70	0.51	1.52	0.00	0.44	0.99	0.00
time (sec)	N/A	0.411	1.030	1.014	0.122	0.086	0.000	0.145	0.272	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	174	105	146	102	320	0	87	211	0
N.S.	1	0.85	0.51	0.72	0.50	1.57	0.00	0.43	1.03	0.00
time (sec)	N/A	0.436	1.028	0.681	0.109	0.077	0.000	0.137	0.174	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	172	115	162	115	334	0	109	219	0
N.S.	1	0.70	0.47	0.66	0.47	1.36	0.00	0.44	0.89	0.00
time (sec)	N/A	0.464	1.035	1.023	0.110	0.075	0.000	0.117	0.167	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	189	127	175	130	370	0	123	235	0
N.S.	1	0.65	0.43	0.60	0.44	1.26	0.00	0.42	0.80	0.00
time (sec)	N/A	0.486	1.035	1.539	0.119	0.076	0.000	0.112	0.166	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	63	44	39	25	26	0	45	22	0
N.S.	1	0.68	0.47	0.42	0.27	0.28	0.00	0.48	0.24	0.00
time (sec)	N/A	0.345	0.019	0.115	0.030	0.069	0.000	0.152	0.169	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	63	44	39	25	22	0	42	20	0
N.S.	1	0.68	0.47	0.42	0.27	0.24	0.00	0.45	0.22	0.00
time (sec)	N/A	0.346	0.019	0.114	0.037	0.067	0.000	0.122	0.164	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	63	44	39	25	18	0	37	17	0
N.S.	1	0.68	0.47	0.42	0.27	0.19	0.00	0.40	0.18	0.00
time (sec)	N/A	0.351	0.017	0.068	0.033	0.064	0.000	0.105	0.166	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	61	43	38	24	19	0	40	18	47
N.S.	1	0.67	0.47	0.42	0.26	0.21	0.00	0.44	0.20	0.52
time (sec)	N/A	0.344	0.018	0.070	0.036	0.077	0.000	0.116	0.167	18.581

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	61	44	39	25	22	0	41	20	52
N.S.	1	0.67	0.48	0.43	0.27	0.24	0.00	0.45	0.22	0.57
time (sec)	N/A	0.337	0.021	0.078	0.027	0.068	0.000	0.131	0.163	18.884

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	61	42	37	24	23	0	47	24	53
N.S.	1	0.67	0.46	0.41	0.26	0.25	0.00	0.52	0.26	0.58
time (sec)	N/A	0.342	0.018	0.082	0.027	0.070	0.000	0.107	0.171	18.192

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	61	42	37	25	21	0	44	24	56
N.S.	1	0.67	0.46	0.41	0.27	0.23	0.00	0.48	0.26	0.62
time (sec)	N/A	0.340	0.019	0.084	0.027	0.070	0.000	0.129	0.167	17.626

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	103	66	61	83	54	0	99	44	0
N.S.	1	0.53	0.34	0.31	0.43	0.28	0.00	0.51	0.23	0.00
time (sec)	N/A	0.401	0.029	0.122	0.042	0.064	0.000	0.148	0.167	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	103	66	61	83	46	0	90	42	0
N.S.	1	0.53	0.34	0.31	0.43	0.24	0.00	0.46	0.22	0.00
time (sec)	N/A	0.393	0.026	0.125	0.037	0.068	0.000	0.116	0.168	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	103	66	61	83	40	0	85	39	0
N.S.	1	0.53	0.34	0.31	0.43	0.21	0.00	0.44	0.20	0.00
time (sec)	N/A	0.392	0.022	0.121	0.035	0.077	0.000	0.123	0.167	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	101	66	61	87	42	0	89	41	76
N.S.	1	0.52	0.34	0.32	0.45	0.22	0.00	0.46	0.21	0.39
time (sec)	N/A	0.403	0.023	0.081	0.037	0.068	0.000	0.128	0.166	18.787

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	99	66	61	87	45	0	102	43	87
N.S.	1	0.52	0.35	0.32	0.46	0.24	0.00	0.53	0.23	0.46
time (sec)	N/A	0.428	0.027	0.086	0.039	0.066	0.000	0.124	0.173	18.875

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	101	66	61	86	45	0	107	46	88
N.S.	1	0.52	0.34	0.32	0.45	0.23	0.00	0.55	0.24	0.46
time (sec)	N/A	0.406	0.031	0.090	0.042	0.066	0.000	0.120	0.165	18.275

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	99	66	61	86	45	0	107	46	91
N.S.	1	0.52	0.35	0.32	0.45	0.24	0.00	0.56	0.24	0.48
time (sec)	N/A	0.394	0.028	0.092	0.045	0.063	0.000	0.128	0.164	18.203

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	143	88	83	147	82	0	153	66	0
N.S.	1	0.48	0.30	0.28	0.49	0.28	0.00	0.52	0.22	0.00
time (sec)	N/A	0.460	0.036	0.131	0.051	0.068	0.000	0.110	0.166	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	143	88	83	147	70	0	138	64	0
N.S.	1	0.48	0.30	0.28	0.49	0.24	0.00	0.46	0.22	0.00
time (sec)	N/A	0.447	0.030	0.128	0.042	0.069	0.000	0.194	0.169	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	143	88	83	147	62	0	133	61	0
N.S.	1	0.48	0.30	0.28	0.49	0.21	0.00	0.45	0.21	0.00
time (sec)	N/A	0.440	0.030	0.125	0.059	0.071	0.000	0.188	0.170	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	139	88	83	151	64	0	137	63	112
N.S.	1	0.47	0.30	0.28	0.52	0.22	0.00	0.47	0.22	0.38
time (sec)	N/A	0.454	0.030	0.084	0.042	0.067	0.000	0.148	0.164	17.947

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	141	88	83	151	67	0	156	65	116
N.S.	1	0.48	0.30	0.28	0.51	0.23	0.00	0.53	0.22	0.39
time (sec)	N/A	0.449	0.032	0.093	0.044	0.069	0.000	0.135	0.167	18.434

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	139	88	83	151	67	0	161	68	116
N.S.	1	0.47	0.30	0.28	0.52	0.23	0.00	0.55	0.23	0.40
time (sec)	N/A	0.454	0.039	0.093	0.049	0.068	0.000	0.116	0.171	19.030

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	141	88	83	150	67	0	162	68	118
N.S.	1	0.48	0.30	0.28	0.51	0.23	0.00	0.55	0.23	0.40
time (sec)	N/A	0.427	0.042	0.097	0.043	0.067	0.000	0.134	0.162	18.927

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	344	163	208	266	219	0	282	162	0
N.S.	1	0.99	0.47	0.60	0.76	0.63	0.00	0.81	0.47	0.00
time (sec)	N/A	0.961	0.194	0.172	0.116	0.085	0.000	0.148	0.171	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	321	151	200	241	208	0	254	152	0
N.S.	1	1.06	0.50	0.66	0.80	0.69	0.00	0.84	0.50	0.00
time (sec)	N/A	0.934	0.164	0.173	0.113	0.078	0.000	0.155	0.171	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	319	150	195	250	160	0	255	148	0
N.S.	1	1.06	0.50	0.65	0.83	0.53	0.00	0.84	0.49	0.00
time (sec)	N/A	0.907	0.154	0.168	0.127	0.086	0.000	0.146	0.167	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	298	123	183	216	172	0	242	114	0
N.S.	1	1.15	0.47	0.70	0.83	0.66	0.00	0.93	0.44	0.00
time (sec)	N/A	0.876	0.121	0.154	0.135	0.086	0.000	0.162	0.173	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	302	125	182	226	156	0	251	117	0
N.S.	1	1.17	0.48	0.70	0.87	0.60	0.00	0.97	0.45	0.00
time (sec)	N/A	0.853	0.125	0.153	0.114	0.085	0.000	0.178	0.175	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	321	150	196	234	201	0	264	160	0
N.S.	1	1.06	0.50	0.65	0.77	0.66	0.00	0.87	0.53	0.00
time (sec)	N/A	0.892	0.165	0.176	0.112	0.090	0.000	0.132	0.174	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	325	152	200	242	226	0	256	168	0
N.S.	1	1.06	0.50	0.65	0.79	0.74	0.00	0.84	0.55	0.00
time (sec)	N/A	0.921	0.157	0.174	0.116	0.088	0.000	0.120	0.177	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	348	160	208	259	252	0	284	185	0
N.S.	1	0.99	0.46	0.59	0.74	0.72	0.00	0.81	0.53	0.00
time (sec)	N/A	0.944	0.184	0.181	0.129	0.092	0.000	0.131	0.172	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	414	206	256	0	364	0	386	502	0
N.S.	1	0.93	0.46	0.58	0.00	0.82	0.00	0.87	1.13	0.00
time (sec)	N/A	1.095	0.428	0.187	0.000	0.117	0.000	0.186	0.184	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	391	195	244	0	359	0	357	490	0
N.S.	1	0.98	0.49	0.61	0.00	0.90	0.00	0.90	1.23	0.00
time (sec)	N/A	1.041	0.429	0.185	0.000	0.108	0.000	0.133	0.171	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	389	195	244	0	330	0	358	489	0
N.S.	1	0.98	0.49	0.61	0.00	0.83	0.00	0.90	1.23	0.00
time (sec)	N/A	1.017	0.411	0.183	0.000	0.105	0.000	0.161	0.173	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	366	184	612	0	337	0	345	483	0
N.S.	1	1.04	0.52	1.74	0.00	0.96	0.00	0.98	1.37	0.00
time (sec)	N/A	0.966	0.460	0.141	0.000	0.096	0.000	0.174	0.180	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	366	184	666	279	336	0	344	482	0
N.S.	1	1.04	0.52	1.89	0.79	0.95	0.00	0.98	1.37	0.00
time (sec)	N/A	1.018	0.386	0.137	0.134	0.098	0.000	0.145	0.174	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	366	179	617	272	350	0	348	483	0
N.S.	1	1.04	0.51	1.75	0.77	0.99	0.00	0.99	1.37	0.00
time (sec)	N/A	0.978	0.384	0.139	0.139	0.100	0.000	0.177	0.172	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	370	180	668	281	328	0	346	480	0
N.S.	1	1.05	0.51	1.89	0.80	0.93	0.00	0.98	1.36	0.00
time (sec)	N/A	0.985	0.368	0.136	0.134	0.106	0.000	0.130	0.168	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	365	181	617	265	328	0	333	480	0
N.S.	1	1.03	0.51	1.74	0.75	0.93	0.00	0.94	1.36	0.00
time (sec)	N/A	0.994	0.259	0.126	0.133	0.095	0.000	0.171	0.173	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	369	181	638	0	324	0	339	482	0
N.S.	1	1.04	0.51	1.80	0.00	0.92	0.00	0.96	1.36	0.00
time (sec)	N/A	0.989	0.259	0.126	0.000	0.104	0.000	0.181	0.181	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	389	193	241	0	375	0	368	511	0
N.S.	1	0.97	0.48	0.60	0.00	0.94	0.00	0.92	1.28	0.00
time (sec)	N/A	1.030	0.411	0.191	0.000	0.101	0.000	0.124	0.176	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	393	193	246	0	401	0	359	518	0
N.S.	1	0.98	0.48	0.62	0.00	1.00	0.00	0.90	1.30	0.00
time (sec)	N/A	1.023	0.412	0.190	0.000	0.102	0.000	0.123	0.180	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	416	204	253	0	420	0	390	537	0
N.S.	1	0.93	0.46	0.57	0.00	0.94	0.00	0.87	1.20	0.00
time (sec)	N/A	1.073	0.433	0.194	0.000	0.096	0.000	0.152	0.184	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	484	229	288	0	502	0	424	844	0
N.S.	1	0.89	0.42	0.53	0.00	0.93	0.00	0.78	1.56	0.00
time (sec)	N/A	1.202	0.865	0.211	0.000	0.100	0.000	0.161	0.181	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	461	218	276	0	497	0	395	832	0
N.S.	1	0.93	0.44	0.56	0.00	1.01	0.00	0.80	1.68	0.00
time (sec)	N/A	1.185	0.883	0.214	0.000	0.099	0.000	0.151	0.187	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	459	218	276	0	468	0	396	831	0
N.S.	1	0.93	0.44	0.56	0.00	0.95	0.00	0.80	1.68	0.00
time (sec)	N/A	1.111	0.808	0.204	0.000	0.087	0.000	0.130	0.183	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	436	207	1046	0	475	0	383	825	0
N.S.	1	0.97	0.46	2.33	0.00	1.06	0.00	0.85	1.84	0.00
time (sec)	N/A	1.073	0.795	0.151	0.000	0.106	0.000	0.127	0.178	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	436	201	1134	583	474	0	382	824	0
N.S.	1	0.97	0.45	2.53	1.30	1.06	0.00	0.85	1.84	0.00
time (sec)	N/A	1.066	0.671	0.148	0.177	0.087	0.000	0.121	0.182	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	436	201	1051	577	498	0	386	825	0
N.S.	1	0.97	0.45	2.33	1.28	1.10	0.00	0.86	1.83	0.00
time (sec)	N/A	1.132	0.661	0.148	0.166	0.088	0.000	0.173	0.179	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	440	202	1136	595	497	0	385	824	0
N.S.	1	0.98	0.45	2.52	1.32	1.10	0.00	0.85	1.83	0.00
time (sec)	N/A	1.120	0.657	0.149	0.169	0.098	0.000	0.128	0.186	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	435	201	1051	584	508	0	386	825	0
N.S.	1	0.96	0.44	2.31	1.29	1.12	0.00	0.85	1.82	0.00
time (sec)	N/A	1.113	0.582	0.145	0.161	0.087	0.000	0.128	0.179	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	439	201	1136	597	507	0	385	824	0
N.S.	1	0.97	0.44	2.50	1.31	1.12	0.00	0.85	1.81	0.00
time (sec)	N/A	1.108	0.583	0.145	0.159	0.088	0.000	0.139	0.179	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	434	201	1051	582	498	0	386	825	0
N.S.	1	0.96	0.45	2.33	1.29	1.10	0.00	0.86	1.83	0.00
time (sec)	N/A	1.084	0.566	0.145	0.171	0.089	0.000	0.133	0.178	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	438	201	1136	586	473	0	385	822	0
N.S.	1	0.97	0.45	2.52	1.30	1.05	0.00	0.85	1.82	0.00
time (sec)	N/A	1.074	0.540	0.155	0.157	0.091	0.000	0.186	0.177	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	433	201	1051	569	460	0	371	822	0
N.S.	1	0.96	0.45	2.34	1.26	1.02	0.00	0.82	1.83	0.00
time (sec)	N/A	1.131	0.365	0.143	0.166	0.093	0.000	0.126	0.182	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	437	201	1133	0	466	0	377	824	0
N.S.	1	0.97	0.45	2.52	0.00	1.04	0.00	0.84	1.83	0.00
time (sec)	N/A	1.134	0.366	0.142	0.000	0.087	0.000	0.125	0.179	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	457	213	273	0	537	0	406	865	0
N.S.	1	0.92	0.43	0.55	0.00	1.08	0.00	0.82	1.74	0.00
time (sec)	N/A	1.153	0.707	0.214	0.000	0.100	0.000	0.127	0.182	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	461	213	278	0	563	0	397	872	0
N.S.	1	0.93	0.43	0.56	0.00	1.14	0.00	0.80	1.76	0.00
time (sec)	N/A	1.147	0.696	0.212	0.000	0.092	0.000	0.119	0.182	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	484	224	285	0	582	0	428	891	0
N.S.	1	0.89	0.41	0.52	0.00	1.07	0.00	0.79	1.64	0.00
time (sec)	N/A	1.210	0.703	0.222	0.000	0.105	0.000	0.173	0.187	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	285	64	0	0	0	0	0	26	0
N.S.	1	0.96	0.21	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.518	4.428	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	48	0	0	0	0	0	22	0
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.432	4.363	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	272	51	0	0	0	0	0	26	0
N.S.	1	0.94	0.18	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.473	5.116	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	618	582	64	0	0	0	0	0	26	0
N.S.	1	0.94	0.10	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.770	5.221	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	609	576	64	0	0	0	0	0	22	0
N.S.	1	0.95	0.11	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.756	4.860	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	649	598	61	0	0	0	0	0	26	0
N.S.	1	0.92	0.09	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.814	5.551	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	105	602	144	507	3104	847	602	540
N.S.	1	1.00	0.70	4.01	0.96	3.38	20.69	5.65	4.01	3.60
time (sec)	N/A	0.551	0.224	0.454	0.044	0.096	0.825	0.152	0.169	18.449

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	73	292	100	253	1278	415	292	263
N.S.	1	1.00	0.70	2.81	0.96	2.43	12.29	3.99	2.81	2.53
time (sec)	N/A	0.449	0.077	0.184	0.048	0.089	0.493	0.113	0.160	18.425

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	57	56	87	330	135	94	95
N.S.	1	1.00	0.71	0.98	0.97	1.50	5.69	2.33	1.62	1.64
time (sec)	N/A	0.349	0.050	0.089	0.042	0.098	0.281	0.135	0.170	18.889

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0	30	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.298	0.031	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0	52	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.297	0.040	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0	74	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	0.295	0.032	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	159	111	453	243	369	0	900	433	0
N.S.	1	0.51	0.35	1.45	0.78	1.18	0.00	2.88	1.38	0.00
time (sec)	N/A	0.527	0.140	0.095	0.039	0.086	0.000	0.164	0.168	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	113	131	199	119	159	0	384	179	0
N.S.	1	0.55	0.64	0.97	0.58	0.78	0.00	1.87	0.87	0.00
time (sec)	N/A	0.436	0.074	0.076	0.034	0.086	0.000	0.146	0.178	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	67	53	56	34	35	0	83	36	55
N.S.	1	0.69	0.55	0.58	0.35	0.36	0.00	0.86	0.37	0.57
time (sec)	N/A	0.368	0.029	0.063	0.036	0.090	0.000	0.110	0.173	18.386

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	0	0	0	0	0	19	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.334	0.033	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	41	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.321	0.035	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	63	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.324	0.034	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	0	0	0	0	0	218	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	2.83	0.00
time (sec)	N/A	0.333	0.031	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	163	110	150	115	163	0	375	162	206
N.S.	1	0.94	0.63	0.86	0.66	0.94	0.00	2.16	0.93	1.18
time (sec)	N/A	0.510	0.069	0.201	0.037	0.072	0.000	0.156	0.198	17.772

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	128	77	96	79	108	0	235	104	137
N.S.	1	0.98	0.59	0.74	0.61	0.83	0.00	1.81	0.80	1.05
time (sec)	N/A	0.448	0.053	0.145	0.037	0.079	0.000	0.115	0.188	17.899

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	83	51	58	54	70	0	132	67	85
N.S.	1	1.08	0.66	0.75	0.70	0.91	0.00	1.71	0.87	1.10
time (sec)	N/A	0.359	0.045	0.109	0.038	0.076	0.000	0.149	0.184	18.423

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	45	29	31	30	37	0	58	39	46
N.S.	1	1.10	0.71	0.76	0.73	0.90	0.00	1.41	0.95	1.12
time (sec)	N/A	0.293	0.003	0.079	0.045	0.074	0.000	0.116	0.168	18.536

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	0	0	0	0	0	64	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.334	0.033	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	67	55	0	0	0	0	0	69	0
N.S.	1	1.05	0.86	0.00	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	0.336	0.037	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	529	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	8.82	0.00
time (sec)	N/A	0.298	0.038	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	299	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	4.98	0.00
time (sec)	N/A	0.291	0.032	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	46	0	0	0	0	0	127	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	2.31	0.00
time (sec)	N/A	0.289	0.030	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	0	0	0	0	0	148	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	2.55	0.00
time (sec)	N/A	0.296	0.038	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	152	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	2.53	0.00
time (sec)	N/A	0.290	0.041	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	323	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	4.82	0.00
time (sec)	N/A	0.307	0.121	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	142	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	2.12	0.00
time (sec)	N/A	0.300	0.094	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	0	0	0	0	0	144	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	2.22	0.00
time (sec)	N/A	0.304	0.108	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	0	0	0	0	0	161	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	2.48	0.00
time (sec)	N/A	0.301	0.143	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	166	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	2.48	0.00
time (sec)	N/A	0.310	0.157	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	614	246	343	0	237	0	0	334	0
N.S.	1	1.66	0.66	0.93	0.00	0.64	0.00	0.00	0.90	0.00
time (sec)	N/A	1.378	3.098	5.260	0.000	0.086	0.000	0.000	0.390	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	531	198	265	0	161	0	0	174	0
N.S.	1	1.79	0.67	0.90	0.00	0.54	0.00	0.00	0.59	0.00
time (sec)	N/A	1.029	2.537	3.927	0.000	0.075	0.000	0.000	0.308	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	447	119	124	0	117	0	0	46	0
N.S.	1	3.26	0.87	0.91	0.00	0.85	0.00	0.00	0.34	0.00
time (sec)	N/A	0.877	1.705	1.312	0.000	0.077	0.000	0.000	0.189	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	190	83	87	0	42	0	0	43	0
N.S.	1	2.02	0.88	0.93	0.00	0.45	0.00	0.00	0.46	0.00
time (sec)	N/A	0.407	0.029	0.751	0.000	0.081	0.000	0.000	0.170	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	493	143	160	0	112	0	0	46	0
N.S.	1	3.57	1.04	1.16	0.00	0.81	0.00	0.00	0.33	0.00
time (sec)	N/A	0.989	2.141	1.822	0.000	0.072	0.000	0.000	0.203	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	586	226	293	0	162	0	0	46	0
N.S.	1	1.95	0.75	0.97	0.00	0.54	0.00	0.00	0.15	0.00
time (sec)	N/A	1.213	2.720	3.987	0.000	0.078	0.000	0.000	0.230	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	681	293	388	0	239	0	0	224	0
N.S.	1	1.82	0.78	1.03	0.00	0.64	0.00	0.00	0.60	0.00
time (sec)	N/A	1.502	3.311	5.287	0.000	0.081	0.000	0.000	0.737	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	525	311	442	0	269	0	0	407	0
N.S.	1	1.23	0.73	1.03	0.00	0.63	0.00	0.00	0.95	0.00
time (sec)	N/A	1.299	3.839	6.204	0.000	0.085	0.000	0.000	0.404	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	437	246	353	0	184	0	0	216	0
N.S.	1	1.32	0.74	1.07	0.00	0.56	0.00	0.00	0.65	0.00
time (sec)	N/A	0.930	2.794	4.519	0.000	0.090	0.000	0.000	0.327	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	354	148	188	0	130	0	0	60	0
N.S.	1	2.25	0.94	1.20	0.00	0.83	0.00	0.00	0.38	0.00
time (sec)	N/A	0.728	1.899	1.539	0.000	0.074	0.000	0.000	0.194	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	154	104	109	0	51	0	0	57	0
N.S.	1	1.47	0.99	1.04	0.00	0.49	0.00	0.00	0.54	0.00
time (sec)	N/A	0.374	1.726	0.884	0.000	0.080	0.000	0.000	0.175	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	396	180	224	0	144	0	0	60	0
N.S.	1	2.59	1.18	1.46	0.00	0.94	0.00	0.00	0.39	0.00
time (sec)	N/A	0.849	2.378	2.460	0.000	0.075	0.000	0.000	0.204	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	494	295	409	0	214	0	0	60	0
N.S.	1	1.48	0.88	1.22	0.00	0.64	0.00	0.00	0.18	0.00
time (sec)	N/A	1.112	3.333	4.546	0.000	0.076	0.000	0.000	0.221	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	592	379	510	0	300	0	0	262	0
N.S.	1	1.39	0.89	1.20	0.00	0.70	0.00	0.00	0.62	0.00
time (sec)	N/A	1.437	4.155	6.066	0.000	0.076	0.000	0.000	0.655	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	727	343	505	0	451	0	0	1411	0
N.S.	1	1.63	0.77	1.13	0.00	1.01	0.00	0.00	3.16	0.00
time (sec)	N/A	1.707	4.505	5.355	0.000	0.104	0.000	0.000	0.538	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	607	256	424	0	312	0	0	491	0
N.S.	1	1.73	0.73	1.21	0.00	0.89	0.00	0.00	1.40	0.00
time (sec)	N/A	1.142	3.514	4.147	0.000	0.078	0.000	0.000	0.349	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	510	153	202	0	227	0	0	66	0
N.S.	1	2.97	0.89	1.17	0.00	1.32	0.00	0.00	0.38	0.00
time (sec)	N/A	0.952	2.267	1.271	0.000	0.088	0.000	0.000	0.202	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	214	105	132	0	72	0	0	63	0
N.S.	1	2.06	1.01	1.27	0.00	0.69	0.00	0.00	0.61	0.00
time (sec)	N/A	0.455	2.087	0.781	0.000	0.086	0.000	0.000	0.185	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	562	233	252	0	193	0	0	66	0
N.S.	1	3.72	1.54	1.67	0.00	1.28	0.00	0.00	0.44	0.00
time (sec)	N/A	1.104	2.789	2.090	0.000	0.087	0.000	0.000	0.209	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	639	256	442	0	226	0	0	66	0
N.S.	1	2.03	0.81	1.40	0.00	0.72	0.00	0.00	0.21	0.00
time (sec)	N/A	1.358	3.375	4.494	0.000	0.089	0.000	0.000	0.264	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	756	364	535	0	316	0	0	415	0
N.S.	1	1.91	0.92	1.35	0.00	0.80	0.00	0.00	1.05	0.00
time (sec)	N/A	1.826	4.535	5.390	0.000	0.093	0.000	0.000	0.940	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	333	321	411	0	258	0	0	385	0
N.S.	1	0.87	0.84	1.07	0.00	0.67	0.00	0.00	1.00	0.00
time (sec)	N/A	1.099	3.825	20.372	0.000	0.095	0.000	0.000	0.569	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	246	270	346	0	172	0	0	218	0
N.S.	1	0.80	0.88	1.13	0.00	0.56	0.00	0.00	0.71	0.00
time (sec)	N/A	0.745	3.257	16.484	0.000	0.090	0.000	0.000	0.396	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	191	155	178	0	147	0	0	60	0
N.S.	1	0.75	0.61	0.70	0.00	0.57	0.00	0.00	0.23	0.00
time (sec)	N/A	0.604	2.050	3.455	0.000	0.086	0.000	0.000	0.212	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	110	110	0	71	0	0	57	0
N.S.	1	1.00	0.87	0.87	0.00	0.56	0.00	0.00	0.45	0.00
time (sec)	N/A	0.406	1.928	1.859	0.000	0.082	0.000	0.000	0.190	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	259	262	245	0	217	0	0	63	0
N.S.	1	1.46	1.47	1.38	0.00	1.22	0.00	0.00	0.35	0.00
time (sec)	N/A	0.782	3.443	4.619	0.000	0.085	0.000	0.000	0.204	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	338	330	423	0	305	0	0	63	0
N.S.	1	0.89	0.87	1.12	0.00	0.81	0.00	0.00	0.17	0.00
time (sec)	N/A	1.003	4.019	16.559	0.000	0.095	0.000	0.000	0.240	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	31	67	51	0	43	0	0	33	0
N.S.	1	0.62	1.34	1.02	0.00	0.86	0.00	0.00	0.66	0.00
time (sec)	N/A	0.332	10.031	1.043	0.000	0.078	0.000	0.000	0.167	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	21	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.84	0.76
time (sec)	N/A	0.277	0.002	0.042	0.031	0.072	0.016	0.106	0.165	0.032

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	26	25	20	19	19	19	19	21	19
N.S.	1	1.04	1.00	0.80	0.76	0.76	0.76	0.76	0.84	0.76
time (sec)	N/A	0.282	0.002	0.040	0.028	0.061	0.017	0.124	0.166	0.030

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	19	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.95	0.80
time (sec)	N/A	0.258	0.000	0.033	0.035	0.065	0.016	0.117	0.161	0.027

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	20	17	17	20	17	17
N.S.	1	1.00	1.00	0.86	0.95	0.81	0.81	0.95	0.81	0.81
time (sec)	N/A	0.274	0.001	0.063	0.030	0.071	0.031	0.126	0.164	0.027

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	12	16	20	16
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.67	0.89	1.11	0.89
time (sec)	N/A	0.273	0.002	0.043	0.029	0.064	0.031	0.124	0.164	0.032

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	20	22	17	26	22	17
N.S.	1	1.00	1.00	0.86	0.95	1.05	0.81	1.24	1.05	0.81
time (sec)	N/A	0.273	0.002	0.046	0.031	0.073	0.045	0.137	0.159	17.914

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	17	21	17	17	21	18
N.S.	1	1.00	1.00	0.94	0.94	1.17	0.94	0.94	1.17	1.00
time (sec)	N/A	0.273	0.004	0.045	0.031	0.074	0.050	0.106	0.199	0.028

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	21	23	19	27	23	20
N.S.	1	1.00	1.00	0.86	1.00	1.10	0.90	1.29	1.10	0.95
time (sec)	N/A	0.272	0.003	0.045	0.037	0.056	0.101	0.113	0.227	0.047

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	21	21	22	21	21	20
N.S.	1	1.00	1.00	0.87	0.91	0.91	0.96	0.91	0.91	0.87
time (sec)	N/A	0.277	0.002	0.043	0.029	0.059	0.116	0.110	0.230	0.034

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	21	21	22	21	21	21
N.S.	1	1.00	1.00	0.80	0.84	0.84	0.88	0.84	0.84	0.84
time (sec)	N/A	0.277	0.002	0.045	0.027	0.055	0.156	0.132	0.224	0.034

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	21	21	22	21	21	21
N.S.	1	1.00	1.00	0.80	0.84	0.84	0.88	0.84	0.84	0.84
time (sec)	N/A	0.275	0.003	0.046	0.033	0.056	0.148	0.109	0.231	0.034

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	51	46	48	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.94	0.85	0.89	0.83
time (sec)	N/A	0.334	0.006	0.105	0.027	0.059	0.018	0.106	0.224	0.029

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	52	48	45	44	44	46	46	48	45
N.S.	1	0.96	0.89	0.83	0.81	0.81	0.85	0.85	0.89	0.83
time (sec)	N/A	0.359	0.006	0.095	0.033	0.056	0.018	0.108	0.224	0.022

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	45	41	48	43	46	42
N.S.	1	1.00	1.00	0.86	0.92	0.84	0.98	0.88	0.94	0.86
time (sec)	N/A	0.321	0.004	0.042	0.033	0.057	0.019	0.126	0.228	0.021

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	54	47	43	44	41	42	46	43	42
N.S.	1	1.15	1.00	0.91	0.94	0.87	0.89	0.98	0.91	0.89
time (sec)	N/A	0.358	0.010	0.108	0.033	0.063	0.047	0.135	0.233	0.026

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	42	46	44	44	48	43
N.S.	1	1.00	1.00	0.94	0.88	0.96	0.92	0.92	1.00	0.90
time (sec)	N/A	0.337	0.013	0.087	0.032	0.063	0.046	0.107	0.226	0.025

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	49	46	45	44	47	44	53	49	43
N.S.	1	0.96	0.90	0.88	0.86	0.92	0.86	1.04	0.96	0.84
time (sec)	N/A	0.380	0.012	0.058	0.025	0.061	0.062	0.133	0.173	0.028

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	42	42	46	46	42	48	44
N.S.	1	1.00	1.00	0.89	0.89	0.98	0.98	0.89	1.02	0.94
time (sec)	N/A	0.322	0.017	0.056	0.027	0.061	0.069	0.107	0.163	0.043

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	52	41	42	45	47	44	60	51	43
N.S.	1	1.16	0.91	0.93	1.00	1.04	0.98	1.33	1.13	0.96
time (sec)	N/A	0.348	0.014	0.079	0.027	0.058	0.175	0.119	0.165	0.038

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	43	45	46	48	47	48	44
N.S.	1	1.00	1.02	0.90	0.94	0.96	1.00	0.98	1.00	0.92
time (sec)	N/A	0.319	0.016	0.057	0.026	0.056	0.197	0.105	0.168	0.043

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	50	50	44	45	48	48	54	50	46
N.S.	1	0.98	0.98	0.86	0.88	0.94	0.94	1.06	0.98	0.90
time (sec)	N/A	0.343	0.014	0.057	0.030	0.062	0.354	0.128	0.161	18.141

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	42	44	46	46	46	48	45
N.S.	1	1.00	1.04	0.89	0.94	0.98	0.98	0.98	1.02	0.96
time (sec)	N/A	0.314	0.018	0.056	0.026	0.058	0.342	0.112	0.169	0.037

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	54	50	43	48	48	48	58	50	45
N.S.	1	1.12	1.04	0.90	1.00	1.00	1.00	1.21	1.04	0.94
time (sec)	N/A	0.348	0.019	0.056	0.029	0.065	0.621	0.124	0.170	0.056

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	45	46	46	49	48	48	46
N.S.	1	1.00	0.96	0.87	0.88	0.88	0.94	0.92	0.92	0.88
time (sec)	N/A	0.312	0.016	0.057	0.031	0.059	0.667	0.104	0.166	17.918

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	45	46	46	49	48	48	47
N.S.	1	1.00	0.98	0.83	0.85	0.85	0.91	0.89	0.89	0.87
time (sec)	N/A	0.356	0.012	0.059	0.031	0.056	0.911	0.102	0.172	18.498

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	56	45	46	46	49	48	48	47
N.S.	1	1.00	1.04	0.83	0.85	0.85	0.91	0.89	0.89	0.87
time (sec)	N/A	0.324	0.020	0.059	0.034	0.056	0.929	0.113	0.165	0.036

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	58	50	45	46	46	49	48	48	47
N.S.	1	1.07	0.93	0.83	0.85	0.85	0.91	0.89	0.89	0.87
time (sec)	N/A	0.342	0.012	0.062	0.029	0.060	1.199	0.145	0.179	0.036

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	85	81	81	97	87	89	76
N.S.	1	1.00	1.00	0.96	0.91	0.91	1.09	0.98	1.00	0.85
time (sec)	N/A	0.382	0.011	0.114	0.034	0.056	0.024	0.104	0.168	0.037

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	87	79	85	81	81	92	87	89	76
N.S.	1	0.98	0.89	0.96	0.91	0.91	1.03	0.98	1.00	0.85
time (sec)	N/A	0.419	0.012	0.102	0.034	0.060	0.023	0.103	0.163	0.032

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	81	85	77	87	83	87	72
N.S.	1	1.00	1.00	1.00	1.05	0.95	1.07	1.02	1.07	0.89
time (sec)	N/A	0.389	0.007	0.048	0.031	0.078	0.023	0.102	0.169	0.033

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	85	82	82	79	92	87	84	73
N.S.	1	1.05	1.00	0.96	0.96	0.93	1.08	1.02	0.99	0.86
time (sec)	N/A	0.413	0.015	0.093	0.039	0.057	0.072	0.148	0.167	0.036

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	84	78	83	82	83	89	73
N.S.	1	1.00	1.00	1.05	0.98	1.04	1.02	1.04	1.11	0.91
time (sec)	N/A	0.376	0.017	0.099	0.029	0.061	0.072	0.138	0.166	0.034

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	85	78	86	82	85	92	98	91	75
N.S.	1	0.99	0.91	1.00	0.95	0.99	1.07	1.14	1.06	0.87
time (sec)	N/A	0.420	0.024	0.065	0.027	0.066	0.090	0.106	0.176	0.038

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	84	80	83	90	84	89	77
N.S.	1	1.00	1.00	1.01	0.96	1.00	1.08	1.01	1.07	0.93
time (sec)	N/A	0.381	0.019	0.068	0.032	0.057	0.094	0.108	0.166	0.032

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	99	93	105	0	313	391	92	458	842
N.S.	1	0.99	0.93	1.05	0.00	3.13	3.91	0.92	4.58	8.42
time (sec)	N/A	0.506	0.060	0.168	0.000	0.072	1.398	0.390	0.170	0.268

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	80	78	83	0	254	316	75	366	655
N.S.	1	0.99	0.96	1.02	0.00	3.14	3.90	0.93	4.52	8.09
time (sec)	N/A	0.435	0.031	0.125	0.000	0.075	1.026	0.377	0.168	19.151

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	62	60	0	197	223	59	236	118
N.S.	1	1.02	0.98	0.95	0.00	3.13	3.54	0.94	3.75	1.87
time (sec)	N/A	0.400	0.018	0.095	0.000	0.078	0.465	0.356	0.168	18.349

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	36	0	129	131	35	95	41
N.S.	1	1.00	1.08	1.00	0.00	3.58	3.64	0.97	2.64	1.14
time (sec)	N/A	0.296	0.008	0.061	0.000	0.068	0.234	0.353	0.177	0.062

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	113	65	0	223	253	68	247	1014
N.S.	1	1.06	1.64	0.94	0.00	3.23	3.67	0.99	3.58	14.70
time (sec)	N/A	0.415	0.052	0.082	0.000	0.081	2.293	0.402	0.169	18.721

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	95	135	85	0	293	345	94	409	2033
N.S.	1	1.07	1.52	0.96	0.00	3.29	3.88	1.06	4.60	22.84
time (sec)	N/A	0.510	0.086	0.098	0.000	0.084	80.874	0.340	0.168	19.357

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	123	188	134	0	374	423	126	517	2451
N.S.	1	1.08	1.65	1.18	0.00	3.28	3.71	1.11	4.54	21.50
time (sec)	N/A	0.574	0.153	0.117	0.000	0.099	175.638	0.394	0.175	19.410

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	214	250	73	0	1564	194	2457	761	4127
N.S.	1	1.05	1.23	0.36	0.00	7.70	0.96	12.10	3.75	20.33
time (sec)	N/A	0.827	0.108	0.126	0.000	0.131	2.220	0.666	0.262	19.051

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	183	202	57	0	1059	129	2109	547	3026
N.S.	1	1.02	1.13	0.32	0.00	5.92	0.72	11.78	3.06	16.91
time (sec)	N/A	0.560	0.076	0.095	0.000	0.083	1.084	0.624	0.249	0.714

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	158	165	41	0	559	75	503	351	416
N.S.	1	1.05	1.10	0.27	0.00	3.73	0.50	3.35	2.34	2.77
time (sec)	N/A	0.444	0.061	0.093	0.000	0.092	0.377	0.687	0.238	0.293

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	129	38	0	613	87	1026	351	763
N.S.	1	1.00	0.86	0.25	0.00	4.09	0.58	6.84	2.34	5.09
time (sec)	N/A	0.403	0.053	0.066	0.000	0.090	0.565	0.319	0.236	0.490

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	172	191	159	0	1116	148	1839	560	2997
N.S.	1	0.99	1.10	0.91	0.00	6.41	0.85	10.57	3.22	17.22
time (sec)	N/A	0.508	0.277	0.118	0.000	0.102	1.260	0.558	0.260	18.109

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	203	216	179	0	1622	211	1640	810	4160
N.S.	1	1.04	1.10	0.91	0.00	8.28	1.08	8.37	4.13	21.22
time (sec)	N/A	0.679	0.097	0.128	0.000	0.119	4.707	0.744	0.298	0.815

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	106	93	106	0	313	391	95	458	842
N.S.	1	0.99	0.87	0.99	0.00	2.93	3.65	0.89	4.28	7.87
time (sec)	N/A	0.539	0.062	0.174	0.000	0.085	1.420	0.361	0.234	17.653

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	86	78	83	0	256	316	77	366	655
N.S.	1	0.99	0.90	0.95	0.00	2.94	3.63	0.89	4.21	7.53
time (sec)	N/A	0.420	0.032	0.124	0.000	0.079	1.037	0.391	0.267	0.519

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	69	62	60	0	194	223	59	236	118
N.S.	1	1.03	0.93	0.90	0.00	2.90	3.33	0.88	3.52	1.76
time (sec)	N/A	0.375	0.017	0.095	0.000	0.086	0.468	0.383	0.187	17.626

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	0	121	131	35	95	41
N.S.	1	1.00	1.00	0.92	0.00	3.10	3.36	0.90	2.44	1.05
time (sec)	N/A	0.299	0.007	0.062	0.000	0.079	0.244	0.393	0.172	0.063

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	76	113	65	0	222	253	68	247	1012
N.S.	1	1.04	1.55	0.89	0.00	3.04	3.47	0.93	3.38	13.86
time (sec)	N/A	0.416	0.053	0.085	0.000	0.089	2.775	0.349	0.176	17.995

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	101	133	85	0	296	0	96	409	2033
N.S.	1	1.06	1.40	0.89	0.00	3.12	0.00	1.01	4.31	21.40
time (sec)	N/A	0.519	0.108	0.100	0.000	0.098	0.000	0.383	0.168	19.254

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	130	184	134	0	371	0	132	517	2451
N.S.	1	1.06	1.50	1.09	0.00	3.02	0.00	1.07	4.20	19.93
time (sec)	N/A	0.606	0.202	0.115	0.000	0.112	0.000	0.332	0.169	19.910

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	532	282	101	0	2133	262	3064	992	5317
N.S.	1	1.50	0.79	0.28	0.00	6.01	0.74	8.63	2.79	14.98
time (sec)	N/A	1.665	0.172	0.165	0.000	0.159	9.743	0.646	0.179	0.813

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	451	250	73	0	1604	194	2479	761	4127
N.S.	1	1.48	0.82	0.24	0.00	5.28	0.64	8.15	2.50	13.58
time (sec)	N/A	1.318	0.157	0.106	0.000	0.120	1.901	0.606	0.192	0.663

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	402	202	57	0	1119	129	2133	547	3026
N.S.	1	1.50	0.75	0.21	0.00	4.18	0.48	7.96	2.04	11.29
time (sec)	N/A	1.070	0.119	0.097	0.000	0.111	1.081	0.587	0.200	18.534

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	336	165	41	0	639	75	503	351	416
N.S.	1	1.51	0.74	0.18	0.00	2.88	0.34	2.27	1.58	1.87
time (sec)	N/A	0.830	0.057	0.089	0.000	0.085	0.373	0.678	0.175	0.286

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	365	129	38	0	701	87	1038	351	763
N.S.	1	1.64	0.58	0.17	0.00	3.16	0.39	4.68	1.58	3.44
time (sec)	N/A	0.855	0.052	0.082	0.000	0.094	0.567	0.354	0.174	18.005

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	435	191	159	0	1172	148	1857	560	2997
N.S.	1	1.65	0.72	0.60	0.00	4.44	0.56	7.03	2.12	11.35
time (sec)	N/A	1.007	0.269	0.115	0.000	0.109	1.251	0.665	0.198	18.795

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	481	214	179	0	1654	211	1664	810	4162
N.S.	1	1.58	0.70	0.59	0.00	5.42	0.69	5.46	2.66	13.65
time (sec)	N/A	1.223	0.143	0.131	0.000	0.110	3.516	0.819	0.222	18.517

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	139	121	179	0	663	745	152	1427	1336
N.S.	1	1.05	0.92	1.36	0.00	5.02	5.64	1.15	10.81	10.12
time (sec)	N/A	0.558	0.119	0.195	0.000	0.094	11.603	0.321	0.180	18.806

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	79	93	104	0	407	282	96	479	187
N.S.	1	1.01	1.19	1.33	0.00	5.22	3.62	1.23	6.14	2.40
time (sec)	N/A	0.387	0.061	0.122	0.000	0.090	0.741	0.388	0.172	17.856

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	76	79	77	0	360	269	82	457	178
N.S.	1	1.01	1.05	1.03	0.00	4.80	3.59	1.09	6.09	2.37
time (sec)	N/A	0.373	0.047	0.111	0.000	0.083	0.642	0.342	0.175	17.662

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	76	79	75	0	361	267	82	477	172
N.S.	1	1.03	1.07	1.01	0.00	4.88	3.61	1.11	6.45	2.32
time (sec)	N/A	0.358	0.051	0.115	0.000	0.082	0.624	0.351	0.173	0.135

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	153	207	185	0	813	0	166	1533	5048
N.S.	1	1.25	1.70	1.52	0.00	6.66	0.00	1.36	12.57	41.38
time (sec)	N/A	0.624	0.220	0.147	0.000	0.136	0.000	0.374	0.178	22.177

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	187	248	213	0	1007	0	182	2043	5491
N.S.	1	1.15	1.53	1.31	0.00	6.22	0.00	1.12	12.61	33.90
time (sec)	N/A	0.727	0.178	0.154	0.000	0.192	0.000	0.348	0.251	22.488

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	306	327	174	0	2856	0	3335	3084	7599
N.S.	1	1.01	1.08	0.57	0.00	9.43	0.00	11.01	10.18	25.08
time (sec)	N/A	0.980	0.426	0.154	0.000	0.296	0.000	0.656	2.289	19.479

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	253	282	151	0	2257	379	2736	2411	6293
N.S.	1	0.92	1.03	0.55	0.00	8.24	1.38	9.99	8.80	22.97
time (sec)	N/A	0.777	0.326	0.138	0.000	0.150	20.547	0.673	0.335	19.417

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	227	235	123	0	1668	296	2132	1795	4973
N.S.	1	0.96	0.99	0.52	0.00	7.04	1.25	9.00	7.57	20.98
time (sec)	N/A	0.621	0.286	0.129	0.000	0.112	2.226	0.534	0.327	19.661

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	213	222	122	0	1680	298	1970	1795	4854
N.S.	1	0.96	1.00	0.55	0.00	7.60	1.35	8.91	8.12	21.96
time (sec)	N/A	0.566	0.314	0.184	0.000	0.107	7.522	0.522	0.325	19.229

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	236	243	151	0	2309	394	2682	2409	6404
N.S.	1	0.94	0.96	0.60	0.00	9.16	1.56	10.64	9.56	25.41
time (sec)	N/A	0.705	0.294	0.143	0.000	0.162	103.353	0.365	0.347	19.640

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	307	302	294	0	2912	0	3087	3104	7555
N.S.	1	1.00	0.98	0.95	0.00	9.45	0.00	10.02	10.08	24.53
time (sec)	N/A	1.098	0.429	0.174	0.000	0.264	0.000	0.618	1.411	20.537

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	239	244	367	0	1631	0	306	3839	2588
N.S.	1	1.14	1.17	1.76	0.00	7.80	0.00	1.46	18.37	12.38
time (sec)	N/A	0.824	0.212	0.316	0.000	0.112	0.000	1.070	0.265	20.863

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	133	194	267	0	973	554	212	1137	444
N.S.	1	1.10	1.60	2.21	0.00	8.04	4.58	1.75	9.40	3.67
time (sec)	N/A	0.481	0.118	0.173	0.000	0.096	1.985	1.197	0.248	0.284

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	134	137	230	0	892	524	171	1091	423
N.S.	1	1.13	1.15	1.93	0.00	7.50	4.40	1.44	9.17	3.55
time (sec)	N/A	0.461	0.135	0.164	0.000	0.095	1.489	1.109	0.259	18.056

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	145	145	216	0	907	580	161	1694	460
N.S.	1	1.12	1.12	1.66	0.00	6.98	4.46	1.24	13.03	3.54
time (sec)	N/A	0.495	0.094	0.175	0.000	0.087	2.024	1.146	0.252	18.069

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	129	114	128	0	808	491	143	1055	400
N.S.	1	1.14	1.01	1.13	0.00	7.15	4.35	1.27	9.34	3.54
time (sec)	N/A	0.451	0.071	0.165	0.000	0.091	1.207	1.125	0.213	18.042

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	126	106	126	0	809	481	144	1084	386
N.S.	1	1.12	0.94	1.12	0.00	7.16	4.26	1.27	9.59	3.42
time (sec)	N/A	0.441	0.068	0.153	0.000	0.091	1.181	1.209	0.212	0.231

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	254	342	360	0	2017	0	323	4090	9339
N.S.	1	1.27	1.71	1.80	0.00	10.08	0.00	1.62	20.45	46.70
time (sec)	N/A	0.893	0.312	0.211	0.000	0.372	0.000	1.099	0.186	25.323

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	298	402	412	0	2312	0	382	4941	10074
N.S.	1	1.17	1.58	1.62	0.00	9.07	0.00	1.50	19.38	39.51
time (sec)	N/A	1.008	0.383	0.230	0.000	0.569	0.000	1.076	0.186	26.209

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	392	455	331	0	4279	0	2430	6796	10912
N.S.	1	1.03	1.19	0.87	0.00	11.20	0.00	6.36	17.79	28.57
time (sec)	N/A	1.252	0.751	0.185	0.000	0.547	0.000	2.059	4.852	22.979

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	345	381	289	0	3725	0	4558	5719	9575
N.S.	1	1.02	1.13	0.86	0.00	11.02	0.00	13.49	16.92	28.33
time (sec)	N/A	0.983	0.624	0.179	0.000	0.320	0.000	1.404	3.763	19.312

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	304	343	251	0	3128	627	1750	4644	8521
N.S.	1	1.02	1.15	0.84	0.00	10.50	2.10	5.87	15.58	28.59
time (sec)	N/A	0.841	0.546	0.162	0.000	0.151	10.739	1.667	4.530	18.055

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	296	285	251	0	3128	648	1861	4644	8397
N.S.	1	1.02	0.99	0.87	0.00	10.82	2.24	6.44	16.07	29.06
time (sec)	N/A	0.788	0.476	0.169	0.000	0.167	107.160	1.532	4.353	21.999

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	334	334	289	0	3777	0	4270	5719	9731
N.S.	1	1.07	1.07	0.93	0.00	12.14	0.00	13.73	18.39	31.29
time (sec)	N/A	0.897	0.547	0.315	0.000	0.297	0.000	1.423	4.439	22.540

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	361	372	331	0	4323	0	2707	6794	10979
N.S.	1	1.02	1.05	0.93	0.00	12.18	0.00	7.63	19.14	30.93
time (sec)	N/A	1.023	0.661	0.253	0.000	0.595	0.000	0.974	3.745	21.905

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	423	454	517	0	4924	0	5273	7940	12130
N.S.	1	1.00	1.07	1.22	0.00	11.59	0.00	12.41	18.68	28.54
time (sec)	N/A	1.258	1.175	0.242	0.000	1.096	0.000	2.032	3.175	23.513

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	80	80	86	0	259	311	78	360	656
N.S.	1	0.98	0.98	1.05	0.00	3.16	3.79	0.95	4.39	8.00
time (sec)	N/A	0.431	0.040	0.111	0.000	0.088	1.035	0.386	0.229	0.521

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	68	65	63	0	206	223	62	228	120
N.S.	1	1.06	1.02	0.98	0.00	3.22	3.48	0.97	3.56	1.88
time (sec)	N/A	0.392	0.017	0.077	0.000	0.083	0.479	0.333	0.162	18.570

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	38	41	38	0	134	131	37	95	42
N.S.	1	1.09	1.17	1.09	0.00	3.83	3.74	1.06	2.71	1.20
time (sec)	N/A	0.296	0.008	0.054	0.000	0.090	0.233	0.365	0.157	17.606

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	75	117	68	0	230	253	71	239	1015
N.S.	1	1.07	1.67	0.97	0.00	3.29	3.61	1.01	3.41	14.50
time (sec)	N/A	0.419	0.052	0.081	0.000	0.105	2.298	0.344	0.164	0.683

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	95	139	87	0	298	350	95	399	2032
N.S.	1	1.07	1.56	0.98	0.00	3.35	3.93	1.07	4.48	22.83
time (sec)	N/A	0.504	0.106	0.098	0.000	0.109	80.913	0.389	0.177	19.724

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	183	208	58	0	1051	129	2153	535	3000
N.S.	1	1.02	1.16	0.32	0.00	5.87	0.72	12.03	2.99	16.76
time (sec)	N/A	0.630	0.087	0.104	0.000	0.100	1.118	0.569	0.182	18.443

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	160	137	43	0	551	75	513	343	416
N.S.	1	1.07	0.91	0.29	0.00	3.67	0.50	3.42	2.29	2.77
time (sec)	N/A	0.465	0.072	0.086	0.000	0.089	0.380	0.548	0.184	17.877

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	137	40	0	605	87	1050	343	763
N.S.	1	1.00	0.91	0.27	0.00	4.03	0.58	7.00	2.29	5.09
time (sec)	N/A	0.400	0.057	0.075	0.000	0.105	0.575	0.362	0.193	0.528

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	171	199	159	0	1108	148	1877	548	2979
N.S.	1	0.99	1.16	0.92	0.00	6.44	0.86	10.91	3.19	17.32
time (sec)	N/A	0.510	0.273	0.116	0.000	0.123	1.318	0.694	0.179	18.537

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	66	62	63	74	156	138	60	207	166
N.S.	1	0.96	0.90	0.91	1.07	2.26	2.00	0.87	3.00	2.41
time (sec)	N/A	0.413	0.027	0.105	0.107	0.100	0.633	0.140	0.152	0.420

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	65	51	49	60	134	110	46	126	153
N.S.	1	1.16	0.91	0.88	1.07	2.39	1.96	0.82	2.25	2.73
time (sec)	N/A	0.384	0.015	0.069	0.106	0.075	0.320	0.162	0.162	0.180

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	39	31	26	37	91	53	23	62	31
N.S.	1	1.26	1.00	0.84	1.19	2.94	1.71	0.74	2.00	1.00
time (sec)	N/A	0.287	0.007	0.063	0.107	0.073	0.158	0.138	0.165	0.102

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	89	90	64	85	146	184	71	138	183
N.S.	1	1.16	1.17	0.83	1.10	1.90	2.39	0.92	1.79	2.38
time (sec)	N/A	0.471	0.038	0.081	0.105	0.081	2.294	0.139	0.161	19.699

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	105	146	82	123	204	372	126	258	389
N.S.	1	1.08	1.51	0.85	1.27	2.10	3.84	1.30	2.66	4.01
time (sec)	N/A	0.530	0.068	0.098	0.108	0.083	13.097	0.149	0.168	19.505

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	145	144	57	0	603	105	511	202	1097
N.S.	1	1.27	1.26	0.50	0.00	5.29	0.92	4.48	1.77	9.62
time (sec)	N/A	0.513	0.067	0.120	0.000	0.085	0.720	0.168	0.170	19.010

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	126	128	43	0	267	44	199	96	216
N.S.	1	1.16	1.17	0.39	0.00	2.45	0.40	1.83	0.88	1.98
time (sec)	N/A	0.352	0.077	0.080	0.000	0.079	0.272	0.225	0.150	0.304

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	105	40	0	553	63	299	204	322
N.S.	1	1.00	0.96	0.37	0.00	5.07	0.58	2.74	1.87	2.95
time (sec)	N/A	0.327	0.046	0.072	0.000	0.079	0.437	0.175	0.153	20.366

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	149	143	119	0	1612	134	698	323	2774
N.S.	1	1.23	1.18	0.98	0.00	13.32	1.11	5.77	2.67	22.93
time (sec)	N/A	0.422	0.105	0.104	0.000	0.091	1.935	0.214	0.148	19.586

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	65	62	62	58	157	144	58	334	302
N.S.	1	0.94	0.90	0.90	0.84	2.28	2.09	0.84	4.84	4.38
time (sec)	N/A	0.396	0.024	0.102	0.111	0.078	0.623	0.171	0.147	0.190

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	64	49	47	42	131	117	42	196	85
N.S.	1	1.19	0.91	0.87	0.78	2.43	2.17	0.78	3.63	1.57
time (sec)	N/A	0.385	0.013	0.070	0.110	0.076	0.314	0.144	0.155	0.092

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	39	31	26	21	91	60	21	109	24
N.S.	1	1.26	1.00	0.84	0.68	2.94	1.94	0.68	3.52	0.77
time (sec)	N/A	0.288	0.009	0.059	0.109	0.068	0.154	0.180	0.173	18.702

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	82	105	62	61	140	194	61	205	71
N.S.	1	1.19	1.52	0.90	0.88	2.03	2.81	0.88	2.97	1.03
time (sec)	N/A	0.440	0.045	0.078	0.110	0.090	2.217	0.141	0.164	18.585

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	96	163	75	104	206	386	125	375	3313
N.S.	1	1.08	1.83	0.84	1.17	2.31	4.34	1.40	4.21	37.22
time (sec)	N/A	0.500	0.072	0.100	0.109	0.086	12.797	0.153	0.167	21.911

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	490	164	57	0	615	105	533	640	1147
N.S.	1	1.53	0.51	0.18	0.00	1.92	0.33	1.67	2.00	3.58
time (sec)	N/A	1.578	0.074	0.243	0.000	0.089	0.745	0.187	0.196	0.414

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	378	143	41	0	279	44	203	407	222
N.S.	1	1.51	0.57	0.16	0.00	1.12	0.18	0.81	1.63	0.89
time (sec)	N/A	0.980	0.087	0.139	0.000	0.078	0.272	0.180	0.166	0.306

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	411	119	38	0	567	63	307	618	986
N.S.	1	1.52	0.44	0.14	0.00	2.09	0.23	1.13	2.28	3.64
time (sec)	N/A	1.089	0.048	0.161	0.000	0.085	0.432	0.158	0.153	18.630

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	492	174	162	0	1582	134	742	887	2848
N.S.	1	1.50	0.53	0.49	0.00	4.82	0.41	2.26	2.70	8.68
time (sec)	N/A	1.332	0.116	0.181	0.000	0.096	1.859	0.184	0.546	18.538

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	53	47	41	40	40	48	40	61	42
N.S.	1	1.04	0.92	0.80	0.78	0.78	0.94	0.78	1.20	0.82
time (sec)	N/A	0.332	0.011	0.077	0.118	0.066	0.048	0.126	0.154	0.057

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	38	35	30	29	29	36	29	41	31
N.S.	1	1.09	1.00	0.86	0.83	0.83	1.03	0.83	1.17	0.89
time (sec)	N/A	0.323	0.008	0.062	0.112	0.062	0.043	0.120	0.158	0.043

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	42	44	36	35	35	42	35	56	37
N.S.	1	0.95	1.00	0.82	0.80	0.80	0.95	0.80	1.27	0.84
time (sec)	N/A	0.327	0.007	0.062	0.113	0.065	0.052	0.104	0.152	0.042

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	39	37	31	30	30	37	30	51	32
N.S.	1	1.05	1.00	0.84	0.81	0.81	1.00	0.81	1.38	0.86
time (sec)	N/A	0.318	0.007	0.052	0.113	0.070	0.047	0.115	0.154	0.035

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	26	18	29	20
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.30	0.90	1.45	1.00
time (sec)	N/A	0.269	0.004	0.047	0.110	0.076	0.042	0.107	0.157	0.051

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	65	31	36	32	41	36	53	34
N.S.	1	1.10	1.67	0.79	0.92	0.82	1.05	0.92	1.36	0.87
time (sec)	N/A	0.348	0.034	0.072	0.109	0.068	0.058	0.107	0.152	18.522

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	50	78	38	41	49	48	46	76	41
N.S.	1	1.04	1.62	0.79	0.85	1.02	1.00	0.96	1.58	0.85
time (sec)	N/A	0.369	0.029	0.075	0.110	0.065	0.063	0.108	0.154	0.053

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	36	71	31	31	33	37	31	48	32
N.S.	1	1.03	2.03	0.89	0.89	0.94	1.06	0.89	1.37	0.91
time (sec)	N/A	0.360	0.015	0.075	0.115	0.070	0.059	0.152	0.152	17.945

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	82	126	65	64	62	75	64	62	65
N.S.	1	1.01	1.56	0.80	0.79	0.77	0.93	0.79	0.77	0.80
time (sec)	N/A	0.548	0.134	0.082	0.111	0.064	0.052	0.103	0.151	0.042

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	68	112	55	54	52	65	54	52	55
N.S.	1	1.01	1.67	0.82	0.81	0.78	0.97	0.81	0.78	0.82
time (sec)	N/A	0.471	0.110	0.075	0.113	0.065	0.051	0.128	0.151	0.027

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	54	98	45	44	42	54	44	42	45
N.S.	1	1.02	1.85	0.85	0.83	0.79	1.02	0.83	0.79	0.85
time (sec)	N/A	0.387	0.097	0.066	0.109	0.063	0.053	0.130	0.149	0.037

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	40	84	35	34	32	44	34	32	35
N.S.	1	1.03	2.15	0.90	0.87	0.82	1.13	0.87	0.82	0.90
time (sec)	N/A	0.309	0.070	0.056	0.134	0.066	0.052	0.106	0.153	0.031

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	44	88	39	38	37	46	38	39	39
N.S.	1	1.02	2.05	0.91	0.88	0.86	1.07	0.88	0.91	0.91
time (sec)	N/A	0.320	0.066	0.075	0.118	0.065	0.058	0.105	0.155	17.801

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	58	102	49	50	53	60	50	53	49
N.S.	1	1.02	1.79	0.86	0.88	0.93	1.05	0.88	0.93	0.86
time (sec)	N/A	0.397	0.078	0.086	0.129	0.072	0.072	0.114	0.148	0.046

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	72	116	59	60	63	70	60	63	59
N.S.	1	1.01	1.63	0.83	0.85	0.89	0.99	0.85	0.89	0.83
time (sec)	N/A	0.469	0.110	0.099	0.108	0.065	0.086	0.130	0.155	0.057

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	99	109	74	73	73	90	73	71	64
N.S.	1	1.16	1.28	0.87	0.86	0.86	1.06	0.86	0.84	0.75
time (sec)	N/A	0.591	0.112	0.119	0.107	0.072	0.083	0.157	0.152	18.216

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	95	99	72	71	71	87	71	69	67
N.S.	1	1.17	1.22	0.89	0.88	0.88	1.07	0.88	0.85	0.83
time (sec)	N/A	0.533	0.083	0.128	0.109	0.076	0.082	0.110	0.159	0.041

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	85	95	64	63	63	80	63	61	54
N.S.	1	1.20	1.34	0.90	0.89	0.89	1.13	0.89	0.86	0.76
time (sec)	N/A	0.509	0.069	0.064	0.114	0.071	0.084	0.145	0.154	0.040

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	81	81	62	61	61	76	61	59	57
N.S.	1	1.21	1.21	0.93	0.91	0.91	1.13	0.91	0.88	0.85
time (sec)	N/A	0.449	0.068	0.062	0.109	0.078	0.082	0.098	0.156	0.041

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	71	90	54	53	53	70	53	51	44
N.S.	1	1.25	1.58	0.95	0.93	0.93	1.23	0.93	0.89	0.77
time (sec)	N/A	0.421	0.086	0.055	0.112	0.068	0.083	0.111	0.190	0.030

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	71	73	54	53	53	70	53	51	47
N.S.	1	1.25	1.28	0.95	0.93	0.93	1.23	0.93	0.89	0.82
time (sec)	N/A	0.379	0.032	0.053	0.112	0.080	0.087	0.110	0.258	18.206

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	81	91	59	65	76	82	65	74	53
N.S.	1	1.21	1.36	0.88	0.97	1.13	1.22	0.97	1.10	0.79
time (sec)	N/A	0.511	0.089	0.084	0.124	0.066	0.098	0.109	0.220	0.040

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	85	88	64	65	76	82	65	74	58
N.S.	1	1.20	1.24	0.90	0.92	1.07	1.15	0.92	1.04	0.82
time (sec)	N/A	0.470	0.067	0.086	0.110	0.077	0.101	0.105	0.217	0.039

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	95	105	72	75	86	92	75	84	63
N.S.	1	1.17	1.30	0.89	0.93	1.06	1.14	0.93	1.04	0.78
time (sec)	N/A	0.568	0.092	0.094	0.168	0.066	0.111	0.131	0.216	0.042

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	43	46	38	37	37	42	37	53	39
N.S.	1	0.93	1.00	0.83	0.80	0.80	0.91	0.80	1.15	0.85
time (sec)	N/A	0.328	0.011	0.079	0.110	0.062	0.049	0.100	0.220	17.685

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	40	39	33	32	32	37	32	48	34
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.95	0.82	1.23	0.87
time (sec)	N/A	0.346	0.007	0.062	0.105	0.061	0.046	0.105	0.221	0.045

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	20	20	19	18	18	26	18	19	20
N.S.	1	0.95	0.95	0.90	0.86	0.86	1.24	0.86	0.90	0.95
time (sec)	N/A	0.281	0.005	0.061	0.105	0.063	0.040	0.107	0.219	0.043

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	44	41	33	38	34	41	38	50	36
N.S.	1	1.07	1.00	0.80	0.93	0.83	1.00	0.93	1.22	0.88
time (sec)	N/A	0.363	0.028	0.080	0.105	0.064	0.058	0.132	0.261	17.949

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	49	80	38	43	51	48	48	73	41
N.S.	1	1.02	1.67	0.79	0.90	1.06	1.00	1.00	1.52	0.85
time (sec)	N/A	0.369	0.036	0.090	0.111	0.061	0.065	0.127	0.179	0.055

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	38	71	30	31	33	39	31	40	33
N.S.	1	1.09	2.03	0.86	0.89	0.94	1.11	0.89	1.14	0.94
time (sec)	N/A	0.350	0.019	0.098	0.107	0.063	0.059	0.102	0.170	18.317

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	91	92	68	0	73	71	67	63	57
N.S.	1	1.36	1.37	1.01	0.00	1.09	1.06	1.00	0.94	0.85
time (sec)	N/A	0.584	0.015	0.158	0.000	0.067	0.043	0.141	0.158	0.056

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	75	76	56	0	61	61	55	51	45
N.S.	1	1.44	1.46	1.08	0.00	1.17	1.17	1.06	0.98	0.87
time (sec)	N/A	0.497	0.011	0.115	0.000	0.064	0.043	0.107	0.159	0.032

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	63	64	48	0	53	51	47	43	37
N.S.	1	1.62	1.64	1.23	0.00	1.36	1.31	1.21	1.10	0.95
time (sec)	N/A	0.410	0.008	0.091	0.000	0.062	0.044	0.129	0.150	0.039

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	47	48	36	0	41	41	35	31	23
N.S.	1	1.96	2.00	1.50	0.00	1.71	1.71	1.46	1.29	0.96
time (sec)	N/A	0.323	0.007	0.069	0.000	0.064	0.042	0.134	0.151	0.028

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	51	52	40	0	46	42	39	37	27
N.S.	1	1.89	1.93	1.48	0.00	1.70	1.56	1.44	1.37	1.00
time (sec)	N/A	0.332	0.009	0.089	0.000	0.062	0.050	0.113	0.166	0.038

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	63	64	48	0	59	56	73	52	39
N.S.	1	1.58	1.60	1.20	0.00	1.48	1.40	1.82	1.30	0.98
time (sec)	N/A	0.411	0.013	0.115	0.000	0.063	0.065	0.145	0.170	17.727

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	79	80	60	0	69	66	61	62	47
N.S.	1	1.44	1.45	1.09	0.00	1.25	1.20	1.11	1.13	0.85
time (sec)	N/A	0.492	0.017	0.135	0.000	0.060	0.081	0.127	0.155	18.725

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	113	105	54	0	72	80	74	66	65
N.S.	1	1.47	1.36	0.70	0.00	0.94	1.04	0.96	0.86	0.84
time (sec)	N/A	0.571	0.118	0.152	0.000	0.072	0.082	0.108	0.157	0.114

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	92	101	49	0	64	73	66	58	54
N.S.	1	1.37	1.51	0.73	0.00	0.96	1.09	0.99	0.87	0.81
time (sec)	N/A	0.515	0.122	0.109	0.000	0.069	0.082	0.130	0.152	0.048

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	97	85	42	0	60	70	62	54	55
N.S.	1	1.59	1.39	0.69	0.00	0.98	1.15	1.02	0.89	0.90
time (sec)	N/A	0.505	0.070	0.079	0.000	0.066	0.082	0.112	0.153	0.051

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	78	94	38	0	54	63	56	48	44
N.S.	1	1.47	1.77	0.72	0.00	1.02	1.19	1.06	0.91	0.83
time (sec)	N/A	0.445	0.083	0.069	0.000	0.066	0.081	0.123	0.156	0.039

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	89	77	35	0	54	63	56	48	47
N.S.	1	1.68	1.45	0.66	0.00	1.02	1.19	1.06	0.91	0.89
time (sec)	N/A	0.419	0.033	0.068	0.000	0.070	0.088	0.118	0.158	0.027

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	90	99	44	0	76	76	68	70	54
N.S.	1	1.38	1.52	0.68	0.00	1.17	1.17	1.05	1.08	0.83
time (sec)	N/A	0.500	0.086	0.102	0.000	0.068	0.099	0.108	0.154	0.042

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	103	92	44	0	77	76	46	71	61
N.S.	1	1.54	1.37	0.66	0.00	1.15	1.13	0.69	1.06	0.91
time (sec)	N/A	0.485	0.095	0.108	0.000	0.066	0.103	0.135	0.157	0.043

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	102	109	50	0	87	85	78	81	66
N.S.	1	1.32	1.42	0.65	0.00	1.13	1.10	1.01	1.05	0.86
time (sec)	N/A	0.604	0.127	0.120	0.000	0.069	0.110	0.134	0.149	0.046

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	16	14	11	10	10	10	10	63	10
N.S.	1	1.14	1.00	0.79	0.71	0.71	0.71	0.71	4.50	0.71
time (sec)	N/A	0.264	0.005	0.073	0.107	0.059	0.039	0.193	0.154	19.269

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	20	18	17	18
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.87	0.78	0.74	0.78
time (sec)	N/A	0.250	0.010	0.089	0.110	0.080	0.058	0.119	0.156	18.232

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	14	12	12	12
N.S.	1	1.00	1.00	0.72	0.67	0.67	0.78	0.67	0.67	0.67
time (sec)	N/A	0.274	0.007	0.088	0.108	0.065	0.062	0.112	0.156	19.061

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	190	39	36	0	157	24	147	231	101
N.S.	1	1.29	0.27	0.24	0.00	1.07	0.16	1.00	1.57	0.69
time (sec)	N/A	0.671	0.025	0.190	0.000	0.079	0.272	0.266	0.152	0.140

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	190	39	36	0	157	24	147	231	101
N.S.	1	1.29	0.27	0.24	0.00	1.07	0.16	1.00	1.57	0.69
time (sec)	N/A	0.636	0.018	0.071	0.000	0.070	0.274	0.264	0.152	0.041

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	77	74	36	0	87	126	68	53	81
N.S.	1	1.33	1.28	0.62	0.00	1.50	2.17	1.17	0.91	1.40
time (sec)	N/A	0.333	0.045	0.096	0.000	0.093	0.206	0.136	0.153	0.123

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	B	B	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	0	74	36	0	87	126	68	53	81
N.S.	1	0.00	1.28	0.62	0.00	1.50	2.17	1.17	0.91	1.40
time (sec)	N/A	0.000	0.011	0.052	0.000	0.074	0.212	0.122	0.154	0.037

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	208	221	46	0	935	180	1160	592	921
N.S.	1	1.06	1.13	0.23	0.00	4.77	0.92	5.92	3.02	4.70
time (sec)	N/A	0.734	0.119	0.160	0.000	0.080	1.532	2.415	0.206	18.921

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	B	A	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	0	221	46	0	935	180	1160	592	921
N.S.	1	0.00	1.13	0.23	0.00	4.77	0.92	5.92	3.02	4.70
time (sec)	N/A	0.000	0.036	0.073	0.000	0.078	1.524	1.255	0.302	17.348

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	228	241	54	0	1995	376	3423	938	1956
N.S.	1	1.06	1.12	0.25	0.00	9.24	1.74	15.85	4.34	9.06
time (sec)	N/A	0.938	0.252	0.206	0.000	0.092	7.447	38.882	0.689	19.878

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	C	F	B	A	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	0	241	54	0	1995	376	3423	938	1956
N.S.	1	0.00	1.12	0.25	0.00	9.24	1.74	15.85	4.34	9.06
time (sec)	N/A	0.000	0.062	0.082	0.000	0.089	7.391	20.092	0.655	0.933

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	20	19	24	29	19	23	21
N.S.	1	1.00	0.81	0.65	0.61	0.77	0.94	0.61	0.74	0.68
time (sec)	N/A	0.276	0.016	0.085	0.027	0.060	0.346	0.107	0.221	17.234

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	20	19	24	29	19	23	21
N.S.	1	1.00	0.81	0.65	0.61	0.77	0.94	0.61	0.74	0.68
time (sec)	N/A	0.272	0.016	0.072	0.027	0.060	0.219	0.134	0.247	0.036

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	20	19	22	29	19	21	21
N.S.	1	1.00	0.81	0.65	0.61	0.71	0.94	0.61	0.68	0.68
time (sec)	N/A	0.272	0.016	0.069	0.028	0.060	0.435	0.097	0.184	0.035

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	19	21	27	19	20	21
N.S.	1	1.00	0.86	0.69	0.66	0.72	0.93	0.66	0.69	0.72
time (sec)	N/A	0.274	0.016	0.070	0.036	0.063	0.130	0.109	0.165	0.033

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	19	21	27	19	22	21
N.S.	1	1.00	0.86	0.69	0.66	0.72	0.93	0.66	0.76	0.72
time (sec)	N/A	0.269	0.020	0.080	0.040	0.065	0.200	0.110	0.158	17.253

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	19	21	27	19	25	21
N.S.	1	1.00	0.86	0.69	0.66	0.72	0.93	0.66	0.86	0.72
time (sec)	N/A	0.275	0.021	0.081	0.034	0.069	0.219	0.109	0.149	0.038

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	20	21	27	20	25	21
N.S.	1	1.00	0.86	0.69	0.69	0.72	0.93	0.69	0.86	0.72
time (sec)	N/A	0.270	0.024	0.087	0.039	0.064	0.268	0.110	0.151	17.092

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	45	44	49	70	46	50	45
N.S.	1	1.00	0.97	0.70	0.69	0.77	1.09	0.72	0.78	0.70
time (sec)	N/A	0.330	0.034	0.209	0.027	0.066	0.766	0.103	0.154	17.984

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	45	44	49	70	46	50	45
N.S.	1	1.00	0.89	0.70	0.69	0.77	1.09	0.72	0.78	0.70
time (sec)	N/A	0.347	0.032	0.142	0.033	0.065	0.518	0.103	0.156	0.025

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	45	44	47	63	46	48	45
N.S.	1	1.00	0.89	0.70	0.69	0.73	0.98	0.72	0.75	0.70
time (sec)	N/A	0.342	0.032	0.176	0.026	0.062	0.674	0.124	0.168	0.026

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	45	48	46	68	46	47	45
N.S.	1	1.00	0.92	0.73	0.77	0.74	1.10	0.74	0.76	0.73
time (sec)	N/A	0.317	0.030	0.088	0.029	0.066	0.351	0.105	0.153	0.026

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	52	47	44	46	68	46	49	45
N.S.	1	1.00	0.84	0.76	0.71	0.74	1.10	0.74	0.79	0.73
time (sec)	N/A	0.322	0.034	0.100	0.031	0.064	0.437	0.109	0.153	0.027

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	52	47	44	46	68	46	52	45
N.S.	1	1.00	0.84	0.76	0.71	0.74	1.10	0.74	0.84	0.73
time (sec)	N/A	0.321	0.037	0.100	0.027	0.068	0.487	0.106	0.149	0.027

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	52	47	45	46	68	47	52	48
N.S.	1	1.00	0.84	0.76	0.73	0.74	1.10	0.76	0.84	0.77
time (sec)	N/A	0.320	0.041	0.109	0.037	0.067	0.592	0.107	0.160	0.046

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	111	90	81	86	129	87	91	76
N.S.	1	1.00	1.08	0.87	0.79	0.83	1.25	0.84	0.88	0.74
time (sec)	N/A	0.391	0.052	0.389	0.029	0.066	1.571	0.110	0.155	0.043

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	111	90	81	86	129	87	91	76
N.S.	1	1.00	1.08	0.87	0.79	0.83	1.25	0.84	0.88	0.74
time (sec)	N/A	0.380	0.052	0.159	0.032	0.061	1.174	0.108	0.153	0.036

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	107	90	81	84	112	87	89	76
N.S.	1	1.00	1.04	0.87	0.79	0.82	1.09	0.84	0.86	0.74
time (sec)	N/A	0.379	0.053	0.194	0.030	0.065	1.246	0.114	0.161	0.037

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	89	88	83	128	87	88	76
N.S.	1	1.00	0.96	0.88	0.87	0.82	1.27	0.86	0.87	0.75
time (sec)	N/A	0.373	0.057	0.104	0.033	0.064	0.839	0.104	0.151	0.035

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	88	81	83	126	87	90	76
N.S.	1	1.00	0.94	0.89	0.82	0.84	1.27	0.88	0.91	0.77
time (sec)	N/A	0.380	0.063	0.115	0.036	0.063	0.922	0.131	0.151	0.040

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	93	88	81	83	128	87	93	76
N.S.	1	1.00	0.92	0.87	0.80	0.82	1.27	0.86	0.92	0.75
time (sec)	N/A	0.390	0.059	0.116	0.032	0.066	1.017	0.109	0.152	0.038

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	88	82	83	124	88	93	79
N.S.	1	1.00	0.94	0.89	0.83	0.84	1.25	0.89	0.94	0.80
time (sec)	N/A	0.364	0.068	0.125	0.031	0.064	1.205	0.112	0.150	0.039

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	364	80	65	0	7019	0	0	20	12789
N.S.	1	0.94	0.21	0.17	0.00	18.04	0.00	0.00	0.05	32.88
time (sec)	N/A	1.130	0.096	0.213	0.000	1.125	0.000	0.000	200.017	19.017

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	359	80	61	0	4023	0	0	20	10449
N.S.	1	0.93	0.21	0.16	0.00	10.45	0.00	0.00	0.05	27.14
time (sec)	N/A	0.927	0.059	0.135	0.000	0.268	0.000	0.000	200.021	19.610

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	332	48	45	0	4449	0	0	20	8093
N.S.	1	1.00	0.15	0.14	0.00	13.44	0.00	0.00	0.06	24.45
time (sec)	N/A	0.813	0.057	0.149	0.000	0.135	0.000	0.000	200.049	20.350

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	330	46	45	0	2157	0	0	20	8229
N.S.	1	1.00	0.14	0.14	0.00	6.52	0.00	0.00	0.06	24.86
time (sec)	N/A	0.779	0.052	0.106	0.000	0.104	0.000	0.000	200.028	19.133

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	327	47	45	0	3209	0	0	20	6133
N.S.	1	0.99	0.14	0.14	0.00	9.69	0.00	0.00	0.06	18.53
time (sec)	N/A	0.758	0.055	0.100	0.000	0.108	0.000	0.000	200.024	18.544

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	318	49	42	0	3137	0	0	20	10401
N.S.	1	0.96	0.15	0.13	0.00	9.48	0.00	0.00	0.06	31.42
time (sec)	N/A	0.726	0.039	0.125	0.000	0.162	0.000	0.000	200.038	19.429

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	350	78	65	0	5778	0	0	20	10573
N.S.	1	0.94	0.21	0.18	0.00	15.57	0.00	0.00	0.05	28.50
time (sec)	N/A	0.885	0.088	0.109	0.000	0.968	0.000	0.000	200.015	20.096

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	350	82	61	0	5046	0	0	20	16557
N.S.	1	0.94	0.22	0.16	0.00	13.60	0.00	0.00	0.05	44.63
time (sec)	N/A	0.858	0.062	0.114	0.000	0.716	0.000	0.000	200.019	22.174

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	383	106	81	0	8376	0	0	20	15149
N.S.	1	0.93	0.26	0.20	0.00	20.33	0.00	0.00	0.05	36.77
time (sec)	N/A	1.074	0.112	0.125	0.000	6.697	0.000	0.000	200.016	19.640

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	441	232	149	0	14601	0	0	20	28774
N.S.	1	0.82	0.43	0.28	0.00	27.24	0.00	0.00	0.04	53.68
time (sec)	N/A	1.200	0.512	0.519	0.000	20.295	0.000	0.000	200.015	20.292

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	535	431	237	146	0	9180	0	0	20	31964
N.S.	1	0.81	0.44	0.27	0.00	17.16	0.00	0.00	0.04	59.75
time (sec)	N/A	1.191	0.425	0.505	0.000	1.980	0.000	0.000	200.019	26.054

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	405	189	121	0	11817	0	0	20	23808
N.S.	1	0.86	0.40	0.26	0.00	25.09	0.00	0.00	0.04	50.55
time (sec)	N/A	1.042	0.353	0.498	0.000	4.694	0.000	0.000	200.020	19.530

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	409	194	118	0	7244	0	0	20	26432
N.S.	1	0.85	0.40	0.24	0.00	15.00	0.00	0.00	0.04	54.72
time (sec)	N/A	1.039	0.301	0.500	0.000	0.454	0.000	0.000	200.031	23.955

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	393	109	121	0	10601	0	0	20	21913
N.S.	1	0.87	0.24	0.27	0.00	23.56	0.00	0.00	0.04	48.70
time (sec)	N/A	1.039	0.227	0.499	0.000	1.883	0.000	0.000	200.031	19.232

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	385	110	118	0	8211	0	0	20	28713
N.S.	1	0.87	0.25	0.27	0.00	18.58	0.00	0.00	0.05	64.96
time (sec)	N/A	0.997	0.199	0.148	0.000	0.950	0.000	0.000	200.024	23.642

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	489	416	136	149	0	13225	0	0	20	26373
N.S.	1	0.85	0.28	0.30	0.00	27.04	0.00	0.00	0.04	53.93
time (sec)	N/A	1.151	0.268	0.152	0.000	8.481	0.000	0.000	200.019	20.577

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	431	140	144	0	10274	0	0	20	35171
N.S.	1	0.86	0.28	0.29	0.00	20.43	0.00	0.00	0.04	69.92
time (sec)	N/A	1.157	0.232	0.151	0.000	4.173	0.000	0.000	200.022	20.230

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	487	265	172	0	16004	0	0	20	31145
N.S.	1	0.85	0.46	0.30	0.00	27.93	0.00	0.00	0.03	54.35
time (sec)	N/A	1.472	0.486	0.238	0.000	80.482	0.000	0.000	200.027	25.430

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	525	506	275	0	14789	0	0	20	50970
N.S.	1	0.85	0.82	0.45	0.00	23.97	0.00	0.00	0.03	82.61
time (sec)	N/A	1.429	0.859	2.523	0.000	8.919	0.000	0.000	200.046	22.571

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	494	392	242	0	19805	0	0	20	39697
N.S.	1	0.87	0.69	0.43	0.00	34.81	0.00	0.00	0.04	69.77
time (sec)	N/A	1.296	0.758	2.477	0.000	42.023	0.000	0.000	200.020	21.363

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	490	397	241	0	12814	0	0	20	45495
N.S.	1	0.86	0.70	0.42	0.00	22.52	0.00	0.00	0.04	79.96
time (sec)	N/A	1.236	0.638	2.476	0.000	3.089	0.000	0.000	200.020	21.741

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	480	339	244	0	18451	0	0	20	37678
N.S.	1	0.90	0.64	0.46	0.00	34.62	0.00	0.00	0.04	70.69
time (sec)	N/A	1.271	0.692	2.475	0.000	24.475	0.000	0.000	200.026	21.355

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	472	345	237	0	13878	0	0	20	47803
N.S.	1	0.89	0.65	0.44	0.00	26.04	0.00	0.00	0.04	89.69
time (sec)	N/A	1.251	0.609	2.476	0.000	6.002	0.000	0.000	200.033	22.482

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	518	214	281	0	21124	0	0	20	42197
N.S.	1	0.87	0.36	0.47	0.00	35.56	0.00	0.00	0.03	71.04
time (sec)	N/A	1.354	0.370	2.506	0.000	62.227	0.000	0.000	200.024	22.133

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	516	215	270	0	15770	0	0	20	54027
N.S.	1	0.87	0.36	0.45	0.00	26.55	0.00	0.00	0.03	90.95
time (sec)	N/A	1.316	0.319	0.243	0.000	15.976	0.000	0.000	200.022	23.626

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	658	576	255	321	0	23877	0	0	20	46948
N.S.	1	0.88	0.39	0.49	0.00	36.29	0.00	0.00	0.03	71.35
time (sec)	N/A	2.378	0.471	0.250	0.000	162.849	0.000	0.000	200.022	23.591

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	658	573	259	316	0	17801	0	0	20	60099
N.S.	1	0.87	0.39	0.48	0.00	27.05	0.00	0.00	0.03	91.34
time (sec)	N/A	2.388	0.399	0.247	0.000	44.143	0.000	0.000	200.023	23.429

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	180	166	154	0	367	0	170	4009	315
N.S.	1	1.05	0.97	0.90	0.00	2.15	0.00	0.99	23.44	1.84
time (sec)	N/A	0.557	0.316	0.283	0.000	0.092	0.000	0.122	0.264	18.544

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	163	132	122	0	303	0	132	2818	193
N.S.	1	1.07	0.86	0.80	0.00	1.98	0.00	0.86	18.42	1.26
time (sec)	N/A	0.543	0.234	0.205	0.000	0.081	0.000	0.129	0.215	18.330

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	119	103	91	0	237	0	96	1536	87
N.S.	1	1.10	0.95	0.84	0.00	2.19	0.00	0.89	14.22	0.81
time (sec)	N/A	0.427	0.201	0.171	0.000	0.093	0.000	0.121	0.195	19.207

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	87	73	0	197	0	74	816	72
N.S.	1	1.05	1.05	0.88	0.00	2.37	0.00	0.89	9.83	0.87
time (sec)	N/A	0.368	0.283	0.171	0.000	0.106	0.000	0.117	0.177	19.090

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	104	91	0	566	0	0	369	88
N.S.	1	1.00	0.95	0.83	0.00	5.19	0.00	0.00	3.39	0.81
time (sec)	N/A	0.493	0.174	0.151	0.000	0.101	0.000	0.000	0.251	18.668

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	115	107	94	0	601	0	148	111	91
N.S.	1	1.03	0.96	0.84	0.00	5.37	0.00	1.32	0.99	0.81
time (sec)	N/A	0.495	0.171	0.181	0.000	0.101	0.000	0.135	0.283	18.381

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	92	88	81	0	215	0	241	123	0
N.S.	1	1.05	1.00	0.92	0.00	2.44	0.00	2.74	1.40	0.00
time (sec)	N/A	0.381	0.248	0.155	0.000	0.090	0.000	0.128	0.352	0.000

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	127	108	101	0	261	0	359	168	0
N.S.	1	1.09	0.93	0.87	0.00	2.25	0.00	3.09	1.45	0.00
time (sec)	N/A	0.450	0.410	0.197	0.000	0.099	0.000	0.134	23.583	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	171	141	130	0	325	0	617	20	0
N.S.	1	1.06	0.88	0.81	0.00	2.02	0.00	3.83	0.12	0.00
time (sec)	N/A	0.570	0.521	0.230	0.000	0.121	0.000	0.130	200.028	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	216	176	160	0	389	0	842	20	0
N.S.	1	1.09	0.88	0.80	0.00	1.95	0.00	4.23	0.10	0.00
time (sec)	N/A	0.685	0.692	0.245	0.000	0.157	0.000	0.158	200.034	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	400	538	476	0	402	0	0	232	0
N.S.	1	1.02	1.37	1.21	0.00	1.02	0.00	0.00	0.59	0.00
time (sec)	N/A	0.958	8.221	3.523	0.000	0.082	0.000	0.000	0.198	0.000

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	341	479	406	0	355	0	0	150	0
N.S.	1	0.99	1.40	1.18	0.00	1.03	0.00	0.00	0.44	0.00
time (sec)	N/A	0.707	6.889	3.019	0.000	0.081	0.000	0.000	0.179	0.000

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	312	445	379	0	301	0	0	86	0
N.S.	1	1.01	1.44	1.23	0.00	0.97	0.00	0.00	0.28	0.00
time (sec)	N/A	0.630	6.329	2.106	0.000	0.078	0.000	0.000	0.186	0.000

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	305	435	381	0	0	0	0	89	0
N.S.	1	1.01	1.44	1.26	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.637	10.539	2.404	0.000	0.000	0.000	0.000	0.180	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	345	459	398	0	303	0	0	96	0
N.S.	1	1.10	1.47	1.27	0.00	0.97	0.00	0.00	0.31	0.00
time (sec)	N/A	0.735	10.598	2.832	0.000	0.075	0.000	0.000	0.197	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	402	530	428	0	357	0	0	125	0
N.S.	1	1.11	1.46	1.18	0.00	0.98	0.00	0.00	0.34	0.00
time (sec)	N/A	0.913	10.901	3.754	0.000	0.080	0.000	0.000	0.234	0.000

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	227	220	241	0	535	0	663	7599	0
N.S.	1	1.02	0.99	1.08	0.00	2.40	0.00	2.97	34.08	0.00
time (sec)	N/A	0.612	0.690	0.355	0.000	0.097	0.000	0.167	0.605	0.000

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	210	194	199	0	451	0	529	5920	0
N.S.	1	1.03	0.95	0.98	0.00	2.21	0.00	2.59	29.02	0.00
time (sec)	N/A	0.585	0.526	0.299	0.000	0.091	0.000	0.165	0.321	0.000

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	166	140	152	0	361	0	408	3844	223
N.S.	1	1.11	0.93	1.01	0.00	2.41	0.00	2.72	25.63	1.49
time (sec)	N/A	0.481	0.358	0.240	0.000	0.089	0.000	0.158	0.221	17.780

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	134	114	120	0	297	0	311	2869	115
N.S.	1	1.08	0.92	0.97	0.00	2.40	0.00	2.51	23.14	0.93
time (sec)	N/A	0.424	0.640	0.208	0.000	0.085	0.000	0.146	0.194	17.714

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	169	148	192	0	727	0	0	693	0
N.S.	1	1.09	0.95	1.24	0.00	4.69	0.00	0.00	4.47	0.00
time (sec)	N/A	0.649	0.452	0.187	0.000	0.160	0.000	0.000	0.254	0.000

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	157	132	170	0	713	0	184	652	0
N.S.	1	1.05	0.88	1.13	0.00	4.75	0.00	1.23	4.35	0.00
time (sec)	N/A	0.680	0.464	0.250	0.000	0.131	0.000	0.170	0.364	0.000

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	159	131	150	0	713	0	304	633	0
N.S.	1	1.05	0.87	0.99	0.00	4.72	0.00	2.01	4.19	0.00
time (sec)	N/A	0.606	0.496	0.244	0.000	0.131	0.000	0.180	0.501	0.000

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	177	148	142	0	771	0	398	233	0
N.S.	1	1.09	0.91	0.87	0.00	4.73	0.00	2.44	1.43	0.00
time (sec)	N/A	0.655	0.670	0.243	0.000	0.148	0.000	0.201	17.272	0.000

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	143	120	123	0	319	0	606	20	0
N.S.	1	1.08	0.90	0.92	0.00	2.40	0.00	4.56	0.15	0.00
time (sec)	N/A	0.473	0.672	0.236	0.000	0.111	0.000	0.167	200.028	0.000

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	178	160	153	0	383	0	832	20	0
N.S.	1	1.10	0.99	0.94	0.00	2.36	0.00	5.14	0.12	0.00
time (sec)	N/A	0.533	0.980	0.264	0.000	0.168	0.000	0.195	200.036	0.000

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	222	201	189	0	473	0	1235	757	0
N.S.	1	1.03	0.93	0.88	0.00	2.19	0.00	5.72	3.50	0.00
time (sec)	N/A	0.656	1.345	0.319	0.000	0.249	0.000	0.202	12.366	0.000

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	498	657	674	0	528	0	0	435	0
N.S.	1	1.01	1.33	1.37	0.00	1.07	0.00	0.00	0.88	0.00
time (sec)	N/A	1.232	11.441	3.700	0.000	0.082	0.000	0.000	0.258	0.000

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	436	602	545	0	472	0	0	323	0
N.S.	1	0.98	1.35	1.22	0.00	1.06	0.00	0.00	0.73	0.00
time (sec)	N/A	0.968	10.968	4.046	0.000	0.082	0.000	0.000	0.246	0.000

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	383	533	471	0	396	0	0	231	0
N.S.	1	1.01	1.40	1.24	0.00	1.04	0.00	0.00	0.61	0.00
time (sec)	N/A	0.833	9.705	2.910	0.000	0.077	0.000	0.000	0.227	0.000

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	359	505	405	0	0	0	0	200	0
N.S.	1	0.99	1.40	1.12	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.791	10.833	2.498	0.000	0.000	0.000	0.000	0.249	0.000

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	355	473	428	0	0	0	0	139	0
N.S.	1	1.01	1.34	1.21	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.755	10.606	3.470	0.000	0.000	0.000	0.000	0.258	0.000

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	401	527	424	0	0	0	0	215	0
N.S.	1	1.10	1.44	1.16	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.924	10.881	3.217	0.000	0.000	0.000	0.000	0.344	0.000

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	455	572	495	0	403	0	0	165	0
N.S.	1	1.11	1.39	1.20	0.00	0.98	0.00	0.00	0.40	0.00
time (sec)	N/A	1.081	11.066	4.322	0.000	0.083	0.000	0.000	0.474	0.000

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	133	101	91	0	241	0	101	1548	0
N.S.	1	1.10	0.83	0.75	0.00	1.99	0.00	0.83	12.79	0.00
time (sec)	N/A	0.464	0.218	0.198	0.000	0.080	0.000	0.121	0.213	0.000

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	114	91	75	0	203	0	80	862	0
N.S.	1	1.10	0.88	0.72	0.00	1.95	0.00	0.77	8.29	0.00
time (sec)	N/A	0.445	0.189	0.160	0.000	0.080	0.000	0.111	0.191	0.000

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	69	68	56	0	161	0	59	249	55
N.S.	1	1.01	1.00	0.82	0.00	2.37	0.00	0.87	3.66	0.81
time (sec)	N/A	0.359	0.126	0.130	0.000	0.077	0.000	0.118	0.185	17.778

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	35	0	118	0	38	47	34
N.S.	1	1.00	0.95	0.81	0.00	2.74	0.00	0.88	1.09	0.79
time (sec)	N/A	0.285	0.078	0.100	0.000	0.076	0.000	0.115	0.183	18.450

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	39	0	124	0	38	131	44
N.S.	1	1.00	0.93	0.89	0.00	2.82	0.00	0.86	2.98	1.00
time (sec)	N/A	0.297	0.077	0.101	0.000	0.078	0.000	0.114	0.194	17.692

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	74	72	63	0	179	0	114	21	56
N.S.	1	1.03	1.00	0.88	0.00	2.49	0.00	1.58	0.29	0.78
time (sec)	N/A	0.359	0.148	0.141	0.000	0.083	0.000	0.120	0.228	17.748

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	118	91	82	0	221	0	221	21	0
N.S.	1	1.09	0.84	0.76	0.00	2.05	0.00	2.05	0.19	0.00
time (sec)	N/A	0.449	0.242	0.156	0.000	0.094	0.000	0.128	0.248	0.000

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	163	110	101	0	265	0	335	0	0
N.S.	1	1.12	0.76	0.70	0.00	1.83	0.00	2.31	0.00	0.00
time (sec)	N/A	0.600	0.352	0.203	0.000	0.101	0.000	0.132	24.332	0.000

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	317	444	388	0	305	0	0	90	0
N.S.	1	1.01	1.42	1.24	0.00	0.97	0.00	0.00	0.29	0.00
time (sec)	N/A	0.678	10.611	2.830	0.000	0.080	0.000	0.000	0.173	0.000

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	270	278	216	0	273	0	0	33	0
N.S.	1	1.01	1.04	0.81	0.00	1.02	0.00	0.00	0.12	0.00
time (sec)	N/A	0.536	10.089	0.962	0.000	0.073	0.000	0.000	0.162	0.000

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	186	144	0	121	0	0	30	0
N.S.	1	1.00	1.63	1.26	0.00	1.06	0.00	0.00	0.26	0.00
time (sec)	N/A	0.321	0.052	0.645	0.000	0.080	0.000	0.000	0.168	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	300	298	239	0	269	0	0	34	0
N.S.	1	1.02	1.01	0.81	0.00	0.91	0.00	0.00	0.12	0.00
time (sec)	N/A	0.641	10.329	1.310	0.000	0.074	0.000	0.000	0.159	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	350	459	399	0	306	0	0	34	0
N.S.	1	1.10	1.45	1.26	0.00	0.97	0.00	0.00	0.11	0.00
time (sec)	N/A	0.766	10.592	2.845	0.000	0.079	0.000	0.000	0.162	0.000

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	136	107	93	153	249	0	112	178	0
N.S.	1	1.10	0.86	0.75	1.23	2.01	0.00	0.90	1.44	0.00
time (sec)	N/A	0.470	0.351	0.663	0.119	0.176	0.000	0.119	0.159	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	117	91	77	105	211	0	91	101	0
N.S.	1	1.09	0.85	0.72	0.98	1.97	0.00	0.85	0.94	0.00
time (sec)	N/A	0.443	0.256	0.190	0.111	0.081	0.000	0.122	0.171	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	72	72	57	50	169	0	70	49	62
N.S.	1	1.03	1.03	0.81	0.71	2.41	0.00	1.00	0.70	0.89
time (sec)	N/A	0.348	0.175	0.227	0.153	0.080	0.000	0.113	0.157	18.430

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	35	28	124	0	45	28	40
N.S.	1	1.00	0.98	0.80	0.64	2.82	0.00	1.02	0.64	0.91
time (sec)	N/A	0.289	0.124	0.123	0.111	0.072	0.000	0.123	0.152	19.439

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	45	36	129	0	36	40	52
N.S.	1	1.00	0.94	0.96	0.77	2.74	0.00	0.77	0.85	1.11
time (sec)	N/A	0.301	0.094	0.140	0.113	0.074	0.000	0.118	0.166	17.747

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	76	74	62	188	0	111	23	64
N.S.	1	1.01	0.99	0.96	0.81	2.44	0.00	1.44	0.30	0.83
time (sec)	N/A	0.357	0.141	0.151	0.123	0.088	0.000	0.121	0.182	17.802

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	125	95	107	126	230	0	224	23	0
N.S.	1	1.09	0.83	0.93	1.10	2.00	0.00	1.95	0.20	0.00
time (sec)	N/A	0.477	0.209	0.170	0.114	0.093	0.000	0.127	0.200	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	171	114	119	179	272	0	344	23	0
N.S.	1	1.11	0.74	0.77	1.16	1.77	0.00	2.23	0.15	0.00
time (sec)	N/A	0.604	0.318	0.207	0.121	0.103	0.000	0.139	0.295	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	337	459	391	0	313	0	0	95	0
N.S.	1	0.82	1.12	0.96	0.00	0.77	0.00	0.00	0.23	0.00
time (sec)	N/A	1.012	10.530	2.862	0.000	0.079	0.000	0.000	0.211	0.000

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	303	271	217	0	282	0	0	35	0
N.S.	1	0.80	0.72	0.58	0.00	0.75	0.00	0.00	0.09	0.00
time (sec)	N/A	0.786	10.086	0.944	0.000	0.074	0.000	0.000	0.163	0.000

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	177	145	0	127	0	0	32	0
N.S.	1	1.00	1.05	0.86	0.00	0.75	0.00	0.00	0.19	0.00
time (sec)	N/A	0.440	0.049	0.646	0.000	0.067	0.000	0.000	0.166	0.000

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	334	283	241	0	274	0	0	36	0
N.S.	1	0.82	0.69	0.59	0.00	0.67	0.00	0.00	0.09	0.00
time (sec)	N/A	0.862	10.295	1.314	0.000	0.077	0.000	0.000	0.181	0.000

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	372	472	402	0	312	0	0	36	0
N.S.	1	0.84	1.06	0.90	0.00	0.70	0.00	0.00	0.08	0.00
time (sec)	N/A	0.940	10.498	2.880	0.000	0.095	0.000	0.000	0.173	0.000

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	206	166	183	0	591	0	213	5905	0
N.S.	1	1.08	0.87	0.96	0.00	3.11	0.00	1.12	31.08	0.00
time (sec)	N/A	0.699	0.609	0.296	0.000	0.125	0.000	0.129	0.317	0.000

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	150	131	170	0	459	0	152	3725	0
N.S.	1	1.12	0.98	1.27	0.00	3.43	0.00	1.13	27.80	0.00
time (sec)	N/A	0.499	0.439	0.268	0.000	0.118	0.000	0.122	0.217	0.000

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	130	94	106	0	387	0	99	1887	84
N.S.	1	1.13	0.82	0.92	0.00	3.37	0.00	0.86	16.41	0.73
time (sec)	N/A	0.457	0.363	0.190	0.000	0.101	0.000	0.135	0.207	17.367

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	38	0	67	0	44	605	37
N.S.	1	1.00	1.00	1.06	0.00	1.86	0.00	1.22	16.81	1.03
time (sec)	N/A	0.299	0.222	0.121	0.000	0.080	0.000	0.126	0.177	17.180

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	36	0	67	0	45	216	35
N.S.	1	1.00	1.03	1.00	0.00	1.86	0.00	1.25	6.00	0.97
time (sec)	N/A	0.282	0.262	0.110	0.000	0.081	0.000	0.111	0.175	17.942

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	92	91	98	0	389	0	110	887	0
N.S.	1	1.03	1.02	1.10	0.00	4.37	0.00	1.24	9.97	0.00
time (sec)	N/A	0.401	0.381	0.177	0.000	0.106	0.000	0.121	0.195	0.000

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	155	127	146	0	485	0	200	59	0
N.S.	1	1.12	0.91	1.05	0.00	3.49	0.00	1.44	0.42	0.00
time (sec)	N/A	0.582	0.504	0.291	0.000	0.123	0.000	0.130	0.320	0.000

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	209	166	184	0	615	0	350	59	0
N.S.	1	1.07	0.85	0.94	0.00	3.15	0.00	1.79	0.30	0.00
time (sec)	N/A	0.704	0.687	0.312	0.000	0.169	0.000	0.140	0.613	0.000

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	386	489	482	0	616	0	0	451	0
N.S.	1	1.01	1.27	1.26	0.00	1.60	0.00	0.00	1.17	0.00
time (sec)	N/A	0.857	10.925	1.635	0.000	0.084	0.000	0.000	0.268	0.000

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	336	452	450	0	449	0	0	222	0
N.S.	1	0.98	1.32	1.32	0.00	1.31	0.00	0.00	0.65	0.00
time (sec)	N/A	0.686	10.577	1.390	0.000	0.084	0.000	0.000	0.259	0.000

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	333	437	446	0	450	0	0	60	0
N.S.	1	0.98	1.28	1.31	0.00	1.32	0.00	0.00	0.18	0.00
time (sec)	N/A	0.686	10.562	0.980	0.000	0.081	0.000	0.000	0.180	0.000

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	348	456	481	0	452	0	0	57	0
N.S.	1	0.99	1.29	1.36	0.00	1.28	0.00	0.00	0.16	0.00
time (sec)	N/A	0.720	0.528	0.662	0.000	0.080	0.000	0.000	0.234	0.000

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	401	515	536	0	611	0	0	61	0
N.S.	1	1.04	1.34	1.39	0.00	1.59	0.00	0.00	0.16	0.00
time (sec)	N/A	0.905	10.888	6.161	0.000	0.089	0.000	0.000	0.184	0.000

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	32	34	30	0	48	22	33
N.S.	1	1.00	0.68	0.64	0.68	0.60	0.00	0.96	0.44	0.66
time (sec)	N/A	0.297	0.008	0.666	0.043	0.070	0.000	0.104	0.186	17.882

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	83	63	52	114	0	59	41	53
N.S.	1	0.98	1.43	1.09	0.90	1.97	0.00	1.02	0.71	0.91
time (sec)	N/A	0.340	0.029	0.170	0.048	0.073	0.000	0.113	0.173	17.642

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	13	20	0	28	12	20
N.S.	1	1.00	1.00	0.95	0.59	0.91	0.00	1.27	0.55	0.91
time (sec)	N/A	0.253	0.004	0.566	0.041	0.073	0.000	0.107	0.175	18.189

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	52	42	32	74	0	38	25	33
N.S.	1	1.00	1.68	1.35	1.03	2.39	0.00	1.23	0.81	1.06
time (sec)	N/A	0.278	0.006	0.156	0.036	0.070	0.000	0.117	0.162	18.436

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	52	50	0	75	0	46	53	0
N.S.	1	1.00	1.73	1.67	0.00	2.50	0.00	1.53	1.77	0.00
time (sec)	N/A	0.249	0.006	0.510	0.000	0.071	0.000	0.110	0.171	0.000

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	21	0	34	23	21
N.S.	1	1.00	1.00	0.96	0.91	0.91	0.00	1.48	1.00	0.91
time (sec)	N/A	0.246	0.005	0.158	0.034	0.065	0.000	0.109	0.186	17.347

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	76	73	0	128	0	52	79	76
N.S.	1	1.00	1.29	1.24	0.00	2.17	0.00	0.88	1.34	1.29
time (sec)	N/A	0.305	0.014	0.585	0.000	0.071	0.000	0.120	0.172	17.360

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	30	44	31	0	59	42	29
N.S.	1	1.00	0.67	0.58	0.85	0.60	0.00	1.13	0.81	0.56
time (sec)	N/A	0.296	0.009	0.158	0.036	0.072	0.000	0.122	0.190	17.464

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	93	91	94	0	158	0	79	101	0
N.S.	1	1.07	1.05	1.08	0.00	1.82	0.00	0.91	1.16	0.00
time (sec)	N/A	0.365	0.016	0.683	0.000	0.080	0.000	0.120	0.178	0.000

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	62	91	0	45	37	0	39	0
N.S.	1	1.00	0.57	0.84	0.00	0.42	0.34	0.00	0.36	0.00
time (sec)	N/A	0.321	10.025	0.941	0.000	0.073	0.414	0.000	0.261	0.000

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	22	14	43	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.22	0.78	2.39	0.78
time (sec)	N/A	0.237	0.013	0.375	0.028	0.063	0.086	0.107	0.224	18.280

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	213	51	97	0	74	37	0	23	0
N.S.	1	1.01	0.24	0.46	0.00	0.35	0.18	0.00	0.11	0.00
time (sec)	N/A	0.468	10.015	0.648	0.000	0.071	0.397	0.000	0.229	0.000

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	31	23	45	66	20	25	27	0
N.S.	1	1.00	1.03	0.77	1.50	2.20	0.67	0.83	0.90	0.00
time (sec)	N/A	0.258	0.212	0.521	0.105	0.075	0.473	0.111	0.224	0.000

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	29	36	0	20	37
N.S.	1	1.00	0.84	0.80	0.00	0.33	0.41	0.00	0.23	0.42
time (sec)	N/A	0.284	10.027	0.547	0.000	0.069	0.382	0.000	0.231	18.453

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	37	60	22	23	58	19
N.S.	1	1.00	1.00	0.74	1.37	2.22	0.81	0.85	2.15	0.70
time (sec)	N/A	0.261	0.022	0.381	0.113	0.070	0.488	0.106	0.225	18.847

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	238	49	115	0	71	39	0	24	40
N.S.	1	1.03	0.21	0.50	0.00	0.31	0.17	0.00	0.10	0.17
time (sec)	N/A	0.517	10.010	0.868	0.000	0.067	0.430	0.000	0.228	18.899

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	31	50	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.95	1.48	2.38	0.81
time (sec)	N/A	0.236	0.116	0.577	0.027	0.059	0.346	0.115	0.226	18.156

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	51	93	0	48	41	0	24	0
N.S.	1	1.00	0.46	0.85	0.00	0.44	0.37	0.00	0.22	0.00
time (sec)	N/A	0.319	10.010	0.944	0.000	0.185	0.486	0.000	0.224	0.000

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	79	71	51	51	124	95	54	61	0
N.S.	1	1.08	0.97	0.70	0.70	1.70	1.30	0.74	0.84	0.00
time (sec)	N/A	0.312	0.170	0.188	0.032	0.078	2.177	0.113	0.236	0.000

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	27	24	33	23	44	30	22	24
N.S.	1	1.11	0.75	0.67	0.92	0.64	1.22	0.83	0.61	0.67
time (sec)	N/A	0.290	0.020	0.144	0.030	0.072	0.146	0.098	0.202	17.937

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	57	39	31	93	42	40	41	56
N.S.	1	1.00	1.16	0.80	0.63	1.90	0.86	0.82	0.84	1.14
time (sec)	N/A	0.270	0.017	0.177	0.028	0.079	1.031	0.112	0.173	17.938

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	20	13	12	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.80	0.87
time (sec)	N/A	0.220	0.002	0.132	0.025	0.065	0.064	0.096	0.173	17.921

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	37	25	20
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	1.48	1.00	0.80
time (sec)	N/A	0.238	0.006	0.139	0.027	0.074	0.469	0.112	0.180	0.128

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	17	63	19	22	53	19
N.S.	1	1.00	1.00	0.80	0.68	2.52	0.76	0.88	2.12	0.76
time (sec)	N/A	0.257	0.019	0.149	0.027	0.073	0.479	0.111	0.170	18.187

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	19	30	23	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	1.00	1.58	1.21	0.89
time (sec)	N/A	0.229	0.035	0.145	0.033	0.068	0.332	0.113	0.184	0.043

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	49	50	43	36	108	42	48	79	38
N.S.	1	0.98	1.00	0.86	0.72	2.16	0.84	0.96	1.58	0.76
time (sec)	N/A	0.272	0.066	0.178	0.026	0.077	1.065	0.101	0.186	17.641

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	26	36	27	46	55	42	25
N.S.	1	1.00	0.70	0.59	0.82	0.61	1.05	1.25	0.95	0.57
time (sec)	N/A	0.266	0.041	0.154	0.025	0.073	0.421	0.108	0.182	17.982

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	8	10	10
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.50	0.62	0.62
time (sec)	N/A	0.229	0.002	0.308	0.030	0.065	0.185	0.100	0.175	17.463

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	15	12	8	10	10
N.S.	1	1.00	1.00	0.77	0.69	1.15	0.92	0.62	0.77	0.77
time (sec)	N/A	0.234	0.001	0.255	0.028	0.058	0.179	0.112	0.171	17.287

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	12	15	15	8	10	10
N.S.	1	1.00	1.14	0.93	0.86	1.07	1.07	0.57	0.71	0.71
time (sec)	N/A	0.232	0.003	0.261	0.034	0.059	0.240	0.108	0.168	17.243

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	13	12	13	12	8	10	0
N.S.	1	1.00	0.94	0.76	0.71	0.76	0.71	0.47	0.59	0.00
time (sec)	N/A	0.226	0.001	0.316	0.026	0.060	0.213	0.108	0.167	0.000

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	14	10	5	7	0
N.S.	1	1.00	1.00	0.92	0.85	1.08	0.77	0.38	0.54	0.00
time (sec)	N/A	0.219	0.011	0.294	0.036	0.056	0.177	0.104	0.171	0.000

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	13	16	14	14	8	0
N.S.	1	1.00	1.00	0.80	0.87	1.07	0.93	0.93	0.53	0.00
time (sec)	N/A	0.218	0.001	0.303	0.032	0.060	0.231	0.112	0.182	0.000

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	15	10	8	10	13
N.S.	1	1.00	1.00	0.92	0.83	1.25	0.83	0.67	0.83	1.08
time (sec)	N/A	0.219	0.001	0.300	0.030	0.058	0.169	0.101	0.173	17.311

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	15	14	8	10	13
N.S.	1	1.00	1.00	0.81	0.75	0.94	0.88	0.50	0.62	0.81
time (sec)	N/A	0.236	0.001	0.302	0.025	0.057	0.190	0.102	0.164	17.270

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	12	15	15	8	10	13
N.S.	1	1.00	1.06	0.81	0.75	0.94	0.94	0.50	0.62	0.81
time (sec)	N/A	0.226	0.002	0.307	0.028	0.061	0.200	0.109	0.179	17.643

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	13	12	15	15	8	10	13
N.S.	1	1.00	0.94	0.81	0.75	0.94	0.94	0.50	0.62	0.81
time (sec)	N/A	0.215	0.002	0.322	0.026	0.055	0.214	0.103	0.171	17.845

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	8	10	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.83	0.67
time (sec)	N/A	0.201	0.001	0.050	0.030	0.062	0.017	0.109	0.175	0.018

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	8	10	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.83	0.67
time (sec)	N/A	0.203	0.001	0.046	0.029	0.062	0.017	0.106	0.171	0.030

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	8	10	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.83	0.67
time (sec)	N/A	0.203	0.000	0.042	0.025	0.060	0.017	0.107	0.160	0.014

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	8	8	10	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.67	0.67	0.83	0.67
time (sec)	N/A	0.203	0.000	0.038	0.025	0.059	0.017	0.131	0.174	0.019

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	5	5	7	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.71	0.71	1.00	0.71
time (sec)	N/A	0.201	0.000	0.030	0.029	0.061	0.029	0.104	0.164	0.002

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	7	8	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.88	1.00	0.75
time (sec)	N/A	0.203	0.000	0.043	0.031	0.063	0.018	0.107	0.170	17.537

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	10	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	1.00	0.80
time (sec)	N/A	0.208	0.000	0.045	0.031	0.060	0.020	0.109	0.172	0.029

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	12	8	10	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	1.00	0.67	0.83	0.67
time (sec)	N/A	0.205	0.000	0.046	0.029	0.062	0.019	0.104	0.164	17.724

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	12	8	10	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	1.00	0.67	0.83	0.67
time (sec)	N/A	0.209	0.000	0.046	0.025	0.059	0.020	0.101	0.170	0.034

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	365	0	0	0	0	0	166	0
N.S.	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	1.13	0.00
time (sec)	N/A	0.567	11.389	0.000	0.000	0.000	0.000	0.000	1.569	0.000

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	342	0	0	0	0	0	94	0
N.S.	1	1.00	2.33	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.479	11.313	0.000	0.000	0.000	0.000	0.000	0.855	0.000

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	342	0	0	0	0	0	98	0
N.S.	1	1.00	2.36	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.473	11.285	0.000	0.000	0.000	0.000	0.000	1.070	0.000

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	345	0	0	0	0	0	105	0
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.478	11.293	0.000	0.000	0.000	0.000	0.000	1.048	0.000

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	459	0	0	0	0	0	355	0
N.S.	1	1.00	3.10	0.00	0.00	0.00	0.00	0.00	2.40	0.00
time (sec)	N/A	0.482	11.619	0.000	0.000	0.000	0.000	0.000	2.919	0.000

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	417	0	0	0	0	0	250	0
N.S.	1	1.00	2.82	0.00	0.00	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.486	11.529	0.000	0.000	0.000	0.000	0.000	1.750	0.000

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	415	0	0	0	0	0	186	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.492	11.515	0.000	0.000	0.000	0.000	0.000	2.156	0.000

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	384	0	0	0	0	0	192	0
N.S.	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	0.469	11.518	0.000	0.000	0.000	0.000	0.000	2.176	0.000

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	173	0	0	0	0	0	37	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.495	11.083	0.000	0.000	0.000	0.000	0.000	1.256	0.000

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	173	0	0	0	0	0	35	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.480	11.088	0.000	0.000	0.000	0.000	0.000	0.293	0.000

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	171	0	0	0	0	0	40	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.476	11.073	0.000	0.000	0.000	0.000	0.000	0.444	0.000

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	348	0	0	0	0	0	42	0
N.S.	1	1.00	2.40	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.492	11.368	0.000	0.000	0.000	0.000	0.000	0.682	0.000

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	348	0	0	0	0	0	64	0
N.S.	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.482	11.324	0.000	0.000	0.000	0.000	0.000	5.568	0.000

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	367	0	0	0	0	0	62	0
N.S.	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.487	11.418	0.000	0.000	0.000	0.000	0.000	5.992	0.000

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	366	0	0	0	0	0	76	0
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.483	11.408	0.000	0.000	0.000	0.000	0.000	8.354	0.000

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	409	0	0	0	0	0	456	0
N.S.	1	1.00	2.76	0.00	0.00	0.00	0.00	0.00	3.08	0.00
time (sec)	N/A	0.479	11.536	0.000	0.000	0.000	0.000	0.000	10.180	0.000

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	111	782	195	594	4332	1132	782	546
N.S.	1	1.00	0.71	5.01	1.25	3.81	27.77	7.26	5.01	3.50
time (sec)	N/A	0.548	0.444	0.300	0.057	0.082	0.892	0.138	0.167	17.937

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	301	110	241	1435	449	301	260
N.S.	1	1.00	0.69	2.98	1.09	2.39	14.21	4.45	2.98	2.57
time (sec)	N/A	0.426	0.127	0.140	0.041	0.073	0.510	0.111	0.183	17.837

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	51	50	71	299	119	78	89
N.S.	1	1.00	0.67	0.98	0.96	1.37	5.75	2.29	1.50	1.71
time (sec)	N/A	0.324	0.044	0.074	0.036	0.074	0.279	0.110	0.175	17.653

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	82	0	0	0	0	0	24	0
N.S.	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.549	0.165	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	315	322	78	0	0	0	0	0	51	0
N.S.	1	1.02	0.25	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.039	0.209	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	357	0	0	0	0	0	0	0
N.S.	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.517	1.737	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	255	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	1.62	0.00
time (sec)	N/A	0.478	1.564	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	37	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.499	1.805	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	221	0	0	0	0	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.505	10.870	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	257	269	162	0	0	0	0	0	0	0
N.S.	1	1.05	0.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.693	0.466	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	222	162	0	0	0	0	0	1573	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	7.05	0.00
time (sec)	N/A	0.564	0.426	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	162	162	0	0	0	0	0	448	0
N.S.	1	1.01	1.01	0.00	0.00	0.00	0.00	0.00	2.80	0.00
time (sec)	N/A	0.417	0.275	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	135	0	0	0	0	0	283	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	2.25	0.00
time (sec)	N/A	0.370	0.124	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	154	152	0	0	0	0	0	95	0
N.S.	1	1.01	1.00	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.537	0.219	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	164	163	0	0	0	0	0	562	0
N.S.	1	0.99	0.98	0.00	0.00	0.00	0.00	0.00	3.39	0.00
time (sec)	N/A	0.492	0.273	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	159	0	0	0	0	0	270	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	1.65	0.00
time (sec)	N/A	0.511	0.295	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.464	0.423	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	1022	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	7.41	0.00
time (sec)	N/A	0.442	0.408	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	247	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	1.86	0.00
time (sec)	N/A	0.405	0.147	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	164	0	0	0	0	0	280	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	2.06	0.00
time (sec)	N/A	0.440	0.415	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	278	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	2.01	0.00
time (sec)	N/A	0.436	0.416	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	179	0	0	0	0	0	473	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	3.05	0.00
time (sec)	N/A	0.469	0.248	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.598	0.000

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	269	0	0	0	0	0	0	0	694	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.58	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.316	0.000

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	181	187	0	0	0	0	0	456	0
N.S.	1	1.01	1.04	0.00	0.00	0.00	0.00	0.00	2.55	0.00
time (sec)	N/A	0.625	0.147	0.000	0.000	0.000	0.000	0.000	0.287	0.000

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	178	0	0	0	0	0	38	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.476	0.252	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	182	0	0	0	0	0	177	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.467	0.303	0.000	0.000	0.000	0.000	0.000	0.257	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [852] had the largest ratio of [1.142859999999999]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	15	0.200
2	A	3	3	1.00	13	0.231
3	A	1	1	1.00	11	0.091
4	A	3	3	1.06	15	0.200
5	A	2	2	1.00	15	0.133
6	A	3	3	1.00	15	0.200
7	A	3	3	1.00	15	0.200
8	A	3	3	1.00	15	0.200
9	A	3	3	1.00	15	0.200
10	A	3	3	0.89	15	0.200
11	A	3	3	1.00	15	0.200
12	A	5	4	1.13	15	0.267
13	A	5	4	1.13	17	0.235
14	A	2	2	1.00	17	0.118
15	A	5	4	1.30	17	0.235
16	A	5	4	1.04	17	0.235
17	A	5	4	1.25	17	0.235
18	A	2	2	1.00	17	0.118
19	A	5	4	1.13	17	0.235
20	A	3	3	1.00	13	0.231
21	A	3	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.00	17	0.176
23	A	3	3	1.00	17	0.176
24	A	3	3	1.00	17	0.176
25	A	3	3	1.00	17	0.176
26	A	3	3	1.00	17	0.176
27	A	5	4	1.09	17	0.235
28	A	5	4	1.12	17	0.235
29	A	2	2	1.00	17	0.118
30	A	5	4	1.10	17	0.235
31	A	5	4	1.05	17	0.235
32	A	5	4	1.02	17	0.235
33	A	5	4	1.10	17	0.235
34	A	2	2	1.00	17	0.118
35	A	5	4	1.10	17	0.235
36	A	5	4	1.09	17	0.235
37	A	3	3	1.00	17	0.176
38	A	3	3	1.00	17	0.176
39	A	3	3	1.00	17	0.176
40	A	3	3	1.00	17	0.176
41	A	3	3	1.00	17	0.176
42	A	3	3	1.00	17	0.176
43	A	3	3	1.00	17	0.176
44	A	3	3	1.00	17	0.176
45	A	5	4	0.98	17	0.235
46	A	5	4	0.98	17	0.235
47	A	5	4	0.96	17	0.235
48	A	2	2	1.00	17	0.118
49	A	6	5	1.18	15	0.333
50	A	5	4	1.03	17	0.235
51	A	5	4	1.02	17	0.235
52	A	3	3	1.00	17	0.176
53	A	3	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	3	3	1.00	17	0.176
55	A	3	3	1.00	17	0.176
56	A	2	2	1.00	17	0.118
57	A	3	3	1.00	13	0.231
58	A	4	4	1.19	17	0.235
59	A	5	5	1.17	17	0.294
60	A	5	4	0.98	17	0.235
61	A	5	4	0.98	17	0.235
62	A	5	4	0.94	17	0.235
63	A	2	2	1.00	17	0.118
64	A	5	4	1.03	17	0.235
65	A	5	4	1.06	15	0.267
66	A	5	4	1.03	17	0.235
67	A	4	4	1.05	17	0.235
68	A	4	4	1.06	17	0.235
69	A	4	4	1.14	17	0.235
70	A	3	3	1.00	17	0.176
71	A	3	3	1.00	17	0.176
72	A	4	4	1.13	17	0.235
73	A	5	5	1.16	13	0.385
74	A	6	6	1.19	17	0.353
75	A	5	4	1.03	17	0.235
76	A	5	4	0.95	17	0.235
77	A	5	4	1.04	17	0.235
78	A	2	2	1.00	17	0.118
79	A	2	2	1.00	17	0.118
80	A	5	4	1.02	17	0.235
81	A	5	4	1.03	17	0.235
82	A	5	4	1.02	15	0.267
83	A	5	4	1.06	17	0.235
84	A	5	5	1.13	17	0.294
85	A	5	5	1.20	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	4	1.12	17	0.235
87	A	4	4	1.08	17	0.235
88	A	4	4	1.13	17	0.235
89	A	5	5	1.22	17	0.294
90	A	6	6	1.21	17	0.353
91	A	7	7	1.26	13	0.538
92	A	8	8	1.25	17	0.471
93	A	3	3	1.00	17	0.176
94	A	3	3	1.00	17	0.176
95	A	3	3	1.00	17	0.176
96	A	3	3	1.00	17	0.176
97	A	3	3	1.00	17	0.176
98	A	3	3	1.00	17	0.176
99	A	3	3	1.00	17	0.176
100	A	3	3	1.00	17	0.176
101	A	3	3	1.00	19	0.158
102	A	3	3	1.00	19	0.158
103	A	3	3	1.00	19	0.158
104	A	3	3	1.00	19	0.158
105	A	3	3	1.00	19	0.158
106	A	3	3	1.00	19	0.158
107	A	3	3	1.00	19	0.158
108	A	3	3	1.00	19	0.158
109	A	3	3	1.00	19	0.158
110	A	3	3	1.00	19	0.158
111	A	3	3	1.00	19	0.158
112	A	3	3	1.00	19	0.158
113	A	3	3	1.00	19	0.158
114	A	3	3	1.00	19	0.158
115	A	3	3	1.00	19	0.158
116	A	3	3	1.00	19	0.158
117	A	13	12	1.54	19	0.632

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
118	A	13	12	1.56	19	0.632
119	A	12	11	1.56	19	0.579
120	A	12	11	1.56	19	0.579
121	A	11	10	1.56	19	0.526
122	A	11	10	1.57	19	0.526
123	A	12	11	1.57	19	0.579
124	A	12	11	1.55	19	0.579
125	A	13	12	1.55	19	0.632
126	A	13	12	1.55	19	0.632
127	A	14	13	1.53	19	0.684
128	A	14	13	1.46	19	0.684
129	A	13	12	1.47	19	0.632
130	A	13	12	1.46	19	0.632
131	A	12	11	1.46	19	0.579
132	A	12	11	1.46	19	0.579
133	A	12	11	1.46	19	0.579
134	A	12	11	1.46	19	0.579
135	A	13	12	1.46	19	0.632
136	A	13	12	1.47	19	0.632
137	A	14	13	1.46	19	0.684
138	A	14	13	1.47	19	0.684
139	A	15	14	1.44	19	0.737
140	A	14	13	1.45	19	0.684
141	A	13	12	1.45	19	0.632
142	A	13	12	1.45	19	0.632
143	A	13	12	1.43	19	0.632
144	A	13	12	1.43	19	0.632
145	A	13	12	1.45	19	0.632
146	A	13	12	1.45	19	0.632
147	A	14	13	1.45	19	0.684
148	A	14	13	1.46	19	0.684
149	A	15	14	1.45	19	0.737

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
150	A	15	14	1.46	19	0.737
151	A	16	15	1.44	19	0.789
152	A	16	15	1.45	19	0.789
153	A	7	6	1.15	19	0.316
154	A	6	5	1.13	19	0.263
155	A	5	4	1.06	17	0.235
156	A	5	4	0.95	19	0.211
157	A	7	6	1.10	19	0.316
158	A	1	1	1.00	19	0.053
159	A	2	2	1.00	19	0.105
160	A	3	3	1.08	19	0.158
161	A	4	4	1.11	19	0.211
162	A	5	5	1.13	19	0.263
163	A	3	3	1.08	19	0.158
164	A	2	2	1.00	19	0.105
165	A	1	1	1.00	15	0.067
166	A	4	3	1.00	19	0.158
167	A	4	3	1.00	19	0.158
168	A	5	4	1.04	19	0.211
169	A	6	5	1.08	19	0.263
170	A	7	6	1.16	19	0.316
171	A	6	5	1.12	17	0.294
172	A	6	5	1.10	19	0.263
173	A	6	5	1.00	19	0.263
174	A	8	7	1.00	19	0.368
175	A	8	7	1.09	19	0.368
176	A	1	1	1.00	19	0.053
177	A	2	2	1.00	19	0.105
178	A	3	3	1.08	19	0.158
179	A	4	4	1.11	19	0.211
180	A	5	5	1.13	19	0.263
181	A	5	5	1.13	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	4	4	1.11	19	0.211
183	A	3	3	1.08	19	0.158
184	A	2	2	1.00	15	0.133
185	A	1	1	1.00	19	0.053
186	A	5	4	1.03	19	0.211
187	A	5	4	0.99	19	0.211
188	A	5	4	1.04	19	0.211
189	A	6	5	1.06	19	0.263
190	A	7	6	1.09	19	0.316
191	A	8	7	1.11	19	0.368
192	A	7	6	1.10	19	0.316
193	A	6	5	1.06	19	0.263
194	A	5	4	0.98	19	0.211
195	A	4	3	1.00	17	0.176
196	A	1	1	1.00	19	0.053
197	A	2	2	1.00	19	0.105
198	A	3	3	1.08	19	0.158
199	A	4	4	1.11	19	0.211
200	A	4	4	1.11	19	0.211
201	A	3	3	1.08	19	0.158
202	A	2	2	1.00	19	0.105
203	A	1	1	1.00	19	0.053
204	A	3	2	1.00	15	0.133
205	A	4	3	1.00	19	0.158
206	A	5	4	1.07	19	0.211
207	A	8	7	1.03	19	0.368
208	A	7	6	1.05	19	0.316
209	A	5	4	1.09	19	0.211
210	A	1	1	1.00	19	0.053
211	A	3	2	1.00	17	0.118
212	A	4	3	1.00	19	0.158
213	A	5	4	1.07	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
214	A	6	5	1.11	19	0.263
215	A	4	4	1.07	19	0.211
216	A	3	3	1.01	19	0.158
217	A	2	2	1.00	19	0.105
218	A	1	1	1.00	19	0.053
219	A	4	3	1.00	19	0.158
220	A	5	4	1.07	15	0.267
221	A	6	5	1.11	19	0.263
222	A	5	4	1.11	19	0.211
223	A	5	4	1.11	19	0.211
224	A	5	4	1.09	19	0.211
225	A	5	4	1.09	19	0.211
226	A	5	4	0.98	19	0.211
227	A	5	4	0.98	20	0.200
228	A	10	9	1.07	21	0.429
229	A	7	6	1.06	21	0.286
230	A	9	8	1.05	21	0.381
231	A	6	5	1.03	21	0.238
232	A	8	7	1.03	21	0.333
233	A	5	4	1.00	21	0.190
234	A	8	7	1.06	21	0.333
235	A	5	4	1.00	21	0.190
236	A	9	8	1.04	21	0.381
237	A	6	5	1.03	21	0.238
238	A	10	9	1.05	21	0.429
239	A	7	6	1.06	21	0.286
240	A	11	10	1.07	21	0.476
241	A	8	7	1.07	21	0.333
242	A	10	9	1.06	21	0.429
243	A	7	6	1.05	21	0.286
244	A	9	8	1.04	21	0.381
245	A	6	5	1.03	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
246	A	9	8	1.04	21	0.381
247	A	6	5	1.02	21	0.238
248	A	9	8	1.04	21	0.381
249	A	6	5	1.03	21	0.238
250	A	10	9	1.04	21	0.429
251	A	7	6	1.05	21	0.286
252	A	11	10	1.06	21	0.476
253	A	8	7	1.07	21	0.333
254	A	7	6	1.08	21	0.286
255	A	9	8	1.06	21	0.381
256	A	6	5	1.05	21	0.238
257	A	8	7	1.05	21	0.333
258	A	5	4	1.00	21	0.190
259	A	7	6	1.06	21	0.286
260	A	4	3	1.00	21	0.143
261	A	8	7	1.06	21	0.333
262	A	5	4	1.00	21	0.190
263	A	9	8	1.05	21	0.381
264	A	6	5	1.05	21	0.238
265	A	10	9	1.06	21	0.429
266	A	7	6	1.08	21	0.286
267	A	7	6	1.09	21	0.286
268	A	9	8	1.07	21	0.381
269	A	6	5	1.05	21	0.238
270	A	8	7	1.05	21	0.333
271	A	5	4	1.00	21	0.190
272	A	8	7	1.05	21	0.333
273	A	5	4	1.00	21	0.190
274	A	9	8	1.07	21	0.381
275	A	6	5	1.06	21	0.238
276	A	10	9	1.07	21	0.429
277	A	7	6	1.09	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	11	10	1.08	21	0.476
279	A	3	3	1.05	19	0.158
280	A	3	3	1.07	19	0.158
281	A	3	3	1.11	17	0.176
282	A	2	2	1.00	19	0.105
283	A	2	2	1.00	19	0.105
284	A	2	2	1.00	19	0.105
285	A	6	5	1.13	22	0.227
286	A	6	5	1.13	22	0.227
287	A	3	3	1.00	20	0.150
288	A	6	5	1.30	22	0.227
289	A	6	5	1.04	22	0.227
290	A	6	5	1.25	22	0.227
291	A	3	3	1.00	22	0.136
292	A	6	5	1.13	22	0.227
293	A	6	5	1.13	22	0.227
294	A	4	4	1.00	22	0.182
295	A	4	4	1.00	22	0.182
296	A	1	1	1.00	18	0.056
297	A	4	4	1.00	22	0.182
298	A	4	4	1.00	22	0.182
299	A	4	4	1.00	22	0.182
300	A	4	4	1.00	22	0.182
301	A	6	5	1.07	24	0.208
302	A	6	5	1.02	24	0.208
303	A	6	5	1.08	24	0.208
304	A	6	5	1.12	24	0.208
305	A	3	3	1.00	22	0.136
306	A	6	5	1.08	24	0.208
307	A	6	5	1.10	24	0.208
308	A	6	5	1.12	24	0.208
309	A	6	5	1.06	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	6	5	1.08	24	0.208
311	A	3	3	1.00	24	0.125
312	A	6	5	1.10	24	0.208
313	A	6	5	1.04	24	0.208
314	A	6	5	1.04	24	0.208
315	A	6	5	1.07	24	0.208
316	A	4	4	1.00	24	0.167
317	A	4	4	1.00	24	0.167
318	A	4	4	1.00	24	0.167
319	A	3	3	1.18	20	0.150
320	A	4	4	1.00	24	0.167
321	A	4	4	1.00	24	0.167
322	A	4	4	1.00	24	0.167
323	A	4	4	1.00	24	0.167
324	A	4	4	1.00	24	0.167
325	A	4	4	1.00	24	0.167
326	A	4	4	1.00	24	0.167
327	A	6	5	1.05	24	0.208
328	A	6	5	1.04	24	0.208
329	A	6	5	1.06	24	0.208
330	A	6	5	1.08	24	0.208
331	A	6	5	1.12	24	0.208
332	A	3	3	1.00	22	0.136
333	A	6	5	1.08	24	0.208
334	A	6	5	1.00	24	0.208
335	A	6	5	1.10	24	0.208
336	A	6	5	0.97	24	0.208
337	A	6	5	1.08	24	0.208
338	A	6	5	0.99	24	0.208
339	A	6	5	1.08	24	0.208
340	A	3	3	1.00	24	0.125
341	A	6	5	1.10	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	7	6	1.16	24	0.250
343	A	8	7	1.19	24	0.292
344	A	6	5	1.02	24	0.208
345	A	6	5	1.05	24	0.208
346	A	4	4	1.00	24	0.167
347	A	4	4	1.00	24	0.167
348	A	4	4	1.00	24	0.167
349	A	4	4	1.00	24	0.167
350	A	3	3	1.12	20	0.150
351	A	4	4	1.00	24	0.167
352	A	4	4	1.00	24	0.167
353	A	4	4	1.00	24	0.167
354	A	4	4	1.00	24	0.167
355	A	4	4	1.00	24	0.167
356	A	4	4	1.00	24	0.167
357	A	4	4	1.00	24	0.167
358	A	4	4	1.00	24	0.167
359	A	6	5	0.99	24	0.208
360	A	6	5	0.97	24	0.208
361	A	6	5	0.98	24	0.208
362	A	6	5	0.98	24	0.208
363	A	6	5	0.94	24	0.208
364	A	3	3	1.00	22	0.136
365	A	6	5	1.03	24	0.208
366	A	6	5	1.06	24	0.208
367	A	6	5	1.03	24	0.208
368	A	5	5	1.08	24	0.208
369	A	5	5	1.05	24	0.208
370	A	5	5	1.06	24	0.208
371	A	5	5	1.14	24	0.208
372	A	4	4	1.00	24	0.167
373	A	3	3	1.16	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
374	A	5	5	1.13	24	0.208
375	A	6	6	1.16	24	0.250
376	A	7	7	1.19	24	0.292
377	A	6	5	1.04	24	0.208
378	A	6	5	1.03	24	0.208
379	A	6	5	0.99	24	0.208
380	A	3	3	1.00	24	0.125
381	A	6	5	1.12	24	0.208
382	A	3	3	1.00	22	0.136
383	A	6	5	1.01	24	0.208
384	A	6	5	1.00	24	0.208
385	A	6	5	1.06	24	0.208
386	A	7	7	1.15	24	0.292
387	A	7	7	1.17	24	0.292
388	A	7	7	1.23	24	0.292
389	A	6	6	1.19	24	0.250
390	A	6	6	1.15	24	0.250
391	A	6	6	1.12	24	0.250
392	A	5	5	1.44	20	0.250
393	A	7	7	1.28	24	0.292
394	A	8	8	1.23	24	0.333
395	A	9	9	1.24	24	0.375
396	A	6	5	0.98	24	0.208
397	A	6	5	0.97	24	0.208
398	A	6	5	0.97	24	0.208
399	A	3	3	1.00	24	0.125
400	A	6	5	1.10	24	0.208
401	A	6	5	1.08	24	0.208
402	A	6	5	1.12	24	0.208
403	A	3	3	1.00	22	0.136
404	A	6	5	1.01	24	0.208
405	A	6	5	1.00	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
406	A	6	5	1.01	24	0.208
407	A	9	9	1.22	24	0.375
408	A	9	9	1.22	24	0.375
409	A	9	9	1.27	24	0.375
410	A	8	8	1.26	24	0.333
411	A	8	8	1.24	24	0.333
412	A	8	8	1.21	24	0.333
413	A	8	8	1.19	24	0.333
414	A	8	8	1.16	24	0.333
415	A	7	7	1.56	20	0.350
416	A	9	9	1.34	24	0.375
417	A	10	10	1.27	24	0.417
418	A	11	11	1.28	24	0.458
419	A	3	3	1.00	12	0.250
420	A	2	2	1.00	14	0.143
421	A	3	3	1.00	16	0.188
422	A	5	4	0.82	16	0.250
423	A	2	2	1.00	14	0.143
424	A	5	4	1.00	16	0.250
425	A	4	4	1.00	26	0.154
426	A	4	4	1.00	26	0.154
427	A	4	4	1.00	26	0.154
428	A	4	4	1.00	26	0.154
429	A	4	4	1.00	26	0.154
430	A	4	4	1.00	26	0.154
431	A	4	4	1.00	26	0.154
432	A	4	4	1.00	28	0.143
433	A	4	4	1.00	28	0.143
434	A	4	4	1.00	28	0.143
435	A	4	4	1.00	28	0.143
436	A	4	4	1.00	28	0.143
437	A	4	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	4	4	1.00	28	0.143
439	A	4	4	1.00	28	0.143
440	A	4	4	1.00	28	0.143
441	A	4	4	1.00	28	0.143
442	A	4	4	1.00	28	0.143
443	A	4	4	1.00	28	0.143
444	A	4	4	1.00	28	0.143
445	A	4	4	1.00	28	0.143
446	A	16	15	1.46	28	0.536
447	A	15	14	1.48	28	0.500
448	A	15	14	1.47	28	0.500
449	A	14	13	1.48	28	0.464
450	A	14	13	1.48	28	0.464
451	A	14	13	1.46	28	0.464
452	A	14	13	1.48	28	0.464
453	A	15	14	1.46	28	0.500
454	A	15	14	1.48	28	0.500
455	A	16	15	1.46	28	0.536
456	A	18	17	1.44	28	0.607
457	A	17	16	1.45	28	0.571
458	A	17	16	1.44	28	0.571
459	A	16	15	1.45	28	0.536
460	A	16	15	1.45	28	0.536
461	A	16	15	1.43	28	0.536
462	A	16	15	1.45	28	0.536
463	A	16	15	1.43	28	0.536
464	A	16	15	1.45	28	0.536
465	A	16	15	1.43	28	0.536
466	A	16	15	1.45	28	0.536
467	A	17	16	1.43	28	0.571
468	A	17	16	1.44	28	0.571
469	A	18	17	1.43	28	0.607

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	20	19	1.43	28	0.679
471	A	19	18	1.43	28	0.643
472	A	19	18	1.43	28	0.643
473	A	18	17	1.43	28	0.607
474	A	18	17	1.43	28	0.607
475	A	18	17	1.42	28	0.607
476	A	18	17	1.43	28	0.607
477	A	18	17	1.40	28	0.607
478	A	18	17	1.41	28	0.607
479	A	18	17	1.38	28	0.607
480	A	18	17	1.40	28	0.607
481	A	18	17	1.40	28	0.607
482	A	18	17	1.42	28	0.607
483	A	18	17	1.41	28	0.607
484	A	18	17	1.42	28	0.607
485	A	19	18	1.41	28	0.643
486	A	19	18	1.42	28	0.643
487	A	20	19	1.41	28	0.679
488	A	4	4	0.62	26	0.154
489	A	5	4	1.06	26	0.154
490	A	4	3	1.00	24	0.125
491	A	4	4	0.60	26	0.154
492	A	4	4	0.60	26	0.154
493	A	5	4	1.00	26	0.154
494	A	4	4	0.62	26	0.154
495	A	4	4	0.62	26	0.154
496	A	4	4	0.62	26	0.154
497	A	4	4	0.62	26	0.154
498	A	4	4	0.62	26	0.154
499	A	2	2	0.68	22	0.091
500	A	4	4	0.58	26	0.154
501	A	4	4	0.61	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	4	4	0.62	26	0.154
503	A	4	4	0.62	26	0.154
504	A	4	4	0.62	26	0.154
505	A	6	5	0.47	26	0.192
506	A	6	5	0.47	26	0.192
507	A	6	5	0.74	26	0.192
508	A	5	4	1.06	26	0.154
509	A	4	3	1.00	24	0.125
510	A	6	5	0.45	26	0.192
511	A	6	5	0.45	26	0.192
512	A	6	5	0.44	26	0.192
513	A	6	5	0.45	26	0.192
514	A	3	3	1.00	26	0.115
515	A	6	5	0.89	26	0.192
516	A	6	5	0.47	26	0.192
517	A	6	5	0.47	26	0.192
518	A	6	5	0.47	26	0.192
519	A	4	4	0.45	26	0.154
520	A	4	4	0.45	26	0.154
521	A	4	4	0.45	26	0.154
522	A	4	4	0.45	26	0.154
523	A	3	3	0.47	22	0.136
524	A	4	4	0.42	26	0.154
525	A	4	4	0.43	26	0.154
526	A	4	4	0.42	26	0.154
527	A	4	4	0.44	26	0.154
528	A	4	4	0.45	26	0.154
529	A	4	4	0.45	26	0.154
530	A	4	4	0.45	26	0.154
531	A	4	4	0.45	26	0.154
532	A	6	5	0.40	26	0.192
533	A	6	5	0.41	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
534	A	6	5	0.41	26	0.192
535	A	6	5	0.67	26	0.192
536	A	6	5	0.74	26	0.192
537	A	5	4	1.06	26	0.154
538	A	4	3	1.00	24	0.125
539	A	6	5	0.39	26	0.192
540	A	6	5	0.39	26	0.192
541	A	6	5	0.40	26	0.192
542	A	6	5	0.40	26	0.192
543	A	6	5	0.38	26	0.192
544	A	6	5	0.39	26	0.192
545	A	3	3	1.00	26	0.115
546	A	6	5	0.89	26	0.192
547	A	7	6	0.80	26	0.231
548	A	6	5	0.40	26	0.192
549	A	6	5	0.41	26	0.192
550	A	6	5	0.41	26	0.192
551	A	6	5	0.40	26	0.192
552	A	4	4	0.40	26	0.154
553	A	4	4	0.40	26	0.154
554	A	4	4	0.40	26	0.154
555	A	4	4	0.40	26	0.154
556	A	4	4	0.40	26	0.154
557	A	4	4	0.39	26	0.154
558	A	3	3	0.41	22	0.136
559	A	4	4	0.38	26	0.154
560	A	4	4	0.37	26	0.154
561	A	4	4	0.38	26	0.154
562	A	4	4	0.38	26	0.154
563	A	4	4	0.37	26	0.154
564	A	4	4	0.39	26	0.154
565	A	4	4	0.39	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	4	4	0.40	26	0.154
567	A	4	4	0.40	26	0.154
568	A	4	4	0.40	26	0.154
569	A	4	4	0.40	26	0.154
570	A	4	4	0.40	26	0.154
571	A	6	5	0.54	26	0.192
572	A	5	4	0.99	26	0.154
573	A	4	3	1.00	24	0.125
574	A	7	6	0.69	26	0.231
575	A	6	5	0.52	26	0.192
576	A	4	4	0.56	26	0.154
577	A	4	4	0.69	26	0.154
578	A	2	2	1.00	22	0.091
579	A	4	4	0.70	26	0.154
580	A	5	5	0.61	26	0.192
581	A	6	5	0.58	26	0.192
582	A	6	5	0.71	26	0.192
583	A	4	3	1.73	26	0.115
584	A	3	2	1.00	24	0.083
585	A	6	5	0.57	26	0.192
586	A	6	5	0.52	26	0.192
587	A	6	6	0.70	26	0.231
588	A	5	5	0.80	26	0.192
589	A	5	5	0.78	26	0.192
590	A	4	4	0.89	22	0.182
591	A	6	6	0.72	26	0.231
592	A	7	7	0.65	26	0.269
593	A	6	5	0.53	26	0.192
594	A	6	5	0.59	26	0.192
595	A	3	3	1.00	26	0.115
596	A	6	5	1.16	26	0.192
597	A	4	3	1.06	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
598	A	3	2	1.00	24	0.083
599	A	6	5	0.52	26	0.192
600	A	6	5	0.49	26	0.192
601	A	8	8	0.68	26	0.308
602	A	7	7	0.74	26	0.269
603	A	7	7	0.73	26	0.269
604	A	7	7	0.72	26	0.269
605	A	7	7	0.70	26	0.269
606	A	6	6	0.85	22	0.273
607	A	8	8	0.70	26	0.308
608	A	9	9	0.65	26	0.346
609	A	4	4	0.68	30	0.133
610	A	4	4	0.68	30	0.133
611	A	4	4	0.68	30	0.133
612	A	4	4	0.67	30	0.133
613	A	4	4	0.67	30	0.133
614	A	4	4	0.67	30	0.133
615	A	4	4	0.67	30	0.133
616	A	4	4	0.53	30	0.133
617	A	4	4	0.53	30	0.133
618	A	4	4	0.53	30	0.133
619	A	4	4	0.52	30	0.133
620	A	4	4	0.52	30	0.133
621	A	4	4	0.52	30	0.133
622	A	4	4	0.52	30	0.133
623	A	4	4	0.48	30	0.133
624	A	4	4	0.48	30	0.133
625	A	4	4	0.48	30	0.133
626	A	4	4	0.47	30	0.133
627	A	4	4	0.48	30	0.133
628	A	4	4	0.47	30	0.133
629	A	4	4	0.48	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
630	A	15	14	0.99	30	0.467
631	A	14	13	1.06	30	0.433
632	A	14	13	1.06	30	0.433
633	A	13	12	1.15	30	0.400
634	A	13	12	1.17	30	0.400
635	A	14	13	1.06	30	0.433
636	A	14	13	1.06	30	0.433
637	A	15	14	0.99	30	0.467
638	A	17	16	0.93	30	0.533
639	A	16	15	0.98	30	0.500
640	A	16	15	0.98	30	0.500
641	A	15	14	1.04	30	0.467
642	A	15	14	1.04	30	0.467
643	A	15	14	1.04	30	0.467
644	A	15	14	1.05	30	0.467
645	A	15	14	1.03	30	0.467
646	A	15	14	1.04	30	0.467
647	A	16	15	0.97	30	0.500
648	A	16	15	0.98	30	0.500
649	A	17	16	0.93	30	0.533
650	A	19	18	0.89	30	0.600
651	A	18	17	0.93	30	0.567
652	A	18	17	0.93	30	0.567
653	A	17	16	0.97	30	0.533
654	A	17	16	0.97	30	0.533
655	A	17	16	0.97	30	0.533
656	A	17	16	0.98	30	0.533
657	A	17	16	0.96	30	0.533
658	A	17	16	0.97	30	0.533
659	A	17	16	0.96	30	0.533
660	A	17	16	0.97	30	0.533
661	A	17	16	0.96	30	0.533

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
662	A	17	16	0.97	30	0.533
663	A	18	17	0.92	30	0.567
664	A	18	17	0.93	30	0.567
665	A	19	18	0.89	30	0.600
666	A	5	4	0.96	26	0.154
667	A	4	3	1.00	22	0.136
668	A	5	4	0.94	26	0.154
669	A	7	6	0.94	26	0.231
670	A	7	6	0.95	22	0.273
671	A	8	7	0.92	26	0.269
672	A	4	4	1.00	26	0.154
673	A	4	4	1.00	26	0.154
674	A	4	4	1.00	24	0.167
675	A	3	3	1.00	26	0.115
676	A	3	3	1.00	26	0.115
677	A	3	3	1.00	26	0.115
678	A	4	4	0.51	28	0.143
679	A	4	4	0.55	28	0.143
680	A	4	4	0.69	28	0.143
681	A	3	3	1.00	28	0.107
682	A	3	3	1.00	28	0.107
683	A	3	3	1.00	28	0.107
684	A	2	2	1.00	26	0.077
685	A	5	4	0.94	24	0.167
686	A	5	4	0.98	24	0.167
687	A	5	4	1.08	24	0.167
688	A	4	3	1.10	22	0.136
689	A	4	3	1.00	24	0.125
690	A	4	3	1.05	24	0.125
691	A	2	2	1.00	24	0.083
692	A	2	2	1.00	24	0.083
693	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
694	A	2	2	1.00	24	0.083
695	A	2	2	1.00	24	0.083
696	A	2	2	1.00	28	0.071
697	A	2	2	1.00	28	0.071
698	A	2	2	1.00	28	0.071
699	A	2	2	1.00	28	0.071
700	A	2	2	1.00	28	0.071
701	A	7	7	1.66	23	0.304
702	A	6	6	1.79	23	0.261
703	B	5	5	3.26	23	0.217
704	B	2	2	2.02	19	0.105
705	B	7	7	3.57	23	0.304
706	A	10	10	1.95	23	0.435
707	A	11	11	1.82	23	0.478
708	A	6	6	1.23	36	0.167
709	A	5	5	1.32	36	0.139
710	B	4	4	2.25	36	0.111
711	A	1	1	1.47	32	0.031
712	B	6	6	2.59	36	0.167
713	A	9	9	1.48	36	0.250
714	A	10	10	1.39	36	0.278
715	A	6	6	1.63	33	0.182
716	A	5	5	1.73	33	0.152
717	B	4	4	2.97	33	0.121
718	B	1	1	2.06	29	0.034
719	B	6	6	3.72	33	0.182
720	B	10	10	2.03	33	0.303
721	A	11	11	1.91	33	0.333
722	A	8	8	0.87	34	0.235
723	A	7	7	0.80	34	0.206
724	A	4	4	0.75	34	0.118
725	A	2	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	6	6	1.46	34	0.176
727	A	8	8	0.89	34	0.235
728	A	5	5	0.62	20	0.250
729	A	2	2	1.00	16	0.125
730	A	3	2	1.04	14	0.143
731	A	1	1	1.00	12	0.083
732	A	2	2	1.00	16	0.125
733	A	2	2	1.00	16	0.125
734	A	2	2	1.00	16	0.125
735	A	2	2	1.00	16	0.125
736	A	2	2	1.00	16	0.125
737	A	2	2	1.00	16	0.125
738	A	2	2	1.00	16	0.125
739	A	2	2	1.00	16	0.125
740	A	2	2	1.00	18	0.111
741	A	4	3	0.96	16	0.188
742	A	2	2	1.00	14	0.143
743	A	4	3	1.15	18	0.167
744	A	2	2	1.00	18	0.111
745	A	4	3	0.96	18	0.167
746	A	2	2	1.00	18	0.111
747	A	4	3	1.16	18	0.167
748	A	2	2	1.00	18	0.111
749	A	4	3	0.98	18	0.167
750	A	2	2	1.00	18	0.111
751	A	4	3	1.12	18	0.167
752	A	2	2	1.00	18	0.111
753	A	4	3	1.00	18	0.167
754	A	2	2	1.00	18	0.111
755	A	4	3	1.07	18	0.167
756	A	2	2	1.00	18	0.111
757	A	4	3	0.98	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
758	A	2	2	1.00	14	0.143
759	A	4	3	1.05	18	0.167
760	A	2	2	1.00	18	0.111
761	A	4	3	0.99	18	0.167
762	A	2	2	1.00	18	0.111
763	A	4	3	0.99	18	0.167
764	A	4	3	0.99	18	0.167
765	A	6	5	1.02	18	0.278
766	A	4	3	1.00	16	0.188
767	A	8	7	1.06	18	0.389
768	A	6	5	1.07	18	0.278
769	A	6	5	1.08	18	0.278
770	A	5	5	1.05	18	0.278
771	A	3	3	1.02	18	0.167
772	A	2	2	1.05	18	0.111
773	A	2	2	1.00	14	0.143
774	A	4	4	0.99	18	0.222
775	A	5	5	1.04	18	0.278
776	A	4	3	0.99	18	0.167
777	A	4	3	0.99	18	0.167
778	A	6	5	1.03	18	0.278
779	A	4	3	1.00	16	0.188
780	A	8	7	1.04	18	0.389
781	A	6	5	1.06	18	0.278
782	A	6	5	1.06	18	0.278
783	A	15	14	1.50	18	0.778
784	A	12	11	1.48	18	0.611
785	A	10	9	1.50	18	0.500
786	A	8	7	1.51	18	0.389
787	A	9	8	1.64	14	0.571
788	A	11	10	1.65	18	0.556
789	A	13	12	1.58	18	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
790	A	5	4	1.05	18	0.222
791	A	5	4	1.01	18	0.222
792	A	5	4	1.01	18	0.222
793	A	5	4	1.03	16	0.250
794	A	6	5	1.25	18	0.278
795	A	6	5	1.15	18	0.278
796	A	6	6	1.01	18	0.333
797	A	4	4	0.92	18	0.222
798	A	3	3	0.96	18	0.167
799	A	3	3	0.96	18	0.167
800	A	4	4	0.94	14	0.286
801	A	5	5	1.00	18	0.278
802	A	7	6	1.14	18	0.333
803	A	6	5	1.10	18	0.278
804	A	6	5	1.13	18	0.278
805	A	7	6	1.12	18	0.333
806	A	6	5	1.14	18	0.278
807	A	6	5	1.12	16	0.312
808	A	8	7	1.27	18	0.389
809	A	8	7	1.17	18	0.389
810	A	8	8	1.03	18	0.444
811	A	5	5	1.02	18	0.278
812	A	5	5	1.02	18	0.278
813	A	5	5	1.02	18	0.278
814	A	5	5	1.07	18	0.278
815	A	6	6	1.02	14	0.429
816	A	7	7	1.00	18	0.389
817	A	4	3	0.98	19	0.158
818	A	7	6	1.06	19	0.316
819	A	4	3	1.09	17	0.176
820	A	8	7	1.07	19	0.368
821	A	5	4	1.07	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
822	A	3	3	1.02	19	0.158
823	A	2	2	1.07	19	0.105
824	A	2	2	1.00	15	0.133
825	A	3	3	0.99	19	0.158
826	A	4	3	0.96	22	0.136
827	A	7	6	1.16	22	0.273
828	A	4	3	1.26	20	0.150
829	A	10	9	1.16	22	0.409
830	A	7	6	1.08	22	0.273
831	A	3	3	1.27	22	0.136
832	A	2	2	1.16	22	0.091
833	A	2	2	1.00	18	0.111
834	A	5	5	1.23	22	0.227
835	A	4	3	0.94	20	0.150
836	A	7	6	1.19	20	0.300
837	A	4	3	1.26	18	0.167
838	A	10	9	1.19	20	0.450
839	A	7	6	1.08	20	0.300
840	A	10	9	1.53	20	0.450
841	A	8	7	1.51	20	0.350
842	A	9	8	1.52	16	0.500
843	A	12	11	1.50	20	0.550
844	A	4	3	1.04	14	0.214
845	A	4	3	1.09	14	0.214
846	A	4	3	0.95	14	0.214
847	A	6	5	1.05	14	0.357
848	A	4	3	1.00	12	0.250
849	A	8	7	1.10	14	0.500
850	A	6	5	1.04	14	0.357
851	A	6	5	1.03	14	0.357
852	A	17	16	1.01	14	1.143
853	A	13	12	1.01	14	0.857

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
854	A	9	8	1.02	14	0.571
855	A	5	4	1.03	14	0.286
856	A	6	5	1.02	14	0.357
857	A	10	9	1.02	14	0.643
858	A	14	13	1.01	14	0.929
859	A	16	15	1.16	14	1.071
860	A	14	13	1.17	14	0.929
861	A	12	11	1.20	14	0.786
862	A	10	9	1.21	14	0.643
863	A	8	7	1.25	14	0.500
864	A	7	6	1.25	10	0.600
865	A	11	10	1.21	14	0.714
866	A	11	10	1.20	14	0.714
867	A	15	14	1.17	14	1.000
868	A	4	3	0.93	16	0.188
869	A	7	6	1.03	16	0.375
870	A	4	3	0.95	14	0.214
871	A	8	7	1.07	16	0.438
872	A	5	4	1.02	16	0.250
873	A	5	4	1.09	16	0.250
874	A	16	16	1.36	16	1.000
875	A	12	12	1.44	16	0.750
876	A	8	8	1.62	16	0.500
877	A	4	4	1.96	16	0.250
878	A	4	4	1.89	16	0.250
879	A	8	8	1.58	16	0.500
880	A	12	12	1.44	16	0.750
881	A	15	14	1.47	16	0.875
882	A	12	11	1.37	16	0.688
883	A	11	10	1.59	16	0.625
884	A	8	7	1.47	16	0.438
885	A	7	6	1.68	12	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
886	A	11	10	1.38	16	0.625
887	A	11	10	1.54	16	0.625
888	A	15	14	1.32	16	0.875
889	A	4	3	1.14	14	0.214
890	A	2	2	1.00	16	0.125
891	A	3	3	1.00	16	0.188
892	A	8	7	1.29	16	0.438
893	A	9	8	1.29	15	0.533
894	A	4	4	1.33	19	0.211
895	F	0	0	N/A	0.000	N/A
896	A	2	2	1.06	20	0.100
897	F	0	0	N/A	0.000	N/A
898	A	2	2	1.06	22	0.091
899	F	0	0	N/A	0.000	N/A
900	A	2	2	1.00	18	0.111
901	A	2	2	1.00	18	0.111
902	A	2	2	1.00	18	0.111
903	A	2	2	1.00	18	0.111
904	A	2	2	1.00	18	0.111
905	A	2	2	1.00	18	0.111
906	A	2	2	1.00	18	0.111
907	A	2	2	1.00	20	0.100
908	A	2	2	1.00	20	0.100
909	A	2	2	1.00	20	0.100
910	A	2	2	1.00	20	0.100
911	A	2	2	1.00	20	0.100
912	A	2	2	1.00	20	0.100
913	A	2	2	1.00	20	0.100
914	A	2	2	1.00	20	0.100
915	A	2	2	1.00	20	0.100
916	A	2	2	1.00	20	0.100
917	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
918	A	2	2	1.00	20	0.100
919	A	2	2	1.00	20	0.100
920	A	2	2	1.00	20	0.100
921	A	9	8	0.94	20	0.400
922	A	7	6	0.93	20	0.300
923	A	7	6	1.00	20	0.300
924	A	6	5	1.00	20	0.250
925	A	7	6	0.99	20	0.300
926	A	6	5	0.96	20	0.250
927	A	9	8	0.94	20	0.400
928	A	8	7	0.94	20	0.350
929	A	10	9	0.93	20	0.450
930	A	10	9	0.82	20	0.450
931	A	8	7	0.81	20	0.350
932	A	8	7	0.86	20	0.350
933	A	7	6	0.85	20	0.300
934	A	8	7	0.87	20	0.350
935	A	7	6	0.87	20	0.300
936	A	9	8	0.85	20	0.400
937	A	8	7	0.86	20	0.350
938	A	10	9	0.85	20	0.450
939	A	11	10	0.85	20	0.500
940	A	10	9	0.87	20	0.450
941	A	9	8	0.86	20	0.400
942	A	11	10	0.90	20	0.500
943	A	9	8	0.89	20	0.400
944	A	11	10	0.87	20	0.500
945	A	9	8	0.87	20	0.400
946	A	11	10	0.88	20	0.500
947	A	10	9	0.87	20	0.450
948	A	8	7	1.05	20	0.350
949	A	8	7	1.07	20	0.350

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
950	A	6	5	1.10	20	0.250
951	A	5	4	1.05	18	0.222
952	A	9	8	1.00	20	0.400
953	A	8	7	1.03	20	0.350
954	A	5	4	1.05	20	0.200
955	A	6	5	1.09	20	0.250
956	A	8	7	1.06	20	0.350
957	A	10	9	1.09	20	0.450
958	A	6	6	1.02	20	0.300
959	A	5	5	0.99	20	0.250
960	A	5	5	1.01	16	0.312
961	A	5	5	1.01	20	0.250
962	A	8	8	1.10	20	0.400
963	A	9	9	1.11	20	0.450
964	A	9	8	1.02	20	0.400
965	A	9	8	1.03	20	0.400
966	A	7	6	1.11	20	0.300
967	A	6	5	1.08	18	0.278
968	A	11	10	1.09	20	0.500
969	A	11	10	1.05	20	0.500
970	A	10	9	1.05	20	0.450
971	A	10	9	1.09	20	0.450
972	A	6	5	1.08	20	0.250
973	A	7	6	1.10	20	0.300
974	A	9	8	1.03	20	0.400
975	A	9	9	1.01	20	0.450
976	A	7	7	0.98	20	0.350
977	A	7	7	1.01	16	0.438
978	A	7	7	0.99	20	0.350
979	A	7	7	1.01	20	0.350
980	A	10	10	1.10	20	0.500
981	A	10	10	1.11	20	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
982	A	7	6	1.10	20	0.300
983	A	7	6	1.10	20	0.300
984	A	5	4	1.01	20	0.200
985	A	4	3	1.00	18	0.167
986	A	4	3	1.00	20	0.150
987	A	5	4	1.03	20	0.200
988	A	7	6	1.09	20	0.300
989	A	9	8	1.12	20	0.400
990	A	5	5	1.01	20	0.250
991	A	4	4	1.01	20	0.200
992	A	1	1	1.00	16	0.062
993	A	6	6	1.02	20	0.300
994	A	9	9	1.10	20	0.450
995	A	7	6	1.10	21	0.286
996	A	7	6	1.09	21	0.286
997	A	5	4	1.03	21	0.190
998	A	4	3	1.00	19	0.158
999	A	4	3	1.00	22	0.136
1000	A	5	4	1.01	22	0.182
1001	A	7	6	1.09	22	0.273
1002	A	9	8	1.11	22	0.364
1003	A	5	5	0.82	21	0.238
1004	A	4	4	0.80	21	0.190
1005	A	2	2	1.00	17	0.118
1006	A	7	7	0.82	21	0.333
1007	A	8	8	0.84	21	0.381
1008	A	9	8	1.08	20	0.400
1009	A	7	6	1.12	20	0.300
1010	A	6	5	1.13	20	0.250
1011	A	3	2	1.00	20	0.100
1012	A	3	2	1.00	18	0.111
1013	A	6	5	1.03	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1014	A	7	6	1.12	20	0.300
1015	A	9	8	1.07	20	0.400
1016	A	7	7	1.01	20	0.350
1017	A	5	5	0.98	20	0.250
1018	A	5	5	0.98	20	0.250
1019	A	6	6	0.99	16	0.375
1020	A	9	9	1.04	20	0.450
1021	A	3	3	1.00	28	0.107
1022	A	6	5	0.98	28	0.179
1023	A	2	2	1.00	28	0.071
1024	A	5	4	1.00	26	0.154
1025	A	4	3	1.00	24	0.125
1026	A	2	2	1.00	28	0.071
1027	A	5	4	1.00	28	0.143
1028	A	3	3	1.00	28	0.107
1029	A	6	5	1.07	28	0.179
1030	A	3	3	1.00	29	0.103
1031	A	2	2	1.00	29	0.069
1032	A	5	5	1.01	29	0.172
1033	A	5	4	1.00	27	0.148
1034	A	2	2	1.00	25	0.080
1035	A	5	4	1.00	29	0.138
1036	A	6	6	1.03	29	0.207
1037	A	2	2	1.00	29	0.069
1038	A	3	3	1.00	29	0.103
1039	A	6	5	1.08	29	0.172
1040	A	5	4	1.11	29	0.138
1041	A	5	4	1.00	29	0.138
1042	A	2	2	1.00	27	0.074
1043	A	4	3	1.00	25	0.120
1044	A	5	4	1.00	29	0.138
1045	A	2	2	1.00	29	0.069

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1046	A	6	5	0.98	29	0.172
1047	A	3	3	1.00	29	0.103
1048	A	4	3	1.00	23	0.130
1049	A	4	3	1.00	23	0.130
1050	A	4	3	1.00	23	0.130
1051	A	4	4	1.00	23	0.174
1052	A	3	3	1.00	23	0.130
1053	A	3	3	1.00	21	0.143
1054	A	3	3	1.00	19	0.158
1055	A	3	3	1.00	23	0.130
1056	A	3	3	1.00	23	0.130
1057	A	3	3	1.00	23	0.130
1058	A	2	2	1.00	24	0.083
1059	A	2	2	1.00	24	0.083
1060	A	2	2	1.00	24	0.083
1061	A	2	2	1.00	22	0.091
1062	A	2	2	1.00	20	0.100
1063	A	2	2	1.00	24	0.083
1064	A	2	2	1.00	24	0.083
1065	A	2	2	1.00	24	0.083
1066	A	2	2	1.00	24	0.083
1067	A	2	2	1.00	24	0.083
1068	A	2	2	1.00	24	0.083
1069	A	2	2	1.00	24	0.083
1070	A	2	2	1.00	24	0.083
1071	A	2	2	1.00	24	0.083
1072	A	2	2	1.00	24	0.083
1073	A	2	2	1.00	24	0.083
1074	A	2	2	1.00	24	0.083
1075	A	2	2	1.00	24	0.083
1076	A	2	2	1.00	24	0.083
1077	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1078	A	2	2	1.00	24	0.083
1079	A	2	2	1.00	24	0.083
1080	A	2	2	1.00	24	0.083
1081	A	2	2	1.00	24	0.083
1082	A	2	2	1.00	24	0.083
1083	A	2	2	1.00	20	0.100
1084	A	2	2	1.00	20	0.100
1085	A	2	2	1.00	18	0.111
1086	A	3	3	1.00	20	0.150
1087	A	5	5	1.02	20	0.250
1088	A	2	2	1.00	22	0.091
1089	A	2	2	1.00	22	0.091
1090	A	2	2	1.00	22	0.091
1091	A	2	2	1.00	22	0.091
1092	A	6	5	1.05	18	0.278
1093	A	6	5	1.00	18	0.278
1094	A	4	3	1.01	18	0.167
1095	A	3	2	1.00	16	0.125
1096	A	4	3	1.01	18	0.167
1097	A	4	3	0.99	18	0.167
1098	A	4	3	1.00	18	0.167
1099	A	2	2	1.00	18	0.111
1100	A	2	2	1.00	18	0.111
1101	A	2	2	1.00	14	0.143
1102	A	2	2	1.00	18	0.111
1103	A	2	2	1.00	18	0.111
1104	A	2	2	1.00	20	0.100
1105	F	0	0	N/A	0.000	N/A
1106	F	0	0	N/A	0.000	N/A
1107	A	2	2	1.01	24	0.083
1108	A	2	2	1.00	24	0.083
1109	A	2	2	1.00	24	0.083

CHAPTER 3

Listing of integrals

3.1 $\int x^2(bx^2+cx^4)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int x^2\left(bx^2+cx^4\right)\,dx=\frac{bx^5}{5}+\frac{cx^7}{7}$$

output
```
1/5*b*x^5+1/7*c*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2\left(bx^2+cx^4\right)\,dx=\frac{bx^5}{5}+\frac{cx^7}{7}$$

input
```
Integrate[x^2*(b*x^2 + c*x^4),x]
```

output
```
(b*x^5)/5 + (c*x^7)/7
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (bx^2 + cx^4) \, dx$$

$$\Big\downarrow \; 9$$

$$\int x^4 (b + cx^2) \, dx$$

$$\Big\downarrow \; 244$$

$$\int (bx^4 + cx^6) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

input
```
Int[x^2*(b*x^2 + c*x^4),x]
```

output
```
(b*x^5)/5 + (c*x^7)/7
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{5} b\, x^{5}+\frac{1}{7} c\, x^{7}$	14
norman	$\frac{1}{5} b\, x^{5}+\frac{1}{7} c\, x^{7}$	14
risch	$\frac{1}{5} b\, x^{5}+\frac{1}{7} c\, x^{7}$	14
parallelrisch	$\frac{1}{5} b\, x^{5}+\frac{1}{7} c\, x^{7}$	14
gosper	$\frac{x^{5} \left(5 c\, x^{2}+7 b \right)}{35}$	16
orering	$\frac{x^{3} \left(5 c\, x^{2}+7 b \right) \left(c\, x^{4}+b\, x^{2}\right)}{35 c\, x^{2}+35 b}$	36

input
```
int(x^2*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/5*b*x^5+1/7*c*x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(bx^2+cx^4\right)\,dx=\frac{1}{7}\,cx^7+\frac{1}{5}\,bx^5$$

input
```
integrate(x^2*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/7*c*x^7 + 1/5*b*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2\left(bx^2+cx^4\right)\,dx=\frac{bx^5}{5}+\frac{cx^7}{7}$$

input
```
integrate(x**2*(c*x**4+b*x**2),x)
```

output
```
b*x**5/5 + c*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(bx^2+cx^4\right)\,dx=\frac{1}{7}\,cx^7+\frac{1}{5}\,bx^5$$

input
```
integrate(x^2*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/7*c*x^7 + 1/5*b*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(bx^2+cx^4\right)\,dx=\frac{1}{7}\,cx^7+\frac{1}{5}\,bx^5$$

input
```
integrate(x^2*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/7*c*x^7 + 1/5*b*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 \left(bx^2 + cx^4\right) \, dx = \frac{c\,x^7}{7} + \frac{b\,x^5}{5}$$

input
```
int(x^2*(b*x^2 + c*x^4),x)
```

output
```
(b*x^5)/5 + (c*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \left(bx^2 + cx^4\right) \, dx = \frac{x^5(5c\,x^2 + 7b)}{35}$$

input
```
int(x^2*(c*x^4+b*x^2),x)
```

output
```
(x**5*(7*b + 5*c*x**2))/35
```

3.2 $\int x(bx^2 + cx^4)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int x\left(bx^2 + cx^4\right)\,dx = \frac{bx^4}{4} + \frac{cx^6}{6}$$

output
```
1/4*b*x^4+1/6*c*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x\left(bx^2 + cx^4\right)\,dx = \frac{bx^4}{4} + \frac{cx^6}{6}$$

input
```
Integrate[x*(b*x^2 + c*x^4),x]
```

output
```
(b*x^4)/4 + (c*x^6)/6
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(bx^2+cx^4)\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^3(b+cx^2)\,dx$$
$$\Big\downarrow \text{ 244}$$
$$\int (bx^3+cx^5)\,dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

input
```
Int[x*(b*x^2 + c*x^4),x]
```

output
```
(b*x^4)/4 + (c*x^6)/6
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{4} b\, x^{4}+\frac{1}{6} c\, x^{6}$	14
norman	$\frac{1}{4} b\, x^{4}+\frac{1}{6} c\, x^{6}$	14
risch	$\frac{1}{4} b\, x^{4}+\frac{1}{6} c\, x^{6}$	14
parallelrisch	$\frac{1}{4} b\, x^{4}+\frac{1}{6} c\, x^{6}$	14
gosper	$\frac{x^{4} \left(2 c\, x^{2}+3 b \right)}{12}$	16
orering	$\frac{x^{2} \left(2 c\, x^{2}+3 b \right) \left(c\, x^{4}+b\, x^{2}\right)}{12 c\, x^{2}+12 b}$	36

input
```
int(x*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*b*x^4+1/6*c*x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(bx^2+cx^4\right)\,dx=\frac{1}{6}\,cx^{6}+\frac{1}{4}\,bx^{4}$$

input
```
integrate(x*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/6*c*x^6 + 1/4*b*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x\left(bx^2+cx^4\right)\,dx=\frac{bx^4}{4}+\frac{cx^6}{6}$$

input
```
integrate(x*(c*x**4+b*x**2),x)
```

output
```
b*x**4/4 + c*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(bx^2+cx^4\right)\,dx=\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4$$

input
```
integrate(x*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/6*c*x^6 + 1/4*b*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(bx^2+cx^4\right)\,dx=\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4$$

input
```
integrate(x*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/6*c*x^6 + 1/4*b*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(bx^2+cx^4)\ dx=\frac{c\,x^6}{6}+\frac{b\,x^4}{4}$$

input
```
int(x*(b*x^2 + c*x^4),x)
```

output
```
(b*x^4)/4 + (c*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x(bx^2+cx^4)\ dx=\frac{x^4(2c\,x^2+3b)}{12}$$

input
```
int(x*(c*x^4+b*x^2),x)
```

output
```
(x**4*(3*b + 2*c*x**2))/12
```

3.3 $\int (bx^2 + cx^4) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int (bx^2 + cx^4) \, dx = \frac{bx^3}{3} + \frac{cx^5}{5}$$

output
```
1/3*b*x^3+1/5*c*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (bx^2 + cx^4) \, dx = \frac{bx^3}{3} + \frac{cx^5}{5}$$

input
```
Integrate[b*x^2 + c*x^4,x]
```

output
```
(b*x^3)/3 + (c*x^5)/5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2+cx^4\right) dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{bx^3}{3}+\frac{cx^5}{5}$$

input
```
Int[b*x^2 + c*x^4,x]
```

output
```
(b*x^3)/3 + (c*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	14
norman	$\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	14
risch	$\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	14
parallelrisch	$\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	14
parts	$\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	14
gosper	$\frac{x^{3}\left(3c\,x^{2}+5b\right)}{15}$	16
orering	$\frac{x\left(3c\,x^{2}+5b\right)\left(c\,x^{4}+b\,x^{2}\right)}{15c\,x^{2}+15b}$	34

input
```
int(c*x^4+b*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x^3+1/5*c*x^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(bx^2+cx^4\right)\,dx=\frac{1}{5}\,cx^5+\frac{1}{3}\,bx^3$$

input
```
integrate(c*x^4+b*x^2,x, algorithm="fricas")
```

output
```
1/5*c*x^5 + 1/3*b*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(bx^2 + cx^4\right) \, dx = \frac{bx^3}{3} + \frac{cx^5}{5}$$

input
```
integrate(c*x**4+b*x**2,x)
```

output
```
b*x**3/3 + c*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(bx^2 + cx^4\right) \, dx = \frac{1}{5} \, cx^5 + \frac{1}{3} \, bx^3$$

input
```
integrate(c*x^4+b*x^2,x, algorithm="maxima")
```

output
```
1/5*c*x^5 + 1/3*b*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(bx^2 + cx^4\right) \, dx = \frac{1}{5} \, cx^5 + \frac{1}{3} \, bx^3$$

input
```
integrate(c*x^4+b*x^2,x, algorithm="giac")
```

output
```
1/5*c*x^5 + 1/3*b*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (bx^2 + cx^4) \, dx = \frac{c\,x^5}{5} + \frac{b\,x^3}{3}$$

input
```
int(b*x^2 + c*x^4,x)
```

output
```
(b*x^3)/3 + (c*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int (bx^2 + cx^4) \, dx = \frac{x^3(3c\,x^2 + 5b)}{15}$$

input
```
int(c*x^4+b*x^2,x)
```

output
```
(x**3*(5*b + 3*c*x**2))/15
```

3.4 $\int \frac{bx^2+cx^4}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{bx^2+cx^4}{x} dx = \frac{(b+cx^2)^2}{4c}$$

output
```
1/4*(c*x^2+b)^2/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{bx^2+cx^4}{x} dx = \frac{bx^2}{2}+\frac{cx^4}{4}$$

input
```
Integrate[(b*x^2 + c*x^4)/x,x]
```

output
```
(b*x^2)/2 + (c*x^4)/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+cx^4}{x}\,dx$$

$$\Big\downarrow \; 9$$

$$\int x\left(b+cx^2\right)dx$$

$$\Big\downarrow \; 244$$

$$\int \left(bx+cx^3\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{bx^2}{2}+\frac{cx^4}{4}$$

input
```
Int[(b*x^2 + c*x^4)/x,x]
```

output
```
(b*x^2)/2 + (c*x^4)/4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
norman	$\frac{1}{4} c\, x^{4}+\frac{1}{2} b\, x^{2}$	14
parallelrisch	$\frac{1}{4} c\, x^{4}+\frac{1}{2} b\, x^{2}$	14
gosper	$\frac{x^{2} \left(c\, x^{2}+2 b \right)}{4}$	15
default	$\frac{\left(c\, x^{2}+b \right)^{2}}{4 c}$	15
risch	$\frac{c\, x^{4}}{4}+\frac{b\, x^{2}}{2}+\frac{b^{2}}{4 c}$	22
orering	$\frac{\left(c\, x^{2}+2 b \right) \left(c\, x^{4}+b\, x^{2}\right)}{4 c\, x^{2}+4 b}$	32

input
```
int((c*x^4+b*x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*c*x^4+1/2*b*x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{bx^2 + cx^4}{x} \, dx = \frac{1}{4} \, cx^4 + \frac{1}{2} \, bx^2$$

input
```
integrate((c*x^4+b*x^2)/x,x, algorithm="fricas")
```

output
```
1/4*c*x^4 + 1/2*b*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{bx^2 + cx^4}{x} \, dx = \frac{bx^2}{2} + \frac{cx^4}{4}$$

input
```
integrate((c*x**4+b*x**2)/x,x)
```

output
```
b*x**2/2 + c*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{bx^2 + cx^4}{x} \, dx = \frac{1}{4} \, cx^4 + \frac{1}{2} \, bx^2$$

input
```
integrate((c*x^4+b*x^2)/x,x, algorithm="maxima")
```

output
```
1/4*c*x^4 + 1/2*b*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{bx^2 + cx^4}{x} \, dx = \frac{1}{4} \, cx^4 + \frac{1}{2} \, bx^2$$

input
```
integrate((c*x^4+b*x^2)/x,x, algorithm="giac")
```

output
```
1/4*c*x^4 + 1/2*b*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{bx^2 + cx^4}{x}\,dx = \frac{c\,x^4}{4} + \frac{b\,x^2}{2}$$

input
```
int((b*x^2 + c*x^4)/x,x)
```

output
```
(b*x^2)/2 + (c*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{bx^2 + cx^4}{x}\,dx = \frac{x^2(c\,x^2 + 2b)}{4}$$

input
```
int((c*x^4+b*x^2)/x,x)
```

output
```
(x**2*(2*b + c*x**2))/4
```

3.5 $\int \frac{bx^2+cx^4}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{bx^2+cx^4}{x^2}\,dx = bx+\frac{cx^3}{3}$$

output
```
b*x+1/3*c*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{bx^2+cx^4}{x^2}\,dx = bx+\frac{cx^3}{3}$$

input
```
Integrate[(b*x^2 + c*x^4)/x^2,x]
```

output
```
b*x + (c*x^3)/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {9, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2 + cx^4}{x^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \left(b + cx^2\right) dx$$

$$\Big\downarrow \; 2009$$

$$bx + \frac{cx^3}{3}$$

input
```
Int[(b*x^2 + c*x^4)/x^2,x]
```

output
```
b*x + (c*x^3)/3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$bx+\frac{1}{3}c\,x^3$	11
risch	$bx+\frac{1}{3}c\,x^3$	11
parallelrisch	$bx+\frac{1}{3}c\,x^3$	11
parts	$bx+\frac{1}{3}c\,x^3$	11
gosper	$\frac{x(c\,x^2+3b)}{3}$	13
norman	$\frac{b\,x^2+\frac{1}{3}c\,x^4}{x}$	17
orering	$\frac{(c\,x^2+3b)(c\,x^4+b\,x^2)}{3x(c\,x^2+b)}$	35

input
```
int((c*x^4+b*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*x+1/3*c*x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{bx^2+cx^4}{x^2}\,dx=\frac{1}{3}\,cx^3+bx$$

input
```
integrate((c*x^4+b*x^2)/x^2,x, algorithm="fricas")
```

output
```
1/3*c*x^3 + b*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{bx^2+cx^4}{x^2}\,dx = bx + \frac{cx^3}{3}$$

input
```
integrate((c*x**4+b*x**2)/x**2,x)
```

output
```
b*x + c*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{bx^2+cx^4}{x^2}\,dx = \frac{1}{3}\,cx^3 + bx$$

input
```
integrate((c*x^4+b*x^2)/x^2,x, algorithm="maxima")
```

output
```
1/3*c*x^3 + b*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{bx^2+cx^4}{x^2}\,dx = \frac{1}{3}\,cx^3 + bx$$

input
```
integrate((c*x^4+b*x^2)/x^2,x, algorithm="giac")
```

output
```
1/3*c*x^3 + b*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{bx^2 + cx^4}{x^2} \, dx = \frac{c\,x^3}{3} + b\,x$$

input
```
int((b*x^2 + c*x^4)/x^2,x)
```

output
```
b*x + (c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{bx^2 + cx^4}{x^2} \, dx = \frac{x(c\,x^2 + 3b)}{3}$$

input
```
int((c*x^4+b*x^2)/x^2,x)
```

output
```
(x*(3*b + c*x**2))/3
```

3.6 $\int \frac{bx^2+cx^4}{x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{bx^2+cx^4}{x^3} \, dx = \frac{cx^2}{2} + b\log(x)$$

output
```
1/2*c*x^2+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{bx^2+cx^4}{x^3} \, dx = \frac{cx^2}{2} + b\log(x)$$

input
```
Integrate[(b*x^2 + c*x^4)/x^3,x]
```

output
```
(c*x^2)/2 + b*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+cx^4}{x^3}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{b+cx^2}{x}dx$$

$$\Big\downarrow \; 244$$

$$\int \left(\frac{b}{x}+cx\right)dx$$

$$\Big\downarrow \; 2009$$

$$b\log(x)+\frac{cx^2}{2}$$

input
```
Int[(b*x^2 + c*x^4)/x^3,x]
```

output
```
(c*x^2)/2 + b*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{c x^2}{2}+b\ln\left(x\right)$	12
norman	$\frac{c x^2}{2}+b\ln\left(x\right)$	12
risch	$\frac{c x^2}{2}+b\ln\left(x\right)$	12
parallelrisch	$\frac{c x^2}{2}+b\ln\left(x\right)$	12

input
```
int((c*x^4+b*x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*x^2+b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{bx^2+cx^4}{x^3}\,dx=\frac{1}{2}\,cx^2+b\log\left(x\right)$$

input
```
integrate((c*x^4+b*x^2)/x^3,x, algorithm="fricas")
```

output
```
1/2*c*x^2 + b*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{bx^2 + cx^4}{x^3}\,dx = b\log(x) + \frac{cx^2}{2}$$

input
```
integrate((c*x**4+b*x**2)/x**3,x)
```

output
```
b*log(x) + c*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{bx^2 + cx^4}{x^3}\,dx = \frac{1}{2}\,cx^2 + \frac{1}{2}\,b\log(x^2)$$

input
```
integrate((c*x^4+b*x^2)/x^3,x, algorithm="maxima")
```

output
```
1/2*c*x^2 + 1/2*b*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{bx^2 + cx^4}{x^3}\,dx = \frac{1}{2}\,cx^2 + \frac{1}{2}\,b\log(x^2)$$

input
```
integrate((c*x^4+b*x^2)/x^3,x, algorithm="giac")
```

output
```
1/2*c*x^2 + 1/2*b*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{bx^2 + cx^4}{x^3} \, dx = \frac{c\,x^2}{2} + b\,\ln\left(x\right)$$

input
```
int((b*x^2 + c*x^4)/x^3,x)
```

output
```
(c*x^2)/2 + b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{bx^2 + cx^4}{x^3} \, dx = \log(x)\,b + \frac{c\,x^2}{2}$$

input
```
int((c*x^4+b*x^2)/x^3,x)
```

output
```
(2*log(x)*b + c*x**2)/2
```

3.7 $\int \frac{bx^2+cx^4}{x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 10

$$\int \frac{bx^2+cx^4}{x^4} \, dx = -\frac{b}{x}+cx$$

output
```
-b/x+c*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{bx^2+cx^4}{x^4} \, dx = -\frac{b}{x}+cx$$

input
```
Integrate[(b*x^2 + c*x^4)/x^4,x]
```

output
```
-(b/x) + c*x
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+cx^4}{x^4}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{b+cx^2}{x^2}dx$$
$$\Big\downarrow \text{ 244}$$
$$\int \left(\frac{b}{x^2}+c\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$cx-\frac{b}{x}$$

input
```
Int[(b*x^2 + c*x^4)/x^4,x]
```

output
```
-(b/x) + c*x
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{b}{x}+cx$	11
risch	$-\frac{b}{x}+cx$	11
gosper	$-\frac{-c\,x^{2}+b}{x}$	14
parallelrisch	$\frac{c\,x^{2}-b}{x}$	14
norman	$\frac{c\,x^{4}-b\,x^{2}}{x^{3}}$	17
orering	$-\frac{\left(-c\,x^{2}+b\right)\left(c\,x^{4}+b\,x^{2}\right)}{x^{3}\left(c\,x^{2}+b\right)}$	34

input
```
int((c*x^4+b*x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-b/x+c*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{bx^2+cx^4}{x^4}\,dx=\frac{cx^2-b}{x}$$

input
```
integrate((c*x^4+b*x^2)/x^4,x, algorithm="fricas")
```

output
```
(c*x^2 - b)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.50

$$\int \frac{bx^2 + cx^4}{x^4} \, dx = -\frac{b}{x} + cx$$

input
```
integrate((c*x**4+b*x**2)/x**4,x)
```

output
```
-b/x + c*x
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{bx^2 + cx^4}{x^4} \, dx = cx - \frac{b}{x}$$

input
```
integrate((c*x^4+b*x^2)/x^4,x, algorithm="maxima")
```

output
```
c*x - b/x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{bx^2 + cx^4}{x^4} \, dx = cx - \frac{b}{x}$$

input
```
integrate((c*x^4+b*x^2)/x^4,x, algorithm="giac")
```

output
```
c*x - b/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{bx^2 + cx^4}{x^4} \, dx = c\,x - \frac{b}{x}$$

input
```
int((b*x^2 + c*x^4)/x^4,x)
```

output
```
c*x - b/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{bx^2 + cx^4}{x^4} \, dx = \frac{c\,x^2 - b}{x}$$

input
```
int((c*x^4+b*x^2)/x^4,x)
```

output
```
( - b + c*x**2)/x
```

3.8 $\int \frac{bx^2+cx^4}{x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{bx^2+cx^4}{x^5} \, dx = -\frac{b}{2x^2}+c\log(x)$$

output
```
-1/2*b/x^2+c*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{bx^2+cx^4}{x^5} \, dx = -\frac{b}{2x^2}+c\log(x)$$

input
```
Integrate[(b*x^2 + c*x^4)/x^5,x]
```

output
```
-1/2*b/x^2 + c*Log[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2 + cx^4}{x^5} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{b + cx^2}{x^3} dx$$

$$\Big\downarrow \; 244$$

$$\int \left(\frac{b}{x^3} + \frac{c}{x}\right) dx$$

$$\Big\downarrow \; 2009$$

$$c \log(x) - \frac{b}{2x^2}$$

input
```
Int[(b*x^2 + c*x^4)/x^5,x]
```

output
```
-1/2*b/x^2 + c*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{b}{2x^2}+c\ln\left(x\right)$	12
norman	$-\frac{b}{2x^2}+c\ln\left(x\right)$	12
risch	$-\frac{b}{2x^2}+c\ln\left(x\right)$	12
parallelrisch	$\frac{2c\ln(x)x^2-b}{2x^2}$	18

input
```
int((c*x^4+b*x^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/x^2+c*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{bx^2+cx^4}{x^5}\,dx=\frac{2\,cx^2\log\left(x\right)-b}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2)/x^5,x, algorithm="fricas")
```

output
```
1/2*(2*c*x^2*log(x) - b)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{bx^2 + cx^4}{x^5} \, dx = -\frac{b}{2x^2} + c \log(x)$$

input
```
integrate((c*x**4+b*x**2)/x**5,x)
```

output
```
-b/(2*x**2) + c*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{bx^2 + cx^4}{x^5} \, dx = \frac{1}{2} c \log(x^2) - \frac{b}{2 x^2}$$

input
```
integrate((c*x^4+b*x^2)/x^5,x, algorithm="maxima")
```

output
```
1/2*c*log(x^2) - 1/2*b/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{bx^2 + cx^4}{x^5} \, dx = \frac{1}{2} c \log(x^2) - \frac{cx^2 + b}{2 x^2}$$

input
```
integrate((c*x^4+b*x^2)/x^5,x, algorithm="giac")
```

output
```
1/2*c*log(x^2) - 1/2*(c*x^2 + b)/x^2
```

Mupad [B] (verification not implemented)

Time = 17.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{bx^2 + cx^4}{x^5}\,dx = c\,\ln(x) - \frac{b}{2\,x^2}$$

input
```
int((b*x^2 + c*x^4)/x^5,x)
```

output
```
c*log(x) - b/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{bx^2 + cx^4}{x^5}\,dx = \frac{2\log(x)\,c\,x^2 - b}{2x^2}$$

input
```
int((c*x^4+b*x^2)/x^5,x)
```

output
```
(2*log(x)*c*x**2 - b)/(2*x**2)
```

3.9 $\int \frac{bx^2+cx^4}{x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{bx^2+cx^4}{x^6} \, dx = -\frac{b}{3x^3}-\frac{c}{x}$$

output
```
-1/3*b/x^3-c/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{bx^2+cx^4}{x^6} \, dx = -\frac{b}{3x^3}-\frac{c}{x}$$

input
```
Integrate[(b*x^2 + c*x^4)/x^6,x]
```

output
```
-1/3*b/x^3 - c/x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+cx^4}{x^6}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{b+cx^2}{x^4}dx$$
$$\Big\downarrow \; 244$$
$$\int \left(\frac{b}{x^4}+\frac{c}{x^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{b}{3x^3}-\frac{c}{x}$$

input
```
Int[(b*x^2 + c*x^4)/x^6,x]
```

output
```
-1/3*b/x^3 - c/x
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{3c\,x^2+b}{3x^3}$	14
default	$-\frac{b}{3x^3}-\frac{c}{x}$	14
risch	$\frac{-c\,x^2-\frac{b}{3}}{x^3}$	15
parallelrisch	$\frac{-3c\,x^2-b}{3x^3}$	16
norman	$\frac{-\frac{1}{3}b\,x^2-c\,x^4}{x^5}$	18
orering	$-\frac{(3c\,x^2+b)(c\,x^4+b\,x^2)}{3x^5(c\,x^2+b)}$	34

input
```
int((c*x^4+b*x^2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3*(3*c*x^2+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{bx^2+cx^4}{x^6}\,dx=-\frac{3\,cx^2+b}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2)/x^6,x, algorithm="fricas")
```

output
```
-1/3*(3*c*x^2 + b)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{bx^2 + cx^4}{x^6} \, dx = \frac{-b - 3cx^2}{3x^3}$$

input
```
integrate((c*x**4+b*x**2)/x**6,x)
```

output
```
(-b - 3*c*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{bx^2 + cx^4}{x^6} \, dx = -\frac{3 \, cx^2 + b}{3 \, x^3}$$

input
```
integrate((c*x^4+b*x^2)/x^6,x, algorithm="maxima")
```

output
```
-1/3*(3*c*x^2 + b)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{bx^2 + cx^4}{x^6} \, dx = -\frac{3 \, cx^2 + b}{3 \, x^3}$$

input
```
integrate((c*x^4+b*x^2)/x^6,x, algorithm="giac")
```

output
```
-1/3*(3*c*x^2 + b)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{bx^2+cx^4}{x^6}\,dx=-\frac{3\,c\,x^2+b}{3\,x^3}$$

input
```
int((b*x^2 + c*x^4)/x^6,x)
```

output
```
-(b + 3*c*x^2)/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{bx^2+cx^4}{x^6}\,dx=\frac{-3c\,x^2-b}{3x^3}$$

input
```
int((c*x^4+b*x^2)/x^6,x)
```

output
```
( - b - 3*c*x**2)/(3*x**3)
```

3.10 $\int \frac{bx^2+cx^4}{x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{bx^2+cx^4}{x^7}\,dx = -\frac{(b+cx^2)^2}{4bx^4}$$

output
```
-1/4*(c*x^2+b)^2/b/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{bx^2+cx^4}{x^7}\,dx = -\frac{b}{4x^4} - \frac{c}{2x^2}$$

input
```
Integrate[(b*x^2 + c*x^4)/x^7,x]
```

output
```
-1/4*b/x^4 - c/(2*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+cx^4}{x^7}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{b+cx^2}{x^5}dx$$

$$\Big\downarrow \; 244$$

$$\int \left(\frac{b}{x^5}+\frac{c}{x^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

input
```
Int[(b*x^2 + c*x^4)/x^7,x]
```

output
```
-1/4*b/x^4 - c/(2*x^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
gosper	$-\frac{2c\,x^{2}+b}{4x^{4}}$	14
default	$-\frac{b}{4x^{4}}-\frac{c}{2x^{2}}$	14
risch	$\frac{-\frac{c\,x^{2}}{2}-\frac{b}{4}}{x^{4}}$	15
parallelrisch	$\frac{-2c\,x^{2}-b}{4x^{4}}$	16
norman	$\frac{-\frac{1}{4}b\,x^{2}-\frac{1}{2}c\,x^{4}}{x^{6}}$	18
orering	$-\frac{\left(2c\,x^{2}+b\right)\left(c\,x^{4}+b\,x^{2}\right)}{4x^{6}\left(c\,x^{2}+b\right)}$	34

input
```
int((c*x^4+b*x^2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4*(2*c*x^2+b)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{bx^2+cx^4}{x^7}\,dx=-\frac{2\,cx^2+b}{4\,x^4}$$

input
```
integrate((c*x^4+b*x^2)/x^7,x, algorithm="fricas")
```

output
```
-1/4*(2*c*x^2 + b)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{bx^2 + cx^4}{x^7} \, dx = \frac{-b - 2cx^2}{4x^4}$$

input
```
integrate((c*x**4+b*x**2)/x**7,x)
```

output
```
(-b - 2*c*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{bx^2 + cx^4}{x^7} \, dx = -\frac{2\,cx^2 + b}{4\,x^4}$$

input
```
integrate((c*x^4+b*x^2)/x^7,x, algorithm="maxima")
```

output
```
-1/4*(2*c*x^2 + b)/x^4
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{bx^2 + cx^4}{x^7} \, dx = -\frac{2\,cx^2 + b}{4\,x^4}$$

input
```
integrate((c*x^4+b*x^2)/x^7,x, algorithm="giac")
```

output
```
-1/4*(2*c*x^2 + b)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{bx^2+cx^4}{x^7}\,dx = -\frac{2\,c\,x^2+b}{4\,x^4}$$

input
```
int((b*x^2 + c*x^4)/x^7,x)
```

output
```
-(b + 2*c*x^2)/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{bx^2+cx^4}{x^7}\,dx = \frac{-2c\,x^2-b}{4x^4}$$

input
```
int((c*x^4+b*x^2)/x^7,x)
```

output
```
( - b - 2*c*x**2)/(4*x**4)
```

3.11 $\int \frac{bx^2+cx^4}{x^8} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{bx^2+cx^4}{x^8} \, dx = -\frac{b}{5x^5} - \frac{c}{3x^3}$$

output
```
-1/5*b/x^5-1/3*c/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{bx^2+cx^4}{x^8} \, dx = -\frac{b}{5x^5} - \frac{c}{3x^3}$$

input
```
Integrate[(b*x^2 + c*x^4)/x^8,x]
```

output
```
-1/5*b/x^5 - c/(3*x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+cx^4}{x^8}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{b+cx^2}{x^6}dx$$
$$\Big\downarrow \; 244$$
$$\int \left(\frac{b}{x^6}+\frac{c}{x^4}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{b}{5x^5}-\frac{c}{3x^3}$$

input
```
Int[(b*x^2 + c*x^4)/x^8,x]
```

output
```
-1/5*b/x^5 - c/(3*x^3)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{b}{5x^5}-\frac{c}{3x^3}$	14
risch	$\frac{-\frac{c\,x^2}{3}-\frac{b}{5}}{x^5}$	15
gosper	$-\frac{5c\,x^2+3b}{15x^5}$	16
parallelrisch	$\frac{-5c\,x^2-3b}{15x^5}$	16
norman	$\frac{-\frac{1}{5}b\,x^2-\frac{1}{3}c\,x^4}{x^7}$	18
orering	$-\frac{(5c\,x^2+3b)\,(c\,x^4+b\,x^2)}{15x^7(c\,x^2+b)}$	36

input
```
int((c*x^4+b*x^2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/5*b/x^5-1/3*c/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{bx^2+cx^4}{x^8}\,dx=-\frac{5\,cx^2+3\,b}{15\,x^5}$$

input
```
integrate((c*x^4+b*x^2)/x^8,x, algorithm="fricas")
```

output
```
-1/15*(5*c*x^2 + 3*b)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{bx^2 + cx^4}{x^8} \, dx = \frac{-3b - 5cx^2}{15x^5}$$

input
```
integrate((c*x**4+b*x**2)/x**8,x)
```

output
```
(-3*b - 5*c*x**2)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{bx^2 + cx^4}{x^8} \, dx = -\frac{5\,cx^2 + 3\,b}{15\,x^5}$$

input
```
integrate((c*x^4+b*x^2)/x^8,x, algorithm="maxima")
```

output
```
-1/15*(5*c*x^2 + 3*b)/x^5
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{bx^2 + cx^4}{x^8} \, dx = -\frac{5\,cx^2 + 3\,b}{15\,x^5}$$

input
```
integrate((c*x^4+b*x^2)/x^8,x, algorithm="giac")
```

output
```
-1/15*(5*c*x^2 + 3*b)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{bx^2 + cx^4}{x^8} \, dx = -\frac{5\,c\,x^2 + 3\,b}{15\,x^5}$$

input
```
int((b*x^2 + c*x^4)/x^8,x)
```

output
```
-(3*b + 5*c*x^2)/(15*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{bx^2 + cx^4}{x^8} \, dx = \frac{-5c\,x^2 - 3b}{15x^5}$$

input
```
int((c*x^4+b*x^2)/x^8,x)
```

output
```
( - 3*b - 5*c*x**2)/(15*x**5)
```

3.12 $\int x(bx^2 + cx^4)^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int x\left(bx^2+cx^4\right)^2 \, dx = \frac{b^2x^6}{6} + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}$$

output
```
1/6*b^2*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x\left(bx^2+cx^4\right)^2 \, dx = \frac{b^2x^6}{6} + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}$$

input
```
Integrate[x*(b*x^2 + c*x^4)^2,x]
```

output
```
(b^2*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10)/10
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \; 9$$

$$\int x^5\left(b+cx^2\right)^2 dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int x^4\left(cx^2+b\right)^2 dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(c^2x^8+2bcx^6+b^2x^4\right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{b^2x^6}{3}+\frac{1}{2}bcx^8+\frac{c^2x^{10}}{5}\right)$$

input
```
Int[x*(b*x^2 + c*x^4)^2,x]
```

output
```
((b^2*x^6)/3 + (b*c*x^8)/2 + (c^2*x^10)/5)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{6}b^2x^6+\frac{1}{4}bc\,x^8+\frac{1}{10}c^2x^{10}$	25
norman	$\frac{1}{6}b^2x^6+\frac{1}{4}bc\,x^8+\frac{1}{10}c^2x^{10}$	25
risch	$\frac{1}{6}b^2x^6+\frac{1}{4}bc\,x^8+\frac{1}{10}c^2x^{10}$	25
parallelrisch	$\frac{1}{6}b^2x^6+\frac{1}{4}bc\,x^8+\frac{1}{10}c^2x^{10}$	25
gosper	$\frac{x^6(6c^2x^4+15bc\,x^2+10b^2)}{60}$	27
orering	$\frac{x^2(6c^2x^4+15bc\,x^2+10b^2)\left(c\,x^4+b\,x^2\right)^2}{60(c\,x^2+b)^2}$	49

input
```
int(x*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(bx^2+cx^4\right)^2\,dx=\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,b^2x^6$$

input
```
integrate(x*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(bx^2+cx^4\right)^2\,dx=\frac{b^2x^6}{6}+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}$$

input
```
integrate(x*(c*x**4+b*x**2)**2,x)
```

output
```
b**2*x**6/6 + b*c*x**8/4 + c**2*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(bx^2+cx^4\right)^2\,dx=\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,b^2x^6$$

input
```
integrate(x*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(bx^2+cx^4)^2\,dx = \frac{1}{10}\,c^2x^{10} + \frac{1}{4}\,bcx^8 + \frac{1}{6}\,b^2x^6$$

input
```
integrate(x*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6
```

Mupad [B] (verification not implemented)

Time = 17.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(bx^2+cx^4)^2\,dx = \frac{b^2\,x^6}{6} + \frac{b\,c\,x^8}{4} + \frac{c^2\,x^{10}}{10}$$

input
```
int(x*(b*x^2 + c*x^4)^2,x)
```

output
```
(b^2*x^6)/6 + (c^2*x^10)/10 + (b*c*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x(bx^2+cx^4)^2\,dx = \frac{x^6(6c^2x^4+15bc\,x^2+10b^2)}{60}$$

input
```
int(x*(c*x^4+b*x^2)^2,x)
```

output
```
(x**6*(10*b**2 + 15*b*c*x**2 + 6*c**2*x**4))/60
```

3.13 $\int \frac{(bx^2+cx^4)^2}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{(bx^2+cx^4)^2}{x} dx = \frac{b^2x^4}{4} + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

output
```
1/4*b^2*x^4+1/3*b*c*x^6+1/8*c^2*x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x} dx = \frac{b^2x^4}{4} + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x,x]
```

output
```
(b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^2+cx^4\right)^2}{x}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^3\left(b+cx^2\right)^2dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int x^2\left(cx^2+b\right)^2dx^2$$
$$\Big\downarrow \text{ 49}$$
$$\frac{1}{2}\int \left(c^2x^6+2bcx^4+b^2x^2\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(\frac{b^2x^4}{2}+\frac{2}{3}bcx^6+\frac{c^2x^8}{4}\right)$$

input
```
Int[(b*x^2 + c*x^4)^2/x,x]
```

output
```
((b^2*x^4)/2 + (2*b*c*x^6)/3 + (c^2*x^8)/4)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{4}b^2x^4+\frac{1}{3}bc\,x^6+\frac{1}{8}c^2x^8$	25
norman	$\frac{1}{4}b^2x^4+\frac{1}{3}bc\,x^6+\frac{1}{8}c^2x^8$	25
risch	$\frac{1}{4}b^2x^4+\frac{1}{3}bc\,x^6+\frac{1}{8}c^2x^8$	25
parallelrisch	$\frac{1}{4}b^2x^4+\frac{1}{3}bc\,x^6+\frac{1}{8}c^2x^8$	25
gosper	$\frac{x^4(3c^2x^4+8bc\,x^2+6b^2)}{24}$	27
orering	$\frac{(3c^2x^4+8bc\,x^2+6b^2)(c\,x^4+b\,x^2)^2}{24(c\,x^2+b)^2}$	46

input
```
int((c*x^4+b*x^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*x^4+1/3*b*c*x^6+1/8*c^2*x^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2 + cx^4)^2}{x} \, dx = \frac{1}{8} \, c^2 x^8 + \frac{1}{3} \, bcx^6 + \frac{1}{4} \, b^2 x^4$$

input
```
integrate((c*x^4+b*x^2)^2/x,x, algorithm="fricas")
```

output
```
1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2 + cx^4)^2}{x} \, dx = \frac{b^2 x^4}{4} + \frac{bcx^6}{3} + \frac{c^2 x^8}{8}$$

input
```
integrate((c*x**4+b*x**2)**2/x,x)
```

output
```
b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2 + cx^4)^2}{x} \, dx = \frac{1}{8} \, c^2 x^8 + \frac{1}{3} \, bcx^6 + \frac{1}{4} \, b^2 x^4$$

input
```
integrate((c*x^4+b*x^2)^2/x,x, algorithm="maxima")
```

output
```
1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2+cx^4)^2}{x}\,dx = \frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}b^2x^4$$

input
```
integrate((c*x^4+b*x^2)^2/x,x, algorithm="giac")
```

output
```
1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2+cx^4)^2}{x}\,dx = \frac{b^2\,x^4}{4} + \frac{b\,c\,x^6}{3} + \frac{c^2\,x^8}{8}$$

input
```
int((b*x^2 + c*x^4)^2/x,x)
```

output
```
(b^2*x^4)/4 + (c^2*x^8)/8 + (b*c*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2+cx^4)^2}{x}\,dx = \frac{x^4(3c^2x^4+8bc\,x^2+6b^2)}{24}$$

input
```
int((c*x^4+b*x^2)^2/x,x)
```

output
```
(x**4*(6*b**2 + 8*b*c*x**2 + 3*c**2*x**4))/24
```

3.14 $\int \frac{(bx^2+cx^4)^2}{x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{(bx^2+cx^4)^2}{x^3} dx = \frac{(b+cx^2)^3}{6c}$$

output
```
1/6*(c*x^2+b)^3/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x^3} dx = \frac{(b+cx^2)^3}{6c}$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^3,x]
```

output
```
(b + c*x^2)^3/(6*c)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {9, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^3} \, dx$$

$$\Big\downarrow \; 9$$

$$\int x\left(b+cx^2\right)^2 dx$$

$$\Big\downarrow \; 241$$

$$\frac{(b+cx^2)^3}{6c}$$

input
```
Int[(b*x^2 + c*x^4)^2/x^3,x]
```

output
```
(b + c*x^2)^3/(6*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(c\,x^2+b)^3}{6c}$	15
parallelrisch	$\frac{1}{6}c^2x^6+\frac{1}{2}bc\,x^4+\frac{1}{2}b^2x^2$	25
gosper	$\frac{x^2(c^2x^4+3bc\,x^2+3b^2)}{6}$	26
norman	$\frac{\frac{1}{2}b^2x^4+\frac{1}{6}c^2x^8+\frac{1}{2}bc\,x^6}{x^2}$	29
risch	$\frac{c^2x^6}{6}+\frac{bc\,x^4}{2}+\frac{b^2x^2}{2}+\frac{b^3}{6c}$	33
orering	$\frac{(c^2x^4+3bc\,x^2+3b^2)(c\,x^4+b\,x^2)^2}{6x^2(c\,x^2+b)^2}$	48

input
```
int((c*x^4+b*x^2)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*(c*x^2+b)^3/c
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{6}\,c^2x^6+\frac{1}{2}\,bcx^4+\frac{1}{2}\,b^2x^2$$

input
```
integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="fricas")
```

output
```
1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(bx^2+cx^4)^2}{x^3}\,dx = \frac{b^2x^2}{2} + \frac{bcx^4}{2} + \frac{c^2x^6}{6}$$

input
```
integrate((c*x**4+b*x**2)**2/x**3,x)
```

output
```
b**2*x**2/2 + b*c*x**4/2 + c**2*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{6}\,c^2x^6 + \frac{1}{2}\,bcx^4 + \frac{1}{2}\,b^2x^2$$

input
```
integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="maxima")
```

output
```
1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{6}\,c^2x^6 + \frac{1}{2}\,bcx^4 + \frac{1}{2}\,b^2x^2$$

input
```
integrate((c*x^4+b*x^2)^2/x^3,x, algorithm="giac")
```

output
```
1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(bx^2 + cx^4)^2}{x^3} \, dx = \frac{b^2 \, x^2}{2} + \frac{b \, c \, x^4}{2} + \frac{c^2 \, x^6}{6}$$

input
```
int((b*x^2 + c*x^4)^2/x^3,x)
```

output
```
(b^2*x^2)/2 + (c^2*x^6)/6 + (b*c*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{(bx^2 + cx^4)^2}{x^3} \, dx = \frac{x^2(c^2x^4 + 3bc \, x^2 + 3b^2)}{6}$$

input
```
int((c*x^4+b*x^2)^2/x^3,x)
```

output
```
(x**2*(3*b**2 + 3*b*c*x**2 + c**2*x**4))/6
```

3.15 $\int \frac{(bx^2+cx^4)^2}{x^5} dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{(bx^2 + cx^4)^2}{x^5} dx = bcx^2 + \frac{c^2x^4}{4} + b^2 \log(x)$$

output
```
b*c*x^2+1/4*c^2*x^4+b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2 + cx^4)^2}{x^5} dx = bcx^2 + \frac{c^2x^4}{4} + b^2 \log(x)$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^5,x]
```

output
```
b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^5}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(b+cx^2)^2}{x}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{(cx^2+b)^2}{x^2}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(\frac{b^2}{x^2}+2cb+c^2x^2\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(b^2\log\left(x^2\right)+2bcx^2+\frac{c^2x^4}{2}\right)$$

input
```
Int[(b*x^2 + c*x^4)^2/x^5,x]
```

output
```
(2*b*c*x^2 + (c^2*x^4)/2 + b^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$bc\,x^2+\frac{c^2x^4}{4}+b^2\ln\left(x\right)$	22
parallelrisch	$bc\,x^2+\frac{c^2x^4}{4}+b^2\ln\left(x\right)$	22
risch	$\frac{c^2x^4}{4}+bc\,x^2+b^2+b^2\ln\left(x\right)$	25
norman	$\frac{bc\,x^6+\frac{1}{4}c^2x^8}{x^4}+b^2\ln\left(x\right)$	27

input
```
int((c*x^4+b*x^2)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
b*c*x^2+1/4*c^2*x^4+b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{4}\,c^2x^4 + bcx^2 + b^2\log(x)$$

input
```
integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="fricas")
```

output
```
1/4*c^2*x^4 + b*c*x^2 + b^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2+cx^4)^2}{x^5}\,dx = b^2\log(x) + bcx^2 + \frac{c^2x^4}{4}$$

input
```
integrate((c*x**4+b*x**2)**2/x**5,x)
```

output
```
b**2*log(x) + b*c*x**2 + c**2*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{4}\,c^2x^4 + bcx^2 + \frac{1}{2}\,b^2\log(x^2)$$

input
```
integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="maxima")
```

output
```
1/4*c^2*x^4 + b*c*x^2 + 1/2*b^2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(bx^2 + cx^4)^2}{x^5}\,dx = \frac{1}{4}\,c^2x^4 + bcx^2 + \frac{1}{2}\,b^2\log\left(x^2\right)$$

input
```
integrate((c*x^4+b*x^2)^2/x^5,x, algorithm="giac")
```

output
```
1/4*c^2*x^4 + b*c*x^2 + 1/2*b^2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 17.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(bx^2 + cx^4)^2}{x^5}\,dx = b^2\,\ln\left(x\right) + \frac{c^2\,x^4}{4} + b\,c\,x^2$$

input
```
int((b*x^2 + c*x^4)^2/x^5,x)
```

output
```
b^2*log(x) + (c^2*x^4)/4 + b*c*x^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(bx^2 + cx^4)^2}{x^5}\,dx = \log(x)\,b^2 + bc\,x^2 + \frac{c^2x^4}{4}$$

input
```
int((c*x^4+b*x^2)^2/x^5,x)
```

output
```
(4*log(x)*b**2 + 4*b*c*x**2 + c**2*x**4)/4
```

3.16 $\int \frac{(bx^2+cx^4)^2}{x^7}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{(bx^2+cx^4)^2}{x^7}\,dx = -\frac{b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)$$

output
```
-1/2*b^2/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x^7}\,dx = -\frac{b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\log(x)$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^7,x]
```

output
```
-1/2*b^2/x^2 + (c^2*x^2)/2 + 2*b*c*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^7}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(b+cx^2)^2}{x^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{(cx^2+b)^2}{x^4}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{b^2}{x^4}+\frac{2cb}{x^2}+c^2\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{b^2}{x^2}+2bc\log\left(x^2\right)+c^2x^2\right)$$

input
```
Int[(b*x^2 + c*x^4)^2/x^7,x]
```

output
```
(-(b^2/x^2) + c^2*x^2 + 2*b*c*Log[x^2])/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\ln(x)$	24
risch	$-\frac{b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\ln(x)$	24
parallelrisch	$\frac{c^2x^4+4bc\ln(x)x^2-b^2}{2x^2}$	28
norman	$\frac{-\frac{1}{2}b^2x^4+\frac{1}{2}c^2x^8}{x^6}+2bc\ln(x)$	29

input
```
int((c*x^4+b*x^2)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x^7}\,dx = \frac{c^2x^4+4\,bcx^2\log(x)-b^2}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="fricas")
```

output
```
1/2*(c^2*x^4 + 4*b*c*x^2*log(x) - b^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(bx^2+cx^4)^2}{x^7}\,dx = -\frac{b^2}{2x^2}+2bc\log(x)+\frac{c^2x^2}{2}$$

input
```
integrate((c*x**4+b*x**2)**2/x**7,x)
```

output
```
-b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(bx^2+cx^4)^2}{x^7}\,dx = \frac{1}{2}\,c^2x^2+bc\log(x^2)-\frac{b^2}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="maxima")
```

output
```
1/2*c^2*x^2 + b*c*log(x^2) - 1/2*b^2/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{(bx^2+cx^4)^2}{x^7}\,dx = \frac{1}{2}\,c^2x^2 + bc\log\left(x^2\right) - \frac{2\,bcx^2+b^2}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2)^2/x^7,x, algorithm="giac")
```

output
```
1/2*c^2*x^2 + b*c*log(x^2) - 1/2*(2*b*c*x^2 + b^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(bx^2+cx^4)^2}{x^7}\,dx = \frac{c^2\,x^2}{2} - \frac{b^2}{2\,x^2} + 2\,b\,c\,\ln\left(x\right)$$

input
```
int((b*x^2 + c*x^4)^2/x^7,x)
```

output
```
(c^2*x^2)/2 - b^2/(2*x^2) + 2*b*c*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x^7}\,dx = \frac{4\log(x)\,bc\,x^2 - b^2 + c^2x^4}{2x^2}$$

input
```
int((c*x^4+b*x^2)^2/x^7,x)
```

output
```
(4*log(x)*b*c*x**2 - b**2 + c**2*x**4)/(2*x**2)
```

3.17 $\int \frac{(bx^2+cx^4)^2}{x^9} dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \frac{(bx^2+cx^4)^2}{x^9} dx = -\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)$$

output
```
-1/4*b^2/x^4-b*c/x^2+c^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x^9} dx = -\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^9,x]
```

output
```
-1/4*b^2/x^4 - (b*c)/x^2 + c^2*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^9}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(b+cx^2)^2}{x^5}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{(cx^2+b)^2}{x^6}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(\frac{b^2}{x^6}+\frac{2cb}{x^4}+\frac{c^2}{x^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{b^2}{2x^4}-\frac{2bc}{x^2}+c^2\log\left(x^2\right)\right)$$

input
```
Int[(b*x^2 + c*x^4)^2/x^9,x]
```

output
```
(-1/2*b^2/x^4 - (2*b*c)/x^2 + c^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{b^2}{4x^4}-\frac{bc}{x^2}+c^2\ln(x)$	23
risch	$\frac{-bc\,x^2-\frac{1}{4}b^2}{x^4}+c^2\ln(x)$	25
norman	$\frac{-\frac{1}{4}b^2x^4-bc\,x^6}{x^8}+c^2\ln(x)$	28
parallelrisch	$\frac{4c^2\ln(x)x^4-4bc\,x^2-b^2}{4x^4}$	29

input
```
int((c*x^4+b*x^2)^2/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^2/x^4-b*c/x^2+c^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{(bx^2 + cx^4)^2}{x^9} \, dx = \frac{4 \, c^2 x^4 \log(x) - 4 \, bcx^2 - b^2}{4 \, x^4}$$

input
```
integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="fricas")
```

output
```
1/4*(4*c^2*x^4*log(x) - 4*b*c*x^2 - b^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2 + cx^4)^2}{x^9} \, dx = c^2 \log(x) + \frac{-b^2 - 4bcx^2}{4x^4}$$

input
```
integrate((c*x**4+b*x**2)**2/x**9,x)
```

output
```
c**2*log(x) + (-b**2 - 4*b*c*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(bx^2 + cx^4)^2}{x^9} \, dx = \frac{1}{2} \, c^2 \log(x^2) - \frac{4 \, bcx^2 + b^2}{4 \, x^4}$$

input
```
integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="maxima")
```

output
```
1/2*c^2*log(x^2) - 1/4*(4*b*c*x^2 + b^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{(bx^2+cx^4)^2}{x^9}\,dx=\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{3\,c^2x^4+4\,bcx^2+b^2}{4\,x^4}$$

input
```
integrate((c*x^4+b*x^2)^2/x^9,x, algorithm="giac")
```

output
```
1/2*c^2*log(x^2) - 1/4*(3*c^2*x^4 + 4*b*c*x^2 + b^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 17.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x^9}\,dx=c^2\,\ln\left(x\right)-\frac{\frac{b^2}{4}+c\,b\,x^2}{x^4}$$

input
```
int((b*x^2 + c*x^4)^2/x^9,x)
```

output
```
c^2*log(x) - (b^2/4 + b*c*x^2)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{(bx^2+cx^4)^2}{x^9}\,dx=\frac{4\log(x)\,c^2x^4-b^2-4bc\,x^2}{4x^4}$$

input
```
int((c*x^4+b*x^2)^2/x^9,x)
```

output
```
(4*log(x)*c**2*x**4 - b**2 - 4*b*c*x**2)/(4*x**4)
```

3.18 $\int \frac{(bx^2+cx^4)^2}{x^{11}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{(bx^2+cx^4)^2}{x^{11}}\,dx = -\frac{(b+cx^2)^3}{6bx^6}$$

output
```
-1/6*(c*x^2+b)^3/b/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{(bx^2+cx^4)^2}{x^{11}}\,dx = -\frac{b^2}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^11,x]
```

output
```
-1/6*b^2/x^6 - (b*c)/(2*x^4) - c^2/(2*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {9, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2 + cx^4)^2}{x^{11}} \, dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{(b + cx^2)^2}{x^7} dx$$
$$\Big\downarrow \; 242$$
$$-\frac{(b + cx^2)^3}{6bx^6}$$

input
```
Int[(b*x^2 + c*x^4)^2/x^11,x]
```

output
```
-1/6*(b + c*x^2)^3/(b*x^6)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

method	result	size
gosper	$-\frac{3c^2x^4+3bc\,x^2+b^2}{6x^6}$	25
default	$-\frac{bc}{2x^4}-\frac{b^2}{6x^6}-\frac{c^2}{2x^2}$	25
risch	$\frac{-\frac{1}{2}c^2x^4-\frac{1}{2}bc\,x^2-\frac{1}{6}b^2}{x^6}$	26
parallelrisch	$\frac{-3c^2x^4-3bc\,x^2-b^2}{6x^6}$	27
norman	$\frac{-\frac{1}{6}b^2x^4-\frac{1}{2}c^2x^8-\frac{1}{2}bc\,x^6}{x^{10}}$	29
orering	$-\frac{(3c^2x^4+3bc\,x^2+b^2)\left(c\,x^4+b\,x^2\right)^2}{6x^{10}(c\,x^2+b)^2}$	47

input
```
int((c*x^4+b*x^2)^2/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^6*(3*c^2*x^4+3*b*c*x^2+b^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(bx^2+cx^4)^2}{x^{11}}\,dx = -\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

input
```
integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="fricas")
```

output
```
-1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(bx^2+cx^4)^2}{x^{11}}\,dx = \frac{-b^2-3bcx^2-3c^2x^4}{6x^6}$$

input
```
integrate((c*x**4+b*x**2)**2/x**11,x)
```

output
```
(-b**2 - 3*b*c*x**2 - 3*c**2*x**4)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(bx^2+cx^4)^2}{x^{11}}\,dx = -\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

input
```
integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="maxima")
```

output
```
-1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(bx^2+cx^4)^2}{x^{11}}\,dx = -\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

input
```
integrate((c*x^4+b*x^2)^2/x^11,x, algorithm="giac")
```

output
```
-1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(bx^2+cx^4)^2}{x^{11}}\,dx = -\frac{\frac{b^2}{6}+\frac{b\,c\,x^2}{2}+\frac{c^2\,x^4}{2}}{x^6}$$

input
```
int((b*x^2 + c*x^4)^2/x^11,x)
```

output
```
-(b^2/6 + (c^2*x^4)/2 + (b*c*x^2)/2)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(bx^2+cx^4)^2}{x^{11}}\,dx = \frac{-3c^2x^4-3bc\,x^2-b^2}{6x^6}$$

input
```
int((c*x^4+b*x^2)^2/x^11,x)
```

output
```
( - b**2 - 3*b*c*x**2 - 3*c**2*x**4)/(6*x**6)
```

3.19 $\int \frac{(bx^2+cx^4)^2}{x^{13}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{(bx^2+cx^4)^2}{x^{13}}\,dx = -\frac{b^2}{8x^8}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

output
```
-1/8*b^2/x^8-1/3*b*c/x^6-1/4*c^2/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x^{13}}\,dx = -\frac{b^2}{8x^8}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^13,x]
```

output
```
-1/8*b^2/x^8 - (b*c)/(3*x^6) - c^2/(4*x^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {9, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^{13}} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(b+cx^2)^2}{x^9}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{(cx^2+b)^2}{x^{10}}dx^2$$

$$\Big\downarrow \; 53$$

$$\frac{1}{2}\int \left(\frac{b^2}{x^{10}}+\frac{2cb}{x^8}+\frac{c^2}{x^6}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{b^2}{4x^8}-\frac{2bc}{3x^6}-\frac{c^2}{2x^4}\right)$$

input
```
Int[(b*x^2 + c*x^4)^2/x^13,x]
```

output
```
(-1/4*b^2/x^8 - (2*b*c)/(3*x^6) - c^2/(2*x^4))/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{b^{2}}{8x^{8}}-\frac{bc}{3x^{6}}-\frac{c^{2}}{4x^{4}}$	25
risch	$\frac{-\frac{1}{4}c^{2}x^{4}-\frac{1}{3}bc\,x^{2}-\frac{1}{8}b^{2}}{x^{8}}$	26
gosper	$-\frac{6c^{2}x^{4}+8bc\,x^{2}+3b^{2}}{24x^{8}}$	27
parallelrisch	$\frac{-6c^{2}x^{4}-8bc\,x^{2}-3b^{2}}{24x^{8}}$	27
norman	$\frac{-\frac{1}{8}b^{2}x^{4}-\frac{1}{4}c^{2}x^{8}-\frac{1}{3}bc\,x^{6}}{x^{12}}$	29
orering	$-\frac{\left(6c^{2}x^{4}+8bc\,x^{2}+3b^{2}\right)\left(c\,x^{4}+b\,x^{2}\right)^{2}}{24x^{12}\left(c\,x^{2}+b\right)^{2}}$	49

input
```
int((c*x^4+b*x^2)^2/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/8*b^2/x^8-1/3*b*c/x^6-1/4*c^2/x^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2 + cx^4)^2}{x^{13}} dx = -\frac{6 c^2 x^4 + 8 bcx^2 + 3 b^2}{24 x^8}$$

input
```
integrate((c*x^4+b*x^2)^2/x^13,x, algorithm="fricas")
```

output
```
-1/24*(6*c^2*x^4 + 8*b*c*x^2 + 3*b^2)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{(bx^2 + cx^4)^2}{x^{13}} dx = \frac{-3b^2 - 8bcx^2 - 6c^2x^4}{24x^8}$$

input
```
integrate((c*x**4+b*x**2)**2/x**13,x)
```

output
```
(-3*b**2 - 8*b*c*x**2 - 6*c**2*x**4)/(24*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2 + cx^4)^2}{x^{13}} dx = -\frac{6 c^2 x^4 + 8 bcx^2 + 3 b^2}{24 x^8}$$

input
```
integrate((c*x^4+b*x^2)^2/x^13,x, algorithm="maxima")
```

output
```
-1/24*(6*c^2*x^4 + 8*b*c*x^2 + 3*b^2)/x^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2+cx^4)^2}{x^{13}}\,dx = -\frac{6\,c^2x^4+8\,bcx^2+3\,b^2}{24\,x^8}$$

input
```
integrate((c*x^4+b*x^2)^2/x^13,x, algorithm="giac")
```

output
```
-1/24*(6*c^2*x^4 + 8*b*c*x^2 + 3*b^2)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2+cx^4)^2}{x^{13}}\,dx = -\frac{\frac{b^2}{8}+\frac{b\,c\,x^2}{3}+\frac{c^2\,x^4}{4}}{x^8}$$

input
```
int((b*x^2 + c*x^4)^2/x^13,x)
```

output
```
-(b^2/8 + (c^2*x^4)/4 + (b*c*x^2)/3)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2+cx^4)^2}{x^{13}}\,dx = \frac{-6c^2x^4-8bc\,x^2-3b^2}{24x^8}$$

input
```
int((c*x^4+b*x^2)^2/x^13,x)
```

output
```
( - 3*b**2 - 8*b*c*x**2 - 6*c**2*x**4)/(24*x**8)
```

3.20 $\int (bx^2 + cx^4)^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int (bx^2 + cx^4)^2 \, dx = \frac{b^2x^5}{5} + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

output
```
1/5*b^2*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (bx^2 + cx^4)^2 \, dx = \frac{b^2x^5}{5} + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

input
```
Integrate[(b*x^2 + c*x^4)^2,x]
```

output
```
(b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1397, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \ 1397$$

$$\int x^4\left(b+cx^2\right)^2 dx$$

$$\Big\downarrow \ 244$$

$$\int \left(b^2x^4+2bcx^6+c^2x^8\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

input
```
Int[(b*x^2 + c*x^4)^2,x]
```

output
```
(b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1397
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[x^(2*p)*(b + c*x^2
)^p, x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{5}x^5b^2+\frac{2}{7}bc\,x^7+\frac{1}{9}c^2x^9$	25
norman	$\frac{1}{5}x^5b^2+\frac{2}{7}bc\,x^7+\frac{1}{9}c^2x^9$	25
risch	$\frac{1}{5}x^5b^2+\frac{2}{7}bc\,x^7+\frac{1}{9}c^2x^9$	25
parallelrisch	$\frac{1}{5}x^5b^2+\frac{2}{7}bc\,x^7+\frac{1}{9}c^2x^9$	25
gosper	$\frac{x^5(35c^2x^4+90bc\,x^2+63b^2)}{315}$	27
orering	$\frac{x(35c^2x^4+90bc\,x^2+63b^2)(c\,x^4+b\,x^2)^2}{315(c\,x^2+b)^2}$	47

input
```
int((c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*b^2+2/7*b*c*x^7+1/9*c^2*x^9
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx^2+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5$$

input
```
integrate((c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(bx^2+cx^4\right)^2\,dx=\frac{b^2x^5}{5}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}$$

input
```
integrate((c*x**4+b*x**2)**2,x)
```

output
```
b**2*x**5/5 + 2*b*c*x**7/7 + c**2*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx^2+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5$$

input
```
integrate((c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx^2+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5$$

input
```
integrate((c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx^2+cx^4\right)^2 \, dx = \frac{b^2\,x^5}{5}+\frac{2\,b\,c\,x^7}{7}+\frac{c^2\,x^9}{9}$$

input
```
int((b*x^2 + c*x^4)^2,x)
```

output
```
(b^2*x^5)/5 + (c^2*x^9)/9 + (2*b*c*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(bx^2+cx^4\right)^2 \, dx = \frac{x^5(35c^2x^4+90bc\,x^2+63b^2)}{315}$$

input
```
int((c*x^4+b*x^2)^2,x)
```

output
```
(x**5*(63*b**2 + 90*b*c*x**2 + 35*c**2*x**4))/315
```

3.21 $\int \frac{(bx^2+cx^4)^2}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{(bx^2+cx^4)^2}{x^2}\,dx = \frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

output
```
1/3*b^2*x^3+2/5*b*c*x^5+1/7*c^2*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x^2}\,dx = \frac{b^2x^3}{3} + \frac{2}{5}bcx^5 + \frac{c^2x^7}{7}$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^2,x]
```

output
```
(b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^2\left(b+cx^2\right)^2dx$$

$$\Big\downarrow \; 244$$

$$\int \left(b^2x^2+2bcx^4+c^2x^6\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{b^2x^3}{3}+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

input
```
Int[(b*x^2 + c*x^4)^2/x^2,x]
```

output
```
(b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{3}b^2x^3+\frac{2}{5}bc\,x^5+\frac{1}{7}c^2x^7$	25
risch	$\frac{1}{3}b^2x^3+\frac{2}{5}bc\,x^5+\frac{1}{7}c^2x^7$	25
parallelrisch	$\frac{1}{3}b^2x^3+\frac{2}{5}bc\,x^5+\frac{1}{7}c^2x^7$	25
gosper	$\frac{x^3(15c^2x^4+42bc\,x^2+35b^2)}{105}$	27
norman	$\frac{\frac{1}{3}b^2x^4+\frac{1}{7}c^2x^8+\frac{2}{5}bc\,x^6}{x}$	29
orering	$\frac{(15c^2x^4+42bc\,x^2+35b^2)(c\,x^4+b\,x^2)^2}{105x(c\,x^2+b)^2}$	49

input
```
int((c*x^4+b*x^2)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^2*x^3+2/5*b*c*x^5+1/7*c^2*x^7
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{7}\,c^2x^7+\frac{2}{5}\,bcx^5+\frac{1}{3}\,b^2x^3$$

input
```
integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="fricas")
```

output
```
1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2 + cx^4)^2}{x^2} dx = \frac{b^2x^3}{3} + \frac{2bcx^5}{5} + \frac{c^2x^7}{7}$$

input
```
integrate((c*x**4+b*x**2)**2/x**2,x)
```

output
```
b**2*x**3/3 + 2*b*c*x**5/5 + c**2*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2 + cx^4)^2}{x^2} dx = \frac{1}{7} c^2x^7 + \frac{2}{5} bcx^5 + \frac{1}{3} b^2x^3$$

input
```
integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="maxima")
```

output
```
1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2 + cx^4)^2}{x^2} dx = \frac{1}{7} c^2x^7 + \frac{2}{5} bcx^5 + \frac{1}{3} b^2x^3$$

input
```
integrate((c*x^4+b*x^2)^2/x^2,x, algorithm="giac")
```

output
```
1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2+cx^4)^2}{x^2}\,dx=\frac{b^2\,x^3}{3}+\frac{2\,b\,c\,x^5}{5}+\frac{c^2\,x^7}{7}$$

input
```
int((b*x^2 + c*x^4)^2/x^2,x)
```

output
```
(b^2*x^3)/3 + (c^2*x^7)/7 + (2*b*c*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2+cx^4)^2}{x^2}\,dx=\frac{x^3(15c^2x^4+42bc\,x^2+35b^2)}{105}$$

input
```
int((c*x^4+b*x^2)^2/x^2,x)
```

output
```
(x**3*(35*b**2 + 42*b*c*x**2 + 15*c**2*x**4))/105
```

3.22 $\int \frac{(bx^2+cx^4)^2}{x^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int \frac{(bx^2 + cx^4)^2}{x^4} \, dx = b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

output
```
b^2*x+2/3*b*c*x^3+1/5*c^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2 + cx^4)^2}{x^4} \, dx = b^2x + \frac{2}{3}bcx^3 + \frac{c^2x^5}{5}$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^4,x]
```

output
```
b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^4} \, dx$$
$$\Big\downarrow \text{ 9}$$
$$\int (b+cx^2)^2 \, dx$$
$$\Big\downarrow \text{ 210}$$
$$\int (b^2+2bcx^2+c^2x^4) \, dx$$
$$\Big\downarrow \text{ 2009}$$
$$b^2x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

input
```
Int[(b*x^2 + c*x^4)^2/x^4,x]
```

output
```
b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$b^2x+\frac{2}{3}bc\,x^3+\frac{1}{5}c^2x^5$	22
risch	$b^2x+\frac{2}{3}bc\,x^3+\frac{1}{5}c^2x^5$	22
parallelrisch	$b^2x+\frac{2}{3}bc\,x^3+\frac{1}{5}c^2x^5$	22
gosper	$\frac{x(3c^2x^4+10bc\,x^2+15b^2)}{15}$	25
norman	$\frac{b^2x^4+\frac{1}{5}c^2x^8+\frac{2}{3}bc\,x^6}{x^3}$	28
orering	$\frac{(3c^2x^4+10bc\,x^2+15b^2)(c\,x^4+b\,x^2)^2}{15x^3(c\,x^2+b)^2}$	49

input
```
int((c*x^4+b*x^2)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
b^2*x+2/3*b*c*x^3+1/5*c^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{5}\,c^2x^5+\frac{2}{3}\,bcx^3+b^2x$$

input
```
integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="fricas")
```

output
```
1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(bx^2+cx^4)^2}{x^4}\,dx = b^2x + \frac{2bcx^3}{3} + \frac{c^2x^5}{5}$$

input
```
integrate((c*x**4+b*x**2)**2/x**4,x)
```

output
```
b**2*x + 2*b*c*x**3/3 + c**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x$$

input
```
integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="maxima")
```

output
```
1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x$$

input
```
integrate((c*x^4+b*x^2)^2/x^4,x, algorithm="giac")
```

output
```
1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(bx^2 + cx^4)^2}{x^4} \, dx = b^2 \, x + \frac{2 \, b \, c \, x^3}{3} + \frac{c^2 \, x^5}{5}$$

input
```
int((b*x^2 + c*x^4)^2/x^4,x)
```

output
```
b^2*x + (c^2*x^5)/5 + (2*b*c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(bx^2 + cx^4)^2}{x^4} \, dx = \frac{x(3c^2x^4 + 10bc \, x^2 + 15b^2)}{15}$$

input
```
int((c*x^4+b*x^2)^2/x^4,x)
```

output
```
(x*(15*b**2 + 10*b*c*x**2 + 3*c**2*x**4))/15
```

3.23 $\int \frac{(bx^2+cx^4)^2}{x^6}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \frac{(bx^2+cx^4)^2}{x^6}\,dx = -\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$$

output
```
-b^2/x+2*b*c*x+1/3*c^2*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x^6}\,dx = -\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^6,x]
```

output
```
-(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^6}dx$$

$$\Big\downarrow 9$$

$$\int \frac{(b+cx^2)^2}{x^2}dx$$

$$\Big\downarrow 244$$

$$\int \left(\frac{b^2}{x^2}+2bc+c^2x^2\right)dx$$

$$\Big\downarrow 2009$$

$$-\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$$

input
```
Int[(b*x^2 + c*x^4)^2/x^6,x]
```

output
```
-(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$	23
risch	$-\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$	23
parallelrisch	$\frac{c^2x^4+6bc\,x^2-3b^2}{3x}$	26
gosper	$-\frac{-c^2x^4-6bc\,x^2+3b^2}{3x}$	27
norman	$\frac{-b^2x^4+\frac{1}{3}c^2x^8+2bc\,x^6}{x^5}$	29
orering	$-\frac{\left(-c^2x^4-6bc\,x^2+3b^2\right)\left(c\,x^4+b\,x^2\right)^2}{3x^5\left(c\,x^2+b\right)^2}$	49

input
```
int((c*x^4+b*x^2)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-b^2/x+2*b*c*x+1/3*c^2*x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{(bx^2+cx^4)^2}{x^6}\,dx=\frac{c^2x^4+6\,bcx^2-3\,b^2}{3\,x}$$

input
```
integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="fricas")
```

output
```
1/3*(c^2*x^4 + 6*b*c*x^2 - 3*b^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{(bx^2+cx^4)^2}{x^6}\,dx = -\frac{b^2}{x}+2bcx+\frac{c^2x^3}{3}$$

input
```
integrate((c*x**4+b*x**2)**2/x**6,x)
```

output
```
-b**2/x + 2*b*c*x + c**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2+cx^4)^2}{x^6}\,dx = \frac{1}{3}\,c^2x^3+2\,bcx-\frac{b^2}{x}$$

input
```
integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="maxima")
```

output
```
1/3*c^2*x^3 + 2*b*c*x - b^2/x
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2+cx^4)^2}{x^6}\,dx = \frac{1}{3}\,c^2x^3+2\,bcx-\frac{b^2}{x}$$

input
```
integrate((c*x^4+b*x^2)^2/x^6,x, algorithm="giac")
```

output
```
1/3*c^2*x^3 + 2*b*c*x - b^2/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2 + cx^4)^2}{x^6} \, dx = \frac{c^2 \, x^3}{3} - \frac{b^2}{x} + 2 \, b \, c \, x$$

input
```
int((b*x^2 + c*x^4)^2/x^6,x)
```

output
```
(c^2*x^3)/3 - b^2/x + 2*b*c*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{(bx^2 + cx^4)^2}{x^6} \, dx = \frac{c^2 x^4 + 6bc \, x^2 - 3b^2}{3x}$$

input
```
int((c*x^4+b*x^2)^2/x^6,x)
```

output
```
( - 3*b**2 + 6*b*c*x**2 + c**2*x**4)/(3*x)
```

3.24 $\int \frac{(bx^2+cx^4)^2}{x^8} dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{(bx^2+cx^4)^2}{x^8} dx = -\frac{b^2}{3x^3} - \frac{2bc}{x} + c^2x$$

output
```
-1/3*b^2/x^3-2*b*c/x+c^2*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x^8} dx = -\frac{b^2}{3x^3} - \frac{2bc}{x} + c^2x$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^8,x]
```

output
```
-1/3*b^2/x^3 - (2*b*c)/x + c^2*x
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^8}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{(b+cx^2)^2}{x^4}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{b^2}{x^4}+\frac{2bc}{x^2}+c^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$$

input
```
Int[(b*x^2 + c*x^4)^2/x^8,x]
```

output
```
-1/3*b^2/x^3 - (2*b*c)/x + c^2*x
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$	22
risch	$c^2x+\frac{-2bc\,x^2-\frac{1}{3}b^2}{x^3}$	24
gosper	$-\frac{-3c^2x^4+6bc\,x^2+b^2}{3x^3}$	25
parallelrisch	$\frac{3c^2x^4-6bc\,x^2-b^2}{3x^3}$	27
norman	$\frac{c^2x^8-\frac{1}{3}b^2x^4-2bc\,x^6}{x^7}$	28
orering	$-\frac{\left(-3c^2x^4+6bc\,x^2+b^2\right)\left(c\,x^4+b\,x^2\right)^2}{3x^7\left(c\,x^2+b\right)^2}$	47

input
```
int((c*x^4+b*x^2)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^2/x^3-2/x*b*c+c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\left(bx^2+cx^4\right)^2}{x^8}\,dx=\frac{3\,c^2x^4-6\,bcx^2-b^2}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="fricas")
```

output
```
1/3*(3*c^2*x^4 - 6*b*c*x^2 - b^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(bx^2+cx^4)^2}{x^8}\,dx = c^2x + \frac{-b^2-6bcx^2}{3x^3}$$

input
```
integrate((c*x**4+b*x**2)**2/x**8,x)
```

output
```
c**2*x + (-b**2 - 6*b*c*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(bx^2+cx^4)^2}{x^8}\,dx = c^2x - \frac{6\,bcx^2+b^2}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="maxima")
```

output
```
c^2*x - 1/3*(6*b*c*x^2 + b^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(bx^2+cx^4)^2}{x^8}\,dx = c^2x - \frac{6\,bcx^2+b^2}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2)^2/x^8,x, algorithm="giac")
```

output
```
c^2*x - 1/3*(6*b*c*x^2 + b^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(bx^2+cx^4)^2}{x^8}\,dx = c^2\,x-\frac{\frac{b^2}{3}+2\,c\,b\,x^2}{x^3}$$

input
```
int((b*x^2 + c*x^4)^2/x^8,x)
```

output
```
c^2*x - (b^2/3 + 2*b*c*x^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{(bx^2+cx^4)^2}{x^8}\,dx = \frac{3c^2x^4-6bc\,x^2-b^2}{3x^3}$$

input
```
int((c*x^4+b*x^2)^2/x^8,x)
```

output
```
( - b**2 - 6*b*c*x**2 + 3*c**2*x**4)/(3*x**3)
```

3.25 $\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx = -\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

output
```
-1/5*b^2/x^5-2/3*b*c/x^3-c^2/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx = -\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^10,x]
```

output
```
-1/5*b^2/x^5 - (2*b*c)/(3*x^3) - c^2/x
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{(b+cx^2)^2}{x^6}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{b^2}{x^6}+\frac{2bc}{x^4}+\frac{c^2}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

input
```
Int[(b*x^2 + c*x^4)^2/x^10,x]
```

output
```
-1/5*b^2/x^5 - (2*b*c)/(3*x^3) - c^2/x
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$	25
risch	$\frac{-c^2x^4-\frac{2}{3}bc\,x^2-\frac{1}{5}b^2}{x^5}$	26
gosper	$-\frac{15c^2x^4+10bc\,x^2+3b^2}{15x^5}$	27
parallelrisch	$\frac{-15c^2x^4-10bc\,x^2-3b^2}{15x^5}$	27
norman	$\frac{-\frac{1}{5}b^2x^4-c^2x^8-\frac{2}{3}bc\,x^6}{x^9}$	29
orering	$-\frac{(15c^2x^4+10bc\,x^2+3b^2)(c\,x^4+b\,x^2)^2}{15x^9(c\,x^2+b)^2}$	49

input
```
int((c*x^4+b*x^2)^2/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/5*b^2/x^5-2/3*b*c/x^3-c^2/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx=-\frac{15\,c^2x^4+10\,bcx^2+3\,b^2}{15\,x^5}$$

input
```
integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="fricas")
```

output
```
-1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(bx^2 + cx^4)^2}{x^{10}}\,dx = \frac{-3b^2 - 10bcx^2 - 15c^2x^4}{15x^5}$$

input
```
integrate((c*x**4+b*x**2)**2/x**10,x)
```

output
```
(-3*b**2 - 10*b*c*x**2 - 15*c**2*x**4)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(bx^2 + cx^4)^2}{x^{10}}\,dx = -\frac{15\,c^2x^4 + 10\,bcx^2 + 3\,b^2}{15\,x^5}$$

input
```
integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="maxima")
```

output
```
-1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(bx^2 + cx^4)^2}{x^{10}}\,dx = -\frac{15\,c^2x^4 + 10\,bcx^2 + 3\,b^2}{15\,x^5}$$

input
```
integrate((c*x^4+b*x^2)^2/x^10,x, algorithm="giac")
```

output
```
-1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx = -\frac{\frac{b^2}{5}+\frac{2\,b\,c\,x^2}{3}+c^2\,x^4}{x^5}$$

input
```
int((b*x^2 + c*x^4)^2/x^10,x)
```

output
```
-(b^2/5 + c^2*x^4 + (2*b*c*x^2)/3)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx = \frac{-15c^2x^4-10bc\,x^2-3b^2}{15x^5}$$

input
```
int((c*x^4+b*x^2)^2/x^10,x)
```

output
```
( - 3*b**2 - 10*b*c*x**2 - 15*c**2*x**4)/(15*x**5)
```

3.26 $\int \frac{(bx^2+cx^4)^2}{x^{12}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{(bx^2+cx^4)^2}{x^{12}}\,dx=-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

output
```
-1/7*b^2/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^2}{x^{12}}\,dx=-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^12,x]
```

output
```
-1/7*b^2/x^7 - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^{12}} \, dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{(b+cx^2)^2}{x^8}dx$$
$$\Big\downarrow \text{ 244}$$
$$\int \left(\frac{b^2}{x^8}+\frac{2bc}{x^6}+\frac{c^2}{x^4}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

input
```
Int[(b*x^2 + c*x^4)^2/x^12,x]
```

output
```
-1/7*b^2/x^7 - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$	25
risch	$\frac{-\frac{1}{3}c^2x^4-\frac{2}{5}bc\,x^2-\frac{1}{7}b^2}{x^7}$	26
gosper	$-\frac{35c^2x^4+42bc\,x^2+15b^2}{105x^7}$	27
parallelrisch	$\frac{-35c^2x^4-42bc\,x^2-15b^2}{105x^7}$	27
norman	$\frac{-\frac{1}{7}b^2x^4-\frac{1}{3}c^2x^8-\frac{2}{5}bc\,x^6}{x^{11}}$	29
orering	$-\frac{(35c^2x^4+42bc\,x^2+15b^2)(c\,x^4+b\,x^2)^2}{105x^{11}(c\,x^2+b)^2}$	49

input
```
int((c*x^4+b*x^2)^2/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/7*b^2/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{35\,c^2x^4+42\,bcx^2+15\,b^2}{105\,x^7}$$

input
```
integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="fricas")
```

output
```
-1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{(bx^2 + cx^4)^2}{x^{12}} dx = \frac{-15b^2 - 42bcx^2 - 35c^2x^4}{105x^7}$$

input
```
integrate((c*x**4+b*x**2)**2/x**12,x)
```

output
```
(-15*b**2 - 42*b*c*x**2 - 35*c**2*x**4)/(105*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2 + cx^4)^2}{x^{12}} dx = -\frac{35 c^2x^4 + 42 bcx^2 + 15 b^2}{105 x^7}$$

input
```
integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="maxima")
```

output
```
-1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2 + cx^4)^2}{x^{12}} dx = -\frac{35 c^2x^4 + 42 bcx^2 + 15 b^2}{105 x^7}$$

input
```
integrate((c*x^4+b*x^2)^2/x^12,x, algorithm="giac")
```

output
```
-1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2+cx^4)^2}{x^{12}}\,dx=-\frac{\frac{b^2}{7}+\frac{2\,b\,c\,x^2}{5}+\frac{c^2\,x^4}{3}}{x^7}$$

input
```
int((b*x^2 + c*x^4)^2/x^12,x)
```

output
```
-(b^2/7 + (c^2*x^4)/3 + (2*b*c*x^2)/5)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2+cx^4)^2}{x^{12}}\,dx=\frac{-35c^2x^4-42bc\,x^2-15b^2}{105x^7}$$

input
```
int((c*x^4+b*x^2)^2/x^12,x)
```

output
```
( - 15*b**2 - 42*b*c*x**2 - 35*c**2*x**4)/(105*x**7)
```

3.27 $\int \frac{(bx^2+cx^4)^3}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{(bx^2+cx^4)^3}{x} dx = \frac{b^3x^6}{6} + \frac{3}{8}b^2cx^8 + \frac{3}{10}bc^2x^{10} + \frac{c^3x^{12}}{12}$$

output
```
1/6*b^3*x^6+3/8*b^2*c*x^8+3/10*b*c^2*x^10+1/12*c^3*x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x} dx = \frac{b^3x^6}{6} + \frac{3}{8}b^2cx^8 + \frac{3}{10}bc^2x^{10} + \frac{c^3x^{12}}{12}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x,x]
```

output
```
(b^3*x^6)/6 + (3*b^2*c*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x} \, dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^5 (b+cx^2)^3 \, dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int x^4 (cx^2+b)^3 \, dx^2$$
$$\Big\downarrow \text{ 49}$$
$$\frac{1}{2}\int \left(c^3x^{10}+3bc^2x^8+3b^2cx^6+b^3x^4\right) dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(\frac{b^3x^6}{3}+\frac{3}{4}b^2cx^8+\frac{3}{5}bc^2x^{10}+\frac{c^3x^{12}}{6}\right)$$

input
```
Int[(b*x^2 + c*x^4)^3/x,x]
```

output
```
((b^3*x^6)/3 + (3*b^2*c*x^8)/4 + (3*b*c^2*x^10)/5 + (c^3*x^12)/6)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{1}{6}b^3x^6+\frac{3}{8}b^2c\,x^8+\frac{3}{10}b\,c^2x^{10}+\frac{1}{12}c^3x^{12}$	36
norman	$\frac{1}{6}b^3x^6+\frac{3}{8}b^2c\,x^8+\frac{3}{10}b\,c^2x^{10}+\frac{1}{12}c^3x^{12}$	36
risch	$\frac{1}{6}b^3x^6+\frac{3}{8}b^2c\,x^8+\frac{3}{10}b\,c^2x^{10}+\frac{1}{12}c^3x^{12}$	36
parallelrisch	$\frac{1}{6}b^3x^6+\frac{3}{8}b^2c\,x^8+\frac{3}{10}b\,c^2x^{10}+\frac{1}{12}c^3x^{12}$	36
gosper	$\frac{x^6\left(10c^3x^6+36b\,c^2x^4+45b^2c\,x^2+20b^3\right)}{120}$	38
orering	$\frac{\left(10c^3x^6+36b\,c^2x^4+45b^2c\,x^2+20b^3\right)\left(c\,x^4+b\,x^2\right)^3}{120\left(c\,x^2+b\right)^3}$	57

input
```
int((c*x^4+b*x^2)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^3*x^6+3/8*b^2*c*x^8+3/10*b*c^2*x^10+1/12*c^3*x^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2 + cx^4)^3}{x} dx = \frac{1}{12} c^3 x^{12} + \frac{3}{10} bc^2 x^{10} + \frac{3}{8} b^2 c x^8 + \frac{1}{6} b^3 x^6$$

input
```
integrate((c*x^4+b*x^2)^3/x,x, algorithm="fricas")
```

output
```
1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*b^2*c*x^8 + 1/6*b^3*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{(bx^2 + cx^4)^3}{x} dx = \frac{b^3 x^6}{6} + \frac{3b^2 c x^8}{8} + \frac{3bc^2 x^{10}}{10} + \frac{c^3 x^{12}}{12}$$

input
```
integrate((c*x**4+b*x**2)**3/x,x)
```

output
```
b**3*x**6/6 + 3*b**2*c*x**8/8 + 3*b*c**2*x**10/10 + c**3*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2 + cx^4)^3}{x} dx = \frac{1}{12} c^3 x^{12} + \frac{3}{10} bc^2 x^{10} + \frac{3}{8} b^2 c x^8 + \frac{1}{6} b^3 x^6$$

input
```
integrate((c*x^4+b*x^2)^3/x,x, algorithm="maxima")
```

output
```
1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*b^2*c*x^8 + 1/6*b^3*x^6
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2+cx^4)^3}{x}\,dx=\frac{1}{12}\,c^3x^{12}+\frac{3}{10}\,bc^2x^{10}+\frac{3}{8}\,b^2cx^8+\frac{1}{6}\,b^3x^6$$

input
```
integrate((c*x^4+b*x^2)^3/x,x, algorithm="giac")
```

output
```
1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*b^2*c*x^8 + 1/6*b^3*x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2+cx^4)^3}{x}\,dx=\frac{b^3\,x^6}{6}+\frac{3\,b^2\,c\,x^8}{8}+\frac{3\,b\,c^2\,x^{10}}{10}+\frac{c^3\,x^{12}}{12}$$

input
```
int((b*x^2 + c*x^4)^3/x,x)
```

output
```
(b^3*x^6)/6 + (c^3*x^12)/12 + (3*b^2*c*x^8)/8 + (3*b*c^2*x^10)/10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x}\,dx=\frac{x^6(10c^3x^6+36b\,c^2x^4+45b^2c\,x^2+20b^3)}{120}$$

input
```
int((c*x^4+b*x^2)^3/x,x)
```

output
```
(x**6*(20*b**3 + 45*b**2*c*x**2 + 36*b*c**2*x**4 + 10*c**3*x**6))/120
```

3.28 $\int \frac{(bx^2+cx^4)^3}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 34

$$\int \frac{(bx^2+cx^4)^3}{x^3}\,dx = -\frac{b(b+cx^2)^4}{8c^2}+\frac{(b+cx^2)^5}{10c^2}$$

output
```
-1/8*b*(c*x^2+b)^4/c^2+1/10*(c*x^2+b)^5/c^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{(bx^2+cx^4)^3}{x^3}\,dx = \frac{b^3x^4}{4}+\frac{1}{2}b^2cx^6+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^3,x]
```

output
```
(b^3*x^4)/4 + (b^2*c*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^3}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^3 (b+cx^2)^3\,dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int x^2 (cx^2+b)^3\,dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{(cx^2+b)^4}{c}-\frac{b(cx^2+b)^3}{c}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{(b+cx^2)^5}{5c^2}-\frac{b(b+cx^2)^4}{4c^2}\right)$$

input
```
Int[(b*x^2 + c*x^4)^3/x^3,x]
```

output
```
(-1/4*(b*(b + c*x^2)^4)/c^2 + (b + c*x^2)^5/(5*c^2))/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{1}{10}c^3x^{10}+\frac{3}{8}b\,c^2x^8+\frac{1}{2}b^2c\,x^6+\frac{1}{4}b^3x^4$	36
risch	$\frac{1}{10}c^3x^{10}+\frac{3}{8}b\,c^2x^8+\frac{1}{2}b^2c\,x^6+\frac{1}{4}b^3x^4$	36
parallelrisch	$\frac{1}{10}c^3x^{10}+\frac{3}{8}b\,c^2x^8+\frac{1}{2}b^2c\,x^6+\frac{1}{4}b^3x^4$	36
gosper	$\frac{x^4\left(4c^3x^6+15b\,c^2x^4+20b^2c\,x^2+10b^3\right)}{40}$	38
norman	$\frac{\frac{1}{4}b^3x^6+\frac{1}{10}c^3x^{12}+\frac{3}{8}b\,c^2x^{10}+\frac{1}{2}b^2c\,x^8}{x^2}$	40
orering	$\frac{\left(4c^3x^6+15b\,c^2x^4+20b^2c\,x^2+10b^3\right)\left(c\,x^4+b\,x^2\right)^3}{40x^2\left(c\,x^2+b\right)^3}$	60

input
```
int((c*x^4+b*x^2)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*c^3*x^10+3/8*b*c^2*x^8+1/2*b^2*c*x^6+1/4*b^3*x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{(bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{10}\,c^3x^{10} + \frac{3}{8}\,bc^2x^8 + \frac{1}{2}\,b^2cx^6 + \frac{1}{4}\,b^3x^4$$

input
```
integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="fricas")
```

output
```
1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{(bx^2+cx^4)^3}{x^3}\,dx = \frac{b^3x^4}{4} + \frac{b^2cx^6}{2} + \frac{3bc^2x^8}{8} + \frac{c^3x^{10}}{10}$$

input
```
integrate((c*x**4+b*x**2)**3/x**3,x)
```

output
```
b**3*x**4/4 + b**2*c*x**6/2 + 3*b*c**2*x**8/8 + c**3*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{(bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{10}\,c^3x^{10} + \frac{3}{8}\,bc^2x^8 + \frac{1}{2}\,b^2cx^6 + \frac{1}{4}\,b^3x^4$$

input
```
integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="maxima")
```

output
```
1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{(bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{10}c^3x^{10} + \frac{3}{8}bc^2x^8 + \frac{1}{2}b^2cx^6 + \frac{1}{4}b^3x^4$$

input
```
integrate((c*x^4+b*x^2)^3/x^3,x, algorithm="giac")
```

output
```
1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{(bx^2+cx^4)^3}{x^3}\,dx = \frac{b^3\,x^4}{4} + \frac{b^2\,c\,x^6}{2} + \frac{3\,b\,c^2\,x^8}{8} + \frac{c^3\,x^{10}}{10}$$

input
```
int((b*x^2 + c*x^4)^3/x^3,x)
```

output
```
(b^3*x^4)/4 + (c^3*x^10)/10 + (b^2*c*x^6)/2 + (3*b*c^2*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{(bx^2+cx^4)^3}{x^3}\,dx = \frac{x^4(4c^3x^6+15b\,c^2x^4+20b^2c\,x^2+10b^3)}{40}$$

input
```
int((c*x^4+b*x^2)^3/x^3,x)
```

output
```
(x**4*(10*b**3 + 20*b**2*c*x**2 + 15*b*c**2*x**4 + 4*c**3*x**6))/40
```

3.29 $\int \frac{(bx^2+cx^4)^3}{x^5} dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{(bx^2 + cx^4)^3}{x^5} dx = \frac{(b + cx^2)^4}{8c}$$

output
```
1/8*(c*x^2+b)^4/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2 + cx^4)^3}{x^5} dx = \frac{(b + cx^2)^4}{8c}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^5,x]
```

output
```
(b + c*x^2)^4/(8*c)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^5}\,dx$$
$$\Big\downarrow \text{9}$$
$$\int x(b+cx^2)^3\,dx$$
$$\Big\downarrow \text{241}$$
$$\frac{(b+cx^2)^4}{8c}$$

input
```
Int[(b*x^2 + c*x^4)^3/x^5,x]
```

output
```
(b + c*x^2)^4/(8*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(c\,x^2+b)^4}{8c}$	15
parallelrisch	$\frac{1}{8}c^3x^8+\frac{1}{2}b\,c^2x^6+\frac{3}{4}b^2c\,x^4+\frac{1}{2}b^3x^2$	36
gosper	$\frac{x^2(c^3x^6+4b\,c^2x^4+6b^2c\,x^2+4b^3)}{8}$	37
norman	$\frac{\frac{1}{2}b^3x^6+\frac{1}{8}c^3x^{12}+\frac{1}{2}b\,c^2x^{10}+\frac{3}{4}b^2c\,x^8}{x^4}$	40
risch	$\frac{c^3x^8}{8}+\frac{b\,c^2x^6}{2}+\frac{3b^2c\,x^4}{4}+\frac{b^3x^2}{2}+\frac{b^4}{8c}$	44
orering	$\frac{(c^3x^6+4b\,c^2x^4+6b^2c\,x^2+4b^3)(c\,x^4+b\,x^2)^3}{8x^4(c\,x^2+b)^3}$	59

input
```
int((c*x^4+b*x^2)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/8*(c*x^2+b)^4/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{(bx^2+cx^4)^3}{x^5}\,dx = \frac{1}{8}\,c^3x^8+\frac{1}{2}\,bc^2x^6+\frac{3}{4}\,b^2cx^4+\frac{1}{2}\,b^3x^2$$

input
```
integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="fricas")
```

output
```
1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \frac{(bx^2 + cx^4)^3}{x^5}\, dx = \frac{b^3x^2}{2} + \frac{3b^2cx^4}{4} + \frac{bc^2x^6}{2} + \frac{c^3x^8}{8}$$

input
```
integrate((c*x**4+b*x**2)**3/x**5,x)
```

output
```
b**3*x**2/2 + 3*b**2*c*x**4/4 + b*c**2*x**6/2 + c**3*x**8/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{(bx^2 + cx^4)^3}{x^5}\, dx = \frac{1}{8}\, c^3x^8 + \frac{1}{2}\, bc^2x^6 + \frac{3}{4}\, b^2cx^4 + \frac{1}{2}\, b^3x^2$$

input
```
integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="maxima")
```

output
```
1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{(bx^2 + cx^4)^3}{x^5}\, dx = \frac{1}{8}\, c^3x^8 + \frac{1}{2}\, bc^2x^6 + \frac{3}{4}\, b^2cx^4 + \frac{1}{2}\, b^3x^2$$

input
```
integrate((c*x^4+b*x^2)^3/x^5,x, algorithm="giac")
```

output
```
1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{(bx^2 + cx^4)^3}{x^5} \, dx = \frac{b^3 \, x^2}{2} + \frac{3 \, b^2 \, c \, x^4}{4} + \frac{b \, c^2 \, x^6}{2} + \frac{c^3 \, x^8}{8}$$

input
```
int((b*x^2 + c*x^4)^3/x^5,x)
```

output
```
(b^3*x^2)/2 + (c^3*x^8)/8 + (3*b^2*c*x^4)/4 + (b*c^2*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{(bx^2 + cx^4)^3}{x^5} \, dx = \frac{x^2(c^3x^6 + 4b \, c^2x^4 + 6b^2c \, x^2 + 4b^3)}{8}$$

input
```
int((c*x^4+b*x^2)^3/x^5,x)
```

output
```
(x**2*(4*b**3 + 6*b**2*c*x**2 + 4*b*c**2*x**4 + c**3*x**6))/8
```

3.30 $\int \frac{(bx^2+cx^4)^3}{x^7} dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{(bx^2+cx^4)^3}{x^7} dx = \frac{3}{2}b^2cx^2 + \frac{3}{4}bc^2x^4 + \frac{c^3x^6}{6} + b^3\log(x)$$

output
```
3/2*b^2*c*x^2+3/4*b*c^2*x^4+1/6*c^3*x^6+b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^7} dx = \frac{3}{2}b^2cx^2 + \frac{3}{4}bc^2x^4 + \frac{c^3x^6}{6} + b^3\log(x)$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^7,x]
```

output
```
(3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^7} dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(b+cx^2)^3}{x} dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{(cx^2+b)^3}{x^2} dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(c^3x^4+3bc^2x^2+3b^2c+\frac{b^3}{x^2}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(b^3\log\left(x^2\right)+3b^2cx^2+\frac{3}{2}bc^2x^4+\frac{c^3x^6}{3}\right)$$

input
```
Int[(b*x^2 + c*x^4)^3/x^7,x]
```

output
```
(3*b^2*c*x^2 + (3*b*c^2*x^4)/2 + (c^3*x^6)/3 + b^3*Log[x^2])/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{3b^2 c x^2}{2}+\frac{3b c^2 x^4}{4}+\frac{c^3 x^6}{6}+b^3 \ln \left(x \right)$	34
risch	$\frac{3b^2 c x^2}{2}+\frac{3b c^2 x^4}{4}+\frac{c^3 x^6}{6}+b^3 \ln \left(x \right)$	34
parallelrisch	$\frac{3b^2 c x^2}{2}+\frac{3b c^2 x^4}{4}+\frac{c^3 x^6}{6}+b^3 \ln \left(x \right)$	34
norman	$\frac{\frac{1}{6} c^3 x^{12}+\frac{3}{4} b c^2 x^{10}+\frac{3}{2} b^2 c x^8}{x^6}+b^3 \ln \left(x \right)$	39

input
```
int((c*x^4+b*x^2)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
3/2*b^2*c*x^2+3/4*b*c^2*x^4+1/6*c^3*x^6+b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(bx^2 + cx^4)^3}{x^7} dx = \frac{1}{6}c^3x^6 + \frac{3}{4}bc^2x^4 + \frac{3}{2}b^2cx^2 + b^3 \log(x)$$

input
```
integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="fricas")
```

output
```
1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + b^3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(bx^2 + cx^4)^3}{x^7} dx = b^3 \log(x) + \frac{3b^2cx^2}{2} + \frac{3bc^2x^4}{4} + \frac{c^3x^6}{6}$$

input
```
integrate((c*x**4+b*x**2)**3/x**7,x)
```

output
```
b**3*log(x) + 3*b**2*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2 + cx^4)^3}{x^7} dx = \frac{1}{6}c^3x^6 + \frac{3}{4}bc^2x^4 + \frac{3}{2}b^2cx^2 + \frac{1}{2}b^3 \log(x^2)$$

input
```
integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="maxima")
```

output
```
1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + 1/2*b^3*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2+cx^4)^3}{x^7}\,dx = \frac{1}{6}\,c^3x^6 + \frac{3}{4}\,bc^2x^4 + \frac{3}{2}\,b^2cx^2 + \frac{1}{2}\,b^3\log\left(x^2\right)$$

input
```
integrate((c*x^4+b*x^2)^3/x^7,x, algorithm="giac")
```

output
```
1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + 1/2*b^3*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(bx^2+cx^4)^3}{x^7}\,dx = b^3\,\ln(x) + \frac{c^3\,x^6}{6} + \frac{3\,b^2\,c\,x^2}{2} + \frac{3\,b\,c^2\,x^4}{4}$$

input
```
int((b*x^2 + c*x^4)^3/x^7,x)
```

output
```
b^3*log(x) + (c^3*x^6)/6 + (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(bx^2+cx^4)^3}{x^7}\,dx = \log(x)\,b^3 + \frac{3b^2c\,x^2}{2} + \frac{3b\,c^2x^4}{4} + \frac{c^3x^6}{6}$$

input
```
int((c*x^4+b*x^2)^3/x^7,x)
```

output
```
(12*log(x)*b**3 + 18*b**2*c*x**2 + 9*b*c**2*x**4 + 2*c**3*x**6)/12
```

3.31 $\int \frac{(bx^2+cx^4)^3}{x^9} dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{(bx^2+cx^4)^3}{x^9} dx = -\frac{b^3}{2x^2} + \frac{3}{2}bc^2x^2 + \frac{c^3x^4}{4} + 3b^2c\log(x)$$

output
```
-1/2*b^3/x^2+3/2*b*c^2*x^2+1/4*c^3*x^4+3*b^2*c*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^9} dx = -\frac{b^3}{2x^2} + \frac{3}{2}bc^2x^2 + \frac{c^3x^4}{4} + 3b^2c\log(x)$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^9,x]
```

output
```
-1/2*b^3/x^2 + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^9}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{(b+cx^2)^3}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(cx^2+b)^3}{x^4}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{b^3}{x^4}+\frac{3cb^2}{x^2}+3c^2b+c^3x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{b^3}{x^2}+3b^2c\log\left(x^2\right)+3bc^2x^2+\frac{c^3x^4}{2}\right)$$

input
```
Int[(b*x^2 + c*x^4)^3/x^9,x]
```

output
```
(-(b^3/x^2) + 3*b*c^2*x^2 + (c^3*x^4)/2 + 3*b^2*c*Log[x^2])/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{b^3}{2x^2}+\frac{3b\,c^2x^2}{2}+\frac{c^3x^4}{4}+3b^2c\ln\left(x\right)$	35
parallelrisch	$\frac{c^3x^6+6b\,c^2x^4+12b^2c\ln(x)x^2-2b^3}{4x^2}$	39
norman	$\frac{-\frac{1}{2}b^3x^6+\frac{1}{4}c^3x^{12}+\frac{3}{2}b\,c^2x^{10}}{x^8}+3b^2c\ln\left(x\right)$	40
risch	$\frac{c^3x^4}{4}+\frac{3b\,c^2x^2}{2}+\frac{9b^2c}{4}-\frac{b^3}{2x^2}+3b^2c\ln\left(x\right)$	41

input
```
int((c*x^4+b*x^2)^3/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^3/x^2+3/2*b*c^2*x^2+1/4*c^3*x^4+3*b^2*c*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{(bx^2 + cx^4)^3}{x^9} dx = \frac{c^3x^6 + 6\,bc^2x^4 + 12\,b^2cx^2 \log(x) - 2\,b^3}{4\,x^2}$$

input
```
integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="fricas")
```

output
```
1/4*(c^3*x^6 + 6*b*c^2*x^4 + 12*b^2*c*x^2*log(x) - 2*b^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2 + cx^4)^3}{x^9} dx = -\frac{b^3}{2x^2} + 3b^2c \log(x) + \frac{3bc^2x^2}{2} + \frac{c^3x^4}{4}$$

input
```
integrate((c*x**4+b*x**2)**3/x**9,x)
```

output
```
-b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{(bx^2 + cx^4)^3}{x^9} dx = \frac{1}{4}\,c^3x^4 + \frac{3}{2}\,bc^2x^2 + \frac{3}{2}\,b^2c \log(x^2) - \frac{b^3}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="maxima")
```

output
```
1/4*c^3*x^4 + 3/2*b*c^2*x^2 + 3/2*b^2*c*log(x^2) - 1/2*b^3/x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(bx^2+cx^4)^3}{x^9}\,dx = \frac{1}{4}\,c^3x^4 + \frac{3}{2}\,bc^2x^2 + \frac{3}{2}\,b^2c\log\left(x^2\right) - \frac{3\,b^2cx^2+b^3}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2)^3/x^9,x, algorithm="giac")
```

output
```
1/4*c^3*x^4 + 3/2*b*c^2*x^2 + 3/2*b^2*c*log(x^2) - 1/2*(3*b^2*c*x^2 + b^3)
/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{(bx^2+cx^4)^3}{x^9}\,dx = \frac{c^3\,x^4}{4} - \frac{b^3}{2\,x^2} + \frac{3\,b\,c^2\,x^2}{2} + 3\,b^2\,c\,\ln\left(x\right)$$

input
```
int((b*x^2 + c*x^4)^3/x^9,x)
```

output
```
(c^3*x^4)/4 - b^3/(2*x^2) + (3*b*c^2*x^2)/2 + 3*b^2*c*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{(bx^2+cx^4)^3}{x^9}\,dx = \frac{12\log(x)\,b^2c\,x^2 - 2b^3 + 6b\,c^2x^4 + c^3x^6}{4x^2}$$

input
```
int((c*x^4+b*x^2)^3/x^9,x)
```

output
```
(12*log(x)*b**2*c*x**2 - 2*b**3 + 6*b*c**2*x**4 + c**3*x**6)/(4*x**2)
```

3.32 $\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx = -\frac{b^3}{4x^4} - \frac{3b^2c}{2x^2} + \frac{c^3x^2}{2} + 3bc^2\log(x)$$

output
```
-1/4*b^3/x^4-3/2*b^2*c/x^2+1/2*c^3*x^2+3*b*c^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx = -\frac{b^3}{4x^4} - \frac{3b^2c}{2x^2} + \frac{c^3x^2}{2} + 3bc^2\log(x)$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^11,x]
```

output
```
-1/4*b^3/x^4 - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^{11}}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{(b+cx^2)^3}{x^5}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{(cx^2+b)^3}{x^6}dx^2$$
$$\Big\downarrow \text{ 49}$$
$$\frac{1}{2}\int \left(\frac{b^3}{x^6}+\frac{3cb^2}{x^4}+\frac{3c^2b}{x^2}+c^3\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(-\frac{b^3}{2x^4}-\frac{3b^2c}{x^2}+3bc^2\log\left(x^2\right)+c^3x^2\right)$$

input
```
Int[(b*x^2 + c*x^4)^3/x^11,x]
```

output
```
(-1/2*b^3/x^4 - (3*b^2*c)/x^2 + c^3*x^2 + 3*b*c^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{b^{3}}{4x^{4}}-\frac{3b^{2}c}{2x^{2}}+\frac{c^{3}x^{2}}{2}+3b\,c^{2}\ln\left(x\right)$	35
risch	$\frac{c^{3}x^{2}}{2}+\frac{-\frac{3}{2}b^{2}c\,x^{2}-\frac{1}{4}b^{3}}{x^{4}}+3b\,c^{2}\ln\left(x\right)$	37
norman	$\frac{-\frac{1}{4}b^{3}x^{6}+\frac{1}{2}c^{3}x^{12}-\frac{3}{2}b^{2}c\,x^{8}}{x^{10}}+3b\,c^{2}\ln\left(x\right)$	40
parallelrisch	$\frac{2c^{3}x^{6}+12b\,c^{2}\ln\left(x\right)x^{4}-6b^{2}c\,x^{2}-b^{3}}{4x^{4}}$	40

input
```
int((c*x^4+b*x^2)^3/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^3/x^4-3/2*b^2*c/x^2+1/2*c^3*x^2+3*b*c^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx=\frac{2\,c^3x^6+12\,bc^2x^4\log(x)-6\,b^2cx^2-b^3}{4\,x^4}$$

input
```
integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="fricas")
```

output
```
1/4*(2*c^3*x^6 + 12*b*c^2*x^4*log(x) - 6*b^2*c*x^2 - b^3)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx=3bc^2\log(x)+\frac{c^3x^2}{2}+\frac{-b^3-6b^2cx^2}{4x^4}$$

input
```
integrate((c*x**4+b*x**2)**3/x**11,x)
```

output
```
3*b*c**2*log(x) + c**3*x**2/2 + (-b**3 - 6*b**2*c*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx=\frac{1}{2}\,c^3x^2+\frac{3}{2}\,bc^2\log(x^2)-\frac{6\,b^2cx^2+b^3}{4\,x^4}$$

input
```
integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="maxima")
```

output
```
1/2*c^3*x^2 + 3/2*b*c^2*log(x^2) - 1/4*(6*b^2*c*x^2 + b^3)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx = \frac{1}{2}\,c^3x^2 + \frac{3}{2}\,bc^2\log\left(x^2\right) - \frac{9\,bc^2x^4+6\,b^2cx^2+b^3}{4\,x^4}$$

input
```
integrate((c*x^4+b*x^2)^3/x^11,x, algorithm="giac")
```

output
```
1/2*c^3*x^2 + 3/2*b*c^2*log(x^2) - 1/4*(9*b*c^2*x^4 + 6*b^2*c*x^2 + b^3)/x
^4
```

Mupad [B] (verification not implemented)

Time = 17.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx = \frac{c^3\,x^2}{2} - \frac{\frac{b^3}{4}+\frac{3\,c\,b^2\,x^2}{2}}{x^4} + 3\,b\,c^2\,\ln\left(x\right)$$

input
```
int((b*x^2 + c*x^4)^3/x^11,x)
```

output
```
(c^3*x^2)/2 - (b^3/4 + (3*b^2*c*x^2)/2)/x^4 + 3*b*c^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx = \frac{12\log(x)\,b\,c^2x^4 - b^3 - 6b^2c\,x^2 + 2c^3x^6}{4x^4}$$

input
```
int((c*x^4+b*x^2)^3/x^11,x)
```

output
```
(12*log(x)*b*c**2*x**4 - b**3 - 6*b**2*c*x**2 + 2*c**3*x**6)/(4*x**4)
```

3.33 $\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx=-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

output
```
-1/6*b^3/x^6-3/4*b^2*c/x^4-3/2*b*c^2/x^2+c^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx=-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^13,x]
```

output
```
-1/6*b^3/x^6 - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^2+cx^4\right)^3}{x^{13}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{\left(b+cx^2\right)^3}{x^7}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{\left(cx^2+b\right)^3}{x^8}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(\frac{b^3}{x^8}+\frac{3cb^2}{x^6}+\frac{3c^2b}{x^4}+\frac{c^3}{x^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{b^3}{3x^6}-\frac{3b^2c}{2x^4}-\frac{3bc^2}{x^2}+c^3\log\left(x^2\right)\right)$$

input
```
Int[(b*x^2 + c*x^4)^3/x^13,x]
```

output
```
(-1/3*b^3/x^6 - (3*b^2*c)/(2*x^4) - (3*b*c^2)/x^2 + c^3*Log[x^2])/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{b^{3}}{6x^{6}}-\frac{3b^{2}c}{4x^{4}}-\frac{3b\,c^{2}}{2x^{2}}+c^{3}\ln\left(x\right)$	34
risch	$\frac{-\frac{3}{2}b\,c^{2}x^{4}-\frac{3}{4}b^{2}c\,x^{2}-\frac{1}{6}b^{3}}{x^{6}}+c^{3}\ln\left(x\right)$	36
norman	$\frac{-\frac{1}{6}b^{3}x^{6}-\frac{3}{2}b\,c^{2}x^{10}-\frac{3}{4}b^{2}c\,x^{8}}{x^{12}}+c^{3}\ln\left(x\right)$	39
parallelrisch	$\frac{12c^{3}\ln(x)x^{6}-18b\,c^{2}x^{4}-9b^{2}c\,x^{2}-2b^{3}}{12x^{6}}$	40

input
```
int((c*x^4+b*x^2)^3/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/6*b^3/x^6-3/4*b^2*c/x^4-3/2*b*c^2/x^2+c^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx=\frac{12\,c^3x^6\log(x)-18\,bc^2x^4-9\,b^2cx^2-2\,b^3}{12\,x^6}$$

input
```
integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="fricas")
```

output
```
1/12*(12*c^3*x^6*log(x) - 18*b*c^2*x^4 - 9*b^2*c*x^2 - 2*b^3)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx=c^3\log(x)+\frac{-2b^3-9b^2cx^2-18bc^2x^4}{12x^6}$$

input
```
integrate((c*x**4+b*x**2)**3/x**13,x)
```

output
```
c**3*log(x) + (-2*b**3 - 9*b**2*c*x**2 - 18*b*c**2*x**4)/(12*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx=\frac{1}{2}\,c^3\log\left(x^2\right)-\frac{18\,bc^2x^4+9\,b^2cx^2+2\,b^3}{12\,x^6}$$

input
```
integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="maxima")
```

output
```
1/2*c^3*log(x^2) - 1/12*(18*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3)/x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx = \frac{1}{2}\,c^3\log\left(x^2\right) - \frac{11\,c^3x^6 + 18\,bc^2x^4 + 9\,b^2cx^2 + 2\,b^3}{12\,x^6}$$

input
```
integrate((c*x^4+b*x^2)^3/x^13,x, algorithm="giac")
```

output
```
1/2*c^3*log(x^2) - 1/12*(11*c^3*x^6 + 18*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3)/
x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx = c^3\,\ln(x) - \frac{\frac{b^3}{6} + \frac{3\,b^2\,c\,x^2}{4} + \frac{3\,b\,c^2\,x^4}{2}}{x^6}$$

input
```
int((b*x^2 + c*x^4)^3/x^13,x)
```

output
```
c^3*log(x) - (b^3/6 + (3*b^2*c*x^2)/4 + (3*b*c^2*x^4)/2)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx = \frac{12\log(x)\,c^3x^6 - 2b^3 - 9b^2c\,x^2 - 18b\,c^2x^4}{12x^6}$$

input
```
int((c*x^4+b*x^2)^3/x^13,x)
```

output
```
(12*log(x)*c**3*x**6 - 2*b**3 - 9*b**2*c*x**2 - 18*b*c**2*x**4)/(12*x**6)
```

3.34 $\int \frac{(bx^2+cx^4)^3}{x^{15}} dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{(bx^2+cx^4)^3}{x^{15}} dx = -\frac{(b+cx^2)^4}{8bx^8}$$

output
```
-1/8*(c*x^2+b)^4/b/x^8
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \frac{(bx^2+cx^4)^3}{x^{15}} dx = -\frac{b^3}{8x^8} - \frac{b^2c}{2x^6} - \frac{3bc^2}{4x^4} - \frac{c^3}{2x^2}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^15,x]
```

output
```
-1/8*b^3/x^8 - (b^2*c)/(2*x^6) - (3*b*c^2)/(4*x^4) - c^3/(2*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^{15}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(b+cx^2)^3}{x^9}dx$$

$$\Big\downarrow \; 242$$

$$-\frac{(b+cx^2)^4}{8bx^8}$$

input
```
Int[(b*x^2 + c*x^4)^3/x^15,x]
```

output
```
-1/8*(b + c*x^2)^4/(b*x^8)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

method	result	size
gosper	$-\frac{4c^3x^6+6b\,c^2x^4+4b^2c\,x^2+b^3}{8x^8}$	36
default	$-\frac{3b\,c^2}{4x^4}-\frac{b^2c}{2x^6}-\frac{c^3}{2x^2}-\frac{b^3}{8x^8}$	36
risch	$\frac{-\frac{1}{2}c^3x^6-\frac{3}{4}b\,c^2x^4-\frac{1}{2}b^2c\,x^2-\frac{1}{8}b^3}{x^8}$	37
parallelrisch	$\frac{-4c^3x^6-6b\,c^2x^4-4b^2c\,x^2-b^3}{8x^8}$	38
norman	$\frac{-\frac{1}{8}b^3x^6-\frac{1}{2}c^3x^{12}-\frac{3}{4}b\,c^2x^{10}-\frac{1}{2}b^2c\,x^8}{x^{14}}$	40
orering	$-\frac{(4c^3x^6+6b\,c^2x^4+4b^2c\,x^2+b^3)(c\,x^4+b\,x^2)^3}{8x^{14}(c\,x^2+b)^3}$	58

input
```
int((c*x^4+b*x^2)^3/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/8/x^8*(4*c^3*x^6+6*b*c^2*x^4+4*b^2*c*x^2+b^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{(bx^2+cx^4)^3}{x^{15}}\,dx = -\frac{4\,c^3x^6+6\,bc^2x^4+4\,b^2cx^2+b^3}{8\,x^8}$$

input
```
integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="fricas")
```

output
```
-1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{(bx^2 + cx^4)^3}{x^{15}}\,dx = \frac{-b^3 - 4b^2cx^2 - 6bc^2x^4 - 4c^3x^6}{8x^8}$$

input
```
integrate((c*x**4+b*x**2)**3/x**15,x)
```

output
```
(-b**3 - 4*b**2*c*x**2 - 6*b*c**2*x**4 - 4*c**3*x**6)/(8*x**8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{(bx^2 + cx^4)^3}{x^{15}}\,dx = -\frac{4\,c^3x^6 + 6\,bc^2x^4 + 4\,b^2cx^2 + b^3}{8\,x^8}$$

input
```
integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="maxima")
```

output
```
-1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{(bx^2 + cx^4)^3}{x^{15}}\,dx = -\frac{4\,c^3x^6 + 6\,bc^2x^4 + 4\,b^2cx^2 + b^3}{8\,x^8}$$

input
```
integrate((c*x^4+b*x^2)^3/x^15,x, algorithm="giac")
```

output
```
-1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

Mupad [B] (verification not implemented)

Time = 16.96 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{(bx^2 + cx^4)^3}{x^{15}}\,dx = -\frac{\frac{b^3}{8} + \frac{b^2\,c\,x^2}{2} + \frac{3\,b\,c^2\,x^4}{4} + \frac{c^3\,x^6}{2}}{x^8}$$

input
```
int((b*x^2 + c*x^4)^3/x^15,x)
```

output
```
-(b^3/8 + (c^3*x^6)/2 + (b^2*c*x^2)/2 + (3*b*c^2*x^4)/4)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{(bx^2 + cx^4)^3}{x^{15}}\,dx = \frac{-4c^3x^6 - 6b\,c^2x^4 - 4b^2c\,x^2 - b^3}{8x^8}$$

input
```
int((c*x^4+b*x^2)^3/x^15,x)
```

output
```
( - b**3 - 4*b**2*c*x**2 - 6*b*c**2*x**4 - 4*c**3*x**6)/(8*x**8)
```

3.35 $\int \frac{(bx^2+cx^4)^3}{x^{17}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{(bx^2+cx^4)^3}{x^{17}}\,dx = -\frac{(b+cx^2)^4}{10bx^{10}} + \frac{c(b+cx^2)^4}{40b^2x^8}$$

output
```
-1/10*(c*x^2+b)^4/b/x^10+1/40*c*(c*x^2+b)^4/b^2/x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{(bx^2+cx^4)^3}{x^{17}}\,dx = -\frac{b^3}{10x^{10}} - \frac{3b^2c}{8x^8} - \frac{bc^2}{2x^6} - \frac{c^3}{4x^4}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^17,x]
```

output
```
-1/10*b^3/x^10 - (3*b^2*c)/(8*x^8) - (b*c^2)/(2*x^6) - c^3/(4*x^4)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {9, 243, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^{17}}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{(b+cx^2)^3}{x^{11}}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{(cx^2+b)^3}{x^{12}}dx^2$$

$$\Big\downarrow \ 55$$

$$\frac{1}{2}\left(-\frac{c\int \frac{(cx^2+b)^3}{x^{10}}dx^2}{5b}-\frac{(b+cx^2)^4}{5bx^{10}}\right)$$

$$\Big\downarrow \ 48$$

$$\frac{1}{2}\left(\frac{c(b+cx^2)^4}{20b^2x^8}-\frac{(b+cx^2)^4}{5bx^{10}}\right)$$

input
```
Int[(b*x^2 + c*x^4)^3/x^17,x]
```

output
```
(-1/5*(b + c*x^2)^4/(b*x^10) + (c*(b + c*x^2)^4)/(20*b^2*x^8))/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{c^{3}}{4x^{4}}-\frac{b\,c^{2}}{2x^{6}}-\frac{3b^{2}c}{8x^{8}}-\frac{b^{3}}{10x^{10}}$	36
risch	$\frac{-\frac{1}{4}c^{3}x^{6}-\frac{1}{2}b\,c^{2}x^{4}-\frac{3}{8}b^{2}c\,x^{2}-\frac{1}{10}b^{3}}{x^{10}}$	37
gosper	$-\frac{10c^{3}x^{6}+20b\,c^{2}x^{4}+15b^{2}c\,x^{2}+4b^{3}}{40x^{10}}$	38
parallelrisch	$\frac{-10c^{3}x^{6}-20b\,c^{2}x^{4}-15b^{2}c\,x^{2}-4b^{3}}{40x^{10}}$	38
norman	$\frac{-\frac{1}{10}b^{3}x^{6}-\frac{1}{4}c^{3}x^{12}-\frac{1}{2}b\,c^{2}x^{10}-\frac{3}{8}b^{2}c\,x^{8}}{x^{16}}$	40
orering	$-\frac{\left(10c^{3}x^{6}+20b\,c^{2}x^{4}+15b^{2}c\,x^{2}+4b^{3}\right)\left(c\,x^{4}+b\,x^{2}\right)^{3}}{40x^{16}\left(c\,x^{2}+b\right)^{3}}$	60

input
```
int((c*x^4+b*x^2)^3/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/4*c^3/x^4-1/2*b*c^2/x^6-3/8*b^2*c/x^8-1/10*b^3/x^10
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2+cx^4)^3}{x^{17}}\,dx = -\frac{10\,c^3x^6+20\,bc^2x^4+15\,b^2cx^2+4\,b^3}{40\,x^{10}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="fricas")
```

output
```
-1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(bx^2+cx^4)^3}{x^{17}}\,dx = \frac{-4b^3-15b^2cx^2-20bc^2x^4-10c^3x^6}{40x^{10}}$$

input
```
integrate((c*x**4+b*x**2)**3/x**17,x)
```

output
```
(-4*b**3 - 15*b**2*c*x**2 - 20*b*c**2*x**4 - 10*c**3*x**6)/(40*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2 + cx^4)^3}{x^{17}}\,dx = -\frac{10\,c^3x^6 + 20\,bc^2x^4 + 15\,b^2cx^2 + 4\,b^3}{40\,x^{10}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="maxima")
```

output
```
-1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2 + cx^4)^3}{x^{17}}\,dx = -\frac{10\,c^3x^6 + 20\,bc^2x^4 + 15\,b^2cx^2 + 4\,b^3}{40\,x^{10}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^17,x, algorithm="giac")
```

output
```
-1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

Mupad [B] (verification not implemented)

Time = 16.98 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2 + cx^4)^3}{x^{17}}\,dx = -\frac{\frac{b^3}{10} + \frac{3\,b^2\,c\,x^2}{8} + \frac{b\,c^2\,x^4}{2} + \frac{c^3\,x^6}{4}}{x^{10}}$$

input
```
int((b*x^2 + c*x^4)^3/x^17,x)
```

output
```
-(b^3/10 + (c^3*x^6)/4 + (3*b^2*c*x^2)/8 + (b*c^2*x^4)/2)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2 + cx^4)^3}{x^{17}} \, dx = \frac{-10c^3x^6 - 20b\,c^2x^4 - 15b^2c\,x^2 - 4b^3}{40x^{10}}$$

input
```
int((c*x^4+b*x^2)^3/x^17,x)
```

output
```
( - 4*b**3 - 15*b**2*c*x**2 - 20*b*c**2*x**4 - 10*c**3*x**6)/(40*x**10)
```

3.36 $\int \frac{(bx^2+cx^4)^3}{x^{19}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{(bx^2 + cx^4)^3}{x^{19}} \, dx = -\frac{b^3}{12x^{12}} - \frac{3b^2c}{10x^{10}} - \frac{3bc^2}{8x^8} - \frac{c^3}{6x^6}$$

output
```
-1/12*b^3/x^12-3/10*b^2*c/x^10-3/8*b*c^2/x^8-1/6*c^3/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2 + cx^4)^3}{x^{19}} \, dx = -\frac{b^3}{12x^{12}} - \frac{3b^2c}{10x^{10}} - \frac{3bc^2}{8x^8} - \frac{c^3}{6x^6}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^19,x]
```

output
```
-1/12*b^3/x^12 - (3*b^2*c)/(10*x^10) - (3*b*c^2)/(8*x^8) - c^3/(6*x^6)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {9, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^{19}}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{(b+cx^2)^3}{x^{13}}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{(cx^2+b)^3}{x^{14}}dx^2$$
$$\Big\downarrow \text{ 53}$$
$$\frac{1}{2}\int \left(\frac{b^3}{x^{14}}+\frac{3cb^2}{x^{12}}+\frac{3c^2b}{x^{10}}+\frac{c^3}{x^8}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(-\frac{b^3}{6x^{12}}-\frac{3b^2c}{5x^{10}}-\frac{3bc^2}{4x^8}-\frac{c^3}{3x^6}\right)$$

input
```
Int[(b*x^2 + c*x^4)^3/x^19,x]
```

output
```
(-1/6*b^3/x^12 - (3*b^2*c)/(5*x^10) - (3*b*c^2)/(4*x^8) - c^3/(3*x^6))/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{b^{3}}{12x^{12}}-\frac{3b^{2}c}{10x^{10}}-\frac{3b\,c^{2}}{8x^{8}}-\frac{c^{3}}{6x^{6}}$	36
risch	$\frac{-\frac{1}{6}c^{3}x^{6}-\frac{3}{8}b\,c^{2}x^{4}-\frac{3}{10}b^{2}c\,x^{2}-\frac{1}{12}b^{3}}{x^{12}}$	37
gosper	$-\frac{20c^{3}x^{6}+45b\,c^{2}x^{4}+36b^{2}c\,x^{2}+10b^{3}}{120x^{12}}$	38
parallelrisch	$\frac{-20c^{3}x^{6}-45b\,c^{2}x^{4}-36b^{2}c\,x^{2}-10b^{3}}{120x^{12}}$	38
norman	$\frac{-\frac{1}{12}b^{3}x^{6}-\frac{1}{6}c^{3}x^{12}-\frac{3}{8}b\,c^{2}x^{10}-\frac{3}{10}b^{2}c\,x^{8}}{x^{18}}$	40
orering	$-\frac{\left(20c^{3}x^{6}+45b\,c^{2}x^{4}+36b^{2}c\,x^{2}+10b^{3}\right)\left(c\,x^{4}+b\,x^{2}\right)^{3}}{120x^{18}\left(c\,x^{2}+b\right)^{3}}$	60

input
```
int((c*x^4+b*x^2)^3/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/12*b^3/x^12-3/10*b^2*c/x^10-3/8*b*c^2/x^8-1/6*c^3/x^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x^{19}}\,dx = -\frac{20\,c^3x^6+45\,bc^2x^4+36\,b^2cx^2+10\,b^3}{120\,x^{12}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^19,x, algorithm="fricas")
```

output
```
-1/120*(20*c^3*x^6 + 45*b*c^2*x^4 + 36*b^2*c*x^2 + 10*b^3)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{(bx^2+cx^4)^3}{x^{19}}\,dx = \frac{-10b^3-36b^2cx^2-45bc^2x^4-20c^3x^6}{120x^{12}}$$

input
```
integrate((c*x**4+b*x**2)**3/x**19,x)
```

output
```
(-10*b**3 - 36*b**2*c*x**2 - 45*b*c**2*x**4 - 20*c**3*x**6)/(120*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x^{19}}\,dx = -\frac{20\,c^3x^6+45\,bc^2x^4+36\,b^2cx^2+10\,b^3}{120\,x^{12}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^19,x, algorithm="maxima")
```

output
```
-1/120*(20*c^3*x^6 + 45*b*c^2*x^4 + 36*b^2*c*x^2 + 10*b^3)/x^12
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x^{19}}\,dx = -\frac{20\,c^3x^6+45\,bc^2x^4+36\,b^2cx^2+10\,b^3}{120\,x^{12}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^19,x, algorithm="giac")
```

output
```
-1/120*(20*c^3*x^6 + 45*b*c^2*x^4 + 36*b^2*c*x^2 + 10*b^3)/x^12
```

Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x^{19}}\,dx = -\frac{\frac{b^3}{12}+\frac{3\,b^2\,c\,x^2}{10}+\frac{3\,b\,c^2\,x^4}{8}+\frac{c^3\,x^6}{6}}{x^{12}}$$

input
```
int((b*x^2 + c*x^4)^3/x^19,x)
```

output
```
-(b^3/12 + (c^3*x^6)/6 + (3*b^2*c*x^2)/10 + (3*b*c^2*x^4)/8)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x^{19}}\,dx = \frac{-20c^3x^6-45b\,c^2x^4-36b^2c\,x^2-10b^3}{120x^{12}}$$

input
```
int((c*x^4+b*x^2)^3/x^19,x)
```

output
```
( - 10*b**3 - 36*b**2*c*x**2 - 45*b*c**2*x**4 - 20*c**3*x**6)/(120*x**12)
```

3.37 $\int \frac{(bx^2+cx^4)^3}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{(bx^2+cx^4)^3}{x^2}\,dx = \frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

output
```
1/5*b^3*x^5+3/7*b^2*c*x^7+1/3*b*c^2*x^9+1/11*c^3*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^2}\,dx = \frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^2,x]
```

output
```
(b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^2} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^4(b+cx^2)^3 \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int (b^3x^4+3b^2cx^6+3bc^2x^8+c^3x^{10}) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

input
```
Int[(b*x^2 + c*x^4)^3/x^2,x]
```

output
```
(b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{1}{5}b^3x^5+\frac{3}{7}b^2c\,x^7+\frac{1}{3}b\,c^2x^9+\frac{1}{11}c^3x^{11}$	36
risch	$\frac{1}{5}b^3x^5+\frac{3}{7}b^2c\,x^7+\frac{1}{3}b\,c^2x^9+\frac{1}{11}c^3x^{11}$	36
parallelrisch	$\frac{1}{5}b^3x^5+\frac{3}{7}b^2c\,x^7+\frac{1}{3}b\,c^2x^9+\frac{1}{11}c^3x^{11}$	36
gosper	$\frac{x^5(105c^3x^6+385b\,c^2x^4+495b^2c\,x^2+231b^3)}{1155}$	38
norman	$\frac{\frac{1}{5}b^3x^6+\frac{1}{11}c^3x^{12}+\frac{1}{3}b\,c^2x^{10}+\frac{3}{7}b^2c\,x^8}{x}$	40
orering	$\frac{(105c^3x^6+385b\,c^2x^4+495b^2c\,x^2+231b^3)(c\,x^4+b\,x^2)^3}{1155x(c\,x^2+b)^3}$	60

input
```
int((c*x^4+b*x^2)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^3*x^5+3/7*b^2*c*x^7+1/3*b*c^2*x^9+1/11*c^3*x^11
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{11}\,c^3x^{11} + \frac{1}{3}\,bc^2x^9 + \frac{3}{7}\,b^2cx^7 + \frac{1}{5}\,b^3x^5$$

input
```
integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="fricas")
```

output
```
1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x^2}\,dx = \frac{b^3x^5}{5} + \frac{3b^2cx^7}{7} + \frac{bc^2x^9}{3} + \frac{c^3x^{11}}{11}$$

input
```
integrate((c*x**4+b*x**2)**3/x**2,x)
```

output
```
b**3*x**5/5 + 3*b**2*c*x**7/7 + b*c**2*x**9/3 + c**3*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{11}\,c^3x^{11} + \frac{1}{3}\,bc^2x^9 + \frac{3}{7}\,b^2cx^7 + \frac{1}{5}\,b^3x^5$$

input
```
integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="maxima")
```

output
```
1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{11}\,c^3x^{11} + \frac{1}{3}\,bc^2x^9 + \frac{3}{7}\,b^2cx^7 + \frac{1}{5}\,b^3x^5$$

input
```
integrate((c*x^4+b*x^2)^3/x^2,x, algorithm="giac")
```

output
```
1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2+cx^4)^3}{x^2}\,dx = \frac{b^3\,x^5}{5}+\frac{3\,b^2\,c\,x^7}{7}+\frac{b\,c^2\,x^9}{3}+\frac{c^3\,x^{11}}{11}$$

input
```
int((b*x^2 + c*x^4)^3/x^2,x)
```

output
```
(b^3*x^5)/5 + (c^3*x^11)/11 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x^2}\,dx = \frac{x^5(105c^3x^6+385b\,c^2x^4+495b^2c\,x^2+231b^3)}{1155}$$

input
```
int((c*x^4+b*x^2)^3/x^2,x)
```

output
```
(x**5*(231*b**3 + 495*b**2*c*x**2 + 385*b*c**2*x**4 + 105*c**3*x**6))/1155
```

3.38 $\int \frac{(bx^2+cx^4)^3}{x^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{(bx^2+cx^4)^3}{x^4} dx = \frac{b^3x^3}{3} + \frac{3}{5}b^2cx^5 + \frac{3}{7}bc^2x^7 + \frac{c^3x^9}{9}$$

output
```
1/3*b^3*x^3+3/5*b^2*c*x^5+3/7*b*c^2*x^7+1/9*c^3*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^4} dx = \frac{b^3x^3}{3} + \frac{3}{5}b^2cx^5 + \frac{3}{7}bc^2x^7 + \frac{c^3x^9}{9}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^4,x]
```

output
```
(b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^4} \, dx$$

$$\Big\downarrow \; 9$$

$$\int x^2 (b+cx^2)^3 \, dx$$

$$\Big\downarrow \; 244$$

$$\int \left(b^3x^2+3b^2cx^4+3bc^2x^6+c^3x^8\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

input
```
Int[(b*x^2 + c*x^4)^3/x^4,x]
```

output
```
(b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p, x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{1}{3}b^3x^3+\frac{3}{5}b^2c\,x^5+\frac{3}{7}b\,c^2x^7+\frac{1}{9}c^3x^9$	36
risch	$\frac{1}{3}b^3x^3+\frac{3}{5}b^2c\,x^5+\frac{3}{7}b\,c^2x^7+\frac{1}{9}c^3x^9$	36
parallelrisch	$\frac{1}{3}b^3x^3+\frac{3}{5}b^2c\,x^5+\frac{3}{7}b\,c^2x^7+\frac{1}{9}c^3x^9$	36
gosper	$\frac{x^3(35c^3x^6+135b\,c^2x^4+189b^2c\,x^2+105b^3)}{315}$	38
norman	$\frac{\frac{1}{3}b^3x^6+\frac{1}{9}c^3x^{12}+\frac{3}{7}b\,c^2x^{10}+\frac{3}{5}b^2c\,x^8}{x^3}$	40
orering	$\frac{(35c^3x^6+135b\,c^2x^4+189b^2c\,x^2+105b^3)(c\,x^4+b\,x^2)^3}{315x^3(c\,x^2+b)^3}$	60

input
```
int((c*x^4+b*x^2)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^3*x^3+3/5*b^2*c*x^5+3/7*b*c^2*x^7+1/9*c^3*x^9
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2+cx^4)^3}{x^4}\,dx = \frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{1}{3}\,b^3x^3$$

input
```
integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="fricas")
```

output
```
1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{(bx^2+cx^4)^3}{x^4}\,dx = \frac{b^3x^3}{3}+\frac{3b^2cx^5}{5}+\frac{3bc^2x^7}{7}+\frac{c^3x^9}{9}$$

input
```
integrate((c*x**4+b*x**2)**3/x**4,x)
```

output
```
b**3*x**3/3 + 3*b**2*c*x**5/5 + 3*b*c**2*x**7/7 + c**3*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2+cx^4)^3}{x^4}\,dx = \frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{1}{3}\,b^3x^3$$

input
```
integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="maxima")
```

output
```
1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2+cx^4)^3}{x^4}\,dx = \frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{1}{3}\,b^3x^3$$

input
```
integrate((c*x^4+b*x^2)^3/x^4,x, algorithm="giac")
```

output
```
1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2 + cx^4)^3}{x^4}\,dx = \frac{b^3\,x^3}{3} + \frac{3\,b^2\,c\,x^5}{5} + \frac{3\,b\,c^2\,x^7}{7} + \frac{c^3\,x^9}{9}$$

input
```
int((b*x^2 + c*x^4)^3/x^4,x)
```

output
```
(b^3*x^3)/3 + (c^3*x^9)/9 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2 + cx^4)^3}{x^4}\,dx = \frac{x^3(35c^3x^6 + 135b\,c^2x^4 + 189b^2c\,x^2 + 105b^3)}{315}$$

input
```
int((c*x^4+b*x^2)^3/x^4,x)
```

output
```
(x**3*(105*b**3 + 189*b**2*c*x**2 + 135*b*c**2*x**4 + 35*c**3*x**6))/315
```

3.39 $\int \frac{(bx^2+cx^4)^3}{x^6}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 35

$$\int \frac{(bx^2+cx^4)^3}{x^6}\,dx = b^3x+b^2cx^3+\frac{3}{5}bc^2x^5+\frac{c^3x^7}{7}$$

output
```
b^3*x+b^2*c*x^3+3/5*b*c^2*x^5+1/7*c^3*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^6}\,dx = b^3x+b^2cx^3+\frac{3}{5}bc^2x^5+\frac{c^3x^7}{7}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^6,x]
```

output
```
b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^6} \, dx$$
$$\Big\downarrow \text{ 9}$$
$$\int (b+cx^2)^3 \, dx$$
$$\Big\downarrow \text{ 210}$$
$$\int (b^3+3b^2cx^2+3bc^2x^4+c^3x^6) \, dx$$
$$\Big\downarrow \text{ 2009}$$
$$b^3x+b^2cx^3+\frac{3}{5}bc^2x^5+\frac{c^3x^7}{7}$$

input
```
Int[(b*x^2 + c*x^4)^3/x^6,x]
```

output
```
b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$b^{3}x+b^{2}c\,x^{3}+\frac{3}{5}b\,c^{2}x^{5}+\frac{1}{7}c^{3}x^{7}$	32
risch	$b^{3}x+b^{2}c\,x^{3}+\frac{3}{5}b\,c^{2}x^{5}+\frac{1}{7}c^{3}x^{7}$	32
parallelrisch	$b^{3}x+b^{2}c\,x^{3}+\frac{3}{5}b\,c^{2}x^{5}+\frac{1}{7}c^{3}x^{7}$	32
gosper	$\frac{x\left(5c^{3}x^{6}+21b\,c^{2}x^{4}+35b^{2}c\,x^{2}+35b^{3}\right)}{35}$	36
norman	$\frac{b^{3}x^{6}+b^{2}c\,x^{8}+\frac{1}{7}c^{3}x^{12}+\frac{3}{5}b\,c^{2}x^{10}}{x^{5}}$	38
orering	$\frac{\left(5c^{3}x^{6}+21b\,c^{2}x^{4}+35b^{2}c\,x^{2}+35b^{3}\right)\left(c\,x^{4}+b\,x^{2}\right)^{3}}{35x^{5}\left(c\,x^{2}+b\right)^{3}}$	60

input
```
int((c*x^4+b*x^2)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
b^3*x+b^2*c*x^3+3/5*b*c^2*x^5+1/7*c^3*x^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(bx^2+cx^4)^3}{x^6}\,dx=\frac{1}{7}\,c^3x^7+\frac{3}{5}\,bc^2x^5+b^2cx^3+b^3x$$

input
```
integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="fricas")
```

output
```
1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{(bx^2 + cx^4)^3}{x^6} dx = b^3x + b^2cx^3 + \frac{3bc^2x^5}{5} + \frac{c^3x^7}{7}$$

input
```
integrate((c*x**4+b*x**2)**3/x**6,x)
```

output
```
b**3*x + b**2*c*x**3 + 3*b*c**2*x**5/5 + c**3*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(bx^2 + cx^4)^3}{x^6} dx = \frac{1}{7}c^3x^7 + \frac{3}{5}bc^2x^5 + b^2cx^3 + b^3x$$

input
```
integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="maxima")
```

output
```
1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(bx^2 + cx^4)^3}{x^6} dx = \frac{1}{7}c^3x^7 + \frac{3}{5}bc^2x^5 + b^2cx^3 + b^3x$$

input
```
integrate((c*x^4+b*x^2)^3/x^6,x, algorithm="giac")
```

output
```
1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(bx^2+cx^4)^3}{x^6}\,dx = b^3\,x+b^2\,c\,x^3+\frac{3\,b\,c^2\,x^5}{5}+\frac{c^3\,x^7}{7}$$

input
```
int((b*x^2 + c*x^4)^3/x^6,x)
```

output
```
b^3*x + (c^3*x^7)/7 + b^2*c*x^3 + (3*b*c^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^6}\,dx = \frac{x(5c^3x^6+21b\,c^2x^4+35b^2c\,x^2+35b^3)}{35}$$

input
```
int((c*x^4+b*x^2)^3/x^6,x)
```

output
```
(x*(35*b**3 + 35*b**2*c*x**2 + 21*b*c**2*x**4 + 5*c**3*x**6))/35
```

3.40 $\int \frac{(bx^2+cx^4)^3}{x^8} dx$

Optimal result

Integrand size = 17, antiderivative size = 34

$$\int \frac{(bx^2+cx^4)^3}{x^8} dx = -\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

output
```
-b^3/x+3*b^2*c*x+b*c^2*x^3+1/5*c^3*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^8} dx = -\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^8,x]
```

output
```
-(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^8}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(b+cx^2)^3}{x^2}dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{b^3}{x^2}+3b^2c+3bc^2x^2+c^3x^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^3}{x}+3b^2cx+bc^2x^3+\frac{c^3x^5}{5}$$

input
```
Int[(b*x^2 + c*x^4)^3/x^8,x]
```

output
```
-(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{b^3}{x}+3b^2cx+b\,c^2x^3+\frac{c^3x^5}{5}$	33
risch	$-\frac{b^3}{x}+3b^2cx+b\,c^2x^3+\frac{c^3x^5}{5}$	33
parallelrisch	$\frac{c^3x^6+5b\,c^2x^4+15b^2c\,x^2-5b^3}{5x}$	37
gosper	$-\frac{-c^3x^6-5b\,c^2x^4-15b^2c\,x^2+5b^3}{5x}$	38
norman	$\frac{b\,c^2x^{10}-b^3x^6+\frac{1}{5}c^3x^{12}+3b^2c\,x^8}{x^7}$	39
orering	$-\frac{\left(-c^3x^6-5b\,c^2x^4-15b^2c\,x^2+5b^3\right)\left(c\,x^4+b\,x^2\right)^3}{5x^7\left(c\,x^2+b\right)^3}$	60

input
```
int((c*x^4+b*x^2)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-b^3/x+3*b^2*c*x+b*c^2*x^3+1/5*c^3*x^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(bx^2+cx^4)^3}{x^8}\,dx=\frac{c^3x^6+5\,bc^2x^4+15\,b^2cx^2-5\,b^3}{5\,x}$$

input
```
integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="fricas")
```

output
```
1/5*(c^3*x^6 + 5*b*c^2*x^4 + 15*b^2*c*x^2 - 5*b^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{(bx^2 + cx^4)^3}{x^8} \, dx = -\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

input
```
integrate((c*x**4+b*x**2)**3/x**8,x)
```

output
```
-b**3/x + 3*b**2*c*x + b*c**2*x**3 + c**3*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(bx^2 + cx^4)^3}{x^8} \, dx = \frac{1}{5} c^3x^5 + bc^2x^3 + 3\, b^2cx - \frac{b^3}{x}$$

input
```
integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="maxima")
```

output
```
1/5*c^3*x^5 + b*c^2*x^3 + 3*b^2*c*x - b^3/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(bx^2 + cx^4)^3}{x^8} \, dx = \frac{1}{5} c^3x^5 + bc^2x^3 + 3\, b^2cx - \frac{b^3}{x}$$

input
```
integrate((c*x^4+b*x^2)^3/x^8,x, algorithm="giac")
```

output
```
1/5*c^3*x^5 + b*c^2*x^3 + 3*b^2*c*x - b^3/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(bx^2 + cx^4)^3}{x^8} \, dx = \frac{c^3 \, x^5}{5} - \frac{b^3}{x} + b \, c^2 \, x^3 + 3 \, b^2 \, c \, x$$

input
```
int((b*x^2 + c*x^4)^3/x^8,x)
```

output
```
(c^3*x^5)/5 - b^3/x + b*c^2*x^3 + 3*b^2*c*x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(bx^2 + cx^4)^3}{x^8} \, dx = \frac{c^3x^6 + 5b \, c^2x^4 + 15b^2c \, x^2 - 5b^3}{5x}$$

input
```
int((c*x^4+b*x^2)^3/x^8,x)
```

output
```
( - 5*b**3 + 15*b**2*c*x**2 + 5*b*c**2*x**4 + c**3*x**6)/(5*x)
```

3.41 $\int \frac{(bx^2+cx^4)^3}{x^{10}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{(bx^2+cx^4)^3}{x^{10}} \, dx = -\frac{b^3}{3x^3} - \frac{3b^2c}{x} + 3bc^2x + \frac{c^3x^3}{3}$$

output
```
-1/3*b^3/x^3-3*b^2*c/x+3*b*c^2*x+1/3*c^3*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^{10}} \, dx = -\frac{b^3}{3x^3} - \frac{3b^2c}{x} + 3bc^2x + \frac{c^3x^3}{3}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^10,x]
```

output
```
-1/3*b^3/x^3 - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^{10}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(b+cx^2)^3}{x^4}dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{b^3}{x^4}+\frac{3b^2c}{x^2}+3bc^2+c^3x^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

input
```
Int[(b*x^2 + c*x^4)^3/x^10,x]
```

output
```
-1/3*b^3/x^3 - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3b\,c^2x+\frac{c^3x^3}{3}$	34
gosper	$-\frac{-c^3x^6-9b\,c^2x^4+9b^2c\,x^2+b^3}{3x^3}$	36
risch	$\frac{c^3x^3}{3}+3b\,c^2x+\frac{-3b^2c\,x^2-\frac{1}{3}b^3}{x^3}$	36
parallelrisch	$\frac{c^3x^6+9b\,c^2x^4-9b^2c\,x^2-b^3}{3x^3}$	37
norman	$\frac{-\frac{1}{3}b^3x^6+\frac{1}{3}c^3x^{12}+3b\,c^2x^{10}-3b^2c\,x^8}{x^9}$	40
orering	$-\frac{\left(-c^3x^6-9b\,c^2x^4+9b^2c\,x^2+b^3\right)\left(c\,x^4+b\,x^2\right)^3}{3x^9\left(c\,x^2+b\right)^3}$	58

input
```
int((c*x^4+b*x^2)^3/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^3/x^3-3*b^2*c/x+3*b*c^2*x+1/3*c^3*x^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{(bx^2+cx^4)^3}{x^{10}}\,dx=\frac{c^3x^6+9\,bc^2x^4-9\,b^2cx^2-b^3}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="fricas")
```

output
```
1/3*(c^3*x^6 + 9*b*c^2*x^4 - 9*b^2*c*x^2 - b^3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{(bx^2 + cx^4)^3}{x^{10}}\,dx = 3bc^2x + \frac{c^3x^3}{3} + \frac{-b^3 - 9b^2cx^2}{3x^3}$$

input
```
integrate((c*x**4+b*x**2)**3/x**10,x)
```

output
```
3*b*c**2*x + c**3*x**3/3 + (-b**3 - 9*b**2*c*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2 + cx^4)^3}{x^{10}}\,dx = \frac{1}{3}\,c^3x^3 + 3\,bc^2x - \frac{9\,b^2cx^2 + b^3}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="maxima")
```

output
```
1/3*c^3*x^3 + 3*b*c^2*x - 1/3*(9*b^2*c*x^2 + b^3)/x^3
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2 + cx^4)^3}{x^{10}}\,dx = \frac{1}{3}\,c^3x^3 + 3\,bc^2x - \frac{9\,b^2cx^2 + b^3}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2)^3/x^10,x, algorithm="giac")
```

output
```
1/3*c^3*x^3 + 3*b*c^2*x - 1/3*(9*b^2*c*x^2 + b^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{(bx^2+cx^4)^3}{x^{10}}\,dx = \frac{c^3\,x^3}{3} - \frac{\frac{b^3}{3}+3\,c\,b^2\,x^2}{x^3} + 3\,b\,c^2\,x$$

input
```
int((b*x^2 + c*x^4)^3/x^10,x)
```

output
```
(c^3*x^3)/3 - (b^3/3 + 3*b^2*c*x^2)/x^3 + 3*b*c^2*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{(bx^2+cx^4)^3}{x^{10}}\,dx = \frac{c^3x^6+9b\,c^2x^4-9b^2c\,x^2-b^3}{3x^3}$$

input
```
int((c*x^4+b*x^2)^3/x^10,x)
```

output
```
( - b**3 - 9*b**2*c*x**2 + 9*b*c**2*x**4 + c**3*x**6)/(3*x**3)
```

3.42 $\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 34

$$\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx = -\frac{b^3}{5x^5} - \frac{b^2c}{x^3} - \frac{3bc^2}{x} + c^3x$$

output
```
-1/5*b^3/x^5-b^2*c/x^3-3*b*c^2/x+c^3*x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx = -\frac{b^3}{5x^5} - \frac{b^2c}{x^3} - \frac{3bc^2}{x} + c^3x$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^12,x]
```

output
```
-1/5*b^3/x^5 - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(b+cx^2)^3}{x^6}dx$$

$$\Big\downarrow \; 244$$

$$\int \left(\frac{b^3}{x^6}+\frac{3b^2c}{x^4}+\frac{3bc^2}{x^2}+c^3\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x$$

input
```
Int[(b*x^2 + c*x^4)^3/x^12,x]
```

output
```
-1/5*b^3/x^5 - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3b\,c^2}{x}+c^3x$	33
risch	$c^3x+\frac{-3b\,c^2x^4-b^2c\,x^2-\frac{1}{5}b^3}{x^5}$	35
gosper	$-\frac{-5c^3x^6+15b\,c^2x^4+5b^2c\,x^2+b^3}{5x^5}$	36
parallelrisch	$\frac{5c^3x^6-15b\,c^2x^4-5b^2c\,x^2-b^3}{5x^5}$	38
norman	$\frac{c^3x^{12}-\frac{1}{5}b^3x^6-3b\,c^2x^{10}-b^2c\,x^8}{x^{11}}$	39
orering	$-\frac{\left(-5c^3x^6+15b\,c^2x^4+5b^2c\,x^2+b^3\right)\left(c\,x^4+b\,x^2\right)^3}{5x^{11}\left(c\,x^2+b\right)^3}$	58

input
```
int((c*x^4+b*x^2)^3/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/5*b^3/x^5-b^2*c/x^3-3*b*c^2/x+c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx=\frac{5\,c^3x^6-15\,bc^2x^4-5\,b^2cx^2-b^3}{5\,x^5}$$

input
```
integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="fricas")
```

output
```
1/5*(5*c^3*x^6 - 15*b*c^2*x^4 - 5*b^2*c*x^2 - b^3)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx = c^3x + \frac{-b^3-5b^2cx^2-15bc^2x^4}{5x^5}$$

input
```
integrate((c*x**4+b*x**2)**3/x**12,x)
```

output
```
c**3*x + (-b**3 - 5*b**2*c*x**2 - 15*b*c**2*x**4)/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx = c^3x - \frac{15\,bc^2x^4+5\,b^2cx^2+b^3}{5\,x^5}$$

input
```
integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="maxima")
```

output
```
c^3*x - 1/5*(15*b*c^2*x^4 + 5*b^2*c*x^2 + b^3)/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx = c^3x - \frac{15\,bc^2x^4+5\,b^2cx^2+b^3}{5\,x^5}$$

input
```
integrate((c*x^4+b*x^2)^3/x^12,x, algorithm="giac")
```

output
```
c^3*x - 1/5*(15*b*c^2*x^4 + 5*b^2*c*x^2 + b^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx = c^3\,x - \frac{\frac{b^3}{5} + b^2\,c\,x^2 + 3\,b\,c^2\,x^4}{x^5}$$

input
```
int((b*x^2 + c*x^4)^3/x^12,x)
```

output
```
c^3*x - (b^3/5 + b^2*c*x^2 + 3*b*c^2*x^4)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx = \frac{5c^3x^6 - 15b\,c^2x^4 - 5b^2c\,x^2 - b^3}{5x^5}$$

input
```
int((c*x^4+b*x^2)^3/x^12,x)
```

output
```
( - b**3 - 5*b**2*c*x**2 - 15*b*c**2*x**4 + 5*c**3*x**6)/(5*x**5)
```

3.43 $\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx = -\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

output
```
-1/7*b^3/x^7-3/5*b^2*c/x^5-b*c^2/x^3-c^3/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx = -\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^14,x]
```

output
```
-1/7*b^3/x^7 - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^{14}} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(b+cx^2)^3}{x^8} dx$$

$$\Big\downarrow \; 244$$

$$\int \left(\frac{b^3}{x^8}+\frac{3b^2c}{x^6}+\frac{3bc^2}{x^4}+\frac{c^3}{x^2}\right) dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

input
```
Int[(b*x^2 + c*x^4)^3/x^14,x]
```

output
```
-1/7*b^3/x^7 - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{b\,c^2}{x^3}-\frac{c^3}{x}$	36
risch	$\frac{-c^3x^6-b\,c^2x^4-\frac{3}{5}b^2c\,x^2-\frac{1}{7}b^3}{x^7}$	37
gosper	$-\frac{35c^3x^6+35b\,c^2x^4+21b^2c\,x^2+5b^3}{35x^7}$	38
parallelrisch	$\frac{-35c^3x^6-35b\,c^2x^4-21b^2c\,x^2-5b^3}{35x^7}$	38
norman	$\frac{-\frac{1}{7}b^3x^6-c^3x^{12}-b\,c^2x^{10}-\frac{3}{5}b^2c\,x^8}{x^{13}}$	40
orering	$-\frac{\left(35c^3x^6+35b\,c^2x^4+21b^2c\,x^2+5b^3\right)\left(c\,x^4+b\,x^2\right)^3}{35x^{13}\left(c\,x^2+b\right)^3}$	60

input
```
int((c*x^4+b*x^2)^3/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/7*b^3/x^7-3/5*b^2*c/x^5-b*c^2/x^3-c^3/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx=-\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

input
```
integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="fricas")
```

output
```
-1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx = \frac{-5b^3-21b^2cx^2-35bc^2x^4-35c^3x^6}{35x^7}$$

input
```
integrate((c*x**4+b*x**2)**3/x**14,x)
```

output
```
(-5*b**3 - 21*b**2*c*x**2 - 35*b*c**2*x**4 - 35*c**3*x**6)/(35*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx = -\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

input
```
integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="maxima")
```

output
```
-1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx = -\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

input
```
integrate((c*x^4+b*x^2)^3/x^14,x, algorithm="giac")
```

output
```
-1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx = -\frac{\frac{b^3}{7}+\frac{3\,b^2\,c\,x^2}{5}+b\,c^2\,x^4+c^3\,x^6}{x^7}$$

input
```
int((b*x^2 + c*x^4)^3/x^14,x)
```

output
```
-(b^3/7 + c^3*x^6 + (3*b^2*c*x^2)/5 + b*c^2*x^4)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx = \frac{-35c^3x^6-35b\,c^2x^4-21b^2c\,x^2-5b^3}{35x^7}$$

input
```
int((c*x^4+b*x^2)^3/x^14,x)
```

output
```
( - 5*b**3 - 21*b**2*c*x**2 - 35*b*c**2*x**4 - 35*c**3*x**6)/(35*x**7)
```

3.44 $\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

output
```
-1/9*b^3/x^9-3/7*b^2*c/x^7-3/5*b*c^2/x^5-1/3*c^3/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^16,x]
```

output
```
-1/9*b^3/x^9 - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{(b+cx^2)^3}{x^{10}}dx$$

$$\Big\downarrow \ 244$$

$$\int \left(\frac{b^3}{x^{10}}+\frac{3b^2c}{x^8}+\frac{3bc^2}{x^6}+\frac{c^3}{x^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

input
```
Int[(b*x^2 + c*x^4)^3/x^16,x]
```

output
```
-1/9*b^3/x^9 - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3b\,c^2}{5x^5}-\frac{c^3}{3x^3}$	36
risch	$\frac{-\frac{1}{3}c^3x^6-\frac{3}{5}b\,c^2x^4-\frac{3}{7}b^2c\,x^2-\frac{1}{9}b^3}{x^9}$	37
gosper	$-\frac{105c^3x^6+189b\,c^2x^4+135b^2c\,x^2+35b^3}{315x^9}$	38
parallelrisch	$\frac{-105c^3x^6-189b\,c^2x^4-135b^2c\,x^2-35b^3}{315x^9}$	38
norman	$\frac{-\frac{1}{9}b^3x^6-\frac{1}{3}c^3x^{12}-\frac{3}{5}b\,c^2x^{10}-\frac{3}{7}b^2c\,x^8}{x^{15}}$	40
orering	$-\frac{(105c^3x^6+189b\,c^2x^4+135b^2c\,x^2+35b^3)\left(c\,x^4+b\,x^2\right)^3}{315x^{15}\left(c\,x^2+b\right)^3}$	60

input
```
int((c*x^4+b*x^2)^3/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/9*b^3/x^9-3/7*b^2*c/x^7-3/5*b*c^2/x^5-1/3*c^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{105\,c^3x^6+189\,bc^2x^4+135\,b^2cx^2+35\,b^3}{315\,x^9}$$

input
```
integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="fricas")
```

output
```
-1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx=\frac{-35b^3-135b^2cx^2-189bc^2x^4-105c^3x^6}{315x^9}$$

input
```
integrate((c*x**4+b*x**2)**3/x**16,x)
```

output
```
(-35*b**3 - 135*b**2*c*x**2 - 189*b*c**2*x**4 - 105*c**3*x**6)/(315*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx=-\frac{105\,c^3x^6+189\,bc^2x^4+135\,b^2cx^2+35\,b^3}{315\,x^9}$$

input
```
integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="maxima")
```

output
```
-1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx=-\frac{105\,c^3x^6+189\,bc^2x^4+135\,b^2cx^2+35\,b^3}{315\,x^9}$$

input
```
integrate((c*x^4+b*x^2)^3/x^16,x, algorithm="giac")
```

output
```
-1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

Mupad [B] (verification not implemented)

Time = 16.96 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{\frac{b^3}{9}+\frac{3\,b^2\,c\,x^2}{7}+\frac{3\,b\,c^2\,x^4}{5}+\frac{c^3\,x^6}{3}}{x^9}$$

input
```
int((b*x^2 + c*x^4)^3/x^16,x)
```

output
```
-(b^3/9 + (c^3*x^6)/3 + (3*b^2*c*x^2)/7 + (3*b*c^2*x^4)/5)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx = \frac{-105c^3x^6-189b\,c^2x^4-135b^2c\,x^2-35b^3}{315x^9}$$

input
```
int((c*x^4+b*x^2)^3/x^16,x)
```

output
```
( - 35*b**3 - 135*b**2*c*x**2 - 189*b*c**2*x**4 - 105*c**3*x**6)/(315*x**9
)
```

3.45 $\int \frac{x^9}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{x^9}{bx^2+cx^4}\,dx=\frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}-\frac{b^3\log\left(b+cx^2\right)}{2c^4}$$

output
```
1/2*b^2*x^2/c^3-1/4*b*x^4/c^2+1/6*x^6/c-1/2*b^3*ln(c*x^2+b)/c^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{bx^2+cx^4}\,dx=\frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}-\frac{b^3\log\left(b+cx^2\right)}{2c^4}$$

input
```
Integrate[x^9/(b*x^2 + c*x^4),x]
```

output
```
(b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*x^2])/(2*
c^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^7}{b+cx^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^6}{cx^2+b}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{x^4}{c}-\frac{bx^2}{c^2}-\frac{b^3}{c^3\left(cx^2+b\right)}+\frac{b^2}{c^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{b^3\log\left(b+cx^2\right)}{c^4}+\frac{b^2x^2}{c^3}-\frac{bx^4}{2c^2}+\frac{x^6}{3c}\right)$$

input
```
Int[x^9/(b*x^2 + c*x^4),x]
```

output
```
((b^2*x^2)/c^3 - (b*x^4)/(2*c^2) + x^6/(3*c) - (b^3*Log[b + c*x^2])/c^4)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\frac{1}{3}c^{2}x^{6}-\frac{1}{2}b c\,x^{4}+b^{2}x^{2}}{2c^{3}}-\frac{b^{3}\ln\left(c\,x^{2}+b\right)}{2c^{4}}$	46
risch	$\frac{b^{2}x^{2}}{2c^{3}}-\frac{b\,x^{4}}{4c^{2}}+\frac{x^{6}}{6c}-\frac{b^{3}\ln\left(c\,x^{2}+b\right)}{2c^{4}}$	46
parallelrisch	$-\frac{-2c^{3}x^{6}+3b\,c^{2}x^{4}-6b^{2}c\,x^{2}+6b^{3}\ln\left(c\,x^{2}+b\right)}{12c^{4}}$	46
norman	$\frac{\frac{x^{7}}{6c}-\frac{b\,x^{5}}{4c^{2}}+\frac{b^{2}x^{3}}{2c^{3}}}{x}-\frac{b^{3}\ln\left(c\,x^{2}+b\right)}{2c^{4}}$	51

input
```
int(x^9/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c^3*(1/3*c^2*x^6-1/2*b*c*x^4+b^2*x^2)-1/2*b^3*ln(c*x^2+b)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^9}{bx^2+cx^4}\,dx = \frac{2\,c^3x^6 - 3\,bc^2x^4 + 6\,b^2cx^2 - 6\,b^3\log\left(cx^2+b\right)}{12\,c^4}$$

input
```
integrate(x^9/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/12*(2*c^3*x^6 - 3*b*c^2*x^4 + 6*b^2*c*x^2 - 6*b^3*log(c*x^2 + b))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{x^9}{bx^2+cx^4}\,dx = -\frac{b^3\log\left(b+cx^2\right)}{2c^4} + \frac{b^2x^2}{2c^3} - \frac{bx^4}{4c^2} + \frac{x^6}{6c}$$

input
```
integrate(x**9/(c*x**4+b*x**2),x)
```

output
```
-b**3*log(b + c*x**2)/(2*c**4) + b**2*x**2/(2*c**3) - b*x**4/(4*c**2) + x*
*6/(6*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{bx^2+cx^4}\,dx = -\frac{b^3\log\left(cx^2+b\right)}{2\,c^4} + \frac{2\,c^2x^6 - 3\,bcx^4 + 6\,b^2x^2}{12\,c^3}$$

input
```
integrate(x^9/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/2*b^3*log(c*x^2 + b)/c^4 + 1/12*(2*c^2*x^6 - 3*b*c*x^4 + 6*b^2*x^2)/c^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x^9}{bx^2+cx^4}\,dx = -\frac{b^3\log\left(|cx^2+b|\right)}{2\,c^4} + \frac{2\,c^2x^6-3\,bcx^4+6\,b^2x^2}{12\,c^3}$$

input
```
integrate(x^9/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-1/2*b^3*log(abs(c*x^2 + b))/c^4 + 1/12*(2*c^2*x^6 - 3*b*c*x^4 + 6*b^2*x^2
)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^9}{bx^2+cx^4}\,dx = \frac{x^6}{6\,c} - \frac{b\,x^4}{4\,c^2} - \frac{b^3\,\ln\left(c\,x^2+b\right)}{2\,c^4} + \frac{b^2\,x^2}{2\,c^3}$$

input
```
int(x^9/(b*x^2 + c*x^4),x)
```

output
```
x^6/(6*c) - (b*x^4)/(4*c^2) - (b^3*log(b + c*x^2))/(2*c^4) + (b^2*x^2)/(2*
c^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^9}{bx^2+cx^4}\,dx = \frac{-6\log(c\,x^2+b)\,b^3+6b^2c\,x^2-3b\,c^2x^4+2c^3x^6}{12c^4}$$

input
```
int(x^9/(c*x^4+b*x^2),x)
```

output
```
( - 6*log(b + c*x**2)*b**3 + 6*b**2*c*x**2 - 3*b*c**2*x**4 + 2*c**3*x**6)/
(12*c**4)
```

3.46 $\int \frac{x^7}{bx^2+cx^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{x^7}{bx^2+cx^4} \, dx = -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b^2 \log (b+cx^2)}{2c^3}$$

output
```
-1/2*b*x^2/c^2+1/4*x^4/c+1/2*b^2*ln(c*x^2+b)/c^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{bx^2+cx^4} \, dx = -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b^2 \log (b+cx^2)}{2c^3}$$

input
```
Integrate[x^7/(b*x^2 + c*x^4),x]
```

output
```
-1/2*(b*x^2)/c^2 + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^5}{b+cx^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^4}{cx^2+b}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{b^2}{c^2\,(cx^2+b)}-\frac{b}{c^2}+\frac{x^2}{c}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{b^2\log\left(b+cx^2\right)}{c^3}-\frac{bx^2}{c^2}+\frac{x^4}{2c}\right)$$

input
```
Int[x^7/(b*x^2 + c*x^4),x]
```

output
```
(-((b*x^2)/c^2) + x^4/(2*c) + (b^2*Log[b + c*x^2])/c^3)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{c^{2} x^{4}-2 b c \,x^{2}+2 b^{2} \ln \left(c \,x^{2}+b \right)}{4 c^{3}}$	34
default	$-\frac{-\frac{1}{2} c \,x^{4}+b \,x^{2}}{2 c^{2}}+\frac{b^{2} \ln \left(c \,x^{2}+b \right)}{2 c^{3}}$	35
norman	$\frac{\frac{x^{5}}{4 c}-\frac{b \,x^{3}}{2 c^{2}}}{x}+\frac{b^{2} \ln \left(c \,x^{2}+b \right)}{2 c^{3}}$	40
risch	$\frac{x^{4}}{4 c}-\frac{b \,x^{2}}{2 c^{2}}+\frac{b^{2}}{4 c^{3}}+\frac{b^{2} \ln \left(c \,x^{2}+b \right)}{2 c^{3}}$	43

input
```
int(x^7/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(c^2*x^4-2*b*c*x^2+2*b^2*ln(c*x^2+b))/c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{bx^2+cx^4}\,dx = \frac{c^2x^4 - 2\,bcx^2 + 2\,b^2\log\left(cx^2+b\right)}{4\,c^3}$$

input
```
integrate(x^7/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/4*(c^2*x^4 - 2*b*c*x^2 + 2*b^2*log(c*x^2 + b))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^7}{bx^2+cx^4}\,dx = \frac{b^2\log\left(b+cx^2\right)}{2c^3} - \frac{bx^2}{2c^2} + \frac{x^4}{4c}$$

input
```
integrate(x**7/(c*x**4+b*x**2),x)
```

output
```
b**2*log(b + c*x**2)/(2*c**3) - b*x**2/(2*c**2) + x**4/(4*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{bx^2+cx^4}\,dx = \frac{b^2\log\left(cx^2+b\right)}{2\,c^3} + \frac{cx^4 - 2\,bx^2}{4\,c^2}$$

input
```
integrate(x^7/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/2*b^2*log(c*x^2 + b)/c^3 + 1/4*(c*x^4 - 2*b*x^2)/c^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{bx^2+cx^4}\,dx = \frac{b^2\log(|cx^2+b|)}{2\,c^3} + \frac{cx^4-2\,bx^2}{4\,c^2}$$

input
```
integrate(x^7/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*b^2*log(abs(c*x^2 + b))/c^3 + 1/4*(c*x^4 - 2*b*x^2)/c^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{bx^2+cx^4}\,dx = \frac{2\,b^2\,\ln(c\,x^2+b)+c^2\,x^4-2\,b\,c\,x^2}{4\,c^3}$$

input
```
int(x^7/(b*x^2 + c*x^4),x)
```

output
```
(2*b^2*log(b + c*x^2) + c^2*x^4 - 2*b*c*x^2)/(4*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{bx^2+cx^4}\,dx = \frac{2\log(c\,x^2+b)\,b^2-2bc\,x^2+c^2x^4}{4c^3}$$

input
```
int(x^7/(c*x^4+b*x^2),x)
```

output
```
(2*log(b + c*x**2)*b**2 - 2*b*c*x**2 + c**2*x**4)/(4*c**3)
```

3.47 $\int \frac{x^5}{bx^2+cx^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{x^5}{bx^2+cx^4} \, dx = \frac{x^2}{2c} - \frac{b \log (b+cx^2)}{2c^2}$$

output
```
1/2*x^2/c-1/2*b*ln(c*x^2+b)/c^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{bx^2+cx^4} \, dx = \frac{x^2}{2c} - \frac{b \log (b+cx^2)}{2c^2}$$

input
```
Integrate[x^5/(b*x^2 + c*x^4),x]
```

output
```
x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^3}{b+cx^2}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{x^2}{cx^2+b}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\frac{1}{c}-\frac{b}{c\,(cx^2+b)}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{x^2}{c}-\frac{b\log\left(b+cx^2\right)}{c^2}\right)$$

input
```
Int[x^5/(b*x^2 + c*x^4),x]
```

output
```
(x^2/c - (b*Log[b + c*x^2])/c^2)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$-\frac{-c\,x^{2}+b\ln\left(c\,x^{2}+b\right)}{2c^{2}}$	23
default	$\frac{x^{2}}{2c}-\frac{b\ln\left(c\,x^{2}+b\right)}{2c^{2}}$	24
norman	$\frac{x^{2}}{2c}-\frac{b\ln\left(c\,x^{2}+b\right)}{2c^{2}}$	24
risch	$\frac{x^{2}}{2c}-\frac{b\ln\left(c\,x^{2}+b\right)}{2c^{2}}$	24

input
```
int(x^5/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-c*x^2+b*ln(c*x^2+b))/c^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{bx^2 + cx^4}\, dx = \frac{cx^2 - b\log\left(cx^2 + b\right)}{2\,c^2}$$

input
```
integrate(x^5/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/2*(c*x^2 - b*log(c*x^2 + b))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{bx^2 + cx^4}\, dx = -\frac{b\log\left(b + cx^2\right)}{2c^2} + \frac{x^2}{2c}$$

input
```
integrate(x**5/(c*x**4+b*x**2),x)
```

output
```
-b*log(b + c*x**2)/(2*c**2) + x**2/(2*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{bx^2 + cx^4}\, dx = \frac{x^2}{2\,c} - \frac{b\log\left(cx^2 + b\right)}{2\,c^2}$$

input
```
integrate(x^5/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/2*x^2/c - 1/2*b*log(c*x^2 + b)/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{bx^2+cx^4}\,dx = \frac{x^2}{2\,c} - \frac{b\log\left(|cx^2+b|\right)}{2\,c^2}$$

input
```
integrate(x^5/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*x^2/c - 1/2*b*log(abs(c*x^2 + b))/c^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{bx^2+cx^4}\,dx = -\frac{b\,\ln\left(c\,x^2+b\right)-c\,x^2}{2\,c^2}$$

input
```
int(x^5/(b*x^2 + c*x^4),x)
```

output
```
-(b*log(b + c*x^2) - c*x^2)/(2*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{bx^2+cx^4}\,dx = \frac{-\log(c\,x^2+b)\,b+c\,x^2}{2c^2}$$

input
```
int(x^5/(c*x^4+b*x^2),x)
```

output
```
( - log(b + c*x**2)*b + c*x**2)/(2*c**2)
```

3.48 $\int \frac{x^3}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{x^3}{bx^2+cx^4}\,dx=\frac{\log(b+cx^2)}{2c}$$

output
```
1/2*ln(c*x^2+b)/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{bx^2+cx^4}\,dx=\frac{\log(b+cx^2)}{2c}$$

input
```
Integrate[x^3/(b*x^2 + c*x^4),x]
```

output
```
Log[b + c*x^2]/(2*c)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{bx^2+cx^4}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x}{b+cx^2}dx$$
$$\Big\downarrow \; 240$$
$$\frac{\log\left(b+cx^2\right)}{2c}$$

input
```
Int[x^3/(b*x^2 + c*x^4),x]
```

output
```
Log[b + c*x^2]/(2*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(c\,x^2+b)}{2c}$	14
norman	$\frac{\ln(c\,x^2+b)}{2c}$	14
risch	$\frac{\ln(c\,x^2+b)}{2c}$	14
parallelrisch	$\frac{\ln(c\,x^2+b)}{2c}$	14

input
```
int(x^3/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*x^2+b)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{bx^2+cx^4}\,dx=\frac{\log\left(cx^2+b\right)}{2\,c}$$

input
```
integrate(x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/2*log(c*x^2 + b)/c
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{bx^2+cx^4}\,dx=\frac{\log\left(b+cx^2\right)}{2c}$$

input
```
integrate(x**3/(c*x**4+b*x**2),x)
```

output
```
log(b + c*x**2)/(2*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{bx^2 + cx^4} \, dx = \frac{\log (cx^2 + b)}{2 \, c}$$

input
```
integrate(x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/2*log(c*x^2 + b)/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{bx^2 + cx^4} \, dx = \frac{\log (|cx^2 + b|)}{2 \, c}$$

input
```
integrate(x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*log(abs(c*x^2 + b))/c
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{bx^2 + cx^4} \, dx = \frac{\ln (c \, x^2 + b)}{2 \, c}$$

input
```
int(x^3/(b*x^2 + c*x^4),x)
```

output
```
log(b + c*x^2)/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{bx^2 + cx^4}\, dx = \frac{\log(c\, x^2 + b)}{2c}$$

input
```
int(x^3/(c*x^4+b*x^2),x)
```

output
```
log(b + c*x**2)/(2*c)
```

3.49 $\int \frac{x}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{x}{bx^2+cx^4}\,dx=\frac{\log(x)}{b}-\frac{\log(b+cx^2)}{2b}$$

output
```
ln(x)/b-1/2*ln(c*x^2+b)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{bx^2+cx^4}\,dx=\frac{\log(x)}{b}-\frac{\log(b+cx^2)}{2b}$$

input
```
Integrate[x/(b*x^2 + c*x^4),x]
```

output
```
Log[x]/b - Log[b + c*x^2]/(2*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {9, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{bx^2+cx^4}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{1}{x(b+cx^2)}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{1}{x^2(cx^2+b)}dx^2$$

$$\Big\downarrow \ 47$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2}dx^2}{b}-\frac{c\int \frac{1}{cx^2+b}dx^2}{b}\right)$$

$$\Big\downarrow \ 14$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{b}-\frac{c\int \frac{1}{cx^2+b}dx^2}{b}\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{b}-\frac{\log(b+cx^2)}{b}\right)$$

input
```
Int[x/(b*x^2 + c*x^4),x]
```

output
```
(Log[x^2]/b - Log[b + c*x^2]/b)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(x)}{b}-\frac{\ln(c\,x^{2}+b)}{2b}$	21
norman	$\frac{\ln(x)}{b}-\frac{\ln(c\,x^{2}+b)}{2b}$	21
risch	$\frac{\ln(x)}{b}-\frac{\ln(c\,x^{2}+b)}{2b}$	21
parallelrisch	$\frac{2\ln(x)-\ln(c\,x^{2}+b)}{2b}$	21

input
```
int(x/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/b-1/2*ln(c*x^2+b)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x}{bx^2 + cx^4} \, dx = -\frac{\log (cx^2 + b) - 2 \log (x)}{2 \, b}$$

input
```
integrate(x/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/2*(log(c*x^2 + b) - 2*log(x))/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{x}{bx^2 + cx^4} \, dx = \frac{\log (x)}{b} - \frac{\log \left(\frac{b}{c} + x^2\right)}{2b}$$

input
```
integrate(x/(c*x**4+b*x**2),x)
```

output
```
log(x)/b - log(b/c + x**2)/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x}{bx^2 + cx^4} \, dx = -\frac{\log (cx^2 + b)}{2 \, b} + \frac{\log (x^2)}{2 \, b}$$

input
```
integrate(x/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/2*log(c*x^2 + b)/b + 1/2*log(x^2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{bx^2 + cx^4}\,dx = -\frac{\log\left(|cx^2 + b|\right)}{2\,b} + \frac{\log\left(|x|\right)}{b}$$

input
```
integrate(x/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-1/2*log(abs(c*x^2 + b))/b + log(abs(x))/b
```

Mupad [B] (verification not implemented)

Time = 17.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x}{bx^2 + cx^4}\,dx = -\frac{\ln\left(c\,x^2 + b\right) - 2\,\ln\left(x\right)}{2\,b}$$

input
```
int(x/(b*x^2 + c*x^4),x)
```

output
```
-(log(b + c*x^2) - 2*log(x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{bx^2 + cx^4}\,dx = \frac{-\log(c\,x^2 + b) + 2\log(x)}{2b}$$

input
```
int(x/(c*x^4+b*x^2),x)
```

output
```
( - log(b + c*x**2) + 2*log(x))/(2*b)
```

3.50 $\int \frac{1}{x(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 35

$$\int \frac{1}{x\left(bx^2+cx^4\right)}\,dx=-\frac{1}{2bx^2}-\frac{c\log(x)}{b^2}+\frac{c\log\left(b+cx^2\right)}{2b^2}$$

output
```
-1/2/b/x^2-c*ln(x)/b^2+1/2*c*ln(c*x^2+b)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(bx^2+cx^4\right)}\,dx=-\frac{1}{2bx^2}-\frac{c\log(x)}{b^2}+\frac{c\log\left(b+cx^2\right)}{2b^2}$$

input
```
Integrate[1/(x*(b*x^2 + c*x^4)),x]
```

output
```
-1/2*1/(b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(bx^2+cx^4\right)}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{x^3\left(b+cx^2\right)}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^4\left(cx^2+b\right)}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(\frac{c^2}{b^2\left(cx^2+b\right)}-\frac{c}{b^2x^2}+\frac{1}{bx^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{c\log\left(x^2\right)}{b^2}+\frac{c\log\left(b+cx^2\right)}{b^2}-\frac{1}{bx^2}\right)$$

input
```
Int[1/(x*(b*x^2 + c*x^4)),x]
```

output
```
(-(1/(b*x^2)) - (c*Log[x^2])/b^2 + (c*Log[b + c*x^2])/b^2)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{2b\,x^{2}}-\frac{c\ln\left(x\right)}{b^{2}}+\frac{c\ln\left(c\,x^{2}+b\right)}{2b^{2}}$	32
norman	$-\frac{1}{2b\,x^{2}}-\frac{c\ln\left(x\right)}{b^{2}}+\frac{c\ln\left(c\,x^{2}+b\right)}{2b^{2}}$	32
parallelrisch	$-\frac{2c\ln\left(x\right)x^{2}-c\ln\left(c\,x^{2}+b\right)x^{2}+b}{2x^{2}b^{2}}$	33
risch	$-\frac{1}{2b\,x^{2}}-\frac{c\ln\left(x\right)}{b^{2}}+\frac{c\ln\left(-c\,x^{2}-b\right)}{2b^{2}}$	35

input
```
int(1/x/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/x^2-c*ln(x)/b^2+1/2*c*ln(c*x^2+b)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\,(bx^2+cx^4)}\,dx = \frac{cx^2\log\left(cx^2+b\right)-2\,cx^2\log\left(x\right)-b}{2\,b^2x^2}$$

input
```
integrate(1/x/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/2*(c*x^2*log(c*x^2 + b) - 2*c*x^2*log(x) - b)/(b^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\,(bx^2+cx^4)}\,dx = -\frac{1}{2bx^2}-\frac{c\log\left(x\right)}{b^2}+\frac{c\log\left(\frac{b}{c}+x^2\right)}{2b^2}$$

input
```
integrate(1/x/(c*x**4+b*x**2),x)
```

output
```
-1/(2*b*x**2) - c*log(x)/b**2 + c*log(b/c + x**2)/(2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\,(bx^2+cx^4)}\,dx = \frac{c\log\left(cx^2+b\right)}{2\,b^2}-\frac{c\log\left(x^2\right)}{2\,b^2}-\frac{1}{2\,bx^2}$$

input
```
integrate(1/x/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/2*c*log(c*x^2 + b)/b^2 - 1/2*c*log(x^2)/b^2 - 1/2/(b*x^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\,(bx^2+cx^4)}\,dx = -\frac{c\log(x^2)}{2\,b^2} + \frac{c\log(|cx^2+b|)}{2\,b^2} + \frac{cx^2-b}{2\,b^2x^2}$$

input
```
integrate(1/x/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-1/2*c*log(x^2)/b^2 + 1/2*c*log(abs(c*x^2 + b))/b^2 + 1/2*(c*x^2 - b)/(b^2
*x^2)
```

Mupad [B] (verification not implemented)

Time = 16.99 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\,(bx^2+cx^4)}\,dx = \frac{c\,\ln(c\,x^2+b)}{2\,b^2} - \frac{1}{2\,b\,x^2} - \frac{c\,\ln(x)}{b^2}$$

input
```
int(1/(x*(b*x^2 + c*x^4)),x)
```

output
```
(c*log(b + c*x^2))/(2*b^2) - 1/(2*b*x^2) - (c*log(x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\,(bx^2+cx^4)}\,dx = \frac{\log(c\,x^2+b)\,c\,x^2 - 2\log(x)\,c\,x^2 - b}{2b^2x^2}$$

input
```
int(1/x/(c*x^4+b*x^2),x)
```

output
```
(log(b + c*x**2)*c*x**2 - 2*log(x)*c*x**2 - b)/(2*b**2*x**2)
```

3.51 $\int \frac{1}{x^3(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{1}{x^3\left(bx^2+cx^4\right)}\,dx=-\frac{1}{4bx^4}+\frac{c}{2b^2x^2}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log\left(b+cx^2\right)}{2b^3}$$

output
```
-1/4/b/x^4+1/2*c/b^2/x^2+c^2*ln(x)/b^3-1/2*c^2*ln(c*x^2+b)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(bx^2+cx^4\right)}\,dx=-\frac{1}{4bx^4}+\frac{c}{2b^2x^2}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log\left(b+cx^2\right)}{2b^3}$$

input
```
Integrate[1/(x^3*(b*x^2 + c*x^4)),x]
```

output
```
-1/4*1/(b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^2])/(
2*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (bx^2 + cx^4)} \, dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{x^5 (b + cx^2)} dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2} \int \frac{1}{x^6 (cx^2 + b)} dx^2$$
$$\Big\downarrow \text{ 54}$$
$$\frac{1}{2} \int \left(-\frac{c^3}{b^3 (cx^2 + b)} + \frac{c^2}{b^3 x^2} - \frac{c}{b^2 x^4} + \frac{1}{bx^6} \right) dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2} \left(\frac{c^2 \log (x^2)}{b^3} - \frac{c^2 \log (b + cx^2)}{b^3} + \frac{c}{b^2 x^2} - \frac{1}{2bx^4} \right)$$

input
```
Int[1/(x^3*(b*x^2 + c*x^4)),x]
```

output
```
(-1/2*1/(b*x^4) + c/(b^2*x^2) + (c^2*Log[x^2])/b^3 - (c^2*Log[b + c*x^2])/
b^3)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{1}{4b\,x^{4}}+\frac{c}{2b^{2}x^{2}}+\frac{c^{2}\ln\left(x\right)}{b^{3}}-\frac{c^{2}\ln\left(c\,x^{2}+b\right)}{2b^{3}}$	44
norman	$\frac{-\frac{1}{4b}+\frac{c\,x^{2}}{2b^{2}}}{x^{4}}+\frac{c^{2}\ln\left(x\right)}{b^{3}}-\frac{c^{2}\ln\left(c\,x^{2}+b\right)}{2b^{3}}$	46
risch	$\frac{-\frac{1}{4b}+\frac{c\,x^{2}}{2b^{2}}}{x^{4}}+\frac{c^{2}\ln\left(x\right)}{b^{3}}-\frac{c^{2}\ln\left(c\,x^{2}+b\right)}{2b^{3}}$	46
parallelrisch	$\frac{4c^{2}\ln\left(x\right)x^{4}-2c^{2}\ln\left(c\,x^{2}+b\right)x^{4}+2bc\,x^{2}-b^{2}}{4b^{3}x^{4}}$	48

input
```
int(1/x^3/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b/x^4+1/2*c/b^2/x^2+c^2*ln(x)/b^3-1/2*c^2*ln(c*x^2+b)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 (bx^2 + cx^4)} \, dx = -\frac{2 \, c^2 x^4 \log \left(cx^2 + b\right) - 4 \, c^2 x^4 \log \left(x\right) - 2 \, bcx^2 + b^2}{4 \, b^3 x^4}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/4*(2*c^2*x^4*log(c*x^2 + b) - 4*c^2*x^4*log(x) - 2*b*c*x^2 + b^2)/(b^3*
x^4)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3 (bx^2 + cx^4)} \, dx = \frac{-b + 2cx^2}{4b^2x^4} + \frac{c^2 \log \left(x\right)}{b^3} - \frac{c^2 \log \left(\frac{b}{c} + x^2\right)}{2b^3}$$

input
```
integrate(1/x**3/(c*x**4+b*x**2),x)
```

output
```
(-b + 2*c*x**2)/(4*b**2*x**4) + c**2*log(x)/b**3 - c**2*log(b/c + x**2)/(2
*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3 (bx^2 + cx^4)} \, dx = -\frac{c^2 \log \left(cx^2 + b\right)}{2 \, b^3} + \frac{c^2 \log \left(x^2\right)}{2 \, b^3} + \frac{2 \, cx^2 - b}{4 \, b^2 x^4}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/2*c^2*log(c*x^2 + b)/b^3 + 1/2*c^2*log(x^2)/b^3 + 1/4*(2*c*x^2 - b)/(b^
2*x^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3 (bx^2 + cx^4)} \, dx = \frac{c^2 \log(x^2)}{2 b^3} - \frac{c^2 \log(|cx^2 + b|)}{2 b^3} - \frac{3 c^2 x^4 - 2 bcx^2 + b^2}{4 b^3 x^4}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*c^2*log(x^2)/b^3 - 1/2*c^2*log(abs(c*x^2 + b))/b^3 - 1/4*(3*c^2*x^4 -
2*b*c*x^2 + b^2)/(b^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 16.97 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 (bx^2 + cx^4)} \, dx = \frac{c^2 \ln(x)}{b^3} - \frac{c^2 \ln(c x^2 + b)}{2 b^3} - \frac{\frac{1}{4 b} - \frac{c x^2}{2 b^2}}{x^4}$$

input
```
int(1/(x^3*(b*x^2 + c*x^4)),x)
```

output
```
(c^2*log(x))/b^3 - (c^2*log(b + c*x^2))/(2*b^3) - (1/(4*b) - (c*x^2)/(2*b^
2))/x^4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3 (bx^2 + cx^4)} \, dx = \frac{-2 \log(c x^2 + b) c^2 x^4 + 4 \log(x) c^2 x^4 - b^2 + 2bc x^2}{4b^3 x^4}$$

input
```
int(1/x^3/(c*x^4+b*x^2),x)
```

output
```
( - 2*log(b + c*x**2)*c**2*x**4 + 4*log(x)*c**2*x**4 - b**2 + 2*b*c*x**2)/
(4*b**3*x**4)
```

3.52 $\int \frac{x^{10}}{bx^2+cx^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{x^{10}}{bx^2+cx^4}\,dx = -\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}+\frac{b^{7/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}$$

output
```
-b^3*x/c^4+1/3*b^2*x^3/c^3-1/5*b*x^5/c^2+1/7*x^7/c+b^(7/2)*arctan(c^(1/2)*
x/b^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{x^{10}}{bx^2+cx^4}\,dx = -\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}+\frac{b^{7/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}$$

input
```
Integrate[x^10/(b*x^2 + c*x^4),x]
```

output
```
-((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c) + (b^(7/2
)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{bx^2+cx^4}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x^8}{b+cx^2}dx$$
$$\Big\downarrow \; 254$$
$$\int \left(\frac{b^4}{c^4\left(b+cx^2\right)}-\frac{b^3}{c^4}+\frac{b^2x^2}{c^3}-\frac{bx^4}{c^2}+\frac{x^6}{c}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{b^{7/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}$$

input
```
Int[x^10/(b*x^2 + c*x^4),x]
```

output
```
-((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c) + (b^(7/2
)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{-\frac{1}{7}c^{3}x^{7}+\frac{1}{5}b\,c^{2}x^{5}-\frac{1}{3}b^{2}c\,x^{3}+b^{3}x}{c^{4}}+\frac{b^{4}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^{4}\sqrt{bc}}$	60
risch	$\frac{x^{7}}{7c}-\frac{b\,x^{5}}{5c^{2}}+\frac{b^{2}x^{3}}{3c^{3}}-\frac{b^{3}x}{c^{4}}+\frac{\sqrt{-bc}\,b^{3}\ln\left(-\sqrt{-bc}\,x+b\right)}{2c^{5}}-\frac{\sqrt{-bc}\,b^{3}\ln\left(\sqrt{-bc}\,x+b\right)}{2c^{5}}$	90

input
```
int(x^10/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/c^4*(-1/7*c^3*x^7+1/5*b*c^2*x^5-1/3*b^2*c*x^3+b^3*x)+b^4/c^4/(b*c)^(1/2
)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.18

$$\int \frac{x^{10}}{bx^2+cx^4}\,dx = \left[\frac{30\,c^3x^7-42\,bc^2x^5+70\,b^2cx^3+105\,b^3\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-210\,b^3x}{210\,c^4}, \frac{15\,c^3x^7-21\,bc^2x^5+35\,b^2c\right.$$

input
```
integrate(x^10/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/210*(30*c^3*x^7 - 42*b*c^2*x^5 + 70*b^2*c*x^3 + 105*b^3*sqrt(-b/c)*log(
(c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) - 210*b^3*x)/c^4, 1/105*(15*c^
3*x^7 - 21*b*c^2*x^5 + 35*b^2*c*x^3 + 105*b^3*sqrt(b/c)*arctan(c*x*sqrt(b/
c)/b) - 105*b^3*x)/c^4]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.57

$$\int \frac{x^{10}}{bx^2+cx^4}\,dx = -\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}-\frac{\sqrt{-\frac{b^7}{c^9}}\log\left(x-\frac{c^4\sqrt{-\frac{b^7}{c^9}}}{b^3}\right)}{2}+\frac{\sqrt{-\frac{b^7}{c^9}}\log\left(x+\frac{c^4\sqrt{-\frac{b^7}{c^9}}}{b^3}\right)}{2}+\frac{x^7}{7c}$$

input
```
integrate(x**10/(c*x**4+b*x**2),x)
```

output
```
-b**3*x/c**4 + b**2*x**3/(3*c**3) - b*x**5/(5*c**2) - sqrt(-b**7/c**9)*log
(x - c**4*sqrt(-b**7/c**9)/b**3)/2 + sqrt(-b**7/c**9)*log(x + c**4*sqrt(-b
**7/c**9)/b**3)/2 + x**7/(7*c)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{x^{10}}{bx^2+cx^4}\,dx = \frac{b^4\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4}+\frac{15\,c^3x^7-21\,bc^2x^5+35\,b^2cx^3-105\,b^3x}{105\,c^4}$$

input
```
integrate(x^10/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
b^4*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*c^3*x^7 - 21*b*c^2*x
^5 + 35*b^2*c*x^3 - 105*b^3*x)/c^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{x^{10}}{bx^2+cx^4}\,dx=\frac{b^4\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4}+\frac{15\,c^6x^7-21\,bc^5x^5+35\,b^2c^4x^3-105\,b^3c^3x}{105\,c^7}$$

input
```
integrate(x^10/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
b^4*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*c^6*x^7 - 21*b*c^5*x
^5 + 35*b^2*c^4*x^3 - 105*b^3*c^3*x)/c^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \frac{x^{10}}{bx^2+cx^4}\,dx=\frac{x^7}{7\,c}-\frac{b\,x^5}{5\,c^2}-\frac{b^3\,x}{c^4}+\frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{9/2}}+\frac{b^2\,x^3}{3\,c^3}$$

input
```
int(x^10/(b*x^2 + c*x^4),x)
```

output
```
x^7/(7*c) - (b*x^5)/(5*c^2) - (b^3*x)/c^4 + (b^(7/2)*atan((c^(1/2)*x)/b^(1
/2)))/c^(9/2) + (b^2*x^3)/(3*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x^{10}}{bx^2+cx^4}\,dx=\frac{105\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^3-105b^3cx+35b^2c^2x^3-21b\,c^3x^5+15c^4x^7}{105c^5}$$

input
```
int(x^10/(c*x^4+b*x^2),x)
```

output
```
(105*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**3 - 105*b**3*c*x + 3
5*b**2*c**2*x**3 - 21*b*c**3*x**5 + 15*c**4*x**7)/(105*c**5)
```

3.53 $\int \frac{x^8}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{x^8}{bx^2+cx^4}\,dx = \frac{b^2x}{c^3} - \frac{bx^3}{3c^2} + \frac{x^5}{5c} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}$$

output
```
b^2*x/c^3-1/3*b*x^3/c^2+1/5*x^5/c-b^(5/2)*arctan(c^(1/2)*x/b^(1/2))/c^(7/2
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{bx^2+cx^4}\,dx = \frac{b^2x}{c^3} - \frac{bx^3}{3c^2} + \frac{x^5}{5c} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}$$

input
```
Integrate[x^8/(b*x^2 + c*x^4),x]
```

output
```
(b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/c^(7/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{bx^2+cx^4}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{x^6}{b+cx^2}dx$$
$$\Big\downarrow \; 254$$
$$\int \left(-\frac{b^3}{c^3\left(b+cx^2\right)}+\frac{b^2}{c^3}-\frac{bx^2}{c^2}+\frac{x^4}{c}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{b^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}$$

input
```
Int[x^8/(b*x^2 + c*x^4),x]
```

output
```
(b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/c^(7/2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\frac{1}{5}c^2x^5-\frac{1}{3}bc\,x^3+b^2x}{c^3}-\frac{b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^3\sqrt{bc}}$	49
risch	$\frac{x^5}{5c}-\frac{b\,x^3}{3c^2}+\frac{b^2x}{c^3}+\frac{\sqrt{-bc}\,b^2\ln\left(-\sqrt{-bc}\,x-b\right)}{2c^4}-\frac{\sqrt{-bc}\,b^2\ln\left(\sqrt{-bc}\,x-b\right)}{2c^4}$	82

input
```
int(x^8/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/5*c^2*x^5-1/3*b*c*x^3+b^2*x)-b^3/c^3/(b*c)^(1/2)*arctan(c*x/(b*c)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int \frac{x^8}{bx^2+cx^4}\,dx = \left[\frac{6\,c^2x^5-10\,bcx^3+15\,b^2\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+30\,b^2x}{30\,c^3}, \frac{3\,c^2x^5-5\,bcx^3-15\,b^2\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)}{15\,c^3}\right.$$

input
```
integrate(x^8/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/30*(6*c^2*x^5 - 10*b*c*x^3 + 15*b^2*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(
-b/c) - b)/(c*x^2 + b)) + 30*b^2*x)/c^3, 1/15*(3*c^2*x^5 - 5*b*c*x^3 - 15*
b^2*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 15*b^2*x)/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

$$\int \frac{x^8}{bx^2 + cx^4}\,dx = \frac{b^2x}{c^3} - \frac{bx^3}{3c^2} + \frac{\sqrt{-\frac{b^5}{c^7}}\log\left(x - \frac{c^3\sqrt{-\frac{b^5}{c^7}}}{b^2}\right)}{2} - \frac{\sqrt{-\frac{b^5}{c^7}}\log\left(x + \frac{c^3\sqrt{-\frac{b^5}{c^7}}}{b^2}\right)}{2} + \frac{x^5}{5c}$$

input
```
integrate(x**8/(c*x**4+b*x**2),x)
```

output
```
b**2*x/c**3 - b*x**3/(3*c**2) + sqrt(-b**5/c**7)*log(x - c**3*sqrt(-b**5/c
**7)/b**2)/2 - sqrt(-b**5/c**7)*log(x + c**3*sqrt(-b**5/c**7)/b**2)/2 + x*
*5/(5*c)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{x^8}{bx^2 + cx^4}\,dx = -\frac{b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{3\,c^2x^5 - 5\,bcx^3 + 15\,b^2x}{15\,c^3}$$

input
```
integrate(x^8/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*c^2*x^5 - 5*b*c*x^3 +
 15*b^2*x)/c^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{bx^2+cx^4}\,dx = -\frac{b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{3\,c^4x^5-5\,bc^3x^3+15\,b^2c^2x}{15\,c^5}$$

input
```
integrate(x^8/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*c^4*x^5 - 5*b*c^3*x^3
 + 15*b^2*c^2*x)/c^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{x^8}{bx^2+cx^4}\,dx = \frac{x^5}{5\,c} - \frac{b\,x^3}{3\,c^2} + \frac{b^2\,x}{c^3} - \frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{7/2}}$$

input
```
int(x^8/(b*x^2 + c*x^4),x)
```

output
```
x^5/(5*c) - (b*x^3)/(3*c^2) + (b^2*x)/c^3 - (b^(5/2)*atan((c^(1/2)*x)/b^(1
/2)))/c^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{bx^2+cx^4}\,dx = \frac{-15\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^2+15b^2cx-5b\,c^2x^3+3c^3x^5}{15c^4}$$

input
```
int(x^8/(c*x^4+b*x^2),x)
```

output
```
( - 15*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**2 + 15*b**2*c*x -
5*b*c**2*x**3 + 3*c**3*x**5)/(15*c**4)
```

3.54 $\int \frac{x^6}{bx^2+cx^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{x^6}{bx^2+cx^4} \, dx = -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}$$

output
```
-b*x/c^2+1/3*x^3/c+b^(3/2)*arctan(c^(1/2)*x/b^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{bx^2+cx^4} \, dx = -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}$$

input
```
Integrate[x^6/(b*x^2 + c*x^4),x]
```

output
```
-((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^4}{b+cx^2}dx$$

$$\Big\downarrow \text{ 254}$$

$$\int \left(\frac{b^2}{c^2(b+cx^2)}-\frac{b}{c^2}+\frac{x^2}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

input
```
Int[x^6/(b*x^2 + c*x^4),x]
```

output
```
-((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{-\frac{1}{3}c\,x^{3}+bx}{c^{2}}+\frac{b^{2}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^{2}\sqrt{bc}}$	38
risch	$\frac{x^{3}}{3c}-\frac{bx}{c^{2}}+\frac{\sqrt{-bc}\,b\ln\left(-\sqrt{-bc}\,x+b\right)}{2c^{3}}-\frac{\sqrt{-bc}\,b\ln\left(\sqrt{-bc}\,x+b\right)}{2c^{3}}$	64

input
```
int(x^6/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(-1/3*c*x^3+b*x)+1/c^2*b^2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.36

$$\int \frac{x^6}{bx^2+cx^4}\,dx = \left[\frac{2\,cx^{3}+3\,b\sqrt{-\frac{b}{c}}\log\left(\frac{cx^{2}+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^{2}+b}\right)-6\,bx}{6\,c^{2}},\ \frac{cx^{3}+3\,b\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-3\,bx}{3\,c^{2}}\right]$$

input
```
integrate(x^6/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/6*(2*c*x^3 + 3*b*sqrt(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 +
 b)) - 6*b*x)/c^2, 1/3*(c*x^3 + 3*b*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 3*
b*x)/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.90

$$\int \frac{x^6}{bx^2 + cx^4}\,dx = -\frac{bx}{c^2} - \frac{\sqrt{-\frac{b^3}{c^5}}\log\left(x - \frac{c^2\sqrt{-\frac{b^3}{c^5}}}{b}\right)}{2} + \frac{\sqrt{-\frac{b^3}{c^5}}\log\left(x + \frac{c^2\sqrt{-\frac{b^3}{c^5}}}{b}\right)}{2} + \frac{x^3}{3c}$$

input
```
integrate(x**6/(c*x**4+b*x**2),x)
```

output
```
-b*x/c**2 - sqrt(-b**3/c**5)*log(x - c**2*sqrt(-b**3/c**5)/b)/2 + sqrt(-b*
*3/c**5)*log(x + c**2*sqrt(-b**3/c**5)/b)/2 + x**3/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{bx^2 + cx^4}\,dx = \frac{b^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^2} + \frac{cx^3 - 3\,bx}{3\,c^2}$$

input
```
integrate(x^6/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(c*x^3 - 3*b*x)/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{x^6}{bx^2+cx^4}\,dx=\frac{b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^2}+\frac{c^2x^3-3\,bcx}{3\,c^3}$$

input
```
integrate(x^6/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{x^6}{bx^2+cx^4}\,dx=\frac{x^3}{3\,c}+\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{b\,x}{c^2}$$

input
```
int(x^6/(b*x^2 + c*x^4),x)
```

output
```
x^3/(3*c) + (b^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/c^(5/2) - (b*x)/c^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{bx^2+cx^4}\,dx=\frac{3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b-3bcx+c^2x^3}{3c^3}$$

input
```
int(x^6/(c*x^4+b*x^2),x)
```

output
```
(3*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b - 3*b*c*x + c**2*x**3)/
(3*c**3)
```

3.55 $\int \frac{x^4}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{x^4}{bx^2+cx^4}\,dx=\frac{x}{c}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}$$

output
```
x/c-b^(1/2)*arctan(c^(1/2)*x/b^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{bx^2+cx^4}\,dx=\frac{x}{c}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}$$

input
```
Integrate[x^4/(b*x^2 + c*x^4),x]
```

output
```
x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{bx^2+cx^4}\,dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{x^2}{b+cx^2}dx$$
$$\Big\downarrow \ 262$$
$$\frac{x}{c}-\frac{b\int \frac{1}{cx^2+b}dx}{c}$$
$$\Big\downarrow \ 218$$
$$\frac{x}{c}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}$$

input
```
Int[x^4/(b*x^2 + c*x^4),x]
```

output
```
x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x}{c}-\frac{b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c\sqrt{bc}}$	27
risch	$\frac{x}{c}+\frac{\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)}{2c^{2}}-\frac{\sqrt{-bc}\,\ln\left(\sqrt{-bc}\,x-b\right)}{2c^{2}}$	56

input
```
int(x^4/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
x/c-1/c*b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.65

$$\int \frac{x^4}{bx^2+cx^4}\,dx=\left[\frac{\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+2\,x}{2\,c},-\frac{\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)-x}{c}\right]$$

input
```
integrate(x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 2*x)/c,
 -(sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - x)/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{x^4}{bx^2 + cx^4}\,dx = \frac{\sqrt{-\frac{b}{c^3}}\log\left(-c\sqrt{-\frac{b}{c^3}} + x\right)}{2} - \frac{\sqrt{-\frac{b}{c^3}}\log\left(c\sqrt{-\frac{b}{c^3}} + x\right)}{2} + \frac{x}{c}$$

input
```
integrate(x**4/(c*x**4+b*x**2),x)
```

output
```
sqrt(-b/c**3)*log(-c*sqrt(-b/c**3) + x)/2 - sqrt(-b/c**3)*log(c*sqrt(-b/c*
*3) + x)/2 + x/c
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{bx^2 + cx^4}\,dx = -\frac{b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c} + \frac{x}{c}$$

input
```
integrate(x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) + x/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{bx^2+cx^4}\,dx=-\frac{b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c}+\frac{x}{c}$$

input
```
integrate(x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) + x/c
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{bx^2+cx^4}\,dx=\frac{x}{c}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{c^{3/2}}$$

input
```
int(x^4/(b*x^2 + c*x^4),x)
```

output
```
x/c - (b^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/c^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{bx^2+cx^4}\,dx=\frac{-\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)+cx}{c^2}$$

input
```
int(x^4/(c*x^4+b*x^2),x)
```

output
```
( - sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b))) + c*x)/c**2
```

3.56 $\int \frac{x^2}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \frac{x^2}{bx^2+cx^4}\,dx=\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}$$

output
```
arctan(c^(1/2)*x/b^(1/2))/b^(1/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{bx^2+cx^4}\,dx=\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}$$

input
```
Integrate[x^2/(b*x^2 + c*x^4),x]
```

output
```
ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {9, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{bx^2+cx^4}\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{b+cx^2}dx$$
$$\Big\downarrow \text{ 218}$$
$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}$$

input
```
Int[x^2/(b*x^2 + c*x^4),x]
```

output
```
ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$	16
risch	$-\frac{\ln(cx+\sqrt{-bc})}{2\sqrt{-bc}}+\frac{\ln(-cx+\sqrt{-bc})}{2\sqrt{-bc}}$	41

input
```
int(x^2/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{x^2}{bx^2+cx^4}\,dx = \left[-\frac{\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x-b}{cx^2+b}\right)}{2\,bc}, \frac{\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{bc}\right]$$

input
```
integrate(x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b))/(b*c), sqrt
(b*c)*arctan(sqrt(b*c)*x/b)/(b*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{x^2}{bx^2+cx^4}\,dx = -\frac{\sqrt{-\frac{1}{bc}}\log\left(-b\sqrt{-\frac{1}{bc}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{bc}}\log\left(b\sqrt{-\frac{1}{bc}}+x\right)}{2}$$

input
```
integrate(x**2/(c*x**4+b*x**2),x)
```

output
```
-sqrt(-1/(b*c))*log(-b*sqrt(-1/(b*c)) + x)/2 + sqrt(-1/(b*c))*log(b*sqrt(-
1/(b*c)) + x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{bx^2 + cx^4}\,dx = \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$$

input
```
integrate(x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
arctan(c*x/sqrt(b*c))/sqrt(b*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{bx^2 + cx^4}\,dx = \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$$

input
```
integrate(x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
arctan(c*x/sqrt(b*c))/sqrt(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{bx^2 + cx^4} \, dx = \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}}$$

input
```
int(x^2/(b*x^2 + c*x^4),x)
```

output
```
atan((c^(1/2)*x)/b^(1/2))/(b^(1/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{bx^2 + cx^4} \, dx = \frac{\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)}{bc}$$

input
```
int(x^2/(c*x^4+b*x^2),x)
```

output
```
(sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b))))/(b*c)
```

3.57 $\int \frac{1}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{1}{bx^2+cx^4}\,dx=-\frac{1}{bx}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}$$

output
```
-1/b/x-c^(1/2)*arctan(c^(1/2)*x/b^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{bx^2+cx^4}\,dx=-\frac{1}{bx}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(-1),x]
```

output
```
-(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1397, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx^2+cx^4}dx$$
$$\Big\downarrow \text{1397}$$
$$\int \frac{1}{x^2(b+cx^2)}dx$$
$$\Big\downarrow \text{264}$$
$$-\frac{c\int \frac{1}{cx^2+b}dx}{b}-\frac{1}{bx}$$
$$\Big\downarrow \text{218}$$
$$-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

input
```
Int[(b*x^2 + c*x^4)^(-1),x]
```

output
```
-(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1397
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[x^(2*p)*(b + c*x^2
)^p, x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{1}{bx}-\frac{c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b\sqrt{bc}}$	30
risch	$-\frac{1}{bx}+\frac{\sqrt{-bc}\,\ln\left(-cx+\sqrt{-bc}\right)}{2b^{2}}-\frac{\sqrt{-bc}\,\ln\left(-cx-\sqrt{-bc}\right)}{2b^{2}}$	58

input
```
int(1/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/x-c/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int \frac{1}{bx^2+cx^4}\,dx = \left[\frac{x\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-2}{2\,bx}, -\frac{x\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+1}{bx}\right]$$

input
```
integrate(1/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/2*(x*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) - 2)/(b
*x), -(x*sqrt(c/b)*arctan(x*sqrt(c/b)) + 1)/(b*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{1}{bx^2 + cx^4}\,dx = \frac{\sqrt{-\frac{c}{b^3}}\log\left(-\frac{b^2\sqrt{-\frac{c}{b^3}}}{c} + x\right)}{2} - \frac{\sqrt{-\frac{c}{b^3}}\log\left(\frac{b^2\sqrt{-\frac{c}{b^3}}}{c} + x\right)}{2} - \frac{1}{bx}$$

input
```
integrate(1/(c*x**4+b*x**2),x)
```

output
```
sqrt(-c/b**3)*log(-b**2*sqrt(-c/b**3)/c + x)/2 - sqrt(-c/b**3)*log(b**2*sq
rt(-c/b**3)/c + x)/2 - 1/(b*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{bx^2 + cx^4}\,dx = -\frac{c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b} - \frac{1}{bx}$$

input
```
integrate(1/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - 1/(b*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{bx^2+cx^4}\,dx = -\frac{c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b} - \frac{1}{bx}$$

input
```
integrate(1/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - 1/(b*x)
```

Mupad [B] (verification not implemented)

Time = 16.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{bx^2+cx^4}\,dx = -\frac{1}{b\,x} - \frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{3/2}}$$

input
```
int(1/(b*x^2 + c*x^4),x)
```

output
```
- 1/(b*x) - (c^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/b^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{bx^2+cx^4}\,dx = \frac{-\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)x - b}{b^2x}$$

input
```
int(1/(c*x^4+b*x^2),x)
```

output
```
( - (sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*x + b))/(b**2*x)
```

3.58 $\int \frac{1}{x^2(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{1}{x^2 (bx^2 + cx^4)}\,dx = -\frac{1}{3bx^3} + \frac{c}{b^2x} + \frac{c^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}$$

output
```
-1/3/b/x^3+c/b^2/x+c^(3/2)*arctan(c^(1/2)*x/b^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (bx^2 + cx^4)}\,dx = -\frac{1}{3bx^3} + \frac{c}{b^2x} + \frac{c^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}$$

input
```
Integrate[1/(x^2*(b*x^2 + c*x^4)),x]
```

output
```
-1/3*1/(b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (bx^2+cx^4)} \, dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{x^4 (b+cx^2)}dx$$
$$\Big\downarrow \text{ 264}$$
$$-\frac{c\int \frac{1}{x^2(cx^2+b)}dx}{b}-\frac{1}{3bx^3}$$
$$\Big\downarrow \text{ 264}$$
$$-\frac{c\left(-\frac{c\int \frac{1}{cx^2+b}dx}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}$$
$$\Big\downarrow \text{ 218}$$
$$-\frac{c\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}$$

input
```
Int[1/(x^2*(b*x^2 + c*x^4)),x]
```

output
```
-1/3*1/(b*x^3) - (c*(-(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^
(3/2)))/b
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{3b\,x^{3}}+\frac{c}{b^{2}x}+\frac{c^{2}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b^{2}\sqrt{bc}}$	39
risch	$\frac{\frac{c\,x^{2}}{b^{2}}-\frac{1}{3b}}{x^{3}}+\frac{\sqrt{-bc}\,c\ln\left(-cx-\sqrt{-bc}\right)}{2b^{3}}-\frac{\sqrt{-bc}\,c\ln\left(-cx+\sqrt{-bc}\right)}{2b^{3}}$	70

input
```
int(1/x^2/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/x^3+c/b^2/x+c^2/b^2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{1}{x^2\,(bx^2+cx^4)}\,dx = \left[\frac{3\,cx^3\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+6\,cx^2-2\,b}{6\,b^2x^3},\ \frac{3\,cx^3\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+3\,cx^2-b}{3\,b^2x^3}\right]$$

input
```
integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/6*(3*c*x^3*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) +
 6*c*x^2 - 2*b)/(b^2*x^3), 1/3*(3*c*x^3*sqrt(c/b)*arctan(x*sqrt(c/b)) + 3*
c*x^2 - b)/(b^2*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int \frac{1}{x^2\,(bx^2+cx^4)}\,dx = -\frac{\sqrt{-\frac{c^3}{b^5}}\log\left(-\frac{b^3\sqrt{-\frac{c^3}{b^5}}}{c^2}+x\right)}{2} + \frac{\sqrt{-\frac{c^3}{b^5}}\log\left(\frac{b^3\sqrt{-\frac{c^3}{b^5}}}{c^2}+x\right)}{2} + \frac{-b+3cx^2}{3b^2x^3}$$

input
```
integrate(1/x**2/(c*x**4+b*x**2),x)
```

output
```
-sqrt(-c**3/b**5)*log(-b**3*sqrt(-c**3/b**5)/c**2 + x)/2 + sqrt(-c**3/b**5
)*log(b**3*sqrt(-c**3/b**5)/c**2 + x)/2 + (-b + 3*c*x**2)/(3*b**2*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2(bx^2+cx^4)}\,dx = \frac{c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^2} + \frac{3\,cx^2-b}{3\,b^2x^3}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/3*(3*c*x^2 - b)/(b^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2(bx^2+cx^4)}\,dx = \frac{c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^2} + \frac{3\,cx^2-b}{3\,b^2x^3}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/3*(3*c*x^2 - b)/(b^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 18.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2(bx^2+cx^4)}\,dx = \frac{c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{5/2}} - \frac{\frac{1}{3\,b}-\frac{c\,x^2}{b^2}}{x^3}$$

input
```
int(1/(x^2*(b*x^2 + c*x^4)),x)
```

output
```
(c^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/b^(5/2) - (1/(3*b) - (c*x^2)/b^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (bx^2 + cx^4)} \, dx = \frac{3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right) c\,x^3 - b^2 + 3bc\,x^2}{3b^3x^3}$$

input
```
int(1/x^2/(c*x^4+b*x^2),x)
```

output
```
(3*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*c*x**3 - b**2 + 3*b*c*x**
2)/(3*b**3*x**3)
```

3.59 $\int \frac{1}{x^4(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \frac{1}{x^4\left(bx^2+cx^4\right)}\,dx=-\frac{1}{5bx^5}+\frac{c}{3b^2x^3}-\frac{c^2}{b^3x}-\frac{c^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}$$

output
```
-1/5/b/x^5+1/3*c/b^2/x^3-c^2/b^3/x-c^(5/2)*arctan(c^(1/2)*x/b^(1/2))/b^(7/
2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(bx^2+cx^4\right)}\,dx=-\frac{1}{5bx^5}+\frac{c}{3b^2x^3}-\frac{c^2}{b^3x}-\frac{c^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}$$

input
```
Integrate[1/(x^4*(b*x^2 + c*x^4)),x]
```

output
```
-1/5*1/(b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqrt[c]*x)
/Sqrt[b]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {9, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (bx^2+cx^4)} \, dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{x^6 (b+cx^2)}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{c\int \frac{1}{x^4(cx^2+b)}dx}{b}-\frac{1}{5bx^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{c\left(-\frac{c\int \frac{1}{x^2\left(cx^2+b\right)}dx}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{1}{5bx^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{c\left(-\frac{c\left(-\frac{c\int \frac{1}{cx^2+b}dx}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{1}{5bx^5}$$

$$\Big\downarrow \text{218}$$

$$-\frac{c\left(-\frac{c\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{1}{5bx^5}$$

input
```
Int[1/(x^4*(b*x^2 + c*x^4)),x]
```

output
```
-1/5*1/(b*x^5) - (c*(-1/3*1/(b*x^3) - (c*(-(1/(b*x)) - (Sqrt[c]*ArcTan[(Sq
rt[c]*x)/Sqrt[b]])/b^(3/2)))/b))/b
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{1}{5b\,x^{5}}-\frac{c^{2}}{b^{3}x}+\frac{c}{3b^{2}x^{3}}-\frac{c^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b^{3}\sqrt{bc}}$	52
risch	$\frac{-\frac{c^{2}x^{4}}{b^{3}}+\frac{c\,x^{2}}{3b^{2}}-\frac{1}{5b}}{x^{5}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{7}\_Z^{2}+c^{5}\right)}\_R\ln\left(\left(3\_R^{2}b^{7}+2c^{5}\right)x+b^{4}c^{2}\_R\right)\right)}{2}$	77

input
```
int(1/x^4/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/b/x^5-c^2/b^3/x+1/3*c/b^2/x^3-c^3/b^3/(b*c)^(1/2)*arctan(c*x/(b*c)^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.28

$$\int \frac{1}{x^4 (bx^2 + cx^4)} dx = \left[\frac{15 c^2 x^5 \sqrt{-\frac{c}{b}} \log \left(\frac{cx^2 - 2 bx \sqrt{-\frac{c}{b}} - b}{cx^2 + b} \right) - 30 c^2 x^4 + 10 bcx^2 - 6 b^2}{30 b^3 x^5}, \right. \\ \left. -\frac{15 c^2 x^5 \sqrt{\frac{c}{b}} \arctan \left(x \sqrt{\frac{c}{b}} \right) + 15 c^2 x^4 - 5 bcx^2 + 3 b^2}{15 b^3 x^5} \right]$$

input
```
integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
[1/30*(15*c^2*x^5*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b
)) - 30*c^2*x^4 + 10*b*c*x^2 - 6*b^2)/(b^3*x^5), -1/15*(15*c^2*x^5*sqrt(c/
b)*arctan(x*sqrt(c/b)) + 15*c^2*x^4 - 5*b*c*x^2 + 3*b^2)/(b^3*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.72

$$\int \frac{1}{x^4 (bx^2 + cx^4)} dx = \frac{\sqrt{-\frac{c^5}{b^7}} \log \left(-\frac{b^4 \sqrt{-\frac{c^5}{b^7}}}{c^3} + x \right)}{2} \\ - \frac{\sqrt{-\frac{c^5}{b^7}} \log \left(\frac{b^4 \sqrt{-\frac{c^5}{b^7}}}{c^3} + x \right)}{2} + \frac{-3b^2 + 5bcx^2 - 15c^2 x^4}{15b^3 x^5}$$

input
```
integrate(1/x**4/(c*x**4+b*x**2),x)
```

output
```
sqrt(-c**5/b**7)*log(-b**4*sqrt(-c**5/b**7)/c**3 + x)/2 - sqrt(-c**5/b**7)
*log(b**4*sqrt(-c**5/b**7)/c**3 + x)/2 + (-3*b**2 + 5*b*c*x**2 - 15*c**2*x
**4)/(15*b**3*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4 (bx^2 + cx^4)} \, dx = -\frac{c^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^3} - \frac{15\,c^2x^4 - 5\,bcx^2 + 3\,b^2}{15\,b^3x^5}$$

input
```
integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/15*(15*c^2*x^4 - 5*b*c*x^2
+ 3*b^2)/(b^3*x^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4 (bx^2 + cx^4)} \, dx = -\frac{c^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^3} - \frac{15\,c^2x^4 - 5\,bcx^2 + 3\,b^2}{15\,b^3x^5}$$

input
```
integrate(1/x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/15*(15*c^2*x^4 - 5*b*c*x^2
+ 3*b^2)/(b^3*x^5)
```

Mupad [B] (verification not implemented)

Time = 18.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^4 (bx^2 + cx^4)} \, dx = -\frac{\frac{1}{5\,b} - \frac{c\,x^2}{3\,b^2} + \frac{c^2\,x^4}{b^3}}{x^5} - \frac{c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}$$

input
```
int(1/(x^4*(b*x^2 + c*x^4)),x)
```

output
```
- (1/(5*b) - (c*x^2)/(3*b^2) + (c^2*x^4)/b^3)/x^5 - (c^(5/2)*atan((c^(1/2)
*x)/b^(1/2)))/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^4 (bx^2 + cx^4)} \, dx = \frac{-15\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right) c^2x^5 - 3b^3 + 5b^2c\,x^2 - 15b\,c^2x^4}{15b^4x^5}$$

input
```
int(1/x^4/(c*x^4+b*x^2),x)
```

output
```
( - 15*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*c**2*x**5 - 3*b**3 +
5*b**2*c*x**2 - 15*b*c**2*x**4)/(15*b**4*x**5)
```

3.60 $\int \frac{x^{11}}{(bx^2+cx^4)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{x^{11}}{(bx^2+cx^4)^2} \, dx = -\frac{bx^2}{c^3} + \frac{x^4}{4c^2} + \frac{b^3}{2c^4(b+cx^2)} + \frac{3b^2\log(b+cx^2)}{2c^4}$$

output
```
-b*x^2/c^3+1/4*x^4/c^2+1/2*b^3/c^4/(c*x^2+b)+3/2*b^2*ln(c*x^2+b)/c^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{(bx^2+cx^4)^2} \, dx = \frac{-4bcx^2 + c^2x^4 + \frac{2b^3}{b+cx^2} + 6b^2\log(b+cx^2)}{4c^4}$$

input
```
Integrate[x^11/(b*x^2 + c*x^4)^2,x]
```

output
```
(-4*b*c*x^2 + c^2*x^4 + (2*b^3)/(b + c*x^2) + 6*b^2*Log[b + c*x^2])/(4*c^4
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(bx^2+cx^4)^2}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{x^7}{(b+cx^2)^2}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{x^6}{(cx^2+b)^2}dx^2$$
$$\Big\downarrow \text{ 49}$$
$$\frac{1}{2}\int \left(-\frac{b^3}{c^3(cx^2+b)^2}+\frac{3b^2}{c^3(cx^2+b)}-\frac{2b}{c^3}+\frac{x^2}{c^2}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(\frac{b^3}{c^4(b+cx^2)}+\frac{3b^2\log(b+cx^2)}{c^4}-\frac{2bx^2}{c^3}+\frac{x^4}{2c^2}\right)$$

input
```
Int[x^11/(b*x^2 + c*x^4)^2,x]
```

output
```
((-2*b*x^2)/c^3 + x^4/(2*c^2) + b^3/(c^4*(b + c*x^2)) + (3*b^2*Log[b + c*x
^2])/c^4)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\left(-c\,x^{2}+2b\right)^{2}}{4c^{4}}+\frac{b^{2}\left(\frac{3\ln\left(c\,x^{2}+b\right)}{c}+\frac{b}{c\left(c\,x^{2}+b\right)}\right)}{2c^{3}}$	55
risch	$\frac{x^{4}}{4c^{2}}-\frac{b\,x^{2}}{c^{3}}+\frac{b^{2}}{c^{4}}+\frac{b^{3}}{2c^{4}\left(c\,x^{2}+b\right)}+\frac{3b^{2}\ln\left(c\,x^{2}+b\right)}{2c^{4}}$	59
norman	$\frac{\frac{x^{9}}{4c}-\frac{3b\,x^{7}}{4c^{2}}+\frac{3b^{3}x^{3}}{2c^{4}}}{x^{3}\left(c\,x^{2}+b\right)}+\frac{3b^{2}\ln\left(c\,x^{2}+b\right)}{2c^{4}}$	60
parallelrisch	$\frac{c^{3}x^{6}-3b\,c^{2}x^{4}+6\ln\left(c\,x^{2}+b\right)x^{2}b^{2}c+6b^{3}\ln\left(c\,x^{2}+b\right)+6b^{3}}{4c^{4}\left(c\,x^{2}+b\right)}$	67

input
```
int(x^11/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c*x^2+2*b)^2/c^4+1/2*b^2/c^3*(3*ln(c*x^2+b)/c+1/c*b/(c*x^2+b))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{x^{11}}{(bx^2+cx^4)^2}\,dx = \frac{c^3x^6 - 3\,bc^2x^4 - 4\,b^2cx^2 + 2\,b^3 + 6\,(b^2cx^2 + b^3)\log(cx^2+b)}{4\,(c^5x^2 + bc^4)}$$

input
```
integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/4*(c^3*x^6 - 3*b*c^2*x^4 - 4*b^2*c*x^2 + 2*b^3 + 6*(b^2*c*x^2 + b^3)*log
(c*x^2 + b))/(c^5*x^2 + b*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}}{(bx^2+cx^4)^2}\,dx = \frac{b^3}{2bc^4 + 2c^5x^2} + \frac{3b^2\log(b+cx^2)}{2c^4} - \frac{bx^2}{c^3} + \frac{x^4}{4c^2}$$

input
```
integrate(x**11/(c*x**4+b*x**2)**2,x)
```

output
```
b**3/(2*b*c**4 + 2*c**5*x**2) + 3*b**2*log(b + c*x**2)/(2*c**4) - b*x**2/c
**3 + x**4/(4*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}}{(bx^2+cx^4)^2}\,dx = \frac{b^3}{2\,(c^5x^2 + bc^4)} + \frac{3\,b^2\log(cx^2+b)}{2\,c^4} + \frac{cx^4 - 4\,bx^2}{4\,c^3}$$

input
```
integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*b^3/(c^5*x^2 + b*c^4) + 3/2*b^2*log(c*x^2 + b)/c^4 + 1/4*(c*x^4 - 4*b*
x^2)/c^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{x^{11}}{(bx^2+cx^4)^2}\,dx = \frac{3\,b^2\log\left(|cx^2+b|\right)}{2\,c^4} + \frac{c^2x^4-4\,bcx^2}{4\,c^4} - \frac{3\,b^2cx^2+2\,b^3}{2\,(cx^2+b)c^4}$$

input
```
integrate(x^11/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
3/2*b^2*log(abs(c*x^2 + b))/c^4 + 1/4*(c^2*x^4 - 4*b*c*x^2)/c^4 - 1/2*(3*b
^2*c*x^2 + 2*b^3)/((c*x^2 + b)*c^4)
```

Mupad [B] (verification not implemented)

Time = 17.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{(bx^2+cx^4)^2}\,dx = \frac{x^4}{4\,c^2} + \frac{b^3}{2\,c\,\left(c^4\,x^2+b\,c^3\right)} - \frac{b\,x^2}{c^3} + \frac{3\,b^2\,\ln\left(c\,x^2+b\right)}{2\,c^4}$$

input
```
int(x^11/(b*x^2 + c*x^4)^2,x)
```

output
```
x^4/(4*c^2) + b^3/(2*c*(b*c^3 + c^4*x^2)) - (b*x^2)/c^3 + (3*b^2*log(b + c
*x^2))/(2*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{x^{11}}{(bx^2+cx^4)^2}\,dx = \frac{6\log(c\,x^2+b)\,b^3+6\log(c\,x^2+b)\,b^2c\,x^2-6b^2c\,x^2-3b\,c^2x^4+c^3x^6}{4c^4\,(c\,x^2+b)}$$

input
```
int(x^11/(c*x^4+b*x^2)^2,x)
```

output
```
(6*log(b + c*x**2)*b**3 + 6*log(b + c*x**2)*b**2*c*x**2 - 6*b**2*c*x**2 -
3*b*c**2*x**4 + c**3*x**6)/(4*c**4*(b + c*x**2))
```

3.61 $\int \frac{x^9}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{x^9}{(bx^2+cx^4)^2} dx = \frac{x^2}{2c^2} - \frac{b^2}{2c^3(b+cx^2)} - \frac{b\log(b+cx^2)}{c^3}$$

output
```
1/2*x^2/c^2-1/2*b^2/c^3/(c*x^2+b)-b*ln(c*x^2+b)/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{x^9}{(bx^2+cx^4)^2} dx = \frac{cx^2 - \frac{b^2}{b+cx^2} - 2b\log(b+cx^2)}{2c^3}$$

input
```
Integrate[x^9/(b*x^2 + c*x^4)^2,x]
```

output
```
(c*x^2 - b^2/(b + c*x^2) - 2*b*Log[b + c*x^2])/(2*c^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^5}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^4}{(cx^2+b)^2}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{b^2}{c^2(cx^2+b)^2}-\frac{2b}{c^2(cx^2+b)}+\frac{1}{c^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{b^2}{c^3(b+cx^2)}-\frac{2b\log(b+cx^2)}{c^3}+\frac{x^2}{c^2}\right)$$

input
```
Int[x^9/(b*x^2 + c*x^4)^2,x]
```

output
```
(x^2/c^2 - b^2/(c^3*(b + c*x^2)) - (2*b*Log[b + c*x^2])/c^3)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{x^2}{2c^2}-\frac{b^2}{2c^3(c\,x^2+b)}-\frac{b\ln(c\,x^2+b)}{c^3}$	41
default	$\frac{x^2}{2c^2}-\frac{b\left(\frac{2\ln(c\,x^2+b)}{c}+\frac{b}{c(c\,x^2+b)}\right)}{2c^2}$	44
norman	$\frac{\frac{x^7}{2c}-\frac{b^2x^3}{c^3}}{x^3(c\,x^2+b)}-\frac{b\ln(c\,x^2+b)}{c^3}$	49
parallelrisch	$-\frac{-c^2x^4+2\ln(c\,x^2+b)x^2bc+2b^2\ln(c\,x^2+b)+2b^2}{2c^3(c\,x^2+b)}$	57

input
```
int(x^9/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c^2-1/2*b^2/c^3/(c*x^2+b)-b*ln(c*x^2+b)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{x^9}{(bx^2+cx^4)^2}\,dx = \frac{c^2x^4+bcx^2-b^2-2\,(bcx^2+b^2)\log(cx^2+b)}{2\,(c^4x^2+bc^3)}$$

input
```
integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/2*(c^2*x^4 + b*c*x^2 - b^2 - 2*(b*c*x^2 + b^2)*log(c*x^2 + b))/(c^4*x^2
+ b*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{x^9}{(bx^2+cx^4)^2}\,dx = -\frac{b^2}{2bc^3+2c^4x^2} - \frac{b\log(b+cx^2)}{c^3} + \frac{x^2}{2c^2}$$

input
```
integrate(x**9/(c*x**4+b*x**2)**2,x)
```

output
```
-b**2/(2*b*c**3 + 2*c**4*x**2) - b*log(b + c*x**2)/c**3 + x**2/(2*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{x^9}{(bx^2+cx^4)^2}\,dx = -\frac{b^2}{2\,(c^4x^2+bc^3)} + \frac{x^2}{2\,c^2} - \frac{b\log(cx^2+b)}{c^3}$$

input
```
integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*b^2/(c^4*x^2 + b*c^3) + 1/2*x^2/c^2 - b*log(c*x^2 + b)/c^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{x^9}{(bx^2+cx^4)^2}\,dx = \frac{x^2}{2\,c^2} - \frac{b\log\left(|cx^2+b|\right)}{c^3} + \frac{2\,bcx^2+b^2}{2\,(cx^2+b)c^3}$$

input
```
integrate(x^9/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*x^2/c^2 - b*log(abs(c*x^2 + b))/c^3 + 1/2*(2*b*c*x^2 + b^2)/((c*x^2 +
b)*c^3)
```

Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{x^9}{(bx^2+cx^4)^2}\,dx = \frac{x^2}{2\,c^2} - \frac{b^2}{2\,\left(c^4\,x^2+b\,c^3\right)} - \frac{b\,\ln\left(c\,x^2+b\right)}{c^3}$$

input
```
int(x^9/(b*x^2 + c*x^4)^2,x)
```

output
```
x^2/(2*c^2) - b^2/(2*(b*c^3 + c^4*x^2)) - (b*log(b + c*x^2))/c^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{x^9}{(bx^2+cx^4)^2}\,dx = \frac{-2\log(c\,x^2+b)\,b^2 - 2\log(c\,x^2+b)\,bc\,x^2 + 2bc\,x^2 + c^2x^4}{2c^3\,(c\,x^2+b)}$$

input
```
int(x^9/(c*x^4+b*x^2)^2,x)
```

output
```
( - 2*log(b + c*x**2)*b**2 - 2*log(b + c*x**2)*b*c*x**2 + 2*b*c*x**2 + c**
2*x**4)/(2*c**3*(b + c*x**2))
```

3.62 $\int \frac{x^7}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{x^7}{(bx^2+cx^4)^2}\,dx=\frac{b}{2c^2(b+cx^2)}+\frac{\log(b+cx^2)}{2c^2}$$

output
```
1/2*b/c^2/(c*x^2+b)+1/2*ln(c*x^2+b)/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{(bx^2+cx^4)^2}\,dx=\frac{\frac{b}{b+cx^2}+\log(b+cx^2)}{2c^2}$$

input
```
Integrate[x^7/(b*x^2 + c*x^4)^2,x]
```

output
```
(b/(b + c*x^2) + Log[b + c*x^2])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^3}{(b+cx^2)^2}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{x^2}{(cx^2+b)^2}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\frac{1}{c\,(cx^2+b)}-\frac{b}{c\,(cx^2+b)^2}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{b}{c^2\,(b+cx^2)}+\frac{\log\left(b+cx^2\right)}{c^2}\right)$$

input
```
Int[x^7/(b*x^2 + c*x^4)^2,x]
```

output
```
(b/(c^2*(b + c*x^2)) + Log[b + c*x^2]/c^2)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{b}{2c^2(c\,x^2+b)}+\frac{\ln(c\,x^2+b)}{2c^2}$	30
norman	$\frac{b}{2c^2(c\,x^2+b)}+\frac{\ln(c\,x^2+b)}{2c^2}$	30
risch	$\frac{b}{2c^2(c\,x^2+b)}+\frac{\ln(c\,x^2+b)}{2c^2}$	30
parallelrisch	$\frac{c\ln(c\,x^2+b)x^2+b\ln(c\,x^2+b)+b}{2c^2(c\,x^2+b)}$	40

input
```
int(x^7/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b/c^2/(c*x^2+b)+1/2*ln(c*x^2+b)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{x^7}{(bx^2+cx^4)^2}\,dx = \frac{(cx^2+b)\log(cx^2+b)+b}{2(c^3x^2+bc^2)}$$

input
```
integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/2*((c*x^2 + b)*log(c*x^2 + b) + b)/(c^3*x^2 + b*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{(bx^2+cx^4)^2}\,dx = \frac{b}{2bc^2+2c^3x^2} + \frac{\log(b+cx^2)}{2c^2}$$

input
```
integrate(x**7/(c*x**4+b*x**2)**2,x)
```

output
```
b/(2*b*c**2 + 2*c**3*x**2) + log(b + c*x**2)/(2*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{x^7}{(bx^2+cx^4)^2}\,dx = \frac{b}{2(c^3x^2+bc^2)} + \frac{\log(cx^2+b)}{2c^2}$$

input
```
integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*b/(c^3*x^2 + b*c^2) + 1/2*log(c*x^2 + b)/c^2
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{x^7}{(bx^2+cx^4)^2}\,dx = -\frac{x^2}{2\,(cx^2+b)c} + \frac{\log\left(|cx^2+b|\right)}{2\,c^2}$$

input
```
integrate(x^7/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*x^2/((c*x^2 + b)*c) + 1/2*log(abs(c*x^2 + b))/c^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{(bx^2+cx^4)^2}\,dx = \frac{\ln\left(c\,x^2+b\right)}{2\,c^2} + \frac{b}{2\,c^2\,\left(c\,x^2+b\right)}$$

input
```
int(x^7/(b*x^2 + c*x^4)^2,x)
```

output
```
log(b + c*x^2)/(2*c^2) + b/(2*c^2*(b + c*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{x^7}{(bx^2+cx^4)^2}\,dx = \frac{\log(c\,x^2+b)\,b+\log(c\,x^2+b)\,c\,x^2-c\,x^2}{2c^2\,(c\,x^2+b)}$$

input
```
int(x^7/(c*x^4+b*x^2)^2,x)
```

output
```
(log(b + c*x**2)*b + log(b + c*x**2)*c*x**2 - c*x**2)/(2*c**2*(b + c*x**2)
)
```

3.63 $\int \frac{x^5}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{x^5}{(bx^2+cx^4)^2}\,dx = -\frac{1}{2c\,(b+cx^2)}$$

output
```
-1/2/c/(c*x^2+b)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(bx^2+cx^4)^2}\,dx = -\frac{1}{2c\,(b+cx^2)}$$

input
```
Integrate[x^5/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*1/(c*(b + c*x^2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x}{(b+cx^2)^2}dx$$

$$\Big\downarrow \; 241$$

$$-\frac{1}{2c\,(b+cx^2)}$$

input
```
Int[x^5/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*1/(c*(b + c*x^2))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{2c(c\,x^2+b)}$	15
default	$-\frac{1}{2c(c\,x^2+b)}$	15
norman	$-\frac{1}{2c(c\,x^2+b)}$	15
risch	$-\frac{1}{2c(c\,x^2+b)}$	15
parallelrisch	$-\frac{1}{2c(c\,x^2+b)}$	15
orering	$-\frac{(c\,x^2+b)\,x^4}{2c(c\,x^4+b\,x^2)^2}$	29

input
```
int(x^5/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c/(c*x^2+b)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(bx^2+cx^4)^2}\,dx = -\frac{1}{2\,(c^2x^2+bc)}$$

input
```
integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/2/(c^2*x^2 + b*c)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(bx^2+cx^4)^2}\,dx = -\frac{1}{2bc+2c^2x^2}$$

input
```
integrate(x**5/(c*x**4+b*x**2)**2,x)
```

output
```
-1/(2*b*c + 2*c**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(bx^2+cx^4)^2}\,dx = -\frac{1}{2\,(c^2x^2+bc)}$$

input
```
integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2/(c^2*x^2 + b*c)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{(bx^2+cx^4)^2}\,dx = -\frac{1}{2\,(cx^2+b)c}$$

input
```
integrate(x^5/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2/((c*x^2 + b)*c)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{(bx^2+cx^4)^2}\,dx = -\frac{1}{2\,c\,\left(c\,x^2+b\right)}$$

input
```
int(x^5/(b*x^2 + c*x^4)^2,x)
```

output
```
-1/(2*c*(b + c*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^5}{(bx^2+cx^4)^2}\,dx = \frac{x^2}{2b\left(c\,x^2+b\right)}$$

input
```
int(x^5/(c*x^4+b*x^2)^2,x)
```

output
```
x**2/(2*b*(b + c*x**2))
```

3.64 $\int \frac{x^3}{(bx^2+cx^4)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \frac{x^3}{(bx^2+cx^4)^2} \, dx = \frac{1}{2b(b+cx^2)} + \frac{\log(x)}{b^2} - \frac{\log(b+cx^2)}{2b^2}$$

output
```
1/2/b/(c*x^2+b)+ln(x)/b^2-1/2*ln(c*x^2+b)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{(bx^2+cx^4)^2} \, dx = \frac{\frac{b}{b+cx^2} + 2\log(x) - \log(b+cx^2)}{2b^2}$$

input
```
Integrate[x^3/(b*x^2 + c*x^4)^2,x]
```

output
```
(b/(b + c*x^2) + 2*Log[x] - Log[b + c*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{x(b+cx^2)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2(cx^2+b)^2}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(-\frac{c}{b^2(cx^2+b)}-\frac{c}{b(cx^2+b)^2}+\frac{1}{b^2x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\log(b+cx^2)}{b^2}+\frac{\log(x^2)}{b^2}+\frac{1}{b(b+cx^2)}\right)$$

input
```
Int[x^3/(b*x^2 + c*x^4)^2,x]
```

output
```
(1/(b*(b + c*x^2)) + Log[x^2]/b^2 - Log[b + c*x^2]/b^2)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{1}{2b(c\,x^2+b)}+\frac{\ln(x)}{b^2}-\frac{\ln(c\,x^2+b)}{2b^2}$	35
norman	$-\frac{c\,x^2}{2b^2(c\,x^2+b)}+\frac{\ln(x)}{b^2}-\frac{\ln(c\,x^2+b)}{2b^2}$	39
default	$\frac{\ln(x)}{b^2}-\frac{c\left(\frac{\ln\left(c\,x^2+b\right)}{c}-\frac{b}{c\left(c\,x^2+b\right)}\right)}{2b^2}$	42
parallelrisch	$\frac{2c\ln(x)x^2-c\ln(c\,x^2+b)x^2-c\,x^2+2b\ln(x)-b\ln(c\,x^2+b)}{2b^2(c\,x^2+b)}$	60

input
```
int(x^3/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b/(c*x^2+b)+ln(x)/b^2-1/2*ln(c*x^2+b)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{x^3}{(bx^2 + cx^4)^2} \, dx = -\frac{(cx^2 + b) \log(cx^2 + b) - 2(cx^2 + b) \log(x) - b}{2(b^2cx^2 + b^3)}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/2*((c*x^2 + b)*log(c*x^2 + b) - 2*(c*x^2 + b)*log(x) - b)/(b^2*c*x^2 +
b^3)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{(bx^2 + cx^4)^2} \, dx = \frac{1}{2b^2 + 2bcx^2} + \frac{\log(x)}{b^2} - \frac{\log\left(\frac{b}{c} + x^2\right)}{2b^2}$$

input
```
integrate(x**3/(c*x**4+b*x**2)**2,x)
```

output
```
1/(2*b**2 + 2*b*c*x**2) + log(x)/b**2 - log(b/c + x**2)/(2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(bx^2 + cx^4)^2} \, dx = \frac{1}{2(bcx^2 + b^2)} - \frac{\log(cx^2 + b)}{2b^2} + \frac{\log(x^2)}{2b^2}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2/(b*c*x^2 + b^2) - 1/2*log(c*x^2 + b)/b^2 + 1/2*log(x^2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{(bx^2+cx^4)^2}\,dx = -\frac{\log(|cx^2+b|)}{2\,b^2} + \frac{\log(|x|)}{b^2} + \frac{1}{2\,(cx^2+b)b}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*log(abs(c*x^2 + b))/b^2 + log(abs(x))/b^2 + 1/2/((c*x^2 + b)*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{(bx^2+cx^4)^2}\,dx = \frac{\ln(x)}{b^2} + \frac{1}{2\,b\,(c\,x^2+b)} - \frac{\ln(c\,x^2+b)}{2\,b^2}$$

input
```
int(x^3/(b*x^2 + c*x^4)^2,x)
```

output
```
log(x)/b^2 + 1/(2*b*(b + c*x^2)) - log(b + c*x^2)/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

$$\int \frac{x^3}{(bx^2+cx^4)^2}\,dx = \frac{-\log(c\,x^2+b)\,b - \log(c\,x^2+b)\,c\,x^2 + 2\log(x)\,b + 2\log(x)\,c\,x^2 - c\,x^2}{2b^2\,(c\,x^2+b)}$$

input
```
int(x^3/(c*x^4+b*x^2)^2,x)
```

output
```
( - log(b + c*x**2)*b - log(b + c*x**2)*c*x**2 + 2*log(x)*b + 2*log(x)*c*x
**2 - c*x**2)/(2*b**2*(b + c*x**2))
```

3.65 $\int \frac{x}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{x}{\left(bx^2+cx^4\right)^2}\,dx=-\frac{1}{2b^2x^2}-\frac{c}{2b^2\left(b+cx^2\right)}-\frac{2c\log(x)}{b^3}+\frac{c\log\left(b+cx^2\right)}{b^3}$$

output
```
-1/2/b^2/x^2-1/2*c/b^2/(c*x^2+b)-2*c*ln(x)/b^3+c*ln(c*x^2+b)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{x}{\left(bx^2+cx^4\right)^2}\,dx=-\frac{b\left(\frac{1}{x^2}+\frac{c}{b+cx^2}\right)+4c\log(x)-2c\log\left(b+cx^2\right)}{2b^3}$$

input
```
Integrate[x/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(b*(x^(-2) + c/(b + c*x^2)) + 4*c*Log[x] - 2*c*Log[b + c*x^2])/b^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^3(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^4(cx^2+b)^2}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2}\int \left(\frac{2c^2}{b^3(cx^2+b)}+\frac{c^2}{b^2(cx^2+b)^2}-\frac{2c}{b^3x^2}+\frac{1}{b^2x^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{2c\log(x^2)}{b^3}+\frac{2c\log(b+cx^2)}{b^3}-\frac{c}{b^2(b+cx^2)}-\frac{1}{b^2x^2}\right)$$

input
```
Int[x/(b*x^2 + c*x^4)^2,x]
```

output
```
(-(1/(b^2*x^2)) - c/(b^2*(b + c*x^2)) - (2*c*Log[x^2])/b^3 + (2*c*Log[b +
c*x^2])/b^3)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

method	result	size
norman	$\frac{\frac{c^{2} x^{5}}{b^{3}}-\frac{x}{2 b}}{x^{3} \left(c \,x^{2}+b \right)}+\frac{c \ln \left(c \,x^{2}+b \right)}{b^{3}}-\frac{2 c \ln \left(x \right)}{b^{3}}$	53
risch	$\frac{-\frac{c \,x^{2}}{b^{2}}-\frac{1}{2 b}}{x^{2} \left(c \,x^{2}+b \right)}-\frac{2 c \ln \left(x \right)}{b^{3}}+\frac{c \ln \left(-c \,x^{2}-b \right)}{b^{3}}$	54
default	$-\frac{1}{2 b^{2} x^{2}}-\frac{2 c \ln \left(x \right)}{b^{3}}+\frac{c^{2} \left(\frac{2 \ln \left(c \,x^{2}+b \right)}{c}-\frac{b}{c \left(c \,x^{2}+b \right)}\right)}{2 b^{3}}$	55
parallelrisch	$-\frac{4 c^{2} \ln \left(x \right) x^{4}-2 c^{2} \ln \left(c \,x^{2}+b \right) x^{4}-2 c^{2} x^{4}+4 b c \ln \left(x \right) x^{2}-2 \ln \left(c \,x^{2}+b \right) x^{2} b c +b^{2}}{2 b^{3} x^{2} \left(c \,x^{2}+b \right)}$	80

input
```
int(x/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(c^2/b^3*x^5-1/2/b*x)/x^3/(c*x^2+b)+c*ln(c*x^2+b)/b^3-2*c*ln(x)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int \frac{x}{(bx^2+cx^4)^2}\,dx = -\frac{2\,bcx^2+b^2-2\,(c^2x^4+bcx^2)\log(cx^2+b)+4\,(c^2x^4+bcx^2)\log(x)}{2\,(b^3cx^4+b^4x^2)}$$

input
```
integrate(x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*c*x^2 + b^2 - 2*(c^2*x^4 + b*c*x^2)*log(c*x^2 + b) + 4*(c^2*x^4
+ b*c*x^2)*log(x))/(b^3*c*x^4 + b^4*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{x}{(bx^2+cx^4)^2}\,dx = \frac{-b-2cx^2}{2b^3x^2+2b^2cx^4} - \frac{2c\log(x)}{b^3} + \frac{c\log\left(\frac{b}{c}+x^2\right)}{b^3}$$

input
```
integrate(x/(c*x**4+b*x**2)**2,x)
```

output
```
(-b - 2*c*x**2)/(2*b**3*x**2 + 2*b**2*c*x**4) - 2*c*log(x)/b**3 + c*log(b/
c + x**2)/b**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x}{(bx^2+cx^4)^2}\,dx = -\frac{2\,cx^2+b}{2\,(b^2cx^4+b^3x^2)} + \frac{c\log(cx^2+b)}{b^3} - \frac{c\log(x^2)}{b^3}$$

input
```
integrate(x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c*x^2 + b)/(b^2*c*x^4 + b^3*x^2) + c*log(c*x^2 + b)/b^3 - c*log(x^
2)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{x}{(bx^2+cx^4)^2}\,dx = \frac{c\log(|cx^2+b|)}{b^3} - \frac{2\,c\log(|x|)}{b^3} - \frac{2\,cx^2+b}{2\,(cx^4+bx^2)b^2}$$

input
```
integrate(x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
c*log(abs(c*x^2 + b))/b^3 - 2*c*log(abs(x))/b^3 - 1/2*(2*c*x^2 + b)/((c*x^
4 + b*x^2)*b^2)
```

Mupad [B] (verification not implemented)

Time = 17.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{x}{(bx^2+cx^4)^2}\,dx = \frac{c\,\ln(c\,x^2+b)}{b^3} - \frac{\frac{1}{2\,b}+\frac{c\,x^2}{b^2}}{c\,x^4+b\,x^2} - \frac{2\,c\,\ln(x)}{b^3}$$

input
```
int(x/(b*x^2 + c*x^4)^2,x)
```

output
```
(c*log(b + c*x^2))/b^3 - (1/(2*b) + (c*x^2)/b^2)/(b*x^2 + c*x^4) - (2*c*lo
g(x))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.65

$$\int \frac{x}{(bx^2 + cx^4)^2}\,dx = \frac{2\log(c\,x^2+b)\,bc\,x^2 + 2\log(c\,x^2+b)\,c^2x^4 - 4\log(x)\,bc\,x^2 - 4\log(x)\,c^2x^4 - b^2 + 2c^2x^4}{2b^3x^2\,(c\,x^2+b)}$$

input
```
int(x/(c*x^4+b*x^2)^2,x)
```

output
```
(2*log(b + c*x**2)*b*c*x**2 + 2*log(b + c*x**2)*c**2*x**4 - 4*log(x)*b*c*x
**2 - 4*log(x)*c**2*x**4 - b**2 + 2*c**2*x**4)/(2*b**3*x**2*(b + c*x**2))
```

3.66 $\int \frac{1}{x(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \frac{1}{x(bx^2+cx^4)^2} dx = -\frac{1}{4b^2x^4} + \frac{c}{b^3x^2} + \frac{c^2}{2b^3(b+cx^2)} + \frac{3c^2\log(x)}{b^4} - \frac{3c^2\log(b+cx^2)}{2b^4}$$

output
```
-1/4/b^2/x^4+c/b^3/x^2+1/2*c^2/b^3/(c*x^2+b)+3*c^2*ln(x)/b^4-3/2*c^2*ln(c*
x^2+b)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \frac{1}{x(bx^2+cx^4)^2} dx = \frac{b\left(-\frac{b}{x^4} + \frac{4c}{x^2} + \frac{2c^2}{b+cx^2}\right) + 12c^2\log(x) - 6c^2\log(b+cx^2)}{4b^4}$$

input
```
Integrate[1/(x*(b*x^2 + c*x^4)^2),x]
```

output
```
(b*(-(b/x^4) + (4*c)/x^2 + (2*c^2)/(b + c*x^2)) + 12*c^2*Log[x] - 6*c^2*Lo
g[b + c*x^2])/(4*b^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^5\left(b+cx^2\right)^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^6\left(cx^2+b\right)^2}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2}\int \left(-\frac{3c^3}{b^4\left(cx^2+b\right)}-\frac{c^3}{b^3\left(cx^2+b\right)^2}+\frac{3c^2}{b^4x^2}-\frac{2c}{b^3x^4}+\frac{1}{b^2x^6}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{3c^2\log\left(x^2\right)}{b^4}-\frac{3c^2\log\left(b+cx^2\right)}{b^4}+\frac{c^2}{b^3\left(b+cx^2\right)}+\frac{2c}{b^3x^2}-\frac{1}{2b^2x^4}\right)$$

input
```
Int[1/(x*(b*x^2 + c*x^4)^2),x]
```

output
```
(-1/2*1/(b^2*x^4) + (2*c)/(b^3*x^2) + c^2/(b^3*(b + c*x^2)) + (3*c^2*Log[x
^2])/b^4 - (3*c^2*Log[b + c*x^2])/b^4)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{1}{4b^2x^4}+\frac{c}{b^3x^2}+\frac{3c^2\ln(x)}{b^4}-\frac{c^3\left(\frac{3\ln\left(c\,x^2+b\right)}{c}-\frac{b}{c\left(c\,x^2+b\right)}\right)}{2b^4}$	65
norman	$\frac{-\frac{1}{4b}+\frac{3c\,x^2}{4b^2}-\frac{3c^3x^6}{2b^4}}{x^4(c\,x^2+b)}+\frac{3c^2\ln(x)}{b^4}-\frac{3c^2\ln\left(c\,x^2+b\right)}{2b^4}$	67
risch	$\frac{\frac{3c^2x^4}{2b^3}+\frac{3c\,x^2}{4b^2}-\frac{1}{4b}}{x^4(c\,x^2+b)}+\frac{3c^2\ln(x)}{b^4}-\frac{3c^2\ln\left(c\,x^2+b\right)}{2b^4}$	67
parallelrisch	$\frac{12c^3\ln(x)x^6-6\ln\left(c\,x^2+b\right)x^6c^3-6c^3x^6+12b\,c^2\ln(x)x^4-6\ln\left(c\,x^2+b\right)x^4b\,c^2+3b^2c\,x^2-b^3}{4b^4x^4(c\,x^2+b)}$	95

input
```
int(1/x/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^2/x^4+c/b^3/x^2+3*c^2*ln(x)/b^4-1/2*c^3/b^4*(3*ln(c*x^2+b)/c-1/c*b/
(c*x^2+b))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(bx^2+cx^4\right)^2}\,dx = \frac{6\,bc^2x^4+3\,b^2cx^2-b^3-6\,(c^3x^6+bc^2x^4)\log\left(cx^2+b\right)+12\,(c^3x^6+bc^2x^4)\log\left(x\right)}{4\,(b^4cx^6+b^5x^4)}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/4*(6*b*c^2*x^4 + 3*b^2*c*x^2 - b^3 - 6*(c^3*x^6 + b*c^2*x^4)*log(c*x^2 +
 b) + 12*(c^3*x^6 + b*c^2*x^4)*log(x))/(b^4*c*x^6 + b^5*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\left(bx^2+cx^4\right)^2}\,dx = \frac{-b^2+3bcx^2+6c^2x^4}{4b^4x^4+4b^3cx^6}+\frac{3c^2\log\left(x\right)}{b^4}-\frac{3c^2\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

input
```
integrate(1/x/(c*x**4+b*x**2)**2,x)
```

output
```
(-b**2 + 3*b*c*x**2 + 6*c**2*x**4)/(4*b**4*x**4 + 4*b**3*c*x**6) + 3*c**2*
log(x)/b**4 - 3*c**2*log(b/c + x**2)/(2*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(bx^2+cx^4\right)^2}\,dx = \frac{6\,c^2x^4+3\,bcx^2-b^2}{4\,\left(b^3cx^6+b^4x^4\right)} - \frac{3\,c^2\log\left(cx^2+b\right)}{2\,b^4} + \frac{3\,c^2\log\left(x^2\right)}{2\,b^4}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/4*(6*c^2*x^4 + 3*b*c*x^2 - b^2)/(b^3*c*x^6 + b^4*x^4) - 3/2*c^2*log(c*x^
2 + b)/b^4 + 3/2*c^2*log(x^2)/b^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.30

$$\int \frac{1}{x\left(bx^2+cx^4\right)^2}\,dx = \frac{3\,c^2\log\left(x^2\right)}{2\,b^4} - \frac{3\,c^2\log\left(|cx^2+b|\right)}{2\,b^4} + \frac{3\,c^3x^2+4\,bc^2}{2\,(cx^2+b)b^4} - \frac{9\,c^2x^4-4\,bcx^2+b^2}{4\,b^4x^4}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
3/2*c^2*log(x^2)/b^4 - 3/2*c^2*log(abs(c*x^2 + b))/b^4 + 1/2*(3*c^3*x^2 +
4*b*c^2)/((c*x^2 + b)*b^4) - 1/4*(9*c^2*x^4 - 4*b*c*x^2 + b^2)/(b^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(bx^2+cx^4\right)^2}\,dx = \frac{\frac{3\,c\,x^2}{4\,b^2}-\frac{1}{4\,b}+\frac{3\,c^2\,x^4}{2\,b^3}}{c\,x^6+b\,x^4} - \frac{3\,c^2\,\ln\left(c\,x^2+b\right)}{2\,b^4} + \frac{3\,c^2\,\ln\left(x\right)}{b^4}$$

input
```
int(1/(x*(b*x^2 + c*x^4)^2),x)
```

output
```
((3*c*x^2)/(4*b^2) - 1/(4*b) + (3*c^2*x^4)/(2*b^3))/(b*x^4 + c*x^6) - (3*c
^2*log(b + c*x^2))/(2*b^4) + (3*c^2*log(x))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \frac{1}{x(bx^2+cx^4)^2}\,dx = \frac{-6\log(c\,x^2+b)\,b\,c^2x^4-6\log(c\,x^2+b)\,c^3x^6+12\log(x)\,b\,c^2x^4+12\log(x)\,c^3x^6-b^3+3b^2c\,x^2-6c^3x^6}{4b^4x^4\,(c\,x^2+b)}$$

input
```
int(1/x/(c*x^4+b*x^2)^2,x)
```

output
```
( - 6*log(b + c*x**2)*b*c**2*x**4 - 6*log(b + c*x**2)*c**3*x**6 + 12*log(x
)*b*c**2*x**4 + 12*log(x)*c**3*x**6 - b**3 + 3*b**2*c*x**2 - 6*c**3*x**6)/
(4*b**4*x**4*(b + c*x**2))
```

3.67 $\int \frac{x^{12}}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \frac{x^{12}}{(bx^2+cx^4)^2} dx = \frac{3b^2x}{c^4} - \frac{2bx^3}{3c^3} + \frac{x^5}{5c^2} + \frac{b^3x}{2c^4(b+cx^2)} - \frac{7b^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}$$

output
```
3*b^2*x/c^4-2/3*b*x^3/c^3+1/5*x^5/c^2+1/2*b^3*x/c^4/(c*x^2+b)-7/2*b^(5/2)*
arctan(c^(1/2)*x/b^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{x^{12}}{(bx^2+cx^4)^2} dx = \frac{x\left(90b^2 - 20bcx^2 + 6c^2x^4 + \frac{15b^3}{b+cx^2}\right)}{30c^4} - \frac{7b^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}$$

input
```
Integrate[x^12/(b*x^2 + c*x^4)^2,x]
```

output
```
(x*(90*b^2 - 20*b*c*x^2 + 6*c^2*x^4 + (15*b^3)/(b + c*x^2)))/(30*c^4) - (7
*b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {9, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^8}{(b+cx^2)^2}dx$$

$$\Big\downarrow \; 252$$

$$\frac{7\int \frac{x^6}{cx^2+b}dx}{2c}-\frac{x^7}{2c\,(b+cx^2)}$$

$$\Big\downarrow \; 254$$

$$\frac{7\int \left(\frac{x^4}{c}-\frac{bx^2}{c^2}-\frac{b^3}{c^3(cx^2+b)}+\frac{b^2}{c^3}\right)dx}{2c}-\frac{x^7}{2c\,(b+cx^2)}$$

$$\Big\downarrow \; 2009$$

$$\frac{7\left(-\frac{b^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}\right)}{2c}-\frac{x^7}{2c\,(b+cx^2)}$$

input
```
Int[x^12/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*x^7/(c*(b + c*x^2)) + (7*((b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) -
 (b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(7/2)))/(2*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{1}{5}c^2x^5-\frac{2}{3}bc\,x^3+3b^2x}{c^4}-\frac{b^3\left(-\frac{x}{2\left(c\,x^2+b\right)}+\frac{7\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^4}$	65
risch	$\frac{x^5}{5c^2}-\frac{2b\,x^3}{3c^3}+\frac{3b^2x}{c^4}+\frac{b^3x}{2c^4(c\,x^2+b)}+\frac{7\sqrt{-bc}\,b^2\ln\left(-\sqrt{-bc}\,x-b\right)}{4c^5}-\frac{7\sqrt{-bc}\,b^2\ln\left(\sqrt{-bc}\,x-b\right)}{4c^5}$	101

input
```
int(x^12/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/5*c^2*x^5-2/3*b*c*x^3+3*b^2*x)-b^3/c^4*(-1/2*x/(c*x^2+b)+7/2/(b*c
)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.44

$$\int \frac{x^{12}}{(bx^2+cx^4)^2}\,dx = \left[\frac{12\,c^3x^7-28\,bc^2x^5+140\,b^2cx^3+210\,b^3x+105\,(b^2cx^2+b^3)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{60\,(c^5x^2+bc^4)}, \frac{6\,c^3x^7-14\,bc^2}{}\right.$$

input
```
integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*c^3*x^7 - 28*b*c^2*x^5 + 140*b^2*c*x^3 + 210*b^3*x + 105*(b^2*c*
x^2 + b^3)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^
5*x^2 + b*c^4), 1/30*(6*c^3*x^7 - 14*b*c^2*x^5 + 70*b^2*c*x^3 + 105*b^3*x
- 105*(b^2*c*x^2 + b^3)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b))/(c^5*x^2 + b*c^
4)]
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.59

$$\int \frac{x^{12}}{(bx^2+cx^4)^2}\,dx = \frac{b^3x}{2bc^4+2c^5x^2}+\frac{3b^2x}{c^4}-\frac{2bx^3}{3c^3}+\frac{7\sqrt{-\frac{b^5}{c^9}}\log\left(x-\frac{c^4\sqrt{-\frac{b^5}{c^9}}}{b^2}\right)}{4} - \frac{7\sqrt{-\frac{b^5}{c^9}}\log\left(x+\frac{c^4\sqrt{-\frac{b^5}{c^9}}}{b^2}\right)}{4}+\frac{x^5}{5c^2}$$

input
```
integrate(x**12/(c*x**4+b*x**2)**2,x)
```

output
```
b**3*x/(2*b*c**4 + 2*c**5*x**2) + 3*b**2*x/c**4 - 2*b*x**3/(3*c**3) + 7*sq
rt(-b**5/c**9)*log(x - c**4*sqrt(-b**5/c**9)/b**2)/4 - 7*sqrt(-b**5/c**9)*
log(x + c**4*sqrt(-b**5/c**9)/b**2)/4 + x**5/(5*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{x^{12}}{(bx^2+cx^4)^2}\,dx = \frac{b^3x}{2\,(c^5x^2+bc^4)} - \frac{7\,b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^4} + \frac{3\,c^2x^5-10\,bcx^3+45\,b^2x}{15\,c^4}$$

input
```
integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*b^3*x/(c^5*x^2 + b*c^4) - 7/2*b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4
) + 1/15*(3*c^2*x^5 - 10*b*c*x^3 + 45*b^2*x)/c^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{x^{12}}{(bx^2+cx^4)^2}\,dx = -\frac{7\,b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^4} + \frac{b^3x}{2\,(cx^2+b)c^4} + \frac{3\,c^8x^5-10\,bc^7x^3+45\,b^2c^6x}{15\,c^{10}}$$

input
```
integrate(x^12/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-7/2*b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/2*b^3*x/((c*x^2 + b)*c^
4) + 1/15*(3*c^8*x^5 - 10*b*c^7*x^3 + 45*b^2*c^6*x)/c^10
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{x^{12}}{(bx^2+cx^4)^2}\,dx = \frac{x^5}{5\,c^2} - \frac{2\,b\,x^3}{3\,c^3} + \frac{3\,b^2\,x}{c^4} - \frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,c^{9/2}} + \frac{b^3\,x}{2\,(c^5\,x^2+b\,c^4)}$$

input
```
int(x^12/(b*x^2 + c*x^4)^2,x)
```

output
```
x^5/(5*c^2) - (2*b*x^3)/(3*c^3) + (3*b^2*x)/c^4 - (7*b^(5/2)*atan((c^(1/2)
*x)/b^(1/2)))/(2*c^(9/2)) + (b^3*x)/(2*(b*c^4 + c^5*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{x^{12}}{(bx^2+cx^4)^2}\,dx$$
$$= \frac{-105\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^3 - 105\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^2c\,x^2 + 105b^3cx + 70b^2c^2x^3 - 14b\,c^3x^5 + 6c^4x^7}{30c^5\,(c\,x^2+b)}$$

input
```
int(x^12/(c*x^4+b*x^2)^2,x)
```

output
```
( - 105*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**3 - 105*sqrt(c)*s
qrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**2*c*x**2 + 105*b**3*c*x + 70*b**2*
c**2*x**3 - 14*b*c**3*x**5 + 6*c**4*x**7)/(30*c**5*(b + c*x**2))
```

3.68 $\int \frac{x^{10}}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{x^{10}}{(bx^2+cx^4)^2}\,dx = -\frac{2bx}{c^3}+\frac{x^3}{3c^2}-\frac{b^2x}{2c^3(b+cx^2)}+\frac{5b^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}$$

output
```
-2*b*x/c^3+1/3*x^3/c^2-1/2*b^2*x/c^3/(c*x^2+b)+5/2*b^(3/2)*arctan(c^(1/2)*
x/b^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{x^{10}}{(bx^2+cx^4)^2}\,dx = \frac{x\left(-12b+2cx^2-\frac{3b^2}{b+cx^2}\right)}{6c^3}+\frac{5b^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}$$

input
```
Integrate[x^10/(b*x^2 + c*x^4)^2,x]
```

output
```
(x*(-12*b + 2*c*x^2 - (3*b^2)/(b + c*x^2)))/(6*c^3) + (5*b^(3/2)*ArcTan[(S
qrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^6}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{5\int \frac{x^4}{cx^2+b}dx}{2c}-\frac{x^5}{2c\,(b+cx^2)}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{5\int \left(\frac{b^2}{c^2(cx^2+b)}-\frac{b}{c^2}+\frac{x^2}{c}\right)dx}{2c}-\frac{x^5}{2c\,(b+cx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5\left(\frac{b^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}\right)}{2c}-\frac{x^5}{2c\,(b+cx^2)}$$

input
```
Int[x^10/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*x^5/(c*(b + c*x^2)) + (5*(-((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)))/(2*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{-\frac{1}{3}c\,x^{3}+2bx}{c^{3}}+\frac{b^{2}\left(-\frac{x}{2\left(c\,x^{2}+b\right)}+\frac{5\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^{3}}$	54
risch	$\frac{x^{3}}{3c^{2}}-\frac{2bx}{c^{3}}-\frac{b^{2}x}{2c^{3}\left(c\,x^{2}+b\right)}+\frac{5\sqrt{-bc}\,b\ln\left(-\sqrt{-bc}\,x+b\right)}{4c^{4}}-\frac{5\sqrt{-bc}\,b\ln\left(\sqrt{-bc}\,x+b\right)}{4c^{4}}$	82

input
```
int(x^10/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/3*c*x^3+2*b*x)+b^2/c^3*(-1/2*x/(c*x^2+b)+5/2/(b*c)^(1/2)*arctan
(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.52

$$\int \frac{x^{10}}{(bx^2+cx^4)^2}\,dx = \left[\frac{4\,c^2x^5-20\,bcx^3-30\,b^2x+15\,(bcx^2+b^2)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{12\,(c^4x^2+bc^3)}, \frac{2\,c^2x^5-10\,bcx^3-15\,b^2x+15\,(b}{6\,(c^4x^2+}\right.$$

input
```
integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*c^2*x^5 - 20*b*c*x^3 - 30*b^2*x + 15*(b*c*x^2 + b^2)*sqrt(-b/c)*l
og((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^4*x^2 + b*c^3), 1/6*(2*
c^2*x^5 - 10*b*c*x^3 - 15*b^2*x + 15*(b*c*x^2 + b^2)*sqrt(b/c)*arctan(c*x*
sqrt(b/c)/b))/(c^4*x^2 + b*c^3)]
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{x^{10}}{(bx^2+cx^4)^2}\,dx = -\frac{b^2x}{2bc^3+2c^4x^2}-\frac{2bx}{c^3}-\frac{5\sqrt{-\frac{b^3}{c^7}}\log\left(x-\frac{c^3\sqrt{-\frac{b^3}{c^7}}}{b}\right)}{4} + \frac{5\sqrt{-\frac{b^3}{c^7}}\log\left(x+\frac{c^3\sqrt{-\frac{b^3}{c^7}}}{b}\right)}{4}+\frac{x^3}{3c^2}$$

input
```
integrate(x**10/(c*x**4+b*x**2)**2,x)
```

output
```
-b**2*x/(2*b*c**3 + 2*c**4*x**2) - 2*b*x/c**3 - 5*sqrt(-b**3/c**7)*log(x -
 c**3*sqrt(-b**3/c**7)/b)/4 + 5*sqrt(-b**3/c**7)*log(x + c**3*sqrt(-b**3/c
**7)/b)/4 + x**3/(3*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{x^{10}}{(bx^2+cx^4)^2}\,dx = -\frac{b^2x}{2\,(c^4x^2+bc^3)} + \frac{5\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^3} + \frac{cx^3-6\,bx}{3\,c^3}$$

input
```
integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*b^2*x/(c^4*x^2 + b*c^3) + 5/2*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^
3) + 1/3*(c*x^3 - 6*b*x)/c^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{x^{10}}{(bx^2+cx^4)^2}\,dx = \frac{5\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^3} - \frac{b^2x}{2\,(cx^2+b)c^3} + \frac{c^4x^3-6\,bc^3x}{3\,c^6}$$

input
```
integrate(x^10/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
5/2*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) - 1/2*b^2*x/((c*x^2 + b)*c^3
) + 1/3*(c^4*x^3 - 6*b*c^3*x)/c^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{x^{10}}{(bx^2+cx^4)^2}\,dx = \frac{x^3}{3\,c^2} + \frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,c^{7/2}} - \frac{b^2\,x}{2\,\left(c^4\,x^2+b\,c^3\right)} - \frac{2\,b\,x}{c^3}$$

input
```
int(x^10/(b*x^2 + c*x^4)^2,x)
```

output
```
x^3/(3*c^2) + (5*b^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/(2*c^(7/2)) - (b^2*x)/
(2*(b*c^3 + c^4*x^2)) - (2*b*x)/c^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

$$\int \frac{x^{10}}{(bx^2+cx^4)^2}\,dx = \frac{15\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^2+15\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)bc\,x^2-15b^2cx-10b\,c^2x^3+2c^3x^5}{6c^4\,(c\,x^2+b)}$$

input
```
int(x^10/(c*x^4+b*x^2)^2,x)
```

output
```
(15*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**2 + 15*sqrt(c)*sqrt(b
)*atan((c*x)/(sqrt(c)*sqrt(b)))*b*c*x**2 - 15*b**2*c*x - 10*b*c**2*x**3 +
2*c**3*x**5)/(6*c**4*(b + c*x**2))
```

3.69 $\int \frac{x^8}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{x^8}{\left(bx^2+cx^4\right)^2}\,dx=\frac{x}{c^2}+\frac{bx}{2c^2\left(b+cx^2\right)}-\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{5/2}}$$

output
```
x/c^2+1/2*b*x/c^2/(c*x^2+b)-3/2*b^(1/2)*arctan(c^(1/2)*x/b^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{\left(bx^2+cx^4\right)^2}\,dx=\frac{x}{c^2}+\frac{bx}{2c^2\left(b+cx^2\right)}-\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{5/2}}$$

input
```
Integrate[x^8/(b*x^2 + c*x^4)^2,x]
```

output
```
x/c^2 + (b*x)/(2*c^2*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]]
)/(2*c^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^4}{(b+cx^2)^2}dx$$

$$\Big\downarrow \; 252$$

$$\frac{3\int \frac{x^2}{cx^2+b}dx}{2c}-\frac{x^3}{2c\,(b+cx^2)}$$

$$\Big\downarrow \; 262$$

$$\frac{3\left(\frac{x}{c}-\frac{b\int \frac{1}{cx^2+b}dx}{c}\right)}{2c}-\frac{x^3}{2c\,(b+cx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{3\left(\frac{x}{c}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}\right)}{2c}-\frac{x^3}{2c\,(b+cx^2)}$$

input
```
Int[x^8/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*x^3/(c*(b + c*x^2)) + (3*(x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])
/c^(3/2)))/(2*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x}{c^{2}}-\frac{b \left(-\frac{x}{2 \left(c \,x^{2}+b \right)}+\frac{3 \arctan \left(\frac{c x}{\sqrt{b c}}\right)}{2 \sqrt{b c}}\right)}{c^{2}}$	42
risch	$\frac{x}{c^{2}}+\frac{b x}{2 c^{2} \left(c \,x^{2}+b \right)}+\frac{3 \sqrt{-b c}\, \ln \left(-\sqrt{-b c}\, x -b \right)}{4 c^{3}}-\frac{3 \sqrt{-b c}\, \ln \left(\sqrt{-b c}\, x -b \right)}{4 c^{3}}$	72

input
```
int(x^8/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2-b/c^2*(-1/2*x/(c*x^2+b)+3/2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.67

$$\int \frac{x^8}{(bx^2+cx^4)^2}\,dx = \left[\frac{4\,cx^3+3\,(cx^2+b)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+6\,bx}{4\,(c^3x^2+bc^2)}, \frac{2\,cx^3-3\,(cx^2+b)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right)+3\,bx}{2\,(c^3x^2+bc^2)}\right]$$

input
```
integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*c*x^3 + 3*(c*x^2 + b)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b
)/(c*x^2 + b)) + 6*b*x)/(c^3*x^2 + b*c^2), 1/2*(2*c*x^3 - 3*(c*x^2 + b)*sq
rt(b/c)*arctan(c*x*sqrt(b/c)/b) + 3*b*x)/(c^3*x^2 + b*c^2)]
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{x^8}{(bx^2+cx^4)^2}\,dx = \frac{bx}{2bc^2+2c^3x^2}+\frac{3\sqrt{-\frac{b}{c^5}}\log\left(-c^2\sqrt{-\frac{b}{c^5}}+x\right)}{4} - \frac{3\sqrt{-\frac{b}{c^5}}\log\left(c^2\sqrt{-\frac{b}{c^5}}+x\right)}{4}+\frac{x}{c^2}$$

input
```
integrate(x**8/(c*x**4+b*x**2)**2,x)
```

output
```
b*x/(2*b*c**2 + 2*c**3*x**2) + 3*sqrt(-b/c**5)*log(-c**2*sqrt(-b/c**5) + x
)/4 - 3*sqrt(-b/c**5)*log(c**2*sqrt(-b/c**5) + x)/4 + x/c**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{(bx^2+cx^4)^2}\,dx = \frac{bx}{2\,(c^3x^2+bc^2)} - \frac{3\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^2} + \frac{x}{c^2}$$

input
```
integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*b*x/(c^3*x^2 + b*c^2) - 3/2*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) +
x/c^2
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(bx^2+cx^4)^2}\,dx = -\frac{3\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^2} + \frac{bx}{2\,(cx^2+b)c^2} + \frac{x}{c^2}$$

input
```
integrate(x^8/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-3/2*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/2*b*x/((c*x^2 + b)*c^2) +
 x/c^2
```

Mupad [B] (verification not implemented)

Time = 18.60 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{x^8}{(bx^2+cx^4)^2}\,dx = \frac{x}{c^2} + \frac{b\,x}{2\,(c^3\,x^2+b\,c^2)} - \frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,c^{5/2}}$$

input
```
int(x^8/(b*x^2 + c*x^4)^2,x)
```

output
```
x/c^2 + (b*x)/(2*(b*c^2 + c^3*x^2)) - (3*b^(1/2)*atan((c^(1/2)*x)/b^(1/2))
)/(2*c^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int \frac{x^8}{(bx^2+cx^4)^2}\,dx = \frac{-3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b-3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)c\,x^2+3bcx+2c^2x^3}{2c^3\,(c\,x^2+b)}$$

input
```
int(x^8/(c*x^4+b*x^2)^2,x)
```

output
```
( - 3*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b - 3*sqrt(c)*sqrt(b)*
atan((c*x)/(sqrt(c)*sqrt(b)))*c*x**2 + 3*b*c*x + 2*c**2*x**3)/(2*c**3*(b +
 c*x**2))
```

3.70 $\int \frac{x^6}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{x^6}{(bx^2+cx^4)^2} dx = -\frac{x}{2c(b+cx^2)} + \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{3/2}}$$

output
```
-1/2*x/c/(c*x^2+b)+1/2*arctan(c^(1/2)*x/b^(1/2))/b^(1/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(bx^2+cx^4)^2} dx = -\frac{x}{2c(b+cx^2)} + \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{3/2}}$$

input
```
Integrate[x^6/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*x/(c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^2}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\int \frac{1}{cx^2+b}dx}{2c}-\frac{x}{2c(b+cx^2)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{3/2}}-\frac{x}{2c(b+cx^2)}$$

input
```
Int[x^6/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*x/(c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(3/2))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{x}{2c(c\,x^2+b)}+\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2c\sqrt{bc}}$	36
risch	$-\frac{x}{2c(c\,x^2+b)}-\frac{\ln\left(cx+\sqrt{-bc}\right)}{4\sqrt{-bc}\,c}+\frac{\ln\left(-cx+\sqrt{-bc}\right)}{4\sqrt{-bc}\,c}$	62

input
```
int(x^6/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/c/(c*x^2+b)+1/2/c/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{x^6}{(bx^2+cx^4)^2}\,dx = \left[-\frac{2\,bcx+(cx^2+b)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x-b}{cx^2+b}\right)}{4\,(bc^3x^2+b^2c^2)},\ -\frac{bcx-(cx^2+b)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{2\,(bc^3x^2+b^2c^2)}\right]$$

input
```
integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*b*c*x + (c*x^2 + b)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(
c*x^2 + b)))/(b*c^3*x^2 + b^2*c^2), -1/2*(b*c*x - (c*x^2 + b)*sqrt(b*c)*ar
ctan(sqrt(b*c)*x/b))/(b*c^3*x^2 + b^2*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{x^6}{(bx^2+cx^4)^2}\,dx = -\frac{x}{2bc+2c^2x^2} - \frac{\sqrt{-\frac{1}{bc^3}}\log\left(-bc\sqrt{-\frac{1}{bc^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{bc^3}}\log\left(bc\sqrt{-\frac{1}{bc^3}}+x\right)}{4}$$

input
```
integrate(x**6/(c*x**4+b*x**2)**2,x)
```

output
```
-x/(2*b*c + 2*c**2*x**2) - sqrt(-1/(b*c**3))*log(-b*c*sqrt(-1/(b*c**3)) +
x)/4 + sqrt(-1/(b*c**3))*log(b*c*sqrt(-1/(b*c**3)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{x^6}{(bx^2+cx^4)^2}\,dx = -\frac{x}{2\,(c^2x^2+bc)} + \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c}$$

input
```
integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(c^2*x^2 + b*c) + 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{(bx^2+cx^4)^2}\,dx = \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c} - \frac{x}{2\left(cx^2+b\right)c}$$

input
```
integrate(x^6/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) - 1/2*x/((c*x^2 + b)*c)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{(bx^2+cx^4)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\sqrt{b}\,c^{3/2}} - \frac{x}{2\,c\,\left(c\,x^2+b\right)}$$

input
```
int(x^6/(b*x^2 + c*x^4)^2,x)
```

output
```
atan((c^(1/2)*x)/b^(1/2))/(2*b^(1/2)*c^(3/2)) - x/(2*c*(b + c*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int \frac{x^6}{(bx^2+cx^4)^2}\,dx = \frac{\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b + \sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)c\,x^2 - bcx}{2b\,c^2\left(c\,x^2+b\right)}$$

input
```
int(x^6/(c*x^4+b*x^2)^2,x)
```

output
```
(sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b + sqrt(c)*sqrt(b)*atan((c
*x)/(sqrt(c)*sqrt(b)))*c*x**2 - b*c*x)/(2*b*c**2*(b + c*x**2))
```

3.71 $\int \frac{x^4}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{x^4}{(bx^2+cx^4)^2} dx = \frac{x}{2b(b+cx^2)} + \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}$$

output
```
1/2*x/b/(c*x^2+b)+1/2*arctan(c^(1/2)*x/b^(1/2))/b^(3/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(bx^2+cx^4)^2} dx = \frac{x}{2b(b+cx^2)} + \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}$$

input
```
Integrate[x^4/(b*x^2 + c*x^4)^2,x]
```

output
```
x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{1}{cx^2+b}dx}{2b}+\frac{x}{2b\,(b+cx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b\,(b+cx^2)}$$

input
```
Int[x^4/(b*x^2 + c*x^4)^2,x]
```

output
```
x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt[c])
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x}{2b(c\,x^2+b)}+\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2b\sqrt{bc}}$	36
risch	$\frac{x}{2b(c\,x^2+b)}-\frac{\ln\left(cx+\sqrt{-bc}\right)}{4\sqrt{-bc}\,b}+\frac{\ln\left(-cx+\sqrt{-bc}\right)}{4\sqrt{-bc}\,b}$	62

input
```
int(x^4/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/b/(c*x^2+b)+1/2/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{x^4}{(bx^2+cx^4)^2}\,dx = \left[\frac{2\,bcx-(cx^2+b)\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x-b}{cx^2+b}\right)}{4\,(b^2c^2x^2+b^3c)}, \frac{bcx+(cx^2+b)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{2\,(b^2c^2x^2+b^3c)}\right]$$

input
```
integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*b*c*x - (c*x^2 + b)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c
*x^2 + b)))/(b^2*c^2*x^2 + b^3*c), 1/2*(b*c*x + (c*x^2 + b)*sqrt(b*c)*arct
an(sqrt(b*c)*x/b))/(b^2*c^2*x^2 + b^3*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{x^4}{(bx^2 + cx^4)^2}\,dx = \frac{x}{2b^2 + 2bcx^2} - \frac{\sqrt{-\frac{1}{b^3c}}\log\left(-b^2\sqrt{-\frac{1}{b^3c}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{b^3c}}\log\left(b^2\sqrt{-\frac{1}{b^3c}} + x\right)}{4}$$

input
```
integrate(x**4/(c*x**4+b*x**2)**2,x)
```

output
```
x/(2*b**2 + 2*b*c*x**2) - sqrt(-1/(b**3*c))*log(-b**2*sqrt(-1/(b**3*c)) +
x)/4 + sqrt(-1/(b**3*c))*log(b**2*sqrt(-1/(b**3*c)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{(bx^2 + cx^4)^2}\,dx = \frac{x}{2\,(bcx^2 + b^2)} + \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*x/(b*c*x^2 + b^2) + 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{\left(bx^2+cx^4\right)^2}\,dx = \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}b} + \frac{x}{2\left(cx^2+b\right)b}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) + 1/2*x/((c*x^2 + b)*b)
```

Mupad [B] (verification not implemented)

Time = 13.92 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x^4}{\left(bx^2+cx^4\right)^2}\,dx = \frac{x}{2\,b\,\left(c\,x^2+b\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{3/2}\,\sqrt{c}}$$

input
```
int(x^4/(b*x^2 + c*x^4)^2,x)
```

output
```
x/(2*b*(b + c*x^2)) + atan((c^(1/2)*x)/b^(1/2))/(2*b^(3/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \frac{x^4}{\left(bx^2+cx^4\right)^2}\,dx = \frac{\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b+\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)c\,x^2+bcx}{2b^2c\left(c\,x^2+b\right)}$$

input
```
int(x^4/(c*x^4+b*x^2)^2,x)
```

output
```
(sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b + sqrt(c)*sqrt(b)*atan((c
*x)/(sqrt(c)*sqrt(b)))*c*x**2 + b*c*x)/(2*b**2*c*(b + c*x**2))
```

3.72 $\int \frac{x^2}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{x^2}{(bx^2+cx^4)^2}\,dx = -\frac{1}{b^2x}-\frac{cx}{2b^2(b+cx^2)}-\frac{3\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}}$$

output
```
-1/b^2/x-1/2*c*x/b^2/(c*x^2+b)-3/2*c^(1/2)*arctan(c^(1/2)*x/b^(1/2))/b^(5/
2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(bx^2+cx^4)^2}\,dx = -\frac{1}{b^2x}-\frac{cx}{2b^2(b+cx^2)}-\frac{3\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}}$$

input
```
Integrate[x^2/(b*x^2 + c*x^4)^2,x]
```

output
```
-(1/(b^2*x)) - (c*x)/(2*b^2*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]*x)/S
qrt[b]])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {9, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{x^2(b+cx^2)^2}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{3\int \frac{1}{x^2(cx^2+b)}dx}{2b}+\frac{1}{2bx(b+cx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{3\left(-\frac{c\int \frac{1}{cx^2+b}dx}{b}-\frac{1}{bx}\right)}{2b}+\frac{1}{2bx(b+cx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{2b}+\frac{1}{2bx(b+cx^2)}$$

input
```
Int[x^2/(b*x^2 + c*x^4)^2,x]
```

output
```
1/(2*b*x*(b + c*x^2)) + (3*(-(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[
b]])/b^(3/2)))/(2*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{1}{b^{2} x}-\frac{c \left(\frac{x}{2 c \,x^{2}+2 b}+\frac{3 \arctan \left(\frac{c x}{\sqrt{b c}}\right)}{2 \sqrt{b c}}\right)}{b^{2}}$	45
risch	$\frac{-\frac{3 c \,x^{2}}{2 b^{2}}-\frac{1}{b}}{x \left(c \,x^{2}+b \right)}+\frac{3 \left(\sum_{\_R =\operatorname{RootOf}\left(b^{5} \_Z^{2}+c \right)}\_R \ln \left(\left(3 \_R^{2} b^{5}+2 c \right) x +b^{3} \_R \right)\right)}{4}$	68

input
```
int(x^2/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2/x-c/b^2*(1/2*x/(c*x^2+b)+3/2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.52

$$\int \frac{x^2}{(bx^2+cx^4)^2}\,dx = \left[-\frac{6\,cx^2 - 3\,(cx^3+bx)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+4\,b}{4\,(b^2cx^3+b^3x)},\right.$$
$$\left.-\frac{3\,cx^2+3\,(cx^3+bx)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+2\,b}{2\,(b^2cx^3+b^3x)}\right]$$

input
```
integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(6*c*x^2 - 3*(c*x^3 + b*x)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b)
- b)/(c*x^2 + b)) + 4*b)/(b^2*c*x^3 + b^3*x), -1/2*(3*c*x^2 + 3*(c*x^3 + b
*x)*sqrt(c/b)*arctan(x*sqrt(c/b)) + 2*b)/(b^2*c*x^3 + b^3*x)]
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{(bx^2+cx^4)^2}\,dx = \frac{3\sqrt{-\frac{c}{b^5}}\log\left(-\frac{b^3\sqrt{-\frac{c}{b^5}}}{c}+x\right)}{4}$$
$$-\frac{3\sqrt{-\frac{c}{b^5}}\log\left(\frac{b^3\sqrt{-\frac{c}{b^5}}}{c}+x\right)}{4}+\frac{-2b-3cx^2}{2b^3x+2b^2cx^3}$$

input
```
integrate(x**2/(c*x**4+b*x**2)**2,x)
```

output
```
3*sqrt(-c/b**5)*log(-b**3*sqrt(-c/b**5)/c + x)/4 - 3*sqrt(-c/b**5)*log(b**
3*sqrt(-c/b**5)/c + x)/4 + (-2*b - 3*c*x**2)/(2*b**3*x + 2*b**2*c*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{(bx^2+cx^4)^2}\,dx = -\frac{3\,cx^2+2\,b}{2\,(b^2cx^3+b^3x)} - \frac{3\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^2}$$

input
```
integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*c*x^2 + 2*b)/(b^2*c*x^3 + b^3*x) - 3/2*c*arctan(c*x/sqrt(b*c))/(sq
rt(b*c)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(bx^2+cx^4)^2}\,dx = -\frac{3\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^2} - \frac{3\,cx^2+2\,b}{2\,(cx^3+bx)b^2}$$

input
```
integrate(x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-3/2*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/2*(3*c*x^2 + 2*b)/((c*x^3
 + b*x)*b^2)
```

Mupad [B] (verification not implemented)

Time = 16.75 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{(bx^2+cx^4)^2}\,dx = -\frac{\frac{1}{b}+\frac{3\,c\,x^2}{2\,b^2}}{c\,x^3+b\,x}-\frac{3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{5/2}}$$

input
```
int(x^2/(b*x^2 + c*x^4)^2,x)
```

output
```
- (1/b + (3*c*x^2)/(2*b^2))/(b*x + c*x^3) - (3*c^(1/2)*atan((c^(1/2)*x)/b^
(1/2)))/(2*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(bx^2+cx^4)^2}\,dx = \frac{-3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)bx-3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)c\,x^3-2b^2-3bc\,x^2}{2b^3x\,(c\,x^2+b)}$$

input
```
int(x^2/(c*x^4+b*x^2)^2,x)
```

output
```
( - 3*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b*x - 3*sqrt(c)*sqrt(b
)*atan((c*x)/(sqrt(c)*sqrt(b)))*c*x**3 - 2*b**2 - 3*b*c*x**2)/(2*b**3*x*(b
 + c*x**2))
```

3.73 $\int \frac{1}{\left(bx^2+cx^4\right)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{1}{\left(bx^2+cx^4\right)^2} \, dx = -\frac{1}{3b^2x^3}+\frac{2c}{b^3x}+\frac{c^2x}{2b^3\left(b+cx^2\right)}+\frac{5c^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}$$

output
```
-1/3/b^2/x^3+2*c/b^3/x+1/2*c^2*x/b^3/(c*x^2+b)+5/2*c^(3/2)*arctan(c^(1/2)*
x/b^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(bx^2+cx^4\right)^2} \, dx = -\frac{1}{3b^2x^3}+\frac{2c}{b^3x}+\frac{c^2x}{2b^3\left(b+cx^2\right)}+\frac{5c^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(-2),x]
```

output
```
-1/3*1/(b^2*x^3) + (2*c)/(b^3*x) + (c^2*x)/(2*b^3*(b + c*x^2)) + (5*c^(3/2
)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1397, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 1397}$$

$$\int \frac{1}{x^4(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{5\int \frac{1}{x^4(cx^2+b)}dx}{2b}+\frac{1}{2bx^3(b+cx^2)}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{5\left(-\frac{c\int \frac{1}{x^2(cx^2+b)}dx}{b}-\frac{1}{3bx^3}\right)}{2b}+\frac{1}{2bx^3(b+cx^2)}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{5\left(-\frac{c\left(-\frac{c\int \frac{1}{cx^2+b}dx}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{2b}+\frac{1}{2bx^3(b+cx^2)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{5\left(-\frac{c\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{2b}+\frac{1}{2bx^3(b+cx^2)}$$

input
```
Int[(b*x^2 + c*x^4)^(-2),x]
```

output
```
1/(2*b*x^3*(b + c*x^2)) + (5*(-1/3*1/(b*x^3) - (c*(-(1/(b*x)) - (Sqrt[c]*A
rcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)))/b))/(2*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1397
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[x^(2*p)*(b + c*x^2
)^p, x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{3b^{2}x^{3}}+\frac{2c}{b^{3}x}+\frac{c^{2}\left(\frac{x}{2c\,x^{2}+2b}+\frac{5\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^{3}}$	55
risch	$\frac{\frac{5c^{2}x^{4}}{2b^{3}}+\frac{5c\,x^{2}}{3b^{2}}-\frac{1}{3b}}{x^{3}\left(c\,x^{2}+b\right)}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(b^{7}\_Z^{2}+c^{3}\right)}\_R\ln\left(\left(3\_R^{2}b^{7}+2c^{3}\right)x-b^{4}c\_R\right)\right)}{4}$	85

input
```
int(1/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^2/x^3+2*c/b^3/x+c^2/b^3*(1/2*x/(c*x^2+b)+5/2/(b*c)^(1/2)*arctan(c*x
/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.57

$$\int \frac{1}{(bx^2+cx^4)^2}\,dx$$
$$= \left[\frac{30\,c^2x^4+20\,bcx^2+15\,(c^2x^5+bcx^3)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-4\,b^2}{12\,(b^3cx^5+b^4x^3)}, \frac{15\,c^2x^4+10\,bcx^2+15\,(c^2x^5+b}{6\,(b^3cx^5-}\right.$$

input
```
integrate(1/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[1/12*(30*c^2*x^4 + 20*b*c*x^2 + 15*(c^2*x^5 + b*c*x^3)*sqrt(-c/b)*log((c*
x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) - 4*b^2)/(b^3*c*x^5 + b^4*x^3), 1
/6*(15*c^2*x^4 + 10*b*c*x^2 + 15*(c^2*x^5 + b*c*x^3)*sqrt(c/b)*arctan(x*sq
rt(c/b)) - 2*b^2)/(b^3*c*x^5 + b^4*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{1}{(bx^2+cx^4)^2}\,dx = -\frac{5\sqrt{-\frac{c^3}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c^3}{b^7}}}{c^2}+x\right)}{4} + \frac{5\sqrt{-\frac{c^3}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c^3}{b^7}}}{c^2}+x\right)}{4} + \frac{-2b^2+10bcx^2+15c^2x^4}{6b^4x^3+6b^3cx^5}$$

input
```
integrate(1/(c*x**4+b*x**2)**2,x)
```

output
```
-5*sqrt(-c**3/b**7)*log(-b**4*sqrt(-c**3/b**7)/c**2 + x)/4 + 5*sqrt(-c**3/
b**7)*log(b**4*sqrt(-c**3/b**7)/c**2 + x)/4 + (-2*b**2 + 10*b*c*x**2 + 15*
c**2*x**4)/(6*b**4*x**3 + 6*b**3*c*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{1}{(bx^2+cx^4)^2}\,dx = \frac{15\,c^2x^4+10\,bcx^2-2\,b^2}{6\,(b^3cx^5+b^4x^3)}+\frac{5\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^3}$$

input
```
integrate(1/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/6*(15*c^2*x^4 + 10*b*c*x^2 - 2*b^2)/(b^3*c*x^5 + b^4*x^3) + 5/2*c^2*arct
an(c*x/sqrt(b*c))/(sqrt(b*c)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{1}{(bx^2+cx^4)^2}\,dx = \frac{5\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^3}+\frac{c^2x}{2\,(cx^2+b)b^3}+\frac{6\,cx^2-b}{3\,b^3x^3}$$

input
```
integrate(1/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
5/2*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/2*c^2*x/((c*x^2 + b)*b^3
) + 1/3*(6*c*x^2 - b)/(b^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 18.48 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1}{(bx^2+cx^4)^2}\,dx=\frac{\frac{5\,c\,x^2}{3\,b^2}-\frac{1}{3\,b}+\frac{5\,c^2\,x^4}{2\,b^3}}{c\,x^5+b\,x^3}+\frac{5\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{7/2}}$$

input
```
int(1/(b*x^2 + c*x^4)^2,x)
```

output
```
((5*c*x^2)/(3*b^2) - 1/(3*b) + (5*c^2*x^4)/(2*b^3))/(b*x^3 + c*x^5) + (5*c
^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/(2*b^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{1}{(bx^2+cx^4)^2}\,dx$$
$$=\frac{15\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)bc\,x^3+15\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)c^2x^5-2b^3+10b^2c\,x^2+15b\,c^2x^4}{6b^4x^3\,(c\,x^2+b)}$$

input
```
int(1/(c*x^4+b*x^2)^2,x)
```

output
```
(15*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b*c*x**3 + 15*sqrt(c)*sq
rt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*c**2*x**5 - 2*b**3 + 10*b**2*c*x**2 +
15*b*c**2*x**4)/(6*b**4*x**3*(b + c*x**2))
```

3.74 $\int \frac{1}{x^2(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \frac{1}{x^2\left(bx^2+cx^4\right)^2}\,dx=-\frac{1}{5b^2x^5}+\frac{2c}{3b^3x^3}-\frac{3c^2}{b^4x}-\frac{c^3x}{2b^4\left(b+cx^2\right)}-\frac{7c^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}$$

output
```
-1/5/b^2/x^5+2/3*c/b^3/x^3-3*c^2/b^4/x-1/2*c^3*x/b^4/(c*x^2+b)-7/2*c^(5/2)
*arctan(c^(1/2)*x/b^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(bx^2+cx^4\right)^2}\,dx=-\frac{1}{5b^2x^5}+\frac{2c}{3b^3x^3}-\frac{3c^2}{b^4x}-\frac{c^3x}{2b^4\left(b+cx^2\right)}-\frac{7c^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}$$

input
```
Integrate[1/(x^2*(b*x^2 + c*x^4)^2),x]
```

output
```
-1/5*1/(b^2*x^5) + (2*c)/(3*b^3*x^3) - (3*c^2)/(b^4*x) - (c^3*x)/(2*b^4*(b
 + c*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {9, 253, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (bx^2 + cx^4)^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^6 (b + cx^2)^2} dx$$

$$\Big\downarrow \; 253$$

$$\frac{7 \int \frac{1}{x^6 (cx^2+b)} dx}{2b} + \frac{1}{2bx^5 (b + cx^2)}$$

$$\Big\downarrow \; 264$$

$$\frac{7 \left(-\frac{c \int \frac{1}{x^4 \left(cx^2+b\right)} dx}{b} - \frac{1}{5bx^5} \right)}{2b} + \frac{1}{2bx^5 (b + cx^2)}$$

$$\Big\downarrow \; 264$$

$$\frac{7 \left(-\frac{c \left(-\frac{c \int \frac{1}{x^2 \left(cx^2+b\right)} dx}{b} - \frac{1}{3bx^3} \right)}{b} - \frac{1}{5bx^5} \right)}{2b} + \frac{1}{2bx^5 (b + cx^2)}$$

$$\Big\downarrow \; 264$$

$$\frac{7\left(-\frac{c\left(-\frac{c\left(-\frac{c\int \frac{1}{cx^2+b}dx}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{1}{5bx^5}\right)}{2b}+\frac{1}{2bx^5(b+cx^2)}$$

$\downarrow$ 218

$$\frac{7\left(-\frac{c\left(-\frac{c\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{1}{5bx^5}\right)}{2b}+\frac{1}{2bx^5(b+cx^2)}$$

input
```
Int[1/(x^2*(b*x^2 + c*x^4)^2),x]
```

output
```
1/(2*b*x^5*(b + c*x^2)) + (7*(-1/5*1/(b*x^5) - (c*(-1/3*1/(b*x^3) - (c*(-(
1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)))/b))/b))/(2*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{1}{5b^2x^5}-\frac{3c^2}{b^4x}+\frac{2c}{3b^3x^3}-\frac{c^3\left(\frac{x}{2c\,x^2+2b}+\frac{7\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^4}$	67
risch	$\frac{-\frac{7c^3x^6}{2b^4}-\frac{7c^2x^4}{3b^3}+\frac{7c\,x^2}{15b^2}-\frac{1}{5b}}{x^5(c\,x^2+b)}+\frac{7\sqrt{-bc}\,c^2\ln\left(-cx+\sqrt{-bc}\right)}{4b^5}-\frac{7\sqrt{-bc}\,c^2\ln\left(-cx-\sqrt{-bc}\right)}{4b^5}$	106

input
```
int(1/x^2/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5/b^2/x^5-3*c^2/b^4/x+2/3*c/b^3/x^3-c^3/b^4*(1/2*x/(c*x^2+b)+7/2/(b*c)^
(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.48

$$\int \frac{1}{x^2 (bx^2+cx^4)^2}\,dx = \left[-\frac{210\,c^3x^6+140\,bc^2x^4-28\,b^2cx^2+12\,b^3-105\,(c^3x^7+bc^2x^5)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{60\,(b^4cx^7+b^5x^5)}, -\frac{105\,c^3x^6+70\,bc^2x^4-14\,b^2cx^2+6\,b^3+105\,(c^3x^7+bc^2x^5)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)}{30\,(b^4cx^7+b^5x^5)}\right]$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/60*(210*c^3*x^6 + 140*b*c^2*x^4 - 28*b^2*c*x^2 + 12*b^3 - 105*(c^3*x^7
 + b*c^2*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/
(b^4*c*x^7 + b^5*x^5), -1/30*(105*c^3*x^6 + 70*b*c^2*x^4 - 14*b^2*c*x^2 +
6*b^3 + 105*(c^3*x^7 + b*c^2*x^5)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^4*c*x^
7 + b^5*x^5)]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^2 (bx^2+cx^4)^2}\,dx = \frac{7\sqrt{-\frac{c^5}{b^9}}\log\left(-\frac{b^5\sqrt{-\frac{c^5}{b^9}}}{c^3}+x\right)}{4} - \frac{7\sqrt{-\frac{c^5}{b^9}}\log\left(\frac{b^5\sqrt{-\frac{c^5}{b^9}}}{c^3}+x\right)}{4} + \frac{-6b^3+14b^2cx^2-70bc^2x^4-105c^3x^6}{30b^5x^5+30b^4cx^7}$$

input
```
integrate(1/x**2/(c*x**4+b*x**2)**2,x)
```

output
```
7*sqrt(-c**5/b**9)*log(-b**5*sqrt(-c**5/b**9)/c**3 + x)/4 - 7*sqrt(-c**5/b
**9)*log(b**5*sqrt(-c**5/b**9)/c**3 + x)/4 + (-6*b**3 + 14*b**2*c*x**2 - 7
0*b*c**2*x**4 - 105*c**3*x**6)/(30*b**5*x**5 + 30*b**4*c*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 (bx^2 + cx^4)^2} dx = -\frac{105 c^3x^6 + 70 bc^2x^4 - 14 b^2cx^2 + 6 b^3}{30 (b^4cx^7 + b^5x^5)} - \frac{7 c^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2 \sqrt{bc}b^4}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/30*(105*c^3*x^6 + 70*b*c^2*x^4 - 14*b^2*c*x^2 + 6*b^3)/(b^4*c*x^7 + b^5
*x^5) - 7/2*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2 (bx^2 + cx^4)^2} dx = -\frac{7 c^3 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2 \sqrt{bc}b^4} - \frac{c^3x}{2 (cx^2 + b)b^4} - \frac{45 c^2x^4 - 10 bcx^2 + 3 b^2}{15 b^4x^5}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-7/2*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) - 1/2*c^3*x/((c*x^2 + b)*b^
4) - 1/15*(45*c^2*x^4 - 10*b*c*x^2 + 3*b^2)/(b^4*x^5)
```

Mupad [B] (verification not implemented)

Time = 17.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2 (bx^2 + cx^4)^2} \, dx = -\frac{\frac{1}{5\,b} - \frac{7\,c\,x^2}{15\,b^2} + \frac{7\,c^2\,x^4}{3\,b^3} + \frac{7\,c^3\,x^6}{2\,b^4}}{c\,x^7 + b\,x^5} - \frac{7\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{2\,b^{9/2}}$$

input
```
int(1/(x^2*(b*x^2 + c*x^4)^2),x)
```

output
```
- (1/(5*b) - (7*c*x^2)/(15*b^2) + (7*c^2*x^4)/(3*b^3) + (7*c^3*x^6)/(2*b^4
))/(b*x^5 + c*x^7) - (7*c^(5/2)*atan((c^(1/2)*x)/b^(1/2)))/(2*b^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^2 (bx^2 + cx^4)^2} \, dx = \frac{-105\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right) b\,c^2x^5 - 105\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right) c^3x^7 - 6b^4 + 14b^3c\,x^2 - 70b^2c^2x^4 - 105b\,c^3x^6}{30b^5x^5\,(c\,x^2 + b)}$$

input
```
int(1/x^2/(c*x^4+b*x^2)^2,x)
```

output
```
( - 105*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b*c**2*x**5 - 105*sq
rt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*c**3*x**7 - 6*b**4 + 14*b**3*c
*x**2 - 70*b**2*c**2*x**4 - 105*b*c**3*x**6)/(30*b**5*x**5*(b + c*x**2))
```

3.75 $\int \frac{x^{15}}{\left(bx^2+cx^4\right)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{x^{15}}{\left(bx^2+cx^4\right)^3} dx = -\frac{3bx^2}{2c^4}+\frac{x^4}{4c^3}-\frac{b^4}{4c^5\left(b+cx^2\right)^2}+\frac{2b^3}{c^5\left(b+cx^2\right)}+\frac{3b^2\log\left(b+cx^2\right)}{c^5}$$

output
```
-3/2*b*x^2/c^4+1/4*x^4/c^3-1/4*b^4/c^5/(c*x^2+b)^2+2*b^3/c^5/(c*x^2+b)+3*b
^2*ln(c*x^2+b)/c^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{x^{15}}{\left(bx^2+cx^4\right)^3} dx = \frac{-6bcx^2+c^2x^4-\frac{b^4}{\left(b+cx^2\right)^2}+\frac{8b^3}{b+cx^2}+12b^2\log\left(b+cx^2\right)}{4c^5}$$

input
```
Integrate[x^15/(b*x^2 + c*x^4)^3,x]
```

output
```
(-6*b*c*x^2 + c^2*x^4 - b^4/(b + c*x^2)^2 + (8*b^3)/(b + c*x^2) + 12*b^2*L
og[b + c*x^2])/(4*c^5)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^9}{(b+cx^2)^3}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{x^8}{(cx^2+b)^3}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(\frac{b^4}{c^4(cx^2+b)^3}-\frac{4b^3}{c^4(cx^2+b)^2}+\frac{6b^2}{c^4(cx^2+b)}-\frac{3b}{c^4}+\frac{x^2}{c^3}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{b^4}{2c^5(b+cx^2)^2}+\frac{4b^3}{c^5(b+cx^2)}+\frac{6b^2\log(b+cx^2)}{c^5}-\frac{3bx^2}{c^4}+\frac{x^4}{2c^3}\right)$$

input
```
Int[x^15/(b*x^2 + c*x^4)^3,x]
```

output
```
((-3*b*x^2)/c^4 + x^4/(2*c^3) - b^4/(2*c^5*(b + c*x^2)^2) + (4*b^3)/(c^5*(
b + c*x^2)) + (6*b^2*Log[b + c*x^2])/c^5)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

method	result	size
norman	$\frac{\frac{x^{13}}{4c}-\frac{b\,x^{11}}{c^{2}}+\frac{6b^{3}x^{7}}{c^{4}}+\frac{9b^{4}x^{5}}{2c^{5}}}{x^{5}\left(c\,x^{2}+b\right)^{2}}+\frac{3b^{2}\ln\left(c\,x^{2}+b\right)}{c^{5}}$	71
default	$\frac{\left(-c\,x^{2}+3b\right)^{2}}{4c^{5}}+\frac{b^{2}\left(\frac{6\ln\left(c\,x^{2}+b\right)}{c}+\frac{4b}{c\left(c\,x^{2}+b\right)}-\frac{b^{2}}{2c\left(c\,x^{2}+b\right)^{2}}\right)}{2c^{4}}$	73
risch	$\frac{x^{4}}{4c^{3}}-\frac{3b\,x^{2}}{2c^{4}}+\frac{9b^{2}}{4c^{5}}+\frac{2b^{3}x^{2}+\frac{7b^{4}}{4c}}{c^{4}\left(c\,x^{2}+b\right)^{2}}+\frac{3b^{2}\ln\left(c\,x^{2}+b\right)}{c^{5}}$	73
parallelrisch	$\frac{c^{4}x^{8}-4b\,x^{6}c^{3}+12\ln\left(c\,x^{2}+b\right)x^{4}b^{2}c^{2}+24\ln\left(c\,x^{2}+b\right)x^{2}b^{3}c+24x^{2}b^{3}c+12\ln\left(c\,x^{2}+b\right)b^{4}+18b^{4}}{4c^{5}\left(c\,x^{2}+b\right)^{2}}$	95

input
```
int(x^15/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/4/c*x^13-b/c^2*x^11+6*b^3/c^4*x^7+9/2*b^4/c^5*x^5)/x^5/(c*x^2+b)^2+3*b^
2*ln(c*x^2+b)/c^5
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.39

$$\int \frac{x^{15}}{(bx^2+cx^4)^3}\,dx = \frac{c^4x^8 - 4\,bc^3x^6 - 11\,b^2c^2x^4 + 2\,b^3cx^2 + 7\,b^4 + 12\,(b^2c^2x^4 + 2\,b^3cx^2 + b^4)\log(cx^2+b)}{4\,(c^7x^4 + 2\,bc^6x^2 + b^2c^5)}$$

input
```
integrate(x^15/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/4*(c^4*x^8 - 4*b*c^3*x^6 - 11*b^2*c^2*x^4 + 2*b^3*c*x^2 + 7*b^4 + 12*(b^
2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*log(c*x^2 + b))/(c^7*x^4 + 2*b*c^6*x^2 + b^
2*c^5)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{x^{15}}{(bx^2+cx^4)^3}\,dx = \frac{3b^2\log(b+cx^2)}{c^5} - \frac{3bx^2}{2c^4} + \frac{7b^4+8b^3cx^2}{4b^2c^5+8bc^6x^2+4c^7x^4} + \frac{x^4}{4c^3}$$

input
```
integrate(x**15/(c*x**4+b*x**2)**3,x)
```

output
```
3*b**2*log(b + c*x**2)/c**5 - 3*b*x**2/(2*c**4) + (7*b**4 + 8*b**3*c*x**2)
/(4*b**2*c**5 + 8*b*c**6*x**2 + 4*c**7*x**4) + x**4/(4*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \frac{x^{15}}{(bx^2+cx^4)^3}\,dx = \frac{8\,b^3cx^2+7\,b^4}{4\,(c^7x^4+2\,bc^6x^2+b^2c^5)}+\frac{3\,b^2\log\left(cx^2+b\right)}{c^5}+\frac{cx^4-6\,bx^2}{4\,c^4}$$

input
```
integrate(x^15/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/4*(8*b^3*c*x^2 + 7*b^4)/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5) + 3*b^2*log(c*
x^2 + b)/c^5 + 1/4*(c*x^4 - 6*b*x^2)/c^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{x^{15}}{(bx^2+cx^4)^3}\,dx = \frac{3\,b^2\log\left(|cx^2+b|\right)}{c^5}+\frac{c^3x^4-6\,bc^2x^2}{4\,c^6}-\frac{18\,b^2c^2x^4+28\,b^3cx^2+11\,b^4}{4\,(cx^2+b)^2c^5}$$

input
```
integrate(x^15/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
3*b^2*log(abs(c*x^2 + b))/c^5 + 1/4*(c^3*x^4 - 6*b*c^2*x^2)/c^6 - 1/4*(18*
b^2*c^2*x^4 + 28*b^3*c*x^2 + 11*b^4)/((c*x^2 + b)^2*c^5)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{x^{15}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{7\,b^4}{4\,c}+2\,b^3\,x^2}{b^2\,c^4+2\,b\,c^5\,x^2+c^6\,x^4}+\frac{x^4}{4\,c^3}-\frac{3\,b\,x^2}{2\,c^4}+\frac{3\,b^2\,\ln\left(c\,x^2+b\right)}{c^5}$$

input
```
int(x^15/(b*x^2 + c*x^4)^3,x)
```

output
```
((7*b^4)/(4*c) + 2*b^3*x^2)/(b^2*c^4 + c^6*x^4 + 2*b*c^5*x^2) + x^4/(4*c^3
) - (3*b*x^2)/(2*c^4) + (3*b^2*log(b + c*x^2))/c^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{x^{15}}{(bx^2+cx^4)^3}\,dx = \frac{12\log(c\,x^2+b)\,b^4+24\log(c\,x^2+b)\,b^3c\,x^2+12\log(c\,x^2+b)\,b^2c^2x^4+6b^4-12b^2c^2x^4-4b\,c^3x^6+c^4x^8}{4c^5\,(c^2x^4+2bc\,x^2+b^2)}$$

input
```
int(x^15/(c*x^4+b*x^2)^3,x)
```

output
```
(12*log(b + c*x**2)*b**4 + 24*log(b + c*x**2)*b**3*c*x**2 + 12*log(b + c*x
**2)*b**2*c**2*x**4 + 6*b**4 - 12*b**2*c**2*x**4 - 4*b*c**3*x**6 + c**4*x*
*8)/(4*c**5*(b**2 + 2*b*c*x**2 + c**2*x**4))
```

3.76 $\int \frac{x^{13}}{(bx^2+cx^4)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{x^{13}}{(bx^2+cx^4)^3} \, dx = \frac{x^2}{2c^3} + \frac{b^3}{4c^4(b+cx^2)^2} - \frac{3b^2}{2c^4(b+cx^2)} - \frac{3b\log(b+cx^2)}{2c^4}$$

output
```
1/2*x^2/c^3+1/4*b^3/c^4/(c*x^2+b)^2-3/2*b^2/c^4/(c*x^2+b)-3/2*b*ln(c*x^2+b
)/c^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{x^{13}}{(bx^2+cx^4)^3} \, dx = -\frac{-2cx^2 + \frac{b^2(5b+6cx^2)}{(b+cx^2)^2} + 6b\log(b+cx^2)}{4c^4}$$

input
```
Integrate[x^13/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(-2*c*x^2 + (b^2*(5*b + 6*c*x^2))/(b + c*x^2)^2 + 6*b*Log[b + c*x^2])
/c^4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^7}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^6}{(cx^2+b)^3}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(-\frac{b^3}{c^3(cx^2+b)^3}+\frac{3b^2}{c^3(cx^2+b)^2}-\frac{3b}{c^3(cx^2+b)}+\frac{1}{c^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{b^3}{2c^4(b+cx^2)^2}-\frac{3b^2}{c^4(b+cx^2)}-\frac{3b\log(b+cx^2)}{c^4}+\frac{x^2}{c^3}\right)$$

input
```
Int[x^13/(b*x^2 + c*x^4)^3,x]
```

output
```
(x^2/c^3 + b^3/(2*c^4*(b + c*x^2)^2) - (3*b^2)/(c^4*(b + c*x^2)) - (3*b*Lo
g[b + c*x^2])/c^4)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{x^{2}}{2 c^{3}}+\frac{-\frac{3 b^{2} x^{2}}{2}-\frac{5 b^{3}}{4 c}}{c^{3} \left(c \,x^{2}+b \right)^{2}}-\frac{3 b \ln \left(c \,x^{2}+b \right)}{2 c^{4}}$	54
norman	$\frac{\frac{x^{11}}{2 c}-\frac{3 b^{2} x^{7}}{c^{3}}-\frac{9 b^{3} x^{5}}{4 c^{4}}}{x^{5} \left(c \,x^{2}+b \right)^{2}}-\frac{3 b \ln \left(c \,x^{2}+b \right)}{2 c^{4}}$	60
default	$\frac{x^{2}}{2 c^{3}}-\frac{b \left(\frac{3 \ln \left(c \,x^{2}+b \right)}{c}+\frac{3 b}{c \left(c \,x^{2}+b \right)}-\frac{b^{2}}{2 c \left(c \,x^{2}+b \right)^{2}}\right)}{2 c^{3}}$	62
parallelrisch	$-\frac{-2 c^{3} x^{6}+6 \ln \left(c \,x^{2}+b \right) x^{4} b \,c^{2}+12 \ln \left(c \,x^{2}+b \right) x^{2} b^{2} c +12 b^{2} c \,x^{2}+6 b^{3} \ln \left(c \,x^{2}+b \right)+9 b^{3}}{4 c^{4} \left(c \,x^{2}+b \right)^{2}}$	85

input
```
int(x^13/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c^3+(-3/2*b^2*x^2-5/4*b^3/c)/c^3/(c*x^2+b)^2-3/2*b*ln(c*x^2+b)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.40

$$\int \frac{x^{13}}{(bx^2+cx^4)^3}\,dx = \frac{2\,c^3x^6+4\,bc^2x^4-4\,b^2cx^2-5\,b^3-6\,(bc^2x^4+2\,b^2cx^2+b^3)\log{(cx^2+b)}}{4\,(c^6x^4+2\,bc^5x^2+b^2c^4)}$$

input
```
integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*c^3*x^6 + 4*b*c^2*x^4 - 4*b^2*c*x^2 - 5*b^3 - 6*(b*c^2*x^4 + 2*b^2*
c*x^2 + b^3)*log(c*x^2 + b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{x^{13}}{(bx^2+cx^4)^3}\,dx = -\frac{3b\log{(b+cx^2)}}{2c^4}+\frac{-5b^3-6b^2cx^2}{4b^2c^4+8bc^5x^2+4c^6x^4}+\frac{x^2}{2c^3}$$

input
```
integrate(x**13/(c*x**4+b*x**2)**3,x)
```

output
```
-3*b*log(b + c*x**2)/(2*c**4) + (-5*b**3 - 6*b**2*c*x**2)/(4*b**2*c**4 + 8
*b*c**5*x**2 + 4*c**6*x**4) + x**2/(2*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{x^{13}}{(bx^2+cx^4)^3}\,dx = -\frac{6\,b^2cx^2+5\,b^3}{4\,(c^6x^4+2\,bc^5x^2+b^2c^4)}+\frac{x^2}{2\,c^3}-\frac{3\,b\log{(cx^2+b)}}{2\,c^4}$$

input
```
integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(6*b^2*c*x^2 + 5*b^3)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 1/2*x^2/c^3
 - 3/2*b*log(c*x^2 + b)/c^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{x^{13}}{(bx^2+cx^4)^3}\,dx = \frac{x^2}{2\,c^3} - \frac{3\,b\log\left(|cx^2+b|\right)}{2\,c^4} + \frac{9\,bc^2x^4+12\,b^2cx^2+4\,b^3}{4\,(cx^2+b)^2c^4}$$

input
```
integrate(x^13/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/2*x^2/c^3 - 3/2*b*log(abs(c*x^2 + b))/c^4 + 1/4*(9*b*c^2*x^4 + 12*b^2*c*
x^2 + 4*b^3)/((c*x^2 + b)^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 18.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{x^{13}}{(bx^2+cx^4)^3}\,dx = \frac{x^2}{2\,c^3} - \frac{\frac{5\,b^3}{4\,c}+\frac{3\,b^2\,x^2}{2}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4} - \frac{3\,b\,\ln\left(c\,x^2+b\right)}{2\,c^4}$$

input
```
int(x^13/(b*x^2 + c*x^4)^3,x)
```

output
```
x^2/(2*c^3) - ((5*b^3)/(4*c) + (3*b^2*x^2)/2)/(b^2*c^3 + c^5*x^4 + 2*b*c^4
*x^2) - (3*b*log(b + c*x^2))/(2*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.46

$$\int \frac{x^{13}}{(bx^2+cx^4)^3}\,dx = \frac{-6\log(c\,x^2+b)\,b^3-12\log(c\,x^2+b)\,b^2c\,x^2-6\log(c\,x^2+b)\,b\,c^2x^4-3b^3+6b\,c^2x^4+2c^3x^6}{4c^4\,(c^2x^4+2bc\,x^2+b^2)}$$

input
```
int(x^13/(c*x^4+b*x^2)^3,x)
```

output
```
( - 6*log(b + c*x**2)*b**3 - 12*log(b + c*x**2)*b**2*c*x**2 - 6*log(b + c*
x**2)*b*c**2*x**4 - 3*b**3 + 6*b*c**2*x**4 + 2*c**3*x**6)/(4*c**4*(b**2 +
2*b*c*x**2 + c**2*x**4))
```

3.77 $\int \frac{x^{11}}{(bx^2+cx^4)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{x^{11}}{(bx^2+cx^4)^3} \, dx = -\frac{b^2}{4c^3(b+cx^2)^2} + \frac{b}{c^3(b+cx^2)} + \frac{\log(b+cx^2)}{2c^3}$$

output
```
-1/4*b^2/c^3/(c*x^2+b)^2+b/c^3/(c*x^2+b)+1/2*ln(c*x^2+b)/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{(bx^2+cx^4)^3} \, dx = \frac{\frac{b(3b+4cx^2)}{(b+cx^2)^2} + 2\log(b+cx^2)}{4c^3}$$

input
```
Integrate[x^11/(b*x^2 + c*x^4)^3,x]
```

output
```
((b*(3*b + 4*c*x^2))/(b + c*x^2)^2 + 2*Log[b + c*x^2])/(4*c^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^5}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^4}{(cx^2+b)^3}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{b^2}{c^2(cx^2+b)^3}-\frac{2b}{c^2(cx^2+b)^2}+\frac{1}{c^2(cx^2+b)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{b^2}{2c^3(b+cx^2)^2}+\frac{2b}{c^3(b+cx^2)}+\frac{\log(b+cx^2)}{c^3}\right)$$

input
```
Int[x^11/(b*x^2 + c*x^4)^3,x]
```

output
```
(-1/2*b^2/(c^3*(b + c*x^2)^2) + (2*b)/(c^3*(b + c*x^2)) + Log[b + c*x^2]/c
^3)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{\frac{b\,x^{2}}{c^{2}}+\frac{3b^{2}}{4c^{3}}}{\left(c\,x^{2}+b\right)^{2}}+\frac{\ln\left(c\,x^{2}+b\right)}{2c^{3}}$	42
default	$-\frac{b^{2}}{4c^{3}\left(c\,x^{2}+b\right)^{2}}+\frac{b}{c^{3}\left(c\,x^{2}+b\right)}+\frac{\ln\left(c\,x^{2}+b\right)}{2c^{3}}$	46
norman	$\frac{\frac{b\,x^{7}}{c^{2}}+\frac{3b^{2}x^{5}}{4c^{3}}}{x^{5}\left(c\,x^{2}+b\right)^{2}}+\frac{\ln\left(c\,x^{2}+b\right)}{2c^{3}}$	48
parallelrisch	$\frac{2c^{2}\ln\left(c\,x^{2}+b\right)x^{4}+4\ln\left(c\,x^{2}+b\right)x^{2}bc+4bc\,x^{2}+2b^{2}\ln\left(c\,x^{2}+b\right)+3b^{2}}{4c^{3}\left(c\,x^{2}+b\right)^{2}}$	72

input
```
int(x^11/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(b*x^2/c^2+3/4*b^2/c^3)/(c*x^2+b)^2+1/2*ln(c*x^2+b)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.41

$$\int \frac{x^{11}}{(bx^2+cx^4)^3}\,dx = \frac{4\,bcx^2+3\,b^2+2\,(c^2x^4+2\,bcx^2+b^2)\log(cx^2+b)}{4\,(c^5x^4+2\,bc^4x^2+b^2c^3)}$$

input
```
integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/4*(4*b*c*x^2 + 3*b^2 + 2*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(c*x^2 + b))/(c^
5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{x^{11}}{(bx^2+cx^4)^3}\,dx = \frac{3b^2+4bcx^2}{4b^2c^3+8bc^4x^2+4c^5x^4}+\frac{\log(b+cx^2)}{2c^3}$$

input
```
integrate(x**11/(c*x**4+b*x**2)**3,x)
```

output
```
(3*b**2 + 4*b*c*x**2)/(4*b**2*c**3 + 8*b*c**4*x**2 + 4*c**5*x**4) + log(b
+ c*x**2)/(2*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{x^{11}}{(bx^2+cx^4)^3}\,dx = \frac{4\,bcx^2+3\,b^2}{4\,(c^5x^4+2\,bc^4x^2+b^2c^3)}+\frac{\log(cx^2+b)}{2\,c^3}$$

input
```
integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/4*(4*b*c*x^2 + 3*b^2)/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) + 1/2*log(c*x^2
+ b)/c^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{(bx^2+cx^4)^3}\,dx = \frac{\log(|cx^2+b|)}{2\,c^3} - \frac{3\,cx^4+2\,bx^2}{4\,(cx^2+b)^2c^2}$$

input
```
integrate(x^11/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/2*log(abs(c*x^2 + b))/c^3 - 1/4*(3*c*x^4 + 2*b*x^2)/((c*x^2 + b)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{3\,b^2}{4\,c^3}+\frac{b\,x^2}{c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4} + \frac{\ln(c\,x^2+b)}{2\,c^3}$$

input
```
int(x^11/(b*x^2 + c*x^4)^3,x)
```

output
```
((3*b^2)/(4*c^3) + (b*x^2)/c^2)/(b^2 + c^2*x^4 + 2*b*c*x^2) + log(b + c*x^
2)/(2*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.65

$$\int \frac{x^{11}}{(bx^2+cx^4)^3}\,dx = \frac{2\log(c\,x^2+b)\,b^2+4\log(c\,x^2+b)\,bc\,x^2+2\log(c\,x^2+b)\,c^2x^4+b^2-2c^2x^4}{4c^3\,(c^2x^4+2bc\,x^2+b^2)}$$

input
```
int(x^11/(c*x^4+b*x^2)^3,x)
```

output
```
(2*log(b + c*x**2)*b**2 + 4*log(b + c*x**2)*b*c*x**2 + 2*log(b + c*x**2)*c
**2*x**4 + b**2 - 2*c**2*x**4)/(4*c**3*(b**2 + 2*b*c*x**2 + c**2*x**4))
```

3.78 $\int \frac{x^9}{(bx^2+cx^4)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{x^9}{(bx^2+cx^4)^3} \, dx = \frac{x^4}{4b(b+cx^2)^2}$$

output
```
1/4*x^4/b/(c*x^2+b)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x^9}{(bx^2+cx^4)^3} \, dx = -\frac{b+2cx^2}{4c^2(b+cx^2)^2}$$

input
```
Integrate[x^9/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(b + 2*c*x^2)/(c^2*(b + c*x^2)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(bx^2+cx^4)^3} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^3}{(b+cx^2)^3}dx$$

$$\Big\downarrow \; 242$$

$$\frac{x^4}{4b\,(b+cx^2)^2}$$

input
```
Int[x^9/(b*x^2 + c*x^4)^3,x]
```

output
```
x^4/(4*b*(b + c*x^2)^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2c\,x^2+b}{4(c\,x^2+b)^2c^2}$	23
parallelrisch	$\frac{-2c\,x^2-b}{4c^2(c\,x^2+b)^2}$	25
risch	$\frac{-\frac{x^2}{2c}-\frac{b}{4c^2}}{(c\,x^2+b)^2}$	26
default	$-\frac{1}{2c^2(c\,x^2+b)}+\frac{b}{4c^2(c\,x^2+b)^2}$	31
norman	$\frac{-\frac{x^7}{2c}-\frac{b\,x^5}{4c^2}}{x^5(c\,x^2+b)^2}$	32
orering	$-\frac{(2c\,x^2+b)x^6(c\,x^2+b)}{4c^2(c\,x^4+b\,x^2)^3}$	37

input
```
int(x^9/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*c*x^2+b)/(c*x^2+b)^2/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^9}{(bx^2+cx^4)^3}\,dx = -\frac{2\,cx^2+b}{4\,(c^4x^4+2\,bc^3x^2+b^2c^2)}$$

input
```
integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*c*x^2 + b)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^9}{(bx^2 + cx^4)^3} \, dx = \frac{-b - 2cx^2}{4b^2c^2 + 8bc^3x^2 + 4c^4x^4}$$

input
```
integrate(x**9/(c*x**4+b*x**2)**3,x)
```

output
```
(-b - 2*c*x**2)/(4*b**2*c**2 + 8*b*c**3*x**2 + 4*c**4*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^9}{(bx^2 + cx^4)^3} \, dx = -\frac{2\,cx^2 + b}{4\,(c^4x^4 + 2\,bc^3x^2 + b^2c^2)}$$

input
```
integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*c*x^2 + b)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{x^9}{(bx^2 + cx^4)^3} \, dx = -\frac{2\,cx^2 + b}{4\,(cx^2 + b)^2c^2}$$

input
```
integrate(x^9/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-1/4*(2*c*x^2 + b)/((c*x^2 + b)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 18.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{x^9}{(bx^2+cx^4)^3}\,dx = -\frac{\frac{b}{4\,c^2}+\frac{x^2}{2\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

input
```
int(x^9/(b*x^2 + c*x^4)^3,x)
```

output
```
-(b/(4*c^2) + x^2/(2*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{x^9}{(bx^2+cx^4)^3}\,dx = \frac{x^4}{4b\,(c^2x^4+2bc\,x^2+b^2)}$$

input
```
int(x^9/(c*x^4+b*x^2)^3,x)
```

output
```
x**4/(4*b*(b**2 + 2*b*c*x**2 + c**2*x**4))
```

3.79 $\int \frac{x^7}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{x^7}{(bx^2+cx^4)^3}\,dx = -\frac{1}{4c\,(b+cx^2)^2}$$

output
```
-1/4/c/(c*x^2+b)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{(bx^2+cx^4)^3}\,dx = -\frac{1}{4c\,(b+cx^2)^2}$$

input
```
Integrate[x^7/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*1/(c*(b + c*x^2)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {9, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(bx^2+cx^4)^3} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x}{(b+cx^2)^3}dx$$

$$\Big\downarrow \; 241$$

$$-\frac{1}{4c\,(b+cx^2)^2}$$

input
```
Int[x^7/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*1/(c*(b + c*x^2)^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{4c(c\,x^2+b)^2}$	15
default	$-\frac{1}{4c(c\,x^2+b)^2}$	15
norman	$-\frac{1}{4c(c\,x^2+b)^2}$	15
risch	$-\frac{1}{4c(c\,x^2+b)^2}$	15
parallelrisch	$-\frac{1}{4c(c\,x^2+b)^2}$	15
orering	$-\frac{(c\,x^2+b)\,x^6}{4c(c\,x^4+b\,x^2)^3}$	29

input
```
int(x^7/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/c/(c*x^2+b)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{x^7}{(bx^2+cx^4)^3}\,dx = -\frac{1}{4\,(c^3x^4+2\,bc^2x^2+b^2c)}$$

input
```
integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/4/(c^3*x^4 + 2*b*c^2*x^2 + b^2*c)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{x^7}{(bx^2 + cx^4)^3}\,dx = -\frac{1}{4b^2c + 8bc^2x^2 + 4c^3x^4}$$

input
```
integrate(x**7/(c*x**4+b*x**2)**3,x)
```

output
```
-1/(4*b**2*c + 8*b*c**2*x**2 + 4*c**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{x^7}{(bx^2 + cx^4)^3}\,dx = -\frac{1}{4\,(c^3x^4 + 2\,bc^2x^2 + b^2c)}$$

input
```
integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/4/(c^3*x^4 + 2*b*c^2*x^2 + b^2*c)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{(bx^2 + cx^4)^3}\,dx = -\frac{1}{4\,(cx^2 + b)^2c}$$

input
```
integrate(x^7/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-1/4/((c*x^2 + b)^2*c)
```

Mupad [B] (verification not implemented)

Time = 18.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{x^7}{(bx^2+cx^4)^3}\,dx = -\frac{1}{4\,b^2\,c+8\,b\,c^2\,x^2+4\,c^3\,x^4}$$

input
```
int(x^7/(b*x^2 + c*x^4)^3,x)
```

output
```
-1/(4*b^2*c + 4*c^3*x^4 + 8*b*c^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{x^7}{(bx^2+cx^4)^3}\,dx = -\frac{1}{4c\,(c^2x^4+2bc\,x^2+b^2)}$$

input
```
int(x^7/(c*x^4+b*x^2)^3,x)
```

output
```
( - 1)/(4*c*(b**2 + 2*b*c*x**2 + c**2*x**4))
```

3.80 $\int \frac{x^5}{(bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{x^5}{(bx^2+cx^4)^3} dx = \frac{1}{4b(b+cx^2)^2} + \frac{1}{2b^2(b+cx^2)} + \frac{\log(x)}{b^3} - \frac{\log(b+cx^2)}{2b^3}$$

output
```
1/4/b/(c*x^2+b)^2+1/2/b^2/(c*x^2+b)+ln(x)/b^3-1/2*ln(c*x^2+b)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{(bx^2+cx^4)^3} dx = \frac{\frac{b(3b+2cx^2)}{(b+cx^2)^2} + 4\log(x) - 2\log(b+cx^2)}{4b^3}$$

input
```
Integrate[x^5/(b*x^2 + c*x^4)^3,x]
```

output
```
((b*(3*b + 2*c*x^2))/(b + c*x^2)^2 + 4*Log[x] - 2*Log[b + c*x^2])/(4*b^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x\,(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^2\,(cx^2+b)^3}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2}\int \left(-\frac{c}{b^3\,(cx^2+b)}-\frac{c}{b^2\,(cx^2+b)^2}-\frac{c}{b\,(cx^2+b)^3}+\frac{1}{b^3x^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{\log(b+cx^2)}{b^3}+\frac{\log(x^2)}{b^3}+\frac{1}{b^2\,(b+cx^2)}+\frac{1}{2b\,(b+cx^2)^2}\right)$$

input
```
Int[x^5/(b*x^2 + c*x^4)^3,x]
```

output
```
(1/(2*b*(b + c*x^2)^2) + 1/(b^2*(b + c*x^2)) + Log[x^2]/b^3 - Log[b + c*x^
2]/b^3)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{\frac{c\,x^{2}}{2 b^{2}}+\frac{3}{4 b}}{\left(c\,x^{2}+b\right)^{2}}+\frac{\ln \left(x \right)}{b^{3}}-\frac{\ln \left(c\,x^{2}+b \right)}{2 b^{3}}$	46
norman	$\frac{-\frac{c\,x^{7}}{b^{2}}-\frac{3 c^{2} x^{9}}{4 b^{3}}}{x^{5} \left(c\,x^{2}+b\right)^{2}}+\frac{\ln \left(x \right)}{b^{3}}-\frac{\ln \left(c\,x^{2}+b \right)}{2 b^{3}}$	55
default	$\frac{\ln \left(x \right)}{b^{3}}-\frac{c \left(-\frac{b^{2}}{2 c \left(c\,x^{2}+b \right)^{2}}+\frac{\ln \left(c\,x^{2}+b \right)}{c}-\frac{b}{c \left(c\,x^{2}+b \right)}\right)}{2 b^{3}}$	59
parallelrisch	$-\frac{2 c^{2} \ln \left(c\,x^{2}+b \right) x^{4}-4 c^{2} \ln \left(x \right) x^{4}+3 c^{2} x^{4}+4 \ln \left(c\,x^{2}+b \right) x^{2} b c -8 b c \ln \left(x \right) x^{2}+4 b c\,x^{2}+2 b^{2} \ln \left(c\,x^{2}+b \right)-4 b^{2} \ln \left(x \right)}{4 b^{3} \left(c\,x^{2}+b \right)^{2}}$	101

input
```
int(x^5/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2*c/b^2*x^2+3/4/b)/(c*x^2+b)^2+ln(x)/b^3-1/2*ln(c*x^2+b)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

$$\int \frac{x^5}{(bx^2+cx^4)^3}\,dx = \frac{2\,bcx^2+3\,b^2-2\,(c^2x^4+2\,bcx^2+b^2)\log(cx^2+b)+4\,(c^2x^4+2\,bcx^2+b^2)\log(x)}{4\,(b^3c^2x^4+2\,b^4cx^2+b^5)}$$

input
```
integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b*c*x^2 + 3*b^2 - 2*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(c*x^2 + b) + 4*
(c^2*x^4 + 2*b*c*x^2 + b^2)*log(x))/(b^3*c^2*x^4 + 2*b^4*c*x^2 + b^5)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{(bx^2+cx^4)^3}\,dx = \frac{3b+2cx^2}{4b^4+8b^3cx^2+4b^2c^2x^4}+\frac{\log(x)}{b^3}-\frac{\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

input
```
integrate(x**5/(c*x**4+b*x**2)**3,x)
```

output
```
(3*b + 2*c*x**2)/(4*b**4 + 8*b**3*c*x**2 + 4*b**2*c**2*x**4) + log(x)/b**3
 - log(b/c + x**2)/(2*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{x^5}{(bx^2+cx^4)^3}\,dx = \frac{2\,cx^2+3\,b}{4\,(b^2c^2x^4+2\,b^3cx^2+b^4)}-\frac{\log(cx^2+b)}{2\,b^3}+\frac{\log(x^2)}{2\,b^3}$$

input
```
integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*c*x^2 + 3*b)/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) - 1/2*log(c*x^2 + b)
/b^3 + 1/2*log(x^2)/b^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(bx^2+cx^4)^3}\,dx = \frac{\log(x^2)}{2\,b^3} - \frac{\log(|cx^2+b|)}{2\,b^3} + \frac{3\,c^2x^4+8\,bcx^2+6\,b^2}{4\,(cx^2+b)^2b^3}$$

input
```
integrate(x^5/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/2*log(x^2)/b^3 - 1/2*log(abs(c*x^2 + b))/b^3 + 1/4*(3*c^2*x^4 + 8*b*c*x^
2 + 6*b^2)/((c*x^2 + b)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 17.81 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{(bx^2+cx^4)^3}\,dx = \frac{\ln(x)}{b^3} + \frac{\frac{3}{4\,b}+\frac{c\,x^2}{2\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4} - \frac{\ln(c\,x^2+b)}{2\,b^3}$$

input
```
int(x^5/(b*x^2 + c*x^4)^3,x)
```

output
```
log(x)/b^3 + (3/(4*b) + (c*x^2)/(2*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) - log
(b + c*x^2)/(2*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.02

$$\int \frac{x^5}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{-2\log(c\,x^2+b)\,b^2-4\log(c\,x^2+b)\,bc\,x^2-2\log(c\,x^2+b)\,c^2x^4+4\log(x)\,b^2+8\log(x)\,bc\,x^2+4\log(x)\,c}{4b^3\,(c^2x^4+2bc\,x^2+b^2)}$$

input
```
int(x^5/(c*x^4+b*x^2)^3,x)
```

output
```
( - 2*log(b + c*x**2)*b**2 - 4*log(b + c*x**2)*b*c*x**2 - 2*log(b + c*x**2
)*c**2*x**4 + 4*log(x)*b**2 + 8*log(x)*b*c*x**2 + 4*log(x)*c**2*x**4 + 2*b
**2 - c**2*x**4)/(4*b**3*(b**2 + 2*b*c*x**2 + c**2*x**4))
```

3.81 $\int \frac{x^3}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \frac{x^3}{(bx^2+cx^4)^3}\,dx = -\frac{1}{2b^3x^2}-\frac{c}{4b^2(b+cx^2)^2}-\frac{c}{b^3(b+cx^2)}-\frac{3c\log(x)}{b^4}+\frac{3c\log(b+cx^2)}{2b^4}$$

output
```
-1/2/b^3/x^2-1/4*c/b^2/(c*x^2+b)^2-c/b^3/(c*x^2+b)-3*c*ln(x)/b^4+3/2*c*ln(
c*x^2+b)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(bx^2+cx^4)^3}\,dx = -\frac{\frac{b(2b^2+9bcx^2+6c^2x^4)}{x^2(b+cx^2)^2}+12c\log(x)-6c\log(b+cx^2)}{4b^4}$$

input
```
Integrate[x^3/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*((b*(2*b^2 + 9*b*c*x^2 + 6*c^2*x^4))/(x^2*(b + c*x^2)^2) + 12*c*Log[x
] - 6*c*Log[b + c*x^2])/b^4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^3 (b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^4 (cx^2+b)^3}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2}\int \left(\frac{3c^2}{b^4 (cx^2+b)}+\frac{2c^2}{b^3 (cx^2+b)^2}+\frac{c^2}{b^2 (cx^2+b)^3}-\frac{3c}{b^4x^2}+\frac{1}{b^3x^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{3c\log (x^2)}{b^4}+\frac{3c\log (b+cx^2)}{b^4}-\frac{2c}{b^3 (b+cx^2)}-\frac{1}{b^3x^2}-\frac{c}{2b^2 (b+cx^2)^2}\right)$$

input
```
Int[x^3/(b*x^2 + c*x^4)^3,x]
```

output
```
(-(1/(b^3*x^2)) - c/(2*b^2*(b + c*x^2)^2) - (2*c)/(b^3*(b + c*x^2)) - (3*c
*Log[x^2])/b^4 + (3*c*Log[b + c*x^2])/b^4)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

method	result
risch	$\frac{-\frac{3c^{2}x^{4}}{2b^{3}}-\frac{9c\,x^{2}}{4b^{2}}-\frac{1}{2b}}{x^{2}\left(c\,x^{2}+b\right)^{2}}-\frac{3c\ln\left(x\right)}{b^{4}}+\frac{3c\ln\left(-c\,x^{2}-b\right)}{2b^{4}}$
norman	$\frac{\frac{3c^{2}x^{7}}{b^{3}}-\frac{x^{3}}{2b}+\frac{9c^{3}x^{9}}{4b^{4}}}{x^{5}\left(c\,x^{2}+b\right)^{2}}-\frac{3c\ln\left(x\right)}{b^{4}}+\frac{3c\ln\left(c\,x^{2}+b\right)}{2b^{4}}$
default	$-\frac{1}{2b^{3}x^{2}}-\frac{3c\ln\left(x\right)}{b^{4}}+\frac{c^{2}\left(-\frac{b^{2}}{2c\left(c\,x^{2}+b\right)^{2}}+\frac{3\ln\left(c\,x^{2}+b\right)}{c}-\frac{2b}{c\left(c\,x^{2}+b\right)}\right)}{2b^{4}}$
parallelrisch	$-\frac{12c^{3}\ln\left(x\right)x^{6}-6\ln\left(c\,x^{2}+b\right)x^{6}c^{3}-9c^{3}x^{6}+24b\,c^{2}\ln\left(x\right)x^{4}-12\ln\left(c\,x^{2}+b\right)x^{4}b\,c^{2}-12b\,c^{2}x^{4}+12b^{2}c\ln\left(x\right)x^{2}-6\ln\left(c\,x^{2}+b\right)x^{2}b^{2}}{4b^{4}x^{2}\left(c\,x^{2}+b\right)^{2}}$

input
```
int(x^3/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/2*c^2/b^3*x^4-9/4*c/b^2*x^2-1/2/b)/x^2/(c*x^2+b)^2-3*c*ln(x)/b^4+3/2/b
^4*c*ln(-c*x^2-b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int \frac{x^3}{(bx^2+cx^4)^3}\,dx = -\frac{6\,bc^2x^4+9\,b^2cx^2+2\,b^3-6\,(c^3x^6+2\,bc^2x^4+b^2cx^2)\log\left(cx^2+b\right)+12\,(c^3x^6+2\,bc^2x^4+b^2cx^2)\log\left(x\right)}{4\,(b^4c^2x^6+2\,b^5cx^4+b^6x^2)}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/4*(6*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3 - 6*(c^3*x^6 + 2*b*c^2*x^4 + b^2*c
*x^2)*log(c*x^2 + b) + 12*(c^3*x^6 + 2*b*c^2*x^4 + b^2*c*x^2)*log(x))/(b^4
*c^2*x^6 + 2*b^5*c*x^4 + b^6*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{(bx^2+cx^4)^3}\,dx = \frac{-2b^2-9bcx^2-6c^2x^4}{4b^5x^2+8b^4cx^4+4b^3c^2x^6}-\frac{3c\log\left(x\right)}{b^4}+\frac{3c\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

input
```
integrate(x**3/(c*x**4+b*x**2)**3,x)
```

output
```
(-2*b**2 - 9*b*c*x**2 - 6*c**2*x**4)/(4*b**5*x**2 + 8*b**4*c*x**4 + 4*b**3
*c**2*x**6) - 3*c*log(x)/b**4 + 3*c*log(b/c + x**2)/(2*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{x^3}{(bx^2+cx^4)^3}\,dx = -\frac{6\,c^2x^4+9\,bcx^2+2\,b^2}{4\,(b^3c^2x^6+2\,b^4cx^4+b^5x^2)}+\frac{3\,c\log(cx^2+b)}{2\,b^4}-\frac{3\,c\log(x^2)}{2\,b^4}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(6*c^2*x^4 + 9*b*c*x^2 + 2*b^2)/(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)
 + 3/2*c*log(c*x^2 + b)/b^4 - 3/2*c*log(x^2)/b^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(bx^2+cx^4)^3}\,dx = \frac{3\,c\log(|cx^2+b|)}{2\,b^4}-\frac{3\,c\log(|x|)}{b^4}-\frac{6\,bc^2x^4+9\,b^2cx^2+2\,b^3}{4\,(cx^2+b)^2b^4x^2}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
3/2*c*log(abs(c*x^2 + b))/b^4 - 3*c*log(abs(x))/b^4 - 1/4*(6*b*c^2*x^4 + 9
*b^2*c*x^2 + 2*b^3)/((c*x^2 + b)^2*b^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 17.64 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{(bx^2+cx^4)^3}\,dx = \frac{3\,c\,\ln(c\,x^2+b)}{2\,b^4}-\frac{\frac{1}{2\,b}+\frac{9\,c\,x^2}{4\,b^2}+\frac{3\,c^2\,x^4}{2\,b^3}}{b^2\,x^2+2\,b\,c\,x^4+c^2\,x^6}-\frac{3\,c\,\ln(x)}{b^4}$$

input
```
int(x^3/(b*x^2 + c*x^4)^3,x)
```

output
```
(3*c*log(b + c*x^2))/(2*b^4) - (1/(2*b) + (9*c*x^2)/(4*b^2) + (3*c^2*x^4)/
(2*b^3))/(b^2*x^2 + c^2*x^6 + 2*b*c*x^4) - (3*c*log(x))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.99

$$\int \frac{x^3}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{6\log(c\,x^2+b)\,b^2c\,x^2+12\log(c\,x^2+b)\,b\,c^2x^4+6\log(c\,x^2+b)\,c^3x^6-12\log(x)\,b^2c\,x^2-24\log(x)\,b\,c^2x^4 -}{4b^4x^2\,(c^2x^4+2bc\,x^2+b^2)}$$

input
```
int(x^3/(c*x^4+b*x^2)^3,x)
```

output
```
(6*log(b + c*x**2)*b**2*c*x**2 + 12*log(b + c*x**2)*b*c**2*x**4 + 6*log(b
+ c*x**2)*c**3*x**6 - 12*log(x)*b**2*c*x**2 - 24*log(x)*b*c**2*x**4 - 12*l
og(x)*c**3*x**6 - 2*b**3 - 6*b**2*c*x**2 + 3*c**3*x**6)/(4*b**4*x**2*(b**2
 + 2*b*c*x**2 + c**2*x**4))
```

3.82 $\int \frac{x}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 86

$$\int \frac{x}{(bx^2+cx^4)^3}\,dx = -\frac{1}{4b^3x^4}+\frac{3c}{2b^4x^2}+\frac{c^2}{4b^3(b+cx^2)^2}+\frac{3c^2}{2b^4(b+cx^2)}+\frac{6c^2\log(x)}{b^5}-\frac{3c^2\log(b+cx^2)}{b^5}$$

output
```
-1/4/b^3/x^4+3/2*c/b^4/x^2+1/4*c^2/b^3/(c*x^2+b)^2+3/2*c^2/b^4/(c*x^2+b)+6
*c^2*ln(x)/b^5-3*c^2*ln(c*x^2+b)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{x}{(bx^2+cx^4)^3}\,dx = \frac{\frac{b(-b^3+4b^2cx^2+18bc^2x^4+12c^3x^6)}{x^4(b+cx^2)^2}+24c^2\log(x)-12c^2\log(b+cx^2)}{4b^5}$$

input
```
Integrate[x/(b*x^2 + c*x^4)^3,x]
```

output
```
((b*(-b^3 + 4*b^2*c*x^2 + 18*b*c^2*x^4 + 12*c^3*x^6))/(x^4*(b + c*x^2)^2)
+ 24*c^2*Log[x] - 12*c^2*Log[b + c*x^2])/(4*b^5)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^5 (b+cx^2)^3}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{1}{x^6 (cx^2+b)^3}dx^2$$

$$\Big\downarrow \; 54$$

$$\frac{1}{2}\int \left(-\frac{6c^3}{b^5 (cx^2+b)}-\frac{3c^3}{b^4 (cx^2+b)^2}-\frac{c^3}{b^3 (cx^2+b)^3}+\frac{6c^2}{b^5x^2}-\frac{3c}{b^4x^4}+\frac{1}{b^3x^6}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{6c^2\log (x^2)}{b^5}-\frac{6c^2\log (b+cx^2)}{b^5}+\frac{3c^2}{b^4 (b+cx^2)}+\frac{3c}{b^4x^2}+\frac{c^2}{2b^3 (b+cx^2)^2}-\frac{1}{2b^3x^4}\right)$$

input
```
Int[x/(b*x^2 + c*x^4)^3,x]
```

output
```
(-1/2*1/(b^3*x^4) + (3*c)/(b^4*x^2) + c^2/(2*b^3*(b + c*x^2)^2) + (3*c^2)/
(b^4*(b + c*x^2)) + (6*c^2*Log[x^2])/b^5 - (6*c^2*Log[b + c*x^2])/b^5)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\frac{3c^{3}x^{6}}{b^{4}}+\frac{9c^{2}x^{4}}{2b^{3}}+\frac{c\,x^{2}}{b^{2}}-\frac{1}{4b}}{x^{4}\left(c\,x^{2}+b\right)^{2}}+\frac{6c^{2}\ln\left(x\right)}{b^{5}}-\frac{3c^{2}\ln\left(c\,x^{2}+b\right)}{b^{5}}$
norman	$\frac{\frac{c\,x^{3}}{b^{2}}-\frac{x}{4b}-\frac{6c^{3}x^{7}}{b^{4}}-\frac{9c^{4}x^{9}}{2b^{5}}}{x^{5}\left(c\,x^{2}+b\right)^{2}}+\frac{6c^{2}\ln\left(x\right)}{b^{5}}-\frac{3c^{2}\ln\left(c\,x^{2}+b\right)}{b^{5}}$
default	$-\frac{1}{4b^{3}x^{4}}+\frac{3c}{2b^{4}x^{2}}+\frac{6c^{2}\ln\left(x\right)}{b^{5}}-\frac{c^{3}\left(-\frac{b^{2}}{2c\left(c\,x^{2}+b\right)^{2}}+\frac{6\ln\left(c\,x^{2}+b\right)}{c}-\frac{3b}{c\left(c\,x^{2}+b\right)}\right)}{2b^{5}}$
parallelrisch	$\frac{24\ln\left(x\right)x^{8}c^{4}-12\ln\left(c\,x^{2}+b\right)x^{8}c^{4}-18c^{4}x^{8}+48\ln\left(x\right)x^{6}b\,c^{3}-24\ln\left(c\,x^{2}+b\right)x^{6}b\,c^{3}-24b\,x^{6}c^{3}+24\ln\left(x\right)x^{4}b^{2}c^{2}-12\ln\left(c\,x^{2}+b\right)x^{4}}{4b^{5}x^{4}\left(c\,x^{2}+b\right)^{2}}$

input
```
int(x/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(3*c^3/b^4*x^6+9/2*c^2/b^3*x^4+c/b^2*x^2-1/4/b)/x^4/(c*x^2+b)^2+6*c^2*ln(x
)/b^5-3*c^2*ln(c*x^2+b)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int \frac{x}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{12\,bc^3x^6 + 18\,b^2c^2x^4 + 4\,b^3cx^2 - b^4 - 12\,(c^4x^8 + 2\,bc^3x^6 + b^2c^2x^4)\log(cx^2+b) + 24\,(c^4x^8 + 2\,bc^3x^6 + b^2}{4\,(b^5c^2x^8 + 2\,b^6cx^6 + b^7x^4)}$$

input
```
integrate(x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/4*(12*b*c^3*x^6 + 18*b^2*c^2*x^4 + 4*b^3*c*x^2 - b^4 - 12*(c^4*x^8 + 2*b
*c^3*x^6 + b^2*c^2*x^4)*log(c*x^2 + b) + 24*(c^4*x^8 + 2*b*c^3*x^6 + b^2*c
^2*x^4)*log(x))/(b^5*c^2*x^8 + 2*b^6*c*x^6 + b^7*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int \frac{x}{(bx^2+cx^4)^3}\,dx = \frac{-b^3+4b^2cx^2+18bc^2x^4+12c^3x^6}{4b^6x^4+8b^5cx^6+4b^4c^2x^8} + \frac{6c^2\log(x)}{b^5} - \frac{3c^2\log\left(\frac{b}{c}+x^2\right)}{b^5}$$

input
```
integrate(x/(c*x**4+b*x**2)**3,x)
```

output
```
(-b**3 + 4*b**2*c*x**2 + 18*b*c**2*x**4 + 12*c**3*x**6)/(4*b**6*x**4 + 8*b
**5*c*x**6 + 4*b**4*c**2*x**8) + 6*c**2*log(x)/b**5 - 3*c**2*log(b/c + x**
2)/b**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int \frac{x}{(bx^2+cx^4)^3}\,dx = \frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2-b^3}{4\,(b^4c^2x^8+2\,b^5cx^6+b^6x^4)} - \frac{3\,c^2\log\left(cx^2+b\right)}{b^5} + \frac{3\,c^2\log\left(x^2\right)}{b^5}$$

input
```
integrate(x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/4*(12*c^3*x^6 + 18*b*c^2*x^4 + 4*b^2*c*x^2 - b^3)/(b^4*c^2*x^8 + 2*b^5*c
*x^6 + b^6*x^4) - 3*c^2*log(c*x^2 + b)/b^5 + 3*c^2*log(x^2)/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{x}{(bx^2+cx^4)^3}\,dx = -\frac{3\,c^2\log\left(|cx^2+b|\right)}{b^5} + \frac{6\,c^2\log\left(|x|\right)}{b^5} + \frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2-b^3}{4\,(cx^4+bx^2)^2b^4}$$

input
```
integrate(x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-3*c^2*log(abs(c*x^2 + b))/b^5 + 6*c^2*log(abs(x))/b^5 + 1/4*(12*c^3*x^6 +
 18*b*c^2*x^4 + 4*b^2*c*x^2 - b^3)/((c*x^4 + b*x^2)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int \frac{x}{(bx^2+cx^4)^3}\,dx = \frac{\frac{c\,x^2}{b^2}-\frac{1}{4\,b}+\frac{9\,c^2\,x^4}{2\,b^3}+\frac{3\,c^3\,x^6}{b^4}}{b^2\,x^4+2\,b\,c\,x^6+c^2\,x^8} - \frac{3\,c^2\,\ln\left(c\,x^2+b\right)}{b^5} + \frac{6\,c^2\,\ln\left(x\right)}{b^5}$$

input
```
int(x/(b*x^2 + c*x^4)^3,x)
```

output
```
((c*x^2)/b^2 - 1/(4*b) + (9*c^2*x^4)/(2*b^3) + (3*c^3*x^6)/b^4)/(b^2*x^4 +
 c^2*x^8 + 2*b*c*x^6) - (3*c^2*log(b + c*x^2))/b^5 + (6*c^2*log(x))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.72

$$\int \frac{x}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{-12\log(c\,x^2+b)\,b^2c^2x^4-24\log(c\,x^2+b)\,b\,c^3x^6-12\log(c\,x^2+b)\,c^4x^8+24\log(x)\,b^2c^2x^4+48\log(x)\,b\,c}{4b^5x^4\,(c^2x^4+2bc\,x^2+b^2)}$$

input
```
int(x/(c*x^4+b*x^2)^3,x)
```

output
```
( - 12*log(b + c*x**2)*b**2*c**2*x**4 - 24*log(b + c*x**2)*b*c**3*x**6 - 1
2*log(b + c*x**2)*c**4*x**8 + 24*log(x)*b**2*c**2*x**4 + 48*log(x)*b*c**3*
x**6 + 24*log(x)*c**4*x**8 - b**4 + 4*b**3*c*x**2 + 12*b**2*c**2*x**4 - 6*
c**4*x**8)/(4*b**5*x**4*(b**2 + 2*b*c*x**2 + c**2*x**4))
```

3.83 $\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx=-\frac{1}{6b^3x^6}+\frac{3c}{4b^4x^4}-\frac{3c^2}{b^5x^2}-\frac{c^3}{4b^4\left(b+cx^2\right)^2}-\frac{2c^3}{b^5\left(b+cx^2\right)}-\frac{10c^3\log(x)}{b^6}+\frac{5c^3\log\left(b+cx^2\right)}{b^6}$$

output
```
-1/6/b^3/x^6+3/4*c/b^4/x^4-3*c^2/b^5/x^2-1/4*c^3/b^4/(c*x^2+b)^2-2*c^3/b^5
/(c*x^2+b)-10*c^3*ln(x)/b^6+5*c^3*ln(c*x^2+b)/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx=-\frac{\frac{b\left(2b^4-5b^3cx^2+20b^2c^2x^4+90bc^3x^6+60c^4x^8\right)}{x^6\left(b+cx^2\right)^2}+120c^3\log(x)-60c^3\log\left(b+cx^2\right)}{12b^6}$$

input
```
Integrate[1/(x*(b*x^2 + c*x^4)^3),x]
```

output
```
-1/12*((b*(2*b^4 - 5*b^3*c*x^2 + 20*b^2*c^2*x^4 + 90*b*c^3*x^6 + 60*c^4*x^
8))/(x^6*(b + c*x^2)^2) + 120*c^3*Log[x] - 60*c^3*Log[b + c*x^2])/b^6
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {9, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(bx^2+cx^4\right)^3}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{x^7\left(b+cx^2\right)^3}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{1}{x^8\left(cx^2+b\right)^3}dx^2$$
$$\Big\downarrow \text{ 54}$$
$$\frac{1}{2}\int \left(\frac{10c^4}{b^6\left(cx^2+b\right)}+\frac{4c^4}{b^5\left(cx^2+b\right)^2}+\frac{c^4}{b^4\left(cx^2+b\right)^3}-\frac{10c^3}{b^6x^2}+\frac{6c^2}{b^5x^4}-\frac{3c}{b^4x^6}+\frac{1}{b^3x^8}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(-\frac{10c^3\log\left(x^2\right)}{b^6}+\frac{10c^3\log\left(b+cx^2\right)}{b^6}-\frac{4c^3}{b^5\left(b+cx^2\right)}-\frac{6c^2}{b^5x^2}-\frac{c^3}{2b^4\left(b+cx^2\right)^2}+\frac{3c}{2b^4x^4}-\frac{1}{3b^3x^6}\right)$$

input
```
Int[1/(x*(b*x^2 + c*x^4)^3),x]
```

output
```
(-1/3*1/(b^3*x^6) + (3*c)/(2*b^4*x^4) - (6*c^2)/(b^5*x^2) - c^3/(2*b^4*(b
+ c*x^2)^2) - (4*c^3)/(b^5*(b + c*x^2)) - (10*c^3*Log[x^2])/b^6 + (10*c^3*
Log[b + c*x^2])/b^6)/2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

method	result
norman	$\frac{-\frac{1}{6b}+\frac{5c\,x^{2}}{12b^{2}}-\frac{5c^{2}x^{4}}{3b^{3}}+\frac{10c^{4}x^{8}}{b^{5}}+\frac{15c^{5}x^{10}}{2b^{6}}}{x^{6}\left(c\,x^{2}+b\right)^{2}}-\frac{10c^{3}\ln\left(x\right)}{b^{6}}+\frac{5c^{3}\ln\left(c\,x^{2}+b\right)}{b^{6}}$
risch	$\frac{-\frac{5c^{4}x^{8}}{b^{5}}-\frac{15c^{3}x^{6}}{2b^{4}}-\frac{5c^{2}x^{4}}{3b^{3}}+\frac{5c\,x^{2}}{12b^{2}}-\frac{1}{6b}}{x^{6}\left(c\,x^{2}+b\right)^{2}}-\frac{10c^{3}\ln\left(x\right)}{b^{6}}+\frac{5c^{3}\ln\left(-c\,x^{2}-b\right)}{b^{6}}$
default	$-\frac{1}{6b^{3}x^{6}}+\frac{3c}{4b^{4}x^{4}}-\frac{3c^{2}}{b^{5}x^{2}}-\frac{10c^{3}\ln\left(x\right)}{b^{6}}+\frac{c^{4}\left(-\frac{b^{2}}{2c\left(c\,x^{2}+b\right)^{2}}+\frac{10\ln\left(c\,x^{2}+b\right)}{c}-\frac{4b}{c\left(c\,x^{2}+b\right)}\right)}{2b^{6}}$
parallelrisch	$-\frac{120\ln\left(x\right)x^{10}c^{5}-60\ln\left(c\,x^{2}+b\right)x^{10}c^{5}-90c^{5}x^{10}+240\ln\left(x\right)x^{8}b\,c^{4}-120\ln\left(c\,x^{2}+b\right)x^{8}b\,c^{4}-120c^{4}x^{8}b+120\ln\left(x\right)x^{6}b^{2}c^{3}-60\ln}{12b^{6}x^{6}\left(c\,x^{2}+b\right)^{2}}$

input
```
int(1/x/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/6/b+5/12*c/b^2*x^2-5/3*c^2/b^3*x^4+10*c^4/b^5*x^8+15/2*c^5/b^6*x^10)/x
^6/(c*x^2+b)^2-10*c^3*ln(x)/b^6+5*c^3*ln(c*x^2+b)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.53

$$\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx = -\frac{60\,bc^4x^8+90\,b^2c^3x^6+20\,b^3c^2x^4-5\,b^4cx^2+2\,b^5-60\left(c^5x^{10}+2\,bc^4x^8+b^2c^3x^6\right)\log\left(cx^2+b\right)+120\,(c}{12\left(b^6c^2x^{10}+2\,b^7cx^8+b^8x^6\right)}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/12*(60*b*c^4*x^8 + 90*b^2*c^3*x^6 + 20*b^3*c^2*x^4 - 5*b^4*c*x^2 + 2*b^
5 - 60*(c^5*x^10 + 2*b*c^4*x^8 + b^2*c^3*x^6)*log(c*x^2 + b) + 120*(c^5*x^
10 + 2*b*c^4*x^8 + b^2*c^3*x^6)*log(x))/(b^6*c^2*x^10 + 2*b^7*c*x^8 + b^8*
x^6)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx = \frac{-2b^4+5b^3cx^2-20b^2c^2x^4-90bc^3x^6-60c^4x^8}{12b^7x^6+24b^6cx^8+12b^5c^2x^{10}} - \frac{10c^3\log\left(x\right)}{b^6} + \frac{5c^3\log\left(\frac{b}{c}+x^2\right)}{b^6}$$

input
```
integrate(1/x/(c*x**4+b*x**2)**3,x)
```

output
```
(-2*b**4 + 5*b**3*c*x**2 - 20*b**2*c**2*x**4 - 90*b*c**3*x**6 - 60*c**4*x*
*8)/(12*b**7*x**6 + 24*b**6*c*x**8 + 12*b**5*c**2*x**10) - 10*c**3*log(x)/
b**6 + 5*c**3*log(b/c + x**2)/b**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx = -\frac{60\,c^4x^8+90\,bc^3x^6+20\,b^2c^2x^4-5\,b^3cx^2+2\,b^4}{12\left(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6\right)} + \frac{5\,c^3\log\left(cx^2+b\right)}{b^6} - \frac{5\,c^3\log\left(x^2\right)}{b^6}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/12*(60*c^4*x^8 + 90*b*c^3*x^6 + 20*b^2*c^2*x^4 - 5*b^3*c*x^2 + 2*b^4)/(
b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6) + 5*c^3*log(c*x^2 + b)/b^6 - 5*c^3*l
og(x^2)/b^6
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

$$\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx = -\frac{5\,c^3\log\left(x^2\right)}{b^6} + \frac{5\,c^3\log\left(\left|cx^2+b\right|\right)}{b^6} - \frac{30\,c^5x^4+68\,bc^4x^2+39\,b^2c^3}{4\left(cx^2+b\right)^2b^6} + \frac{110\,c^3x^6-36\,bc^2x^4+9\,b^2cx^2-2\,b^3}{12\,b^6x^6}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-5*c^3*log(x^2)/b^6 + 5*c^3*log(abs(c*x^2 + b))/b^6 - 1/4*(30*c^5*x^4 + 68
*b*c^4*x^2 + 39*b^2*c^3)/((c*x^2 + b)^2*b^6) + 1/12*(110*c^3*x^6 - 36*b*c^
2*x^4 + 9*b^2*c*x^2 - 2*b^3)/(b^6*x^6)
```

Mupad [B] (verification not implemented)

Time = 17.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx = \frac{5\,c^3\,\ln\left(c\,x^2+b\right)}{b^6} - \frac{\frac{1}{6\,b}-\frac{5\,c\,x^2}{12\,b^2}+\frac{5\,c^2\,x^4}{3\,b^3}+\frac{15\,c^3\,x^6}{2\,b^4}+\frac{5\,c^4\,x^8}{b^5}}{b^2\,x^6+2\,b\,c\,x^8+c^2\,x^{10}} - \frac{10\,c^3\,\ln\left(x\right)}{b^6}$$

input
```
int(1/(x*(b*x^2 + c*x^4)^3),x)
```

output
```
(5*c^3*log(b + c*x^2))/b^6 - (1/(6*b) - (5*c*x^2)/(12*b^2) + (5*c^2*x^4)/(
3*b^3) + (15*c^3*x^6)/(2*b^4) + (5*c^4*x^8)/b^5)/(b^2*x^6 + c^2*x^10 + 2*b
*c*x^8) - (10*c^3*log(x))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\left(bx^2+cx^4\right)^3}\,dx = \frac{60\log(c\,x^2+b)\,b^2c^3x^6+120\log(c\,x^2+b)\,b\,c^4x^8+60\log(c\,x^2+b)\,c^5x^{10}-120\log(x)\,b^2c^3x^6-240\log(x)}{12b^6x^6\left(c^2x^4+2bc\,x^2+b^2\right)}$$

input
```
int(1/x/(c*x^4+b*x^2)^3,x)
```

output
```
(60*log(b + c*x**2)*b**2*c**3*x**6 + 120*log(b + c*x**2)*b*c**4*x**8 + 60*
log(b + c*x**2)*c**5*x**10 - 120*log(x)*b**2*c**3*x**6 - 240*log(x)*b*c**4
*x**8 - 120*log(x)*c**5*x**10 - 2*b**5 + 5*b**4*c*x**2 - 20*b**3*c**2*x**4
 - 60*b**2*c**3*x**6 + 30*c**5*x**10)/(12*b**6*x**6*(b**2 + 2*b*c*x**2 + c
**2*x**4))
```

3.84 $\int \frac{x^{14}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \frac{x^{14}}{(bx^2+cx^4)^3}\,dx = -\frac{3bx}{c^4}+\frac{x^3}{3c^3}+\frac{b^3x}{4c^4(b+cx^2)^2}-\frac{13b^2x}{8c^4(b+cx^2)}+\frac{35b^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}$$

output
```
-3*b*x/c^4+1/3*x^3/c^3+1/4*b^3*x/c^4/(c*x^2+b)^2-13/8*b^2*x/c^4/(c*x^2+b)+
35/8*b^(3/2)*arctan(c^(1/2)*x/b^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{x^{14}}{(bx^2+cx^4)^3}\,dx = -\frac{105b^3x+175b^2cx^3+56bc^2x^5-8c^3x^7}{24c^4(b+cx^2)^2}+\frac{35b^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}$$

input
```
Integrate[x^14/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/24*(105*b^3*x + 175*b^2*c*x^3 + 56*b*c^2*x^5 - 8*c^3*x^7)/(c^4*(b + c*x
^2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {9, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^8}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{7\int \frac{x^6}{(cx^2+b)^2}dx}{4c}-\frac{x^7}{4c(b+cx^2)^2}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{7\left(\frac{5\int \frac{x^4}{cx^2+b}dx}{2c}-\frac{x^5}{2c(b+cx^2)}\right)}{4c}-\frac{x^7}{4c(b+cx^2)^2}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{7\left(\frac{5\int \left(\frac{b^2}{c^2(cx^2+b)}-\frac{b}{c^2}+\frac{x^2}{c}\right)dx}{2c}-\frac{x^5}{2c(b+cx^2)}\right)}{4c}-\frac{x^7}{4c(b+cx^2)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{7\left(\frac{5\left(\frac{b^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}\right)}{2c}-\frac{x^5}{2c(b+cx^2)}\right)}{4c}-\frac{x^7}{4c(b+cx^2)^2}$$

input
```
Int[x^14/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*x^7/(c*(b + c*x^2)^2) + (7*(-1/2*x^5/(c*(b + c*x^2)) + (5*(-((b*x)/c^
2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(5/2)))/(2*c)))/(
4*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{-\frac{1}{3} c \,x^{3}+3 b x}{c^{4}}+\frac{b^{2} \left(\frac{-\frac{13}{8} c \,x^{3}-\frac{11}{8} b x}{\left(c \,x^{2}+b \right)^{2}}+\frac{35 \arctan \left(\frac{c x}{\sqrt{b c}}\right)}{8 \sqrt{b c}}\right)}{c^{4}}$	63
risch	$\frac{x^{3}}{3 c^{3}}-\frac{3 b x}{c^{4}}+\frac{-\frac{13}{8} b^{2} c \,x^{3}-\frac{11}{8} b^{3} x}{c^{4} \left(c \,x^{2}+b \right)^{2}}+\frac{35 \sqrt{-b c}\, b \ln \left(-\sqrt{-b c}\, x +b \right)}{16 c^{5}}-\frac{35 \sqrt{-b c}\, b \ln \left(\sqrt{-b c}\, x +b \right)}{16 c^{5}}$	93

input
```
int(x^14/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/c^4*(-1/3*c*x^3+3*b*x)+b^2/c^4*((-13/8*c*x^3-11/8*b*x)/(c*x^2+b)^2+35/8
/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.71

$$\int \frac{x^{14}}{(bx^2+cx^4)^3}\,dx$$
$$= \left[\frac{16\,c^3x^7 - 112\,bc^2x^5 - 350\,b^2cx^3 - 210\,b^3x + 105\,(bc^2x^4 + 2\,b^2cx^2 + b^3)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{48\,(c^6x^4 + 2\,bc^5x^2 + b^2c^4)}, \frac{8\,c^3 \ldots\right.$$

input
```
integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*c^3*x^7 - 112*b*c^2*x^5 - 350*b^2*c*x^3 - 210*b^3*x + 105*(b*c^2
*x^4 + 2*b^2*c*x^2 + b^3)*sqrt(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c
*x^2 + b)))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4), 1/24*(8*c^3*x^7 - 56*b*c^2*
x^5 - 175*b^2*c*x^3 - 105*b^3*x + 105*(b*c^2*x^4 + 2*b^2*c*x^2 + b^3)*sqrt
(b/c)*arctan(c*x*sqrt(b/c)/b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int \frac{x^{14}}{(bx^2+cx^4)^3}\,dx = -\frac{3bx}{c^4} - \frac{35\sqrt{-\frac{b^3}{c^9}}\log\left(x - \frac{c^4\sqrt{-\frac{b^3}{c^9}}}{b}\right)}{16} + \frac{35\sqrt{-\frac{b^3}{c^9}}\log\left(x + \frac{c^4\sqrt{-\frac{b^3}{c^9}}}{b}\right)}{16} + \frac{-11b^3x - 13b^2cx^3}{8b^2c^4 + 16bc^5x^2 + 8c^6x^4} + \frac{x^3}{3c^3}$$

input
```
integrate(x**14/(c*x**4+b*x**2)**3,x)
```

output
```
-3*b*x/c**4 - 35*sqrt(-b**3/c**9)*log(x - c**4*sqrt(-b**3/c**9)/b)/16 + 35
*sqrt(-b**3/c**9)*log(x + c**4*sqrt(-b**3/c**9)/b)/16 + (-11*b**3*x - 13*b
**2*c*x**3)/(8*b**2*c**4 + 16*b*c**5*x**2 + 8*c**6*x**4) + x**3/(3*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{x^{14}}{(bx^2+cx^4)^3}\,dx = -\frac{13\,b^2cx^3+11\,b^3x}{8\,(c^6x^4+2\,bc^5x^2+b^2c^4)} + \frac{35\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^4} + \frac{cx^3-9\,bx}{3\,c^4}$$

input
```
integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(13*b^2*c*x^3 + 11*b^3*x)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 35/8*b^
2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/3*(c*x^3 - 9*b*x)/c^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{x^{14}}{(bx^2+cx^4)^3}\,dx = \frac{35\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^4} - \frac{13\,b^2cx^3+11\,b^3x}{8\,(cx^2+b)^2c^4} + \frac{c^6x^3-9\,bc^5x}{3\,c^9}$$

input
```
integrate(x^14/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
35/8*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) - 1/8*(13*b^2*c*x^3 + 11*b^
3*x)/((c*x^2 + b)^2*c^4) + 1/3*(c^6*x^3 - 9*b*c^5*x)/c^9
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{x^{14}}{(bx^2+cx^4)^3}\,dx = \frac{x^3}{3\,c^3} - \frac{\frac{11\,b^3\,x}{8} + \frac{13\,c\,b^2\,x^3}{8}}{b^2\,c^4 + 2\,b\,c^5\,x^2 + c^6\,x^4} + \frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,c^{9/2}} - \frac{3\,b\,x}{c^4}$$

input
```
int(x^14/(b*x^2 + c*x^4)^3,x)
```

output
```
x^3/(3*c^3) - ((11*b^3*x)/8 + (13*b^2*c*x^3)/8)/(b^2*c^4 + c^6*x^4 + 2*b*c
^5*x^2) + (35*b^(3/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*c^(9/2)) - (3*b*x)/c^4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.55

$$\int \frac{x^{14}}{(bx^2+cx^4)^3}\,dx = \frac{105\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^3 + 210\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^2c\,x^2 + 105\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b\,c^2x^4 - 105b^3cx - 1}{24c^5\,(c^2x^4 + 2bc\,x^2 + b^2)}$$

input
```
int(x^14/(c*x^4+b*x^2)^3,x)
```

output
```
(105*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**3 + 210*sqrt(c)*sqrt
(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**2*c*x**2 + 105*sqrt(c)*sqrt(b)*atan((
c*x)/(sqrt(c)*sqrt(b)))*b*c**2*x**4 - 105*b**3*c*x - 175*b**2*c**2*x**3 -
56*b*c**3*x**5 + 8*c**4*x**7)/(24*c**5*(b**2 + 2*b*c*x**2 + c**2*x**4))
```

3.85 $\int \frac{x^{12}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{x^{12}}{(bx^2+cx^4)^3}\,dx=\frac{x}{c^3}-\frac{b^2x}{4c^3(b+cx^2)^2}+\frac{9bx}{8c^3(b+cx^2)}-\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{7/2}}$$

output
```
x/c^3-1/4*b^2*x/c^3/(c*x^2+b)^2+9/8*b*x/c^3/(c*x^2+b)-15/8*b^(1/2)*arctan(
c^(1/2)*x/b^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{x^{12}}{(bx^2+cx^4)^3}\,dx=\frac{15b^2x+25bcx^3+8c^2x^5}{8c^3(b+cx^2)^2}-\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{7/2}}$$

input
```
Integrate[x^12/(b*x^2 + c*x^4)^3,x]
```

output
```
(15*b^2*x + 25*b*c*x^3 + 8*c^2*x^5)/(8*c^3*(b + c*x^2)^2) - (15*Sqrt[b]*Ar
cTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {9, 252, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^6}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5\int \frac{x^4}{(cx^2+b)^2}dx}{4c}-\frac{x^5}{4c(b+cx^2)^2}$$

$$\Big\downarrow \text{252}$$

$$\frac{5\left(\frac{3\int \frac{x^2}{cx^2+b}dx}{2c}-\frac{x^3}{2c(b+cx^2)}\right)}{4c}-\frac{x^5}{4c(b+cx^2)^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{5\left(\frac{3\left(\frac{x}{c}-\frac{b\int \frac{1}{cx^2+b}dx}{c}\right)}{2c}-\frac{x^3}{2c(b+cx^2)}\right)}{4c}-\frac{x^5}{4c(b+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{5\left(\frac{3\left(\frac{x}{c}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}\right)}{2c}-\frac{x^3}{2c(b+cx^2)}\right)}{4c}-\frac{x^5}{4c(b+cx^2)^2}$$

input
```
Int[x^12/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*x^5/(c*(b + c*x^2)^2) + (5*(-1/2*x^3/(c*(b + c*x^2)) + (3*(x/c - (Sqr
t[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)))/(2*c)))/(4*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{x}{c^3}-\frac{b\left(\frac{-\frac{9}{8}c\,x^3-\frac{7}{8}bx}{\left(c\,x^2+b\right)^2}+\frac{15\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{c^3}$	51
risch	$\frac{x}{c^3}+\frac{\frac{9}{8}bc\,x^3+\frac{7}{8}b^2x}{c^3\left(c\,x^2+b\right)^2}+\frac{15\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)}{16c^4}-\frac{15\sqrt{-bc}\,\ln\left(\sqrt{-bc}\,x-b\right)}{16c^4}$	83

input
```
int(x^12/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
x/c^3-b/c^3*((-9/8*c*x^3-7/8*b*x)/(c*x^2+b)^2+15/8/(b*c)^(1/2)*arctan(c*x/
(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.85

$$\int \frac{x^{12}}{(bx^2+cx^4)^3}\,dx$$
$$=\left[\frac{16\,c^2x^5+50\,bcx^3+30\,b^2x+15\,(c^2x^4+2\,bcx^2+b^2)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{16\,(c^5x^4+2\,bc^4x^2+b^2c^3)},\ \frac{8\,c^2x^5+25\,bcx^3+15\,b\right.$$

input
```
integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*c^2*x^5 + 50*b*c*x^3 + 30*b^2*x + 15*(c^2*x^4 + 2*b*c*x^2 + b^2)
*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^5*x^4 + 2*
b*c^4*x^2 + b^2*c^3), 1/8*(8*c^2*x^5 + 25*b*c*x^3 + 15*b^2*x - 15*(c^2*x^4
 + 2*b*c*x^2 + b^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b))/(c^5*x^4 + 2*b*c^4*
x^2 + b^2*c^3)]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.51

$$\int \frac{x^{12}}{\left(bx^2+cx^4\right)^3}\,dx = \frac{15\sqrt{-\frac{b}{c^7}}\log\left(-c^3\sqrt{-\frac{b}{c^7}}+x\right)}{16} - \frac{15\sqrt{-\frac{b}{c^7}}\log\left(c^3\sqrt{-\frac{b}{c^7}}+x\right)}{16} + \frac{7b^2x+9bcx^3}{8b^2c^3+16bc^4x^2+8c^5x^4} + \frac{x}{c^3}$$

input
```
integrate(x**12/(c*x**4+b*x**2)**3,x)
```

output
```
15*sqrt(-b/c**7)*log(-c**3*sqrt(-b/c**7) + x)/16 - 15*sqrt(-b/c**7)*log(c*
*3*sqrt(-b/c**7) + x)/16 + (7*b**2*x + 9*b*c*x**3)/(8*b**2*c**3 + 16*b*c**
4*x**2 + 8*c**5*x**4) + x/c**3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{x^{12}}{\left(bx^2+cx^4\right)^3}\,dx = \frac{9\,bcx^3+7\,b^2x}{8\,\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)} - \frac{15\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^3} + \frac{x}{c^3}$$

input
```
integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/8*(9*b*c*x^3 + 7*b^2*x)/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) - 15/8*b*arcta
n(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + x/c^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{x^{12}}{(bx^2+cx^4)^3}\,dx = -\frac{15\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^3}+\frac{x}{c^3}+\frac{9\,bcx^3+7\,b^2x}{8\,(cx^2+b)^2c^3}$$

input
```
integrate(x^12/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-15/8*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + x/c^3 + 1/8*(9*b*c*x^3 + 7
*b^2*x)/((c*x^2 + b)^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 17.54 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{x^{12}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{7\,b^2\,x}{8}+\frac{9\,c\,b\,x^3}{8}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{x}{c^3}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,c^{7/2}}$$

input
```
int(x^12/(b*x^2 + c*x^4)^3,x)
```

output
```
((7*b^2*x)/8 + (9*b*c*x^3)/8)/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + x/c^3 -
(15*b^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*c^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int \frac{x^{12}}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{-15\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^2-30\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)bc\,x^2-15\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)c^2x^4+15b^2cx+25b\,c^2}{8c^4\,(c^2x^4+2bc\,x^2+b^2)}$$

input
```
int(x^12/(c*x^4+b*x^2)^3,x)
```

output
```
( - 15*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**2 - 30*sqrt(c)*sqr
t(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b*c*x**2 - 15*sqrt(c)*sqrt(b)*atan((c*x
)/(sqrt(c)*sqrt(b)))*c**2*x**4 + 15*b**2*c*x + 25*b*c**2*x**3 + 8*c**3*x**
5)/(8*c**4*(b**2 + 2*b*c*x**2 + c**2*x**4))
```

3.86 $\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx = -\frac{x^3}{4c(b+cx^2)^2} - \frac{3x}{8c^2(b+cx^2)} + \frac{3\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{5/2}}$$

output
```
-1/4*x^3/c/(c*x^2+b)^2-3/8*x/c^2/(c*x^2+b)+3/8*arctan(c^(1/2)*x/b^(1/2))/b
^(1/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx = -\frac{3bx+5cx^3}{8c^2(b+cx^2)^2} + \frac{3\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{5/2}}$$

input
```
Integrate[x^10/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/8*(3*b*x + 5*c*x^3)/(c^2*(b + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[b]
])/(8*Sqrt[b]*c^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 252, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^4}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{3\int \frac{x^2}{(cx^2+b)^2}dx}{4c}-\frac{x^3}{4c\,(b+cx^2)^2}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{3\left(\frac{\int \frac{1}{cx^2+b}dx}{2c}-\frac{x}{2c(b+cx^2)}\right)}{4c}-\frac{x^3}{4c\,(b+cx^2)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{3/2}}-\frac{x}{2c(b+cx^2)}\right)}{4c}-\frac{x^3}{4c\,(b+cx^2)^2}$$

input
```
Int[x^10/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*x^3/(c*(b + c*x^2)^2) + (3*(-1/2*x/(c*(b + c*x^2)) + ArcTan[(Sqrt[c]*
x)/Sqrt[b]]/(2*Sqrt[b]*c^(3/2))))/(4*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{-\frac{5x^3}{8c}-\frac{3bx}{8c^2}}{\left(c\,x^2+b\right)^2}+\frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8c^2\sqrt{bc}}$	47
risch	$\frac{-\frac{5x^3}{8c}-\frac{3bx}{8c^2}}{\left(c\,x^2+b\right)^2}-\frac{3\ln\left(cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\,c^2}+\frac{3\ln\left(-cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\,c^2}$	73

input
```
int(x^10/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-5/8*x^3/c-3/8*b*x/c^2)/(c*x^2+b)^2+3/8/c^2/(b*c)^(1/2)*arctan(c*x/(b*c)^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.94

$$\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx = \left[-\frac{10\,bc^2x^3+6\,b^2cx+3\,(c^2x^4+2\,bcx^2+b^2)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x-b}{cx^2+b}\right)}{16\,(bc^5x^4+2\,b^2c^4x^2+b^3c^3)}, -\frac{5\,bc^2x^3+3\,b^2cx-3\,(c^2x^4+2\,bcx^2+b^2)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{8\,(bc^5x^4+2\,b^2c^4x^2+b^3c^3)}\right]$$

input
```
integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[-1/16*(10*b*c^2*x^3 + 6*b^2*c*x + 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c
)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b*c^5*x^4 + 2*b^2*c^4*x^
2 + b^3*c^3), -1/8*(5*b*c^2*x^3 + 3*b^2*c*x - 3*(c^2*x^4 + 2*b*c*x^2 + b^2
)*sqrt(b*c)*arctan(sqrt(b*c)*x/b))/(b*c^5*x^4 + 2*b^2*c^4*x^2 + b^3*c^3)]
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.72

$$\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx = -\frac{3\sqrt{-\frac{1}{bc^5}}\log\left(-bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{bc^5}}\log\left(bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{16} + \frac{-3bx-5cx^3}{8b^2c^2+16bc^3x^2+8c^4x^4}$$

input
```
integrate(x**10/(c*x**4+b*x**2)**3,x)
```

output
```
-3*sqrt(-1/(b*c**5))*log(-b*c**2*sqrt(-1/(b*c**5)) + x)/16 + 3*sqrt(-1/(b*
c**5))*log(b*c**2*sqrt(-1/(b*c**5)) + x)/16 + (-3*b*x - 5*c*x**3)/(8*b**2*
c**2 + 16*b*c**3*x**2 + 8*c**4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx = -\frac{5\,cx^3+3\,bx}{8\,(c^4x^4+2\,bc^3x^2+b^2c^2)} + \frac{3\,\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^2}$$

input
```
integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(5*c*x^3 + 3*b*x)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2) + 3/8*arctan(c*x/
sqrt(b*c))/(sqrt(b*c)*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

$$\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx = \frac{3\,\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^2} - \frac{5\,cx^3+3\,bx}{8\,(cx^2+b)^2c^2}$$

input
```
integrate(x^10/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
3/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) - 1/8*(5*c*x^3 + 3*b*x)/((c*x^2
+ b)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 17.74 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,\sqrt{b}\,c^{5/2}} - \frac{\frac{5\,x^3}{8\,c}+\frac{3\,b\,x}{8\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

input
```
int(x^10/(b*x^2 + c*x^4)^3,x)
```

output
```
(3*atan((c^(1/2)*x)/b^(1/2)))/(8*b^(1/2)*c^(5/2)) - ((5*x^3)/(8*c) + (3*b*
x)/(8*c^2))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.77

$$\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^2+6\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)bc\,x^2+3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)c^2x^4-3b^2cx-5b\,c^2x^3}{8b\,c^3\,(c^2x^4+2bc\,x^2+b^2)}$$

input
```
int(x^10/(c*x^4+b*x^2)^3,x)
```

output
```
(3*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**2 + 6*sqrt(c)*sqrt(b)*
atan((c*x)/(sqrt(c)*sqrt(b)))*b*c*x**2 + 3*sqrt(c)*sqrt(b)*atan((c*x)/(sqr
t(c)*sqrt(b)))*c**2*x**4 - 3*b**2*c*x - 5*b*c**2*x**3)/(8*b*c**3*(b**2 + 2
*b*c*x**2 + c**2*x**4))
```

3.87 $\int \frac{x^8}{(bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{x^8}{(bx^2+cx^4)^3} dx = -\frac{x}{4c(b+cx^2)^2} + \frac{x}{8bc(b+cx^2)} + \frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}$$

output
```
-1/4*x/c/(c*x^2+b)^2+1/8*x/b/c/(c*x^2+b)+1/8*arctan(c^(1/2)*x/b^(1/2))/b^(
3/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{(bx^2+cx^4)^3} dx = \frac{\frac{\sqrt{b}\sqrt{c}x(-b+cx^2)}{(b+cx^2)^2} + \arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}$$

input
```
Integrate[x^8/(b*x^2 + c*x^4)^3,x]
```

output
```
((Sqrt[b]*Sqrt[c]*x*(-b + c*x^2))/(b + c*x^2)^2 + ArcTan[(Sqrt[c]*x)/Sqrt[
b]])/(8*b^(3/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {9, 252, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^2}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\int \frac{1}{(cx^2+b)^2}dx}{4c}-\frac{x}{4c(b+cx^2)^2}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{\int \frac{1}{cx^2+b}dx}{2b}+\frac{x}{2b(b+cx^2)}}{4c}-\frac{x}{4c(b+cx^2)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b(b+cx^2)}}{4c}-\frac{x}{4c(b+cx^2)^2}$$

input
```
Int[x^8/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*x/(c*(b + c*x^2)^2) + (x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[
b]]/(2*b^(3/2)*Sqrt[c]))/(4*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\frac{x^3}{8b}-\frac{x}{8c}}{(c\,x^2+b)^2}+\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8cb\sqrt{bc}}$	49
risch	$\frac{\frac{x^3}{8b}-\frac{x}{8c}}{(c\,x^2+b)^2}-\frac{\ln\left(cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\,cb}+\frac{\ln\left(-cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\,cb}$	78

input
```
int(x^8/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8/b*x^3-1/8*x/c)/(c*x^2+b)^2+1/8/c/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.92

$$\int \frac{x^8}{(bx^2+cx^4)^3}\,dx = \left[\frac{2\,bc^2x^3-2\,b^2cx-(c^2x^4+2\,bcx^2+b^2)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x-b}{cx^2+b}\right)}{16\,(b^2c^4x^4+2\,b^3c^3x^2+b^4c^2)}, \frac{bc^2x^3-b^2cx+(c^2x^4+2\,bcx^2+b^2)\sqrt{\ }}{8\,(b^2c^4x^4+2\,b^3c^3x^2+b}\right.$$

input
```
integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*b*c^2*x^3 - 2*b^2*c*x - (c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)*lo
g((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b^2*c^4*x^4 + 2*b^3*c^3*x^2
+ b^4*c^2), 1/8*(b*c^2*x^3 - b^2*c*x + (c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(b*
c)*arctan(sqrt(b*c)*x/b))/(b^2*c^4*x^4 + 2*b^3*c^3*x^2 + b^4*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(51) = 102$.

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{x^8}{(bx^2+cx^4)^3}\,dx = -\frac{\sqrt{-\frac{1}{b^3c^3}}\log\left(-b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{b^3c^3}}\log\left(b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{16} + \frac{-bx+cx^3}{8b^3c+16b^2c^2x^2+8bc^3x^4}$$

input
```
integrate(x**8/(c*x**4+b*x**2)**3,x)
```

output
```
-sqrt(-1/(b**3*c**3))*log(-b**2*c*sqrt(-1/(b**3*c**3)) + x)/16 + sqrt(-1/(
b**3*c**3))*log(b**2*c*sqrt(-1/(b**3*c**3)) + x)/16 + (-b*x + c*x**3)/(8*b
**3*c + 16*b**2*c**2*x**2 + 8*b*c**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{x^8}{(bx^2+cx^4)^3}\,dx = \frac{cx^3-bx}{8\,(bc^3x^4+2\,b^2c^2x^2+b^3c)} + \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}bc}$$

input
```
integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/8*(c*x^3 - b*x)/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c) + 1/8*arctan(c*x/sqr
t(b*c))/(sqrt(b*c)*b*c)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{x^8}{(bx^2+cx^4)^3}\,dx = \frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}bc} + \frac{cx^3-bx}{8\,(cx^2+b)^2bc}$$

input
```
integrate(x^8/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c) + 1/8*(c*x^3 - b*x)/((c*x^2 + b)
^2*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{x^8}{(bx^2+cx^4)^3}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{3/2}\,c^{3/2}} - \frac{\frac{x}{8\,c}-\frac{x^3}{8\,b}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

input
```
int(x^8/(b*x^2 + c*x^4)^3,x)
```

output
```
atan((c^(1/2)*x)/b^(1/2))/(8*b^(3/2)*c^(3/2)) - (x/(8*c) - x^3/(8*b))/(b^2
 + c^2*x^4 + 2*b*c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{x^8}{(bx^2+cx^4)^3}\,dx = \frac{\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^2+2\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)bc\,x^2+\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)c^2x^4-b^2cx+b\,c^2x^3}{8b^2c^2\,(c^2x^4+2bc\,x^2+b^2)}$$

input
```
int(x^8/(c*x^4+b*x^2)^3,x)
```

output
```
(sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**2 + 2*sqrt(c)*sqrt(b)*at
an((c*x)/(sqrt(c)*sqrt(b)))*b*c*x**2 + sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)
*sqrt(b)))*c**2*x**4 - b**2*c*x + b*c**2*x**3)/(8*b**2*c**2*(b**2 + 2*b*c*
x**2 + c**2*x**4))
```

3.88 $\int \frac{x^6}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \frac{x^6}{(bx^2+cx^4)^3}\,dx=\frac{x}{4b(b+cx^2)^2}+\frac{3x}{8b^2(b+cx^2)}+\frac{3\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}$$

output
```
1/4*x/b/(c*x^2+b)^2+3/8*x/b^2/(c*x^2+b)+3/8*arctan(c^(1/2)*x/b^(1/2))/b^(5
/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{(bx^2+cx^4)^3}\,dx=\frac{5bx+3cx^3}{8b^2(b+cx^2)^2}+\frac{3\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}$$

input
```
Integrate[x^6/(b*x^2 + c*x^4)^3,x]
```

output
```
(5*b*x + 3*c*x^3)/(8*b^2*(b + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
(8*b^(5/2)*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {9, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{3\int \frac{1}{(cx^2+b)^2}dx}{4b}+\frac{x}{4b\,(b+cx^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\left(\frac{\int \frac{1}{cx^2+b}dx}{2b}+\frac{x}{2b(b+cx^2)}\right)}{4b}+\frac{x}{4b\,(b+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b(b+cx^2)}\right)}{4b}+\frac{x}{4b\,(b+cx^2)^2}$$

input
```
Int[x^6/(b*x^2 + c*x^4)^3,x]
```

output
```
x/(4*b*(b + c*x^2)^2) + (3*(x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[
b]]/(2*b^(3/2)*Sqrt[c])))/(4*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{x}{4b(c\,x^2+b)^2}+\frac{\frac{3x}{8b\left(c\,x^2+b\right)}+\frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8b\sqrt{bc}}}{b}$	57
risch	$\frac{\frac{3c\,x^3}{8b^2}+\frac{5x}{8b}}{(c\,x^2+b)^2}-\frac{3\ln\left(cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\,b^2}+\frac{3\ln\left(-cx+\sqrt{-bc}\right)}{16\sqrt{-bc}\,b^2}$	73

input
```
int(x^6/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/b/(c*x^2+b)^2+3/4/b*(1/2*x/b/(c*x^2+b)+1/2/b/(b*c)^(1/2)*arctan(c*x/
(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.03

$$\int \frac{x^6}{(bx^2 + cx^4)^3}\, dx$$
$$= \left[\frac{6\, bc^2x^3 + 10\, b^2cx - 3\, (c^2x^4 + 2\, bcx^2 + b^2)\sqrt{-bc}\log\left(\frac{cx^2 - 2\,\sqrt{-bc}x - b}{cx^2 + b}\right)}{16\, (b^3c^3x^4 + 2\, b^4c^2x^2 + b^5c)}, \frac{3\, bc^2x^3 + 5\, b^2cx + 3\, (c^2x^4 + 2\, bcx^2}{8\, (b^3c^3x^4 + 2\, b^4c^2x}\right.$$

input
```
integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(6*b*c^2*x^3 + 10*b^2*c*x - 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(-b*c)
*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))/(b^3*c^3*x^4 + 2*b^4*c^2*x
^2 + b^5*c), 1/8*(3*b*c^2*x^3 + 5*b^2*c*x + 3*(c^2*x^4 + 2*b*c*x^2 + b^2)*
sqrt(b*c)*arctan(sqrt(b*c)*x/b))/(b^3*c^3*x^4 + 2*b^4*c^2*x^2 + b^5*c)]
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.69

$$\int \frac{x^6}{(bx^2 + cx^4)^3}\, dx = -\frac{3\sqrt{-\frac{1}{b^5c}}\log\left(-b^3\sqrt{-\frac{1}{b^5c}} + x\right)}{16} + \frac{3\sqrt{-\frac{1}{b^5c}}\log\left(b^3\sqrt{-\frac{1}{b^5c}} + x\right)}{16} + \frac{5bx + 3cx^3}{8b^4 + 16b^3cx^2 + 8b^2c^2x^4}$$

input
```
integrate(x**6/(c*x**4+b*x**2)**3,x)
```

output
```
-3*sqrt(-1/(b**5*c))*log(-b**3*sqrt(-1/(b**5*c)) + x)/16 + 3*sqrt(-1/(b**5
*c))*log(b**3*sqrt(-1/(b**5*c)) + x)/16 + (5*b*x + 3*c*x**3)/(8*b**4 + 16*
b**3*c*x**2 + 8*b**2*c**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{x^6}{(bx^2+cx^4)^3}\,dx = \frac{3\,cx^3+5\,bx}{8\,(b^2c^2x^4+2\,b^3cx^2+b^4)} + \frac{3\,\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^2}$$

input
```
integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/8*(3*c*x^3 + 5*b*x)/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) + 3/8*arctan(c*x/s
qrt(b*c))/(sqrt(b*c)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{(bx^2+cx^4)^3}\,dx = \frac{3\,\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^2} + \frac{3\,cx^3+5\,bx}{8\,(cx^2+b)^2b^2}$$

input
```
integrate(x^6/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
3/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/8*(3*c*x^3 + 5*b*x)/((c*x^2
+ b)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 17.67 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{(bx^2+cx^4)^3}\,dx = \frac{\frac{5\,x}{8\,b}+\frac{3\,c\,x^3}{8\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{5/2}\,\sqrt{c}}$$

input
```
int(x^6/(b*x^2 + c*x^4)^3,x)
```

output
```
((5*x)/(8*b) + (3*c*x^3)/(8*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) + (3*atan((c
^(1/2)*x)/b^(1/2)))/(8*b^(5/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

$$\int \frac{x^6}{(bx^2 + cx^4)^3} \, dx$$
$$= \frac{3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right) b^2 + 6\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right) bc\,x^2 + 3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right) c^2x^4 + 5b^2cx + 3b\,c^2x^3}{8b^3c\,(c^2x^4 + 2bc\,x^2 + b^2)}$$

input
```
int(x^6/(c*x^4+b*x^2)^3,x)
```

output
```
(3*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**2 + 6*sqrt(c)*sqrt(b)*
atan((c*x)/(sqrt(c)*sqrt(b)))*b*c*x**2 + 3*sqrt(c)*sqrt(b)*atan((c*x)/(sqr
t(c)*sqrt(b)))*c**2*x**4 + 5*b**2*c*x + 3*b*c**2*x**3)/(8*b**3*c*(b**2 + 2
*b*c*x**2 + c**2*x**4))
```

3.89 $\int \frac{x^4}{\left(bx^2+cx^4\right)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{x^4}{\left(bx^2+cx^4\right)^3}\,dx=-\frac{1}{b^3x}-\frac{cx}{4b^2\left(b+cx^2\right)^2}-\frac{7cx}{8b^3\left(b+cx^2\right)}-\frac{15\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}}$$

output
```
-1/b^3/x-1/4*c*x/b^2/(c*x^2+b)^2-7/8*c*x/b^3/(c*x^2+b)-15/8*c^(1/2)*arctan
(c^(1/2)*x/b^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{\left(bx^2+cx^4\right)^3}\,dx=-\frac{8b^2+25bcx^2+15c^2x^4}{8b^3x\left(b+cx^2\right)^2}-\frac{15\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}}$$

input
```
Integrate[x^4/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/8*(8*b^2 + 25*b*c*x^2 + 15*c^2*x^4)/(b^3*x*(b + c*x^2)^2) - (15*Sqrt[c]
*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {9, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{x^2(b+cx^2)^3}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5\int \frac{1}{x^2(cx^2+b)^2}dx}{4b}+\frac{1}{4bx(b+cx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{5\left(\frac{3\int \frac{1}{x^2(cx^2+b)}dx}{2b}+\frac{1}{2bx(b+cx^2)}\right)}{4b}+\frac{1}{4bx(b+cx^2)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{5\left(\frac{3\left(-\frac{c\int \frac{1}{cx^2+b}dx}{b}-\frac{1}{bx}\right)}{2b}+\frac{1}{2bx(b+cx^2)}\right)}{4b}+\frac{1}{4bx(b+cx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{5\left(\frac{3\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{2b}+\frac{1}{2bx(b+cx^2)}\right)}{4b}+\frac{1}{4bx(b+cx^2)^2}$$

input
```
Int[x^4/(b*x^2 + c*x^4)^3,x]
```

output
```
1/(4*b*x*(b + c*x^2)^2) + (5*(1/(2*b*x*(b + c*x^2)) + (3*(-(1/(b*x)) - (Sq
rt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{1}{b^{3}x}-\frac{c\left(\frac{\frac{7}{8}c\,x^{3}+\frac{9}{8}bx}{\left(c\,x^{2}+b\right)^{2}}+\frac{15\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^{3}}$	54
risch	$\frac{-\frac{15c^{2}x^{4}}{8b^{3}}-\frac{25c\,x^{2}}{8b^{2}}-\frac{1}{b}}{x\left(c\,x^{2}+b\right)^{2}}+\frac{15\left(\sum_{\_R=\mathrm{RootOf}\left(b^{7}\_Z^{2}+c\right)}\_R\ln\left(\left(3\_R^{2}b^{7}+2c\right)x+b^{4}\_R\right)\right)}{16}$	79

input
```
int(x^4/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^3/x-1/b^3*c*((7/8*c*x^3+9/8*b*x)/(c*x^2+b)^2+15/8/(b*c)^(1/2)*arctan(
c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.81

$$\int \frac{x^4}{(bx^2+cx^4)^3}\,dx = \left[-\frac{30\,c^2x^4+50\,bcx^2-15\,(c^2x^5+2\,bcx^3+b^2x)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+16\,b^2}{16\,(b^3c^2x^5+2\,b^4cx^3+b^5x)},\; -\frac{15\,c^2x^4+25\,bcx^2+15\,(c^2x^5+2\,bcx^3+b^2x)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+8\,b^2}{8\,(b^3c^2x^5+2\,b^4cx^3+b^5x)}\right]$$

input
```
integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[-1/16*(30*c^2*x^4 + 50*b*c*x^2 - 15*(c^2*x^5 + 2*b*c*x^3 + b^2*x)*sqrt(-c
/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) + 16*b^2)/(b^3*c^2*x^5
 + 2*b^4*c*x^3 + b^5*x), -1/8*(15*c^2*x^4 + 25*b*c*x^2 + 15*(c^2*x^5 + 2*b
*c*x^3 + b^2*x)*sqrt(c/b)*arctan(x*sqrt(c/b)) + 8*b^2)/(b^3*c^2*x^5 + 2*b^
4*c*x^3 + b^5*x)]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.61

$$\int \frac{x^4}{(bx^2+cx^4)^3}\,dx = \frac{15\sqrt{-\frac{c}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c}{b^7}}}{c}+x\right)}{16} - \frac{15\sqrt{-\frac{c}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c}{b^7}}}{c}+x\right)}{16} + \frac{-8b^2-25bcx^2-15c^2x^4}{8b^5x+16b^4cx^3+8b^3c^2x^5}$$

input
```
integrate(x**4/(c*x**4+b*x**2)**3,x)
```

output
```
15*sqrt(-c/b**7)*log(-b**4*sqrt(-c/b**7)/c + x)/16 - 15*sqrt(-c/b**7)*log(
b**4*sqrt(-c/b**7)/c + x)/16 + (-8*b**2 - 25*b*c*x**2 - 15*c**2*x**4)/(8*b
**5*x + 16*b**4*c*x**3 + 8*b**3*c**2*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x^4}{(bx^2+cx^4)^3}\,dx = -\frac{15\,c^2x^4+25\,bcx^2+8\,b^2}{8\,(b^3c^2x^5+2\,b^4cx^3+b^5x)} - \frac{15\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^3}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(15*c^2*x^4 + 25*b*c*x^2 + 8*b^2)/(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)
 - 15/8*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{x^4}{(bx^2+cx^4)^3}\,dx = -\frac{15\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^3} - \frac{7\,c^2x^3+9\,bcx}{8\,(cx^2+b)^2b^3} - \frac{1}{b^3x}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-15/8*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/8*(7*c^2*x^3 + 9*b*c*x)/
((c*x^2 + b)^2*b^3) - 1/(b^3*x)
```

Mupad [B] (verification not implemented)

Time = 17.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(bx^2+cx^4)^3}\,dx = -\frac{\frac{1}{b}+\frac{25\,c\,x^2}{8\,b^2}+\frac{15\,c^2\,x^4}{8\,b^3}}{b^2\,x+2\,b\,c\,x^3+c^2\,x^5} - \frac{15\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{7/2}}$$

input
```
int(x^4/(b*x^2 + c*x^4)^3,x)
```

output
```
- (1/b + (25*c*x^2)/(8*b^2) + (15*c^2*x^4)/(8*b^3))/(b^2*x + c^2*x^5 + 2*b
*c*x^3) - (15*c^(1/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*b^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.68

$$\int \frac{x^4}{(bx^2+cx^4)^3}\,dx = \frac{-15\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^2x - 30\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)bc\,x^3 - 15\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)c^2x^5 - 8b^3 - 25b^2c\,x^2}{8b^4x\,(c^2x^4+2bc\,x^2+b^2)}$$

input
```
int(x^4/(c*x^4+b*x^2)^3,x)
```

output
```
( - 15*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**2*x - 30*sqrt(c)*s
qrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b*c*x**3 - 15*sqrt(c)*sqrt(b)*atan((c
*x)/(sqrt(c)*sqrt(b)))*c**2*x**5 - 8*b**3 - 25*b**2*c*x**2 - 15*b*c**2*x**
4)/(8*b**4*x*(b**2 + 2*b*c*x**2 + c**2*x**4))
```

3.90 $\int \frac{x^2}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \frac{x^2}{(bx^2+cx^4)^3}\,dx = -\frac{1}{3b^3x^3}+\frac{3c}{b^4x}+\frac{c^2x}{4b^3(b+cx^2)^2}+\frac{11c^2x}{8b^4(b+cx^2)}+\frac{35c^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}$$

output
```
-1/3/b^3/x^3+3*c/b^4/x+1/4*c^2*x/b^3/(c*x^2+b)^2+11/8*c^2*x/b^4/(c*x^2+b)+
35/8*c^(3/2)*arctan(c^(1/2)*x/b^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{(bx^2+cx^4)^3}\,dx = \frac{-8b^3+56b^2cx^2+175bc^2x^4+105c^3x^6}{24b^4x^3(b+cx^2)^2}+\frac{35c^{3/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}$$

input
```
Integrate[x^2/(b*x^2 + c*x^4)^3,x]
```

output
```
(-8*b^3 + 56*b^2*c*x^2 + 175*b*c^2*x^4 + 105*c^3*x^6)/(24*b^4*x^3*(b + c*x
^2)^2) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {9, 253, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(bx^2+cx^4)^3} \, dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{1}{x^4 (b+cx^2)^3} dx$$

$$\Big\downarrow \ 253$$

$$\frac{7\int \frac{1}{x^4(cx^2+b)^2}dx}{4b} + \frac{1}{4bx^3 (b+cx^2)^2}$$

$$\Big\downarrow \ 253$$

$$\frac{7\left(\frac{5\int \frac{1}{x^4\left(cx^2+b\right)}dx}{2b} + \frac{1}{2bx^3(b+cx^2)}\right)}{4b} + \frac{1}{4bx^3 (b+cx^2)^2}$$

$$\Big\downarrow \ 264$$

$$\frac{7\left(\frac{5\left(-\frac{c\int \frac{1}{x^2\left(cx^2+b\right)}dx}{b} - \frac{1}{3bx^3}\right)}{2b} + \frac{1}{2bx^3(b+cx^2)}\right)}{4b} + \frac{1}{4bx^3 (b+cx^2)^2}$$

$$\Big\downarrow \ 264$$

$$\frac{7\left(\frac{5\left(-\frac{c\left(-\frac{c\int\frac{1}{cx^2+b}dx}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{2b}+\frac{1}{2bx^3(b+cx^2)}\right)}{4b}+\frac{1}{4bx^3\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{7\left(\frac{5\left(-\frac{c\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{2b}+\frac{1}{2bx^3(b+cx^2)}\right)}{4b}+\frac{1}{4bx^3\left(b+cx^2\right)^2}$$

input
```
Int[x^2/(b*x^2 + c*x^4)^3,x]
```

output
```
1/(4*b*x^3*(b + c*x^2)^2) + (7*(1/(2*b*x^3*(b + c*x^2)) + (5*(-1/3*1/(b*x^
3) - (c*(-(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)))/b))/
(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{1}{3b^{3}x^{3}}+\frac{3c}{b^{4}x}+\frac{c^{2}\left(\frac{\frac{11}{8}c\,x^{3}+\frac{13}{8}bx}{\left(c\,x^{2}+b\right)^{2}}+\frac{35\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^{4}}$	64
risch	$\frac{\frac{35c^{3}x^{6}}{8b^{4}}+\frac{175c^{2}x^{4}}{24b^{3}}+\frac{7c\,x^{2}}{3b^{2}}-\frac{1}{3b}}{\left(c\,x^{2}+b\right)^{2}x^{3}}+\frac{35\left(\sum_{\_R=\operatorname{RootOf}\left(b^{9}\_Z^{2}+c^{3}\right)}\_R\ln\left(\left(3\_R^{2}b^{9}+2c^{3}\right)x-b^{5}c\_R\right)\right)}{16}$	96

input
```
int(x^2/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^3/x^3+3*c/b^4/x+1/b^4*c^2*((11/8*c*x^3+13/8*b*x)/(c*x^2+b)^2+35/8/(
b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.74

$$\int \frac{x^2}{(bx^2+cx^4)^3}\,dx = \left[\frac{210\,c^3x^6+350\,bc^2x^4+112\,b^2cx^2-16\,b^3+105\,(c^3x^7+2\,bc^2x^5+b^2cx^3)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{48\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)}, \frac{105\right.$$

input
```
integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[1/48*(210*c^3*x^6 + 350*b*c^2*x^4 + 112*b^2*c*x^2 - 16*b^3 + 105*(c^3*x^7
 + 2*b*c^2*x^5 + b^2*c*x^3)*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/
(c*x^2 + b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3), 1/24*(105*c^3*x^6 + 1
75*b*c^2*x^4 + 56*b^2*c*x^2 - 8*b^3 + 105*(c^3*x^7 + 2*b*c^2*x^5 + b^2*c*x
^3)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.59

$$\int \frac{x^2}{(bx^2+cx^4)^3}\,dx = -\frac{35\sqrt{-\frac{c^3}{b^9}}\log\left(-\frac{b^5\sqrt{-\frac{c^3}{b^9}}}{c^2}+x\right)}{16} + \frac{35\sqrt{-\frac{c^3}{b^9}}\log\left(\frac{b^5\sqrt{-\frac{c^3}{b^9}}}{c^2}+x\right)}{16} + \frac{-8b^3+56b^2cx^2+175bc^2x^4+105c^3x^6}{24b^6x^3+48b^5cx^5+24b^4c^2x^7}$$

input
```
integrate(x**2/(c*x**4+b*x**2)**3,x)
```

output
```
-35*sqrt(-c**3/b**9)*log(-b**5*sqrt(-c**3/b**9)/c**2 + x)/16 + 35*sqrt(-c*
*3/b**9)*log(b**5*sqrt(-c**3/b**9)/c**2 + x)/16 + (-8*b**3 + 56*b**2*c*x**
2 + 175*b*c**2*x**4 + 105*c**3*x**6)/(24*b**6*x**3 + 48*b**5*c*x**5 + 24*b
**4*c**2*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{(bx^2+cx^4)^3}\,dx = \frac{105\,c^3x^6+175\,bc^2x^4+56\,b^2cx^2-8\,b^3}{24\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)} + \frac{35\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^4}$$

input
```
integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/24*(105*c^3*x^6 + 175*b*c^2*x^4 + 56*b^2*c*x^2 - 8*b^3)/(b^4*c^2*x^7 + 2
*b^5*c*x^5 + b^6*x^3) + 35/8*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(bx^2+cx^4)^3}\,dx = \frac{35\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^4} + \frac{11\,c^3x^3+13\,bc^2x}{8\,(cx^2+b)^2b^4} + \frac{9\,cx^2-b}{3\,b^4x^3}$$

input
```
integrate(x^2/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
35/8*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) + 1/8*(11*c^3*x^3 + 13*b*c^
2*x)/((c*x^2 + b)^2*b^4) + 1/3*(9*c*x^2 - b)/(b^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 17.79 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(bx^2+cx^4)^3}\,dx = \frac{\frac{7\,c\,x^2}{3\,b^2}-\frac{1}{3\,b}+\frac{175\,c^2\,x^4}{24\,b^3}+\frac{35\,c^3\,x^6}{8\,b^4}}{b^2\,x^3+2\,b\,c\,x^5+c^2\,x^7} + \frac{35\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{9/2}}$$

input
```
int(x^2/(b*x^2 + c*x^4)^3,x)
```

output
```
((7*c*x^2)/(3*b^2) - 1/(3*b) + (175*c^2*x^4)/(24*b^3) + (35*c^3*x^6)/(8*b^
4))/(b^2*x^3 + c^2*x^7 + 2*b*c*x^5) + (35*c^(3/2)*atan((c^(1/2)*x)/b^(1/2)
))/(8*b^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \frac{x^2}{(bx^2 + cx^4)^3} \, dx$$
$$= \frac{105\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right) b^2 c\, x^3 + 210\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right) b\, c^2 x^5 + 105\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right) c^3 x^7 - 8b^4 + 56b}{24b^5 x^3\,(c^2 x^4 + 2bc\, x^2 + b^2)}$$

input
```
int(x^2/(c*x^4+b*x^2)^3,x)
```

output
```
(105*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**2*c*x**3 + 210*sqrt(
c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b*c**2*x**5 + 105*sqrt(c)*sqrt(b)
*atan((c*x)/(sqrt(c)*sqrt(b)))*c**3*x**7 - 8*b**4 + 56*b**3*c*x**2 + 175*b
**2*c**2*x**4 + 105*b*c**3*x**6)/(24*b**5*x**3*(b**2 + 2*b*c*x**2 + c**2*x
**4))
```

3.91 $\int \frac{1}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{1}{(bx^2+cx^4)^3}\,dx = -\frac{1}{5b^3x^5}+\frac{c}{b^4x^3}-\frac{6c^2}{b^5x}-\frac{c^3x}{4b^4(b+cx^2)^2}-\frac{15c^3x}{8b^5(b+cx^2)}-\frac{63c^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}$$

output
```
-1/5/b^3/x^5+c/b^4/x^3-6*c^2/b^5/x-1/4*c^3*x/b^4/(c*x^2+b)^2-15/8*c^3*x/b^
5/(c*x^2+b)-63/8*c^(5/2)*arctan(c^(1/2)*x/b^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{1}{(bx^2+cx^4)^3}\,dx = -\frac{8b^4-24b^3cx^2+168b^2c^2x^4+525bc^3x^6+315c^4x^8}{40b^5x^5(b+cx^2)^2}-\frac{63c^{5/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(-3),x]
```

output
```
-1/40*(8*b^4 - 24*b^3*c*x^2 + 168*b^2*c^2*x^4 + 525*b*c^3*x^6 + 315*c^4*x^
8)/(b^5*x^5*(b + c*x^2)^2) - (63*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b
^(11/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {1397, 253, 253, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{1397}$$

$$\int \frac{1}{x^6(b+cx^2)^3}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{9\int \frac{1}{x^6(cx^2+b)^2}dx}{4b}+\frac{1}{4bx^5(b+cx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{9\left(\frac{7\int \frac{1}{x^6(cx^2+b)}dx}{2b}+\frac{1}{2bx^5(b+cx^2)}\right)}{4b}+\frac{1}{4bx^5(b+cx^2)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{9\left(\frac{7\left(-\frac{c\int \frac{1}{x^4(cx^2+b)}dx}{b}-\frac{1}{5bx^5}\right)}{2b}+\frac{1}{2bx^5(b+cx^2)}\right)}{4b}+\frac{1}{4bx^5(b+cx^2)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{9\left(\frac{7\left(-\frac{c\left(-\frac{c\int \frac{1}{x^2\left(cx^2+b\right)}dx}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{1}{5bx^5}\right)}{2b}+\frac{1}{2bx^5(b+cx^2)}\right)}{4b}+\frac{1}{4bx^5\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 264

$$\frac{9\left(\frac{7\left(-\frac{c\left(-\frac{c\left(-\frac{c\int \frac{1}{cx^2+b}dx}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{1}{5bx^5}\right)}{2b}+\frac{1}{2bx^5(b+cx^2)}\right)}{4b}+\frac{1}{4bx^5\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 218

$$\frac{9\left(\frac{7\left(-\frac{c\left(-\frac{c\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{1}{5bx^5}\right)}{2b}+\frac{1}{2bx^5(b+cx^2)}\right)}{4b}+\frac{1}{4bx^5\left(b+cx^2\right)^2}$$

input
```
Int[(b*x^2 + c*x^4)^(-3),x]
```

output
```
1/(4*b*x^5*(b + c*x^2)^2) + (9*(1/(2*b*x^5*(b + c*x^2)) + (7*(-1/5*1/(b*x^
5) - (c*(-1/3*1/(b*x^3) - (c*(-(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqr
t[b]])/b^(3/2)))/b))/b))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1397
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[x^(2*p)*(b + c*x^2
)^p, x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1}{5b^3x^5}+\frac{c}{b^4x^3}-\frac{6c^2}{b^5x}-\frac{c^3\left(\frac{\frac{15}{8}c\,x^3+\frac{17}{8}bx}{\left(c\,x^2+b\right)^2}+\frac{63\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^5}$	75
risch	$\frac{-\frac{63c^4x^8}{8b^5}-\frac{105c^3x^6}{8b^4}-\frac{21c^2x^4}{5b^3}+\frac{3c\,x^2}{5b^2}-\frac{1}{5b}}{x^5\left(c\,x^2+b\right)^2}+\frac{63\left(\sum_{\_R=\operatorname{RootOf}\left(b^{11}\_Z^2+c^5\right)}\_R\ln\left(\left(3\_R^2b^{11}+2c^5\right)x+b^6c^2\_R\right)\right)}{16}$	108

input
```
int(1/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5/b^3/x^5+c/b^4/x^3-6*c^2/b^5/x-1/b^5*c^3*((15/8*c*x^3+17/8*b*x)/(c*x^2
+b)^2+63/8/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.72

$$\int \frac{1}{(bx^2+cx^4)^3}\,dx$$
$$= \left[-\frac{630\,c^4x^8+1050\,bc^3x^6+336\,b^2c^2x^4-48\,b^3cx^2+16\,b^4-315\,(c^4x^9+2\,bc^3x^7+b^2c^2x^5)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-\cdots}{\cdots}\right.}{80\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)} \right.$$

$$-\frac{315\,c^4x^8+525\,bc^3x^6+168\,b^2c^2x^4-24\,b^3cx^2+8\,b^4+315\,(c^4x^9+2\,bc^3x^7+b^2c^2x^5)\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)}{40\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)}$$

input
```
integrate(1/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[-1/80*(630*c^4*x^8 + 1050*b*c^3*x^6 + 336*b^2*c^2*x^4 - 48*b^3*c*x^2 + 16
*b^4 - 315*(c^4*x^9 + 2*b*c^3*x^7 + b^2*c^2*x^5)*sqrt(-c/b)*log((c*x^2 - 2
*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5),
-1/40*(315*c^4*x^8 + 525*b*c^3*x^6 + 168*b^2*c^2*x^4 - 24*b^3*c*x^2 + 8*b^
4 + 315*(c^4*x^9 + 2*b*c^3*x^7 + b^2*c^2*x^5)*sqrt(c/b)*arctan(x*sqrt(c/b)
))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)]
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

$$\int \frac{1}{(bx^2+cx^4)^3}\,dx = \frac{63\sqrt{-\frac{c^5}{b^{11}}}\log\left(-\frac{b^6\sqrt{-\frac{c^5}{b^{11}}}}{c^3}+x\right)}{16} - \frac{63\sqrt{-\frac{c^5}{b^{11}}}\log\left(\frac{b^6\sqrt{-\frac{c^5}{b^{11}}}}{c^3}+x\right)}{16} + \frac{-8b^4+24b^3cx^2-168b^2c^2x^4-525bc^3x^6-315c^4x^8}{40b^7x^5+80b^6cx^7+40b^5c^2x^9}$$

input
```
integrate(1/(c*x**4+b*x**2)**3,x)
```

output
```
63*sqrt(-c**5/b**11)*log(-b**6*sqrt(-c**5/b**11)/c**3 + x)/16 - 63*sqrt(-c
**5/b**11)*log(b**6*sqrt(-c**5/b**11)/c**3 + x)/16 + (-8*b**4 + 24*b**3*c*
x**2 - 168*b**2*c**2*x**4 - 525*b*c**3*x**6 - 315*c**4*x**8)/(40*b**7*x**5
 + 80*b**6*c*x**7 + 40*b**5*c**2*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bx^2+cx^4)^3}\,dx = -\frac{315\,c^4x^8+525\,bc^3x^6+168\,b^2c^2x^4-24\,b^3cx^2+8\,b^4}{40\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)} - \frac{63\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^5}$$

input
```
integrate(1/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/40*(315*c^4*x^8 + 525*b*c^3*x^6 + 168*b^2*c^2*x^4 - 24*b^3*c*x^2 + 8*b^
4)/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5) - 63/8*c^3*arctan(c*x/sqrt(b*c))/
(sqrt(b*c)*b^5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{1}{(bx^2+cx^4)^3}\,dx = -\frac{63\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^5} - \frac{15\,c^4x^3+17\,bc^3x}{8\,(cx^2+b)^2b^5} - \frac{30\,c^2x^4-5\,bcx^2+b^2}{5\,b^5x^5}$$

input
```
integrate(1/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-63/8*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^5) - 1/8*(15*c^4*x^3 + 17*b*c
^3*x)/((c*x^2 + b)^2*b^5) - 1/5*(30*c^2*x^4 - 5*b*c*x^2 + b^2)/(b^5*x^5)
```

Mupad [B] (verification not implemented)

Time = 17.69 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(bx^2+cx^4\right)^3}\,dx=-\frac{\frac{1}{5\,b}-\frac{3\,c\,x^2}{5\,b^2}+\frac{21\,c^2\,x^4}{5\,b^3}+\frac{105\,c^3\,x^6}{8\,b^4}+\frac{63\,c^4\,x^8}{8\,b^5}}{b^2\,x^5+2\,b\,c\,x^7+c^2\,x^9}-\frac{63\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{11/2}}$$

input
```
int(1/(b*x^2 + c*x^4)^3,x)
```

output
```
- (1/(5*b) - (3*c*x^2)/(5*b^2) + (21*c^2*x^4)/(5*b^3) + (105*c^3*x^6)/(8*b
^4) + (63*c^4*x^8)/(8*b^5))/(b^2*x^5 + c^2*x^9 + 2*b*c*x^7) - (63*c^(5/2)*
atan((c^(1/2)*x)/b^(1/2)))/(8*b^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

$$\int \frac{1}{\left(bx^2+cx^4\right)^3}\,dx$$
$$=\frac{-315\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^2c^2x^5-630\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b\,c^3x^7-315\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)c^4x^9-8b^5+2}{40b^6x^5\left(c^2x^4+2bc\,x^2+b^2\right)}$$

input
```
int(1/(c*x^4+b*x^2)^3,x)
```

output
```
( - 315*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**2*c**2*x**5 - 630
*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b*c**3*x**7 - 315*sqrt(c)*s
qrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*c**4*x**9 - 8*b**5 + 24*b**4*c*x**2 -
 168*b**3*c**2*x**4 - 525*b**2*c**3*x**6 - 315*b*c**4*x**8)/(40*b**6*x**5*
(b**2 + 2*b*c*x**2 + c**2*x**4))
```

3.92 $\int \frac{1}{x^2(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 111

$$\int \frac{1}{x^2\left(bx^2+cx^4\right)^3}\,dx=-\frac{1}{7b^3x^7}+\frac{3c}{5b^4x^5}-\frac{2c^2}{b^5x^3}+\frac{10c^3}{b^6x}+\frac{c^4x}{4b^5\left(b+cx^2\right)^2}+\frac{19c^4x}{8b^6\left(b+cx^2\right)}+\frac{99c^{7/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{13/2}}$$

output
```
-1/7/b^3/x^7+3/5*c/b^4/x^5-2*c^2/b^5/x^3+10*c^3/b^6/x+1/4*c^4*x/b^5/(c*x^2
+b)^2+19/8*c^4*x/b^6/(c*x^2+b)+99/8*c^(7/2)*arctan(c^(1/2)*x/b^(1/2))/b^(1
3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(bx^2+cx^4\right)^3}\,dx=\frac{-40b^5+88b^4cx^2-264b^3c^2x^4+1848b^2c^3x^6+5775bc^4x^8+3465c^5x^{10}}{280b^6x^7\left(b+cx^2\right)^2}+\frac{99c^{7/2}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{13/2}}$$

input
```
Integrate[1/(x^2*(b*x^2 + c*x^4)^3),x]
```

output
```
(-40*b^5 + 88*b^4*c*x^2 - 264*b^3*c^2*x^4 + 1848*b^2*c^3*x^6 + 5775*b*c^4*
x^8 + 3465*c^5*x^10)/(280*b^6*x^7*(b + c*x^2)^2) + (99*c^(7/2)*ArcTan[(Sqr
t[c]*x)/Sqrt[b]])/(8*b^(13/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {9, 253, 253, 264, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(bx^2+cx^4\right)^3} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^8 \left(b+cx^2\right)^3}dx$$

$$\Big\downarrow \; 253$$

$$\frac{11 \int \frac{1}{x^8 (cx^2+b)^2}dx}{4b}+\frac{1}{4bx^7 \left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 253$$

$$\frac{11 \left(\frac{9 \int \frac{1}{x^8 \left(cx^2+b\right)}dx}{2b}+\frac{1}{2bx^7 (b+cx^2)}\right)}{4b}+\frac{1}{4bx^7 \left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 264$$

$$\frac{11 \left(\frac{9 \left(-\frac{c \int \frac{1}{x^6 \left(cx^2+b\right)}dx}{b}-\frac{1}{7bx^7}\right)}{2b}+\frac{1}{2bx^7 (b+cx^2)}\right)}{4b}+\frac{1}{4bx^7 \left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{11\left(\frac{9\left(-\frac{c\left(-\frac{c\int \frac{1}{x^4\left(cx^2+b\right)}dx}{b}-\frac{1}{5bx^5}\right)}{b}-\frac{1}{7bx^7}\right)}{2b}+\frac{1}{2bx^7(b+cx^2)}\right)}{4b}+\frac{1}{4bx^7\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{11\left(\frac{9\left(-\frac{c\left(-\frac{c\left(-\frac{c\int \frac{1}{x^2\left(cx^2+b\right)}dx}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{1}{5bx^5}\right)}{b}-\frac{1}{7bx^7}\right)}{2b}+\frac{1}{2bx^7(b+cx^2)}\right)}{4b}+\frac{1}{4bx^7\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{11\left(\frac{9\left(-\frac{c\left(-\frac{c\left(-\frac{c\left(-\frac{c\int\frac{1}{cx^2+b}dx}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{1}{5bx^5}\right)}{b}-\frac{1}{7bx^7}\right)}{2b}+\frac{1}{2bx^7(b+cx^2)}\right)}{4b}+\frac{1}{4bx^7\left(b+cx^2\right)^2}$$

$\downarrow$ 218

$$\frac{11\left(\frac{9\left(-\frac{c\left(-\frac{c\left(-\frac{c\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{b}-\frac{1}{5bx^5}\right)}{b}-\frac{1}{7bx^7}\right)}{2b}+\frac{1}{2bx^7(b+cx^2)}\right)}{4b}+\frac{1}{4bx^7\left(b+cx^2\right)^2}$$

input
```
Int[1/(x^2*(b*x^2 + c*x^4)^3),x]
```

output
```
1/(4*b*x^7*(b + c*x^2)^2) + (11*(1/(2*b*x^7*(b + c*x^2)) + (9*(-1/7*1/(b*x
^7) - (c*(-1/5*1/(b*x^5) - (c*(-1/3*1/(b*x^3) - (c*(-(1/(b*x)) - (Sqrt[c]*
ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)))/b))/b))/b))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px, 
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p, 
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] && 
 !MonomialQ[Px, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

method	result	s
default	$-\frac{1}{7b^3x^7}+\frac{10c^3}{b^6x}-\frac{2c^2}{b^5x^3}+\frac{3c}{5b^4x^5}+\frac{c^4\left(\frac{\frac{19}{8}c\,x^3+\frac{21}{8}bx}{\left(c\,x^2+b\right)^2}+\frac{99\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^6}$	8
risch	$\frac{\frac{99c^5x^{10}}{8b^6}+\frac{165c^4x^8}{8b^5}+\frac{33c^3x^6}{5b^4}-\frac{33c^2x^4}{35b^3}+\frac{11c\,x^2}{35b^2}-\frac{1}{7b}}{x^7\left(c\,x^2+b\right)^2}+\frac{99\left(\sum_{\_R=\mathrm{RootOf}\left(b^{13}\_Z^2+c^7\right)}\_R\ln\left(\left(3\_R^2b^{13}+2c^7\right)x-b^7c^3\_R\right)\right)}{16}$	1

input
```
int(1/x^2/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/7/b^3/x^7+10*c^3/b^6/x-2*c^2/b^5/x^3+3/5*c/b^4/x^5+1/b^6*c^4*((19/8*c*x
^3+21/8*b*x)/(c*x^2+b)^2+99/8/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.58

$$\int \frac{1}{x^2 (bx^2+cx^4)^3}\,dx$$
$$= \left[\frac{6930\,c^5x^{10} + 11550\,bc^4x^8 + 3696\,b^2c^3x^6 - 528\,b^3c^2x^4 + 176\,b^4cx^2 - 80\,b^5 + 3465\,(c^5x^{11} + 2\,bc^4x^9 + b^2c^{3}\cdots}{560\,(b^6c^2x^{11} + 2\,b^7cx^9 + b^8x^7)}\right.$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
[1/560*(6930*c^5*x^10 + 11550*b*c^4*x^8 + 3696*b^2*c^3*x^6 - 528*b^3*c^2*x
^4 + 176*b^4*c*x^2 - 80*b^5 + 3465*(c^5*x^11 + 2*b*c^4*x^9 + b^2*c^3*x^7)*
sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^6*c^2*x^11
+ 2*b^7*c*x^9 + b^8*x^7), 1/280*(3465*c^5*x^10 + 5775*b*c^4*x^8 + 1848*b^2
*c^3*x^6 - 264*b^3*c^2*x^4 + 88*b^4*c*x^2 - 40*b^5 + 3465*(c^5*x^11 + 2*b*
c^4*x^9 + b^2*c^3*x^7)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^6*c^2*x^11 + 2*b^
7*c*x^9 + b^8*x^7)]
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^2 (bx^2+cx^4)^3}\,dx$$
$$= -\frac{99\sqrt{-\frac{c^7}{b^{13}}}\log\left(-\frac{b^7\sqrt{-\frac{c^7}{b^{13}}}}{c^4}+x\right)}{16} + \frac{99\sqrt{-\frac{c^7}{b^{13}}}\log\left(\frac{b^7\sqrt{-\frac{c^7}{b^{13}}}}{c^4}+x\right)}{16} + \frac{-40b^5+88b^4cx^2-264b^3c^2x^4+1848b^2c^3x^6+5775bc^4x^8+3465c^5x^{10}}{280b^8x^7+560b^7cx^9+280b^6c^2x^{11}}$$

input
```
integrate(1/x**2/(c*x**4+b*x**2)**3,x)
```

output
```
-99*sqrt(-c**7/b**13)*log(-b**7*sqrt(-c**7/b**13)/c**4 + x)/16 + 99*sqrt(-
c**7/b**13)*log(b**7*sqrt(-c**7/b**13)/c**4 + x)/16 + (-40*b**5 + 88*b**4*
c*x**2 - 264*b**3*c**2*x**4 + 1848*b**2*c**3*x**6 + 5775*b*c**4*x**8 + 346
5*c**5*x**10)/(280*b**8*x**7 + 560*b**7*c*x**9 + 280*b**6*c**2*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2 (bx^2 + cx^4)^3} \, dx = \frac{3465 \, c^5 x^{10} + 5775 \, bc^4 x^8 + 1848 \, b^2 c^3 x^6 - 264 \, b^3 c^2 x^4 + 88 \, b^4 c x^2 - 40 \, b^5}{280 \, (b^6 c^2 x^{11} + 2 \, b^7 c x^9 + b^8 x^7)} + \frac{99 \, c^4 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8 \, \sqrt{bc} b^6}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/280*(3465*c^5*x^10 + 5775*b*c^4*x^8 + 1848*b^2*c^3*x^6 - 264*b^3*c^2*x^4
 + 88*b^4*c*x^2 - 40*b^5)/(b^6*c^2*x^11 + 2*b^7*c*x^9 + b^8*x^7) + 99/8*c^
4*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2 (bx^2 + cx^4)^3} \, dx = \frac{99 \, c^4 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8 \, \sqrt{bc} b^6} + \frac{19 \, c^5 x^3 + 21 \, bc^4 x}{8 \, (cx^2 + b)^2 b^6} + \frac{350 \, c^3 x^6 - 70 \, bc^2 x^4 + 21 \, b^2 c x^2 - 5 \, b^3}{35 \, b^6 x^7}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
99/8*c^4*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^6) + 1/8*(19*c^5*x^3 + 21*b*c^
4*x)/((c*x^2 + b)^2*b^6) + 1/35*(350*c^3*x^6 - 70*b*c^2*x^4 + 21*b^2*c*x^2
 - 5*b^3)/(b^6*x^7)
```

Mupad [B] (verification not implemented)

Time = 17.87 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\left(bx^2+cx^4\right)^3}\,dx = \frac{\frac{11\,c\,x^2}{35\,b^2}-\frac{1}{7\,b}-\frac{33\,c^2\,x^4}{35\,b^3}+\frac{33\,c^3\,x^6}{5\,b^4}+\frac{165\,c^4\,x^8}{8\,b^5}+\frac{99\,c^5\,x^{10}}{8\,b^6}}{b^2\,x^7+2\,b\,c\,x^9+c^2\,x^{11}} + \frac{99\,c^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{8\,b^{13/2}}$$

input
```
int(1/(x^2*(b*x^2 + c*x^4)^3),x)
```

output
```
((11*c*x^2)/(35*b^2) - 1/(7*b) - (33*c^2*x^4)/(35*b^3) + (33*c^3*x^6)/(5*b
^4) + (165*c^4*x^8)/(8*b^5) + (99*c^5*x^10)/(8*b^6))/(b^2*x^7 + c^2*x^11 +
 2*b*c*x^9) + (99*c^(7/2)*atan((c^(1/2)*x)/b^(1/2)))/(8*b^(13/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^2\left(bx^2+cx^4\right)^3}\,dx = \frac{3465\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b^2c^3x^7+6930\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)b\,c^4x^9+3465\sqrt{c}\,\sqrt{b}\,atan\left(\frac{cx}{\sqrt{c}\,\sqrt{b}}\right)c^5x^{11}-40b^6}{280b^7x^7\left(c^2x^4+2bc\,x^2+b^2\right)}$$

input
```
int(1/x^2/(c*x^4+b*x^2)^3,x)
```

output

```
(3465*sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b**2*c**3*x**7 + 6930*
sqrt(c)*sqrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*b*c**4*x**9 + 3465*sqrt(c)*s
qrt(b)*atan((c*x)/(sqrt(c)*sqrt(b)))*c**5*x**11 - 40*b**6 + 88*b**5*c*x**2
 - 264*b**4*c**2*x**4 + 1848*b**3*c**3*x**6 + 5775*b**2*c**4*x**8 + 3465*b
*c**5*x**10)/(280*b**7*x**7*(b**2 + 2*b*c*x**2 + c**2*x**4))
```

3.93 $\int x^{7/2}(bx^2+cx^4)\ dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int x^{7/2}(bx^2+cx^4)\ dx=\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

output
```
2/13*b*x^(13/2)+2/17*c*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{7/2}(bx^2+cx^4)\ dx=\frac{2}{221}\left(17bx^{13/2}+13cx^{17/2}\right)$$

input
```
Integrate[x^(7/2)*(b*x^2 + c*x^4),x]
```

output
```
(2*(17*b*x^(13/2) + 13*c*x^(17/2)))/221
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(bx^2+cx^4\right)\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^{11/2}\left(b+cx^2\right)dx$$
$$\Big\downarrow \text{ 244}$$
$$\int \left(bx^{11/2}+cx^{15/2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

input
```
Int[x^(7/2)*(b*x^2 + c*x^4),x]
```

output
```
(2*b*x^(13/2))/13 + (2*c*x^(17/2))/17
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{13}{2}}}{13}+\frac{2c\,x^{\frac{17}{2}}}{17}$	14
default	$\frac{2b\,x^{\frac{13}{2}}}{13}+\frac{2c\,x^{\frac{17}{2}}}{17}$	14
gosper	$\frac{2x^{\frac{13}{2}}\left(13c\,x^{2}+17b\right)}{221}$	16
trager	$\frac{2x^{\frac{13}{2}}\left(13c\,x^{2}+17b\right)}{221}$	16
risch	$\frac{2x^{\frac{13}{2}}\left(13c\,x^{2}+17b\right)}{221}$	16
orering	$\frac{2\left(13c\,x^{2}+17b\right)x^{\frac{9}{2}}\left(c\,x^{4}+b\,x^{2}\right)}{221\left(c\,x^{2}+b\right)}$	36

input
```
int(x^(7/2)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/13*b*x^(13/2)+2/17*c*x^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^{7/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{221}\left(13\,cx^8+17\,bx^6\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
2/221*(13*c*x^8 + 17*b*x^6)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{7/2}\left(bx^2+cx^4\right)\,dx=\frac{2bx^{\frac{13}{2}}}{13}+\frac{2cx^{\frac{17}{2}}}{17}$$

input
```
integrate(x**(7/2)*(c*x**4+b*x**2),x)
```

output
```
2*b*x**(13/2)/13 + 2*c*x**(17/2)/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{7/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{17}\,cx^{\frac{17}{2}}+\frac{2}{13}\,bx^{\frac{13}{2}}$$

input
```
integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
2/17*c*x^(17/2) + 2/13*b*x^(13/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{7/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{17}\,cx^{\frac{17}{2}}+\frac{2}{13}\,bx^{\frac{13}{2}}$$

input
```
integrate(x^(7/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
2/17*c*x^(17/2) + 2/13*b*x^(13/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x^{7/2}\left(bx^2+cx^4\right)\,dx=\frac{2\,x^{13/2}\,\left(13\,c\,x^2+17\,b\right)}{221}$$

input
```
int(x^(7/2)*(b*x^2 + c*x^4),x)
```

output
```
(2*x^(13/2)*(17*b + 13*c*x^2))/221
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{7/2}\left(bx^2+cx^4\right)\,dx=\frac{2\sqrt{x}\,x^6(13c\,x^2+17b)}{221}$$

input
```
int(x^(7/2)*(c*x^4+b*x^2),x)
```

output
```
(2*sqrt(x)*x**6*(17*b + 13*c*x**2))/221
```

3.94 $\int x^{5/2}(bx^2+cx^4)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int x^{5/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

output
```
2/11*b*x^(11/2)+2/15*c*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{5/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{165}\left(15bx^{11/2}+11cx^{15/2}\right)$$

input
```
Integrate[x^(5/2)*(b*x^2 + c*x^4),x]
```

output
```
(2*(15*b*x^(11/2) + 11*c*x^(15/2)))/165
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2} \left(bx^2+cx^4\right) \, dx$$
$$\Big\downarrow \; 9$$
$$\int x^{9/2} \left(b+cx^2\right) dx$$
$$\Big\downarrow \; 244$$
$$\int \left(bx^{9/2}+cx^{13/2}\right) dx$$
$$\Big\downarrow \; 2009$$
$$\frac{2}{11} b x^{11/2}+\frac{2}{15} c x^{15/2}$$

input
```
Int[x^(5/2)*(b*x^2 + c*x^4),x]
```

output
```
(2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{11}{2}}}{11}+\frac{2c\,x^{\frac{15}{2}}}{15}$	14
default	$\frac{2b\,x^{\frac{11}{2}}}{11}+\frac{2c\,x^{\frac{15}{2}}}{15}$	14
gosper	$\frac{2x^{\frac{11}{2}}\left(11c\,x^{2}+15b\right)}{165}$	16
trager	$\frac{2x^{\frac{11}{2}}\left(11c\,x^{2}+15b\right)}{165}$	16
risch	$\frac{2x^{\frac{11}{2}}\left(11c\,x^{2}+15b\right)}{165}$	16
orering	$\frac{2\left(11c\,x^{2}+15b\right)x^{\frac{7}{2}}\left(c\,x^{4}+b\,x^{2}\right)}{165\left(c\,x^{2}+b\right)}$	36

input
```
int(x^(5/2)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/11*b*x^(11/2)+2/15*c*x^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^{5/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{165}\left(11\,cx^7+15\,bx^5\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
2/165*(11*c*x^7 + 15*b*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{5/2}\left(bx^2+cx^4\right)\,dx=\frac{2bx^{\frac{11}{2}}}{11}+\frac{2cx^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(5/2)*(c*x**4+b*x**2),x)
```

output
```
2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
2/15*c*x^(15/2) + 2/11*b*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
2/15*c*x^(15/2) + 2/11*b*x^(11/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x^{5/2}\left(bx^2+cx^4\right)\,dx=\frac{2\,x^{11/2}\,\left(11\,c\,x^2+15\,b\right)}{165}$$

input
```
int(x^(5/2)*(b*x^2 + c*x^4),x)
```

output
```
(2*x^(11/2)*(15*b + 11*c*x^2))/165
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(bx^2+cx^4\right)\,dx=\frac{2\sqrt{x}\,x^5(11c\,x^2+15b)}{165}$$

input
```
int(x^(5/2)*(c*x^4+b*x^2),x)
```

output
```
(2*sqrt(x)*x**5*(15*b + 11*c*x**2))/165
```

3.95 $\int x^{3/2}(bx^2+cx^4)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int x^{3/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

output
```
2/9*b*x^(9/2)+2/13*c*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{3/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{117}\left(13bx^{9/2}+9cx^{13/2}\right)$$

input
```
Integrate[x^(3/2)*(b*x^2 + c*x^4),x]
```

output
```
(2*(13*b*x^(9/2) + 9*c*x^(13/2)))/117
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(bx^2+cx^4\right) dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^{7/2}\left(b+cx^2\right)dx$$
$$\Big\downarrow \text{ 244}$$
$$\int \left(bx^{7/2}+cx^{11/2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

input
```
Int[x^(3/2)*(b*x^2 + c*x^4),x]
```

output
```
(2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2 b \,x^{\frac{9}{2}}}{9}+\frac{2 c \,x^{\frac{13}{2}}}{13}$	14
default	$\frac{2 b \,x^{\frac{9}{2}}}{9}+\frac{2 c \,x^{\frac{13}{2}}}{13}$	14
gosper	$\frac{2 x^{\frac{9}{2}} \left(9 c \,x^{2}+13 b \right)}{117}$	16
trager	$\frac{2 x^{\frac{9}{2}} \left(9 c \,x^{2}+13 b \right)}{117}$	16
risch	$\frac{2 x^{\frac{9}{2}} \left(9 c \,x^{2}+13 b \right)}{117}$	16
orering	$\frac{2 \left(9 c \,x^{2}+13 b \right) x^{\frac{5}{2}} \left(c \,x^{4}+b \,x^{2}\right)}{117 \left(c \,x^{2}+b \right)}$	36

input
```
int(x^(3/2)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/9*b*x^(9/2)+2/13*c*x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^{3/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{117}\left(9\,cx^6+13\,bx^4\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
2/117*(9*c*x^6 + 13*b*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{3/2}\left(bx^2+cx^4\right)\,dx=\frac{2bx^{\frac{9}{2}}}{9}+\frac{2cx^{\frac{13}{2}}}{13}$$

input
```
integrate(x**(3/2)*(c*x**4+b*x**2),x)
```

output
```
2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{3/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
2/13*c*x^(13/2) + 2/9*b*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{3/2}\left(bx^2+cx^4\right)\,dx=\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
2/13*c*x^(13/2) + 2/9*b*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x^{3/2}\left(bx^2+cx^4\right)\,dx=\frac{2\,x^{9/2}\,\left(9\,c\,x^2+13\,b\right)}{117}$$

input
```
int(x^(3/2)*(b*x^2 + c*x^4),x)
```

output
```
(2*x^(9/2)*(13*b + 9*c*x^2))/117
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(bx^2+cx^4\right)\,dx=\frac{2\sqrt{x}\,x^4(9c\,x^2+13b)}{117}$$

input
```
int(x^(3/2)*(c*x^4+b*x^2),x)
```

output
```
(2*sqrt(x)*x**4*(13*b + 9*c*x**2))/117
```

3.96 $\int \sqrt{x}(bx^2+cx^4)\ dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \sqrt{x}\left(bx^2+cx^4\right)\ dx=\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

output
```
2/7*b*x^(7/2)+2/11*c*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sqrt{x}\left(bx^2+cx^4\right)\ dx=\frac{2}{77}x^{7/2}\left(11b+7cx^2\right)$$

input
```
Integrate[Sqrt[x]*(b*x^2 + c*x^4),x]
```

output
```
(2*x^(7/2)*(11*b + 7*c*x^2))/77
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(bx^2+cx^4\right) dx$$
$$\Big\downarrow \; 9$$
$$\int x^{5/2}\left(b+cx^2\right) dx$$
$$\Big\downarrow \; 244$$
$$\int \left(bx^{5/2}+cx^{9/2}\right) dx$$
$$\Big\downarrow \; 2009$$
$$\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

input
```
Int[Sqrt[x]*(b*x^2 + c*x^4),x]
```

output
```
(2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{7}{2}}}{7}+\frac{2c\,x^{\frac{11}{2}}}{11}$	14
default	$\frac{2b\,x^{\frac{7}{2}}}{7}+\frac{2c\,x^{\frac{11}{2}}}{11}$	14
gosper	$\frac{2x^{\frac{7}{2}}\left(7c\,x^{2}+11b\right)}{77}$	16
trager	$\frac{2x^{\frac{7}{2}}\left(7c\,x^{2}+11b\right)}{77}$	16
risch	$\frac{2x^{\frac{7}{2}}\left(7c\,x^{2}+11b\right)}{77}$	16
orering	$\frac{2\left(7c\,x^{2}+11b\right)x^{\frac{3}{2}}\left(c\,x^{4}+b\,x^{2}\right)}{77\left(c\,x^{2}+b\right)}$	36

input
```
int(x^(1/2)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/7*b*x^(7/2)+2/11*c*x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \sqrt{x}\left(bx^2+cx^4\right)\,dx=\frac{2}{77}\left(7\,cx^5+11\,bx^3\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
2/77*(7*c*x^5 + 11*b*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sqrt{x}\left(bx^2+cx^4\right)\,dx=\frac{2bx^{\frac{7}{2}}}{7}+\frac{2cx^{\frac{11}{2}}}{11}$$

input
```
integrate(x**(1/2)*(c*x**4+b*x**2),x)
```

output
```
2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{x}\left(bx^2+cx^4\right)\,dx=\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
2/11*c*x^(11/2) + 2/7*b*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{x}\left(bx^2+cx^4\right)\,dx=\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
2/11*c*x^(11/2) + 2/7*b*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{x}\left(bx^2+cx^4\right)\,dx=\frac{2\,x^{7/2}\,\left(7\,c\,x^2+11\,b\right)}{77}$$

input
```
int(x^(1/2)*(b*x^2 + c*x^4),x)
```

output
```
(2*x^(7/2)*(11*b + 7*c*x^2))/77
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\left(bx^2+cx^4\right)\,dx=\frac{2\sqrt{x}\,x^3(7c\,x^2+11b)}{77}$$

input
```
int(x^(1/2)*(c*x^4+b*x^2),x)
```

output
```
(2*sqrt(x)*x**3*(11*b + 7*c*x**2))/77
```

3.97 $\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx = \frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

output
```
2/5*b*x^(5/2)+2/9*c*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx = \frac{2}{45}\left(9bx^{5/2}+5cx^{9/2}\right)$$

input
```
Integrate[(b*x^2 + c*x^4)/Sqrt[x],x]
```

output
```
(2*(9*b*x^(5/2) + 5*c*x^(9/2)))/45
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx$$

$$\Big\downarrow 9$$

$$\int x^{3/2}\left(b+cx^2\right)dx$$

$$\Big\downarrow 244$$

$$\int \left(bx^{3/2}+cx^{7/2}\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

input
```
Int[(b*x^2 + c*x^4)/Sqrt[x],x]
```

output
```
(2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{5}{2}}}{5}+\frac{2c\,x^{\frac{9}{2}}}{9}$	14
default	$\frac{2b\,x^{\frac{5}{2}}}{5}+\frac{2c\,x^{\frac{9}{2}}}{9}$	14
gosper	$\frac{2x^{\frac{5}{2}}\left(5c\,x^{2}+9b\right)}{45}$	16
trager	$\frac{2x^{\frac{5}{2}}\left(5c\,x^{2}+9b\right)}{45}$	16
risch	$\frac{2x^{\frac{5}{2}}\left(5c\,x^{2}+9b\right)}{45}$	16
orering	$\frac{2\left(5c\,x^{2}+9b\right)\sqrt{x}\,\left(c\,x^{4}+b\,x^{2}\right)}{45\left(c\,x^{2}+b\right)}$	36

input
```
int((c*x^4+b*x^2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*b*x^(5/2)+2/9*c*x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx=\frac{2}{45}\left(5\,cx^4+9\,bx^2\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

output
```
2/45*(5*c*x^4 + 9*b*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx=\frac{2bx^{\frac{5}{2}}}{5}+\frac{2cx^{\frac{9}{2}}}{9}$$

input
```
integrate((c*x**4+b*x**2)/x**(1/2),x)
```

output
```
2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx=\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}$$

input
```
integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

output
```
2/9*c*x^(9/2) + 2/5*b*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx=\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}$$

input
```
integrate((c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

output
```
2/9*c*x^(9/2) + 2/5*b*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{bx^2 + cx^4}{\sqrt{x}}\, dx = \frac{2\, x^{5/2}\, (5\, c\, x^2 + 9\, b)}{45}$$

input
```
int((b*x^2 + c*x^4)/x^(1/2),x)
```

output
```
(2*x^(5/2)*(9*b + 5*c*x^2))/45
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{bx^2 + cx^4}{\sqrt{x}}\, dx = \frac{2\sqrt{x}\, x^2 (5c\, x^2 + 9b)}{45}$$

input
```
int((c*x^4+b*x^2)/x^(1/2),x)
```

output
```
(2*sqrt(x)*x**2*(9*b + 5*c*x**2))/45
```

3.98 $\int \frac{bx^2+cx^4}{x^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{bx^2+cx^4}{x^{3/2}}\,dx = \frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

output
```
2/3*b*x^(3/2)+2/7*c*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{bx^2+cx^4}{x^{3/2}}\,dx = \frac{2}{21}\left(7bx^{3/2}+3cx^{7/2}\right)$$

input
```
Integrate[(b*x^2 + c*x^4)/x^(3/2),x]
```

output
```
(2*(7*b*x^(3/2) + 3*c*x^(7/2)))/21
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+cx^4}{x^{3/2}}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \sqrt{x}\left(b+cx^2\right)dx$$
$$\Big\downarrow \; 244$$
$$\int \left(b\sqrt{x}+cx^{5/2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

input
```
Int[(b*x^2 + c*x^4)/x^(3/2),x]
```

output
```
(2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2b\,x^{\frac{3}{2}}}{3}+\frac{2c\,x^{\frac{7}{2}}}{7}$	14
default	$\frac{2b\,x^{\frac{3}{2}}}{3}+\frac{2c\,x^{\frac{7}{2}}}{7}$	14
gosper	$\frac{2x^{\frac{3}{2}}\left(3c\,x^{2}+7b\right)}{21}$	16
trager	$\frac{2x^{\frac{3}{2}}\left(3c\,x^{2}+7b\right)}{21}$	16
risch	$\frac{2x^{\frac{3}{2}}\left(3c\,x^{2}+7b\right)}{21}$	16
orering	$\frac{2\left(3c\,x^{2}+7b\right)\left(c\,x^{4}+b\,x^{2}\right)}{21\sqrt{x}\,\left(c\,x^{2}+b\right)}$	36

input
```
int((c*x^4+b*x^2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*b*x^(3/2)+2/7*c*x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{bx^2+cx^4}{x^{3/2}}\,dx=\frac{2}{21}\,\left(3\,cx^3+7\,bx\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="fricas")
```

output
```
2/21*(3*c*x^3 + 7*b*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{bx^2 + cx^4}{x^{3/2}}\,dx = \frac{2bx^{\frac{3}{2}}}{3} + \frac{2cx^{\frac{7}{2}}}{7}$$

input
```
integrate((c*x**4+b*x**2)/x**(3/2),x)
```

output
```
2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{bx^2 + cx^4}{x^{3/2}}\,dx = \frac{2}{7}\,cx^{\frac{7}{2}} + \frac{2}{3}\,bx^{\frac{3}{2}}$$

input
```
integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="maxima")
```

output
```
2/7*c*x^(7/2) + 2/3*b*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{bx^2 + cx^4}{x^{3/2}}\,dx = \frac{2}{7}\,cx^{\frac{7}{2}} + \frac{2}{3}\,bx^{\frac{3}{2}}$$

input
```
integrate((c*x^4+b*x^2)/x^(3/2),x, algorithm="giac")
```

output
```
2/7*c*x^(7/2) + 2/3*b*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{bx^2 + cx^4}{x^{3/2}}\,dx = \frac{2\,x^{3/2}\,(3\,c\,x^2 + 7\,b)}{21}$$

input
```
int((b*x^2 + c*x^4)/x^(3/2),x)
```

output
```
(2*x^(3/2)*(7*b + 3*c*x^2))/21
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{bx^2 + cx^4}{x^{3/2}}\,dx = \frac{2\sqrt{x}\,x(3c\,x^2 + 7b)}{21}$$

input
```
int((c*x^4+b*x^2)/x^(3/2),x)
```

output
```
(2*sqrt(x)*x*(7*b + 3*c*x**2))/21
```

3.99 $\int \frac{bx^2+cx^4}{x^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{bx^2+cx^4}{x^{5/2}}\,dx=2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

output
```
2*b*x^(1/2)+2/5*c*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{bx^2+cx^4}{x^{5/2}}\,dx=\frac{2}{5}\left(5b\sqrt{x}+cx^{5/2}\right)$$

input
```
Integrate[(b*x^2 + c*x^4)/x^(5/2),x]
```

output
```
(2*(5*b*Sqrt[x] + c*x^(5/2)))/5
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+cx^4}{x^{5/2}}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{b+cx^2}{\sqrt{x}}dx$$

$$\Big\downarrow \ 244$$

$$\int \left(\frac{b}{\sqrt{x}}+cx^{3/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

input
```
Int[(b*x^2 + c*x^4)/x^(5/2),x]
```

output
```
2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$2b\sqrt{x}+\frac{2c\,x^{\frac{5}{2}}}{5}$	14
default	$2b\sqrt{x}+\frac{2c\,x^{\frac{5}{2}}}{5}$	14
gosper	$\frac{2\sqrt{x}\,(c\,x^{2}+5b)}{5}$	15
trager	$\left(\frac{2c\,x^{2}}{5}+2b\right)\sqrt{x}$	15
risch	$\frac{2\sqrt{x}\,(c\,x^{2}+5b)}{5}$	15
orering	$\frac{2(c\,x^{2}+5b)\,(c\,x^{4}+b\,x^{2})}{5x^{\frac{3}{2}}\,(c\,x^{2}+b)}$	35

input
```
int((c*x^4+b*x^2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*b*x^(1/2)+2/5*c*x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{bx^2+cx^4}{x^{5/2}}\,dx=\frac{2}{5}\left(cx^2+5\,b\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="fricas")
```

output
```
2/5*(c*x^2 + 5*b)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{bx^2 + cx^4}{x^{5/2}}\,dx = 2b\sqrt{x} + \frac{2cx^{\frac{5}{2}}}{5}$$

input
```
integrate((c*x**4+b*x**2)/x**(5/2),x)
```

output
```
2*b*sqrt(x) + 2*c*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{bx^2 + cx^4}{x^{5/2}}\,dx = \frac{2}{5}\,cx^{\frac{5}{2}} + 2\,b\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="maxima")
```

output
```
2/5*c*x^(5/2) + 2*b*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{bx^2 + cx^4}{x^{5/2}}\,dx = \frac{2}{5}\,cx^{\frac{5}{2}} + 2\,b\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)/x^(5/2),x, algorithm="giac")
```

output
```
2/5*c*x^(5/2) + 2*b*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{bx^2+cx^4}{x^{5/2}}\,dx=\frac{2\,\sqrt{x}\,\left(c\,x^2+5\,b\right)}{5}$$

input
```
int((b*x^2 + c*x^4)/x^(5/2),x)
```

output
```
(2*x^(1/2)*(5*b + c*x^2))/5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{bx^2+cx^4}{x^{5/2}}\,dx=\frac{2\sqrt{x}\,\left(c\,x^2+5b\right)}{5}$$

input
```
int((c*x^4+b*x^2)/x^(5/2),x)
```

output
```
(2*sqrt(x)*(5*b + c*x**2))/5
```

3.100 $\int \frac{bx^2+cx^4}{x^{7/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{bx^2+cx^4}{x^{7/2}} \, dx = -\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}$$

output
```
-2*b/x^(1/2)+2/3*c*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{bx^2+cx^4}{x^{7/2}} \, dx = -\frac{2(3b-cx^2)}{3\sqrt{x}}$$

input
```
Integrate[(b*x^2 + c*x^4)/x^(7/2),x]
```

output
```
(-2*(3*b - c*x^2))/(3*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx^2+cx^4}{x^{7/2}}dx$$

$$\Big\downarrow 9$$

$$\int \frac{b+cx^2}{x^{3/2}}dx$$

$$\Big\downarrow 244$$

$$\int \left(\frac{b}{x^{3/2}}+c\sqrt{x}\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{2}{3}cx^{3/2}-\frac{2b}{\sqrt{x}}$$

input
```
Int[(b*x^2 + c*x^4)/x^(7/2),x]
```

output
```
(-2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{2b}{\sqrt{x}}+\frac{2c\,x^{\frac{3}{2}}}{3}$	14
default	$-\frac{2b}{\sqrt{x}}+\frac{2c\,x^{\frac{3}{2}}}{3}$	14
gosper	$-\frac{2\left(-c\,x^{2}+3b\right)}{3\sqrt{x}}$	16
trager	$-\frac{2\left(-c\,x^{2}+3b\right)}{3\sqrt{x}}$	16
risch	$-\frac{2\left(-c\,x^{2}+3b\right)}{3\sqrt{x}}$	16
orering	$-\frac{2\left(-c\,x^{2}+3b\right)\left(c\,x^{4}+b\,x^{2}\right)}{3x^{\frac{5}{2}}\left(c\,x^{2}+b\right)}$	36

input
```
int((c*x^4+b*x^2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b/x^(1/2)+2/3*c*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{bx^2+cx^4}{x^{7/2}}\,dx=\frac{2\,(cx^2-3\,b)}{3\,\sqrt{x}}$$

input
```
integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="fricas")
```

output
```
2/3*(c*x^2 - 3*b)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{bx^2 + cx^4}{x^{7/2}}\,dx = -\frac{2b}{\sqrt{x}} + \frac{2cx^{\frac{3}{2}}}{3}$$

input
```
integrate((c*x**4+b*x**2)/x**(7/2),x)
```

output
```
-2*b/sqrt(x) + 2*c*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{bx^2 + cx^4}{x^{7/2}}\,dx = \frac{2}{3}\,cx^{\frac{3}{2}} - \frac{2\,b}{\sqrt{x}}$$

input
```
integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="maxima")
```

output
```
2/3*c*x^(3/2) - 2*b/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{bx^2 + cx^4}{x^{7/2}}\,dx = \frac{2}{3}\,cx^{\frac{3}{2}} - \frac{2\,b}{\sqrt{x}}$$

input
```
integrate((c*x^4+b*x^2)/x^(7/2),x, algorithm="giac")
```

output
```
2/3*c*x^(3/2) - 2*b/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{bx^2 + cx^4}{x^{7/2}} \, dx = -\frac{6\,b - 2\,c\,x^2}{3\,\sqrt{x}}$$

input
```
int((b*x^2 + c*x^4)/x^(7/2),x)
```

output
```
-(6*b - 2*c*x^2)/(3*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{bx^2 + cx^4}{x^{7/2}} \, dx = \frac{\frac{2c\,x^2}{3} - 2b}{\sqrt{x}}$$

input
```
int((c*x^4+b*x^2)/x^(7/2),x)
```

output
```
(2*( - 3*b + c*x**2))/(3*sqrt(x))
```

3.101 $\int x^{7/2}(bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{17}b^2x^{17/2}+\frac{4}{21}bcx^{21/2}+\frac{2}{25}c^2x^{25/2}$$

output
```
2/17*b^2*x^(17/2)+4/21*b*c*x^(21/2)+2/25*c^2*x^(25/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2x^{17/2}(525b^2+850bcx^2+357c^2x^4)}{8925}$$

input
```
Integrate[x^(7/2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*x^(17/2)*(525*b^2 + 850*b*c*x^2 + 357*c^2*x^4))/8925
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2} \left(bx^2+cx^4\right)^2 \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{15/2} \left(b+cx^2\right)^2 dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(b^2 x^{15/2}+2bcx^{19/2}+c^2 x^{23/2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{17} b^2 x^{17/2}+\frac{4}{21} bcx^{21/2}+\frac{2}{25} c^2 x^{25/2}$$

input
```
Int[x^(7/2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*b^2*x^(17/2))/17 + (4*b*c*x^(21/2))/21 + (2*c^2*x^(25/2))/25
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{17}{2}}}{17}+\frac{4bc\,x^{\frac{21}{2}}}{21}+\frac{2c^2x^{\frac{25}{2}}}{25}$	25
default	$\frac{2b^2x^{\frac{17}{2}}}{17}+\frac{4bc\,x^{\frac{21}{2}}}{21}+\frac{2c^2x^{\frac{25}{2}}}{25}$	25
gosper	$\frac{2x^{\frac{17}{2}}\left(357c^2x^4+850bc\,x^2+525b^2\right)}{8925}$	27
trager	$\frac{2x^{\frac{17}{2}}\left(357c^2x^4+850bc\,x^2+525b^2\right)}{8925}$	27
risch	$\frac{2x^{\frac{17}{2}}\left(357c^2x^4+850bc\,x^2+525b^2\right)}{8925}$	27
orering	$\frac{2\left(357c^2x^4+850bc\,x^2+525b^2\right)x^{\frac{9}{2}}\left(c\,x^4+b\,x^2\right)^2}{8925\left(c\,x^2+b\right)^2}$	49

input
```
int(x^(7/2)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/17*b^2*x^(17/2)+4/21*b*c*x^(21/2)+2/25*c^2*x^(25/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{8925}\left(357\,c^2x^{12}+850\,bcx^{10}+525\,b^2x^8\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
2/8925*(357*c^2*x^12 + 850*b*c*x^10 + 525*b^2*x^8)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2b^2x^{\frac{17}{2}}}{17}+\frac{4bcx^{\frac{21}{2}}}{21}+\frac{2c^2x^{\frac{25}{2}}}{25}$$

input
```
integrate(x**(7/2)*(c*x**4+b*x**2)**2,x)
```

output
```
2*b**2*x**(17/2)/17 + 4*b*c*x**(21/2)/21 + 2*c**2*x**(25/2)/25
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{25}\,c^2x^{\frac{25}{2}}+\frac{4}{21}\,bcx^{\frac{21}{2}}+\frac{2}{17}\,b^2x^{\frac{17}{2}}$$

input
```
integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
2/25*c^2*x^(25/2) + 4/21*b*c*x^(21/2) + 2/17*b^2*x^(17/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{25}\,c^2x^{\frac{25}{2}}+\frac{4}{21}\,bcx^{\frac{21}{2}}+\frac{2}{17}\,b^2x^{\frac{17}{2}}$$

input
```
integrate(x^(7/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
2/25*c^2*x^(25/2) + 4/21*b*c*x^(21/2) + 2/17*b^2*x^(17/2)
```

Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx=x^{17/2}\left(\frac{2\,b^2}{17}+\frac{4\,b\,c\,x^2}{21}+\frac{2\,c^2\,x^4}{25}\right)$$

input
```
int(x^(7/2)*(b*x^2 + c*x^4)^2,x)
```

output
```
x^(17/2)*((2*b^2)/17 + (2*c^2*x^4)/25 + (4*b*c*x^2)/21)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{7/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2\sqrt{x}\,x^8(357c^2x^4+850bc\,x^2+525b^2)}{8925}$$

input
```
int(x^(7/2)*(c*x^4+b*x^2)^2,x)
```

output
```
(2*sqrt(x)*x**8*(525*b**2 + 850*b*c*x**2 + 357*c**2*x**4))/8925
```

3.102 $\int x^{5/2}(bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int x^{5/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

output
```
2/15*b^2*x^(15/2)+4/19*b*c*x^(19/2)+2/23*c^2*x^(23/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int x^{5/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2x^{15/2}(437b^2+690bcx^2+285c^2x^4)}{6555}$$

input
```
Integrate[x^(5/2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*x^(15/2)*(437*b^2 + 690*b*c*x^2 + 285*c^2*x^4))/6555
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(bx^2+cx^4\right)^2\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{13/2}\left(b+cx^2\right)^2dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(b^2x^{13/2}+2bcx^{17/2}+c^2x^{21/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{15}b^2x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

input
```
Int[x^(5/2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*b^2*x^(15/2))/15 + (4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{15}{2}}}{15}+\frac{4bc\,x^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}$	25
default	$\frac{2b^2x^{\frac{15}{2}}}{15}+\frac{4bc\,x^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}$	25
gosper	$\frac{2x^{\frac{15}{2}}\left(285c^2x^4+690bc\,x^2+437b^2\right)}{6555}$	27
trager	$\frac{2x^{\frac{15}{2}}\left(285c^2x^4+690bc\,x^2+437b^2\right)}{6555}$	27
risch	$\frac{2x^{\frac{15}{2}}\left(285c^2x^4+690bc\,x^2+437b^2\right)}{6555}$	27
orering	$\frac{2\left(285c^2x^4+690bc\,x^2+437b^2\right)x^{\frac{7}{2}}\left(c\,x^4+b\,x^2\right)^2}{6555\left(c\,x^2+b\right)^2}$	49

input
```
int(x^(5/2)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/15*b^2*x^(15/2)+4/19*b*c*x^(19/2)+2/23*c^2*x^(23/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{6555}\left(285\,c^2x^{11}+690\,bcx^9+437\,b^2x^7\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
2/6555*(285*c^2*x^11 + 690*b*c*x^9 + 437*b^2*x^7)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{5/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2b^2x^{\frac{15}{2}}}{15}+\frac{4bcx^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}$$

input
```
integrate(x**(5/2)*(c*x**4+b*x**2)**2,x)
```

output
```
2*b**2*x**(15/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^2x^{\frac{15}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^2x^{\frac{15}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2)
```

Mupad [B] (verification not implemented)

Time = 17.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int x^{5/2}(bx^2+cx^4)^2\,dx=x^{15/2}\left(\frac{2\,b^2}{15}+\frac{4\,b\,c\,x^2}{19}+\frac{2\,c^2\,x^4}{23}\right)$$

input
```
int(x^(5/2)*(b*x^2 + c*x^4)^2,x)
```

output
```
x^(15/2)*((2*b^2)/15 + (2*c^2*x^4)/23 + (4*b*c*x^2)/19)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{5/2}(bx^2+cx^4)^2\,dx=\frac{2\sqrt{x}\,x^7(285c^2x^4+690bc\,x^2+437b^2)}{6555}$$

input
```
int(x^(5/2)*(c*x^4+b*x^2)^2,x)
```

output
```
(2*sqrt(x)*x**7*(437*b**2 + 690*b*c*x**2 + 285*c**2*x**4))/6555
```

3.103 $\int x^{3/2}(bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int x^{3/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{13}b^2x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

output
```
2/13*b^2*x^(13/2)+4/17*b*c*x^(17/2)+2/21*c^2*x^(21/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int x^{3/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2x^{13/2}(357b^2+546bcx^2+221c^2x^4)}{4641}$$

input
```
Integrate[x^(3/2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*x^(13/2)*(357*b^2 + 546*b*c*x^2 + 221*c^2*x^4))/4641
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{11/2}\left(b+cx^2\right)^2 dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(b^2x^{11/2}+2bcx^{15/2}+c^2x^{19/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{13}b^2x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

input
```
Int[x^(3/2)*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*b^2*x^(13/2))/13 + (4*b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bc\,x^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$	25
default	$\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bc\,x^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$	25
gosper	$\frac{2x^{\frac{13}{2}}\left(221c^2x^4+546bc\,x^2+357b^2\right)}{4641}$	27
trager	$\frac{2x^{\frac{13}{2}}\left(221c^2x^4+546bc\,x^2+357b^2\right)}{4641}$	27
risch	$\frac{2x^{\frac{13}{2}}\left(221c^2x^4+546bc\,x^2+357b^2\right)}{4641}$	27
orering	$\frac{2\left(221c^2x^4+546bc\,x^2+357b^2\right)x^{\frac{5}{2}}\left(c\,x^4+b\,x^2\right)^2}{4641\left(c\,x^2+b\right)^2}$	49

input
```
int(x^(3/2)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/13*b^2*x^(13/2)+4/17*b*c*x^(17/2)+2/21*c^2*x^(21/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{4641}\left(221\,c^2x^{10}+546\,bcx^8+357\,b^2x^6\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
2/4641*(221*c^2*x^10 + 546*b*c*x^8 + 357*b^2*x^6)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{3/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bcx^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(3/2)*(c*x**4+b*x**2)**2,x)
```

output
```
2*b**2*x**(13/2)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int x^{3/2}\left(bx^2+cx^4\right)^2\,dx=x^{13/2}\left(\frac{2\,b^2}{13}+\frac{4\,b\,c\,x^2}{17}+\frac{2\,c^2\,x^4}{21}\right)$$

input
```
int(x^(3/2)*(b*x^2 + c*x^4)^2,x)
```

output
```
x^(13/2)*((2*b^2)/13 + (2*c^2*x^4)/21 + (4*b*c*x^2)/17)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{3/2}\left(bx^2+cx^4\right)^2\,dx=\frac{2\sqrt{x}\,x^6(221c^2x^4+546bc\,x^2+357b^2)}{4641}$$

input
```
int(x^(3/2)*(c*x^4+b*x^2)^2,x)
```

output
```
(2*sqrt(x)*x**6*(357*b**2 + 546*b*c*x**2 + 221*c**2*x**4))/4641
```

3.104 $\int \sqrt{x}(bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \sqrt{x}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

output
```
2/11*b^2*x^(11/2)+4/15*b*c*x^(15/2)+2/19*c^2*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \sqrt{x}\left(bx^2+cx^4\right)^2\,dx=\frac{2x^{11/2}(285b^2+418bcx^2+165c^2x^4)}{3135}$$

input
```
Integrate[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*x^(11/2)*(285*b^2 + 418*b*c*x^2 + 165*c^2*x^4))/3135
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(bx^2+cx^4\right)^2\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^{9/2}\left(b+cx^2\right)^2dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(b^2x^{9/2}+2bcx^{13/2}+c^2x^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{11}b^2x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

input
```
Int[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
```

output
```
(2*b^2*x^(11/2))/11 + (4*b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p, x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] && !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{11}{2}}}{11}+\frac{4bc\,x^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}$	25
default	$\frac{2b^2x^{\frac{11}{2}}}{11}+\frac{4bc\,x^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}$	25
gosper	$\frac{2x^{\frac{11}{2}}\left(165c^2x^4+418bc\,x^2+285b^2\right)}{3135}$	27
trager	$\frac{2x^{\frac{11}{2}}\left(165c^2x^4+418bc\,x^2+285b^2\right)}{3135}$	27
risch	$\frac{2x^{\frac{11}{2}}\left(165c^2x^4+418bc\,x^2+285b^2\right)}{3135}$	27
orering	$\frac{2\left(165c^2x^4+418bc\,x^2+285b^2\right)x^{\frac{3}{2}}\left(c\,x^4+b\,x^2\right)^2}{3135\left(c\,x^2+b\right)^2}$	49

input
```
int(x^(1/2)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/11*b^2*x^(11/2)+4/15*b*c*x^(15/2)+2/19*c^2*x^(19/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{3135}\left(165\,c^2x^9+418\,bcx^7+285\,b^2x^5\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
2/3135*(165*c^2*x^9 + 418*b*c*x^7 + 285*b^2*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \sqrt{x}\left(bx^2+cx^4\right)^2\,dx=\frac{2b^2x^{\frac{11}{2}}}{11}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}$$

input
```
integrate(x**(1/2)*(c*x**4+b*x**2)**2,x)
```

output
```
2*b**2*x**(11/2)/11 + 4*b*c*x**(15/2)/15 + 2*c**2*x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt{x}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt{x}\left(bx^2+cx^4\right)^2\,dx=\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2)
```

Mupad [B] (verification not implemented)

Time = 17.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \sqrt{x}\left(bx^2+cx^4\right)^2\,dx=x^{11/2}\left(\frac{2\,b^2}{11}+\frac{4\,b\,c\,x^2}{15}+\frac{2\,c^2\,x^4}{19}\right)$$

input
```
int(x^(1/2)*(b*x^2 + c*x^4)^2,x)
```

output
```
x^(11/2)*((2*b^2)/11 + (2*c^2*x^4)/19 + (4*b*c*x^2)/15)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \sqrt{x}\left(bx^2+cx^4\right)^2\,dx=\frac{2\sqrt{x}\,x^5(165c^2x^4+418bc\,x^2+285b^2)}{3135}$$

input
```
int(x^(1/2)*(c*x^4+b*x^2)^2,x)
```

output
```
(2*sqrt(x)*x**5*(285*b**2 + 418*b*c*x**2 + 165*c**2*x**4))/3135
```

3.105 $\int \frac{(bx^2+cx^4)^2}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{(bx^2+cx^4)^2}{\sqrt{x}} \, dx = \frac{2}{9}b^2x^{9/2} + \frac{4}{13}bcx^{13/2} + \frac{2}{17}c^2x^{17/2}$$

output
```
2/9*b^2*x^(9/2)+4/13*b*c*x^(13/2)+2/17*c^2*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{(bx^2+cx^4)^2}{\sqrt{x}} \, dx = \frac{2x^{9/2}(221b^2 + 306bcx^2 + 117c^2x^4)}{1989}$$

input
```
Integrate[(b*x^2 + c*x^4)^2/Sqrt[x],x]
```

output
```
(2*x^(9/2)*(221*b^2 + 306*b*c*x^2 + 117*c^2*x^4))/1989
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{\sqrt{x}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^{7/2}\left(b+cx^2\right)^2dx$$

$$\Big\downarrow \; 244$$

$$\int \left(b^2x^{7/2}+2bcx^{11/2}+c^2x^{15/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{9}b^2x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

input
```
Int[(b*x^2 + c*x^4)^2/Sqrt[x],x]
```

output
```
(2*b^2*x^(9/2))/9 + (4*b*c*x^(13/2))/13 + (2*c^2*x^(17/2))/17
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{9}{2}}}{9}+\frac{4bc\,x^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{17}{2}}}{17}$	25
default	$\frac{2b^2x^{\frac{9}{2}}}{9}+\frac{4bc\,x^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{17}{2}}}{17}$	25
gosper	$\frac{2x^{\frac{9}{2}}\left(117c^2x^4+306bc\,x^2+221b^2\right)}{1989}$	27
trager	$\frac{2x^{\frac{9}{2}}\left(117c^2x^4+306bc\,x^2+221b^2\right)}{1989}$	27
risch	$\frac{2x^{\frac{9}{2}}\left(117c^2x^4+306bc\,x^2+221b^2\right)}{1989}$	27
orering	$\frac{2(117c^2x^4+306bc\,x^2+221b^2)\sqrt{x}\,\left(c\,x^4+b\,x^2\right)^2}{1989(c\,x^2+b)^2}$	49

input
```
int((c*x^4+b*x^2)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*b^2*x^(9/2)+4/13*b*c*x^(13/2)+2/17*c^2*x^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{\left(bx^2+cx^4\right)^2}{\sqrt{x}}\,dx=\frac{2}{1989}\left(117\,c^2x^8+306\,bcx^6+221\,b^2x^4\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/1989*(117*c^2*x^8 + 306*b*c*x^6 + 221*b^2*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(bx^2 + cx^4)^2}{\sqrt{x}}\,dx = \frac{2b^2x^{\frac{9}{2}}}{9} + \frac{4bcx^{\frac{13}{2}}}{13} + \frac{2c^2x^{\frac{17}{2}}}{17}$$

input
```
integrate((c*x**4+b*x**2)**2/x**(1/2),x)
```

output
```
2*b**2*x**(9/2)/9 + 4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(bx^2 + cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{17}\,c^2x^{\frac{17}{2}} + \frac{4}{13}\,bcx^{\frac{13}{2}} + \frac{2}{9}\,b^2x^{\frac{9}{2}}$$

input
```
integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

output
```
2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(bx^2 + cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{17}\,c^2x^{\frac{17}{2}} + \frac{4}{13}\,bcx^{\frac{13}{2}} + \frac{2}{9}\,b^2x^{\frac{9}{2}}$$

input
```
integrate((c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

output
```
2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2 + cx^4)^2}{\sqrt{x}}\, dx = x^{9/2}\left(\frac{2\,b^2}{9} + \frac{4\,b\,c\,x^2}{13} + \frac{2\,c^2\,x^4}{17}\right)$$

input
```
int((b*x^2 + c*x^4)^2/x^(1/2),x)
```

output
```
x^(9/2)*((2*b^2)/9 + (2*c^2*x^4)/17 + (4*b*c*x^2)/13)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{(bx^2 + cx^4)^2}{\sqrt{x}}\, dx = \frac{2\sqrt{x}\,x^4(117c^2x^4 + 306bc\,x^2 + 221b^2)}{1989}$$

input
```
int((c*x^4+b*x^2)^2/x^(1/2),x)
```

output
```
(2*sqrt(x)*x**4*(221*b**2 + 306*b*c*x**2 + 117*c**2*x**4))/1989
```

3.106 $\int \frac{(bx^2+cx^4)^2}{x^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{(bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

output
```
2/7*b^2*x^(7/2)+4/11*b*c*x^(11/2)+2/15*c^2*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{(bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2x^{7/2}(165b^2+210bcx^2+77c^2x^4)}{1155}$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^(3/2),x]
```

output
```
(2*x^(7/2)*(165*b^2 + 210*b*c*x^2 + 77*c^2*x^4))/1155
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^{3/2}}dx$$

$$\Big\downarrow \ 9$$

$$\int x^{5/2}(b+cx^2)^2dx$$

$$\Big\downarrow \ 244$$

$$\int \left(b^2x^{5/2}+2bcx^{9/2}+c^2x^{13/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

input
```
Int[(b*x^2 + c*x^4)^2/x^(3/2),x]
```

output
```
(2*b^2*x^(7/2))/7 + (4*b*c*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4bc\,x^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{15}{2}}}{15}$	25
default	$\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4bc\,x^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{15}{2}}}{15}$	25
gosper	$\frac{2x^{\frac{7}{2}}\left(77c^2x^4+210bc\,x^2+165b^2\right)}{1155}$	27
trager	$\frac{2x^{\frac{7}{2}}\left(77c^2x^4+210bc\,x^2+165b^2\right)}{1155}$	27
risch	$\frac{2x^{\frac{7}{2}}\left(77c^2x^4+210bc\,x^2+165b^2\right)}{1155}$	27
orering	$\frac{2\left(77c^2x^4+210bc\,x^2+165b^2\right)\left(c\,x^4+b\,x^2\right)^2}{1155\sqrt{x}\,\left(c\,x^2+b\right)^2}$	49

input
```
int((c*x^4+b*x^2)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*b^2*x^(7/2)+4/11*b*c*x^(11/2)+2/15*c^2*x^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2}{1155}\left(77\,c^2x^7+210\,bcx^5+165\,b^2x^3\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(77*c^2*x^7 + 210*b*c*x^5 + 165*b^2*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(bx^2 + cx^4)^2}{x^{3/2}}\,dx = \frac{2b^2x^{\frac{7}{2}}}{7} + \frac{4bcx^{\frac{11}{2}}}{11} + \frac{2c^2x^{\frac{15}{2}}}{15}$$

input
```
integrate((c*x**4+b*x**2)**2/x**(3/2),x)
```

output
```
2*b**2*x**(7/2)/7 + 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(bx^2 + cx^4)^2}{x^{3/2}}\,dx = \frac{2}{15}\,c^2x^{\frac{15}{2}} + \frac{4}{11}\,bcx^{\frac{11}{2}} + \frac{2}{7}\,b^2x^{\frac{7}{2}}$$

input
```
integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="maxima")
```

output
```
2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(bx^2 + cx^4)^2}{x^{3/2}}\,dx = \frac{2}{15}\,c^2x^{\frac{15}{2}} + \frac{4}{11}\,bcx^{\frac{11}{2}} + \frac{2}{7}\,b^2x^{\frac{7}{2}}$$

input
```
integrate((c*x^4+b*x^2)^2/x^(3/2),x, algorithm="giac")
```

output
```
2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{(bx^2 + cx^4)^2}{x^{3/2}}\,dx = \frac{2\,x^{7/2}\,(165\,b^2 + 210\,b\,c\,x^2 + 77\,c^2\,x^4)}{1155}$$

input
```
int((b*x^2 + c*x^4)^2/x^(3/2),x)
```

output
```
(2*x^(7/2)*(165*b^2 + 77*c^2*x^4 + 210*b*c*x^2))/1155
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{(bx^2 + cx^4)^2}{x^{3/2}}\,dx = \frac{2\sqrt{x}\,x^3(77c^2x^4 + 210bc\,x^2 + 165b^2)}{1155}$$

input
```
int((c*x^4+b*x^2)^2/x^(3/2),x)
```

output
```
(2*sqrt(x)*x**3*(165*b**2 + 210*b*c*x**2 + 77*c**2*x**4))/1155
```

3.107 $\int \frac{(bx^2+cx^4)^2}{x^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

output
```
2/5*b^2*x^(5/2)+4/9*b*c*x^(9/2)+2/13*c^2*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{585}x^{5/2}\left(117b^2+130bcx^2+45c^2x^4\right)$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^(5/2),x]
```

output
```
(2*x^(5/2)*(117*b^2 + 130*b*c*x^2 + 45*c^2*x^4))/585
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^2}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{3/2}\left(b+cx^2\right)^2dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(b^2x^{3/2}+2bcx^{7/2}+c^2x^{11/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

input
```
Int[(b*x^2 + c*x^4)^2/x^(5/2),x]
```

output
```
(2*b^2*x^(5/2))/5 + (4*b*c*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{4bc\,x^{\frac{9}{2}}}{9}+\frac{2c^2x^{\frac{13}{2}}}{13}$	25
default	$\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{4bc\,x^{\frac{9}{2}}}{9}+\frac{2c^2x^{\frac{13}{2}}}{13}$	25
gosper	$\frac{2x^{\frac{5}{2}}\left(45c^2x^4+130bc\,x^2+117b^2\right)}{585}$	27
trager	$\frac{2x^{\frac{5}{2}}\left(45c^2x^4+130bc\,x^2+117b^2\right)}{585}$	27
risch	$\frac{2x^{\frac{5}{2}}\left(45c^2x^4+130bc\,x^2+117b^2\right)}{585}$	27
orering	$\frac{2\left(45c^2x^4+130bc\,x^2+117b^2\right)\left(c\,x^4+b\,x^2\right)^2}{585x^{\frac{3}{2}}\left(c\,x^2+b\right)^2}$	49

input
```
int((c*x^4+b*x^2)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*b^2*x^(5/2)+4/9*b*c*x^(9/2)+2/13*c^2*x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{\left(bx^2+cx^4\right)^2}{x^{5/2}}\,dx = \frac{2}{585}\left(45\,c^2x^6+130\,bcx^4+117\,b^2x^2\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="fricas")
```

output
```
2/585*(45*c^2*x^6 + 130*b*c*x^4 + 117*b^2*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(bx^2 + cx^4)^2}{x^{5/2}}\,dx = \frac{2b^2x^{\frac{5}{2}}}{5} + \frac{4bcx^{\frac{9}{2}}}{9} + \frac{2c^2x^{\frac{13}{2}}}{13}$$

input
```
integrate((c*x**4+b*x**2)**2/x**(5/2),x)
```

output
```
2*b**2*x**(5/2)/5 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(bx^2 + cx^4)^2}{x^{5/2}}\,dx = \frac{2}{13}\,c^2x^{\frac{13}{2}} + \frac{4}{9}\,bcx^{\frac{9}{2}} + \frac{2}{5}\,b^2x^{\frac{5}{2}}$$

input
```
integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="maxima")
```

output
```
2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(bx^2 + cx^4)^2}{x^{5/2}}\,dx = \frac{2}{13}\,c^2x^{\frac{13}{2}} + \frac{4}{9}\,bcx^{\frac{9}{2}} + \frac{2}{5}\,b^2x^{\frac{5}{2}}$$

input
```
integrate((c*x^4+b*x^2)^2/x^(5/2),x, algorithm="giac")
```

output
```
2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2\,x^{5/2}\,(117\,b^2+130\,b\,c\,x^2+45\,c^2\,x^4)}{585}$$

input
```
int((b*x^2 + c*x^4)^2/x^(5/2),x)
```

output
```
(2*x^(5/2)*(117*b^2 + 45*c^2*x^4 + 130*b*c*x^2))/585
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2\sqrt{x}\,x^2(45c^2x^4+130bc\,x^2+117b^2)}{585}$$

input
```
int((c*x^4+b*x^2)^2/x^(5/2),x)
```

output
```
(2*sqrt(x)*x**2*(117*b**2 + 130*b*c*x**2 + 45*c**2*x**4))/585
```

3.108 $\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2}{3}b^2x^{3/2} + \frac{4}{7}bcx^{7/2} + \frac{2}{11}c^2x^{11/2}$$

output
```
2/3*b^2*x^(3/2)+4/7*b*c*x^(7/2)+2/11*c^2*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2}{231}x^{3/2}\left(77b^2+66bcx^2+21c^2x^4\right)$$

input
```
Integrate[(b*x^2 + c*x^4)^2/x^(7/2),x]
```

output
```
(2*x^(3/2)*(77*b^2 + 66*b*c*x^2 + 21*c^2*x^4))/231
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^2+cx^4\right)^2}{x^{7/2}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \sqrt{x}\left(b+cx^2\right)^2dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(b^2\sqrt{x}+2bcx^{5/2}+c^2x^{9/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}b^2x^{3/2}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

input
```
Int[(b*x^2 + c*x^4)^2/x^(7/2),x]
```

output
```
(2*b^2*x^(3/2))/3 + (4*b*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2b^2x^{\frac{3}{2}}}{3}+\frac{4bc\,x^{\frac{7}{2}}}{7}+\frac{2c^2x^{\frac{11}{2}}}{11}$	25
default	$\frac{2b^2x^{\frac{3}{2}}}{3}+\frac{4bc\,x^{\frac{7}{2}}}{7}+\frac{2c^2x^{\frac{11}{2}}}{11}$	25
gosper	$\frac{2x^{\frac{3}{2}}\left(21c^2x^4+66bc\,x^2+77b^2\right)}{231}$	27
trager	$\frac{2x^{\frac{3}{2}}\left(21c^2x^4+66bc\,x^2+77b^2\right)}{231}$	27
risch	$\frac{2x^{\frac{3}{2}}\left(21c^2x^4+66bc\,x^2+77b^2\right)}{231}$	27
orering	$\frac{2\left(21c^2x^4+66bc\,x^2+77b^2\right)\left(c\,x^4+b\,x^2\right)^2}{231x^{\frac{5}{2}}\left(c\,x^2+b\right)^2}$	49

input
```
int((c*x^4+b*x^2)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*b^2*x^(3/2)+4/7*b*c*x^(7/2)+2/11*c^2*x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx=\frac{2}{231}\left(21\,c^2x^5+66\,bcx^3+77\,b^2x\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="fricas")
```

output
```
2/231*(21*c^2*x^5 + 66*b*c*x^3 + 77*b^2*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2b^2x^{\frac{3}{2}}}{3} + \frac{4bcx^{\frac{7}{2}}}{7} + \frac{2c^2x^{\frac{11}{2}}}{11}$$

input
```
integrate((c*x**4+b*x**2)**2/x**(7/2),x)
```

output
```
2*b**2*x**(3/2)/3 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{7}\,bcx^{\frac{7}{2}} + \frac{2}{3}\,b^2x^{\frac{3}{2}}$$

input
```
integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="maxima")
```

output
```
2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{7}\,bcx^{\frac{7}{2}} + \frac{2}{3}\,b^2x^{\frac{3}{2}}$$

input
```
integrate((c*x^4+b*x^2)^2/x^(7/2),x, algorithm="giac")
```

output
```
2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 17.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2\,x^{3/2}\,\left(77\,b^2+66\,b\,c\,x^2+21\,c^2\,x^4\right)}{231}$$

input
```
int((b*x^2 + c*x^4)^2/x^(7/2),x)
```

output
```
(2*x^(3/2)*(77*b^2 + 21*c^2*x^4 + 66*b*c*x^2))/231
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2\sqrt{x}\,x(21c^2x^4+66bc\,x^2+77b^2)}{231}$$

input
```
int((c*x^4+b*x^2)^2/x^(7/2),x)
```

output
```
(2*sqrt(x)*x*(77*b**2 + 66*b*c*x**2 + 21*c**2*x**4))/231
```

3.109 $\int x^{7/2}(bx^2+cx^4)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int x^{7/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

output
```
2/21*b^3*x^(21/2)+6/25*b^2*c*x^(25/2)+6/29*b*c^2*x^(29/2)+2/33*c^3*x^(33/2
)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^{7/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

input
```
Integrate[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/29 + (2*c
^3*x^(33/2))/33
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(bx^2+cx^4\right)^3\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{19/2}\left(b+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(b^3x^{19/2}+3b^2cx^{23/2}+3bc^2x^{27/2}+c^3x^{31/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

input
```
Int[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/29 + (2*c
^3*x^(33/2))/33
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{21}{2}}}{21}+\frac{6b^2c\,x^{\frac{25}{2}}}{25}+\frac{6b\,c^2x^{\frac{29}{2}}}{29}+\frac{2c^3x^{\frac{33}{2}}}{33}$	36
default	$\frac{2b^3x^{\frac{21}{2}}}{21}+\frac{6b^2c\,x^{\frac{25}{2}}}{25}+\frac{6b\,c^2x^{\frac{29}{2}}}{29}+\frac{2c^3x^{\frac{33}{2}}}{33}$	36
gosper	$\frac{2x^{\frac{21}{2}}\left(5075c^3x^6+17325b\,c^2x^4+20097b^2c\,x^2+7975b^3\right)}{167475}$	38
trager	$\frac{2x^{\frac{21}{2}}\left(5075c^3x^6+17325b\,c^2x^4+20097b^2c\,x^2+7975b^3\right)}{167475}$	38
risch	$\frac{2x^{\frac{21}{2}}\left(5075c^3x^6+17325b\,c^2x^4+20097b^2c\,x^2+7975b^3\right)}{167475}$	38
orering	$\frac{2\left(5075c^3x^6+17325b\,c^2x^4+20097b^2c\,x^2+7975b^3\right)x^{\frac{9}{2}}\left(c\,x^4+b\,x^2\right)^3}{167475\left(c\,x^2+b\right)^3}$	60

input
```
int(x^(7/2)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/21*b^3*x^(21/2)+6/25*b^2*c*x^(25/2)+6/29*b*c^2*x^(29/2)+2/33*c^3*x^(33/2
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^{7/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{167475}\left(5075\,c^3x^{16}+17325\,bc^2x^{14}+20097\,b^2cx^{12}+7975\,b^3x^{10}\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
2/167475*(5075*c^3*x^16 + 17325*b*c^2*x^14 + 20097*b^2*c*x^12 + 7975*b^3*x
^10)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{7/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2b^3x^{\frac{21}{2}}}{21}+\frac{6b^2cx^{\frac{25}{2}}}{25}+\frac{6bc^2x^{\frac{29}{2}}}{29}+\frac{2c^3x^{\frac{33}{2}}}{33}$$

input
```
integrate(x**(7/2)*(c*x**4+b*x**2)**3,x)
```

output
```
2*b**3*x**(21/2)/21 + 6*b**2*c*x**(25/2)/25 + 6*b*c**2*x**(29/2)/29 + 2*c*
*3*x**(33/2)/33
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{7/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{33}\,c^3x^{\frac{33}{2}}+\frac{6}{29}\,bc^2x^{\frac{29}{2}}+\frac{6}{25}\,b^2cx^{\frac{25}{2}}+\frac{2}{21}\,b^3x^{\frac{21}{2}}$$

input
```
integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
2/33*c^3*x^(33/2) + 6/29*b*c^2*x^(29/2) + 6/25*b^2*c*x^(25/2) + 2/21*b^3*x
^(21/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{7/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{33}\,c^3x^{\frac{33}{2}}+\frac{6}{29}\,bc^2x^{\frac{29}{2}}+\frac{6}{25}\,b^2cx^{\frac{25}{2}}+\frac{2}{21}\,b^3x^{\frac{21}{2}}$$

input
```
integrate(x^(7/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
2/33*c^3*x^(33/2) + 6/29*b*c^2*x^(29/2) + 6/25*b^2*c*x^(25/2) + 2/21*b^3*x
^(21/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{7/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2\,b^3\,x^{21/2}}{21}+\frac{2\,c^3\,x^{33/2}}{33}+\frac{6\,b^2\,c\,x^{25/2}}{25}+\frac{6\,b\,c^2\,x^{29/2}}{29}$$

input
```
int(x^(7/2)*(b*x^2 + c*x^4)^3,x)
```

output
```
(2*b^3*x^(21/2))/21 + (2*c^3*x^(33/2))/33 + (6*b^2*c*x^(25/2))/25 + (6*b*c
^2*x^(29/2))/29
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{7/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2\sqrt{x}\,x^{10}(5075c^3x^6+17325b\,c^2x^4+20097b^2c\,x^2+7975b^3)}{167475}$$

input
```
int(x^(7/2)*(c*x^4+b*x^2)^3,x)
```

output
```
(2*sqrt(x)*x**10*(7975*b**3 + 20097*b**2*c*x**2 + 17325*b*c**2*x**4 + 5075
*c**3*x**6))/167475
```

3.110 $\int x^{5/2}(bx^2+cx^4)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

output
```
2/19*b^3*x^(19/2)+6/23*b^2*c*x^(23/2)+2/9*b*c^2*x^(27/2)+2/31*c^3*x^(31/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

input
```
Integrate[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^
3*x^(31/2))/31
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{17/2}\left(b+cx^2\right)^3dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(b^3x^{17/2}+3b^2cx^{21/2}+3bc^2x^{25/2}+c^3x^{29/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{19}b^3x^{19/2}+\frac{6}{23}b^2cx^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

input
```
Int[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^
3*x^(31/2))/31
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{19}{2}}}{19}+\frac{6b^2c\,x^{\frac{23}{2}}}{23}+\frac{2b\,c^2x^{\frac{27}{2}}}{9}+\frac{2c^3x^{\frac{31}{2}}}{31}$	36
default	$\frac{2b^3x^{\frac{19}{2}}}{19}+\frac{6b^2c\,x^{\frac{23}{2}}}{23}+\frac{2b\,c^2x^{\frac{27}{2}}}{9}+\frac{2c^3x^{\frac{31}{2}}}{31}$	36
gosper	$\frac{2x^{\frac{19}{2}}\left(3933c^3x^6+13547b\,c^2x^4+15903b^2c\,x^2+6417b^3\right)}{121923}$	38
trager	$\frac{2x^{\frac{19}{2}}\left(3933c^3x^6+13547b\,c^2x^4+15903b^2c\,x^2+6417b^3\right)}{121923}$	38
risch	$\frac{2x^{\frac{19}{2}}\left(3933c^3x^6+13547b\,c^2x^4+15903b^2c\,x^2+6417b^3\right)}{121923}$	38
orering	$\frac{2\left(3933c^3x^6+13547b\,c^2x^4+15903b^2c\,x^2+6417b^3\right)x^{\frac{7}{2}}\left(c\,x^4+b\,x^2\right)^3}{121923\left(c\,x^2+b\right)^3}$	60

input
```
int(x^(5/2)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/19*b^3*x^(19/2)+6/23*b^2*c*x^(23/2)+2/9*b*c^2*x^(27/2)+2/31*c^3*x^(31/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{121923}\left(3933\,c^3x^{15}+13547\,bc^2x^{13}+15903\,b^2cx^{11}+6417\,b^3x^9\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
2/121923*(3933*c^3*x^15 + 13547*b*c^2*x^13 + 15903*b^2*c*x^11 + 6417*b^3*x
^9)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2b^3x^{\frac{19}{2}}}{19}+\frac{6b^2cx^{\frac{23}{2}}}{23}+\frac{2bc^2x^{\frac{27}{2}}}{9}+\frac{2c^3x^{\frac{31}{2}}}{31}$$

input
```
integrate(x**(5/2)*(c*x**4+b*x**2)**3,x)
```

output
```
2*b**3*x**(19/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x**(27/2)/9 + 2*c**
3*x**(31/2)/31
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,b^2cx^{\frac{23}{2}}+\frac{2}{19}\,b^3x^{\frac{19}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 2/19*b^3*x^
(19/2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,b^2cx^{\frac{23}{2}}+\frac{2}{19}\,b^3x^{\frac{19}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 2/19*b^3*x^
(19/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2\,b^3\,x^{19/2}}{19}+\frac{2\,c^3\,x^{31/2}}{31}+\frac{6\,b^2\,c\,x^{23/2}}{23}+\frac{2\,b\,c^2\,x^{27/2}}{9}$$

input
```
int(x^(5/2)*(b*x^2 + c*x^4)^3,x)
```

output
```
(2*b^3*x^(19/2))/19 + (2*c^3*x^(31/2))/31 + (6*b^2*c*x^(23/2))/23 + (2*b*c
^2*x^(27/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{5/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2\sqrt{x}\,x^9\left(3933c^3x^6+13547b\,c^2x^4+15903b^2c\,x^2+6417b^3\right)}{121923}$$

input
```
int(x^(5/2)*(c*x^4+b*x^2)^3,x)
```

output
```
(2*sqrt(x)*x**9*(6417*b**3 + 15903*b**2*c*x**2 + 13547*b*c**2*x**4 + 3933*
c**3*x**6))/121923
```

3.111 $\int x^{3/2}(bx^2+cx^4)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int x^{3/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{17}b^3x^{17/2}+\frac{2}{7}b^2cx^{21/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

output
```
2/17*b^3*x^(17/2)+2/7*b^2*c*x^(21/2)+6/25*b*c^2*x^(25/2)+2/29*c^3*x^(29/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int x^{3/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2\left(5075b^3x^{17/2}+12325b^2cx^{21/2}+10353bc^2x^{25/2}+2975c^3x^{29/2}\right)}{86275}$$

input
```
Integrate[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*(5075*b^3*x^(17/2) + 12325*b^2*c*x^(21/2) + 10353*b*c^2*x^(25/2) + 2975
*c^3*x^(29/2)))/86275
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(bx^2+cx^4\right)^3\,dx$$
$$\Big\downarrow \ 9$$
$$\int x^{15/2}\left(b+cx^2\right)^3dx$$
$$\Big\downarrow \ 244$$
$$\int \left(b^3x^{15/2}+3b^2cx^{19/2}+3bc^2x^{23/2}+c^3x^{27/2}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{2}{17}b^3x^{17/2}+\frac{2}{7}b^2cx^{21/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

input
```
Int[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*b^3*x^(17/2))/17 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2*x^(25/2))/25 + (2*c^
3*x^(29/2))/29
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{17}{2}}}{17}+\frac{2b^2c\,x^{\frac{21}{2}}}{7}+\frac{6b\,c^2x^{\frac{25}{2}}}{25}+\frac{2c^3x^{\frac{29}{2}}}{29}$	36
default	$\frac{2b^3x^{\frac{17}{2}}}{17}+\frac{2b^2c\,x^{\frac{21}{2}}}{7}+\frac{6b\,c^2x^{\frac{25}{2}}}{25}+\frac{2c^3x^{\frac{29}{2}}}{29}$	36
gosper	$\frac{2x^{\frac{17}{2}}\left(2975c^3x^6+10353b\,c^2x^4+12325b^2c\,x^2+5075b^3\right)}{86275}$	38
trager	$\frac{2x^{\frac{17}{2}}\left(2975c^3x^6+10353b\,c^2x^4+12325b^2c\,x^2+5075b^3\right)}{86275}$	38
risch	$\frac{2x^{\frac{17}{2}}\left(2975c^3x^6+10353b\,c^2x^4+12325b^2c\,x^2+5075b^3\right)}{86275}$	38
orering	$\frac{2\left(2975c^3x^6+10353b\,c^2x^4+12325b^2c\,x^2+5075b^3\right)x^{\frac{5}{2}}\left(c\,x^4+b\,x^2\right)^3}{86275\left(c\,x^2+b\right)^3}$	60

input
```
int(x^(3/2)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/17*b^3*x^(17/2)+2/7*b^2*c*x^(21/2)+6/25*b*c^2*x^(25/2)+2/29*c^3*x^(29/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^{3/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{86275}\left(2975\,c^3x^{14}+10353\,bc^2x^{12}+12325\,b^2cx^{10}+5075\,b^3x^8\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
2/86275*(2975*c^3*x^14 + 10353*b*c^2*x^12 + 12325*b^2*c*x^10 + 5075*b^3*x^
8)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{3/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2b^3x^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{21}{2}}}{7}+\frac{6bc^2x^{\frac{25}{2}}}{25}+\frac{2c^3x^{\frac{29}{2}}}{29}$$

input
```
integrate(x**(3/2)*(c*x**4+b*x**2)**3,x)
```

output
```
2*b**3*x**(17/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(25/2)/25 + 2*c**
3*x**(29/2)/29
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{3/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,b^2cx^{\frac{21}{2}}+\frac{2}{17}\,b^3x^{\frac{17}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/17*b^3*x^
(17/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{3/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,b^2cx^{\frac{21}{2}}+\frac{2}{17}\,b^3x^{\frac{17}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/17*b^3*x^
(17/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{3/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2\,b^3\,x^{17/2}}{17}+\frac{2\,c^3\,x^{29/2}}{29}+\frac{2\,b^2\,c\,x^{21/2}}{7}+\frac{6\,b\,c^2\,x^{25/2}}{25}$$

input
```
int(x^(3/2)*(b*x^2 + c*x^4)^3,x)
```

output
```
(2*b^3*x^(17/2))/17 + (2*c^3*x^(29/2))/29 + (2*b^2*c*x^(21/2))/7 + (6*b*c^
2*x^(25/2))/25
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{3/2}\left(bx^2+cx^4\right)^3\,dx=\frac{2\sqrt{x}\,x^8(2975c^3x^6+10353b\,c^2x^4+12325b^2c\,x^2+5075b^3)}{86275}$$

input
```
int(x^(3/2)*(c*x^4+b*x^2)^3,x)
```

output
```
(2*sqrt(x)*x**8*(5075*b**3 + 12325*b**2*c*x**2 + 10353*b*c**2*x**4 + 2975*
c**3*x**6))/86275
```

3.112 $\int \sqrt{x}(bx^2+cx^4)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \sqrt{x}(bx^2+cx^4)^3\,dx=\frac{2}{15}b^3x^{15/2}+\frac{6}{19}b^2cx^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

output
```
2/15*b^3*x^(15/2)+6/19*b^2*c*x^(19/2)+6/23*b*c^2*x^(23/2)+2/27*c^3*x^(27/2
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \sqrt{x}(bx^2+cx^4)^3\,dx=\frac{2x^{15/2}(3933b^3+9315b^2cx^2+7695bc^2x^4+2185c^3x^6)}{58995}$$

input
```
Integrate[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*x^(15/2)*(3933*b^3 + 9315*b^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x^6))/5
8995
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(bx^2+cx^4\right)^3\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{13/2}\left(b+cx^2\right)^3dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(b^3x^{13/2}+3b^2cx^{17/2}+3bc^2x^{21/2}+c^3x^{25/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{15}b^3x^{15/2}+\frac{6}{19}b^2cx^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

input
```
Int[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
```

output
```
(2*b^3*x^(15/2))/15 + (6*b^2*c*x^(19/2))/19 + (6*b*c^2*x^(23/2))/23 + (2*c
^3*x^(27/2))/27
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{15}{2}}}{15}+\frac{6b^2c\,x^{\frac{19}{2}}}{19}+\frac{6b\,c^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{27}{2}}}{27}$	36
default	$\frac{2b^3x^{\frac{15}{2}}}{15}+\frac{6b^2c\,x^{\frac{19}{2}}}{19}+\frac{6b\,c^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{27}{2}}}{27}$	36
gosper	$\frac{2x^{\frac{15}{2}}\left(2185c^3x^6+7695b\,c^2x^4+9315b^2c\,x^2+3933b^3\right)}{58995}$	38
trager	$\frac{2x^{\frac{15}{2}}\left(2185c^3x^6+7695b\,c^2x^4+9315b^2c\,x^2+3933b^3\right)}{58995}$	38
risch	$\frac{2x^{\frac{15}{2}}\left(2185c^3x^6+7695b\,c^2x^4+9315b^2c\,x^2+3933b^3\right)}{58995}$	38
orering	$\frac{2(2185c^3x^6+7695b\,c^2x^4+9315b^2c\,x^2+3933b^3)x^{\frac{3}{2}}\left(c\,x^4+b\,x^2\right)^3}{58995(c\,x^2+b)^3}$	60

input
```
int(x^(1/2)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/15*b^3*x^(15/2)+6/19*b^2*c*x^(19/2)+6/23*b*c^2*x^(23/2)+2/27*c^3*x^(27/2
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \sqrt{x}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{58995}\left(2185\,c^3x^{13}+7695\,bc^2x^{11}+9315\,b^2cx^9+3933\,b^3x^7\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
2/58995*(2185*c^3*x^13 + 7695*b*c^2*x^11 + 9315*b^2*c*x^9 + 3933*b^3*x^7)*
sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \sqrt{x}\left(bx^2+cx^4\right)^3\,dx=\frac{2b^3x^{\frac{15}{2}}}{15}+\frac{6b^2cx^{\frac{19}{2}}}{19}+\frac{6bc^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{27}{2}}}{27}$$

input
```
integrate(x**(1/2)*(c*x**4+b*x**2)**3,x)
```

output
```
2*b**3*x**(15/2)/15 + 6*b**2*c*x**(19/2)/19 + 6*b*c**2*x**(23/2)/23 + 2*c*
*3*x**(27/2)/27
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\,b^2cx^{\frac{19}{2}}+\frac{2}{15}\,b^3x^{\frac{15}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 2/15*b^3*x
^(15/2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}\left(bx^2+cx^4\right)^3\,dx=\frac{2}{27}c^3x^{\frac{27}{2}}+\frac{6}{23}bc^2x^{\frac{23}{2}}+\frac{6}{19}b^2cx^{\frac{19}{2}}+\frac{2}{15}b^3x^{\frac{15}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 2/15*b^3*x
^(15/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}\left(bx^2+cx^4\right)^3\,dx=\frac{2\,b^3\,x^{15/2}}{15}+\frac{2\,c^3\,x^{27/2}}{27}+\frac{6\,b^2\,c\,x^{19/2}}{19}+\frac{6\,b\,c^2\,x^{23/2}}{23}$$

input
```
int(x^(1/2)*(b*x^2 + c*x^4)^3,x)
```

output
```
(2*b^3*x^(15/2))/15 + (2*c^3*x^(27/2))/27 + (6*b^2*c*x^(19/2))/19 + (6*b*c
^2*x^(23/2))/23
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \sqrt{x}\left(bx^2+cx^4\right)^3\,dx=\frac{2\sqrt{x}\,x^7\left(2185c^3x^6+7695b\,c^2x^4+9315b^2c\,x^2+3933b^3\right)}{58995}$$

input
```
int(x^(1/2)*(c*x^4+b*x^2)^3,x)
```

output
```
(2*sqrt(x)*x**7*(3933*b**3 + 9315*b**2*c*x**2 + 7695*b*c**2*x**4 + 2185*c*
*3*x**6))/58995
```

3.113 $\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{13}b^3x^{13/2} + \frac{6}{17}b^2cx^{17/2} + \frac{2}{7}bc^2x^{21/2} + \frac{2}{25}c^3x^{25/2}$$

output
```
2/13*b^3*x^(13/2)+6/17*b^2*c*x^(17/2)+2/7*b*c^2*x^(21/2)+2/25*c^3*x^(25/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2x^{13/2}(2975b^3 + 6825b^2cx^2 + 5525bc^2x^4 + 1547c^3x^6)}{38675}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/Sqrt[x],x]
```

output
```
(2*x^(13/2)*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x^6))/3
8675
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^{11/2}(b+cx^2)^3\,dx$$
$$\Big\downarrow \text{ 244}$$
$$\int \left(b^3x^{11/2}+3b^2cx^{15/2}+3bc^2x^{19/2}+c^3x^{23/2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2}{13}b^3x^{13/2}+\frac{6}{17}b^2cx^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

input
```
Int[(b*x^2 + c*x^4)^3/Sqrt[x],x]
```

output
```
(2*b^3*x^(13/2))/13 + (6*b^2*c*x^(17/2))/17 + (2*b*c^2*x^(21/2))/7 + (2*c^
3*x^(25/2))/25
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{13}{2}}}{13}+\frac{6b^2c\,x^{\frac{17}{2}}}{17}+\frac{2b\,c^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{25}{2}}}{25}$	36
default	$\frac{2b^3x^{\frac{13}{2}}}{13}+\frac{6b^2c\,x^{\frac{17}{2}}}{17}+\frac{2b\,c^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{25}{2}}}{25}$	36
gosper	$\frac{2x^{\frac{13}{2}}\left(1547c^3x^6+5525b\,c^2x^4+6825b^2c\,x^2+2975b^3\right)}{38675}$	38
trager	$\frac{2x^{\frac{13}{2}}\left(1547c^3x^6+5525b\,c^2x^4+6825b^2c\,x^2+2975b^3\right)}{38675}$	38
risch	$\frac{2x^{\frac{13}{2}}\left(1547c^3x^6+5525b\,c^2x^4+6825b^2c\,x^2+2975b^3\right)}{38675}$	38
orering	$\frac{2\left(1547c^3x^6+5525b\,c^2x^4+6825b^2c\,x^2+2975b^3\right)\sqrt{x}\,\left(c\,x^4+b\,x^2\right)^3}{38675\left(c\,x^2+b\right)^3}$	60

input
```
int((c*x^4+b*x^2)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/13*b^3*x^(13/2)+6/17*b^2*c*x^(17/2)+2/7*b*c^2*x^(21/2)+2/25*c^3*x^(25/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx=\frac{2}{38675}\left(1547\,c^3x^{12}+5525\,bc^2x^{10}+6825\,b^2cx^8+2975\,b^3x^6\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/38675*(1547*c^3*x^12 + 5525*b*c^2*x^10 + 6825*b^2*c*x^8 + 2975*b^3*x^6)*
sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{\left(bx^2+cx^4\right)^3}{\sqrt{x}}\,dx=\frac{2b^3x^{\frac{13}{2}}}{13}+\frac{6b^2cx^{\frac{17}{2}}}{17}+\frac{2bc^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{25}{2}}}{25}$$

input
```
integrate((c*x**4+b*x**2)**3/x**(1/2),x)
```

output
```
2*b**3*x**(13/2)/13 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)/7 + 2*c**
3*x**(25/2)/25
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{\left(bx^2+cx^4\right)^3}{\sqrt{x}}\,dx=\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

output
```
2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^
(13/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{25}\,c^3x^{\frac{25}{2}} + \frac{2}{7}\,bc^2x^{\frac{21}{2}} + \frac{6}{17}\,b^2cx^{\frac{17}{2}} + \frac{2}{13}\,b^3x^{\frac{13}{2}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

output
```
2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^
(13/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2\,b^3\,x^{13/2}}{13} + \frac{2\,c^3\,x^{25/2}}{25} + \frac{6\,b^2\,c\,x^{17/2}}{17} + \frac{2\,b\,c^2\,x^{21/2}}{7}$$

input
```
int((b*x^2 + c*x^4)^3/x^(1/2),x)
```

output
```
(2*b^3*x^(13/2))/13 + (2*c^3*x^(25/2))/25 + (6*b^2*c*x^(17/2))/17 + (2*b*c
^2*x^(21/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,x^6(1547c^3x^6 + 5525b\,c^2x^4 + 6825b^2c\,x^2 + 2975b^3)}{38675}$$

input
```
int((c*x^4+b*x^2)^3/x^(1/2),x)
```

output
```
(2*sqrt(x)*x**6*(2975*b**3 + 6825*b**2*c*x**2 + 5525*b*c**2*x**4 + 1547*c*
*3*x**6))/38675
```

3.114 $\int \frac{(bx^2+cx^4)^3}{x^{3/2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{(bx^2 + cx^4)^3}{x^{3/2}} dx = \frac{2}{11}b^3x^{11/2} + \frac{2}{5}b^2cx^{15/2} + \frac{6}{19}bc^2x^{19/2} + \frac{2}{23}c^3x^{23/2}$$

output
```
2/11*b^3*x^(11/2)+2/5*b^2*c*x^(15/2)+6/19*b*c^2*x^(19/2)+2/23*c^3*x^(23/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2 + cx^4)^3}{x^{3/2}} dx = \frac{2x^{11/2}(2185b^3 + 4807b^2cx^2 + 3795bc^2x^4 + 1045c^3x^6)}{24035}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^(3/2),x]
```

output
```
(2*x^(11/2)*(2185*b^3 + 4807*b^2*c*x^2 + 3795*b*c^2*x^4 + 1045*c^3*x^6))/2
4035
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^{3/2}}\,dx$$
$$\Big\downarrow \ 9$$
$$\int x^{9/2}(b+cx^2)^3\,dx$$
$$\Big\downarrow \ 244$$
$$\int \left(b^3x^{9/2}+3b^2cx^{13/2}+3bc^2x^{17/2}+c^3x^{21/2}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{2}{11}b^3x^{11/2}+\frac{2}{5}b^2cx^{15/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

input
```
Int[(b*x^2 + c*x^4)^3/x^(3/2),x]
```

output
```
(2*b^3*x^(11/2))/11 + (2*b^2*c*x^(15/2))/5 + (6*b*c^2*x^(19/2))/19 + (2*c^
3*x^(23/2))/23
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{2b^2c\,x^{\frac{15}{2}}}{5}+\frac{6b\,c^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{23}{2}}}{23}$	36
default	$\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{2b^2c\,x^{\frac{15}{2}}}{5}+\frac{6b\,c^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{23}{2}}}{23}$	36
gosper	$\frac{2x^{\frac{11}{2}}\left(1045c^3x^6+3795b\,c^2x^4+4807b^2c\,x^2+2185b^3\right)}{24035}$	38
trager	$\frac{2x^{\frac{11}{2}}\left(1045c^3x^6+3795b\,c^2x^4+4807b^2c\,x^2+2185b^3\right)}{24035}$	38
risch	$\frac{2x^{\frac{11}{2}}\left(1045c^3x^6+3795b\,c^2x^4+4807b^2c\,x^2+2185b^3\right)}{24035}$	38
orering	$\frac{2\left(1045c^3x^6+3795b\,c^2x^4+4807b^2c\,x^2+2185b^3\right)\left(c\,x^4+b\,x^2\right)^3}{24035\sqrt{x}\,\left(c\,x^2+b\right)^3}$	60

input
```
int((c*x^4+b*x^2)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*b^3*x^(11/2)+2/5*b^2*c*x^(15/2)+6/19*b*c^2*x^(19/2)+2/23*c^3*x^(23/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{24035}\left(1045\,c^3x^{11}+3795\,bc^2x^9+4807\,b^2cx^7+2185\,b^3x^5\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="fricas")
```

output
```
2/24035*(1045*c^3*x^11 + 3795*b*c^2*x^9 + 4807*b^2*c*x^7 + 2185*b^3*x^5)*s
qrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2b^3x^{\frac{11}{2}}}{11} + \frac{2b^2cx^{\frac{15}{2}}}{5} + \frac{6bc^2x^{\frac{19}{2}}}{19} + \frac{2c^3x^{\frac{23}{2}}}{23}$$

input
```
integrate((c*x**4+b*x**2)**3/x**(3/2),x)
```

output
```
2*b**3*x**(11/2)/11 + 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/19 + 2*c**
3*x**(23/2)/23
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{23}\,c^3x^{\frac{23}{2}} + \frac{6}{19}\,bc^2x^{\frac{19}{2}} + \frac{2}{5}\,b^2cx^{\frac{15}{2}} + \frac{2}{11}\,b^3x^{\frac{11}{2}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="maxima")
```

output
```
2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/11*b^3*x^
(11/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{23}\,c^3x^{\frac{23}{2}} + \frac{6}{19}\,bc^2x^{\frac{19}{2}} + \frac{2}{5}\,b^2cx^{\frac{15}{2}} + \frac{2}{11}\,b^3x^{\frac{11}{2}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^(3/2),x, algorithm="giac")
```

output
```
2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/11*b^3*x^
(11/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2\,b^3\,x^{11/2}}{11} + \frac{2\,c^3\,x^{23/2}}{23} + \frac{2\,b^2\,c\,x^{15/2}}{5} + \frac{6\,b\,c^2\,x^{19/2}}{19}$$

input
```
int((b*x^2 + c*x^4)^3/x^(3/2),x)
```

output
```
(2*b^3*x^(11/2))/11 + (2*c^3*x^(23/2))/23 + (2*b^2*c*x^(15/2))/5 + (6*b*c^
2*x^(19/2))/19
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2\sqrt{x}\,x^5(1045c^3x^6+3795b\,c^2x^4+4807b^2c\,x^2+2185b^3)}{24035}$$

input
```
int((c*x^4+b*x^2)^3/x^(3/2),x)
```

output
```
(2*sqrt(x)*x**5*(2185*b**3 + 4807*b**2*c*x**2 + 3795*b*c**2*x**4 + 1045*c*
*3*x**6))/24035
```

3.115 $\int \frac{(bx^2+cx^4)^3}{x^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{9}b^3x^{9/2}+\frac{6}{13}b^2cx^{13/2}+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

output
```
2/9*b^3*x^(9/2)+6/13*b^2*c*x^(13/2)+6/17*b*c^2*x^(17/2)+2/21*c^3*x^(21/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2x^{9/2}(1547b^3+3213b^2cx^2+2457bc^2x^4+663c^3x^6)}{13923}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^(5/2),x]
```

output
```
(2*x^(9/2)*(1547*b^3 + 3213*b^2*c*x^2 + 2457*b*c^2*x^4 + 663*c^3*x^6))/139
23
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^{5/2}}\,dx$$

$$\Big\downarrow \ 9$$

$$\int x^{7/2}(b+cx^2)^3\,dx$$

$$\Big\downarrow \ 244$$

$$\int \left(b^3x^{7/2}+3b^2cx^{11/2}+3bc^2x^{15/2}+c^3x^{19/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{9}b^3x^{9/2}+\frac{6}{13}b^2cx^{13/2}+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

input
```
Int[(b*x^2 + c*x^4)^3/x^(5/2),x]
```

output
```
(2*b^3*x^(9/2))/9 + (6*b^2*c*x^(13/2))/13 + (6*b*c^2*x^(17/2))/17 + (2*c^3
*x^(21/2))/21
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6b^2c\,x^{\frac{13}{2}}}{13}+\frac{6b\,c^2x^{\frac{17}{2}}}{17}+\frac{2c^3x^{\frac{21}{2}}}{21}$	36
default	$\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6b^2c\,x^{\frac{13}{2}}}{13}+\frac{6b\,c^2x^{\frac{17}{2}}}{17}+\frac{2c^3x^{\frac{21}{2}}}{21}$	36
gosper	$\frac{2x^{\frac{9}{2}}\left(663c^3x^6+2457b\,c^2x^4+3213b^2c\,x^2+1547b^3\right)}{13923}$	38
trager	$\frac{2x^{\frac{9}{2}}\left(663c^3x^6+2457b\,c^2x^4+3213b^2c\,x^2+1547b^3\right)}{13923}$	38
risch	$\frac{2x^{\frac{9}{2}}\left(663c^3x^6+2457b\,c^2x^4+3213b^2c\,x^2+1547b^3\right)}{13923}$	38
orering	$\frac{2\left(663c^3x^6+2457b\,c^2x^4+3213b^2c\,x^2+1547b^3\right)\left(c\,x^4+b\,x^2\right)^3}{13923x^{\frac{3}{2}}\left(c\,x^2+b\right)^3}$	60

input
```
int((c*x^4+b*x^2)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*b^3*x^(9/2)+6/13*b^2*c*x^(13/2)+6/17*b*c^2*x^(17/2)+2/21*c^3*x^(21/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{(bx^2+cx^4)^3}{x^{5/2}}\,dx=\frac{2}{13923}\left(663\,c^3x^{10}+2457\,bc^2x^8+3213\,b^2cx^6+1547\,b^3x^4\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="fricas")
```

output
```
2/13923*(663*c^3*x^10 + 2457*b*c^2*x^8 + 3213*b^2*c*x^6 + 1547*b^3*x^4)*sq
rt(x)
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(bx^2 + cx^4)^3}{x^{5/2}} \, dx = \frac{2b^3x^{\frac{9}{2}}}{9} + \frac{6b^2cx^{\frac{13}{2}}}{13} + \frac{6bc^2x^{\frac{17}{2}}}{17} + \frac{2c^3x^{\frac{21}{2}}}{21}$$

input
```
integrate((c*x**4+b*x**2)**3/x**(5/2),x)
```

output
```
2*b**3*x**(9/2)/9 + 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/17 + 2*c**3
*x**(21/2)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2 + cx^4)^3}{x^{5/2}} \, dx = \frac{2}{21} \, c^3x^{\frac{21}{2}} + \frac{6}{17} \, bc^2x^{\frac{17}{2}} + \frac{6}{13} \, b^2cx^{\frac{13}{2}} + \frac{2}{9} \, b^3x^{\frac{9}{2}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="maxima")
```

output
```
2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 2/9*b^3*x^
(9/2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{21}\,c^3x^{\frac{21}{2}} + \frac{6}{17}\,bc^2x^{\frac{17}{2}} + \frac{6}{13}\,b^2cx^{\frac{13}{2}} + \frac{2}{9}\,b^3x^{\frac{9}{2}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^(5/2),x, algorithm="giac")
```

output
```
2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 2/9*b^3*x^
(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2\,b^3\,x^{9/2}}{9} + \frac{2\,c^3\,x^{21/2}}{21} + \frac{6\,b^2\,c\,x^{13/2}}{13} + \frac{6\,b\,c^2\,x^{17/2}}{17}$$

input
```
int((b*x^2 + c*x^4)^3/x^(5/2),x)
```

output
```
(2*b^3*x^(9/2))/9 + (2*c^3*x^(21/2))/21 + (6*b^2*c*x^(13/2))/13 + (6*b*c^2
*x^(17/2))/17
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2\sqrt{x}\,x^4(663c^3x^6+2457b\,c^2x^4+3213b^2c\,x^2+1547b^3)}{13923}$$

input
```
int((c*x^4+b*x^2)^3/x^(5/2),x)
```

output
```
(2*sqrt(x)*x**4*(1547*b**3 + 3213*b**2*c*x**2 + 2457*b*c**2*x**4 + 663*c**
3*x**6))/13923
```

3.116 $\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{7}b^3x^{7/2}+\frac{6}{11}b^2cx^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

output
```
2/7*b^3*x^(7/2)+6/11*b^2*c*x^(11/2)+2/5*b*c^2*x^(15/2)+2/19*c^3*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2x^{7/2}(1045b^3+1995b^2cx^2+1463bc^2x^4+385c^3x^6)}{7315}$$

input
```
Integrate[(b*x^2 + c*x^4)^3/x^(7/2),x]
```

output
```
(2*x^(7/2)*(1045*b^3 + 1995*b^2*c*x^2 + 1463*b*c^2*x^4 + 385*c^3*x^6))/731
5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{5/2}\left(b+cx^2\right)^3dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(b^3x^{5/2}+3b^2cx^{9/2}+3bc^2x^{13/2}+c^3x^{17/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{7}b^3x^{7/2}+\frac{6}{11}b^2cx^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

input
```
Int[(b*x^2 + c*x^4)^3/x^(7/2),x]
```

output
```
(2*b^3*x^(7/2))/7 + (6*b^2*c*x^(11/2))/11 + (2*b*c^2*x^(15/2))/5 + (2*c^3*
x^(19/2))/19
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2c\,x^{\frac{11}{2}}}{11}+\frac{2b\,c^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$	36
default	$\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2c\,x^{\frac{11}{2}}}{11}+\frac{2b\,c^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$	36
gosper	$\frac{2x^{\frac{7}{2}}\left(385c^3x^6+1463b\,c^2x^4+1995b^2c\,x^2+1045b^3\right)}{7315}$	38
trager	$\frac{2x^{\frac{7}{2}}\left(385c^3x^6+1463b\,c^2x^4+1995b^2c\,x^2+1045b^3\right)}{7315}$	38
risch	$\frac{2x^{\frac{7}{2}}\left(385c^3x^6+1463b\,c^2x^4+1995b^2c\,x^2+1045b^3\right)}{7315}$	38
orering	$\frac{2\left(385c^3x^6+1463b\,c^2x^4+1995b^2c\,x^2+1045b^3\right)\left(c\,x^4+b\,x^2\right)^3}{7315x^{\frac{5}{2}}\left(c\,x^2+b\right)^3}$	60

input
```
int((c*x^4+b*x^2)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*b^3*x^(7/2)+6/11*b^2*c*x^(11/2)+2/5*b*c^2*x^(15/2)+2/19*c^3*x^(19/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx=\frac{2}{7315}\left(385\,c^3x^9+1463\,bc^2x^7+1995\,b^2cx^5+1045\,b^3x^3\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="fricas")
```

output
```
2/7315*(385*c^3*x^9 + 1463*b*c^2*x^7 + 1995*b^2*c*x^5 + 1045*b^3*x^3)*sqrt
(x)
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2b^3x^{\frac{7}{2}}}{7} + \frac{6b^2cx^{\frac{11}{2}}}{11} + \frac{2bc^2x^{\frac{15}{2}}}{5} + \frac{2c^3x^{\frac{19}{2}}}{19}$$

input
```
integrate((c*x**4+b*x**2)**3/x**(7/2),x)
```

output
```
2*b**3*x**(7/2)/7 + 6*b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5 + 2*c**3*
x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{19}\,c^3x^{\frac{19}{2}} + \frac{2}{5}\,bc^2x^{\frac{15}{2}} + \frac{6}{11}\,b^2cx^{\frac{11}{2}} + \frac{2}{7}\,b^3x^{\frac{7}{2}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="maxima")
```

output
```
2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 2/7*b^3*x^(
7/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{19}\,c^3x^{\frac{19}{2}} + \frac{2}{5}\,bc^2x^{\frac{15}{2}} + \frac{6}{11}\,b^2cx^{\frac{11}{2}} + \frac{2}{7}\,b^3x^{\frac{7}{2}}$$

input
```
integrate((c*x^4+b*x^2)^3/x^(7/2),x, algorithm="giac")
```

output
```
2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 2/7*b^3*x^(
7/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2\,b^3\,x^{7/2}}{7} + \frac{2\,c^3\,x^{19/2}}{19} + \frac{6\,b^2\,c\,x^{11/2}}{11} + \frac{2\,b\,c^2\,x^{15/2}}{5}$$

input
```
int((b*x^2 + c*x^4)^3/x^(7/2),x)
```

output
```
(2*b^3*x^(7/2))/7 + (2*c^3*x^(19/2))/19 + (6*b^2*c*x^(11/2))/11 + (2*b*c^2
*x^(15/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2\sqrt{x}\,x^3(385c^3x^6 + 1463b\,c^2x^4 + 1995b^2c\,x^2 + 1045b^3)}{7315}$$

input
```
int((c*x^4+b*x^2)^3/x^(7/2),x)
```

output
```
(2*sqrt(x)*x**3*(1045*b**3 + 1995*b**2*c*x**2 + 1463*b*c**2*x**4 + 385*c**
3*x**6))/7315
```

3.117 $\int \frac{x^{13/2}}{bx^2+cx^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 162

$$\int \frac{x^{13/2}}{bx^2+cx^4} dx = -\frac{2bx^{3/2}}{3c^2}+\frac{2x^{7/2}}{7c}-\frac{b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}} +\frac{b^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}}-\frac{b^{7/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{11/4}}$$

output
```
-2/3*b*x^(3/2)/c^2+2/7*x^(7/2)/c-1/2*b^(7/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1
/2)/b^(1/4))*2^(1/2)/c^(11/4)+1/2*b^(7/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)
/b^(1/4))*2^(1/2)/c^(11/4)-1/2*b^(7/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(
1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/c^(11/4)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.80

$$\int \frac{x^{13/2}}{bx^2+cx^4}\,dx = \frac{4c^{3/4}x^{3/2}(-7b+3cx^2)-21\sqrt{2}b^{7/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-21\sqrt{2}b^{7/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{42c^{11/4}}$$

input
```
Integrate[x^(13/2)/(b*x^2 + c*x^4),x]
```

output
```
(4*c^(3/4)*x^(3/2)*(-7*b + 3*c*x^2) - 21*Sqrt[2]*b^(7/4)*ArcTan[(Sqrt[b] -
 Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 21*Sqrt[2]*b^(7/4)*ArcTan
h[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(42*c^(11/4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.54, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {9, 262, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^{9/2}}{b+cx^2}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^{7/2}}{7c}-\frac{b\int \frac{x^{5/2}}{cx^2+b}dx}{c}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\int \frac{\sqrt{x}}{cx^2+b}dx}{c}\right)}{c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\int\frac{x}{cx^2+b}d\sqrt{x}}{c}\right)}{c}$$

$$\Big\downarrow \text{826}$$

$$\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

$$\Big\downarrow \text{217}$$

$$\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 1479

$$\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 25

$$\frac{\frac{2x^{7/2}}{7c}-b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

$\downarrow$ 27

$$\frac{\frac{2x^{7/2}}{7c}-b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

$\downarrow$ 1103

$$\frac{2x^{7/2}}{7c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{c}$$

input
```
Int[x^(13/2)/(b*x^2 + c*x^4),x]
```

output
```
(2*x^(7/2))/(7*c) - (b*((2*x^(3/2))/(3*c) - (2*b*((-(ArcTan[1 - (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)
*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*
Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/c))/c
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{2\left(-\frac{c\,x^{\frac{7}{2}}}{7}+\frac{b\,x^{\frac{3}{2}}}{3}\right)}{c^{2}}+\frac{b^{2}\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	128
default	$-\frac{2\left(-\frac{c\,x^{\frac{7}{2}}}{7}+\frac{b\,x^{\frac{3}{2}}}{3}\right)}{c^{2}}+\frac{b^{2}\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	128
risch	$-\frac{2x^{\frac{3}{2}}\left(-3c\,x^{2}+7b\right)}{21c^{2}}+\frac{b^{2}\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	128

input
```
int(x^(13/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/c^2*(-1/7*c*x^(7/2)+1/3*b*x^(3/2))+1/4/c^3*b^2/(1/c*b)^(1/4)*2^(1/2)*(l
n((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)
*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arcta
n(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.09

$$\int \frac{x^{13/2}}{bx^2 + cx^4}\,dx = \frac{21\,c^2\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\log\left(c^8\left(-\frac{b^7}{c^{11}}\right)^{\frac{3}{4}} + b^5\sqrt{x}\right) - 21i\,c^2\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\log\left(i\,c^8\left(-\frac{b^7}{c^{11}}\right)^{\frac{3}{4}} + b^5\sqrt{x}\right) +$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/42*(21*c^2*(-b^7/c^11)^(1/4)*log(c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) -
21*I*c^2*(-b^7/c^11)^(1/4)*log(I*c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) + 21
*I*c^2*(-b^7/c^11)^(1/4)*log(-I*c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) - 21*
c^2*(-b^7/c^11)^(1/4)*log(-c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) + 4*(3*c*x
^3 - 7*b*x)*sqrt(x))/c^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{bx^2 + cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)/(c*x**4+b*x**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.22

$$\int \frac{x^{13/2}}{bx^2+cx^4}\,dx = \frac{b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{4c^2} + \frac{2\left(3cx^{\frac{7}{2}}-7bx^{\frac{3}{2}}\right)}{21c^2}$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)
*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(
sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^
(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*l
og(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4
)))/c^2 + 2/21*(3*c*x^(7/2) - 7*b*x^(3/2))/c^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

$$\int \frac{x^{13/2}}{bx^2+cx^4}\,dx = \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^5} + \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^5} - \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\,b\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^5} + \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\,b\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^5} + \frac{2\left(3\,c^6x^{\frac{7}{2}}-7\,bc^5x^{\frac{3}{2}}\right)}{21\,c^7}$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(b*c^3)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/c^5 + 1/2*sqrt(2)*(b*c^3)^(3/4)*b*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/4*sqrt(2)*(b*c^3)^(
3/4)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/4*sqrt(2)*
(b*c^3)^(3/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 2/
21*(3*c^6*x^(7/2) - 7*b*c^5*x^(3/2))/c^7
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.41

$$\int \frac{x^{13/2}}{bx^2+cx^4}\,dx = \frac{2\,x^{7/2}}{7\,c} - \frac{2\,b\,x^{3/2}}{3\,c^2} + \frac{(-b)^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{c^{11/4}} + \frac{(-b)^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,1\mathrm{i}}{c^{11/4}}$$

input
```
int(x^(13/2)/(b*x^2 + c*x^4),x)
```

output
```
(2*x^(7/2))/(7*c) - (2*b*x^(3/2))/(3*c^2) + ((-b)^(7/4)*atan((c^(1/4)*x^(1
/2))/(-b)^(1/4)))/c^(11/4) + ((-b)^(7/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1
/4))*1i)/c^(11/4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98

$$\int \frac{x^{13/2}}{bx^2 + cx^4}\,dx = \frac{-42c^{\frac{1}{4}}b^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right) + 42c^{\frac{1}{4}}b^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right) + 21c^{\frac{1}{4}}b^{\frac{7}{4}}\sqrt{2}\log(}{}$$

input
```
int(x^(13/2)/(c*x^4+b*x^2),x)
```

output
```
( - 42*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b + 42*c**(1/4)*b**(3/4)*sqrt(2)*
atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sq
rt(2)))*b + 21*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*
sqrt(2) + sqrt(b) + sqrt(c)*x)*b - 21*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x
)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b - 56*sqrt(x)*b*c*x +
24*sqrt(x)*c**2*x**3)/(84*c**3)
```

3.118 $\int \frac{x^{11/2}}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{x^{11/2}}{bx^2+cx^4}\,dx = -\frac{2b\sqrt{x}}{c^2}+\frac{2x^{5/2}}{5c}-\frac{b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}}+\frac{b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}}+\frac{b^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{9/4}}$$

output
```
-2*b*x^(1/2)/c^2+2/5*x^(5/2)/c-1/2*b^(5/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1/2
)/b^(1/4))*2^(1/2)/c^(9/4)+1/2*b^(5/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^
(1/4))*2^(1/2)/c^(9/4)+1/2*b^(5/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)
/(b^(1/2)+c^(1/2)*x))*2^(1/2)/c^(9/4)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

$$\int \frac{x^{11/2}}{bx^2+cx^4}\,dx = \frac{4\sqrt[4]{c}\sqrt{x}(-5b+cx^2)-5\sqrt{2}b^{5/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+5\sqrt{2}b^{5/4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{10c^{9/4}}$$

input
```
Integrate[x^(11/2)/(b*x^2 + c*x^4),x]
```

output
```
(4*c^(1/4)*Sqrt[x]*(-5*b + c*x^2) - 5*Sqrt[2]*b^(5/4)*ArcTan[(Sqrt[b] - Sq
rt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sqrt[2]*b^(5/4)*ArcTanh[(S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(10*c^(9/4))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.56, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {9, 262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{bx^2+cx^4}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{7/2}}{b+cx^2}dx$$

$$\Big\downarrow \; 262$$

$$\frac{2x^{5/2}}{5c}-\frac{b\int \frac{x^{3/2}}{cx^2+b}dx}{c}$$

$$\Big\downarrow \; 262$$

$$\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{b\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{c}\right)}{c}$$

$$\Big\downarrow \; 266$$

$$\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{cx^2+b}d\sqrt{x}}{c}\right)}{c}$$

$$\Big\downarrow \; 755$$

$$\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$$\Big\downarrow \; 1476$$

$$\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$$\Big\downarrow \; 1082$$

$$\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$$\Big\downarrow \; 217$$

$$\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$\downarrow$ 1479

$$\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$\downarrow$ 25

$$\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

$\downarrow$ 27

$$\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

Note: in both expressions above, the numerator also begins with $\frac{2x^{5/2}}{5c}-$ printed above the bracket.

$\downarrow$ 1103

$$\frac{\frac{2x^{5/2}}{5c}-b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}$$

input
```
Int[x^(11/2)/(b*x^2 + c*x^4),x]
```

output
```
(2*x^(5/2))/(5*c) - (b*((2*Sqrt[x])/c - (2*b*((-(ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[S
qrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(
1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt
[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/c))/c
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{2\left(-\frac{c\,x^{\frac{5}{2}}}{5}+b\sqrt{x}\right)}{c^2}+\frac{b\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^2}$
default	$-\frac{2\left(-\frac{c\,x^{\frac{5}{2}}}{5}+b\sqrt{x}\right)}{c^2}+\frac{b\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^2}$
risch	$-\frac{2\left(-c\,x^2+5b\right)\sqrt{x}}{5c^2}+\frac{b\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^2}$

input
```
int(x^(11/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/c^2*(-1/5*c*x^(5/2)+b*x^(1/2))+1/4/c^2*b*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(
1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/
2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1
/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.05

$$\int \frac{x^{11/2}}{bx^2+cx^4}\,dx=\frac{5\,c^2\left(-\frac{b^5}{c^9}\right)^{\frac{1}{4}}\log\left(c^2\left(-\frac{b^5}{c^9}\right)^{\frac{1}{4}}+b\sqrt{x}\right)+5i\,c^2\left(-\frac{b^5}{c^9}\right)^{\frac{1}{4}}\log\left(i\,c^2\left(-\frac{b^5}{c^9}\right)^{\frac{1}{4}}+b\sqrt{x}\right)-5i\,c^2\left(-\right.}{}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/10*(5*c^2*(-b^5/c^9)^(1/4)*log(c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) + 5*I*c
^2*(-b^5/c^9)^(1/4)*log(I*c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) - 5*I*c^2*(-b^
5/c^9)^(1/4)*log(-I*c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) - 5*c^2*(-b^5/c^9)^(
1/4)*log(-c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) + 4*(c*x^2 - 5*b)*sqrt(x))/c^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{bx^2 + cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)/(c*x**4+b*x**2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.22

$$\int \frac{x^{11/2}}{bx^2 + cx^4}\,dx = \frac{2\left(cx^{\frac{5}{2}} - 5\,b\sqrt{x}\right)}{5\,c^2} + \frac{\frac{2\,\sqrt{2}b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\,\sqrt{2}b^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}b^{\frac{5}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{c^{\frac{1}{4}}} - \frac{\sqrt{2}b^{\frac{5}{4}}\log}{}}{4\,c^2}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
2/5*(c*x^(5/2) - 5*b*sqrt(x))/c^2 + 1/4*(2*sqrt(2)*b^(3/2)*arctan(1/2*sqrt
(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/s
qrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*b^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1
/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(
c)) + sqrt(2)*b^(5/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sq
rt(b))/c^(1/4) - sqrt(2)*b^(5/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sq
rt(c)*x + sqrt(b))/c^(1/4))/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.23

$$\int \frac{x^{11/2}}{bx^2+cx^4}\,dx = \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^3} + \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^3} + \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\,b\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^3} - \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\,b\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^3} + \frac{2\left(c^4x^{\frac{5}{2}}-5\,bc^3\sqrt{x}\right)}{5\,c^5}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(b*c^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/c^3 + 1/2*sqrt(2)*(b*c^3)^(1/4)*b*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^3 + 1/4*sqrt(2)*(b*c^3)^(
1/4)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 - 1/4*sqrt(2)*
(b*c^3)^(1/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 2/
5*(c^4*x^(5/2) - 5*b*c^3*sqrt(x))/c^5
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.42

$$\int \frac{x^{11/2}}{bx^2+cx^4}\,dx=\frac{2\,x^{5/2}}{5\,c}-\frac{2\,b\,\sqrt{x}}{c^2}-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{c^{9/4}}+\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,1\mathrm{i}}{c^{9/4}}$$

input
```
int(x^(11/2)/(b*x^2 + c*x^4),x)
```

output
```
(2*x^(5/2))/(5*c) - (2*b*x^(1/2))/c^2 - ((-b)^(5/4)*atan((c^(1/4)*x^(1/2))
/(-b)^(1/4)))/c^(9/4) + ((-b)^(5/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*
1i)/c^(9/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int \frac{x^{11/2}}{bx^2+cx^4}\,dx=\frac{-10c^{\frac{3}{4}}b^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)+10c^{\frac{3}{4}}b^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)-5c^{\frac{3}{4}}b^{\frac{5}{4}}\sqrt{2}\log\left(\cdots\right.}{}$$

input
```
int(x^(11/2)/(c*x^4+b*x^2),x)
```

output
```
( - 10*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b + 10*c**(3/4)*b**(1/4)*sqrt(2)*
atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sq
rt(2)))*b - 5*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*s
qrt(2) + sqrt(b) + sqrt(c)*x)*b + 5*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*
c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b - 40*sqrt(x)*b*c + 8*sq
rt(x)*c**2*x**2)/(20*c**3)
```

3.119 $\int \frac{x^{9/2}}{bx^2+cx^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 148

$$\int \frac{x^{9/2}}{bx^2+cx^4} dx = \frac{2x^{3/2}}{3c} + \frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{7/4}} - \frac{b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{7/4}} + \frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{7/4}}$$

output
```
2/3*x^(3/2)/c+1/2*b^(3/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2
)/c^(7/4)-1/2*b^(3/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/c^
(7/4)+1/2*b^(3/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)
*x))*2^(1/2)/c^(7/4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \frac{x^{9/2}}{bx^2+cx^4}\,dx = \frac{4c^{3/4}x^{3/2}+3\sqrt{2}b^{3/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+3\sqrt{2}b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{6c^{7/4}}$$

input
```
Integrate[x^(9/2)/(b*x^2 + c*x^4),x]
```

output
```
(4*c^(3/4)*x^(3/2) + 3*Sqrt[2]*b^(3/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 3*Sqrt[2]*b^(3/4)*ArcTanh[(Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(6*c^(7/4))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.56, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {9, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{bx^2+cx^4}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^{5/2}}{b+cx^2}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^{3/2}}{3c}-\frac{b\int \frac{\sqrt{x}}{cx^2+b}dx}{c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2x^{3/2}}{3c}-\frac{2b\int \frac{x}{cx^2+b}d\sqrt{x}}{c}$$

$$\Big\downarrow \text{826}$$

$$\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}$$

$$\Big\downarrow \text{217}$$

$$\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}$$

$\Big\downarrow$ 25

$$\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}$$

$\Big\downarrow$ 27

$$\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{c}$$

$\Big\downarrow$ 1103

$$\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}$$

input
```
Int[x^(9/2)/(b*x^2 + c*x^4),x]
```

output
```
(2*x^(3/2))/(3*c) - (2*b*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b]
+ Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4))
)/(2*Sqrt[c])))/c
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	116
default	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	116
risch	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	116

input
```
int(x^(9/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)/c-1/4/c^2*b/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)
*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2
*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^
(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.07

$$\int \frac{x^{9/2}}{bx^2+cx^4}\,dx =$$

$$-\frac{3\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\log\left(c^5\left(-\frac{b^3}{c^7}\right)^{\frac{3}{4}}+b^2\sqrt{x}\right)-3i\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\log\left(i\,c^5\left(-\frac{b^3}{c^7}\right)^{\frac{3}{4}}+b^2\sqrt{x}\right)+3i\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\log\left(-i\,c^5\left(\right.\right.}{6\,c}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/6*(3*c*(-b^3/c^7)^(1/4)*log(c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) - 3*I*c
*(-b^3/c^7)^(1/4)*log(I*c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) + 3*I*c*(-b^3/
c^7)^(1/4)*log(-I*c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) - 3*c*(-b^3/c^7)^(1/
4)*log(-c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) - 4*x^(3/2))/c
```

Sympy [A] (verification not implemented)

Time = 91.92 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{x^{9/2}}{bx^2+cx^4}\,dx = \begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } b=0 \wedge c=0 \\ \frac{2x^{\frac{7}{2}}}{7b} & \text{for } c=0 \\ \frac{2x^{\frac{3}{2}}}{3c} & \text{for } b=0 \\ -\frac{b\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2c^2\sqrt[4]{-\frac{b}{c}}}+\frac{b\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2c^2\sqrt[4]{-\frac{b}{c}}}-\frac{b\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c^2\sqrt[4]{-\frac{b}{c}}}+\frac{2x^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(9/2)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(b, 0) & Eq(c, 0)), (2*x**(7/2)/(7*b), Eq(c, 0)
), (2*x**(3/2)/(3*c), Eq(b, 0)), (-b*log(sqrt(x) - (-b/c)**(1/4))/(2*c**2*
(-b/c)**(1/4)) + b*log(sqrt(x) + (-b/c)**(1/4))/(2*c**2*(-b/c)**(1/4)) - b
*atan(sqrt(x)/(-b/c)**(1/4))/(c**2*(-b/c)**(1/4)) + 2*x**(3/2)/(3*c), True
))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.26

$$\int \frac{x^{9/2}}{bx^2+cx^4}\,dx = -\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{4c} + \frac{2x^{\frac{3}{2}}}{3c}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(
1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*lo
g(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)
))/c + 2/3*x^(3/2)/c
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.20

$$\int \frac{x^{9/2}}{bx^2+cx^4}\,dx = \frac{2x^{\frac{3}{2}}}{3c} - \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^4} - \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^4} + \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^4} - \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^4}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
2/3*x^(3/2)/c - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/4*sqrt(
2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/
4*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/
c^4
```

Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.36

$$\int \frac{x^{9/2}}{bx^2+cx^4}\,dx = \frac{2\,x^{3/2}}{3\,c} + \frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{c^{7/4}} - \frac{(-b)^{3/4}\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{c^{7/4}}$$

input
```
int(x^(9/2)/(b*x^2 + c*x^4),x)
```

output
```
(2*x^(3/2))/(3*c) + ((-b)^(3/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(7/4
) - ((-b)^(3/4)*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(7/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99

$$\int \frac{x^{9/2}}{bx^2+cx^4}\,dx = \frac{6c^{\frac{1}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right) - 6c^{\frac{1}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right) - 3c^{\frac{1}{4}}b^{\frac{3}{4}}\sqrt{2}\log\left(-\sqrt{\right.}{12c^2}$$

input
```
int(x^(9/2)/(c*x^4+b*x^2),x)
```

output
```
(6*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*s
qrt(c))/(c**(1/4)*b**(1/4)*sqrt(2))) - 6*c**(1/4)*b**(3/4)*sqrt(2)*atan((c
**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))
 - 3*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) +
sqrt(b) + sqrt(c)*x) + 3*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b*
*(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x) + 8*sqrt(x)*c*x)/(12*c**2)
```

3.120 $\int \frac{x^{7/2}}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 147

$$\int \frac{x^{7/2}}{bx^2+cx^4}\,dx = \frac{2\sqrt{x}}{c}+\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{5/4}} -\frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{5/4}}-\frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{5/4}}$$

output
```
2*x^(1/2)/c+1/2*b^(1/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/
c^(5/4)-1/2*b^(1/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/c^(5
/4)-1/2*b^(1/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x
))*2^(1/2)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.80

$$\int \frac{x^{7/2}}{bx^2+cx^4}\,dx=\frac{4\sqrt[4]{c}\sqrt{x}+\sqrt{2}\sqrt[4]{b}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-\sqrt{2}\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{2c^{5/4}}$$

input
```
Integrate[x^(7/2)/(b*x^2 + c*x^4),x]
```

output
```
(4*c^(1/4)*Sqrt[x] + Sqrt[2]*b^(1/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x])] - Sqrt[2]*b^(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(2*c^(5/4))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.56, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {9, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{bx^2+cx^4}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^{3/2}}{b+cx^2}dx$$

$$\Big\downarrow \ 262$$

$$\frac{2\sqrt{x}}{c}-\frac{b\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{c}$$

$$\Big\downarrow \ 266$$

$$\frac{2\sqrt{x}}{c}-\frac{2b\int \frac{1}{cx^2+b}d\sqrt{x}}{c}$$

$$\Big\downarrow \text{755}$$

$$\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow \; 25$$

$$\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}$$

$$\Big\downarrow \; 1103$$

$$\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}$$

input
```
Int[x^(7/2)/(b*x^2 + c*x^4),x]
```

output
```
(2*Sqrt[x])/c - (2*b*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sq
rt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(S
qrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2
*Sqrt[b])))/c
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c}$	115
default	$\frac{2\sqrt{x}}{c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c}$	115
risch	$\frac{2\sqrt{x}}{c}-\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c}$	115

input
```
int(x^(7/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/c-1/4/c*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/
2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arcta
n(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-
1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.80

$$\int \frac{x^{7/2}}{bx^2+cx^4}\,dx =$$

$$-\frac{c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)+i\,c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(i\,c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-i\,c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(-i\,c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-}{2\,c}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/2*(c*(-b/c^5)^(1/4)*log(c*(-b/c^5)^(1/4) + sqrt(x)) + I*c*(-b/c^5)^(1/4
)*log(I*c*(-b/c^5)^(1/4) + sqrt(x)) - I*c*(-b/c^5)^(1/4)*log(-I*c*(-b/c^5)
^(1/4) + sqrt(x)) - c*(-b/c^5)^(1/4)*log(-c*(-b/c^5)^(1/4) + sqrt(x)) - 4*
sqrt(x))/c
```

Sympy [A] (verification not implemented)

Time = 46.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.75

$$\int \frac{x^{7/2}}{bx^2+cx^4}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} & \text{for } b=0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for } c=0 \\ \frac{2\sqrt{x}}{c} & \text{for } b=0 \\ \frac{2\sqrt{x}}{c} + \frac{\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2c} - \frac{\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2c} - \frac{\sqrt[4]{-\frac{b}{c}}\,\text{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c} & \text{otherwis} \end{cases}$$

input
```
integrate(x**(7/2)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(b, 0) & Eq(c, 0)), (2*x**(5/2)/(5*b), Eq(c, 0))
, (2*sqrt(x)/c, Eq(b, 0)), (2*sqrt(x)/c + (-b/c)**(1/4)*log(sqrt(x) - (-b/
c)**(1/4))/(2*c) - (-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*c) - (-b/
c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/c, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.26

$$\int \frac{x^{7/2}}{bx^2+cx^4}\,dx =$$

$$-\frac{\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{c^{\frac{1}{4}}} - \frac{\sqrt{2}b^{\frac{1}{4}}\,\text{lo}}{}}{4c}$$

$$+ \frac{2\sqrt{x}}{c}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sq
rt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*sq
rt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sq
rt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + sqrt(2)*b^(1/4)*log(sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4) - sqrt(2)*b^(1/4)*lo
g(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4))/c + 2*s
qrt(x)/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.21

$$\int \frac{x^{7/2}}{bx^2+cx^4}\,dx = -\frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^2} - \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^2} - \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^2} + \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^2} + \frac{2\sqrt{x}}{c}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^2 - 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^2 - 1/4*sqrt(2)*(b*c^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^2 + 1/4*sqrt(2)*(b*c^
3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^2 + 2*sqrt(x)
/c
```

Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.37

$$\int \frac{x^{7/2}}{bx^2+cx^4}\,dx=\frac{2\sqrt{x}}{c}-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{c^{5/4}}-\frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{c^{5/4}}$$

input
```
int(x^(7/2)/(b*x^2 + c*x^4),x)
```

output
```
(2*x^(1/2))/c - ((-b)^(1/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(5/4) -
((-b)^(1/4)*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/c^(5/4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{x^{7/2}}{bx^2+cx^4}\,dx=\frac{2c^{\frac{3}{4}}b^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)-2c^{\frac{3}{4}}b^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)+c^{\frac{3}{4}}b^{\frac{1}{4}}\sqrt{2}\log\left(-\sqrt{x}\right.}{4c^2}$$

input
```
int(x^(7/2)/(c*x^4+b*x^2),x)
```

output
```
(2*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*s
qrt(c))/(c**(1/4)*b**(1/4)*sqrt(2))) - 2*c**(3/4)*b**(1/4)*sqrt(2)*atan((c
**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))
 + c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sq
rt(b) + sqrt(c)*x) - c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/
4)*sqrt(2) + sqrt(b) + sqrt(c)*x) + 8*sqrt(x)*c)/(4*c**2)
```

3.121 $\int \frac{x^{5/2}}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{x^{5/2}}{bx^2+cx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}}$$

output
```
-1/2*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(1/4)/c^(3/4)+1/2
*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(1/4)/c^(3/4)-1/2*arc
tanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(1/4)/
c^(3/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.66

$$\int \frac{x^{5/2}}{bx^2+cx^4}\,dx=-\frac{\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}}$$

input

```
Integrate[x^(5/2)/(b*x^2 + c*x^4),x]
```

output

```
-((ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + ArcTa
nh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(1
/4)*c^(3/4)))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.56, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.526$, Rules used = {9, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{\sqrt{x}}{b+cx^2}dx$$

$$\Big\downarrow \text{266}$$

$$2\int \frac{x}{cx^2+b}d\sqrt{x}$$

$$\Big\downarrow \text{826}$$

$$2\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \ 1103$$

$$2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)$$

input
```
Int[x^(5/2)/(b*x^2 + c*x^4),x]
```

output
```
2*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/
4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1
/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	106
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	106

input
```
int(x^(5/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(
1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1
/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int \frac{x^{5/2}}{bx^2 + cx^4}\,dx = \frac{1}{2}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\log\left(bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}} + \sqrt{x}\right) - \frac{1}{2}i\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\log\left(i\,bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}} + \sqrt{x}\right) + \frac{1}{2}i\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\log\left(-i\,bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}} + \sqrt{x}\right) - \frac{1}{2}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\log\left(-bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}} + \sqrt{x}\right)$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/2*(-1/(b*c^3))^(1/4)*log(b*c^2*(-1/(b*c^3))^(3/4) + sqrt(x)) - 1/2*I*(-1
/(b*c^3))^(1/4)*log(I*b*c^2*(-1/(b*c^3))^(3/4) + sqrt(x)) + 1/2*I*(-1/(b*c
^3))^(1/4)*log(-I*b*c^2*(-1/(b*c^3))^(3/4) + sqrt(x)) - 1/2*(-1/(b*c^3))^(
1/4)*log(-b*c^2*(-1/(b*c^3))^(3/4) + sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 26.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{x^{5/2}}{bx^2 + cx^4}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\ b = 0 \wedge c = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for}\ c = 0 \\ -\frac{2}{c\sqrt{x}} & \text{for}\ b = 0 \\ \frac{\log\left(\sqrt{x} - \sqrt[4]{-\frac{b}{c}}\right)}{2c\sqrt[4]{-\frac{b}{c}}} - \frac{\log\left(\sqrt{x} + \sqrt[4]{-\frac{b}{c}}\right)}{2c\sqrt[4]{-\frac{b}{c}}} + \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c\sqrt[4]{-\frac{b}{c}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(5/2)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(b, 0) & Eq(c, 0)), (2*x**(3/2)/(3*b), Eq(c, 0))
, (-2/(c*sqrt(x)), Eq(b, 0)), (log(sqrt(x) - (-b/c)**(1/4))/(2*c*(-b/c)**(
1/4)) - log(sqrt(x) + (-b/c)**(1/4))/(2*c*(-b/c)**(1/4)) + atan(sqrt(x)/(-
b/c)**(1/4))/(c*(-b/c)**(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int \frac{x^{5/2}}{bx^2+cx^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{4\,b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{4\,b^{\frac{1}{4}}c^{\frac{3}{4}}}$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x
))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 1/2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(
b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - 1/4*sqrt(2)*log(sqrt(2)*b^(
1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + 1/4*sqrt(2
)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(
3/4))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.33

$$\int \frac{x^{5/2}}{bx^2+cx^4}\,dx = \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^3} + \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^3} - \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^3} + \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^3}$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b*c^3) + 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/4*sqrt(2)*(b*c^
3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) + 1/4*sq
rt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c
^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.28

$$\int \frac{x^{5/2}}{bx^2+cx^4}\,dx = \frac{\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)-\mathrm{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{(-b)^{1/4}\,c^{3/4}}$$

input
```
int(x^(5/2)/(b*x^2 + c*x^4),x)
```

output
```
(atan((c^(1/4)*x^(1/2))/(-b)^(1/4)) - atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))
/((-b)^(1/4)*c^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{x^{5/2}}{bx^2+cx^4}\,dx = \frac{\sqrt{2}\left(-2atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)+\log\left(-\sqrt{x}\,c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+\sqrt{b}+\right.\right.}{4c^{\frac{3}{4}}b^{\frac{1}{4}}}$$

input
```
int(x^(5/2)/(c*x^4+b*x^2),x)
```

output
```
(c**(1/4)*b**(3/4)*sqrt(2)*( - 2*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2))) + 2*atan((c**(1/4)*b**(1/4)*sqrt(
2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2))) + log( - sqrt(x)*c**(
1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x) - log(sqrt(x)*c**(1/4)*b**(1/
4)*sqrt(2) + sqrt(b) + sqrt(c)*x)))/(4*b*c)
```

3.122 $\int \frac{x^{3/2}}{bx^2+cx^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int \frac{x^{3/2}}{bx^2+cx^4} \, dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}}$$

output
```
-1/2*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(3/4)/c^(1/4)+1/2
*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(3/4)/c^(1/4)+1/2*arc
tanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(3/4)/
c^(1/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.68

$$\int \frac{x^{3/2}}{bx^2+cx^4}\,dx=\frac{-\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}}$$

input
```
Integrate[x^(3/2)/(b*x^2 + c*x^4),x]
```

output
```
(-ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + ArcTan
h[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(3/
4)*c^(1/4))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.57, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {9, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{\sqrt{x}\,(b+cx^2)}dx$$

$$\Big\downarrow \text{ 266}$$

$$2\int \frac{1}{cx^2+b}d\sqrt{x}$$

$$\Big\downarrow \text{ 755}$$

$$2\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \; 1103$$

$$2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)$$

input

```
Int[x^(3/2)/(b*x^2 + c*x^4),x]
```

output

```
2*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/
4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1
/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b}$	106
default	$\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b}$	106

input
```
int(x^(3/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(
1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1
/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{x^{3/2}}{bx^2+cx^4}\,dx = \frac{1}{2}\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}+\sqrt{x}\right) + \frac{1}{2}i\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\log\left(i\,b\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}+\sqrt{x}\right) - \frac{1}{2}i\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\log\left(-i\,b\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}+\sqrt{x}\right) - \frac{1}{2}\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}+\sqrt{x}\right)$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/2*(-1/(b^3*c))^(1/4)*log(b*(-1/(b^3*c))^(1/4) + sqrt(x)) + 1/2*I*(-1/(b^
3*c))^(1/4)*log(I*b*(-1/(b^3*c))^(1/4) + sqrt(x)) - 1/2*I*(-1/(b^3*c))^(1/
4)*log(-I*b*(-1/(b^3*c))^(1/4) + sqrt(x)) - 1/2*(-1/(b^3*c))^(1/4)*log(-b*
(-1/(b^3*c))^(1/4) + sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{x^{3/2}}{bx^2+cx^4}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } b=0 \wedge c= \\ -\frac{2}{3cx^{\frac{3}{2}}} & \text{for } b=0 \\ \frac{2\sqrt{x}}{b} & \text{for } c=0 \\ -\frac{\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b} + \frac{\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b} + \frac{\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(3/2)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(b, 0) & Eq(c, 0)), (-2/(3*c*x**(3/2)), Eq(b, 0
)), (2*sqrt(x)/b, Eq(c, 0)), (-(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/
(2*b) + (-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b) + (-b/c)**(1/4)*a
tan(sqrt(x)/(-b/c)**(1/4))/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int \frac{x^{3/2}}{bx^2+cx^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{2\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{2\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{4\,b^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{4\,b^{\frac{3}{4}}c^{\frac{1}{4}}}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x
))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 1/2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(
b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 1/4*sqrt(2)*log(sqrt(2)*b^(
1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - 1/4*sqrt(2
)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(
1/4))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.34

$$\int \frac{x^{3/2}}{bx^2+cx^4}\,dx = \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc} + \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc} + \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc} - \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b*c) + 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c) + 1/4*sqrt(2)*(b*c^3)^(
1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c) - 1/4*sqrt(2)*
(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.27

$$\int \frac{x^{3/2}}{bx^2+cx^4}\,dx = -\frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)+\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{(-b)^{3/4}\,c^{1/4}}$$

input
```
int(x^(3/2)/(b*x^2 + c*x^4),x)
```

output
```
-(atan((c^(1/4)*x^(1/2))/(-b)^(1/4)) + atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))
)/((-b)^(3/4)*c^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{x^{3/2}}{bx^2+cx^4}\,dx = \frac{\sqrt{2}\left(-2atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)-\log\left(-\sqrt{x}\,c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+\sqrt{b}+\right.\right.}{4c^{\frac{1}{4}}b^{\frac{3}{4}}}$$

input
```
int(x^(3/2)/(c*x^4+b*x^2),x)
```

output
```
(c**(3/4)*b**(1/4)*sqrt(2)*( - 2*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2))) + 2*atan((c**(1/4)*b**(1/4)*sqrt(
2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2))) - log( - sqrt(x)*c**(
1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x) + log(sqrt(x)*c**(1/4)*b**(1/
4)*sqrt(2) + sqrt(b) + sqrt(c)*x)))/(4*b*c)
```

3.123 $\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 146

$$\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx = -\frac{2}{b\sqrt{x}}+\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{c}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{5/4}}$$

output
```
-2/b/x^(1/2)+1/2*c^(1/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)
/b^(5/4)-1/2*c^(1/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(
5/4)+1/2*c^(1/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*
x))*2^(1/2)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx = \frac{-\frac{4\sqrt[4]{b}}{\sqrt{x}}+\sqrt{2}\sqrt[4]{c}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+\sqrt{2}\sqrt[4]{c}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{2b^{5/4}}$$

input
```
Integrate[Sqrt[x]/(b*x^2 + c*x^4),x]
```

output
```
((-4*b^(1/4))/Sqrt[x] + Sqrt[2]*c^(1/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + Sqrt[2]*c^(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(2*b^(5/4))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.57, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {9, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{bx^2+cx^4}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{x^{3/2}(b+cx^2)}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}$$

$\big\downarrow$ 25

$$-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}$$

$\big\downarrow$ 27

$$-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}$$

$\big\downarrow$ 1103

$$-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}$$

input
```
Int[Sqrt[x]/(b*x^2 + c*x^4),x]
```

output
```
-2/(b*Sqrt[x]) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(S
qrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(
Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(
2*Sqrt[c])))/b
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{2}{b\sqrt{x}}-\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	115
default	$-\frac{2}{b\sqrt{x}}-\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	115
risch	$-\frac{2}{b\sqrt{x}}-\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	115

input
```
int(x^(1/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/b/x^(1/2)-1/4/b/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1
/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arct
an(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)
-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx =$$
$$-\frac{bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(b^4\left(-\frac{c}{b^5}\right)^{\frac{3}{4}}+c\sqrt{x}\right)-i\,bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(i\,b^4\left(-\frac{c}{b^5}\right)^{\frac{3}{4}}+c\sqrt{x}\right)+i\,bx\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(-i\,b^4\left(-\frac{c}{b^5}\right.\right.}{2\,bx}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/2*(b*x*(-c/b^5)^(1/4)*log(b^4*(-c/b^5)^(3/4) + c*sqrt(x)) - I*b*x*(-c/b
^5)^(1/4)*log(I*b^4*(-c/b^5)^(3/4) + c*sqrt(x)) + I*b*x*(-c/b^5)^(1/4)*log
(-I*b^4*(-c/b^5)^(3/4) + c*sqrt(x)) - b*x*(-c/b^5)^(1/4)*log(-b^4*(-c/b^5)
^(3/4) + c*sqrt(x)) + 4*sqrt(x))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 7.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{x}}{bx^2 + cx^4}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{5cx^{\frac{5}{2}}} & \text{for } b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } c = 0 \\ -\frac{\log\left(\sqrt{x} - \sqrt[4]{-\frac{b}{c}}\right)}{2b\sqrt[4]{-\frac{b}{c}}} + \frac{\log\left(\sqrt{x} + \sqrt[4]{-\frac{b}{c}}\right)}{2b\sqrt[4]{-\frac{b}{c}}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b\sqrt[4]{-\frac{b}{c}}} - \frac{2}{b\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(b, 0) & Eq(c, 0)), (-2/(5*c*x**(5/2)), Eq(b, 0
)), (-2/(b*sqrt(x)), Eq(c, 0)), (-log(sqrt(x) - (-b/c)**(1/4))/(2*b*(-b/c)
**(1/4)) + log(sqrt(x) + (-b/c)**(1/4))/(2*b*(-b/c)**(1/4)) - atan(sqrt(x)
/(-b/c)**(1/4))/(b*(-b/c)**(1/4)) - 2/(b*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{x}}{bx^2 + cx^4}\,dx = -\frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\right.}{}\right.}{4b} - \frac{2}{b\sqrt{x}}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/4*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(
1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*lo
g(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)
))/b - 2/(b*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx = -\frac{2}{b\sqrt{x}} - \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^2} - \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^2} + \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2c^2} - \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2c^2}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-2/(b*sqrt(x)) - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/
c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) - 1/2*sqrt(2)*(b*c^3)^(3/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2
) + 1/4*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b
/c))/(b^2*c^2) - 1/4*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4
) + x + sqrt(b/c))/(b^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 17.88 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx = \frac{(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{5/4}} - \frac{(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{5/4}} - \frac{2}{b\,\sqrt{x}}$$

input
```
int(x^(1/2)/(b*x^2 + c*x^4),x)
```

output
```
((-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(5/4) - ((-c)^(1/4)*ata
n(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(5/4) - 2/(b*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx$$
$$= \frac{2\sqrt{x}\,c^{\frac{1}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)-2\sqrt{x}\,c^{\frac{1}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)-\sqrt{x}\,c^{\frac{1}{4}}b^{\frac{3}{4}}\sqrt{2}\log\left(-\sqrt{x}\,c^{\frac{1}{4}}b^{\frac{1}{4}} \ldots\right.}{4\sqrt{x}\,b^2}$$

input
```
int(x^(1/2)/(c*x^4+b*x^2),x)
```

output
```
(2*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*s
qrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2))) - 2*sqrt(x)*c**(1/4)*b**(3/4)
*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b*
*(1/4)*sqrt(2))) - sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/
4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x) + sqrt(x)*c**(1/4)*b**(3/4)*sqr
t(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x) - 8*b)/(
4*sqrt(x)*b**2)
```

3.124 $\int \frac{1}{\sqrt{x}(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 149

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)}\,dx = -\frac{2}{3bx^{3/2}} + \frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}} - \frac{c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}} - \frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{7/4}}$$

output
```
-2/3/b/x^(3/2)+1/2*c^(3/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/
2)/b^(7/4)-1/2*c^(3/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b
^(7/4)-1/2*c^(3/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2
)*x))*2^(1/2)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)}\,dx = \frac{-\frac{4b^{3/4}}{x^{3/2}}+3\sqrt{2}c^{3/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-3\sqrt{2}c^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{6b^{7/4}}$$

input
```
Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

output
```
((-4*b^(3/4))/x^(3/2) + 3*Sqrt[2]*c^(3/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 3*Sqrt[2]*c^(3/4)*ArcTanh[(Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(6*b^(7/4))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.55, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {9, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^{5/2}\,(b+cx^2)}dx$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{c\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{b}-\frac{2}{3bx^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{2c\int \frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}$$

$$\Big\downarrow \text{755}$$

$$-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}$$

input
```
Int[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

output
```
-2/(3*b*x^(3/2)) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/
(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))
/(2*Sqrt[b])))/b
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{c\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}}-\frac{2}{3b\,x^{\frac{3}{2}}}$	116
default	$-\frac{c\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}}-\frac{2}{3b\,x^{\frac{3}{2}}}$	116
risch	$-\frac{c\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}}-\frac{2}{3b\,x^{\frac{3}{2}}}$	116

input
```
int(1/x^(1/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*c/b^2*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c
*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/
2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))-2/3
/b/x^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)}\,dx = -\frac{3\,bx^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}+c\sqrt{x}\right)+3i\,bx^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\log\left(i\,b^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}+c\sqrt{x}\right)-3i\,bx^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\log}{6\,bx^2}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/6*(3*b*x^2*(-c^3/b^7)^(1/4)*log(b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) + 3*I
*b*x^2*(-c^3/b^7)^(1/4)*log(I*b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) - 3*I*b*x^
2*(-c^3/b^7)^(1/4)*log(-I*b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) - 3*b*x^2*(-c^
3/b^7)^(1/4)*log(-b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) + 4*sqrt(x))/(b*x^2)
```

Sympy [A] (verification not implemented)

Time = 12.58 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } b=0 \wedge c=0 \\ -\frac{2}{7cx^{\frac{7}{2}}} & \text{for } b=0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } c=0 \\ -\frac{2}{3bx^{\frac{3}{2}}} + \frac{c\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^2} - \frac{c\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^2} - \frac{c\sqrt[4]{-\frac{b}{c}}\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(b, 0) & Eq(c, 0)), (-2/(7*c*x**(7/2)), Eq(b, 0
)), (-2/(3*b*x**(3/2)), Eq(c, 0)), (-2/(3*b*x**(3/2)) + c*(-b/c)**(1/4)*lo
g(sqrt(x) - (-b/c)**(1/4))/(2*b**2) - c*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)
**(1/4))/(2*b**2) - c*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/b**2, True
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)}\,dx = -\frac{\frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}c^{\frac{3}{4}}\log}{}}{4b} - \frac{2}{3bx^{\frac{3}{2}}}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2
)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(3/4)*log(s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^
(3/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))
/b - 2/3/(b*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)}\,dx = -\frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2} - \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2} - \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2} + \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2} - \frac{2}{3\,bx^{\frac{3}{2}}}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/b^2 - 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^2 - 1/4*sqrt(2)*(b*c^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^2 + 1/4*sqrt(2)*(b*c^
3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^2 - 2/3/(b*x^
(3/2))
```

Mupad [B] (verification not implemented)

Time = 17.72 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)}\,dx = \frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{7/4}} - \frac{2}{3\,b\,x^{3/2}} + \frac{(-c)^{3/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{7/4}}$$

input
```
int(1/(x^(1/2)*(b*x^2 + c*x^4)),x)
```

output
```
((-c)^(3/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(7/4) - 2/(3*b*x^(3/2))
+ ((-c)^(3/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(7/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)}\,dx = \frac{6\sqrt{x}\,c^{\frac{3}{4}}b^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x-6\sqrt{x}\,c^{\frac{3}{4}}b^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x+3\sqrt{x}\,c^{\frac{3}{4}}b^{\frac{1}{4}}\sqrt{2}\log\left(-\sqrt{x}\,c\right.}{12\sqrt{x}\,b^2x}$$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2),x)
```

output
```
(6*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*s
qrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*x - 6*sqrt(x)*c**(3/4)*b**(1/
4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*
b**(1/4)*sqrt(2)))*x + 3*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*
c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*x - 3*sqrt(x)*c**(3/4)*b*
*(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x
)*x - 8*b)/(12*sqrt(x)*b**2*x)
```

3.125 $\int \frac{1}{x^{3/2}(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)}\,dx=-\frac{2}{5bx^{5/2}}+\frac{2c}{b^2\sqrt{x}}-\frac{c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}}+\frac{c^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}}-\frac{c^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{9/4}}$$

output
```
-2/5/b/x^(5/2)+2*c/b^2/x^(1/2)-1/2*c^(5/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1/2
)/b^(1/4))*2^(1/2)/b^(9/4)+1/2*c^(5/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^
(1/4))*2^(1/2)/b^(9/4)-1/2*c^(5/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)
/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{3/2}(bx^2+cx^4)}\,dx = \frac{-\frac{4\sqrt[4]{b}(b-5cx^2)}{x^{5/2}} - 5\sqrt{2}c^{5/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - 5\sqrt{2}c^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{10b^{9/4}}$$

input
```
Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

output
```
((-4*b^(1/4)*(b - 5*c*x^2))/x^(5/2) - 5*Sqrt[2]*c^(5/4)*ArcTan[(Sqrt[b] -
Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 5*Sqrt[2]*c^(5/4)*ArcTanh[
(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(10*b^(9/4))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.55, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {9, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(bx^2+cx^4)}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^{7/2}(b+cx^2)}dx$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{c\int \frac{1}{x^{3/2}(cx^2+b)}dx}{b} - \frac{2}{5bx^{5/2}}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{c\left(-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b} - \frac{2}{b\sqrt{x}}\right)}{b} - \frac{2}{5bx^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{c\left(-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

$$\Big\downarrow \text{826}$$

$$-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

$\Big\downarrow$ 1479

$$-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

$\Big\downarrow$ 25

$$-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

input
```
Int[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

output
```
-2/(5*b*x^(5/2)) - (c*(-2/(b*Sqrt[x]) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[S
qrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(
1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt
[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b))/b
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{2(-5c\,x^2+b)}{5b^2x^{\frac{5}{2}}}+\frac{c\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	124
derivativedivides	$-\frac{2}{5b\,x^{\frac{5}{2}}}+\frac{2c}{b^2\sqrt{x}}+\frac{c\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	125
default	$-\frac{2}{5b\,x^{\frac{5}{2}}}+\frac{2c}{b^2\sqrt{x}}+\frac{c\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	125

input
```
int(1/x^(3/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-5*c*x^2+b)/b^2/x^(5/2)+1/4*c/b^2/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*
b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(
1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/
(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = \frac{5\,b^2x^3\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}\log\left(b^7\left(-\frac{c^5}{b^9}\right)^{\frac{3}{4}}+c^4\sqrt{x}\right)-5i\,b^2x^3\left(-\frac{c^5}{b^9}\right)^{\frac{1}{4}}\log\left(i\,b^7\left(-\frac{c^5}{b^9}\right)^{\frac{3}{4}}+c^4\sqrt{\right.}$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/10*(5*b^2*x^3*(-c^5/b^9)^(1/4)*log(b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) -
 5*I*b^2*x^3*(-c^5/b^9)^(1/4)*log(I*b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) +
5*I*b^2*x^3*(-c^5/b^9)^(1/4)*log(-I*b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) -
5*b^2*x^3*(-c^5/b^9)^(1/4)*log(-b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) + 4*(5
*c*x^2 - b)*sqrt(x))/(b^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 24.62 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } b = \\ -\frac{2}{9cx^{\frac{9}{2}}} & \text{for } b = \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } c = \\ -\frac{2}{5bx^{\frac{5}{2}}} + \frac{c\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^2\sqrt[4]{-\frac{b}{c}}} - \frac{c\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^2\sqrt[4]{-\frac{b}{c}}} + \frac{c\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^2\sqrt[4]{-\frac{b}{c}}} + \frac{2c}{b^2\sqrt{x}} & \text{otherwi} \end{cases}$$

input
```
integrate(1/x**(3/2)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo/x**(9/2), Eq(b, 0) & Eq(c, 0)), (-2/(9*c*x**(9/2)), Eq(b, 0
)), (-2/(5*b*x**(5/2)), Eq(c, 0)), (-2/(5*b*x**(5/2)) + c*log(sqrt(x) - (-
b/c)**(1/4))/(2*b**2*(-b/c)**(1/4)) - c*log(sqrt(x) + (-b/c)**(1/4))/(2*b*
*2*(-b/c)**(1/4)) + c*atan(sqrt(x)/(-b/c)**(1/4))/(b**2*(-b/c)**(1/4)) + 2
*c/(b**2*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = \frac{c^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\right.}{b^{\frac{1}{4}}c}\right.}{4b^2} + \frac{2\left(5cx^2-b\right)}{5b^2x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/4*c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)
*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(
sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^
(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*l
og(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4
)))/b^2 + 2/5*(5*c*x^2 - b)/(b^2*x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^{3/2}(bx^2+cx^4)}\,dx = \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c} + \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c} - \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3c} + \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3c} + \frac{2\,(5\,cx^2-b)}{5\,b^2x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b^3*c) + 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) - 1/4*sqrt(2)*(b*c^
3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) + 1/4*sq
rt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3
*c) + 2/5*(5*c*x^2 - b)/(b^2*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{3/2}(bx^2+cx^4)}\,dx = \frac{(-c)^{5/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{9/4}} - \frac{(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{9/4}} - \frac{\frac{2}{5\,b}-\frac{2\,c\,x^2}{b^2}}{x^{5/2}}$$

input
```
int(1/(x^(3/2)*(b*x^2 + c*x^4)),x)
```

output
```
((-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(9/4) - ((-c)^(5/4)*ata
n(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(9/4) - (2/(5*b) - (2*c*x^2)/b^2)/x^(5/
2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = \frac{-10\sqrt{x}\,c^{\frac{5}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^2+10\sqrt{x}\,c^{\frac{5}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x}{}$$

input
```
int(1/x^(3/2)/(c*x^4+b*x^2),x)
```

output
```
( - 10*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c*x**2 + 10*sqrt(x)*c**(1
/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/
(c**(1/4)*b**(1/4)*sqrt(2)))*c*x**2 + 5*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*
log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**2 - 5
*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) +
 sqrt(b) + sqrt(c)*x)*c*x**2 - 8*b**2 + 40*b*c*x**2)/(20*sqrt(x)*b**3*x**2
)
```

3.126 $\int \frac{1}{x^{5/2}(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)}\,dx=-\frac{2}{7bx^{7/2}}+\frac{2c}{3b^2x^{3/2}}-\frac{c^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}}+\frac{c^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}}+\frac{c^{7/4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{11/4}}$$

output
```
-2/7/b/x^(7/2)+2/3*c/b^2/x^(3/2)-1/2*c^(7/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1
/2)/b^(1/4))*2^(1/2)/b^(11/4)+1/2*c^(7/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)
/b^(1/4))*2^(1/2)/b^(11/4)+1/2*c^(7/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(
1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)}\,dx=\frac{\frac{4b^{3/4}\left(-3b+7cx^2\right)}{x^{7/2}}-21\sqrt{2}c^{7/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+21\sqrt{2}c^{7/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{42b^{11/4}}$$

input
```
Integrate[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

output
```
((4*b^(3/4)*(-3*b + 7*c*x^2))/x^(7/2) - 21*Sqrt[2]*c^(7/4)*ArcTan[(Sqrt[b]
 - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 21*Sqrt[2]*c^(7/4)*ArcT
anh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(42*b^(11/4)
)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.55, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {9, 264, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^{9/2}\left(b+cx^2\right)}dx$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{c\int \frac{1}{x^{5/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{7bx^{7/2}}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{c\left(-\frac{c\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{c\left(-\frac{2c\int \frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}$$

$$\Big\downarrow \text{ 755}$$

$$-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}$$

$$\Big\downarrow \text{ 1476}$$

$$-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}$$

$$\Big\downarrow \text{ 1082}$$

$$-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}$$

$$\Big\downarrow \; 217$$

$$-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}$$

$$\Big\downarrow \; 1479$$

$$-\frac{c\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}$$

$$\frac{2}{7bx^{7/2}}$$

$\downarrow$ 27

$$-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-$$

$$\frac{2}{7bx^{7/2}}$$

$\downarrow$ 1103

$$-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}$$

input
```
Int[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

output
```
-2/(7*b*x^(7/2)) - (c*(-2/(3*b*x^(3/2)) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^
(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log
[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c
^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sq
rt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/b))/b
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

method	result	si
derivativedivides	$\frac{c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{3}}-\frac{2}{7b\,x^{\frac{7}{2}}}+\frac{2c}{3b^{2}x^{\frac{3}{2}}}$	12
default	$\frac{c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{3}}-\frac{2}{7b\,x^{\frac{7}{2}}}+\frac{2c}{3b^{2}x^{\frac{3}{2}}}$	12
risch	$-\frac{2\left(-7c\,x^{2}+3b\right)}{21b^{2}x^{\frac{7}{2}}}+\frac{c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{3}}$	12

input
```
int(1/x^(5/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*c^2/b^3*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/
c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1
/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))-2/
7/b/x^(7/2)+2/3*c/b^2/x^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)}\,dx=\frac{21\,b^{2}x^{4}\left(-\frac{c^{7}}{b^{11}}\right)^{\frac{1}{4}}\log\left(b^{3}\left(-\frac{c^{7}}{b^{11}}\right)^{\frac{1}{4}}+c^{2}\sqrt{x}\right)+21i\,b^{2}x^{4}\left(-\frac{c^{7}}{b^{11}}\right)^{\frac{1}{4}}\log\left(i\,b^{3}\left(-\frac{c^{7}}{b^{11}}\right)^{\frac{1}{4}}+\right.}{}$$

input
```
integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
1/42*(21*b^2*x^4*(-c^7/b^11)^(1/4)*log(b^3*(-c^7/b^11)^(1/4) + c^2*sqrt(x)
) + 21*I*b^2*x^4*(-c^7/b^11)^(1/4)*log(I*b^3*(-c^7/b^11)^(1/4) + c^2*sqrt(
x)) - 21*I*b^2*x^4*(-c^7/b^11)^(1/4)*log(-I*b^3*(-c^7/b^11)^(1/4) + c^2*sq
rt(x)) - 21*b^2*x^4*(-c^7/b^11)^(1/4)*log(-b^3*(-c^7/b^11)^(1/4) + c^2*sqr
t(x)) + 4*(7*c*x^2 - 3*b)*sqrt(x))/(b^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 61.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{2}}} \\ -\frac{2}{11cx^{\frac{11}{2}}} \\ -\frac{2}{7bx^{\frac{7}{2}}} \\ -\frac{2}{7bx^{\frac{7}{2}}} + \frac{2c}{3b^2x^{\frac{3}{2}}} - \frac{c^2\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^3} + \frac{c^2\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^3} + \frac{c^2\sqrt[4]{-\frac{b}{c}}\operatorname{ata}}{b} \end{cases}$$

input
```
integrate(1/x**(5/2)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo/x**(11/2), Eq(b, 0) & Eq(c, 0)), (-2/(11*c*x**(11/2)), Eq(b
, 0)), (-2/(7*b*x**(7/2)), Eq(c, 0)), (-2/(7*b*x**(7/2)) + 2*c/(3*b**2*x**
(3/2)) - c**2*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b**3) + c**2*(
-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b**3) + c**2*(-b/c)**(1/4)*at
an(sqrt(x)/(-b/c)**(1/4))/b**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^{5/2}\,(bx^2+cx^4)}\,dx = \frac{\frac{2\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}c^{\frac{7}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}c^{\frac{7}{4}}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}}{4\,b^2} + \frac{2\,(7\,cx^2-3\,b)}{21\,b^2x^{\frac{7}{2}}}$$

input
```
integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)
*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(
2)*c^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/s
qrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(7/4)*lo
g(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)
*c^(7/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/
4))/b^2 + 2/21*(7*c*x^2 - 3*b)/(b^2*x^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^{5/2}\,(bx^2+cx^4)}\,dx = \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3} + \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3} + \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\,c\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3} - \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\,c\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3} + \frac{2\,(7\,cx^2-3\,b)}{21\,b^2x^{\frac{7}{2}}}$$

input
```
integrate(1/x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/b^3 + 1/2*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^3 + 1/4*sqrt(2)*(b*c^3)^(
1/4)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 - 1/4*sqrt(2)*
(b*c^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 2/
21*(7*c*x^2 - 3*b)/(b^2*x^(7/2))
```

Mupad [B] (verification not implemented)

Time = 17.72 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^{5/2}\,(bx^2+cx^4)}\,dx=\frac{(-c)^{7/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{11/4}}-\frac{\frac{2}{7\,b}-\frac{2\,c\,x^2}{3\,b^2}}{x^{7/2}}+\frac{(-c)^{7/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{b^{11/4}}$$

input
```
int(1/(x^(5/2)*(b*x^2 + c*x^4)),x)
```

output
```
((-c)^(7/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(11/4) - (2/(7*b) - (2*c
*x^2)/(3*b^2))/x^(7/2) + ((-c)^(7/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/
b^(11/4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^{5/2}\,(bx^2+cx^4)}\,dx=\frac{-42\sqrt{x}\,c^{\frac{7}{4}}b^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^3+42\sqrt{x}\,c^{\frac{7}{4}}b^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x}{}$$

input
```
int(1/x^(5/2)/(c*x^4+b*x^2),x)
```

output
```
( - 42*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c*x**3 + 42*sqrt(x)*c**(3
/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/
(c**(1/4)*b**(1/4)*sqrt(2)))*c*x**3 - 21*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)
*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**3 +
21*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2)
 + sqrt(b) + sqrt(c)*x)*c*x**3 - 24*b**2 + 56*b*c*x**2)/(84*sqrt(x)*b**3*x
**3)
```

3.127 $\int \frac{1}{x^{7/2}(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 174

$$\int \frac{1}{x^{7/2}(bx^2+cx^4)}\,dx = -\frac{2}{9bx^{9/2}}+\frac{2c}{5b^2x^{5/2}}-\frac{2c^2}{b^3\sqrt{x}}+\frac{c^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}} - \frac{c^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}}+\frac{c^{9/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{13/4}}$$

output

```
-2/9/b/x^(9/2)+2/5*c/b^2/x^(5/2)-2*c^2/b^3/x^(1/2)+1/2*c^(9/4)*arctan(1-2^
(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(13/4)-1/2*c^(9/4)*arctan(1+2^(1/
2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(13/4)+1/2*c^(9/4)*arctanh(2^(1/2)*b
^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(13/4)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{7/2}\left(bx^2+cx^4\right)}\,dx=\frac{-\frac{4\sqrt[4]{b}(5b^2-9bcx^2+45c^2x^4)}{x^{9/2}}+45\sqrt{2}c^{9/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+45\sqrt{2}c^{9/4}\text{arctanh}\left(\frac{\sqrt{2}}{}\right.}{90b^{13/4}}$$

input
```
Integrate[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

output
```
((-4*b^(1/4)*(5*b^2 - 9*b*c*x^2 + 45*c^2*x^4))/x^(9/2) + 45*Sqrt[2]*c^(9/4
)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 45*Sqr
t[2]*c^(9/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*
x)])/(90*b^(13/4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.53, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {9, 264, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\left(bx^2+cx^4\right)}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^{11/2}\left(b+cx^2\right)}dx$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{c\int \frac{1}{x^{7/2}(cx^2+b)}dx}{b}-\frac{2}{9bx^{9/2}}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{c\left(-\frac{c\int \frac{1}{x^{3/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}$$

$\Big\downarrow$ 264

$$-\frac{c\left(-\frac{c\left(-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}$$

$\Big\downarrow$ 266

$$-\frac{c\left(-\frac{c\left(-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}$$

$\Big\downarrow$ 826

$$-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}$$

$\Big\downarrow$ 1476

$$-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}$$

$$\Big\downarrow \ 1082$$

$$-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}$$

$$\Big\downarrow \; 217$$

$$-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}$$

$\Big\downarrow$ 1479

$$-\frac{c\Bigg(-\frac{c\bigg(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{\cdots}{b}\cdots}{b}\cdots}{b}\cdots$$

$$\frac{2}{9bx^{9/2}}$$

$\downarrow$ 25

$$-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}\right.}{b}$$

$$\frac{2}{9bx^{9/2}}$$

$\downarrow$ 27

$$-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}.$$

$$\frac{2}{9bx^{9/2}}$$

$\downarrow$ 1103

$$-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\right)}{b}$$

$$\frac{2}{9bx^{9/2}}$$

input
```
Int[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

output
```
-2/(9*b*x^(9/2)) - (c*(-2/(5*b*x^(5/2)) - (c*(-2/(b*Sqrt[x]) - (2*c*((-(Ar
cTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + A
rcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2
*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b))/b))/b
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{c^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{2}{9b\,x^{\frac{9}{2}}}-\frac{2c^{2}}{b^{3}\sqrt{x}}+\frac{2c}{5b^{2}x^{\frac{5}{2}}}$
default	$-\frac{c^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{2}{9b\,x^{\frac{9}{2}}}-\frac{2c^{2}}{b^{3}\sqrt{x}}+\frac{2c}{5b^{2}x^{\frac{5}{2}}}$
risch	$-\frac{2\left(45c^{2}x^{4}-9bc\,x^{2}+5b^{2}\right)}{45b^{3}x^{\frac{9}{2}}}-\frac{c^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

input
```
int(1/x^(7/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*c^2/b^3/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1
/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(
1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))-2
/9/b/x^(9/2)-2*c^2/b^3/x^(1/2)+2/5*c/b^2/x^(5/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^{7/2}\,(bx^2+cx^4)}\,dx =$$

$$-\frac{45\,b^3x^5\left(-\frac{c^9}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^{10}\left(-\frac{c^9}{b^{13}}\right)^{\frac{3}{4}}+c^7\sqrt{x}\right)-45i\,b^3x^5\left(-\frac{c^9}{b^{13}}\right)^{\frac{1}{4}}\log\left(i\,b^{10}\left(-\frac{c^9}{b^{13}}\right)^{\frac{3}{4}}+c^7\sqrt{x}\right)+45i\,b^3x^5\left(\cdots\right.}{}$$

input
```
integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
-1/90*(45*b^3*x^5*(-c^9/b^13)^(1/4)*log(b^10*(-c^9/b^13)^(3/4) + c^7*sqrt(
x)) - 45*I*b^3*x^5*(-c^9/b^13)^(1/4)*log(I*b^10*(-c^9/b^13)^(3/4) + c^7*sq
rt(x)) + 45*I*b^3*x^5*(-c^9/b^13)^(1/4)*log(-I*b^10*(-c^9/b^13)^(3/4) + c^
7*sqrt(x)) - 45*b^3*x^5*(-c^9/b^13)^(1/4)*log(-b^10*(-c^9/b^13)^(3/4) + c^
7*sqrt(x)) + 4*(45*c^2*x^4 - 9*b*c*x^2 + 5*b^2)*sqrt(x))/(b^3*x^5)
```

Sympy [A] (verification not implemented)

Time = 143.90 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{7/2}\left(bx^2+cx^4\right)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{13}{2}}} \\ -\frac{2}{13cx^{\frac{13}{2}}} \\ -\frac{2}{9bx^{\frac{9}{2}}} \\ -\frac{2}{9bx^{\frac{9}{2}}}+\frac{2c}{5b^2x^{\frac{5}{2}}}-\frac{c^2\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^3\sqrt[4]{-\frac{b}{c}}}+\frac{c^2\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^3\sqrt[4]{-\frac{b}{c}}}-\frac{c^2\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^3\sqrt[4]{-\frac{b}{c}}}-\frac{2c^2}{b^3\sqrt{x}} \end{cases}$$

input
```
integrate(1/x**(7/2)/(c*x**4+b*x**2),x)
```

output
```
Piecewise((zoo/x**(13/2), Eq(b, 0) & Eq(c, 0)), (-2/(13*c*x**(13/2)), Eq(b
, 0)), (-2/(9*b*x**(9/2)), Eq(c, 0)), (-2/(9*b*x**(9/2)) + 2*c/(5*b**2*x**
(5/2)) - c**2*log(sqrt(x) - (-b/c)**(1/4))/(2*b**3*(-b/c)**(1/4)) + c**2*l
og(sqrt(x) + (-b/c)**(1/4))/(2*b**3*(-b/c)**(1/4)) - c**2*atan(sqrt(x)/(-b
/c)**(1/4))/(b**3*(-b/c)**(1/4)) - 2*c**2/(b**3*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^{7/2}\left(bx^2+cx^4\right)}\,dx =$$

$$-\frac{c^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{\right.}}{}\right)}{4b^3}$$

$$-\frac{2\left(45c^2x^4-9bcx^2+5b^2\right)}{45b^3x^{\frac{9}{2}}}$$

input
```
integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
-1/4*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c
)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*
log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/
4)))/b^3 - 2/45*(45*c^2*x^4 - 9*b*c*x^2 + 5*b^2)/(b^3*x^(9/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^{7/2}(bx^2+cx^4)}\,dx = -\frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4} - \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4} + \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4} - \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4} - \frac{2\,(45\,c^2x^4-9\,bcx^2+5\,b^2)}{45\,b^3x^{\frac{9}{2}}}$$

input
```
integrate(1/x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/b^4 - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/4*sqrt(2)*(b*c^3)^(3/4
)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/4*sqrt(2)*(b*c^
3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 2/45*(45*
c^2*x^4 - 9*b*c*x^2 + 5*b^2)/(b^3*x^(9/2))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^{7/2}\,(bx^2+cx^4)}\,dx = \frac{(-c)^{9/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{13/4}} - \frac{(-c)^{9/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)}{b^{13/4}} - \frac{\frac{2}{9\,b}-\frac{2\,c\,x^2}{5\,b^2}+\frac{2\,c^2\,x^4}{b^3}}{x^{9/2}}$$

input
```
int(1/(x^(7/2)*(b*x^2 + c*x^4)),x)
```

output
```
((-c)^(9/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(13/4) - ((-c)^(9/4)*at
an(((-c)^(1/4)*x^(1/2))/b^(1/4)))/b^(13/4) - (2/(9*b) - (2*c*x^2)/(5*b^2)
+ (2*c^2*x^4)/b^3)/x^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^{7/2}\,(bx^2+cx^4)}\,dx = \frac{90\sqrt{x}\,c^{\frac{9}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^4 - 90\sqrt{x}\,c^{\frac{9}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^4}{}$$

input
```
int(1/x^(7/2)/(c*x^4+b*x^2),x)
```

output
```
(90*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*
sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 - 90*sqrt(x)*c**(1
/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/
(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 - 45*sqrt(x)*c**(1/4)*b**(3/4)*sqrt
(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x
**4 + 45*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*s
qrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4 - 40*b**3 + 72*b**2*c*x**2 - 360*b
*c**2*x**4)/(180*sqrt(x)*b**4*x**4)
```

3.128 $\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 190

$$\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^2}\,dx = -\frac{9b\sqrt{x}}{2c^3}+\frac{9x^{5/2}}{10c^2}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}-\frac{9b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}c^{13/4}}$$

output
```
-9/2*b*x^(1/2)/c^3+9/10*x^(5/2)/c^2-1/2*x^(9/2)/c/(c*x^2+b)-9/8*b^(5/4)*ar
ctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/c^(13/4)+9/8*b^(5/4)*arcta
n(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/c^(13/4)+9/8*b^(5/4)*arctanh(
2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/c^(13/4)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^2}\,dx = \frac{\frac{4\sqrt[4]{c}\sqrt{x}\left(-45b^2-36bcx^2+4c^2x^4\right)}{b+cx^2}-45\sqrt{2}b^{5/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+45\sqrt{2}b^{5/4}\operatorname{arctanh}\left(\frac{\sqrt{2}\,\cdots}{\cdots}\right.}{40c^{13/4}}$$

input

```
Integrate[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```

output

```
((4*c^(1/4)*Sqrt[x]*(-45*b^2 - 36*b*c*x^2 + 4*c^2*x^4))/(b + c*x^2) - 45*S
qrt[2]*b^(5/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x])] + 45*Sqrt[2]*b^(5/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[
b] + Sqrt[c]*x)])/(40*c^(13/4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.46, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {9, 252, 262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19/2}}{\left(bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^{11/2}}{\left(b+cx^2\right)^2}dx$$

$$\Big\downarrow \ 252$$

$$\frac{9\int \frac{x^{7/2}}{cx^2+b}dx}{4c}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \ 262$$

$$\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\int\frac{x^{3/2}}{cx^2+b}dx}{c}\right)}{4c}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 262

$$\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{b\int\frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 266

$$\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{cx^2+b}d\sqrt{x}}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 755

$$\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}$$

$\downarrow$ 1479

$$\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\cdots}{\sqrt[4]{b}}\cdots\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots}{2\sqrt{b}}\cdots\right)}{c}\cdots\right)}{c}\cdots\right)}{4c}\cdots$$

$$\frac{x^{9/2}}{2c\,(b+cx^2)}$$

$\downarrow$ 25

$$\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right.}{c}\right.}{c}\right.}{4c}$$

$$\frac{x^{9/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4c}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{9\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{c}\right)}{4c}$$

$$\frac{x^{9/2}}{2c\left(b+cx^2\right)}$$

input
```
Int[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*x^(9/2)/(c*(b + c*x^2)) + (9*((2*x^(5/2))/(5*c) - (b*((2*Sqrt[x])/c -
 (2*b*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c
^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*
c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/c))
/c))/(4*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{2(-c\,x^{2}+10b)\sqrt{x}}{5c^{3}}+\frac{b^{2}\left(-\frac{\sqrt{x}}{2\left(c\,x^{2}+b\right)}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{16b}\right.}{c^{3}}$
derivativedivides	$-\frac{2\left(-\frac{c\,x^{\frac{5}{2}}}{5}+2b\sqrt{x}\right)}{c^{3}}+\frac{2b^{2}\left(-\frac{\sqrt{x}}{4\left(c\,x^{2}+b\right)}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{b}{c}\right)}\right.\right.}{32b}\right.}{c^{3}}$
default	$-\frac{2\left(-\frac{c\,x^{\frac{5}{2}}}{5}+2b\sqrt{x}\right)}{c^{3}}+\frac{2b^{2}\left(-\frac{\sqrt{x}}{4\left(c\,x^{2}+b\right)}+\frac{9\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{b}{c}\right)}\right.\right.}{32b}\right.}{c^{3}}$

input
```
int(x^(19/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-c*x^2+10*b)*x^(1/2)/c^3+b^2/c^3*(-1/2*x^(1/2)/(c*x^2+b)+9/16*(1/c*b
)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(
1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4
)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.26

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^2}\,dx = \frac{45\,(c^4x^2+bc^3)\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}\log\left(9\,c^3\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}+9\,b\sqrt{x}\right)-45\,(-i\,c^4x^2-i\,bc^3)\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}\mathrm{l}}{}$$

input
```
integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/40*(45*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*log(9*c^3*(-b^5/c^13)^(1/4) +
 9*b*sqrt(x)) - 45*(-I*c^4*x^2 - I*b*c^3)*(-b^5/c^13)^(1/4)*log(9*I*c^3*(-
b^5/c^13)^(1/4) + 9*b*sqrt(x)) - 45*(I*c^4*x^2 + I*b*c^3)*(-b^5/c^13)^(1/4
)*log(-9*I*c^3*(-b^5/c^13)^(1/4) + 9*b*sqrt(x)) - 45*(c^4*x^2 + b*c^3)*(-b
^5/c^13)^(1/4)*log(-9*c^3*(-b^5/c^13)^(1/4) + 9*b*sqrt(x)) + 4*(4*c^2*x^4
- 36*b*c*x^2 - 45*b^2)*sqrt(x))/(c^4*x^2 + b*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(19/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.14

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^2}\,dx = -\frac{b^2\sqrt{x}}{2\,(c^4x^2+bc^3)} + \frac{2\left(cx^{\frac{5}{2}}-10\,b\sqrt{x}\right)}{5\,c^3} + \frac{9\left(\frac{2\,\sqrt{2}b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\,\sqrt{2}b^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}b^{\frac{5}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{c^{\frac{1}{4}}} - \frac{\sqrt{2}b}{}\right.}{16\,c^3}$$

input
```
integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*b^2*sqrt(x)/(c^4*x^2 + b*c^3) + 2/5*(c*x^(5/2) - 10*b*sqrt(x))/c^3 +
9/16*(2*sqrt(2)*b^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sq
rt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*b^
(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sq
rt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + sqrt(2)*b^(5/4)*log(sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4) - sqrt(2)*b^(5/4)*lo
g(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4))/c^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.14

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^2}\,dx = \frac{9\sqrt{2}(bc^3)^{\frac{1}{4}}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^4} + \frac{9\sqrt{2}(bc^3)^{\frac{1}{4}}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^4} + \frac{9\sqrt{2}(bc^3)^{\frac{1}{4}}\,b\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^4} - \frac{9\sqrt{2}(bc^3)^{\frac{1}{4}}\,b\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^4} - \frac{b^2\sqrt{x}}{2\,(cx^2+b)c^3} + \frac{2\left(c^8x^{\frac{5}{2}}-10\,bc^7\sqrt{x}\right)}{5\,c^{10}}$$

input
```
integrate(x^(19/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
9/8*sqrt(2)*(b*c^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/c^4 + 9/8*sqrt(2)*(b*c^3)^(1/4)*b*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 + 9/16*sqrt(2)*(b*c^3)^
(1/4)*b*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 9/16*sqrt(2
)*(b*c^3)^(1/4)*b*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 -
1/2*b^2*sqrt(x)/((c*x^2 + b)*c^3) + 2/5*(c^8*x^(5/2) - 10*b*c^7*sqrt(x))/c
^10
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.48

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^2}\,dx = \frac{2\,x^{5/2}}{5\,c^2} - \frac{b^2\,\sqrt{x}}{2\,(c^4\,x^2+b\,c^3)} - \frac{4\,b\,\sqrt{x}}{c^3} - \frac{9\,(-b)^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{4\,c^{13/4}} + \frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,9\mathrm{i}}{4\,c^{13/4}}$$

input
```
int(x^(19/2)/(b*x^2 + c*x^4)^2,x)
```

output
```
(2*x^(5/2))/(5*c^2) - (b^2*x^(1/2))/(2*(b*c^3 + c^4*x^2)) - (4*b*x^(1/2))/
c^3 - (9*(-b)^(5/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*c^(13/4)) + ((-
b)^(5/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*9i)/(4*c^(13/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.72

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^2}\,dx = \frac{-90c^{\frac{3}{4}}b^{\frac{9}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right) - 90c^{\frac{7}{4}}b^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^2 + 90c^{\frac{3}{4}}b^{\frac{9}{4}}\sqrt{}}{}$$

input
```
int(x^(19/2)/(c*x^4+b*x^2)^2,x)
```

output
```
( - 90*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 - 90*c**(3/4)*b**(1/4)*sqrt(
2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)
*sqrt(2)))*b*c*x**2 + 90*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)
*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 + 90*c**(3
/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/
(c**(1/4)*b**(1/4)*sqrt(2)))*b*c*x**2 - 45*c**(3/4)*b**(1/4)*sqrt(2)*log(
- sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b**2 - 45*c**(3
/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) +
sqrt(c)*x)*b*c*x**2 + 45*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b*
*(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b**2 + 45*c**(3/4)*b**(1/4)*sqrt(2)*
log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 - 36
0*sqrt(x)*b**2*c - 288*sqrt(x)*b*c**2*x**2 + 32*sqrt(x)*c**3*x**4)/(80*c**
4*(b + c*x**2))
```

3.129 $\int \frac{x^{17/2}}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 177

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^2}\,dx = \frac{7x^{3/2}}{6c^2} - \frac{x^{7/2}}{2c(b+cx^2)} + \frac{7b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{11/4}} - \frac{7b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{11/4}} + \frac{7b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}c^{11/4}}$$

output
```
7/6*x^(3/2)/c^2-1/2*x^(7/2)/c/(c*x^2+b)+7/8*b^(3/4)*arctan(1-2^(1/2)*c^(1/
4)*x^(1/2)/b^(1/4))*2^(1/2)/c^(11/4)-7/8*b^(3/4)*arctan(1+2^(1/2)*c^(1/4)*
x^(1/2)/b^(1/4))*2^(1/2)/c^(11/4)+7/8*b^(3/4)*arctanh(2^(1/2)*b^(1/4)*c^(1
/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/c^(11/4)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4c^{3/4}x^{3/2}(7b+4cx^2)}{b+cx^2} + 21\sqrt{2}b^{3/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + 21\sqrt{2}b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{24c^{11/4}}$$

input
```
Integrate[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*c^(3/4)*x^(3/2)*(7*b + 4*c*x^2))/(b + c*x^2) + 21*Sqrt[2]*b^(3/4)*ArcT
an[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 21*Sqrt[2]*b
^(3/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(
24*c^(11/4))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.47, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {9, 252, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^{9/2}}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{7\int \frac{x^{5/2}}{cx^2+b}dx}{4c} - \frac{x^{7/2}}{2c(b+cx^2)}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{b\int \frac{\sqrt{x}}{cx^2+b}dx}{c}\right)}{4c}-\frac{x^{7/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 266

$$\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\int \frac{x}{cx^2+b}d\sqrt{x}}{c}\right)}{4c}-\frac{x^{7/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 826

$$\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^{7/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^{7/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^{7/2}}{2c\left(b+cx^2\right)}$$

$\downarrow$ 217

$$\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^{7/2}}{2c\left(b+cx^2\right)}$$

$\downarrow$ 1479

$$\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{4c}$$

$$\frac{x^{7/2}}{2c\left(b+cx^2\right)}$$

$\downarrow$ 25

$$\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^{7/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^{7/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{7\left(\frac{2x^{3/2}}{3c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^{7/2}}{2c\left(b+cx^2\right)}$$

input
```
Int[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*x^(7/2)/(c*(b + c*x^2)) + (7*((2*x^(3/2))/(3*c) - (2*b*((-(ArcTan[1 -
 (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1
+ (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]
) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt
[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqr
t[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/c))/(4*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3c^{2}}-\frac{2b\left(-\frac{x^{\frac{3}{2}}}{4\left(c\,x^{2}+b\right)}+\frac{7\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{c^{2}}$	136
default	$\frac{2x^{\frac{3}{2}}}{3c^{2}}-\frac{2b\left(-\frac{x^{\frac{3}{2}}}{4\left(c\,x^{2}+b\right)}+\frac{7\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{c^{2}}$	136
risch	$\frac{2x^{\frac{3}{2}}}{3c^{2}}-\frac{b\left(-\frac{x^{\frac{3}{2}}}{2\left(c\,x^{2}+b\right)}+\frac{7\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{c^{2}}$	136

input
```
int(x^(17/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)/c^2-2*b/c^2*(-1/4*x^(3/2)/(c*x^2+b)+7/32/c/(1/c*b)^(1/4)*2^(1/
2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^
(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*
arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.34

$$\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^2}\,dx =$$

$$-\frac{21\left(c^3x^2+bc^2\right)\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}\log\left(343\,c^8\left(-\frac{b^3}{c^{11}}\right)^{\frac{3}{4}}+343\,b^2\sqrt{x}\right)+21\left(-i\,c^3x^2-i\,bc^2\right)\left(-\frac{b^3}{c^{11}}\right)^{\frac{1}{4}}\log\left(343i\,c^8\left(-\right.\right.}{}$$

input
```
integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/24*(21*(c^3*x^2 + b*c^2)*(-b^3/c^11)^(1/4)*log(343*c^8*(-b^3/c^11)^(3/4
) + 343*b^2*sqrt(x)) + 21*(-I*c^3*x^2 - I*b*c^2)*(-b^3/c^11)^(1/4)*log(343
*I*c^8*(-b^3/c^11)^(3/4) + 343*b^2*sqrt(x)) + 21*(I*c^3*x^2 + I*b*c^2)*(-b
^3/c^11)^(1/4)*log(-343*I*c^8*(-b^3/c^11)^(3/4) + 343*b^2*sqrt(x)) - 21*(c
^3*x^2 + b*c^2)*(-b^3/c^11)^(1/4)*log(-343*c^8*(-b^3/c^11)^(3/4) + 343*b^2
*sqrt(x)) - 4*(4*c*x^3 + 7*b*x)*sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(17/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.17

$$\int \frac{x^{17/2}}{\left(bx^2+cx^4\right)^2}\,dx = \frac{bx^{\frac{3}{2}}}{2\,(c^3x^2+bc^2)} - \frac{7\,b\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{\right.}}{}\right)}{16\,c^2} + \frac{2\,x^{\frac{3}{2}}}{3\,c^2}$$

input

```
integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output

```
1/2*b*x^(3/2)/(c^3*x^2 + b*c^2) - 7/16*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sq
rt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sq
rt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c
^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*
sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(
b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqr
t(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^2 + 2/3*x^(3/2)/c^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.11

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^2}\,dx = \frac{bx^{\frac{3}{2}}}{2\,(cx^2+b)c^2} + \frac{2\,x^{\frac{3}{2}}}{3\,c^2} - \frac{7\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^5} - \frac{7\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^5} + \frac{7\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^5} - \frac{7\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^5}$$

input
```
integrate(x^(17/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*b*x^(3/2)/((c*x^2 + b)*c^2) + 2/3*x^(3/2)/c^2 - 7/8*sqrt(2)*(b*c^3)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5
- 7/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*s
qrt(x))/(b/c)^(1/4))/c^5 + 7/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*
(b/c)^(1/4) + x + sqrt(b/c))/c^5 - 7/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5
```

Mupad [B] (verification not implemented)

Time = 18.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^2}\,dx = \frac{2\,x^{3/2}}{3\,c^2} + \frac{7\,(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{4\,c^{11/4}} + \frac{b\,x^{3/2}}{2\,\left(c^3\,x^2+b\,c^2\right)} + \frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,7\mathrm{i}}{4\,c^{11/4}}$$

input
```
int(x^(17/2)/(b*x^2 + c*x^4)^2,x)
```

output
```
(2*x^(3/2))/(3*c^2) + (7*(-b)^(3/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4
*c^(11/4)) + ((-b)^(3/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*7i)/(4*c^(1
1/4)) + (b*x^(3/2))/(2*(b*c^2 + c^3*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.77

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^2}\,dx = \frac{42c^{\frac{1}{4}}b^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)+42c^{\frac{5}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^2-42c^{\frac{1}{4}}b^{\frac{7}{4}}\sqrt{2}}{}$$

input
```
int(x^(17/2)/(c*x^4+b*x^2)^2,x)
```

output
```
(42*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*
sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b + 42*c**(1/4)*b**(3/4)*sqrt(2)*ata
n((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(
2)))*c*x**2 - 42*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b - 42*c**(1/4)*b**(3/4
)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b
**(1/4)*sqrt(2)))*c*x**2 - 21*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**
(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b - 21*c**(1/4)*b**(3/4)*sqr
t(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**
2 + 21*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + s
qrt(b) + sqrt(c)*x)*b + 21*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*
b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**2 + 56*sqrt(x)*b*c*x + 32*sqr
t(x)*c**2*x**3)/(48*c**3*(b + c*x**2))
```

3.130 $\int \frac{x^{15/2}}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 177

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^2}\,dx = \frac{5\sqrt{x}}{2c^2} - \frac{x^{5/2}}{2c(b+cx^2)} + \frac{5\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{9/4}} - \frac{5\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{9/4}} - \frac{5\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}c^{9/4}}$$

output
```
5/2*x^(1/2)/c^2-1/2*x^(5/2)/c/(c*x^2+b)+5/8*b^(1/4)*arctan(1-2^(1/2)*c^(1/
4)*x^(1/2)/b^(1/4))*2^(1/2)/c^(9/4)-5/8*b^(1/4)*arctan(1+2^(1/2)*c^(1/4)*x
^(1/2)/b^(1/4))*2^(1/2)/c^(9/4)-5/8*b^(1/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4
)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/c^(9/4)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4\sqrt[4]{c}\sqrt{x}(5b+4cx^2)}{b+cx^2}+5\sqrt{2}\sqrt[4]{b}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-5\sqrt{2}\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{8c^{9/4}}$$

input
```
Integrate[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*c^(1/4)*Sqrt[x]*(5*b + 4*c*x^2))/(b + c*x^2) + 5*Sqrt[2]*b^(1/4)*ArcTa
n[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 5*Sqrt[2]*b^(
1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(8*
c^(9/4))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.46, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {9, 252, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{7/2}}{(b+cx^2)^2}dx$$

$$\Big\downarrow \; 252$$

$$\frac{5\int \frac{x^{3/2}}{cx^2+b}dx}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}$$

$$\Big\downarrow \; 262$$

$$\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{b\int\frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{c}\right)}{4c}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{cx^2+b}d\sqrt{x}}{c}\right)}{4c}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 217

$$\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 1479

$$\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}$$

$$\frac{x^{5/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 25

$$\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}$$

$\big\downarrow$ 27

$$\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}$$

$\big\downarrow$ 1103

$$\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c\left(b+cx^2\right)}$$

input
```
Int[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*x^(5/2)/(c*(b + c*x^2)) + (5*((2*Sqrt[x])/c - (2*b*((-(ArcTan[1 - (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) +
(-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*
b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/c))/(4*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

method	result	siz
derivativedivides	$\frac{2\sqrt{x}}{c^2}-\frac{2b\left(-\frac{\sqrt{x}}{4\left(c\,x^2+b\right)}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{c^2}$	13
default	$\frac{2\sqrt{x}}{c^2}-\frac{2b\left(-\frac{\sqrt{x}}{4\left(c\,x^2+b\right)}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{c^2}$	13
risch	$\frac{2\sqrt{x}}{c^2}-\frac{b\left(-\frac{\sqrt{x}}{2\left(c\,x^2+b\right)}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b}\right)}{c^2}$	13

input
```
int(x^(15/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/c^2-2*b/c^2*(-1/4*x^(1/2)/(c*x^2+b)+5/32*(1/c*b)^(1/4)/b*2^(1/2)
*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1
/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*ar
ctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.18

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^2}\,dx = -\frac{5\,(c^3x^2+bc^2)\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}\log\left(5\,c^2\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}+5\,\sqrt{x}\right)+5\,(i\,c^3x^2+i\,bc^2)\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}\log\left(5i\,c^2\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}+5\,\sqrt{x}\right)+5}{}$$

input
```
integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/8*(5*(c^3*x^2 + b*c^2)*(-b/c^9)^(1/4)*log(5*c^2*(-b/c^9)^(1/4) + 5*sqrt
(x)) + 5*(I*c^3*x^2 + I*b*c^2)*(-b/c^9)^(1/4)*log(5*I*c^2*(-b/c^9)^(1/4) +
 5*sqrt(x)) + 5*(-I*c^3*x^2 - I*b*c^2)*(-b/c^9)^(1/4)*log(-5*I*c^2*(-b/c^9
)^(1/4) + 5*sqrt(x)) - 5*(c^3*x^2 + b*c^2)*(-b/c^9)^(1/4)*log(-5*c^2*(-b/c
^9)^(1/4) + 5*sqrt(x)) - 4*(4*c*x^2 + 5*b)*sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(15/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^2}\,dx = \frac{b\sqrt{x}}{2\,(c^3x^2+bc^2)} - \frac{5\left(\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{c^{\frac{1}{4}}} - \cdots\right.}{16\,c^2} + \frac{2\sqrt{x}}{c^2}$$

input
```
integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*b*sqrt(x)/(c^3*x^2 + b*c^2) - 5/16*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(
2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sq
rt(sqrt(b)*sqrt(c)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/
4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c
)) + sqrt(2)*b^(1/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqr
t(b))/c^(1/4) - sqrt(2)*b^(1/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqr
t(c)*x + sqrt(b))/c^(1/4))/c^2 + 2*sqrt(x)/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.11

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^2}\,dx = -\frac{5\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^3} - \frac{5\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^3} - \frac{5\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^3} + \frac{5\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^3} + \frac{b\sqrt{x}}{2\,(cx^2+b)c^2} + \frac{2\sqrt{x}}{c^2}$$

input
```
integrate(x^(15/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-5/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/c^3 - 5/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^3 - 5/16*sqrt(2)*(b*c^3)^(1/
4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 5/16*sqrt(2)*(b*
c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 1/2*b*s
qrt(x)/((c*x^2 + b)*c^2) + 2*sqrt(x)/c^2
```

Mupad [B] (verification not implemented)

Time = 18.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^2}\,dx = \frac{2\sqrt{x}}{c^2} - \frac{5\,(-b)^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{4\,c^{9/4}} + \frac{b\sqrt{x}}{2\,\left(c^3\,x^2+b\,c^2\right)} + \frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,5\mathrm{i}}{4\,c^{9/4}}$$

input
```
int(x^(15/2)/(b*x^2 + c*x^4)^2,x)
```

output
```
(2*x^(1/2))/c^2 - (5*(-b)^(1/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*c^(
9/4)) + ((-b)^(1/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*5i)/(4*c^(9/4))
+ (b*x^(1/2))/(2*(b*c^2 + c^3*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.77

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^2}\,dx = \frac{10c^{\frac{3}{4}}b^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)+10c^{\frac{7}{4}}b^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^2-10c^{\frac{3}{4}}b^{\frac{5}{4}}\sqrt{2}}{}$$

input
```
int(x^(15/2)/(c*x^4+b*x^2)^2,x)
```

output
```
(10*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*
sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b + 10*c**(3/4)*b**(1/4)*sqrt(2)*ata
n((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(
2)))*c*x**2 - 10*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b - 10*c**(3/4)*b**(1/4
)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b
**(1/4)*sqrt(2)))*c*x**2 + 5*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(
1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b + 5*c**(3/4)*b**(1/4)*sqrt(
2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**2
- 5*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt
(b) + sqrt(c)*x)*b - 5*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(
1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**2 + 40*sqrt(x)*b*c + 32*sqrt(x)*c
**2*x**2)/(16*c**3*(b + c*x**2))
```

3.131 $\int \frac{x^{13/2}}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 165

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^2}\,dx = -\frac{x^{3/2}}{2c(b+cx^2)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}} - \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}}$$

output
```
-1/2*x^(3/2)/c/(c*x^2+b)-3/8*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(
1/2)/b^(1/4)/c^(7/4)+3/8*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)
/b^(1/4)/c^(7/4)-3/8*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1
/2)*x))*2^(1/2)/b^(1/4)/c^(7/4)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.78

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^2}\,dx = \frac{-\frac{4c^{3/4}x^{3/2}}{b+cx^2} - \frac{3\sqrt{2}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{b}} - \frac{3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt[4]{b}}}{8c^{7/4}}$$

input
```
Integrate[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
((-4*c^(3/4)*x^(3/2))/(b + c*x^2) - (3*Sqrt[2]*ArcTan[(Sqrt[b] - Sqrt[c]*x
)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(1/4) - (3*Sqrt[2]*ArcTanh[(Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^(1/4))/(8*c^(7/4))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {9, 252, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^{5/2}}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{3\int \frac{\sqrt{x}}{cx^2+b}dx}{4c} - \frac{x^{3/2}}{2c(b+cx^2)}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{3\int \frac{x}{cx^2+b}d\sqrt{x}}{2c}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 826

$$\frac{3\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{3\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{3\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 217

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 1479

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

$\big\downarrow$ 25

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

$\big\downarrow$ 27

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

$\big\downarrow$ 1103

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

input
```
Int[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*x^(3/2)/(c*(b + c*x^2)) + (3*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log
[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)
*c^(1/4)))/(2*Sqrt[c])))/(2*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{x^{\frac{3}{2}}}{2c(c\,x^{2}+b)}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	124
default	$-\frac{x^{\frac{3}{2}}}{2c(c\,x^{2}+b)}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16c^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	124

input
```
int(x^(13/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(3/2)/c/(c*x^2+b)+3/16/c^2/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/
4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)
^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b
)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.21

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^2}\,dx = \frac{3\left(c^2x^2+bc\right)\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\log\left(bc^5\left(-\frac{1}{bc^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)-3\left(i\,c^2x^2+i\,bc\right)\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\log\left(i\,bc^5\left(-\frac{1}{b}\right.\right.}{}$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/8*(3*(c^2*x^2 + b*c)*(-1/(b*c^7))^(1/4)*log(b*c^5*(-1/(b*c^7))^(3/4) + s
qrt(x)) - 3*(I*c^2*x^2 + I*b*c)*(-1/(b*c^7))^(1/4)*log(I*b*c^5*(-1/(b*c^7)
)^(3/4) + sqrt(x)) - 3*(-I*c^2*x^2 - I*b*c)*(-1/(b*c^7))^(1/4)*log(-I*b*c^
5*(-1/(b*c^7))^(3/4) + sqrt(x)) - 3*(c^2*x^2 + b*c)*(-1/(b*c^7))^(1/4)*log
(-b*c^5*(-1/(b*c^7))^(3/4) + sqrt(x)) - 4*x^(3/2))/(c^2*x^2 + b*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.18

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^2}\,dx = -\frac{x^{\frac{3}{2}}}{2\,(c^2x^2+bc)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{16\,c}$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*x^(3/2)/(c^2*x^2 + b*c) + 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)
*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)
*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4
) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(
c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(
b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*
x + sqrt(b))/(b^(1/4)*c^(3/4)))/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.21

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^2}\,dx = -\frac{x^{\frac{3}{2}}}{2\,(cx^2+b)c} + \frac{3\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^4} + \frac{3\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^4} - \frac{3\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^4} + \frac{3\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^4}$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*x^(3/2)/((c*x^2 + b)*c) + 3/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) + 3/8*sqrt(2)*(b*
c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/
4))/(b*c^4) - 3/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) +
 x + sqrt(b/c))/(b*c^4) + 3/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(b/c)^(1/4) + x + sqrt(b/c))/(b*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.39

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^2}\,dx = \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{4\,(-b)^{1/4}\,c^{7/4}} - \frac{x^{3/2}}{2\,c\,(c\,x^2+b)} - \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{4\,(-b)^{1/4}\,c^{7/4}}$$

input
```
int(x^(13/2)/(b*x^2 + c*x^4)^2,x)
```

output
```
(3*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^(1/4)*c^(7/4)) - x^(3/2)/(2
*c*(b + c*x^2)) - (3*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^(1/4)*c^
(7/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.86

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^2}\,dx = \frac{-6c^{\frac{1}{4}}b^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right) - 6c^{\frac{5}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^2 + 6c^{\frac{1}{4}}b^{\frac{7}{4}}\sqrt{2}\,at}{}$$

input
```
int(x^(13/2)/(c*x^4+b*x^2)^2,x)
```

output
```
( - 6*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b - 6*c**(1/4)*b**(3/4)*sqrt(2)*at
an((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt
(2)))*c*x**2 + 6*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b + 6*c**(1/4)*b**(3/4)
*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b*
*(1/4)*sqrt(2)))*c*x**2 + 3*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1
/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b + 3*c**(1/4)*b**(3/4)*sqrt(2
)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**2 -
 3*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(
b) + sqrt(c)*x)*b - 3*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1
/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**2 - 8*sqrt(x)*b*c*x)/(16*b*c**2*(b
 + c*x**2))
```

3.132 $\int \frac{x^{11/2}}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 165

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^2} dx = -\frac{\sqrt{x}}{2c(b+cx^2)} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{5/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{3/4}c^{5/4}}$$

output
```
-1/2*x^(1/2)/c/(c*x^2+b)-1/8*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(
1/2)/b^(3/4)/c^(5/4)+1/8*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)
/b^(3/4)/c^(5/4)+1/8*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1
/2)*x))*2^(1/2)/b^(3/4)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^2}\,dx = \frac{-\frac{4\sqrt[4]{c}\sqrt{x}}{b+cx^2}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{b^{3/4}}+\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{b^{3/4}}}{8c^{5/4}}$$

input
```
Integrate[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
((-4*c^(1/4)*Sqrt[x])/(b + c*x^2) - (Sqrt[2]*ArcTan[(Sqrt[b] - Sqrt[c]*x)/
(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(3/4) + (Sqrt[2]*ArcTanh[(Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^(3/4))/(8*c^(5/4))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {9, 252, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{3/2}}{(b+cx^2)^2}dx$$

$$\Big\downarrow \; 252$$

$$\frac{\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{4c}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \; 266$$

$$\frac{\int \frac{1}{cx^2+b}d\sqrt{x}}{2c}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \; 755$$

$$\frac{\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

input

```
Int[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

output

```
-1/2*Sqrt[x]/(c*(b + c*x^2)) + ((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sq
rt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^
(1/4)))/(2*Sqrt[b]))/(2*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{2c(c\,x^2+b)}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16cb}$	127
default	$-\frac{\sqrt{x}}{2c(c\,x^2+b)}+\frac{\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16cb}$	127

input
```
int(x^(11/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(1/2)/c/(c*x^2+b)+1/16/c*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/
4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)
^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b
)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.16

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^2}\,dx = \frac{(c^2x^2+bc)\left(-\frac{1}{b^3c^5}\right)^{\frac{1}{4}}\log\left(bc\left(-\frac{1}{b^3c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-(-i\,c^2x^2-i\,bc)\left(-\frac{1}{b^3c^5}\right)^{\frac{1}{4}}\log\left(i\,bc\left(-\frac{1}{b^3}\right.\right.}{}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/8*((c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*log(b*c*(-1/(b^3*c^5))^(1/4) + s
qrt(x)) - (-I*c^2*x^2 - I*b*c)*(-1/(b^3*c^5))^(1/4)*log(I*b*c*(-1/(b^3*c^5
))^(1/4) + sqrt(x)) - (I*c^2*x^2 + I*b*c)*(-1/(b^3*c^5))^(1/4)*log(-I*b*c*
(-1/(b^3*c^5))^(1/4) + sqrt(x)) - (c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*log
(-b*c*(-1/(b^3*c^5))^(1/4) + sqrt(x)) - 4*sqrt(x))/(c^2*x^2 + b*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.18

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^2}\,dx = \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,c} - \frac{\sqrt{x}}{2\,(c^2x^2+bc)}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sq
rt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/
4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(
-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))
/c - 1/2*sqrt(x)/(c^2*x^2 + b*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.21

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^2}\,dx = \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^2} + \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^2} + \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^2} - \frac{\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^2} - \frac{\sqrt{x}}{2\,(cx^2+b)c}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b*c^2) + 1/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) + 1/16*sqrt(2)*(b*c
^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2) - 1/16*
sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b
*c^2) - 1/2*sqrt(x)/((c*x^2 + b)*c)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.39

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^2}\,dx = -\frac{\sqrt{x}}{2\,c\,(c\,x^2+b)} - \frac{\operatorname{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{4\,(-b)^{3/4}\,c^{5/4}} - \frac{\operatorname{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{4\,(-b)^{3/4}\,c^{5/4}}$$

input
```
int(x^(11/2)/(b*x^2 + c*x^4)^2,x)
```

output
```
- x^(1/2)/(2*c*(b + c*x^2)) - atan((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(
3/4)*c^(5/4)) - atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(3/4)*c^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.84

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^2}\,dx = \frac{-2c^{\frac{3}{4}}b^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right) - 2c^{\frac{7}{4}}b^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^2 + 2c^{\frac{3}{4}}b^{\frac{5}{4}}\sqrt{2}\,a}{}$$

input
```
int(x^(11/2)/(c*x^4+b*x^2)^2,x)
```

output
```
( - 2*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b - 2*c**(3/4)*b**(1/4)*sqrt(2)*at
an((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt
(2)))*c*x**2 + 2*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b + 2*c**(3/4)*b**(1/4)
*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b*
*(1/4)*sqrt(2)))*c*x**2 - c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4
)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b - c**(3/4)*b**(1/4)*sqrt(2)*lo
g( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**2 + c**
(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + s
qrt(c)*x)*b + c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt
(2) + sqrt(b) + sqrt(c)*x)*c*x**2 - 8*sqrt(x)*b*c)/(16*b*c**2*(b + c*x**2)
)
```

3.133 $\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 165

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx = \frac{x^{3/2}}{2b(b+cx^2)} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{3/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{3/4}} - \frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{5/4}c^{3/4}}$$

output
```
1/2*x^(3/2)/b/(c*x^2+b)-1/8*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1
/2)/b^(5/4)/c^(3/4)+1/8*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/
b^(5/4)/c^(3/4)-1/8*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/
2)*x))*2^(1/2)/b^(5/4)/c^(3/4)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.78

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx=\frac{\frac{4\sqrt[4]{b}x^{3/2}}{b+cx^2}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{c^{3/4}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{c^{3/4}}}{8b^{5/4}}$$

input
```
Integrate[x^(9/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*b^(1/4)*x^(3/2))/(b + c*x^2) - (Sqrt[2]*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(3/4) - (Sqrt[2]*ArcTanh[(Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(3/4))/(8*b^(5/4))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.579$, Rules used = {9, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{\sqrt{x}}{(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{\int \frac{\sqrt{x}}{cx^2+b}dx}{4b}+\frac{x^{3/2}}{2b(b+cx^2)}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\int \frac{x}{cx^2+b}d\sqrt{x}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}$$

$$\Big\downarrow \text{826}$$

$$\frac{\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

input
```
Int[x^(9/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
x^(3/2)/(2*b*(b + c*x^2)) + ((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[
b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/
4)))/(2*Sqrt[c]))/(2*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{x^{\frac{3}{2}}}{2b(c\,x^{2}+b)}+\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	127
default	$\frac{x^{\frac{3}{2}}}{2b(c\,x^{2}+b)}+\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	127

input
```
int(x^(9/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(3/2)/b/(c*x^2+b)+1/16/b/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4
)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^
(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)
^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx = \frac{(bcx^2+b^2)\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}\log\left(b^4c^2\left(-\frac{1}{b^5c^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-(i\,bcx^2+i\,b^2)\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}\log\left(i\,b^4c^2\left(-\right.\right.}{}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/8*((b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*log(b^4*c^2*(-1/(b^5*c^3))^(3/4)
 + sqrt(x)) - (I*b*c*x^2 + I*b^2)*(-1/(b^5*c^3))^(1/4)*log(I*b^4*c^2*(-1/(
b^5*c^3))^(3/4) + sqrt(x)) - (-I*b*c*x^2 - I*b^2)*(-1/(b^5*c^3))^(1/4)*log
(-I*b^4*c^2*(-1/(b^5*c^3))^(3/4) + sqrt(x)) - (b*c*x^2 + b^2)*(-1/(b^5*c^3
))^(1/4)*log(-b^4*c^2*(-1/(b^5*c^3))^(3/4) + sqrt(x)) + 4*x^(3/2))/(b*c*x^
2 + b^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.18

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx = \frac{x^{\frac{3}{2}}}{2\,(bcx^2+b^2)} + \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\cdots\right)}{b^{\frac{1}{4}}c\cdots}}{16\,b}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/2*x^(3/2)/(b*c*x^2 + b^2) + 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*
sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
 - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c
)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b
^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
 + sqrt(b))/(b^(1/4)*c^(3/4)))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx &= \frac{x^{\frac{3}{2}}}{2\,(cx^2+b)b} + \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^3} \\ &+ \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^3} \\ &- \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^3} \\ &+ \frac{\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^3}\end{aligned}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*x^(3/2)/((c*x^2 + b)*b) + 1/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/8*sqrt(2)*(b
*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1
/4))/(b^2*c^3) - 1/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4
) + x + sqrt(b/c))/(b^2*c^3) + 1/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqr
t(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.39

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx = \frac{x^{3/2}}{2\,b\,(c\,x^2+b)} - \frac{\operatorname{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{4\,(-b)^{5/4}\,c^{3/4}} + \frac{\operatorname{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{4\,(-b)^{5/4}\,c^{3/4}}$$

input
```
int(x^(9/2)/(b*x^2 + c*x^4)^2,x)
```

output
```
x^(3/2)/(2*b*(b + c*x^2)) - atan((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(5/
4)*c^(3/4)) + atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))/(4*(-b)^(5/4)*c^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.85

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx = \frac{-2c^{\frac{1}{4}}b^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right) - 2c^{\frac{5}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^2 + 2c^{\frac{1}{4}}b^{\frac{7}{4}}\sqrt{2}\,a}{}$$

input
```
int(x^(9/2)/(c*x^4+b*x^2)^2,x)
```

output
```
( - 2*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b - 2*c**(1/4)*b**(3/4)*sqrt(2)*at
an((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt
(2)))*c*x**2 + 2*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b + 2*c**(1/4)*b**(3/4)
*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b*
*(1/4)*sqrt(2)))*c*x**2 + c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4
)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b + c**(1/4)*b**(3/4)*sqrt(2)*lo
g( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**2 - c**
(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + s
qrt(c)*x)*b - c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt
(2) + sqrt(b) + sqrt(c)*x)*c*x**2 + 8*sqrt(x)*b*c*x)/(16*b**2*c*(b + c*x**
2))
```

3.134 $\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 165

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx=\frac{\sqrt{x}}{2b(b+cx^2)}-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

output
```
1/2*x^(1/2)/b/(c*x^2+b)-3/8*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1
/2)/b^(7/4)/c^(1/4)+3/8*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/
b^(7/4)/c^(1/4)+3/8*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/
2)*x))*2^(1/2)/b^(7/4)/c^(1/4)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.78

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx=\frac{\frac{4b^{3/4}\sqrt{x}}{b+cx^2}-\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt[4]{c}}}{8b^{7/4}}$$

input
```
Integrate[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*b^(3/4)*Sqrt[x])/(b + c*x^2) - (3*Sqrt[2]*ArcTan[(Sqrt[b] - Sqrt[c]*x)
/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(1/4) + (3*Sqrt[2]*ArcTanh[(Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(1/4))/(8*b^(7/4))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.579$, Rules used = {9, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{1}{\sqrt{x}\,(b+cx^2)^2}dx$$

$$\Big\downarrow \ 253$$

$$\frac{3\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{4b}+\frac{\sqrt{x}}{2b\,(b+cx^2)}$$

$$\Big\downarrow \ 266$$

$$\frac{3\int \frac{1}{cx^2+b}d\sqrt{x}}{2b}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

$\Big\downarrow$ 755

$$\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

$\Big\downarrow$ 217

$$\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

$\Big\downarrow$ 1479

$$\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

$\downarrow$ 25

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

$\downarrow$ 27

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

$\downarrow$ 1103

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

input
```
Int[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
Sqrt[x]/(2*b*(b + c*x^2)) + (3*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sq
rt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^
(1/4)))/(2*Sqrt[b])))/(2*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\sqrt{x}}{2b(c\,x^{2}+b)}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b^{2}}$	124
default	$\frac{\sqrt{x}}{2b(c\,x^{2}+b)}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b^{2}}$	124

input
```
int(x^(7/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(1/2)/b/(c*x^2+b)+3/16/b^2*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4
)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^
(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)
^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.17

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx = \frac{3\,(bcx^2+b^2)\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\,(-i\,bcx^2-i\,b^2)\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}\log\left(i\,b^2\left(-\frac{1}{b^7c}\right.\right.}{}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/8*(3*(b*c*x^2 + b^2)*(-1/(b^7*c))^(1/4)*log(b^2*(-1/(b^7*c))^(1/4) + sqr
t(x)) - 3*(-I*b*c*x^2 - I*b^2)*(-1/(b^7*c))^(1/4)*log(I*b^2*(-1/(b^7*c))^(
1/4) + sqrt(x)) - 3*(I*b*c*x^2 + I*b^2)*(-1/(b^7*c))^(1/4)*log(-I*b^2*(-1/
(b^7*c))^(1/4) + sqrt(x)) - 3*(b*c*x^2 + b^2)*(-1/(b^7*c))^(1/4)*log(-b^2*
(-1/(b^7*c))^(1/4) + sqrt(x)) + 4*sqrt(x))/(b*c*x^2 + b^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.18

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^2}\,dx = \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{\vphantom{x}}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{16\,b} + \frac{\sqrt{x}}{2\left(bcx^2+b^2\right)}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sq
rt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/
4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(
-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))
/b + 1/2*sqrt(x)/(b*c*x^2 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.21

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx = \frac{3\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c} + \frac{3\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c} + \frac{3\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c} - \frac{3\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c} + \frac{\sqrt{x}}{2\,(cx^2+b)b}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
3/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b^2*c) + 3/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c) + 3/16*sqrt(2)*(b*c
^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c) - 3/16*
sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b
^2*c) + 1/2*sqrt(x)/((c*x^2 + b)*b)
```

Mupad [B] (verification not implemented)

Time = 17.57 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.39

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx = \frac{\sqrt{x}}{2\,b\,\left(c\,x^2+b\right)} + \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{7/4}\,c^{1/4}} + \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{\left(-b\right)}^{1/4}}\right)}{4\,{\left(-b\right)}^{7/4}\,c^{1/4}}$$

input
```
int(x^(7/2)/(b*x^2 + c*x^4)^2,x)
```

output
```
x^(1/2)/(2*b*(b + c*x^2)) + (3*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)
^(7/4)*c^(1/4)) + (3*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(4*(-b)^(7/4)*c^
(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.85

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx = \frac{-6c^{\frac{3}{4}}b^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)-6c^{\frac{7}{4}}b^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^2+6c^{\frac{3}{4}}b^{\frac{5}{4}}\sqrt{2}\,at}{}$$

input
```
int(x^(7/2)/(c*x^4+b*x^2)^2,x)
```

output
```
( - 6*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b - 6*c**(3/4)*b**(1/4)*sqrt(2)*at
an((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt
(2)))*c*x**2 + 6*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b + 6*c**(3/4)*b**(1/4)
*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b*
*(1/4)*sqrt(2)))*c*x**2 - 3*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1
/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b - 3*c**(3/4)*b**(1/4)*sqrt(2
)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**2 +
 3*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(
b) + sqrt(c)*x)*b + 3*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1
/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**2 + 8*sqrt(x)*b*c)/(16*b**2*c*(b +
 c*x**2))
```

3.135 $\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 177

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx = -\frac{5}{2b^2\sqrt{x}}+\frac{1}{2b\sqrt{x}\,(b+cx^2)}+\frac{5\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}} - \frac{5\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{c}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{9/4}}$$

output

```
-5/2/b^2/x^(1/2)+1/2/b/x^(1/2)/(c*x^2+b)+5/8*c^(1/4)*arctan(1-2^(1/2)*c^(1
/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(9/4)-5/8*c^(1/4)*arctan(1+2^(1/2)*c^(1/4)*
x^(1/2)/b^(1/4))*2^(1/2)/b^(9/4)+5/8*c^(1/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/
4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx = \frac{-\frac{4\sqrt[4]{b}(4b+5cx^2)}{\sqrt{x}(b+cx^2)}+5\sqrt{2}\sqrt[4]{c}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+5\sqrt{2}\sqrt[4]{c}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{8b^{9/4}}$$

input
```
Integrate[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
((-4*b^(1/4)*(4*b + 5*c*x^2))/(Sqrt[x]*(b + c*x^2)) + 5*Sqrt[2]*c^(1/4)*Ar
cTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sqrt[2]*
c^(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/
(8*b^(9/4))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.46, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {9, 253, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^{3/2}(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{5\int \frac{1}{x^{3/2}(cx^2+b)}dx}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{5\left(-\frac{c\int\frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}\,(b+cx^2)}$$

$\Big\downarrow$ 266

$$\frac{5\left(-\frac{2c\int\frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}\,(b+cx^2)}$$

$\Big\downarrow$ 826

$$\frac{5\left(-\frac{2c\left(\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}\,(b+cx^2)}$$

$\Big\downarrow$ 1476

$$\frac{5\left(-\frac{2c\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}\,(b+cx^2)}$$

$\Big\downarrow$ 1082

$$\frac{5\left(-\frac{2c\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}\,(b+cx^2)}$$

$\Big\downarrow$ 217

$$\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}\left(b+cx^2\right)}$$

$\Big\downarrow$ 1479

$$\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}$$

$$\frac{1}{2b\sqrt{x}\left(b+cx^2\right)}$$

$\Big\downarrow$ 25

$$\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}\left(b+cx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}\left(b+cx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}\left(b+cx^2\right)}$$

input
```
Int[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
1/(2*b*Sqrt[x]*(b + c*x^2)) + (5*(-2/(b*Sqrt[x]) - (2*c*((-(ArcTan[1 - (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) -
(-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*
b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b))/(4*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{2c\left(\frac{x^{\frac{3}{2}}}{4c\,x^{2}+4b}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{2}}-\frac{2}{b^{2}\sqrt{x}}$	136
default	$-\frac{2c\left(\frac{x^{\frac{3}{2}}}{4c\,x^{2}+4b}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{2}}-\frac{2}{b^{2}\sqrt{x}}$	136
risch	$-\frac{2}{b^{2}\sqrt{x}}-\frac{c\left(\frac{x^{\frac{3}{2}}}{2c\,x^{2}+2b}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{2}}$	136

input
```
int(x^(5/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*c/b^2*(1/4*x^(3/2)/(c*x^2+b)+5/32/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b
)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1
/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(
1/c*b)^(1/4)*x^(1/2)-1)))-2/b^2/x^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.23

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx =$$
$$-\frac{5\,(b^2cx^3+b^3x)\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}\log\left(125\,b^7\left(-\frac{c}{b^9}\right)^{\frac{3}{4}}+125\,c\sqrt{x}\right)+5\,(-i\,b^2cx^3-i\,b^3x)\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}\log\left(125i\,b^7\left(-\frac{c}{b^9}\right)^{\frac{3}{4}}\right.}{}$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/8*(5*(b^2*c*x^3 + b^3*x)*(-c/b^9)^(1/4)*log(125*b^7*(-c/b^9)^(3/4) + 12
5*c*sqrt(x)) + 5*(-I*b^2*c*x^3 - I*b^3*x)*(-c/b^9)^(1/4)*log(125*I*b^7*(-c
/b^9)^(3/4) + 125*c*sqrt(x)) + 5*(I*b^2*c*x^3 + I*b^3*x)*(-c/b^9)^(1/4)*lo
g(-125*I*b^7*(-c/b^9)^(3/4) + 125*c*sqrt(x)) - 5*(b^2*c*x^3 + b^3*x)*(-c/b
^9)^(1/4)*log(-125*b^7*(-c/b^9)^(3/4) + 125*c*sqrt(x)) + 4*(5*c*x^2 + 4*b)
*sqrt(x))/(b^2*c*x^3 + b^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx = -\frac{5\,cx^2+4\,b}{2\left(b^2cx^{\frac{5}{2}}+b^3\sqrt{x}\right)} - \frac{5\,c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{\right.}}{}\right.}{16\,b^2}$$

input

```
integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output

```
-1/2*(5*c*x^2 + 4*b)/(b^2*c*x^(5/2) + b^3*sqrt(x)) - 5/16*c*(2*sqrt(2)*arc
tan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)
*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqr
t(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^
(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^2
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.19

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx = -\frac{5\,cx^2+4\,b}{2\left(cx^{\frac{5}{2}}+b\sqrt{x}\right)b^2} - \frac{5\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^2} - \frac{5\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^2} + \frac{5\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c^2} - \frac{5\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c^2}$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*(5*c*x^2 + 4*b)/((c*x^(5/2) + b*sqrt(x))*b^2) - 5/8*sqrt(2)*(b*c^3)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^
3*c^2) - 5/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 5/16*sqrt(2)*(b*c^3)^(3/4)*log(sqr
t(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) - 5/16*sqrt(2)*(b*c^3)
^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.44

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx = \frac{5\,(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{9/4}} - \frac{5\,(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{9/4}} - \frac{\frac{2}{b}+\frac{5\,c\,x^2}{2\,b^2}}{b\,\sqrt{x}+c\,x^{5/2}}$$

input
```
int(x^(5/2)/(b*x^2 + c*x^4)^2,x)
```

output
```
(5*(-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(9/4)) - (5*(-c)^(
1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(9/4)) - (2/b + (5*c*x^2)/(2
*b^2))/(b*x^(1/2) + c*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.86

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx = \frac{10\sqrt{x}\,c^{\frac{1}{4}}b^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)+10\sqrt{x}\,c^{\frac{5}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^2-10\sqrt{}}{}$$

input
```
int(x^(5/2)/(c*x^4+b*x^2)^2,x)
```

output
```
(10*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*
sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b + 10*sqrt(x)*c**(1/4)*b**(
3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4
)*b**(1/4)*sqrt(2)))*c*x**2 - 10*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c
**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))
*b - 10*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2)
+ 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c*x**2 - 5*sqrt(x)*c**(1
/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) +
sqrt(c)*x)*b - 5*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)
*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**2 + 5*sqrt(x)*c**(1/4)*b**(3
/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b
 + 5*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(
2) + sqrt(b) + sqrt(c)*x)*c*x**2 - 32*b**2 - 40*b*c*x**2)/(16*sqrt(x)*b**3
*(b + c*x**2))
```

3.136 $\int \frac{x^{3/2}}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 177

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^2} dx = -\frac{7}{6b^2x^{3/2}} + \frac{1}{2bx^{3/2}(b+cx^2)} + \frac{7c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}} - \frac{7c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}} - \frac{7c^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{11/4}}$$

output
```
-7/6/b^2/x^(3/2)+1/2/b/x^(3/2)/(c*x^2+b)+7/8*c^(3/4)*arctan(1-2^(1/2)*c^(1
/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(11/4)-7/8*c^(3/4)*arctan(1+2^(1/2)*c^(1/4)
*x^(1/2)/b^(1/4))*2^(1/2)/b^(11/4)-7/8*c^(3/4)*arctanh(2^(1/2)*b^(1/4)*c^(
1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^2}\,dx = \frac{-\frac{4b^{3/4}(4b+7cx^2)}{x^{3/2}(b+cx^2)}+21\sqrt{2}c^{3/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-21\sqrt{2}c^{3/4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{24b^{11/4}}$$

input
```
Integrate[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
((-4*b^(3/4)*(4*b + 7*c*x^2))/(x^(3/2)*(b + c*x^2)) + 21*Sqrt[2]*c^(3/4)*A
rcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 21*Sqrt[2
]*c^(3/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)]
)/(24*b^(11/4))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.47, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {9, 253, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^{5/2}(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{7\int \frac{1}{x^{5/2}(cx^2+b)}dx}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{7\left(-\frac{c\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 266

$$\frac{7\left(-\frac{2c\int \frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 755

$$\frac{7\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{7\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{7\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{7\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{7\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/}}\right)}{4b}$$

$$\frac{1}{2bx^{3/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{7\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}$$

$$\frac{1}{2bx^{3/2}\left(b+cx^2\right)}$$

$\downarrow$ 27

$$\frac{7\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+$$

$$\frac{1}{2bx^{3/2}\left(b+cx^2\right)}$$

$\downarrow$ 1103

$$\frac{7\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}\left(b+cx^2\right)}$$

input
```
Int[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```

output
```
1/(2*b*x^(3/2)*(b + c*x^2)) + (7*(-2/(3*b*x^(3/2)) - (2*c*((-(ArcTan[1 - (
Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])
+ (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2
]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/b))/(4*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

method	result	si
derivativedivides	$-\frac{2c\left(\frac{\sqrt{x}}{4c\,x^{2}+4b}+\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{b^{2}}-\frac{2}{3b^{2}x^{\frac{3}{2}}}$	1
default	$-\frac{2c\left(\frac{\sqrt{x}}{4c\,x^{2}+4b}+\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{b^{2}}-\frac{2}{3b^{2}x^{\frac{3}{2}}}$	1
risch	$-\frac{2}{3b^{2}x^{\frac{3}{2}}}-\frac{c\left(\frac{\sqrt{x}}{2c\,x^{2}+2b}+\frac{7\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b}\right)}{b^{2}}$	1

input
```
int(x^(3/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*c/b^2*(1/4*x^(1/2)/(c*x^2+b)+7/32*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b
)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1
/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(
1/c*b)^(1/4)*x^(1/2)-1)))-2/3/b^2/x^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.38

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^2}\,dx =$$

$$-\frac{21\,(b^2cx^4+b^3x^2)\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}\log\left(7\,b^3\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}+7\,c\sqrt{x}\right)+21\,(i\,b^2cx^4+i\,b^3x^2)\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}\log\left(7i\,b^3\left(-\frac{c^3}{b^{11}}\right)^{\frac{1}{4}}\right.}{}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/24*(21*(b^2*c*x^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*log(7*b^3*(-c^3/b^11)^(1
/4) + 7*c*sqrt(x)) + 21*(I*b^2*c*x^4 + I*b^3*x^2)*(-c^3/b^11)^(1/4)*log(7*
I*b^3*(-c^3/b^11)^(1/4) + 7*c*sqrt(x)) + 21*(-I*b^2*c*x^4 - I*b^3*x^2)*(-c
^3/b^11)^(1/4)*log(-7*I*b^3*(-c^3/b^11)^(1/4) + 7*c*sqrt(x)) - 21*(b^2*c*x
^4 + b^3*x^2)*(-c^3/b^11)^(1/4)*log(-7*b^3*(-c^3/b^11)^(1/4) + 7*c*sqrt(x)
) + 4*(7*c*x^2 + 4*b)*sqrt(x))/(b^2*c*x^4 + b^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.18

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^2}\,dx = -\frac{7\,cx^2+4\,b}{6\left(b^2cx^{\frac{7}{2}}+b^3x^{\frac{3}{2}}\right)} - \frac{7\left(\frac{2\,\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\,\sqrt{2}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}-\frac{\sqrt{2}c^{\frac{3}{4}}\,\mathrm{l}}{}\right.}{16\,b^2}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(7*c*x^2 + 4*b)/(b^2*c*x^(7/2) + b^3*x^(3/2)) - 7/16*(2*sqrt(2)*c*arc
tan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)
*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*c*arctan(-1/2*sqrt(
2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(s
qrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(3/4)*log(sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(3/4)*log(-sqrt(2)*b^
(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.11

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^2}\,dx = -\frac{7\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3} - \frac{7\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3} - \frac{7\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3} + \frac{7\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3} - \frac{c\sqrt{x}}{2\,(cx^2+b)b^2} - \frac{2}{3\,b^2x^{\frac{3}{2}}}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-7/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/b^3 - 7/8*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^3 - 7/16*sqrt(2)*(b*c^3)^(1/
4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 7/16*sqrt(2)*(b*
c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 - 1/2*c*s
qrt(x)/((c*x^2 + b)*b^2) - 2/3/(b^2*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 17.58 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.44

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^2}\,dx = \frac{7\,(-c)^{3/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{11/4}} - \frac{\frac{2}{3\,b}+\frac{7\,c\,x^2}{6\,b^2}}{b\,x^{3/2}+c\,x^{7/2}} + \frac{7\,(-c)^{3/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{11/4}}$$

input
```
int(x^(3/2)/(b*x^2 + c*x^4)^2,x)
```

output
```
(7*(-c)^(3/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(11/4)) - (2/(3*b)
+ (7*c*x^2)/(6*b^2))/(b*x^(3/2) + c*x^(7/2)) + (7*(-c)^(3/4)*atanh(((-c)^(
1/4)*x^(1/2))/b^(1/4)))/(4*b^(11/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.90

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^2}\,dx = \frac{42\sqrt{x}\,c^{\frac{3}{4}}b^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x+42\sqrt{x}\,c^{\frac{7}{4}}b^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^3-42}{}$$

input
```
int(x^(3/2)/(c*x^4+b*x^2)^2,x)
```

output
```
(42*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*
sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b*x + 42*sqrt(x)*c**(3/4)*b*
*(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1
/4)*b**(1/4)*sqrt(2)))*c*x**3 - 42*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan(
(c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)
))*b*x - 42*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt
(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c*x**3 + 21*sqrt(x)*
c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(
b) + sqrt(c)*x)*b*x + 21*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*
c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**3 - 21*sqrt(x)*c**(3
/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqr
t(c)*x)*b*x - 21*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b*
*(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c*x**3 - 32*b**2 - 56*b*c*x**2)/(48*
sqrt(x)*b**3*x*(b + c*x**2))
```

3.137 $\int \frac{\sqrt{x}}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 190

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^2}\,dx = -\frac{9}{10b^2x^{5/2}}+\frac{9c}{2b^3\sqrt{x}}+\frac{1}{2bx^{5/2}(b+cx^2)} - \frac{9c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}} + \frac{9c^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}} - \frac{9c^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{13/4}}$$

output
```
-9/10/b^2/x^(5/2)+9/2*c/b^3/x^(1/2)+1/2/b/x^(5/2)/(c*x^2+b)-9/8*c^(5/4)*ar
ctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(13/4)+9/8*c^(5/4)*arcta
n(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(13/4)-9/8*c^(5/4)*arctanh(
2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(13/4)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^2}\,dx = \frac{\frac{4\sqrt[4]{b}\left(-4b^2+36bcx^2+45c^2x^4\right)}{x^{5/2}(b+cx^2)}-45\sqrt{2}c^{5/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-45\sqrt{2}c^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{40b^{13/4}}$$

input
```
Integrate[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```

output
```
((4*b^(1/4)*(-4*b^2 + 36*b*c*x^2 + 45*c^2*x^4))/(x^(5/2)*(b + c*x^2)) - 45
*Sqrt[2]*c^(5/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x])] - 45*Sqrt[2]*c^(5/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqr
t[b] + Sqrt[c]*x)])/(40*b^(13/4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.46, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {9, 253, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{x^{7/2}\left(b+cx^2\right)^2}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{9\int \frac{1}{x^{7/2}(cx^2+b)}dx}{4b}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{9\left(-\frac{c\int \frac{1}{x^{3/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \textbf{264}$$

$$\frac{9\left(-\frac{c\left(-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \textbf{266}$$

$$\frac{9\left(-\frac{c\left(-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \textbf{826}$$

$$\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \textbf{1476}$$

$$\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

$\downarrow$ 217

$$\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

$\downarrow$ 1479

$$\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}\cdots}\right)}{b}\cdots\right)}{4b}$$

$$\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\cdots\right)}{4b}$$

$$\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+$$

$$\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

$\downarrow$ 1103

$$\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{}{5}\right.}{4b}$$

$$\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

input

```
Int[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```

output

```
1/(2*b*x^(5/2)*(b + c*x^2)) + (9*(-2/(5*b*x^(5/2)) - (c*(-2/(b*Sqrt[x]) -
(2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^
(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c
^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b))/
b))/(4*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{2(-10c\,x^2+b)}{5b^3x^{\frac{5}{2}}}+\frac{c^2\left(\frac{x^{\frac{3}{2}}}{2c\,x^2+2b}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^3}$
derivativedivides	$-\frac{2}{5b^2x^{\frac{5}{2}}}+\frac{4c}{b^3\sqrt{x}}+\frac{2c^2\left(\frac{x^{\frac{3}{2}}}{4c\,x^2+4b}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^3}$
default	$-\frac{2}{5b^2x^{\frac{5}{2}}}+\frac{4c}{b^3\sqrt{x}}+\frac{2c^2\left(\frac{x^{\frac{3}{2}}}{4c\,x^2+4b}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^3}$

input
```
int(x^(1/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-10*c*x^2+b)/b^3/x^(5/2)+1/b^3*c^2*(1/2*x^(3/2)/(c*x^2+b)+9/16/c/(1/
c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+
(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/
4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^2}\,dx$$

$$= \frac{45\left(b^3cx^5+b^4x^3\right)\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}\log\left(729\,b^{10}\left(-\frac{c^5}{b^{13}}\right)^{\frac{3}{4}}+729\,c^4\sqrt{x}\right)-45\left(i\,b^3cx^5+i\,b^4x^3\right)\left(-\frac{c^5}{b^{13}}\right)^{\frac{1}{4}}\log\left(729i\,b\right.}{}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/40*(45*(b^3*c*x^5 + b^4*x^3)*(-c^5/b^13)^(1/4)*log(729*b^10*(-c^5/b^13)^
(3/4) + 729*c^4*sqrt(x)) - 45*(I*b^3*c*x^5 + I*b^4*x^3)*(-c^5/b^13)^(1/4)*
log(729*I*b^10*(-c^5/b^13)^(3/4) + 729*c^4*sqrt(x)) - 45*(-I*b^3*c*x^5 - I
*b^4*x^3)*(-c^5/b^13)^(1/4)*log(-729*I*b^10*(-c^5/b^13)^(3/4) + 729*c^4*sq
rt(x)) - 45*(b^3*c*x^5 + b^4*x^3)*(-c^5/b^13)^(1/4)*log(-729*b^10*(-c^5/b^
13)^(3/4) + 729*c^4*sqrt(x)) + 4*(45*c^2*x^4 + 36*b*c*x^2 - 4*b^2)*sqrt(x)
)/(b^3*c*x^5 + b^4*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**(1/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{x}}{\left(bx^2+cx^4\right)^2}\,dx=\frac{45\,c^2x^4+36\,bcx^2-4\,b^2}{10\,\left(b^3cx^{\frac{9}{2}}+b^4x^{\frac{5}{2}}\right)}+\frac{9\,c^2\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log}{}\right.}{16\,b^3}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/10*(45*c^2*x^4 + 36*b*c*x^2 - 4*b^2)/(b^3*c*x^(9/2) + b^4*x^(5/2)) + 9/1
6*c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sq
rt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1
/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log
(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4))
)/b^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^2}\,dx = \frac{c^2x^{\frac{3}{2}}}{2\,(cx^2+b)b^3} + \frac{9\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c} + \frac{9\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c} - \frac{9\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4c} + \frac{9\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4c} + \frac{2\,(10\,cx^2-b)}{5\,b^3x^{\frac{5}{2}}}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
1/2*c^2*x^(3/2)/((c*x^2 + b)*b^3) + 9/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*s
qrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 9/8*sqrt(2
)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c
)^(1/4))/(b^4*c) - 9/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1
/4) + x + sqrt(b/c))/(b^4*c) + 9/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqr
t(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) + 2/5*(10*c*x^2 - b)/(b^3*x^(5/2
))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^2}\,dx = \frac{\frac{18\,c\,x^2}{5\,b^2}-\frac{2}{5\,b}+\frac{9\,c^2\,x^4}{2\,b^3}}{b\,x^{5/2}+c\,x^{9/2}}-\frac{9\,(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{13/4}}+\frac{9\,(-c)^{5/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{13/4}}$$

input
```
int(x^(1/2)/(b*x^2 + c*x^4)^2,x)
```

output
```
((18*c*x^2)/(5*b^2) - 2/(5*b) + (9*c^2*x^4)/(2*b^3))/(b*x^(5/2) + c*x^(9/2
)) - (9*(-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(13/4)) + (9*(
-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(13/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^2}\,dx = \frac{-90\sqrt{x}\,c^{\frac{5}{4}}b^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^2-90\sqrt{x}\,c^{\frac{9}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^4+90\sqrt{x}\,c^{\frac{5}{4}}b^{\frac{7}{4}}\sqrt{2}\,ata}{}$$

input
```
int(x^(1/2)/(c*x^4+b*x^2)^2,x)
```

output
```
( - 90*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b*c*x**2 - 90*sqrt(x)*c**
(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c)
)/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 + 90*sqrt(x)*c**(1/4)*b**(3/4)*sq
rt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1
/4)*sqrt(2)))*b*c*x**2 + 90*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/
4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2
*x**4 + 45*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1
/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 + 45*sqrt(x)*c**(1/4)*b**(3/4)
*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c
**2*x**4 - 45*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1
/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 - 45*sqrt(x)*c**(1/4)*b**(3/4)
*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2
*x**4 - 32*b**3 + 288*b**2*c*x**2 + 360*b*c**2*x**4)/(80*sqrt(x)*b**4*x**2
*(b + c*x**2))
```

3.138 $\int \frac{1}{\sqrt{x}(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 190

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx &= -\frac{11}{14b^2x^{7/2}}+\frac{11c}{6b^3x^{3/2}}+\frac{1}{2bx^{7/2}\,(b+cx^2)} \\ &\quad -\frac{11c^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}} \\ &\quad +\frac{11c^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}} \\ &\quad +\frac{11c^{7/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{15/4}}\end{aligned}$$

output
```
-11/14/b^2/x^(7/2)+11/6*c/b^3/x^(3/2)+1/2/b/x^(7/2)/(c*x^2+b)-11/8*c^(7/4)
*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(15/4)+11/8*c^(7/4)*a
rctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(15/4)+11/8*c^(7/4)*arc
tanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(15/4)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx = \frac{\frac{4b^{3/4}\left(-12b^2+44bcx^2+77c^2x^4\right)}{x^{7/2}\left(b+cx^2\right)}-231\sqrt{2}c^{7/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+231\sqrt{2}c^{7/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{168b^{15/4}}$$

input
```
Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

output
```
((4*b^(3/4)*(-12*b^2 + 44*b*c*x^2 + 77*c^2*x^4))/(x^(7/2)*(b + c*x^2)) - 2
31*Sqrt[2]*c^(7/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*S
qrt[x])] + 231*Sqrt[2]*c^(7/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(
Sqrt[b] + Sqrt[c]*x)])/(168*b^(15/4))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.47, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {9, 253, 264, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^{9/2}\left(b+cx^2\right)^2}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{11\int \frac{1}{x^{9/2}\left(cx^2+b\right)}dx}{4b}+\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{11\left(-\frac{c\int \frac{1}{x^{5/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 264

$$\frac{11\left(-\frac{c\left(-\frac{c\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 266

$$\frac{11\left(-\frac{c\left(-\frac{2c\int \frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 755

$$\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

$\downarrow$ 1479

$$\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{}{3b}\right)}{b}\right)}{4b}$$

$$\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

$$\downarrow 25$$

11

$$\frac{\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right.}{b}\right.}{4b}$$

$$\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

$$\downarrow 27$$

$$\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}$$

$$\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

$\downarrow$ 1103

$$\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}\cdots\right)}{4b}$$

$$\frac{1}{2bx^{7/2}\,(b+cx^2)}$$

input
```
Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

output
```
1/(2*b*x^(7/2)*(b + c*x^2)) + (11*(-2/(7*b*x^(7/2)) - (c*(-2/(3*b*x^(3/2))
 - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)
*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4
)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/b
))/b))/(4*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{2}{7b^2x^{\frac{7}{2}}}+\frac{4c}{3b^3x^{\frac{3}{2}}}+\frac{2c^2\left(\frac{\sqrt{x}}{4c\,x^2+4b}+\frac{11\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{32b}\right.}{b^3}$
default	$-\frac{2}{7b^2x^{\frac{7}{2}}}+\frac{4c}{3b^3x^{\frac{3}{2}}}+\frac{2c^2\left(\frac{\sqrt{x}}{4c\,x^2+4b}+\frac{11\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{32b}\right.}{b^3}$
risch	$-\frac{2\left(-14c\,x^2+3b\right)}{21b^3x^{\frac{7}{2}}}+\frac{c^2\left(\frac{\sqrt{x}}{2c\,x^2+2b}+\frac{11\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right.}{16b}\right.}{b^3}$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/7/b^2/x^(7/2)+4/3*c/b^3/x^(3/2)+2*c^2/b^3*(1/4*x^(1/2)/(c*x^2+b)+11/32*
(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)
)/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b
)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx$$
$$= \frac{231\left(b^3cx^6+b^4x^4\right)\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}\log\left(11\,b^4\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}+11\,c^2\sqrt{x}\right)-231\left(-i\,b^3cx^6-i\,b^4x^4\right)\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}\log\left(11i\,b\right.}{}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
1/168*(231*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*log(11*b^4*(-c^7/b^15)^
(1/4) + 11*c^2*sqrt(x)) - 231*(-I*b^3*c*x^6 - I*b^4*x^4)*(-c^7/b^15)^(1/4)
*log(11*I*b^4*(-c^7/b^15)^(1/4) + 11*c^2*sqrt(x)) - 231*(I*b^3*c*x^6 + I*b
^4*x^4)*(-c^7/b^15)^(1/4)*log(-11*I*b^4*(-c^7/b^15)^(1/4) + 11*c^2*sqrt(x)
) - 231*(b^3*c*x^6 + b^4*x^4)*(-c^7/b^15)^(1/4)*log(-11*b^4*(-c^7/b^15)^(1
/4) + 11*c^2*sqrt(x)) + 4*(77*c^2*x^4 + 44*b*c*x^2 - 12*b^2)*sqrt(x))/(b^3
*c*x^6 + b^4*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(1/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = \frac{77\,c^2x^4+44\,bcx^2-12\,b^2}{42\left(b^3cx^{\frac{11}{2}}+b^4x^{\frac{7}{2}}\right)} + \frac{11\left(\frac{2\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}c^{\frac{7}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}} - \right.}{16\,b^3}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
1/42*(77*c^2*x^4 + 44*b*c*x^2 - 12*b^2)/(b^3*c*x^(11/2) + b^4*x^(7/2)) + 1
1/16*(2*sqrt(2)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c
)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt
(2)*c^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/
sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(7/4)*l
og(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2
)*c^(7/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3
/4))/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = \frac{11\,\sqrt{2}(bc^3)^{\frac{1}{4}}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4} + \frac{11\,\sqrt{2}(bc^3)^{\frac{1}{4}}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4} + \frac{11\,\sqrt{2}(bc^3)^{\frac{1}{4}}\,c\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4} - \frac{11\,\sqrt{2}(bc^3)^{\frac{1}{4}}\,c\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4} + \frac{c^2\sqrt{x}}{2\,(cx^2+b)b^3} + \frac{2\,(14\,cx^2-3\,b)}{21\,b^3x^{\frac{7}{2}}}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
11/8*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/b^4 + 11/8*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 + 11/16*sqrt(2)*(b*c^
3)^(1/4)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 11/16*sq
rt(2)*(b*c^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^
4 + 1/2*c^2*sqrt(x)/((c*x^2 + b)*b^3) + 2/21*(14*c*x^2 - 3*b)/(b^3*x^(7/2)
)
```

Mupad [B] (verification not implemented)

Time = 17.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = \frac{\frac{22\,c\,x^2}{21\,b^2}-\frac{2}{7\,b}+\frac{11\,c^2\,x^4}{6\,b^3}}{b\,x^{7/2}+c\,x^{11/2}}+\frac{11\,{(-c)}^{7/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{15/4}} + \frac{11\,{(-c)}^{7/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{15/4}}$$

input
```
int(1/(x^(1/2)*(b*x^2 + c*x^4)^2),x)
```

output
```
((22*c*x^2)/(21*b^2) - 2/(7*b) + (11*c^2*x^4)/(6*b^3))/(b*x^(7/2) + c*x^(1
1/2)) + (11*(-c)^(7/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(15/4)) +
(11*(-c)^(7/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(15/4))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.87

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = \frac{-462\sqrt{x}\,c^{\frac{7}{4}}b^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^3-462\sqrt{x}\,c^{\frac{11}{4}}b^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^5+462\sqrt{x}\,c^{\frac{7}{4}}b^{\frac{5}{4}}\sqrt{2}}{}$$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2)^2,x)
```

output

```
( - 462*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2)
- 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b*c*x**3 - 462*sqrt(x)*c
**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(
c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**5 + 462*sqrt(x)*c**(3/4)*b**(1/4)
*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b*
*(1/4)*sqrt(2)))*b*c*x**3 + 462*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c*
*(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*
c**2*x**5 - 231*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*
b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**3 - 231*sqrt(x)*c**(3/4)*b*
*(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c
)*x)*c**2*x**5 + 231*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4
)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**3 + 231*sqrt(x)*c**(3/4)*
b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)
*x)*c**2*x**5 - 96*b**3 + 352*b**2*c*x**2 + 616*b*c**2*x**4)/(336*sqrt(x)*
b**4*x**3*(b + c*x**2))
```

3.139 $\int \frac{1}{x^{3/2}(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 205

$$\int \frac{1}{x^{3/2}(bx^2+cx^4)^2} dx = -\frac{13}{18b^2x^{9/2}} + \frac{13c}{10b^3x^{5/2}} - \frac{13c^2}{2b^4\sqrt{x}} + \frac{1}{2bx^{9/2}(b+cx^2)} + \frac{13c^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}} - \frac{13c^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}} + \frac{13c^{9/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{4\sqrt{2}b^{17/4}}$$

output
```
-13/18/b^2/x^(9/2)+13/10*c/b^3/x^(5/2)-13/2*c^2/b^4/x^(1/2)+1/2/b/x^(9/2)/
(c*x^2+b)+13/8*c^(9/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b
^(17/4)-13/8*c^(9/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(
17/4)+13/8*c^(9/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2
)*x))*2^(1/2)/b^(17/4)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{3/2}(bx^2+cx^4)^2}\,dx = \frac{-\frac{4\sqrt[4]{b}(20b^3-52b^2cx^2+468bc^2x^4+585c^3x^6)}{x^{9/2}(b+cx^2)}+585\sqrt{2}c^{9/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+585\sqrt{2}c}{360b^{17/4}}$$

input
```
Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

output
```
((-4*b^(1/4)*(20*b^3 - 52*b^2*c*x^2 + 468*b*c^2*x^4 + 585*c^3*x^6))/(x^(9/
2)*(b + c*x^2)) + 585*Sqrt[2]*c^(9/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x])] + 585*Sqrt[2]*c^(9/4)*ArcTanh[(Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(360*b^(17/4))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.44, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {9, 253, 264, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^{11/2}(b+cx^2)^2}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{13\int \frac{1}{x^{11/2}(cx^2+b)}dx}{4b}+\frac{1}{2bx^{9/2}(b+cx^2)}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{13\left(-\frac{c\int \frac{1}{x^{7/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}$$

$\downarrow$ 264

$$\frac{13\left(-\frac{c\left(-\frac{c\int \frac{1}{x^{3/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}$$

$\downarrow$ 264

$$\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}$$

$\downarrow$ 266

$$\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}$$

$\downarrow$ 826

$$\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}$$

$\downarrow$ 1476

$$\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}$$

↓ 1082

$$\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+$$

$$\frac{1}{2bx^{9/2}\left(b+cx^2\right)}$$

$\downarrow$ 217

$$\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}$$

↓ 1479

$$13\left(-\frac{c\left(c-\frac{c\left(c-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right.}{b}\right.}{b}\right.}{b}\right.$$

↓ 25

13 $$-\frac{\left(c-\frac{\left(c-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{\cdots}}\cdots\right)}{b}\cdots\right)}{b}\cdots$$

$$\Big\downarrow \text{27}$$

13

$$-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}$$

↓ 1103

$$\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}}{b}}{4b}$$

$$\frac{1}{2bx^{9/2}\left(b+cx^{2}\right)}$$

input
```
Int[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

output
```
1/(2*b*x^(9/2)*(b + c*x^2)) + (13*(-2/(9*b*x^(9/2)) - (c*(-2/(5*b*x^(5/2))
 - (c*(-2/(b*Sqrt[x]) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt
[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1
/4)))/(2*Sqrt[c])))/b))/b))/b))/(4*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{2c^3\left(\frac{x^{\frac{3}{2}}}{4c\,x^2+4b}+\frac{13\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^4}-\frac{2}{9b^2x^{\frac{9}{2}}}-\frac{6c^2}{b^4\sqrt{}}$
default	$-\frac{2c^3\left(\frac{x^{\frac{3}{2}}}{4c\,x^2+4b}+\frac{13\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^4}-\frac{2}{9b^2x^{\frac{9}{2}}}-\frac{6c^2}{b^4\sqrt{}}$
risch	$-\frac{2\left(135c^2x^4-18bc\,x^2+5b^2\right)}{45b^4x^{\frac{9}{2}}}-\frac{c^3\left(\frac{x^{\frac{3}{2}}}{2c\,x^2+2b}+\frac{13\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{16c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^4}$

input
```
int(1/x^(3/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*c^3/b^4*(1/4*x^(3/2)/(c*x^2+b)+13/32/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/
c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)
+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2
)/(1/c*b)^(1/4)*x^(1/2)-1)))-2/9/b^2/x^(9/2)-6*c^2/b^4/x^(1/2)+4/5*c/b^3/x
^(5/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^{3/2} (bx^2 + cx^4)^2} dx =$$
$$-\frac{585 (b^4cx^7 + b^5x^5) \left(-\frac{c^9}{b^{17}}\right)^{\frac{1}{4}} \log \left(2197 b^{13} \left(-\frac{c^9}{b^{17}}\right)^{\frac{3}{4}} + 2197 c^7 \sqrt{x}\right) + 585 (-i b^4cx^7 - i b^5x^5) \left(-\frac{c^9}{b^{17}}\right)^{\frac{1}{4}} \log \left(\right.}{}$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
-1/360*(585*(b^4*c*x^7 + b^5*x^5)*(-c^9/b^17)^(1/4)*log(2197*b^13*(-c^9/b^
17)^(3/4) + 2197*c^7*sqrt(x)) + 585*(-I*b^4*c*x^7 - I*b^5*x^5)*(-c^9/b^17)
^(1/4)*log(2197*I*b^13*(-c^9/b^17)^(3/4) + 2197*c^7*sqrt(x)) + 585*(I*b^4*
c*x^7 + I*b^5*x^5)*(-c^9/b^17)^(1/4)*log(-2197*I*b^13*(-c^9/b^17)^(3/4) +
2197*c^7*sqrt(x)) - 585*(b^4*c*x^7 + b^5*x^5)*(-c^9/b^17)^(1/4)*log(-2197*
b^13*(-c^9/b^17)^(3/4) + 2197*c^7*sqrt(x)) + 4*(585*c^3*x^6 + 468*b*c^2*x^
4 - 52*b^2*c*x^2 + 20*b^3)*sqrt(x))/(b^4*c*x^7 + b^5*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2} (bx^2 + cx^4)^2} dx = \text{Timed out}$$

input
```
integrate(1/x**(3/2)/(c*x**4+b*x**2)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^{3/2} (bx^2 + cx^4)^2} \, dx = -\frac{585 \, c^3 x^6 + 468 \, bc^2 x^4 - 52 \, b^2 cx^2 + 20 \, b^3}{90 \left(b^4 cx^{\frac{13}{2}} + b^5 x^{\frac{9}{2}}\right)} - \frac{13 \, c^3 \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} + 2 \sqrt{c}\sqrt{x}\right)}{2 \sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} - 2 \sqrt{c}\sqrt{x}\right)}{2 \sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2} b^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{1}{4}} c^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(\right.}{} \right.}{16 \, b^4}$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
-1/90*(585*c^3*x^6 + 468*b*c^2*x^4 - 52*b^2*c*x^2 + 20*b^3)/(b^4*c*x^(13/2
) + b^5*x^(9/2)) - 13/16*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4
)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c
))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*s
qrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - s
qrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)
*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqr
t(b))/(b^(1/4)*c^(3/4)))/b^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = -\frac{c^3x^{\frac{3}{2}}}{2\left(cx^2+b\right)b^4} - \frac{13\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5} - \frac{13\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5} + \frac{13\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^5} - \frac{13\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^5} - \frac{2\left(135\,c^2x^4-18\,bcx^2+5\,b^2\right)}{45\,b^4x^{\frac{9}{2}}}$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
-1/2*c^3*x^(3/2)/((c*x^2 + b)*b^4) - 13/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2
*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^5 - 13/8*sqrt(2)
*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)
^(1/4))/b^5 + 13/16*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4)
+ x + sqrt(b/c))/b^5 - 13/16*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/b^5 - 2/45*(135*c^2*x^4 - 18*b*c*x^2 + 5*b^2)/(
b^4*x^(9/2))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx=\frac{13\,{\left(-c\right)}^{9/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{17/4}}-\frac{13\,{\left(-c\right)}^{9/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{4\,b^{17/4}}-\frac{\frac{2}{9\,b}-\frac{26\,c\,x^2}{45\,b^2}+\frac{26\,c^2\,x^4}{5\,b^3}+\frac{13\,c^3\,x^6}{2\,b^4}}{b\,x^{9/2}+c\,x^{13/2}}$$

input
```
int(1/(x^(3/2)*(b*x^2 + c*x^4)^2),x)
```

output
```
(13*(-c)^(9/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(17/4)) - (13*(-c
)^(9/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(4*b^(17/4)) - (2/(9*b) - (26*
c*x^2)/(45*b^2) + (26*c^2*x^4)/(5*b^3) + (13*c^3*x^6)/(2*b^4))/(b*x^(9/2)
+ c*x^(13/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.79

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx=\frac{1170\sqrt{x}\,c^{\frac{9}{4}}b^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)x^4+1170\sqrt{x}\,c^{\frac{13}{4}}b^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{c}}{c^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{2}}\right)\cdots}{\cdots}$$

input
```
int(1/x^(3/2)/(c*x^4+b*x^2)^2,x)
```

output
```
(1170*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) -
2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b*c**2*x**4 + 1170*sqrt(x)
*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqr
t(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**3*x**6 - 1170*sqrt(x)*c**(1/4)*b**(3
/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)
*b**(1/4)*sqrt(2)))*b*c**2*x**4 - 1170*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*a
tan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqr
t(2)))*c**3*x**6 - 585*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c*
*(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c**2*x**4 - 585*sqrt(x)*c
**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b
) + sqrt(c)*x)*c**3*x**6 + 585*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(
x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c**2*x**4 + 585*sqrt
(x)*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt
(b) + sqrt(c)*x)*c**3*x**6 - 160*b**4 + 416*b**3*c*x**2 - 3744*b**2*c**2*x
**4 - 4680*b*c**3*x**6)/(720*sqrt(x)*b**5*x**4*(b + c*x**2))
```

3.140 $\int \frac{x^{23/2}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 198

$$\int \frac{x^{23/2}}{(bx^2+cx^4)^3}\,dx = \frac{45\sqrt{x}}{16c^3} - \frac{x^{9/2}}{4c(b+cx^2)^2} - \frac{9x^{5/2}}{16c^2(b+cx^2)} + \frac{45\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{13/4}} - \frac{45\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{13/4}} - \frac{45\sqrt[4]{b}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}c^{13/4}}$$

output
```
45/16*x^(1/2)/c^3-1/4*x^(9/2)/c/(c*x^2+b)^2-9/16*x^(5/2)/c^2/(c*x^2+b)+45/
64*b^(1/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/c^(13/4)-45/6
4*b^(1/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/c^(13/4)-45/64
*b^(1/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1
/2)/c^(13/4)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.75

$$\int \frac{x^{23/2}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4\sqrt[4]{c}\sqrt{x}(45b^2+81bcx^2+32c^2x^4)}{(b+cx^2)^2}+45\sqrt{2}\sqrt[4]{b}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-45\sqrt{2}\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{64c^{13/4}}$$

input
```
Integrate[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
((4*c^(1/4)*Sqrt[x]*(45*b^2 + 81*b*c*x^2 + 32*c^2*x^4))/(b + c*x^2)^2 + 45
*Sqrt[2]*b^(1/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x])] - 45*Sqrt[2]*b^(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqr
t[b] + Sqrt[c]*x)])/(64*c^(13/4))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.45, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {9, 252, 252, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{23/2}}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^{11/2}}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{9\int \frac{x^{7/2}}{(cx^2+b)^2}dx}{8c}-\frac{x^{9/2}}{4c(b+cx^2)^2}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{9\left(\frac{5\int\frac{x^{3/2}}{cx^2+b}dx}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 262

$$\frac{9\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{b\int\frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 266

$$\frac{9\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{cx^2+b}d\sqrt{x}}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 755

$$\frac{9\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{9\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 1082$$

$$\frac{9\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{9\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1479

$$\frac{9\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}\right)}{8c}$$

$$\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}$$

$\big\downarrow$ 25

$$\frac{9\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{\ldots}{2c\ldots}\right.}{8c}$$

$$\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}$$

$\downarrow$ 27

$$\frac{9\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^{5/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{9\left(\frac{5\left(\frac{2\sqrt{x}}{c}-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{c}\right)}{4c}-\frac{x^5}{2c(b+}\right)}{8c}$$

$$\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}$$

input

```
Int[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```

output

```
-1/4*x^(9/2)/(c*(b + c*x^2)^2) + (9*(-1/2*x^(5/2)/(c*(b + c*x^2)) + (5*((2
*Sqrt[x])/c - (2*b*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt
[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqr
t[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c
^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*S
qrt[b])))/c))/(4*c)))/(8*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2\sqrt{x}}{c^{3}}-\frac{2b\left(\frac{-\frac{17c\,x^{\frac{5}{2}}}{32}-\frac{13b\sqrt{x}}{32}}{\left(c\,x^{2}+b\right)^{2}}+\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256b}\right)}{c^{3}}$
default	$\frac{2\sqrt{x}}{c^{3}}-\frac{2b\left(\frac{-\frac{17c\,x^{\frac{5}{2}}}{32}-\frac{13b\sqrt{x}}{32}}{\left(c\,x^{2}+b\right)^{2}}+\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256b}\right)}{c^{3}}$
risch	$\frac{2\sqrt{x}}{c^{3}}-\frac{b\left(\frac{-\frac{17c\,x^{\frac{5}{2}}}{16}-\frac{13b\sqrt{x}}{16}}{\left(c\,x^{2}+b\right)^{2}}+\frac{45\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128b}\right)}{c^{3}}$

input
```
int(x^(23/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/c^3-2/c^3*b*((-17/32*c*x^(5/2)-13/32*b*x^(1/2))/(c*x^2+b)^2+45/2
56*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1
/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/
c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.39

$$\int \frac{x^{23/2}}{\left(bx^2+cx^4\right)^3}\,dx =$$

$$-\frac{45\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\log\left(45\,c^3\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}+45\sqrt{x}\right)+45\left(i\,c^5x^4+2i\,bc^4x^2+i\,b^2c^3\right)\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\text{lo}}{}$$

input
```
integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/64*(45*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1/4)*log(45*c^3*(-b
/c^13)^(1/4) + 45*sqrt(x)) + 45*(I*c^5*x^4 + 2*I*b*c^4*x^2 + I*b^2*c^3)*(-
b/c^13)^(1/4)*log(45*I*c^3*(-b/c^13)^(1/4) + 45*sqrt(x)) + 45*(-I*c^5*x^4
- 2*I*b*c^4*x^2 - I*b^2*c^3)*(-b/c^13)^(1/4)*log(-45*I*c^3*(-b/c^13)^(1/4)
 + 45*sqrt(x)) - 45*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1/4)*log(
-45*c^3*(-b/c^13)^(1/4) + 45*sqrt(x)) - 4*(32*c^2*x^4 + 81*b*c*x^2 + 45*b^
2)*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{23/2}}{(bx^2 + cx^4)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**(23/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.16

$$\int \frac{x^{23/2}}{(bx^2 + cx^4)^3} \, dx = \frac{17\, bcx^{\frac{5}{2}} + 13\, b^2\sqrt{x}}{16\,(c^5x^4 + 2\, bc^4x^2 + b^2c^3)} - \frac{45\left(\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{c^{\frac{1}{4}}} - \cdots\right)}{128\, c^3} + \frac{2\sqrt{x}}{c^3}$$

input
```
integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/16*(17*b*c*x^(5/2) + 13*b^2*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) -
 45/128*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + 2*sqrt(2)
*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/sqrt(sqrt(b)*sqrt(c)) + sqrt(2)*b^(1/4)*log(sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4) - sqrt(2)*b^(1/4)
*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/c^(1/4))/c^3
+ 2*sqrt(x)/c^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.05

$$\int \frac{x^{23/2}}{(bx^2+cx^4)^3}\,dx = -\frac{45\,\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^4} - \frac{45\,\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^4} - \frac{45\,\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,c^4} + \frac{45\,\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,c^4} + \frac{2\,\sqrt{x}}{c^3} + \frac{17\,bcx^{\frac{5}{2}}+13\,b^2\sqrt{x}}{16\,(cx^2+b)^2c^3}$$

input
```
integrate(x^(23/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-45/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/c^4 - 45/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^4 - 45/128*sqrt(2)*(b*c^
3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 45/128*sqr
t(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 +
 2*sqrt(x)/c^3 + 1/16*(17*b*c*x^(5/2) + 13*b^2*sqrt(x))/((c*x^2 + b)^2*c^3
)
```

Mupad [B] (verification not implemented)

Time = 17.67 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.51

$$\int \frac{x^{23/2}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{13\,b^2\,\sqrt{x}}{16}+\frac{17\,b\,c\,x^{5/2}}{16}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{2\,\sqrt{x}}{c^3} - \frac{45\,(-b)^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,c^{13/4}}+\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,45\mathrm{i}}{32\,c^{13/4}}$$

input
```
int(x^(23/2)/(b*x^2 + c*x^4)^3,x)
```

output
```
((13*b^2*x^(1/2))/16 + (17*b*c*x^(5/2))/16)/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x
^2) + (2*x^(1/2))/c^3 - (45*(-b)^(1/4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))
/(32*c^(13/4)) + ((-b)^(1/4)*atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*45i)/(3
2*c^(13/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.44

$$\int \frac{x^{23/2}}{(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(23/2)/(c*x^4+b*x^2)^3,x)
```

output
```
(90*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*
sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 + 180*c**(3/4)*b**(1/4)*sqrt(2)
*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*s
qrt(2)))*b*c*x**2 + 90*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 - 90*c*
*(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c
))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 - 180*c**(3/4)*b**(1/4)*sqrt(2)*atan(
(c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)
))*b*c*x**2 - 90*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 + 45*c**(3/4)
*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqr
t(c)*x)*b**2 + 90*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/
4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 + 45*c**(3/4)*b**(1/4)*sqrt(2)*
log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4
- 45*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqr
t(b) + sqrt(c)*x)*b**2 - 90*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)
*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 - 45*c**(3/4)*b**(1/4)*s
qrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x
**4 + 360*sqrt(x)*b**2*c + 648*sqrt(x)*b*c**2*x**2 + 256*sqrt(x)*c**3*x**4
)/(128*c**4*(b**2 + 2*b*c*x**2 + c**2*x**4))
```

3.141 $\int \frac{x^{21/2}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 186

$$\int \frac{x^{21/2}}{(bx^2+cx^4)^3}\,dx = -\frac{x^{7/2}}{4c(b+cx^2)^2} - \frac{7x^{3/2}}{16c^2(b+cx^2)} - \frac{21\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}} + \frac{21\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}} - \frac{21\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}$$

output
```
-1/4*x^(7/2)/c/(c*x^2+b)^2-7/16*x^(3/2)/c^2/(c*x^2+b)-21/64*arctan(1-2^(1/
2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(1/4)/c^(11/4)+21/64*arctan(1+2^(1/2
)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(1/4)/c^(11/4)-21/64*arctanh(2^(1/2)*
b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(1/4)/c^(11/4)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.74

$$\int \frac{x^{21/2}}{(bx^2+cx^4)^3}\,dx=\frac{-\frac{4c^{3/4}x^{3/2}(7b+11cx^2)}{(b+cx^2)^2}-\frac{21\sqrt{2}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{b}}-\frac{21\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt[4]{b}}}{64c^{11/4}}$$

input
```
Integrate[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*c^(3/4)*x^(3/2)*(7*b + 11*c*x^2))/(b + c*x^2)^2 - (21*Sqrt[2]*ArcTan[
(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(1/4) - (21*Sq
rt[2]*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^
(1/4))/(64*c^(11/4))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.45, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {9, 252, 252, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{21/2}}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^{9/2}}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{7\int \frac{x^{5/2}}{(cx^2+b)^2}dx}{8c}-\frac{x^{7/2}}{4c(b+cx^2)^2}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{7\left(\frac{3\int \frac{\sqrt{x}}{cx^2+b}dx}{4c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 266$$

$$\frac{7\left(\frac{3\int \frac{x}{cx^2+b}d\sqrt{x}}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 826$$

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 1476$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 1082$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 217

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1479

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8c}$$

$$\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 25

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8c}$$

$$\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

$\downarrow$ 27

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8c}-$$

$$\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

$\downarrow$ 1103

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{2c}-\frac{x^{3/2}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

input
```
Int[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*x^(7/2)/(c*(b + c*x^2)^2) + (7*(-1/2*x^(3/2)/(c*(b + c*x^2)) + (3*((-
(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))
+ ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))
/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/(2*c)))/(8*c
)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{-\frac{11 x^{\frac{7}{2}}}{16 c}-\frac{7 b \,x^{\frac{3}{2}}}{16 c^{2}}}{\left(c \,x^{2}+b \right)^{2}}+\frac{21 \sqrt{2}\, \left(\ln \left(\frac{x -\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{b}{c}}}{x +\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128 c^{3} \left(\frac{b}{c}\right)^{\frac{1}{4}}}$	136
default	$\frac{-\frac{11 x^{\frac{7}{2}}}{16 c}-\frac{7 b \,x^{\frac{3}{2}}}{16 c^{2}}}{\left(c \,x^{2}+b \right)^{2}}+\frac{21 \sqrt{2}\, \left(\ln \left(\frac{x -\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{b}{c}}}{x +\left(\frac{b}{c}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128 c^{3} \left(\frac{b}{c}\right)^{\frac{1}{4}}}$	136

input
```
int(x^(21/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-11/32*x^(7/2)/c-7/32*b*x^(3/2)/c^2)/(c*x^2+b)^2+21/128/c^3/(1/c*b)^(1/
4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^
(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/
2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.48

$$\int \frac{x^{21/2}}{(bx^2 + cx^4)^3}\, dx = \frac{21\,(c^4x^4 + 2\,bc^3x^2 + b^2c^2)\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\log\left(bc^8\left(-\frac{1}{bc^{11}}\right)^{\frac{3}{4}} + \sqrt{x}\right) - 21\,(i\,c^4x^4 + 2i\,bc^3x^2 +$$

input
```
integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/64*(21*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^11))^(1/4)*log(b*c^8*(
-1/(b*c^11))^(3/4) + sqrt(x)) - 21*(I*c^4*x^4 + 2*I*b*c^3*x^2 + I*b^2*c^2)
*(-1/(b*c^11))^(1/4)*log(I*b*c^8*(-1/(b*c^11))^(3/4) + sqrt(x)) - 21*(-I*c
^4*x^4 - 2*I*b*c^3*x^2 - I*b^2*c^2)*(-1/(b*c^11))^(1/4)*log(-I*b*c^8*(-1/(
b*c^11))^(3/4) + sqrt(x)) - 21*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^
11))^(1/4)*log(-b*c^8*(-1/(b*c^11))^(3/4) + sqrt(x)) - 4*(11*c*x^3 + 7*b*x
)*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{21/2}}{(bx^2 + cx^4)^3}\, dx = \text{Timed out}$$

input
```
integrate(x**(21/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.17

$$\int \frac{x^{21/2}}{(bx^2+cx^4)^3}\,dx = -\frac{11\,cx^{\frac{7}{2}}+7\,bx^{\frac{3}{2}}}{16\,(c^4x^4+2\,bc^3x^2+b^2c^2)} + \frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{\right.}}{}\right.}{128\,c^2}$$

input
```
integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/16*(11*c*x^(7/2) + 7*b*x^(3/2))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2) + 21/
128*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-
sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/
c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.12

$$\int \frac{x^{21/2}}{(bx^2+cx^4)^3}\,dx = -\frac{11\,cx^{\frac{7}{2}}+7\,bx^{\frac{3}{2}}}{16\,(cx^2+b)^2c^2} + \frac{21\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^5} + \frac{21\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^5} - \frac{21\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^5} + \frac{21\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^5}$$

input
```
integrate(x^(21/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-1/16*(11*c*x^(7/2) + 7*b*x^(3/2))/((c*x^2 + b)^2*c^2) + 21/64*sqrt(2)*(b*
c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4
))/(b*c^5) + 21/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) - 21/128*sqrt(2)*(b*c^3)^(3/4)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5) + 21/128*sqrt(2)*(
b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
```

Mupad [B] (verification not implemented)

Time = 17.74 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.47

$$\int \frac{x^{21/2}}{(bx^2+cx^4)^3}\,dx = \frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{1/4}\,c^{11/4}} - \frac{\frac{11\,x^{7/2}}{16\,c}+\frac{7\,b\,x^{3/2}}{16\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4} - \frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{1/4}\,c^{11/4}}$$

input
```
int(x^(21/2)/(b*x^2 + c*x^4)^3,x)
```

output
```
(21*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(1/4)*c^(11/4)) - ((11*x^
(7/2))/(16*c) + (7*b*x^(3/2))/(16*c^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) - (21*
atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(1/4)*c^(11/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.57

$$\int \frac{x^{21/2}}{(bx^2 + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int(x^(21/2)/(c*x^4+b*x^2)^3,x)
```

output
```
( - 42*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 - 84*c**(1/4)*b**(3/4)*sqrt(
2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)
*sqrt(2)))*b*c*x**2 - 42*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 + 42*
c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 + 84*c**(1/4)*b**(3/4)*sqrt(2)*atan
((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2
)))*b*c*x**2 + 42*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2
) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 + 21*c**(1/4
)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sq
rt(c)*x)*b**2 + 42*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1
/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 + 21*c**(1/4)*b**(3/4)*sqrt(2)
*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4
 - 21*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sq
rt(b) + sqrt(c)*x)*b**2 - 42*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4
)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 - 21*c**(1/4)*b**(3/4)*
sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*
x**4 - 56*sqrt(x)*b**2*c*x - 88*sqrt(x)*b*c**2*x**3)/(128*b*c**3*(b**2 + 2
*b*c*x**2 + c**2*x**4))
```

3.142 $\int \frac{x^{19/2}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 186

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^3}\,dx = -\frac{x^{5/2}}{4c(b+cx^2)^2} - \frac{5\sqrt{x}}{16c^2(b+cx^2)} - \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{9/4}} + \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{9/4}} + \frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{3/4}c^{9/4}}$$

output
```
-1/4*x^(5/2)/c/(c*x^2+b)^2-5/16*x^(1/2)/c^2/(c*x^2+b)-5/64*arctan(1-2^(1/2
)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(3/4)/c^(9/4)+5/64*arctan(1+2^(1/2)*c
^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(3/4)/c^(9/4)+5/64*arctanh(2^(1/2)*b^(1/
4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(3/4)/c^(9/4)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.74

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4\sqrt[4]{c}\sqrt{x}(5b+9cx^2)}{(b+cx^2)^2} - \frac{5\sqrt{2}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{b^{3/4}} + \frac{5\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{b^{3/4}}}{64c^{9/4}}$$

input
```
Integrate[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*c^(1/4)*Sqrt[x]*(5*b + 9*c*x^2))/(b + c*x^2)^2 - (5*Sqrt[2]*ArcTan[(S
qrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(3/4) + (5*Sqrt[
2]*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^(3/
4))/(64*c^(9/4))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.45, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {9, 252, 252, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^{7/2}}{(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{5\int \frac{x^{3/2}}{(cx^2+b)^2}dx}{8c} - \frac{x^{5/2}}{4c(b+cx^2)^2}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{5\left(\frac{\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{4c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{5/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{5\left(\frac{\int \frac{1}{cx^2+b}d\sqrt{x}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{5/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{5/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{5/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{5/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{5/2}}{4c\left(b+cx^2\right)^2}$$

$\downarrow$ 1479

$$\frac{5\left(\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8c}$$

$$\frac{x^{5/2}}{4c\left(b+cx^2\right)^2}$$

$\downarrow$ 25

$$\frac{5\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8c}-$$

$$\frac{x^{5/2}}{4c\left(b+cx^2\right)^2}$$

$\downarrow$ 27

$$\frac{5\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{5/2}}{4c\,(b+cx^2)^2}$$

$\Big\downarrow$ 1103

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}}{2c}-\frac{\sqrt{x}}{2c(b+cx^2)}\right)}{8c}-\frac{x^{5/2}}{4c\,(b+cx^2)^2}$$

input

```
Int[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```

output

```
-1/4*x^(5/2)/(c*(b + c*x^2)^2) + (5*(-1/2*Sqrt[x]/(c*(b + c*x^2)) + ((-(Ar
cTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + A
rcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2
*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]))/(2*c)))/(8*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{-\frac{9x^{\frac{5}{2}}}{16c}-\frac{5b\sqrt{x}}{16c^2}}{(c\,x^2+b)^2}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128c^2b}$	139
default	$\frac{-\frac{9x^{\frac{5}{2}}}{16c}-\frac{5b\sqrt{x}}{16c^2}}{(c\,x^2+b)^2}+\frac{5\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128c^2b}$	139

input
```
int(x^(19/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-9/32*x^(5/2)/c-5/32*b*x^(1/2)/c^2)/(c*x^2+b)^2+5/128/c^2*(1/c*b)^(1/4)
/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^
(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/
2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.48

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^3}\,dx = \frac{5\,(c^4x^4+2\,bc^3x^2+b^2c^2)\left(-\frac{1}{b^3c^9}\right)^{\frac{1}{4}}\log\left(bc^2\left(-\frac{1}{b^3c^9}\right)^{\frac{1}{4}}+\sqrt{x}\right)-5\,(-i\,c^4x^4-2i\,bc^3x^2-$$

input
```
integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/64*(5*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^3*c^9))^(1/4)*log(b*c^2*(
-1/(b^3*c^9))^(1/4) + sqrt(x)) - 5*(-I*c^4*x^4 - 2*I*b*c^3*x^2 - I*b^2*c^2
)*(-1/(b^3*c^9))^(1/4)*log(I*b*c^2*(-1/(b^3*c^9))^(1/4) + sqrt(x)) - 5*(I*
c^4*x^4 + 2*I*b*c^3*x^2 + I*b^2*c^2)*(-1/(b^3*c^9))^(1/4)*log(-I*b*c^2*(-1
/(b^3*c^9))^(1/4) + sqrt(x)) - 5*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^
3*c^9))^(1/4)*log(-b*c^2*(-1/(b^3*c^9))^(1/4) + sqrt(x)) - 4*(9*c*x^2 + 5*
b)*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(19/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.17

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^3}\,dx = -\frac{9\,cx^{\frac{5}{2}}+5\,b\sqrt{x}}{16\,(c^4x^4+2\,bc^3x^2+b^2c^2)} + \frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{128\,c^2}$$

input
```
integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/16*(9*c*x^(5/2) + 5*b*sqrt(x))/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2) + 5/12
8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*
c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c^
2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.12

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^3}\,dx = \frac{5\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^3} + \frac{5\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^3} + \frac{5\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^3} - \frac{5\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^3} - \frac{9\,cx^{\frac{5}{2}}+5\,b\sqrt{x}}{16\,(cx^2+b)^2c^2}$$

input
```
integrate(x^(19/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
5/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/(b*c^3) + 5/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) + 5/128*sqrt(2)*(
b*c^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) - 5/
128*sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c)
)/(b*c^3) - 1/16*(9*c*x^(5/2) + 5*b*sqrt(x))/((c*x^2 + b)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.47

$$\int \frac{x^{19/2}}{(bx^2+cx^4)^3}\,dx = -\frac{\frac{9\,x^{5/2}}{16\,c}+\frac{5\,b\,\sqrt{x}}{16\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4} - \frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{3/4}\,c^{9/4}} - \frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{3/4}\,c^{9/4}}$$

input
```
int(x^(19/2)/(b*x^2 + c*x^4)^3,x)
```

output
```
- ((9*x^(5/2))/(16*c) + (5*b*x^(1/2))/(16*c^2))/(b^2 + c^2*x^4 + 2*b*c*x^2
) - (5*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(3/4)*c^(9/4)) - (5*at
anh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(3/4)*c^(9/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.56

$$\int \frac{x^{19/2}}{(bx^2 + cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(19/2)/(c*x^4+b*x^2)^3,x)
```

output
```
( - 10*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 - 20*c**(3/4)*b**(1/4)*sqrt(
2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)
*sqrt(2)))*b*c*x**2 - 10*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 + 10*
c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 + 20*c**(3/4)*b**(1/4)*sqrt(2)*atan
((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2
)))*b*c*x**2 + 10*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2
) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 - 5*c**(3/4)
*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqr
t(c)*x)*b**2 - 10*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/
4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 - 5*c**(3/4)*b**(1/4)*sqrt(2)*l
og( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4 +
 5*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(
b) + sqrt(c)*x)*b**2 + 10*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b
**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 + 5*c**(3/4)*b**(1/4)*sqrt
(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4
 - 40*sqrt(x)*b**2*c - 72*sqrt(x)*b*c**2*x**2)/(128*b*c**3*(b**2 + 2*b*c*x
**2 + c**2*x**4))
```

3.143 $\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 189

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx = -\frac{x^{3/2}}{4c(b+cx^2)^2}+\frac{3x^{3/2}}{16bc(b+cx^2)}-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{7/4}}+\frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{7/4}}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{5/4}c^{7/4}}$$

output
```
-1/4*x^(3/2)/c/(c*x^2+b)^2+3/16*x^(3/2)/b/c/(c*x^2+b)-3/64*arctan(1-2^(1/2
)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(5/4)/c^(7/4)+3/64*arctan(1+2^(1/2)*c
^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(5/4)/c^(7/4)-3/64*arctanh(2^(1/2)*b^(1/
4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(5/4)/c^(7/4)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.72

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4\sqrt[4]{b}c^{3/4}x^{3/2}(b-3cx^2)}{(b+cx^2)^2}-3\sqrt{2}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{64b^{5/4}c^{7/4}}$$

input
```
Integrate[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*b^(1/4)*c^(3/4)*x^(3/2)*(b - 3*c*x^2))/(b + c*x^2)^2 - 3*Sqrt[2]*ArcT
an[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 3*Sqrt[2]*Ar
cTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(5/4
)*c^(7/4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.43, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {9, 252, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{5/2}}{(b+cx^2)^3}dx$$

$$\Big\downarrow \; 252$$

$$\frac{3\int \frac{\sqrt{x}}{(cx^2+b)^2}dx}{8c}-\frac{x^{3/2}}{4c\,(b+cx^2)^2}$$

$$\Big\downarrow \; 253$$

$$\frac{3\left(\frac{\int \frac{\sqrt{x}}{cx^2+b}dx}{4b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8c}-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{3\left(\frac{\int \frac{x}{cx^2+b}d\sqrt{x}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8c}-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8c}-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3\left(\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8c}-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\left(\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8c}-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8c}-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1479

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8c}$$

$$\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 25

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8c}-$$

$$\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8c}-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8c}-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

input

```
Int[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```

output

```
-1/4*x^(3/2)/(c*(b + c*x^2)^2) + (3*(x^(3/2)/(2*b*(b + c*x^2)) + ((-(ArcTa
n[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sq
rt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/
(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]))/(2*b)))/(8*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{3x^{\frac{7}{2}}}{16b}-\frac{x^{\frac{3}{2}}}{16c}}{(cx^2+b)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128c^2b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	138
default	$\frac{\frac{3x^{\frac{7}{2}}}{16b}-\frac{x^{\frac{3}{2}}}{16c}}{(cx^2+b)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128c^2b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	138

input
```
int(x^(17/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(3/32/b*x^(7/2)-1/32*x^(3/2)/c)/(c*x^2+b)^2+3/128/c^2/b/(1/c*b)^(1/4)*2^
(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)
*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)
+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.53

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx = \frac{3\,(bc^3x^4+2\,b^2c^2x^2+b^3c)\left(-\frac{1}{b^5c^7}\right)^{\frac{1}{4}}\log\left(b^4c^5\left(-\frac{1}{b^5c^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)-3\,(i\,bc^3x^4+2i\,b^2c^2x^2}{}$$

input
```
integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^5*c^7))^(1/4)*log(b^4*c
^5*(-1/(b^5*c^7))^(3/4) + sqrt(x)) - 3*(I*b*c^3*x^4 + 2*I*b^2*c^2*x^2 + I*
b^3*c)*(-1/(b^5*c^7))^(1/4)*log(I*b^4*c^5*(-1/(b^5*c^7))^(3/4) + sqrt(x))
- 3*(-I*b*c^3*x^4 - 2*I*b^2*c^2*x^2 - I*b^3*c)*(-1/(b^5*c^7))^(1/4)*log(-I
*b^4*c^5*(-1/(b^5*c^7))^(3/4) + sqrt(x)) - 3*(b*c^3*x^4 + 2*b^2*c^2*x^2 +
b^3*c)*(-1/(b^5*c^7))^(1/4)*log(-b^4*c^5*(-1/(b^5*c^7))^(3/4) + sqrt(x)) +
 4*(3*c*x^3 - b*x)*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(17/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.17

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx = \frac{3\,cx^{\frac{7}{2}} - bx^{\frac{3}{2}}}{16\,(bc^3x^4 + 2\,b^2c^2x^2 + b^3c)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{128\,bc}$$

input
```
integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/16*(3*c*x^(7/2) - b*x^(3/2))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c) + 3/128
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x
))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)
*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b*
c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.12

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx = \frac{3\,cx^{\frac{7}{2}}-bx^{\frac{3}{2}}}{16\,(cx^2+b)^2bc} + \frac{3\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^4} + \frac{3\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^4} - \frac{3\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^4} + \frac{3\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^4}$$

input
```
integrate(x^(17/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/16*(3*c*x^(7/2) - b*x^(3/2))/((c*x^2 + b)^2*b*c) + 3/64*sqrt(2)*(b*c^3)^
(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b
^2*c^4) + 3/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^4) - 3/128*sqrt(2)*(b*c^3)^(3/4)*log(
sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4) + 3/128*sqrt(2)*(b*
c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 17.52 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.45

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{3\,x^{7/2}}{16\,b}-\frac{x^{3/2}}{16\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{5/4}\,c^{7/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{5/4}\,c^{7/4}}$$

input
```
int(x^(17/2)/(b*x^2 + c*x^4)^3,x)
```

output
```
((3*x^(7/2))/(16*b) - x^(3/2)/(16*c))/(b^2 + c^2*x^4 + 2*b*c*x^2) - (3*ata
n((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(5/4)*c^(7/4)) + (3*atanh((c^(1/
4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(5/4)*c^(7/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.53

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(17/2)/(c*x^4+b*x^2)^3,x)
```

output
```
( - 6*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 - 12*c**(1/4)*b**(3/4)*sqrt(2
)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*
sqrt(2)))*b*c*x**2 - 6*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 + 6*c**
(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c)
)/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 + 12*c**(1/4)*b**(3/4)*sqrt(2)*atan((c
**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))
*b*c*x**2 + 6*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) +
2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 + 3*c**(1/4)*b**
(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)
*x)*b**2 + 6*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sq
rt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 + 3*c**(1/4)*b**(3/4)*sqrt(2)*log( -
 sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4 - 3*c*
*(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) +
sqrt(c)*x)*b**2 - 6*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4
)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 - 3*c**(1/4)*b**(3/4)*sqrt(2)*lo
g(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4 - 8*s
qrt(x)*b**2*c*x + 24*sqrt(x)*b*c**2*x**3)/(128*b**2*c**2*(b**2 + 2*b*c*x**
2 + c**2*x**4))
```

3.144 $\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 189

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx = -\frac{\sqrt{x}}{4c(b+cx^2)^2} + \frac{\sqrt{x}}{16bc(b+cx^2)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{5/4}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{5/4}} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{7/4}c^{5/4}}$$

output
```
-1/4*x^(1/2)/c/(c*x^2+b)^2+1/16*x^(1/2)/b/c/(c*x^2+b)-3/64*arctan(1-2^(1/2
)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(7/4)/c^(5/4)+3/64*arctan(1+2^(1/2)*c
^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(7/4)/c^(5/4)+3/64*arctanh(2^(1/2)*b^(1/
4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(7/4)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx=\frac{\frac{4b^{3/4}\sqrt[4]{c}\sqrt{x}(-3b+cx^2)}{(b+cx^2)^2}-3\sqrt{2}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{64b^{7/4}c^{5/4}}$$

input
```
Integrate[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
((4*b^(3/4)*c^(1/4)*Sqrt[x]*(-3*b + c*x^2))/(b + c*x^2)^2 - 3*Sqrt[2]*ArcT
an[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 3*Sqrt[2]*Ar
cTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(7/4
)*c^(5/4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.43, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {9, 252, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^{3/2}}{(b+cx^2)^3}dx$$

$$\Big\downarrow \ 252$$

$$\frac{\int \frac{1}{\sqrt{x}(cx^2+b)^2}dx}{8c}-\frac{\sqrt{x}}{4c(b+cx^2)^2}$$

$$\Big\downarrow \ 253$$

$$\frac{\frac{3\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{4b}+\frac{\sqrt{x}}{2b(b+cx^2)}}{8c}-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 266

$$\frac{\frac{3\int \frac{1}{cx^2+b}d\sqrt{x}}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}}{8c}-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 755

$$\frac{\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}}{8c}-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}}{8c}-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}}{8c}-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 217

$$\frac{\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}}{8c}-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1479

$$\frac{\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}}{8c}-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}}{8c}-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}}{8c}-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}}{8c}-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}$$

input
```
Int[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*Sqrt[x]/(c*(b + c*x^2)^2) + (Sqrt[x]/(2*b*(b + c*x^2)) + (3*((-(ArcTa
n[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sq
rt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/
(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/(2*b))/(8*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{x^{\frac{5}{2}}}{16b}-\frac{3\sqrt{x}}{16c}}{\left(c\,x^{2}+b\right)^{2}}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128c\,b^{2}}$	138
default	$\frac{\frac{x^{\frac{5}{2}}}{16b}-\frac{3\sqrt{x}}{16c}}{\left(c\,x^{2}+b\right)^{2}}+\frac{3\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128c\,b^{2}}$	138

input
```
int(x^(15/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/32/b*x^(5/2)-3/32*x^(1/2)/c)/(c*x^2+b)^2+3/128/c/b^2*(1/c*b)^(1/4)*2^
(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)
*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)
+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.48

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx = \frac{3\,(bc^3x^4+2\,b^2c^2x^2+b^3c)\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}\log\left(b^2c\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\,(-i\,bc^3x^4-2i\,b^2c^2x^2}{}$$

input
```
integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^7*c^5))^(1/4)*log(b^2*c
*(-1/(b^7*c^5))^(1/4) + sqrt(x)) - 3*(-I*b*c^3*x^4 - 2*I*b^2*c^2*x^2 - I*b
^3*c)*(-1/(b^7*c^5))^(1/4)*log(I*b^2*c*(-1/(b^7*c^5))^(1/4) + sqrt(x)) - 3
*(I*b*c^3*x^4 + 2*I*b^2*c^2*x^2 + I*b^3*c)*(-1/(b^7*c^5))^(1/4)*log(-I*b^2
*c*(-1/(b^7*c^5))^(1/4) + sqrt(x)) - 3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)
*(-1/(b^7*c^5))^(1/4)*log(-b^2*c*(-1/(b^7*c^5))^(1/4) + sqrt(x)) + 4*(c*x^
2 - 3*b)*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(15/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.17

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx = \frac{cx^{\frac{5}{2}}-3\,b\sqrt{x}}{16\,(bc^3x^4+2\,b^2c^2x^2+b^3c)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}c^{\frac14}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}c^{\frac14}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac14}c^{\frac14}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac34}c^{\frac14}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{128\,bc}$$

input
```
integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/16*(c*x^(5/2) - 3*b*sqrt(x))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c) + 3/128
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x
))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)
*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/(b*
c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.12

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx = \frac{3\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^2} + \frac{3\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^2} + \frac{3\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^2} - \frac{3\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^2} + \frac{cx^{\frac{5}{2}}-3\,b\sqrt{x}}{16\,(cx^2+b)^2bc}$$

input
```
integrate(x^(15/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
3/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqr
t(x))/(b/c)^(1/4))/(b^2*c^2) + 3/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 3/128*sqrt(
2)*(b*c^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2
) - 3/128*sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/(b^2*c^2) + 1/16*(c*x^(5/2) - 3*b*sqrt(x))/((c*x^2 + b)^2*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.45

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{x^{5/2}}{16\,b}-\frac{3\sqrt{x}}{16\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4} + \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{7/4}\,c^{5/4}} + \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{7/4}\,c^{5/4}}$$

input
```
int(x^(15/2)/(b*x^2 + c*x^4)^3,x)
```

output
```
(x^(5/2)/(16*b) - (3*x^(1/2))/(16*c))/(b^2 + c^2*x^4 + 2*b*c*x^2) + (3*ata
n((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(7/4)*c^(5/4)) + (3*atanh((c^(1/
4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(7/4)*c^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.52

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(15/2)/(c*x^4+b*x^2)^3,x)
```

output
```
( - 6*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 - 12*c**(3/4)*b**(1/4)*sqrt(2
)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*
sqrt(2)))*b*c*x**2 - 6*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 + 6*c**
(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c)
)/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 + 12*c**(3/4)*b**(1/4)*sqrt(2)*atan((c
**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))
*b*c*x**2 + 6*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) +
2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 - 3*c**(3/4)*b**
(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)
*x)*b**2 - 6*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sq
rt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 - 3*c**(3/4)*b**(1/4)*sqrt(2)*log( -
 sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4 + 3*c*
*(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) +
sqrt(c)*x)*b**2 + 6*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4
)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 + 3*c**(3/4)*b**(1/4)*sqrt(2)*lo
g(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4 - 24*
sqrt(x)*b**2*c + 8*sqrt(x)*b*c**2*x**2)/(128*b**2*c**2*(b**2 + 2*b*c*x**2
+ c**2*x**4))
```

3.145 $\int \frac{x^{13/2}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 186

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^3}\,dx = \frac{x^{3/2}}{4b(b+cx^2)^2}+\frac{5x^{3/2}}{16b^2(b+cx^2)}-\frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{3/4}} + \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{3/4}}-\frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{9/4}c^{3/4}}$$

output
```
1/4*x^(3/2)/b/(c*x^2+b)^2+5/16*x^(3/2)/b^2/(c*x^2+b)-5/64*arctan(1-2^(1/2)
*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(9/4)/c^(3/4)+5/64*arctan(1+2^(1/2)*c^
(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(9/4)/c^(3/4)-5/64*arctanh(2^(1/2)*b^(1/4
)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(9/4)/c^(3/4)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.74

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4\sqrt[4]{b}x^{3/2}(9b+5cx^2)}{(b+cx^2)^2} - \frac{5\sqrt{2}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{c^{3/4}} - \frac{5\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{c^{3/4}}}{64b^{9/4}}$$

input
```
Integrate[x^(13/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
((4*b^(1/4)*x^(3/2)*(9*b + 5*c*x^2))/(b + c*x^2)^2 - (5*Sqrt[2]*ArcTan[(Sq
rt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(3/4) - (5*Sqrt[2
]*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(3/4
))/(64*b^(9/4))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.45, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {9, 253, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{\sqrt{x}}{(b+cx^2)^3}dx$$

$$\Big\downarrow \; 253$$

$$\frac{5\int \frac{\sqrt{x}}{(cx^2+b)^2}dx}{8b}+\frac{x^{3/2}}{4b(b+cx^2)^2}$$

$$\Big\downarrow \; 253$$

$$\frac{5\left(\frac{\int \frac{\sqrt{x}}{cx^2+b}dx}{4b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8b}+\frac{x^{3/2}}{4b\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 266

$$\frac{5\left(\frac{\int \frac{x}{cx^2+b}d\sqrt{x}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8b}+\frac{x^{3/2}}{4b\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 826

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8b}+\frac{x^{3/2}}{4b\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{5\left(\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8b}+\frac{x^{3/2}}{4b\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{5\left(\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8b}+\frac{x^{3/2}}{4b\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 217

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8b}+\frac{x^{3/2}}{4b(b+cx^2)^2}$$

$\big\downarrow$ 1479

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8b}+\frac{x^{3/2}}{4b(b+cx^2)^2}$$

$\big\downarrow$ 25

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8b}+\frac{x^{3/2}}{4b(b+cx^2)^2}$$

$\big\downarrow$ 27

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8b}+\frac{x^{3/2}}{4b\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}}{2b}+\frac{x^{3/2}}{2b(b+cx^2)}\right)}{8b}+\frac{x^{3/2}}{4b\left(b+cx^2\right)^2}$$

input
```
Int[x^(13/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
x^(3/2)/(4*b*(b + c*x^2)^2) + (5*(x^(3/2)/(2*b*(b + c*x^2)) + ((-(ArcTan[1
 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[
1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[
c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sq
rt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + S
qrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]))/(2*b)))/(8*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{x^{\frac{3}{2}}}{4b(c\,x^{2}+b)^{2}}+\frac{\frac{5x^{\frac{3}{2}}}{16b\left(c\,x^{2}+b\right)}+\frac{5\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b}$	150
default	$\frac{x^{\frac{3}{2}}}{4b(c\,x^{2}+b)^{2}}+\frac{\frac{5x^{\frac{3}{2}}}{16b\left(c\,x^{2}+b\right)}+\frac{5\sqrt{2}\,\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b}$	150

input
```
int(x^(13/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^(3/2)/b/(c*x^2+b)^2+5/4/b*(1/4*x^(3/2)/b/(c*x^2+b)+1/32/b/c/(1/c*b)^
(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*
b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^
(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.51

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^3}\,dx = \frac{5\,(b^2c^2x^4+2\,b^3cx^2+b^4)\left(-\frac{1}{b^9c^3}\right)^{\frac{1}{4}}\log\left(b^7c^2\left(-\frac{1}{b^9c^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-5\,(i\,b^2c^2x^4+2i\,b^3cx^2+$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/64*(5*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9*c^3))^(1/4)*log(b^7*c^2
*(-1/(b^9*c^3))^(3/4) + sqrt(x)) - 5*(I*b^2*c^2*x^4 + 2*I*b^3*c*x^2 + I*b^
4)*(-1/(b^9*c^3))^(1/4)*log(I*b^7*c^2*(-1/(b^9*c^3))^(3/4) + sqrt(x)) - 5*
(-I*b^2*c^2*x^4 - 2*I*b^3*c*x^2 - I*b^4)*(-1/(b^9*c^3))^(1/4)*log(-I*b^7*c
^2*(-1/(b^9*c^3))^(3/4) + sqrt(x)) - 5*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(
-1/(b^9*c^3))^(1/4)*log(-b^7*c^2*(-1/(b^9*c^3))^(3/4) + sqrt(x)) + 4*(5*c*
x^3 + 9*b*x)*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^3}\,dx = \frac{5\,cx^{\frac{7}{2}}+9\,bx^{\frac{3}{2}}}{16\,(b^2c^2x^4+2\,b^3cx^2+b^4)} + \frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{128\,b^2}$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/16*(5*c*x^(7/2) + 9*b*x^(3/2))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) + 5/128
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x
))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)
*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c
^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.12

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^3}\,dx = \frac{5\,cx^{\frac{7}{2}}+9\,bx^{\frac{3}{2}}}{16\,\left(cx^2+b\right)^2b^2} + \frac{5\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^3} + \frac{5\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^3} - \frac{5\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c^3} + \frac{5\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c^3}$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
1/16*(5*c*x^(7/2) + 9*b*x^(3/2))/((c*x^2 + b)^2*b^2) + 5/64*sqrt(2)*(b*c^3
)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/
(b^3*c^3) + 5/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^
(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^3) - 5/128*sqrt(2)*(b*c^3)^(3/4)*lo
g(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3) + 5/128*sqrt(2)*(
b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 17.57 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.46

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^3}\,dx = \frac{\frac{9\,x^{3/2}}{16\,b}+\frac{5\,c\,x^{7/2}}{16\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4} + \frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{32\,{(-b)}^{9/4}\,c^{3/4}} - \frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{{(-b)}^{1/4}}\right)}{32\,{(-b)}^{9/4}\,c^{3/4}}$$

input
```
int(x^(13/2)/(b*x^2 + c*x^4)^3,x)
```

output
```
((9*x^(3/2))/(16*b) + (5*c*x^(7/2))/(16*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2)
+ (5*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(9/4)*c^(3/4)) - (5*atan
h((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(9/4)*c^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.57

$$\int \frac{x^{13/2}}{(bx^2 + cx^4)^3} dx = \text{Too large to display}$$

input
```
int(x^(13/2)/(c*x^4+b*x^2)^3,x)
```

output
```
( - 10*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 - 20*c**(1/4)*b**(3/4)*sqrt(
2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)
*sqrt(2)))*b*c*x**2 - 10*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 + 10*
c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 + 20*c**(1/4)*b**(3/4)*sqrt(2)*atan
((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2
)))*b*c*x**2 + 10*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2
) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 + 5*c**(1/4)
*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqr
t(c)*x)*b**2 + 10*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/
4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 + 5*c**(1/4)*b**(3/4)*sqrt(2)*l
og( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4 -
 5*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(
b) + sqrt(c)*x)*b**2 - 10*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b
**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 - 5*c**(1/4)*b**(3/4)*sqrt
(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4
 + 72*sqrt(x)*b**2*c*x + 40*sqrt(x)*b*c**2*x**3)/(128*b**3*c*(b**2 + 2*b*c
*x**2 + c**2*x**4))
```

3.146 $\int \frac{x^{11/2}}{(bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 186

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^3} dx = \frac{\sqrt{x}}{4b(b+cx^2)^2} + \frac{7\sqrt{x}}{16b^2(b+cx^2)} - \frac{21\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}\sqrt[4]{c}} + \frac{21\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}\sqrt[4]{c}} + \frac{21\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

output
```
1/4*x^(1/2)/b/(c*x^2+b)^2+7/16*x^(1/2)/b^2/(c*x^2+b)-21/64*arctan(1-2^(1/2
)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(11/4)/c^(1/4)+21/64*arctan(1+2^(1/2)
*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(11/4)/c^(1/4)+21/64*arctanh(2^(1/2)*b
^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(11/4)/c^(1/4)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.74

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4b^{3/4}\sqrt{x}(11b+7cx^2)}{(b+cx^2)^2} - \frac{21\sqrt{2}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{c}} + \frac{21\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt[4]{c}}}{64b^{11/4}}$$

input
```
Integrate[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
((4*b^(3/4)*Sqrt[x]*(11*b + 7*c*x^2))/(b + c*x^2)^2 - (21*Sqrt[2]*ArcTan[(
Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(1/4) + (21*Sqr
t[2]*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(
1/4))/(64*b^(11/4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.45, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {9, 253, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{\sqrt{x}\,(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{7\int \frac{1}{\sqrt{x}(cx^2+b)^2}dx}{8b} + \frac{\sqrt{x}}{4b\,(b+cx^2)^2}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{7\left(\frac{3\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{4b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8b}+\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

$\downarrow$ 266

$$\frac{7\left(\frac{3\int \frac{1}{cx^2+b}d\sqrt{x}}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8b}+\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

$\downarrow$ 755

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8b}+\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

$\downarrow$ 1476

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8b}+\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

$\downarrow$ 1082

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8b}+\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8b}+\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

$$\Big\downarrow \textbf{1479}$$

$$\frac{7\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8b}$$

$$\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 25

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8b}$$

$$\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8b}+\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{2b}+\frac{\sqrt{x}}{2b(b+cx^2)}\right)}{8b}+\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

input
```
Int[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
Sqrt[x]/(4*b*(b + c*x^2)^2) + (7*(Sqrt[x]/(2*b*(b + c*x^2)) + (3*((-(ArcTa
n[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sq
rt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/
(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/(2*b)))/(8*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.79

method	result	s
derivativedivides	$\frac{\sqrt{x}}{4b(c\,x^2+b)^2}+\frac{\frac{7\sqrt{x}}{16b(c\,x^2+b)}+\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128b^2}}{b}$	1
default	$\frac{\sqrt{x}}{4b(c\,x^2+b)^2}+\frac{\frac{7\sqrt{x}}{16b(c\,x^2+b)}+\frac{21\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128b^2}}{b}$	1

input
```
int(x^(11/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^(1/2)/b/(c*x^2+b)^2+7/4/b*(1/4*x^(1/2)/b/(c*x^2+b)+3/32/b^2*(1/c*b)^
(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*
b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^
(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.44

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^3}\,dx = \frac{21\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}+\sqrt{x}\right)-21\left(-i\,b^2c^2x^4-2i\,b^3cx^2\right.}{}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/64*(21*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^11*c))^(1/4)*log(b^3*(-1
/(b^11*c))^(1/4) + sqrt(x)) - 21*(-I*b^2*c^2*x^4 - 2*I*b^3*c*x^2 - I*b^4)*
(-1/(b^11*c))^(1/4)*log(I*b^3*(-1/(b^11*c))^(1/4) + sqrt(x)) - 21*(I*b^2*c
^2*x^4 + 2*I*b^3*c*x^2 + I*b^4)*(-1/(b^11*c))^(1/4)*log(-I*b^3*(-1/(b^11*c
))^(1/4) + sqrt(x)) - 21*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^11*c))^(
1/4)*log(-b^3*(-1/(b^11*c))^(1/4) + sqrt(x)) + 4*(7*c*x^2 + 11*b)*sqrt(x))
/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^3}\,dx = \frac{7\,cx^{\frac{5}{2}}+11\,b\sqrt{x}}{16\,(b^2c^2x^4+2\,b^3cx^2+b^4)} + \frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(-\sqrt{}\right.}{}\right.}{128\,b^2}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/16*(7*c*x^(5/2) + 11*b*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) + 21/1
28*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt
(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(
b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*log(-s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b
^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.12

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^3}\,dx = \frac{21\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c} + \frac{21\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c} + \frac{21\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c} - \frac{21\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c} + \frac{7\,cx^{\frac{5}{2}}+11\,b\sqrt{x}}{16\,(cx^2+b)^2b^2}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
21/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sq
rt(x))/(b/c)^(1/4))/(b^3*c) + 21/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 21/128*sqrt(2
)*(b*c^3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) -
 21/128*sqrt(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(
b/c))/(b^3*c) + 1/16*(7*c*x^(5/2) + 11*b*sqrt(x))/((c*x^2 + b)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.46

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{11\sqrt{x}}{16\,b}+\frac{7\,c\,x^{5/2}}{16\,b^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4}-\frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{11/4}\,c^{1/4}}-\frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)}{32\,(-b)^{11/4}\,c^{1/4}}$$

input
```
int(x^(11/2)/(b*x^2 + c*x^4)^3,x)
```

output
```
((11*x^(1/2))/(16*b) + (7*c*x^(5/2))/(16*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2)
 - (21*atan((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(11/4)*c^(1/4)) - (21*
atanh((c^(1/4)*x^(1/2))/(-b)^(1/4)))/(32*(-b)^(11/4)*c^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.56

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(11/2)/(c*x^4+b*x^2)^3,x)
```

output
```
( - 42*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 - 84*c**(3/4)*b**(1/4)*sqrt(
2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)
*sqrt(2)))*b*c*x**2 - 42*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 + 42*
c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 + 84*c**(3/4)*b**(1/4)*sqrt(2)*atan
((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2
)))*b*c*x**2 + 42*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2
) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 - 21*c**(3/4
)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sq
rt(c)*x)*b**2 - 42*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1
/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 - 21*c**(3/4)*b**(1/4)*sqrt(2)
*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4
 + 21*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sq
rt(b) + sqrt(c)*x)*b**2 + 42*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4
)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 + 21*c**(3/4)*b**(1/4)*
sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*
x**4 + 88*sqrt(x)*b**2*c + 56*sqrt(x)*b*c**2*x**2)/(128*b**3*c*(b**2 + 2*b
*c*x**2 + c**2*x**4))
```

3.147 $\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 198

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx = -\frac{45}{16b^3\sqrt{x}} + \frac{1}{4b\sqrt{x}\,(b+cx^2)^2} + \frac{9}{16b^2\sqrt{x}\,(b+cx^2)} + \frac{45\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}} - \frac{45\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}} + \frac{45\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{13/4}}$$

output

```
-45/16/b^3/x^(1/2)+1/4/b/x^(1/2)/(c*x^2+b)^2+9/16/b^2/x^(1/2)/(c*x^2+b)+45
/64*c^(1/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(13/4)-45/
64*c^(1/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(13/4)+45/6
4*c^(1/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(
1/2)/b^(13/4)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.75

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4\sqrt[4]{b}(32b^2+81bcx^2+45c^2x^4)}{\sqrt{x}(b+cx^2)^2}+45\sqrt{2}\sqrt[4]{c}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+45\sqrt{2}\sqrt[4]{c}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\ldots}{\sqrt{b}\ldots}\right.}{64b^{13/4}}$$

input
```
Integrate[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*b^(1/4)*(32*b^2 + 81*b*c*x^2 + 45*c^2*x^4))/(Sqrt[x]*(b + c*x^2)^2) +
 45*Sqrt[2]*c^(1/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x])] + 45*Sqrt[2]*c^(1/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(
Sqrt[b] + Sqrt[c]*x)])/(64*b^(13/4))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.45, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {9, 253, 253, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^{3/2}(b+cx^2)^3}dx$$

$$\Big\downarrow \; 253$$

$$\frac{9\int \frac{1}{x^{3/2}(cx^2+b)^2}dx}{8b}+\frac{1}{4b\sqrt{x}(b+cx^2)^2}$$

$$\Big\downarrow \; 253$$

$$\frac{9\left(\frac{5\int \frac{1}{x^{3/2}\left(cx^2+b\right)}dx}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8b}+\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{9\left(\frac{5\left(-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8b}+\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{9\left(\frac{5\left(-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8b}+\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{9\left(\frac{5\left(-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8b}+\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{9\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8b}+\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{9\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8b}+\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}$$

$\downarrow$ 217

$$\frac{9\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8b}+\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}$$

$\big\downarrow$ 1479

$$\frac{9\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}\right.}{8b}$$

$$\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{9\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\right.}{8b}$$

$$\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{9\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{1}{2b\sqrt{x}(b+cx^2)}\right)}{8b}$$

$$\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}$$

$\downarrow$ 1103

$$\frac{9\left(\frac{5\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{4b}+\frac{}{2b\sqrt{\;}}\right.}{8b}$$

$$\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}$$

input

```
Int[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```

output

```
1/(4*b*Sqrt[x]*(b + c*x^2)^2) + (9*(1/(2*b*Sqrt[x]*(b + c*x^2)) + (5*(-2/(
b*Sqrt[x]) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[
2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt
[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sq
rt[c])))/b))/(4*b)))/(8*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{2}{b^{3}\sqrt{x}}-\frac{2c\left(\frac{\frac{13c\,x^{\frac{7}{2}}}{32}+\frac{17b\,x^{\frac{3}{2}}}{32}}{\left(c\,x^{2}+b\right)^{2}}+\frac{45\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{3}}$
default	$-\frac{2}{b^{3}\sqrt{x}}-\frac{2c\left(\frac{\frac{13c\,x^{\frac{7}{2}}}{32}+\frac{17b\,x^{\frac{3}{2}}}{32}}{\left(c\,x^{2}+b\right)^{2}}+\frac{45\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{3}}$
risch	$-\frac{2}{b^{3}\sqrt{x}}-\frac{c\left(\frac{\frac{13c\,x^{\frac{7}{2}}}{16}+\frac{17b\,x^{\frac{3}{2}}}{16}}{\left(c\,x^{2}+b\right)^{2}}+\frac{45\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{3}}$

input
```
int(x^(9/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b^3/x^(1/2)-2/b^3*c*((13/32*c*x^(7/2)+17/32*b*x^(3/2))/(c*x^2+b)^2+45/2
56/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1
/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/
c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.43

$$\int \frac{x^{9/2}}{\left(bx^2+cx^4\right)^3}\,dx =$$

$$-\frac{45\left(b^3c^2x^5+2\,b^4cx^3+b^5x\right)\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}\log\left(91125\,b^{10}\left(-\frac{c}{b^{13}}\right)^{\frac{3}{4}}+91125\,c\sqrt{x}\right)+45\left(-i\,b^3c^2x^5-2i\,b^4cx^3-i\,b\right.}{}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/64*(45*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-c/b^13)^(1/4)*log(91125*b^
10*(-c/b^13)^(3/4) + 91125*c*sqrt(x)) + 45*(-I*b^3*c^2*x^5 - 2*I*b^4*c*x^3
 - I*b^5*x)*(-c/b^13)^(1/4)*log(91125*I*b^10*(-c/b^13)^(3/4) + 91125*c*sqr
t(x)) + 45*(I*b^3*c^2*x^5 + 2*I*b^4*c*x^3 + I*b^5*x)*(-c/b^13)^(1/4)*log(-
91125*I*b^10*(-c/b^13)^(3/4) + 91125*c*sqrt(x)) - 45*(b^3*c^2*x^5 + 2*b^4*
c*x^3 + b^5*x)*(-c/b^13)^(1/4)*log(-91125*b^10*(-c/b^13)^(3/4) + 91125*c*s
qrt(x)) + 4*(45*c^2*x^4 + 81*b*c*x^2 + 32*b^2)*sqrt(x))/(b^3*c^2*x^5 + 2*b
^4*c*x^3 + b^5*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.16

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx = -\frac{45\,c^2x^4+81\,bcx^2+32\,b^2}{16\left(b^3c^2x^{\frac{9}{2}}+2\,b^4cx^{\frac{5}{2}}+b^5\sqrt{x}\right)} - \frac{45\,c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\right.}{}\right.}{128\,b^3}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/16*(45*c^2*x^4 + 81*b*c*x^2 + 32*b^2)/(b^3*c^2*x^(9/2) + 2*b^4*c*x^(5/2
) + b^5*sqrt(x)) - 45/128*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)
*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c)
)*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sq
rt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sq
rt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*
c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt
(b))/(b^(1/4)*c^(3/4)))/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx &= -\frac{2}{b^3\sqrt{x}} - \frac{45\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c^2} \\ &\quad - \frac{45\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c^2} \\ &\quad + \frac{45\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4c^2} \\ &\quad - \frac{45\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4c^2} - \frac{13\,c^2x^{\frac{7}{2}}+17\,bcx^{\frac{3}{2}}}{16\,(cx^2+b)^2b^3}\end{aligned}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-2/(b^3*sqrt(x)) - 45/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)
*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c^2) - 45/64*sqrt(2)*(b*c^3)^(
3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b
^4*c^2) + 45/128*sqrt(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
 + sqrt(b/c))/(b^4*c^2) - 45/128*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x
)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 1/16*(13*c^2*x^(7/2) + 17*b*c*x
^(3/2))/((c*x^2 + b)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 17.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.50

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx = \frac{45\,(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{13/4}} - \frac{45\,(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{13/4}} - \frac{\frac{2}{b}+\frac{81\,c\,x^2}{16\,b^2}+\frac{45\,c^2\,x^4}{16\,b^3}}{b^2\,\sqrt{x}+c^2\,x^{9/2}+2\,b\,c\,x^{5/2}}$$

input
```
int(x^(9/2)/(b*x^2 + c*x^4)^3,x)
```

output
```
(45*(-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(13/4)) - (45*(-
c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(13/4)) - (2/b + (81*c*
x^2)/(16*b^2) + (45*c^2*x^4)/(16*b^3))/(b^2*x^(1/2) + c^2*x^(9/2) + 2*b*c*
x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.56

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(9/2)/(c*x^4+b*x^2)^3,x)
```

output
```
(90*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*
sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 + 180*sqrt(x)*c**(1/4)*
b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**
(1/4)*b**(1/4)*sqrt(2)))*b*c*x**2 + 90*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*a
tan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqr
t(2)))*c**2*x**4 - 90*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**
(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2 - 180
*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqr
t(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b*c*x**2 - 90*sqrt(x)*c**(1/4)*
b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**
(1/4)*b**(1/4)*sqrt(2)))*c**2*x**4 - 45*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*
log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b**2 - 90*
sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2)
 + sqrt(b) + sqrt(c)*x)*b*c*x**2 - 45*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*lo
g( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**4 +
45*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2)
 + sqrt(b) + sqrt(c)*x)*b**2 + 90*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log(sq
rt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**2 + 45*sqrt(
x)*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(
b) + sqrt(c)*x)*c**2*x**4 - 256*b**3 - 648*b**2*c*x**2 - 360*b*c**2*x**...
```

3.148 $\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 198

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx = -\frac{77}{48b^3x^{3/2}}+\frac{1}{4bx^{3/2}(b+cx^2)^2}+\frac{11}{16b^2x^{3/2}(b+cx^2)}+\frac{77c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}}-\frac{77c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}}-\frac{77c^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{15/4}}$$

output
```
-77/48/b^3/x^(3/2)+1/4/b/x^(3/2)/(c*x^2+b)^2+11/16/b^2/x^(3/2)/(c*x^2+b)+7
7/64*c^(3/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(15/4)-77
/64*c^(3/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(15/4)-77/
64*c^(3/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^
(1/2)/b^(15/4)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.75

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4b^{3/4}(32b^2+121bcx^2+77c^2x^4)}{x^{3/2}(b+cx^2)^2}+231\sqrt{2}c^{3/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-231\sqrt{2}c^{3/4}\text{arctanh}\left(\cdots\right.}{192b^{15/4}}$$

input
```
Integrate[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*b^(3/4)*(32*b^2 + 121*b*c*x^2 + 77*c^2*x^4))/(x^(3/2)*(b + c*x^2)^2)
+ 231*Sqrt[2]*c^(3/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x])] - 231*Sqrt[2]*c^(3/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
)/(Sqrt[b] + Sqrt[c]*x)])/(192*b^(15/4))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.46, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {9, 253, 253, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^{5/2}(b+cx^2)^3}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{11\int \frac{1}{x^{5/2}(cx^2+b)^2}dx}{8b}+\frac{1}{4bx^{3/2}(b+cx^2)^2}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{11\left(\frac{7\int \frac{1}{x^{5/2}\left(cx^2+b\right)}dx}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 264

$$\frac{11\left(\frac{7\left(-\frac{c\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 266

$$\frac{11\left(\frac{7\left(-\frac{2c\int \frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 755

$$\frac{11\left(\frac{7\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{11\left(\frac{7\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

$\downarrow$ 1082

$$\frac{11\left(\frac{7\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \ 217$$

$$\frac{11\left(\frac{7\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1479

$$11\left(\frac{7\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3}\cdots}\right.}{4b}\right.}{8b}$$

$$\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{11\left(\frac{7\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}\right)}{8b}$$

$$\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{11\left(\frac{7\left(-\frac{2c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\frac{1}{2bx^{3/2}(b+c}\right.}{8b}$$

$$\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{11\left(\frac{7\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{4b}+\right.}{8b}$$

$$\frac{1}{4bx^{3/2}\left(b+cx^2\right)^2}$$

input
```
Int[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
1/(4*b*x^(3/2)*(b + c*x^2)^2) + (11*(1/(2*b*x^(3/2)*(b + c*x^2)) + (7*(-2/
(3*b*x^(3/2)) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sq
rt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(S
qrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2
*Sqrt[b])))/b))/(4*b)))/(8*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{2}{3b^3x^{\frac{3}{2}}}-\frac{2c\left(\frac{\frac{15c\,x^{\frac{5}{2}}}{32}+\frac{19b\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{77\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256b}\right)}{b^3}$
default	$-\frac{2}{3b^3x^{\frac{3}{2}}}-\frac{2c\left(\frac{\frac{15c\,x^{\frac{5}{2}}}{32}+\frac{19b\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{77\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256b}\right)}{b^3}$
risch	$-\frac{2}{3b^3x^{\frac{3}{2}}}-\frac{c\left(\frac{\frac{15c\,x^{\frac{5}{2}}}{16}+\frac{19b\sqrt{x}}{16}}{\left(c\,x^2+b\right)^2}+\frac{77\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{128b}\right)}{b^3}$

input
```
int(x^(7/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3/b^3/x^(3/2)-2/b^3*c*((15/32*c*x^(5/2)+19/32*b*x^(1/2))/(c*x^2+b)^2+77
/256*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^
(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(
1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.57

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx =$$

$$-\frac{231\,(b^3c^2x^6+2\,b^4cx^4+b^5x^2)\left(-\frac{c^3}{b^{15}}\right)^{\frac{1}{4}}\log\left(77\,b^4\left(-\frac{c^3}{b^{15}}\right)^{\frac{1}{4}}+77\,c\sqrt{x}\right)+231\,(i\,b^3c^2x^6+2i\,b^4cx^4+i\,b^5x^2)}{}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/192*(231*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-c^3/b^15)^(1/4)*log(77
*b^4*(-c^3/b^15)^(1/4) + 77*c*sqrt(x)) + 231*(I*b^3*c^2*x^6 + 2*I*b^4*c*x^
4 + I*b^5*x^2)*(-c^3/b^15)^(1/4)*log(77*I*b^4*(-c^3/b^15)^(1/4) + 77*c*sqr
t(x)) + 231*(-I*b^3*c^2*x^6 - 2*I*b^4*c*x^4 - I*b^5*x^2)*(-c^3/b^15)^(1/4)
*log(-77*I*b^4*(-c^3/b^15)^(1/4) + 77*c*sqrt(x)) - 231*(b^3*c^2*x^6 + 2*b^
4*c*x^4 + b^5*x^2)*(-c^3/b^15)^(1/4)*log(-77*b^4*(-c^3/b^15)^(1/4) + 77*c*
sqrt(x)) + 4*(77*c^2*x^4 + 121*b*c*x^2 + 32*b^2)*sqrt(x))/(b^3*c^2*x^6 + 2
*b^4*c*x^4 + b^5*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.17

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx = -\frac{77\,c^2x^4+121\,bcx^2+32\,b^2}{48\left(b^3c^2x^{\frac{11}{2}}+2\,b^4cx^{\frac{7}{2}}+b^5x^{\frac{3}{2}}\right)} - \frac{77\left(\frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}} - \frac{\sqrt{2}c^{\frac{3}{4}}\cdots}{\cdots}\right)}{128\,b^3}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/48*(77*c^2*x^4 + 121*b*c*x^2 + 32*b^2)/(b^3*c^2*x^(11/2) + 2*b^4*c*x^(7
/2) + b^5*x^(3/2)) - 77/128*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/
4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(
b)*sqrt(c))) + 2*sqrt(2)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) -
2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c)))
+ sqrt(2)*c^(3/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b
))/b^(3/4) - sqrt(2)*c^(3/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c
)*x + sqrt(b))/b^(3/4))/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.05

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx = -\frac{77\,\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4} - \frac{77\,\sqrt{2}(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4} - \frac{77\,\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4} + \frac{77\,\sqrt{2}(bc^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4} - \frac{15\,c^2x^{\frac{5}{2}}+19\,bc\sqrt{x}}{16\,(cx^2+b)^2b^3} - \frac{2}{3\,b^3x^{\frac{3}{2}}}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-77/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/b^4 - 77/64*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 - 77/128*sqrt(2)*(b*c^
3)^(1/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 77/128*sqr
t(2)*(b*c^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 -
 1/16*(15*c^2*x^(5/2) + 19*b*c*sqrt(x))/((c*x^2 + b)^2*b^3) - 2/3/(b^3*x^(
3/2))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.50

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx = \frac{77\,(-c)^{3/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{15/4}} - \frac{\frac{2}{3\,b}+\frac{121\,c\,x^2}{48\,b^2}+\frac{77\,c^2\,x^4}{48\,b^3}}{b^2\,x^{3/2}+c^2\,x^{11/2}+2\,b\,c\,x^{7/2}} + \frac{77\,(-c)^{3/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{15/4}}$$

input
```
int(x^(7/2)/(b*x^2 + c*x^4)^3,x)
```

output
```
(77*(-c)^(3/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(15/4)) - (2/(3*b
) + (121*c*x^2)/(48*b^2) + (77*c^2*x^4)/(48*b^3))/(b^2*x^(3/2) + c^2*x^(11
/2) + 2*b*c*x^(7/2)) + (77*(-c)^(3/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))
/(32*b^(15/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.59

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(7/2)/(c*x^4+b*x^2)^3,x)
```

output
```
(462*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2
*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2*x + 924*sqrt(x)*c**(3/
4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(
c**(1/4)*b**(1/4)*sqrt(2)))*b*c*x**3 + 462*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(
2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)
*sqrt(2)))*c**2*x**5 - 462*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4
)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2*
x - 924*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2)
+ 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b*c*x**3 - 462*sqrt(x)*c
**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(
c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**2*x**5 + 231*sqrt(x)*c**(3/4)*b**(1/4)
*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b
**2*x + 462*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(
1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c*x**3 + 231*sqrt(x)*c**(3/4)*b**(1/
4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)
*c**2*x**5 - 231*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b*
*(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b**2*x - 462*sqrt(x)*c**(3/4)*b**(1/
4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*
c*x**3 - 231*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/
4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**2*x**5 - 256*b**3 - 968*b**2*c*x**...
```

3.149 $\int \frac{x^{5/2}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 211

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^3}\,dx = -\frac{117}{80b^3x^{5/2}}+\frac{117c}{16b^4\sqrt{x}}+\frac{1}{4bx^{5/2}(b+cx^2)^2}+\frac{13}{16b^2x^{5/2}(b+cx^2)}-\frac{117c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}}+\frac{117c^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}}-\frac{117c^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{17/4}}$$

output

```
-117/80/b^3/x^(5/2)+117/16*c/b^4/x^(1/2)+1/4/b/x^(5/2)/(c*x^2+b)^2+13/16/b
^2/x^(5/2)/(c*x^2+b)-117/64*c^(5/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/
4))*2^(1/2)/b^(17/4)+117/64*c^(5/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/
4))*2^(1/2)/b^(17/4)-117/64*c^(5/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2
)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(17/4)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.76

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4\sqrt[4]{b}(-32b^3+416b^2cx^2+1053bc^2x^4+585c^3x^6)}{x^{5/2}(b+cx^2)^2} - 585\sqrt{2}c^{5/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - 585\sqrt{2}c^{5/4}\text{a}}{320b^{17/4}}$$

input
```
Integrate[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
((4*b^(1/4)*(-32*b^3 + 416*b^2*c*x^2 + 1053*b*c^2*x^4 + 585*c^3*x^6))/(x^(
5/2)*(b + c*x^2)^2) - 585*Sqrt[2]*c^(5/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 585*Sqrt[2]*c^(5/4)*ArcTanh[(Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(320*b^(17/4))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.45, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {9, 253, 253, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^3}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^{7/2}(b+cx^2)^3}dx$$

$$\Big\downarrow \; 253$$

$$\frac{13\int \frac{1}{x^{7/2}(cx^2+b)^2}dx}{8b} + \frac{1}{4bx^{5/2}(b+cx^2)^2}$$

$$\Big\downarrow \; 253$$

$$\frac{13\left(\frac{9\int \frac{1}{x^{7/2}\left(cx^2+b\right)}dx}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \textbf{264}$$

$$\frac{13\left(\frac{9\left(-\frac{c\int \frac{1}{x^{3/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \textbf{264}$$

$$\frac{13\left(\frac{9\left(-\frac{c\left(-\frac{c\int \frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \textbf{266}$$

$$\frac{13\left(\frac{9\left(-\frac{c\left(-\frac{2c\int \frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \textbf{826}$$

$$\frac{13\left(\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{13\left(\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}$$

↓ 1082

$$\frac{13\left(\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8b}+$$

$$\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}$$

↓ 217

$$\frac{13\left(\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\frac{1}{2bx^{5/2}(b+cx^2)}\right)}{8b}+$$

$$\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}$$

↓ 1479

13

$$\frac{\left(9 - \frac{\left(c - \frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}} - \frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}\,d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}\,d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b} - \cdots}{b} \cdots\right.}{4b \cdots}$$

↓ 25

$$
13 \quad \frac{\left(9 \quad -\frac{\left(c \quad -\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}}{b}}{4b}
$$

$$\Big\downarrow \; 27$$

$$13\left(\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{4b}+\right.$$

↓ 1103

$$13\left(\frac{9\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\cdots\right)}{4b}\cdots\right.}{8b}$$

$$\frac{1}{4bx^{5/2}\left(b+cx^2\right)^2}$$

input
```
Int[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
1/(4*b*x^(5/2)*(b + c*x^2)^2) + (13*(1/(2*b*x^(5/2)*(b + c*x^2)) + (9*(-2/
(5*b*x^(5/2)) - (c*(-2/(b*Sqrt[x]) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*
Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt
[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4
)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]
*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b))/b))/(4*b)))/(8*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{2(-15c\,x^2+b)}{5b^4x^{\frac{5}{2}}}+\frac{c^2\left(\frac{\frac{21c\,x^{\frac{7}{2}}}{16}+\frac{25b\,x^{\frac{3}{2}}}{16}}{(c\,x^2+b)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right.}{128c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^4}$
derivativedivides	$\frac{2c^2\left(\frac{\frac{21c\,x^{\frac{7}{2}}}{32}+\frac{25b\,x^{\frac{3}{2}}}{32}}{(c\,x^2+b)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^4}-\frac{2}{5b^3x^{\frac{5}{2}}}+$
default	$\frac{2c^2\left(\frac{\frac{21c\,x^{\frac{7}{2}}}{32}+\frac{25b\,x^{\frac{3}{2}}}{32}}{(c\,x^2+b)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^4}-\frac{2}{5b^3x^{\frac{5}{2}}}+$

input
```
int(x^(5/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-15*c*x^2+b)/b^4/x^(5/2)+1/b^4*c^2*(2*(21/32*c*x^(7/2)+25/32*b*x^(3/
2))/(c*x^2+b)^2+117/128/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/
2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))
+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*
x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.56

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^3}\,dx = \frac{585\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)\left(-\frac{c^5}{b^{17}}\right)^{\frac{1}{4}}\log\left(1601613\,b^{13}\left(-\frac{c^5}{b^{17}}\right)^{\frac{3}{4}}+1601613\,c^4\sqrt{x}\right)-}{}$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/320*(585*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)*log(160
1613*b^13*(-c^5/b^17)^(3/4) + 1601613*c^4*sqrt(x)) - 585*(I*b^4*c^2*x^7 +
2*I*b^5*c*x^5 + I*b^6*x^3)*(-c^5/b^17)^(1/4)*log(1601613*I*b^13*(-c^5/b^17
)^(3/4) + 1601613*c^4*sqrt(x)) - 585*(-I*b^4*c^2*x^7 - 2*I*b^5*c*x^5 - I*b
^6*x^3)*(-c^5/b^17)^(1/4)*log(-1601613*I*b^13*(-c^5/b^17)^(3/4) + 1601613*
c^4*sqrt(x)) - 585*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-c^5/b^17)^(1/4)
*log(-1601613*b^13*(-c^5/b^17)^(3/4) + 1601613*c^4*sqrt(x)) + 4*(585*c^3*x
^6 + 1053*b*c^2*x^4 + 416*b^2*c*x^2 - 32*b^3)*sqrt(x))/(b^4*c^2*x^7 + 2*b^
5*c*x^5 + b^6*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(bx^2 + cx^4)^3}\, dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.15

$$\int \frac{x^{5/2}}{(bx^2 + cx^4)^3}\, dx = \frac{585\, c^3x^6 + 1053\, bc^2x^4 + 416\, b^2cx^2 - 32\, b^3}{80\, \left(b^4c^2x^{\frac{13}{2}} + 2\, b^5cx^{\frac{9}{2}} + b^6x^{\frac{5}{2}}\right)} + \frac{117\, c^2 \left(\frac{2\, \sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} + 2\, \sqrt{c}\sqrt{x}\right)}{2\, \sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\, \sqrt{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} - 2\, \sqrt{c}\sqrt{x}\right)}{2\, \sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2} \log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x} + \sqrt{c}x + \sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2} \log}{}\right.}{128\, b^4}$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/80*(585*c^3*x^6 + 1053*b*c^2*x^4 + 416*b^2*c*x^2 - 32*b^3)/(b^4*c^2*x^(1
3/2) + 2*b^5*c*x^(9/2) + b^6*x^(5/2)) + 117/128*c^2*(2*sqrt(2)*arctan(1/2*
sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)
))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2
)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b
)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c
)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sq
rt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.10

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^3}\,dx = \frac{117\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c} + \frac{117\,\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c} - \frac{117\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5c} + \frac{117\,\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5c} + \frac{21\,c^3x^{\frac{7}{2}}+25\,bc^2x^{\frac{3}{2}}}{16\,(cx^2+b)^2b^4} + \frac{2\,(15\,cx^2-b)}{5\,b^4x^{\frac{5}{2}}}$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
117/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/(b^5*c) + 117/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) - 117/128*sqr
t(2)*(b*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c
) + 117/128*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + s
qrt(b/c))/(b^5*c) + 1/16*(21*c^3*x^(7/2) + 25*b*c^2*x^(3/2))/((c*x^2 + b)^
2*b^4) + 2/5*(15*c*x^2 - b)/(b^4*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 17.53 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.52

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{26\,c\,x^2}{5\,b^2}-\frac{2}{5\,b}+\frac{1053\,c^2\,x^4}{80\,b^3}+\frac{117\,c^3\,x^6}{16\,b^4}}{b^2\,x^{5/2}+c^2\,x^{13/2}+2\,b\,c\,x^{9/2}} - \frac{117\,(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{17/4}} + \frac{117\,(-c)^{5/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{17/4}}$$

input
```
int(x^(5/2)/(b*x^2 + c*x^4)^3,x)
```

output
```
((26*c*x^2)/(5*b^2) - 2/(5*b) + (1053*c^2*x^4)/(80*b^3) + (117*c^3*x^6)/(1
6*b^4))/(b^2*x^(5/2) + c^2*x^(13/2) + 2*b*c*x^(9/2)) - (117*(-c)^(5/4)*ata
n(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(17/4)) + (117*(-c)^(5/4)*atanh(((-
c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(17/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.52

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(5/2)/(c*x^4+b*x^2)^3,x)
```

output

```
( - 1170*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2)
 - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2*c*x**2 - 2340*sqrt
(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*
sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b*c**2*x**4 - 1170*sqrt(x)*c**(1/4)*
b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**
(1/4)*b**(1/4)*sqrt(2)))*c**3*x**6 + 1170*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2
)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*
sqrt(2)))*b**2*c*x**2 + 2340*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1
/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b*c
**2*x**4 + 1170*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*
sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**3*x**6 + 585*
sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2)
 + sqrt(b) + sqrt(c)*x)*b**2*c*x**2 + 1170*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(
2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c**2*
x**4 + 585*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1
/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**3*x**6 - 585*sqrt(x)*c**(1/4)*b**(3/
4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*
*2*c*x**2 - 1170*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b*
*(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c**2*x**4 - 585*sqrt(x)*c**(1/4)*b
**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(...
```

3.150 $\int \frac{x^{3/2}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 211

$$\begin{aligned}\int \frac{x^{3/2}}{(bx^2+cx^4)^3}\,dx = &-\frac{165}{112b^3x^{7/2}}+\frac{55c}{16b^4x^{3/2}}+\frac{1}{4bx^{7/2}(b+cx^2)^2}\\ &+\frac{15}{16b^2x^{7/2}(b+cx^2)}-\frac{165c^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}\\ &+\frac{165c^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}+\frac{165c^{7/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{19/4}}\end{aligned}$$

output
```
-165/112/b^3/x^(7/2)+55/16*c/b^4/x^(3/2)+1/4/b/x^(7/2)/(c*x^2+b)^2+15/16/b
^2/x^(7/2)/(c*x^2+b)-165/64*c^(7/4)*arctan(1-2^(1/2)*c^(1/4)*x^(1/2)/b^(1/
4))*2^(1/2)/b^(19/4)+165/64*c^(7/4)*arctan(1+2^(1/2)*c^(1/4)*x^(1/2)/b^(1/
4))*2^(1/2)/b^(19/4)+165/64*c^(7/4)*arctanh(2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2
)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(19/4)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.76

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4b^{3/4}(-32b^3+160b^2cx^2+605bc^2x^4+385c^3x^6)}{x^{7/2}(b+cx^2)^2} - 1155\sqrt{2}c^{7/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + 1155\sqrt{2}c^{7/}}{448b^{19/4}}$$

input
```
Integrate[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
((4*b^(3/4)*(-32*b^3 + 160*b^2*c*x^2 + 605*b*c^2*x^4 + 385*c^3*x^6))/(x^(7
/2)*(b + c*x^2)^2) - 1155*Sqrt[2]*c^(7/4)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 1155*Sqrt[2]*c^(7/4)*ArcTanh[(Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(448*b^(19/4))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.46, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {9, 253, 253, 264, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{x^{9/2}(b+cx^2)^3}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{15\int \frac{1}{x^{9/2}(cx^2+b)^2}dx}{8b} + \frac{1}{4bx^{7/2}(b+cx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{15\left(\frac{11\int \frac{1}{x^{9/2}\left(cx^2+b\right)}dx}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{15\left(\frac{11\left(-\frac{c\int \frac{1}{x^{5/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{15\left(\frac{11\left(-\frac{c\left(-\frac{c\int \frac{1}{\sqrt{x}\left(cx^2+b\right)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{15\left(\frac{11\left(-\frac{c\left(-\frac{2c\int \frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{15\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{15\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}$$

↓ 1082

$$\frac{15\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}$$

↓ 217

$$\frac{15\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}+\frac{1}{2bx^{7/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}$$

↓ 1479

15

$$\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}\cdots\right)}{b}\cdots\right)}{4b}\cdots$$

↓ 25

$$15\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)$$

$$\downarrow 27$$

$$15\,\frac{\left(11\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{4b}$$

↓ 1103

$$\frac{15\left(\frac{11\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8b}$$

$$\frac{1}{4bx^{7/2}\left(b+cx^2\right)^2}$$

input
```
Int[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```

output
```
1/(4*b*x^(7/2)*(b + c*x^2)^2) + (15*(1/(2*b*x^(7/2)*(b + c*x^2)) + (11*(-2
/(7*b*x^(7/2)) - (c*(-2/(3*b*x^(3/2)) - (2*c*((-(ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[S
qrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(Sqrt[2]*b^(1/4)*c^(
1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt
[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/b))/b))/(4*b)))/(8*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{2(-7cx^2+b)}{7b^4x^{\frac{7}{2}}}+\frac{c^2\left(\frac{\frac{23c\,x^{\frac{5}{2}}}{16}+\frac{27b\sqrt{x}}{16}}{\left(c\,x^2+b\right)^2}+\frac{165\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)}\right.\right.}{128b}\right.}{b^4}$
derivativedivides	$\frac{2c^2\left(\frac{\frac{23c\,x^{\frac{5}{2}}}{32}+\frac{27b\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{165\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256b}\right)}{b^4}-\frac{2}{7b^3x}$
default	$\frac{2c^2\left(\frac{\frac{23c\,x^{\frac{5}{2}}}{32}+\frac{27b\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{165\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256b}\right)}{b^4}-\frac{2}{7b^3x}$

input
```
int(x^(3/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/7*(-7*c*x^2+b)/b^4/x^(7/2)+1/b^4*c^2*(2*(23/32*c*x^(5/2)+27/32*b*x^(1/2
))/(c*x^2+b)^2+165/128*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2
)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+
2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x
^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.56

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^3}\,dx = \frac{1155\,(b^4c^2x^8+2\,b^5cx^6+b^6x^4)\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}\log\left(165\,b^5\left(-\frac{c^7}{b^{19}}\right)^{\frac{1}{4}}+165\,c^2\sqrt{x}\right)-1155\,(-}{}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
1/448*(1155*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-c^7/b^19)^(1/4)*log(16
5*b^5*(-c^7/b^19)^(1/4) + 165*c^2*sqrt(x)) - 1155*(-I*b^4*c^2*x^8 - 2*I*b^
5*c*x^6 - I*b^6*x^4)*(-c^7/b^19)^(1/4)*log(165*I*b^5*(-c^7/b^19)^(1/4) + 1
65*c^2*sqrt(x)) - 1155*(I*b^4*c^2*x^8 + 2*I*b^5*c*x^6 + I*b^6*x^4)*(-c^7/b
^19)^(1/4)*log(-165*I*b^5*(-c^7/b^19)^(1/4) + 165*c^2*sqrt(x)) - 1155*(b^4
*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-c^7/b^19)^(1/4)*log(-165*b^5*(-c^7/b^1
9)^(1/4) + 165*c^2*sqrt(x)) + 4*(385*c^3*x^6 + 605*b*c^2*x^4 + 160*b^2*c*x
^2 - 32*b^3)*sqrt(x))/(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.17

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^3}\,dx = \frac{385\,c^3x^6+605\,bc^2x^4+160\,b^2cx^2-32\,b^3}{112\,\left(b^4c^2x^{\frac{15}{2}}+2\,b^5cx^{\frac{11}{2}}+b^6x^{\frac{7}{2}}\right)} + \frac{165\left(\frac{2\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}c^{\frac{7}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}} - \cdots\right)}{128\,b^4}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
1/112*(385*c^3*x^6 + 605*b*c^2*x^4 + 160*b^2*c*x^2 - 32*b^3)/(b^4*c^2*x^(1
5/2) + 2*b^5*c*x^(11/2) + b^6*x^(7/2)) + 165/128*(2*sqrt(2)*c^2*arctan(1/2
*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c
)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*c^2*arctan(-1/2*sqrt(2)*(s
qrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b
)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(7/4)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqr
t(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c^(7/4)*log(-sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4))/b^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.06

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^3}\,dx = \frac{165\sqrt{2}(bc^3)^{\frac{1}{4}}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5} + \frac{165\sqrt{2}(bc^3)^{\frac{1}{4}}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5} + \frac{165\sqrt{2}(bc^3)^{\frac{1}{4}}\,c\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5} - \frac{165\sqrt{2}(bc^3)^{\frac{1}{4}}\,c\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5} + \frac{23\,c^3x^{\frac{5}{2}}+27\,bc^2\sqrt{x}}{16\,(cx^2+b)^2b^4} + \frac{2\,(7\,cx^2-b)}{7\,b^4x^{\frac{7}{2}}}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
165/64*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2
*sqrt(x))/(b/c)^(1/4))/b^5 + 165/64*sqrt(2)*(b*c^3)^(1/4)*c*arctan(-1/2*sq
rt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^5 + 165/128*sqrt(2)
*(b*c^3)^(1/4)*c*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^5 - 16
5/128*sqrt(2)*(b*c^3)^(1/4)*c*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(
b/c))/b^5 + 1/16*(23*c^3*x^(5/2) + 27*b*c^2*sqrt(x))/((c*x^2 + b)^2*b^4) +
 2/7*(7*c*x^2 - b)/(b^4*x^(7/2))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.52

$$\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^3}\,dx = \frac{\frac{10\,c\,x^2}{7\,b^2}-\frac{2}{7\,b}+\frac{605\,c^2\,x^4}{112\,b^3}+\frac{55\,c^3\,x^6}{16\,b^4}}{b^2\,x^{7/2}+c^2\,x^{15/2}+2\,b\,c\,x^{11/2}} + \frac{165\,{\left(-c\right)}^{7/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{19/4}}+\frac{165\,{\left(-c\right)}^{7/4}\,\mathrm{atanh}\left(\frac{{\left(-c\right)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{19/4}}$$

input
```
int(x^(3/2)/(b*x^2 + c*x^4)^3,x)
```

output
```
((10*c*x^2)/(7*b^2) - 2/(7*b) + (605*c^2*x^4)/(112*b^3) + (55*c^3*x^6)/(16
*b^4))/(b^2*x^(7/2) + c^2*x^(15/2) + 2*b*c*x^(11/2)) + (165*(-c)^(7/4)*ata
n(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(19/4)) + (165*(-c)^(7/4)*atanh(((-
c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(19/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.52

$$\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(3/2)/(c*x^4+b*x^2)^3,x)
```

output
```
( - 2310*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2)
 - 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2*c*x**3 - 4620*sqrt
(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*
sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b*c**2*x**5 - 2310*sqrt(x)*c**(3/4)*
b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(c**
(1/4)*b**(1/4)*sqrt(2)))*c**3*x**7 + 2310*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2
)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*
sqrt(2)))*b**2*c*x**3 + 4620*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1
/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b*c
**2*x**5 + 2310*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*
sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**3*x**7 - 1155
*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2
) + sqrt(b) + sqrt(c)*x)*b**2*c*x**3 - 2310*sqrt(x)*c**(3/4)*b**(1/4)*sqrt
(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c**2
*x**5 - 1155*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**
(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**3*x**7 + 1155*sqrt(x)*c**(3/4)*b**
(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)
*b**2*c*x**3 + 2310*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)
*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c**2*x**5 + 1155*sqrt(x)*c**(3/
4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + s...
```

3.151 $\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 226

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx = -\frac{221}{144b^3x^{9/2}}+\frac{221c}{80b^4x^{5/2}}-\frac{221c^2}{16b^5\sqrt{x}}+\frac{1}{4bx^{9/2}(b+cx^2)^2}+\frac{17}{16b^2x^{9/2}(b+cx^2)}+\frac{221c^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}}-\frac{221c^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{21/4}}$$

output
```
-221/144/b^3/x^(9/2)+221/80*c/b^4/x^(5/2)-221/16*c^2/b^5/x^(1/2)+1/4/b/x^(
9/2)/(c*x^2+b)^2+17/16/b^2/x^(9/2)/(c*x^2+b)+221/64*c^(9/4)*arctan(1-2^(1/
2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(21/4)-221/64*c^(9/4)*arctan(1+2^(1/
2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(21/4)+221/64*c^(9/4)*arctanh(2^(1/2
)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(21/4)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{-\frac{4\sqrt[4]{b}(160b^4-544b^3cx^2+7072b^2c^2x^4+17901bc^3x^6+9945c^4x^8)}{x^{9/2}(b+cx^2)^2}+9945\sqrt{2}c^{9/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+9945\sqrt{2}c^{9/4}\text{arcta}}{2880b^{21/4}}$$

input
```
Integrate[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```

output
```
((-4*b^(1/4)*(160*b^4 - 544*b^3*c*x^2 + 7072*b^2*c^2*x^4 + 17901*b*c^3*x^6
 + 9945*c^4*x^8))/(x^(9/2)*(b + c*x^2)^2) + 9945*Sqrt[2]*c^(9/4)*ArcTan[(S
qrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 9945*Sqrt[2]*c^(9
/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(288
0*b^(21/4))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.44, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {9, 253, 253, 264, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{1}{x^{11/2}(b+cx^2)^3}dx$$
$$\Big\downarrow \ 253$$
$$\frac{17\int \frac{1}{x^{11/2}(cx^2+b)^2}dx}{8b}+\frac{1}{4bx^{9/2}(b+cx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{17\left(\frac{13\int \frac{1}{x^{11/2}\left(cx^2+b\right)}dx}{4b}+\frac{1}{2bx^{9/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{9/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{17\left(\frac{13\left(-\frac{c\int \frac{1}{x^{7/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{9/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{17\left(\frac{13\left(-\frac{c\left(-\frac{c\int \frac{1}{x^{3/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{9/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{17\left(\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{c\int\frac{\sqrt{x}}{cx^2+b}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{9/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{17\left(\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\int\frac{x}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{9/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{17\left(\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{9/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \; 1476$$

17 $$-\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}(b+cx^2)}$$

↓ 1082

$$17\,\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}(b+cx}$$

↓ 217

$$\left(17\,\frac{13\left(-\frac{c\left(c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}\,d\sqrt{x}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)\right)}{b}-\frac{2}{9bx^{9/2}}\right)}{4b}+\frac{1}{2bx^{9/2}(b+cx^2)}\right.$$

↓ 1479

$$13 \quad -\frac{\left(c-\frac{\left(c-\frac{\left(2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{\cdots}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\cdots}{b}\cdots}{b}\cdots}{b}\cdots$$

↓ 25

$$13\left(-\frac{c\left(c-\frac{c\left(c-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\cdots}{b}\cdots}{b}\cdots\right)$$

$$\downarrow 27$$

$$\left(13 \quad \left(-\frac{\left(c\left(-\frac{\left(c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{x}}\right)\right)}{b}-\frac{2}{5bx^{5/2}} \right)\right)}{b}\right.\right.$$

↓ 1103

$$17\left(\frac{13\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{b}-\frac{2}{b\sqrt{\cdots}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right.$$

input
```
Int[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```

output
```
1/(4*b*x^(9/2)*(b + c*x^2)^2) + (17*(1/(2*b*x^(9/2)*(b + c*x^2)) + (13*(-2
/(9*b*x^(9/2)) - (c*(-2/(5*b*x^(5/2)) - (c*(-2/(b*Sqrt[x]) - (2*c*((-(ArcT
an[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))) + Arc
Tan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))/(2*S
qrt[c]) - (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[c])))/b))/b))/b))/(4*b)
))/(8*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{2c^3\left(\frac{\frac{29c\,x^{\frac{7}{2}}}{32}+\frac{33b\,x^{\frac{3}{2}}}{32}}{(c\,x^2+b)^2}+\frac{221\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^5}-\frac{2}{9b^3x^{\frac{9}{2}}}$
default	$-\frac{2c^3\left(\frac{\frac{29c\,x^{\frac{7}{2}}}{32}+\frac{33b\,x^{\frac{3}{2}}}{32}}{(c\,x^2+b)^2}+\frac{221\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^5}-\frac{2}{9b^3x^{\frac{9}{2}}}$
risch	$-\frac{2(270c^2x^4-27bc\,x^2+5b^2)}{45b^5x^{\frac{9}{2}}}-\frac{c^3\left(\frac{\frac{29c\,x^{\frac{7}{2}}}{16}+\frac{33b\,x^{\frac{3}{2}}}{16}}{(c\,x^2+b)^2}+\frac{221\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arcta}\right.}{128c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^5}$

input

```
int(x^(1/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output

```
-2/b^5*c^3*((29/32*c*x^(7/2)+33/32*b*x^(3/2))/(c*x^2+b)^2+221/256/c/(1/c*b
)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/
c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*
x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))-2/9/b^3/x^(9/2)-12*
c^2/b^5/x^(1/2)+6/5*c/b^4/x^(5/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx = -\frac{9945\left(b^5c^2x^9+2\,b^6cx^7+b^7x^5\right)\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}}\log\left(10793861\,b^{16}\left(-\frac{c^9}{b^{21}}\right)^{\frac{3}{4}}+10793861\,c^7\sqrt{x}\right)+9945\left(-i\,b^5c\right.}{}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/2880*(9945*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9/b^21)^(1/4)*log(
10793861*b^16*(-c^9/b^21)^(3/4) + 10793861*c^7*sqrt(x)) + 9945*(-I*b^5*c^2
*x^9 - 2*I*b^6*c*x^7 - I*b^7*x^5)*(-c^9/b^21)^(1/4)*log(10793861*I*b^16*(-
c^9/b^21)^(3/4) + 10793861*c^7*sqrt(x)) + 9945*(I*b^5*c^2*x^9 + 2*I*b^6*c*
x^7 + I*b^7*x^5)*(-c^9/b^21)^(1/4)*log(-10793861*I*b^16*(-c^9/b^21)^(3/4)
+ 10793861*c^7*sqrt(x)) - 9945*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-c^9
/b^21)^(1/4)*log(-10793861*b^16*(-c^9/b^21)^(3/4) + 10793861*c^7*sqrt(x))
+ 4*(9945*c^4*x^8 + 17901*b*c^3*x^6 + 7072*b^2*c^2*x^4 - 544*b^3*c*x^2 + 1
60*b^4)*sqrt(x))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx = -\frac{9945\,c^4x^8+17901\,bc^3x^6+7072\,b^2c^2x^4-544\,b^3cx^2+160\,b^4}{720\left(b^5c^2x^{\frac{17}{2}}+2\,b^6cx^{\frac{13}{2}}+b^7x^{\frac{9}{2}}\right)}$$

$$-\frac{221\,c^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\cdots}{\cdots}\right.}{128\,b^5}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/720*(9945*c^4*x^8 + 17901*b*c^3*x^6 + 7072*b^2*c^2*x^4 - 544*b^3*c*x^2
+ 160*b^4)/(b^5*c^2*x^(17/2) + 2*b^6*c*x^(13/2) + b^7*x^(9/2)) - 221/128*c
^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt
(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(
b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b
^5
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx = -\frac{221\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6} - \frac{221\sqrt{2}(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6} + \frac{221\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6} - \frac{221\sqrt{2}(bc^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6} - \frac{29\,c^4x^{\frac{7}{2}}+33\,bc^3x^{\frac{3}{2}}}{16\,(cx^2+b)^2b^5} - \frac{2\,(270\,c^2x^4-27\,bcx^2+5\,b^2)}{45\,b^5x^{\frac{9}{2}}}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-221/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*
sqrt(x))/(b/c)^(1/4))/b^6 - 221/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(
2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 + 221/128*sqrt(2)*(b
*c^3)^(3/4)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 - 221/128
*sqrt(2)*(b*c^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b
^6 - 1/16*(29*c^4*x^(7/2) + 33*b*c^3*x^(3/2))/((c*x^2 + b)^2*b^5) - 2/45*(
270*c^2*x^4 - 27*b*c*x^2 + 5*b^2)/(b^5*x^(9/2))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx = \frac{221\,{(-c)}^{9/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{21/4}} - \frac{221\,{(-c)}^{9/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{21/4}} - \frac{\frac{2}{9\,b}-\frac{34\,c\,x^2}{45\,b^2}+\frac{442\,c^2\,x^4}{45\,b^3}+\frac{1989\,c^3\,x^6}{80\,b^4}+\frac{221\,c^4\,x^8}{16\,b^5}}{b^2\,x^{9/2}+c^2\,x^{17/2}+2\,b\,c\,x^{13/2}}$$

input
```
int(x^(1/2)/(b*x^2 + c*x^4)^3,x)
```

output
```
(221*(-c)^(9/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(21/4)) - (221*
(-c)^(9/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(21/4)) - (2/(9*b) -
(34*c*x^2)/(45*b^2) + (442*c^2*x^4)/(45*b^3) + (1989*c^3*x^6)/(80*b^4) + (
221*c^4*x^8)/(16*b^5))/(b^2*x^(9/2) + c^2*x^(17/2) + 2*b*c*x^(13/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)/(c*x^4+b*x^2)^3,x)
```

output
```
(19890*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2*c**2*x**4 + 39780*sq
rt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b*c**3*x**6 + 19890*sqrt(x)*c**(1/
4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(
c**(1/4)*b**(1/4)*sqrt(2)))*c**4*x**8 - 19890*sqrt(x)*c**(1/4)*b**(3/4)*sq
rt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1
/4)*sqrt(2)))*b**2*c**2*x**4 - 39780*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*ata
n((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(
2)))*b*c**3*x**6 - 19890*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*atan((c**(1/4)*
b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**4*x*
*8 - 9945*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/
4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b**2*c**2*x**4 - 19890*sqrt(x)*c**(1/4)*
b**(3/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt
(c)*x)*b*c**3*x**6 - 9945*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log( - sqrt(x)
*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**4*x**8 + 9945*sqrt(x)
*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b)
 + sqrt(c)*x)*b**2*c**2*x**4 + 19890*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log
(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c**3*x**6 + 99
45*sqrt(x)*c**(1/4)*b**(3/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt...
```

3.152 $\int \frac{1}{\sqrt{x}(bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 226

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx =& -\frac{285}{176b^3x^{11/2}}+\frac{285c}{112b^4x^{7/2}}-\frac{95c^2}{16b^5x^{3/2}}+\frac{1}{4bx^{11/2}\,(b+cx^2)^2}\\&+\frac{19}{16b^2x^{11/2}\,(b+cx^2)}+\frac{285c^{11/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}}\\&-\frac{285c^{11/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}}\\&-\frac{285c^{11/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{32\sqrt{2}b^{23/4}}\end{aligned}$$

output
```
-285/176/b^3/x^(11/2)+285/112*c/b^4/x^(7/2)-95/16*c^2/b^5/x^(3/2)+1/4/b/x^
(11/2)/(c*x^2+b)^2+19/16/b^2/x^(11/2)/(c*x^2+b)+285/64*c^(11/4)*arctan(1-2
^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(23/4)-285/64*c^(11/4)*arctan(1+
2^(1/2)*c^(1/4)*x^(1/2)/b^(1/4))*2^(1/2)/b^(23/4)-285/64*c^(11/4)*arctanh(
2^(1/2)*b^(1/4)*c^(1/4)*x^(1/2)/(b^(1/2)+c^(1/2)*x))*2^(1/2)/b^(23/4)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = \frac{-\frac{4b^{3/4}(224b^4-608b^3cx^2+3040b^2c^2x^4+11495bc^3x^6+7315c^4x^8)}{x^{11/2}(b+cx^2)^2}+21945\sqrt{2}c^{11/4}\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)-21945\sqrt{2}c^{11/4}\text{a}}{4928b^{23/4}}$$

input
```
Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

output
```
((-4*b^(3/4)*(224*b^4 - 608*b^3*c*x^2 + 3040*b^2*c^2*x^4 + 11495*b*c^3*x^6
 + 7315*c^4*x^8))/(x^(11/2)*(b + c*x^2)^2) + 21945*Sqrt[2]*c^(11/4)*ArcTan
[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 21945*Sqrt[2]*
c^(11/4)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])
/(4928*b^(23/4))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.45, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {9, 253, 253, 264, 264, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^{13/2}\,(b+cx^2)^3}dx$$

$$\Big\downarrow \; 253$$

$$\frac{19\int \frac{1}{x^{13/2}(cx^2+b)^2}dx}{8b}+\frac{1}{4bx^{11/2}\,(b+cx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{19\left(\frac{15\int \frac{1}{x^{13/2}\left(cx^2+b\right)}dx}{4b}+\frac{1}{2bx^{11/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{19\left(\frac{15\left(-\frac{c\int \frac{1}{x^{9/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{19\left(\frac{15\left(-\frac{c\left(-\frac{c\int \frac{1}{x^{5/2}\left(cx^2+b\right)}dx}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{19\left(\frac{15\left(-\frac{c\left(-\frac{c\left(-\frac{c\int \frac{1}{\sqrt{x}(cx^2+b)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{11/2}(b+cx^2)^2}$$

$$\Big\downarrow \; 266$$

$$\frac{19\left(\frac{15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\int \frac{1}{cx^2+b}d\sqrt{x}}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{11/2}(b+cx^2)^2}$$

$$\Big\downarrow \; 755$$

$$\frac{19\left(\frac{15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{c}x+\sqrt{b}}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}(b+cx^2)}\right)}{8b}+\frac{1}{4bx^{11/2}\left(b+cx^2\right)^2}$$

$$\Big\downarrow \text{ 1476}$$

$$19\left(\frac{15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}\,d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}\,d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}\,d\sqrt{x}}{2\sqrt{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}(b

↓ 1082

$$19\left(\frac{15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}(\cdots}\right.$$

↓ 217

$$19\ \frac{15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{cx^2+b}d\sqrt{x}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx^{7/2}}\right)}{b}-\frac{2}{11bx^{11/2}}\right)}{4b}+\frac{1}{2bx^{11/2}(b+cx^2}$$

↓ 1479

$$15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\cdots\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}\cdots}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{b}\cdots\right.$$

$\downarrow$ 25

$$15 - \frac{c - \frac{c - \frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{c}\left(x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}\right)}{\sqrt[4]{c}\left(x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b} - \cdots}{b}}{b}$$

$$\Big\downarrow \text{27}$$

$$\left(15\left(-\frac{c\left(-\frac{c\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}+\sqrt[4]{b}}{x+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{c}}+\frac{\sqrt{b}}{\sqrt{c}}}d\sqrt{x}}{2\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{7bx}}{b}\right.\right.$$

↓ 1103

$$19 \quad \frac{\left(15 - \frac{\left(c - \frac{\left(c\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}}}{2\sqrt{b}}\right)}{b}\right)-\frac{\cdots}{3bx\cdots}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4b}\cdots$$

input
```
Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

output
```
1/(4*b*x^(11/2)*(b + c*x^2)^2) + (19*(1/(2*b*x^(11/2)*(b + c*x^2)) + (15*(
-2/(11*b*x^(11/2)) - (c*(-2/(7*b*x^(7/2)) - (c*(-2/(3*b*x^(3/2)) - (2*c*((
-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4)))
 + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*c^(1/4))
)/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt
[c]*x]/(Sqrt[2]*b^(1/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*S
qrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(1/4)))/(2*Sqrt[b])))/b))/b))/b))
/(4*b)))/(8*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{2c^3\left(\frac{\frac{31c\,x^{\frac{5}{2}}}{32}+\frac{35b\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{285\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256b}\right)}{b^5}-\frac{}{11}$
default	$-\frac{2c^3\left(\frac{\frac{31c\,x^{\frac{5}{2}}}{32}+\frac{35b\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{285\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256b}\right)}{b^5}-\frac{}{11}$
risch	$-\frac{2\left(154c^2x^4-33bc\,x^2+7b^2\right)}{77b^5x^{\frac{11}{2}}}-\frac{c^3\left(\frac{\frac{31c\,x^{\frac{5}{2}}}{16}+\frac{35b\sqrt{x}}{16}}{\left(c\,x^2+b\right)^2}+\frac{285\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\right.}{128b}\right.}{b^5}$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b^5*c^3*((31/32*c*x^(5/2)+35/32*b*x^(1/2))/(c*x^2+b)^2+285/256*(1/c*b)^
(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/
c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*
x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))-2/11/b^3/x^(11/2)-4
*c^2/b^5/x^(3/2)+6/7*c/b^4/x^(7/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = -\frac{21945\,(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6)\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}\log\left(285\,b^6\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}+285\,c^3\sqrt{x}\right)+21945\,(i\,b^5c^2x^{10}+2i\,b^6}{}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
-1/4928*(21945*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-c^11/b^23)^(1/4)*l
og(285*b^6*(-c^11/b^23)^(1/4) + 285*c^3*sqrt(x)) + 21945*(I*b^5*c^2*x^10 +
 2*I*b^6*c*x^8 + I*b^7*x^6)*(-c^11/b^23)^(1/4)*log(285*I*b^6*(-c^11/b^23)^
(1/4) + 285*c^3*sqrt(x)) + 21945*(-I*b^5*c^2*x^10 - 2*I*b^6*c*x^8 - I*b^7*
x^6)*(-c^11/b^23)^(1/4)*log(-285*I*b^6*(-c^11/b^23)^(1/4) + 285*c^3*sqrt(x
)) - 21945*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-c^11/b^23)^(1/4)*log(-
285*b^6*(-c^11/b^23)^(1/4) + 285*c^3*sqrt(x)) + 4*(7315*c^4*x^8 + 11495*b*
c^3*x^6 + 3040*b^2*c^2*x^4 - 608*b^3*c*x^2 + 224*b^4)*sqrt(x))/(b^5*c^2*x^
10 + 2*b^6*c*x^8 + b^7*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(1/2)/(c*x**4+b*x**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = -\frac{7315\,c^4x^8+11495\,bc^3x^6+3040\,b^2c^2x^4-608\,b^3cx^2+224\,b^4}{1232\left(b^5c^2x^{\frac{19}{2}}+2\,b^6cx^{\frac{15}{2}}+b^7x^{\frac{11}{2}}\right)} - \frac{285\left(\frac{2\sqrt{2}c^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}c^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}c^{\frac{11}{4}}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{3}{4}}}\right.}{128\,b^5}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
-1/1232*(7315*c^4*x^8 + 11495*b*c^3*x^6 + 3040*b^2*c^2*x^4 - 608*b^3*c*x^2
 + 224*b^4)/(b^5*c^2*x^(19/2) + 2*b^6*c*x^(15/2) + b^7*x^(11/2)) - 285/128
*(2*sqrt(2)*c^3*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sq
rt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*
c^3*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*c^(11/4)*log(
sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4) - sqrt(2)*c
^(11/4)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/b^(3/4
))/b^5
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = -\frac{285\,\sqrt{2}(bc^3)^{\frac{1}{4}}\,c^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6} - \frac{285\,\sqrt{2}(bc^3)^{\frac{1}{4}}\,c^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6} - \frac{285\,\sqrt{2}(bc^3)^{\frac{1}{4}}\,c^2\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6} + \frac{285\,\sqrt{2}(bc^3)^{\frac{1}{4}}\,c^2\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6} - \frac{31\,c^4x^{\frac{5}{2}}+35\,bc^3\sqrt{x}}{16\,(cx^2+b)^2b^5} - \frac{2\,(154\,c^2x^4-33\,bcx^2+7\,b^2)}{77\,b^5x^{\frac{11}{2}}}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
-285/64*sqrt(2)*(b*c^3)^(1/4)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
+ 2*sqrt(x))/(b/c)^(1/4))/b^6 - 285/64*sqrt(2)*(b*c^3)^(1/4)*c^2*arctan(-1
/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 - 285/128*sq
rt(2)*(b*c^3)^(1/4)*c^2*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b
^6 + 285/128*sqrt(2)*(b*c^3)^(1/4)*c^2*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) +
x + sqrt(b/c))/b^6 - 1/16*(31*c^4*x^(5/2) + 35*b*c^3*sqrt(x))/((c*x^2 + b)
^2*b^5) - 2/77*(154*c^2*x^4 - 33*b*c*x^2 + 7*b^2)/(b^5*x^(11/2))
```

Mupad [B] (verification not implemented)

Time = 18.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = \frac{285\,(-c)^{11/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{23/4}} - \frac{\frac{2}{11\,b}-\frac{38\,c\,x^2}{77\,b^2}+\frac{190\,c^2\,x^4}{77\,b^3}+\frac{1045\,c^3\,x^6}{112\,b^4}+\frac{95\,c^4\,x^8}{16\,b^5}}{b^2\,x^{11/2}+c^2\,x^{19/2}+2\,b\,c\,x^{15/2}} + \frac{285\,(-c)^{11/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)}{32\,b^{23/4}}$$

input
```
int(1/(x^(1/2)*(b*x^2 + c*x^4)^3),x)
```

output
```
(285*(-c)^(11/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(23/4)) - (2/(1
1*b) - (38*c*x^2)/(77*b^2) + (190*c^2*x^4)/(77*b^3) + (1045*c^3*x^6)/(112*
b^4) + (95*c^4*x^8)/(16*b^5))/(b^2*x^(11/2) + c^2*x^(19/2) + 2*b*c*x^(15/2
)) + (285*(-c)^(11/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4)))/(32*b^(23/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2)^3,x)
```

output
```
(43890*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b**2*c**2*x**5 + 87780*sq
rt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*b*c**3*x**7 + 43890*sqrt(x)*c**(3/
4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(c))/(
c**(1/4)*b**(1/4)*sqrt(2)))*c**4*x**9 - 43890*sqrt(x)*c**(3/4)*b**(1/4)*sq
rt(2)*atan((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1
/4)*sqrt(2)))*b**2*c**2*x**5 - 87780*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*ata
n((c**(1/4)*b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(
2)))*b*c**3*x**7 - 43890*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*atan((c**(1/4)*
b**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(c))/(c**(1/4)*b**(1/4)*sqrt(2)))*c**4*x*
*9 + 21945*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1
/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b**2*c**2*x**5 + 43890*sqrt(x)*c**(3/4)
*b**(1/4)*sqrt(2)*log( - sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqr
t(c)*x)*b*c**3*x**7 + 21945*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log( - sqrt(
x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*c**4*x**9 - 21945*sqrt
(x)*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt
(b) + sqrt(c)*x)*b**2*c**2*x**5 - 43890*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*
log(sqrt(x)*c**(1/4)*b**(1/4)*sqrt(2) + sqrt(b) + sqrt(c)*x)*b*c**3*x**7 -
 21945*sqrt(x)*c**(3/4)*b**(1/4)*sqrt(2)*log(sqrt(x)*c**(1/4)*b**(1/4)*...
```

3.153 $\int x^5\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 119

$$\int x^5\sqrt{bx^2+cx^4}\,dx=\frac{5b^2(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b(bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2(bx^2+cx^4)^{3/2}}{8c}-\frac{5b^4\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}$$

output
```
5/128*b^2*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^3-5/48*b*(c*x^4+b*x^2)^(3/2)/c
^2+1/8*x^2*(c*x^4+b*x^2)^(3/2)/c-5/128*b^4*arctanh(c^(1/2)*x^2/(c*x^4+b*x^
2)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int x^5\sqrt{bx^2+cx^4}\,dx=\frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}(15b^3-10b^2cx^2+8bc^2x^4+48c^3x^6)+30b^4\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}-\sqrt{b+cx^2}}\right)\right)}{384c^{7/2}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^5*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(15*b^3 - 10*b^2*c*x^2 + 8*b
*c^2*x^4 + 48*c^3*x^6) + 30*b^4*ArcTanh[(Sqrt[c]*x)/(Sqrt[b] - Sqrt[b + c*
x^2])]))/(384*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1424, 1134, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int x^4\sqrt{cx^4+bx^2}dx^2$$

$$\Big\downarrow \text{1134}$$

$$\frac{1}{2}\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\int x^2\sqrt{cx^4+bx^2}dx^2}{8c}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\int \sqrt{cx^4+bx^2}dx^2}{2c}\right)}{8c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)}{2c}\right)}{8c}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)}{2c}\right)}{8c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)}{2c}\right)}{8c}\right)$$

input
```
Int[x^5*Sqrt[b*x^2 + c*x^4],x]
```

output
```
((x^2*(b*x^2 + c*x^4)^(3/2))/(4*c) - (5*b*((b*x^2 + c*x^4)^(3/2)/(3*c) - (
b*(((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/
Sqrt[b*x^2 + c*x^4]])/(4*c^(3/2))))/(2*c)))/(8*c))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$-\frac{5\left(\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b^4+\left(-\frac{32c^{\frac{7}{2}}x^6}{5}-\frac{16c^{\frac{5}{2}}b\,x^4}{15}+\frac{4c^{\frac{3}{2}}b^2x^2}{3}-2\sqrt{c}\,b^3\right)\sqrt{x^2(c\,x^2+b)}-\ln(2)b^4\right)}{256c^{\frac{7}{2}}}$
risch	$\frac{\left(48c^3x^6+8b\,c^2x^4-10b^2c\,x^2+15b^3\right)\sqrt{x^2(c\,x^2+b)}}{384c^3}-\frac{5b^4\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)\sqrt{x^2(c\,x^2+b)}}{128c^{\frac{7}{2}}x\sqrt{c\,x^2+b}}$
default	$\frac{\sqrt{c\,x^4+b\,x^2}\,\left(48x^5\left(c\,x^2+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}-40c^{\frac{3}{2}}\left(c\,x^2+b\right)^{\frac{3}{2}}b\,x^3+30\sqrt{c}\left(c\,x^2+b\right)^{\frac{3}{2}}b^2x-15\sqrt{c}\sqrt{c\,x^2+b}\,b^3x-15\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)b}{384x\sqrt{c\,x^2+b}\,c^{\frac{7}{2}}}$

input
```
int(x^5*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/256/c^(7/2)*(ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1/2))*b^
4+(-32/5*c^(7/2)*x^6-16/15*c^(5/2)*b*x^4+4/3*c^(3/2)*b^2*x^2-2*c^(1/2)*b^3
)*(x^2*(c*x^2+b))^(1/2)-ln(2)*b^4)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.58

$$\int x^5\sqrt{bx^2+cx^4}\,dx$$
$$=\left[\frac{15\,b^4\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,(48\,c^4x^6+8\,bc^3x^4-10\,b^2c^2x^2+15\,b^3c)\sqrt{cx^4+bx^2}}{768\,c^4},\,-\right.$$

input
```
integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(15*b^4*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) +
 2*(48*c^4*x^6 + 8*b*c^3*x^4 - 10*b^2*c^2*x^2 + 15*b^3*c)*sqrt(c*x^4 + b*x
^2))/c^4, 1/384*(15*b^4*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^
2 + b)) + (48*c^4*x^6 + 8*b*c^3*x^4 - 10*b^2*c^2*x^2 + 15*b^3*c)*sqrt(c*x^
4 + b*x^2))/c^4]
```

Sympy [F]

$$\int x^5\sqrt{bx^2+cx^4}\,dx=\int x^5\sqrt{x^2\left(b+cx^2\right)}\,dx$$

input
```
integrate(x**5*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**5*sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int x^5\sqrt{bx^2+cx^4}\,dx=\frac{5\sqrt{cx^4+bx^2}b^2x^2}{64\,c^2}+\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}{8\,c}-\frac{5\,b^4\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{256\,c^{\frac{7}{2}}}+\frac{5\sqrt{cx^4+bx^2}b^3}{128\,c^3}-\frac{5\left(cx^4+bx^2\right)^{\frac{3}{2}}b}{48\,c^2}$$

input
```
integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
5/64*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^2 + 1/8*(c*x^4 + b*x^2)^(3/2)*x^2/c - 5
/256*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 5/128*
sqrt(c*x^4 + b*x^2)*b^3/c^3 - 5/48*(c*x^4 + b*x^2)^(3/2)*b/c^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.85

$$\int x^5\sqrt{bx^2+cx^4}\,dx = \frac{1}{384}\left(2\left(4\left(6\,x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x^2-\frac{5\,b^2\mathrm{sgn}(x)}{c^2}\right)x^2+\frac{15\,b^3\mathrm{sgn}(x)}{c^3}\right)\sqrt{cx^2+b}x + \frac{5\,b^4\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{128\,c^{\frac{7}{2}}} - \frac{5\,b^4\log\left(|b|\right)\mathrm{sgn}(x)}{256\,c^{\frac{7}{2}}}$$

input
```
integrate(x^5*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*x^2*sgn(x) + b*sgn(x)/c)*x^2 - 5*b^2*sgn(x)/c^2)*x^2 + 15*b
^3*sgn(x)/c^3)*sqrt(c*x^2 + b)*x + 5/128*b^4*log(abs(-sqrt(c)*x + sqrt(c*x
^2 + b)))*sgn(x)/c^(7/2) - 5/256*b^4*log(abs(b))*sgn(x)/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 18.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int x^5\sqrt{bx^2+cx^4}\,dx = \frac{x^2\left(c\,x^4+b\,x^2\right)^{3/2}}{8\,c} - \frac{5\,b\left(\frac{b^3\,\ln\left(\frac{2\,c\,x^2+b}{\sqrt{c}}+2\,\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{16\,c}$$

input
```
int(x^5*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
(x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*b*((b^3*log((b + 2*c*x^2)/c^(1/2) +
 2*(b*x^2 + c*x^4)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^
4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int x^5\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{15\sqrt{c\,x^2+b}\,b^3cx-10\sqrt{c\,x^2+b}\,b^2c^2x^3+8\sqrt{c\,x^2+b}\,b\,c^3x^5+48\sqrt{c\,x^2+b}\,c^4x^7-15\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)}{384c^4}$$

input
```
int(x^5*(c*x^4+b*x^2)^(1/2),x)
```

output
```
(15*sqrt(b + c*x**2)*b**3*c*x - 10*sqrt(b + c*x**2)*b**2*c**2*x**3 + 8*sqr
t(b + c*x**2)*b*c**3*x**5 + 48*sqrt(b + c*x**2)*c**4*x**7 - 15*sqrt(c)*log
((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b))*b**4)/(384*c**4)
```

3.154 $\int x^3\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int x^3\sqrt{bx^2+cx^4}\,dx=-\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c^2}+\frac{(bx^2+cx^4)^{3/2}}{6c}+\frac{b^3\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}$$

output
```
-1/16*b*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^2+1/6*(c*x^4+b*x^2)^(3/2)/c+1/16
*b^3*arctanh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int x^3\sqrt{bx^2+cx^4}\,dx=\frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}(-3b^2+2bcx^2+8c^2x^4)+6b^3\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)\right)}{48c^{5/2}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^3*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(-3*b^2 + 2*b*c*x^2 + 8*c^2*
x^4) + 6*b^3*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))/(48*c^(5/
2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1424, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int x^2\sqrt{cx^4+bx^2}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{(bx^2+cx^4)^{3/2}}{3c}-\frac{b\int \sqrt{cx^4+bx^2}dx^2}{2c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{(bx^2+cx^4)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)}{2c}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{(bx^2+cx^4)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)}{2c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(bx^2+cx^4)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)}{2c}\right)$$

input
```
Int[x^3*Sqrt[b*x^2 + c*x^4],x]
```

output
```
((b*x^2 + c*x^4)^(3/2)/(3*c) - (b*(((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(4*
c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(4*c^(3/2))))/(2*c))
/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

method	result	si
risch	$-\frac{\left(-8c^{2}x^{4}-2bc\,x^{2}+3b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{48c^{2}}+\frac{b^{3}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{16c^{\frac{5}{2}}x\sqrt{c\,x^{2}+b}}$	90
default	$\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(8x^{3}\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}-6\sqrt{c}\,\left(c\,x^{2}+b\right)^{\frac{3}{2}}bx+3\sqrt{c}\,\sqrt{c\,x^{2}+b}\,b^{2}x+3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)b^{3}\right)}{48x\sqrt{c\,x^{2}+b}\,c^{\frac{5}{2}}}$	10
pseudoelliptic	$\frac{16c^{\frac{5}{2}}x^{4}\sqrt{x^{2}\left(c\,x^{2}+b\right)}+4\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,c^{\frac{3}{2}}b\,x^{2}-6\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}\,b^{2}-3\ln\left(2\right)b^{3}+3\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b^{3}}{96c^{\frac{5}{2}}}$	11

input
```
int(x^3*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-8*c^2*x^4-2*b*c*x^2+3*b^2)/c^2*(x^2*(c*x^2+b))^(1/2)+1/16*b^3/c^(5
/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.84

$$\int x^3\sqrt{bx^2+cx^4}\,dx$$
$$=\left[\frac{3\,b^3\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,(8\,c^3x^4+2\,bc^2x^2-3\,b^2c)\sqrt{cx^4+bx^2}}{96\,c^3},\right.$$
$$\left.-\frac{3\,b^3\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(8\,c^3x^4+2\,bc^2x^2-3\,b^2c)\sqrt{cx^4+bx^2}}{48\,c^3}\right]$$

input
```
integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*b^3*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*(8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^3, -1/48*(3*b^
3*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (8*c^3*x^4 +
 2*b*c^2*x^2 - 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^3]
```

Sympy [F]

$$\int x^3\sqrt{bx^2+cx^4}\,dx=\int x^3\sqrt{x^2\left(b+cx^2\right)}\,dx$$

input
```
integrate(x**3*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**3*sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

$$\int x^3\sqrt{bx^2+cx^4}\,dx=-\frac{\sqrt{cx^4+bx^2}bx^2}{8\,c}+\frac{b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{32\,c^{\frac{5}{2}}}-\frac{\sqrt{cx^4+bx^2}b^2}{16\,c^2}+\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{6\,c}$$

input
```
integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(c*x^4 + b*x^2)*b*x^2/c + 1/32*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4
 + b*x^2)*sqrt(c))/c^(5/2) - 1/16*sqrt(c*x^4 + b*x^2)*b^2/c^2 + 1/6*(c*x^4
 + b*x^2)^(3/2)/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

$$\int x^3\sqrt{bx^2+cx^4}\,dx = \frac{1}{48}\left(2\left(4\,x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x^2-\frac{3\,b^2\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}x - \frac{b^3\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}} + \frac{b^3\log\left(|b|\right)\mathrm{sgn}(x)}{32\,c^{\frac{5}{2}}}$$

input
```
integrate(x^3*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*x^2*sgn(x) + b*sgn(x)/c)*x^2 - 3*b^2*sgn(x)/c^2)*sqrt(c*x^2 + b
)*x - 1/16*b^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(5/2) + 1/3
2*b^3*log(abs(b))*sgn(x)/c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

$$\int x^3\sqrt{bx^2+cx^4}\,dx = \frac{b^3\ \ln\left(\frac{2\,c\,x^2+b}{\sqrt{c}}+2\,\sqrt{c\,x^4+b\,x^2}\right)}{32\,c^{5/2}} + \frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{48\,c^2}$$

input
```
int(x^3*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
(b^3*log((b + 2*c*x^2)/c^(1/2) + 2*(b*x^2 + c*x^4)^(1/2)))/(32*c^(5/2)) +
((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(48*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int x^3\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{-3\sqrt{c\,x^2+b}\,b^2cx+2\sqrt{c\,x^2+b}\,b\,c^2x^3+8\sqrt{c\,x^2+b}\,c^3x^5+3\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)b^3}{48c^3}$$

input
```
int(x^3*(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - 3*sqrt(b + c*x**2)*b**2*c*x + 2*sqrt(b + c*x**2)*b*c**2*x**3 + 8*sqrt(
b + c*x**2)*c**3*x**5 + 3*sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(
b))*b**3)/(48*c**3)
```

3.155 $\int x\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int x\sqrt{bx^2+cx^4}\,dx=\frac{(b+2cx^2)\,\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

output
```
1/8*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c-1/8*b^2*arctanh(c^(1/2)*x^2/(c*x^4+b
*x^2)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.43

$$\int x\sqrt{bx^2+cx^4}\,dx=\frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}(b+2cx^2)+2b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}-\sqrt{b+cx^2}}\right)\right)}{8c^{3/2}\sqrt{x^2\,(b+cx^2)}}$$

input
```
Integrate[x*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(b + 2*c*x^2) + 2*b^2*ArcTan
h[(Sqrt[c]*x)/(Sqrt[b] - Sqrt[b + c*x^2])]))/(8*c^(3/2)*Sqrt[x^2*(b + c*x^
2)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1424, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \sqrt{cx^4+bx^2}dx^2$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)$$

input
```
Int[x*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/Sq
rt[b*x^2 + c*x^4]])/(4*c^(3/2)))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13

method	result	size
risch	$\frac{(2c\,x^2+b)\sqrt{x^2(c\,x^2+b)}}{8c}-\frac{b^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)\sqrt{x^2(c\,x^2+b)}}{8c^{\frac{3}{2}}x\sqrt{c\,x^2+b}}$	77
default	$\frac{\sqrt{c\,x^4+b\,x^2}\left(2x(c\,x^2+b)^{\frac{3}{2}}\sqrt{c}-\sqrt{c}\sqrt{c\,x^2+b}\,bx-\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)b^2\right)}{8x\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}}$	84
pseudoelliptic	$\frac{4c^{\frac{3}{2}}x^2\sqrt{x^2(c\,x^2+b)}+\ln(2)b^2-\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b^2+2b\sqrt{x^2(c\,x^2+b)}\,\sqrt{c}}{16c^{\frac{3}{2}}}$	89

input
```
int(x*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*c*x^2+b)*(x^2*(c*x^2+b))^(1/2)/c-1/8*b^2/c^(3/2)*ln(c^(1/2)*x+(c*x^
2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.06

$$\int x\sqrt{bx^2+cx^4}\,dx$$
$$= \left[\frac{b^2\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}(2\,c^2x^2+bc)}{16\,c^2}, \frac{b^2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+}{8\,c^2}\right.$$

input
```
integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(b^2*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*s
qrt(c*x^4 + b*x^2)*(2*c^2*x^2 + b*c))/c^2, 1/8*(b^2*sqrt(-c)*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + b*c
))/c^2]
```

Sympy [F]

$$\int x\sqrt{bx^2+cx^4}\,dx = \int x\sqrt{x^2\left(b+cx^2\right)}\,dx$$

input
```
integrate(x*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x*sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int x\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{1}{4}\sqrt{cx^4+bx^2}x^2-\frac{b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{16\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2}b}{8\,c}$$

input
```
integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(c*x^4 + b*x^2)*x^2 - 1/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*
x^2)*sqrt(c))/c^(3/2) + 1/8*sqrt(c*x^4 + b*x^2)*b/c
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int x\sqrt{bx^2+cx^4}\,dx=\frac{1}{8}\sqrt{cx^2+b}\left(2\,x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x$$
$$+\frac{b^2\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\mathrm{sgn}(x)}{8\,c^{\frac{3}{2}}}-\frac{b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}$$

input
```
integrate(x*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(c*x^2 + b)*(2*x^2*sgn(x) + b*sgn(x)/c)*x + 1/8*b^2*log(abs(-sqrt(
c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(3/2) - 1/16*b^2*log(abs(b))*sgn(x)/c^(3
/2)
```

Mupad [B] (verification not implemented)

Time = 18.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int x\sqrt{bx^2+cx^4}\,dx=\frac{\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2}}{2}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{3/2}}$$

input
```
int(x*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
((b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2))/2 - (b^2*log((b/2 + c*x^2)/c^(1/
2) + (b*x^2 + c*x^4)^(1/2)))/(16*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int x\sqrt{bx^2+cx^4}\,dx=\frac{\sqrt{c\,x^2+b}\,bcx+2\sqrt{c\,x^2+b}\,c^2x^3-\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)b^2}{8c^2}$$

input
```
int(x*(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(b + c*x**2)*b*c*x + 2*sqrt(b + c*x**2)*c**2*x**3 - sqrt(c)*log((sqrt
(b + c*x**2) + sqrt(c)*x)/sqrt(b))*b**2)/(8*c**2)
```

3.156 $\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx=\frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

output
```
1/2*(c*x^4+b*x^2)^(1/2)+1/2*b*arctanh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/c^(
1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx=\frac{1}{2}\sqrt{x^2(b+cx^2)}\left(1-\frac{b\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)}{\sqrt{c}x\sqrt{b+cx^2}}\right)$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x,x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(1 - (b*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]])/(Sqrt[
c]*x*Sqrt[b + c*x^2])))/2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1424, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{\sqrt{cx^4+bx^2}}{x^2}dx^2$$

$$\Big\downarrow \text{1131}$$

$$\frac{1}{2}\left(\frac{1}{2}b\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2+\sqrt{bx^2+cx^4}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(b\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}+\sqrt{bx^2+cx^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}+\sqrt{bx^2+cx^4}\right)$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x,x]
```

output
```
(Sqrt[b*x^2 + c*x^4] + (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/Sqrt
[c])/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

method	result	size
pseudoelliptic	$\frac{b \ln\left(\frac{2 c \,x^{2}+2 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \sqrt{c}+b}{\sqrt{c}}\right)-b \ln\left(2\right)+2 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \sqrt{c}}{4 \sqrt{c}}$	63
default	$\frac{\sqrt{c \,x^{4}+b \,x^{2}}\, \left(x \sqrt{c \,x^{2}+b}\, \sqrt{c}+b \ln\left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right)\right)}{2 x \sqrt{c \,x^{2}+b}\, \sqrt{c}}$	64
risch	$\frac{\sqrt{x^{2} \left(c \,x^{2}+b \right)}}{2}+\frac{b \ln\left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{2 \sqrt{c}\, x \sqrt{c \,x^{2}+b}}$	64

input
```
int((c*x^4+b*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1/2))-b*ln(2)+2*(
x^2*(c*x^2+b))^(1/2)*c^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx = \left[\frac{b\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}c}{4\,c}, -\frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}c}{2\,c}\right]$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*(b*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt
(c*x^4 + b*x^2)*c)/c, -1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)
/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*c)/c]
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx = \int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx = \frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{4\,\sqrt{c}} + \frac{1}{2}\,\sqrt{cx^4+bx^2}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 1/2*sqrt(
c*x^4 + b*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx = \frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{4\,\sqrt{c}} + \frac{1}{2}\left(\sqrt{cx^2+b}x - \frac{b\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{\sqrt{c}}\right)\operatorname{sgn}(x)$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/4*b*log(abs(b))*sgn(x)/sqrt(c) + 1/2*(sqrt(c*x^2 + b)*x - b*log(abs(-sqr
t(c)*x + sqrt(c*x^2 + b)))/sqrt(c))*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 18.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}}{2} + \frac{b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,\sqrt{c}}$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x,x)
```

output
```
(b*x^2 + c*x^4)^(1/2)/2 + (b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(
1/2)))/(4*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{bx^2 + cx^4}}{x} \, dx = \frac{\sqrt{c\,x^2 + b}\,cx + \sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)b}{2c}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x,x)
```

output
```
(sqrt(b + c*x**2)*c*x + sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b)
)*b)/(2*c)
```

3.157 $\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx=-\frac{\sqrt{bx^2+cx^4}}{x^2}+\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

output
```
-(c*x^4+b*x^2)^(1/2)/x^2+c^(1/2)*arctanh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx=-\frac{\sqrt{b+cx^2}\left(\sqrt{b+cx^2}+\sqrt{c}x\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

output
```
-((Sqrt[b + c*x^2]*(Sqrt[b + c*x^2] + Sqrt[c]*x*Log[-(Sqrt[c]*x) + Sqrt[b
+ c*x^2]]))/Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1424, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^3} \, dx$$

$$\Big\downarrow \text{ 1424}$$

$$\frac{1}{2} \int \frac{\sqrt{cx^4+bx^2}}{x^4}dx^2$$

$$\Big\downarrow \text{ 1125}$$

$$\frac{1}{2} \left(-\int -\frac{c}{\sqrt{cx^4+bx^2}}dx^2 - \frac{2\sqrt{bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2} \left(\int \frac{c}{\sqrt{cx^4+bx^2}}dx^2 - \frac{2\sqrt{bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2} \left(c\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2 - \frac{2\sqrt{bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{1}{2} \left(2c\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}} - \frac{2\sqrt{bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2} \left(2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right) - \frac{2\sqrt{bx^2+cx^4}}{x^2}\right)$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

output
```
((-2*Sqrt[b*x^2 + c*x^4])/x^2 + 2*Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2
 + c*x^4]])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

method	result	size
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{x^{2}}+\frac{\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{x\sqrt{c\,x^{2}+b}}$	65
pseudoelliptic	$\frac{-\ln\left(2\right)x^{2}\sqrt{c}+\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)x^{2}\sqrt{c}-2\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{2x^{2}}$	70
default	$\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(c^{\frac{3}{2}}\sqrt{c\,x^{2}+b}\,x^{2}-\left(c\,x^{2}+b\right)^{\frac{3}{2}}\sqrt{c}+\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)bcx\right)}{x^{2}\sqrt{c\,x^{2}+b}\,b\sqrt{c}}$	84

input
```
int((c*x^4+b*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-(x^2*(c*x^2+b))^(1/2)/x^2+c^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x
^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx = \left[\frac{\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}}{2\,x^2},\ -\frac{\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}}{x^2}\right]$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sq
rt(c*x^4 + b*x^2))/x^2, -(sqrt(-c)*x^2*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)
/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2))/x^2]
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx = \int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^3}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx = \frac{1}{2}\sqrt{c}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right) - \frac{\sqrt{cx^4+bx^2}}{x^2}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
1/2*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - sqrt(c*x^4
+ b*x^2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx = -\frac{1}{2}\sqrt{c}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\operatorname{sgn}(x) + \frac{2\,b\sqrt{c}\operatorname{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/2*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2*b*sqrt(c)*sgn
(x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^3}\,dx$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^3,x)
```

output
```
int((b*x^2 + c*x^4)^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx=\frac{-\sqrt{c\,x^2+b}+\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)x-\sqrt{c}\,x}{x}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^3,x)
```

output
```
( - sqrt(b + c*x**2) + sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b))
*x - sqrt(c)*x)/x
```

3.158 $\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx=-\frac{\left(bx^2+cx^4\right)^{3/2}}{3bx^6}$$

output
```
-1/3*(c*x^4+b*x^2)^(3/2)/b/x^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx=-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}}{3bx^6}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^5,x]
```

output
```
-1/3*(x^2*(b + c*x^2))^(3/2)/(b*x^6)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx$$

$$\Big\downarrow \text{1422}$$

$$-\frac{\left(bx^2+cx^4\right)^{3/2}}{3bx^6}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^5,x]
```

output
```
-1/3*(b*x^2 + c*x^4)^(3/2)/(b*x^6)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$-\frac{(c x^2+b)\sqrt{c x^4+b x^2}}{3x^4 b}$	29
default	$-\frac{(c x^2+b)\sqrt{c x^4+b x^2}}{3x^4 b}$	29
trager	$-\frac{(c x^2+b)\sqrt{c x^4+b x^2}}{3x^4 b}$	29
risch	$-\frac{\sqrt{x^2(c x^2+b)}\,(c x^2+b)}{3x^4 b}$	29
pseudoelliptic	$-\frac{\sqrt{x^2(c x^2+b)}\,(c x^2+b)}{3x^4 b}$	29
orering	$-\frac{(c x^2+b)\sqrt{c x^4+b x^2}}{3x^4 b}$	29

input
```
int((c*x^4+b*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^4*(c*x^2+b)/b*(c*x^4+b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx=-\frac{\sqrt{cx^4+bx^2}(cx^2+b)}{3\,bx^4}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/(b*x^4)
```

Sympy [F]

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^5} \, dx = \int \frac{\sqrt{x^2 (b + cx^2)}}{x^5} \, dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^5} \, dx = -\frac{\sqrt{cx^4 + bx^2}c}{3\,bx^2} - \frac{\sqrt{cx^4 + bx^2}}{3\,x^4}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/3*sqrt(c*x^4 + b*x^2)*c/(b*x^2) - 1/3*sqrt(c*x^4 + b*x^2)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(21) = 42.

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^5} \, dx = \frac{2 \left(3 \left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^4 c^{\frac{3}{2}} \mathrm{sgn}(x) + b^2 c^{\frac{3}{2}} \mathrm{sgn}(x)\right)}{3 \left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 - b\right)^3}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^4*c^(3/2)*sgn(x) + b^2*c^(3/2)*sgn(x)
)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3
```

Mupad [B] (verification not implemented)

Time = 18.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx = -\frac{(c\,x^2+b)\,\sqrt{c\,x^4+b\,x^2}}{3\,b\,x^4}$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^5,x)
```

output
```
-((b + c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{bx^2+cx^4}}{x^5}\,dx = \frac{-\sqrt{c\,x^2+b}\,b-\sqrt{c\,x^2+b}\,c\,x^2-\sqrt{c}\,c\,x^3}{3b\,x^3}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^5,x)
```

output
```
( - (sqrt(b + c*x**2)*b + sqrt(b + c*x**2)*c*x**2 + sqrt(c)*c*x**3))/(3*b*
x**3)
```

3.159 $\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx = -\frac{(bx^2+cx^4)^{3/2}}{5bx^8}+\frac{2c(bx^2+cx^4)^{3/2}}{15b^2x^6}$$

output
```
-1/5*(c*x^4+b*x^2)^(3/2)/b/x^8+2/15*c*(c*x^4+b*x^2)^(3/2)/b^2/x^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx = \frac{\sqrt{x^2(b+cx^2)}(-3b^2-bcx^2+2c^2x^4)}{15b^2x^6}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^7,x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-3*b^2 - b*c*x^2 + 2*c^2*x^4))/(15*b^2*x^6)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1423, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx$$

$$\Big\downarrow \; 1423$$

$$-\frac{2c\int \frac{\sqrt{cx^4+bx^2}}{x^5}dx}{5b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

$$\Big\downarrow \; 1422$$

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^7,x]
```

output
```
-1/5*(b*x^2 + c*x^4)^(3/2)/(b*x^8) + (2*c*(b*x^2 + c*x^4)^(3/2))/(15*b^2*x
^6)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{(c\,x^2+b)\left(-\frac{2c\,x^2}{3}+b\right)\sqrt{x^2(c\,x^2+b)}}{5b^2x^6}$	37
gosper	$-\frac{(c\,x^2+b)(-2c\,x^2+3b)\sqrt{c\,x^4+b\,x^2}}{15b^2x^6}$	39
default	$-\frac{(c\,x^2+b)(-2c\,x^2+3b)\sqrt{c\,x^4+b\,x^2}}{15b^2x^6}$	39
orering	$-\frac{(c\,x^2+b)(-2c\,x^2+3b)\sqrt{c\,x^4+b\,x^2}}{15b^2x^6}$	39
trager	$-\frac{(-2c^2x^4+bc\,x^2+3b^2)\sqrt{c\,x^4+b\,x^2}}{15b^2x^6}$	42
risch	$-\frac{\sqrt{x^2(c\,x^2+b)}\,(-2c^2x^4+bc\,x^2+3b^2)}{15x^6b^2}$	42

input
```
int((c*x^4+b*x^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(c*x^2+b)*(-2/3*c*x^2+b)*(x^2*(c*x^2+b))^(1/2)/b^2/x^6
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx = \frac{(2\,c^2x^4-bcx^2-3\,b^2)\sqrt{cx^4+bx^2}}{15\,b^2x^6}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
1/15*(2*c^2*x^4 - b*c*x^2 - 3*b^2)*sqrt(c*x^4 + b*x^2)/(b^2*x^6)
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx = \int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^7}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**7,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx = \frac{2\sqrt{cx^4+bx^2}c^2}{15\,b^2x^2} - \frac{\sqrt{cx^4+bx^2}c}{15\,bx^4} - \frac{\sqrt{cx^4+bx^2}}{5\,x^6}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
2/15*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^2) - 1/15*sqrt(c*x^4 + b*x^2)*c/(b*x^4
) - 1/5*sqrt(c*x^4 + b*x^2)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(44) = 88.

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.31

$$\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx = \frac{4\left(15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6c^{\frac{5}{2}}\mathrm{sgn}(x)+5\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4bc^{\frac{5}{2}}\mathrm{sgn}(x)+5\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2b^2c^{\frac{5}{2}}\mathrm{sgn}(x)}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^5}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="giac")
```

output
```
4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^6*c^(5/2)*sgn(x) + 5*(sqrt(c)*x - s
qrt(c*x^2 + b))^4*b*c^(5/2)*sgn(x) + 5*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^2
*c^(5/2)*sgn(x) - b^3*c^(5/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b
)^5
```

Mupad [B] (verification not implemented)

Time = 18.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,\left(3\,b^2+b\,c\,x^2-2\,c^2\,x^4\right)}{15\,b^2\,x^6}$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^7,x)
```

output
```
-((b*x^2 + c*x^4)^(1/2)*(3*b^2 - 2*c^2*x^4 + b*c*x^2))/(15*b^2*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{bx^2+cx^4}}{x^7}\,dx = \frac{-3\sqrt{c\,x^2+b}\,b^2-\sqrt{c\,x^2+b}\,bc\,x^2+2\sqrt{c\,x^2+b}\,c^2x^4-2\sqrt{c}\,c^2x^5}{15b^2x^5}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^7,x)
```

output
```
( - 3*sqrt(b + c*x**2)*b**2 - sqrt(b + c*x**2)*b*c*x**2 + 2*sqrt(b + c*x**
2)*c**2*x**4 - 2*sqrt(c)*c**2*x**5)/(15*b**2*x**5)
```

3.160 $\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx = -\frac{(bx^2+cx^4)^{3/2}}{7bx^{10}}+\frac{4c(bx^2+cx^4)^{3/2}}{35b^2x^8}-\frac{8c^2(bx^2+cx^4)^{3/2}}{105b^3x^6}$$

output
```
-1/7*(c*x^4+b*x^2)^(3/2)/b/x^10+4/35*c*(c*x^4+b*x^2)^(3/2)/b^2/x^8-8/105*c
^2*(c*x^4+b*x^2)^(3/2)/b^3/x^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{\sqrt{x^2(b+cx^2)}(-15b^3-3b^2cx^2+4bc^2x^4-8c^3x^6)}{105b^3x^8}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-15*b^3 - 3*b^2*c*x^2 + 4*b*c^2*x^4 - 8*c^3*x^6))/
(105*b^3*x^8)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1423, 1423, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^9}dx$$

$$\Big\downarrow \text{1423}$$

$$-\frac{4c\int \frac{\sqrt{cx^4+bx^2}}{x^7}dx}{7b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

$$\Big\downarrow \text{1423}$$

$$-\frac{4c\left(-\frac{2c\int \frac{\sqrt{cx^4+bx^2}}{x^5}dx}{5b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}\right)}{7b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

$$\Big\downarrow \text{1422}$$

$$-\frac{4c\left(\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}\right)}{7b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

output
```
-1/7*(b*x^2 + c*x^4)^(3/2)/(b*x^10) - (4*c*(-1/5*(b*x^2 + c*x^4)^(3/2)/(b*
x^8) + (2*c*(b*x^2 + c*x^4)^(3/2))/(15*b^2*x^6)))/(7*b)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$-\frac{\left(\frac{8}{15}c^{2}x^{4}-\frac{4}{5}bc\,x^{2}+b^{2}\right)\left(c\,x^{2}+b\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{7x^{8}b^{3}}$	48
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(8c^{2}x^{4}-12bc\,x^{2}+15b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{105x^{8}b^{3}}$	50
default	$-\frac{\left(c\,x^{2}+b\right)\left(8c^{2}x^{4}-12bc\,x^{2}+15b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{105x^{8}b^{3}}$	50
orering	$-\frac{\left(c\,x^{2}+b\right)\left(8c^{2}x^{4}-12bc\,x^{2}+15b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{105x^{8}b^{3}}$	50
trager	$-\frac{\left(8c^{3}x^{6}-4b\,c^{2}x^{4}+3b^{2}c\,x^{2}+15b^{3}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{105x^{8}b^{3}}$	54
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(8c^{3}x^{6}-4b\,c^{2}x^{4}+3b^{2}c\,x^{2}+15b^{3}\right)}{105x^{8}b^{3}}$	54

input
```
int((c*x^4+b*x^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/7*(8/15*c^2*x^4-4/5*b*c*x^2+b^2)*(c*x^2+b)*(x^2*(c*x^2+b))^(1/2)/x^8/b^
3
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx = -\frac{(8\,c^3x^6-4\,bc^2x^4+3\,b^2cx^2+15\,b^3)\sqrt{cx^4+bx^2}}{105\,b^3x^8}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
-1/105*(8*c^3*x^6 - 4*b*c^2*x^4 + 3*b^2*c*x^2 + 15*b^3)*sqrt(c*x^4 + b*x^2
)/(b^3*x^8)
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}}{x^9}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**9,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**9, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx = -\frac{8\,\sqrt{cx^4+bx^2}c^3}{105\,b^3x^2}+\frac{4\,\sqrt{cx^4+bx^2}c^2}{105\,b^2x^4}-\frac{\sqrt{cx^4+bx^2}c}{35\,bx^6}-\frac{\sqrt{cx^4+bx^2}}{7\,x^8}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-8/105*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^2) + 4/105*sqrt(c*x^4 + b*x^2)*c^2/(
b^2*x^4) - 1/35*sqrt(c*x^4 + b*x^2)*c/(b*x^6) - 1/7*sqrt(c*x^4 + b*x^2)/x^
8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(68) = 136$.

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{16\left(70\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8 c^{\frac{7}{2}}\operatorname{sgn}(x)+35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6 bc^{\frac{7}{2}}\operatorname{sgn}(x)+21\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4 b^2c^{\frac{7}{2}}\operatorname{sgn}}{105\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^7}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="giac")
```

output
```
16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*c^(7/2)*sgn(x) + 35*(sqrt(c)*x
- sqrt(c*x^2 + b))^6*b*c^(7/2)*sgn(x) + 21*(sqrt(c)*x - sqrt(c*x^2 + b))^4
*b^2*c^(7/2)*sgn(x) - 7*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^3*c^(7/2)*sgn(x)
 + b^4*c^(7/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

Mupad [B] (verification not implemented)

Time = 18.67 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{4\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^4}-\frac{c\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^6}-\frac{\sqrt{c\,x^4+b\,x^2}}{7\,x^8}-\frac{8\,c^3\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^2}$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^9,x)
```

output
```
(4*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (c*(b*x^2 + c*x^4)^(1/2))/(3
5*b*x^6) - (b*x^2 + c*x^4)^(1/2)/(7*x^8) - (8*c^3*(b*x^2 + c*x^4)^(1/2))/(
105*b^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{bx^2+cx^4}}{x^9}\,dx$$
$$= \frac{-15\sqrt{c\,x^2+b}\,b^3-3\sqrt{c\,x^2+b}\,b^2c\,x^2+4\sqrt{c\,x^2+b}\,b\,c^2x^4-8\sqrt{c\,x^2+b}\,c^3x^6+8\sqrt{c}\,c^3x^7}{105b^3x^7}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^9,x)
```

output
```
( - 15*sqrt(b + c*x**2)*b**3 - 3*sqrt(b + c*x**2)*b**2*c*x**2 + 4*sqrt(b +
 c*x**2)*b*c**2*x**4 - 8*sqrt(b + c*x**2)*c**3*x**6 + 8*sqrt(c)*c**3*x**7)
/(105*b**3*x**7)
```

3.161 $\int \frac{\sqrt{bx^2+cx^4}}{x^{11}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11}} \, dx = -\frac{(bx^2+cx^4)^{3/2}}{9bx^{12}} + \frac{2c(bx^2+cx^4)^{3/2}}{21b^2x^{10}} - \frac{8c^2(bx^2+cx^4)^{3/2}}{105b^3x^8} + \frac{16c^3(bx^2+cx^4)^{3/2}}{315b^4x^6}$$

output
```
-1/9*(c*x^4+b*x^2)^(3/2)/b/x^12+2/21*c*(c*x^4+b*x^2)^(3/2)/b^2/x^10-8/105*
c^2*(c*x^4+b*x^2)^(3/2)/b^3/x^8+16/315*c^3*(c*x^4+b*x^2)^(3/2)/b^4/x^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11}} \, dx = \frac{\sqrt{x^2(b+cx^2)}(-35b^4-5b^3cx^2+6b^2c^2x^4-8bc^3x^6+16c^4x^8)}{315b^4x^{10}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^11,x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-35*b^4 - 5*b^3*c*x^2 + 6*b^2*c^2*x^4 - 8*b*c^3*x^
6 + 16*c^4*x^8))/(315*b^4*x^10)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1423, 1423, 1423, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx$$

$$\Big\downarrow \text{ 1423}$$

$$-\frac{2c\int \frac{\sqrt{cx^4+bx^2}}{x^9}dx}{3b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

$$\Big\downarrow \text{ 1423}$$

$$-\frac{2c\left(-\frac{4c\int \frac{\sqrt{cx^4+bx^2}}{x^7}dx}{7b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}\right)}{3b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

$$\Big\downarrow \text{ 1423}$$

$$-\frac{2c\left(-\frac{4c\left(-\frac{2c\int \frac{\sqrt{cx^4+bx^2}}{x^5}dx}{5b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}\right)}{7b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}\right)}{3b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

$$\Big\downarrow \text{ 1422}$$

$$-\frac{2c\left(-\frac{4c\left(\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}\right)}{7b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}\right)}{3b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^11,x]
```

output
```
-1/9*(b*x^2 + c*x^4)^(3/2)/(b*x^12) - (2*c*(-1/7*(b*x^2 + c*x^4)^(3/2)/(b*
x^10) - (4*c*(-1/5*(b*x^2 + c*x^4)^(3/2)/(b*x^8) + (2*c*(b*x^2 + c*x^4)^(3
/2))/(15*b^2*x^6)))/(7*b)))/(3*b)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-16 c^{3} x^{6}+24 b\,c^{2} x^{4}-30 b^{2} c\,x^{2}+35 b^{3}\right) \sqrt{c\,x^{4}+b\,x^{2}}}{315 x^{10} b^{4}}$	61
default	$-\frac{\left(c\,x^{2}+b\right)\left(-16 c^{3} x^{6}+24 b\,c^{2} x^{4}-30 b^{2} c\,x^{2}+35 b^{3}\right) \sqrt{c\,x^{4}+b\,x^{2}}}{315 x^{10} b^{4}}$	61
orering	$-\frac{\left(c\,x^{2}+b\right)\left(-16 c^{3} x^{6}+24 b\,c^{2} x^{4}-30 b^{2} c\,x^{2}+35 b^{3}\right) \sqrt{c\,x^{4}+b\,x^{2}}}{315 x^{10} b^{4}}$	61
trager	$-\frac{\left(-16 c^{4} x^{8}+8 b\,c^{3} x^{6}-6 b^{2} c^{2} x^{4}+5 x^{2} b^{3} c+35 b^{4}\right) \sqrt{c\,x^{4}+b\,x^{2}}}{315 x^{10} b^{4}}$	65
risch	$-\frac{\sqrt{x^{2} \left(c\,x^{2}+b\right)}\,\left(-16 c^{4} x^{8}+8 b\,c^{3} x^{6}-6 b^{2} c^{2} x^{4}+5 x^{2} b^{3} c+35 b^{4}\right)}{315 x^{10} b^{4}}$	65
pseudoelliptic	$\frac{\left(16 c^{4} x^{8}-8 b\,c^{3} x^{6}+6 b^{2} c^{2} x^{4}-5 x^{2} b^{3} c-35 b^{4}\right) \sqrt{x^{2} \left(c\,x^{2}+b\right)}}{315 x^{10} b^{4}}$	65

input
```
int((c*x^4+b*x^2)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(c*x^2+b)*(-16*c^3*x^6+24*b*c^2*x^4-30*b^2*c*x^2+35*b^3)*(c*x^4+b*x
^2)^(1/2)/x^10/b^4
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx = \frac{\left(16\,c^4x^8 - 8\,bc^3x^6 + 6\,b^2c^2x^4 - 5\,b^3cx^2 - 35\,b^4\right)\sqrt{cx^4+bx^2}}{315\,b^4x^{10}}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="fricas")
```

output
```
1/315*(16*c^4*x^8 - 8*b*c^3*x^6 + 6*b^2*c^2*x^4 - 5*b^3*c*x^2 - 35*b^4)*sq
rt(c*x^4 + b*x^2)/(b^4*x^10)
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}}{x^{11}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**11,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**11, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx = \frac{16\,\sqrt{cx^4+bx^2}c^4}{315\,b^4x^2} - \frac{8\,\sqrt{cx^4+bx^2}c^3}{315\,b^3x^4} + \frac{2\,\sqrt{cx^4+bx^2}c^2}{105\,b^2x^6} - \frac{\sqrt{cx^4+bx^2}c}{63\,bx^8} - \frac{\sqrt{cx^4+bx^2}}{9\,x^{10}}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="maxima")
```

output
```
16/315*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^2) - 8/315*sqrt(c*x^4 + b*x^2)*c^3/(
b^3*x^4) + 2/105*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^6) - 1/63*sqrt(c*x^4 + b*x
^2)*c/(b*x^8) - 1/9*sqrt(c*x^4 + b*x^2)/x^10
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx = \frac{32\left(315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}c^{\frac{9}{2}}\operatorname{sgn}(x)+189\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}bc^{\frac{9}{2}}\operatorname{sgn}(x)+84\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}b^2c^{\frac{9}{2}}\right.}{315\left(\left(\sqrt{c}x-\sqrt{cx^2+\right.\right.}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="giac")
```

output
```
32/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*c^(9/2)*sgn(x) + 189*(sqrt(c)
*x - sqrt(c*x^2 + b))^8*b*c^(9/2)*sgn(x) + 84*(sqrt(c)*x - sqrt(c*x^2 + b)
)^6*b^2*c^(9/2)*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^3*c^(9/2)*sg
n(x) + 9*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^4*c^(9/2)*sgn(x) - b^5*c^(9/2)*
sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^9
```

Mupad [B] (verification not implemented)

Time = 18.82 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx = \frac{2\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^6}-\frac{c\,\sqrt{c\,x^4+b\,x^2}}{63\,b\,x^8}-\frac{\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{8\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^4}+\frac{16\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^4\,x^2}$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^11,x)
```

output
```
(2*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^6) - (c*(b*x^2 + c*x^4)^(1/2))/(6
3*b*x^8) - (b*x^2 + c*x^4)^(1/2)/(9*x^10) - (8*c^3*(b*x^2 + c*x^4)^(1/2))/
(315*b^3*x^4) + (16*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx$$
$$= \frac{-35\sqrt{c\,x^2+b}\,b^4 - 5\sqrt{c\,x^2+b}\,b^3c\,x^2 + 6\sqrt{c\,x^2+b}\,b^2c^2x^4 - 8\sqrt{c\,x^2+b}\,b\,c^3x^6 + 16\sqrt{c\,x^2+b}\,c^4x^8 - 16\sqrt{c}}{315b^4x^9}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^11,x)
```

output
```
( - 35*sqrt(b + c*x**2)*b**4 - 5*sqrt(b + c*x**2)*b**3*c*x**2 + 6*sqrt(b +
 c*x**2)*b**2*c**2*x**4 - 8*sqrt(b + c*x**2)*b*c**3*x**6 + 16*sqrt(b + c*x
**2)*c**4*x**8 - 16*sqrt(c)*c**4*x**9)/(315*b**4*x**9)
```

3.162 $\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx = -\frac{(bx^2+cx^4)^{3/2}}{11bx^{14}}+\frac{8c(bx^2+cx^4)^{3/2}}{99b^2x^{12}}-\frac{16c^2(bx^2+cx^4)^{3/2}}{231b^3x^{10}}+\frac{64c^3(bx^2+cx^4)^{3/2}}{1155b^4x^8}-\frac{128c^4(bx^2+cx^4)^{3/2}}{3465b^5x^6}$$

output
```
-1/11*(c*x^4+b*x^2)^(3/2)/b/x^14+8/99*c*(c*x^4+b*x^2)^(3/2)/b^2/x^12-16/23
1*c^2*(c*x^4+b*x^2)^(3/2)/b^3/x^10+64/1155*c^3*(c*x^4+b*x^2)^(3/2)/b^4/x^8
-128/3465*c^4*(c*x^4+b*x^2)^(3/2)/b^5/x^6
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx = \frac{\sqrt{x^2(b+cx^2)}(-315b^5-35b^4cx^2+40b^3c^2x^4-48b^2c^3x^6+64bc^4x^8-128c^5x^{10})}{3465b^5x^{12}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^13,x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-315*b^5 - 35*b^4*c*x^2 + 40*b^3*c^2*x^4 - 48*b^2*
c^3*x^6 + 64*b*c^4*x^8 - 128*c^5*x^10))/(3465*b^5*x^12)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1423, 1423, 1423, 1423, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}dx$$

$$\Big\downarrow \text{1423}$$

$$-\frac{8c\int \frac{\sqrt{cx^4+bx^2}}{x^{11}}dx}{11b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

$$\Big\downarrow \text{1423}$$

$$-\frac{8c\left(-\frac{2c\int \frac{\sqrt{cx^4+bx^2}}{x^9}dx}{3b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}\right)}{11b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

$$\Big\downarrow \text{1423}$$

$$-\frac{8c\left(-\frac{2c\left(-\frac{4c\int \frac{\sqrt{cx^4+bx^2}}{x^7}dx}{7b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}\right)}{3b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}\right)}{11b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

$$\Big\downarrow \text{1423}$$

$$-\frac{8c\left(-\frac{2c\left(-\frac{4c\left(-\frac{2c\int\frac{\sqrt{cx^4+bx^2}}{x^5}dx}{5b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}\right)}{7b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}\right)}{3b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}\right)}{11b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

$$\Big\downarrow \text{1422}$$

$$-\frac{8c\left(-\frac{2c\left(-\frac{4c\left(\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}\right)}{7b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}\right)}{3b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}\right)}{11b}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^13,x]
```

output
```
-1/11*(b*x^2 + c*x^4)^(3/2)/(b*x^14) - (8*c*(-1/9*(b*x^2 + c*x^4)^(3/2)/(b
*x^12) - (2*c*(-1/7*(b*x^2 + c*x^4)^(3/2)/(b*x^10) - (4*c*(-1/5*(b*x^2 + c
*x^4)^(3/2)/(b*x^8) + (2*c*(b*x^2 + c*x^4)^(3/2))/(15*b^2*x^6)))/(7*b)))/(
3*b)))/(11*b)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(128c^{4}x^{8}-192b\,c^{3}x^{6}+240b^{2}c^{2}x^{4}-280x^{2}b^{3}c+315b^{4}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{3465x^{12}b^{5}}$	72
default	$-\frac{\left(c\,x^{2}+b\right)\left(128c^{4}x^{8}-192b\,c^{3}x^{6}+240b^{2}c^{2}x^{4}-280x^{2}b^{3}c+315b^{4}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{3465x^{12}b^{5}}$	72
orering	$-\frac{\left(c\,x^{2}+b\right)\left(128c^{4}x^{8}-192b\,c^{3}x^{6}+240b^{2}c^{2}x^{4}-280x^{2}b^{3}c+315b^{4}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{3465x^{12}b^{5}}$	72
trager	$-\frac{\left(128c^{5}x^{10}-64c^{4}x^{8}b+48b^{2}c^{3}x^{6}-40c^{2}x^{4}b^{3}+35x^{2}c\,b^{4}+315b^{5}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{3465x^{12}b^{5}}$	76
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(128c^{5}x^{10}-64c^{4}x^{8}b+48b^{2}c^{3}x^{6}-40c^{2}x^{4}b^{3}+35x^{2}c\,b^{4}+315b^{5}\right)}{3465x^{12}b^{5}}$	76
pseudoelliptic	$\frac{\left(-128c^{5}x^{10}+64c^{4}x^{8}b-48b^{2}c^{3}x^{6}+40c^{2}x^{4}b^{3}-35x^{2}c\,b^{4}-315b^{5}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{3465x^{12}b^{5}}$	76

input
```
int((c*x^4+b*x^2)^(1/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/3465*(c*x^2+b)*(128*c^4*x^8-192*b*c^3*x^6+240*b^2*c^2*x^4-280*b^3*c*x^2
+315*b^4)*(c*x^4+b*x^2)^(1/2)/x^12/b^5
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx = -\frac{(128\,c^5x^{10}-64\,bc^4x^8+48\,b^2c^3x^6-40\,b^3c^2x^4+35\,b^4cx^2+315\,b^5)\sqrt{cx^4+bx^2}}{3465\,b^5x^{12}}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="fricas")
```

output
```
-1/3465*(128*c^5*x^10 - 64*b*c^4*x^8 + 48*b^2*c^3*x^6 - 40*b^3*c^2*x^4 + 3
5*b^4*c*x^2 + 315*b^5)*sqrt(c*x^4 + b*x^2)/(b^5*x^12)
```

Sympy [F]

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^{13}}\,dx = \int \frac{\sqrt{x^2 (b + cx^2)}}{x^{13}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**13,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**13, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^{13}}\,dx = -\frac{128\,\sqrt{cx^4 + bx^2}c^5}{3465\,b^5x^2} + \frac{64\,\sqrt{cx^4 + bx^2}c^4}{3465\,b^4x^4} - \frac{16\,\sqrt{cx^4 + bx^2}c^3}{1155\,b^3x^6} + \frac{8\,\sqrt{cx^4 + bx^2}c^2}{693\,b^2x^8} - \frac{\sqrt{cx^4 + bx^2}c}{99\,bx^{10}} - \frac{\sqrt{cx^4 + bx^2}}{11\,x^{12}}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="maxima")
```

output
```
-128/3465*sqrt(c*x^4 + b*x^2)*c^5/(b^5*x^2) + 64/3465*sqrt(c*x^4 + b*x^2)*
c^4/(b^4*x^4) - 16/1155*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^6) + 8/693*sqrt(c*x
^4 + b*x^2)*c^2/(b^2*x^8) - 1/99*sqrt(c*x^4 + b*x^2)*c/(b*x^10) - 1/11*sqr
t(c*x^4 + b*x^2)/x^12
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx = \frac{256\left(1386\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}c^{\frac{11}{2}}\operatorname{sgn}(x)+924\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{11}{2}}\operatorname{sgn}(x)+330\left(\sqrt{c}x-\sqrt{cx^2+b}\right)\right.}{}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="giac")
```

output
```
256/3465*(1386*(sqrt(c)*x - sqrt(c*x^2 + b))^12*c^(11/2)*sgn(x) + 924*(sqr
t(c)*x - sqrt(c*x^2 + b))^10*b*c^(11/2)*sgn(x) + 330*(sqrt(c)*x - sqrt(c*x
^2 + b))^8*b^2*c^(11/2)*sgn(x) - 165*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^3*c
^(11/2)*sgn(x) + 55*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^4*c^(11/2)*sgn(x) -
11*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^5*c^(11/2)*sgn(x) + b^6*c^(11/2)*sgn(
x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^11
```

Mupad [B] (verification not implemented)

Time = 19.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx = \frac{8\,c^2\,\sqrt{c\,x^4+b\,x^2}}{693\,b^2\,x^8} - \frac{c\,\sqrt{c\,x^4+b\,x^2}}{99\,b\,x^{10}} - \frac{\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}} - \frac{16\,c^3\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^6} + \frac{64\,c^4\,\sqrt{c\,x^4+b\,x^2}}{3465\,b^4\,x^4} - \frac{128\,c^5\,\sqrt{c\,x^4+b\,x^2}}{3465\,b^5\,x^2}$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^13,x)
```

output
```
(8*c^2*(b*x^2 + c*x^4)^(1/2))/(693*b^2*x^8) - (c*(b*x^2 + c*x^4)^(1/2))/(9
9*b*x^10) - (b*x^2 + c*x^4)^(1/2)/(11*x^12) - (16*c^3*(b*x^2 + c*x^4)^(1/2
))/(1155*b^3*x^6) + (64*c^4*(b*x^2 + c*x^4)^(1/2))/(3465*b^4*x^4) - (128*c
^5*(b*x^2 + c*x^4)^(1/2))/(3465*b^5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx$$
$$= \frac{-315\sqrt{c\,x^2+b}\,b^5-35\sqrt{c\,x^2+b}\,b^4c\,x^2+40\sqrt{c\,x^2+b}\,b^3c^2x^4-48\sqrt{c\,x^2+b}\,b^2c^3x^6+64\sqrt{c\,x^2+b}\,b\,c^4x^8}{3465b^5x^{11}}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^13,x)
```

output
```
( - 315*sqrt(b + c*x**2)*b**5 - 35*sqrt(b + c*x**2)*b**4*c*x**2 + 40*sqrt(
b + c*x**2)*b**3*c**2*x**4 - 48*sqrt(b + c*x**2)*b**2*c**3*x**6 + 64*sqrt(
b + c*x**2)*b*c**4*x**8 - 128*sqrt(b + c*x**2)*c**5*x**10 + 128*sqrt(c)*c*
*5*x**11)/(3465*b**5*x**11)
```

3.163 $\int x^4\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int x^4\sqrt{bx^2+cx^4}\,dx=\frac{8b^2(bx^2+cx^4)^{3/2}}{105c^3x^3}-\frac{4b(bx^2+cx^4)^{3/2}}{35c^2x}+\frac{x(bx^2+cx^4)^{3/2}}{7c}$$

output
```
8/105*b^2*(c*x^4+b*x^2)^(3/2)/c^3/x^3-4/35*b*(c*x^4+b*x^2)^(3/2)/c^2/x+1/7
*x*(c*x^4+b*x^2)^(3/2)/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int x^4\sqrt{bx^2+cx^4}\,dx=\frac{\sqrt{x^2(b+cx^2)}(8b^3-4b^2cx^2+3bc^2x^4+15c^3x^6)}{105c^3x}$$

input
```
Integrate[x^4*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(8*b^3 - 4*b^2*c*x^2 + 3*b*c^2*x^4 + 15*c^3*x^6))/(
105*c^3*x)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1421, 1421, 1398}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1421}$$

$$\frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}-\frac{4b\int x^2\sqrt{cx^4+bx^2}dx}{7c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}-\frac{4b\left(\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{2b\int\sqrt{cx^4+bx^2}dx}{5c}\right)}{7c}$$

$$\Big\downarrow \text{1398}$$

$$\frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}-\frac{4b\left(\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{2b\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}\right)}{7c}$$

input
```
Int[x^4*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*(b*x^2 + c*x^4)^(3/2))/(7*c) - (4*b*((-2*b*(b*x^2 + c*x^4)^(3/2))/(15*c
^2*x^3) + (b*x^2 + c*x^4)^(3/2)/(5*c*x)))/(7*c)
```

Defintions of rubi rules used

rule 1398
```
Int[Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> Simp[(b*x^2 + c*x^4)^(3
/2)/(3*c*x^3), x] /; FreeQ[{b, c}, x]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\left(c\,x^{2}+b\right)\left(15c^{2}x^{4}-12bc\,x^{2}+8b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{105c^{3}x}$	50
default	$\frac{\left(c\,x^{2}+b\right)\left(15c^{2}x^{4}-12bc\,x^{2}+8b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{105c^{3}x}$	50
orering	$\frac{\left(c\,x^{2}+b\right)\left(15c^{2}x^{4}-12bc\,x^{2}+8b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{105c^{3}x}$	50
trager	$\frac{\left(15c^{3}x^{6}+3b\,c^{2}x^{4}-4b^{2}c\,x^{2}+8b^{3}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{105c^{3}x}$	54
risch	$\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(15c^{3}x^{6}+3b\,c^{2}x^{4}-4b^{2}c\,x^{2}+8b^{3}\right)}{105x\,c^{3}}$	54

input
```
int(x^4*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(c*x^2+b)*(15*c^2*x^4-12*b*c*x^2+8*b^2)*(c*x^4+b*x^2)^(1/2)/c^3/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68

$$\int x^4\sqrt{bx^2+cx^4}\,dx = \frac{(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3)\sqrt{cx^4+bx^2}}{105\,c^3x}$$

input
```
integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2)
/(c^3*x)
```

Sympy [F]

$$\int x^4\sqrt{bx^2+cx^4}\,dx = \int x^4\sqrt{x^2\,(b+cx^2)}\,dx$$

input
```
integrate(x**4*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**4*sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59

$$\int x^4\sqrt{bx^2+cx^4}\,dx = \frac{(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3)\sqrt{cx^2+b}}{105\,c^3}$$

input
```
integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)/c^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

$$\int x^4\sqrt{bx^2+cx^4}\,dx$$
$$= -\frac{8\,b^{\frac{7}{2}}\mathrm{sgn}(x)}{105\,c^3} + \frac{15\,(cx^2+b)^{\frac{7}{2}}\mathrm{sgn}(x) - 42\,(cx^2+b)^{\frac{5}{2}}b\mathrm{sgn}(x) + 35\,(cx^2+b)^{\frac{3}{2}}b^2\mathrm{sgn}(x)}{105\,c^3}$$

input
```
integrate(x^4*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-8/105*b^(7/2)*sgn(x)/c^3 + 1/105*(15*(c*x^2 + b)^(7/2)*sgn(x) - 42*(c*x^2
 + b)^(5/2)*b*sgn(x) + 35*(c*x^2 + b)^(3/2)*b^2*sgn(x))/c^3
```

Mupad [B] (verification not implemented)

Time = 17.94 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68

$$\int x^4\sqrt{bx^2+cx^4}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,(8\,b^3-4\,b^2\,c\,x^2+3\,b\,c^2\,x^4+15\,c^3\,x^6)}{105\,c^3\,x}$$

input
```
int(x^4*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*(8*b^3 + 15*c^3*x^6 - 4*b^2*c*x^2 + 3*b*c^2*x^4))/(
105*c^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.58

$$\int x^4\sqrt{bx^2+cx^4}\,dx = \frac{\sqrt{c\,x^2+b}\,(15c^3x^6+3b\,c^2x^4-4b^2c\,x^2+8b^3)}{105c^3}$$

input
```
int(x^4*(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(b + c*x**2)*(8*b**3 - 4*b**2*c*x**2 + 3*b*c**2*x**4 + 15*c**3*x**6))
/(105*c**3)
```

3.164 $\int x^2\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int x^2\sqrt{bx^2+cx^4}\,dx=-\frac{2b(bx^2+cx^4)^{3/2}}{15c^2x^3}+\frac{(bx^2+cx^4)^{3/2}}{5cx}$$

output
```
-2/15*b*(c*x^4+b*x^2)^(3/2)/c^2/x^3+1/5*(c*x^4+b*x^2)^(3/2)/c/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int x^2\sqrt{bx^2+cx^4}\,dx=\frac{\sqrt{x^2(b+cx^2)}(-2b^2+bcx^2+3c^2x^4)}{15c^2x}$$

input
```
Integrate[x^2*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-2*b^2 + b*c*x^2 + 3*c^2*x^4))/(15*c^2*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1421, 1398}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{bx^2 + cx^4} \, dx$$

$$\Big\downarrow \text{1421}$$

$$\frac{\left(bx^2 + cx^4\right)^{3/2}}{5cx} - \frac{2b \int \sqrt{cx^4 + bx^2} dx}{5c}$$

$$\Big\downarrow \text{1398}$$

$$\frac{\left(bx^2 + cx^4\right)^{3/2}}{5cx} - \frac{2b\left(bx^2 + cx^4\right)^{3/2}}{15c^2x^3}$$

input
```
Int[x^2*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(-2*b*(b*x^2 + c*x^4)^(3/2))/(15*c^2*x^3) + (b*x^2 + c*x^4)^(3/2)/(5*c*x)
```

Defintions of rubi rules used

rule 1398
```
Int[Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> Simp[(b*x^2 + c*x^4)^(3
/2)/(3*c*x^3), x] /; FreeQ[{b, c}, x]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{(c\,x^{2}+b)\,(-3c\,x^{2}+2b)\,\sqrt{c\,x^{4}+b\,x^{2}}}{15c^{2}x}$	39
default	$-\frac{(c\,x^{2}+b)\,(-3c\,x^{2}+2b)\,\sqrt{c\,x^{4}+b\,x^{2}}}{15c^{2}x}$	39
orering	$-\frac{(c\,x^{2}+b)\,(-3c\,x^{2}+2b)\,\sqrt{c\,x^{4}+b\,x^{2}}}{15c^{2}x}$	39
trager	$-\frac{(-3c^{2}x^{4}-bc\,x^{2}+2b^{2})\,\sqrt{c\,x^{4}+b\,x^{2}}}{15c^{2}x}$	43
risch	$-\frac{\sqrt{x^{2}(c\,x^{2}+b)}\,(-3c^{2}x^{4}-bc\,x^{2}+2b^{2})}{15x\,c^{2}}$	43

input
```
int(x^2*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(c*x^2+b)*(-3*c*x^2+2*b)*(c*x^4+b*x^2)^(1/2)/c^2/x
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int x^2\sqrt{bx^2+cx^4}\,dx=\frac{(3\,c^2x^4+bcx^2-2\,b^2)\sqrt{cx^4+bx^2}}{15\,c^2x}$$

input
```
integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^4 + b*x^2)/(c^2*x)
```

Sympy [F]

$$\int x^2\sqrt{bx^2+cx^4}\,dx = \int x^2\sqrt{x^2\left(b+cx^2\right)}\,dx$$

input
```
integrate(x**2*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int x^2\sqrt{bx^2+cx^4}\,dx = \frac{\left(3\,c^2x^4+bcx^2-2\,b^2\right)\sqrt{cx^2+b}}{15\,c^2}$$

input
```
integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 + b)/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int x^2\sqrt{bx^2+cx^4}\,dx = \frac{2\,b^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,c^2} + \frac{3\,\left(cx^2+b\right)^{\frac{5}{2}}\mathrm{sgn}(x)-5\,\left(cx^2+b\right)^{\frac{3}{2}}b\mathrm{sgn}(x)}{15\,c^2}$$

input
```
integrate(x^2*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/15*b^(5/2)*sgn(x)/c^2 + 1/15*(3*(c*x^2 + b)^(5/2)*sgn(x) - 5*(c*x^2 + b)
^(3/2)*b*sgn(x))/c^2
```

Mupad [B] (verification not implemented)

Time = 17.92 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int x^2\sqrt{bx^2+cx^4}\,dx=\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-2\,b^2+b\,c\,x^2+3\,c^2\,x^4\right)}{15\,c^2\,x}$$

input
```
int(x^2*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*(3*c^2*x^4 - 2*b^2 + b*c*x^2))/(15*c^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

$$\int x^2\sqrt{bx^2+cx^4}\,dx=\frac{\sqrt{c\,x^2+b}\,(3c^2x^4+bc\,x^2-2b^2)}{15c^2}$$

input
```
int(x^2*(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(b + c*x**2)*( - 2*b**2 + b*c*x**2 + 3*c**2*x**4))/(15*c**2)
```

3.165 $\int \sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \sqrt{bx^2+cx^4}\,dx=\frac{(bx^2+cx^4)^{3/2}}{3cx^3}$$

output
```
1/3*(c*x^4+b*x^2)^(3/2)/c/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{bx^2+cx^4}\,dx=\frac{(x^2(b+cx^2))^{3/2}}{3cx^3}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x^2*(b + c*x^2))^(3/2)/(3*c*x^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1398}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1398}$$

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

input
```
Int[Sqrt[b*x^2 + c*x^4],x]
```

output
```
(b*x^2 + c*x^4)^(3/2)/(3*c*x^3)
```

Defintions of rubi rules used

rule 1398
```
Int[Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> Simp[(b*x^2 + c*x^4)^(3
/2)/(3*c*x^3), x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{(c\,x^2+b)\sqrt{c\,x^4+b\,x^2}}{3cx}$	29
default	$\frac{(c\,x^2+b)\sqrt{c\,x^4+b\,x^2}}{3cx}$	29
trager	$\frac{(c\,x^2+b)\sqrt{c\,x^4+b\,x^2}}{3cx}$	29
risch	$\frac{\sqrt{x^2(c\,x^2+b)}\,(c\,x^2+b)}{3xc}$	29
orering	$\frac{(c\,x^2+b)\sqrt{c\,x^4+b\,x^2}}{3cx}$	29

input
```
int((c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*x^2+b)/c/x*(c*x^4+b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \sqrt{bx^2 + cx^4}\,dx = \frac{\sqrt{cx^4 + bx^2}(cx^2 + b)}{3\,cx}$$

input
```
integrate((c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/(c*x)
```

Sympy [F]

$$\int \sqrt{bx^2 + cx^4}\, dx = \int \sqrt{bx^2 + cx^4}\, dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(sqrt(b*x**2 + c*x**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt{bx^2 + cx^4}\, dx = \frac{(cx^2 + b)^{\frac{3}{2}}}{3\,c}$$

input
```
integrate((c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(c*x^2 + b)^(3/2)/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt{bx^2 + cx^4}\, dx = \frac{(cx^2 + b)^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,c} - \frac{b^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,c}$$

input
```
integrate((c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(c*x^2 + b)^(3/2)*sgn(x)/c - 1/3*b^(3/2)*sgn(x)/c
```

Mupad [B] (verification not implemented)

Time = 18.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \sqrt{bx^2 + cx^4}\,dx = \frac{\left(\frac{b}{3\,c} + \frac{x^2}{3}\right)\sqrt{c\,x^4 + b\,x^2}}{x}$$

input
```
int((b*x^2 + c*x^4)^(1/2),x)
```

output
```
((b/(3*c) + x^2/3)*(b*x^2 + c*x^4)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \sqrt{bx^2 + cx^4}\,dx = \frac{\sqrt{c\,x^2 + b}\,(c\,x^2 + b)}{3c}$$

input
```
int((c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(b + c*x**2)*(b + c*x**2))/(3*c)
```

3.166 $\int \frac{\sqrt{bx^2+cx^4}}{x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{\sqrt{bx^2+cx^4}}{x^2} \, dx = \frac{\sqrt{bx^2+cx^4}}{x} - \sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

output
```
(c*x^4+b*x^2)^(1/2)/x-b^(1/2)*arctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{bx^2+cx^4}}{x^2} \, dx = \frac{x\left(b+cx^2-\sqrt{b}\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^2,x]
```

output
```
(x*(b + c*x^2 - Sqrt[b]*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))
/Sqrt[x^2*(b + c*x^2)]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1426, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^2}dx$$

$$\Big\downarrow \text{1426}$$

$$b\int \frac{1}{\sqrt{cx^4+bx^2}}dx+\frac{\sqrt{bx^2+cx^4}}{x}$$

$$\Big\downarrow \text{1400}$$

$$\frac{\sqrt{bx^2+cx^4}}{x}-b\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^2,x]
```

output
```
Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

method	result	size
default	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}\,\left(\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)-\sqrt{c\,x^{2}+b}\right)}{x\sqrt{c\,x^{2}+b}}$	65

input
```
int((c*x^4+b*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)-(c*x^2+b
)^(1/2))/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.24

$$\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx = \left[\frac{\sqrt{b}x\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}}{2\,x}, \frac{\sqrt{-b}x\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{bx}\right)+\sqrt{cx^4+bx^2}}{x}\right]$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b)*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)
+ 2*sqrt(c*x^4 + b*x^2))/x, (sqrt(-b)*x*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b
)/(b*x)) + sqrt(c*x^4 + b*x^2))/x]
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}}{x^2}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx = \frac{b\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} + \sqrt{cx^2+b}\operatorname{sgn}(x) - \frac{\left(b\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\sqrt{b}\right)\operatorname{sgn}(x)}{\sqrt{-b}}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
b*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + sqrt(c*x^2 + b)*sgn(x
) - (b*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b)*sqrt(b))*sgn(x)/sqrt(-b)
```

Mupad [B] (verification not implemented)

Time = 18.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}}{x} + \frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c\,x^4+b\,x^2}\,1\mathrm{i}}{\sqrt{c}\,x^2\,\sqrt{\frac{b}{c\,x^2}+1}}$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^2,x)
```

output
```
(b*x^2 + c*x^4)^(1/2)/x + (b^(1/2)*asin((b^(1/2)*1i)/(c^(1/2)*x))*(b*x^2 +
 c*x^4)^(1/2)*1i)/(c^(1/2)*x^2*(b/(c*x^2) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx = \sqrt{c\,x^2+b} + \sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right) - \sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^2,x)
```

output
```
sqrt(b + c*x**2) + sqrt(b)*log((sqrt(b + c*x**2) - sqrt(b) + sqrt(c)*x)/sq
rt(b)) - sqrt(b)*log((sqrt(b + c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))
```

3.167 $\int \frac{\sqrt{bx^2+cx^4}}{x^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{\sqrt{bx^2+cx^4}}{x^4} \, dx = -\frac{\sqrt{bx^2+cx^4}}{2x^3} - \frac{c \operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}$$

output
```
-1/2*(c*x^4+b*x^2)^(1/2)/x^3-1/2*c*arctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))/
b^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{bx^2+cx^4}}{x^4} \, dx = -\frac{\sqrt{x^2(b+cx^2)}\left(1+\frac{cx^2 \operatorname{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+cx^2}}\right)}{2x^3}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^4,x]
```

output
```
-1/2*(Sqrt[x^2*(b + c*x^2)]*(1 + (c*x^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/
(Sqrt[b]*Sqrt[b + c*x^2])))/x^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1425, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^4} \, dx$$

$$\Big\downarrow \; 1425$$

$$\frac{1}{2} c \int \frac{1}{\sqrt{cx^4+bx^2}} dx - \frac{\sqrt{bx^2+cx^4}}{2x^3}$$

$$\Big\downarrow \; 1400$$

$$-\frac{1}{2} c \int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}} d\frac{x}{\sqrt{cx^4+bx^2}} - \frac{\sqrt{bx^2+cx^4}}{2x^3}$$

$$\Big\downarrow \; 219$$

$$-\frac{c\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt{bx^2+cx^4}}{2x^3}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^4,x]
```

output
```
-1/2*Sqrt[b*x^2 + c*x^4]/x^3 - (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
)/(2*Sqrt[b])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{2 x^{3}}-\frac{c \ln\left(\frac{2 b +2 \sqrt{b}\, \sqrt{c\,x^{2}+b}}{x}\right) \sqrt{x^{2}\left(c\,x^{2}+b\right)}}{2 \sqrt{b}\, x \sqrt{c\,x^{2}+b}}$	74
default	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}\, \left(\sqrt{b}\, \ln\left(\frac{2 b +2 \sqrt{b}\, \sqrt{c\,x^{2}+b}}{x}\right) c\,x^{2}-\sqrt{c\,x^{2}+b}\, c\,x^{2}+\left(c\,x^{2}+b \right)^{\frac{3}{2}}\right)}{2 x^{3} \sqrt{c\,x^{2}+b}\, b}$	85

input
```
int((c*x^4+b*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^2*(c*x^2+b))^(1/2)/x^3-1/2*c/b^(1/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(
1/2))/x)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.30

$$\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx = \left[\frac{\sqrt{b}cx^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}b}{4\,bx^3}, \frac{\sqrt{-b}cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{bx}\right)-\sqrt{cx^4+bx^2}b}{2\,bx^3}\right]$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x
^3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b*x^3), 1/2*(sqrt(-b)*c*x^3*arctan(sqrt(c*
x^4 + b*x^2)*sqrt(-b)/(b*x)) - sqrt(c*x^4 + b*x^2)*b)/(b*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^4}\,dx = \int \frac{\sqrt{x^2\,(b + cx^2)}}{x^4}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4 + bx^2}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^4}\,dx = \frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} - \frac{\sqrt{cx^2 + b}\operatorname{sgn}(x)}{cx^2}\right)c$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/2*(arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - sqrt(c*x^2 + b)*sg
n(x)/(c*x^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^4}\,dx$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^4,x)
```

output
```
int((b*x^2 + c*x^4)^(1/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx = \frac{-\sqrt{c\,x^2+b}\,b+\sqrt{b}\,\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c\,x^2-\sqrt{b}\,\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c\,x^2}{2b\,x^2}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^4,x)
```

output
```
( - sqrt(b + c*x**2)*b + sqrt(b)*log((sqrt(b + c*x**2) - sqrt(b) + sqrt(c)
*x)/sqrt(b))*c*x**2 - sqrt(b)*log((sqrt(b + c*x**2) + sqrt(b) + sqrt(c)*x)
/sqrt(b))*c*x**2)/(2*b*x**2)
```

3.168 $\int \frac{\sqrt{bx^2+cx^4}}{x^6} dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx=-\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}+\frac{c^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}$$

output
```
-1/4*(c*x^4+b*x^2)^(1/2)/x^5-1/8*c*(c*x^4+b*x^2)^(1/2)/b/x^3+1/8*c^2*arcta
nh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx=\frac{\sqrt{x^2(b+cx^2)}\left(-\sqrt{b}\sqrt{b+cx^2}(2b+cx^2)+c^2x^4\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{8b^{3/2}x^5\sqrt{b+cx^2}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-(Sqrt[b]*Sqrt[b + c*x^2]*(2*b + c*x^2)) + c^2*x^4
*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(8*b^(3/2)*x^5*Sqrt[b + c*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1425, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^6} \, dx$$

$$\Big\downarrow \; 1425$$

$$\frac{1}{4} c \int \frac{1}{x^2 \sqrt{cx^4+bx^2}} dx - \frac{\sqrt{bx^2+cx^4}}{4x^5}$$

$$\Big\downarrow \; 1430$$

$$\frac{1}{4} c \left(-\frac{c \int \frac{1}{\sqrt{cx^4+bx^2}} dx}{2b} - \frac{\sqrt{bx^2+cx^4}}{2bx^3} \right) - \frac{\sqrt{bx^2+cx^4}}{4x^5}$$

$$\Big\downarrow \; 1400$$

$$\frac{1}{4} c \left(\frac{c \int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}} d\frac{x}{\sqrt{cx^4+bx^2}}}{2b} - \frac{\sqrt{bx^2+cx^4}}{2bx^3} \right) - \frac{\sqrt{bx^2+cx^4}}{4x^5}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{4} c \left(\frac{c \operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}} - \frac{\sqrt{bx^2+cx^4}}{2bx^3} \right) - \frac{\sqrt{bx^2+cx^4}}{4x^5}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

output
```
-1/4*Sqrt[b*x^2 + c*x^4]/x^5 + (c*(-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*A
rcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(3/2))))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

method	result	size
risch	$-\frac{(c\,x^2+2b)\sqrt{x^2(c\,x^2+b)}}{8x^5b}+\frac{c^2\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)\sqrt{x^2(c\,x^2+b)}}{8b^{\frac{3}{2}}x\sqrt{c\,x^2+b}}$	88
default	$\frac{\sqrt{c\,x^4+b\,x^2}\left(\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c^2x^4-\sqrt{c\,x^2+b}\,c^2x^4+(c\,x^2+b)^{\frac{3}{2}}c\,x^2-2(c\,x^2+b)^{\frac{3}{2}}b\right)}{8x^5\sqrt{c\,x^2+b}\,b^2}$	106

input
```
int((c*x^4+b*x^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(c*x^2+2*b)/x^5/b*(x^2*(c*x^2+b))^(1/2)+1/8*c^2/b^(3/2)*ln((2*b+2*b^(
1/2)*(c*x^2+b)^(1/2))/x)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx = \left[\frac{\sqrt{b}c^2x^5\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}(bcx^2+2\,b^2)}{16\,b^2x^5}, -\frac{\sqrt{-b}c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{bx}\right)+\sqrt{cx^4+bx^2}(bcx^2+2\,b^2)}{8\,b^2x^5}\right]$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
[1/16*(sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b)
)/x^3) - 2*sqrt(c*x^4 + b*x^2)*(b*c*x^2 + 2*b^2))/(b^2*x^5), -1/8*(sqrt(-b
)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(b*x)) + sqrt(c*x^4 + b*x^2)
*(b*c*x^2 + 2*b^2))/(b^2*x^5)]
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}}{x^6}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**6,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^6, x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx = -\frac{\frac{c^3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}b} + \frac{(cx^2+b)^{\frac{3}{2}}c^3\mathrm{sgn}(x)+\sqrt{cx^2+b}bc^3\mathrm{sgn}(x)}{bc^2x^4}}{8\,c}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="giac")
```

output
```
-1/8*(c^3*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + ((c*x^2 +
 b)^(3/2)*c^3*sgn(x) + sqrt(c*x^2 + b)*b*c^3*sgn(x))/(b*c^2*x^4))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2}}{x^6}\,dx$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^6,x)
```

output
```
int((b*x^2 + c*x^4)^(1/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^6}\,dx$$
$$= \frac{-2\sqrt{c\,x^2+b}\,b^2 - \sqrt{c\,x^2+b}\,bc\,x^2 - \sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c^2x^4 + \sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c^2x^4}{8b^2x^4}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^6,x)
```

output
```
( - 2*sqrt(b + c*x**2)*b**2 - sqrt(b + c*x**2)*b*c*x**2 - sqrt(b)*log((sqr
t(b + c*x**2) - sqrt(b) + sqrt(c)*x)/sqrt(b))*c**2*x**4 + sqrt(b)*log((sqr
t(b + c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))*c**2*x**4)/(8*b**2*x**4)
```

3.169 $\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx=-\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}$$

output
```
-1/6*(c*x^4+b*x^2)^(1/2)/x^7-1/24*c*(c*x^4+b*x^2)^(1/2)/b/x^5+1/16*c^2*(c*
x^4+b*x^2)^(1/2)/b^2/x^3-1/16*c^3*arctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))/b
^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}(8b^2+2bcx^2-3c^2x^4)+3c^3x^6\operatorname{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{48b^{5/2}x^7\sqrt{b+cx^2}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

output
```
-1/48*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(8*b^2 + 2*b*c*x^2 -
 3*c^2*x^4) + 3*c^3*x^6*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(b^(5/2)*x^7*Sq
rt[b + c*x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1425, 1430, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^8}dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{1}{6}c\int \frac{1}{x^4\sqrt{cx^4+bx^2}}dx-\frac{\sqrt{bx^2+cx^4}}{6x^7}$$

$$\Big\downarrow \text{1430}$$

$$\frac{1}{6}c\left(-\frac{3c\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)-\frac{\sqrt{bx^2+cx^4}}{6x^7}$$

$$\Big\downarrow \text{1430}$$

$$\frac{1}{6}c\left(-\frac{3c\left(-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)-\frac{\sqrt{bx^2+cx^4}}{6x^7}$$

$$\Big\downarrow \text{1400}$$

$$\frac{1}{6}c\left(-\frac{3c\left(\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)-\frac{\sqrt{bx^2+cx^4}}{6x^7}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}c\left(-\frac{3c\left(\frac{c\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)-\frac{\sqrt{bx^2+cx^4}}{6x^7}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

output
```
-1/6*Sqrt[b*x^2 + c*x^4]/x^7 + (c*(-1/4*Sqrt[b*x^2 + c*x^4]/(b*x^5) - (3*c
*(-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c
*x^4]])/(2*b^(3/2))))/(4*b)))/6
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{\left(-3c^{2}x^{4}+2bc\,x^{2}+8b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{48x^{7}b^{2}}-\frac{c^{3}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{16b^{\frac{5}{2}}x\sqrt{c\,x^{2}+b}}$	100
default	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}\left(3\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c^{3}x^{6}-3\sqrt{c\,x^{2}+b}\,c^{3}x^{6}+3\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{2}x^{4}-6\left(c\,x^{2}+b\right)^{\frac{3}{2}}bc\,x^{2}+8\left(c\,x^{2}+b\right)^{\frac{3}{2}}b^{2}\right)}{48x^{7}\sqrt{c\,x^{2}+b}\,b^{3}}$	128

input
```
int((c*x^4+b*x^2)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-3*c^2*x^4+2*b*c*x^2+8*b^2)/x^7/b^2*(x^2*(c*x^2+b))^(1/2)-1/16*c^3/
b^(5/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)*(x^2*(c*x^2+b))^(1/2)/x/(c*x
^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.61

$$\int\frac{\sqrt{bx^2+cx^4}}{x^8}\,dx$$
$$=\left[\frac{3\sqrt{b}c^3x^7\log\left(-\frac{cx^3+2\,bx-2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\left(3\,bc^2x^4-2\,b^2cx^2-8\,b^3\right)\sqrt{cx^4+bx^2}}{96\,b^3x^7},\frac{3\sqrt{-b}c^3x^7\arctan\left(\frac{\sqrt{c}\right.$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
[1/96*(3*sqrt(b)*c^3*x^7*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(
b))/x^3) + 2*(3*b*c^2*x^4 - 2*b^2*c*x^2 - 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^3
*x^7), 1/48*(3*sqrt(-b)*c^3*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(b*x))
 + (3*b*c^2*x^4 - 2*b^2*c*x^2 - 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^3*x^7)]
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}}{x^8}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**8,x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**8, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^8}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^8, x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx = \frac{1}{48}\,c^3\left(\frac{3\,\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}b^2}+\frac{3\,(cx^2+b)^{\frac{5}{2}}\operatorname{sgn}(x)-8\,(cx^2+b)^{\frac{3}{2}}b\operatorname{sgn}(x)-3\,\sqrt{cx^2+b}b^2\operatorname{sgn}(x)}{b^2c^3x^6}\right)$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^8,x, algorithm="giac")
```

output
```
1/48*c^3*(3*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^2) + (3*(c
*x^2 + b)^(5/2)*sgn(x) - 8*(c*x^2 + b)^(3/2)*b*sgn(x) - 3*sqrt(c*x^2 + b)*
b^2*sgn(x))/(b^2*c^3*x^6))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^8}\,dx$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^8,x)
```

output
```
int((b*x^2 + c*x^4)^(1/2)/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx = \frac{-8\sqrt{c\,x^2+b}\,b^3-2\sqrt{c\,x^2+b}\,b^2c\,x^2+3\sqrt{c\,x^2+b}\,b\,c^2x^4+3\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c^3x^6-3\sqrt{b}\log\left(\frac{\sqrt{c\,x}}{}\right.}{48b^3x^6}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^8,x)
```

output
```
( - 8*sqrt(b + c*x**2)*b**3 - 2*sqrt(b + c*x**2)*b**2*c*x**2 + 3*sqrt(b +
c*x**2)*b*c**2*x**4 + 3*sqrt(b)*log((sqrt(b + c*x**2) - sqrt(b) + sqrt(c)*
x)/sqrt(b))*c**3*x**6 - 3*sqrt(b)*log((sqrt(b + c*x**2) + sqrt(b) + sqrt(c
)*x)/sqrt(b))*c**3*x**6)/(48*b**3*x**6)
```

3.170 $\int x^3(bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int x^3\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{3b^3(b+2cx^2)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{3b^5\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}$$

output
```
3/256*b^3*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^3-1/32*b*(2*c*x^2+b)*(c*x^4+b*
x^2)^(3/2)/c^2+1/10*(c*x^4+b*x^2)^(5/2)/c-3/256*b^5*arctanh(c^(1/2)*x^2/(c
*x^4+b*x^2)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06

$$\int x^3\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}(15b^4-10b^3cx^2+8b^2c^2x^4+176bc^3x^6+128c^4x^8)+30b^5\operatorname{arctanh}\left(\right.\right.}{1280c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(15*b^4 - 10*b^3*c*x^2 + 8*b
^2*c^2*x^4 + 176*b*c^3*x^6 + 128*c^4*x^8) + 30*b^5*ArcTanh[(Sqrt[c]*x)/(Sq
rt[b] - Sqrt[b + c*x^2])]))/(1280*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1424, 1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int x^2 \left(cx^4+bx^2\right)^{3/2} dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{b\int \left(cx^4+bx^2\right)^{3/2} dx^2}{2c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{b\left(\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^4+bx^2}dx^2}{16c}\right)}{2c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{5c}-\frac{b\left(\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)}{16c}\right)}{2c}\right)$$

$\downarrow$ 1091

$$\frac{1}{2}\left(\frac{(bx^2+cx^4)^{5/2}}{5c}-\frac{b\left(\frac{(b+2cx^2)(bx^2+cx^4)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)}{16c}\right)}{2c}\right)$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{(bx^2+cx^4)^{5/2}}{5c}-\frac{b\left(\frac{(b+2cx^2)(bx^2+cx^4)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)}{16c}\right)}{2c}\right)$$

input
```
Int[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((b*x^2 + c*x^4)^(5/2)/(5*c) - (b*(((b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(
8*c) - (3*b^2*(((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(4*c) - (b^2*ArcTanh[(S
qrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(4*c^(3/2))))/(16*c)))/(2*c))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] && !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{3\left(\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b^{5}+\left(-\frac{256c^{\frac{9}{2}}x^{8}}{15}-\frac{352c^{\frac{7}{2}}b\,x^{6}}{15}-\frac{16c^{\frac{5}{2}}b^{2}x^{4}}{15}+\frac{4c^{\frac{3}{2}}b^{3}x^{2}}{3}-2\sqrt{c}\,b^{4}\right)\sqrt{x^{2}(c\,x^{2}+b)}-\ln(2)b^{5}}{512c^{\frac{7}{2}}}$
risch	$\frac{\left(128c^{4}x^{8}+176b\,c^{3}x^{6}+8b^{2}c^{2}x^{4}-10x^{2}b^{3}c+15b^{4}\right)\sqrt{x^{2}(c\,x^{2}+b)}}{1280c^{3}}-\frac{3b^{5}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)\sqrt{x^{2}(c\,x^{2}+b)}}{256c^{\frac{7}{2}}x\sqrt{c\,x^{2}+b}}$
default	$\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(128x^{5}\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{5}{2}}-80c^{\frac{3}{2}}\left(c\,x^{2}+b\right)^{\frac{5}{2}}b\,x^{3}+40\sqrt{c}\left(c\,x^{2}+b\right)^{\frac{5}{2}}b^{2}x-10\sqrt{c}\left(c\,x^{2}+b\right)^{\frac{3}{2}}b^{3}x-15\sqrt{c}\sqrt{c\,x^{2}+b}\,b^{4}x-}{1280x^{3}\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{7}{2}}}$

input
```
int(x^3*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3/512/c^(7/2)*(ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1/2))*b^
5+(-256/15*c^(9/2)*x^8-352/15*c^(7/2)*b*x^6-16/15*c^(5/2)*b^2*x^4+4/3*c^(3
/2)*b^3*x^2-2*c^(1/2)*b^4)*(x^2*(c*x^2+b))^(1/2)-ln(2)*b^5)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.69

$$\int x^3 (bx^2 + cx^4)^{3/2} \, dx = \left[\frac{15\,b^5\sqrt{c}\log\left(-2\,cx^2 - b + 2\,\sqrt{cx^4 + bx^2}\sqrt{c}\right) + 2\,(128\,c^5x^8 + 176\,bc^4x^6 + 8\,b^2c^3x^4 - 10\,b^3c^2}{2560\,c^4} \right.$$

input
```
integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2560*(15*b^5*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
+ 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 8*b^2*c^3*x^4 - 10*b^3*c^2*x^2 + 15*b^4
*c)*sqrt(c*x^4 + b*x^2))/c^4, 1/1280*(15*b^5*sqrt(-c)*arctan(sqrt(c*x^4 +
b*x^2)*sqrt(-c)/(c*x^2 + b)) + (128*c^5*x^8 + 176*b*c^4*x^6 + 8*b^2*c^3*x^
4 - 10*b^3*c^2*x^2 + 15*b^4*c)*sqrt(c*x^4 + b*x^2))/c^4]
```

Sympy [F]

$$\int x^3 (bx^2 + cx^4)^{3/2} \, dx = \int x^3 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**3*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**3*(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int x^3 (bx^2 + cx^4)^{3/2} \, dx = \frac{3\sqrt{cx^4 + bx^2} b^3 x^2}{128\, c^2} - \frac{(cx^4 + bx^2)^{\frac{3}{2}} b x^2}{16\, c} - \frac{3\, b^5 \log\left(2\, cx^2 + b + 2\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{512\, c^{\frac{7}{2}}} + \frac{3\sqrt{cx^4 + bx^2} b^4}{256\, c^3} - \frac{(cx^4 + bx^2)^{\frac{3}{2}} b^2}{32\, c^2} + \frac{(cx^4 + bx^2)^{\frac{5}{2}}}{10\, c}$$

input
```
integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
3/128*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^2 - 1/16*(c*x^4 + b*x^2)^(3/2)*b*x^2/c
 - 3/512*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 3/
256*sqrt(c*x^4 + b*x^2)*b^4/c^3 - 1/32*(c*x^4 + b*x^2)^(3/2)*b^2/c^2 + 1/1
0*(c*x^4 + b*x^2)^(5/2)/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int x^3 (bx^2 + cx^4)^{3/2} \, dx = \frac{3\, b^5 \log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + b}\right|\right) \mathrm{sgn}(x)}{256\, c^{\frac{7}{2}}} - \frac{3\, b^5 \log\left(|b|\right) \mathrm{sgn}(x)}{512\, c^{\frac{7}{2}}} + \frac{1}{1280}\left(2\left(4\left(2\left(8\, cx^2 \mathrm{sgn}(x) + 11\, b \mathrm{sgn}(x)\right)x^2 + \frac{b^2 \mathrm{sgn}(x)}{c}\right)x^2 - \frac{5\, b^3 \mathrm{sgn}(x)}{c^2}\right)x^2 + \frac{15\, b^4 \mathrm{sgn}(x)}{c^3}\right)\sqrt{cx^2 + b}$$

input
```
integrate(x^3*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
3/256*b^5*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(7/2) - 3/512*b^
5*log(abs(b))*sgn(x)/c^(7/2) + 1/1280*(2*(4*(2*(8*c*x^2*sgn(x) + 11*b*sgn(
x))*x^2 + b^2*sgn(x)/c)*x^2 - 5*b^3*sgn(x)/c^2)*x^2 + 15*b^4*sgn(x)/c^3)*s
qrt(c*x^2 + b)*x
```

Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.08

$$\int x^3 \left(bx^2+cx^4\right)^{3/2}\,dx = \frac{\left(c\,x^4+b\,x^2\right)^{5/2}}{10\,c} - \frac{b\left(\frac{x^2\,\left(c\,x^4+b\,x^2\right)^{3/2}}{4}-\frac{3\,b^2\,\left(\frac{\left(2\,c\,x^2+b\right)\,\sqrt{c\,x^4+b\,x^2}}{4\,c}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{16\,c}+\frac{b\,\left(c\,x^4+b\,x^2\right)^{3/2}}{8\,c}\right)}{4\,c}$$

input
```
int(x^3*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
(b*x^2 + c*x^4)^(5/2)/(10*c) - (b*((x^2*(b*x^2 + c*x^4)^(3/2))/4 - (3*b^2*
(((b + 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(4*c) - (b^2*log((b/2 + c*x^2)/c^(1
/2) + (b*x^2 + c*x^4)^(1/2)))/(8*c^(3/2))))/(16*c) + (b*(b*x^2 + c*x^4)^(3
/2))/(8*c)))/(4*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\int x^3 \left(bx^2 +cx^4\right)^{3/2}\,dx = \frac{15\sqrt{c\,x^2+b}\,b^4cx-10\sqrt{c\,x^2+b}\,b^3c^2x^3+8\sqrt{c\,x^2+b}\,b^2c^3x^5+176\sqrt{c\,x^2+b}\,b\,c^4x^7+128\sqrt{c\,x}}{1280c^4}$$

input
```
int(x^3*(c*x^4+b*x^2)^(3/2),x)
```

output
```
(15*sqrt(b + c*x**2)*b**4*c*x - 10*sqrt(b + c*x**2)*b**3*c**2*x**3 + 8*sqr
t(b + c*x**2)*b**2*c**3*x**5 + 176*sqrt(b + c*x**2)*b*c**4*x**7 + 128*sqrt
(b + c*x**2)*c**5*x**9 - 15*sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqr
t(b))*b**5)/(1280*c**4)
```

3.171 $\int x(bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int x\left(bx^2+cx^4\right)^{3/2}\,dx=-\frac{3b^2(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2}+\frac{(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{16c}+\frac{3b^4\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}$$

output
```
-3/128*b^2*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^2+1/16*(2*c*x^2+b)*(c*x^4+b*x
^2)^(3/2)/c+3/128*b^4*arctanh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

$$\int x\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}(-3b^3+2b^2cx^2+24bc^2x^4+16c^3x^6)+6b^4\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)\right)}{128c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(-3*b^3 + 2*b^2*c*x^2 + 24*b
*c^2*x^4 + 16*c^3*x^6) + 6*b^4*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*
x^2])]))/(128*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1424, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(bx^2+cx^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \left(cx^4+bx^2\right)^{3/2}dx^2$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^4+bx^2}dx^2}{16c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)}{16c}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{(b+2cx^2)\left(bx^2+cx^4\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)}{16c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(b+2cx^2)\,(bx^2+cx^4)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)}{16c}\right)$$

input
```
Int[x*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(((b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x^2)*Sqrt
[b*x^2 + c*x^4])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/
(4*c^(3/2))))/(16*c))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] && !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{\frac{3 \ln\left(\frac{2 c x^{2}+2 \sqrt{x^{2} \left(c x^{2}+b\right)}\, \sqrt{c}+b}{\sqrt{c}}\right) b^{4}}{256}+\frac{3 \left(\frac{32 c^{\frac{7}{2}} x^{6}}{3}+16 c^{\frac{5}{2}} b x^{4}+\frac{4 c^{\frac{3}{2}} b^{2} x^{2}}{3}-2 \sqrt{c}\, b^{3}\right) \sqrt{x^{2} \left(c x^{2}+b\right)}}{256}-\frac{3 \ln \left(2\right) b^{4}}{256}}{c^{\frac{5}{2}}}$
risch	$-\frac{\left(-16 c^{3} x^{6}-24 b c^{2} x^{4}-2 b^{2} c x^{2}+3 b^{3}\right) \sqrt{x^{2} \left(c x^{2}+b\right)}}{128 c^{2}}+\frac{3 b^{4} \ln \left(\sqrt{c}\, x+\sqrt{c x^{2}+b}\right) \sqrt{x^{2} \left(c x^{2}+b\right)}}{128 c^{\frac{5}{2}} x \sqrt{c x^{2}+b}}$
default	$\frac{\left(c x^{4}+b x^{2}\right)^{\frac{3}{2}} \left(16 x^{3} \left(c x^{2}+b\right)^{\frac{5}{2}} c^{\frac{3}{2}}-8 \sqrt{c}\, \left(c x^{2}+b\right)^{\frac{5}{2}} b x+2 \sqrt{c}\, \left(c x^{2}+b\right)^{\frac{3}{2}} b^{2} x+3 \sqrt{c}\, \sqrt{c x^{2}+b}\, b^{3} x+3 \ln \left(\sqrt{c}\, x+\sqrt{c x^{2}+b}\right) b^{4}\right)}{128 x^{3} \left(c x^{2}+b\right)^{\frac{3}{2}} c^{\frac{5}{2}}}$

input
```
int(x*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/256*(ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1/2))*b^4+(32/3*c
^(7/2)*x^6+16*c^(5/2)*b*x^4+4/3*c^(3/2)*b^2*x^2-2*c^(1/2)*b^3)*(x^2*(c*x^2
+b))^(1/2)-ln(2)*b^4)/c^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.87

$$\int x\left(bx^2+cx^4\right)^{3/2}\,dx = \left[\frac{3\,b^4\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,(16\,c^4x^6+24\,bc^3x^4+2\,b^2c^2x^2-3\,b^3c)\sqrt{cx^4+bx^2}}{256\,c^3}, -\frac{3\,b^4\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(16\,c^4x^6+24\,bc^3x^4+2\,b^2c^2x^2-3\,b^3c)\sqrt{cx^4+bx^2}}{128\,c^3}\right]$$

input
```
integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/256*(3*b^4*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) +
2*(16*c^4*x^6 + 24*b*c^3*x^4 + 2*b^2*c^2*x^2 - 3*b^3*c)*sqrt(c*x^4 + b*x^2
))/c^3, -1/128*(3*b^4*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2
+ b)) - (16*c^4*x^6 + 24*b*c^3*x^4 + 2*b^2*c^2*x^2 - 3*b^3*c)*sqrt(c*x^4 +
 b*x^2))/c^3]
```

Sympy [F]

$$\int x\left(bx^2+cx^4\right)^{3/2}\,dx=\int x\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x*(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int x\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{8}\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2-\frac{3\sqrt{cx^4+bx^2}b^2x^2}{64\,c}+\frac{3\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{256\,c^{\frac{5}{2}}}-\frac{3\sqrt{cx^4+bx^2}b^3}{128\,c^2}+\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}b}{16\,c}$$

input
```
integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(c*x^4 + b*x^2)^(3/2)*x^2 - 3/64*sqrt(c*x^4 + b*x^2)*b^2*x^2/c + 3/256
*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2) - 3/128*sqrt
(c*x^4 + b*x^2)*b^3/c^2 + 1/16*(c*x^4 + b*x^2)^(3/2)*b/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

$$\int x(bx^2+cx^4)^{3/2}\,dx = -\frac{3\,b^4\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\operatorname{sgn}(x)}{128\,c^{\frac{5}{2}}}+\frac{3\,b^4\log\left(|b|\right)\operatorname{sgn}(x)}{256\,c^{\frac{5}{2}}} + \frac{1}{128}\left(2\left(4\left(2\,cx^2\operatorname{sgn}(x)+3\,b\operatorname{sgn}(x)\right)x^2+\frac{b^2\operatorname{sgn}(x)}{c}\right)x^2-\frac{3\,b^3\operatorname{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}x$$

input
```
integrate(x*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-3/128*b^4*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(5/2) + 3/256*b
^4*log(abs(b))*sgn(x)/c^(5/2) + 1/128*(2*(4*(2*c*x^2*sgn(x) + 3*b*sgn(x))*
x^2 + b^2*sgn(x)/c)*x^2 - 3*b^3*sgn(x)/c^2)*sqrt(c*x^2 + b)*x
```

Mupad [B] (verification not implemented)

Time = 18.90 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

$$\int x(bx^2+cx^4)^{3/2}\,dx = \frac{\left(c\,x^4+b\,x^2\right)^{3/2}\left(c\,x^2+\frac{b}{2}\right)}{8\,c} - \frac{3\,b^2\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{32\,c}$$

input
```
int(x*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
((b*x^2 + c*x^4)^(3/2)*(b/2 + c*x^2))/(8*c) - (3*b^2*((b/(4*c) + x^2/2)*(b
*x^2 + c*x^4)^(1/2) - (b^2*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/
2)))/(8*c^(3/2))))/(32*c)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

$$\int x(bx^2+cx^4)^{3/2}\,dx = \frac{-3\sqrt{c x^2+b}\,b^3cx+2\sqrt{c x^2+b}\,b^2c^2x^3+24\sqrt{c x^2+b}\,b\,c^3x^5+16\sqrt{c x^2+b}\,c^4x^7+3\sqrt{c}\log\left(\cdots\right.}{128c^3}$$

input
```
int(x*(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - 3*sqrt(b + c*x**2)*b**3*c*x + 2*sqrt(b + c*x**2)*b**2*c**2*x**3 + 24*s
qrt(b + c*x**2)*b*c**3*x**5 + 16*sqrt(b + c*x**2)*c**4*x**7 + 3*sqrt(c)*lo
g((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b))*b**4)/(128*c**3)
```

3.172 $\int \frac{(bx^2+cx^4)^{3/2}}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 88

$$\int \frac{(bx^2+cx^4)^{3/2}}{x} dx = \frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c} + \frac{1}{6}(bx^2+cx^4)^{3/2} - \frac{b^3 \text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}$$

output
```
1/16*b*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c+1/6*(c*x^4+b*x^2)^(3/2)-1/16*b^3*
arctanh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int \frac{(bx^2+cx^4)^{3/2}}{x} dx = \frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}(3b^2+14bcx^2+8c^2x^4)+6b^3 \text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}-\sqrt{b+cx^2}}\right)\right)}{48c^{3/2}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x,x]
```

output
```
(x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(3*b^2 + 14*b*c*x^2 + 8*c^2*
x^4) + 6*b^3*ArcTanh[(Sqrt[c]*x)/(Sqrt[b] - Sqrt[b + c*x^2])]))/(48*c^(3/2
)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1424, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2)^{3/2}}{x^2}dx^2$$

$$\Big\downarrow \text{1131}$$

$$\frac{1}{2}\left(\frac{1}{2}b\int \sqrt{cx^4+bx^2}dx^2+\frac{1}{3}(bx^2+cx^4)^{3/2}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{8c}\right)+\frac{1}{3}(bx^2+cx^4)^{3/2}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{4c}\right)+\frac{1}{3}(bx^2+cx^4)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{4c^{3/2}}\right)+\frac{1}{3}(bx^2+cx^4)^{3/2}\right)$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x,x]
```

output
```
((b*x^2 + c*x^4)^(3/2)/3 + (b*(((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(4*c) -
 (b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(4*c^(3/2))))/2)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

method	result	s
risch	$\frac{(8c^2x^4+14bc\,x^2+3b^2)\sqrt{x^2(c\,x^2+b)}}{48c}-\frac{b^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)\sqrt{x^2(c\,x^2+b)}}{16c^{\frac{3}{2}}\,x\sqrt{c\,x^2+b}}$	9
default	$\frac{(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(8x(c\,x^2+b)^{\frac{5}{2}}\sqrt{c}-2\sqrt{c}\,(c\,x^2+b)^{\frac{3}{2}}bx-3\sqrt{c}\,\sqrt{c\,x^2+b}\,b^2x-3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)b^3\right)}{48x^3(c\,x^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}}$	1
pseudoelliptic	$\frac{16c^{\frac{5}{2}}x^4\sqrt{x^2(c\,x^2+b)}+28\sqrt{x^2(c\,x^2+b)}\,c^{\frac{3}{2}}b\,x^2+3\ln(2)b^3-3\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b^3+6\sqrt{x^2(c\,x^2+b)}\,\sqrt{c}\,b^2}{96c^{\frac{3}{2}}}$	1

input
```
int((c*x^4+b*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/48*(8*c^2*x^4+14*b*c*x^2+3*b^2)/c*(x^2*(c*x^2+b))^(1/2)-1/16*b^3/c^(3/2)
*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.89

$$\int \frac{(bx^2+cx^4)^{3/2}}{x}\,dx = \left[\frac{3\,b^3\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,(8\,c^3x^4+14\,bc^2x^2+3\,b^2c)\sqrt{cx^4+}}{96\,c^2}\right.$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/96*(3*b^3*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^2, 1/48*(3*b^
3*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*c^3*x^4 +
 14*b*c^2*x^2 + 3*b^2*c)*sqrt(c*x^4 + b*x^2))/c^2]
```

Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.90

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x}\, dx = \frac{b \left(\begin{cases} -\frac{b^2 \left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for}\ \frac{b^2}{c} \neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{8c} + \left(\frac{b}{4c} + \frac{x^2}{2}\right)\sqrt{bx^2 + cx^4} & \text{for}\ c \neq 0 \\ \frac{2(bx^2)^{\frac{3}{2}}}{3b} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{2}$$

$$+ \frac{c \left(\begin{cases} \frac{b^3 \left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for}\ \frac{b^2}{c} \neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{16c^2} + \sqrt{bx^2 + cx^4}\left(-\frac{b^2}{8c^2} + \frac{bx^2}{12c} + \frac{x^4}{3}\right) & \text{for}\ c \neq 0 \\ \frac{2(bx^2)^{\frac{5}{2}}}{5b^2} & \text{for}\ b \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{2}$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x,x)
```

output
```
b*Piecewise((-b**2*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*
c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqr
t(c*(b/(2*c) + x**2)**2), True))/(8*c) + (b/(4*c) + x**2/2)*sqrt(b*x**2 +
c*x**4), Ne(c, 0)), (2*(b*x**2)**(3/2)/(3*b), Ne(b, 0)), (0, True))/2 + c*
Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x
**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c
*(b/(2*c) + x**2)**2), True))/(16*c**2) + sqrt(b*x**2 + c*x**4)*(-b**2/(8*
c**2) + b*x**2/(12*c) + x**4/3), Ne(c, 0)), (2*(b*x**2)**(5/2)/(5*b**2), N
e(b, 0)), (0, True))/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \frac{(bx^2+cx^4)^{3/2}}{x}\,dx = \frac{1}{8}\sqrt{cx^4+bx^2}bx^2 - \frac{b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{32\,c^{\frac{3}{2}}} + \frac{1}{6}\left(cx^4+bx^2\right)^{\frac{3}{2}} + \frac{\sqrt{cx^4+bx^2}b^2}{16\,c}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/8*sqrt(c*x^4 + b*x^2)*b*x^2 - 1/32*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
b*x^2)*sqrt(c))/c^(3/2) + 1/6*(c*x^4 + b*x^2)^(3/2) + 1/16*sqrt(c*x^4 + b*
x^2)*b^2/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\int \frac{(bx^2+cx^4)^{3/2}}{x}\,dx = \frac{b^3\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\text{sgn}(x)}{16\,c^{\frac{3}{2}}} - \frac{b^3\log\left(|b|\right)\text{sgn}(x)}{32\,c^{\frac{3}{2}}} + \frac{1}{48}\left(2\left(4\,cx^2\text{sgn}(x)+7\,b\text{sgn}(x)\right)x^2+\frac{3\,b^2\text{sgn}(x)}{c}\right)\sqrt{cx^2+b}x$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/16*b^3*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/c^(3/2) - 1/32*b^3*
log(abs(b))*sgn(x)/c^(3/2) + 1/48*(2*(4*c*x^2*sgn(x) + 7*b*sgn(x))*x^2 + 3
*b^2*sgn(x)/c)*sqrt(c*x^2 + b)*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x}\,dx=\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x,x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int \frac{(bx^2+cx^4)^{3/2}}{x}\,dx=\frac{3\sqrt{c\,x^2+b}\,b^2cx+14\sqrt{c\,x^2+b}\,b\,c^2x^3+8\sqrt{c\,x^2+b}\,c^3x^5-3\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)b^3}{48c^2}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x,x)
```

output
```
(3*sqrt(b + c*x**2)*b**2*c*x + 14*sqrt(b + c*x**2)*b*c**2*x**3 + 8*sqrt(b
+ c*x**2)*c**3*x**5 - 3*sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b)
)*b**3)/(48*c**2)
```

3.173 $\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}+\frac{3b^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}$$

output
```
3/8*b*(c*x^4+b*x^2)^(1/2)+1/4*(c*x^4+b*x^2)^(3/2)/x^2+3/8*b^2*arctanh(c^(1
/2)*x^2/(c*x^4+b*x^2)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{1}{8}\sqrt{x^2(b+cx^2)}\left(5b+2cx^2-\frac{3b^2\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)}{\sqrt{c}x\sqrt{b+cx^2}}\right)$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(5*b + 2*c*x^2 - (3*b^2*Log[-(Sqrt[c]*x) + Sqrt[b +
 c*x^2]])/(Sqrt[c]*x*Sqrt[b + c*x^2])))/8
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1424, 1131, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^3} \, dx$$

$$\Big\downarrow \text{ 1424}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{1}{2}\left(\frac{3}{4}b\int \frac{\sqrt{cx^4+bx^2}}{x^2}dx^2+\frac{(bx^2+cx^4)^{3/2}}{2x^2}\right)$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2+\sqrt{bx^2+cx^4}\right)+\frac{(bx^2+cx^4)^{3/2}}{2x^2}\right)$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(b\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}+\sqrt{bx^2+cx^4}\right)+\frac{(bx^2+cx^4)^{3/2}}{2x^2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}+\sqrt{bx^2+cx^4}\right)+\frac{(bx^2+cx^4)^{3/2}}{2x^2}\right)$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

output
```
((b*x^2 + c*x^4)^(3/2)/(2*x^2) + (3*b*(Sqrt[b*x^2 + c*x^4] + (b*ArcTanh[(S
qrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/Sqrt[c]))/4)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{\left(2 c \,x^{2}+5 b \right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{8}+\frac{3 b^{2} \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{8 \sqrt{c}\, x \sqrt{c \,x^{2}+b}}$	76
default	$\frac{\left(c \,x^{4}+b \,x^{2}\right)^{\frac{3}{2}} \left(2 x \left(c \,x^{2}+b \right)^{\frac{3}{2}} \sqrt{c}+3 \sqrt{c}\, \sqrt{c \,x^{2}+b}\, b x +3 \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) b^{2}\right)}{8 x^{3} \left(c \,x^{2}+b \right)^{\frac{3}{2}} \sqrt{c}}$	84
pseudoelliptic	$\frac{4 c^{\frac{3}{2}} x^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}-3 \ln \left(2\right) b^{2}+3 \ln \left(\frac{2 c \,x^{2}+2 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \sqrt{c}+b}{\sqrt{c}}\right) b^{2}+10 b \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \sqrt{c}}{16 \sqrt{c}}$	90

input
```
int((c*x^4+b*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*c*x^2+5*b)*(x^2*(c*x^2+b))^(1/2)+3/8*b^2*ln(c^(1/2)*x+(c*x^2+b)^(1/
2))/c^(1/2)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.81

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx = \left[\frac{3\,b^2\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}(2\,c^2x^2+5\,bc)}{16\,c},\right.$$
$$\left.-\frac{3\,b^2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}(2\,c^2x^2+5\,bc)}{8\,c}\right]$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/16*(3*b^2*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + 5*b*c))/c, -1/8*(3*b^2*sqrt(-c)*arctan(s
qrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(2*c^2*x^2
+ 5*b*c))/c]
```

Sympy [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**3,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{16\,\sqrt{c}} + \frac{3}{8}\,\sqrt{cx^4+bx^2}b + \frac{(cx^4+bx^2)^{\frac{3}{2}}}{4\,x^2}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 3/8*sq
rt(c*x^4 + b*x^2)*b + 1/4*(c*x^4 + b*x^2)^(3/2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx = -\frac{3\,b^2\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\operatorname{sgn}(x)}{8\,\sqrt{c}} + \frac{3\,b^2\log\left(|b|\right)\operatorname{sgn}(x)}{16\,\sqrt{c}} + \frac{1}{8}\,\left(2\,cx^2\operatorname{sgn}(x)+5\,b\operatorname{sgn}(x)\right)\sqrt{cx^2+b}x$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
-3/8*b^2*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sgn(x)/sqrt(c) + 3/16*b^2*
log(abs(b))*sgn(x)/sqrt(c) + 1/8*(2*c*x^2*sgn(x) + 5*b*sgn(x))*sqrt(c*x^2
+ b)*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx = \int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^3}\,dx$$

input

```
int((b*x^2 + c*x^4)^(3/2)/x^3,x)
```

output

```
int((b*x^2 + c*x^4)^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{5\sqrt{c\,x^2+b}\,bcx+2\sqrt{c\,x^2+b}\,c^2x^3+3\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)b^2}{8c}$$

input

```
int((c*x^4+b*x^2)^(3/2)/x^3,x)
```

output

```
(5*sqrt(b + c*x**2)*b*c*x + 2*sqrt(b + c*x**2)*c**2*x**3 + 3*sqrt(c)*log((
sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b))*b**2)/(8*c)
```

3.174 $\int \frac{(bx^2+cx^4)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^5}\,dx = \frac{1}{2}c\sqrt{bx^2+cx^4} - \frac{b\sqrt{bx^2+cx^4}}{x^2} + \frac{3}{2}b\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

output
```
1/2*c*(c*x^4+b*x^2)^(1/2)-b*(c*x^4+b*x^2)^(1/2)/x^2+3/2*b*c^(1/2)*arctanh(
c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^5}\,dx = \frac{-2b^2-bcx^2+c^2x^4+6b\sqrt{c}x\sqrt{b+cx^2}\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)}{2\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

output
```
(-2*b^2 - b*c*x^2 + c^2*x^4 + 6*b*Sqrt[c]*x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[
c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])])/(2*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1424, 1125, 25, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^6}dx^2$$

$$\Big\downarrow \text{1125}$$

$$\frac{1}{2}\left(-\int -\frac{c\left(cx^2+2b\right)}{\sqrt{cx^4+bx^2}}dx^2-\frac{2b\sqrt{bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\int \frac{c\left(cx^2+2b\right)}{\sqrt{cx^4+bx^2}}dx^2-\frac{2b\sqrt{bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(c\int \frac{cx^2+2b}{\sqrt{cx^4+bx^2}}dx^2-\frac{2b\sqrt{bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(c\left(\frac{3}{2}b\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2+\sqrt{bx^2+cx^4}\right)-\frac{2b\sqrt{bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(c\left(3b\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}+\sqrt{bx^2+cx^4}\right)-\frac{2b\sqrt{bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(c\left(\frac{3b\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}+\sqrt{bx^2+cx^4}\right)-\frac{2b\sqrt{bx^2+cx^4}}{x^2}\right)$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

output
```
((-2*b*Sqrt[b*x^2 + c*x^4])/x^2 + c*(Sqrt[b*x^2 + c*x^4] + (3*b*ArcTanh[(S
qrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/Sqrt[c]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{\left(-c\,x^{2}+2 b \right) \sqrt{x^{2} \left(c\,x^{2}+b \right)}}{2 x^{2}}+\frac{3 b \sqrt{c}\, \ln \left(\sqrt{c}\, x +\sqrt{c\,x^{2}+b}\right) \sqrt{x^{2} \left(c\,x^{2}+b \right)}}{2 x \sqrt{c\,x^{2}+b}}$	77
pseudoelliptic	$\frac{3 x^{2} b \left(-\ln \left(2\right)+\ln \left(\frac{2 c\,x^{2}+2 \sqrt{x^{2} \left(c\,x^{2}+b \right)}\, \sqrt{c}+b}{\sqrt{c}}\right)\right) \sqrt{c}-2 \sqrt{x^{2} \left(c\,x^{2}+b \right)}\, \left(-c\,x^{2}+2 b \right)}{4 x^{2}}$	77
default	$\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}} \left(2 c^{\frac{3}{2}} \left(c\,x^{2}+b \right)^{\frac{3}{2}} x^{2}+3 c^{\frac{3}{2}} \sqrt{c\,x^{2}+b}\, b\,x^{2}-2 \left(c\,x^{2}+b \right)^{\frac{5}{2}} \sqrt{c}+3 \ln \left(\sqrt{c}\, x +\sqrt{c\,x^{2}+b}\right) b^{2} c x \right)}{2 x^{4} \left(c\,x^{2}+b \right)^{\frac{3}{2}} b \sqrt{c}}$	107

input
```
int((c*x^4+b*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-c*x^2+2*b)/x^2*(x^2*(c*x^2+b))^(1/2)+3/2*b*c^(1/2)*ln(c^(1/2)*x+(c*
x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.81

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^5}\,dx = \left[\frac{3\,b\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}(cx^2-2\,b)}{4\,x^2}, -\frac{3\,b\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}(cx^2-2\,b)}{2\,x^2}\right]$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/4*(3*b*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) +
2*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b))/x^2, -1/2*(3*b*sqrt(-c)*x^2*arctan(sq
rt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b
))/x^2]
```

Sympy [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^5}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**5,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^5}\,dx = \frac{3}{4}\,b\sqrt{c}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right) - \frac{3\,\sqrt{cx^4+bx^2}b}{2\,x^2} + \frac{(cx^4+bx^2)^{\frac{3}{2}}}{2\,x^4}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
3/4*b*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 3/2*sqrt(
c*x^4 + b*x^2)*b/x^2 + 1/2*(c*x^4 + b*x^2)^(3/2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^5}\,dx = \frac{1}{2}\,\sqrt{cx^2+b}cx\mathrm{sgn}(x) - \frac{3}{4}\,b\sqrt{c}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\mathrm{sgn}(x) + \frac{2\,b^2\sqrt{c}\mathrm{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + b)*c*x*sgn(x) - 3/4*b*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2
 + b))^2)*sgn(x) + 2*b^2*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 -
 b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^5}\,dx=\int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^5}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^5,x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^5}\,dx=\frac{-8\sqrt{c\,x^2+b}\,b+4\sqrt{c\,x^2+b}\,c\,x^2+12\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)bx-9\sqrt{c}\,bx}{8x}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^5,x)
```

output
```
( - 8*sqrt(b + c*x**2)*b + 4*sqrt(b + c*x**2)*c*x**2 + 12*sqrt(c)*log((sqr
t(b + c*x**2) + sqrt(c)*x)/sqrt(b))*b*x - 9*sqrt(c)*b*x)/(8*x)
```

3.175 $\int \frac{(bx^2+cx^4)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^7} dx = -\frac{c\sqrt{bx^2+cx^4}}{x^2} - \frac{(bx^2+cx^4)^{3/2}}{3x^6} + c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

output
```
-c*(c*x^4+b*x^2)^(1/2)/x^2-1/3*(c*x^4+b*x^2)^(3/2)/x^6+c^(3/2)*arctanh(c^(
1/2)*x^2/(c*x^4+b*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.13

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^7} dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}(b+4cx^2)+3c^{3/2}x^3\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)\right)}{3x^4\sqrt{b+cx^2}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

output
```
-1/3*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(b + 4*c*x^2) + 3*c^(3/2)*x^3
*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(x^4*Sqrt[b + c*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1424, 1130, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2)^{3/2}}{x^8}dx^2$$

$$\Big\downarrow \text{1130}$$

$$\frac{1}{2}\left(c\int \frac{\sqrt{cx^4+bx^2}}{x^4}dx^2-\frac{2(bx^2+cx^4)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{1125}$$

$$\frac{1}{2}\left(c\left(-\int -\frac{c}{\sqrt{cx^4+bx^2}}dx^2-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2(bx^2+cx^4)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(c\left(\int \frac{c}{\sqrt{cx^4+bx^2}}dx^2-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2(bx^2+cx^4)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(c\left(c\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2(bx^2+cx^4)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(c\left(2c\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2(bx^2+cx^4)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(c\left(2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{2\sqrt{bx^2+cx^4}}{x^2}\right)-\frac{2(bx^2+cx^4)^{3/2}}{3x^6}\right)$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

output
```
((-2*(b*x^2 + c*x^4)^(3/2))/(3*x^6) + c*((-2*Sqrt[b*x^2 + c*x^4])/x^2 + 2*
Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{\left(4 c \,x^{2}+b \right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{3 x^{4}}+\frac{c^{\frac{3}{2}} \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{x \sqrt{c \,x^{2}+b}}$
pseudoelliptic	$\frac{3 x^{4} \left(-\ln \left(2\right)+\ln \left(\frac{2 c \,x^{2}+2 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \sqrt{c}+b}{\sqrt{c}}\right)\right) c^{\frac{3}{2}}-2 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \left(4 c \,x^{2}+b \right)}{6 x^{4}}$
default	$\frac{\left(c \,x^{4}+b \,x^{2}\right)^{\frac{3}{2}} \left(2 c^{\frac{5}{2}} \left(c \,x^{2}+b \right)^{\frac{3}{2}} x^{4}+3 c^{\frac{5}{2}} \sqrt{c \,x^{2}+b}\, b \,x^{4}-2 c^{\frac{3}{2}} \left(c \,x^{2}+b \right)^{\frac{5}{2}} x^{2}+3 \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) b^{2} c^{2} x^{3}-\left(c \,x^{2}+b \right)^{\frac{5}{2}} b \sqrt{c}\right)}{3 x^{6} \left(c \,x^{2}+b \right)^{\frac{3}{2}} b^{2} \sqrt{c}}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(4*c*x^2+b)/x^4*(x^2*(c*x^2+b))^(1/2)+c^(3/2)*ln(c^(1/2)*x+(c*x^2+b)^
(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.80

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^7}\,dx = \left[\frac{3\,c^{\frac{3}{2}}x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}(4\,cx^2+b)}{6\,x^4},\right.$$
$$\left.-\frac{3\,\sqrt{-c}cx^4\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}(4\,cx^2+b)}{3\,x^4}\right]$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/6*(3*c^(3/2)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*
sqrt(c*x^4 + b*x^2)*(4*c*x^2 + b))/x^4, -1/3*(3*sqrt(-c)*c*x^4*arctan(sqrt
(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*(4*c*x^2 + b))
/x^4]
```

Sympy [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^7}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**7,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.19

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{1}{2}\,c^{\frac{3}{2}}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right) - \frac{7\,\sqrt{cx^4+bx^2}c}{6\,x^2} - \frac{\sqrt{cx^4+bx^2}b}{6\,x^4} - \frac{(cx^4+bx^2)^{\frac{3}{2}}}{6\,x^6}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
1/2*c^(3/2)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 7/6*sqrt(c*
x^4 + b*x^2)*c/x^2 - 1/6*sqrt(c*x^4 + b*x^2)*b/x^4 - 1/6*(c*x^4 + b*x^2)^(
3/2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.63

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^7}\,dx = -\frac{1}{2}\,c^{\frac{3}{2}}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\mathrm{sgn}(x) + \frac{4\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4bc^{\frac{3}{2}}\mathrm{sgn}(x) - 3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2b^2c^{\frac{3}{2}}\mathrm{sgn}(x) + 2\,b^3c^{\frac{3}{2}}\mathrm{sgn}(x)\right)}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^3}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-1/2*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 4/3*(3*(sqrt(c)
*x - sqrt(c*x^2 + b))^4*b*c^(3/2)*sgn(x) - 3*(sqrt(c)*x - sqrt(c*x^2 + b))
^2*b^2*c^(3/2)*sgn(x) + 2*b^3*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b
))^2 - b)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^7} \, dx = \int \frac{(c\,x^4 + b\,x^2)^{3/2}}{x^7} \, dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^7,x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^7} \, dx = \frac{-\sqrt{c\,x^2 + b}\,b - 4\sqrt{c\,x^2 + b}\,c\,x^2 + 3\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)c\,x^3}{3x^3}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^7,x)
```

output
```
( - sqrt(b + c*x**2)*b - 4*sqrt(b + c*x**2)*c*x**2 + 3*sqrt(c)*log((sqrt(b
 + c*x**2) + sqrt(c)*x)/sqrt(b))*c*x**3)/(3*x**3)
```

3.176 $\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

output
```
-1/5*(c*x^4+b*x^2)^(5/2)/b/x^10
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{(x^2(b+cx^2))^{5/2}}{5bx^{10}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^9,x]
```

output
```
-1/5*(x^2*(b + c*x^2))^(5/2)/(b*x^10)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx$$

$$\Big\downarrow \text{ 1422}$$

$$-\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^9,x]
```

output
```
-1/5*(b*x^2 + c*x^4)^(5/2)/(b*x^10)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$-\frac{(c x^2+b)(c x^4+b x^2)^{\frac{3}{2}}}{5x^8 b}$	29
default	$-\frac{(c x^2+b)(c x^4+b x^2)^{\frac{3}{2}}}{5x^8 b}$	29
orering	$-\frac{(c x^2+b)(c x^4+b x^2)^{\frac{3}{2}}}{5x^8 b}$	29
pseudoelliptic	$-\frac{(c x^2+b)^2 \sqrt{x^2(c x^2+b)}}{5x^6 b}$	31
trager	$-\frac{(c^2 x^4+2bc x^2+b^2)\sqrt{c x^4+b x^2}}{5b x^6}$	40
risch	$-\frac{\sqrt{x^2(c x^2+b)}\,(c^2 x^4+2bc x^2+b^2)}{5x^6 b}$	40

input
```
int((c*x^4+b*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^8*(c*x^2+b)/b*(c*x^4+b*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{(c^2x^4+2\,bcx^2+b^2)\sqrt{cx^4+bx^2}}{5\,bx^6}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
-1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^4 + b*x^2)/(b*x^6)
```

Sympy [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**9,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**9, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(21) = 42$.

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.24

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{\sqrt{cx^4+bx^2}c^2}{5\,bx^2} + \frac{\sqrt{cx^4+bx^2}c}{10\,x^4} + \frac{3\,\sqrt{cx^4+bx^2}b}{10\,x^6} - \frac{(cx^4+bx^2)^{\frac{3}{2}}}{2\,x^8}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
-1/5*sqrt(c*x^4 + b*x^2)*c^2/(b*x^2) + 1/10*sqrt(c*x^4 + b*x^2)*c/x^4 + 3/
10*sqrt(c*x^4 + b*x^2)*b/x^6 - 1/2*(c*x^4 + b*x^2)^(3/2)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(21) = 42$.

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.68

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx = \frac{2\left(5\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8 c^{\frac{5}{2}}\mathrm{sgn}(x) + 10\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4 b^2 c^{\frac{5}{2}}\mathrm{sgn}(x) + b^4 c^{\frac{5}{2}}\mathrm{sgn}(x)\right.}{5\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^5}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
2/5*(5*(sqrt(c)*x - sqrt(c*x^2 + b))^8*c^(5/2)*sgn(x) + 10*(sqrt(c)*x - sq
rt(c*x^2 + b))^4*b^2*c^(5/2)*sgn(x) + b^4*c^(5/2)*sgn(x))/((sqrt(c)*x - sq
rt(c*x^2 + b))^2 - b)^5
```

Mupad [B] (verification not implemented)

Time = 20.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{(c\,x^2+b)^2\,\sqrt{c\,x^4+b\,x^2}}{5\,b\,x^6}$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^9,x)
```

output
```
-((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2))/(5*b*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx = \frac{-\sqrt{c\,x^2+b}\,b^2-2\sqrt{c\,x^2+b}\,bc\,x^2-\sqrt{c\,x^2+b}\,c^2x^4-\sqrt{c}\,c^2x^5}{5b\,x^5}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^9,x)
```

output
```
( - sqrt(b + c*x**2)*b**2 - 2*sqrt(b + c*x**2)*b*c*x**2 - sqrt(b + c*x**2)
*c**2*x**4 - sqrt(c)*c**2*x**5)/(5*b*x**5)
```

3.177 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}} dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}} dx = -\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}} + \frac{2c(bx^2+cx^4)^{5/2}}{35b^2x^{10}}$$

output
```
-1/7*(c*x^4+b*x^2)^(5/2)/b/x^12+2/35*c*(c*x^4+b*x^2)^(5/2)/b^2/x^10
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}} dx = \frac{(x^2(b+cx^2))^{5/2}(-5b+2cx^2)}{35b^2x^{12}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^11,x]
```

output
```
((x^2*(b + c*x^2))^(5/2)*(-5*b + 2*c*x^2))/(35*b^2*x^12)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1423, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}dx$$

$$\Big\downarrow \; 1423$$

$$-\frac{2c\int \frac{(cx^4+bx^2)^{3/2}}{x^9}dx}{7b}-\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

$$\Big\downarrow \; 1422$$

$$\frac{2c(bx^2+cx^4)^{5/2}}{35b^2x^{10}}-\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^11,x]
```

output
```
-1/7*(b*x^2 + c*x^4)^(5/2)/(b*x^12) + (2*c*(b*x^2 + c*x^4)^(5/2))/(35*b^2*
x^10)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{(c\,x^2+b)\,(-2c\,x^2+5b)\,(c\,x^4+b\,x^2)^{\frac{3}{2}}}{35b^2x^{10}}$	39
default	$-\frac{(c\,x^2+b)\,(-2c\,x^2+5b)\,(c\,x^4+b\,x^2)^{\frac{3}{2}}}{35b^2x^{10}}$	39
pseudoelliptic	$-\frac{\left(-\frac{2c\,x^2}{5}+b\right)(c\,x^2+b)^2\sqrt{x^2(c\,x^2+b)}}{7x^8b^2}$	39
orering	$-\frac{(c\,x^2+b)\,(-2c\,x^2+5b)\,(c\,x^4+b\,x^2)^{\frac{3}{2}}}{35b^2x^{10}}$	39
trager	$-\frac{(-2c^3x^6+b\,c^2x^4+8b^2c\,x^2+5b^3)\sqrt{c\,x^4+b\,x^2}}{35b^2x^8}$	53
risch	$-\frac{\sqrt{x^2(c\,x^2+b)}\,(-2c^3x^6+b\,c^2x^4+8b^2c\,x^2+5b^3)}{35x^8b^2}$	53

input
```
int((c*x^4+b*x^2)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/35*(c*x^2+b)*(-2*c*x^2+5*b)*(c*x^4+b*x^2)^(3/2)/b^2/x^10
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = \frac{(2\,c^3x^6-bc^2x^4-8\,b^2cx^2-5\,b^3)\sqrt{cx^4+bx^2}}{35\,b^2x^8}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="fricas")
```

output
```
1/35*(2*c^3*x^6 - b*c^2*x^4 - 8*b^2*c*x^2 - 5*b^3)*sqrt(c*x^4 + b*x^2)/(b^
2*x^8)
```

Sympy [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{11}}\,dx = \int \frac{(x^2(b + cx^2))^{\frac{3}{2}}}{x^{11}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**11,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**11, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(44) = 88.

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.02

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{11}}\,dx = \frac{2\sqrt{cx^4 + bx^2}c^3}{35\,b^2x^2} - \frac{\sqrt{cx^4 + bx^2}c^2}{35\,bx^4} + \frac{3\sqrt{cx^4 + bx^2}c}{140\,x^6} + \frac{3\sqrt{cx^4 + bx^2}b}{28\,x^8} - \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{4\,x^{10}}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="maxima")
```

output
```
2/35*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^2) - 1/35*sqrt(c*x^4 + b*x^2)*c^2/(b*x
^4) + 3/140*sqrt(c*x^4 + b*x^2)*c/x^6 + 3/28*sqrt(c*x^4 + b*x^2)*b/x^8 - 1
/4*(c*x^4 + b*x^2)^(3/2)/x^10
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(44) = 88.

Time = 0.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.42

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{11}}\,dx = \frac{4\left(35\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^{10} c^{\frac{7}{2}}\operatorname{sgn}(x) + 35\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^{8} bc^{\frac{7}{2}}\operatorname{sgn}(x) + 70\left(\sqrt{c}x - \right.\right.}{3}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="giac")
```

output
```
4/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^10*c^(7/2)*sgn(x) + 35*(sqrt(c)*x -
 sqrt(c*x^2 + b))^8*b*c^(7/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^6*
b^2*c^(7/2)*sgn(x) + 14*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^3*c^(7/2)*sgn(x)
 + 7*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^4*c^(7/2)*sgn(x) - b^5*c^(7/2)*sgn(
x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

Mupad [B] (verification not implemented)

Time = 20.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.67

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = \frac{2\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^2} - \frac{8\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,x^6} - \frac{c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^4} - \frac{b\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8}$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^11,x)
```

output
```
(2*c^3*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^2) - (8*c*(b*x^2 + c*x^4)^(1/2))/(
35*x^6) - (c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) - (b*(b*x^2 + c*x^4)^(1/2
))/(7*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.58

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = \frac{-5\sqrt{c\,x^2+b}\,b^3-8\sqrt{c\,x^2+b}\,b^2c\,x^2-\sqrt{c\,x^2+b}\,b\,c^2x^4+2\sqrt{c\,x^2+b}\,c^3x^6-2\sqrt{c}\,c^3x}{35b^2x^7}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^11,x)
```

output
```
( - 5*sqrt(b + c*x**2)*b**3 - 8*sqrt(b + c*x**2)*b**2*c*x**2 - sqrt(b + c*
x**2)*b*c**2*x**4 + 2*sqrt(b + c*x**2)*c**3*x**6 - 2*sqrt(c)*c**3*x**7)/(3
5*b**2*x**7)
```

3.178 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}} \, dx = -\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}} + \frac{4c(bx^2+cx^4)^{5/2}}{63b^2x^{12}} - \frac{8c^2(bx^2+cx^4)^{5/2}}{315b^3x^{10}}$$

output
```
-1/9*(c*x^4+b*x^2)^(5/2)/b/x^14+4/63*c*(c*x^4+b*x^2)^(5/2)/b^2/x^12-8/315*
c^2*(c*x^4+b*x^2)^(5/2)/b^3/x^10
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}} \, dx = \frac{(x^2(b+cx^2))^{5/2}(-35b^2+20bcx^2-8c^2x^4)}{315b^3x^{14}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^13,x]
```

output
```
((x^2*(b + c*x^2))^(5/2)*(-35*b^2 + 20*b*c*x^2 - 8*c^2*x^4))/(315*b^3*x^14
)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1423, 1423, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx$$
$$\Big\downarrow \text{1423}$$
$$-\frac{4c\int \frac{(cx^4+bx^2)^{3/2}}{x^{11}}dx}{9b}-\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}}$$
$$\Big\downarrow \text{1423}$$
$$-\frac{4c\left(-\frac{2c\int \frac{(cx^4+bx^2)^{3/2}}{x^9}dx}{7b}-\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}}$$
$$\Big\downarrow \text{1422}$$
$$-\frac{4c\left(\frac{2c(bx^2+cx^4)^{5/2}}{35b^2x^{10}}-\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^13,x]
```

output
```
-1/9*(b*x^2 + c*x^4)^(5/2)/(b*x^14) - (4*c*(-1/7*(b*x^2 + c*x^4)^(5/2)/(b*
x^12) + (2*c*(b*x^2 + c*x^4)^(5/2))/(35*b^2*x^10)))/(9*b)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] && !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] && !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(8c^{2}x^{4}-20bc\,x^{2}+35b^{2}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{315x^{12}b^{3}}$	50
default	$-\frac{\left(c\,x^{2}+b\right)\left(8c^{2}x^{4}-20bc\,x^{2}+35b^{2}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{315x^{12}b^{3}}$	50
pseudoelliptic	$-\frac{\left(\frac{8}{35}c^{2}x^{4}-\frac{4}{7}bc\,x^{2}+b^{2}\right)\left(c\,x^{2}+b\right)^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{9x^{10}b^{3}}$	50
orering	$-\frac{\left(c\,x^{2}+b\right)\left(8c^{2}x^{4}-20bc\,x^{2}+35b^{2}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{315x^{12}b^{3}}$	50
trager	$-\frac{\left(8c^{4}x^{8}-4b\,c^{3}x^{6}+3b^{2}c^{2}x^{4}+50x^{2}b^{3}c+35b^{4}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{315b^{3}x^{10}}$	65
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(8c^{4}x^{8}-4b\,c^{3}x^{6}+3b^{2}c^{2}x^{4}+50x^{2}b^{3}c+35b^{4}\right)}{315x^{10}b^{3}}$	65

input
```
int((c*x^4+b*x^2)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(c*x^2+b)*(8*c^2*x^4-20*b*c*x^2+35*b^2)*(c*x^4+b*x^2)^(3/2)/x^12/b^
3
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = -\frac{(8\,c^4x^8-4\,bc^3x^6+3\,b^2c^2x^4+50\,b^3cx^2+35\,b^4)\sqrt{cx^4+bx^2}}{315\,b^3x^{10}}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="fricas")
```

output
```
-1/315*(8*c^4*x^8 - 4*b*c^3*x^6 + 3*b^2*c^2*x^4 + 50*b^3*c*x^2 + 35*b^4)*s
qrt(c*x^4 + b*x^2)/(b^3*x^10)
```

Sympy [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^{13}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**13,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**13, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.61

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = -\frac{8\,\sqrt{cx^4+bx^2}c^4}{315\,b^3x^2}+\frac{4\,\sqrt{cx^4+bx^2}c^3}{315\,b^2x^4} - \frac{\sqrt{cx^4+bx^2}c^2}{105\,bx^6}+\frac{\sqrt{cx^4+bx^2}c}{126\,x^8}+\frac{\sqrt{cx^4+bx^2}b}{18\,x^{10}}-\frac{(cx^4+bx^2)^{\frac{3}{2}}}{6\,x^{12}}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="maxima")
```

output
```
-8/315*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^2) + 4/315*sqrt(c*x^4 + b*x^2)*c^3/(
b^2*x^4) - 1/105*sqrt(c*x^4 + b*x^2)*c^2/(b*x^6) + 1/126*sqrt(c*x^4 + b*x^
2)*c/x^8 + 1/18*sqrt(c*x^4 + b*x^2)*b/x^10 - 1/6*(c*x^4 + b*x^2)^(3/2)/x^1
2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(68) = 136$.

Time = 0.18 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.58

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}c^{\frac{9}{2}}\text{sgn}(x)+315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{9}{2}}\text{sgn}(x)+441\left(\right.}{}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="giac")
```

output
```
16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + b))^12*c^(9/2)*sgn(x) + 315*(sqrt(c)
*x - sqrt(c*x^2 + b))^10*b*c^(9/2)*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 +
b))^8*b^2*c^(9/2)*sgn(x) + 126*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^3*c^(9/2)
*sgn(x) + 36*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^4*c^(9/2)*sgn(x) - 9*(sqrt(
c)*x - sqrt(c*x^2 + b))^2*b^5*c^(9/2)*sgn(x) + b^6*c^(9/2)*sgn(x))/((sqrt(
c)*x - sqrt(c*x^2 + b))^2 - b)^9
```

Mupad [B] (verification not implemented)

Time = 19.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.39

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{4\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^2\,x^4} - \frac{10\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,x^8} - \frac{c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b\,x^6} - \frac{b\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}} - \frac{8\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^2}$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^13,x)
```

output
```
(4*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (10*c*(b*x^2 + c*x^4)^(1/2))
/(63*x^8) - (c^2*(b*x^2 + c*x^4)^(1/2))/(105*b*x^6) - (b*(b*x^2 + c*x^4)^(
1/2))/(9*x^10) - (8*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{-35\sqrt{c\,x^2+b}\,b^4-50\sqrt{c\,x^2+b}\,b^3c\,x^2-3\sqrt{c\,x^2+b}\,b^2c^2x^4+4\sqrt{c\,x^2+b}\,b\,c^3x^6-8\sqrt{\ }}{315b^3x^9}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^13,x)
```

output
```
( - 35*sqrt(b + c*x**2)*b**4 - 50*sqrt(b + c*x**2)*b**3*c*x**2 - 3*sqrt(b
+ c*x**2)*b**2*c**2*x**4 + 4*sqrt(b + c*x**2)*b*c**3*x**6 - 8*sqrt(b + c*x
**2)*c**4*x**8 + 8*sqrt(c)*c**4*x**9)/(315*b**3*x**9)
```

3.179 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = -\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}} + \frac{2c(bx^2+cx^4)^{5/2}}{33b^2x^{14}} - \frac{8c^2(bx^2+cx^4)^{5/2}}{231b^3x^{12}} + \frac{16c^3(bx^2+cx^4)^{5/2}}{1155b^4x^{10}}$$

output
```
-1/11*(c*x^4+b*x^2)^(5/2)/b/x^16+2/33*c*(c*x^4+b*x^2)^(5/2)/b^2/x^14-8/231
*c^2*(c*x^4+b*x^2)^(5/2)/b^3/x^12+16/1155*c^3*(c*x^4+b*x^2)^(5/2)/b^4/x^10
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = \frac{(x^2(b+cx^2))^{5/2}\,(-105b^3+70b^2cx^2-40bc^2x^4+16c^3x^6)}{1155b^4x^{16}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

output
```
((x^2*(b + c*x^2))^(5/2)*(-105*b^3 + 70*b^2*c*x^2 - 40*b*c^2*x^4 + 16*c^3*
x^6))/(1155*b^4*x^16)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1423, 1423, 1423, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}dx$$

$$\Big\downarrow \text{ 1423}$$

$$-\frac{6c\int \frac{(cx^4+bx^2)^{3/2}}{x^{13}}dx}{11b}-\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}}$$

$$\Big\downarrow \text{ 1423}$$

$$-\frac{6c\left(-\frac{4c\int \frac{(cx^4+bx^2)^{3/2}}{x^{11}}dx}{9b}-\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}}$$

$$\Big\downarrow \text{ 1423}$$

$$-\frac{6c\left(-\frac{4c\left(-\frac{2c\int \frac{(cx^4+bx^2)^{3/2}}{x^9}dx}{7b}-\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}}$$

$$\Big\downarrow \text{ 1422}$$

$$-\frac{6c\left(-\frac{4c\left(\frac{2c(bx^2+cx^4)^{5/2}}{35b^2x^{10}}-\frac{(bx^2+cx^4)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{(bx^2+cx^4)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{(bx^2+cx^4)^{5/2}}{11bx^{16}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

output
```
-1/11*(b*x^2 + c*x^4)^(5/2)/(b*x^16) - (6*c*(-1/9*(b*x^2 + c*x^4)^(5/2)/(b
*x^14) - (4*c*(-1/7*(b*x^2 + c*x^4)^(5/2)/(b*x^12) + (2*c*(b*x^2 + c*x^4)^
(5/2))/(35*b^2*x^10)))/(9*b)))/(11*b)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] && !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] && !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-16 c^{3} x^{6}+40 b\,c^{2} x^{4}-70 b^{2} c\,x^{2}+105 b^{3}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{1155 x^{14} b^{4}}$	61
default	$-\frac{\left(c\,x^{2}+b\right)\left(-16 c^{3} x^{6}+40 b\,c^{2} x^{4}-70 b^{2} c\,x^{2}+105 b^{3}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{1155 x^{14} b^{4}}$	61
orering	$-\frac{\left(c\,x^{2}+b\right)\left(-16 c^{3} x^{6}+40 b\,c^{2} x^{4}-70 b^{2} c\,x^{2}+105 b^{3}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{1155 x^{14} b^{4}}$	61
pseudoelliptic	$-\frac{\left(c\,x^{2}+b\right)^{2}\left(-16 c^{3} x^{6}+40 b\,c^{2} x^{4}-70 b^{2} c\,x^{2}+105 b^{3}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{1155 x^{12} b^{4}}$	63
trager	$-\frac{\left(-16 c^{5} x^{10}+8 c^{4} x^{8} b-6 b^{2} c^{3} x^{6}+5 c^{2} x^{4} b^{3}+140 x^{2} c\,b^{4}+105 b^{5}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{1155 b^{4} x^{12}}$	76
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\left(-16 c^{5} x^{10}+8 c^{4} x^{8} b-6 b^{2} c^{3} x^{6}+5 c^{2} x^{4} b^{3}+140 x^{2} c\,b^{4}+105 b^{5}\right)}{1155 x^{12} b^{4}}$	76

input
```
int((c*x^4+b*x^2)^(3/2)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(c*x^2+b)*(-16*c^3*x^6+40*b*c^2*x^4-70*b^2*c*x^2+105*b^3)*(c*x^4+b
*x^2)^(3/2)/x^14/b^4
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = \frac{(16\,c^5x^{10}-8\,bc^4x^8+6\,b^2c^3x^6-5\,b^3c^2x^4-140\,b^4cx^2-105\,b^5)\sqrt{cx^4+bx^2}}{1155\,b^4x^{12}}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="fricas")
```

output
```
1/1155*(16*c^5*x^10 - 8*b*c^4*x^8 + 6*b^2*c^3*x^6 - 5*b^3*c^2*x^4 - 140*b^
4*c*x^2 - 105*b^5)*sqrt(c*x^4 + b*x^2)/(b^4*x^12)
```

Sympy [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^{15}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**15,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**15, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.42

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = \frac{16\,\sqrt{cx^4+bx^2}c^5}{1155\,b^4x^2} - \frac{8\,\sqrt{cx^4+bx^2}c^4}{1155\,b^3x^4} + \frac{2\,\sqrt{cx^4+bx^2}c^3}{385\,b^2x^6} - \frac{\sqrt{cx^4+bx^2}c^2}{231\,bx^8} + \frac{\sqrt{cx^4+bx^2}c}{264\,x^{10}} + \frac{3\,\sqrt{cx^4+bx^2}b}{88\,x^{12}} - \frac{(cx^4+bx^2)^{\frac{3}{2}}}{8\,x^{14}}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="maxima")
```

output
```
16/1155*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^2) - 8/1155*sqrt(c*x^4 + b*x^2)*c^4
/(b^3*x^4) + 2/385*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^6) - 1/231*sqrt(c*x^4 +
b*x^2)*c^2/(b*x^8) + 1/264*sqrt(c*x^4 + b*x^2)*c/x^10 + 3/88*sqrt(c*x^4 +
b*x^2)*b/x^12 - 1/8*(c*x^4 + b*x^2)^(3/2)/x^14
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(92) = 184$.

Time = 0.20 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.19

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{15}}\,dx = \frac{32\left(1155\left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{14}c^{\frac{11}{2}}\operatorname{sgn}(x) + 2079\left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{12}bc^{\frac{11}{2}}\operatorname{sgn}(x) + 2\right.}{}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="giac")
```

output
```
32/1155*(1155*(sqrt(c)*x - sqrt(c*x^2 + b))^14*c^(11/2)*sgn(x) + 2079*(sqr
t(c)*x - sqrt(c*x^2 + b))^12*b*c^(11/2)*sgn(x) + 2541*(sqrt(c)*x - sqrt(c*
x^2 + b))^10*b^2*c^(11/2)*sgn(x) + 825*(sqrt(c)*x - sqrt(c*x^2 + b))^8*b^3
*c^(11/2)*sgn(x) + 165*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^4*c^(11/2)*sgn(x)
 - 55*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^5*c^(11/2)*sgn(x) + 11*(sqrt(c)*x
- sqrt(c*x^2 + b))^2*b^6*c^(11/2)*sgn(x) - b^7*c^(11/2)*sgn(x))/((sqrt(c)*
x - sqrt(c*x^2 + b))^2 - b)^11
```

Mupad [B] (verification not implemented)

Time = 19.78 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.25

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{15}}\,dx = \frac{2\,c^3\,\sqrt{c\,x^4+b\,x^2}}{385\,b^2\,x^6} - \frac{4\,c\,\sqrt{c\,x^4+b\,x^2}}{33\,x^{10}} - \frac{c^2\,\sqrt{c\,x^4+b\,x^2}}{231\,b\,x^8} - \frac{b\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}} - \frac{8\,c^4\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^4} + \frac{16\,c^5\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^4\,x^2}$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^15,x)
```

output
```
(2*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (4*c*(b*x^2 + c*x^4)^(1/2))/
(33*x^10) - (c^2*(b*x^2 + c*x^4)^(1/2))/(231*b*x^8) - (b*(b*x^2 + c*x^4)^(
1/2))/(11*x^12) - (8*c^4*(b*x^2 + c*x^4)^(1/2))/(1155*b^3*x^4) + (16*c^5*(
b*x^2 + c*x^4)^(1/2))/(1155*b^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = \frac{-105\sqrt{c\,x^2+b}\,b^5-140\sqrt{c\,x^2+b}\,b^4c\,x^2-5\sqrt{c\,x^2+b}\,b^3c^2x^4+6\sqrt{c\,x^2+b}\,b^2c^3x^6-}{1155b^4x^{11}}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^15,x)
```

output
```
( - 105*sqrt(b + c*x**2)*b**5 - 140*sqrt(b + c*x**2)*b**4*c*x**2 - 5*sqrt(
b + c*x**2)*b**3*c**2*x**4 + 6*sqrt(b + c*x**2)*b**2*c**3*x**6 - 8*sqrt(b
+ c*x**2)*b*c**4*x**8 + 16*sqrt(b + c*x**2)*c**5*x**10 - 16*sqrt(c)*c**5*x
**11)/(1155*b**4*x**11)
```

3.180 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{17}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17}}\,dx=-\frac{(bx^2+cx^4)^{5/2}}{13bx^{18}}+\frac{8c(bx^2+cx^4)^{5/2}}{143b^2x^{16}}-\frac{16c^2(bx^2+cx^4)^{5/2}}{429b^3x^{14}}+\frac{64c^3(bx^2+cx^4)^{5/2}}{3003b^4x^{12}}-\frac{128c^4(bx^2+cx^4)^{5/2}}{15015b^5x^{10}}$$

output
```
-1/13*(c*x^4+b*x^2)^(5/2)/b/x^18+8/143*c*(c*x^4+b*x^2)^(5/2)/b^2/x^16-16/4
29*c^2*(c*x^4+b*x^2)^(5/2)/b^3/x^14+64/3003*c^3*(c*x^4+b*x^2)^(5/2)/b^4/x^
12-128/15015*c^4*(c*x^4+b*x^2)^(5/2)/b^5/x^10
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17}}\,dx=\frac{(x^2(b+cx^2))^{5/2}(-1155b^4+840b^3cx^2-560b^2c^2x^4+320bc^3x^6-128c^4x^8)}{15015b^5x^{18}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

output
```
((x^2*(b + c*x^2))^(5/2)*(-1155*b^4 + 840*b^3*c*x^2 - 560*b^2*c^2*x^4 + 32
0*b*c^3*x^6 - 128*c^4*x^8))/(15015*b^5*x^18)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1423, 1423, 1423, 1423, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{17}}dx$$

$$\Big\downarrow \text{1423}$$

$$-\frac{8c\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^{15}}dx}{13b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

$$\Big\downarrow \text{1423}$$

$$-\frac{8c\left(-\frac{6c\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^{13}}dx}{11b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}\right)}{13b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

$$\Big\downarrow \text{1423}$$

$$-\frac{8c\left(-\frac{6c\left(-\frac{4c\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^{11}}dx}{9b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}\right)}{13b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

$$\Big\downarrow \text{1423}$$

$$-\frac{8c\left(-\frac{6c\left(-\frac{4c\left(-\frac{2c\int\frac{\left(cx^4+bx^2\right)^{3/2}}{x^9}dx}{7b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}\right)}{13b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

$$\Big\downarrow \text{1422}$$

$$-\frac{8c\left(-\frac{6c\left(-\frac{4c\left(\frac{2c\left(bx^2+cx^4\right)^{5/2}}{35b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}\right)}{9b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}\right)}{11b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}\right)}{13b}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

input

```
Int[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

output

```
-1/13*(b*x^2 + c*x^4)^(5/2)/(b*x^18) - (8*c*(-1/11*(b*x^2 + c*x^4)^(5/2)/(
b*x^16) - (6*c*(-1/9*(b*x^2 + c*x^4)^(5/2)/(b*x^14) - (4*c*(-1/7*(b*x^2 +
c*x^4)^(5/2)/(b*x^12) + (2*c*(b*x^2 + c*x^4)^(5/2))/(35*b^2*x^10)))/(9*b))
)/(11*b)))/(13*b)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{(c\,x^2+b)\left(128c^4x^8-320b\,c^3x^6+560b^2c^2x^4-840x^2b^3c+1155b^4\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{15015x^{16}b^5}$	72
default	$-\frac{(c\,x^2+b)\left(128c^4x^8-320b\,c^3x^6+560b^2c^2x^4-840x^2b^3c+1155b^4\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{15015x^{16}b^5}$	72
orering	$-\frac{(c\,x^2+b)\left(128c^4x^8-320b\,c^3x^6+560b^2c^2x^4-840x^2b^3c+1155b^4\right)\left(c\,x^4+b\,x^2\right)^{\frac{3}{2}}}{15015x^{16}b^5}$	72
pseudoelliptic	$-\frac{(c\,x^2+b)^2\left(128c^4x^8-320b\,c^3x^6+560b^2c^2x^4-840x^2b^3c+1155b^4\right)\sqrt{x^2(c\,x^2+b)}}{15015x^{14}b^5}$	74
trager	$-\frac{\left(128c^6x^{12}-64b\,c^5x^{10}+48b^2c^4x^8-40b^3c^3x^6+35b^4c^2x^4+1470b^5c\,x^2+1155b^6\right)\sqrt{c\,x^4+b\,x^2}}{15015b^5x^{14}}$	87
risch	$-\frac{\sqrt{x^2(c\,x^2+b)}\left(128c^6x^{12}-64b\,c^5x^{10}+48b^2c^4x^8-40b^3c^3x^6+35b^4c^2x^4+1470b^5c\,x^2+1155b^6\right)}{15015x^{14}b^5}$	87

input
```
int((c*x^4+b*x^2)^(3/2)/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/15015*(c*x^2+b)*(128*c^4*x^8-320*b*c^3*x^6+560*b^2*c^2*x^4-840*b^3*c*x^
2+1155*b^4)*(c*x^4+b*x^2)^(3/2)/x^16/b^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.63

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = -\frac{(128\,c^6x^{12}-64\,bc^5x^{10}+48\,b^2c^4x^8-40\,b^3c^3x^6+35\,b^4c^2x^4+1470\,b^5cx^2+1155\,b^6)\sqrt{cx^4+bx^2}}{15015\,b^5x^{14}}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="fricas")
```

output
```
-1/15015*(128*c^6*x^12 - 64*b*c^5*x^10 + 48*b^2*c^4*x^8 - 40*b^3*c^3*x^6 +
 35*b^4*c^2*x^4 + 1470*b^5*c*x^2 + 1155*b^6)*sqrt(c*x^4 + b*x^2)/(b^5*x^14
)
```

Sympy [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^{17}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**17,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**17, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.30

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = -\frac{128\,\sqrt{cx^4+bx^2}c^6}{15015\,b^5x^2}+\frac{64\,\sqrt{cx^4+bx^2}c^5}{15015\,b^4x^4}-\frac{16\,\sqrt{cx^4+bx^2}c^4}{5005\,b^3x^6}+\frac{8\,\sqrt{cx^4+bx^2}c^3}{3003\,b^2x^8}-\frac{\sqrt{cx^4+bx^2}c^2}{429\,bx^{10}}+\frac{3\,\sqrt{cx^4+bx^2}c}{1430\,x^{12}}+\frac{3\,\sqrt{cx^4+bx^2}b}{130\,x^{14}}-\frac{(cx^4+bx^2)^{\frac{3}{2}}}{10\,x^{16}}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="maxima")
```

output
```
-128/15015*sqrt(c*x^4 + b*x^2)*c^6/(b^5*x^2) + 64/15015*sqrt(c*x^4 + b*x^2
)*c^5/(b^4*x^4) - 16/5005*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^6) + 8/3003*sqrt(
c*x^4 + b*x^2)*c^3/(b^2*x^8) - 1/429*sqrt(c*x^4 + b*x^2)*c^2/(b*x^10) + 3/
1430*sqrt(c*x^4 + b*x^2)*c/x^12 + 3/130*sqrt(c*x^4 + b*x^2)*b/x^14 - 1/10*
(c*x^4 + b*x^2)^(3/2)/x^16
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(116) = 232$.

Time = 0.15 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.94

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{17}} dx = \frac{256 \left(6006 \left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^{16} c^{\frac{13}{2}} \operatorname{sgn}(x) + 12012 \left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^{14} bc^{\frac{13}{2}} \operatorname{sgn}(x) + \right.}{}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="giac")
```

output
```
256/15015*(6006*(sqrt(c)*x - sqrt(c*x^2 + b))^16*c^(13/2)*sgn(x) + 12012*(
sqrt(c)*x - sqrt(c*x^2 + b))^14*b*c^(13/2)*sgn(x) + 13728*(sqrt(c)*x - sqr
t(c*x^2 + b))^12*b^2*c^(13/2)*sgn(x) + 4719*(sqrt(c)*x - sqrt(c*x^2 + b))^
10*b^3*c^(13/2)*sgn(x) + 715*(sqrt(c)*x - sqrt(c*x^2 + b))^8*b^4*c^(13/2)*
sgn(x) - 286*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b^5*c^(13/2)*sgn(x) + 78*(sqr
t(c)*x - sqrt(c*x^2 + b))^4*b^6*c^(13/2)*sgn(x) - 13*(sqrt(c)*x - sqrt(c*x
^2 + b))^2*b^7*c^(13/2)*sgn(x) + b^8*c^(13/2)*sgn(x))/((sqrt(c)*x - sqrt(c
*x^2 + b))^2 - b)^13
```

Mupad [B] (verification not implemented)

Time = 19.87 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.17

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{17}} dx = \frac{8\,c^3\,\sqrt{c\,x^4 + b\,x^2}}{3003\,b^2\,x^8} - \frac{14\,c\,\sqrt{c\,x^4 + b\,x^2}}{143\,x^{12}} - \frac{c^2\,\sqrt{c\,x^4 + b\,x^2}}{429\,b\,x^{10}} - \frac{b\,\sqrt{c\,x^4 + b\,x^2}}{13\,x^{14}} - \frac{16\,c^4\,\sqrt{c\,x^4 + b\,x^2}}{5005\,b^3\,x^6} + \frac{64\,c^5\,\sqrt{c\,x^4 + b\,x^2}}{15015\,b^4\,x^4} - \frac{128\,c^6\,\sqrt{c\,x^4 + b\,x^2}}{15015\,b^5\,x^2}$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^17,x)
```

output
```
(8*c^3*(b*x^2 + c*x^4)^(1/2))/(3003*b^2*x^8) - (14*c*(b*x^2 + c*x^4)^(1/2)
)/(143*x^12) - (c^2*(b*x^2 + c*x^4)^(1/2))/(429*b*x^10) - (b*(b*x^2 + c*x^
4)^(1/2))/(13*x^14) - (16*c^4*(b*x^2 + c*x^4)^(1/2))/(5005*b^3*x^6) + (64*
c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^4) - (128*c^6*(b*x^2 + c*x^4)^(1/2
))/(15015*b^5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.02

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \frac{-1155\sqrt{c\,x^2+b}\,b^6-1470\sqrt{c\,x^2+b}\,b^5c\,x^2-35\sqrt{c\,x^2+b}\,b^4c^2x^4+40\sqrt{c\,x^2+b}\,b^3c^3}{15015}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^17,x)
```

output
```
( - 1155*sqrt(b + c*x**2)*b**6 - 1470*sqrt(b + c*x**2)*b**5*c*x**2 - 35*sq
rt(b + c*x**2)*b**4*c**2*x**4 + 40*sqrt(b + c*x**2)*b**3*c**3*x**6 - 48*sq
rt(b + c*x**2)*b**2*c**4*x**8 + 64*sqrt(b + c*x**2)*b*c**5*x**10 - 128*sqr
t(b + c*x**2)*c**6*x**12 + 128*sqrt(c)*c**6*x**13)/(15015*b**5*x**13)
```

3.181 $\int x^6(bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\int x^6\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{128b^4(bx^2+cx^4)^{5/2}}{15015c^5x^5}-\frac{64b^3(bx^2+cx^4)^{5/2}}{3003c^4x^3}+\frac{16b^2(bx^2+cx^4)^{5/2}}{429c^3x}-\frac{8bx(bx^2+cx^4)^{5/2}}{143c^2}+\frac{x^3(bx^2+cx^4)^{5/2}}{13c}$$

output
```
128/15015*b^4*(c*x^4+b*x^2)^(5/2)/c^5/x^5-64/3003*b^3*(c*x^4+b*x^2)^(5/2)/
c^4/x^3+16/429*b^2*(c*x^4+b*x^2)^(5/2)/c^3/x-8/143*b*x*(c*x^4+b*x^2)^(5/2)
/c^2+1/13*x^3*(c*x^4+b*x^2)^(5/2)/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.51

$$\int x^6\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\left(x^2(b+cx^2)\right)^{5/2}\left(128b^4-320b^3cx^2+560b^2c^2x^4-840bc^3x^6+1155c^4x^8\right)}{15015c^5x^5}$$

input
```
Integrate[x^6*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((x^2*(b + c*x^2))^(5/2)*(128*b^4 - 320*b^3*c*x^2 + 560*b^2*c^2*x^4 - 840*
b*c^3*x^6 + 1155*c^4*x^8))/(15015*c^5*x^5)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1421, 1421, 1421, 1399, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^3 \left(bx^2+cx^4\right)^{5/2}}{13c}-\frac{8b\int x^4 \left(cx^4+bx^2\right)^{3/2}dx}{13c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^3 \left(bx^2+cx^4\right)^{5/2}}{13c}-\frac{8b\left(\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{6b\int x^2 \left(cx^4+bx^2\right)^{3/2}dx}{11c}\right)}{13c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^3 \left(bx^2+cx^4\right)^{5/2}}{13c}-\frac{8b\left(\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{6b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b\int \left(cx^4+bx^2\right)^{3/2}dx}{9c}\right)}{11c}\right)}{13c}$$

$$\Big\downarrow \text{1399}$$

$$\frac{x^3(bx^2+cx^4)^{5/2}}{13c}-\frac{8b\left(\frac{x(bx^2+cx^4)^{5/2}}{11c}-\frac{6b\left(\frac{(bx^2+cx^4)^{5/2}}{9cx}-\frac{4b\left(\frac{(bx^2+cx^4)^{5/2}}{7cx^3}-\frac{2b\int\frac{(cx^4+bx^2)^{3/2}}{x^2}dx}{7c}\right)}{9c}\right)}{11c}\right)}{13c}$$

$\Big\downarrow$ 1420

$$\frac{x^3(bx^2+cx^4)^{5/2}}{13c}-\frac{8b\left(\frac{x(bx^2+cx^4)^{5/2}}{11c}-\frac{6b\left(\frac{(bx^2+cx^4)^{5/2}}{9cx}-\frac{4b\left(\frac{(bx^2+cx^4)^{5/2}}{7cx^3}-\frac{2b(bx^2+cx^4)^{5/2}}{35c^2x^5}\right)}{9c}\right)}{11c}\right)}{13c}$$

input
```
Int[x^6*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^3*(b*x^2 + c*x^4)^(5/2))/(13*c) - (8*b*((x*(b*x^2 + c*x^4)^(5/2))/(11*c
) - (6*b*((b*x^2 + c*x^4)^(5/2)/(9*c*x) - (4*b*((-2*b*(b*x^2 + c*x^4)^(5/2
))/(35*c^2*x^5) + (b*x^2 + c*x^4)^(5/2)/(7*c*x^3)))/(9*c)))/(11*c)))/(13*c
)
```

Defintions of rubi rules used

rule 1399
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(b*x^2 + c*x^4)^(
p + 1)/(c*(4*p + 1)*x^3), x] - Simp[b*((2*p - 1)/(c*(4*p + 1)))   Int[(b*x^
2 + c*x^4)^p/x^2, x], x] /; FreeQ[{b, c, p}, x] && IGtQ[p - 1/2, 0]
```

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] && !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] && !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.54

method	result	size
gosper	$\frac{\left(c\,x^{2}+b\right)\left(1155 c^{4} x^{8}-840 b\,c^{3} x^{6}+560 b^{2} c^{2} x^{4}-320 x^{2} b^{3} c+128 b^{4}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{15015 c^{5} x^{3}}$	72
default	$\frac{\left(c\,x^{2}+b\right)\left(1155 c^{4} x^{8}-840 b\,c^{3} x^{6}+560 b^{2} c^{2} x^{4}-320 x^{2} b^{3} c+128 b^{4}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{15015 c^{5} x^{3}}$	72
orering	$\frac{\left(c\,x^{2}+b\right)\left(1155 c^{4} x^{8}-840 b\,c^{3} x^{6}+560 b^{2} c^{2} x^{4}-320 x^{2} b^{3} c+128 b^{4}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{15015 c^{5} x^{3}}$	72
trager	$\frac{\left(1155 c^{6} x^{12}+1470 b\,c^{5} x^{10}+35 b^{2} c^{4} x^{8}-40 b^{3} c^{3} x^{6}+48 b^{4} c^{2} x^{4}-64 b^{5} c\,x^{2}+128 b^{6}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{15015 c^{5} x}$	87
risch	$\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(1155 c^{6} x^{12}+1470 b\,c^{5} x^{10}+35 b^{2} c^{4} x^{8}-40 b^{3} c^{3} x^{6}+48 b^{4} c^{2} x^{4}-64 b^{5} c\,x^{2}+128 b^{6}\right)}{15015 x\,c^{5}}$	87

input
```
int(x^6*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15015*(c*x^2+b)*(1155*c^4*x^8-840*b*c^3*x^6+560*b^2*c^2*x^4-320*b^3*c*x^
2+128*b^4)*(c*x^4+b*x^2)^(3/2)/c^5/x^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.64

$$\int x^6(bx^2+cx^4)^{3/2}\,dx = \frac{(1155\,c^6x^{12}+1470\,bc^5x^{10}+35\,b^2c^4x^8-40\,b^3c^3x^6+48\,b^4c^2x^4-64\,b^5cx^2+128\,b^6)\sqrt{cx^4+b}}{15015\,c^5x}$$

input
```
integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15015*(1155*c^6*x^12 + 1470*b*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6
 + 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*sqrt(c*x^4 + b*x^2)/(c^5*x)
```

Sympy [F]

$$\int x^6(bx^2+cx^4)^{3/2}\,dx = \int x^6\left(x^2(b+cx^2)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**6*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**6*(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.59

$$\int x^6(bx^2+cx^4)^{3/2}\,dx = \frac{(1155\,c^6x^{12}+1470\,bc^5x^{10}+35\,b^2c^4x^8-40\,b^3c^3x^6+48\,b^4c^2x^4-64\,b^5cx^2+128\,b^6)\sqrt{cx^2+b}}{15015\,c^5}$$

input
```
integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/15015*(1155*c^6*x^12 + 1470*b*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6
 + 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*sqrt(c*x^2 + b)/c^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.69

$$\int x^6 (bx^2 + cx^4)^{3/2} \, dx = -\frac{128 \, b^{\frac{13}{2}} \mathrm{sgn}(x)}{15015 \, c^5} + \frac{1155 \, (cx^2 + b)^{\frac{13}{2}} \mathrm{sgn}(x) - 5460 \, (cx^2 + b)^{\frac{11}{2}} b \mathrm{sgn}(x) + 10010 \, (cx^2 + b)^{\frac{9}{2}} b^2 \mathrm{sgn}(x) - 8580 \, (cx^2 + b)^{\frac{7}{2}} b^3 \mathrm{sgn}(x)}{15015 \, c^5}$$

input
```
integrate(x^6*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-128/15015*b^(13/2)*sgn(x)/c^5 + 1/15015*(1155*(c*x^2 + b)^(13/2)*sgn(x) -
 5460*(c*x^2 + b)^(11/2)*b*sgn(x) + 10010*(c*x^2 + b)^(9/2)*b^2*sgn(x) - 8
580*(c*x^2 + b)^(7/2)*b^3*sgn(x) + 3003*(c*x^2 + b)^(5/2)*b^4*sgn(x))/c^5
```

Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.54

$$\int x^6 (bx^2 + cx^4)^{3/2} \, dx = \frac{(c \, x^2 + b)^2 \sqrt{c \, x^4 + b \, x^2} \, (128 \, b^4 - 320 \, b^3 \, c \, x^2 + 560 \, b^2 \, c^2 \, x^4 - 840 \, b \, c^3 \, x^6 + 1155 \, c^4 \, x^8)}{15015 \, c^5 \, x}$$

input
```
int(x^6*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(128*b^4 + 1155*c^4*x^8 - 320*b^3*c*x
^2 - 840*b*c^3*x^6 + 560*b^2*c^2*x^4))/(15015*c^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.58

$$\int x^6 \left(bx^2+cx^4\right)^{3/2}\,dx = \frac{\sqrt{c\,x^2+b}\,(1155c^6x^{12}+1470b\,c^5x^{10}+35b^2c^4x^8-40b^3c^3x^6+48b^4c^2x^4-64b^5c\,x^2+128b^6)}{15015c^5}$$

input
```
int(x^6*(c*x^4+b*x^2)^(3/2),x)
```

output
```
(sqrt(b + c*x**2)*(128*b**6 - 64*b**5*c*x**2 + 48*b**4*c**2*x**4 - 40*b**3
*c**3*x**6 + 35*b**2*c**4*x**8 + 1470*b*c**5*x**10 + 1155*c**6*x**12))/(15
015*c**5)
```

3.182 $\int x^4 (bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 106

$$\int x^4 (bx^2+cx^4)^{3/2}\,dx = -\frac{16b^3(bx^2+cx^4)^{5/2}}{1155c^4x^5} + \frac{8b^2(bx^2+cx^4)^{5/2}}{231c^3x^3} - \frac{2b(bx^2+cx^4)^{5/2}}{33c^2x} + \frac{x(bx^2+cx^4)^{5/2}}{11c}$$

output
```
-16/1155*b^3*(c*x^4+b*x^2)^(5/2)/c^4/x^5+8/231*b^2*(c*x^4+b*x^2)^(5/2)/c^3
/x^3-2/33*b*(c*x^4+b*x^2)^(5/2)/c^2/x+1/11*x*(c*x^4+b*x^2)^(5/2)/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54

$$\int x^4 (bx^2+cx^4)^{3/2}\,dx = \frac{(x^2(b+cx^2))^{5/2}(-16b^3+40b^2cx^2-70bc^2x^4+105c^3x^6)}{1155c^4x^5}$$

input
```
Integrate[x^4*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((x^2*(b + c*x^2))^(5/2)*(-16*b^3 + 40*b^2*c*x^2 - 70*b*c^2*x^4 + 105*c^3*
x^6))/(1155*c^4*x^5)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1421, 1421, 1399, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(bx^2+cx^4\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1421}$$

$$\frac{x \left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{6b \int x^2 \left(cx^4+bx^2\right)^{3/2}dx}{11c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{x \left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{6b \left(\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b \int \left(cx^4+bx^2\right)^{3/2}dx}{9c}\right)}{11c}$$

$$\Big\downarrow \text{1399}$$

$$\frac{x \left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{6b \left(\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b \left(\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b \int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^2}dx}{7c}\right)}{9c}\right)}{11c}$$

$$\Big\downarrow \text{1420}$$

$$\frac{x \left(bx^2+cx^4\right)^{5/2}}{11c}-\frac{6b \left(\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b \left(\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b \left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}\right)}{9c}\right)}{11c}$$

input
```
Int[x^4*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(b*x^2 + c*x^4)^(5/2))/(11*c) - (6*b*((b*x^2 + c*x^4)^(5/2)/(9*c*x) - (
4*b*((-2*b*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (b*x^2 + c*x^4)^(5/2)/(7*
c*x^3)))/(9*c)))/(11*c)
```

Defintions of rubi rules used

rule 1399
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(b*x^2 + c*x^4)^(
p + 1)/(c*(4*p + 1)*x^3), x] - Simp[b*((2*p - 1)/(c*(4*p + 1)))   Int[(b*x^
2 + c*x^4)^p/x^2, x], x] /; FreeQ[{b, c, p}, x] && IGtQ[p - 1/2, 0]
```

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{(c\,x^{2}+b)\left(-105c^{3}x^{6}+70b\,c^{2}x^{4}-40b^{2}c\,x^{2}+16b^{3}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{1155c^{4}x^{3}}$	61
default	$-\frac{(c\,x^{2}+b)\left(-105c^{3}x^{6}+70b\,c^{2}x^{4}-40b^{2}c\,x^{2}+16b^{3}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{1155c^{4}x^{3}}$	61
orering	$-\frac{(c\,x^{2}+b)\left(-105c^{3}x^{6}+70b\,c^{2}x^{4}-40b^{2}c\,x^{2}+16b^{3}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{1155c^{4}x^{3}}$	61
trager	$-\frac{\left(-105c^{5}x^{10}-140c^{4}x^{8}b-5b^{2}c^{3}x^{6}+6c^{2}x^{4}b^{3}-8x^{2}c\,b^{4}+16b^{5}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{1155c^{4}x}$	76
risch	$-\frac{\sqrt{x^{2}(c\,x^{2}+b)}\,\left(-105c^{5}x^{10}-140c^{4}x^{8}b-5b^{2}c^{3}x^{6}+6c^{2}x^{4}b^{3}-8x^{2}c\,b^{4}+16b^{5}\right)}{1155x\,c^{4}}$	76

input
```
int(x^4*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(c*x^2+b)*(-105*c^3*x^6+70*b*c^2*x^4-40*b^2*c*x^2+16*b^3)*(c*x^4+b
*x^2)^(3/2)/c^4/x^3
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

$$\int x^4(bx^2+cx^4)^{3/2}\,dx=\frac{(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5)\sqrt{cx^4+bx^2}}{1155\,c^4x}$$

input
```
integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b
^4*c*x^2 - 16*b^5)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

Sympy [F]

$$\int x^4(bx^2+cx^4)^{3/2}\,dx=\int x^4\left(x^2(b+cx^2)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**4*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**4*(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int x^4(bx^2+cx^4)^{3/2}\,dx = \frac{(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5)\sqrt{cx^2+b}}{1155\,c^4}$$

input
```
integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b
^4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)/c^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int x^4(bx^2+cx^4)^{3/2}\,dx = \frac{16\,b^{\frac{11}{2}}\mathrm{sgn}(x)}{1155\,c^4} + \frac{105\,(cx^2+b)^{\frac{11}{2}}\mathrm{sgn}(x)-385\,(cx^2+b)^{\frac{9}{2}}b\mathrm{sgn}(x)+495\,(cx^2+b)^{\frac{7}{2}}b^2\mathrm{sgn}(x)-231\,(cx^2+b)^{\frac{5}{2}}b^3\mathrm{sgn}(x)}{1155\,c^4}$$

input
```
integrate(x^4*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
16/1155*b^(11/2)*sgn(x)/c^4 + 1/1155*(105*(c*x^2 + b)^(11/2)*sgn(x) - 385*
(c*x^2 + b)^(9/2)*b*sgn(x) + 495*(c*x^2 + b)^(7/2)*b^2*sgn(x) - 231*(c*x^2
 + b)^(5/2)*b^3*sgn(x))/c^4
```

Mupad [B] (verification not implemented)

Time = 17.80 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int x^4 \left(bx^2+cx^4\right)^{3/2} \, dx = -\frac{\left(c\,x^2+b\right)^2\,\sqrt{c\,x^4+b\,x^2}\,\left(16\,b^3-40\,b^2\,c\,x^2+70\,b\,c^2\,x^4-105\,c^3\,x^6\right)}{1155\,c^4\,x}$$

input
```
int(x^4*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
-((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(16*b^3 - 105*c^3*x^6 - 40*b^2*c*x^2
 + 70*b*c^2*x^4))/(1155*c^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

$$\int x^4 \left(bx^2 + cx^4\right)^{3/2} \, dx = \frac{\sqrt{c\,x^2+b}\,(105c^5x^{10}+140b\,c^4x^8+5b^2c^3x^6-6b^3c^2x^4+8b^4c\,x^2-16b^5)}{1155c^4}$$

input
```
int(x^4*(c*x^4+b*x^2)^(3/2),x)
```

output
```
(sqrt(b + c*x**2)*( - 16*b**5 + 8*b**4*c*x**2 - 6*b**3*c**2*x**4 + 5*b**2*
c**3*x**6 + 140*b*c**4*x**8 + 105*c**5*x**10))/(1155*c**4)
```

3.183 $\int x^2(bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int x^2\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{8b^2(bx^2+cx^4)^{5/2}}{315c^3x^5}-\frac{4b(bx^2+cx^4)^{5/2}}{63c^2x^3}+\frac{(bx^2+cx^4)^{5/2}}{9cx}$$

output
```
8/315*b^2*(c*x^4+b*x^2)^(5/2)/c^3/x^5-4/63*b*(c*x^4+b*x^2)^(5/2)/c^2/x^3+1
/9*(c*x^4+b*x^2)^(5/2)/c/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int x^2\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\left(x^2(b+cx^2)\right)^{5/2}\left(8b^2-20bcx^2+35c^2x^4\right)}{315c^3x^5}$$

input
```
Integrate[x^2*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((x^2*(b + c*x^2))^(5/2)*(8*b^2 - 20*b*c*x^2 + 35*c^2*x^4))/(315*c^3*x^5)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1421, 1399, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(bx^2+cx^4\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1421}$$

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b\int \left(cx^4+bx^2\right)^{3/2}dx}{9c}$$

$$\Big\downarrow \text{1399}$$

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^2}dx}{7c}\right)}{9c}$$

$$\Big\downarrow \text{1420}$$

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}-\frac{4b\left(\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}\right)}{9c}$$

input
```
Int[x^2*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(b*x^2 + c*x^4)^(5/2)/(9*c*x) - (4*b*((-2*b*(b*x^2 + c*x^4)^(5/2))/(35*c^2
*x^5) + (b*x^2 + c*x^4)^(5/2)/(7*c*x^3)))/(9*c)
```

Defintions of rubi rules used

rule 1399
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(b*x^2 + c*x^4)^(
p + 1)/(c*(4*p + 1)*x^3), x] - Simp[b*((2*p - 1)/(c*(4*p + 1)))   Int[(b*x^
2 + c*x^4)^p/x^2, x], x] /; FreeQ[{b, c, p}, x] && IGtQ[p - 1/2, 0]
```

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{(c\,x^{2}+b)\left(35c^{2}x^{4}-20bc\,x^{2}+8b^{2}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{315c^{3}x^{3}}$	50
default	$\frac{(c\,x^{2}+b)\left(35c^{2}x^{4}-20bc\,x^{2}+8b^{2}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{315c^{3}x^{3}}$	50
orering	$\frac{(c\,x^{2}+b)\left(35c^{2}x^{4}-20bc\,x^{2}+8b^{2}\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{315c^{3}x^{3}}$	50
trager	$\frac{\left(35c^{4}x^{8}+50b\,c^{3}x^{6}+3b^{2}c^{2}x^{4}-4x^{2}b^{3}c+8b^{4}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{315c^{3}x}$	65
risch	$\frac{\sqrt{x^{2}(c\,x^{2}+b)}\,\left(35c^{4}x^{8}+50b\,c^{3}x^{6}+3b^{2}c^{2}x^{4}-4x^{2}b^{3}c+8b^{4}\right)}{315x\,c^{3}}$	65

input
```
int(x^2*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(c*x^2+b)*(35*c^2*x^4-20*b*c*x^2+8*b^2)*(c*x^4+b*x^2)^(3/2)/c^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^2 \left(bx^2+cx^4\right)^{3/2} \, dx = \frac{(35\, c^4x^8 + 50\, bc^3x^6 + 3\, b^2c^2x^4 - 4\, b^3cx^2 + 8\, b^4)\sqrt{cx^4+bx^2}}{315\, c^3x}$$

input
```
integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sq
rt(c*x^4 + b*x^2)/(c^3*x)
```

Sympy [F]

$$\int x^2 \left(bx^2+cx^4\right)^{3/2} \, dx = \int x^2 \left(x^2 \left(b+cx^2\right)\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**2*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**2*(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^2 \left(bx^2+cx^4\right)^{3/2} \, dx = \frac{(35\, c^4x^8 + 50\, bc^3x^6 + 3\, b^2c^2x^4 - 4\, b^3cx^2 + 8\, b^4)\sqrt{cx^2+b}}{315\, c^3}$$

input
```
integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sq
rt(c*x^2 + b)/c^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int x^2 \left(bx^2+cx^4\right)^{3/2} dx = -\frac{8 b^{\frac{9}{2}} \text{sgn}(x)}{315 c^3} + \frac{35 \left(cx^2+b\right)^{\frac{9}{2}} \text{sgn}(x) - 90 \left(cx^2+b\right)^{\frac{7}{2}} b\text{sgn}(x) + 63 \left(cx^2+b\right)^{\frac{5}{2}} b^2 \text{sgn}(x)}{315 c^3}$$

input
```
integrate(x^2*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-8/315*b^(9/2)*sgn(x)/c^3 + 1/315*(35*(c*x^2 + b)^(9/2)*sgn(x) - 90*(c*x^2
 + b)^(7/2)*b*sgn(x) + 63*(c*x^2 + b)^(5/2)*b^2*sgn(x))/c^3
```

Mupad [B] (verification not implemented)

Time = 18.71 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int x^2 \left(bx^2+cx^4\right)^{3/2} dx = \frac{\left(c x^2+b\right)^2 \sqrt{c x^4+b x^2} \left(8 b^2-20 b c x^2+35 c^2 x^4\right)}{315 c^3 x}$$

input
```
int(x^2*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(8*b^2 + 35*c^2*x^4 - 20*b*c*x^2))/(3
15*c^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int x^2 \left(bx^2+cx^4\right)^{3/2} dx = \frac{\sqrt{c x^2+b} \left(35c^4x^8+50b c^3x^6+3b^2c^2x^4-4b^3c x^2+8b^4\right)}{315c^3}$$

input
```
int(x^2*(c*x^4+b*x^2)^(3/2),x)
```

output
```
(sqrt(b + c*x**2)*(8*b**4 - 4*b**3*c*x**2 + 3*b**2*c**2*x**4 + 50*b*c**3*x
**6 + 35*c**4*x**8))/(315*c**3)
```

3.184 $\int (bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \left(bx^2+cx^4\right)^{3/2}\,dx=-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}+\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}$$

output
```
-2/35*b*(c*x^4+b*x^2)^(5/2)/c^2/x^5+1/7*(c*x^4+b*x^2)^(5/2)/c/x^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(-2b+5cx^2\right)}{35c^2x^5}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((x^2*(b + c*x^2))^(5/2)*(-2*b + 5*c*x^2))/(35*c^2*x^5)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1399, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2+cx^4\right)^{3/2}\,dx$$

$$\Big\downarrow \ 1399$$

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\int \frac{\left(cx^4+bx^2\right)^{3/2}}{x^2}dx}{7c}$$

$$\Big\downarrow \ 1420$$

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-2*b*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (b*x^2 + c*x^4)^(5/2)/(7*c*x^3
)
```

Defintions of rubi rules used

rule 1399
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(b*x^2 + c*x^4)^(
p + 1)/(c*(4*p + 1)*x^3), x] - Simp[b*((2*p - 1)/(c*(4*p + 1)))   Int[(b*x^
2 + c*x^4)^p/x^2, x], x] /; FreeQ[{b, c, p}, x] && IGtQ[p - 1/2, 0]
```

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-5c\,x^{2}+2b\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{35c^{2}x^{3}}$	39
default	$-\frac{\left(c\,x^{2}+b\right)\left(-5c\,x^{2}+2b\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{35c^{2}x^{3}}$	39
orering	$-\frac{\left(c\,x^{2}+b\right)\left(-5c\,x^{2}+2b\right)\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}{35c^{2}x^{3}}$	39
trager	$-\frac{\left(-5c^{3}x^{6}-8b\,c^{2}x^{4}-b^{2}c\,x^{2}+2b^{3}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{35c^{2}x}$	54
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(-5c^{3}x^{6}-8b\,c^{2}x^{4}-b^{2}c\,x^{2}+2b^{3}\right)}{35x\,c^{2}}$	54

input
```
int((c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(c*x^2+b)*(-5*c*x^2+2*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \left(bx^2+cx^4\right)^{3/2}\,dx=\frac{(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3)\sqrt{cx^4+bx^2}}{35\,c^2x}$$

input
```
integrate((c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^4 + b*x^2)/(c^
2*x)
```

Sympy [F]

$$\int \left(bx^2+cx^4\right)^{3/2}\,dx=\int \left(bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral((b*x**2 + c*x**4)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\left(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3\right)\sqrt{cx^2+b}}{35\,c^2}$$

input
```
integrate((c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2\,b^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,c^2}+\frac{5\,(cx^2+b)^{\frac{7}{2}}\mathrm{sgn}(x)-7\,(cx^2+b)^{\frac{5}{2}}b\mathrm{sgn}(x)}{35\,c^2}$$

input
```
integrate((c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
2/35*b^(7/2)*sgn(x)/c^2 + 1/35*(5*(c*x^2 + b)^(7/2)*sgn(x) - 7*(c*x^2 + b)
^(5/2)*b*sgn(x))/c^2
```

Mupad [B] (verification not implemented)

Time = 19.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \left(bx^2+cx^4\right)^{3/2} dx = -\frac{\left(c\,x^2+b\right)^2\,\sqrt{c\,x^4+b\,x^2}\,\left(2\,b-5\,c\,x^2\right)}{35\,c^2\,x}$$

input
```
int((b*x^2 + c*x^4)^(3/2),x)
```

output
```
-((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2)*(2*b - 5*c*x^2))/(35*c^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(bx^2+cx^4\right)^{3/2} dx = \frac{\sqrt{c\,x^2+b}\,(5c^3x^6+8b\,c^2x^4+b^2c\,x^2-2b^3)}{35c^2}$$

input
```
int((c*x^4+b*x^2)^(3/2),x)
```

output
```
(sqrt(b + c*x**2)*( - 2*b**3 + b**2*c*x**2 + 8*b*c**2*x**4 + 5*c**3*x**6))
/(35*c**2)
```

3.185 $\int \frac{(bx^2+cx^4)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^2} dx = \frac{(bx^2+cx^4)^{5/2}}{5cx^5}$$

output
```
1/5*(c*x^4+b*x^2)^(5/2)/c/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^2} dx = \frac{(x^2(b+cx^2))^{5/2}}{5cx^5}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^2,x]
```

output
```
(x^2*(b + c*x^2))^(5/2)/(5*c*x^5)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^2} \, dx$$

$$\Big\downarrow \; 1420$$

$$\frac{(bx^2 + cx^4)^{5/2}}{5cx^5}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^2,x]
```

output
```
(b*x^2 + c*x^4)^(5/2)/(5*c*x^5)
```

Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{(c x^2+b)(c x^4+b x^2)^{\frac{3}{2}}}{5c x^3}$	29
default	$\frac{(c x^2+b)(c x^4+b x^2)^{\frac{3}{2}}}{5c x^3}$	29
orering	$\frac{(c x^2+b)(c x^4+b x^2)^{\frac{3}{2}}}{5c x^3}$	29
trager	$\frac{(c^2 x^4+2bc x^2+b^2)\sqrt{c x^4+b x^2}}{5cx}$	40
risch	$\frac{\sqrt{x^2(c x^2+b)}\,(c^2 x^4+2bc x^2+b^2)}{5xc}$	40

input
```
int((c*x^4+b*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*(c*x^2+b)/c/x^3*(c*x^4+b*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{(c^2x^4+2\,bcx^2+b^2)\sqrt{cx^4+bx^2}}{5\,cx}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^4 + b*x^2)/(c*x)
```

Sympy [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**2,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^2}\, dx = \frac{(c^2x^4 + 2\,bcx^2 + b^2)\sqrt{cx^2 + b}}{5\,c}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^2 + b)/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^2}\, dx = \frac{(cx^2 + b)^{\frac{5}{2}}\text{sgn}(x)}{5\,c} - \frac{b^{\frac{5}{2}}\text{sgn}(x)}{5\,c}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/5*(c*x^2 + b)^(5/2)*sgn(x)/c - 1/5*b^(5/2)*sgn(x)/c
```

Mupad [B] (verification not implemented)

Time = 18.93 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^2}\, dx = \frac{(c\,x^2 + b)^2\,\sqrt{c\,x^4 + b\,x^2}}{5\,c\,x}$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^2,x)
```

output
```
((b + c*x^2)^2*(b*x^2 + c*x^4)^(1/2))/(5*c*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^2}\,dx = \frac{\sqrt{c\,x^2 + b}\,(c^2x^4 + 2bc\,x^2 + b^2)}{5c}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^2,x)
```

output
```
(sqrt(b + c*x**2)*(b**2 + 2*b*c*x**2 + c**2*x**4))/(5*c)
```

3.186 $\int \frac{(bx^2+cx^4)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^4} dx = \frac{b\sqrt{bx^2+cx^4}}{x} + \frac{(bx^2+cx^4)^{3/2}}{3x^3} - b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

output
```
b*(c*x^4+b*x^2)^(1/2)/x+1/3*(c*x^4+b*x^2)^(3/2)/x^3-b^(3/2)*arctanh(b^(1/2
)*x/(c*x^4+b*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^4} dx = \frac{x\left(4b^2+5bcx^2+c^2x^4-3b^{3/2}\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{3\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```

output
```
(x*(4*b^2 + 5*b*c*x^2 + c^2*x^4 - 3*b^(3/2)*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b
 + c*x^2]/Sqrt[b]]))/(3*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1426, 1426, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^4}dx$$
$$\Big\downarrow \text{ 1426}$$
$$b\int \frac{\sqrt{cx^4+bx^2}}{x^2}dx+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$
$$\Big\downarrow \text{ 1426}$$
$$b\left(b\int \frac{1}{\sqrt{cx^4+bx^2}}dx+\frac{\sqrt{bx^2+cx^4}}{x}\right)+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$
$$\Big\downarrow \text{ 1400}$$
$$b\left(\frac{\sqrt{bx^2+cx^4}}{x}-b\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}\right)+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$
$$\Big\downarrow \text{ 219}$$
$$b\left(\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```

output
```
(b*x^2 + c*x^4)^(3/2)/(3*x^3) + b*(Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh
[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}} \left(3 b^{\frac{3}{2}} \ln \left(\frac{2 b+2 \sqrt{b}\, \sqrt{c\,x^{2}+b}}{x}\right)-\left(c\,x^{2}+b\right)^{\frac{3}{2}}-3 \sqrt{c\,x^{2}+b}\, b\right)}{3 x^{3} \left(c\,x^{2}+b\right)^{\frac{3}{2}}}$	78

input
```
int((c*x^4+b*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c*x^4+b*x^2)^(3/2)*(3*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)-(c
*x^2+b)^(3/2)-3*(c*x^2+b)^(1/2)*b)/x^3/(c*x^2+b)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.85

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx = \left[\frac{3\,b^{\frac{3}{2}}x\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}(cx^2+4\,b)}{6\,x},\ \frac{3\,\sqrt{-b}bx\arctan\left(\right.}{}\right.$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/6*(3*b^(3/2)*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3
) + 2*sqrt(c*x^4 + b*x^2)*(c*x^2 + 4*b))/x, 1/3*(3*sqrt(-b)*b*x*arctan(sqr
t(c*x^4 + b*x^2)*sqrt(-b)/(b*x)) + sqrt(c*x^4 + b*x^2)*(c*x^2 + 4*b))/x]
```

Sympy [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**4,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx = \frac{b^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} + \frac{1}{3}\left(cx^2+b\right)^{\frac{3}{2}}\operatorname{sgn}(x) + \sqrt{cx^2+b}\,b\operatorname{sgn}(x) - \frac{\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\sqrt{-b}b^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{3\sqrt{-b}}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
b^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 1/3*(c*x^2 + b)^(3/
2)*sgn(x) + sqrt(c*x^2 + b)*b*sgn(x) - 1/3*(3*b^2*arctan(sqrt(b)/sqrt(-b))
 + 4*sqrt(-b)*b^(3/2))*sgn(x)/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^4}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^4,x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx = \frac{4\sqrt{c\,x^2+b}\,b}{3} + \frac{\sqrt{c\,x^2+b}\,c\,x^2}{3} + \sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)b - \sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)b$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^4,x)
```

output
```
(4*sqrt(b + c*x**2)*b + sqrt(b + c*x**2)*c*x**2 + 3*sqrt(b)*log((sqrt(b +
c*x**2) - sqrt(b) + sqrt(c)*x)/sqrt(b))*b - 3*sqrt(b)*log((sqrt(b + c*x**2
) + sqrt(b) + sqrt(c)*x)/sqrt(b))*b)/3
```

3.187 $\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx=\frac{3c\sqrt{bx^2+cx^4}}{2x}-\frac{(bx^2+cx^4)^{3/2}}{2x^5}-\frac{3}{2}\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

output
```
3/2*c*(c*x^4+b*x^2)^(1/2)/x-1/2*(c*x^4+b*x^2)^(3/2)/x^5-3/2*b^(1/2)*c*arct
anh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx=-\frac{\sqrt{x^2(b+cx^2)}\left((b-2cx^2)\sqrt{b+cx^2}+3\sqrt{b}cx^2\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{2x^3\sqrt{b+cx^2}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```

output
```
-1/2*(Sqrt[x^2*(b + c*x^2)]*((b - 2*c*x^2)*Sqrt[b + c*x^2] + 3*Sqrt[b]*c*x
^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(x^3*Sqrt[b + c*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1425, 1426, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{3}{2}c\int \frac{\sqrt{cx^4+bx^2}}{x^2}dx-\frac{(bx^2+cx^4)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{1426}$$

$$\frac{3}{2}c\left(b\int \frac{1}{\sqrt{cx^4+bx^2}}dx+\frac{\sqrt{bx^2+cx^4}}{x}\right)-\frac{(bx^2+cx^4)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{1400}$$

$$\frac{3}{2}c\left(\frac{\sqrt{bx^2+cx^4}}{x}-b\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}\right)-\frac{(bx^2+cx^4)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{2}c\left(\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)\right)-\frac{(bx^2+cx^4)^{3/2}}{2x^5}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```

output
```
-1/2*(b*x^2 + c*x^4)^(3/2)/x^5 + (3*c*(Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*Arc
Tanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.11

method	result	size
risch	$-\frac{b\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{2x^{3}}+\frac{\left(-\frac{3\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c}{2}+\sqrt{c\,x^{2}+b}\,c\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{x\sqrt{c\,x^{2}+b}}$	88
default	$-\frac{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)c\,x^{2}-\left(c\,x^{2}+b\right)^{\frac{3}{2}}c\,x^{2}+\left(c\,x^{2}+b\right)^{\frac{5}{2}}-3\sqrt{c\,x^{2}+b}\,b\,c\,x^{2}\right)}{2x^{5}\left(c\,x^{2}+b\right)^{\frac{3}{2}}b}$	102

input
```
int((c*x^4+b*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/x^3*(x^2*(c*x^2+b))^(1/2)+(-3/2*b^(1/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)
^(1/2))/x)*c+(c*x^2+b)^(1/2)*c)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.80

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^6} \, dx = \left[\frac{3\sqrt{b}cx^3 \log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) + 2\,\sqrt{cx^4+bx^2}(2\,cx^2-b)}{4\,x^3}, \frac{3\,\sqrt{-b}cx^3 \arctan}{} \right.$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[1/4*(3*sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))
/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b))/x^3, 1/2*(3*sqrt(-b)*c*x^3*ar
ctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(b*x)) + sqrt(c*x^4 + b*x^2)*(2*c*x^2 -
b))/x^3]
```

Sympy [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^6} \, dx = \int \frac{(x^2(b + cx^2))^{\frac{3}{2}}}{x^6} \, dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**6,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^6} \, dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^6} \, dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^6, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx = \frac{1}{2}\left(\frac{3\,b\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}} + 2\sqrt{cx^2+b}\,\mathrm{sgn}(x) - \frac{\sqrt{cx^2+b}\,b\,\mathrm{sgn}(x)}{cx^2}\right)c$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
1/2*(3*b*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2*sqrt(c*x^2 +
 b)*sgn(x) - sqrt(c*x^2 + b)*b*sgn(x)/(c*x^2))*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx = \int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^6}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^6,x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx = \frac{-\sqrt{c\,x^2+b}\,b + 2\sqrt{c\,x^2+b}\,c\,x^2 + 3\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c\,x^2 - 3\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}+\dots}{\sqrt{\dots}}\right.\dots}{2x^2}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^6,x)
```

output

```
( - sqrt(b + c*x**2)*b + 2*sqrt(b + c*x**2)*c*x**2 + 3*sqrt(b)*log((sqrt(b
 + c*x**2) - sqrt(b) + sqrt(c)*x)/sqrt(b))*c*x**2 - 3*sqrt(b)*log((sqrt(b
+ c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))*c*x**2)/(2*x**2)
```

3.188 $\int \frac{(bx^2+cx^4)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^8}\,dx=-\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{(bx^2+cx^4)^{3/2}}{4x^7}-\frac{3c^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}$$

output
```
-3/8*c*(c*x^4+b*x^2)^(1/2)/x^3-1/4*(c*x^4+b*x^2)^(3/2)/x^7-3/8*c^2*arctanh
(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^8}\,dx=-\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}(2b+5cx^2)+3c^2x^4\operatorname{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{8\sqrt{b}x^5\sqrt{b+cx^2}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

output
```
-1/8*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(2*b + 5*c*x^2) + 3*c
^2*x^4*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(Sqrt[b]*x^5*Sqrt[b + c*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1425, 1425, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^8} \, dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{3}{4}c\int \frac{\sqrt{cx^4+bx^2}}{x^4}dx-\frac{(bx^2+cx^4)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1425}$$

$$\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{cx^4+bx^2}}dx-\frac{\sqrt{bx^2+cx^4}}{2x^3}\right)-\frac{(bx^2+cx^4)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1400}$$

$$\frac{3}{4}c\left(-\frac{1}{2}c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}-\frac{\sqrt{bx^2+cx^4}}{2x^3}\right)-\frac{(bx^2+cx^4)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}c\left(-\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt{bx^2+cx^4}}{2x^3}\right)-\frac{(bx^2+cx^4)^{3/2}}{4x^7}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

output
```
-1/4*(b*x^2 + c*x^4)^(3/2)/x^7 + (3*c*(-1/2*Sqrt[b*x^2 + c*x^4]/x^3 - (c*A
rcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[b])))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{\left(5 c \,x^{2}+2 b \right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{8 x^{5}}-\frac{3 c^{2} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{8 \sqrt{b}\, x \sqrt{c \,x^{2}+b}}$	86
default	$-\frac{\left(c \,x^{4}+b \,x^{2}\right)^{\frac{3}{2}} \left(3 b^{\frac{3}{2}} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) c^{2} x^{4}-\left(c \,x^{2}+b \right)^{\frac{3}{2}} c^{2} x^{4}+\left(c \,x^{2}+b \right)^{\frac{5}{2}} c \,x^{2}-3 \sqrt{c \,x^{2}+b}\, b \,c^{2} x^{4}+2 \left(c \,x^{2}+b \right)^{\frac{5}{2}} b \right)}{8 x^{7} \left(c \,x^{2}+b \right)^{\frac{3}{2}} b^{2}}$	125

input
```
int((c*x^4+b*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(5*c*x^2+2*b)/x^5*(x^2*(c*x^2+b))^(1/2)-3/8*c^2/b^(1/2)*ln((2*b+2*b^(
1/2)*(c*x^2+b)^(1/2))/x)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.96

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^8}\,dx = \left[\frac{3\,\sqrt{b}c^2x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}(5\,bcx^2+2\,b^2)}{16\,bx^5},\ \frac{3\,\sqrt{-b}c^2x^5\,\mathrm{a}\ldots}{\ldots}\right.$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(
b))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(5*b*c*x^2 + 2*b^2))/(b*x^5), 1/8*(3*sqrt
(-b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(b*x)) - sqrt(c*x^4 + b*x
^2)*(5*b*c*x^2 + 2*b^2))/(b*x^5)]
```

Sympy [F]

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^8}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**8,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**8, x)
```

Maxima [F]

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^8}\,dx = \int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^8, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^8}\,dx = \frac{\frac{3\,c^3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} - \frac{5\,(cx^2+b)^{\frac{3}{2}}c^3\operatorname{sgn}(x) - 3\,\sqrt{cx^2+b}bc^3\operatorname{sgn}(x)}{c^2x^4}}{8\,c}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
1/8*(3*c^3*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - (5*(c*x^2 +
b)^(3/2)*c^3*sgn(x) - 3*sqrt(c*x^2 + b)*b*c^3*sgn(x))/(c^2*x^4))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^8}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^8,x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^8}\,dx = \frac{-2\sqrt{c\,x^2+b}\,b^2 - 5\sqrt{c\,x^2+b}\,bc\,x^2 + 3\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c^2x^4 - 3\sqrt{b}\log\left(\frac{\sqrt{c\,x}}{}\right.}{8b\,x^4}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^8,x)
```

output
```
( - 2*sqrt(b + c*x**2)*b**2 - 5*sqrt(b + c*x**2)*b*c*x**2 + 3*sqrt(b)*log(
(sqrt(b + c*x**2) - sqrt(b) + sqrt(c)*x)/sqrt(b))*c**2*x**4 - 3*sqrt(b)*lo
g((sqrt(b + c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))*c**2*x**4)/(8*b*x**4)
```

3.189 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = -\frac{c\sqrt{bx^2+cx^4}}{8x^5} - \frac{c^2\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(bx^2+cx^4)^{3/2}}{6x^9} + \frac{c^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}$$

output
```
-1/8*c*(c*x^4+b*x^2)^(1/2)/x^5-1/16*c^2*(c*x^4+b*x^2)^(1/2)/b/x^3-1/6*(c*x
^4+b*x^2)^(3/2)/x^9+1/16*c^3*arctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))/b^(3/2
)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{x^2(b+cx^2)}\left(-\sqrt{b}\sqrt{b+cx^2}(8b^2+14bcx^2+3c^2x^4)+3c^3x^6\operatorname{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{48b^{3/2}x^7\sqrt{b+cx^2}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-(Sqrt[b]*Sqrt[b + c*x^2]*(8*b^2 + 14*b*c*x^2 + 3*
c^2*x^4)) + 3*c^3*x^6*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(48*b^(3/2)*x^7*S
qrt[b + c*x^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1425, 1425, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{1}{2}c\int \frac{\sqrt{cx^4+bx^2}}{x^6}dx - \frac{(bx^2+cx^4)^{3/2}}{6x^9}$$

$$\Big\downarrow \text{1425}$$

$$\frac{1}{2}c\left(\frac{1}{4}c\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx - \frac{\sqrt{bx^2+cx^4}}{4x^5}\right) - \frac{(bx^2+cx^4)^{3/2}}{6x^9}$$

$$\Big\downarrow \text{1430}$$

$$\frac{1}{2}c\left(\frac{1}{4}c\left(-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b} - \frac{\sqrt{bx^2+cx^4}}{2bx^3}\right) - \frac{\sqrt{bx^2+cx^4}}{4x^5}\right) - \frac{(bx^2+cx^4)^{3/2}}{6x^9}$$

$$\Big\downarrow \text{1400}$$

$$\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b} - \frac{\sqrt{bx^2+cx^4}}{2bx^3}\right) - \frac{\sqrt{bx^2+cx^4}}{4x^5}\right) - \frac{(bx^2+cx^4)^{3/2}}{6x^9}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}} - \frac{\sqrt{bx^2+cx^4}}{2bx^3}\right) - \frac{\sqrt{bx^2+cx^4}}{4x^5}\right) - \frac{(bx^2+cx^4)^{3/2}}{6x^9}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

output
```
-1/6*(b*x^2 + c*x^4)^(3/2)/x^9 + (c*(-1/4*Sqrt[b*x^2 + c*x^4]/x^5 + (c*(-1
/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4
]])/(2*b^(3/2))))/4))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{(3c^2x^4+14bc\,x^2+8b^2)\sqrt{x^2(c\,x^2+b)}}{48x^7b}+\frac{c^3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)\sqrt{x^2(c\,x^2+b)}}{16b^{\frac{3}{2}}x\sqrt{c\,x^2+b}}$
default	$\frac{(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c^3x^6-(c\,x^2+b)^{\frac{3}{2}}c^3x^6+(c\,x^2+b)^{\frac{5}{2}}c^2x^4-3\sqrt{c\,x^2+b}\,b\,c^3x^6+2(c\,x^2+b)^{\frac{5}{2}}bc\,x^2-8(c\,x^2+b)^{\frac{5}{2}}}{48x^9(c\,x^2+b)^{\frac{3}{2}}b^3}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(3*c^2*x^4+14*b*c*x^2+8*b^2)/x^7/b*(x^2*(c*x^2+b))^(1/2)+1/16*c^3/b^
(3/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2
+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.65

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \left[\frac{3\sqrt{b}c^3x^7\log\left(-\frac{cx^3+2\,bx+2\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\left(3\,bc^2x^4+14\,b^2cx^2+8\,b^3\right)\sqrt{cx^4+bx^2}}{96\,b^2x^7},\right.$$
$$\left.-\frac{3\sqrt{-b}c^3x^7\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{bx}\right)+\left(3\,bc^2x^4+14\,b^2cx^2+8\,b^3\right)\sqrt{cx^4+bx^2}}{48\,b^2x^7}\right]$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
[1/96*(3*sqrt(b)*c^3*x^7*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(
b))/x^3) - 2*(3*b*c^2*x^4 + 14*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^
2*x^7), -1/48*(3*sqrt(-b)*c^3*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(b*x
)) + (3*b*c^2*x^4 + 14*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4 + b*x^2))/(b^2*x^7)]
```

Sympy [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**10,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**10, x)
```

Maxima [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^10, x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = -\frac{1}{48}c^3\left(\frac{3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}b} + \frac{3\,(cx^2+b)^{\frac{5}{2}}\mathrm{sgn}(x) + 8\,(cx^2+b)^{\frac{3}{2}}b\mathrm{sgn}(x) - 3\sqrt{cx^2+b}b^2\mathrm{sgn}(x)}{bc^3x^6}\right)$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="giac")
```

output
```
-1/48*c^3*(3*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + (3*(c*
x^2 + b)^(5/2)*sgn(x) + 8*(c*x^2 + b)^(3/2)*b*sgn(x) - 3*sqrt(c*x^2 + b)*b
^2*sgn(x))/(b*c^3*x^6))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{10}} \, dx = \int \frac{(c\,x^4 + b\,x^2)^{3/2}}{x^{10}} \, dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^10,x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^10, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{10}} \, dx = \frac{-8\sqrt{c\,x^2+b}\,b^3 - 14\sqrt{c\,x^2+b}\,b^2c\,x^2 - 3\sqrt{c\,x^2+b}\,b\,c^2x^4 - 3\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right.}{48b^2x^6}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^10,x)
```

output
```
( - 8*sqrt(b + c*x**2)*b**3 - 14*sqrt(b + c*x**2)*b**2*c*x**2 - 3*sqrt(b +
 c*x**2)*b*c**2*x**4 - 3*sqrt(b)*log((sqrt(b + c*x**2) - sqrt(b) + sqrt(c)
*x)/sqrt(b))*c**3*x**6 + 3*sqrt(b)*log((sqrt(b + c*x**2) + sqrt(b) + sqrt(
c)*x)/sqrt(b))*c**3*x**6)/(48*b**2*x**6)
```

3.190 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = -\frac{c\sqrt{bx^2+cx^4}}{16x^7} - \frac{c^2\sqrt{bx^2+cx^4}}{64bx^5} + \frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(bx^2+cx^4)^{3/2}}{8x^{11}} - \frac{3c^4\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}$$

output
```
-1/16*c*(c*x^4+b*x^2)^(1/2)/x^7-1/64*c^2*(c*x^4+b*x^2)^(1/2)/b/x^5+3/128*c
^3*(c*x^4+b*x^2)^(1/2)/b^2/x^3-1/8*(c*x^4+b*x^2)^(3/2)/x^11-3/128*c^4*arct
anh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}(16b^3+24b^2cx^2+2bc^2x^4-3c^3x^6)+3c^4x^8\operatorname{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{128b^{5/2}x^9\sqrt{b+cx^2}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

output
```
-1/128*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(16*b^3 + 24*b^2*c*
x^2 + 2*b*c^2*x^4 - 3*c^3*x^6) + 3*c^4*x^8*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]
]))/(b^(5/2)*x^9*Sqrt[b + c*x^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1425, 1425, 1430, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{12}}dx$$

$$\Big\downarrow \text{ 1425}$$

$$\frac{3}{8}c\int \frac{\sqrt{cx^4+bx^2}}{x^8}dx-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}$$

$$\Big\downarrow \text{ 1425}$$

$$\frac{3}{8}c\left(\frac{1}{6}c\int \frac{1}{x^4\sqrt{cx^4+bx^2}}dx-\frac{\sqrt{bx^2+cx^4}}{6x^7}\right)-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}$$

$$\Big\downarrow \text{ 1430}$$

$$\frac{3}{8}c\left(\frac{1}{6}c\left(-\frac{3c\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)-\frac{\sqrt{bx^2+cx^4}}{6x^7}\right)-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}$$

$$\Big\downarrow \text{ 1430}$$

$$\frac{3}{8}c\left(\frac{1}{6}c\left(-\frac{3c\left(-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)-\frac{\sqrt{bx^2+cx^4}}{6x^7}\right)-\frac{\left(bx^2+cx^4\right)^{3/2}}{8x^{11}}$$

$$\Big\downarrow \text{1400}$$

$$\frac{3}{8}c\left(\frac{1}{6}c\left(-\frac{3c\left(\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)-\frac{\sqrt{bx^2+cx^4}}{6x^7}\right)-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{8}c\left(\frac{1}{6}c\left(-\frac{3c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)-\frac{\sqrt{bx^2+cx^4}}{6x^7}\right)-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

output
```
-1/8*(b*x^2 + c*x^4)^(3/2)/x^11 + (3*c*(-1/6*Sqrt[b*x^2 + c*x^4]/x^7 + (c*
(-1/4*Sqrt[b*x^2 + c*x^4]/(b*x^5) - (3*c*(-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3)
 + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(3/2))))/(4*b)))/6))/
8
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\left(-3 c^{3} x^{6}+2 b \,c^{2} x^{4}+24 b^{2} c \,x^{2}+16 b^{3}\right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{128 x^{9} b^{2}}-\frac{3 c^{4} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{128 b^{\frac{5}{2}} x \sqrt{c \,x^{2}+b}}$
default	$-\frac{\left(c \,x^{4}+b \,x^{2}\right)^{\frac{3}{2}} \left(3 b^{\frac{3}{2}} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) c^{4} x^{8}-\left(c \,x^{2}+b \right)^{\frac{3}{2}} c^{4} x^{8}+\left(c \,x^{2}+b \right)^{\frac{5}{2}} c^{3} x^{6}-3 \sqrt{c \,x^{2}+b}\, b \,c^{4} x^{8}+2 \left(c \,x^{2}+b \right)^{\frac{5}{2}} b \,c^{2} x^{4}-8 \left(c \,x^{2}+\right.}{128 x^{11} \left(c \,x^{2}+b \right)^{\frac{3}{2}} b^{4}}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/128*(-3*c^3*x^6+2*b*c^2*x^4+24*b^2*c*x^2+16*b^3)/x^9/b^2*(x^2*(c*x^2+b)
)^(1/2)-3/128*c^4/b^(5/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)*(x^2*(c*x^
2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.47

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{12}}\,dx = \left[\frac{3\sqrt{b}c^4x^9\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right) + 2\,(3\,bc^3x^6 - 2\,b^2c^2x^4 - 24\,b^3cx^2 - 16\,b^4)\sqrt{}}{256\,b^3x^9}\right.$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="fricas")
```

output
```
[1/256*(3*sqrt(b)*c^4*x^9*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt
(b))/x^3) + 2*(3*b*c^3*x^6 - 2*b^2*c^2*x^4 - 24*b^3*c*x^2 - 16*b^4)*sqrt(c
*x^4 + b*x^2))/(b^3*x^9), 1/128*(3*sqrt(-b)*c^4*x^9*arctan(sqrt(c*x^4 + b*
x^2)*sqrt(-b)/(b*x)) + (3*b*c^3*x^6 - 2*b^2*c^2*x^4 - 24*b^3*c*x^2 - 16*b^
4)*sqrt(c*x^4 + b*x^2))/(b^3*x^9)]
```

Sympy [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(x^2(b + cx^2))^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**12,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**12, x)
```

Maxima [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^12, x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = \frac{\frac{3\,c^5\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}b^2} + \frac{3\,(cx^2+b)^{\frac{7}{2}}c^5\mathrm{sgn}(x) - 11\,(cx^2+b)^{\frac{5}{2}}bc^5\mathrm{sgn}(x) - 11\,(cx^2+b)^{\frac{3}{2}}b^2c^5\mathrm{sgn}(x) + 3\,\sqrt{cx^2}}{b^2c^4x^8}}{128\,c}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="giac")
```

output
```
1/128*(3*c^5*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^2) + (3*(
c*x^2 + b)^(7/2)*c^5*sgn(x) - 11*(c*x^2 + b)^(5/2)*b*c^5*sgn(x) - 11*(c*x^
2 + b)^(3/2)*b^2*c^5*sgn(x) + 3*sqrt(c*x^2 + b)*b^3*c^5*sgn(x))/(b^2*c^4*x
^8))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^{12}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^12,x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^12, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = \frac{-16\sqrt{c\,x^2+b}\,b^4 - 24\sqrt{c\,x^2+b}\,b^3c\,x^2 - 2\sqrt{c\,x^2+b}\,b^2c^2x^4 + 3\sqrt{c\,x^2+b}\,b\,c^3x^6 + 3\sqrt{}}{128b^3x^8}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^12,x)
```

output
```
( - 16*sqrt(b + c*x**2)*b**4 - 24*sqrt(b + c*x**2)*b**3*c*x**2 - 2*sqrt(b
+ c*x**2)*b**2*c**2*x**4 + 3*sqrt(b + c*x**2)*b*c**3*x**6 + 3*sqrt(b)*log(
(sqrt(b + c*x**2) - sqrt(b) + sqrt(c)*x)/sqrt(b))*c**4*x**8 - 3*sqrt(b)*lo
g((sqrt(b + c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))*c**4*x**8)/(128*b**3*x
**8)
```

3.191 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 165

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = -\frac{3c\sqrt{bx^2+cx^4}}{80x^9} - \frac{c^2\sqrt{bx^2+cx^4}}{160bx^7} + \frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{(bx^2+cx^4)^{3/2}}{10x^{13}} + \frac{3c^5\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}$$

output
```
-3/80*c*(c*x^4+b*x^2)^(1/2)/x^9-1/160*c^2*(c*x^4+b*x^2)^(1/2)/b/x^7+1/128*
c^3*(c*x^4+b*x^2)^(1/2)/b^2/x^5-3/256*c^4*(c*x^4+b*x^2)^(1/2)/b^3/x^3-1/10
*(c*x^4+b*x^2)^(3/2)/x^13+3/256*c^5*arctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))
/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.76

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \frac{\sqrt{b+cx^2}\left(-\sqrt{b}\sqrt{b+cx^2}(128b^4+176b^3cx^2+8b^2c^2x^4-10bc^3x^6+15c^4x^8)+15c^5x\right.}{1280b^{7/2}x^9\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```

output
```
(Sqrt[b + c*x^2]*(-(Sqrt[b]*Sqrt[b + c*x^2]*(128*b^4 + 176*b^3*c*x^2 + 8*b
^2*c^2*x^4 - 10*b*c^3*x^6 + 15*c^4*x^8)) + 15*c^5*x^10*ArcTanh[Sqrt[b + c*
x^2]/Sqrt[b]]))/(1280*b^(7/2)*x^9*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1425, 1425, 1430, 1430, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{3}{10}c\int \frac{\sqrt{cx^4+bx^2}}{x^{10}}dx-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{3}{10}c\left(\frac{1}{8}c\int \frac{1}{x^6\sqrt{cx^4+bx^2}}dx-\frac{\sqrt{bx^2+cx^4}}{8x^9}\right)-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{3}{10}c\left(\frac{1}{8}c\left(-\frac{5c\int \frac{1}{x^4\sqrt{cx^4+bx^2}}dx}{6b}-\frac{\sqrt{bx^2+cx^4}}{6bx^7}\right)-\frac{\sqrt{bx^2+cx^4}}{8x^9}\right)-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{3}{10}c\left(\frac{1}{8}c\left(-\frac{5c\left(-\frac{3c\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)}{6b}-\frac{\sqrt{bx^2+cx^4}}{6bx^7}\right)-\frac{\sqrt{bx^2+cx^4}}{8x^9}\right)-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{3}{10}c\left(\frac{1}{8}c\left(-\frac{5c\left(-\frac{3c\left(-\frac{c\int\frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)}{6b}-\frac{\sqrt{bx^2+cx^4}}{6bx^7}\right)-\frac{\sqrt{bx^2+cx^4}}{8x^9}\right)-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}$$

$\Big\downarrow$ 1400

$$\frac{3}{10}c\left(\frac{1}{8}c\left(-\frac{5c\left(-\frac{3c\left(\frac{c\int\frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)}{6b}-\frac{\sqrt{bx^2+cx^4}}{6bx^7}\right)-\frac{\sqrt{bx^2+cx^4}}{8x^9}\right)-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}$$

$\Big\downarrow$ 219

$$\frac{3}{10}c\left(\frac{1}{8}c\left(-\frac{5c\left(-\frac{3c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)}{6b}-\frac{\sqrt{bx^2+cx^4}}{6bx^7}\right)-\frac{\sqrt{bx^2+cx^4}}{8x^9}\right)-\frac{\left(bx^2+cx^4\right)^{3/2}}{10x^{13}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```

output
```
-1/10*(b*x^2 + c*x^4)^(3/2)/x^13 + (3*c*(-1/8*Sqrt[b*x^2 + c*x^4]/x^9 + (c
*(-1/6*Sqrt[b*x^2 + c*x^4]/(b*x^7) - (5*c*(-1/4*Sqrt[b*x^2 + c*x^4]/(b*x^5
) - (3*c*(-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b
*x^2 + c*x^4]])/(2*b^(3/2))))/(4*b)))/(6*b)))/8))/10
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(15c^4x^8-10b\,c^3x^6+8b^2c^2x^4+176x^2b^3c+128b^4)\sqrt{x^2(c\,x^2+b)}}{1280x^{11}b^3}+\frac{3c^5\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)\sqrt{x^2(c\,x^2+b)}}{256b^{\frac{7}{2}}x\sqrt{c\,x^2+b}}$
default	$\frac{(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(15b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c^5x^{10}-5(c\,x^2+b)^{\frac{3}{2}}c^5x^{10}+5(c\,x^2+b)^{\frac{5}{2}}c^4x^8-15\sqrt{c\,x^2+b}\,b\,c^5x^{10}+10(c\,x^2+b)^{\frac{5}{2}}b\,c^3x^6-4\right.}{1280x^{13}(c\,x^2+b)^{\frac{3}{2}}b^5}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/1280*(15*c^4*x^8-10*b*c^3*x^6+8*b^2*c^2*x^4+176*b^3*c*x^2+128*b^4)/x^11
/b^3*(x^2*(c*x^2+b))^(1/2)+3/256*c^5/b^(7/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(
1/2))/x)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.36

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \left[\frac{15\,\sqrt{b}c^5x^{11}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\,(15\,bc^4x^8-10\,b^2c^3x^6+8\,b^3c^2x^4+176}{2560\,b^4x^{11}}\right.$$
$$\left.-\frac{15\,\sqrt{-b}c^5x^{11}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{bx}\right)+(15\,bc^4x^8-10\,b^2c^3x^6+8\,b^3c^2x^4+176\,b^4cx^2+128\,b^5)\sqrt{cx^4+bx^2}}{1280\,b^4x^{11}}\right]$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="fricas")
```

output
```
[1/2560*(15*sqrt(b)*c^5*x^11*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*s
qrt(b))/x^3) - 2*(15*b*c^4*x^8 - 10*b^2*c^3*x^6 + 8*b^3*c^2*x^4 + 176*b^4*
c*x^2 + 128*b^5)*sqrt(c*x^4 + b*x^2))/(b^4*x^11), -1/1280*(15*sqrt(-b)*c^5
*x^11*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(b*x)) + (15*b*c^4*x^8 - 10*b^2*
c^3*x^6 + 8*b^3*c^2*x^4 + 176*b^4*c*x^2 + 128*b^5)*sqrt(c*x^4 + b*x^2))/(b
^4*x^11)]
```

Sympy [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{x^{14}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**14,x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**14, x)
```

Maxima [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{x^{14}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^14, x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.73

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx =$$
$$-\frac{1}{1280}c^5\left(\frac{15\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}b^3}+\frac{15(cx^2+b)^{\frac{9}{2}}\operatorname{sgn}(x)-70(cx^2+b)^{\frac{7}{2}}b\operatorname{sgn}(x)+128(cx^2+b)^{\frac{5}{2}}b^2\operatorname{sg}}{b^3c^5x^{10}}\right.$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="giac")
```

output
```
-1/1280*c^5*(15*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^3) + (
15*(c*x^2 + b)^(9/2)*sgn(x) - 70*(c*x^2 + b)^(7/2)*b*sgn(x) + 128*(c*x^2 +
 b)^(5/2)*b^2*sgn(x) + 70*(c*x^2 + b)^(3/2)*b^3*sgn(x) - 15*sqrt(c*x^2 + b
)*b^4*sgn(x))/(b^3*c^5*x^10))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^{14}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^14,x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^14, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \frac{-128\sqrt{c\,x^2+b}\,b^5-176\sqrt{c\,x^2+b}\,b^4c\,x^2-8\sqrt{c\,x^2+b}\,b^3c^2x^4+10\sqrt{c\,x^2+b}\,b^2c^3x^6}{1}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^14,x)
```

output
```
( - 128*sqrt(b + c*x**2)*b**5 - 176*sqrt(b + c*x**2)*b**4*c*x**2 - 8*sqrt(
b + c*x**2)*b**3*c**2*x**4 + 10*sqrt(b + c*x**2)*b**2*c**3*x**6 - 15*sqrt(
b + c*x**2)*b*c**4*x**8 - 15*sqrt(b)*log((sqrt(b + c*x**2) - sqrt(b) + sqr
t(c)*x)/sqrt(b))*c**5*x**10 + 15*sqrt(b)*log((sqrt(b + c*x**2) + sqrt(b) +
 sqrt(c)*x)/sqrt(b))*c**5*x**10)/(1280*b**4*x**10)
```

3.192 $\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 114

$$\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx = \frac{5b^2\sqrt{bx^2+cx^4}}{16c^3} - \frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{x^4\sqrt{bx^2+cx^4}}{6c} - \frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}$$

output
```
5/16*b^2*(c*x^4+b*x^2)^(1/2)/c^3-5/24*b*x^2*(c*x^4+b*x^2)^(1/2)/c^2+1/6*x^
4*(c*x^4+b*x^2)^(1/2)/c-5/16*b^3*arctanh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/
c^(7/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx = \frac{x\left(\sqrt{c}x(15b^3+5b^2cx^2-2bc^2x^4+8c^3x^6)+30b^3\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}-\sqrt{b+cx^2}}\right)\right)}{48c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x^7/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*(Sqrt[c]*x*(15*b^3 + 5*b^2*c*x^2 - 2*b*c^2*x^4 + 8*c^3*x^6) + 30*b^3*Sq
rt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/(Sqrt[b] - Sqrt[b + c*x^2])]))/(48*c^(7/
2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1424, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{x^6}{\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{1134}$$

$$\frac{1}{2}\left(\frac{x^4\sqrt{bx^2+cx^4}}{3c}-\frac{5b\int \frac{x^4}{\sqrt{cx^4+bx^2}}dx^2}{6c}\right)$$

$$\Big\downarrow \text{1134}$$

$$\frac{1}{2}\left(\frac{x^4\sqrt{bx^2+cx^4}}{3c}-\frac{5b\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx^2}{4c}\right)}{6c}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{x^4\sqrt{bx^2+cx^4}}{3c}-\frac{5b\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\int\frac{1}{\sqrt{cx^4+bx^2}}dx^2}{2c}\right)}{4c}\right)}{6c}\right)$$

$\downarrow$ 1091

$$\frac{1}{2}\left(\frac{x^4\sqrt{bx^2+cx^4}}{3c}-\frac{5b\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{c}\right)}{4c}\right)}{6c}\right)$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{x^4\sqrt{bx^2+cx^4}}{3c}-\frac{5b\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}\right)}{4c}\right)}{6c}\right)$$

input
```
Int[x^7/Sqrt[b*x^2 + c*x^4],x]
```

output
```
((x^4*Sqrt[b*x^2 + c*x^4])/(3*c) - (5*b*((x^2*Sqrt[b*x^2 + c*x^4])/(2*c) -
 (3*b*(Sqrt[b*x^2 + c*x^4]/c - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4
]])/c^(3/2)))/(4*c)))/(6*c))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$-\frac{5\left(\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b^{3}+\left(-\frac{16c^{\frac{5}{2}}x^{4}}{15}+\frac{4c^{\frac{3}{2}}b\,x^{2}}{3}-2\sqrt{c}\,b^{2}\right)\sqrt{x^{2}(c\,x^{2}+b)}-\ln(2)b^{3}\right)}{32c^{\frac{7}{2}}}$	89
risch	$\frac{x^{2}\left(8c^{2}x^{4}-10bc\,x^{2}+15b^{2}\right)\left(c\,x^{2}+b\right)}{48c^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{5b^{3}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)x\sqrt{c\,x^{2}+b}}{16c^{\frac{7}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	98
default	$\frac{x\sqrt{c\,x^{2}+b}\,\left(8x^{5}\sqrt{c\,x^{2}+b}\,c^{\frac{7}{2}}-10c^{\frac{5}{2}}\sqrt{c\,x^{2}+b}\,b\,x^{3}+15c^{\frac{3}{2}}\sqrt{c\,x^{2}+b}\,b^{2}x-15\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)b^{3}c\right)}{48\sqrt{c\,x^{4}+b\,x^{2}}\,c^{\frac{9}{2}}}$	105

input
```
int(x^7/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/32/c^(7/2)*(ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1/2))*b^3
+(-16/15*c^(5/2)*x^4+4/3*c^(3/2)*b*x^2-2*c^(1/2)*b^2)*(x^2*(c*x^2+b))^(1/2
)-ln(2)*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.46

$$\int\frac{x^{7}}{\sqrt{bx^{2}+cx^{4}}}\,dx=\left[\frac{15\,b^{3}\sqrt{c}\log\left(-2\,cx^{2}-b+2\,\sqrt{cx^{4}+bx^{2}}\sqrt{c}\right)+2\,\left(8\,c^{3}x^{4}-10\,bc^{2}x^{2}+15\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}}{96\,c^{4}},\ \frac{15\,b^{3}\sqrt{-c}\,\mathrm{arct}$$

input
```
integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(15*b^3*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) +
2*(8*c^3*x^4 - 10*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 + b*x^2))/c^4, 1/48*(15
*b^3*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*c^3*x^
4 - 10*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 + b*x^2))/c^4]
```

Sympy [F]

$$\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^7}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**7/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**7/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{cx^4+bx^2}x^4}{6\,c} - \frac{5\,\sqrt{cx^4+bx^2}bx^2}{24\,c^2} - \frac{5\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{32\,c^{\frac{7}{2}}} + \frac{5\,\sqrt{cx^4+bx^2}b^2}{16\,c^3}$$

input
```
integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(c*x^4 + b*x^2)*x^4/c - 5/24*sqrt(c*x^4 + b*x^2)*b*x^2/c^2 - 5/32*
b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 5/16*sqrt(c
*x^4 + b*x^2)*b^2/c^3
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{48}\,\sqrt{cx^2+b}\left(2\,x^2\left(\frac{4\,x^2}{c\mathrm{sgn}\,(x)} - \frac{5\,b}{c^2\mathrm{sgn}\,(x)}\right) + \frac{15\,b^2}{c^3\mathrm{sgn}\,(x)}\right)x - \frac{5\,b^3\log\left(|b|\right)\mathrm{sgn}(x)}{32\,c^{\frac{7}{2}}} + \frac{5\,b^3\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{16\,c^{\frac{7}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/48*sqrt(c*x^2 + b)*(2*x^2*(4*x^2/(c*sgn(x)) - 5*b/(c^2*sgn(x))) + 15*b^2
/(c^3*sgn(x)))*x - 5/32*b^3*log(abs(b))*sgn(x)/c^(7/2) + 5/16*b^3*log(abs(
-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(7/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^7}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(x^7/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^7/(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx = \frac{15\sqrt{c\,x^2+b}\,b^2cx - 10\sqrt{c\,x^2+b}\,b\,c^2x^3 + 8\sqrt{c\,x^2+b}\,c^3x^5 - 15\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)b^3}{48c^4}$$

input
```
int(x^7/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(15*sqrt(b + c*x**2)*b**2*c*x - 10*sqrt(b + c*x**2)*b*c**2*x**3 + 8*sqrt(b
 + c*x**2)*c**3*x**5 - 15*sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(
b))*b**3)/(48*c**4)
```

3.193 $\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx = -\frac{3b\sqrt{bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{bx^2+cx^4}}{4c}+\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}$$

output
```
-3/8*b*(c*x^4+b*x^2)^(1/2)/c^2+1/4*x^2*(c*x^4+b*x^2)^(1/2)/c+3/8*b^2*arcta
nh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx = \frac{x\left(\sqrt{c}x(-3b^2-bcx^2+2c^2x^4)+6b^2\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)\right)}{8c^{5/2}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^5/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*(Sqrt[c]*x*(-3*b^2 - b*c*x^2 + 2*c^2*x^4) + 6*b^2*Sqrt[b + c*x^2]*ArcTa
nh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))/(8*c^(5/2)*Sqrt[x^2*(b + c*
x^2)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1424, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{1134}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx^2}{4c}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{2c}\right)}{4c}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{c}\right)}{4c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{bx^2+cx^4}}{2c}-\frac{3b\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}\right)}{4c}\right)$$

input
```
Int[x^5/Sqrt[b*x^2 + c*x^4],x]
```

output
```
((x^2*Sqrt[b*x^2 + c*x^4])/(2*c) - (3*b*(Sqrt[b*x^2 + c*x^4]/c - (b*ArcTan
h[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/c^(3/2)))/(4*c))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{x\sqrt{c\,x^{2}+b}\,\left(2x^{3}\sqrt{c\,x^{2}+b}\,c^{\frac{5}{2}}-3c^{\frac{3}{2}}\sqrt{c\,x^{2}+b}\,bx+3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)b^{2}c\right)}{8\sqrt{c\,x^{4}+b\,x^{2}}\,c^{\frac{7}{2}}}$	85
risch	$-\frac{x^{2}\left(-2c\,x^{2}+3b\right)\left(c\,x^{2}+b\right)}{8c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{3b^{2}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)x\sqrt{c\,x^{2}+b}}{8c^{\frac{5}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	87
pseudoelliptic	$\frac{4c^{\frac{3}{2}}x^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}-6b\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+3\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b^{2}-3\ln\left(2\right)b^{2}}{16c^{\frac{5}{2}}}$	90

input
```
int(x^5/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(c*x^2+b)^(1/2)*(2*x^3*(c*x^2+b)^(1/2)*c^(5/2)-3*c^(3/2)*(c*x^2+b)^(
1/2)*b*x+3*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*c)/(c*x^4+b*x^2)^(1/2)/c^(7/2
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.69

$$\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx = \left[\frac{3\,b^{2}\sqrt{c}\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\sqrt{c}\right)+2\,\sqrt{cx^{4}+bx^{2}}(2\,c^{2}x^{2}-3\,bc)}{16\,c^{3}},\ -\frac{3\,b^{2}\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}}\sqrt{-c}}{cx^{2}+b}\right)-\sqrt{cx^{4}+bx^{2}}(2\,c^{2}x^{2}-3\,bc)}{8\,c^{3}}\right]$$

input
```
integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*b^2*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2
*sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 - 3*b*c))/c^3, -1/8*(3*b^2*sqrt(-c)*arctan
(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*x^4 + b*x^2)*(2*c^2*x^
2 - 3*b*c))/c^3]
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^5}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**5/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**5/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{cx^4+bx^2}x^2}{4\,c} + \frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{16\,c^{\frac{5}{2}}} - \frac{3\,\sqrt{cx^4+bx^2}b}{8\,c^2}$$

input
```
integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(c*x^4 + b*x^2)*x^2/c + 3/16*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
b*x^2)*sqrt(c))/c^(5/2) - 3/8*sqrt(c*x^4 + b*x^2)*b/c^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{8}\sqrt{cx^2+b}x\left(\frac{2\,x^2}{c\mathrm{sgn}\,(x)}-\frac{3\,b}{c^2\mathrm{sgn}\,(x)}\right) + \frac{3\,b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}} - \frac{3\,b^2\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{8\,c^{\frac{5}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(c*x^2 + b)*x*(2*x^2/(c*sgn(x)) - 3*b/(c^2*sgn(x))) + 3/16*b^2*log
(abs(b))*sgn(x)/c^(5/2) - 3/8*b^2*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(
c^(5/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^5}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(x^5/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^5/(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{\sqrt{bx^2+cx^4}}\,dx = \frac{-3\sqrt{c\,x^2+b}\,bcx+2\sqrt{c\,x^2+b}\,c^2x^3+3\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)b^2}{8c^3}$$

input
```
int(x^5/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - 3*sqrt(b + c*x**2)*b*c*x + 2*sqrt(b + c*x**2)*c**2*x**3 + 3*sqrt(c)*lo
g((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b))*b**2)/(8*c**3)
```

3.194 $\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

output
```
1/2*(c*x^4+b*x^2)^(1/2)/c-1/2*b*arctanh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/c
^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.43

$$\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx=\frac{x\left(\sqrt{c}x(b+cx^2)+2b\sqrt{b+cx^2}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}-\sqrt{b+cx^2}}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x^3/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*(Sqrt[c]*x*(b + c*x^2) + 2*b*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/(Sqrt[
b] - Sqrt[b + c*x^2])]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1424, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{2c}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}\right)$$

input
```
Int[x^3/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(Sqrt[b*x^2 + c*x^4]/c - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/c^
(3/2))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

method	result	size
pseudoelliptic	$\frac{2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}-\ln\left(\frac{2 c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right) b+b\ln\left(2\right)}{4 c^{\frac{3}{2}}}$	63
default	$\frac{x\sqrt{c\,x^{2}+b}\,\left(x\sqrt{c\,x^{2}+b}\,c^{\frac{3}{2}}-b\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right) c\right)}{2\sqrt{c\,x^{4}+b\,x^{2}}\,c^{\frac{5}{2}}}$	64
risch	$\frac{x^{2}\left(c\,x^{2}+b\right)}{2 c\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right) x\sqrt{c\,x^{2}+b}}{2 c^{\frac{3}{2}}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	75

input
```
int(x^3/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)-ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c
^(1/2)+b)/c^(1/2))*b+b*ln(2))/c^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.97

$$\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \left[\frac{b\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}c}{4\,c^2}, \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}c}{2\,c^2}\right.$$

input
```
integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(b*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt
(c*x^4 + b*x^2)*c)/c^2, 1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c
)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/c^2]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^3}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx = -\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2}}{2\,c}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) + 1/2*sqrt
(c*x^4 + b*x^2)/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx = -\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^2+b}x}{2\,c\operatorname{sgn}\left(x\right)}+\frac{b\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{2\,c^{\frac{3}{2}}\operatorname{sgn}\left(x\right)}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*b*log(abs(b))*sgn(x)/c^(3/2) + 1/2*sqrt(c*x^2 + b)*x/(c*sgn(x)) + 1/2
*b*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 18.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}}{2\,c}-\frac{b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,c^{3/2}}$$

input
```
int(x^3/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
(b*x^2 + c*x^4)^(1/2)/(2*c) - (b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^
4)^(1/2)))/(4*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^2+b}\,cx-\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)b}{2c^2}$$

input
```
int(x^3/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(b + c*x**2)*c*x - sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b)
)*b)/(2*c**2)
```

3.195 $\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

output
```
arctanh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

$$\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx=\frac{x\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)}{\sqrt{c}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]])/(Sqrt[c]*Sqrt[x^2
*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1424, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{1091}$$

$$\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

input
```
Int[x/Sqrt[b*x^2 + c*x^4],x]
```

output
```
ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] && !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

method	result	size
pseudoelliptic	$\frac{-\ln(2)+\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)}{2\sqrt{c}}$	42
default	$\frac{x\sqrt{c\,x^2+b}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)}{\sqrt{c\,x^4+b\,x^2}\,\sqrt{c}}$	44

input
```
int(x/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-ln(2)+ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1/2)))/c^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.39

$$\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx = \left[\frac{\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)}{c}\right]$$

input
```
integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c), -sqrt(-c)*
arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))/c]
```

Sympy [F]

$$\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx = \frac{\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{2\,\sqrt{c}}$$

input
```
integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx = \frac{\log\left(|b|\right)\text{sgn}(x)}{2\,\sqrt{c}} - \frac{\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{\sqrt{c}\text{sgn}\left(x\right)}$$

input
```
integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*log(abs(b))*sgn(x)/sqrt(c) - log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(s
qrt(c)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 18.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx=\frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,\sqrt{c}}$$

input
```
int(x/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2))/(2*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)}{c}$$

input
```
int(x/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b)))/c
```

3.196 $\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 23

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

output
```
-(c*x^4+b*x^2)^(1/2)/b/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{x^2(b+cx^2)}}{bx^2}$$

input
```
Integrate[1/(x*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
-(Sqrt[x^2*(b + c*x^2)]/(b*x^2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx$$
$$\Big\downarrow \text{1422}$$
$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

input
```
Int[1/(x*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
-(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
trager	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{b\,x^{2}}$	22
gosper	$-\frac{c\,x^{2}+b}{b\sqrt{c\,x^{4}+b\,x^{2}}}$	26
default	$-\frac{c\,x^{2}+b}{b\sqrt{c\,x^{4}+b\,x^{2}}}$	26
risch	$-\frac{c\,x^{2}+b}{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,b}$	26
pseudoelliptic	$-\frac{c\,x^{2}+b}{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,b}$	26
orering	$-\frac{c\,x^{2}+b}{b\sqrt{c\,x^{4}+b\,x^{2}}}$	26

input
```
int(1/x/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^4+b*x^2)^(1/2)/b/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx=-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(c*x^4 + b*x^2)/(b*x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx=\int \frac{1}{x\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(1/x/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx=-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(c*x^4 + b*x^2)/(b*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx=\frac{2\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)\operatorname{sgn}(x)}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}}{b\,x^2}$$

input
```
int(1/(x*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
-(b*x^2 + c*x^4)^(1/2)/(b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx = \frac{-\sqrt{c\,x^2+b}-\sqrt{c}\,x}{bx}$$

input
```
int(1/x/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - (sqrt(b + c*x**2) + sqrt(c)*x))/(b*x)
```

3.197 $\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{3bx^4}+\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}$$

output
```
-1/3*(c*x^4+b*x^2)^(1/2)/b/x^4+2/3*c*(c*x^4+b*x^2)^(1/2)/b^2/x^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{x^2(b+cx^2)}(-b+2cx^2)}{3b^2x^4}$$

input
```
Integrate[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-b + 2*c*x^2))/(3*b^2*x^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1423, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 1423$$

$$-\frac{2c\int \frac{1}{x\sqrt{cx^4+bx^2}}dx}{3b}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

$$\Big\downarrow \; 1422$$

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

input
```
Int[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
-1/3*Sqrt[b*x^2 + c*x^4]/(b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

method	result	size
trager	$-\frac{\left(-2 c \,x^{2}+b \right) \sqrt{c \,x^{4}+b \,x^{2}}}{3 b^{2} x^{4}}$	30
gosper	$-\frac{\left(c \,x^{2}+b \right) \left(-2 c \,x^{2}+b \right)}{3 x^{2} b^{2} \sqrt{c \,x^{4}+b \,x^{2}}}$	37
default	$-\frac{\left(c \,x^{2}+b \right) \left(-2 c \,x^{2}+b \right)}{3 x^{2} b^{2} \sqrt{c \,x^{4}+b \,x^{2}}}$	37
risch	$-\frac{\left(c \,x^{2}+b \right) \left(-2 c \,x^{2}+b \right)}{3 x^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, b^{2}}$	37
pseudoelliptic	$-\frac{\left(c \,x^{2}+b \right) \left(-2 c \,x^{2}+b \right)}{3 x^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, b^{2}}$	37
orering	$-\frac{\left(c \,x^{2}+b \right) \left(-2 c \,x^{2}+b \right)}{3 x^{2} b^{2} \sqrt{c \,x^{4}+b \,x^{2}}}$	37

input
```
int(1/x^3/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-2*c*x^2+b)/b^2/x^4*(c*x^4+b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt{cx^4+bx^2}(2\,cx^2-b)}{3\,b^2x^4}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b)/(b^2*x^4)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^3\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx = \frac{2\sqrt{cx^4+bx^2}c}{3\,b^2x^2} - \frac{\sqrt{cx^4+bx^2}}{3\,bx^4}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - 1/3*sqrt(c*x^4 + b*x^2)/(b*x^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx = \frac{4\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)c^{\frac{3}{2}}}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^3\operatorname{sgn}\left(x\right)}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
4/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*c^(3/2)/(((sqrt(c)*x - sqrt(c*
x^2 + b))^2 - b)^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx = -\frac{(b-2\,c\,x^2)\,\sqrt{c\,x^4+b\,x^2}}{3\,b^2\,x^4}$$

input
```
int(1/(x^3*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
-((b - 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*b^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx = \frac{-\sqrt{c\,x^2+b}\,b+2\sqrt{c\,x^2+b}\,c\,x^2-2\sqrt{c}\,c\,x^3}{3b^2x^3}$$

input
```
int(1/x^3/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - sqrt(b + c*x**2)*b + 2*sqrt(b + c*x**2)*c*x**2 - 2*sqrt(c)*c*x**3)/(3*
b**2*x**3)
```

3.198 $\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx=-\frac{\sqrt{bx^2+cx^4}}{5bx^6}+\frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}$$

output
```
-1/5*(c*x^4+b*x^2)^(1/2)/b/x^6+4/15*c*(c*x^4+b*x^2)^(1/2)/b^2/x^4-8/15*c^2
*(c*x^4+b*x^2)^(1/2)/b^3/x^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx=-\frac{\sqrt{x^2\left(b+cx^2\right)}(3b^2-4bcx^2+8c^2x^4)}{15b^3x^6}$$

input
```
Integrate[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
-1/15*(Sqrt[x^2*(b + c*x^2)]*(3*b^2 - 4*b*c*x^2 + 8*c^2*x^4))/(b^3*x^6)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1423, 1423, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1423}$$

$$-\frac{4c\int \frac{1}{x^3\sqrt{cx^4+bx^2}}dx}{5b}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}$$

$$\Big\downarrow \text{1423}$$

$$-\frac{4c\left(-\frac{2c\int \frac{1}{x\sqrt{cx^4+bx^2}}dx}{3b}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}\right)}{5b}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}$$

$$\Big\downarrow \text{1422}$$

$$-\frac{4c\left(\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}\right)}{5b}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}$$

input
```
Int[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
-1/5*Sqrt[b*x^2 + c*x^4]/(b*x^6) - (4*c*(-1/3*Sqrt[b*x^2 + c*x^4]/(b*x^4)
+ (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)))/(5*b)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.54

method	result	size
trager	$-\frac{\left(8 c^{2} x^{4}-4 b c \,x^{2}+3 b^{2}\right) \sqrt{c \,x^{4}+b \,x^{2}}}{15 b^{3} x^{6}}$	43
pseudoelliptic	$-\frac{\left(c \,x^{2}+b \right) \left(\frac{8}{3} c^{2} x^{4}-\frac{4}{3} b c \,x^{2}+b^{2}\right)}{5 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, x^{4} b^{3}}$	48
gosper	$-\frac{\left(c \,x^{2}+b \right) \left(8 c^{2} x^{4}-4 b c \,x^{2}+3 b^{2}\right)}{15 x^{4} b^{3} \sqrt{c \,x^{4}+b \,x^{2}}}$	50
default	$-\frac{\left(c \,x^{2}+b \right) \left(8 c^{2} x^{4}-4 b c \,x^{2}+3 b^{2}\right)}{15 x^{4} b^{3} \sqrt{c \,x^{4}+b \,x^{2}}}$	50
risch	$-\frac{\left(c \,x^{2}+b \right) \left(8 c^{2} x^{4}-4 b c \,x^{2}+3 b^{2}\right)}{15 x^{4} \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, b^{3}}$	50
orering	$-\frac{\left(c \,x^{2}+b \right) \left(8 c^{2} x^{4}-4 b c \,x^{2}+3 b^{2}\right)}{15 x^{4} b^{3} \sqrt{c \,x^{4}+b \,x^{2}}}$	50

input
```
int(1/x^5/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(8*c^2*x^4-4*b*c*x^2+3*b^2)/b^3/x^6*(c*x^4+b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx = -\frac{(8\,c^2x^4-4\,bcx^2+3\,b^2)\sqrt{cx^4+bx^2}}{15\,b^3x^6}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(8*c^2*x^4 - 4*b*c*x^2 + 3*b^2)*sqrt(c*x^4 + b*x^2)/(b^3*x^6)
```

Sympy [F]

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^5\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(1/x**5/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx = -\frac{8\,\sqrt{cx^4+bx^2}c^2}{15\,b^3x^2}+\frac{4\,\sqrt{cx^4+bx^2}c}{15\,b^2x^4}-\frac{\sqrt{cx^4+bx^2}}{5\,bx^6}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-8/15*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^2) + 4/15*sqrt(c*x^4 + b*x^2)*c/(b^2*
x^4) - 1/5*sqrt(c*x^4 + b*x^2)/(b*x^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx = \frac{16\left(10\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4-5\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2b+b^2\right)c^{\frac{5}{2}}}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^5\operatorname{sgn}(x)}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
16/15*(10*(sqrt(c)*x - sqrt(c*x^2 + b))^4 - 5*(sqrt(c)*x - sqrt(c*x^2 + b)
)^2*b + b^2)*c^(5/2)/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,\left(3\,b^2-4\,b\,c\,x^2+8\,c^2\,x^4\right)}{15\,b^3\,x^6}$$

input
```
int(1/(x^5*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
-((b*x^2 + c*x^4)^(1/2)*(3*b^2 + 8*c^2*x^4 - 4*b*c*x^2))/(15*b^3*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx = \frac{-3\sqrt{c\,x^2+b}\,b^2+4\sqrt{c\,x^2+b}\,bc\,x^2-8\sqrt{c\,x^2+b}\,c^2x^4+8\sqrt{c}\,c^2x^5}{15b^3x^5}$$

input
```
int(1/x^5/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - 3*sqrt(b + c*x**2)*b**2 + 4*sqrt(b + c*x**2)*b*c*x**2 - 8*sqrt(b + c*x
**2)*c**2*x**4 + 8*sqrt(c)*c**2*x**5)/(15*b**3*x**5)
```

3.199 $\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx=-\frac{\sqrt{bx^2+cx^4}}{7bx^8}+\frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6}-\frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4}+\frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}$$

output
```
-1/7*(c*x^4+b*x^2)^(1/2)/b/x^8+6/35*c*(c*x^4+b*x^2)^(1/2)/b^2/x^6-8/35*c^2
*(c*x^4+b*x^2)^(1/2)/b^3/x^4+16/35*c^3*(c*x^4+b*x^2)^(1/2)/b^4/x^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt{x^2(b+cx^2)}(-5b^3+6b^2cx^2-8bc^2x^4+16c^3x^6)}{35b^4x^8}$$

input
```
Integrate[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-5*b^3 + 6*b^2*c*x^2 - 8*b*c^2*x^4 + 16*c^3*x^6))/
(35*b^4*x^8)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1423, 1423, 1423, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1423}$$

$$-\frac{6c\int \frac{1}{x^5\sqrt{cx^4+bx^2}}dx}{7b}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

$$\Big\downarrow \text{1423}$$

$$-\frac{6c\left(-\frac{4c\int \frac{1}{x^3\sqrt{cx^4+bx^2}}dx}{5b}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}\right)}{7b}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

$$\Big\downarrow \text{1423}$$

$$-\frac{6c\left(-\frac{4c\left(-\frac{2c\int \frac{1}{x\sqrt{cx^4+bx^2}}dx}{3b}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}\right)}{5b}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}\right)}{7b}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

$$\Big\downarrow \text{1422}$$

$$-\frac{6c\left(-\frac{4c\left(\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}\right)}{5b}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}\right)}{7b}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

input
```
Int[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
-1/7*Sqrt[b*x^2 + c*x^4]/(b*x^8) - (6*c*(-1/5*Sqrt[b*x^2 + c*x^4]/(b*x^6)
- (4*c*(-1/3*Sqrt[b*x^2 + c*x^4]/(b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^
2*x^2)))/(5*b)))/(7*b)
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] && !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] && !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

method	result	size
trager	$-\frac{(-16c^3x^6+8b\,c^2x^4-6b^2c\,x^2+5b^3)\sqrt{c\,x^4+b\,x^2}}{35b^4x^8}$	54
gosper	$-\frac{(c\,x^2+b)(-16c^3x^6+8b\,c^2x^4-6b^2c\,x^2+5b^3)}{35x^6b^4\sqrt{c\,x^4+b\,x^2}}$	61
default	$-\frac{(c\,x^2+b)(-16c^3x^6+8b\,c^2x^4-6b^2c\,x^2+5b^3)}{35x^6b^4\sqrt{c\,x^4+b\,x^2}}$	61
risch	$-\frac{(c\,x^2+b)(-16c^3x^6+8b\,c^2x^4-6b^2c\,x^2+5b^3)}{35x^6\sqrt{x^2(c\,x^2+b)}\,b^4}$	61
orering	$-\frac{(c\,x^2+b)(-16c^3x^6+8b\,c^2x^4-6b^2c\,x^2+5b^3)}{35x^6b^4\sqrt{c\,x^4+b\,x^2}}$	61
pseudoelliptic	$\frac{16c^4x^8+8b\,c^3x^6-2b^2c^2x^4+x^2b^3c-5b^4}{35\sqrt{x^2(c\,x^2+b)}\,x^6b^4}$	64

input
```
int(1/x^7/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(-16*c^3*x^6+8*b*c^2*x^4-6*b^2*c*x^2+5*b^3)/b^4/x^8*(c*x^4+b*x^2)^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx = \frac{(16\,c^3x^6-8\,bc^2x^4+6\,b^2cx^2-5\,b^3)\sqrt{cx^4+bx^2}}{35\,b^4x^8}$$

input
```
integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/35*(16*c^3*x^6 - 8*b*c^2*x^4 + 6*b^2*c*x^2 - 5*b^3)*sqrt(c*x^4 + b*x^2)/
(b^4*x^8)
```

Sympy [F]

$$\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^7\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(1/x**7/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**7*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx = \frac{16\,\sqrt{cx^4+bx^2}c^3}{35\,b^4x^2} - \frac{8\,\sqrt{cx^4+bx^2}c^2}{35\,b^3x^4} + \frac{6\,\sqrt{cx^4+bx^2}c}{35\,b^2x^6} - \frac{\sqrt{cx^4+bx^2}}{7\,bx^8}$$

input
```
integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
16/35*sqrt(c*x^4 + b*x^2)*c^3/(b^4*x^2) - 8/35*sqrt(c*x^4 + b*x^2)*c^2/(b^
3*x^4) + 6/35*sqrt(c*x^4 + b*x^2)*c/(b^2*x^6) - 1/7*sqrt(c*x^4 + b*x^2)/(b
*x^8)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx = \frac{32\left(35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6-21\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4b+7\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2b^2-b^3\right)c^{\frac{7}{2}}}{35\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^7\operatorname{sgn}(x)}$$

input
```
integrate(1/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
32/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^6 - 21*(sqrt(c)*x - sqrt(c*x^2 + b
))^4*b + 7*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^2 - b^3)*c^(7/2)/(((sqrt(c)*x
 - sqrt(c*x^2 + b))^2 - b)^7*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 18.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx = \frac{6\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^6}-\frac{\sqrt{c\,x^4+b\,x^2}}{7\,b\,x^8}-\frac{8\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b^3\,x^4}+\frac{16\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^4\,x^2}$$

input
```
int(1/(x^7*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
(6*c*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^6) - (b*x^2 + c*x^4)^(1/2)/(7*b*x^8)
 - (8*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b^3*x^4) + (16*c^3*(b*x^2 + c*x^4)^(1
/2))/(35*b^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{-5\sqrt{c\,x^2+b}\,b^3+6\sqrt{c\,x^2+b}\,b^2c\,x^2-8\sqrt{c\,x^2+b}\,b\,c^2x^4+16\sqrt{c\,x^2+b}\,c^3x^6-16\sqrt{c}\,c^3x^7}{35b^4x^7}$$

input
```
int(1/x^7/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - 5*sqrt(b + c*x**2)*b**3 + 6*sqrt(b + c*x**2)*b**2*c*x**2 - 8*sqrt(b +
c*x**2)*b*c**2*x**4 + 16*sqrt(b + c*x**2)*c**3*x**6 - 16*sqrt(c)*c**3*x**7
)/(35*b**4*x**7)
```

3.200 $\int \frac{x^8}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 106

$$\int \frac{x^8}{\sqrt{bx^2+cx^4}}\,dx = -\frac{16b^3\sqrt{bx^2+cx^4}}{35c^4x}+\frac{8b^2x\sqrt{bx^2+cx^4}}{35c^3}-\frac{6bx^3\sqrt{bx^2+cx^4}}{35c^2}+\frac{x^5\sqrt{bx^2+cx^4}}{7c}$$

output
```
-16/35*b^3*(c*x^4+b*x^2)^(1/2)/c^4/x+8/35*b^2*x*(c*x^4+b*x^2)^(1/2)/c^3-6/
35*b*x^3*(c*x^4+b*x^2)^(1/2)/c^2+1/7*x^5*(c*x^4+b*x^2)^(1/2)/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54

$$\int \frac{x^8}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{x^2(b+cx^2)}(-16b^3+8b^2cx^2-6bc^2x^4+5c^3x^6)}{35c^4x}$$

input
```
Integrate[x^8/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-16*b^3 + 8*b^2*c*x^2 - 6*b*c^2*x^4 + 5*c^3*x^6))/
(35*c^4*x)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1421, 1421, 1421, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^5\sqrt{bx^2+cx^4}}{7c}-\frac{6b\int \frac{x^6}{\sqrt{cx^4+bx^2}}dx}{7c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^5\sqrt{bx^2+cx^4}}{7c}-\frac{6b\left(\frac{x^3\sqrt{bx^2+cx^4}}{5c}-\frac{4b\int \frac{x^4}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{7c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^5\sqrt{bx^2+cx^4}}{7c}-\frac{6b\left(\frac{x^3\sqrt{bx^2+cx^4}}{5c}-\frac{4b\left(\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{5c}\right)}{7c}$$

$$\Big\downarrow \text{1420}$$

$$\frac{x^5\sqrt{bx^2+cx^4}}{7c}-\frac{6b\left(\frac{x^3\sqrt{bx^2+cx^4}}{5c}-\frac{4b\left(\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}\right)}{5c}\right)}{7c}$$

input
```
Int[x^8/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x^5*Sqrt[b*x^2 + c*x^4])/(7*c) - (6*b*((x^3*Sqrt[b*x^2 + c*x^4])/(5*c) -
(4*b*((-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
))/(5*c)))/(7*c)
```

Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.51

method	result	size
trager	$-\frac{\left(-5c^{3}x^{6}+6b\,c^{2}x^{4}-8b^{2}c\,x^{2}+16b^{3}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{35c^{4}x}$	54
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-5c^{3}x^{6}+6b\,c^{2}x^{4}-8b^{2}c\,x^{2}+16b^{3}\right)x}{35c^{4}\sqrt{c\,x^{4}+b\,x^{2}}}$	59
default	$-\frac{\left(c\,x^{2}+b\right)\left(-5c^{3}x^{6}+6b\,c^{2}x^{4}-8b^{2}c\,x^{2}+16b^{3}\right)x}{35c^{4}\sqrt{c\,x^{4}+b\,x^{2}}}$	59
risch	$-\frac{x\left(c\,x^{2}+b\right)\left(-5c^{3}x^{6}+6b\,c^{2}x^{4}-8b^{2}c\,x^{2}+16b^{3}\right)}{35\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,c^{4}}$	59
orering	$-\frac{\left(c\,x^{2}+b\right)\left(-5c^{3}x^{6}+6b\,c^{2}x^{4}-8b^{2}c\,x^{2}+16b^{3}\right)x}{35c^{4}\sqrt{c\,x^{4}+b\,x^{2}}}$	59

input
```
int(x^8/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(-5*c^3*x^6+6*b*c^2*x^4-8*b^2*c*x^2+16*b^3)/c^4/x*(c*x^4+b*x^2)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int \frac{x^8}{\sqrt{bx^2+cx^4}}\,dx = \frac{(5\,c^3x^6-6\,bc^2x^4+8\,b^2cx^2-16\,b^3)\sqrt{cx^4+bx^2}}{35\,c^4x}$$

input
```
integrate(x^8/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/35*(5*c^3*x^6 - 6*b*c^2*x^4 + 8*b^2*c*x^2 - 16*b^3)*sqrt(c*x^4 + b*x^2)/
(c^4*x)
```

Sympy [F]

$$\int \frac{x^8}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^8}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**8/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**8/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54

$$\int \frac{x^8}{\sqrt{bx^2+cx^4}}\,dx = \frac{5\,c^4x^8-bc^3x^6+2\,b^2c^2x^4-8\,b^3cx^2-16\,b^4}{35\,\sqrt{cx^2+b}c^4}$$

input
```
integrate(x^8/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/35*(5*c^4*x^8 - b*c^3*x^6 + 2*b^2*c^2*x^4 - 8*b^3*c*x^2 - 16*b^4)/(sqrt(
c*x^2 + b)*c^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

$$\int \frac{x^8}{\sqrt{bx^2+cx^4}}\,dx = \frac{16\,b^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,c^4} - \frac{\sqrt{cx^2+b}b^3}{c^4\mathrm{sgn}\,(x)} + \frac{5\,(cx^2+b)^{\frac{7}{2}} - 21\,(cx^2+b)^{\frac{5}{2}}b + 35\,(cx^2+b)^{\frac{3}{2}}b^2}{35\,c^4\mathrm{sgn}\,(x)}$$

input
```
integrate(x^8/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
16/35*b^(7/2)*sgn(x)/c^4 - sqrt(c*x^2 + b)*b^3/(c^4*sgn(x)) + 1/35*(5*(c*x
^2 + b)^(7/2) - 21*(c*x^2 + b)^(5/2)*b + 35*(c*x^2 + b)^(3/2)*b^2)/(c^4*sg
n(x))
```

Mupad [B] (verification not implemented)

Time = 17.69 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int \frac{x^8}{\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,(16\,b^3-8\,b^2\,c\,x^2+6\,b\,c^2\,x^4-5\,c^3\,x^6)}{35\,c^4\,x}$$

input
```
int(x^8/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
-((b*x^2 + c*x^4)^(1/2)*(16*b^3 - 5*c^3*x^6 - 8*b^2*c*x^2 + 6*b*c^2*x^4))/
(35*c^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int \frac{x^8}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^2+b}\,(5c^3x^6-6b\,c^2x^4+8b^2c\,x^2-16b^3)}{35c^4}$$

input
```
int(x^8/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(b + c*x**2)*( - 16*b**3 + 8*b**2*c*x**2 - 6*b*c**2*x**4 + 5*c**3*x**
6))/(35*c**4)
```

3.201 $\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx=\frac{8b^2\sqrt{bx^2+cx^4}}{15c^3x}-\frac{4bx\sqrt{bx^2+cx^4}}{15c^2}+\frac{x^3\sqrt{bx^2+cx^4}}{5c}$$

output
```
8/15*b^2*(c*x^4+b*x^2)^(1/2)/c^3/x-4/15*b*x*(c*x^4+b*x^2)^(1/2)/c^2+1/5*x^
3*(c*x^4+b*x^2)^(1/2)/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59

$$\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt{x^2\left(b+cx^2\right)}(8b^2-4bcx^2+3c^2x^4)}{15c^3x}$$

input
```
Integrate[x^6/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(8*b^2 - 4*b*c*x^2 + 3*c^2*x^4))/(15*c^3*x)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1421, 1421, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^3\sqrt{bx^2+cx^4}}{5c}-\frac{4b\int \frac{x^4}{\sqrt{cx^4+bx^2}}dx}{5c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^3\sqrt{bx^2+cx^4}}{5c}-\frac{4b\left(\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{5c}$$

$$\Big\downarrow \text{1420}$$

$$\frac{x^3\sqrt{bx^2+cx^4}}{5c}-\frac{4b\left(\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}\right)}{5c}$$

input
```
Int[x^6/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x^3*Sqrt[b*x^2 + c*x^4])/(5*c) - (4*b*((-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*
x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)))/(5*c)
```

Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.55

method	result	size
trager	$\frac{\left(3 c^{2} x^{4}-4 b c \,x^{2}+8 b^{2}\right) \sqrt{c \,x^{4}+b \,x^{2}}}{15 c^{3} x}$	43
gosper	$\frac{\left(c \,x^{2}+b \right) \left(3 c^{2} x^{4}-4 b c \,x^{2}+8 b^{2}\right) x}{15 c^{3} \sqrt{c \,x^{4}+b \,x^{2}}}$	48
default	$\frac{\left(c \,x^{2}+b \right) \left(3 c^{2} x^{4}-4 b c \,x^{2}+8 b^{2}\right) x}{15 c^{3} \sqrt{c \,x^{4}+b \,x^{2}}}$	48
risch	$\frac{x \left(c \,x^{2}+b \right) \left(3 c^{2} x^{4}-4 b c \,x^{2}+8 b^{2}\right)}{15 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, c^{3}}$	48
orering	$\frac{\left(c \,x^{2}+b \right) \left(3 c^{2} x^{4}-4 b c \,x^{2}+8 b^{2}\right) x}{15 c^{3} \sqrt{c \,x^{4}+b \,x^{2}}}$	48

input
```
int(x^6/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*c^2*x^4-4*b*c*x^2+8*b^2)/c^3/x*(c*x^4+b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.54

$$\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx = \frac{(3\,c^2x^4-4\,bcx^2+8\,b^2)\sqrt{cx^4+bx^2}}{15\,c^3x}$$

input
```
integrate(x^6/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*c^2*x^4 - 4*b*c*x^2 + 8*b^2)*sqrt(c*x^4 + b*x^2)/(c^3*x)
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^6}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**6/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**6/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59

$$\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx = \frac{3\,c^3x^6-bc^2x^4+4\,b^2cx^2+8\,b^3}{15\,\sqrt{cx^2+b}c^3}$$

input
```
integrate(x^6/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(3*c^3*x^6 - b*c^2*x^4 + 4*b^2*c*x^2 + 8*b^3)/(sqrt(c*x^2 + b)*c^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx = -\frac{8\,b^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,c^3} + \frac{\sqrt{cx^2+b}b^2}{c^3\mathrm{sgn}\,(x)} + \frac{3\,(cx^2+b)^{\frac{5}{2}} - 10\,(cx^2+b)^{\frac{3}{2}}b}{15\,c^3\mathrm{sgn}\,(x)}$$

input
```
integrate(x^6/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-8/15*b^(5/2)*sgn(x)/c^3 + sqrt(c*x^2 + b)*b^2/(c^3*sgn(x)) + 1/15*(3*(c*x
^2 + b)^(5/2) - 10*(c*x^2 + b)^(3/2)*b)/(c^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 16.99 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.54

$$\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,(8\,b^2-4\,b\,c\,x^2+3\,c^2\,x^4)}{15\,c^3\,x}$$

input
```
int(x^6/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*(8*b^2 + 3*c^2*x^4 - 4*b*c*x^2))/(15*c^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.44

$$\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^2+b}\,(3c^2x^4-4bc\,x^2+8b^2)}{15c^3}$$

input
```
int(x^6/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(b + c*x**2)*(8*b**2 - 4*b*c*x**2 + 3*c**2*x**4))/(15*c**3)
```

3.202 $\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx=-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}+\frac{x\sqrt{bx^2+cx^4}}{3c}$$

output
```
-2/3*b*(c*x^4+b*x^2)^(1/2)/c^2/x+1/3*x*(c*x^4+b*x^2)^(1/2)/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx=\frac{(-2b+cx^2)\sqrt{x^2(b+cx^2)}}{3c^2x}$$

input
```
Integrate[x^4/Sqrt[b*x^2 + c*x^4],x]
```

output
```
((-2*b + c*x^2)*Sqrt[x^2*(b + c*x^2)])/(3*c^2*x)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1421, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{bx^2+cx^4}}dx$$
$$\Big\downarrow \text{ 1421}$$
$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx}{3c}$$
$$\Big\downarrow \text{ 1420}$$
$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

input
```
Int[x^4/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
```

Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

method	result	size
trager	$-\frac{\left(-c\,x^{2}+2 b \right) \sqrt{c\,x^{4}+b\,x^{2}}}{3 c^{2} x}$	32
gosper	$-\frac{\left(c\,x^{2}+b \right) \left(-c\,x^{2}+2 b \right) x}{3 c^{2} \sqrt{c\,x^{4}+b\,x^{2}}}$	37
default	$-\frac{\left(c\,x^{2}+b \right) \left(-c\,x^{2}+2 b \right) x}{3 c^{2} \sqrt{c\,x^{4}+b\,x^{2}}}$	37
risch	$-\frac{x \left(c\,x^{2}+b \right) \left(-c\,x^{2}+2 b \right)}{3 \sqrt{x^{2} \left(c\,x^{2}+b \right)}\, c^{2}}$	37
orering	$-\frac{\left(c\,x^{2}+b \right) \left(-c\,x^{2}+2 b \right) x}{3 c^{2} \sqrt{c\,x^{4}+b\,x^{2}}}$	37

input
```
int(x^4/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-c*x^2+2*b)/c^2/x*(c*x^4+b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.60

$$\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt{cx^4+bx^2}(cx^2-2\,b)}{3\,c^2x}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b)/(c^2*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^4}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(x**4/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**4/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx=\frac{c^2x^4-bcx^2-2\,b^2}{3\,\sqrt{cx^2+b}c^2}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)/(sqrt(c*x^2 + b)*c^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx=\frac{2\,b^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,c^2}+\frac{\left(cx^2+b\right)^{\frac{3}{2}}}{3\,c^2\mathrm{sgn}\left(x\right)}-\frac{\sqrt{cx^2+b}b}{c^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*b^(3/2)*sgn(x)/c^2 + 1/3*(c*x^2 + b)^(3/2)/(c^2*sgn(x)) - sqrt(c*x^2 +
 b)*b/(c^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.63 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

$$\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{2\,b}{3\,c^2}-\frac{x^2}{3\,c}\right)}{x}$$

input
```
int(x^4/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
-((b*x^2 + c*x^4)^(1/2)*((2*b)/(3*c^2) - x^2/(3*c)))/x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.44

$$\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^2+b}\,(c\,x^2-2b)}{3c^2}$$

input
```
int(x^4/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(b + c*x**2)*( - 2*b + c*x**2))/(3*c**2)
```

3.203 $\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{bx^2+cx^4}}{cx}$$

output
```
(c*x^4+b*x^2)^(1/2)/c/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{x^2(b+cx^2)}}{cx}$$

input
```
Integrate[x^2/Sqrt[b*x^2 + c*x^4],x]
```

output
```
Sqrt[x^2*(b + c*x^2)]/(c*x)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}dx$$
$$\Big\downarrow \text{ 1420}$$
$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

input
```
Int[x^2/Sqrt[b*x^2 + c*x^4],x]
```

output
```
Sqrt[b*x^2 + c*x^4]/(c*x)
```

Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
trager	$\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{c x}$	21
gosper	$\frac{\left(c\,x^{2}+b\right)x}{c\sqrt{c\,x^{4}+b\,x^{2}}}$	26
default	$\frac{\left(c\,x^{2}+b\right)x}{c\sqrt{c\,x^{4}+b\,x^{2}}}$	26
risch	$\frac{x\left(c\,x^{2}+b\right)}{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,c}$	26
orering	$\frac{\left(c\,x^{2}+b\right)x}{c\sqrt{c\,x^{4}+b\,x^{2}}}$	26

input
```
int(x^2/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x^4+b*x^2)^(1/2)/c/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt{cx^4+bx^2}}{cx}$$

input
```
integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^4 + b*x^2)/(c*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^2}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**2/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{cx^2+b}}{c}$$

input
```
integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(c*x^2 + b)/c
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{b}\mathrm{sgn}(x)}{c} + \frac{\sqrt{cx^2+b}}{c\mathrm{sgn}\,(x)}$$

input
```
integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(b)*sgn(x)/c + sqrt(c*x^2 + b)/(c*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 18.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt{c\,x^4+b\,x^2}}{c\,x}$$

input
```
int(x^2/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
(b*x^2 + c*x^4)^(1/2)/(c*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt{c\,x^2+b}}{c}$$

input
```
int(x^2/(c*x^4+b*x^2)^(1/2),x)
```

output
```
sqrt(b + c*x**2)/c
```

3.204 $\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

output
```
-arctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx=-\frac{x\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[1/Sqrt[b*x^2 + c*x^4],x]
```

output
```
-((x*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[x^2*(
b + c*x^2)]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1400}$$

$$-\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[b*x^2 + c*x^4],x]
```

output
```
-(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.

Time = 0.48 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

method	result	size
default	$-\frac{x\sqrt{c\,x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)}{\sqrt{c\,x^4+b\,x^2}\,\sqrt{b}}$	50

input
```
int(1/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(
1/2)+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx = \left[\frac{\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{bx}\right)}{b}\right]$$

input
```
integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)/sqrt(b), sq
rt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(b*x))/b]
```

Sympy [F]

$$\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/sqrt(b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx=-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}+\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\operatorname{sgn}(x)}$$

input
```
integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + arctan(sqrt(c*x^2 + b)/sqrt(-b
))/(sqrt(-b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(1/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(1/(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt{b}\left(\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)-\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)\right)}{b}$$

input
```
int(1/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(b)*(log((sqrt(b + c*x**2) - sqrt(b) + sqrt(c)*x)/sqrt(b)) - log((sqr
t(b + c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))))/b
```

3.205 $\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{2bx^3} + \frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}$$

output
```
-1/2*(c*x^4+b*x^2)^(1/2)/b/x^3+1/2*c*arctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2)
)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx = \frac{-\sqrt{b}(b+cx^2)+cx^2\sqrt{b+cx^2}\,\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{2b^{3/2}x\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-(Sqrt[b]*(b + c*x^2)) + c*x^2*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sq
rt[b]])/(2*b^(3/2)*x*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1430}$$

$$-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

$$\Big\downarrow \text{1400}$$

$$\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

input
```
Int[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]])/(2*b^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\sqrt{c\,x^{2}+b}\,\left(-c\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)b\,x^{2}+\sqrt{c\,x^{2}+b}\,b\,b^{\frac{3}{2}}\right)}{2x\sqrt{c\,x^{4}+b\,x^{2}}\,b^{\frac{5}{2}}}$	73
risch	$-\frac{c\,x^{2}+b}{2bx\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{c\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)x\sqrt{c\,x^{2}+b}}{2b^{\frac{3}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	82

input
```
int(1/x^2/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x*(c*x^2+b)^(1/2)*(-c*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*x^2+(c*x^
2+b)^(1/2)*b^(3/2))/(c*x^4+b*x^2)^(1/2)/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx = \left[\frac{\sqrt{b}cx^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}b}{4\,b^2x^3},\ -\frac{\sqrt{-b}cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{bx}\right)+\sqrt{cx^4+bx^2}b}{2\,b^2x^3}\right]$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x
^3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3), -1/2*(sqrt(-b)*c*x^3*arctan(sqrt
(c*x^4 + b*x^2)*sqrt(-b)/(b*x)) + sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^2\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2}x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx = -\frac{c\left(\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{\sqrt{cx^2+b}}{bcx^2}\right)}{2\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*c*(arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(c*x^2 + b)/(b
*c*x^2))/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 18.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx = -\frac{\left(\frac{\sqrt{c}\,x^2\,\sqrt{c+\frac{b}{x^2}}}{2\,b}+\frac{c^{3/2}\,x^3\,\mathrm{asin}\left(\frac{\sqrt{b}\,1{}\mathrm{i}}{\sqrt{c}\,x}\right)\,1{}\mathrm{i}}{2\,b^{3/2}}\right)\,\sqrt{\frac{b}{c\,x^2}+1}}{x\,\sqrt{c\,x^4+b\,x^2}}$$

input
```
int(1/(x^2*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
-(((c^(1/2)*x^2*(c + b/x^2)^(1/2))/(2*b) + (c^(3/2)*x^3*asin((b^(1/2)*1i)/
(c^(1/2)*x))*1i)/(2*b^(3/2)))*(b/(c*x^2) + 1)^(1/2))/(x*(b*x^2 + c*x^4)^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{-\sqrt{c\,x^2+b}\,b-\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c\,x^2+\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c\,x^2}{2b^2x^2}$$

input
```
int(1/x^2/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - sqrt(b + c*x**2)*b - sqrt(b)*log((sqrt(b + c*x**2) - sqrt(b) + sqrt(c)
*x)/sqrt(b))*c*x**2 + sqrt(b)*log((sqrt(b + c*x**2) + sqrt(b) + sqrt(c)*x)
/sqrt(b))*c*x**2)/(2*b**2*x**2)
```

3.206 $\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{3c^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}$$

output
```
-1/4*(c*x^4+b*x^2)^(1/2)/b/x^5+3/8*c*(c*x^4+b*x^2)^(1/2)/b^2/x^3-3/8*c^2*a
rctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{b}(-2b^2+bcx^2+3c^2x^4)-3c^2x^4\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{8b^{5/2}x^3\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[b]*(-2*b^2 + b*c*x^2 + 3*c^2*x^4) - 3*c^2*x^4*Sqrt[b + c*x^2]*ArcTan
h[Sqrt[b + c*x^2]/Sqrt[b]])/(8*b^(5/2)*x^3*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1430, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1430}$$

$$-\frac{3c\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

$$\Big\downarrow \text{1430}$$

$$-\frac{3c\left(-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

$$\Big\downarrow \text{1400}$$

$$-\frac{3c\left(\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

input
```
Int[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
-1/4*Sqrt[b*x^2 + c*x^4]/(b*x^5) - (3*c*(-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3)
+ (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(3/2))))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{\sqrt{c\,x^{2}+b}\,\left(3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)b\,c^{2}x^{4}-3b^{\frac{3}{2}}\sqrt{c\,x^{2}+b}\,c\,x^{2}+2\sqrt{c\,x^{2}+b}\,b^{\frac{5}{2}}\right)}{8x^{3}\sqrt{c\,x^{4}+b\,x^{2}}\,b^{\frac{7}{2}}}$	94
risch	$-\frac{\left(c\,x^{2}+b\right)\left(-3c\,x^{2}+2b\right)}{8b^{2}x^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{3c^{2}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)x\sqrt{c\,x^{2}+b}}{8b^{\frac{5}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	94

input
```
int(1/x^4/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/x^3*(c*x^2+b)^(1/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*c^2*x^4-
3*b^(3/2)*(c*x^2+b)^(1/2)*c*x^2+2*(c*x^2+b)^(1/2)*b^(5/2))/(c*x^4+b*x^2)^(
1/2)/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx$$
$$= \left[\frac{3\sqrt{b}c^2x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}(3\,bcx^2-2\,b^2)}{16\,b^3x^5}, \frac{3\,\sqrt{-b}c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{bx}\right) \cdots}{8\,b^3x^5 \cdots}\right.$$

input
```
integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(
b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5), 1/8*(3*sq
rt(-b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(b*x)) + sqrt(c*x^4 + b
*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5)]
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^4\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2}x^4}\,dx$$

input
```
integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx = \frac{\frac{3\,c^3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2} + \frac{3\,(cx^2+b)^{\frac{3}{2}}c^3-5\,\sqrt{cx^2+b}bc^3}{b^2c^2x^4}}{8\,c\mathrm{sgn}\,(x)}$$

input
```
integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(3*c^3*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*(c*x^2 + b
)^(3/2)*c^3 - 5*sqrt(c*x^2 + b)*b*c^3)/(b^2*c^2*x^4))/(c*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^4\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(1/(x^4*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx = \frac{-2\sqrt{c\,x^2+b}\,b^2+3\sqrt{c\,x^2+b}\,bc\,x^2+3\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c^2x^4-3\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c^2x^4}{8b^3x^4}$$

input
```
int(1/x^4/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(b + c*x**2)*b**2 + 3*sqrt(b + c*x**2)*b*c*x**2 + 3*sqrt(b)*log(
(sqrt(b + c*x**2) - sqrt(b) + sqrt(c)*x)/sqrt(b))*c**2*x**4 - 3*sqrt(b)*lo
g((sqrt(b + c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))*c**2*x**4)/(8*b**3*x**
4)
```

3.207 $\int \frac{x^9}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{x^9}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{b^2x^2}{c^3\sqrt{bx^2+cx^4}} - \frac{7b\sqrt{bx^2+cx^4}}{8c^3} + \frac{x^2\sqrt{bx^2+cx^4}}{4c^2} + \frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}$$

output
```
-b^2*x^2/c^3/(c*x^4+b*x^2)^(1/2)-7/8*b*(c*x^4+b*x^2)^(1/2)/c^3+1/4*x^2*(c*
x^4+b*x^2)^(1/2)/c^2+15/8*b^2*arctanh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/c^(
7/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{x^9}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{x\left(\sqrt{c}x\left(-15b^2-5bcx^2+2c^2x^4\right)+30b^2\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)\right)}{8c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x^9/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(Sqrt[c]*x*(-15*b^2 - 5*b*c*x^2 + 2*c^2*x^4) + 30*b^2*Sqrt[b + c*x^2]*A
rcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))/(8*c^(7/2)*Sqrt[x^2*(b
+ c*x^2)])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1424, 1124, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{x^8}{(cx^4+bx^2)^{3/2}}dx^2$$

$$\Big\downarrow \text{1124}$$

$$\frac{1}{2}\left(\frac{\int \frac{c^2x^4-bcx^2+b^2}{\sqrt{cx^4+bx^2}}dx^2}{c^3}-\frac{2b^2x^2}{c^3\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{bc(4b-7cx^2)}{2\sqrt{cx^4+bx^2}}dx^2}{2c}+\frac{1}{2}cx^2\sqrt{bx^2+cx^4}}{c^3}-\frac{2b^2x^2}{c^3\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{1}{4}b\int \frac{4b-7cx^2}{\sqrt{cx^4+bx^2}}dx^2+\frac{1}{2}cx^2\sqrt{bx^2+cx^4}}{c^3}-\frac{2b^2x^2}{c^3\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{\frac{1}{4}b\left(\frac{15}{2}b\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2-7\sqrt{bx^2+cx^4}\right)+\frac{1}{2}cx^2\sqrt{bx^2+cx^4}}{c^3}-\frac{2b^2x^2}{c^3\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \; 1091$$

$$\frac{1}{2}\left(\frac{\frac{1}{4}b\left(15b\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}-7\sqrt{bx^2+cx^4}\right)+\frac{1}{2}cx^2\sqrt{bx^2+cx^4}}{c^3}-\frac{2b^2x^2}{c^3\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{\frac{1}{4}b\left(\frac{15b\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}-7\sqrt{bx^2+cx^4}\right)+\frac{1}{2}cx^2\sqrt{bx^2+cx^4}}{c^3}-\frac{2b^2x^2}{c^3\sqrt{bx^2+cx^4}}\right)$$

input
```
Int[x^9/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((-2*b^2*x^2)/(c^3*Sqrt[b*x^2 + c*x^4]) + ((c*x^2*Sqrt[b*x^2 + c*x^4])/2 +
 (b*(-7*Sqrt[b*x^2 + c*x^4] + (15*b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]])/Sqrt[c]))/4)/c^3)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x^{3} \left(c \,x^{2}+b \right) \left(2 c^{\frac{7}{2}} x^{5}-5 c^{\frac{5}{2}} b \,x^{3}-15 c^{\frac{3}{2}} b^{2} x +15 \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) \sqrt{c \,x^{2}+b}\, b^{2} c \right)}{8 \left(c \,x^{4}+b \,x^{2}\right)^{\frac{3}{2}} c^{\frac{9}{2}}}$	87
risch	$-\frac{x^{2} \left(-2 c \,x^{2}+7 b \right) \left(c \,x^{2}+b \right)}{8 c^{3} \sqrt{x^{2} \left(c \,x^{2}+b \right)}}+\frac{\left(-\frac{b^{2} x}{c^{3} \sqrt{c \,x^{2}+b}}+\frac{15 b^{2} \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right)}{8 c^{\frac{7}{2}}}\right) x \sqrt{c \,x^{2}+b}}{\sqrt{x^{2} \left(c \,x^{2}+b \right)}}$	107
pseudoelliptic	$\frac{\frac{c^{\frac{5}{2}} x^{6}}{4}-\frac{5 b \,c^{\frac{3}{2}} x^{4}}{8}-\frac{15 b^{2} x^{2} \sqrt{c}}{8}+\frac{15 \ln \left(\frac{2 c \,x^{2}+2 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \sqrt{c}+b}{\sqrt{c}}\right) b^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{16}-\frac{15 \ln \left(2\right) b^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{16}}{c^{\frac{7}{2}} \sqrt{x^{2} \left(c \,x^{2}+b \right)}}$	116

input
```
int(x^9/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^3*(c*x^2+b)*(2*c^(7/2)*x^5-5*c^(5/2)*b*x^3-15*c^(3/2)*b^2*x+15*ln(c^
(1/2)*x+(c*x^2+b)^(1/2))*(c*x^2+b)^(1/2)*b^2*c)/(c*x^4+b*x^2)^(3/2)/c^(9/2
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.87

$$\int \frac{x^9}{(bx^2+cx^4)^{3/2}}\,dx = \left[\frac{15\,(b^2cx^2+b^3)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,(2\,c^3x^4-5\,bc^2x^2-15\,b\cdots}{16\,(c^5x^2+bc^4)}\right.$$
$$\left. -\frac{15\,(b^2cx^2+b^3)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(2\,c^3x^4-5\,bc^2x^2-15\,b^2c)\sqrt{cx^4+bx^2}}{8\,(c^5x^2+bc^4)}\right]$$

input
```
integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(15*(b^2*c*x^2 + b^3)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^
2)*sqrt(c)) + 2*(2*c^3*x^4 - 5*b*c^2*x^2 - 15*b^2*c)*sqrt(c*x^4 + b*x^2))/
(c^5*x^2 + b*c^4), -1/8*(15*(b^2*c*x^2 + b^3)*sqrt(-c)*arctan(sqrt(c*x^4 +
 b*x^2)*sqrt(-c)/(c*x^2 + b)) - (2*c^3*x^4 - 5*b*c^2*x^2 - 15*b^2*c)*sqrt(
c*x^4 + b*x^2))/(c^5*x^2 + b*c^4)]
```

Sympy [F]

$$\int \frac{x^9}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^9}{\left(x^2\,(b+cx^2)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**9/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**9/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.92

$$\int \frac{x^9}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^6}{4\sqrt{cx^4+bx^2}c} - \frac{5\,bx^4}{8\sqrt{cx^4+bx^2}c^2} - \frac{15\,b^2x^2}{8\sqrt{cx^4+bx^2}c^3} + \frac{15\,b^2\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{16\,c^{\frac{7}{2}}}$$

input
```
integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*x^6/(sqrt(c*x^4 + b*x^2)*c) - 5/8*b*x^4/(sqrt(c*x^4 + b*x^2)*c^2) - 15
/8*b^2*x^2/(sqrt(c*x^4 + b*x^2)*c^3) + 15/16*b^2*log(2*c*x^2 + b + 2*sqrt(
c*x^4 + b*x^2)*sqrt(c))/c^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int \frac{x^9}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\left(x^2\left(\frac{2\,x^2}{c\text{sgn}(x)} - \frac{5\,b}{c^2\text{sgn}(x)}\right) - \frac{15\,b^2}{c^3\text{sgn}(x)}\right)x}{8\sqrt{cx^2+b}} + \frac{15\,b^2\log\left(|b|\right)\text{sgn}(x)}{16\,c^{\frac{7}{2}}} - \frac{15\,b^2\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{8\,c^{\frac{7}{2}}\text{sgn}\left(x\right)}$$

input
```
integrate(x^9/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(x^2*(2*x^2/(c*sgn(x)) - 5*b/(c^2*sgn(x))) - 15*b^2/(c^3*sgn(x)))*x/sq
rt(c*x^2 + b) + 15/16*b^2*log(abs(b))*sgn(x)/c^(7/2) - 15/8*b^2*log(abs(-s
qrt(c)*x + sqrt(c*x^2 + b)))/(c^(7/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^9}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int(x^9/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^9/(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.22

$$\int \frac{x^9}{(bx^2+cx^4)^{3/2}}\,dx = \frac{-15\sqrt{c\,x^2+b}\,b^2cx - 5\sqrt{c\,x^2+b}\,b\,c^2x^3 + 2\sqrt{c\,x^2+b}\,c^3x^5 + 15\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)}{8c^4\,(c\,x^2+b)}$$

input
```
int(x^9/(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - 15*sqrt(b + c*x**2)*b**2*c*x - 5*sqrt(b + c*x**2)*b*c**2*x**3 + 2*sqrt
(b + c*x**2)*c**3*x**5 + 15*sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqr
t(b))*b**3 + 15*sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b))*b**2*c
*x**2 - 10*sqrt(c)*b**3 - 10*sqrt(c)*b**2*c*x**2)/(8*c**4*(b + c*x**2))
```

3.208 $\int \frac{x^7}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{x^7}{(bx^2+cx^4)^{3/2}}\,dx=\frac{bx^2}{c^2\sqrt{bx^2+cx^4}}+\frac{\sqrt{bx^2+cx^4}}{2c^2}-\frac{3b\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}$$

output
```
b*x^2/c^2/(c*x^4+b*x^2)^(1/2)+1/2*(c*x^4+b*x^2)^(1/2)/c^2-3/2*b*arctanh(c^
(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{x^7}{(bx^2+cx^4)^{3/2}}\,dx=\frac{x\left(\sqrt{c}x(3b+cx^2)+6b\sqrt{b+cx^2}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}-\sqrt{b+cx^2}}\right)\right)}{2c^{5/2}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(Sqrt[c]*x*(3*b + c*x^2) + 6*b*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/(Sqr
t[b] - Sqrt[b + c*x^2])]))/(2*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1424, 1124, 25, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{x^6}{(cx^4+bx^2)^{3/2}}dx^2$$

$$\Big\downarrow \text{1124}$$

$$\frac{1}{2}\left(\frac{\int -\frac{b-cx^2}{\sqrt{cx^4+bx^2}}dx^2}{c^2}+\frac{2bx^2}{c^2\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{2bx^2}{c^2\sqrt{bx^2+cx^4}}-\frac{\int \frac{b-cx^2}{\sqrt{cx^4+bx^2}}dx^2}{c^2}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{2bx^2}{c^2\sqrt{bx^2+cx^4}}-\frac{\frac{3}{2}b\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2-\sqrt{bx^2+cx^4}}{c^2}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{2bx^2}{c^2\sqrt{bx^2+cx^4}}-\frac{3b\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}-\sqrt{bx^2+cx^4}}{c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2bx^2}{c^2\sqrt{bx^2+cx^4}}-\frac{\frac{3b\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}-\sqrt{bx^2+cx^4}}{c^2}\right)$$

input
```
Int[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((2*b*x^2)/(c^2*Sqrt[b*x^2 + c*x^4]) - (-Sqrt[b*x^2 + c*x^4] + (3*b*ArcTan
h[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/Sqrt[c])/c^2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{x^{3}\left(c\,x^{2}+b\right)\left(c^{\frac{5}{2}}x^{3}+3c^{\frac{3}{2}}bx-3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)\sqrt{c\,x^{2}+b}\,bc\right)}{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}c^{\frac{7}{2}}}$	73
risch	$\frac{x^{2}\left(c\,x^{2}+b\right)}{2c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(\frac{bx}{c^{2}\sqrt{c\,x^{2}+b}}-\frac{3b\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)}{2c^{\frac{5}{2}}}\right)x\sqrt{c\,x^{2}+b}}{\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	92
pseudoelliptic	$-\frac{3\left(-\frac{2c^{\frac{3}{2}}x^{4}}{3}-2b\,x^{2}\sqrt{c}+\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b\sqrt{x^{2}\left(c\,x^{2}+b\right)}-\ln\left(2\right)b\sqrt{x^{2}\left(c\,x^{2}+b\right)}\right)}{4c^{\frac{5}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	101

input
```
int(x^7/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^3*(c*x^2+b)*(c^(5/2)*x^3+3*c^(3/2)*b*x-3*ln(c^(1/2)*x+(c*x^2+b)^(1/2
))*(c*x^2+b)^(1/2)*b*c)/(c*x^4+b*x^2)^(3/2)/c^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.22

$$\int \frac{x^7}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \left[\frac{3\left(bcx^2+b^2\right)\sqrt{c}\log\left(-2\,cx^2-b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\sqrt{cx^4+bx^2}(c^2x^2+3\,bc)}{4\left(c^4x^2+bc^3\right)},\right.$$

input
```
integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(b*c*x^2 + b^2)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*s
qrt(c)) + 2*sqrt(c*x^4 + b*x^2)*(c^2*x^2 + 3*b*c))/(c^4*x^2 + b*c^3), 1/2*
(3*(b*c*x^2 + b^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b
)) + sqrt(c*x^4 + b*x^2)*(c^2*x^2 + 3*b*c))/(c^4*x^2 + b*c^3)]
```

Sympy [F]

$$\int \frac{x^7}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7}{(x^2(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**7/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**7/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^4}{2\sqrt{cx^4+bx^2}c} + \frac{3\,bx^2}{2\sqrt{cx^4+bx^2}c^2} - \frac{3\,b\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{4\,c^{\frac{5}{2}}}$$

input
```
integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^4/(sqrt(c*x^4 + b*x^2)*c) + 3/2*b*x^2/(sqrt(c*x^4 + b*x^2)*c^2) - 3/
4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{x^7}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x\left(\frac{x^2}{c\mathrm{sgn}(x)} + \frac{3\,b}{c^2\mathrm{sgn}(x)}\right)}{2\sqrt{cx^2+b}} - \frac{3\,b\log\left(|b|\right)\mathrm{sgn}(x)}{4\,c^{\frac{5}{2}}} + \frac{3\,b\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{2\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^7/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*x*(x^2/(c*sgn(x)) + 3*b/(c^2*sgn(x)))/sqrt(c*x^2 + b) - 3/4*b*log(abs(
b))*sgn(x)/c^(5/2) + 3/2*b*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(5/2)
*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int(x^7/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^7/(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int \frac{x^7}{(bx^2+cx^4)^{3/2}}\,dx = \frac{12\sqrt{c\,x^2+b}\,bcx+4\sqrt{c\,x^2+b}\,c^2x^3-12\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)b^2-12\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}}{\sqrt{b}}\right.}{8c^3\,(c\,x^2+b)}$$

input
```
int(x^7/(c*x^4+b*x^2)^(3/2),x)
```

output
```
(12*sqrt(b + c*x**2)*b*c*x + 4*sqrt(b + c*x**2)*c**2*x**3 - 12*sqrt(c)*log
((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b))*b**2 - 12*sqrt(c)*log((sqrt(b + c
*x**2) + sqrt(c)*x)/sqrt(b))*b*c*x**2 + 9*sqrt(c)*b**2 + 9*sqrt(c)*b*c*x**
2)/(8*c**3*(b + c*x**2))
```

3.209 $\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x^2}{c\sqrt{bx^2+cx^4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}$$

output
```
-x^2/c/(c*x^4+b*x^2)^(1/2)+arctanh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/c^(3/2
)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x\left(\sqrt{c}x + 2\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}-\sqrt{b+cx^2}}\right)\right)}{c^{3/2}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^5/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-((x*(Sqrt[c]*x + 2*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/(Sqrt[b] - Sqrt[b
+ c*x^2])]))/(c^(3/2)*Sqrt[x^2*(b + c*x^2)]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1424, 1124, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{x^4}{(cx^4+bx^2)^{3/2}}dx^2$$

$$\Big\downarrow \text{1124}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{c}-\frac{2x^2}{c\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{c}-\frac{2x^2}{c\sqrt{bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{2x^2}{c\sqrt{bx^2+cx^4}}\right)$$

input
```
Int[x^5/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
((-2*x^2)/(c*Sqrt[b*x^2 + c*x^4]) + (2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
c*x^4]])/c^(3/2))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

method	result	size
default	$-\frac{x^{3}\left(c\,x^{2}+b\right)\left(x\,c^{\frac{3}{2}}-\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)c\sqrt{c\,x^{2}+b}\right)}{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}}$	63
pseudoelliptic	$\frac{-2\sqrt{c}\,x^{2}+\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)-\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\ln\left(2\right)}{2c^{\frac{3}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	90

input
```
int(x^5/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-x^3*(c*x^2+b)*(x*c^(3/2)-ln(c^(1/2)*x+(c*x^2+b)^(1/2))*c*(c*x^2+b)^(1/2))
/(c*x^4+b*x^2)^(3/2)/c^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.73

$$\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx = \left[\frac{(cx^2+b)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}c}{2\,(c^3x^2+bc^2)}, -\frac{(cx^2+b)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}c}{c^3x^2+bc^2}\right]$$

input
```
integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((c*x^2 + b)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)
) - 2*sqrt(c*x^4 + b*x^2)*c)/(c^3*x^2 + b*c^2), -((c*x^2 + b)*sqrt(-c)*arc
tan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/(c^
3*x^2 + b*c^2)]
```

Sympy [F]

$$\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^5}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**5/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x^2}{\sqrt{cx^4+bx^2}c} + \frac{\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{2\,c^{\frac{3}{2}}}$$

input
```
integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-x^2/(sqrt(c*x^4 + b*x^2)*c) + 1/2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)
*sqrt(c))/c^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\log(|b|)\,\mathrm{sgn}(x)}{2\,c^{\frac{3}{2}}} - \frac{x}{\sqrt{cx^2+b}c\mathrm{sgn}\,(x)} - \frac{\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{c^{\frac{3}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*log(abs(b))*sgn(x)/c^(3/2) - x/(sqrt(c*x^2 + b)*c*sgn(x)) - log(abs(-s
qrt(c)*x + sqrt(c*x^2 + b)))/(c^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 18.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,c^{3/2}} - \frac{x^2}{c\,\sqrt{c\,x^4+b\,x^2}}$$

input
```
int(x^5/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2))/(2*c^(3/2)) - x^2/(c*(b
*x^2 + c*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int \frac{x^5}{(bx^2 + cx^4)^{3/2}}\,dx = \frac{-\sqrt{c\,x^2+b}\,cx + \sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)b + \sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)c\,x^2 - \sqrt{c}\,b - \sqrt{c}\,c\,x}{c^2\,(c\,x^2+b)}$$

input
```
int(x^5/(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - sqrt(b + c*x**2)*c*x + sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt
(b))*b + sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b))*c*x**2 - sqrt
(c)*b - sqrt(c)*c*x**2)/(c**2*(b + c*x**2))
```

3.210 $\int \frac{x^3}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{x^3}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^2}{b\sqrt{bx^2+cx^4}}$$

output
```
x^2/b/(c*x^4+b*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^2}{b\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^3/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
x^2/(b*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 1422$$

$$\frac{x^2}{b\sqrt{bx^2+cx^4}}$$

input
```
Int[x^3/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
x^2/(b*Sqrt[b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] && !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
pseudoelliptic	$\frac{x^2}{b\sqrt{x^2(c\,x^2+b)}}$	21
trager	$\frac{\sqrt{c\,x^4+b\,x^2}}{b(c\,x^2+b)}$	27
gosper	$\frac{(c\,x^2+b)x^4}{b(c\,x^4+b\,x^2)^{\frac{3}{2}}}$	28
default	$\frac{(c\,x^2+b)x^4}{b(c\,x^4+b\,x^2)^{\frac{3}{2}}}$	28
orering	$\frac{(c\,x^2+b)x^4}{b(c\,x^4+b\,x^2)^{\frac{3}{2}}}$	28

input
```
int(x^3/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*x^2/(x^2*(c*x^2+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{cx^4+bx^2}}{bcx^2+b^2}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^4 + b*x^2)/(b*c*x^2 + b^2)
```

Sympy [F]

$$\int \frac{x^3}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(x^2(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**3/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{(bx^2 + cx^4)^{3/2}}\,dx = \frac{x^2}{\sqrt{cx^4 + bx^2}b}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
x^2/(sqrt(c*x^4 + b*x^2)*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{(bx^2 + cx^4)^{3/2}}\,dx = \frac{x}{\sqrt{cx^2 + b}b\mathrm{sgn}\,(x)}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
x/(sqrt(c*x^2 + b)*b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(bx^2 + cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4 + b\,x^2}}{b\,\left(c\,x^2 + b\right)}$$

input
```
int(x^3/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
(b*x^2 + c*x^4)^(1/2)/(b*(b + c*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^3}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^2+b}\,cx+\sqrt{c}\,b+\sqrt{c}\,c\,x^2}{bc\,(c\,x^2+b)}$$

input
```
int(x^3/(c*x^4+b*x^2)^(3/2),x)
```

output
```
(sqrt(b + c*x**2)*c*x + sqrt(c)*b + sqrt(c)*c*x**2)/(b*c*(b + c*x**2))
```

3.211 $\int \frac{x}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{x}{\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

output
```
-(2*c*x^2+b)/b^2/(c*x^4+b*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{x}{\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{-b-2cx^2}{b^2\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-b - 2*c*x^2)/(b^2*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1424, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(bx^2+cx^4)^{3/2}}\,dx$$
$$\Big\downarrow \text{1424}$$
$$\frac{1}{2}\int \frac{1}{(cx^4+bx^2)^{3/2}}dx^2$$
$$\Big\downarrow \text{1088}$$
$$-\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

input
```
Int[x/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-((b + 2*c*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2 Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m, p}, x] && !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
pseudoelliptic	$-\frac{2c\,x^{2}+b}{b^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	27
gosper	$-\frac{x^{2}\left(c\,x^{2}+b\right)\left(2c\,x^{2}+b\right)}{b^{2}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	37
default	$-\frac{x^{2}\left(c\,x^{2}+b\right)\left(2c\,x^{2}+b\right)}{b^{2}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	37
orering	$-\frac{x^{2}\left(c\,x^{2}+b\right)\left(2c\,x^{2}+b\right)}{b^{2}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	37
trager	$-\frac{\left(2c\,x^{2}+b\right)\sqrt{c\,x^{4}+b\,x^{2}}}{\left(c\,x^{2}+b\right)b^{2}x^{2}}$	39
risch	$-\frac{c\,x^{2}+b}{b^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{c\,x^{2}}{b^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	49

input
```
int(x/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(2*c*x^2+b)/b^2/(x^2*(c*x^2+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{x}{\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{\sqrt{cx^4+bx^2}\left(2\,cx^2+b\right)}{b^2cx^4+b^3x^2}$$

input
```
integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(c*x^4 + b*x^2)*(2*c*x^2 + b)/(b^2*c*x^4 + b^3*x^2)
```

Sympy [F]

$$\int \frac{x}{(bx^2 + cx^4)^{3/2}} \, dx = \int \frac{x}{(x^2 (b + cx^2))^{\frac{3}{2}}} \, dx$$

input
```
integrate(x/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{x}{(bx^2 + cx^4)^{3/2}} \, dx = -\frac{2\,cx^2}{\sqrt{cx^4 + bx^2}b^2} - \frac{1}{\sqrt{cx^4 + bx^2}b}$$

input
```
integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) - 1/(sqrt(c*x^4 + b*x^2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.07

$$\int \frac{x}{(bx^2 + cx^4)^{3/2}} \, dx = -\frac{cx}{\sqrt{cx^2 + b}b^2 \mathrm{sgn}\,(x)} + \frac{2\,\sqrt{c}}{\left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 - b\right) b \mathrm{sgn}\,(x)}$$

input
```
integrate(x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-c*x/(sqrt(c*x^2 + b)*b^2*sgn(x)) + 2*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 +
b))^2 - b)*b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 16.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{2\,c\,x^2+b}{b^2\,\sqrt{c\,x^4+b\,x^2}}$$

input
```
int(x/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
-(b + 2*c*x^2)/(b^2*(b*x^2 + c*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{x}{(bx^2+cx^4)^{3/2}}\,dx = \frac{-\sqrt{c\,x^2+b}\,b-2\sqrt{c\,x^2+b}\,c\,x^2-2\sqrt{c}\,bx-2\sqrt{c}\,c\,x^3}{b^2x\,(c\,x^2+b)}$$

input
```
int(x/(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - sqrt(b + c*x**2)*b - 2*sqrt(b + c*x**2)*c*x**2 - 2*sqrt(c)*b*x - 2*sqr
t(c)*c*x**3)/(b**2*x*(b + c*x**2))
```

3.212 $\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{1}{3bx^2\sqrt{bx^2+cx^4}}+\frac{4c(b+2cx^2)}{3b^3\sqrt{bx^2+cx^4}}$$

output
```
-1/3/b/x^2/(c*x^4+b*x^2)^(1/2)+4/3*c*(2*c*x^2+b)/b^3/(c*x^4+b*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{-b^2+4bcx^2+8c^2x^4}{3b^3x^2\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[1/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-b^2 + 4*b*c*x^2 + 8*c^2*x^4)/(3*b^3*x^2*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1423, 1424, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \; 1423$$

$$-\frac{4c\int \frac{x}{\left(cx^4+bx^2\right)^{3/2}}dx}{3b}-\frac{1}{3bx^2\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 1424$$

$$-\frac{2c\int \frac{1}{\left(cx^4+bx^2\right)^{3/2}}dx^2}{3b}-\frac{1}{3bx^2\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 1088$$

$$\frac{4c\left(b+2cx^2\right)}{3b^3\sqrt{bx^2+cx^4}}-\frac{1}{3bx^2\sqrt{bx^2+cx^4}}$$

input
```
Int[1/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-1/3*1/(b*x^2*Sqrt[b*x^2 + c*x^4]) + (4*c*(b + 2*c*x^2))/(3*b^3*Sqrt[b*x^2
 + c*x^4])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$-\frac{-8c^{2}x^{4}-4bc\,x^{2}+b^{2}}{3b^{3}x^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	41
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-8c^{2}x^{4}-4bc\,x^{2}+b^{2}\right)}{3b^{3}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	45
default	$-\frac{\left(c\,x^{2}+b\right)\left(-8c^{2}x^{4}-4bc\,x^{2}+b^{2}\right)}{3b^{3}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	45
orering	$-\frac{\left(c\,x^{2}+b\right)\left(-8c^{2}x^{4}-4bc\,x^{2}+b^{2}\right)}{3b^{3}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	45
trager	$-\frac{\left(-8c^{2}x^{4}-4bc\,x^{2}+b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{3\left(c\,x^{2}+b\right)b^{3}x^{4}}$	50
risch	$-\frac{\left(c\,x^{2}+b\right)\left(-5c\,x^{2}+b\right)}{3b^{3}x^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{x^{2}c^{2}}{b^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	61

input
```
int(1/x/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-8*c^2*x^4-4*b*c*x^2+b^2)/b^3/x^2/(x^2*(c*x^2+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{\left(8\,c^2x^4+4\,bcx^2-b^2\right)\sqrt{cx^4+bx^2}}{3\left(b^3cx^6+b^4x^4\right)}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(8*c^2*x^4 + 4*b*c*x^2 - b^2)*sqrt(c*x^4 + b*x^2)/(b^3*c*x^6 + b^4*x^4
)
```

Sympy [F]

$$\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(1/(x*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{8\,c^2x^2}{3\,\sqrt{cx^4+bx^2}b^3}+\frac{4\,c}{3\,\sqrt{cx^4+bx^2}b^2}-\frac{1}{3\,\sqrt{cx^4+bx^2}bx^2}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
8/3*c^2*x^2/(sqrt(c*x^4 + b*x^2)*b^3) + 4/3*c/(sqrt(c*x^4 + b*x^2)*b^2) -
1/3/(sqrt(c*x^4 + b*x^2)*b*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(49) = 98.

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{c^2x}{\sqrt{cx^2+b}b^3\mathrm{sgn}\left(x\right)} - \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4c^{\frac{3}{2}}-12\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2bc^{\frac{3}{2}}+5\,b^2c^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^3b^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
c^2*x/(sqrt(c*x^2 + b)*b^3*sgn(x)) - 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^
4*c^(3/2) - 12*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b*c^(3/2) + 5*b^2*c^(3/2))/
(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3*b^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\left(-b^2+4\,b\,c\,x^2+8\,c^2\,x^4\right)}{3\,b^3\,x^4\,\left(c\,x^2+b\right)}$$

input
```
int(1/(x*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - b^2 + 4*b*c*x^2))/(3*b^3*x^4*(b + c*x^
2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.42

$$\int \frac{1}{x\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{-\sqrt{c\,x^2+b}\,b^2+4\sqrt{c\,x^2+b}\,bc\,x^2+8\sqrt{c\,x^2+b}\,c^2x^4-8\sqrt{c}\,bc\,x^3-8\sqrt{c}\,c^2x^5}{3b^3x^3\left(c\,x^2+b\right)}$$

input
```
int(1/x/(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - sqrt(b + c*x**2)*b**2 + 4*sqrt(b + c*x**2)*b*c*x**2 + 8*sqrt(b + c*x**
2)*c**2*x**4 - 8*sqrt(c)*b*c*x**3 - 8*sqrt(c)*c**2*x**5)/(3*b**3*x**3*(b +
 c*x**2))
```

3.213 $\int \frac{1}{x^3 (bx^2+cx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \frac{1}{x^3 (bx^2+cx^4)^{3/2}} \, dx = -\frac{1}{5bx^4\sqrt{bx^2+cx^4}} + \frac{2c}{5b^2x^2\sqrt{bx^2+cx^4}} - \frac{8c^2(b+2cx^2)}{5b^4\sqrt{bx^2+cx^4}}$$

output
```
-1/5/b/x^4/(c*x^4+b*x^2)^(1/2)+2/5*c/b^2/x^2/(c*x^4+b*x^2)^(1/2)-8/5*c^2*(
2*c*x^2+b)/b^4/(c*x^4+b*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3 (bx^2+cx^4)^{3/2}} \, dx = \frac{-b^3+2b^2cx^2-8bc^2x^4-16c^3x^6}{5b^4x^4\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-b^3 + 2*b^2*c*x^2 - 8*b*c^2*x^4 - 16*c^3*x^6)/(5*b^4*x^4*Sqrt[x^2*(b + c
*x^2)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1423, 1423, 1424, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1423}$$

$$-\frac{6c\int \frac{1}{x(cx^4+bx^2)^{3/2}}dx}{5b}-\frac{1}{5bx^4\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1423}$$

$$-\frac{6c\left(-\frac{4c\int \frac{x}{\left(cx^4+bx^2\right)^{3/2}}dx}{3b}-\frac{1}{3bx^2\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{1}{5bx^4\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1424}$$

$$-\frac{6c\left(-\frac{2c\int \frac{1}{\left(cx^4+bx^2\right)^{3/2}}dx^2}{3b}-\frac{1}{3bx^2\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{1}{5bx^4\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1088}$$

$$-\frac{6c\left(\frac{4c(b+2cx^2)}{3b^3\sqrt{bx^2+cx^4}}-\frac{1}{3bx^2\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{1}{5bx^4\sqrt{bx^2+cx^4}}$$

input
```
Int[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-1/5*1/(b*x^4*Sqrt[b*x^2 + c*x^4]) - (6*c*(-1/3*1/(b*x^2*Sqrt[b*x^2 + c*x^
4]) + (4*c*(b + 2*c*x^2))/(3*b^3*Sqrt[b*x^2 + c*x^4])))/(5*b)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{16c^{3}x^{6}+8b\,c^{2}x^{4}-2b^{2}c\,x^{2}+b^{3}}{5x^{4}b^{4}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	52
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(16c^{3}x^{6}+8b\,c^{2}x^{4}-2b^{2}c\,x^{2}+b^{3}\right)}{5x^{2}b^{4}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	59
default	$-\frac{\left(c\,x^{2}+b\right)\left(16c^{3}x^{6}+8b\,c^{2}x^{4}-2b^{2}c\,x^{2}+b^{3}\right)}{5x^{2}b^{4}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	59
orering	$-\frac{\left(c\,x^{2}+b\right)\left(16c^{3}x^{6}+8b\,c^{2}x^{4}-2b^{2}c\,x^{2}+b^{3}\right)}{5x^{2}b^{4}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	59
trager	$-\frac{\left(16c^{3}x^{6}+8b\,c^{2}x^{4}-2b^{2}c\,x^{2}+b^{3}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{5\left(c\,x^{2}+b\right)b^{4}x^{6}}$	61
risch	$-\frac{\left(c\,x^{2}+b\right)\left(11c^{2}x^{4}-3bc\,x^{2}+b^{2}\right)}{5b^{4}x^{4}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{x^{2}c^{3}}{b^{4}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	73

input
```
int(1/x^3/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(16*c^3*x^6+8*b*c^2*x^4-2*b^2*c*x^2+b^3)/x^4/b^4/(x^2*(c*x^2+b))^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3 (bx^2+cx^4)^{3/2}}\,dx = -\frac{(16\,c^3x^6+8\,bc^2x^4-2\,b^2cx^2+b^3)\sqrt{cx^4+bx^2}}{5\,(b^4cx^8+b^5x^6)}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/5*(16*c^3*x^6 + 8*b*c^2*x^4 - 2*b^2*c*x^2 + b^3)*sqrt(c*x^4 + b*x^2)/(b
^4*c*x^8 + b^5*x^6)
```

Sympy [F]

$$\int \frac{1}{x^3 (bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{x^3 (x^2 (b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(1/(x**3*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3 (bx^2+cx^4)^{3/2}}\,dx = -\frac{16\,c^3x^2}{5\,\sqrt{cx^4+bx^2}b^4} - \frac{8\,c^2}{5\,\sqrt{cx^4+bx^2}b^3} + \frac{2\,c}{5\,\sqrt{cx^4+bx^2}b^2x^2} - \frac{1}{5\,\sqrt{cx^4+bx^2}bx^4}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-16/5*c^3*x^2/(sqrt(c*x^4 + b*x^2)*b^4) - 8/5*c^2/(sqrt(c*x^4 + b*x^2)*b^3
) + 2/5*c/(sqrt(c*x^4 + b*x^2)*b^2*x^2) - 1/5/(sqrt(c*x^4 + b*x^2)*b*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(73) = 146$.

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.99

$$\int \frac{1}{x^3 (bx^2+cx^4)^{3/2}}\,dx = -\frac{c^3x}{\sqrt{cx^2+b}b^4\mathrm{sgn}(x)}$$
$$+\frac{2\left(5\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8c^{\frac{5}{2}}-30\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6bc^{\frac{5}{2}}+80\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4b^2c^{\frac{5}{2}}-50\left(\sqrt{c}x-\sqrt{cx^2}\right.\right.}{5\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^5b^3\mathrm{sgn}(x)}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-c^3*x/(sqrt(c*x^2 + b)*b^4*sgn(x)) + 2/5*(5*(sqrt(c)*x - sqrt(c*x^2 + b))
^8*c^(5/2) - 30*(sqrt(c)*x - sqrt(c*x^2 + b))^6*b*c^(5/2) + 80*(sqrt(c)*x
- sqrt(c*x^2 + b))^4*b^2*c^(5/2) - 50*(sqrt(c)*x - sqrt(c*x^2 + b))^2*b^3*
c^(5/2) + 11*b^4*c^(5/2))/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5*b^3*sgn
(x))
```

Mupad [B] (verification not implemented)

Time = 17.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3 (bx^2+cx^4)^{3/2}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,\left(b^3-2\,b^2\,c\,x^2+8\,b\,c^2\,x^4+16\,c^3\,x^6\right)}{5\,b^4\,x^6\,\left(c\,x^2+b\right)}$$

input
```
int(1/(x^3*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
-((b*x^2 + c*x^4)^(1/2)*(b^3 + 16*c^3*x^6 - 2*b^2*c*x^2 + 8*b*c^2*x^4))/(5
*b^4*x^6*(b + c*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^3 (bx^2 + cx^4)^{3/2}} \, dx = \frac{-\sqrt{c\,x^2+b}\,b^3 + 2\sqrt{c\,x^2+b}\,b^2c\,x^2 - 8\sqrt{c\,x^2+b}\,b\,c^2x^4 - 16\sqrt{c\,x^2+b}\,c^3x^6 + 16\sqrt{}}{5b^4x^5\,(c\,x^2+b)}$$

input
```
int(1/x^3/(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - sqrt(b + c*x**2)*b**3 + 2*sqrt(b + c*x**2)*b**2*c*x**2 - 8*sqrt(b + c*
x**2)*b*c**2*x**4 - 16*sqrt(b + c*x**2)*c**3*x**6 + 16*sqrt(c)*b*c**2*x**5
 + 16*sqrt(c)*c**3*x**7)/(5*b**4*x**5*(b + c*x**2))
```

3.214 $\int \frac{1}{x^5(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{1}{x^5 (bx^2+cx^4)^{3/2}}\,dx = -\frac{1}{7bx^6\sqrt{bx^2+cx^4}} + \frac{8c}{35b^2x^4\sqrt{bx^2+cx^4}} - \frac{16c^2}{35b^3x^2\sqrt{bx^2+cx^4}} + \frac{64c^3(b+2cx^2)}{35b^5\sqrt{bx^2+cx^4}}$$

output
```
-1/7/b/x^6/(c*x^4+b*x^2)^(1/2)+8/35*c/b^2/x^4/(c*x^4+b*x^2)^(1/2)-16/35*c^
2/b^3/x^2/(c*x^4+b*x^2)^(1/2)+64/35*c^3*(2*c*x^2+b)/b^5/(c*x^4+b*x^2)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^5 (bx^2+cx^4)^{3/2}}\,dx = \frac{-5b^4+8b^3cx^2-16b^2c^2x^4+64bc^3x^6+128c^4x^8}{35b^5x^6\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-5*b^4 + 8*b^3*c*x^2 - 16*b^2*c^2*x^4 + 64*b*c^3*x^6 + 128*c^4*x^8)/(35*b
^5*x^6*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1423, 1423, 1423, 1424, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (bx^2+cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1423}$$

$$-\frac{8c \int \frac{1}{x^3 (cx^4+bx^2)^{3/2}}dx}{7b}-\frac{1}{7bx^6\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1423}$$

$$-\frac{8c \left(-\frac{6c \int \frac{1}{x \left(cx^4+bx^2\right)^{3/2}}dx}{5b}-\frac{1}{5bx^4\sqrt{bx^2+cx^4}}\right)}{7b}-\frac{1}{7bx^6\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1423}$$

$$-\frac{8c \left(-\frac{6c \left(-\frac{4c \int \frac{x}{\left(cx^4+bx^2\right)^{3/2}}dx}{3b}-\frac{1}{3bx^2\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{1}{5bx^4\sqrt{bx^2+cx^4}}\right)}{7b}-\frac{1}{7bx^6\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1424}$$

$$-\frac{8c\left(-\frac{6c\left(-\frac{2c\int \frac{1}{\left(cx^4+bx^2\right)^{3/2}}dx^2}{3b}-\frac{1}{3bx^2\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{1}{5bx^4\sqrt{bx^2+cx^4}}\right)}{7b}-\frac{1}{7bx^6\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{8c\left(-\frac{6c\left(\frac{4c\left(b+2cx^2\right)}{3b^3\sqrt{bx^2+cx^4}}-\frac{1}{3bx^2\sqrt{bx^2+cx^4}}\right)}{5b}-\frac{1}{5bx^4\sqrt{bx^2+cx^4}}\right)}{7b}-\frac{1}{7bx^6\sqrt{bx^2+cx^4}}$$

input
```
Int[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-1/7*1/(b*x^6*Sqrt[b*x^2 + c*x^4]) - (8*c*(-1/5*1/(b*x^4*Sqrt[b*x^2 + c*x^
4]) - (6*c*(-1/3*1/(b*x^2*Sqrt[b*x^2 + c*x^4]) + (4*c*(b + 2*c*x^2))/(3*b^
3*Sqrt[b*x^2 + c*x^4])))/(5*b)))/(7*b)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

method	result	size
pseudoelliptic	$\frac{128c^{4}x^{8}+64b\,c^{3}x^{6}-16b^{2}c^{2}x^{4}+8x^{2}b^{3}c-5b^{4}}{35x^{6}b^{5}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	65
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-128c^{4}x^{8}-64b\,c^{3}x^{6}+16b^{2}c^{2}x^{4}-8x^{2}b^{3}c+5b^{4}\right)}{35x^{4}b^{5}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	72
default	$-\frac{\left(c\,x^{2}+b\right)\left(-128c^{4}x^{8}-64b\,c^{3}x^{6}+16b^{2}c^{2}x^{4}-8x^{2}b^{3}c+5b^{4}\right)}{35x^{4}b^{5}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	72
orering	$-\frac{\left(c\,x^{2}+b\right)\left(-128c^{4}x^{8}-64b\,c^{3}x^{6}+16b^{2}c^{2}x^{4}-8x^{2}b^{3}c+5b^{4}\right)}{35x^{4}b^{5}\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	72
trager	$-\frac{\left(-128c^{4}x^{8}-64b\,c^{3}x^{6}+16b^{2}c^{2}x^{4}-8x^{2}b^{3}c+5b^{4}\right)\sqrt{c\,x^{4}+b\,x^{2}}}{35\left(c\,x^{2}+b\right)b^{5}x^{8}}$	74
risch	$-\frac{\left(c\,x^{2}+b\right)\left(-93c^{3}x^{6}+29b\,c^{2}x^{4}-13b^{2}c\,x^{2}+5b^{3}\right)}{35b^{5}x^{6}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{x^{2}c^{4}}{b^{5}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	85

input
```
int(1/x^5/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(128*c^4*x^8+64*b*c^3*x^6-16*b^2*c^2*x^4+8*b^3*c*x^2-5*b^4)/x^6/b^5/(
x^2*(c*x^2+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^5 \left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{\left(128\,c^4x^8+64\,bc^3x^6-16\,b^2c^2x^4+8\,b^3cx^2-5\,b^4\right)\sqrt{cx^4+bx^2}}{35\left(b^5cx^{10}+b^6x^8\right)}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/35*(128*c^4*x^8 + 64*b*c^3*x^6 - 16*b^2*c^2*x^4 + 8*b^3*c*x^2 - 5*b^4)*s
qrt(c*x^4 + b*x^2)/(b^5*c*x^10 + b^6*x^8)
```

Sympy [F]

$$\int \frac{1}{x^5 (bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{x^5 \left(x^2 (b+cx^2)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**5/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(1/(x**5*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (bx^2+cx^4)^{3/2}}\,dx = \frac{128\,c^4x^2}{35\,\sqrt{cx^4+bx^2}b^5} + \frac{64\,c^3}{35\,\sqrt{cx^4+bx^2}b^4} - \frac{16\,c^2}{35\,\sqrt{cx^4+bx^2}b^3x^2} + \frac{8\,c}{35\,\sqrt{cx^4+bx^2}b^2x^4} - \frac{1}{7\,\sqrt{cx^4+bx^2}bx^6}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
128/35*c^4*x^2/(sqrt(c*x^4 + b*x^2)*b^5) + 64/35*c^3/(sqrt(c*x^4 + b*x^2)*
b^4) - 16/35*c^2/(sqrt(c*x^4 + b*x^2)*b^3*x^2) + 8/35*c/(sqrt(c*x^4 + b*x^
2)*b^2*x^4) - 1/7/(sqrt(c*x^4 + b*x^2)*b*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(97) = 194.

Time = 0.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.96

$$\int \frac{1}{x^5 (bx^2+cx^4)^{3/2}}\,dx = \frac{c^4x}{\sqrt{cx^2+b}b^5\mathrm{sgn}(x)} - \frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}c^{\frac{7}{2}} - 280\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{7}{2}} + 1015\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}b^2c^{\frac{7}{2}} - 2240\left(\sqrt{c}\right.\right.}{35\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right.}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
c^4*x/(sqrt(c*x^2 + b)*b^5*sgn(x)) - 2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b)
)^12*c^(7/2) - 280*(sqrt(c)*x - sqrt(c*x^2 + b))^10*b*c^(7/2) + 1015*(sqrt
(c)*x - sqrt(c*x^2 + b))^8*b^2*c^(7/2) - 2240*(sqrt(c)*x - sqrt(c*x^2 + b)
)^6*b^3*c^(7/2) + 1673*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b^4*c^(7/2) - 616*(
sqrt(c)*x - sqrt(c*x^2 + b))^2*b^5*c^(7/2) + 93*b^6*c^(7/2))/(((sqrt(c)*x
- sqrt(c*x^2 + b))^2 - b)^7*b^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 18.55 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^5 (bx^2+cx^4)^{3/2}}\,dx = \frac{13\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,b^3\,x^6} - \frac{\sqrt{c\,x^4+b\,x^2}}{7\,b^2\,x^8} - \frac{29\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b^4\,x^4} + \frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{93\,c^3}{35\,b^4}+\frac{128\,c^4\,x^2}{35\,b^5}\right)}{x^2\,\left(c\,x^2+b\right)}$$

input
```
int(1/(x^5*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
(13*c*(b*x^2 + c*x^4)^(1/2))/(35*b^3*x^6) - (b*x^2 + c*x^4)^(1/2)/(7*b^2*x
^8) - (29*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b^4*x^4) + ((b*x^2 + c*x^4)^(1/2)
*((93*c^3)/(35*b^4) + (128*c^4*x^2)/(35*b^5)))/(x^2*(b + c*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^5 (bx^2+cx^4)^{3/2}}\,dx = \frac{-5\sqrt{c\,x^2+b}\,b^4+8\sqrt{c\,x^2+b}\,b^3c\,x^2-16\sqrt{c\,x^2+b}\,b^2c^2x^4+64\sqrt{c\,x^2+b}\,b\,c^3x^6+}{35b^5x^7\,(c\,x^2+b)}$$

input
```
int(1/x^5/(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - 5*sqrt(b + c*x**2)*b**4 + 8*sqrt(b + c*x**2)*b**3*c*x**2 - 16*sqrt(b +
 c*x**2)*b**2*c**2*x**4 + 64*sqrt(b + c*x**2)*b*c**3*x**6 + 128*sqrt(b + c
*x**2)*c**4*x**8 - 128*sqrt(c)*b*c**3*x**7 - 128*sqrt(c)*c**4*x**9)/(35*b*
*5*x**7*(b + c*x**2))
```

3.215 $\int \frac{x^{10}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 106

$$\int \frac{x^{10}}{(bx^2+cx^4)^{3/2}}\,dx=\frac{16b^3x}{5c^4\sqrt{bx^2+cx^4}}+\frac{8b^2x^3}{5c^3\sqrt{bx^2+cx^4}}-\frac{2bx^5}{5c^2\sqrt{bx^2+cx^4}}+\frac{x^7}{5c\sqrt{bx^2+cx^4}}$$

output
```
16/5*b^3*x/c^4/(c*x^4+b*x^2)^(1/2)+8/5*b^2*x^3/c^3/(c*x^4+b*x^2)^(1/2)-2/5
*b*x^5/c^2/(c*x^4+b*x^2)^(1/2)+1/5*x^7/c/(c*x^4+b*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.51

$$\int \frac{x^{10}}{(bx^2+cx^4)^{3/2}}\,dx=\frac{x(16b^3+8b^2cx^2-2bc^2x^4+c^3x^6)}{5c^4\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^10/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(16*b^3 + 8*b^2*c*x^2 - 2*b*c^2*x^4 + c^3*x^6))/(5*c^4*Sqrt[x^2*(b + c*
x^2)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1421, 1421, 1421, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^7}{5c\sqrt{bx^2+cx^4}}-\frac{6b\int \frac{x^8}{(cx^4+bx^2)^{3/2}}dx}{5c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^7}{5c\sqrt{bx^2+cx^4}}-\frac{6b\left(\frac{x^5}{3c\sqrt{bx^2+cx^4}}-\frac{4b\int \frac{x^6}{\left(cx^4+bx^2\right)^{3/2}}dx}{3c}\right)}{5c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^7}{5c\sqrt{bx^2+cx^4}}-\frac{6b\left(\frac{x^5}{3c\sqrt{bx^2+cx^4}}-\frac{4b\left(\frac{x^3}{c\sqrt{bx^2+cx^4}}-\frac{2b\int \frac{x^4}{\left(cx^4+bx^2\right)^{3/2}}dx}{c}\right)}{3c}\right)}{5c}$$

$$\Big\downarrow \text{1420}$$

$$\frac{x^7}{5c\sqrt{bx^2+cx^4}}-\frac{6b\left(\frac{x^5}{3c\sqrt{bx^2+cx^4}}-\frac{4b\left(\frac{2bx}{c^2\sqrt{bx^2+cx^4}}+\frac{x^3}{c\sqrt{bx^2+cx^4}}\right)}{3c}\right)}{5c}$$

input
```
Int[x^10/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
x^7/(5*c*Sqrt[b*x^2 + c*x^4]) - (6*b*(x^5/(3*c*Sqrt[b*x^2 + c*x^4]) - (4*b
*((2*b*x)/(c^2*Sqrt[b*x^2 + c*x^4]) + x^3/(c*Sqrt[b*x^2 + c*x^4])))/(3*c))
)/(5*c)
```

Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{\left(c\,x^{2}+b\right)\left(c^{3} x^{6}-2 b\,c^{2} x^{4}+8 b^{2} c\,x^{2}+16 b^{3}\right) x^{3}}{5 c^{4} \left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	60
default	$\frac{\left(c\,x^{2}+b\right)\left(c^{3} x^{6}-2 b\,c^{2} x^{4}+8 b^{2} c\,x^{2}+16 b^{3}\right) x^{3}}{5 c^{4} \left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	60
orering	$\frac{\left(c\,x^{2}+b\right)\left(c^{3} x^{6}-2 b\,c^{2} x^{4}+8 b^{2} c\,x^{2}+16 b^{3}\right) x^{3}}{5 c^{4} \left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	60
trager	$\frac{\left(c^{3} x^{6}-2 b\,c^{2} x^{4}+8 b^{2} c\,x^{2}+16 b^{3}\right) \sqrt{c\,x^{4}+b\,x^{2}}}{5 \left(c\,x^{2}+b\right) c^{4} x}$	62
risch	$\frac{\left(c^{2} x^{4}-3 b c\,x^{2}+11 b^{2}\right)\left(c\,x^{2}+b\right) x}{5 c^{4} \sqrt{x^{2} \left(c\,x^{2}+b\right)}}+\frac{b^{3} x}{c^{4} \sqrt{x^{2} \left(c\,x^{2}+b\right)}}$	69

input
```
int(x^10/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(c*x^2+b)*(c^3*x^6-2*b*c^2*x^4+8*b^2*c*x^2+16*b^3)*x^3/c^4/(c*x^4+b*x^
2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{x^{10}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(c^3x^6-2\,bc^2x^4+8\,b^2cx^2+16\,b^3)\sqrt{cx^4+bx^2}}{5\,(c^5x^3+bc^4x)}$$

input
```
integrate(x^10/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(c^3*x^6 - 2*b*c^2*x^4 + 8*b^2*c*x^2 + 16*b^3)*sqrt(c*x^4 + b*x^2)/(c^
5*x^3 + b*c^4*x)
```

Sympy [F]

$$\int \frac{x^{10}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{10}}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**10/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**10/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int \frac{x^{10}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{c^3x^6-2\,bc^2x^4+8\,b^2cx^2+16\,b^3}{5\,\sqrt{cx^2+b}c^4}$$

input
```
integrate(x^10/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/5*(c^3*x^6 - 2*b*c^2*x^4 + 8*b^2*c*x^2 + 16*b^3)/(sqrt(c*x^2 + b)*c^4)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.81

$$\int \frac{x^{10}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{16\,b^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,c^4} + \frac{b^3}{\sqrt{cx^2+b}c^4\mathrm{sgn}\left(x\right)} + \frac{(cx^2+b)^{\frac{5}{2}}c^{16} - 5\,(cx^2+b)^{\frac{3}{2}}bc^{16} + 15\,\sqrt{cx^2+b}b^2c^{16}}{5\,c^{20}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^10/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-16/5*b^(5/2)*sgn(x)/c^4 + b^3/(sqrt(c*x^2 + b)*c^4*sgn(x)) + 1/5*((c*x^2
+ b)^(5/2)*c^16 - 5*(c*x^2 + b)^(3/2)*b*c^16 + 15*sqrt(c*x^2 + b)*b^2*c^16
)/(c^20*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 18.61 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.58

$$\int \frac{x^{10}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,\left(16\,b^3+8\,b^2\,c\,x^2-2\,b\,c^2\,x^4+c^3\,x^6\right)}{5\,c^4\,x\,\left(c\,x^2+b\right)}$$

input
```
int(x^10/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*(16*b^3 + c^3*x^6 + 8*b^2*c*x^2 - 2*b*c^2*x^4))/(5*
c^4*x*(b + c*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int \frac{x^{10}}{(bx^2 + cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^2 + b}\,(c^3x^6 - 2b\,c^2x^4 + 8b^2c\,x^2 + 16b^3)}{5c^4\,(c\,x^2 + b)}$$

input
```
int(x^10/(c*x^4+b*x^2)^(3/2),x)
```

output
```
(sqrt(b + c*x**2)*(16*b**3 + 8*b**2*c*x**2 - 2*b*c**2*x**4 + c**3*x**6))/(
5*c**4*(b + c*x**2))
```

3.216 $\int \frac{x^8}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{x^8}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{8b^2x}{3c^3\sqrt{bx^2+cx^4}} - \frac{4bx^3}{3c^2\sqrt{bx^2+cx^4}} + \frac{x^5}{3c\sqrt{bx^2+cx^4}}$$

output
```
-8/3*b^2*x/c^3/(c*x^4+b*x^2)^(1/2)-4/3*b*x^3/c^2/(c*x^4+b*x^2)^(1/2)+1/3*x
^5/c/(c*x^4+b*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.55

$$\int \frac{x^8}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x(-8b^2-4bcx^2+c^2x^4)}{3c^3\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^8/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(-8*b^2 - 4*b*c*x^2 + c^2*x^4))/(3*c^3*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1421, 1421, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\left(bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^5}{3c\sqrt{bx^2+cx^4}}-\frac{4b\int \frac{x^6}{\left(cx^4+bx^2\right)^{3/2}}dx}{3c}$$

$$\Big\downarrow \text{1421}$$

$$\frac{x^5}{3c\sqrt{bx^2+cx^4}}-\frac{4b\left(\frac{x^3}{c\sqrt{bx^2+cx^4}}-\frac{2b\int \frac{x^4}{\left(cx^4+bx^2\right)^{3/2}}dx}{c}\right)}{3c}$$

$$\Big\downarrow \text{1420}$$

$$\frac{x^5}{3c\sqrt{bx^2+cx^4}}-\frac{4b\left(\frac{2bx}{c^2\sqrt{bx^2+cx^4}}+\frac{x^3}{c\sqrt{bx^2+cx^4}}\right)}{3c}$$

input
```
Int[x^8/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
x^5/(3*c*Sqrt[b*x^2 + c*x^4]) - (4*b*((2*b*x)/(c^2*Sqrt[b*x^2 + c*x^4]) +
x^3/(c*Sqrt[b*x^2 + c*x^4])))/(3*c)
```

Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-c^{2} x^{4}+4 b c\,x^{2}+8 b^{2}\right) x^{3}}{3 c^{3} \left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	50
default	$-\frac{\left(c\,x^{2}+b\right)\left(-c^{2} x^{4}+4 b c\,x^{2}+8 b^{2}\right) x^{3}}{3 c^{3} \left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	50
orering	$-\frac{\left(c\,x^{2}+b\right)\left(-c^{2} x^{4}+4 b c\,x^{2}+8 b^{2}\right) x^{3}}{3 c^{3} \left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	50
trager	$-\frac{\left(-c^{2} x^{4}+4 b c\,x^{2}+8 b^{2}\right) \sqrt{c\,x^{4}+b\,x^{2}}}{3 \left(c\,x^{2}+b\right) c^{3} x}$	52
risch	$-\frac{\left(-c\,x^{2}+5 b\right)\left(c\,x^{2}+b\right) x}{3 c^{3} \sqrt{x^{2} \left(c\,x^{2}+b\right)}}-\frac{b^{2} x}{c^{3} \sqrt{x^{2} \left(c\,x^{2}+b\right)}}$	60

input
```
int(x^8/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c*x^2+b)*(-c^2*x^4+4*b*c*x^2+8*b^2)*x^3/c^3/(c*x^4+b*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{x^8}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(c^2x^4-4\,bcx^2-8\,b^2)\sqrt{cx^4+bx^2}}{3\,(c^4x^3+bc^3x)}$$

input
```
integrate(x^8/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(c^2*x^4 - 4*b*c*x^2 - 8*b^2)*sqrt(c*x^4 + b*x^2)/(c^4*x^3 + b*c^3*x)
```

Sympy [F]

$$\int \frac{x^8}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^8}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**8/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**8/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.44

$$\int \frac{x^8}{(bx^2+cx^4)^{3/2}}\,dx = \frac{c^2x^4-4\,bcx^2-8\,b^2}{3\,\sqrt{cx^2+b}c^3}$$

input
```
integrate(x^8/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*(c^2*x^4 - 4*b*c*x^2 - 8*b^2)/(sqrt(c*x^2 + b)*c^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{x^8}{(bx^2+cx^4)^{3/2}}\,dx = \frac{8\,b^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,c^3} - \frac{\frac{3\,b^2}{\sqrt{cx^2+b}c\mathrm{sgn}(x)} - \frac{(cx^2+b)^{\frac{3}{2}}c^2-6\,\sqrt{cx^2+b}bc^2}{c^3\mathrm{sgn}(x)}}{3\,c^2}$$

input
```
integrate(x^8/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
8/3*b^(3/2)*sgn(x)/c^3 - 1/3*(3*b^2/(sqrt(c*x^2 + b)*c*sgn(x)) - ((c*x^2 +
 b)^(3/2)*c^2 - 6*sqrt(c*x^2 + b)*b*c^2)/(c^3*sgn(x)))/c^2
```

Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{x^8}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,\left(8\,b^2+4\,b\,c\,x^2-c^2\,x^4\right)}{3\,c^3\,x\,\left(c\,x^2+b\right)}$$

input
```
int(x^8/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
-((b*x^2 + c*x^4)^(1/2)*(8*b^2 - c^2*x^4 + 4*b*c*x^2))/(3*c^3*x*(b + c*x^2
))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.54

$$\int \frac{x^8}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^2+b}\,(c^2x^4-4bc\,x^2-8b^2)}{3c^3\,(c\,x^2+b)}$$

input
```
int(x^8/(c*x^4+b*x^2)^(3/2),x)
```

output
```
(sqrt(b + c*x**2)*( - 8*b**2 - 4*b*c*x**2 + c**2*x**4))/(3*c**3*(b + c*x**
2))
```

3.217 $\int \frac{x^6}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{x^6}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2bx}{c^2\sqrt{bx^2+cx^4}}+\frac{x^3}{c\sqrt{bx^2+cx^4}}$$

output
```
2*b*x/c^2/(c*x^4+b*x^2)^(1/2)+x^3/c/(c*x^4+b*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{x^6}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x(2b+cx^2)}{c^2\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^6/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(2*b + c*x^2))/(c^2*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1421, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(bx^2+cx^4)^{3/2}} \, dx$$
$$\Big\downarrow \text{1421}$$
$$\frac{x^3}{c\sqrt{bx^2+cx^4}}-\frac{2b\int \frac{x^4}{(cx^4+bx^2)^{3/2}}dx}{c}$$
$$\Big\downarrow \text{1420}$$
$$\frac{2bx}{c^2\sqrt{bx^2+cx^4}}+\frac{x^3}{c\sqrt{bx^2+cx^4}}$$

input
```
Int[x^6/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*b*x)/(c^2*Sqrt[b*x^2 + c*x^4]) + x^3/(c*Sqrt[b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{(c\,x^2+b)\,(c\,x^2+2b)\,x^3}{c^2(c\,x^4+b\,x^2)^{\frac{3}{2}}}$	37
default	$\frac{(c\,x^2+b)\,(c\,x^2+2b)\,x^3}{c^2(c\,x^4+b\,x^2)^{\frac{3}{2}}}$	37
orering	$\frac{(c\,x^2+b)\,(c\,x^2+2b)\,x^3}{c^2(c\,x^4+b\,x^2)^{\frac{3}{2}}}$	37
trager	$\frac{(c\,x^2+2b)\sqrt{c\,x^4+b\,x^2}}{(c\,x^2+b)c^2x}$	39
risch	$\frac{(c\,x^2+b)x}{c^2\sqrt{x^2(c\,x^2+b)}}+\frac{bx}{c^2\sqrt{x^2(c\,x^2+b)}}$	46

input
```
int(x^6/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b)*(c*x^2+2*b)*x^3/c^2/(c*x^4+b*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{x^6}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{cx^4+bx^2}(cx^2+2\,b)}{c^3x^3+bc^2x}$$

input
```
integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^4 + b*x^2)*(c*x^2 + 2*b)/(c^3*x^3 + b*c^2*x)
```

Sympy [F]

$$\int \frac{x^6}{\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{x^6}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**6/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int \frac{x^6}{\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{cx^2+2\,b}{\sqrt{cx^2+b}c^2}$$

input
```
integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
(c*x^2 + 2*b)/(sqrt(c*x^2 + b)*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{x^6}{\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{\frac{\sqrt{cx^2+b}}{c\operatorname{sgn}(x)}+\frac{b}{\sqrt{cx^2+b}c\operatorname{sgn}(x)}}{c}-\frac{2\sqrt{b}\operatorname{sgn}(x)}{c^2}$$

input
```
integrate(x^6/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(c*x^2 + b)/(c*sgn(x)) + b/(sqrt(c*x^2 + b)*c*sgn(x)))/c - 2*sqrt(b)*
sgn(x)/c^2
```

Mupad [B] (verification not implemented)

Time = 18.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{x^6}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,(c\,x^2+2\,b)}{c^2\,x\,\left(c\,x^2+b\right)}$$

input
```
int(x^6/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
((b*x^2 + c*x^4)^(1/2)*(2*b + c*x^2))/(c^2*x*(b + c*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

$$\int \frac{x^6}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^2+b}\,(c\,x^2+2b)}{c^2\,(c\,x^2+b)}$$

input
```
int(x^6/(c*x^4+b*x^2)^(3/2),x)
```

output
```
(sqrt(b + c*x**2)*(2*b + c*x**2))/(c**2*(b + c*x**2))
```

3.218 $\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x}{c\sqrt{bx^2+cx^4}}$$

output
```
-x/c/(c*x^4+b*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x}{c\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^4/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(x/(c*Sqrt[x^2*(b + c*x^2)]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1420}$$

$$-\frac{x}{c\sqrt{bx^2+cx^4}}$$

input
```
Int[x^4/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(x/(c*Sqrt[b*x^2 + c*x^4]))
```

Defintions of rubi rules used

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] && !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

method	result	size
gosper	$-\frac{(c\,x^{2}+b)\,x^{3}}{c\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	29
default	$-\frac{(c\,x^{2}+b)\,x^{3}}{c\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	29
orering	$-\frac{(c\,x^{2}+b)\,x^{3}}{c\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}}$	29
trager	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{(c\,x^{2}+b)\,c x}$	31

input
```
int(x^4/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+b)/c*x^3/(c*x^4+b*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{\sqrt{cx^4+bx^2}}{c^2x^3+bcx}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(c*x^4 + b*x^2)/(c^2*x^3 + b*c*x)
```

Sympy [F]

$$\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^4}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**4/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{1}{\sqrt{cx^2+b}c}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/(sqrt(c*x^2 + b)*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\operatorname{sgn}(x)}{\sqrt{b}c} - \frac{1}{\sqrt{cx^2+b}c\operatorname{sgn}(x)}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
sgn(x)/(sqrt(b)*c) - 1/(sqrt(c*x^2 + b)*c*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 18.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}}{c\,x\,\left(c\,x^2+b\right)}$$

input
```
int(x^4/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
-(b*x^2 + c*x^4)^(1/2)/(c*x*(b + c*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{\sqrt{c\,x^2+b}}{c\,(c\,x^2+b)}$$

input
```
int(x^4/(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - sqrt(b + c*x**2))/(c*(b + c*x**2))
```

3.219 $\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx=\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

output
```
x/b/(c*x^4+b*x^2)^(1/2)-arctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx=\frac{x\left(\sqrt{b}-\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{b^{3/2}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^2/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(Sqrt[b] - Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(b^(3/2)*
Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1428, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1428}$$

$$\frac{\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{b}+\frac{x}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1400}$$

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

input
```
Int[x^2/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
x/(b*Sqrt[b*x^2 + c*x^4]) - ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/b^(3/
2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{x^{3}\left(c\,x^{2}+b\right)\left(b^{\frac{3}{2}}-\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)b\sqrt{c\,x^{2}+b}\right)}{\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$	65

input
```
int(x^2/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^3*(c*x^2+b)*(b^(3/2)-ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*(c*x^2+b)^(1/
2))/(c*x^4+b*x^2)^(3/2)/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.08

$$\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx = \left[\frac{(cx^3+bx)\sqrt{b}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}b}{2\,(b^2cx^3+b^3x)}, \frac{(cx^3+bx)\sqrt{-b}\,\mathrm{arcta}\ldots}{\ldots}\right.$$

input
```
integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((c*x^3 + b*x)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sq
rt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*c*x^3 + b^3*x), ((c*x^3 + b*x)
*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(b*x)) + sqrt(c*x^4 + b*x^2)
*b)/(b^2*c*x^3 + b^3*x)]
```

Sympy [F]

$$\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**2/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.55

$$\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{\left(\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\right)\operatorname{sgn}(x)}{\sqrt{-b}b^{\frac{3}{2}}} + \frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b\operatorname{sgn}(x)} + \frac{1}{\sqrt{cx^2+b}b\operatorname{sgn}(x)}$$

input
```
integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-(sqrt(b)*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b))*sgn(x)/(sqrt(-b)*b^(3/2)) +
 arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b*sgn(x)) + 1/(sqrt(c*x^2 + b)
*b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int(x^2/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^2/(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.67

$$\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^2+b}\,b+\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)b+\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c\,x^2-\sqrt{b}\log\left(\right.}{b^2\left(c\,x^2+b\right)}$$

input
```
int(x^2/(c*x^4+b*x^2)^(3/2),x)
```

output
```
(sqrt(b + c*x**2)*b + sqrt(b)*log((sqrt(b + c*x**2) - sqrt(b) + sqrt(c)*x)
/sqrt(b))*b + sqrt(b)*log((sqrt(b + c*x**2) - sqrt(b) + sqrt(c)*x)/sqrt(b)
)*c*x**2 - sqrt(b)*log((sqrt(b + c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))*b
 - sqrt(b)*log((sqrt(b + c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))*c*x**2)/(
b**2*(b + c*x**2))
```

3.220 $\int \frac{1}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{1}{(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{bx\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{2b^2x^3} + \frac{3c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}$$

output
```
1/b/x/(c*x^4+b*x^2)^(1/2)-3/2*(c*x^4+b*x^2)^(1/2)/b^2/x^3+3/2*c*arctanh(b^
(1/2)*x/(c*x^4+b*x^2)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{1}{(bx^2+cx^4)^{3/2}}\,dx = \frac{-\sqrt{b}(b+3cx^2)+3cx^2\sqrt{b+cx^2}\,\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{2b^{5/2}x\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(-3/2),x]
```

output
```
(-(Sqrt[b]*(b + 3*c*x^2)) + 3*c*x^2*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2
]/Sqrt[b]])/(2*b^(5/2)*x*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1401, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1401}$$

$$\frac{3\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx}{b}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{3\left(-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{b}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1400}$$

$$\frac{3\left(\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{b}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{b}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

input
```
Int[(b*x^2 + c*x^4)^(-3/2),x]
```

output
```
1/(b*x*Sqrt[b*x^2 + c*x^4]) + (3*(-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*Ar
cTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(3/2))))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1401
```
Int[((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[-(b*x^2 + c*x^4)^
(p + 1)/(2*b*(p + 1)*x), x] + Simp[(4*p + 3)/(2*b*(p + 1))   Int[(b*x^2 + c
*x^4)^(p + 1)/x^2, x], x] /; FreeQ[{b, c}, x] &&  !IntegerQ[p] && LtQ[p, -1
]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{x\left(c\,x^{2}+b\right)\left(3 b^{\frac{3}{2}} c\,x^{2}-3 \ln\left(\frac{2 b+2 \sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right) \sqrt{c\,x^{2}+b}\, b c\,x^{2}+b^{\frac{5}{2}}\right)}{2 \left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}} b^{\frac{7}{2}}}$	77
risch	$-\frac{c\,x^{2}+b}{2 b^{2} x \sqrt{x^{2} \left(c\,x^{2}+b\right)}}+\frac{\left(-\frac{c}{b^{2} \sqrt{c\,x^{2}+b}}+\frac{3 c \ln\left(\frac{2 b+2 \sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)}{2 b^{\frac{5}{2}}}\right) x \sqrt{c\,x^{2}+b}}{\sqrt{x^{2} \left(c\,x^{2}+b\right)}}$	99

input
```
int(1/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(c*x^2+b)*(3*b^(3/2)*c*x^2-3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*(c
*x^2+b)^(1/2)*b*c*x^2+b^(5/2))/(c*x^4+b*x^2)^(3/2)/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.40

$$\int \frac{1}{(bx^2+cx^4)^{3/2}}\,dx = \left[\frac{3\left(c^2x^5+bcx^3\right)\sqrt{b}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\left(3\,bcx^2+b^2\right)}{4\left(b^3cx^5+b^4x^3\right)},\right.$$
$$\left.-\frac{3\left(c^2x^5+bcx^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{bx}\right)+\sqrt{cx^4+bx^2}\left(3\,bcx^2+b^2\right)}{2\left(b^3cx^5+b^4x^3\right)}\right]$$

input
```
integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(c^2*x^5 + b*c*x^3)*sqrt(b)*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b
*x^2)*sqrt(b))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 + b^2))/(b^3*c*x^5
+ b^4*x^3), -1/2*(3*(c^2*x^5 + b*c*x^3)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2
)*sqrt(-b)/(b*x)) + sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 + b^2))/(b^3*c*x^5 + b^
4*x^3)]
```

Sympy [F]

$$\int \frac{1}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral((b*x**2 + c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(bx^2 + cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4 + bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(-3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{1}{(bx^2 + cx^4)^{3/2}}\,dx = -\frac{3\,c\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{2\,\sqrt{-b}b^2\mathrm{sgn}\,(x)} - \frac{3\,(cx^2+b)c - 2\,bc}{2\left((cx^2+b)^{\frac{3}{2}} - \sqrt{cx^2+b}b\right)b^2\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
-3/2*c*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2*sgn(x)) - 1/2*(3*(c*
x^2 + b)*c - 2*b*c)/(((c*x^2 + b)^(3/2) - sqrt(c*x^2 + b)*b)*b^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{1}{(bx^2 + cx^4)^{3/2}}\,dx = -\frac{x\left(\frac{b}{c\,x^2}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{b}{c\,x^2}\right)}{5\,(c\,x^4+b\,x^2)^{3/2}}$$

input
```
int(1/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
-(x*(b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -b/(c*x^2)))/(5*(b*x^
2 + c*x^4)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.12

$$\int \frac{1}{(bx^2+cx^4)^{3/2}}\,dx = \frac{-\sqrt{c\,x^2+b}\,b^2 - 3\sqrt{c\,x^2+b}\,bc\,x^2 - 3\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)bc\,x^2 - 3\sqrt{b}\log\left(\frac{\sqrt{c\,x^2}\cdots}{}\right.\cdots}{2b^3x^2\,(c\cdots}$$

input
```
int(1/(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - sqrt(b + c*x**2)*b**2 - 3*sqrt(b + c*x**2)*b*c*x**2 - 3*sqrt(b)*log((s
qrt(b + c*x**2) - sqrt(b) + sqrt(c)*x)/sqrt(b))*b*c*x**2 - 3*sqrt(b)*log((
sqrt(b + c*x**2) - sqrt(b) + sqrt(c)*x)/sqrt(b))*c**2*x**4 + 3*sqrt(b)*log
((sqrt(b + c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))*b*c*x**2 + 3*sqrt(b)*lo
g((sqrt(b + c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))*c**2*x**4)/(2*b**3*x**
2*(b + c*x**2))
```

3.221 $\int \frac{1}{x^2(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{x^2\,(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{bx^3\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{4b^2x^5} + \frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3} - \frac{15c^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}$$

output
```
1/b/x^3/(c*x^4+b*x^2)^(1/2)-5/4*(c*x^4+b*x^2)^(1/2)/b^2/x^5+15/8*c*(c*x^4+
b*x^2)^(1/2)/b^3/x^3-15/8*c^2*arctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))/b^(7/
2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\,(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{b}(-2b^2+5bcx^2+15c^2x^4) - 15c^2x^4\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{8b^{7/2}x^3\sqrt{x^2\,(b+cx^2)}}$$

input
```
Integrate[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(Sqrt[b]*(-2*b^2 + 5*b*c*x^2 + 15*c^2*x^4) - 15*c^2*x^4*Sqrt[b + c*x^2]*Ar
cTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(8*b^(7/2)*x^3*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1428, 1430, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1428}$$

$$\frac{5\int \frac{1}{x^4\sqrt{cx^4+bx^2}}dx}{b}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{5\left(-\frac{3c\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)}{b}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{5\left(-\frac{3c\left(-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)}{b}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1400}$$

$$\frac{5\left(-\frac{3c\left(\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)}{b}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

$$\downarrow 219$$

$$\frac{5\left(-\frac{3c\left(\frac{c\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}\right)}{b}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

input
```
Int[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
1/(b*x^3*Sqrt[b*x^2 + c*x^4]) + (5*(-1/4*Sqrt[b*x^2 + c*x^4]/(b*x^5) - (3*
c*(-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 +
c*x^4]])/(2*b^(3/2))))/(4*b)))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{(c x^2+b)\left(-15 b^{\frac{3}{2}} c^2 x^4+15 \ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right)\sqrt{c x^2+b}\, b c^2 x^4-5 b^{\frac{5}{2}} c x^2+2 b^{\frac{7}{2}}\right)}{8 x (c x^4+b x^2)^{\frac{3}{2}} b^{\frac{9}{2}}}$	94
risch	$-\frac{(c x^2+b)(-7 c x^2+2b)}{8 b^3 x^3 \sqrt{x^2 (c x^2+b)}}+\frac{\left(\frac{c^2}{b^3 \sqrt{c x^2+b}}-\frac{15 c^2 \ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c x^2+b}}{x}\right)}{8 b^{\frac{7}{2}}}\right) x \sqrt{c x^2+b}}{\sqrt{x^2 (c x^2+b)}}$	112

input
```
int(1/x^2/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/x*(c*x^2+b)*(-15*b^(3/2)*c^2*x^4+15*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/
x)*(c*x^2+b)^(1/2)*b*c^2*x^4-5*b^(5/2)*c*x^2+2*b^(7/2))/(c*x^4+b*x^2)^(3/2
)/b^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.06

$$\int \frac{1}{x^2 (bx^2+cx^4)^{3/2}}\, dx = \left[\frac{15 (c^3 x^7 + b c^2 x^5)\sqrt{b} \log\left(-\frac{c x^3+2 b x-2 \sqrt{c x^4+b x^2}\sqrt{b}}{x^3}\right) + 2 (15 b c^2 x^4 + 5 b^2 c x^2 - 2 b^3)}{16 (b^4 c x^7 + b^5 x^5)}\right.$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(15*(c^3*x^7 + b*c^2*x^5)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4
 + b*x^2)*sqrt(b))/x^3) + 2*(15*b*c^2*x^4 + 5*b^2*c*x^2 - 2*b^3)*sqrt(c*x^
4 + b*x^2))/(b^4*c*x^7 + b^5*x^5), 1/8*(15*(c^3*x^7 + b*c^2*x^5)*sqrt(-b)*
arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(b*x)) + (15*b*c^2*x^4 + 5*b^2*c*x^2 -
 2*b^3)*sqrt(c*x^4 + b*x^2))/(b^4*c*x^7 + b^5*x^5)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(1/(x**2*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{15\,c^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{8\,\sqrt{-b}b^3\mathrm{sgn}\left(x\right)}+\frac{c^2}{\sqrt{cx^2+b}b^3\mathrm{sgn}\left(x\right)}+\frac{7\left(cx^2+b\right)^{\frac{3}{2}}c^2-9\,\sqrt{cx^2+b}bc^2}{8\,b^3c^2x^4\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
15/8*c^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^3*sgn(x)) + c^2/(sqr
t(c*x^2 + b)*b^3*sgn(x)) + 1/8*(7*(c*x^2 + b)^(3/2)*c^2 - 9*sqrt(c*x^2 + b
)*b*c^2)/(b^3*c^2*x^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^2 (bx^2 + cx^4)^{3/2}} \, dx = -\frac{\left(\frac{b}{c\,x^2}+1\right)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{7}{2};\ \frac{9}{2};\ -\frac{b}{c\,x^2}\right)}{7\,x\,\left(c\,x^4+b\,x^2\right)^{3/2}}$$

input
```
int(1/(x^2*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
-((b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -b/(c*x^2)))/(7*x*(b*x^
2 + c*x^4)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.79

$$\int \frac{1}{x^2 (bx^2 + cx^4)^{3/2}} \, dx = \frac{-2\sqrt{c\,x^2+b}\,b^3 + 5\sqrt{c\,x^2+b}\,b^2 c\,x^2 + 15\sqrt{c\,x^2+b}\,b\,c^2 x^4 + 15\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+}{\sqrt{b}}\right.}{}$$

input
```
int(1/x^2/(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - 2*sqrt(b + c*x**2)*b**3 + 5*sqrt(b + c*x**2)*b**2*c*x**2 + 15*sqrt(b +
 c*x**2)*b*c**2*x**4 + 15*sqrt(b)*log((sqrt(b + c*x**2) - sqrt(b) + sqrt(c
)*x)/sqrt(b))*b*c**2*x**4 + 15*sqrt(b)*log((sqrt(b + c*x**2) - sqrt(b) + s
qrt(c)*x)/sqrt(b))*c**3*x**6 - 15*sqrt(b)*log((sqrt(b + c*x**2) + sqrt(b)
+ sqrt(c)*x)/sqrt(b))*b*c**2*x**4 - 15*sqrt(b)*log((sqrt(b + c*x**2) + sqr
t(b) + sqrt(c)*x)/sqrt(b))*c**3*x**6)/(8*b**4*x**4*(b + c*x**2))
```

3.222 $\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx = -\frac{1}{8}\sqrt{3x^2-4x^4}-\frac{3}{32}\arcsin\left(\frac{1}{3}(3-8x^2)\right)$$

output
```
-1/8*(-4*x^4+3*x^2)^(1/2)+3/32*arcsin(8/3*x^2-1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx = \frac{x\left(-6x+8x^3-3\sqrt{-3+4x^2}\log\left(-2x+\sqrt{-3+4x^2}\right)\right)}{16\sqrt{3x^2-4x^4}}$$

input
```
Integrate[x^3/Sqrt[3*x^2 - 4*x^4],x]
```

output
```
(x*(-6*x + 8*x^3 - 3*Sqrt[-3 + 4*x^2]*Log[-2*x + Sqrt[-3 + 4*x^2]]))/(16*S
qrt[3*x^2 - 4*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1424, 1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{3x^2-4x^4}}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{3x^2-4x^4}}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{3}{8}\int \frac{1}{\sqrt{3x^2-4x^4}}dx^2-\frac{1}{4}\sqrt{3x^2-4x^4}\right)$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}\left(-\frac{1}{16}\int \frac{1}{\sqrt{1-\frac{x^4}{9}}}d(3-8x^2)-\frac{1}{4}\sqrt{3x^2-4x^4}\right)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\left(-\frac{3}{16}\arcsin\left(\frac{1}{3}(3-8x^2)\right)-\frac{1}{4}\sqrt{3x^2-4x^4}\right)$$

input
```
Int[x^3/Sqrt[3*x^2 - 4*x^4],x]
```

output
```
(-1/4*Sqrt[3*x^2 - 4*x^4] - (3*ArcSin[(3 - 8*x^2)/3])/16)/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$-\frac{\sqrt{-4x^4+3x^2}}{8}+\frac{3\arcsin\left(\frac{8x^2}{3}-1\right)}{32}$	27
meijerg	$\frac{3i\left(\frac{2i\sqrt{\pi}\,x\sqrt{3}\,\sqrt{-\frac{4x^2}{3}+1}}{3}-i\sqrt{\pi}\,\arcsin\left(\frac{2\sqrt{3}\,x}{3}\right)\right)}{16\sqrt{\pi}}$	40
default	$\frac{x\sqrt{-4x^2+3}\,\left(-2x\sqrt{-4x^2+3}+3\arcsin\left(\frac{2\sqrt{3}\,x}{3}\right)\right)}{16\sqrt{-4x^4+3x^2}}$	48
trager	$-\frac{\sqrt{-4x^4+3x^2}}{8}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2+\sqrt{-4x^4+3x^2}}{x}\right)}{16}$	55
risch	$\frac{x^2\left(4x^2-3\right)}{8\sqrt{-x^2\left(4x^2-3\right)}}+\frac{3\arcsin\left(\frac{2\sqrt{3}\,x}{3}\right)x\sqrt{-4x^2+3}}{16\sqrt{-x^2\left(4x^2-3\right)}}$	61

input
```
int(x^3/(-4*x^4+3*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-4*x^4+3*x^2)^(1/2)+3/32*arcsin(8/3*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx = -\frac{1}{8}\sqrt{-4\,x^4+3\,x^2} - \frac{3}{16}\arctan\left(\frac{2\sqrt{-4\,x^4+3\,x^2}}{4\,x^2-3}\right)$$

input
```
integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-4*x^4 + 3*x^2) - 3/16*arctan(2*sqrt(-4*x^4 + 3*x^2)/(4*x^2 - 3)
)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx = \int \frac{x^3}{\sqrt{-x^2\cdot(4x^2-3)}}\,dx$$

input
```
integrate(x**3/(-4*x**4+3*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(-x**2*(4*x**2 - 3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx = -\frac{1}{8}\sqrt{-4\,x^4+3\,x^2}-\frac{3}{32}\,\arcsin\left(-\frac{8}{3}\,x^2+1\right)$$

input
```
integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(-4*x^4 + 3*x^2) - 3/32*arcsin(-8/3*x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx = -\frac{\sqrt{-4\,x^2+3}x}{8\,\mathrm{sgn}\,(x)}+\frac{3\,\arcsin\left(\frac{2}{3}\,\sqrt{3}x\right)}{16\,\mathrm{sgn}\,(x)}$$

input
```
integrate(x^3/(-4*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(-4*x^2 + 3)*x/sgn(x) + 3/16*arcsin(2/3*sqrt(3)*x)/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 17.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx = -\frac{\sqrt{3\,x^2-4\,x^4}}{8}-\frac{\ln\left(x^2-\frac{3}{8}-\frac{\sqrt{3-4\,x^2}\,\sqrt{x^2}\,1\mathrm{i}}{2}\right)\,3\mathrm{i}}{32}$$

input
```
int(x^3/(3*x^2 - 4*x^4)^(1/2),x)
```

output
```
- (log(x^2 - ((3 - 4*x^2)^(1/2)*(x^2)^(1/2)*1i)/2 - 3/8)*3i)/32 - (3*x^2 -
4*x^4)^(1/2)/8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx = \frac{3asin\left(\frac{2x}{\sqrt{3}}\right)}{16} - \frac{\sqrt{-4x^2+3}\,x}{8}$$

input
```
int(x^3/(-4*x^4+3*x^2)^(1/2),x)
```

output
```
(3*asin((2*x)/sqrt(3)) - 2*sqrt( - 4*x**2 + 3)*x)/16
```

3.223 $\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx=-\frac{1}{8}\sqrt{-3x^2-4x^4}-\frac{3}{32}\arcsin\left(\frac{1}{3}(3+8x^2)\right)$$

output
```
-1/8*(-4*x^4-3*x^2)^(1/2)-3/32*arcsin(8/3*x^2+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx=\frac{x\left(6x+8x^3+3\sqrt{3+4x^2}\log\left(-2x+\sqrt{3+4x^2}\right)\right)}{16\sqrt{-x^2(3+4x^2)}}$$

input
```
Integrate[x^3/Sqrt[-3*x^2 - 4*x^4],x]
```

output
```
(x*(6*x + 8*x^3 + 3*Sqrt[3 + 4*x^2]*Log[-2*x + Sqrt[3 + 4*x^2]]))/(16*Sqrt
[-(x^2*(3 + 4*x^2))])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1424, 1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{-4x^4-3x^2}}\,dx$$

$$\Big\downarrow \text{ 1424}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{-4x^4-3x^2}}dx^2$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{2}\left(-\frac{3}{8}\int \frac{1}{\sqrt{-4x^4-3x^2}}dx^2-\frac{1}{4}\sqrt{-4x^4-3x^2}\right)$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{2}\left(\frac{1}{16}\int \frac{1}{\sqrt{1-\frac{x^4}{9}}}d\left(-8x^2-3\right)-\frac{1}{4}\sqrt{-4x^4-3x^2}\right)$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{2}\left(\frac{3}{16}\arcsin\left(\frac{1}{3}\left(-8x^2-3\right)\right)-\frac{1}{4}\sqrt{-4x^4-3x^2}\right)$$

input
```
Int[x^3/Sqrt[-3*x^2 - 4*x^4],x]
```

output
```
(-1/4*Sqrt[-3*x^2 - 4*x^4] + (3*ArcSin[(-3 - 8*x^2)/3])/16)/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$-\frac{\sqrt{-4x^4-3x^2}}{8}-\frac{3\arcsin\left(\frac{8x^2}{3}+1\right)}{32}$	27
meijerg	$-\frac{3i\left(\frac{2\sqrt{\pi}\,x\sqrt{3}\,\sqrt{\frac{4x^2}{3}+1}}{3}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,x}{3}\right)\right)}{16\sqrt{\pi}}$	38
default	$-\frac{x\sqrt{-4x^2-3}\,\left(2x\sqrt{-4x^2-3}+3\arctan\left(\frac{2x}{\sqrt{-4x^2-3}}\right)\right)}{16\sqrt{-4x^4-3x^2}}$	54
trager	$-\frac{\sqrt{-4x^4-3x^2}}{8}+\frac{3\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2+1\right)x^2+\sqrt{-4x^4-3x^2}}{x}\right)}{16}$	55
risch	$\frac{x^2\left(4x^2+3\right)}{8\sqrt{-x^2\left(4x^2+3\right)}}-\frac{3\arctan\left(\frac{2x}{\sqrt{-4x^2-3}}\right)x\sqrt{-4x^2-3}}{16\sqrt{-x^2\left(4x^2+3\right)}}$	67

input
```
int(x^3/(-4*x^4-3*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-4*x^4-3*x^2)^(1/2)-3/32*arcsin(8/3*x^2+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx = -\frac{1}{8}\sqrt{-4\,x^2-3}x - \frac{3}{32}i\,\log\left(-\frac{4\left(2\,x+i\,\sqrt{-4\,x^2-3}\right)}{x}\right) + \frac{3}{32}i\,\log\left(-\frac{4\left(2\,x-i\,\sqrt{-4\,x^2-3}\right)}{x}\right)$$

input
```
integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-4*x^2 - 3)*x - 3/32*I*log(-4*(2*x + I*sqrt(-4*x^2 - 3))/x) + 3/
32*I*log(-4*(2*x - I*sqrt(-4*x^2 - 3))/x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx = \int \frac{x^3}{\sqrt{-x^2\cdot(4x^2+3)}}\,dx$$

input
```
integrate(x**3/(-4*x**4-3*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(-x**2*(4*x**2 + 3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx = -\frac{1}{8}\,\sqrt{-4\,x^4-3\,x^2}+\frac{3}{32}\,\arcsin\left(-\frac{8}{3}\,x^2-1\right)$$

input
```
integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(-4*x^4 - 3*x^2) + 3/32*arcsin(-8/3*x^2 - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx = \frac{3}{32}i\,\log\left(3\right)\mathrm{sgn}(x)-\frac{i\,\sqrt{4\,x^2+3}x}{8\,\mathrm{sgn}\left(x\right)}-\frac{3i\,\log\left(-2\,x+\sqrt{4\,x^2+3}\right)}{16\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3/(-4*x^4-3*x^2)^(1/2),x, algorithm="giac")
```

output
```
3/32*I*log(3)*sgn(x) - 1/8*I*sqrt(4*x^2 + 3)*x/sgn(x) - 3/16*I*log(-2*x +
sqrt(4*x^2 + 3))/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 17.49 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx = -\frac{\sqrt{-4\,x^4-3\,x^2}}{8}+\frac{\ln\left(\frac{\sqrt{4\,x^2+3}\,\sqrt{x^2}}{2}+x^2+\frac{3}{8}\right)\,3{}\mathrm{i}}{32}$$

input
```
int(x^3/(- 3*x^2 - 4*x^4)^(1/2),x)
```

output
```
(log(((4*x^2 + 3)^(1/2)*(x^2)^(1/2))/2 + x^2 + 3/8)*3i)/32 - (- 3*x^2 - 4*
x^4)^(1/2)/8
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx = \frac{3asinh\left(\frac{2x}{\sqrt{3}}\right)i}{16} + \frac{\sqrt{-4x^2-3}\,x}{8}$$

input
```
int(x^3/(-4*x^4-3*x^2)^(1/2),x)
```

output
```
(3*asinh((2*x)/sqrt(3))*i + 2*sqrt( - 4*x**2 - 3)*x)/16
```

3.224 $\int \frac{x^3}{\sqrt{3x^2+4x^4}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{x^3}{\sqrt{3x^2+4x^4}} \, dx = \frac{1}{8}\sqrt{3x^2+4x^4} - \frac{3}{16}\text{arctanh}\left(\frac{2x^2}{\sqrt{3x^2+4x^4}}\right)$$

output
```
1/8*(4*x^4+3*x^2)^(1/2)-3/16*arctanh(2*x^2/(4*x^4+3*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \frac{x^3}{\sqrt{3x^2+4x^4}} \, dx = \frac{x\left(6x+8x^3+3\sqrt{3+4x^2}\log\left(-2x+\sqrt{3+4x^2}\right)\right)}{16\sqrt{x^2(3+4x^2)}}$$

input
```
Integrate[x^3/Sqrt[3*x^2 + 4*x^4],x]
```

output
```
(x*(6*x + 8*x^3 + 3*Sqrt[3 + 4*x^2]*Log[-2*x + Sqrt[3 + 4*x^2]]))/(16*Sqrt
[x^2*(3 + 4*x^2)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1424, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{4x^4+3x^2}}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{4x^4+3x^2}}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{1}{4}\sqrt{4x^4+3x^2}-\frac{3}{8}\int \frac{1}{\sqrt{4x^4+3x^2}}dx^2\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{1}{4}\sqrt{4x^4+3x^2}-\frac{3}{4}\int \frac{1}{1-4x^4}d\frac{x^2}{\sqrt{4x^4+3x^2}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{4}\sqrt{4x^4+3x^2}-\frac{3}{8}\text{arctanh}\left(\frac{2x^2}{\sqrt{4x^4+3x^2}}\right)\right)$$

input
```
Int[x^3/Sqrt[3*x^2 + 4*x^4],x]
```

output
```
(Sqrt[3*x^2 + 4*x^4]/4 - (3*ArcTanh[(2*x^2)/Sqrt[3*x^2 + 4*x^4]])/8)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\, x \sqrt{3}\, \sqrt{\frac{4 x^{2}}{3}+1}}{8}-\frac{3 \sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{2 \sqrt{3}\, x}{3}\right)}{16}}{\sqrt{\pi}}$	37
pseudoelliptic	$-\frac{3 \ln \left(8 x^{2}+3+4 \sqrt{4 x^{4}+3 x^{2}}\right)}{32}+\frac{\sqrt{4 x^{4}+3 x^{2}}}{8}$	42
trager	$\frac{\sqrt{4 x^{4}+3 x^{2}}}{8}+\frac{3 \ln \left(\frac{-2 x^{2}+\sqrt{4 x^{4}+3 x^{2}}}{x}\right)}{16}$	43
default	$-\frac{x \sqrt{4 x^{2}+3}\, \left(-2 x \sqrt{4 x^{2}+3}+3 \operatorname{arcsinh}\left(\frac{2 \sqrt{3}\, x}{3}\right)\right)}{16 \sqrt{4 x^{4}+3 x^{2}}}$	48
risch	$\frac{x^{2} \left(4 x^{2}+3\right)}{8 \sqrt{x^{2} \left(4 x^{2}+3\right)}}-\frac{3 \operatorname{arcsinh}\left(\frac{2 \sqrt{3}\, x}{3}\right) x \sqrt{4 x^{2}+3}}{16 \sqrt{x^{2} \left(4 x^{2}+3\right)}}$	59

input
```
int(x^3/(4*x^4+3*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/16/Pi^(1/2)*(2/3*Pi^(1/2)*x*3^(1/2)*(4/3*x^2+1)^(1/2)-Pi^(1/2)*arcsinh(2
/3*3^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx = \frac{1}{8}\sqrt{4\,x^4+3\,x^2} + \frac{3}{16}\log\left(-\frac{2\,x^2-\sqrt{4\,x^4+3\,x^2}}{x}\right)$$

input
```
integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(4*x^4 + 3*x^2) + 3/16*log(-(2*x^2 - sqrt(4*x^4 + 3*x^2))/x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx = \int \frac{x^3}{\sqrt{x^2\cdot(4x^2+3)}}\,dx$$

input
```
integrate(x**3/(4*x**4+3*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**2*(4*x**2 + 3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx = \frac{1}{8}\sqrt{4\,x^4+3\,x^2} - \frac{3}{32}\log\left(8\,x^2+4\sqrt{4\,x^4+3\,x^2}+3\right)$$

input
```
integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/8*sqrt(4*x^4 + 3*x^2) - 3/32*log(8*x^2 + 4*sqrt(4*x^4 + 3*x^2) + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx = -\frac{3}{32}\log(3)\operatorname{sgn}(x) + \frac{\sqrt{4x^2+3}x}{8\operatorname{sgn}(x)} + \frac{3\log\left(-2x+\sqrt{4x^2+3}\right)}{16\operatorname{sgn}(x)}$$

input
```
integrate(x^3/(4*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

output
```
-3/32*log(3)*sgn(x) + 1/8*sqrt(4*x^2 + 3)*x/sgn(x) + 3/16*log(-2*x + sqrt(
4*x^2 + 3))/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 18.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx = \frac{\sqrt{4x^4+3x^2}}{8} - \frac{3\ln\left(\frac{\sqrt{4x^2+3}\sqrt{x^2}}{2}+x^2+\frac{3}{8}\right)}{32}$$

input
```
int(x^3/(3*x^2 + 4*x^4)^(1/2),x)
```

output
```
(3*x^2 + 4*x^4)^(1/2)/8 - (3*log(((4*x^2 + 3)^(1/2)*(x^2)^(1/2))/2 + x^2 +
 3/8))/32
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx = \frac{\sqrt{4x^2+3}\,x}{8} - \frac{3\log\left(\frac{\sqrt{4x^2+3}+2x}{\sqrt{3}}\right)}{16}$$

input
```
int(x^3/(4*x^4+3*x^2)^(1/2),x)
```

output
```
(2*sqrt(4*x**2 + 3)*x - 3*log((sqrt(4*x**2 + 3) + 2*x)/sqrt(3)))/16
```

3.225 $\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx=\frac{1}{8}\sqrt{-3x^2+4x^4}+\frac{3}{16}\operatorname{arctanh}\left(\frac{2x^2}{\sqrt{-3x^2+4x^4}}\right)$$

output
```
1/8*(4*x^4-3*x^2)^(1/2)+3/16*arctanh(2*x^2/(4*x^4-3*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx=\frac{x\left(-6x+8x^3-3\sqrt{-3+4x^2}\log\left(-2x+\sqrt{-3+4x^2}\right)\right)}{16\sqrt{x^2\left(-3+4x^2\right)}}$$

input
```
Integrate[x^3/Sqrt[-3*x^2 + 4*x^4],x]
```

output
```
(x*(-6*x + 8*x^3 - 3*Sqrt[-3 + 4*x^2]*Log[-2*x + Sqrt[-3 + 4*x^2]]))/(16*S
qrt[x^2*(-3 + 4*x^2)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1424, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{4x^4-3x^2}}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{4x^4-3x^2}}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{3}{8}\int \frac{1}{\sqrt{4x^4-3x^2}}dx^2+\frac{1}{4}\sqrt{4x^4-3x^2}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \frac{1}{1-4x^4}d\frac{x^2}{\sqrt{4x^4-3x^2}}+\frac{1}{4}\sqrt{4x^4-3x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{3}{8}\text{arctanh}\left(\frac{2x^2}{\sqrt{4x^4-3x^2}}\right)+\frac{1}{4}\sqrt{4x^4-3x^2}\right)$$

input
```
Int[x^3/Sqrt[-3*x^2 + 4*x^4],x]
```

output
```
(Sqrt[-3*x^2 + 4*x^4]/4 + (3*ArcTanh[(2*x^2)/Sqrt[-3*x^2 + 4*x^4]])/8)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{3 \ln\left(8x^{2}-3+4\sqrt{4x^{4}-3x^{2}}\right)}{32}+\frac{\sqrt{4x^{4}-3x^{2}}}{8}$	42
trager	$\frac{\sqrt{4x^{4}-3x^{2}}}{8}+\frac{3 \ln\left(\frac{2x^{2}+\sqrt{4x^{4}-3x^{2}}}{x}\right)}{16}$	43
default	$\frac{x\sqrt{4x^{2}-3}\,\left(3 \ln\left(x\sqrt{4}+\sqrt{4x^{2}-3}\right)\sqrt{4}+4x\sqrt{4x^{2}-3}\right)}{32\sqrt{4x^{4}-3x^{2}}}$	60
meijerg	$\frac{3i\sqrt{-\operatorname{signum}\left(-1+\frac{4x^{2}}{3}\right)}\,\left(\frac{2i\sqrt{\pi}\,x\sqrt{3}\,\sqrt{-\frac{4x^{2}}{3}+1}}{3}-i\sqrt{\pi}\,\arcsin\left(\frac{2\sqrt{3}\,x}{3}\right)\right)}{16\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(-1+\frac{4x^{2}}{3}\right)}}$	62
risch	$\frac{x^{2}\left(4x^{2}-3\right)}{8\sqrt{x^{2}\left(4x^{2}-3\right)}}+\frac{3 \ln\left(x\sqrt{4}+\sqrt{4x^{2}-3}\right)\sqrt{4}\,x\sqrt{4x^{2}-3}}{32\sqrt{x^{2}\left(4x^{2}-3\right)}}$	71

input
```
int(x^3/(4*x^4-3*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/32*ln(8*x^2-3+4*(4*x^4-3*x^2)^(1/2))+1/8*(4*x^4-3*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx = \frac{1}{8}\sqrt{4\,x^4-3\,x^2} - \frac{3}{16}\log\left(-\frac{2\,x^2-\sqrt{4\,x^4-3\,x^2}}{x}\right)$$

input
```
integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(4*x^4 - 3*x^2) - 3/16*log(-(2*x^2 - sqrt(4*x^4 - 3*x^2))/x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx = \int \frac{x^3}{\sqrt{x^2\cdot(4x^2-3)}}\,dx$$

input
```
integrate(x**3/(4*x**4-3*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**2*(4*x**2 - 3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx = \frac{1}{8}\sqrt{4\,x^4-3\,x^2}+\frac{3}{32}\,\log\left(8\,x^2+4\,\sqrt{4\,x^4-3\,x^2}-3\right)$$

input
```
integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/8*sqrt(4*x^4 - 3*x^2) + 3/32*log(8*x^2 + 4*sqrt(4*x^4 - 3*x^2) - 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx = \frac{3}{32}\,\log\left(3\right)\mathrm{sgn}(x)+\frac{\sqrt{4\,x^2-3}x}{8\,\mathrm{sgn}\left(x\right)}-\frac{3\,\log\left(\left|-2\,x+\sqrt{4\,x^2-3}\right|\right)}{16\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3/(4*x^4-3*x^2)^(1/2),x, algorithm="giac")
```

output
```
3/32*log(3)*sgn(x) + 1/8*sqrt(4*x^2 - 3)*x/sgn(x) - 3/16*log(abs(-2*x + sq
rt(4*x^2 - 3)))/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 18.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx = \frac{3\,\ln\left(\frac{\sqrt{4\,x^2-3}\,\sqrt{x^2}}{2}+x^2-\frac{3}{8}\right)}{32}+\frac{\sqrt{4\,x^4-3\,x^2}}{8}$$

input
```
int(x^3/(4*x^4 - 3*x^2)^(1/2),x)
```

output
```
(3*log(((4*x^2 - 3)^(1/2)*(x^2)^(1/2))/2 + x^2 - 3/8))/32 + (4*x^4 - 3*x^2
)^(1/2)/8
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx = \frac{\sqrt{4x^2-3}\,x}{8} + \frac{3\log\left(\frac{\sqrt{4x^2-3}+2x}{\sqrt{3}}\right)}{16}$$

input
```
int(x^3/(4*x^4-3*x^2)^(1/2),x)
```

output
```
(2*sqrt(4*x**2 - 3)*x + 3*log((sqrt(4*x**2 - 3) + 2*x)/sqrt(3)))/16
```

3.226 $\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx=\frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}$$

output
```
1/2*(b*x^4+a*x^2)^(1/2)/b-1/2*a*arctanh(b^(1/2)*x^2/(b*x^4+a*x^2)^(1/2))/b
^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.43

$$\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx=\frac{x\left(\sqrt{b}x(a+bx^2)+2a\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}-\sqrt{a+bx^2}}\right)\right)}{2b^{3/2}\sqrt{x^2\left(a+bx^2\right)}}$$

input
```
Integrate[x^3/Sqrt[a*x^2 + b*x^4],x]
```

output
```
(x*(Sqrt[b]*x*(a + b*x^2) + 2*a*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/(Sqrt[
a] - Sqrt[a + b*x^2])]))/(2*b^(3/2)*Sqrt[x^2*(a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1424, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^2+bx^4}}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{bx^4+ax^2}}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{\sqrt{ax^2+bx^4}}{b}-\frac{a\int \frac{1}{\sqrt{bx^4+ax^2}}dx^2}{2b}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{\sqrt{ax^2+bx^4}}{b}-\frac{a\int \frac{1}{1-bx^4}d\frac{x^2}{\sqrt{bx^4+ax^2}}}{b}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\sqrt{ax^2+bx^4}}{b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2+bx^4}}\right)}{b^{3/2}}\right)$$

input
```
Int[x^3/Sqrt[a*x^2 + b*x^4],x]
```

output
```
(Sqrt[a*x^2 + b*x^4]/b - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^2 + b*x^4]])/b^
(3/2))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

method	result	size
pseudoelliptic	$\frac{\ln(2)a-\ln\left(\frac{2b\,x^2+2\sqrt{x^2\left(b\,x^2+a\right)}\,\sqrt{b}+a}{\sqrt{b}}\right)a+2\sqrt{x^2(b\,x^2+a)}\,\sqrt{b}}{4b^{\frac{3}{2}}}$	63
default	$\frac{x\sqrt{b\,x^2+a}\,\left(x\sqrt{b\,x^2+a}\,b^{\frac{3}{2}}-a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)b\right)}{2\sqrt{b\,x^4+a\,x^2}\,b^{\frac{5}{2}}}$	64
risch	$\frac{x^2\left(b\,x^2+a\right)}{2b\sqrt{x^2\left(b\,x^2+a\right)}}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)x\sqrt{b\,x^2+a}}{2b^{\frac{3}{2}}\,\sqrt{x^2\left(b\,x^2+a\right)}}$	75

input
```
int(x^3/(b*x^4+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^(3/2)*(ln(2)*a-ln((2*b*x^2+2*(x^2*(b*x^2+a))^(1/2)*b^(1/2)+a)/b^(1/2
))*a+2*(x^2*(b*x^2+a))^(1/2)*b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.97

$$\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx = \left[\frac{a\sqrt{b}\log\left(-2\,bx^2-a+2\,\sqrt{bx^4+ax^2}\sqrt{b}\right)+2\,\sqrt{bx^4+ax^2}b}{4\,b^2}, \frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{bx^4+ax^2}\sqrt{-b}}{bx^2+a}\right)+\sqrt{bx^4+ax^2}b}{2\,b^2}\right]$$

input
```
integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(a*sqrt(b)*log(-2*b*x^2 - a + 2*sqrt(b*x^4 + a*x^2)*sqrt(b)) + 2*sqrt
(b*x^4 + a*x^2)*b)/b^2, 1/2*(a*sqrt(-b)*arctan(sqrt(b*x^4 + a*x^2)*sqrt(-b
)/(b*x^2 + a)) + sqrt(b*x^4 + a*x^2)*b)/b^2]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx = \int \frac{x^3}{\sqrt{x^2\,(a+bx^2)}}\,dx$$

input
```
integrate(x**3/(b*x**4+a*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**2*(a + b*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx = -\frac{a\log\left(2\,bx^2+a+2\,\sqrt{bx^4+ax^2}\sqrt{b}\right)}{4\,b^{\frac{3}{2}}}+\frac{\sqrt{bx^4+ax^2}}{2\,b}$$

input
```
integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a*log(2*b*x^2 + a + 2*sqrt(b*x^4 + a*x^2)*sqrt(b))/b^(3/2) + 1/2*sqrt
(b*x^4 + a*x^2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx = -\frac{a\log(|a|)\operatorname{sgn}(x)}{4\,b^{\frac{3}{2}}}+\frac{\sqrt{bx^2+a}x}{2\,b\operatorname{sgn}(x)}+\frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}\operatorname{sgn}(x)}$$

input
```
integrate(x^3/(b*x^4+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*a*log(abs(a))*sgn(x)/b^(3/2) + 1/2*sqrt(b*x^2 + a)*x/(b*sgn(x)) + 1/2
*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/(b^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 18.46 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx = \frac{\sqrt{b\,x^4+a\,x^2}}{2\,b}-\frac{a\,\ln\left(\frac{b\,x^2+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^4+a\,x^2}\right)}{4\,b^{3/2}}$$

input
```
int(x^3/(a*x^2 + b*x^4)^(1/2),x)
```

output
```
(a*x^2 + b*x^4)^(1/2)/(2*b) - (a*log((a/2 + b*x^2)/b^(1/2) + (a*x^2 + b*x^
4)^(1/2)))/(4*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx = \frac{\sqrt{b\,x^2+a}\,bx-\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a}{2b^2}$$

input
```
int(x^3/(b*x^4+a*x^2)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*b*x - sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a)/(2*b**2)
```

3.227 $\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx=-\frac{\sqrt{ax^2-bx^4}}{2b}+\frac{a\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}$$

output
```
-1/2*(-b*x^4+a*x^2)^(1/2)/b+1/2*a*arctan(b^(1/2)*x^2/(-b*x^4+a*x^2)^(1/2))
/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.47

$$\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx=\frac{x\left(\sqrt{b}x(-a+bx^2)+2a\sqrt{a-bx^2}\arctan\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a-bx^2}}\right)\right)}{2b^{3/2}\sqrt{x^2\left(a-bx^2\right)}}$$

input
```
Integrate[x^3/Sqrt[a*x^2 - b*x^4],x]
```

output
```
(x*(Sqrt[b]*x*(-a + b*x^2) + 2*a*Sqrt[a - b*x^2]*ArcTan[(Sqrt[b]*x)/(-Sqrt
[a] + Sqrt[a - b*x^2])]))/(2*b^(3/2)*Sqrt[x^2*(a - b*x^2)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1424, 1160, 1091, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^2-bx^4}}dx$$

$$\Big\downarrow \text{1424}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{ax^2-bx^4}}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{\sqrt{ax^2-bx^4}}dx^2}{2b}-\frac{\sqrt{ax^2-bx^4}}{b}\right)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{bx^4+1}d\frac{x^2}{\sqrt{ax^2-bx^4}}}{b}-\frac{\sqrt{ax^2-bx^4}}{b}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{a\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2-bx^4}}\right)}{b^{3/2}}-\frac{\sqrt{ax^2-bx^4}}{b}\right)$$

input
```
Int[x^3/Sqrt[a*x^2 - b*x^4],x]
```

output
```
(-(Sqrt[a*x^2 - b*x^4]/b) + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a*x^2 - b*x^4]])/
b^(3/2))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$-\frac{\arctan\left(\frac{-2 b \,x^{2}+a}{2 \sqrt{b}\, \sqrt{x^{2} \left(-b \,x^{2}+a \right)}}\right) a +2 \sqrt{b}\, \sqrt{x^{2} \left(-b \,x^{2}+a \right)}}{4 b^{\frac{3}{2}}}$	56
default	$-\frac{x \sqrt{-b \,x^{2}+a}\, \left(x \sqrt{-b \,x^{2}+a}\, b^{\frac{3}{2}}-a \arctan\left(\frac{\sqrt{b}\, x}{\sqrt{-b \,x^{2}+a}}\right) b \right)}{2 \sqrt{-b \,x^{4}+a \,x^{2}}\, b^{\frac{5}{2}}}$	67
risch	$-\frac{x^{2} \left(-b \,x^{2}+a \right)}{2 b \sqrt{x^{2} \left(-b \,x^{2}+a \right)}}+\frac{a \arctan\left(\frac{\sqrt{b}\, x}{\sqrt{-b \,x^{2}+a}}\right) x \sqrt{-b \,x^{2}+a}}{2 b^{\frac{3}{2}} \sqrt{x^{2} \left(-b \,x^{2}+a \right)}}$	79

input
```
int(x^3/(-b*x^4+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^(3/2)*(arctan(1/2/b^(1/2)*(-2*b*x^2+a)/(x^2*(-b*x^2+a))^(1/2))*a+2*
b^(1/2)*(x^2*(-b*x^2+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx = \left[-\frac{a\sqrt{-b}\log\left(2\,bx^2-a-2\,\sqrt{-bx^4+ax^2}\sqrt{-b}\right)+2\,\sqrt{-bx^4+ax^2}b}{4\,b^2}, -\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{-bx^4+ax^2}\sqrt{b}}{bx^2-a}\right)+\sqrt{-bx^4+ax^2}b}{2\,b^2}\right]$$

input
```
integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(a*sqrt(-b)*log(2*b*x^2 - a - 2*sqrt(-b*x^4 + a*x^2)*sqrt(-b)) + 2*s
qrt(-b*x^4 + a*x^2)*b)/b^2, -1/2*(a*sqrt(b)*arctan(sqrt(-b*x^4 + a*x^2)*sq
rt(b)/(b*x^2 - a)) + sqrt(-b*x^4 + a*x^2)*b)/b^2]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx = \int \frac{x^3}{\sqrt{-x^2\left(-a+bx^2\right)}}\,dx$$

input
```
integrate(x**3/(-b*x**4+a*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(-x**2*(-a + b*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx = -\frac{a\arcsin\left(-\frac{2\,bx^2-a}{a}\right)}{4\,b^{\frac{3}{2}}} - \frac{\sqrt{-bx^4+ax^2}}{2\,b}$$

input
```
integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a*arcsin(-(2*b*x^2 - a)/a)/b^(3/2) - 1/2*sqrt(-b*x^4 + a*x^2)/b
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx = \frac{a\log\left(|a|\right)\operatorname{sgn}(x)}{4\,\sqrt{-b}b} - \frac{\sqrt{-bx^2+a}x}{2\,b\operatorname{sgn}\left(x\right)} - \frac{a\log\left(\left|-\sqrt{-b}x+\sqrt{-bx^2+a}\right|\right)}{2\,\sqrt{-b}b\operatorname{sgn}\left(x\right)}$$

input
```
integrate(x^3/(-b*x^4+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*a*log(abs(a))*sgn(x)/(sqrt(-b)*b) - 1/2*sqrt(-b*x^2 + a)*x/(b*sgn(x))
- 1/2*a*log(abs(-sqrt(-b)*x + sqrt(-b*x^2 + a)))/(sqrt(-b)*b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.93 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx = -\frac{\sqrt{a\,x^2-b\,x^4}}{2\,b} - \frac{a\,\ln\left(\frac{\frac{a}{2}-b\,x^2}{\sqrt{-b}}+\sqrt{a\,x^2-b\,x^4}\right)}{4\,{(-b)}^{3/2}}$$

input
```
int(x^3/(a*x^2 - b*x^4)^(1/2),x)
```

output
```
- (a*x^2 - b*x^4)^(1/2)/(2*b) - (a*log((a/2 - b*x^2)/(-b)^(1/2) + (a*x^2 -
 b*x^4)^(1/2)))/(4*(-b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{\sqrt{ax^2 - bx^4}}\,dx = \frac{\sqrt{b}\,asin\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)a - \sqrt{-b\,x^2 + a}\,bx}{2b^2}$$

input
```
int(x^3/(-b*x^4+a*x^2)^(1/2),x)
```

output
```
(sqrt(b)*asin((sqrt(b)*x)/sqrt(a))*a - sqrt(a - b*x**2)*b*x)/(2*b**2)
```

3.228 $\int x^{7/2}\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 323

$$\int x^{7/2}\sqrt{bx^2+cx^4}\,dx=\frac{28b^3x^{3/2}(b+cx^2)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}-\frac{28b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}+\frac{14b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}$$

output
```
28/195*b^3*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/
2)-28/585*b^2*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2+4/117*b*x^(5/2)*(c*x^4+b*x^2
)^(1/2)/c+2/13*x^(9/2)*(c*x^4+b*x^2)^(1/2)-28/195*b^(13/4)*x*(b^(1/2)+c^(1
/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/c^(11/4)/(c*x^4+b*x^2)^(1/2)+14/195*b^(
13/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*Invers
eJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/c^(11/4)/(c*x^4+b
*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.32

$$\int x^{7/2}\sqrt{bx^2+cx^4}\,dx=\frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(\sqrt{1+\frac{cx^2}{b}}(-7b^2+2bcx^2+9c^2x^4)+7b^2\,\text{Hypergeometric2F1}\left(\right.\right.}{117c^2\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[x^(7/2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(Sqrt[1 + (c*x^2)/b]*(-7*b^2 + 2*b*c*x^2
+ 9*c^2*x^4) + 7*b^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(11
7*c^2*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1426, 1429, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1426}$$

$$\frac{2}{13}b\int \frac{x^{11/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\int \frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}$$

$\Big\downarrow$ 1431

$$\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}$$

$\Big\downarrow$ 266

$$\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}$$

$\Big\downarrow$ 834

$$\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)+$$

$$\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}$$

$\Big\downarrow$ 27

$$\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)+$$

$$\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}$$

$\downarrow$ 761

$$\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{\ldots}}{\sqrt{cx^2\ldots}}}{\sqrt{\ldots}}\right.}{5c\sqrt{bx^2+cx^4}}\right.}{9c}\right.$$

$$\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}$$

$\downarrow$ 1510

$$\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\sqrt[4]{b}\left(\sqrt{\right.}\cdots}{\cdots}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right.$$

$$\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}$$

input

```
Int[x^(7/2)*Sqrt[b*x^2 + c*x^4],x]
```

output

```
(2*x^(9/2)*Sqrt[b*x^2 + c*x^4])/13 + (2*b*((2*x^(5/2)*Sqrt[b*x^2 + c*x^4])
/(9*c) - (7*b*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) - (6*b*x*Sqrt[b + c*x
^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqr
t[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c])
 + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b +
 c*x^2])))/(5*c*Sqrt[b*x^2 + c*x^4])))/(9*c)))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.73

method	result
default	$\frac{2\sqrt{c\,x^4+b\,x^2}\left(45c^4x^8+55b\,c^3x^6+42b^4\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-21b^4\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\ldots\right.}{585x^{\frac{3}{2}}\left(c\,x^2+b\right)c^3}$
risch	$-\frac{2\sqrt{x}\left(-45c^2x^4-10bc\,x^2+14b^2\right)\sqrt{x^2\left(c\,x^2+b\right)}}{585c^2}+\frac{14b^3\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\text{EllipticE}\left(\sqrt{\ldots}\right.}{c\ldots}\ldots\right.}{195c^3\sqrt{c\,x\ldots}}$

input
```
int(x^(7/2)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/585*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^3*(45*c^4*x^8+55*b*c^3*x^6+4
2*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-21*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)
*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4
*b^2*c^2*x^4-14*x^2*b^3*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.23

$$\int x^{7/2}\sqrt{bx^2+cx^4}\,dx =$$
$$-\frac{2\left(42\,b^3\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(45\,c^3x^4+10\,bc^2x^2-14\,b^2c\right)\sqrt{cx^4}\right.}{585\,c^3}$$

input
```
integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/585*(42*b^3*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b
/c, 0, x)) - (45*c^3*x^4 + 10*b*c^2*x^2 - 14*b^2*c)*sqrt(c*x^4 + b*x^2)*sq
rt(x))/c^3
```

Sympy [F]

$$\int x^{7/2}\sqrt{bx^2+cx^4}\,dx = \int x^{\frac{7}{2}}\sqrt{x^2\left(b+cx^2\right)}\,dx$$

input
```
integrate(x**(7/2)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(7/2)*sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int x^{7/2}\sqrt{bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2}x^{\frac{7}{2}}\,dx$$

input
```
integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

Giac [F]

$$\int x^{7/2}\sqrt{bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2}x^{\frac{7}{2}}\,dx$$

input
```
integrate(x^(7/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{7/2}\sqrt{bx^2+cx^4}\,dx=\int x^{7/2}\,\sqrt{c\,x^4+b\,x^2}\,dx$$

input
```
int(x^(7/2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(7/2)*(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^{7/2}\sqrt{bx^2+cx^4}\,dx=\frac{-\frac{28\sqrt{x}\,\sqrt{c\,x^2+b}\,b^2x}{585}+\frac{4\sqrt{x}\,\sqrt{c\,x^2+b}\,bc\,x^3}{117}+\frac{2\sqrt{x}\,\sqrt{c\,x^2+b}\,c^2x^5}{13}+\frac{14\left(\int\frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^2+b}dx\right)b^3}{195}}{c^2}$$

input
```
int(x^(7/2)*(c*x^4+b*x^2)^(1/2),x)
```

output
```
(2*( - 14*sqrt(x)*sqrt(b + c*x**2)*b**2*x + 10*sqrt(x)*sqrt(b + c*x**2)*b*
c*x**3 + 45*sqrt(x)*sqrt(b + c*x**2)*c**2*x**5 + 21*int((sqrt(x)*sqrt(b +
c*x**2))/(b + c*x**2),x)*b**3))/(585*c**2)
```

3.229 $\int x^{5/2}\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 176

$$\int x^{5/2}\sqrt{bx^2+cx^4}\,dx=-\frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}+\frac{10b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}$$

output
```
-20/231*b^2*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+4/77*b*x^(3/2)*(c*x^4+b*x^2)^(
1/2)/c+2/11*x^(7/2)*(c*x^4+b*x^2)^(1/2)+10/231*b^(11/4)*x*(b^(1/2)+c^(1/2)
*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.58

$$\int x^{5/2}\sqrt{bx^2+cx^4}\,dx=\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{1+\frac{cx^2}{b}}\left(-5b^2+2bcx^2+7c^2x^4\right)+5b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2}\right.\right.}{77c^2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[x^(5/2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*(Sqrt[1 + (c*x^2)/b]*(-5*b^2 + 2*b*c*x^2 + 7*c^2*
x^4) + 5*b^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(77*c^2*Sqr
t[x]*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1426, 1429, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1426}$$

$$\frac{2}{11}b\int \frac{x^{9/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow 266$$

$$\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow 761$$

$$\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}$$

input
```
Int[x^(5/2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/11 + (2*b*((2*x^(3/2)*Sqrt[b*x^2 + c*x^4])
/(7*c) - (5*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])))/(7*c)
))/11
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.89

method	result
default	$\frac{2\sqrt{c\,x^{4}+b\,x^{2}}\,\left(21c^{4}x^{7}+5b^{3}\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+27b\,c^{3}x^{5}-4b^{2}c^{2}x^{3}-10b^{3}cx\right.}{231x^{\frac{3}{2}}\left(c\,x^{2}+b\right)c^{3}}$
risch	$-\frac{2\left(-21c^{2}x^{4}-6bc\,x^{2}+10b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{231c^{2}\sqrt{x}}+\frac{10b^{3}\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\right.}{231c^{3}\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}$

input
```
int(x^(5/2)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/231*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(21*c^4*x^7+5*b^3*(-b*c)^(1/2)
*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2),1/2*2^(1/2))+27*b*c^3*x^5-4*b^2*c^2*x^3-10*b^3*c*x)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.39

$$\int x^{5/2}\sqrt{bx^2+cx^4}\,dx = \frac{2\left(10\,b^3\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(21\,c^3x^4+6\,bc^2x^2-10\,b^2c\right)\sqrt{cx^4+b}\right.}{231\,c^3x}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/231*(10*b^3*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + (21*c^3*x^4 +
6*b*c^2*x^2 - 10*b^2*c)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^3*x)
```

Sympy [F]

$$\int x^{5/2}\sqrt{bx^2+cx^4}\,dx = \int x^{\frac{5}{2}}\sqrt{x^2\left(b+cx^2\right)}\,dx$$

input
```
integrate(x**(5/2)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(5/2)*sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int x^{5/2}\sqrt{bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2}x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

Giac [F]

$$\int x^{5/2}\sqrt{bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2}x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}\sqrt{bx^2+cx^4}\,dx=\int x^{5/2}\,\sqrt{c\,x^4+b\,x^2}\,dx$$

input
```
int(x^(5/2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(5/2)*(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^{5/2}\sqrt{bx^2+cx^4}\,dx=\frac{-\frac{20\sqrt{x}\,\sqrt{c\,x^2+b}\,b^2}{231}+\frac{4\sqrt{x}\,\sqrt{c\,x^2+b}\,bc\,x^2}{77}+\frac{2\sqrt{x}\,\sqrt{c\,x^2+b}\,c^2x^4}{11}+\frac{10\left(\int\frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^3+bx}dx\right)b^3}{231}}{c^2}$$

input
```
int(x^(5/2)*(c*x^4+b*x^2)^(1/2),x)
```

output
```
(2*( - 10*sqrt(x)*sqrt(b + c*x**2)*b**2 + 6*sqrt(x)*sqrt(b + c*x**2)*b*c*x
**2 + 21*sqrt(x)*sqrt(b + c*x**2)*c**2*x**4 + 5*int((sqrt(x)*sqrt(b + c*x*
*2))/(b*x + c*x**3),x)*b**3))/(231*c**2)
```

3.230 $\int x^{3/2}\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 293

$$\int x^{3/2}\sqrt{bx^2+cx^4}\,dx=-\frac{4b^2x^{3/2}(b+cx^2)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}+\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}-\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}$$

output
```
-4/15*b^2*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2
)+4/45*b*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+2/9*x^(5/2)*(c*x^4+b*x^2)^(1/2)+4/1
5*b^(9/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*El
lipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/c^(7/4)/(c*x^4
+b*x^2)^(1/2)-2/15*b^(9/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/
2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/
2))/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.29

$$\int x^{3/2}\sqrt{bx^2+cx^4}\,dx=\frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left((b+cx^2)\sqrt{1+\frac{cx^2}{b}}-b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{9c\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[x^(3/2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b] - b*Hype
rgeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(9*c*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1426, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1426}$$

$$\frac{2}{9}b\int \frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}$$

$\big\downarrow$ 266

$$\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}$$

$\big\downarrow$ 834

$$\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}$$

$\big\downarrow$ 27

$$\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}$$

$\big\downarrow$ 761

$$\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}$$

$\big\downarrow$ 1510

$$\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+c}{\left(\sqrt{b}+\right.}}}{\sqrt[4]{}}\right.}{5c\sqrt{bx^2+cx^4}}\right.$$

$$\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}$$

input
```
Int[x^(3/2)*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(5/2)*Sqrt[b*x^2 + c*x^4])/9 + (2*b*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/
(5*c) - (6*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] +
Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)
*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)]
, 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*c*Sqrt[b*x^2 + c*x^4])))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.77

method	result
default	$-\frac{2\sqrt{c\,x^4+b\,x^2}\left(-5c^3x^6+6b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\cdots\right.}{45x^{\frac{3}{2}}\left(c\,x^2+b\right)c^2}$
risch	$\frac{2\sqrt{x}\,\left(5c\,x^2+2b\right)\sqrt{x^2\left(c\,x^2+b\right)}}{45c}-\frac{2b^2\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}\cdots\right.}{15c^2\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}\left(c\,x^2+b\right)}$

input
```
int(x^(3/2)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^2*(-5*c^3*x^6+6*b^3*((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2),1/2*2^(1/2))-3*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2
)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*Ellip
ticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-7*b*c^2*x^4-2*b^
2*c*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.21

$$\int x^{3/2}\sqrt{bx^2+cx^4}\,dx=\frac{2\left(6\,b^2\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}(5\cdots\right.}{45\,c^2}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(6*b^2*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c,
 0, x)) + sqrt(c*x^4 + b*x^2)*(5*c^2*x^2 + 2*b*c)*sqrt(x))/c^2
```

Sympy [F]

$$\int x^{3/2}\sqrt{bx^2+cx^4}\,dx=\int x^{\frac{3}{2}}\sqrt{x^2\,(b+cx^2)}\,dx$$

input
```
integrate(x**(3/2)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(3/2)*sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int x^{3/2}\sqrt{bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2}x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

Giac [F]

$$\int x^{3/2}\sqrt{bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2}x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\sqrt{bx^2+cx^4}\,dx=\int x^{3/2}\,\sqrt{c\,x^4+b\,x^2}\,dx$$

input
```
int(x^(3/2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(3/2)*(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^{3/2}\sqrt{bx^2+cx^4}\,dx=\frac{\frac{4\sqrt{x}\,\sqrt{c\,x^2+b}\,bx}{45}+\frac{2\sqrt{x}\,\sqrt{c\,x^2+b}\,c\,x^3}{9}-\frac{2\left(\int\frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^2+b}dx\right)b^2}{15}}{c}$$

input
```
int(x^(3/2)*(c*x^4+b*x^2)^(1/2),x)
```

output
```
(2*(2*sqrt(x)*sqrt(b + c*x**2)*b*x + 5*sqrt(x)*sqrt(b + c*x**2)*c*x**3 - 3
*int((sqrt(x)*sqrt(b + c*x**2))/(b + c*x**2),x)*b**2))/(45*c)
```

3.231 $\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx = \frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4} - \frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}$$

output

```
4/21*b*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)+2/7*x^(3/2)*(c*x^4+b*x^2)^(1/2)-2/21*
b^(7/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*Inve
rseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/c^(5/4)/(c*x^4+
b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.82 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.59

$$\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left((b+cx^2)\sqrt{1+\frac{cx^2}{b}}-b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{7c\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[Sqrt[x]*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b] - b*Hypergeometr
ic2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(7*c*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1426, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1426}$$

$$\frac{2}{7}b\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)+$$
$$\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}$$

input
```
Int[Sqrt[x]*Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(3/2)*Sqrt[b*x^2 + c*x^4])/7 + (2*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqr
t[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*S
qrt[b*x^2 + c*x^4])))/7
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

method	result
default	$-\frac{2\sqrt{c\,x^{4}+b\,x^{2}}\,\left(b^{2}\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3c^{3}x^{5}-5b\,c^{2}x^{3}-2b^{2}cx\right)}{21x^{\frac{3}{2}}\left(c\,x^{2}+b\right)c^{2}}$
risch	$\frac{2\left(3c\,x^{2}+2b\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{21c\sqrt{x}}-\frac{2b^{2}\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right.}}{21c^{2}\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}$

input
```
int(x^(1/2)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(b^2*(-b*c)^(1/2)*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))-3*c^3*x^5-5*b*c^2*x^3-2*b^2*c*x)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.40

$$\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx = -\frac{2\left(2\,b^2\sqrt{c}x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}(3\,c^2x^2+2\,bc)\sqrt{x}\right)}{21\,c^2x}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/21*(2*b^2*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*
x^2)*(3*c^2*x^2 + 2*b*c)*sqrt(x))/(c^2*x)
```

Sympy [F]

$$\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx = \int \sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\,dx$$

input
```
integrate(x**(1/2)*(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(sqrt(x)*sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

Giac [F]

$$\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx=\int \sqrt{x}\,\sqrt{c\,x^4+b\,x^2}\,dx$$

input
```
int(x^(1/2)*(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(1/2)*(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx=\frac{\frac{4\sqrt{x}\,\sqrt{c\,x^2+b}\,b}{21}+\frac{2\sqrt{x}\,\sqrt{c\,x^2+b}\,c\,x^2}{7}-\frac{2\left(\int\frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^3+bx}dx\right)b^2}{21}}{c}$$

input
```
int(x^(1/2)*(c*x^4+b*x^2)^(1/2),x)
```

output
```
(2*(2*sqrt(x)*sqrt(b + c*x**2)*b + 3*sqrt(x)*sqrt(b + c*x**2)*c*x**2 - int
((sqrt(x)*sqrt(b + c*x**2))/(b*x + c*x**3),x)*b**2))/(21*c)
```

3.232 $\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 263

$$\begin{aligned}\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx &= \frac{4bx^{3/2}(b+cx^2)}{5\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\\ &\quad-\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}\\ &\quad+\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

output

```
4/5*b*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)+2/
5*x^(1/2)*(c*x^4+b*x^2)^(1/2)-4/5*b^(5/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)
/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/
4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+b*x^2)^(1/2)+2/5*b^(5/4)*x*(b^(1/2)+c^(1/
2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.93 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)}{3\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^
2)/b)])/(3*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1426, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{1426}$$

$$\frac{2}{5}b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2bx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \text{266}$$

$$\frac{4bx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \textbf{834}$$

$$\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \textbf{761}$$

$$\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}$$

$$\Big\downarrow \textbf{1510}$$

$$\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}$$
$$\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 + (4*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*
Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*Sqrt[
b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.81

method	result
default	$\frac{2\sqrt{c x^4+b x^2}\left(2b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\right.}{5x^{\frac{3}{2}}\left(c x^2+b\right)c}$
risch	$\frac{2\sqrt{x}\,\sqrt{x^2\left(c x^2+b\right)}}{5}+\frac{2b\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,\operatorname{Ellipt}}{}\right.}{5c\sqrt{c x^3+bx}\,x^{\frac{3}{2}}\left(c x^2+b\right)}$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c*(2*b^2*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*
c)^(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))-b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+c^2*x^4+b*c*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = -\frac{2\left(2\,b\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}c\sqrt{x}\right)}{5\,c}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(2*b*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0
, x)) - sqrt(c*x^4 + b*x^2)*c*sqrt(x))/c
```

Sympy [F]

$$\int \frac{\sqrt{bx^2 + cx^4}}{\sqrt{x}} \, dx = \int \frac{\sqrt{x^2 (b + cx^2)}}{\sqrt{x}} \, dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**(1/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/sqrt(x), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2 + cx^4}}{\sqrt{x}} \, dx = \int \frac{\sqrt{cx^4 + bx^2}}{\sqrt{x}} \, dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

Giac [F]

$$\int \frac{\sqrt{bx^2 + cx^4}}{\sqrt{x}} \, dx = \int \frac{\sqrt{cx^4 + bx^2}}{\sqrt{x}} \, dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2 + cx^4}}{\sqrt{x}} \, dx = \int \frac{\sqrt{c\,x^4 + b\,x^2}}{\sqrt{x}} \, dx$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^(1/2),x)
```

output
```
int((b*x^2 + c*x^4)^(1/2)/x^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{bx^2 + cx^4}}{\sqrt{x}} \, dx = \frac{2\sqrt{x}\,\sqrt{c\,x^2 + b}\,x}{5} + \frac{2\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^2+b}dx\right) b}{5}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(1/2),x)
```

output
```
(2*(sqrt(x)*sqrt(b + c*x**2)*x + int((sqrt(x)*sqrt(b + c*x**2))/(b + c*x**
2),x)*b))/5
```

3.233 $\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}} + \frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

output
```
2/3*(c*x^4+b*x^2)^(1/2)/x^(1/2)+2/3*b^(3/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+
b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)),1/2*2^(1/2))/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.84 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^2}{b}\right)}{\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^(3/2),x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)])/
(Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1426, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1426}$$

$$\frac{2}{3}b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{2bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{4bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^(3/2),x]
```

output
```
(2*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) + (2*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.10

method	result
default	$\frac{2\sqrt{c\,x^4+b\,x^2}\left(b\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\ \operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+c^2x^3+bcx\right)}{3x^{\frac{3}{2}}\left(c\,x^2+b\right)c}$
risch	$\frac{2\sqrt{x^2\left(c\,x^2+b\right)}}{3\sqrt{x}}+\frac{2b\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\ \operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^2\left(c\,x^2+b\right)}\sqrt{x(c\,x^2+b}}{3c\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}\left(c\,x^2+b\right)}$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(b*(-b*c)^(1/2)*((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
,1/2*2^(1/2))+c^2*x^3+b*c*x)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \frac{2\left(2\,b\sqrt{c}x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}c\sqrt{x}\right)}{3\,cx}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
2/3*(2*b*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^2)
*c*sqrt(x))/(c*x)
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**(3/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{3/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^(3/2),x)
```

output
```
int((b*x^2 + c*x^4)^(1/2)/x^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{c\,x^2+b}}{3} + \frac{2\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^3+bx}dx\right)b}{3}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(3/2),x)
```

output
```
(2*(sqrt(x)*sqrt(b + c*x**2) + int((sqrt(x)*sqrt(b + c*x**2))/(b*x + c*x**
3),x)*b))/3
```

3.234 $\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 254

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = \frac{4\sqrt{c}x^{3/2}(b+cx^2)}{\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{x^{3/2}} - \frac{4\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}} + \frac{2\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}}$$

output
```
4*c^(1/2)*x^(3/2)*(c*x^2+b)/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)-2*(c*x
^4+b*x^2)^(1/2)/x^(3/2)-4*b^(1/4)*c^(1/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)
/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/
4))),1/2*2^(1/2))/(c*x^4+b*x^2)^(1/2)+2*b^(1/4)*c^(1/4)*x*(b^(1/2)+c^(1/2)
*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{cx^2}{b}\right)}{x^{3/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^(5/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((c*x^2)/b)]
)/(x^(3/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1425, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}dx$$

$$\Big\downarrow \text{1425}$$

$$2c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2cx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{4cx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}$$

$\Big\downarrow$ 834

$$\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}$$

$\Big\downarrow$ 761

$$\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}$$

$\Big\downarrow$ 1510

$$\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^(5/2),x]
```

output
```
(-2*Sqrt[b*x^2 + c*x^4])/x^(3/2) + (4*c*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*S
qrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/Sqrt[b*x^
2 + c*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.80

method	result
default	$\frac{2\sqrt{c x^4+b x^2}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b-\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{El}\cdots\right.}{x^{\frac{3}{2}}(c x^2+b)}$
risch	$-\frac{2\sqrt{x^2(c x^2+b)}}{x^{\frac{3}{2}}}+\frac{2\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,\mathrm{Elliptic}\cdots}{\cdots}\right.}{\sqrt{c x^3+bx}\,x^{\frac{3}{2}}(c x^2+b)}$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)
*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b
-((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-c*x^2-b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = -\frac{2\left(2\sqrt{c}x^2\text{weierstrassZeta}\left(-\frac{4b}{c},0,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\sqrt{x}\right)}{x^2}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
-2*(2*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0
, x)) + sqrt(c*x^4 + b*x^2)*sqrt(x))/x^2
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = \int \frac{\sqrt{x^2(b+cx^2)}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**(5/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx=\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{5/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^(5/2),x)
```

output
```
int((b*x^2 + c*x^4)^(1/2)/x^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx=\frac{2\sqrt{c\,x^2+b}+2\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^4+b\,x^2}dx\right)b}{\sqrt{x}}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(5/2),x)
```

output
```
(2*(sqrt(b + c*x**2) + sqrt(x)*int((sqrt(x)*sqrt(b + c*x**2))/(b*x**2 + c*
x**4),x)*b))/sqrt(x)
```

3.235 $\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx=-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}+\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{bx^2+cx^4}}$$

output
```
-2/3*(c*x^4+b*x^2)^(1/2)/x^(5/2)+2/3*c^(3/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2
+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/
b^(1/4)),1/2*2^(1/2))/b^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx=-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{cx^2}{b}\right)}{3x^{5/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^(7/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((c*x^2)/b)]
)/(3*x^(5/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1425, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{2}{3}c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{4cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^(7/2),x]
```

output
```
(-2*Sqrt[b*x^2 + c*x^4])/(3*x^(5/2)) + (2*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

method	result
default	$\frac{2\sqrt{c\,x^{4}+b\,x^{2}}\,\left(\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)x-c\,x^{2}-b\right)}{3x^{\frac{5}{2}}\,(c\,x^{2}+b)}$
risch	$-\frac{2\sqrt{x^{2}(c\,x^{2}+b)}}{3x^{\frac{5}{2}}}+\frac{2\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^{2}(c\,x^{2}+b)}\,\sqrt{x(c\,x^{2}+}}{3\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\,(c\,x^{2}+b)}$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(c*x^4+b*x^2)^(1/2)/x^(5/2)/(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c
/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1
/2*2^(1/2))*x-c*x^2-b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \frac{2\left(2\sqrt{c}x^3\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-\sqrt{cx^4+bx^2}\sqrt{x}\right)}{3x^3}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
2/3*(2*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*x^2)
*sqrt(x))/x^3
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**(7/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{7/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^(7/2),x)
```

output
```
int((b*x^2 + c*x^4)^(1/2)/x^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \frac{-2\sqrt{c\,x^2+b}-2\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^5+b\,x^3}dx\right)bx}{\sqrt{x}\,x}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(7/2),x)
```

output
```
( - 2*(sqrt(b + c*x**2) + sqrt(x)*int((sqrt(x)*sqrt(b + c*x**2))/(b*x**3 +
 c*x**5),x)*b*x))/(sqrt(x)*x)
```

3.236 $\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 293

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = \frac{4c^{3/2}x^{3/2}(b+cx^2)}{5b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}} + \frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

output
```
4/5*c^(3/2)*x^(3/2)*(c*x^2+b)/b/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)-2/
5*(c*x^4+b*x^2)^(1/2)/x^(7/2)-4/5*c*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-4/5*c^(5
/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*Elliptic
E(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/b^(3/4)/(c*x^4+b*x^2
)^(1/2)+2/5*c^(5/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2
)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(
3/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{cx^2}{b}\right)}{5x^{7/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^(9/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((c*x^2)/b)
])/(5*x^(7/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1425, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}dx$$

$$\Big\downarrow \text{ 1425}$$

$$\frac{2}{5}c\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}$$

$$\Big\downarrow \text{ 1430}$$

$$\frac{2}{5}c\left(\frac{c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{2}{5}c\left(\frac{cx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{b\sqrt{bx^2+cx^4}}\right.$$

$$\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}$$

input

```
Int[Sqrt[b*x^2 + c*x^4]/x^(9/2),x]
```

output

```
(-2*Sqrt[b*x^2 + c*x^4])/(5*x^(7/2)) + (2*c*((-2*Sqrt[b*x^2 + c*x^4])/(b*x
^(3/2)) + (2*c*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b]
+ Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/
4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(b*Sqrt[b*x^2 + c*x^4])))/5
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.76

method	result
default	$\frac{2\sqrt{c x^4+b x^2}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc x^2-\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\right.}{5x^{\frac{7}{2}}(c x^2+b)b}$
risch	$-\frac{2(2c x^2+b)\sqrt{x^2(c x^2+b)}}{5x^{\frac{7}{2}}b}+\frac{2c\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\right.}{5b\sqrt{c x^3+bx}\,x^{\frac{3}{2}}(c x^2+b)}$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(c*x^4+b*x^2)^(1/2)/x^(7/2)/(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/
2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*b*c*x^2-((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-2*c^2*x^4-3*b*c*x^2-b^2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = -\frac{2\left(2\,c^{\frac{3}{2}}x^4\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}(2\,cx^2+b)\sqrt{x}\right)}{5\,bx^4}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/5*(2*c^(3/2)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c,
 0, x)) + sqrt(c*x^4 + b*x^2)*(2*c*x^2 + b)*sqrt(x))/(b*x^4)
```

Sympy [F]

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^{9/2}}\,dx = \int \frac{\sqrt{x^2\,(b + cx^2)}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**(9/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**(9/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^{9/2}}\,dx = \int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{bx^2 + cx^4}}{x^{9/2}}\,dx = \int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{9/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^(9/2),x)
```

output
```
int((b*x^2 + c*x^4)^(1/2)/x^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx=\frac{-2\sqrt{c\,x^2+b}-2\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^6+b\,x^4}dx\right)b\,x^2}{3\sqrt{x}\,x^2}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(9/2),x)
```

output
```
( - 2*(sqrt(b + c*x**2) + sqrt(x)*int((sqrt(x)*sqrt(b + c*x**2))/(b*x**4 +
 c*x**6),x)*b*x**2))/(3*sqrt(x)*x**2)
```

3.237 $\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx=-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/7*(c*x^4+b*x^2)^(1/2)/x^(9/2)-4/21*c*(c*x^4+b*x^2)^(1/2)/b/x^(5/2)-2/21
*c^(7/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*Inv
erseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(5/4)/(c*x^4
+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx=-\frac{2\sqrt{x^2\left(b+cx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},-\frac{cx^2}{b}\right)}{7x^{9/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^(11/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-7/4, -1/2, -3/4, -((c*x^2)/b)
])/(7*x^(9/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1425, 1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{2}{7}c\int \frac{1}{x^{3/2}\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{2}{7}c\left(-\frac{c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2}{7}c\left(-\frac{cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{7}c\left(-\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{7}c\left(-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^(11/2),x]
```

output
```
(-2*Sqrt[b*x^2 + c*x^4])/(7*x^(9/2)) + (2*c*((-2*Sqrt[b*x^2 + c*x^4])/(3*b
*x^(5/2)) - (c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/
4)*Sqrt[b*x^2 + c*x^4])))/7
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

method	result
default	$-\frac{2\sqrt{c\,x^{4}+b\,x^{2}}\,\left(\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c\,x^{3}+2c^{2}x^{4}+5bc\,x^{2}+3b^{2}\right)}{21x^{\frac{9}{2}}\left(c\,x^{2}+b\right)b}$
risch	$-\frac{2\left(2c\,x^{2}+3b\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{21x^{\frac{9}{2}}\,b}-\frac{2c\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^{2}\left(c\,x^{2}+\right.}}{21b\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(c*x^4+b*x^2)^(1/2)/x^(9/2)/(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
,1/2*2^(1/2))*c*x^3+2*c^2*x^4+5*b*c*x^2+3*b^2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = -\frac{2\left(2\,c^{\frac{3}{2}}x^5\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}(2\,cx^2+3\,b)\sqrt{x}\right)}{21\,bx^5}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="fricas")
```

output
```
-2/21*(2*c^(3/2)*x^5*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^
2)*(2*c*x^2 + 3*b)*sqrt(x))/(b*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**(11/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**(11/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

Giac [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx=\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{11/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^(11/2),x)
```

output
```
int((b*x^2 + c*x^4)^(1/2)/x^(11/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx=\frac{-2\sqrt{c\,x^2+b}-2\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^7+b\,x^5}dx\right)b\,x^3}{5\sqrt{x}\,x^3}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(11/2),x)
```

output
```
( - 2*(sqrt(b + c*x**2) + sqrt(x)*int((sqrt(x)*sqrt(b + c*x**2))/(b*x**5 +
 c*x**7),x)*b*x**3))/(5*sqrt(x)*x**3)
```

3.238 $\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 323

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = -\frac{4c^{5/2}x^{3/2}(b+cx^2)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} + \frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} + \frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}} - \frac{2c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}$$

output
```
-4/15*c^(5/2)*x^(3/2)*(c*x^2+b)/b^2/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2
)-2/9*(c*x^4+b*x^2)^(1/2)/x^(11/2)-4/45*c*(c*x^4+b*x^2)^(1/2)/b/x^(7/2)+4/
15*c^2*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)+4/15*c^(9/4)*x*(b^(1/2)+c^(1/2)*x)*
((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4))),1/2*2^(1/2))/b^(7/4)/(c*x^4+b*x^2)^(1/2)-2/15*c^(9/4)*x*(b^
(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{1}{2},-\frac{5}{4},-\frac{cx^2}{b}\right)}{9x^{11/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^(13/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-9/4, -1/2, -5/4, -((c*x^2)/b)
])/(9*x^(11/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1425, 1430, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{2}{9}c\int \frac{1}{x^{5/2}\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{2}{9}c\left(-\frac{3c\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{2}{9}c\left(-\frac{3c\left(\frac{c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}$$

$\downarrow$ 1431

$$\frac{2}{9}c\left(-\frac{3c\left(\frac{cx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}$$

$\downarrow$ 266

$$\frac{2}{9}c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}$$

$\downarrow$ 834

$$\frac{2}{9}c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}$$

$\downarrow$ 27

$$\frac{2}{9}c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}$$

$\big\downarrow$ 761

$$\frac{2}{9}c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+c}}{5bx^{7/2}}\right.$$

$$\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}$$

$\big\downarrow$ 1510

$$\frac{2}{9}c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\cdots\right)}{b\sqrt{bx^2+cx^4}}\cdots\right)}{5b}\cdots\right)$$

$$\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^(13/2),x]
```

output
```
(-2*Sqrt[b*x^2 + c*x^4])/(9*x^(11/2)) + (2*c*((-2*Sqrt[b*x^2 + c*x^4])/(5*
b*x^(7/2)) - (3*c*((-2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) + (2*c*x*Sqrt[b +
c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[
c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[
b + c*x^2])))/(b*Sqrt[b*x^2 + c*x^4])))/(5*b)))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.74

method	result
default	$-\frac{2\sqrt{c\,x^{4}+b\,x^{2}}\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^{2}x^{4}-3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\cdots}\right.}{45x^{\frac{11}{2}}\left(c\,x^{2}+b\right)b^{2}}$
risch	$-\frac{2\left(-6c^{2}x^{4}+2bc\,x^{2}+5b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{45x^{\frac{11}{2}}\,b^{2}}-\frac{2c^{2}\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-b\cdots}}{c}\right)}{\sqrt{-b\cdots}}}\cdots\right.}{c}\cdots\right.}{15b^{2}\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\cdots}$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(c*x^4+b*x^2)^(1/2)/x^(11/2)/(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^
(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*b*c^2*x^4-3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-6*c^3*x^6-4*b*c^2*
x^4+7*b^2*c*x^2+5*b^3)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \frac{2\left(6\,c^{\frac{5}{2}}x^6\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(6\,c^2x^4-2\,bcx^2-\right.\right.}{45\,b^2x^6}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="fricas")
```

output
```
2/45*(6*c^(5/2)*x^6*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c,
 0, x)) + (6*c^2*x^4 - 2*b*c*x^2 - 5*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^
2*x^6)
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**(13/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**(13/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

Giac [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{13/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^(13/2),x)
```

output
```
int((b*x^2 + c*x^4)^(1/2)/x^(13/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \frac{-2\sqrt{c\,x^2+b}-2\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^8+b\,x^6}dx\right)b\,x^4}{7\sqrt{x}\,x^4}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(13/2),x)
```

output
```
( - 2*(sqrt(b + c*x**2) + sqrt(x)*int((sqrt(x)*sqrt(b + c*x**2))/(b*x**6 +
 c*x**8),x)*b*x**4))/(7*sqrt(x)*x**4)
```

3.239 $\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx=-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}+\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}+\frac{10c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}$$

output

```
-2/11*(c*x^4+b*x^2)^(1/2)/x^(13/2)-4/77*c*(c*x^4+b*x^2)^(1/2)/b/x^(9/2)+20
/231*c^2*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)+10/231*c^(11/4)*x*(b^(1/2)+c^(1/2
)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx=-\frac{2\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{2},-\frac{7}{4},-\frac{cx^2}{b}\right)}{11x^{13/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[Sqrt[b*x^2 + c*x^4]/x^(15/2),x]
```

output
```
(-2*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-11/4, -1/2, -7/4, -((c*x^2)/b
)])/(11*x^(13/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1425, 1430, 1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{2}{11}c\int \frac{1}{x^{7/2}\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{2}{11}c\left(-\frac{5c\int \frac{1}{x^{3/2}\sqrt{cx^4+bx^2}}dx}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{2}{11}c\left(-\frac{5c\left(-\frac{c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2}{11}c\left(-\frac{5c\left(-\frac{cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{11}c\left(-\frac{5c\left(-\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{11}c\left(-\frac{5c\left(-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

input
```
Int[Sqrt[b*x^2 + c*x^4]/x^(15/2),x]
```

output
```
(-2*Sqrt[b*x^2 + c*x^4])/(11*x^(13/2)) + (2*c*((-2*Sqrt[b*x^2 + c*x^4])/(7
*b*x^(9/2)) - (5*c*((-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*x^4])))
/(7*b)))/11
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.89

method	result
default	$\frac{2\sqrt{c\,x^4+b\,x^2}\left(5\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c^2x^5+10c^3x^6+4b\,c^2x^4-27b^2c\,x^2-21b^3\right.}{231x^{\frac{13}{2}}(c\,x^2+b)b^2}$
risch	$-\frac{2(-10c^2x^4+6bc\,x^2+21b^2)\sqrt{x^2(c\,x^2+b)}}{231x^{\frac{13}{2}}b^2}+\frac{10c^2\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\right.}{231b^2\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}(c\,x^2+b)}$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

output
```
2/231*(c*x^4+b*x^2)^(1/2)/x^(13/2)/(c*x^2+b)*(5*(-b*c)^(1/2)*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*c^2*x^5+10*c^3*x^6+4*b*c^2*x^4-27*b^2*c*x^2-21*b^3)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \frac{2\left(10\,c^{\frac{5}{2}}x^7\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(10\,c^2x^4-6\,bcx^2-21\,b^2\right)\sqrt{cx^4+bx^2}\sqrt{x}\right)}{231\,b^2x^7}$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="fricas")
```

output
```
2/231*(10*c^(5/2)*x^7*weierstrassPInverse(-4*b/c, 0, x) + (10*c^2*x^4 - 6*
b*c*x^2 - 21*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^2*x^7)
```

Sympy [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(1/2)/x**(15/2),x)
```

output
```
Integral(sqrt(x**2*(b + c*x**2))/x**(15/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

Giac [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2}}{x^{15/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(1/2)/x^(15/2),x)
```

output
```
int((b*x^2 + c*x^4)^(1/2)/x^(15/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \frac{-2\sqrt{c\,x^2+b}-2\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^9+b\,x^7}dx\right)b\,x^5}{9\sqrt{x}\,x^5}$$

input
```
int((c*x^4+b*x^2)^(1/2)/x^(15/2),x)
```

output
```
( - 2*(sqrt(b + c*x**2) + sqrt(x)*int((sqrt(x)*sqrt(b + c*x**2))/(b*x**7 +
 c*x**9),x)*b*x**5))/(9*sqrt(x)*x**5)
```

3.240 $\int x^{3/2}(bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 350

$$\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{56b^4x^{3/2}(b+cx^2)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}$$
$$-\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}$$
$$+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}-\frac{56b^{17/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}+\frac{28b^{17/4}x\left(\sqrt{b}+\sqrt{c}\ldots\right.}{\ldots}$$

output
```
56/1105*b^4*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1
/2)-56/3315*b^3*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2+8/663*b^2*x^(5/2)*(c*x^4+b
*x^2)^(1/2)/c+12/221*b*x^(9/2)*(c*x^4+b*x^2)^(1/2)+2/17*x^(5/2)*(c*x^4+b*x
^2)^(3/2)-56/1105*b^(17/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/
2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2
))/c^(11/4)/(c*x^4+b*x^2)^(1/2)+28/1105*b^(17/4)*x*(b^(1/2)+c^(1/2)*x)*((c
*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)),1/2*2^(1/2))/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.29

$$\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(-\left(\left(7b-13cx^2\right)\left(b+cx^2\right)^2\sqrt{1+\frac{cx^2}{b}}\right)+7b^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\right.\right.}{221c^2\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[x^(3/2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((7*b - 13*c*x^2)*(b + c*x^2)^2*Sqrt[1
+ (c*x^2)/b]) + 7*b^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(2
21*c^2*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.476$, Rules used = {1426, 1426, 1429, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1426}$$

$$\frac{6}{17}b\int x^{7/2}\sqrt{cx^4+bx^2}dx+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{1426}$$

$$\frac{6}{17}b\left(\frac{2}{13}b\int\frac{x^{11/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{1429}$$

$$\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\int\frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{1429}$$

$$\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{1431}$$

$$\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{834}$$

$$\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{\right.$$

$$\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

$\downarrow$ 27

$$\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right)+\frac{2}{13}x^{9/2}\sqrt{bx^2+\right.$$

$$\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

$\downarrow$ 761

$$\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\cdots\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}\right.\right.$$

$$\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

$\downarrow$ 1510

$$\frac{6}{17}b\left(\frac{2}{13}b\left(\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\cdots\right)}{5c\sqrt{bx^2+c\cdots}}\right)}{9c}\right.\right.$$

$$\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

input
```
Int[x^(3/2)*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*x^(5/2)*(b*x^2 + c*x^4)^(3/2))/17 + (6*b*((2*x^(9/2)*Sqrt[b*x^2 + c*x^4
])/13 + (2*b*((2*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*c) - (7*b*((2*Sqrt[x]*Sqr
t[b*x^2 + c*x^4])/(5*c) - (6*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c
*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*c*Sqrt[b*x^2 +
c*x^4])))/(9*c)))/13))/17
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.71

method	result
default	$\frac{2(cx^4+bx^2)^{\frac{3}{2}}\left(195c^5x^{10}+480c^4x^8b+305b^2c^3x^6+84b^5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-42b^5\right.}{3315x^{\frac{7}{2}}(cx^2+b)^2c^3}$
risch	$-\frac{2\sqrt{x}\left(-195c^3x^6-285bc^2x^4-20b^2cx^2+28b^3\right)\sqrt{x^2(cx^2+b)}}{3315c^2}+\frac{28b^4\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-b}}{}\right.}{}$

input
```
int(x^(3/2)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3315*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^3*(195*c^5*x^10+480*c^4*x
^8*b+305*b^2*c^3*x^6+84*b^5*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2
)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*Ellip
ticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-42*b^5*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2),1/2*2^(1/2))-8*c^2*x^4*b^3-28*x^2*c*b^4)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.24

$$\int x^{3/2}(bx^2+cx^4)^{3/2}\,dx = -\frac{2\left(84\,b^4\sqrt{c}\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(195\,c^4x^6+285\,bc^3x^4+20\,b^2c^2x^2\right.\right.}{3315\,c^3}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-2/3315*(84*b^4*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*
b/c, 0, x)) - (195*c^4*x^6 + 285*b*c^3*x^4 + 20*b^2*c^2*x^2 - 28*b^3*c)*sq
rt(c*x^4 + b*x^2)*sqrt(x))/c^3
```

Sympy [F]

$$\int x^{3/2}(bx^2+cx^4)^{3/2}\,dx = \int x^{\frac{3}{2}}\left(x^2(b+cx^2)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**(3/2)*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**(3/2)*(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2), x)
```

Giac [F]

$$\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx=\int x^{3/2}\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

input
```
int(x^(3/2)*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(3/2)*(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int x^{3/2}\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{-\frac{56\sqrt{x}\,\sqrt{c\,x^2+b}\,b^3x}{3315}+\frac{8\sqrt{x}\,\sqrt{c\,x^2+b}\,b^2c\,x^3}{663}+\frac{38\sqrt{x}\,\sqrt{c\,x^2+b}\,b\,c^2x^5}{221}+\frac{2\sqrt{x}\,\sqrt{c\,x^2+b}\,c^3x^7}{17}+\frac{28\left(\int\frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^2+b}dx\right)b^4}{1105}}{c^2}$$

input
```
int(x^(3/2)*(c*x^4+b*x^2)^(3/2),x)
```

output
```
(2*( - 28*sqrt(x)*sqrt(b + c*x**2)*b**3*x + 20*sqrt(x)*sqrt(b + c*x**2)*b*
*2*c*x**3 + 285*sqrt(x)*sqrt(b + c*x**2)*b*c**2*x**5 + 195*sqrt(x)*sqrt(b
 + c*x**2)*c**3*x**7 + 42*int((sqrt(x)*sqrt(b + c*x**2))/(b + c*x**2),x)*b*
*4))/(3315*c**2)
```

3.241 $\int \sqrt{x}(bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 203

$$\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx=-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}+\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}$$

output
```
-8/231*b^3*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+8/385*b^2*x^(3/2)*(c*x^4+b*x^2)
^(1/2)/c+4/55*b*x^(7/2)*(c*x^4+b*x^2)^(1/2)+2/15*x^(3/2)*(c*x^4+b*x^2)^(3/
2)+4/231*b^(15/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^
(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/c^(9/
4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

$$\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2\sqrt{x^2\left(b+cx^2\right)}\left(-\left(\left(5b-11cx^2\right)\left(b+cx^2\right)^2\sqrt{1+\frac{cx^2}{b}}\right)+5b^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\ldots\right)\right.}{165c^2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[Sqrt[x]*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*(-((5*b - 11*c*x^2)*(b + c*x^2)^2*Sqrt[1 + (c*x^2
)/b]) + 5*b^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(165*c^2*S
qrt[x]*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1426, 1426, 1429, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{1426}$$

$$\frac{2}{5}b\int x^{5/2}\sqrt{cx^4+bx^2}dx+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{1426}$$

$$\frac{2}{5}b\left(\frac{2}{11}b\int \frac{x^{9/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2}{5}b\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2}{5}b\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2}{5}b\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{5}b\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)+\frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{5}b\left(\frac{2}{11}b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{7c}\right)+\frac{2}{11}x\right.$$

$$\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}$$

input
```
Int[Sqrt[x]*(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*x^(3/2)*(b*x^2 + c*x^4)^(3/2))/15 + (2*b*((2*x^(7/2)*Sqrt[b*x^2 + c*x^4
])/11 + (2*b*((2*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c) - (5*b*((2*Sqrt[b*x^2
+ c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])))/(7*c)))/11))/5
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.83

method	result
default	$\frac{2(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(77c^5x^9+196c^4x^7b+10b^4\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+131b^2c^3x^5-8\right.}{1155x^{\frac{7}{2}}(c\,x^2+b)^2c^3}$
risch	$-\frac{2(-77c^3x^6-119b\,c^2x^4-12b^2c\,x^2+20b^3)\sqrt{x^2(c\,x^2+b)}}{1155c^2\sqrt{x}}+\frac{4b^4\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\right.\right.}{231c^3\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}(c\,x^2+b}$

input
```
int(x^(1/2)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(77*c^5*x^9+196*c^4*x^7*b+1
0*b^4*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+131*b^2*c^3*x^5-8*c^2*x^3
*b^3-20*x*c*b^4)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.39

$$\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2\left(20\,b^4\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(77\,c^4x^6+119\,bc^3x^4+12\,b^2c^2x^2-20\,b^3c\right)\sqrt{cx}\right.}{1155\,c^3x}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(20*b^4*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + (77*c^4*x^6 +
 119*b*c^3*x^4 + 12*b^2*c^2*x^2 - 20*b^3*c)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(
c^3*x)
```

Sympy [F]

$$\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx=\int \sqrt{x}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**(1/2)*(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(sqrt(x)*(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)
```

Giac [F]

$$\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx=\int \sqrt{x}\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

input
```
int(x^(1/2)*(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(1/2)*(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{x}\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{-\frac{8\sqrt{x}\,\sqrt{c\,x^2+b}\,b^3}{231}+\frac{8\sqrt{x}\,\sqrt{c\,x^2+b}\,b^2c\,x^2}{385}+\frac{34\sqrt{x}\,\sqrt{c\,x^2+b}\,b\,c^2x^4}{165}+\frac{2\sqrt{x}\,\sqrt{c\,x^2+b}\,c^3x^6}{15}+\frac{4\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^3+bx}dx\right)b^4}{231}}{c^2}$$

input
```
int(x^(1/2)*(c*x^4+b*x^2)^(3/2),x)
```

output
```
(2*( - 20*sqrt(x)*sqrt(b + c*x**2)*b**3 + 12*sqrt(x)*sqrt(b + c*x**2)*b**2
*c*x**2 + 119*sqrt(x)*sqrt(b + c*x**2)*b*c**2*x**4 + 77*sqrt(x)*sqrt(b + c
*x**2)*c**3*x**6 + 10*int((sqrt(x)*sqrt(b + c*x**2))/(b*x + c*x**3),x)*b**
4))/(1155*c**2)
```

3.242 $\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 320

$$\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = -\frac{8b^3x^{3/2}(b+cx^2)}{65c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c} + \frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4} + \frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2} + \frac{8b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65c^{7/4}\sqrt{bx^2+cx^4}} - \frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)}{}$$

output
```
-8/65*b^3*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2
)+8/195*b^2*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+4/39*b*x^(5/2)*(c*x^4+b*x^2)^(1/
2)+2/13*x^(1/2)*(c*x^4+b*x^2)^(3/2)+8/65*b^(13/4)*x*(b^(1/2)+c^(1/2)*x)*((
c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4))),1/2*2^(1/2))/c^(7/4)/(c*x^4+b*x^2)^(1/2)-4/65*b^(13/4)*x*(b^(
1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.28

$$\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left((b+cx^2)^2\sqrt{1+\frac{cx^2}{b}} - b^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)\right.}{13c\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b^2*
Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(13*c*Sqrt[1 + (c*x^2)/b
])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1426, 1426, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 1426}$$

$$\frac{6}{13}b\int x^{3/2}\sqrt{cx^4+bx^2}dx + \frac{2}{13}\sqrt{x}(bx^2+cx^4)^{3/2}$$

$$\Big\downarrow \text{ 1426}$$

$$\frac{6}{13}b\left(\frac{2}{9}b\int \frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx + \frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right) + \frac{2}{13}\sqrt{x}(bx^2+cx^4)^{3/2}$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}$$

$\Big\downarrow$ 1431

$$\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}$$

$\Big\downarrow$ 266

$$\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}$$

$\Big\downarrow$ 834

$$\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}\right)+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}$$

$\Big\downarrow$ 761

$$\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right.}{5c\sqrt{bx^2+cx^4}}\right.\right.$$

$$\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 1510$$

$$\frac{6}{13}b\left(\frac{2}{9}b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{}}{}\right.}{5c\sqrt{bx^2+cx^4}}\right.\right.$$

$$\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/13 + (6*b*((2*x^(5/2)*Sqrt[b*x^2 + c*x^4
])/9 + (2*b*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) - (6*b*x*Sqrt[b + c*x^2
]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) +
 (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*
EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c
*x^2])))/(5*c*Sqrt[b*x^2 + c*x^4])))/9))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.74

method	result
default	$-\frac{2(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(-15c^4x^8-40b\,c^3x^6+12b^4\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\ \text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-6b^4\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{195x^{\frac{7}{2}}(c\,x^2+b)^2c^2}$
risch	$\frac{2\sqrt{x}\,(15c^2x^4+25bc\,x^2+4b^2)\sqrt{x^2(c\,x^2+b)}}{195c}-\frac{4b^3\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\ \text{EllipticE}\left(\sqrt{\frac{\left(x+\right.}{}}\right.}{c}\right.}{65c^2\sqrt{c\,x^3+bx}\,x}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/195*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^2*(-15*c^4*x^8-40*b*c^3*x
^6+12*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-6*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1
/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)-29*b^2*c^2*x^4-4*x^2*b^3*c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.22

$$\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \frac{2\left(12\,b^3\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(15\,c^3x^4+25\right.}{195\,c^2}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

output
```
2/195*(12*b^3*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/
c, 0, x)) + (15*c^3*x^4 + 25*b*c^2*x^2 + 4*b^2*c)*sqrt(c*x^4 + b*x^2)*sqrt
(x))/c^2
```

Sympy [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/sqrt(x), x)
```

Maxima [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x), x)
```

Giac [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(c\,x^4+b\,x^2)^{3/2}}{\sqrt{x}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^(1/2),x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^(1/2), x)
```

Reduce [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \frac{\frac{8\sqrt{x}\,\sqrt{c\,x^2+b}\,b^2x}{195}+\frac{10\sqrt{x}\,\sqrt{c\,x^2+b}\,bc\,x^3}{39}+\frac{2\sqrt{x}\,\sqrt{c\,x^2+b}\,c^2x^5}{13}-\frac{4\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^2+b}dx\right)b^3}{65}}{c}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(1/2),x)
```

output
```
(2*(4*sqrt(x)*sqrt(b + c*x**2)*b**2*x + 25*sqrt(x)*sqrt(b + c*x**2)*b*c*x**
*3 + 15*sqrt(x)*sqrt(b + c*x**2)*c**2*x**5 - 6*int((sqrt(x)*sqrt(b + c*x**
2))/(b + c*x**2),x)*b**3))/(195*c)
```

3.243 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 173

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}} - \frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}$$

output

```
8/77*b^2*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)+12/77*b*x^(3/2)*(c*x^4+b*x^2)^(1/2)
+2/11*(c*x^4+b*x^2)^(3/2)/x^(1/2)-4/77*b^(11/4)*x*(b^(1/2)+c^(1/2)*x)*((c*
x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)),1/2*2^(1/2))/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.52

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left((b+cx^2)^2\sqrt{1+\frac{cx^2}{b}} - b^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{11c\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^(3/2),x]
```

output
```
(2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b^2*Hypergeo
metric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(11*c*Sqrt[x]*Sqrt[1 + (c*x^2)/b
])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1426, 1426, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}dx$$

$$\Big\downarrow \text{1426}$$

$$\frac{6}{11}b\int \sqrt{x}\sqrt{cx^4+bx^2}dx+\frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}}$$

$$\Big\downarrow \text{1426}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)+\frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)+\frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{6}{11}b\left(\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)+\frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}}$$

$$\Big\downarrow \; 266$$

$$\frac{6}{11}b\left(\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}}$$

$$\Big\downarrow \; 761$$

$$\frac{6}{11}b\left(\frac{2}{7}b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)+\frac{2}{7}x^{3/2}\sqrt{bx^2+\right.$$

$$\frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^(3/2),x]
```

output
```
(2*(b*x^2 + c*x^4)^(3/2))/(11*Sqrt[x]) + (6*b*((2*x^(3/2)*Sqrt[b*x^2 + c*x
^4])/7 + (2*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])))/7))/1
1
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

method	result
default	$-\frac{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(-7c^{4}x^{7}+2b^{3}\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-20b\,c^{3}x^{5}-17b^{2}c^{2}x^{3}-\right.}{77x^{\frac{7}{2}}\left(c\,x^{2}+b\right)^{2}c^{2}}$
risch	$\frac{2\left(7c^{2}x^{4}+13bc\,x^{2}+4b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{77c\sqrt{x}}-\frac{4b^{3}\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\ }}{77c^{2}\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(-7*c^4*x^7+2*b^3*(-b*c)^(1/
2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-20*b*c^3*x^5-17*b^2*c^2*x^3-4*b^3*c*x)/c^
2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.40

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{3/2}} \, dx = -\frac{2 \left(4 \, b^3 \sqrt{c} x \text{weierstrassPInverse}\left(-\frac{4 \, b}{c}, 0, x\right) - (7 \, c^3 x^4 + 13 \, bc^2 x^2 + 4 \, b^2 c)\sqrt{cx^4 + bx^2}\sqrt{x}\right)}{77 \, c^2 x}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

output
```
-2/77*(4*b^3*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - (7*c^3*x^4 + 13
*b*c^2*x^2 + 4*b^2*c)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^2*x)
```

Sympy [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{3/2}} \, dx = \int \frac{(x^2(b + cx^2))^{\frac{3}{2}}}{x^{\frac{3}{2}}} \, dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**(3/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**(3/2), x)
```

Maxima [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{3/2}} \, dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{3}{2}}} \, dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2), x)
```

Giac [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^{3/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^(3/2),x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^(3/2), x)
```

Reduce [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \frac{\frac{8\sqrt{x}\,\sqrt{c\,x^2+b}\,b^2}{77} + \frac{26\sqrt{x}\,\sqrt{c\,x^2+b}\,bc\,x^2}{77} + \frac{2\sqrt{x}\,\sqrt{c\,x^2+b}\,c^2x^4}{11} - \frac{4\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^3+bx}dx\right)b^3}{77}}{c}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(3/2),x)
```

output
```
(2*(4*sqrt(x)*sqrt(b + c*x**2)*b**2 + 13*sqrt(x)*sqrt(b + c*x**2)*b*c*x**2
 + 7*sqrt(x)*sqrt(b + c*x**2)*c**2*x**4 - 2*int((sqrt(x)*sqrt(b + c*x**2))
/(b*x + c*x**3),x)*b**3))/(77*c)
```

3.244 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 290

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \frac{8b^2x^{3/2}(b+cx^2)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4}$$
$$+\frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}}-\frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}$$

output
```
8/15*b^2*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)
+4/15*b*x^(1/2)*(c*x^4+b*x^2)^(1/2)+2/9*(c*x^4+b*x^2)^(3/2)/x^(3/2)-8/15*b
^(9/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*Ellip
ticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+b*
x^2)^(1/2)+4/15*b^(9/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*
x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))
/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.20

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \frac{2b\sqrt{x}\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)}{3\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^(5/2),x]
```

output
```
(2*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*
x^2)/b)])/(3*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1426, 1426, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{1426}$$

$$\frac{2}{3}b\int \frac{\sqrt{cx^4+bx^2}}{\sqrt{x}}dx+\frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}}$$

$$\Big\downarrow \text{1426}$$

$$\frac{2}{3}b\left(\frac{2}{5}b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)+\frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2}{3}b\left(\frac{2bx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}$$

$$\Big\downarrow \; 834$$

$$\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}$$

$$\Big\downarrow \; 1510$$

$$\frac{2}{3}b\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{5\sqrt{bx^2+cx^4}}\right.$$

$$\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^(5/2),x]
```

output
```
(2*(b*x^2 + c*x^4)^(3/2))/(9*x^(3/2)) + (2*b*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^
4])/5 + (4*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] +
Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)
*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)]
, 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*Sqrt[b*x^2 + c*x^4])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.78

method	result
default	$\frac{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(5c^{3}x^{6}+12b^{3}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-6b^{3}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-b}}{\sqrt{-bc}}}\right.}{45x^{\frac{7}{2}}\left(c\,x^{2}+b\right)^{2}c}$
risch	$\frac{2\sqrt{x}\,\left(5c\,x^{2}+11b\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{45}+\frac{4b^{2}\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}\right.}{15c\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c*(5*c^3*x^6+12*b^3*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2),1/2*2^(1/2))-6*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)
*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*Ellipt
icF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+16*b*c^2*x^4+11*b
^2*c*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.21

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{5/2}}\,dx = -\frac{2\left(12\,b^2\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}(5\,c^2x^2+11\,bc)\sqrt{x}\right)}{45\,c}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

output
```
-2/45*(12*b^2*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/
c, 0, x)) - sqrt(c*x^4 + b*x^2)*(5*c^2*x^2 + 11*b*c)*sqrt(x))/c
```

Sympy [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{5/2}} \, dx = \int \frac{(x^2(b + cx^2))^{\frac{3}{2}}}{x^{\frac{5}{2}}} \, dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**(5/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**(5/2), x)
```

Maxima [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{5/2}} \, dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{5}{2}}} \, dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2), x)
```

Giac [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{5/2}} \, dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{5}{2}}} \, dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx=\int \frac{{\left(c\,x^4+b\,x^2\right)}^{3/2}}{x^{5/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^(5/2),x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^(5/2), x)
```

Reduce [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx=\frac{22\sqrt{x}\,\sqrt{c\,x^2+b}\,bx}{45}+\frac{2\sqrt{x}\,\sqrt{c\,x^2+b}\,c\,x^3}{9}+\frac{4\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^2+b}dx\right)b^2}{15}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(5/2),x)
```

output
```
(2*(11*sqrt(x)*sqrt(b + c*x**2)*b*x + 5*sqrt(x)*sqrt(b + c*x**2)*c*x**3 +
6*int((sqrt(x)*sqrt(b + c*x**2))/(b + c*x**2),x)*b**2))/45
```

3.245 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2(bx^2+cx^4)^{3/2}}{7x^{5/2}} + \frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{7\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

output
```
4/7*b*(c*x^4+b*x^2)^(1/2)/x^(1/2)+2/7*(c*x^4+b*x^2)^(3/2)/x^(5/2)+4/7*b^(7
/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJ
acobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/c^(1/4)/(c*x^4+b*x^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.39

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \frac{2b\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^2}{b}\right)}{\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^(7/2),x]
```

output
```
(2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]
)/(Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1426, 1426, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{1426}$$

$$\frac{6}{7}b\int \frac{\sqrt{cx^4+bx^2}}{x^{3/2}}dx+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}$$

$$\Big\downarrow \text{1426}$$

$$\frac{6}{7}b\left(\frac{2}{3}b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{6}{7}b\left(\frac{2bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{7}b\left(\frac{4bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{7}b\left(\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^(7/2),x]
```

output
```
(2*(b*x^2 + c*x^4)^(3/2))/(7*x^(5/2)) + (6*b*((2*Sqrt[b*x^2 + c*x^4])/(3*S
qrt[x]) + (2*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(1/
4)*Sqrt[b*x^2 + c*x^4])))/7
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

method	result
default	$\frac{2(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(2b^2\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+c^3x^5+4b\,c^2x^3+3b^2cx\right)}{7x^{\frac{7}{2}}(c\,x^2+b)^2c}$
risch	$\frac{2(c\,x^2+3b)\sqrt{x^2(c\,x^2+b)}}{7\sqrt{x}}+\frac{4b^2\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^2(c\,x^2+b)}}{7c\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}(c\,x^2+b)}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(2*b^2*(-b*c)^(1/2)*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))+c^3*x^5+4*b*c^2*x^3+3*b^2*c*x)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.39

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \frac{2\left(4\,b^2\sqrt{c}x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}(c^2x^2+3\,bc)\sqrt{x}\right)}{7\,cx}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="fricas")
```

output
```
2/7*(4*b^2*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^
2)*(c^2*x^2 + 3*b*c)*sqrt(x))/(c*x)
```

Sympy [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{7/2}}\,dx = \int \frac{(x^2(b + cx^2))^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**(7/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**(7/2), x)
```

Maxima [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{7/2}}\,dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2), x)
```

Giac [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{7/2}}\,dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx=\int \frac{\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{7/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^(7/2),x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^(7/2), x)
```

Reduce [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx=\frac{6\sqrt{x}\,\sqrt{c\,x^2+b}\,b}{7}+\frac{2\sqrt{x}\,\sqrt{c\,x^2+b}\,c\,x^2}{7}+\frac{4\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^3+bx}dx\right)b^2}{7}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(7/2),x)
```

output
```
(2*(3*sqrt(x)*sqrt(b + c*x**2)*b + sqrt(x)*sqrt(b + c*x**2)*c*x**2 + 2*int
((sqrt(x)*sqrt(b + c*x**2))/(b*x + c*x**3),x)*b**2))/7
```

3.246 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 286

$$\begin{aligned}\int \frac{(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx &= \frac{24b\sqrt{c}x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} \\ &+ \frac{12}{5}c\sqrt{x}\sqrt{bx^2+cx^4}-\frac{2(bx^2+cx^4)^{3/2}}{x^{7/2}} \\ &- \frac{24b^{5/4}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} \\ &+ \frac{12b^{5/4}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}\end{aligned}$$

output
```
24/5*b*c^(1/2)*x^(3/2)*(c*x^2+b)/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)+1
2/5*c*x^(1/2)*(c*x^4+b*x^2)^(1/2)-2*(c*x^4+b*x^2)^(3/2)/x^(7/2)-24/5*b^(5/
4)*c^(1/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*E
llipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/(c*x^4+b*x^2)
^(1/2)+12/5*b^(5/4)*c^(1/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1
/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1
/2))/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.20

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx=-\frac{2b\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{cx^2}{b}\right)}{x^{3/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^(9/2),x]
```

output
```
(-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((c*x^2)/b
)])/(x^(3/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1425, 1426, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx$$

$$\Big\downarrow \; 1425$$

$$6c\int \frac{\sqrt{cx^4+bx^2}}{\sqrt{x}}dx-\frac{2(bx^2+cx^4)^{3/2}}{x^{7/2}}$$

$$\Big\downarrow \; 1426$$

$$6c\left(\frac{2}{5}b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)-\frac{2(bx^2+cx^4)^{3/2}}{x^{7/2}}$$

$$\Big\downarrow \; 1431$$

$$6c\left(\frac{2bx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}$$

$$\Big\downarrow \; 266$$

$$6c\left(\frac{4bx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}$$

$$\Big\downarrow \; 834$$

$$6c\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}$$

$$\Big\downarrow \; 27$$

$$6c\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}$$

$$\Big\downarrow \; 761$$

$$6c\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5\sqrt{bx^2+cx^4}}+\frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}$$

$$\Big\downarrow \; 1510$$

$$6c\left(\frac{4bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{5\sqrt{bx^2+cx^4}}\right.$$

$$\frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^(9/2),x]
```

output
```
(-2*(b*x^2 + c*x^4)^(3/2))/x^(7/2) + 6*c*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/
5 + (4*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt
[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqr
t[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*Sqrt[b*x^2 + c*x^4]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.76

method	result
default	$\frac{2(cx^4+bx^2)^{\frac{3}{2}}\left(12b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\ \mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-6b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\cdots}\right.}{5x^{\frac{7}{2}}(cx^2+b)^2}$
risch	$-\frac{2(-cx^2+5b)\sqrt{x^2(cx^2+b)}}{5x^{\frac{3}{2}}}+\frac{12b\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}\cdots\right.}{5\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(cx^2+b)}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(12*b^2*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b
*c)^(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))-6*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+c^2*x^4-4*b*c*x^2-5*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.21

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx =$$
$$-\frac{2\left(12\,b\sqrt{c}x^2\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}(cx^2-5\,b)\sqrt{x}\right)}{5\,x^2}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/5*(12*b*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b
/c, 0, x)) - sqrt(c*x^4 + b*x^2)*(c*x^2 - 5*b)*sqrt(x))/x^2
```

Sympy [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{9/2}}\,dx = \int \frac{(x^2(b + cx^2))^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**(9/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**(9/2), x)
```

Maxima [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{9/2}}\,dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2), x)
```

Giac [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{9/2}}\,dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{9/2}} \, dx = \int \frac{(c\,x^4 + b\,x^2)^{3/2}}{x^{9/2}} \, dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^(9/2),x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^(9/2), x)
```

Reduce [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{9/2}} \, dx = \frac{\frac{14\sqrt{c\,x^2+b}\,b}{5} + \frac{2\sqrt{c\,x^2+b}\,c\,x^2}{5} + \frac{12\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^4+b\,x^2}dx\right)b^2}{5}}{\sqrt{x}}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(9/2),x)
```

output
```
(2*(7*sqrt(b + c*x**2)*b + sqrt(b + c*x**2)*c*x**2 + 6*sqrt(x)*int((sqrt(x
)*sqrt(b + c*x**2))/(b*x**2 + c*x**4),x)*b**2))/(5*sqrt(x))
```

3.247 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}} \, dx = \frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}} - \frac{2(bx^2+cx^4)^{3/2}}{3x^{9/2}} + \frac{4b^{3/4}c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt{bx^2+cx^4}}$$

output
```
4/3*c*(c*x^4+b*x^2)^(1/2)/x^(1/2)-2/3*(c*x^4+b*x^2)^(3/2)/x^(9/2)+4/3*b^(3
/4)*c^(3/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*
InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/(c*x^4+b*x^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.41

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}} \, dx = -\frac{2b\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{cx^2}{b}\right)}{3x^{5/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^(11/2),x]
```

output
```
(-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((c*x^2)/b
)])/(3*x^(5/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1425, 1426, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{11/2}}dx$$

$$\Big\downarrow \text{1425}$$

$$2c\int \frac{\sqrt{cx^4+bx^2}}{x^{3/2}}dx-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}}$$

$$\Big\downarrow \text{1426}$$

$$2c\left(\frac{2}{3}b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}}$$

$$\Big\downarrow \text{1431}$$

$$2c\left(\frac{2bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}}$$

$$\Big\downarrow \text{266}$$

$$2c\left(\frac{4bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}}$$

$$\Big\downarrow \text{761}$$

$$2c\left(\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^(11/2),x]
```

output
```
(-2*(b*x^2 + c*x^4)^(3/2))/(3*x^(9/2)) + 2*c*((2*Sqrt[b*x^2 + c*x^4])/(3*S
qrt[x]) + (2*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(1/
4)*Sqrt[b*x^2 + c*x^4]))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1426
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 4*p + 1))), x] + Simp[2*b*(p/(d^2
*(m + 4*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.91

method	result
default	$\frac{2(c x^4+b x^2)^{\frac{3}{2}}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,bx+c^2x^4-b^2\right)}{3x^{\frac{9}{2}}(c x^2+b)^2}$
risch	$-\frac{2(-c x^2+b)\sqrt{x^2(c x^2+b)}}{3x^{\frac{5}{2}}}+\frac{4b\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^2(c x^2+}}{3\sqrt{c x^3+bx}\,x^{\frac{3}{2}}(c x^2+b)}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(c*x^4+b*x^2)^(3/2)/x^(9/2)/(c*x^2+b)^2*(2*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(
1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*(-b*c)^(1/2)*b*x+c^2*x^4-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.35

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = \frac{2\left(4\,b\sqrt{c}x^3\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}(cx^2-b)\sqrt{x}\right)}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="fricas")
```

output
```
2/3*(4*b*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^
2)*(c*x^2 - b)*sqrt(x))/x^3
```

Sympy [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{11/2}} \, dx = \int \frac{(x^2(b + cx^2))^{\frac{3}{2}}}{x^{\frac{11}{2}}} \, dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**(11/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**(11/2), x)
```

Maxima [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{11/2}} \, dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{11}{2}}} \, dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2), x)
```

Giac [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{11/2}} \, dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{11}{2}}} \, dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx=\int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^{11/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^(11/2),x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^(11/2), x)
```

Reduce [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx=\frac{-\frac{10\sqrt{c\,x^2+b}\,b}{3}+\frac{2\sqrt{c\,x^2+b}\,c\,x^2}{3}-4\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^5+b\,x^3}dx\right)b^2x}{\sqrt{x}\,x}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(11/2),x)
```

output
```
(2*( - 5*sqrt(b + c*x**2)*b + sqrt(b + c*x**2)*c*x**2 - 6*sqrt(x)*int((sqr
t(x)*sqrt(b + c*x**2))/(b*x**3 + c*x**5),x)*b**2*x))/(3*sqrt(x)*x)
```

3.248 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 287

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = \frac{24c^{3/2}x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}} - \frac{24\sqrt[4]{b}c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} + \frac{12\sqrt[4]{b}c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}$$

output
```
24/5*c^(3/2)*x^(3/2)*(c*x^2+b)/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)-12/
5*c*(c*x^4+b*x^2)^(1/2)/x^(3/2)-2/5*(c*x^4+b*x^2)^(3/2)/x^(11/2)-24/5*b^(1
/4)*c^(5/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*
EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/(c*x^4+b*x^2
)^(1/2)+12/5*b^(1/4)*c^(5/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(
1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(
1/2))/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.20

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = -\frac{2b\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},-\frac{cx^2}{b}\right)}{5x^{7/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^(13/2),x]
```

output
```
(-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((c*x^2)/
b)])/(5*x^(7/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1425, 1425, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{6}{5}c\int \frac{\sqrt{cx^4+bx^2}}{x^{5/2}}dx-\frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{6}{5}c\left(2c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{5x^{11/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{6}{5}c\left(\frac{2cx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}}$$

$\downarrow$ 266

$$\frac{6}{5}c\left(\frac{4cx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}}$$

$\downarrow$ 834

$$\frac{6}{5}c\left(\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}}$$

$\downarrow$ 27

$$\frac{6}{5}c\left(\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}}$$

$\downarrow$ 761

$$\frac{6}{5}c\left(\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}}$$

$\downarrow$ 1510

$$\frac{6}{5}c\left(\frac{4cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{\sqrt{bx^2+cx^4}}\right.$$

$$\frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^(13/2),x]
```

output
```
(-2*(b*x^2 + c*x^4)^(3/2))/(5*x^(11/2)) + (6*c*((-2*Sqrt[b*x^2 + c*x^4])/x
^(3/2) + (4*c*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] +
 Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4
)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*
x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/Sqrt[b*x^2 + c*x^4]))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425

```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1431

```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{2(7cx^2+b)\sqrt{x^2(cx^2+b)}}{5x^{\frac{7}{2}}}+\frac{12c\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\right.}{5\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(cx^2+b)}$
default	$\frac{2(cx^4+bx^2)^{\frac{3}{2}}\left(12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bcx^2-6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\right.}{5x^{\frac{11}{2}}(cx^2+b)^2}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(7*c*x^2+b)/x^(7/2)*(x^2*(c*x^2+b))^(1/2)+12/5*c*(-b*c)^(1/2)*((x+1/c
*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/
2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*E
llipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*
c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.20

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = -\frac{2\left(12\,c^{\frac{3}{2}}x^4\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}(7\,cx^2+b)\sqrt{x}\right)}{5\,x^4}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="fricas")
```

output
```
-2/5*(12*c^(3/2)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c
, 0, x)) + sqrt(c*x^4 + b*x^2)*(7*c*x^2 + b)*sqrt(x))/x^4
```

Sympy [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{13/2}}\,dx = \int \frac{(x^2(b + cx^2))^{\frac{3}{2}}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**(13/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**(13/2), x)
```

Maxima [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{13/2}}\,dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2), x)
```

Giac [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{13/2}}\,dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx=\int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^{13/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^(13/2),x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^(13/2), x)
```

Reduce [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx=\frac{-2\sqrt{c\,x^2+b}\,b+2\sqrt{c\,x^2+b}\,c\,x^2-4\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^6+b\,x^4}dx\right)b^2x^2}{\sqrt{x}\,x^2}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(13/2),x)
```

output
```
(2*( - sqrt(b + c*x**2)*b + sqrt(b + c*x**2)*c*x**2 - 2*sqrt(x)*int((sqrt(
x)*sqrt(b + c*x**2))/(b*x**4 + c*x**6),x)*b**2*x**2))/(sqrt(x)*x**2)
```

3.249 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx = -\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{7x^{13/2}} + \frac{4c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{bx^2+cx^4}}$$

output
```
-4/7*c*(c*x^4+b*x^2)^(1/2)/x^(5/2)-2/7*(c*x^4+b*x^2)^(3/2)/x^(13/2)+4/7*c^
(7/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*Invers
eJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(1/4)/(c*x^4+b*
x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.41

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx = -\frac{2b\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{cx^2}{b}\right)}{7x^{9/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^(15/2),x]
```

output
```
(-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((c*x^2)/
b)])/(7*x^(9/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1425, 1425, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{6}{7}c\int \frac{\sqrt{cx^4+bx^2}}{x^{7/2}}dx-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{6}{7}c\left(\frac{2}{3}c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{6}{7}c\left(\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{7}c\left(\frac{4cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{7}c\left(\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^(15/2),x]
```

output
```
(-2*(b*x^2 + c*x^4)^(3/2))/(7*x^(13/2)) + (6*c*((-2*Sqrt[b*x^2 + c*x^4])/(
3*x^(5/2)) + (2*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
+ Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^
(1/4)*Sqrt[b*x^2 + c*x^4])))/7
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.98

method	result
default	$\frac{2(cx^4+bx^2)^{\frac{3}{2}}\left(2\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)cx^3-3c^2x^4-4bcx^2-b^2\right)}{7x^{\frac{13}{2}}(cx^2+b)^2}$
risch	$-\frac{2(3cx^2+b)\sqrt{x^2(cx^2+b)}}{7x^{\frac{9}{2}}}+\frac{4c\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^2(cx^2+b}}{7\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(cx^2+b)}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*(c*x^4+b*x^2)^(3/2)/x^(13/2)/(c*x^2+b)^2*(2*(-b*c)^(1/2)*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*c*x^3-3*c^2*x^4-4*b*c*x^2-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.34

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx=\frac{2\left(4\,c^{\frac{3}{2}}x^5\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}(3\,cx^2+b)\sqrt{x}\right)}{7\,x^5}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

output
```
2/7*(4*c^(3/2)*x^5*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*x^2)
*(3*c*x^2 + b)*sqrt(x))/x^5
```

Sympy [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{15/2}} \, dx = \int \frac{(x^2(b + cx^2))^{\frac{3}{2}}}{x^{\frac{15}{2}}} \, dx$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**(15/2),x)
```

output
```
Integral((x**2*(b + c*x**2))**(3/2)/x**(15/2), x)
```

Maxima [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{15/2}} \, dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{15}{2}}} \, dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2), x)
```

Giac [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{15/2}} \, dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{15}{2}}} \, dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx=\int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^{15/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^(15/2),x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^(15/2), x)
```

Reduce [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx=\frac{\frac{2\sqrt{c\,x^2+b}\,b}{5}-2\sqrt{c\,x^2+b}\,c\,x^2+\frac{12\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^7+b\,x^5}dx\right)b^2x^3}{5}}{\sqrt{x}\,x^3}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(15/2),x)
```

output
```
(2*(sqrt(b + c*x**2)*b - 5*sqrt(b + c*x**2)*c*x**2 + 6*sqrt(x)*int((sqrt(x
)*sqrt(b + c*x**2))/(b*x**5 + c*x**7),x)*b**2*x**3))/(5*sqrt(x)*x**3)
```

3.250 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 320

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = \frac{8c^{5/2}x^{3/2}(b+cx^2)}{15b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{9x^{15/2}} - \frac{8c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}} + \frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}$$

output
```
8/15*c^(5/2)*x^(3/2)*(c*x^2+b)/b/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)-4
/15*c*(c*x^4+b*x^2)^(1/2)/x^(7/2)-8/15*c^2*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-2
/9*(c*x^4+b*x^2)^(3/2)/x^(15/2)-8/15*c^(9/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2
+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4))),1/2*2^(1/2))/b^(3/4)/(c*x^4+b*x^2)^(1/2)+4/15*c^(9/4)*x*(b^(1/2)+c
^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.18

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = -\frac{2b\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2},-\frac{5}{4},-\frac{cx^2}{b}\right)}{9x^{11/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^(17/2),x]
```

output
```
(-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((c*x^2)/
b)])/(9*x^(11/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1425, 1425, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{2}{3}c\int \frac{\sqrt{cx^4+bx^2}}{x^{9/2}}dx-\frac{2(bx^2+cx^4)^{3/2}}{9x^{15/2}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{2}{3}c\left(\frac{2}{5}c\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{9x^{15/2}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{2}{3}c\left(\frac{2}{5}c\left(\frac{c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{2}{3}c\left(\frac{2}{5}c\left(\frac{cx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{3}c\left(\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{2}{3}c\left(\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}c\left(\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2}{3}c\left(\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)\right.$$

$$\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}$$

$$\Big\downarrow \ 1510$$

$$\frac{2}{3}c\left(\frac{2}{5}c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{b\sqrt{bx^2+cx^4}}\right.\right.$$

$$\frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^(17/2),x]
```

output
```
(-2*(b*x^2 + c*x^4)^(3/2))/(9*x^(15/2)) + (2*c*((-2*Sqrt[b*x^2 + c*x^4])/(
5*x^(7/2)) + (2*c*((-2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) + (2*c*x*Sqrt[b +
c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[
c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[
b + c*x^2])))/(b*Sqrt[b*x^2 + c*x^4])))/5))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.75

method	result
default	$\frac{2(cx^4+bx^2)^{\frac{3}{2}}\left(12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc^2x^4-6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\cdots}\right.}{45x^{\frac{15}{2}}(cx^2+b)^2b}$
risch	$-\frac{2(12c^2x^4+11bcx^2+5b^2)\sqrt{x^2(cx^2+b)}}{45x^{\frac{11}{2}}b}+\frac{4c^2\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-}\cdots}{c}\right)}{\sqrt{-b}\cdots}}\right.}{c}\cdots\right.}{15b\sqrt{cx^3+bx}\,x^{\frac{3}{2}}\cdots}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(17/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*(c*x^4+b*x^2)^(3/2)/x^(15/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c
)^(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(
1/2))*b*c^2*x^4-6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-12*c^3*x^6-23*b*
c^2*x^4-16*b^2*c*x^2-5*b^3)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.22

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{17/2}}\,dx =$$
$$-\frac{2\left(12\,c^{\frac{5}{2}}x^6\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(12\,c^2x^4+11\,bcx^2+5\,b^2\right)\sqrt{cx^4+b}\right.}{45\,bx^6}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="fricas")
```

output
```
-2/45*(12*c^(5/2)*x^6*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/
c, 0, x)) + (12*c^2*x^4 + 11*b*c*x^2 + 5*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))
/(b*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**(17/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{17/2}}\,dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{17}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2), x)
```

Giac [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx=\int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{x^{\frac{17}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx=\int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^{17/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^(17/2),x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^(17/2), x)
```

Reduce [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx=\frac{-\frac{2\sqrt{c\,x^2+b}\,b}{21}-\frac{2\sqrt{c\,x^2+b}\,c\,x^2}{3}+\frac{4\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^8+b\,x^6}dx\right)b^2x^4}{7}}{\sqrt{x}\,x^4}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(17/2),x)
```

output
```
(2*( - sqrt(b + c*x**2)*b - 7*sqrt(b + c*x**2)*c*x**2 + 6*sqrt(x)*int((sqr
t(x)*sqrt(b + c*x**2))/(b*x**6 + c*x**8),x)*b**2*x**4))/(21*sqrt(x)*x**4)
```

3.251 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{19/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 173

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{19/2}}\,dx = -\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{11x^{17/2}} - \frac{4c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{5/4}\sqrt{bx^2+cx^4}}$$

output
```
-12/77*c*(c*x^4+b*x^2)^(1/2)/x^(9/2)-8/77*c^2*(c*x^4+b*x^2)^(1/2)/b/x^(5/2
)-2/11*(c*x^4+b*x^2)^(3/2)/x^(17/2)-4/77*c^(11/4)*x*(b^(1/2)+c^(1/2)*x)*((
c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4)),1/2*2^(1/2))/b^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.34

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{19/2}}\,dx = -\frac{2b\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{3}{2},-\frac{7}{4},-\frac{cx^2}{b}\right)}{11x^{13/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^(19/2),x]
```

output
```
(-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-11/4, -3/2, -7/4, -((c*x^2)
/b)])/(11*x^(13/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1425, 1425, 1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{19/2}}dx$$

$$\Big\downarrow \text{ 1425}$$

$$\frac{6}{11}c\int \frac{\sqrt{cx^4+bx^2}}{x^{11/2}}dx-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}$$

$$\Big\downarrow \text{ 1425}$$

$$\frac{6}{11}c\left(\frac{2}{7}c\int \frac{1}{x^{3/2}\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}$$

$$\Big\downarrow \text{ 1430}$$

$$\frac{6}{11}c\left(\frac{2}{7}c\left(-\frac{c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{6}{11}c\left(\frac{2}{7}c\left(-\frac{cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{6}{11}c\left(\frac{2}{7}c\left(-\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{11}c\left(\frac{2}{7}c\left(-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)-\frac{2\sqrt{bx^2+cx}}{7x^{9/2}}\right.$$

$$\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^(19/2),x]
```

output
```
(-2*(b*x^2 + c*x^4)^(3/2))/(11*x^(17/2)) + (6*c*((-2*Sqrt[b*x^2 + c*x^4])/
(7*x^(9/2)) + (2*c*((-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*x^4])))
/7))/11
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

method	result
default	$-\frac{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}\left(2\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c^{2}x^{5}+4c^{3}x^{6}+17b\,c^{2}x^{4}+20b^{2}c\,x^{2}+\right.}{77x^{\frac{17}{2}}\left(c\,x^{2}+b\right)^{2}b}$
risch	$-\frac{2\left(4c^{2}x^{4}+13bc\,x^{2}+7b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{77x^{\frac{13}{2}}\,b}-\frac{4c^{2}\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{77b\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(19/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*(c*x^4+b*x^2)^(3/2)/x^(17/2)/(c*x^2+b)^2*(2*(-b*c)^(1/2)*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))*c^2*x^5+4*c^3*x^6+17*b*c^2*x^4+20*b^2*c*x^2+7*b^3)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.37

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{19/2}} \, dx = -\frac{2 \left(4 c^{\frac{5}{2}} x^7 \text{weierstrassPInverse}\left(-\frac{4 b}{c}, 0, x\right) + (4 c^2 x^4 + 13 bcx^2 + 7 b^2) \sqrt{cx^4 + bx^2} \sqrt{x}\right)}{77 bx^7}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="fricas")
```

output
```
-2/77*(4*c^(5/2)*x^7*weierstrassPInverse(-4*b/c, 0, x) + (4*c^2*x^4 + 13*b
*c*x^2 + 7*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{19/2}} \, dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**(19/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{19/2}} \, dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{19}{2}}} \, dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2), x)
```

Giac [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{19/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{x^{\frac{19}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(19/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{19/2}}\,dx = \int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^{19/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^(19/2),x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^(19/2), x)
```

Reduce [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{19/2}}\,dx = \frac{-\frac{2\sqrt{c\,x^2+b}\,b}{15}-\frac{2\sqrt{c\,x^2+b}\,c\,x^2}{5}+\frac{4\sqrt{x}\left(\int\frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^9+b\,x^7}dx\right)b^2x^5}{15}}{\sqrt{x}\,x^5}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(19/2),x)
```

output
```
(2*( - sqrt(b + c*x**2)*b - 3*sqrt(b + c*x**2)*c*x**2 + 2*sqrt(x)*int((sqr
t(x)*sqrt(b + c*x**2))/(b*x**7 + c*x**9),x)*b**2*x**5))/(15*sqrt(x)*x**5)
```

3.252 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{21/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 350

$$\begin{aligned}\int \frac{(bx^2+cx^4)^{3/2}}{x^{21/2}}\,dx &= -\frac{8c^{7/2}x^{3/2}(b+cx^2)}{65b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} \\ &\quad -\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{2(bx^2+cx^4)^{3/2}}{13x^{19/2}} \\ &\quad +\frac{8c^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{bx^2+cx^4}} \\ &\quad -\frac{4c^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{65b^{7/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

output
```
-8/65*c^(7/2)*x^(3/2)*(c*x^2+b)/b^2/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2
)-4/39*c*(c*x^4+b*x^2)^(1/2)/x^(11/2)-8/195*c^2*(c*x^4+b*x^2)^(1/2)/b/x^(7
/2)+8/65*c^3*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)-2/13*(c*x^4+b*x^2)^(3/2)/x^(1
9/2)+8/65*c^(13/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)
^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/b^(7/
4)/(c*x^4+b*x^2)^(1/2)-4/65*c^(13/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(
1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
,1/2*2^(1/2))/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.17

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{21/2}}\,dx = -\frac{2b\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},-\frac{3}{2},-\frac{9}{4},-\frac{cx^2}{b}\right)}{13x^{15/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^(21/2),x]
```

output
```
(-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-13/4, -3/2, -9/4, -((c*x^2)
/b)])/(13*x^(15/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {1425, 1425, 1430, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{21/2}}dx$$

$$\Big\downarrow \text{ 1425}$$

$$\frac{6}{13}c\int \frac{\sqrt{cx^4+bx^2}}{x^{13/2}}dx-\frac{2(bx^2+cx^4)^{3/2}}{13x^{19/2}}$$

$$\Big\downarrow \text{ 1425}$$

$$\frac{6}{13}c\left(\frac{2}{9}c\int \frac{1}{x^{5/2}\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{13x^{19/2}}$$

$$\Big\downarrow \text{ 1430}$$

$$\frac{6}{13}c\left(\frac{2}{9}c\left(-\frac{3c\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}$$

$$\Big\downarrow \text{ 1430}$$

$$\frac{6}{13}c\left(\frac{2}{9}c\left(-\frac{3c\left(\frac{c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{6}{13}c\left(\frac{2}{9}c\left(-\frac{3c\left(\frac{cx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{6}{13}c\left(\frac{2}{9}c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{6}{13}c\left(\frac{2}{9}c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}$$

$\downarrow$ 27

$$\frac{6}{13}c\left(\frac{2}{9}c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}$$

$\downarrow$ 761

$$\frac{6}{13}c\left(\frac{2}{9}c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{b}\cdots}{5\cdots}\right.\right.$$

$$\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}$$

$$\downarrow \text{1510}$$

$$\frac{6}{13}c\left(\frac{2}{9}c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\cdots}{\sqrt[4]{\cdots}}\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\cdots\right.}{b\sqrt{bx^2+cx^4}}\cdots\right.}{5b}\cdots\right.\right.$$

$$\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^(21/2),x]
```

output
```
(-2*(b*x^2 + c*x^4)^(3/2))/(13*x^(19/2)) + (6*c*((-2*Sqrt[b*x^2 + c*x^4])/
(9*x^(11/2)) + (2*c*((-2*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) - (3*c*((-2*Sq
rt[b*x^2 + c*x^4])/(b*x^(3/2)) + (2*c*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqr
t[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] +
 Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(
c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(b*Sqrt[b*x
^2 + c*x^4])))/(5*b)))/9))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.71

method	result
default	$-\frac{2(c x^4+b x^2)^{\frac{3}{2}}\left(12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\text{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right) b c^3 x^6-6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\cdots\right.}{195 x^{\frac{19}{2}}(c x^2+b)^2 b^2}$
risch	$-\frac{2(-12c^3x^6+4b c^2x^4+25b^2c x^2+15b^3)\sqrt{x^2(c x^2+b)}}{195x^{\frac{15}{2}} b^2}-\frac{4c^3\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\text{Ellipti}\cdots}{\cdots}\right.}{65b\cdots}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(21/2),x,method=_RETURNVERBOSE)
```

output
```
-2/195*(c*x^4+b*x^2)^(3/2)/x^(19/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b
*c)^(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))*b*c^3*x^6-6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^3*x^6-12*c^4*x^8-8*b
*c^3*x^6+29*b^2*c^2*x^4+40*x^2*b^3*c+15*b^4)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.24

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{21/2}}\,dx = \frac{2\left(12\,c^{\frac{7}{2}}x^8\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(12\,c^3x^6-4\,b\right.}{195\,b^2x^8}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="fricas")
```

output
```
2/195*(12*c^(7/2)*x^8*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/
c, 0, x)) + (12*c^3*x^6 - 4*b*c^2*x^4 - 25*b^2*c*x^2 - 15*b^3)*sqrt(c*x^4
+ b*x^2)*sqrt(x))/(b^2*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{21/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**(21/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{21/2}}\,dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{21}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2), x)
```

Giac [F]

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{21/2}}\,dx = \int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{21}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(21/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{21/2}}\,dx = \int \frac{(c\,x^4 + b\,x^2)^{3/2}}{x^{21/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^(21/2),x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^(21/2), x)
```

Reduce [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{21/2}}\,dx = \frac{-\frac{10\sqrt{c\,x^2+b}\,b}{77}-\frac{2\sqrt{c\,x^2+b}\,c\,x^2}{7}+\frac{12\sqrt{x}\,\left(\int\frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^{10}+b\,x^8}dx\right)b^2x^6}{77}}{\sqrt{x}\,x^6}$$

input

```
int((c*x^4+b*x^2)^(3/2)/x^(21/2),x)
```

output

```
(2*( - 5*sqrt(b + c*x**2)*b - 11*sqrt(b + c*x**2)*c*x**2 + 6*sqrt(x)*int((
sqrt(x)*sqrt(b + c*x**2))/(b*x**8 + c*x**10),x)*b**2*x**6))/(77*sqrt(x)*x*
*6)
```

3.253 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 203

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx = -\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}} + \frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}} + \frac{4c^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}$$

output
```
-4/55*c*(c*x^4+b*x^2)^(1/2)/x^(13/2)-8/385*c^2*(c*x^4+b*x^2)^(1/2)/b/x^(9/
2)+8/231*c^3*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)-2/15*(c*x^4+b*x^2)^(3/2)/x^(2
1/2)+4/231*c^(15/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2
)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(
9/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.29

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx = -\frac{2b\sqrt{x^2(b+cx^2)}\,\text{Hypergeometric2F1}\left(-\frac{15}{4},-\frac{3}{2},-\frac{11}{4},-\frac{cx^2}{b}\right)}{15x^{17/2}\sqrt{1+\frac{cx^2}{b}}}$$

input
```
Integrate[(b*x^2 + c*x^4)^(3/2)/x^(23/2),x]
```

output
```
(-2*b*Sqrt[x^2*(b + c*x^2)]*Hypergeometric2F1[-15/4, -3/2, -11/4, -((c*x^2
)/b)])/(15*x^(17/2)*Sqrt[1 + (c*x^2)/b])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1425, 1425, 1430, 1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}dx$$

$$\Big\downarrow \text{1425}$$

$$\frac{2}{5}c\int \frac{\sqrt{cx^4+bx^2}}{x^{15/2}}dx-\frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}}$$

$$\Big\downarrow \text{1425}$$

$$\frac{2}{5}c\left(\frac{2}{11}c\int \frac{1}{x^{7/2}\sqrt{cx^4+bx^2}}dx-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}\right)-\frac{2(bx^2+cx^4)^{3/2}}{15x^{21/2}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{2}{5}c\left(\frac{2}{11}c\left(-\frac{5c\int \frac{1}{x^{3/2}\sqrt{cx^4+bx^2}}dx}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}$$

$\Big\downarrow$ 1430

$$\frac{2}{5}c\left(\frac{2}{11}c\left(-\frac{5c\left(-\frac{c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}$$

$\Big\downarrow$ 1431

$$\frac{2}{5}c\left(\frac{2}{11}c\left(-\frac{5c\left(-\frac{cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}$$

$\Big\downarrow$ 266

$$\frac{2}{5}c\left(\frac{2}{11}c\left(-\frac{5c\left(-\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}\right)-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}$$

$\Big\downarrow$ 761

$$\frac{2}{5}c\left(\frac{2}{11}c\left(-\frac{5c\left(-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)-\frac{2\sqrt{b}}{11}\right.$$

$$\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}$$

input
```
Int[(b*x^2 + c*x^4)^(3/2)/x^(23/2),x]
```

output
```
(-2*(b*x^2 + c*x^4)^(3/2))/(15*x^(21/2)) + (2*c*((-2*Sqrt[b*x^2 + c*x^4])/
(11*x^(13/2)) + (2*c*((-2*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) - (5*c*((-2*S
qrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*x^4])))/(7*b)))/11))/5
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1425
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(d*x)^(m + 1)*((b*x^2 + c*x^4)^p/(d*(m + 2*p + 1))), x] - Simp[2*c*(p/(d^4
*(m + 2*p + 1)))   Int[(d*x)^(m + 4)*(b*x^2 + c*x^4)^(p - 1), x], x] /; Fre
eQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[p, 0] && LtQ[m + 2*p + 1, 0]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.82

method	result
default	$\frac{2(c\,x^4+b\,x^2)^{\frac{3}{2}}\left(10\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c^3x^7+20c^4x^8+8b\,c^3x^6-131b^2c^2x^4-\right.}{1155x^{\frac{21}{2}}(c\,x^2+b)^2b^2}$
risch	$-\frac{2(-20c^3x^6+12b\,c^2x^4+119b^2c\,x^2+77b^3)\sqrt{x^2(c\,x^2+b)}}{1155x^{\frac{17}{2}}b^2}+\frac{4c^3\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\right.}{231b^2\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}(c\,x^2+b}$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(23/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*(c*x^4+b*x^2)^(3/2)/x^(21/2)/(c*x^2+b)^2*(10*(-b*c)^(1/2)*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2),1/2*2^(1/2))*c^3*x^7+20*c^4*x^8+8*b*c^3*x^6-131*b^2*c^2*x^4-196*x^
2*b^3*c-77*b^4)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.37

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx = \frac{2\left(20\,c^{\frac{7}{2}}x^9\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(20\,c^3x^6-12\,bc^2x^4-119\,b^2cx^2-77\,b^3\right.\right.}{1155\,b^2x^9}$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="fricas")
```

output
```
2/1155*(20*c^(7/2)*x^9*weierstrassPInverse(-4*b/c, 0, x) + (20*c^3*x^6 - 1
2*b*c^2*x^4 - 119*b^2*c*x^2 - 77*b^3)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^2*x^
9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2)**(3/2)/x**(23/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{x^{\frac{23}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2), x)
```

Giac [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{x^{\frac{23}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2)^(3/2)/x^(23/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx=\int \frac{(c\,x^4+b\,x^2)^{3/2}}{x^{23/2}}\,dx$$

input
```
int((b*x^2 + c*x^4)^(3/2)/x^(23/2),x)
```

output
```
int((b*x^2 + c*x^4)^(3/2)/x^(23/2), x)
```

Reduce [F]

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx=\frac{-\frac{14\sqrt{c\,x^2+b}\,b}{117}-\frac{2\sqrt{c\,x^2+b}\,c\,x^2}{9}+\frac{4\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^{11}+b\,x^9}dx\right)b^2x^7}{39}}{\sqrt{x}\,x^7}$$

input
```
int((c*x^4+b*x^2)^(3/2)/x^(23/2),x)
```

output
```
(2*( - 7*sqrt(b + c*x**2)*b - 13*sqrt(b + c*x**2)*c*x**2 + 6*sqrt(x)*int((
sqrt(x)*sqrt(b + c*x**2))/(b*x**9 + c*x**11),x)*b**2*x**7))/(117*sqrt(x)*x
**7)
```

3.254 $\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}} - \frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{15b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77c^{13/4}\sqrt{bx^2+cx^4}}$$

output
```
30/77*b^2*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-18/77*b*x^(3/2)*(c*x^4+b*x^2)^(1
/2)/c^2+2/11*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c-15/77*b^(11/4)*x*(b^(1/2)+c^(1/
2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(
1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.54

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{3/2}\left(15b^3+6b^2cx^2-2bc^2x^4+7c^3x^6-15b^3\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\right.\right.}{77c^3\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x^(13/2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(3/2)*(15*b^3 + 6*b^2*c*x^2 - 2*b*c^2*x^4 + 7*c^3*x^6 - 15*b^3*Sqrt[1
 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(77*c^3*Sqr
t[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1429, 1429, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\int \frac{x^{9/2}}{\sqrt{cx^4+bx^2}}dx}{11c}$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}\right)}{11c}$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}\right)}{11c}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}$$

$$\Big\downarrow \; 266$$

$$\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}$$

$$\Big\downarrow \; 761$$

$$\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}-\frac{9b\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{11c}$$

input
```
Int[x^(13/2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(11*c) - (9*b*((2*x^(3/2)*Sqrt[b*x^2 + c*x
^4])/(7*c) - (5*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqr
t[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])))/(
7*c)))/(11*c)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.83

method	result
default	$-\frac{\sqrt{x}\left(-14c^4x^7+15b^3\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+4b c^3x^5-12b^2c^2x^3-30b^3cx\right)}{77\sqrt{c x^4+b x^2}\,c^4}$
risch	$\frac{2(7c^2x^4-9bc x^2+15b^2)x^{\frac{3}{2}}(c x^2+b)}{77c^3\sqrt{x^2(c x^2+b)}}-\frac{15b^3\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\cdots}}{77c^4\sqrt{c x^3+bx}\sqrt{x^2(c x^2+b)}}$

input
```
int(x^(13/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/77/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-14*c^4*x^7+15*b^3*(-b*c)^(1/2)*((c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2),1/2*2^(1/2))+4*b*c^3*x^5-12*b^2*c^2*x^3-30*b^3*c*x)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.39

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(15\,b^3\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\left(7\,c^3x^4-9\,bc^2x^2+15\,b^2c\right)\sqrt{cx^4+bx^2}\sqrt{x}\right)}{77\,c^4x}$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/77*(15*b^3*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - (7*c^3*x^4 - 9
*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x)
```

Sympy [F]

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(x**(13/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(13/2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{13/2}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(x^(13/2)/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(13/2)/(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{30\sqrt{x}\,\sqrt{c\,x^2+b}\,b^2-18\sqrt{x}\,\sqrt{c\,x^2+b}\,bc\,x^2+14\sqrt{x}\,\sqrt{c\,x^2+b}\,c^2x^4-15\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^3+bx}dx\right.}{77c^3}$$

input
```
int(x^(13/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(30*sqrt(x)*sqrt(b + c*x**2)*b**2 - 18*sqrt(x)*sqrt(b + c*x**2)*b*c*x**2 +
 14*sqrt(x)*sqrt(b + c*x**2)*c**2*x**4 - 15*int((sqrt(x)*sqrt(b + c*x**2))
/(b*x + c*x**3),x)*b**3)/(77*c**3)
```

3.255 $\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 296

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{14b^2x^{3/2}(b+cx^2)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}} + \frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}$$

output

```
14/15*b^2*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2
)-14/45*b*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2+2/9*x^(5/2)*(c*x^4+b*x^2)^(1/2)/
c-14/15*b^(9/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1
/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/c^(11/4)
/(c*x^4+b*x^2)^(1/2)+7/15*b^(9/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2
)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/
2*2^(1/2))/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.29

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{5/2}\left(-7b^2-2bcx^2+5c^2x^4+7b^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{45c^2\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^(11/2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(5/2)*(-7*b^2 - 2*b*c*x^2 + 5*c^2*x^4 + 7*b^2*Sqrt[1 + (c*x^2)/b]*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(45*c^2*Sqrt[x^2*(b + c*x^2)
])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1429, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1429}$$

$$\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\int \frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx}{9c}$$

$$\Big\downarrow \text{1429}$$

$$\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{9c}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)}{9c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)}{9c}$$

$$\Big\downarrow \text{834}$$

$$\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}$$

$$\Big\downarrow \text{761}$$

$$\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{9c}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}-\frac{7b\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{5c\sqrt{bx^2+cx^4}}\right.}{9c}$$

input
```
Int[x^(11/2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*c) - (7*b*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^
4])/(5*c) - (6*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b
] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(
1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
 c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1
/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*c*Sqrt[b*x^2 + c*x^4])))/(9*c
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.73

method	result
default	$\frac{\sqrt{x}\left(10c^3x^6+42b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-21b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{c}{\cdots}}\cdots\right.}{45\sqrt{cx^4+bx^2}\,c^3}$
risch	$-\frac{2x^{\frac{5}{2}}\left(-5cx^2+7b\right)\left(cx^2+b\right)}{45c^2\sqrt{x^2\left(cx^2+b\right)}}+\frac{7b^2\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}\cdots\right.}{15c^3\sqrt{cx^3+bx}\,\sqrt{x^2\left(cx^2+b\right)}}$

input
```
int(x^(11/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/45/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^3*(10*c^3*x^6+42*b^3*((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c
/(-b*c)^(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1
/2*2^(1/2))-21*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*b*c^2*x^4-14*b^2*c*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.21

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(21\,b^2\sqrt{c}\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}\left(5\,c^2x^2-7\,bc\right)\sqrt{x}\right)}{45\,c^3}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(21*b^2*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/
c, 0, x)) - sqrt(c*x^4 + b*x^2)*(5*c^2*x^2 - 7*b*c)*sqrt(x))/c^3
```

Sympy [F]

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{11}{2}}}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(11/2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{11}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{11}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^{11/2}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(x^(11/2)/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(11/2)/(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx=\frac{-14\sqrt{x}\,\sqrt{c\,x^2+b}\,bx+10\sqrt{x}\,\sqrt{c\,x^2+b}\,c\,x^3+21\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^2+b}dx\right)b^2}{45c^2}$$

input
```
int(x^(11/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - 14*sqrt(x)*sqrt(b + c*x**2)*b*x + 10*sqrt(x)*sqrt(b + c*x**2)*c*x**3 +
 21*int((sqrt(x)*sqrt(b + c*x**2))/(b + c*x**2),x)*b**2)/(45*c**2)
```

3.256 $\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx = -\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}+\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}$$

output
```
-10/21*b*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+2/7*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c
+5/21*b^(7/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2
)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/c^(9/4)/(
c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.58

$$\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{3/2}\left(-5b^2-2bcx^2+3c^2x^4+5b^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{21c^2\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x^(9/2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(3/2)*(-5*b^2 - 2*b*c*x^2 + 3*c^2*x^4 + 5*b^2*Sqrt[1 + (c*x^2)/b]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(21*c^2*Sqrt[x^2*(b + c*x^2)
])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1429, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{7c}$$

input
```
Int[x^(9/2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c) - (5*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c
*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/
4)*Sqrt[b*x^2 + c*x^4])))/(7*c)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.92

method	result
default	$\frac{\sqrt{x}\left(5b^{2}\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+6c^{3}x^{5}-4b\,c^{2}x^{3}-10b^{2}cx\right)}{21\sqrt{c\,x^{4}+b\,x^{2}}\,c^{3}}$
risch	$-\frac{2(-3c\,x^{2}+5b)x^{\frac{3}{2}}(c\,x^{2}+b)}{21c^{2}\sqrt{x^{2}(c\,x^{2}+b)}}+\frac{5b^{2}\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x}\sqrt{x(c}}{21c^{3}\sqrt{c\,x^{3}+bx}\sqrt{x^{2}(c\,x^{2}+b)}}$

input
```
int(x^(9/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/21/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(5*b^2*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-
b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2))+6*c^3*x^5-4*b*c^2*x^3-10*b^2*c*x)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.38

$$\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(5\,b^2\sqrt{c}x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}(3\,c^2x^2-5\,bc)\sqrt{x}\right)}{21\,c^3x}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/21*(5*b^2*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x
^2)*(3*c^2*x^2 - 5*b*c)*sqrt(x))/(c^3*x)
```

Sympy [F]

$$\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^{\frac{9}{2}}}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(9/2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^{\frac{9}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^{\frac{9}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{9/2}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(x^(9/2)/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(9/2)/(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{-10\sqrt{x}\,\sqrt{c\,x^2+b}\,b+6\sqrt{x}\,\sqrt{c\,x^2+b}\,c\,x^2+5\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^3+bx}dx\right)b^2}{21c^2}$$

input
```
int(x^(9/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - 10*sqrt(x)*sqrt(b + c*x**2)*b + 6*sqrt(x)*sqrt(b + c*x**2)*c*x**2 + 5*
int((sqrt(x)*sqrt(b + c*x**2))/(b*x + c*x**3),x)*b**2)/(21*c**2)
```

3.257 $\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 266

$$\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx = -\frac{6bx^{3/2}(b+cx^2)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c} + \frac{6b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}} - \frac{3b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}$$

output
```
-6/5*b*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)+2
/5*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+6/5*b^(5/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2
+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4))),1/2*2^(1/2))/c^(7/4)/(c*x^4+b*x^2)^(1/2)-3/5*b^(5/4)*x*(b^(1/2)+c^
(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.26

$$\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx=\frac{2x^{5/2}\left(b+cx^2-b\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{5c\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x^(7/2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(5/2)*(b + c*x^2 - b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4,
7/4, -((c*x^2)/b)]))/(5*c*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1429}$$

$$\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \textbf{834}$$

$$\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \textbf{761}$$

$$\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \textbf{1510}$$

$$\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right.}{5c\sqrt{bx^2+cx^4}}$$

input
```
Int[x^(7/2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) - (6*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt
[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*S
qrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sq
rt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*c
*Sqrt[b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.77

method	result
default	$-\frac{\sqrt{x}\left(6b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\text{Ellip}\ldots}{5\sqrt{c\,x^4+b\,x^2}\,c^2}$
risch	$\frac{2x^{\frac{5}{2}}\left(c\,x^2+b\right)}{5c\sqrt{x^2\left(c\,x^2+b\right)}}-\frac{3b\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,\text{EllipticF}\ldots}{\ldots}\ldots\right.}{5c^2\sqrt{c\,x^3+bx}\,\sqrt{x^2\left(c\,x^2+b\right)}}$

input
```
int(x^(7/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^2*(6*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2
)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-
3*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-2*c^2*x^4-2*b*c*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.18

$$\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(3\,b\sqrt{c}\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}c\sqrt{x}\right)}{5\,c^2}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(3*b*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0,
 x)) + sqrt(c*x^4 + b*x^2)*c*sqrt(x))/c^2
```

Sympy [F]

$$\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(7/2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{7/2}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(x^(7/2)/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(7/2)/(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\sqrt{x}\,\sqrt{c\,x^2+b}\,x-3\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^2+b}dx\right)b}{5c}$$

input
```
int(x^(7/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(b + c*x**2)*x - 3*int((sqrt(x)*sqrt(b + c*x**2))/(b + c*x*
*2),x)*b)/(5*c)
```

3.258 $\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx=\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}$$

output
```
2/3*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-1/3*b^(3/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^
2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)
/b^(1/4)),1/2*2^(1/2))/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx=\frac{2x^{3/2}\left(b+cx^2-b\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{3c\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x^(5/2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(3/2)*(b + c*x^2 - b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2,
5/4, -((c*x^2)/b)]))/(3*c*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1429}$$

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}$$

$$\Big\downarrow \text{1431}$$

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}$$

input
```
Int[x^(5/2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{\sqrt{x}\left(b\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\text{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-2c^2x^3-2bcx\right)}{3\sqrt{c\,x^4+b\,x^2}\,c^2}$	123
risch	$\frac{2x^{\frac{3}{2}}\left(c\,x^2+b\right)}{3c\sqrt{x^2\left(c\,x^2+b\right)}}-\frac{b\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x}\,\sqrt{x\left(c\,x^2+b\right)}}{3c^2\sqrt{c\,x^3+bx}\,\sqrt{x^2\left(c\,x^2+b\right)}}$	166

input
```
int(x^(5/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(b*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)
^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))-2*c^2*x^3-2*b*c*x)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.36

$$\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(b\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}c\sqrt{x}\right)}{3\,c^2x}$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(b*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*x^2)*
c*sqrt(x))/(c^2*x)
```

Sympy [F]

$$\int \frac{x^{5/2}}{\sqrt{bx^2 + cx^4}} \, dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{x^2 (b + cx^2)}} \, dx$$

input
```
integrate(x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(5/2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^{5/2}}{\sqrt{bx^2 + cx^4}} \, dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{cx^4 + bx^2}} \, dx$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{x^{5/2}}{\sqrt{bx^2 + cx^4}} \, dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{cx^4 + bx^2}} \, dx$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^{5/2}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(x^(5/2)/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(5/2)/(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx=\frac{2\sqrt{x}\,\sqrt{c\,x^2+b}-\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^3+bx}dx\right)b}{3c}$$

input
```
int(x^(5/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(b + c*x**2) - int((sqrt(x)*sqrt(b + c*x**2))/(b*x + c*x**3
),x)*b)/(3*c)
```

3.259 $\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 231

$$\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{3/2}(b+cx^2)}{\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}} + \frac{\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}$$

output
```
2*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)-2*b^(1
/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*Elliptic
E(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+b*x^2
)^(1/2)+b^(1/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1
/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/c^(3/4)
/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.25

$$\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx=\frac{2x^{5/2}\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)}{3\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x^(3/2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(5/2)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/
b)])/(3*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{x\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2x\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{2x\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 27

$$\frac{2x\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 761

$$\frac{2x\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 1510

$$\frac{2x\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}-}{\sqrt{c}}\right.}{\sqrt{bx^2+cx^4}}$$

input
```
Int[x^(3/2)/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)
) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^
2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b +
c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2
*c^(3/4)*Sqrt[b + c*x^2])))/Sqrt[b*x^2 + c*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{\sqrt{x}\, b \sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}\, \sqrt{2}\, \sqrt{\frac{-c x+\sqrt{-b c}}{\sqrt{-b c}}}\, \sqrt{-\frac{c x}{\sqrt{-b c}}}\, \left(2 \,\mathrm{EllipticE}\left(\sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}, \frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}, \frac{\sqrt{2}}{2}\right)\right)}{\sqrt{c\, x^{4}+b\, x^{2}}\, c}$	131

input
```
int(x^(3/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*b/c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)
*(2*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-Ellipti
cF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.10

$$\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c}, 0, \mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c}, 0, x\right)\right)}{\sqrt{c}}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x))/sqrt(c)
```

Sympy [F]

$$\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**(3/2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{bx^2 + cx^4}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{x^{3/2}}{\sqrt{bx^2 + cx^4}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{bx^2 + cx^4}}\,dx = \int \frac{x^{3/2}}{\sqrt{c\,x^4 + b\,x^2}}\,dx$$

input
```
int(x^(3/2)/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(3/2)/(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^2+b}dx$$

input
```
int(x^(3/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b + c*x**2),x)
```

3.260 $\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx=\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

output
```
x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacob
iAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(1/4)/c^(1/4)/(c*x^4+
b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx=\frac{2x^{3/2}\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^2}{b}\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[Sqrt[x]/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(2*x^(3/2)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/
b)])/Sqrt[x^2*(b + c*x^2)]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1431}$$

$$\frac{x\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2x\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

input
```
Int[Sqrt[x]/Sqrt[b*x^2 + c*x^4],x]
```

output
```
(x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(1/4)*c^(1/4)*Sqrt[b*x^2
+ c*x^4])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\sqrt{x}\,\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}, \frac{\sqrt{2}}{2}\right)}{\sqrt{c\,x^{4}+b\,x^{2}}\,c}$	106

input
```
int(x^(1/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)
*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))/c
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)}{\sqrt{c}}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(-4*b/c, 0, x)/sqrt(c)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(sqrt(x)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{\sqrt{x}}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(x^(1/2)/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^(1/2)/(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^3+bx}dx$$

input
```
int(x^(1/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b*x + c*x**3),x)
```

3.261 $\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 259

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx
&= \frac{2\sqrt{c}x^{3/2}(b+cx^2)}{b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}} \\
&\quad - \frac{2\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}} \\
&\quad + \frac{\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

output
```
2*c^(1/2)*x^(3/2)*(c*x^2+b)/b/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)-2*(c
*x^4+b*x^2)^(1/2)/b/x^(3/2)-2*c^(1/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^
(1/2)+c^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
,1/2*2^(1/2))/b^(3/4)/(c*x^4+b*x^2)^(1/2)+c^(1/4)*x*(b^(1/2)+c^(1/2)*x)*((
c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4)),1/2*2^(1/2))/b^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx=-\frac{2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{cx^2}{b}\right)}{\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[x]*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((c*x^2
)/b)])/Sqrt[x^2*(b + c*x^2)]
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1430}$$

$$\frac{c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{cx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2cx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

$$\Big\downarrow \ 834$$

$$\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

$$\Big\downarrow \ 761$$

$$\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

$$\Big\downarrow \ 1510$$

$$\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) + (2*c*x*Sqrt[b + c*x^2]*(-((-((Sqrt[
x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sq
rt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqr
t[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(b*Sq
rt[b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.75

method	result
default	$\frac{\sqrt{x}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b-\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\right.\right.}{\sqrt{c\,x^{4}+b\,x^{2}}\,b}$
risch	$-\frac{2(c\,x^{2}+b)\sqrt{x}}{b\sqrt{x^{2}(c\,x^{2}+b)}}+\frac{\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,\operatorname{EllipticF}}{}\right.}{b\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}(c\,x^{2}+b)}}$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*
EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^
(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2),1/2*2^(1/2))*b-2*c*x^2-2*b)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.19

$$\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(\sqrt{c}x^2\text{weierstrassZeta}\left(-\frac{4b}{c},0,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}\sqrt{x}\right)}{bx^2}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0,
x)) + sqrt(c*x^4 + b*x^2)*sqrt(x))/(b*x^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{x^2(b+cx^2)}}\,dx$$

input
```
integrate(1/x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(1/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = \int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^4+b\,x^2}dx$$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b*x**2 + c*x**4),x)
```

3.262 $\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/3*(c*x^4+b*x^2)^(1/2)/b/x^(5/2)-1/3*c^(3/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x
^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)),1/2*2^(1/2))/b^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{cx^2}{b}\right)}{3\sqrt{x}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[1/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((c*x^2)/b)])/(
3*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1430}$$

$$-\frac{c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

$$\Big\downarrow \text{ 1431}$$

$$-\frac{cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

input
```
Int[1/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

method	result
default	$-\frac{\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)x+2c\,x^{2}+2b}{3\sqrt{c\,x^{4}+b\,x^{2}}\,\sqrt{x}\,b}$
risch	$-\frac{2\left(c\,x^{2}+b\right)}{3b\sqrt{x}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x}\,\sqrt{x\left(c\,x^{2}+b\right)}}{3b\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$

input
```
int(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(c*x^4+b*x^2)^(1/2)/x^(1/2)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(
1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*x+2*c*x^2+2*b)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx=-\frac{2\left(\sqrt{c}x^3\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}\sqrt{x}\right)}{3\,bx^3}$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^2)*
sqrt(x))/(b*x^3)
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{bx^2 + cx^4}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2(b + cx^2)}}\,dx$$

input
```
integrate(1/x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(3/2)*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\sqrt{bx^2 + cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4 + bx^2}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{3/2}\sqrt{bx^2 + cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4 + bx^2}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{1}{x^{3/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(1/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^5+b\,x^3}dx$$

input
```
int(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b*x**3 + c*x**5),x)
```

3.263 $\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 296

$$\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{6c^{3/2}x^{3/2}(b+cx^2)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{6c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}} - \frac{3c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}$$

output
```
-6/5*c^(3/2)*x^(3/2)*(c*x^2+b)/b^2/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)
-2/5*(c*x^4+b*x^2)^(1/2)/b/x^(7/2)+6/5*c*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)+6
/5*c^(5/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*E
llipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/b^(7/4)/(c*x^
4+b*x^2)^(1/2)-3/5*c^(5/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/
2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/
2))/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},-\frac{cx^2}{b}\right)}{5x^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[1/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((c*x^2)/b)])/
(5*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1430, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1430}$$

$$-\frac{3c\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{ 1430}$$

$$-\frac{3c\left(\frac{c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{ 1431}$$

$$-\frac{3c\left(\frac{cx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{ 834}$$

$$-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

$$\Big\downarrow \text{ 1510}$$

$$-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}-\frac{\sqrt{x}}{\sqrt{}}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}\right)}{5b}$$

$$\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

input

```
Int[1/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output

```
(-2*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) - (3*c*((-2*Sqrt[b*x^2 + c*x^4])/(b
*x^(3/2)) + (2*c*x*Sqrt[b + c*x^2]*(-((-(-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b
] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(
1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
 c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1
/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(b*Sqrt[b*x^2 + c*x^4])))/(5*b)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.73

method	result
default	$-\frac{6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^{2}-3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}}{5\sqrt{c\,x^{4}+b\,x^{2}}\,x^{\frac{3}{2}}\,b^{2}}$
risch	$-\frac{2(c\,x^{2}+b)(-3c\,x^{2}+b)}{5b^{2}x^{\frac{3}{2}}\sqrt{x^{2}(c\,x^{2}+b)}}-\frac{3c\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\right.}{5b^{2}\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}(c\,x^{2}+b)}}$

input
```
int(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/(c*x^4+b*x^2)^(1/2)/x^(3/2)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/
2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-
3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-6*c^2*x^4-4*b*c*x^2+2*b^2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx=\frac{2\left(3\,c^{\frac{3}{2}}x^{4}\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^{4}+bx^{2}}(3\right.}{5\,b^{2}x^{4}}$$

input
```
integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(3*c^(3/2)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c,
0, x)) + sqrt(c*x^4 + b*x^2)*(3*c*x^2 - b)*sqrt(x))/(b^2*x^4)
```

Sympy [F]

$$\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(1/x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(5/2)*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^{5/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(1/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^6+b\,x^4}dx$$

input
```
int(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b*x**4 + c*x**6),x)
```

3.264 $\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} + \frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/7*(c*x^4+b*x^2)^(1/2)/b/x^(9/2)+10/21*c*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)
+5/21*c^(7/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2
)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(9/4)/(
c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},-\frac{cx^2}{b}\right)}{7x^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[1/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((c*x^2)/b)])/
(7*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1430, 1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{ 1430}$$

$$-\frac{5c\int \frac{1}{x^{3/2}\sqrt{cx^4+bx^2}}dx}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

$$\Big\downarrow \text{ 1430}$$

$$-\frac{5c\left(-\frac{c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

$$\Big\downarrow \text{ 1431}$$

$$-\frac{5c\left(-\frac{cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{5c\left(-\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{5c\left(-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

input
```
Int[1/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) - (5*c*((-2*Sqrt[b*x^2 + c*x^4])/(3
*b*x^(5/2)) - (c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
 Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(
5/4)*Sqrt[b*x^2 + c*x^4])))/(7*b)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.90

method	result
default	$\frac{5\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c\,x^{3}+10c^{2}x^{4}+4bc\,x^{2}-6b^{2}}{21\sqrt{c\,x^{4}+b\,x^{2}}\,x^{\frac{5}{2}}\,b^{2}}$
risch	$-\frac{2(c\,x^{2}+b)\,(-5c\,x^{2}+3b)}{21b^{2}x^{\frac{5}{2}}\,\sqrt{x^{2}(c\,x^{2}+b)}}+\frac{5c\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x}\,\sqrt{x(c\,x^{2}+\cdots}}{21b^{2}\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}(c\,x^{2}+b)}}$

input
```
int(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/21/(c*x^4+b*x^2)^(1/2)/x^(5/2)*(5*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)
^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*c*x^3+10*c^2*x^4+4*b*c*x^2-6*b^2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(5\,c^{\frac{3}{2}}x^{5}\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}(5\,cx^2-3\,b)\sqrt{x}\right)}{21\,b^2x^5}$$

input
```
integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/21*(5*c^(3/2)*x^5*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*x^2
)*(5*c*x^2 - 3*b)*sqrt(x))/(b^2*x^5)
```

Sympy [F]

$$\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(1/x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(7/2)*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^{7/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(1/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^7+b\,x^5}dx$$

input
```
int(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b*x**5 + c*x**7),x)
```

3.265 $\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 326

$$\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = \frac{14c^{5/2}x^{3/2}(b+cx^2)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{14c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}} + \frac{7c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}$$

output
```
14/15*c^(5/2)*x^(3/2)*(c*x^2+b)/b^3/(b^(1/2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2
)-2/9*(c*x^4+b*x^2)^(1/2)/b/x^(11/2)+14/45*c*(c*x^4+b*x^2)^(1/2)/b^2/x^(7/
2)-14/15*c^2*(c*x^4+b*x^2)^(1/2)/b^3/x^(3/2)-14/15*c^(9/4)*x*(b^(1/2)+c^(1
/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1
/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/b^(11/4)/(c*x^4+b*x^2)^(1/2)+7/15*c^(9/
4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJa
cobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(11/4)/(c*x^4+b*x^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{1}{2},-\frac{5}{4},-\frac{cx^2}{b}\right)}{9x^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[1/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-9/4, 1/2, -5/4, -((c*x^2)/b)])/
(9*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1430, 1430, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \ 1430$$

$$-\frac{7c\int \frac{1}{x^{5/2}\sqrt{cx^4+bx^2}}dx}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$$\Big\downarrow \ 1430$$

$$-\frac{7c\left(-\frac{3c\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$$\Big\downarrow \ 1430$$

$$-\frac{7c\left(-\frac{3c\left(\frac{c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$\Big\downarrow$ 1431

$$-\frac{7c\left(-\frac{3c\left(\frac{cx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$\Big\downarrow$ 266

$$-\frac{7c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$\Big\downarrow$ 834

$$-\frac{7c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$\Big\downarrow$ 27

$$-\frac{7c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$\Big\downarrow$ 761

$$-\frac{7c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

$\Big\downarrow$ 1510

$$-\frac{7c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}}{b\sqrt{bx^2+cx^4}}}{5b}}{9b}$$

$$\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

input
```
Int[1/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) - (7*c*((-2*Sqrt[b*x^2 + c*x^4])/(
5*b*x^(7/2)) - (3*c*((-2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) + (2*c*x*Sqrt[b
+ c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE
[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqr
t[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqr
t[b + c*x^2])))/(b*Sqrt[b*x^2 + c*x^4])))/(5*b)))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.71

method	result
default	$\frac{42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^{2}x^{4}-21\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}}{45\sqrt{c\,x^{4}+b\,x^{2}}\,x^{\frac{7}{2}}\,b^{3}}$
risch	$-\frac{2(c\,x^{2}+b)\left(21c^{2}x^{4}-7bc\,x^{2}+5b^{2}\right)}{45b^{3}x^{\frac{7}{2}}\sqrt{x^{2}(c\,x^{2}+b)}}+\frac{7c^{2}\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\right.}{c}\right.}{15b^{3}\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}(c\,x^{2}+b)}}$

input
```
int(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/45/(c*x^4+b*x^2)^(1/2)/x^(7/2)*(42*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1
/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x
^4-21*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-42*c^3*x^6-28*b*c^2*x^4+4*b^
2*c*x^2-10*b^3)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(21\,c^{\frac{5}{2}}x^{6}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(21\,c^{2}x^{4}-7\,bcx^{2}+5\,b^{2}\right)\sqrt{cx^{4}+bx}}{45\,b^{3}x^{6}}$$

input
```
integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(21*c^(5/2)*x^6*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/
c, 0, x)) + (21*c^2*x^4 - 7*b*c*x^2 + 5*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/
(b^3*x^6)
```

Sympy [F]

$$\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{1}{x^{\frac{9}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(1/x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(9/2)*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^{9/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(1/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^8+b\,x^6}dx$$

input
```
int(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b*x**6 + c*x**8),x)
```

3.266 $\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}+\frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}-\frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}} - \frac{15c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{13/4}\sqrt{bx^2+cx^4}}$$

output
```
-2/11*(c*x^4+b*x^2)^(1/2)/b/x^(13/2)+18/77*c*(c*x^4+b*x^2)^(1/2)/b^2/x^(9/
2)-30/77*c^2*(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)-15/77*c^(11/4)*x*(b^(1/2)+c^(
1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{1}{2},-\frac{7}{4},-\frac{cx^2}{b}\right)}{11x^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[1/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-11/4, 1/2, -7/4, -((c*x^2)/b)])
/(11*x^(9/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1430, 1430, 1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1430}$$

$$-\frac{9c\int \frac{1}{x^{7/2}\sqrt{cx^4+bx^2}}dx}{11b}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

$$\Big\downarrow \text{1430}$$

$$-\frac{9c\left(-\frac{5c\int \frac{1}{x^{3/2}\sqrt{cx^4+bx^2}}dx}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{11b}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

$$\Big\downarrow \text{1430}$$

$$-\frac{9c\left(-\frac{5c\left(-\frac{c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{11b}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

$$\Big\downarrow \text{1431}$$

$$-\frac{9c\left(-\frac{5c\left(-\frac{cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{11b}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{9c\left(-\frac{5c\left(-\frac{2cx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{11b}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{9c\left(-\frac{5c\left(-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{11b}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

input
```
Int[1/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) - (9*c*((-2*Sqrt[b*x^2 + c*x^4])/
(7*b*x^(9/2)) - (5*c*((-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[
2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*x^4])
))/(7*b)))/(11*b)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.82

method	result
default	$-\frac{15\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c^{2}x^{5}+30c^{3}x^{6}+12b\,c^{2}x^{4}-4b^{2}c\,x^{2}+14b^{3}}{77\sqrt{c\,x^{4}+b\,x^{2}}\,x^{\frac{9}{2}}\,b^{3}}$
risch	$-\frac{2\left(c\,x^{2}+b\right)\left(15c^{2}x^{4}-9bc\,x^{2}+7b^{2}\right)}{77b^{3}x^{\frac{9}{2}}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{15c^{2}\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{}}{77b^{3}\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$

input
```
int(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/77/(c*x^4+b*x^2)^(1/2)/x^(9/2)*(15*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*
c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))*c^2*x^5+30*c^3*x^6+12*b*c^2*x^4-4*b^2*c*x^2+14*b^3)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(15\,c^{\frac{5}{2}}x^7\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(15\,c^2x^4-9\,bcx^2+7\,b^2\right)\sqrt{cx^4+bx^2}\sqrt{x}\right)}{77\,b^3x^7}$$

input
```
integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/77*(15*c^(5/2)*x^7*weierstrassPInverse(-4*b/c, 0, x) + (15*c^2*x^4 - 9*
b*c*x^2 + 7*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^3*x^7)
```

Sympy [F]

$$\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^{\frac{11}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(1/x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**(11/2)*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{1}{x^{11/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(1/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c\,x^9+b\,x^7}dx$$

input
```
int(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b*x**7 + c*x**9),x)
```

3.267 $\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}} - \frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}} + \frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2} + \frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14c^{13/4}\sqrt{bx^2+cx^4}}$$

output
```
-x^(11/2)/c/(c*x^4+b*x^2)^(1/2)-15/7*b*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)+9/7
*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2+15/14*b^(7/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x
^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)),1/2*2^(1/2))/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.49

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^{3/2}\left(-15b^2-6bcx^2+2c^2x^4+15b^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{7c^3\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^(17/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^(3/2)*(-15*b^2 - 6*b*c*x^2 + 2*c^2*x^4 + 15*b^2*Sqrt[1 + (c*x^2)/b]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(7*c^3*Sqrt[x^2*(b + c*x^2)]
)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1427, 1429, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1427}$$

$$\frac{9\int \frac{x^{9/2}}{\sqrt{cx^4+bx^2}}dx}{2c}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{9\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{7c}\right)}{2c}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{9\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{7c}\right)}{2c}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 1431

$$\frac{9\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{2c}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 266

$$\frac{9\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{2c}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 761

$$\frac{9\left(\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}-\frac{5b\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{7c}\right)}{2c}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

input

```
Int[x^(17/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output

```
-(x^(11/2)/(c*Sqrt[b*x^2 + c*x^4])) + (9*((2*x^(3/2)*Sqrt[b*x^2 + c*x^4])/
(7*c) - (5*b*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[
(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x^4])))/(7*c))
)/(2*c)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1427
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] - Simp[d^4*(
(m + 2*p - 1)/(2*c*(p + 1)))   Int[(d*x)^(m - 4)*(b*x^2 + c*x^4)^(p + 1), x
], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1] && GtQ[m
+ 2*p + 1, 2]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.83

method	result
default	$\frac{x^{\frac{5}{2}}(c\,x^{2}+b)\left(15b^{2}\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)+4c^{3}x^{5}-12b\,c^{2}x^{3}-30b^{2}cx\right)}{14(c\,x^{4}+b\,x^{2})^{\frac{3}{2}}c^{4}}$
risch	$-\frac{2(-c\,x^{2}+4b)x^{\frac{3}{2}}(c\,x^{2}+b)}{7c^{3}\sqrt{x^{2}(c\,x^{2}+b)}}+\frac{b^{2}\left(\frac{11\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c\sqrt{c\,x^{3}+bx}}-7b\left(\frac{}{b\sqrt{(x^{2}}}\right.\right.}{7c^{3}}$

input
```
int(x^(17/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/14/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(15*b^2*(-b*c)^(1/2)*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))+4*c^3*x^5-12*b*c^2*x^3-30*b^2*c*x)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.51

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{15\,(b^2cx^3+b^3x)\sqrt{c}\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(2\,c^3x^4-6\,bc^2x^2-15\,b^2c)\sqrt{cx}}{7\,(c^5x^3+bc^4x)}$$

input
```
integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/7*(15*(b^2*c*x^3 + b^3*x)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) + (2
*c^3*x^4 - 6*b*c^2*x^2 - 15*b^2*c)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^5*x^3 +
 b*c^4*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(17/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{17}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{17}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(17/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx=\int \frac{x^{17/2}}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

input
```
int(x^(17/2)/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(17/2)/(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx=\frac{-30\sqrt{x}\,\sqrt{c\,x^2+b}\,b^2-6\sqrt{x}\,\sqrt{c\,x^2+b}\,bc\,x^2+2\sqrt{x}\,\sqrt{c\,x^2+b}\,c^2x^4+15\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2}}{c^2x^5+2bc\,x^3}\right.}{7c^3\left(c\,x^2+b\right)}$$

input
```
int(x^(17/2)/(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - 30*sqrt(x)*sqrt(b + c*x**2)*b**2 - 6*sqrt(x)*sqrt(b + c*x**2)*b*c*x**2
 + 2*sqrt(x)*sqrt(b + c*x**2)*c**2*x**4 + 15*int((sqrt(x)*sqrt(b + c*x**2)
)/(b**2*x + 2*b*c*x**3 + c**2*x**5),x)*b**4 + 15*int((sqrt(x)*sqrt(b + c*x
**2))/(b**2*x + 2*b*c*x**3 + c**2*x**5),x)*b**3*c*x**2)/(7*c**3*(b + c*x**
2))
```

3.268 $\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 291

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}-\frac{21bx^{3/2}(b+cx^2)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2}+\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{11/4}\sqrt{bx^2+cx^4}}-\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}$$

output
```
-x^(9/2)/c/(c*x^4+b*x^2)^(1/2)-21/5*b*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+c
^(1/2)*x)/(c*x^4+b*x^2)^(1/2)+7/5*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2+21/5*b^(
5/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*Ellipti
cE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/c^(11/4)/(c*x^4+b*x
^2)^(1/2)-21/10*b^(5/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*
x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))
/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.25

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2x^{5/2}\left(-7b+cx^2+7b\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{5c^2\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^(15/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*x^(5/2)*(-7*b + c*x^2 + 7*b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4,
3/2, 7/4, -((c*x^2)/b)]))/(5*c^2*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1427, 1429, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1427}$$

$$\frac{7\int \frac{x^{7/2}}{\sqrt{cx^4+bx^2}}dx}{2c}-\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{7\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3b\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{5c}\right)}{2c}-\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{7\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{3bx\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{5c\sqrt{bx^2+cx^4}}\right)}{2c}-\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 266$$

$$\frac{7\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{5c\sqrt{bx^2+cx^4}}\right)}{2c}-\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 834$$

$$\frac{7\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{2c}-\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{7\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{2c}-\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 761$$

$$\frac{7\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{2c}-\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 1510$$

$$\frac{7\left(\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}-\frac{6bx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right)}{5c\sqrt{bx^2+cx^4}}\right)}{2c}$$

$$\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}$$

input
```
Int[x^(15/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(x^(9/2)/(c*Sqrt[b*x^2 + c*x^4])) + (7*((2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(
5*c) - (6*b*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + S
qrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sq
rt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*
Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^
2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(5*c*Sqrt[b*x^2 + c*x^4])))/(2*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1427
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] - Simp[d^4*(
(m + 2*p - 1)/(2*c*(p + 1)))   Int[(d*x)^(m - 4)*(b*x^2 + c*x^4)^(p + 1), x
], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1] && GtQ[m
+ 2*p + 1, 2]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.73

method	result
default	$-\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(42b^{2}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-21b^{2}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-}\right.}{10\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}c^{3}}$
risch	$\frac{2x^{\frac{5}{2}}\left(c\,x^{2}+b\right)}{5c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b\left(\frac{8\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,\mathrm{EllipticF}}{}\right.}{c\sqrt{c\,x^{3}+bx}}\right.}{}$

input

```
int(x^(15/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/10/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(42*b^2*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b
*c)^(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))-21*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*c^2*x^4-14*b*c*x^2)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{21\left(bcx^2+b^2\right)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\sqrt{cx^4-}}{5\left(c^4x^2+bc^3\right)}$$

input

```
integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(21*(b*c*x^2 + b^2)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInv
erse(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + 7*b*c)*sqrt(x))/(c^
4*x^2 + b*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**(15/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{x^{\frac{15}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{15/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{x^{\frac{15}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(15/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^{3/2}}\,dx=\int \frac{x^{15/2}}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int(x^(15/2)/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(15/2)/(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{15/2}}{(bx^2+cx^4)^{3/2}}\,dx=\frac{-14\sqrt{x}\,\sqrt{c\,x^2+b}\,bx+2\sqrt{x}\,\sqrt{c\,x^2+b}\,c\,x^3+21\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c^2x^4+2bc\,x^2+b^2}dx\right)b^3+21\left(\int \frac{\sqrt{x}}{c^2x^4+}\right.}{5c^2\,(c\,x^2+b)}$$

input
```
int(x^(15/2)/(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - 14*sqrt(x)*sqrt(b + c*x**2)*b*x + 2*sqrt(x)*sqrt(b + c*x**2)*c*x**3 +
21*int((sqrt(x)*sqrt(b + c*x**2))/(b**2 + 2*b*c*x**2 + c**2*x**4),x)*b**3
+ 21*int((sqrt(x)*sqrt(b + c*x**2))/(b**2 + 2*b*c*x**2 + c**2*x**4),x)*b**
2*c*x**2)/(5*c**2*(b + c*x**2))
```

3.269 $\int \frac{x^{13/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}+\frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}} - \frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6c^{9/4}\sqrt{bx^2+cx^4}}$$

output
```
-x^(7/2)/c/(c*x^4+b*x^2)^(1/2)+5/3*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)-5/6*b^(
3/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*Inverse
JacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/c^(9/4)/(c*x^4+b*x
^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.50

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^{3/2}\left(5b+2cx^2-5b\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{3c^2\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^(13/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^(3/2)*(5*b + 2*c*x^2 - 5*b*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1
/2, 5/4, -((c*x^2)/b)]))/(3*c^2*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1427, 1429, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1427}$$

$$\frac{5\int \frac{x^{5/2}}{\sqrt{cx^4+bx^2}}dx}{2c}-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1429}$$

$$\frac{5\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3c}\right)}{2c}-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{5\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3c\sqrt{bx^2+cx^4}}\right)}{2c}-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{2bx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3c\sqrt{bx^2+cx^4}}\right)}{2c}-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5\left(\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}\right)}{2c}-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}$$

input
```
Int[x^(13/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(x^(7/2)/(c*Sqrt[b*x^2 + c*x^4])) + (5*((2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt
[x]) - (b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c
]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sq
rt[b*x^2 + c*x^4])))/(2*c)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1427
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] - Simp[d^4*(
(m + 2*p - 1)/(2*c*(p + 1)))   Int[(d*x)^(m - 4)*(b*x^2 + c*x^4)^(p + 1), x
], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1] && GtQ[m
+ 2*p + 1, 2]
```

rule 1429
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && GtQ[m + 2*p - 1,
 0] && NeQ[m + 4*p + 1, 0]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

method	result
default	$-\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(5b\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-4c^{2}x^{3}-10bcx\right)}{6\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}c^{3}}$
risch	$\frac{2x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}{3c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b\left(\frac{4\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c\sqrt{c\,x^{3}+bx}}-3b\left(\frac{x}{b\sqrt{\left(x^{2}+\frac{b}{c}\right)cx}}+\cdots\right)\right)}{3c^{2}\sqrt{x^{2}\left(c\,x^{2}+\cdots\right)}}$

input
```
int(x^(13/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(5*b*(-b*c)^(1/2)*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))-4*c^2*x^3-10*b*c*x)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{5\,(bcx^3+b^2x)\sqrt{c}\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}(2\,c^2x^2+5\,bc)\sqrt{x}}{3\,(c^4x^3+bc^3x)}$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(5*(b*c*x^3 + b^2*x)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) - sqrt
(c*x^4 + b*x^2)*(2*c^2*x^2 + 5*b*c)*sqrt(x))/(c^4*x^3 + b*c^3*x)
```

Sympy [F]

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{13}{2}}}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(13/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**(13/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{x^{\frac{13}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{x^{\frac{13}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{x^{13/2}}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^(13/2)/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(13/2)/(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{13/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{10\sqrt{x}\,\sqrt{c\,x^2+b}\,b+2\sqrt{x}\,\sqrt{c\,x^2+b}\,c\,x^2-5\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c^2x^5+2bc\,x^3+b^2x}dx\right)b^3-5\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2}}{c^2x^5+2bc\,x^3}\right.}{3c^2\,(c\,x^2+b)}$$

input
```
int(x^(13/2)/(c*x^4+b*x^2)^(3/2),x)
```

output
```
(10*sqrt(x)*sqrt(b + c*x**2)*b + 2*sqrt(x)*sqrt(b + c*x**2)*c*x**2 - 5*int
((sqrt(x)*sqrt(b + c*x**2))/(b**2*x + 2*b*c*x**3 + c**2*x**5),x)*b**3 - 5*
int((sqrt(x)*sqrt(b + c*x**2))/(b**2*x + 2*b*c*x**3 + c**2*x**5),x)*b**2*c
*x**2)/(3*c**2*(b + c*x**2))
```

3.270 $\int \frac{x^{11/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 259

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}+\frac{3x^{3/2}(b+cx^2)}{c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{bx^2+cx^4}} + \frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{7/4}\sqrt{bx^2+cx^4}}$$

output
```
-x^(5/2)/c/(c*x^4+b*x^2)^(1/2)+3*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+c^(1/2
)*x)/(c*x^4+b*x^2)^(1/2)-3*b^(1/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/
2)+c^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/
2*2^(1/2))/c^(7/4)/(c*x^4+b*x^2)^(1/2)+3/2*b^(1/4)*x*(b^(1/2)+c^(1/2)*x)*(
(c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4)),1/2*2^(1/2))/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{2x^{5/2}\left(-1+\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{c\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^(11/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-2*x^(5/2)*(-1 + Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((
c*x^2)/b)]))/(c*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1427, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1427}$$

$$\frac{3\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{2c} - \frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{3x\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{2c\sqrt{bx^2+cx^4}} - \frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{3x\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{c\sqrt{bx^2+cx^4}}-\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 834

$$\frac{3x\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{c\sqrt{bx^2+cx^4}}-\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 27

$$\frac{3x\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{c\sqrt{bx^2+cx^4}}-\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 761

$$\frac{3x\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{c\sqrt{bx^2+cx^4}}-\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 1510

$$\frac{3x\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}-}{\sqrt{c}}\right)}{c\sqrt{bx^2+cx^4}}$$

$$\frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}$$

input
```
Int[x^(11/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(x^(5/2)/(c*Sqrt[b*x^2 + c*x^4])) + (3*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*S
qrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(c*Sqrt[b
*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1427
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] - Simp[d^4*(
(m + 2*p - 1)/(2*c*(p + 1)))   Int[(d*x)^(m - 4)*(b*x^2 + c*x^4)^(p + 1), x
], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1] && GtQ[m
+ 2*p + 1, 2]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.77

method	result
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b-3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{El}\right.}{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}c^{2}}$

input
```
int(x^(11/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/
2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*b-3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-2*c*x^2)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.25

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{3\,(cx^2+b)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}c\sqrt{x}}{c^3x^2+bc^2}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-(3*(c*x^2 + b)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*
b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*c*sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [F]

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{11}{2}}}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(11/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**(11/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{11}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{11}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{11/2}}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int(x^(11/2)/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(11/2)/(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{11/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{c\,x^2+b}\,x-3\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c^2x^4+2bc\,x^2+b^2}dx\right)b^2-3\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c^2x^4+2bc\,x^2+b^2}dx\right)bc\,x^2}{c\,(c\,x^2+b)}$$

input
```
int(x^(11/2)/(c*x^4+b*x^2)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(b + c*x**2)*x - 3*int((sqrt(x)*sqrt(b + c*x**2))/(b**2 + 2
*b*c*x**2 + c**2*x**4),x)*b**2 - 3*int((sqrt(x)*sqrt(b + c*x**2))/(b**2 +
2*b*c*x**2 + c**2*x**4),x)*b*c*x**2)/(c*(b + c*x**2))
```

3.271 $\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}} + \frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}$$

output
```
-x^(3/2)/c/(c*x^4+b*x^2)^(1/2)+1/2*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/
2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1
/2*2^(1/2))/b^(1/4)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^{3/2}\left(-1+\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{c\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^(9/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^(3/2)*(-1 + Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x
^2)/b)]))/(c*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1427, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}dx$$
$$\Big\downarrow \text{ 1427}$$
$$\frac{\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{2c}-\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}$$
$$\Big\downarrow \text{ 1431}$$
$$\frac{x\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{2c\sqrt{bx^2+cx^4}}-\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}$$
$$\Big\downarrow \text{ 266}$$
$$\frac{x\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{c\sqrt{bx^2+cx^4}}-\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}$$
$$\Big\downarrow \text{ 761}$$
$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}$$

input
```
Int[x^(9/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
-(x^(3/2)/(c*Sqrt[b*x^2 + c*x^4])) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*
x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(2*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1427
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] - Simp[d^4*(
(m + 2*p - 1)/(2*c*(p + 1)))   Int[(d*x)^(m - 4)*(b*x^2 + c*x^4)^(p + 1), x
], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1] && GtQ[m
+ 2*p + 1, 2]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(\sqrt{-b c}\,\sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}\,\sqrt{2}\,\sqrt{\frac{-c x+\sqrt{-b c}}{\sqrt{-b c}}}\,\sqrt{-\frac{c x}{\sqrt{-b c}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}, \frac{\sqrt{2}}{2}\right)-2 c x\right)}{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}} c^{2}}$	120

input
```
int(x^(9/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c
/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1
/2*2^(1/2))-2*c*x)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(cx^3+bx)\sqrt{c}\,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-\sqrt{cx^4+bx^2}\,c\sqrt{x}}{c^3x^3+bc^2x}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
((c*x^3 + b*x)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*
x^2)*c*sqrt(x))/(c^3*x^3 + b*c^2*x)
```

Sympy [F]

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{9}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(9/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**(9/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{9}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{9}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{9/2}}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int(x^(9/2)/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(9/2)/(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{c\,x^2+b}+\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c^2x^5+2bc\,x^3+b^2x}dx\right)b^2+\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c^2x^5+2bc\,x^3+b^2x}dx\right)bc\,x^2}{c\,(c\,x^2+b)}$$

input
```
int(x^(9/2)/(c*x^4+b*x^2)^(3/2),x)
```

output
```
( - 2*sqrt(x)*sqrt(b + c*x**2) + int((sqrt(x)*sqrt(b + c*x**2))/(b**2*x +
2*b*c*x**3 + c**2*x**5),x)*b**2 + int((sqrt(x)*sqrt(b + c*x**2))/(b**2*x +
 2*b*c*x**3 + c**2*x**5),x)*b*c*x**2)/(c*(b + c*x**2))
```

3.272 $\int \frac{x^{7/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 260

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^{5/2}}{b\sqrt{bx^2+cx^4}} - \frac{x^{3/2}(b+cx^2)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}} - \frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

output
```
x^(5/2)/b/(c*x^4+b*x^2)^(1/2)-x^(3/2)*(c*x^2+b)/b/c^(1/2)/(b^(1/2)+c^(1/2)
*x)/(c*x^4+b*x^2)^(1/2)+x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*
x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/
b^(3/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)-1/2*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(
b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/
4)),1/2*2^(1/2))/b^(3/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.23

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2x^{5/2}\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{cx^2}{b}\right)}{3b\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^(7/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*x^(5/2)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^2)/
b)])/(3*b*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1428, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1428}$$

$$\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{2b}$$

$$\Big\downarrow \text{1431}$$

$$\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x\sqrt{b+cx^2}\int \frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{2b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x\sqrt{b+cx^2}\int \frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 834

$$\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 27

$$\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 761

$$\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int \frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 1510

$$\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}-\ldots}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}$$

input
```
Int[x^(7/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
x^(5/2)/(b*Sqrt[b*x^2 + c*x^4]) - (x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b
 + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
 + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sq
rt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(
1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(b*Sqrt[b*x^2
+ c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.78

method	result
default	$-\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b-\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{E}}{2\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}cb}$

input
```
int(x^(7/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1
/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)*b-((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-2*c*x^2)/c/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.25

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{\left(cx^2+b\right)\sqrt{c}\,\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}}{bc^2x^2+b^2c}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
((c*x^2 + b)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c
, 0, x)) + sqrt(c*x^4 + b*x^2)*c*sqrt(x))/(b*c^2*x^2 + b^2*c)
```

Sympy [F]

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(7/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**(7/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{7/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{7/2}}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int(x^(7/2)/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(7/2)/(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{7/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c^2x^4+2bc\,x^2+b^2}dx$$

input
```
int(x^(7/2)/(c*x^4+b*x^2)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b**2 + 2*b*c*x**2 + c**2*x**4),x)
```

3.273 $\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^{3/2}}{b\sqrt{bx^2+cx^4}} + \frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

output
```
x^(3/2)/b/(c*x^4+b*x^2)^(1/2)+1/2*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2
)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/
2*2^(1/2))/b^(5/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^{3/2}\left(1+\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{b\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^(5/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^(3/2)*(1 + Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^
2)/b)]))/(b*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1428, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1428}$$

$$\frac{\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{2b}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{x\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{2b\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{x\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}$$

input
```
Int[x^(5/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
x^(3/2)/(b*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
1/2])/(2*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{x^{\frac{5}{2}} \left(c\,x^{2}+b \right) \left(\sqrt{-b c}\, \sqrt{\frac{c x +\sqrt{-b c}}{\sqrt{-b c}}}\, \sqrt{2}\, \sqrt{\frac{-c x +\sqrt{-b c}}{\sqrt{-b c}}}\, \sqrt{-\frac{c x}{\sqrt{-b c}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{c x +\sqrt{-b c}}{\sqrt{-b c}}}, \frac{\sqrt{2}}{2}\right)+2 c x \right)}{2 \left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}} c b}$	123

input
```
int(x^(5/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c
/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1
/2*2^(1/2))+2*c*x)/c/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(cx^3+bx)\sqrt{c}\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+\sqrt{cx^4+bx^2}c\sqrt{x}}{bc^2x^3+b^2cx}$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
((c*x^3 + b*x)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x^4 + b*
x^2)*c*sqrt(x))/(b*c^2*x^3 + b^2*c*x)
```

Sympy [F]

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**(5/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{5/2}}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int(x^(5/2)/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(5/2)/(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{5/2}}{(bx^2 + cx^4)^{3/2}}\, dx = \int \frac{\sqrt{x}\,\sqrt{c\,x^2 + b}}{c^2x^5 + 2bc\,x^3 + b^2x} dx$$

input
```
int(x^(5/2)/(c*x^4+b*x^2)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b**2*x + 2*b*c*x**3 + c**2*x**5),x)
```

3.274 $\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 286

$$\int \frac{x^{3/2}}{\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}+\frac{3\sqrt{c}x^{3/2}(b+cx^2)}{b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}}-\frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{bx^2+cx^4}}+\frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{7/4}\sqrt{bx^2+cx^4}}$$

output
```
x^(1/2)/b/(c*x^4+b*x^2)^(1/2)+3*c^(1/2)*x^(3/2)*(c*x^2+b)/b^2/(b^(1/2)+c^(
1/2)*x)/(c*x^4+b*x^2)^(1/2)-3*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)-3*c^(1/4)*x*
(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*EllipticE(sin(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/b^(7/4)/(c*x^4+b*x^2)^(1/2
)+3/2*c^(1/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2
)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(7/4)/(
c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.20

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^{3/2}}\,dx=-\frac{2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{cx^2}{b}\right)}{b\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[x^(3/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-2*Sqrt[x]*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((c*x^2
)/b)])/(b*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1428, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1428}$$

$$\frac{3\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{3\left(\frac{c\int \frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{3\left(\frac{cx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$\downarrow$ 266

$$\frac{3\left(\frac{2cx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$\downarrow$ 834

$$\frac{3\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$\downarrow$ 27

$$\frac{3\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$\downarrow$ 761

$$\frac{3\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{2b}+\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

$\downarrow$ 1510

$$\frac{3\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}-\frac{\sqrt{x}\sqrt{b-}}{\sqrt{b}+\sqrt{}}}{\sqrt{c}}\right.}{b\sqrt{bx^2+cx^4}}\right.}{2b}$$

$$\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

input
```
Int[x^(3/2)/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
Sqrt[x]/(b*Sqrt[b*x^2 + c*x^4]) + (3*((-2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2))
 + (2*c*x*Sqrt[b + c*x^2]*(-((-(-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[
c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c
]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt
[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(
Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(b*Sqrt[b*x^2 + c*x^4])))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.71

method	result
default	$\frac{x^{\frac{5}{2}}(c\,x^2+b)\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b-3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{El}}{2(c\,x^4+b\,x^2)^{\frac{3}{2}}b^2}$
risch	$-\frac{2(c\,x^2+b)\sqrt{x}}{b^2\sqrt{x^2(c\,x^2+b)}}+\frac{c^2\left(\frac{\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,\mathrm{Elliptic}}{}\right.}{c^2\sqrt{c\,x^3+bx}}\right.}{}$

input
```
int(x^(3/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/
2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*b-3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b-6*c*x^2-4*b)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.28

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{3\,(cx^4+bx^2)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}(3\,cx^2+2\,b)\sqrt{x}}{b^2cx^4+b^3x^2}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-(3*(c*x^4 + b*x^2)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse
(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*(3*c*x^2 + 2*b)*sqrt(x))/(b^2*c*x^4
+ b^3*x^2)
```

Sympy [F]

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(x**(3/2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{3/2}}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int(x^(3/2)/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(3/2)/(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{3/2}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c^2x^6+2bc\,x^4+b^2x^2}dx$$

input
```
int(x^(3/2)/(c*x^4+b*x^2)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b**2*x**2 + 2*b*c*x**4 + c**2*x**6),x)
```

3.275 $\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}} - \frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt{bx^2+cx^4}}$$

output
```
1/b/x^(1/2)/(c*x^4+b*x^2)^(1/2)-5/3*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)-5/6*c^
(3/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*Invers
eJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(9/4)/(c*x^4+b*
x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{cx^2}{b}\right)}{3b\sqrt{x}\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[Sqrt[x]/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((c*x^2)/b)])/(
3*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1428, 1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1428}$$

$$\frac{5\int \frac{1}{x^{3/2}\sqrt{cx^4+bx^2}}dx}{2b}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{5\left(-\frac{c\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{2b}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1431}$$

$$\frac{5\left(-\frac{cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{2b}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(-\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{2b}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5\left(-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{2b}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}$$

input
```
Int[Sqrt[x]/(b*x^2 + c*x^4)^(3/2),x]
```

output
```
1/(b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) + (5*((-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5
/2)) - (c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c
]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*Sq
rt[b*x^2 + c*x^4])))/(2*b)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.88

method	result
default	$-\frac{x^{\frac{3}{2}}\left(c\,x^{2}+b\right)\left(5\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)x+10c\,x^{2}+4b\right)}{6\left(c\,x^{4}+b\,x^{2}\right)^{\frac{3}{2}}b^{2}}$
risch	$-\frac{2\left(c\,x^{2}+b\right)}{3b^{2}\sqrt{x}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{c\left(\frac{\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c\sqrt{c\,x^{3}+bx}}+3b\left(\frac{x}{b\sqrt{\left(x^{2}+\frac{b}{c}\right)cx}}\right.\right.}{3b^{2}\sqrt{x^{2}(}}$

input
```
int(x^(1/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(c*x^4+b*x^2)^(3/2)*x^(3/2)*(c*x^2+b)*(5*(-b*c)^(1/2)*((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2))*x+10*c*x^2+4*b)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{5\,(cx^5+bx^3)\sqrt{c}\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}(5\,cx^2+2\,b)\sqrt{x}}{3\,(b^2cx^5+b^3x^3)}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(5*(c*x^5 + b*x^3)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c
*x^4 + b*x^2)*(5*c*x^2 + 2*b)*sqrt(x))/(b^2*c*x^5 + b^3*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{x}}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(1/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(sqrt(x)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{x}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{x}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{x}}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int(x^(1/2)/(b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^(1/2)/(b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c^2x^7+2bc\,x^5+b^2x^3}dx$$

input
```
int(x^(1/2)/(c*x^4+b*x^2)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b**2*x**3 + 2*b*c*x**5 + c**2*x**7),x)
```

3.276 $\int \frac{1}{\sqrt{x}(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 320

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx &= \frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}\\ &- \frac{21c^{3/2}x^{3/2}(b+cx^2)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}} + \frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}\\ &+ \frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{bx^2+cx^4}}\\ &- \frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

output
```
1/b/x^(3/2)/(c*x^4+b*x^2)^(1/2)-21/5*c^(3/2)*x^(3/2)*(c*x^2+b)/b^3/(b^(1/2
)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)-7/5*(c*x^4+b*x^2)^(1/2)/b^2/x^(7/2)+21/5*
c*(c*x^4+b*x^2)^(1/2)/b^3/x^(3/2)+21/5*c^(5/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x
^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/
b^(1/4))),1/2*2^(1/2))/b^(11/4)/(c*x^4+b*x^2)^(1/2)-21/10*c^(5/4)*x*(b^(1/
2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4)),1/2*2^(1/2))/b^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.19

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},-\frac{1}{4},-\frac{cx^2}{b}\right)}{5bx^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((c*x^2)/b)])/
(5*b*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1428, 1430, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1428}$$

$$\frac{7\int \frac{1}{x^{5/2}\sqrt{cx^4+bx^2}}dx}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{7\left(-\frac{3c\int \frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{1430}$$

$$\frac{7\left(-\frac{3c\left(\frac{c\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 1431

$$\frac{7\left(-\frac{3c\left(\frac{cx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 266

$$\frac{7\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 834

$$\frac{7\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 27

$$\frac{7\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 761

$$\frac{7\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{2b}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}$$

$\Big\downarrow$ 1510

$$\frac{7\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}-\cdots}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}\cdots\right)}{5b}\cdots\right)}{2b}$$

$$\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}$$

input
```
Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
1/(b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) + (7*((-2*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7
/2)) - (3*c*((-2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) + (2*c*x*Sqrt[b + c*x^2]
*(-((-((Sqrt[x]*Sqrt[b + c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) +
(b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*
x^2])))/(b*Sqrt[b*x^2 + c*x^4])))/(5*b)))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.69

method	result
default	$-\frac{\sqrt{x}\,(c x^2+b)\left(42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^2-21\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\cdots}\right.}{10(c x^4+b x^2)^{\frac{3}{2}}b^3}$
risch	$-\frac{2(c x^2+b)(-8c x^2+b)}{5b^3x^{\frac{3}{2}}\sqrt{x^2(c x^2+b)}}-\frac{c^2\left(\frac{8\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-b\cdots}}{\cdots}\right.}{c\sqrt{c x^3+bx}}\cdots\right.}{\cdots}$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10/(c*x^4+b*x^2)^(3/2)*x^(1/2)*(c*x^2+b)*(42*((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^
(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*b*c*x^2-21*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-42*c^2*x^4-28*b*c*x^2
+4*b^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx = \frac{21\,(c^2x^6+bcx^4)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4b}{c},0,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+(}{5\,(b^3cx^6+b^4x^4)}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(21*(c^2*x^6 + b*c*x^4)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrass
PInverse(-4*b/c, 0, x)) + (21*c^2*x^4 + 14*b*c*x^2 - 2*b^2)*sqrt(c*x^4 + b
*x^2)*sqrt(x))/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{\sqrt{x}\,(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**(1/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(1/(sqrt(x)*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^2)^{\frac{3}{2}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx=\int \frac{1}{(cx^4+bx^2)^{\frac{3}{2}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx=\int \frac{1}{\sqrt{x}\,(c\,x^4+b\,x^2)^{3/2}}\,dx$$

input
```
int(1/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx=\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c^2x^8+2bc\,x^6+b^2x^4}dx$$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)
```

3.277 $\int \frac{1}{x^{3/2}(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 173

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}-\frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}}+\frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}} + \frac{15c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14b^{13/4}\sqrt{bx^2+cx^4}}$$

output
```
1/b/x^(5/2)/(c*x^4+b*x^2)^(1/2)-9/7*(c*x^4+b*x^2)^(1/2)/b^2/x^(9/2)+15/7*c
*(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)+15/14*c^(7/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x
^2+b)/(b^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)),1/2*2^(1/2))/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{2},-\frac{3}{4},-\frac{cx^2}{b}\right)}{7bx^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-7/4, 3/2, -3/4, -((c*x^2)/b)])/
(7*b*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1428, 1430, 1430, 1431, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1428}$$

$$\frac{9 \int \frac{1}{x^{7/2}\sqrt{cx^4+bx^2}}dx}{2b}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1430}$$

$$\frac{9 \left(-\frac{5c \int \frac{1}{x^{3/2}\sqrt{cx^4+bx^2}}dx}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{2b}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1430}$$

$$\frac{9 \left(-\frac{5c \left(-\frac{c \int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}dx}{3b}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{2b}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1431}$$

$$\frac{9\left(-\frac{5c\left(-\frac{cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{x}\sqrt{cx^2+b}}dx}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{2b}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \textbf{266}$$

$$\frac{9\left(-\frac{5c\left(-\frac{2cx\sqrt{b+cx^2}\int \frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{3b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{2b}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \textbf{761}$$

$$\frac{9\left(-\frac{5c\left(-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}\right)}{7b}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}\right)}{2b}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

input
```
Int[1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
1/(b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) + (9*((-2*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9
/2)) - (5*c*((-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[
(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*x^4])))/(7*b))
)/(2*b)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.82

method	result
default	$\frac{(c\,x^{2}+b)\left(15\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c\,x^{3}+30c^{2}x^{4}+12bc\,x^{2}-4b^{2}\right)}{14(c\,x^{4}+b\,x^{2})^{\frac{3}{2}}\sqrt{x}\,b^{3}}$
risch	$-\frac{2(c\,x^{2}+b)(-4c\,x^{2}+b)}{7b^{3}x^{\frac{5}{2}}\sqrt{x^{2}(c\,x^{2}+b)}}+\frac{c^{2}\left(\frac{4\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c\sqrt{c\,x^{3}+bx}}+7b\left(\frac{x}{b\sqrt{\left(x^{2}+\frac{b}{c}\right)}}\right.\right.}{7b^{3}\sqrt{x}}$

input
```
int(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/14/(c*x^4+b*x^2)^(3/2)/x^(1/2)*(c*x^2+b)*(15*(-b*c)^(1/2)*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*c*x^3+30*c^2*x^4+12*b*c*x^2-4*b^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{3/2}\left(bx^{2}+cx^{4}\right)^{3/2}}\,dx=\frac{15\left(c^{2}x^{7}+bcx^{5}\right)\sqrt{c}\,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)+\left(15\,c^{2}x^{4}+6\,bcx^{2}-2\,b^{2}\right)\sqrt{}}{7\left(b^{3}cx^{7}+b^{4}x^{5}\right)}$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/7*(15*(c^2*x^7 + b*c*x^5)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) + (1
5*c^2*x^4 + 6*b*c*x^2 - 2*b^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^3*c*x^7 + b
^4*x^5)
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x^{\frac{3}{2}}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(1/(x**(3/2)*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x^{3/2}\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{\sqrt{x}\,\sqrt{c\,x^2+b}}{c^2x^9+2bc\,x^7+b^2x^5}dx$$

input
```
int(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b**2*x***5 + 2*b*c*x**7 + c**2*x**9),x)
```

3.278 $\int \frac{1}{x^{5/2}(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 350

$$\begin{aligned}\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}+\frac{77c^{5/2}x^{3/2}(b+cx^2)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} \\ &-\frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}}+\frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}}-\frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} \\ &-\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{15/4}\sqrt{bx^2+cx^4}} \\ &+\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

output
```
1/b/x^(7/2)/(c*x^4+b*x^2)^(1/2)+77/15*c^(5/2)*x^(3/2)*(c*x^2+b)/b^4/(b^(1/
2)+c^(1/2)*x)/(c*x^4+b*x^2)^(1/2)-11/9*(c*x^4+b*x^2)^(1/2)/b^2/x^(11/2)+77
/45*c*(c*x^4+b*x^2)^(1/2)/b^3/x^(7/2)-77/15*c^2*(c*x^4+b*x^2)^(1/2)/b^4/x^
(3/2)-77/15*c^(9/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b^(1/2)+c^(1/2)*x)^2
)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))/b^(1
5/4)/(c*x^4+b*x^2)^(1/2)+77/30*c^(9/4)*x*(b^(1/2)+c^(1/2)*x)*((c*x^2+b)/(b
^(1/2)+c^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)),1/2*2^(1/2))/b^(15/4)/(c*x^4+b*x^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{3}{2},-\frac{5}{4},-\frac{cx^2}{b}\right)}{9bx^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-9/4, 3/2, -5/4, -((c*x^2)/b)])/
(9*b*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {1428, 1430, 1430, 1430, 1431, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1428$$

$$\frac{11\int \frac{1}{x^{9/2}\sqrt{cx^4+bx^2}}dx}{2b}+\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 1430$$

$$\frac{11\left(-\frac{7c\int \frac{1}{x^{5/2}\sqrt{cx^4+bx^2}}dx}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}\right)}{2b}+\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \; 1430$$

$$\frac{11\left(-\frac{7c\left(-\frac{3c\int\frac{1}{\sqrt{x}\sqrt{cx^4+bx^2}}dx}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}\right)}{2b}+\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

$\downarrow$ 1430

$$\frac{11\left(-\frac{7c\left(-\frac{3c\left(\frac{c\int\frac{x^{3/2}}{\sqrt{cx^4+bx^2}}dx}{b}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}\right)}{2b}+\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

$\downarrow$ 1431

$$\frac{11\left(-\frac{7c\left(-\frac{3c\left(\frac{cx\sqrt{b+cx^2}\int\frac{\sqrt{x}}{\sqrt{cx^2+b}}dx}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}\right)}{2b}+\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

$\downarrow$ 266

$$\frac{11\left(-\frac{7c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\int\frac{x}{\sqrt{cx^2+b}}d\sqrt{x}}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}\right)}{2b}+\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

$\downarrow$ 834

$$\frac{11\left(-\frac{7c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\sqrt{b}\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{b}\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}\right)}{2b}+\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

$\big\downarrow$ 27

$$\frac{11\left(-\frac{7c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt{b}\int\frac{1}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}\right)}{2b}+\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

$\big\downarrow$ 761

$$\frac{11\left(-\frac{7c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\int\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{cx^2+b}}d\sqrt{x}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}\right)}{5b}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}\right)}{9b}-\cdots}{2b}$$

$$\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

$$\downarrow 1510$$

$$\frac{11\left(-\frac{7c\left(-\frac{3c\left(\frac{2cx\sqrt{b+cx^2}\left(\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{b+cx^2}}-\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right)}{\sqrt[4]{c}\sqrt{b+cx^2}}}{\sqrt{c}}\right)}{b\sqrt{bx^2+cx^4}}\right)}{5b}\right)}{9b}\right)}{2b}$$

$$\frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

input
```
Int[1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

output
```
1/(b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + (11*((-2*Sqrt[b*x^2 + c*x^4])/(9*b*x^(
11/2)) - (7*c*((-2*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) - (3*c*((-2*Sqrt[b*x
^2 + c*x^4])/(b*x^(3/2)) + (2*c*x*Sqrt[b + c*x^2]*(-((-((Sqrt[x]*Sqrt[b +
c*x^2])/(Sqrt[b] + Sqrt[c]*x)) + (b^(1/4)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/
4)], 1/2])/(c^(1/4)*Sqrt[b + c*x^2]))/Sqrt[c]) + (b^(1/4)*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[b + c*x^2])))/(b*Sqrt[b*x^2 + c
*x^4])))/(5*b)))/(9*b)))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1428
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] + Simp[d^2*
((m + 4*p + 3)/(2*b*(p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[p, -1]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

rule 1431
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(b*x^2 + c*x^4)^p/((d*x)^(2*p)*(b + c*x^2)^p)   Int[(d*x)^(m + 2*p)*(b + c
*x^2)^p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.68

method	result
default	$\frac{(c x^2+b)\left(462\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b c^2 x^4-231\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{}{}}\right.}{90(c x^4+b x^2)^{\frac{3}{2}} x^{\frac{3}{2}} b^4}$
risch	$-\frac{2(c x^2+b)(93c^2x^4-16bc x^2+5b^2)}{45b^4x^{\frac{7}{2}}\sqrt{x^2(c x^2+b)}}+\frac{c^3\left(\frac{31\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\right.}{c}\right.}{c\sqrt{c x^3+bx}}\right.}{}$

input
```
int(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/90/(c*x^4+b*x^2)^(3/2)/x^(3/2)*(c*x^2+b)*(462*((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^
(1/2)*x)^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*b*c^2*x^4-231*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-c/(-b*c)^(1/2)*x)^(1/2)*EllipticF(((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-462*c^3*x^6-308*
b*c^2*x^4+44*b^2*c*x^2-20*b^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx =$$
$$-\frac{231\left(c^3x^8+bc^2x^6\right)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\left(231\,c^3x^6+154\,bc^2x^4-\right.}{45\left(b^4cx^8+b^5x^6\right)}$$

input
```
integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/45*(231*(c^3*x^8 + b*c^2*x^6)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weiers
trassPInverse(-4*b/c, 0, x)) + (231*c^3*x^6 + 154*b*c^2*x^4 - 22*b^2*c*x^2
 + 10*b^3)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^4*c*x^8 + b^5*x^6)
```

Sympy [F]

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x^{\frac{5}{2}}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

output
```
Integral(1/(x**(5/2)*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2} \left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x^{5/2}\,{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,dx$$

input
```
int(1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{5/2} \left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{\sqrt{x}\,\sqrt{c\,x^{2}+b}}{c^{2}x^{10}+2bc\,x^{8}+b^{2}x^{6}}dx$$

input
```
int(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x)
```

output
```
int((sqrt(x)*sqrt(b + c*x**2))/(b**2*x**6 + 2*b*c*x**8 + c**2*x**10),x)
```

3.279 $\int (dx)^m \left(bx^2+cx^4\right)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int (dx)^m \left(bx^2+cx^4\right)^3 \, dx = \frac{b^3(dx)^{7+m}}{d^7(7+m)}+\frac{3b^2c(dx)^{9+m}}{d^9(9+m)}+\frac{3bc^2(dx)^{11+m}}{d^{11}(11+m)}+\frac{c^3(dx)^{13+m}}{d^{13}(13+m)}$$

output
```
b^3*(d*x)^(7+m)/d^7/(7+m)+3*b^2*c*(d*x)^(9+m)/d^9/(9+m)+3*b*c^2*(d*x)^(11+
m)/d^11/(11+m)+c^3*(d*x)^(13+m)/d^13/(13+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int (dx)^m \left(bx^2+cx^4\right)^3 \, dx = x^7(dx)^m \left(\frac{b^3}{7+m}+\frac{3b^2cx^2}{9+m}+\frac{3bc^2x^4}{11+m}+\frac{c^3x^6}{13+m}\right)$$

input
```
Integrate[(d*x)^m*(b*x^2 + c*x^4)^3,x]
```

output
```
x^7*(d*x)^m*(b^3/(7 + m) + (3*b^2*c*x^2)/(9 + m) + (3*b*c^2*x^4)/(11 + m)
+ (c^3*x^6)/(13 + m))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2+cx^4\right)^3 (dx)^m \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\frac{\int (dx)^{m+6} \left(cx^2+b\right)^3 dx}{d^6}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left(b^3 (dx)^{m+6}+\frac{3b^2c(dx)^{m+8}}{d^2}+\frac{3bc^2(dx)^{m+10}}{d^4}+\frac{c^3(dx)^{m+12}}{d^6}\right)dx}{d^6}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b^3(dx)^{m+7}}{d(m+7)}+\frac{3b^2c(dx)^{m+9}}{d^3(m+9)}+\frac{3bc^2(dx)^{m+11}}{d^5(m+11)}+\frac{c^3(dx)^{m+13}}{d^7(m+13)}}{d^6}$$

input
```
Int[(d*x)^m*(b*x^2 + c*x^4)^3,x]
```

output
```
((b^3*(d*x)^(7 + m))/(d*(7 + m)) + (3*b^2*c*(d*x)^(9 + m))/(d^3*(9 + m)) +
 (3*b*c^2*(d*x)^(11 + m))/(d^5*(11 + m)) + (c^3*(d*x)^(13 + m))/(d^7*(13 +
 m)))/d^6
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(81) = 162$.

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.23

method	result
gosper	$\frac{(dx)^m (c^3m^3x^6+27c^3m^2x^6+3b\,c^2m^3x^4+239m\,x^6c^3+87b\,c^2m^2x^4+693c^3x^6+3b^2c\,m^3x^2+813b\,c^2m\,x^4+93b^2c\,m^2x^2+2457b}{(13+m)(11+m)(9+m)(7+m)}$
risch	$\frac{(dx)^m (c^3m^3x^6+27c^3m^2x^6+3b\,c^2m^3x^4+239m\,x^6c^3+87b\,c^2m^2x^4+693c^3x^6+3b^2c\,m^3x^2+813b\,c^2m\,x^4+93b^2c\,m^2x^2+2457b}{(13+m)(11+m)(9+m)(7+m)}$
orering	$\frac{(c^3m^3x^6+27c^3m^2x^6+3b\,c^2m^3x^4+239m\,x^6c^3+87b\,c^2m^2x^4+693c^3x^6+3b^2c\,m^3x^2+813b\,c^2m\,x^4+93b^2c\,m^2x^2+2457b\,c^2x^4+}{(13+m)(11+m)(9+m)(7+m)(c\,x^2+b)^3}$
parallelrisch	$x^{13}(dx)^m c^3m^3+27x^{13}(dx)^m c^3m^2+239x^{13}(dx)^m c^3m+3x^{11}(dx)^m b\,c^2m^3+693x^{13}(dx)^m c^3+87x^{11}(dx)^m b\,c^2m^2+813x^{11}(dx$

input
```
int((d*x)^m*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(d*x)^m*(c^3*m^3*x^6+27*c^3*m^2*x^6+3*b*c^2*m^3*x^4+239*c^3*m*x^6+87*b*c^2
*m^2*x^4+693*c^3*x^6+3*b^2*c*m^3*x^2+813*b*c^2*m*x^4+93*b^2*c*m^2*x^2+2457
*b*c^2*x^4+b^3*m^3+933*b^2*c*m*x^2+33*b^3*m^2+3003*b^2*c*x^2+359*b^3*m+128
7*b^3)*x^7/(13+m)/(11+m)/(9+m)/(7+m)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.99

$$\int (dx)^m \left(bx^2+cx^4\right)^3 dx$$
$$= \frac{((c^3m^3+27\,c^3m^2+239\,c^3m+693\,c^3)x^{13}+3\,(bc^2m^3+29\,bc^2m^2+271\,bc^2m+819\,bc^2)x^{11}+3\,(b^2cm^3+}{m^4+40\,m^3+590\,m^2+380}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
((c^3*m^3 + 27*c^3*m^2 + 239*c^3*m + 693*c^3)*x^13 + 3*(b*c^2*m^3 + 29*b*c
^2*m^2 + 271*b*c^2*m + 819*b*c^2)*x^11 + 3*(b^2*c*m^3 + 31*b^2*c*m^2 + 311
*b^2*c*m + 1001*b^2*c)*x^9 + (b^3*m^3 + 33*b^3*m^2 + 359*b^3*m + 1287*b^3)
*x^7)*(d*x)^m/(m^4 + 40*m^3 + 590*m^2 + 3800*m + 9009)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. 2(73) = 146.

Time = 0.67 (sec) , antiderivative size = 731, normalized size of antiderivative = 9.02

$$\int (dx)^m \left(bx^2+cx^4\right)^3 dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(c*x**4+b*x**2)**3,x)
```

output
```
Piecewise(((-b**3/(6*x**6) - 3*b**2*c/(4*x**4) - 3*b*c**2/(2*x**2) + c**3*
log(x))/d**13, Eq(m, -13)), ((-b**3/(4*x**4) - 3*b**2*c/(2*x**2) + 3*b*c**
2*log(x) + c**3*x**2/2)/d**11, Eq(m, -11)), ((-b**3/(2*x**2) + 3*b**2*c*lo
g(x) + 3*b*c**2*x**2/2 + c**3*x**4/4)/d**9, Eq(m, -9)), ((b**3*log(x) + 3*
b**2*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6)/d**7, Eq(m, -7)), (b**3*m**
3*x**7*(d*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 33*b**3*m**2
*x**7*(d*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 359*b**3*m*x*
*7*(d*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 1287*b**3*x**7*(
d*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3*b**2*c*m**3*x**9*(
d*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 93*b**2*c*m**2*x**9*
(d*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 933*b**2*c*m*x**9*(
d*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3003*b**2*c*x**9*(d*
x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 3*b*c**2*m**3*x**11*(d
*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 87*b*c**2*m**2*x**11*
(d*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 813*b*c**2*m*x**11*
(d*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 2457*b*c**2*x**11*(
d*x)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + c**3*m**3*x**13*(d*x
)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 27*c**3*m**2*x**13*(d*x
)**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 239*c**3*m*x**13*(d*x)*
*m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 693*c**3*x**13*(d*x)**...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int (dx)^m \left(bx^2+cx^4\right)^3 \, dx = \frac{c^3 d^m x^{13} x^m}{m+13} + \frac{3 \, bc^2 d^m x^{11} x^m}{m+11} + \frac{3 \, b^2 c d^m x^9 x^m}{m+9} + \frac{b^3 d^m x^7 x^m}{m+7}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
c^3*d^m*x^13*x^m/(m + 13) + 3*b*c^2*d^m*x^11*x^m/(m + 11) + 3*b^2*c*d^m*x^
9*x^m/(m + 9) + b^3*d^m*x^7*x^m/(m + 7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(81) = 162$.

Time = 0.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.26

$$\int (dx)^m \left(bx^2+cx^4\right)^3 \, dx = \frac{(dx)^m c^3m^3x^{13} + 27\,(dx)^m c^3m^2x^{13} + 3\,(dx)^m bc^2m^3x^{11} + 239\,(dx)^m c^3mx^{13} + 87\,(dx)^m bc^2m^2x^{11} + 693}{}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
((d*x)^m*c^3*m^3*x^13 + 27*(d*x)^m*c^3*m^2*x^13 + 3*(d*x)^m*b*c^2*m^3*x^11
 + 239*(d*x)^m*c^3*m*x^13 + 87*(d*x)^m*b*c^2*m^2*x^11 + 693*(d*x)^m*c^3*x^
13 + 3*(d*x)^m*b^2*c*m^3*x^9 + 813*(d*x)^m*b*c^2*m*x^11 + 93*(d*x)^m*b^2*c
*m^2*x^9 + 2457*(d*x)^m*b*c^2*x^11 + (d*x)^m*b^3*m^3*x^7 + 933*(d*x)^m*b^2
*c*m*x^9 + 33*(d*x)^m*b^3*m^2*x^7 + 3003*(d*x)^m*b^2*c*x^9 + 359*(d*x)^m*b
^3*m*x^7 + 1287*(d*x)^m*b^3*x^7)/(m^4 + 40*m^3 + 590*m^2 + 3800*m + 9009)
```

Mupad [B] (verification not implemented)

Time = 17.80 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.11

$$\int (dx)^m \left(bx^2+cx^4\right)^3 \, dx = (d\,x)^m \left(\frac{b^3\,x^7\,\left(m^3+33\,m^2+359\,m+1287\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009} + \frac{c^3\,x^{13}\,\left(m^3+27\,m^2+239\,m+693\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009} + \frac{3\,b\,c^2\,x^{11}\,\left(m^3+29\,m^2+271\,m+819\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009} + \frac{3\,b^2\,c\,x^9\,\left(m^3+31\,m^2+311\,m+1001\right)}{m^4+40\,m^3+590\,m^2+3800\,m+9009}\right)$$

input
```
int((d*x)^m*(b*x^2 + c*x^4)^3,x)
```

output
```
(d*x)^m*((b^3*x^7*(359*m + 33*m^2 + m^3 + 1287))/(3800*m + 590*m^2 + 40*m^
3 + m^4 + 9009) + (c^3*x^13*(239*m + 27*m^2 + m^3 + 693))/(3800*m + 590*m^
2 + 40*m^3 + m^4 + 9009) + (3*b*c^2*x^11*(271*m + 29*m^2 + m^3 + 819))/(38
00*m + 590*m^2 + 40*m^3 + m^4 + 9009) + (3*b^2*c*x^9*(311*m + 31*m^2 + m^3
 + 1001))/(3800*m + 590*m^2 + 40*m^3 + m^4 + 9009))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.23

$$\int (dx)^m \left(bx^2+cx^4\right)^3 \, dx$$

$$= \frac{x^m d^m x^7 (c^3m^3x^6 + 27c^3m^2x^6 + 3b\,c^2m^3x^4 + 239c^3m\,x^6 + 87b\,c^2m^2x^4 + 693c^3x^6 + 3b^2c\,m^3x^2 + 813b\,c^2m}{m^4 + 40m^3 + 590m^2 +}$$

input
```
int((d*x)^m*(c*x^4+b*x^2)^3,x)
```

output
```
(x**m*d**m*x**7*(b**3*m**3 + 33*b**3*m**2 + 359*b**3*m + 1287*b**3 + 3*b**
2*c*m**3*x**2 + 93*b**2*c*m**2*x**2 + 933*b**2*c*m*x**2 + 3003*b**2*c*x**2
 + 3*b*c**2*m**3*x**4 + 87*b*c**2*m**2*x**4 + 813*b*c**2*m*x**4 + 2457*b*c
**2*x**4 + c**3*m**3*x**6 + 27*c**3*m**2*x**6 + 239*c**3*m*x**6 + 693*c**3
*x**6))/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009)
```

3.280 $\int (dx)^m \left(bx^2+cx^4\right)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (dx)^m \left(bx^2+cx^4\right)^2 \, dx = \frac{b^2(dx)^{5+m}}{d^5(5+m)}+\frac{2bc(dx)^{7+m}}{d^7(7+m)}+\frac{c^2(dx)^{9+m}}{d^9(9+m)}$$

output
```
b^2*(d*x)^(5+m)/d^5/(5+m)+2*b*c*(d*x)^(7+m)/d^7/(7+m)+c^2*(d*x)^(9+m)/d^9/
(9+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int (dx)^m \left(bx^2+cx^4\right)^2 \, dx = x^5(dx)^m \left(\frac{b^2}{5+m}+\frac{2bcx^2}{7+m}+\frac{c^2x^4}{9+m}\right)$$

input
```
Integrate[(d*x)^m*(b*x^2 + c*x^4)^2,x]
```

output
```
x^5*(d*x)^m*(b^2/(5 + m) + (2*b*c*x^2)/(7 + m) + (c^2*x^4)/(9 + m))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2+cx^4\right)^2 (dx)^m \, dx$$

$$\Big\downarrow \ 9$$

$$\frac{\int (dx)^{m+4} \left(cx^2+b\right)^2 dx}{d^4}$$

$$\Big\downarrow \ 244$$

$$\frac{\int \left(b^2 (dx)^{m+4}+\frac{2bc(dx)^{m+6}}{d^2}+\frac{c^2 (dx)^{m+8}}{d^4}\right) dx}{d^4}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{b^2 (dx)^{m+5}}{d(m+5)}+\frac{2bc(dx)^{m+7}}{d^3 (m+7)}+\frac{c^2 (dx)^{m+9}}{d^5 (m+9)}}{d^4}$$

input
```
Int[(d*x)^m*(b*x^2 + c*x^4)^2,x]
```

output
```
((b^2*(d*x)^(5 + m))/(d*(5 + m)) + (2*b*c*(d*x)^(7 + m))/(d^3*(7 + m)) + (
c^2*(d*x)^(9 + m))/(d^5*(9 + m)))/d^4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.66

method	result
gosper	$\frac{(dx)^m (c^2m^2x^4+12m\,x^4c^2+2bc\,m^2x^2+35c^2x^4+28bcm\,x^2+b^2m^2+90bc\,x^2+16b^2m+63b^2)x^5}{(9+m)(7+m)(5+m)}$
risch	$\frac{(dx)^m (c^2m^2x^4+12m\,x^4c^2+2bc\,m^2x^2+35c^2x^4+28bcm\,x^2+b^2m^2+90bc\,x^2+16b^2m+63b^2)x^5}{(9+m)(7+m)(5+m)}$
orering	$\frac{(c^2m^2x^4+12m\,x^4c^2+2bc\,m^2x^2+35c^2x^4+28bcm\,x^2+b^2m^2+90bc\,x^2+16b^2m+63b^2)x(dx)^m(c\,x^4+b\,x^2)^2}{(9+m)(7+m)(5+m)(c\,x^2+b)^2}$
parallelrisch	$\frac{x^9(dx)^m c^2m^2+12x^9(dx)^m c^2m+35x^9(dx)^m c^2+2x^7(dx)^m bc\,m^2+28x^7(dx)^m bcm+90x^7(dx)^m bc+x^5(dx)^m b^2m^2+16x^5(dx)^{\cdots}}{(9+m)(7+m)(5+m)}$

input
```
int((d*x)^m*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(d*x)^m*(c^2*m^2*x^4+12*c^2*m*x^4+2*b*c*m^2*x^2+35*c^2*x^4+28*b*c*m*x^2+b^
2*m^2+90*b*c*x^2+16*b^2*m+63*b^2)*x^5/(9+m)/(7+m)/(5+m)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.53

$$\int (dx)^m \left(bx^2+cx^4\right)^2 \,dx = \frac{((c^2m^2+12\,c^2m+35\,c^2)x^9+2\,(bcm^2+14\,bcm+45\,bc)x^7+(b^2m^2+16\,b^2m+63\,b^2)x^5)(dx)^m}{m^3+21\,m^2+143\,m+315}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
((c^2*m^2 + 12*c^2*m + 35*c^2)*x^9 + 2*(b*c*m^2 + 14*b*c*m + 45*b*c)*x^7 +
 (b^2*m^2 + 16*b^2*m + 63*b^2)*x^5)*(d*x)^m/(m^3 + 21*m^2 + 143*m + 315)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(51) = 102$.

Time = 0.42 (sec) , antiderivative size = 337, normalized size of antiderivative = 5.81

$$\int (dx)^m \left(bx^2 + cx^4\right)^2 \, dx$$
$$= \begin{cases} \frac{-\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)}{d^9} \\ \frac{-\frac{b^2}{2x^2} + 2bc \log(x) + \frac{c^2x^2}{2}}{d^7} \\ \frac{b^2 \log(x) + bcx^2 + \frac{c^2x^4}{4}}{d^5} \\ \frac{b^2m^2x^5(dx)^m}{m^3+21m^2+143m+315} + \frac{16b^2mx^5(dx)^m}{m^3+21m^2+143m+315} + \frac{63b^2x^5(dx)^m}{m^3+21m^2+143m+315} + \frac{2bcm^2x^7(dx)^m}{m^3+21m^2+143m+315} + \frac{28bcmx^7(dx)^m}{m^3+21m^2+143m+315} + \end{cases}$$

input
```
integrate((d*x)**m*(c*x**4+b*x**2)**2,x)
```

output
```
Piecewise(((-b**2/(4*x**4) - b*c/x**2 + c**2*log(x))/d**9, Eq(m, -9)), ((-
b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2)/d**7, Eq(m, -7)), ((b**2*log(x
) + b*c*x**2 + c**2*x**4/4)/d**5, Eq(m, -5)), (b**2*m**2*x**5*(d*x)**m/(m*
*3 + 21*m**2 + 143*m + 315) + 16*b**2*m*x**5*(d*x)**m/(m**3 + 21*m**2 + 14
3*m + 315) + 63*b**2*x**5*(d*x)**m/(m**3 + 21*m**2 + 143*m + 315) + 2*b*c*
m**2*x**7*(d*x)**m/(m**3 + 21*m**2 + 143*m + 315) + 28*b*c*m*x**7*(d*x)**m
/(m**3 + 21*m**2 + 143*m + 315) + 90*b*c*x**7*(d*x)**m/(m**3 + 21*m**2 + 1
43*m + 315) + c**2*m**2*x**9*(d*x)**m/(m**3 + 21*m**2 + 143*m + 315) + 12*
c**2*m*x**9*(d*x)**m/(m**3 + 21*m**2 + 143*m + 315) + 35*c**2*x**9*(d*x)**
m/(m**3 + 21*m**2 + 143*m + 315), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (dx)^m \left(bx^2+cx^4\right)^2\,dx=\frac{c^2d^mx^9x^m}{m+9}+\frac{2\,bcd^mx^7x^m}{m+7}+\frac{b^2d^mx^5x^m}{m+5}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
c^2*d^m*x^9*x^m/(m + 9) + 2*b*c*d^m*x^7*x^m/(m + 7) + b^2*d^m*x^5*x^m/(m +
 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(58) = 116.

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.43

$$\int (dx)^m \left(bx^2+cx^4\right)^2\,dx$$
$$=\frac{(dx)^m\,c^2m^2x^9+12\,(dx)^m\,c^2mx^9+2\,(dx)^m\,bcm^2x^7+35\,(dx)^m\,c^2x^9+28\,(dx)^m\,bcmx^7+(dx)^m\,b^2m^2x^5+}{m^3+21\,m^2+143\,m+315}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
((d*x)^m*c^2*m^2*x^9 + 12*(d*x)^m*c^2*m*x^9 + 2*(d*x)^m*b*c*m^2*x^7 + 35*(
d*x)^m*c^2*x^9 + 28*(d*x)^m*b*c*m*x^7 + (d*x)^m*b^2*m^2*x^5 + 90*(d*x)^m*b
*c*x^7 + 16*(d*x)^m*b^2*m*x^5 + 63*(d*x)^m*b^2*x^5)/(m^3 + 21*m^2 + 143*m
+ 315)
```

Mupad [B] (verification not implemented)

Time = 17.92 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int (dx)^m \left(bx^2+cx^4\right)^2 \, dx = (d\,x)^m \left(\frac{b^2\,x^5\,\left(m^2+16\,m+63\right)}{m^3+21\,m^2+143\,m+315} + \frac{c^2\,x^9\,\left(m^2+12\,m+35\right)}{m^3+21\,m^2+143\,m+315} + \frac{2\,b\,c\,x^7\,\left(m^2+14\,m+45\right)}{m^3+21\,m^2+143\,m+315}\right)$$

input
```
int((d*x)^m*(b*x^2 + c*x^4)^2,x)
```

output
```
(d*x)^m*((b^2*x^5*(16*m + m^2 + 63))/(143*m + 21*m^2 + m^3 + 315) + (c^2*x
^9*(12*m + m^2 + 35))/(143*m + 21*m^2 + m^3 + 315) + (2*b*c*x^7*(14*m + m^
2 + 45))/(143*m + 21*m^2 + m^3 + 315))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.66

$$\int (dx)^m \left(bx^2+cx^4\right)^2 \, dx = \frac{x^m d^m x^5 (c^2 m^2 x^4+12 c^2 m\,x^4+2 b c\,m^2 x^2+35 c^2 x^4+28 b c m\,x^2+b^2 m^2+90 b c\,x^2+16 b^2 m+63 b^2)}{m^3+21 m^2+143 m+315}$$

input
```
int((d*x)^m*(c*x^4+b*x^2)^2,x)
```

output
```
(x**m*d**m*x**5*(b**2*m**2 + 16*b**2*m + 63*b**2 + 2*b*c*m**2*x**2 + 28*b*
c*m*x**2 + 90*b*c*x**2 + c**2*m**2*x**4 + 12*c**2*m*x**4 + 35*c**2*x**4))/
(m**3 + 21*m**2 + 143*m + 315)
```

3.281 $\int (dx)^m (bx^2+cx^4) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 35

$$\int (dx)^m (bx^2+cx^4) \, dx = \frac{b(dx)^{3+m}}{d^3(3+m)} + \frac{c(dx)^{5+m}}{d^5(5+m)}$$

output
```
b*(d*x)^(3+m)/d^3/(3+m)+c*(d*x)^(5+m)/d^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int (dx)^m (bx^2+cx^4) \, dx = x^3(dx)^m \left(\frac{b}{3+m} + \frac{cx^2}{5+m}\right)$$

input
```
Integrate[(d*x)^m*(b*x^2 + c*x^4),x]
```

output
```
x^3*(d*x)^m*(b/(3 + m) + (c*x^2)/(5 + m))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {9, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2+cx^4\right)(dx)^m\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\frac{\int (dx)^{m+2}\left(cx^2+b\right)dx}{d^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left(b(dx)^{m+2}+\frac{c(dx)^{m+4}}{d^2}\right)dx}{d^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b(dx)^{m+3}}{d(m+3)}+\frac{c(dx)^{m+5}}{d^3(m+5)}}{d^2}$$

input
```
Int[(d*x)^m*(b*x^2 + c*x^4),x]
```

output
```
((b*(d*x)^(3 + m))/(d*(3 + m)) + (c*(d*x)^(5 + m))/(d^3*(5 + m)))/d^2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
norman	$\frac{b\,x^{3} {\mathrm e}^{m \ln \left(d x \right)}}{3+m}+\frac{c\,x^{5} {\mathrm e}^{m \ln \left(d x \right)}}{5+m}$	36
gosper	$\frac{\left(d x \right)^{m} \left(c m \,x^{2}+3 c \,x^{2}+b m +5 b \right) x^{3}}{\left(5+m \right) \left(3+m \right)}$	39
risch	$\frac{\left(d x \right)^{m} \left(c m \,x^{2}+3 c \,x^{2}+b m +5 b \right) x^{3}}{\left(5+m \right) \left(3+m \right)}$	39
parallelrisch	$\frac{x^{5} \left(d x \right)^{m} c m +3 x^{5} \left(d x \right)^{m} c +x^{3} \left(d x \right)^{m} b m +5 x^{3} \left(d x \right)^{m} b}{\left(5+m \right) \left(3+m \right)}$	57
orering	$\frac{\left(c m \,x^{2}+3 c \,x^{2}+b m +5 b \right) x \left(d x \right)^{m} \left(c \,x^{4}+b \,x^{2}\right)}{\left(5+m \right) \left(3+m \right) \left(c \,x^{2}+b \right)}$	57

input
```
int((d*x)^m*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

output
```
b/(3+m)*x^3*exp(m*ln(d*x))+c/(5+m)*x^5*exp(m*ln(d*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(bx^2+cx^4\right) \, dx=\frac{{\left({\left(c m + 3 \, c\right)} x^{5} + {\left(b m + 5 \, b\right)} x^{3}\right)} \left(d x\right)^{m}}{m^{2} + 8 \, m + 15}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
((c*m + 3*c)*x^5 + (b*m + 5*b)*x^3)*(d*x)^m/(m^2 + 8*m + 15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(29) = 58$.

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.20

$$\int (dx)^m \left(bx^2+cx^4\right) \, dx = \begin{cases} \frac{-\frac{b}{2x^2}+c\log(x)}{d^5} & \text{for } m=-5 \\ \frac{b\log(x)+\frac{cx^2}{2}}{d^3} & \text{for } m=-3 \\ \frac{bmx^3(dx)^m}{m^2+8m+15} + \frac{5bx^3(dx)^m}{m^2+8m+15} + \frac{cmx^5(dx)^m}{m^2+8m+15} + \frac{3cx^5(dx)^m}{m^2+8m+15} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x)**m*(c*x**4+b*x**2),x)
```

output
```
Piecewise(((-b/(2*x**2) + c*log(x))/d**5, Eq(m, -5)), ((b*log(x) + c*x**2/
2)/d**3, Eq(m, -3)), (b*m*x**3*(d*x)**m/(m**2 + 8*m + 15) + 5*b*x**3*(d*x)
**m/(m**2 + 8*m + 15) + c*m*x**5*(d*x)**m/(m**2 + 8*m + 15) + 3*c*x**5*(d*
x)**m/(m**2 + 8*m + 15), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(bx^2+cx^4\right) \, dx = \frac{cd^m x^5 x^m}{m+5} + \frac{bd^m x^3 x^m}{m+3}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
c*d^m*x^5*x^m/(m + 5) + b*d^m*x^3*x^m/(m + 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int (dx)^m \left(bx^2 + cx^4\right) \, dx = \frac{(dx)^m \, cmx^5 + 3\,(dx)^m \, cx^5 + (dx)^m \, bmx^3 + 5\,(dx)^m \, bx^3}{m^2 + 8\,m + 15}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
((d*x)^m*c*m*x^5 + 3*(d*x)^m*c*x^5 + (d*x)^m*b*m*x^3 + 5*(d*x)^m*b*x^3)/(m
^2 + 8*m + 15)
```

Mupad [B] (verification not implemented)

Time = 18.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int (dx)^m \left(bx^2 + cx^4\right) \, dx = \frac{x^3 \, (d\,x)^m \, (5\,b + b\,m + 3\,c\,x^2 + c\,m\,x^2)}{m^2 + 8\,m + 15}$$

input
```
int((d*x)^m*(b*x^2 + c*x^4),x)
```

output
```
(x^3*(d*x)^m*(5*b + b*m + 3*c*x^2 + c*m*x^2))/(8*m + m^2 + 15)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(bx^2 + cx^4\right) \, dx = \frac{x^m d^m x^3 (cm\,x^2 + 3c\,x^2 + bm + 5b)}{m^2 + 8m + 15}$$

input
```
int((d*x)^m*(c*x^4+b*x^2),x)
```

output
```
(x**m*d**m*x**3*(b*m + 5*b + c*m*x**2 + 3*c*x**2))/(m**2 + 8*m + 15)
```

3.282 $\int \frac{(dx)^m}{bx^2+cx^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{(dx)^m}{bx^2+cx^4} dx = -\frac{d(dx)^{-1+m} \text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+m),\frac{1+m}{2},-\frac{cx^2}{b}\right)}{b(1-m)}$$

output
```
-d*(d*x)^(-1+m)*hypergeom([1, -1/2+1/2*m],[1/2+1/2*m],-c*x^2/b)/b/(1-m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{(dx)^m}{bx^2+cx^4} dx = \frac{(dx)^m \text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+m),\frac{1+m}{2},-\frac{cx^2}{b}\right)}{b(-1+m)x}$$

input
```
Integrate[(d*x)^m/(b*x^2 + c*x^4),x]
```

output
```
((d*x)^m*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*(-1
 + m)*x)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {9, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{bx^2+cx^4}\,dx$$

$$\Big\downarrow \ 9$$

$$d^2\int \frac{(dx)^{m-2}}{cx^2+b}dx$$

$$\Big\downarrow \ 278$$

$$-\frac{d(dx)^{m-1}\,\text{Hypergeometric2F1}\left(1,\frac{m-1}{2},\frac{m+1}{2},-\frac{cx^2}{b}\right)}{b(1-m)}$$

input
```
Int[(d*x)^m/(b*x^2 + c*x^4),x]
```

output
```
-((d*(d*x)^(-1 + m)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((c*x^2)/
b)])/(b*(1 - m)))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(dx)^m}{c\,x^4+b\,x^2}dx$$

input
```
int((d*x)^m/(c*x^4+b*x^2),x)
```

output
```
int((d*x)^m/(c*x^4+b*x^2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{bx^2+cx^4}\,dx = \int \frac{(dx)^m}{cx^4+bx^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c*x^4 + b*x^2), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{bx^2+cx^4}\,dx = \int \frac{(dx)^m}{x^2\,(b+cx^2)}\,dx$$

input
```
integrate((d*x)**m/(c*x**4+b*x**2),x)
```

output
```
Integral((d*x)**m/(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{bx^2+cx^4}\,dx = \int \frac{(dx)^m}{cx^4+bx^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{bx^2+cx^4}\,dx = \int \frac{(dx)^m}{cx^4+bx^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{bx^2+cx^4}\,dx = \int \frac{(d\,x)^m}{c\,x^4+b\,x^2}\,dx$$

input
```
int((d*x)^m/(b*x^2 + c*x^4),x)
```

output
```
int((d*x)^m/(b*x^2 + c*x^4), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{bx^2 + cx^4}\, dx = d^m \left(\int \frac{x^m}{c\, x^4 + b\, x^2} dx \right)$$

input
```
int((d*x)^m/(c*x^4+b*x^2),x)
```

output
```
d**m*int(x**m/(b*x**2 + c*x**4),x)
```

3.283 $\int \frac{(dx)^m}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{(dx)^m}{(bx^2+cx^4)^2} dx = -\frac{d^3(dx)^{-3+m}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),-\frac{cx^2}{b}\right)}{b^2(3-m)}$$

output
```
-d^3*(d*x)^(-3+m)*hypergeom([2, -3/2+1/2*m],[-1/2+1/2*m],-c*x^2/b)/b^2/(3-
m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m}{(bx^2+cx^4)^2} dx = \frac{(dx)^m\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}(-3+m),1+\frac{1}{2}(-3+m),-\frac{cx^2}{b}\right)}{b^2(-3+m)x^3}$$

input
```
Integrate[(d*x)^m/(b*x^2 + c*x^4)^2,x]
```

output
```
((d*x)^m*Hypergeometric2F1[2, (-3 + m)/2, 1 + (-3 + m)/2, -((c*x^2)/b)])/(
b^2*(-3 + m)*x^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {9, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(bx^2+cx^4)^2}dx$$
$$\Big\downarrow \; 9$$
$$d^4\int \frac{(dx)^{m-4}}{(cx^2+b)^2}dx$$
$$\Big\downarrow \; 278$$
$$-\frac{d^3(dx)^{m-3}\,\text{Hypergeometric2F1}\left(2,\frac{m-3}{2},\frac{m-1}{2},-\frac{cx^2}{b}\right)}{b^2(3-m)}$$

input
```
Int[(d*x)^m/(b*x^2 + c*x^4)^2,x]
```

output
```
-((d^3*(d*x)^(-3 + m)*Hypergeometric2F1[2, (-3 + m)/2, (-1 + m)/2, -((c*x^
2)/b)])/(b^2*(3 - m)))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(dx)^m}{(c\,x^4+b\,x^2)^2}dx$$

input
```
int((d*x)^m/(c*x^4+b*x^2)^2,x)
```

output
```
int((d*x)^m/(c*x^4+b*x^2)^2,x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(bx^2+cx^4)^2}\,dx = \int \frac{(dx)^m}{(cx^4+bx^2)^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c^2*x^8 + 2*b*c*x^6 + b^2*x^4), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(bx^2 + cx^4)^2}\, dx = \int \frac{(dx)^m}{x^4 (b + cx^2)^2}\, dx$$

input
```
integrate((d*x)**m/(c*x**4+b*x**2)**2,x)
```

output
```
Integral((d*x)**m/(x**4*(b + c*x**2)**2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(bx^2 + cx^4)^2}\, dx = \int \frac{(dx)^m}{(cx^4 + bx^2)^2}\, dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^4 + b*x^2)^2, x)
```

Giac [F]

$$\int \frac{(dx)^m}{(bx^2 + cx^4)^2}\, dx = \int \frac{(dx)^m}{(cx^4 + bx^2)^2}\, dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^4 + b*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(bx^2+cx^4)^2}\,dx=\int \frac{(d\,x)^m}{(c\,x^4+b\,x^2)^2}\,dx$$

input
```
int((d*x)^m/(b*x^2 + c*x^4)^2,x)
```

output
```
int((d*x)^m/(b*x^2 + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(bx^2+cx^4)^2}\,dx=d^m\left(\int \frac{x^m}{c^2x^8+2bc\,x^6+b^2x^4}dx\right)$$

input
```
int((d*x)^m/(c*x^4+b*x^2)^2,x)
```

output
```
d**m*int(x**m/(b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)
```

3.284 $\int \frac{(dx)^m}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{(dx)^m}{(bx^2+cx^4)^3}\,dx = -\frac{d^5(dx)^{-5+m}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}(-5+m),\frac{1}{2}(-3+m),-\frac{cx^2}{b}\right)}{b^3(5-m)}$$

output
```
-d^5*(d*x)^(-5+m)*hypergeom([3, -5/2+1/2*m],[-3/2+1/2*m],-c*x^2/b)/b^3/(5-
m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m}{(bx^2+cx^4)^3}\,dx = \frac{(dx)^m\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}(-5+m),1+\frac{1}{2}(-5+m),-\frac{cx^2}{b}\right)}{b^3(-5+m)x^5}$$

input
```
Integrate[(d*x)^m/(b*x^2 + c*x^4)^3,x]
```

output
```
((d*x)^m*Hypergeometric2F1[3, (-5 + m)/2, 1 + (-5 + m)/2, -((c*x^2)/b)])/(
b^3*(-5 + m)*x^5)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {9, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(bx^2+cx^4)^3} \, dx$$

$$\Big\downarrow \ 9$$

$$d^6 \int \frac{(dx)^{m-6}}{(cx^2+b)^3} dx$$

$$\Big\downarrow \ 278$$

$$-\frac{d^5 (dx)^{m-5} \text{Hypergeometric2F1}\left(3, \frac{m-5}{2}, \frac{m-3}{2}, -\frac{cx^2}{b}\right)}{b^3 (5-m)}$$

input
```
Int[(d*x)^m/(b*x^2 + c*x^4)^3,x]
```

output
```
-((d^5*(d*x)^(-5 + m)*Hypergeometric2F1[3, (-5 + m)/2, (-3 + m)/2, -((c*x^
2)/b)])/(b^3*(5 - m)))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(dx)^m}{(c\,x^4+b\,x^2)^3}dx$$

input
```
int((d*x)^m/(c*x^4+b*x^2)^3,x)
```

output
```
int((d*x)^m/(c*x^4+b*x^2)^3,x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(bx^2+cx^4)^3}\,dx = \int \frac{(dx)^m}{(cx^4+bx^2)^3}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c^3*x^12 + 3*b*c^2*x^10 + 3*b^2*c*x^8 + b^3*x^6), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(bx^2+cx^4)^3}\,dx = \int \frac{(dx)^m}{x^6\,(b+cx^2)^3}\,dx$$

input
```
integrate((d*x)**m/(c*x**4+b*x**2)**3,x)
```

output
```
Integral((d*x)**m/(x**6*(b + c*x**2)**3), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(bx^2+cx^4)^3}\,dx = \int \frac{(dx)^m}{(cx^4+bx^2)^3}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^4 + b*x^2)^3, x)
```

Giac [F]

$$\int \frac{(dx)^m}{(bx^2+cx^4)^3}\,dx = \int \frac{(dx)^m}{(cx^4+bx^2)^3}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^4 + b*x^2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(bx^2+cx^4)^3}\,dx=\int \frac{(d\,x)^m}{(c\,x^4+b\,x^2)^3}\,dx$$

input
```
int((d*x)^m/(b*x^2 + c*x^4)^3,x)
```

output
```
int((d*x)^m/(b*x^2 + c*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(bx^2+cx^4)^3}\,dx=d^m\left(\int \frac{x^m}{c^3x^{12}+3b\,c^2x^{10}+3b^2c\,x^8+b^3x^6}dx\right)$$

input
```
int((d*x)^m/(c*x^4+b*x^2)^3,x)
```

output
```
d**m*int(x**m/(b**3*x**6 + 3*b**2*c*x**8 + 3*b*c**2*x**10 + c**3*x**12),x)
```

3.285 $\int x^5(a^2+2abx^2+b^2x^4)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 30

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

output
```
1/6*a^2*x^6+1/4*a*b*x^8+1/10*b^2*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(a^2*x^6)/6 + (a*b*x^8)/4 + (b^2*x^10)/10
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right) \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int b^2x^5 \left(bx^2+a\right)^2 dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int x^5 \left(a+bx^2\right)^2 dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int x^4 \left(bx^2+a\right)^2 dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(b^2x^8+2abx^6+a^2x^4\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{a^2x^6}{3}+\frac{1}{2}abx^8+\frac{b^2x^{10}}{5}\right)$$

input
```
Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
((a^2*x^6)/3 + (a*b*x^8)/2 + (b^2*x^10)/5)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{6}a^{2}x^{6}+\frac{1}{4}ab\,x^{8}+\frac{1}{10}b^{2}x^{10}$	25
norman	$\frac{1}{6}a^{2}x^{6}+\frac{1}{4}ab\,x^{8}+\frac{1}{10}b^{2}x^{10}$	25
risch	$\frac{1}{6}a^{2}x^{6}+\frac{1}{4}ab\,x^{8}+\frac{1}{10}b^{2}x^{10}$	25
parallelrisch	$\frac{1}{6}a^{2}x^{6}+\frac{1}{4}ab\,x^{8}+\frac{1}{10}b^{2}x^{10}$	25
gosper	$\frac{x^{6}\left(6b^{2}x^{4}+15ab\,x^{2}+10a^{2}\right)}{60}$	27
orering	$\frac{x^{6}\left(6b^{2}x^{4}+15ab\,x^{2}+10a^{2}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}{60\left(b\,x^{2}+a\right)^{2}}$	54

input
```
int(x^5*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*a^2*x^6+1/4*a*b*x^8+1/10*b^2*x^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^5 \left(a^2 + 2abx^2 + b^2x^4\right) dx = \frac{1}{10} b^2x^{10} + \frac{1}{4} abx^8 + \frac{1}{6} a^2x^6$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^5 \left(a^2 + 2abx^2 + b^2x^4\right) dx = \frac{a^2x^6}{6} + \frac{abx^8}{4} + \frac{b^2x^{10}}{10}$$

input
```
integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
a**2*x**6/6 + a*b*x**8/4 + b**2*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^5 \left(a^2 + 2abx^2 + b^2x^4\right) dx = \frac{1}{10} b^2x^{10} + \frac{1}{4} abx^8 + \frac{1}{6} a^2x^6$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{1}{10}\,b^2x^{10}+\frac{1}{4}\,abx^8+\frac{1}{6}\,a^2x^6$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{a^2\,x^6}{6}+\frac{a\,b\,x^8}{4}+\frac{b^2\,x^{10}}{10}$$

input
```
int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(a^2*x^6)/6 + (b^2*x^10)/10 + (a*b*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{x^6(6b^2x^4+15ab\,x^2+10a^2)}{60}$$

input
```
int(x^5*(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(x**6*(10*a**2 + 15*a*b*x**2 + 6*b**2*x**4))/60
```

3.286 $\int x^3(a^2+2abx^2+b^2x^4)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 30

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

output
```
1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right) \, dx$$

$$\Big\downarrow \ 1380$$

$$\frac{\int b^2x^3\left(bx^2+a\right)^2dx}{b^2}$$

$$\Big\downarrow \ 27$$

$$\int x^3\left(a+bx^2\right)^2dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int x^2\left(bx^2+a\right)^2dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(b^2x^6+2abx^4+a^2x^2\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{a^2x^4}{2}+\frac{2}{3}abx^6+\frac{b^2x^8}{4}\right)$$

input
```
Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
((a^2*x^4)/2 + (2*a*b*x^6)/3 + (b^2*x^8)/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
norman	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
risch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
parallelrisch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
gosper	$\frac{x^4(3b^2x^4+8ab\,x^2+6a^2)}{24}$	27
orering	$\frac{x^4(3b^2x^4+8ab\,x^2+6a^2)(b^2x^4+2ab\,x^2+a^2)}{24(b\,x^2+a)^2}$	54

input
```
int(x^3*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3(a^2+2abx^2+b^2x^4)\,dx=\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3(a^2+2abx^2+b^2x^4)\,dx=\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}$$

input
```
integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3(a^2+2abx^2+b^2x^4)\,dx=\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3 \left(a^2 + 2abx^2 + b^2x^4\right) \, dx = \frac{1}{8} b^2 x^8 + \frac{1}{3} abx^6 + \frac{1}{4} a^2 x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3 \left(a^2 + 2abx^2 + b^2x^4\right) \, dx = \frac{a^2 x^4}{4} + \frac{a\,b\,x^6}{3} + \frac{b^2\,x^8}{8}$$

input
```
int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^3 \left(a^2 + 2abx^2 + b^2x^4\right) \, dx = \frac{x^4 (3b^2 x^4 + 8ab\,x^2 + 6a^2)}{24}$$

input
```
int(x^3*(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(x**4*(6*a**2 + 8*a*b*x**2 + 3*b**2*x**4))/24
```

3.287 $\int x(a^2+2abx^2+b^2x^4)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 16

$$\int x\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{\left(a+bx^2\right)^3}{6b}$$

output
```
1/6*(b*x^2+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{\left(a+bx^2\right)^3}{6b}$$

input
```
Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(a + b*x^2)^3/(6*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1380, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a^2+2abx^2+b^2x^4)\,dx$$
$$\Big\downarrow \text{1380}$$
$$\frac{\int b^2x(bx^2+a)^2dx}{b^2}$$
$$\Big\downarrow \text{27}$$
$$\int x(a+bx^2)^2dx$$
$$\Big\downarrow \text{241}$$
$$\frac{(a+bx^2)^3}{6b}$$

input
```
Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(a + b*x^2)^3/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

method	result	size
default	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
norman	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
risch	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
parallelrisch	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
gosper	$\frac{x^2\left(b^2x^4+3ab\,x^2+3a^2\right)}{6}$	26
orering	$\frac{x^2\left(b^2x^4+3ab\,x^2+3a^2\right)\left(b^2x^4+2ab\,x^2+a^2\right)}{6\left(b\,x^2+a\right)^2}$	53

input
```
int(x*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2*x^6+1/2*a*b*x^4+1/2*a^2*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{1}{6}\,b^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}$$

input
```
integrate(x*(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{1}{6}\,b^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{1}{6}\,b^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{a^2\,x^2}{2}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^6}{6}$$

input
```
int(x*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int x\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{x^2(b^2x^4+3ab\,x^2+3a^2)}{6}$$

input
```
int(x*(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(x**2*(3*a**2 + 3*a*b*x**2 + b**2*x**4))/6
```

3.288 $\int \frac{a^2+2abx^2+b^2x^4}{x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 23

$$\int \frac{a^2+2abx^2+b^2x^4}{x} \, dx = abx^2 + \frac{b^2x^4}{4} + a^2 \log(x)$$

output
```
a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx^2+b^2x^4}{x} \, dx = abx^2 + \frac{b^2x^4}{4} + a^2 \log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]
```

output
```
a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{x}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{x}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^2}+2ba+b^2x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^2\log\left(x^2\right)+2abx^2+\frac{b^2x^4}{2}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]
```

output
```
(2*a*b*x^2 + (b^2*x^4)/2 + a^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$ab\,x^2+\frac{b^2x^4}{4}+a^2\ln\left(x\right)$	22
norman	$ab\,x^2+\frac{b^2x^4}{4}+a^2\ln\left(x\right)$	22
parallelrisch	$ab\,x^2+\frac{b^2x^4}{4}+a^2\ln\left(x\right)$	22
risch	$\frac{b^2x^4}{4}+ab\,x^2+a^2+a^2\ln\left(x\right)$	25

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x,x,method=_RETURNVERBOSE)
```

output
```
a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x} \, dx = \frac{1}{4} b^2 x^4 + abx^2 + a^2 \log(x)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="fricas")
```

output
```
1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x} \, dx = a^2 \log(x) + abx^2 + \frac{b^2x^4}{4}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/x,x)
```

output
```
a**2*log(x) + a*b*x**2 + b**2*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x} \, dx = \frac{1}{4} b^2 x^4 + abx^2 + \frac{1}{2} a^2 \log(x^2)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x} \, dx = \frac{1}{4} b^2 x^4 + abx^2 + \frac{1}{2} a^2 \log \left(x^2\right)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x,x, algorithm="giac")
```

output
```
1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x} \, dx = a^2 \ln(x) + \frac{b^2 x^4}{4} + a \, b \, x^2$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/x,x)
```

output
```
a^2*log(x) + (b^2*x^4)/4 + a*b*x^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x} \, dx = \log(x) \, a^2 + ab \, x^2 + \frac{b^2x^4}{4}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x,x)
```

output
```
(4*log(x)*a**2 + 4*a*b*x**2 + b**2*x**4)/4
```

3.289 $\int \frac{a^2+2abx^2+b^2x^4}{x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \frac{a^2+2abx^2+b^2x^4}{x^3} \, dx = -\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\log(x)$$

output
```
-1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx^2+b^2x^4}{x^3} \, dx = -\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]
```

output
```
-1/2*a^2/x^2 + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{x^3} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{x^3}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^4}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^4}+\frac{2ba}{x^2}+b^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{x^2}+2ab\log\left(x^2\right)+b^2x^2\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]
```

output
```
(-(a^2/x^2) + b^2*x^2 + 2*a*b*Log[x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\ln\left(x\right)$	24
risch	$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\ln\left(x\right)$	24
norman	$\frac{-\frac{a^2}{2}+\frac{b^2x^4}{2}}{x^2}+2ab\ln\left(x\right)$	26
parallelrisch	$\frac{b^2x^4+4ab\ln(x)x^2-a^2}{2x^2}$	28

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^3} \, dx = \frac{b^2x^4 + 4\,abx^2 \log{(x)} - a^2}{2\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(b^2*x^4 + 4*a*b*x^2*log(x) - a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^3} \, dx = -\frac{a^2}{2x^2} + 2ab\log{(x)} + \frac{b^2x^2}{2}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/x**3,x)
```

output
```
-a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^3} \, dx = \frac{1}{2}\,b^2x^2 + ab\log{\left(x^2\right)} - \frac{a^2}{2\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2 + a*b*log(x^2) - 1/2*a^2/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^3} \, dx = \frac{1}{2} b^2x^2 + ab \log \left(x^2\right) - \frac{2 \, abx^2 + a^2}{2 \, x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^3,x, algorithm="giac")
```

output
```
1/2*b^2*x^2 + a*b*log(x^2) - 1/2*(2*a*b*x^2 + a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 17.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^3} \, dx = \frac{b^2 \, x^2}{2} - \frac{a^2}{2 \, x^2} + 2 \, a \, b \, \ln \left(x\right)$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^3,x)
```

output
```
(b^2*x^2)/2 - a^2/(2*x^2) + 2*a*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^3} \, dx = \frac{4 \log(x) \, ab \, x^2 - a^2 + b^2x^4}{2x^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^3,x)
```

output
```
(4*log(x)*a*b*x**2 - a**2 + b**2*x**4)/(2*x**2)
```

3.290 $\int \frac{a^2+2abx^2+b^2x^4}{x^5} dx$

Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^5} dx = -\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

output
```
-1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^5} dx = -\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]
```

output
```
-1/4*a^2/x^4 - (a*b)/x^2 + b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{x^5}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{x^5}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^6}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^6}+\frac{2ba}{x^4}+\frac{b^2}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{2x^4}-\frac{2ab}{x^2}+b^2\log\left(x^2\right)\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]
```

output
```
(-1/2*a^2/x^4 - (2*a*b)/x^2 + b^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^{2}}{4 x^{4}}-\frac{a b}{x^{2}}+b^{2} \ln \left(x \right)$	23
norman	$\frac{-\frac{1}{4} a^{2}-a b \,x^{2}}{x^{4}}+b^{2} \ln \left(x \right)$	25
risch	$\frac{-\frac{1}{4} a^{2}-a b \,x^{2}}{x^{4}}+b^{2} \ln \left(x \right)$	25
parallelrisch	$\frac{4 b^{2} \ln \left(x \right) x^{4}-4 a b \,x^{2}-a^{2}}{4 x^{4}}$	29

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{a^2+2abx^2+b^2x^4}{x^5}\,dx=\frac{4\,b^2x^4\log{(x)}-4\,abx^2-a^2}{4\,x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="fricas")
```

output
```
1/4*(4*b^2*x^4*log(x) - 4*a*b*x^2 - a^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx^2+b^2x^4}{x^5}\,dx=b^2\log{(x)}+\frac{-a^2-4abx^2}{4x^4}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/x**5,x)
```

output
```
b**2*log(x) + (-a**2 - 4*a*b*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{a^2+2abx^2+b^2x^4}{x^5}\,dx=\frac{1}{2}\,b^2\log{(x^2)}-\frac{4\,abx^2+a^2}{4\,x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="maxima")
```

output
```
1/2*b^2*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^5} \, dx = \frac{1}{2} b^2 \log \left(x^2\right) - \frac{3 \, b^2x^4 + 4 \, abx^2 + a^2}{4 \, x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^5,x, algorithm="giac")
```

output
```
1/2*b^2*log(x^2) - 1/4*(3*b^2*x^4 + 4*a*b*x^2 + a^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^5} \, dx = b^2 \, \ln(x) - \frac{\frac{a^2}{4} + b \, a \, x^2}{x^4}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^5,x)
```

output
```
b^2*log(x) - (a^2/4 + a*b*x^2)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^5} \, dx = \frac{4 \log(x) \, b^2x^4 - a^2 - 4ab \, x^2}{4x^4}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^5,x)
```

output
```
(4*log(x)*b**2*x**4 - a**2 - 4*a*b*x**2)/(4*x**4)
```

3.291 $\int \frac{a^2+2abx^2+b^2x^4}{x^7} dx$

Optimal result

Integrand size = 22, antiderivative size = 19

$$\int \frac{a^2+2abx^2+b^2x^4}{x^7} dx = -\frac{(a+bx^2)^3}{6ax^6}$$

output
```
-1/6*(b*x^2+a)^3/a/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{a^2+2abx^2+b^2x^4}{x^7} dx = -\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{b^2}{2x^2}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]
```

output
```
-1/6*a^2/x^6 - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1380, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{x^7}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{x^7}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{x^7}dx$$

$$\Big\downarrow \text{242}$$

$$-\frac{(a+bx^2)^3}{6ax^6}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]
```

output
```
-1/6*(a + b*x^2)^3/(a*x^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

method	result	size
gosper	$-\frac{3b^2x^4+3ab\,x^2+a^2}{6x^6}$	25
default	$-\frac{ab}{2x^4}-\frac{a^2}{6x^6}-\frac{b^2}{2x^2}$	25
norman	$\frac{-\frac{1}{2}b^2x^4-\frac{1}{2}ab\,x^2-\frac{1}{6}a^2}{x^6}$	26
risch	$\frac{-\frac{1}{2}b^2x^4-\frac{1}{2}ab\,x^2-\frac{1}{6}a^2}{x^6}$	26
parallelrisch	$\frac{-3b^2x^4-3ab\,x^2-a^2}{6x^6}$	27
orering	$-\frac{(3b^2x^4+3ab\,x^2+a^2)(b^2x^4+2ab\,x^2+a^2)}{6x^6(b\,x^2+a)^2}$	52

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(3*b^2*x^4+3*a*b*x^2+a^2)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{a^2+2abx^2+b^2x^4}{x^7}\,dx=-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="fricas")
```

output
```
-1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^7} \, dx = \frac{-a^2 - 3abx^2 - 3b^2x^4}{6x^6}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/x**7,x)
```

output
```
(-a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^7} \, dx = -\frac{3\,b^2x^4 + 3\,abx^2 + a^2}{6\,x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="maxima")
```

output
```
-1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^7} \, dx = -\frac{3\,b^2x^4 + 3\,abx^2 + a^2}{6\,x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^7,x, algorithm="giac")
```

output
```
-1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{a^2+2abx^2+b^2x^4}{x^7}\,dx = -\frac{\frac{a^2}{6}+\frac{a\,b\,x^2}{2}+\frac{b^2\,x^4}{2}}{x^6}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^7,x)
```

output
```
-(a^2/6 + (b^2*x^4)/2 + (a*b*x^2)/2)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{a^2+2abx^2+b^2x^4}{x^7}\,dx = \frac{-3b^2x^4-3ab\,x^2-a^2}{6x^6}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^7,x)
```

output
```
( - a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*x**6)
```

3.292 $\int \frac{a^2+2abx^2+b^2x^4}{x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 30

$$\int \frac{a^2+2abx^2+b^2x^4}{x^9}\,dx=-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$$

output
```
-1/8*a^2/x^8-1/3*a*b/x^6-1/4*b^2/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx^2+b^2x^4}{x^9}\,dx=-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^9,x]
```

output
```
-1/8*a^2/x^8 - (a*b)/(3*x^6) - b^2/(4*x^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1380, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{x^9}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{x^9}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{x^9}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^{10}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^{10}}+\frac{2ba}{x^8}+\frac{b^2}{x^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{4x^8}-\frac{2ab}{3x^6}-\frac{b^2}{2x^4}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^9,x]
```

output
```
(-1/4*a^2/x^8 - (2*a*b)/(3*x^6) - b^2/(2*x^4))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^{2}}{8x^{8}}-\frac{ab}{3x^{6}}-\frac{b^{2}}{4x^{4}}$	25
norman	$\frac{-\frac{1}{4}b^{2}x^{4}-\frac{1}{3}ab\,x^{2}-\frac{1}{8}a^{2}}{x^{8}}$	26
risch	$\frac{-\frac{1}{4}b^{2}x^{4}-\frac{1}{3}ab\,x^{2}-\frac{1}{8}a^{2}}{x^{8}}$	26
gosper	$-\frac{6b^{2}x^{4}+8ab\,x^{2}+3a^{2}}{24x^{8}}$	27
parallelrisch	$\frac{-6b^{2}x^{4}-8ab\,x^{2}-3a^{2}}{24x^{8}}$	27
orering	$-\frac{\left(6b^{2}x^{4}+8ab\,x^{2}+3a^{2}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}{24x^{8}\left(b\,x^{2}+a\right)^{2}}$	54

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^2/x^8-1/3*a*b/x^6-1/4*b^2/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^9} \, dx = -\frac{6\, b^2x^4 + 8\, abx^2 + 3\, a^2}{24\, x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^9,x, algorithm="fricas")
```

output
```
-1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^9} \, dx = \frac{-3a^2 - 8abx^2 - 6b^2x^4}{24x^8}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/x**9,x)
```

output
```
(-3*a**2 - 8*a*b*x**2 - 6*b**2*x**4)/(24*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^9} \, dx = -\frac{6\,b^2x^4 + 8\,abx^2 + 3\,a^2}{24\,x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^9,x, algorithm="maxima")
```

output
```
-1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^9} \, dx = -\frac{6\,b^2x^4 + 8\,abx^2 + 3\,a^2}{24\,x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^9,x, algorithm="giac")
```

output
```
-1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^9} \, dx = -\frac{\frac{a^2}{8} + \frac{a\,b\,x^2}{3} + \frac{b^2\,x^4}{4}}{x^8}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^9,x)
```

output
```
-(a^2/8 + (b^2*x^4)/4 + (a*b*x^2)/3)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^9}\,dx = \frac{-6b^2x^4 - 8ab\,x^2 - 3a^2}{24x^8}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^9,x)
```

output
```
( - 3*a**2 - 8*a*b*x**2 - 6*b**2*x**4)/(24*x**8)
```

3.293 $\int \frac{a^2+2abx^2+b^2x^4}{x^{11}} dx$

Optimal result

Integrand size = 22, antiderivative size = 30

$$\int \frac{a^2+2abx^2+b^2x^4}{x^{11}} dx = -\frac{a^2}{10x^{10}} - \frac{ab}{4x^8} - \frac{b^2}{6x^6}$$

output
```
-1/10*a^2/x^10-1/4*a*b/x^8-1/6*b^2/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx^2+b^2x^4}{x^{11}} dx = -\frac{a^2}{10x^{10}} - \frac{ab}{4x^8} - \frac{b^2}{6x^6}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^11,x]
```

output
```
-1/10*a^2/x^10 - (a*b)/(4*x^8) - b^2/(6*x^6)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1380, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{x^{11}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{x^{11}}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{x^{11}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^{12}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^{12}}+\frac{2ba}{x^{10}}+\frac{b^2}{x^8}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{5x^{10}}-\frac{ab}{2x^8}-\frac{b^2}{3x^6}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^11,x]
```

output
```
(-1/5*a^2/x^10 - (a*b)/(2*x^8) - b^2/(3*x^6))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^{2}}{10 x^{10}}-\frac{a b}{4 x^{8}}-\frac{b^{2}}{6 x^{6}}$	25
norman	$\frac{-\frac{1}{6} b^{2} x^{4}-\frac{1}{4} a b \,x^{2}-\frac{1}{10} a^{2}}{x^{10}}$	26
risch	$\frac{-\frac{1}{6} b^{2} x^{4}-\frac{1}{4} a b \,x^{2}-\frac{1}{10} a^{2}}{x^{10}}$	26
gosper	$-\frac{10 b^{2} x^{4}+15 a b \,x^{2}+6 a^{2}}{60 x^{10}}$	27
parallelrisch	$\frac{-10 b^{2} x^{4}-15 a b \,x^{2}-6 a^{2}}{60 x^{10}}$	27
orering	$-\frac{\left(10 b^{2} x^{4}+15 a b \,x^{2}+6 a^{2}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}{60 x^{10} \left(b \,x^{2}+a \right)^{2}}$	54

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^2/x^10-1/4*a*b/x^8-1/6*b^2/x^6
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^{11}} \, dx = -\frac{10\, b^2x^4 + 15\, abx^2 + 6\, a^2}{60\, x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^11,x, algorithm="fricas")
```

output
```
-1/60*(10*b^2*x^4 + 15*a*b*x^2 + 6*a^2)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^{11}} \, dx = \frac{-6a^2 - 15abx^2 - 10b^2x^4}{60x^{10}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/x**11,x)
```

output
```
(-6*a**2 - 15*a*b*x**2 - 10*b**2*x**4)/(60*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^{11}} \, dx = -\frac{10\, b^2x^4 + 15\, abx^2 + 6\, a^2}{60\, x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^11,x, algorithm="maxima")
```

output
```
-1/60*(10*b^2*x^4 + 15*a*b*x^2 + 6*a^2)/x^10
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^{11}} \, dx = -\frac{10\, b^2x^4 + 15\, abx^2 + 6\, a^2}{60\, x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^11,x, algorithm="giac")
```

output
```
-1/60*(10*b^2*x^4 + 15*a*b*x^2 + 6*a^2)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^{11}} \, dx = -\frac{\frac{a^2}{10} + \frac{a\,b\,x^2}{4} + \frac{b^2\,x^4}{6}}{x^{10}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^11,x)
```

output
```
-(a^2/10 + (b^2*x^4)/6 + (a*b*x^2)/4)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^{11}} \, dx = \frac{-10b^2x^4 - 15ab\,x^2 - 6a^2}{60x^{10}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^11,x)
```

output
```
( - 6*a**2 - 15*a*b*x**2 - 10*b**2*x**4)/(60*x**10)
```

3.294 $\int x^4(a^2+2abx^2+b^2x^4)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 30

$$\int x^4(a^2+2abx^2+b^2x^4)\,dx=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

output
```
1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^4(a^2+2abx^2+b^2x^4)\,dx=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a^2+2 a b x^2+b^2 x^4\right) \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int b^2 x^4 \left(b x^2+a\right)^2dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int x^4 \left(a+b x^2\right)^2dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^2 x^4+2 a b x^6+b^2 x^8\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2 x^5}{5}+\frac{2}{7} a b x^7+\frac{b^2 x^9}{9}$$

input
```
Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
norman	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
risch	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
parallelrisch	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
gosper	$\frac{x^5(35b^2x^4+90ab\,x^2+63a^2)}{315}$	27
orering	$\frac{x^5(35b^2x^4+90ab\,x^2+63a^2)(b^2x^4+2ab\,x^2+a^2)}{315(b\,x^2+a)^2}$	54

input
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{a^2x^5}{5}+\frac{2abx^7}{7}+\frac{b^2x^9}{9}$$

input
```
integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{a^2\,x^5}{5}+\frac{2\,a\,b\,x^7}{7}+\frac{b^2\,x^9}{9}$$

input
```
int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(a^2*x^5)/5 + (b^2*x^9)/9 + (2*a*b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{x^5(35b^2x^4+90ab\,x^2+63a^2)}{315}$$

input
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(x**5*(63*a**2 + 90*a*b*x**2 + 35*b**2*x**4))/315
```

3.295 $\int x^2(a^2+2abx^2+b^2x^4)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 30

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

output
```
1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2 a b x^2+b^2 x^4\right) \, dx$$

$$\Big\downarrow \ 1380$$

$$\frac{\int b^2 x^2 \left(b x^2+a\right)^2 dx}{b^2}$$

$$\Big\downarrow \ 27$$

$$\int x^2 \left(a+b x^2\right)^2 dx$$

$$\Big\downarrow \ 244$$

$$\int \left(a^2 x^2+2 a b x^4+b^2 x^6\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2 x^3}{3}+\frac{2}{5} a b x^5+\frac{b^2 x^7}{7}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
norman	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
risch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
parallelrisch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
gosper	$\frac{x^3(15b^2x^4+42ab\,x^2+35a^2)}{105}$	27
orering	$\frac{x^3(15b^2x^4+42ab\,x^2+35a^2)(b^2x^4+2ab\,x^2+a^2)}{105(b\,x^2+a)^2}$	54

input
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 \left(a^2 + 2abx^2 + b^2x^4\right) \, dx = \frac{1}{7} b^2x^7 + \frac{2}{5} abx^5 + \frac{1}{3} a^2x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^2 \left(a^2 + 2abx^2 + b^2x^4\right) \, dx = \frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}$$

input
```
integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 \left(a^2 + 2abx^2 + b^2x^4\right) \, dx = \frac{1}{7} b^2x^7 + \frac{2}{5} abx^5 + \frac{1}{3} a^2x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(a^2+2abx^2+b^2x^4)\ dx = \frac{1}{7}b^2x^7+\frac{2}{5}abx^5+\frac{1}{3}a^2x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(a^2+2abx^2+b^2x^4)\ dx = \frac{a^2\,x^3}{3}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^7}{7}$$

input
```
int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^2(a^2+2abx^2+b^2x^4)\ dx = \frac{x^3(15b^2x^4+42ab\,x^2+35a^2)}{105}$$

input
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(x**3*(35*a**2 + 42*a*b*x**2 + 15*b**2*x**4))/105
```

3.296 $\int (a^2 + 2abx^2 + b^2x^4) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 25

$$\int (a^2 + 2abx^2 + b^2x^4) \, dx = a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

output
```
a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a^2 + 2abx^2 + b^2x^4) \, dx = a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

input
```
Integrate[a^2 + 2*a*b*x^2 + b^2*x^4,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^2+b^2x^4\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

input
```
Int[a^2 + 2*a*b*x^2 + b^2*x^4,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}x^5b^2$	22
norman	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}x^5b^2$	22
risch	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}x^5b^2$	22
parallelrisch	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}x^5b^2$	22
parts	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}x^5b^2$	22
gosper	$\frac{x(3b^2x^4+10ab\,x^2+15a^2)}{15}$	25
orering	$\frac{x(3b^2x^4+10ab\,x^2+15a^2)(b^2x^4+2ab\,x^2+a^2)}{15(b\,x^2+a)^2}$	52

input
```
int(b^2*x^4+2*a*b*x^2+a^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/3*a*b*x^3+1/5*x^5*b^2
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \left(a^2+2abx^2+b^2x^4\right)\,dx=a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}$$

input
```
integrate(b**2*x**4+2*a*b*x**2+a**2,x)
```

output
```
a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate(b^2*x^4+2*a*b*x^2+a^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a^2 + 2abx^2 + b^2x^4\right) dx = a^2 x + \frac{2 a b x^3}{3} + \frac{b^2 x^5}{5}$$

input
```
int(a^2 + b^2*x^4 + 2*a*b*x^2,x)
```

output
```
a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \left(a^2 + 2abx^2 + b^2x^4\right) dx = \frac{x(3b^2x^4 + 10ab\,x^2 + 15a^2)}{15}$$

input
```
int(b^2*x^4+2*a*b*x^2+a^2,x)
```

output
```
(x*(15*a**2 + 10*a*b*x**2 + 3*b**2*x**4))/15
```

3.297 $\int \frac{a^2+2abx^2+b^2x^4}{x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 24

$$\int \frac{a^2+2abx^2+b^2x^4}{x^2} dx = -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

output
```
-a^2/x+2*a*b*x+1/3*b^2*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx^2+b^2x^4}{x^2} dx = -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]
```

output
```
-(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{x^2}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{x^2}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{x^2}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{x^2}+2ab+b^2x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]
```

output
```
-(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
risch	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
norman	$\frac{\frac{1}{3}b^2x^4+2ab\,x^2-a^2}{x}$	26
parallelrisch	$\frac{b^2x^4+6ab\,x^2-3a^2}{3x}$	26
gosper	$-\frac{-b^2x^4-6ab\,x^2+3a^2}{3x}$	27
orering	$-\frac{\left(-b^2x^4-6ab\,x^2+3a^2\right)\left(b^2x^4+2ab\,x^2+a^2\right)}{3x\left(b\,x^2+a\right)^2}$	54

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+2*a*b*x+1/3*b^2*x^3
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{a^2+2abx^2+b^2x^4}{x^2}\,dx=\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="fricas")
```

output
```
1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{a^2+2abx^2+b^2x^4}{x^2}\,dx=-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/x**2,x)
```

output
```
-a**2/x + 2*a*b*x + b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{a^2+2abx^2+b^2x^4}{x^2}\,dx=\frac{1}{3}\,b^2x^3+2\,abx-\frac{a^2}{x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^2} \, dx = \frac{1}{3} b^2x^3 + 2 \, abx - \frac{a^2}{x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^2,x, algorithm="giac")
```

output
```
1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^2} \, dx = \frac{b^2 \, x^3}{3} - \frac{a^2}{x} + 2 \, a \, b \, x$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^2,x)
```

output
```
(b^2*x^3)/3 - a^2/x + 2*a*b*x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^2} \, dx = \frac{b^2x^4 + 6ab \, x^2 - 3a^2}{3x}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^2,x)
```

output
```
( - 3*a**2 + 6*a*b*x**2 + b**2*x**4)/(3*x)
```

3.298 $\int \frac{a^2+2abx^2+b^2x^4}{x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 23

$$\int \frac{a^2+2abx^2+b^2x^4}{x^4} dx = -\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

output
```
-1/3*a^2/x^3-2*a*b/x+b^2*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx^2+b^2x^4}{x^4} dx = -\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]
```

output
```
-1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{x^4} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{x^4}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{x^4}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{x^4}+\frac{2ab}{x^2}+b^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]
```

output
```
-1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^{2}}{3 x^{3}}-\frac{2 a b}{x}+b^{2} x$	22
risch	$b^{2} x+\frac{-2 a b \,x^{2}-\frac{1}{3} a^{2}}{x^{3}}$	24
gosper	$-\frac{-3 b^{2} x^{4}+6 a b \,x^{2}+a^{2}}{3 x^{3}}$	25
norman	$\frac{b^{2} x^{4}-2 a b \,x^{2}-\frac{1}{3} a^{2}}{x^{3}}$	25
parallelrisch	$\frac{3 b^{2} x^{4}-6 a b \,x^{2}-a^{2}}{3 x^{3}}$	27
orering	$-\frac{\left(-3 b^{2} x^{4}+6 a b \,x^{2}+a^{2}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}{3 x^{3} \left(b \,x^{2}+a \right)^{2}}$	52

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3-2*a*b/x+b^2*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{a^2+2abx^2+b^2x^4}{x^4}\,dx=\frac{3\,b^2x^4-6\,abx^2-a^2}{3\,x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*b^2*x^4 - 6*a*b*x^2 - a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a^2+2abx^2+b^2x^4}{x^4}\,dx=b^2x+\frac{-a^2-6abx^2}{3x^3}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/x**4,x)
```

output
```
b**2*x + (-a**2 - 6*a*b*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a^2+2abx^2+b^2x^4}{x^4}\,dx=b^2x-\frac{6\,abx^2+a^2}{3\,x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="maxima")
```

output
```
b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^4} \, dx = b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^4,x, algorithm="giac")
```

output
```
b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^4} \, dx = b^2\,x - \frac{\frac{a^2}{3} + 2\,b\,a\,x^2}{x^3}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^4,x)
```

output
```
b^2*x - (a^2/3 + 2*a*b*x^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^4} \, dx = \frac{3b^2x^4 - 6ab\,x^2 - a^2}{3x^3}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^4,x)
```

output
```
( - a**2 - 6*a*b*x**2 + 3*b**2*x**4)/(3*x**3)
```

3.299 $\int \frac{a^2+2abx^2+b^2x^4}{x^6} dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{a^2+2abx^2+b^2x^4}{x^6} dx = -\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

output
```
-1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx^2+b^2x^4}{x^6} dx = -\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]
```

output
```
-1/5*a^2/x^5 - (2*a*b)/(3*x^3) - b^2/x
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{x^6}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{x^6}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{x^6}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^4}+\frac{b^2}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]
```

output
```
-1/5*a^2/x^5 - (2*a*b)/(3*x^3) - b^2/x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$	25
norman	$\frac{-b^2x^4-\frac{2}{3}ab\,x^2-\frac{1}{5}a^2}{x^5}$	26
risch	$\frac{-b^2x^4-\frac{2}{3}ab\,x^2-\frac{1}{5}a^2}{x^5}$	26
gosper	$-\frac{15b^2x^4+10ab\,x^2+3a^2}{15x^5}$	27
parallelrisch	$\frac{-15b^2x^4-10ab\,x^2-3a^2}{15x^5}$	27
orering	$-\frac{(15b^2x^4+10ab\,x^2+3a^2)(b^2x^4+2ab\,x^2+a^2)}{15x^5(b\,x^2+a)^2}$	54

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^6}\,dx = -\frac{15\,b^2x^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="fricas")
```

output
```
-1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^6}\,dx = \frac{-3a^2 - 10abx^2 - 15b^2x^4}{15x^5}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/x**6,x)
```

output
```
(-3*a**2 - 10*a*b*x**2 - 15*b**2*x**4)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^6}\,dx = -\frac{15\,b^2x^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="maxima")
```

output
```
-1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^6} \, dx = -\frac{15\, b^2x^4 + 10\, abx^2 + 3\, a^2}{15\, x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^6,x, algorithm="giac")
```

output
```
-1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^6} \, dx = -\frac{\frac{a^2}{5} + \frac{2\,a\,b\,x^2}{3} + b^2\,x^4}{x^5}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^6,x)
```

output
```
-(a^2/5 + b^2*x^4 + (2*a*b*x^2)/3)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^6} \, dx = \frac{-15b^2x^4 - 10ab\,x^2 - 3a^2}{15x^5}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^6,x)
```

output
```
( - 3*a**2 - 10*a*b*x**2 - 15*b**2*x**4)/(15*x**5)
```

3.300 $\int \frac{a^2+2abx^2+b^2x^4}{x^8} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 30

$$\int \frac{a^2+2abx^2+b^2x^4}{x^8} \, dx = -\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

output
```
-1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{a^2+2abx^2+b^2x^4}{x^8} \, dx = -\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]
```

output
```
-1/7*a^2/x^7 - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{x^8}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{x^8}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{x^8}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]
```

output
```
-1/7*a^2/x^7 - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$	25
norman	$\frac{-\frac{1}{3}b^2x^4-\frac{2}{5}ab\,x^2-\frac{1}{7}a^2}{x^7}$	26
risch	$\frac{-\frac{1}{3}b^2x^4-\frac{2}{5}ab\,x^2-\frac{1}{7}a^2}{x^7}$	26
gosper	$-\frac{35b^2x^4+42ab\,x^2+15a^2}{105x^7}$	27
parallelrisch	$\frac{-35b^2x^4-42ab\,x^2-15a^2}{105x^7}$	27
orering	$-\frac{(35b^2x^4+42ab\,x^2+15a^2)(b^2x^4+2ab\,x^2+a^2)}{105x^7(b\,x^2+a)^2}$	54

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^8} \, dx = -\frac{35\, b^2x^4 + 42\, abx^2 + 15\, a^2}{105\, x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="fricas")
```

output
```
-1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^8} \, dx = \frac{-15a^2 - 42abx^2 - 35b^2x^4}{105x^7}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/x**8,x)
```

output
```
(-15*a**2 - 42*a*b*x**2 - 35*b**2*x**4)/(105*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^8} \, dx = -\frac{35\, b^2x^4 + 42\, abx^2 + 15\, a^2}{105\, x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="maxima")
```

output
```
-1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^8} \, dx = -\frac{35\, b^2 x^4 + 42\, abx^2 + 15\, a^2}{105\, x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/x^8,x, algorithm="giac")
```

output
```
-1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^8} \, dx = -\frac{\frac{a^2}{7} + \frac{2\,a\,b\,x^2}{5} + \frac{b^2\,x^4}{3}}{x^7}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/x^8,x)
```

output
```
-(a^2/7 + (b^2*x^4)/3 + (2*a*b*x^2)/5)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{x^8} \, dx = \frac{-35b^2x^4 - 42ab\,x^2 - 15a^2}{105x^7}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/x^8,x)
```

output
```
( - 15*a**2 - 42*a*b*x**2 - 35*b**2*x**4)/(105*x**7)
```

3.301 $\int x^9(a^2+2abx^2+b^2x^4)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int x^9\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^{10}}{10}+\frac{1}{3}a^3bx^{12}+\frac{3}{7}a^2b^2x^{14}+\frac{1}{4}ab^3x^{16}+\frac{b^4x^{18}}{18}$$

output
```
1/10*a^4*x^10+1/3*a^3*b*x^12+3/7*a^2*b^2*x^14+1/4*a*b^3*x^16+1/18*b^4*x^18
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int x^9\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^{10}}{10}+\frac{1}{3}a^3bx^{12}+\frac{3}{7}a^2b^2x^{14}+\frac{1}{4}ab^3x^{16}+\frac{b^4x^{18}}{18}$$

input
```
Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^4*x^10)/10 + (a^3*b*x^12)/3 + (3*a^2*b^2*x^14)/7 + (a*b^3*x^16)/4 + (b^
4*x^18)/18
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int b^4x^9 \left(bx^2+a\right)^4 dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int x^9 \left(a+bx^2\right)^4 dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int x^8 \left(bx^2+a\right)^4 dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(b^4x^{16}+4ab^3x^{14}+6a^2b^2x^{12}+4a^3bx^{10}+a^4x^8\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{a^4x^{10}}{5}+\frac{2}{3}a^3bx^{12}+\frac{6}{7}a^2b^2x^{14}+\frac{1}{2}ab^3x^{16}+\frac{b^4x^{18}}{9}\right)$$

input
```
Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
((a^4*x^10)/5 + (2*a^3*b*x^12)/3 + (6*a^2*b^2*x^14)/7 + (a*b^3*x^16)/2 + (
b^4*x^18)/9)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{1}{10}a^4x^{10}+\frac{1}{3}a^3b\,x^{12}+\frac{3}{7}a^2b^2x^{14}+\frac{1}{4}a\,b^3x^{16}+\frac{1}{18}b^4x^{18}$	47
norman	$\frac{1}{10}a^4x^{10}+\frac{1}{3}a^3b\,x^{12}+\frac{3}{7}a^2b^2x^{14}+\frac{1}{4}a\,b^3x^{16}+\frac{1}{18}b^4x^{18}$	47
risch	$\frac{1}{10}a^4x^{10}+\frac{1}{3}a^3b\,x^{12}+\frac{3}{7}a^2b^2x^{14}+\frac{1}{4}a\,b^3x^{16}+\frac{1}{18}b^4x^{18}$	47
parallelrisch	$\frac{1}{10}a^4x^{10}+\frac{1}{3}a^3b\,x^{12}+\frac{3}{7}a^2b^2x^{14}+\frac{1}{4}a\,b^3x^{16}+\frac{1}{18}b^4x^{18}$	47
gosper	$\frac{x^{10}\left(70b^4x^8+315a\,b^3x^6+540a^2b^2x^4+420a^3b\,x^2+126a^4\right)}{1260}$	49
orering	$\frac{x^{10}\left(70b^4x^8+315a\,b^3x^6+540a^2b^2x^4+420a^3b\,x^2+126a^4\right)\left(b^2x^4+2ab\,x^2+a^2\right)^2}{1260\left(b\,x^2+a\right)^4}$	78

input
```
int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*a^4*x^10+1/3*a^3*b*x^12+3/7*a^2*b^2*x^14+1/4*a*b^3*x^16+1/18*b^4*x^18
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{1}{18}\,b^4x^{18}+\frac{1}{4}\,ab^3x^{16}+\frac{3}{7}\,a^2b^2x^{14}+\frac{1}{3}\,a^3bx^{12}+\frac{1}{10}\,a^4x^{10}$$

input
```
integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/18*b^4*x^18 + 1/4*a*b^3*x^16 + 3/7*a^2*b^2*x^14 + 1/3*a^3*b*x^12 + 1/10*
a^4*x^10
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{a^4x^{10}}{10}+\frac{a^3bx^{12}}{3}+\frac{3a^2b^2x^{14}}{7}+\frac{ab^3x^{16}}{4}+\frac{b^4x^{18}}{18}$$

input
```
integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
a**4*x**10/10 + a**3*b*x**12/3 + 3*a**2*b**2*x**14/7 + a*b**3*x**16/4 + b*
*4*x**18/18
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^9 (a^2 + 2abx^2 + b^2x^4)^2 \, dx = \frac{1}{18} b^4x^{18} + \frac{1}{4} ab^3x^{16} + \frac{3}{7} a^2b^2x^{14} + \frac{1}{3} a^3bx^{12} + \frac{1}{10} a^4x^{10}$$

input
```
integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/18*b^4*x^18 + 1/4*a*b^3*x^16 + 3/7*a^2*b^2*x^14 + 1/3*a^3*b*x^12 + 1/10*
a^4*x^10
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^9 (a^2 + 2abx^2 + b^2x^4)^2 \, dx = \frac{1}{18} b^4x^{18} + \frac{1}{4} ab^3x^{16} + \frac{3}{7} a^2b^2x^{14} + \frac{1}{3} a^3bx^{12} + \frac{1}{10} a^4x^{10}$$

input
```
integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/18*b^4*x^18 + 1/4*a*b^3*x^16 + 3/7*a^2*b^2*x^14 + 1/3*a^3*b*x^12 + 1/10*
a^4*x^10
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^9 (a^2 + 2abx^2 + b^2x^4)^2 \, dx = \frac{a^4 x^{10}}{10} + \frac{a^3 b x^{12}}{3} + \frac{3 a^2 b^2 x^{14}}{7} + \frac{a b^3 x^{16}}{4} + \frac{b^4 x^{18}}{18}$$

input
```
int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(a^4*x^10)/10 + (b^4*x^18)/18 + (a^3*b*x^12)/3 + (a*b^3*x^16)/4 + (3*a^2*b
^2*x^14)/7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{x^{10}(70b^4x^8 + 315a\,b^3x^6 + 540a^2b^2x^4 + 420a^3b\,x^2 + 126a^4)}{1260}$$

input
```
int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(x**10*(126*a**4 + 420*a**3*b*x**2 + 540*a**2*b**2*x**4 + 315*a*b**3*x**6
+ 70*b**4*x**8))/1260
```

3.302 $\int x^7(a^2+2abx^2+b^2x^4)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^8}{8}+\frac{2}{5}a^3bx^{10}+\frac{1}{2}a^2b^2x^{12}+\frac{2}{7}ab^3x^{14}+\frac{b^4x^{16}}{16}$$

output
```
1/8*a^4*x^8+2/5*a^3*b*x^10+1/2*a^2*b^2*x^12+2/7*a*b^3*x^14+1/16*b^4*x^16
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^8}{8}+\frac{2}{5}a^3bx^{10}+\frac{1}{2}a^2b^2x^{12}+\frac{2}{7}ab^3x^{14}+\frac{b^4x^{16}}{16}$$

input
```
Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^4*x^8)/8 + (2*a^3*b*x^10)/5 + (a^2*b^2*x^12)/2 + (2*a*b^3*x^14)/7 + (b^
4*x^16)/16
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^2 dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^4x^7(bx^2+a)^4dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int x^7 \left(a+bx^2\right)^4 dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^6 \left(bx^2+a\right)^4 dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(b^4x^{14}+4ab^3x^{12}+6a^2b^2x^{10}+4a^3bx^8+a^4x^6\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^4x^8}{4}+\frac{4}{5}a^3bx^{10}+a^2b^2x^{12}+\frac{4}{7}ab^3x^{14}+\frac{b^4x^{16}}{8}\right)$$

input
```
Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
((a^4*x^8)/4 + (4*a^3*b*x^10)/5 + a^2*b^2*x^12 + (4*a*b^3*x^14)/7 + (b^4*x
^16)/8)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{1}{8}a^4x^8+\frac{2}{5}a^3b\,x^{10}+\frac{1}{2}a^2b^2x^{12}+\frac{2}{7}a\,b^3x^{14}+\frac{1}{16}b^4x^{16}$	47
norman	$\frac{1}{8}a^4x^8+\frac{2}{5}a^3b\,x^{10}+\frac{1}{2}a^2b^2x^{12}+\frac{2}{7}a\,b^3x^{14}+\frac{1}{16}b^4x^{16}$	47
risch	$\frac{1}{8}a^4x^8+\frac{2}{5}a^3b\,x^{10}+\frac{1}{2}a^2b^2x^{12}+\frac{2}{7}a\,b^3x^{14}+\frac{1}{16}b^4x^{16}$	47
parallelrisch	$\frac{1}{8}a^4x^8+\frac{2}{5}a^3b\,x^{10}+\frac{1}{2}a^2b^2x^{12}+\frac{2}{7}a\,b^3x^{14}+\frac{1}{16}b^4x^{16}$	47
gosper	$\frac{x^8\left(35b^4x^8+160a\,b^3x^6+280a^2b^2x^4+224a^3b\,x^2+70a^4\right)}{560}$	49
orering	$\frac{x^8\left(35b^4x^8+160a\,b^3x^6+280a^2b^2x^4+224a^3b\,x^2+70a^4\right)\left(b^2x^4+2ab\,x^2+a^2\right)^2}{560\left(b\,x^2+a\right)^4}$	78

input
```
int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^4*x^8+2/5*a^3*b*x^10+1/2*a^2*b^2*x^12+2/7*a*b^3*x^14+1/16*b^4*x^16
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{1}{16}\,b^4x^{16}+\frac{2}{7}\,ab^3x^{14}+\frac{1}{2}\,a^2b^2x^{12}+\frac{2}{5}\,a^3bx^{10}+\frac{1}{8}\,a^4x^8$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/16*b^4*x^16 + 2/7*a*b^3*x^14 + 1/2*a^2*b^2*x^12 + 2/5*a^3*b*x^10 + 1/8*a
^4*x^8
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^8}{8}+\frac{2a^3bx^{10}}{5}+\frac{a^2b^2x^{12}}{2}+\frac{2ab^3x^{14}}{7}+\frac{b^4x^{16}}{16}$$

input
```
integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
a**4*x**8/8 + 2*a**3*b*x**10/5 + a**2*b**2*x**12/2 + 2*a*b**3*x**14/7 + b*
*4*x**16/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^7 (a^2 + 2abx^2 + b^2x^4)^2 \, dx = \frac{1}{16} b^4 x^{16} + \frac{2}{7} ab^3 x^{14} + \frac{1}{2} a^2 b^2 x^{12} + \frac{2}{5} a^3 b x^{10} + \frac{1}{8} a^4 x^8$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/16*b^4*x^16 + 2/7*a*b^3*x^14 + 1/2*a^2*b^2*x^12 + 2/5*a^3*b*x^10 + 1/8*a
^4*x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^7 (a^2 + 2abx^2 + b^2x^4)^2 \, dx = \frac{1}{16} b^4 x^{16} + \frac{2}{7} ab^3 x^{14} + \frac{1}{2} a^2 b^2 x^{12} + \frac{2}{5} a^3 b x^{10} + \frac{1}{8} a^4 x^8$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/16*b^4*x^16 + 2/7*a*b^3*x^14 + 1/2*a^2*b^2*x^12 + 2/5*a^3*b*x^10 + 1/8*a
^4*x^8
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^7 (a^2 + 2abx^2 + b^2x^4)^2 \, dx = \frac{a^4 x^8}{8} + \frac{2 a^3 b x^{10}}{5} + \frac{a^2 b^2 x^{12}}{2} + \frac{2 a b^3 x^{14}}{7} + \frac{b^4 x^{16}}{16}$$

input
```
int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(a^4*x^8)/8 + (b^4*x^16)/16 + (2*a^3*b*x^10)/5 + (2*a*b^3*x^14)/7 + (a^2*b
^2*x^12)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{x^8(35b^4x^8+160a\,b^3x^6+280a^2b^2x^4+224a^3b\,x^2+70a^4)}{560}$$

input
```
int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(x**8*(70*a**4 + 224*a**3*b*x**2 + 280*a**2*b**2*x**4 + 160*a*b**3*x**6 +
35*b**4*x**8))/560
```

3.303 $\int x^5(a^2+2abx^2+b^2x^4)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 53

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^2(a+bx^2)^5}{10b^3}-\frac{a(a+bx^2)^6}{6b^3}+\frac{(a+bx^2)^7}{14b^3}$$

output
```
1/10*a^2*(b*x^2+a)^5/b^3-1/6*a*(b*x^2+a)^6/b^3+1/14*(b*x^2+a)^7/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^6}{6}+\frac{1}{2}a^3bx^8+\frac{3}{5}a^2b^2x^{10}+\frac{1}{3}ab^3x^{12}+\frac{b^4x^{14}}{14}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^4*x^6)/6 + (a^3*b*x^8)/2 + (3*a^2*b^2*x^10)/5 + (a*b^3*x^12)/3 + (b^4*x
^14)/14
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^2 dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^4x^5 \left(bx^2+a\right)^4dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int x^5 \left(a+bx^2\right)^4dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^4 \left(bx^2+a\right)^4dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^6}{b^2}-\frac{2a\left(bx^2+a\right)^5}{b^2}+\frac{a^2\left(bx^2+a\right)^4}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^2\left(a+bx^2\right)^5}{5b^3}+\frac{\left(a+bx^2\right)^7}{7b^3}-\frac{a\left(a+bx^2\right)^6}{3b^3}\right)$$

input
```
Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
((a^2*(a + b*x^2)^5)/(5*b^3) - (a*(a + b*x^2)^6)/(3*b^3) + (a + b*x^2)^7/(
7*b^3))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{1}{6}a^4x^6+\frac{1}{2}a^3b\,x^8+\frac{3}{5}a^2b^2x^{10}+\frac{1}{3}b^3a\,x^{12}+\frac{1}{14}b^4x^{14}$	47
norman	$\frac{1}{6}a^4x^6+\frac{1}{2}a^3b\,x^8+\frac{3}{5}a^2b^2x^{10}+\frac{1}{3}b^3a\,x^{12}+\frac{1}{14}b^4x^{14}$	47
risch	$\frac{1}{6}a^4x^6+\frac{1}{2}a^3b\,x^8+\frac{3}{5}a^2b^2x^{10}+\frac{1}{3}b^3a\,x^{12}+\frac{1}{14}b^4x^{14}$	47
parallelrisch	$\frac{1}{6}a^4x^6+\frac{1}{2}a^3b\,x^8+\frac{3}{5}a^2b^2x^{10}+\frac{1}{3}b^3a\,x^{12}+\frac{1}{14}b^4x^{14}$	47
gosper	$\frac{x^6(15b^4x^8+70a\,b^3x^6+126a^2b^2x^4+105a^3b\,x^2+35a^4)}{210}$	49
orering	$\frac{x^6(15b^4x^8+70a\,b^3x^6+126a^2b^2x^4+105a^3b\,x^2+35a^4)(b^2x^4+2ab\,x^2+a^2)^2}{210(b\,x^2+a)^4}$	78

input
```
int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^4*x^6+1/2*a^3*b*x^8+3/5*a^2*b^2*x^10+1/3*b^3*a*x^12+1/14*b^4*x^14
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{1}{14}\,b^4x^{14}+\frac{1}{3}\,ab^3x^{12}+\frac{3}{5}\,a^2b^2x^{10}+\frac{1}{2}\,a^3bx^8+\frac{1}{6}\,a^4x^6$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8 + 1/6*a^
4*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^6}{6}+\frac{a^3bx^8}{2}+\frac{3a^2b^2x^{10}}{5}+\frac{ab^3x^{12}}{3}+\frac{b^4x^{14}}{14}$$

input
```
integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
a**4*x**6/6 + a**3*b*x**8/2 + 3*a**2*b**2*x**10/5 + a*b**3*x**12/3 + b**4*
x**14/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{1}{14}\,b^4x^{14}+\frac{1}{3}\,ab^3x^{12}+\frac{3}{5}\,a^2b^2x^{10}+\frac{1}{2}\,a^3bx^8+\frac{1}{6}\,a^4x^6$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8 + 1/6*a^
4*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{1}{14}\,b^4x^{14}+\frac{1}{3}\,ab^3x^{12}+\frac{3}{5}\,a^2b^2x^{10}+\frac{1}{2}\,a^3bx^8+\frac{1}{6}\,a^4x^6$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8 + 1/6*a^
4*x^6
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4\,x^6}{6}+\frac{a^3\,b\,x^8}{2}+\frac{3\,a^2\,b^2\,x^{10}}{5}+\frac{a\,b^3\,x^{12}}{3}+\frac{b^4\,x^{14}}{14}$$

input
```
int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(a^4*x^6)/6 + (b^4*x^14)/14 + (a^3*b*x^8)/2 + (a*b^3*x^12)/3 + (3*a^2*b^2*
x^10)/5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int x^5 (a^2 + 2abx^2 + b^2x^4)^2 \, dx = \frac{x^6(15b^4x^8 + 70a\,b^3x^6 + 126a^2b^2x^4 + 105a^3b\,x^2 + 35a^4)}{210}$$

input
```
int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(x**6*(35*a**4 + 105*a**3*b*x**2 + 126*a**2*b**2*x**4 + 70*a*b**3*x**6 + 1
5*b**4*x**8))/210
```

3.304 $\int x^3(a^2+2abx^2+b^2x^4)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^2\,dx=-\frac{a(a+bx^2)^5}{10b^2}+\frac{(a+bx^2)^6}{12b^2}$$

output
```
-1/10*a*(b*x^2+a)^5/b^2+1/12*(b*x^2+a)^6/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^4}{4}+\frac{2}{3}a^3bx^6+\frac{3}{4}a^2b^2x^8+\frac{2}{5}ab^3x^{10}+\frac{b^4x^{12}}{12}$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^4*x^4)/4 + (2*a^3*b*x^6)/3 + (3*a^2*b^2*x^8)/4 + (2*a*b^3*x^10)/5 + (b^
4*x^12)/12
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int b^4x^3\left(bx^2+a\right)^4dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int x^3\left(a+bx^2\right)^4dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int x^2\left(bx^2+a\right)^4dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^5}{b}-\frac{a\left(bx^2+a\right)^4}{b}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^6}{6b^2}-\frac{a\left(a+bx^2\right)^5}{5b^2}\right)$$

input
```
Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(-1/5*(a*(a + b*x^2)^5)/b^2 + (a + b*x^2)^6/(6*b^2))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{1}{12}b^4x^{12}+\frac{2}{5}a\,b^3x^{10}+\frac{3}{4}a^2b^2x^8+\frac{2}{3}a^3b\,x^6+\frac{1}{4}a^4x^4$	47
norman	$\frac{1}{12}b^4x^{12}+\frac{2}{5}a\,b^3x^{10}+\frac{3}{4}a^2b^2x^8+\frac{2}{3}a^3b\,x^6+\frac{1}{4}a^4x^4$	47
risch	$\frac{1}{12}b^4x^{12}+\frac{2}{5}a\,b^3x^{10}+\frac{3}{4}a^2b^2x^8+\frac{2}{3}a^3b\,x^6+\frac{1}{4}a^4x^4$	47
parallelrisch	$\frac{1}{12}b^4x^{12}+\frac{2}{5}a\,b^3x^{10}+\frac{3}{4}a^2b^2x^8+\frac{2}{3}a^3b\,x^6+\frac{1}{4}a^4x^4$	47
gosper	$\frac{x^4(5b^4x^8+24a\,b^3x^6+45a^2b^2x^4+40a^3b\,x^2+15a^4)}{60}$	49
orering	$\frac{x^4(5b^4x^8+24a\,b^3x^6+45a^2b^2x^4+40a^3b\,x^2+15a^4)(b^2x^4+2ab\,x^2+a^2)^2}{60(b\,x^2+a)^4}$	78

input
```
int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^4*x^12+2/5*a*b^3*x^10+3/4*a^2*b^2*x^8+2/3*a^3*b*x^6+1/4*a^4*x^4
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{1}{12}\,b^4x^{12}+\frac{2}{5}\,ab^3x^{10}+\frac{3}{4}\,a^2b^2x^8+\frac{2}{3}\,a^3bx^6+\frac{1}{4}\,a^4x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6 + 1/4*a^4
*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{a^4x^4}{4}+\frac{2a^3bx^6}{3}+\frac{3a^2b^2x^8}{4}+\frac{2ab^3x^{10}}{5}+\frac{b^4x^{12}}{12}$$

input
```
integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
a**4*x**4/4 + 2*a**3*b*x**6/3 + 3*a**2*b**2*x**8/4 + 2*a*b**3*x**10/5 + b*
*4*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{1}{12}\,b^4x^{12}+\frac{2}{5}\,ab^3x^{10}+\frac{3}{4}\,a^2b^2x^8+\frac{2}{3}\,a^3bx^6+\frac{1}{4}\,a^4x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6 + 1/4*a^4
*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{1}{12}\,b^4x^{12}+\frac{2}{5}\,ab^3x^{10}+\frac{3}{4}\,a^2b^2x^8+\frac{2}{3}\,a^3bx^6+\frac{1}{4}\,a^4x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6 + 1/4*a^4
*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{a^4\,x^4}{4}+\frac{2\,a^3\,b\,x^6}{3}+\frac{3\,a^2\,b^2\,x^8}{4}+\frac{2\,a\,b^3\,x^{10}}{5}+\frac{b^4\,x^{12}}{12}$$

input
```
int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(a^4*x^4)/4 + (b^4*x^12)/12 + (2*a^3*b*x^6)/3 + (2*a*b^3*x^10)/5 + (3*a^2*
b^2*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{x^4(5b^4x^8+24a\,b^3x^6+45a^2b^2x^4+40a^3b\,x^2+15a^4)}{60}$$

input
```
int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(x**4*(15*a**4 + 40*a**3*b*x**2 + 45*a**2*b**2*x**4 + 24*a*b**3*x**6 + 5*b
**4*x**8))/60
```

3.305 $\int x(a^2+2abx^2+b^2x^4)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int x\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{\left(a+bx^2\right)^5}{10b}$$

output
```
1/10*(b*x^2+a)^5/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{\left(a+bx^2\right)^5}{10b}$$

input
```
Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a + b*x^2)^5/(10*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1380, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx^2+b^2x^4\right)^2\,dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^4x\left(bx^2+a\right)^4dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int x\left(a+bx^2\right)^4dx$$

$$\Big\downarrow \text{241}$$

$$\frac{\left(a+bx^2\right)^5}{10b}$$

input
```
Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a + b*x^2)^5/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(14) = 28.

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

method	result	size
default	$\frac{1}{10}b^4x^{10}+\frac{1}{2}a\,b^3x^8+a^2b^2x^6+a^3b\,x^4+\frac{1}{2}a^4x^2$	45
norman	$\frac{1}{10}b^4x^{10}+\frac{1}{2}a\,b^3x^8+a^2b^2x^6+a^3b\,x^4+\frac{1}{2}a^4x^2$	45
risch	$\frac{1}{10}b^4x^{10}+\frac{1}{2}a\,b^3x^8+a^2b^2x^6+a^3b\,x^4+\frac{1}{2}a^4x^2$	45
parallelrisch	$\frac{1}{10}b^4x^{10}+\frac{1}{2}a\,b^3x^8+a^2b^2x^6+a^3b\,x^4+\frac{1}{2}a^4x^2$	45
gosper	$\frac{x^2(b^4x^8+5a\,b^3x^6+10a^2b^2x^4+10a^3b\,x^2+5a^4)}{10}$	48
orering	$\frac{x^2(b^4x^8+5a\,b^3x^6+10a^2b^2x^4+10a^3b\,x^2+5a^4)(b^2x^4+2ab\,x^2+a^2)^2}{10(b\,x^2+a)^4}$	77

input
```
int(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*b^4*x^10+1/2*a*b^3*x^8+a^2*b^2*x^6+a^3*b*x^4+1/2*a^4*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(14) = 28.

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int x\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{1}{10}\,b^4x^{10}+\frac{1}{2}\,ab^3x^8+a^2b^2x^6+a^3bx^4+\frac{1}{2}\,a^4x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int x\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^2}{2}+a^3bx^4+a^2b^2x^6+\frac{ab^3x^8}{2}+\frac{b^4x^{10}}{10}$$

input
```
integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
a**4*x**2/2 + a**3*b*x**4 + a**2*b**2*x**6 + a*b**3*x**8/2 + b**4*x**10/10
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int x\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{1}{10}\,b^4x^{10}+\frac{1}{2}\,ab^3x^8+a^2b^2x^6+a^3bx^4+\frac{1}{2}\,a^4x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int x\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{1}{10}\,b^4x^{10}+\frac{1}{2}\,ab^3x^8+a^2b^2x^6+a^3bx^4+\frac{1}{2}\,a^4x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int x\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4\,x^2}{2}+a^3\,b\,x^4+a^2\,b^2\,x^6+\frac{a\,b^3\,x^8}{2}+\frac{b^4\,x^{10}}{10}$$

input
```
int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(a^4*x^2)/2 + (b^4*x^10)/10 + a^3*b*x^4 + (a*b^3*x^8)/2 + a^2*b^2*x^6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int x\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{x^2(b^4x^8+5a\,b^3x^6+10a^2b^2x^4+10a^3b\,x^2+5a^4)}{10}$$

input
```
int(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(x**2*(5*a**4 + 10*a**3*b*x**2 + 10*a**2*b**2*x**4 + 5*a*b**3*x**6 + b**4*
x**8))/10
```

3.306 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x}\,dx = 2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}+a^4\log(x)$$

output
```
2*a^3*b*x^2+3/2*a^2*b^2*x^4+2/3*a*b^3*x^6+1/8*b^4*x^8+a^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x}\,dx = 2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}+a^4\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

output
```
2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 + a^4*Log[
x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^4}{x^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(b^4x^6+4ab^3x^4+6a^2b^2x^2+4a^3b+\frac{a^4}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^4\log(x^2)+4a^3bx^2+3a^2b^2x^4+\frac{4}{3}ab^3x^6+\frac{b^4x^8}{4}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

output
```
(4*a^3*b*x^2 + 3*a^2*b^2*x^4 + (4*a*b^3*x^6)/3 + (b^4*x^8)/4 + a^4*Log[x^2
])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
default	$2a^{3}b\,x^{2}+\frac{3a^{2}b^{2}x^{4}}{2}+\frac{2a\,b^{3}x^{6}}{3}+\frac{b^{4}x^{8}}{8}+a^{4}\ln\left(x\right)$	45
norman	$2a^{3}b\,x^{2}+\frac{3a^{2}b^{2}x^{4}}{2}+\frac{2a\,b^{3}x^{6}}{3}+\frac{b^{4}x^{8}}{8}+a^{4}\ln\left(x\right)$	45
parallelrisch	$2a^{3}b\,x^{2}+\frac{3a^{2}b^{2}x^{4}}{2}+\frac{2a\,b^{3}x^{6}}{3}+\frac{b^{4}x^{8}}{8}+a^{4}\ln\left(x\right)$	45
risch	$\frac{3a^{2}b^{2}x^{4}}{2}+2a^{3}b\,x^{2}+\frac{4a^{4}}{3}+\frac{2a\,b^{3}x^{6}}{3}+\frac{b^{4}x^{8}}{8}+a^{4}\ln\left(x\right)$	50

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
2*a^3*b*x^2+3/2*a^2*b^2*x^4+2/3*a*b^3*x^6+1/8*b^4*x^8+a^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x} \, dx = \frac{1}{8} \, b^4x^8 + \frac{2}{3} \, ab^3x^6 + \frac{3}{2} \, a^2b^2x^4 + 2 \, a^3bx^2 + a^4 \log(x)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="fricas")
```

output
```
1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x} \, dx = a^4 \log(x) + 2a^3bx^2 + \frac{3a^2b^2x^4}{2} + \frac{2ab^3x^6}{3} + \frac{b^4x^8}{8}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x,x)
```

output
```
a**4*log(x) + 2*a**3*b*x**2 + 3*a**2*b**2*x**4/2 + 2*a*b**3*x**6/3 + b**4*
x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x} \, dx = \frac{1}{8} \, b^4x^8 + \frac{2}{3} \, ab^3x^6 + \frac{3}{2} \, a^2b^2x^4 + 2 \, a^3bx^2 + \frac{1}{2} \, a^4 \log(x^2)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="maxima")
```

output
```
1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + 1/2*a^4*log(
x^2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x}\,dx = \frac{1}{8}\,b^4x^8+\frac{2}{3}\,ab^3x^6+\frac{3}{2}\,a^2b^2x^4+2\,a^3bx^2+\frac{1}{2}\,a^4\log\left(x^2\right)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x,x, algorithm="giac")
```

output
```
1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + 1/2*a^4*log(
x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x}\,dx = a^4\,\ln\left(x\right)+\frac{b^4\,x^8}{8}+2\,a^3\,b\,x^2+\frac{2\,a\,b^3\,x^6}{3}+\frac{3\,a^2\,b^2\,x^4}{2}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x,x)
```

output
```
a^4*log(x) + (b^4*x^8)/8 + 2*a^3*b*x^2 + (2*a*b^3*x^6)/3 + (3*a^2*b^2*x^4)
/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x}\,dx = \log(x)\,a^4+2a^3b\,x^2+\frac{3a^2b^2x^4}{2}+\frac{2a\,b^3x^6}{3}+\frac{b^4x^8}{8}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x,x)
```

output
```
(24*log(x)*a**4 + 48*a**3*b*x**2 + 36*a**2*b**2*x**4 + 16*a*b**3*x**6 + 3*
b**4*x**8)/24
```

3.307 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3} dx = -\frac{a^4}{2x^2} + 3a^2b^2x^2 + ab^3x^4 + \frac{b^4x^6}{6} + 4a^3b\log(x)$$

output
```
-1/2*a^4/x^2+3*a^2*b^2*x^2+a*b^3*x^4+1/6*b^4*x^6+4*a^3*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3} dx = -\frac{a^4}{2x^2} + 3a^2b^2x^2 + ab^3x^4 + \frac{b^4x^6}{6} + 4a^3b\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
```

output
```
-1/2*a^4/x^2 + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*Log[x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^3}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^4}{x^4}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^4}{x^4}+\frac{4ba^3}{x^2}+6b^2a^2+4b^3x^2a+b^4x^4\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^4}{x^2}+4a^3b\log\left(x^2\right)+6a^2b^2x^2+2ab^3x^4+\frac{b^4x^6}{3}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
```

output
```
(-(a^4/x^2) + 6*a^2*b^2*x^2 + 2*a*b^3*x^4 + (b^4*x^6)/3 + 4*a^3*b*Log[x^2]
)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a^{4}}{2 x^{2}}+3 a^{2} b^{2} x^{2}+a\,b^{3} x^{4}+\frac{b^{4} x^{6}}{6}+4 a^{3} b \ln \left(x \right)$	45
risch	$-\frac{a^{4}}{2 x^{2}}+3 a^{2} b^{2} x^{2}+a\,b^{3} x^{4}+\frac{b^{4} x^{6}}{6}+4 a^{3} b \ln \left(x \right)$	45
norman	$\frac{a\,b^{3} x^{6}-\frac{1}{2} a^{4}+\frac{1}{6} b^{4} x^{8}+3 a^{2} b^{2} x^{4}}{x^{2}}+4 a^{3} b \ln \left(x \right)$	47
parallelrisch	$\frac{b^{4} x^{8}+6 a\,b^{3} x^{6}+18 a^{2} b^{2} x^{4}+24 a^{3} b \ln \left(x \right) x^{2}-3 a^{4}}{6 x^{2}}$	50

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^4/x^2+3*a^2*b^2*x^2+a*b^3*x^4+1/6*b^4*x^6+4*a^3*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3}\,dx = \frac{b^4x^8+6\,ab^3x^6+18\,a^2b^2x^4+24\,a^3bx^2\log{(x)}-3\,a^4}{6\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="fricas")
```

output
```
1/6*(b^4*x^8 + 6*a*b^3*x^6 + 18*a^2*b^2*x^4 + 24*a^3*b*x^2*log(x) - 3*a^4)
/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3}\,dx = -\frac{a^4}{2x^2}+4a^3b\log{(x)}+3a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**3,x)
```

output
```
-a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**2 + a*b**3*x**4 + b**4*x
**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3}\,dx = \frac{1}{6}\,b^4x^6+ab^3x^4+3\,a^2b^2x^2+2\,a^3b\log\left(x^2\right)-\frac{a^4}{2\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="maxima")
```

output
```
1/6*b^4*x^6 + a*b^3*x^4 + 3*a^2*b^2*x^2 + 2*a^3*b*log(x^2) - 1/2*a^4/x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3}\,dx = \frac{1}{6}\,b^4x^6+ab^3x^4+3\,a^2b^2x^2+2\,a^3b\log\left(x^2\right)-\frac{4\,a^3bx^2+a^4}{2\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x, algorithm="giac")
```

output
```
1/6*b^4*x^6 + a*b^3*x^4 + 3*a^2*b^2*x^2 + 2*a^3*b*log(x^2) - 1/2*(4*a^3*b*
x^2 + a^4)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3}\,dx = \frac{b^4\,x^6}{6}-\frac{a^4}{2\,x^2}+a\,b^3\,x^4+4\,a^3\,b\,\ln\left(x\right)+3\,a^2\,b^2\,x^2$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^3,x)
```

output
```
(b^4*x^6)/6 - a^4/(2*x^2) + a*b^3*x^4 + 4*a^3*b*log(x) + 3*a^2*b^2*x^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3}\,dx = \frac{24\log(x)\,a^3b\,x^2-3a^4+18a^2b^2x^4+6a\,b^3x^6+b^4x^8}{6x^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x)
```

output
```
(24*log(x)*a**3*b*x**2 - 3*a**4 + 18*a**2*b**2*x**4 + 6*a*b**3*x**6 + b**4
*x**8)/(6*x**2)
```

3.308 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5}\,dx=-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+2ab^3x^2+\frac{b^4x^4}{4}+6a^2b^2\log(x)$$

output
```
-1/4*a^4/x^4-2*a^3*b/x^2+2*a*b^3*x^2+1/4*b^4*x^4+6*a^2*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5}\,dx=-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+2ab^3x^2+\frac{b^4x^4}{4}+6a^2b^2\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
```

output
```
-1/4*a^4/x^4 - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*b^2*Log[x
]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^5}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^4}{x^6}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^4}{x^6}+\frac{4ba^3}{x^4}+\frac{6b^2a^2}{x^2}+4b^3a+b^4x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^4}{2x^4}-\frac{4a^3b}{x^2}+6a^2b^2\log(x^2)+4ab^3x^2+\frac{b^4x^4}{2}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
```

output
```
(-1/2*a^4/x^4 - (4*a^3*b)/x^2 + 4*a*b^3*x^2 + (b^4*x^4)/2 + 6*a^2*b^2*Log[
x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+2a\,b^3x^2+\frac{b^4x^4}{4}+6a^2b^2\ln\left(x\right)$	46
norman	$\frac{-\frac{1}{4}a^4+\frac{1}{4}b^4x^8+2a\,b^3x^6-2a^3b\,x^2}{x^4}+6a^2b^2\ln\left(x\right)$	48
parallelrisch	$\frac{b^4x^8+8a\,b^3x^6+24a^2b^2\ln\left(x\right)x^4-8a^3b\,x^2-a^4}{4x^4}$	50
risch	$\frac{b^4x^4}{4}+2a\,b^3x^2+4a^2b^2+\frac{-2a^3b\,x^2-\frac{1}{4}a^4}{x^4}+6a^2b^2\ln\left(x\right)$	56

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^4/x^4-2*a^3*b/x^2+2*a*b^3*x^2+1/4*b^4*x^4+6*a^2*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^5}\,dx = \frac{b^4x^8 + 8\,ab^3x^6 + 24\,a^2b^2x^4\log(x) - 8\,a^3bx^2 - a^4}{4\,x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="fricas")
```

output
```
1/4*(b^4*x^8 + 8*a*b^3*x^6 + 24*a^2*b^2*x^4*log(x) - 8*a^3*b*x^2 - a^4)/x^
4
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^5}\,dx = 6a^2b^2\log(x) + 2ab^3x^2 + \frac{b^4x^4}{4} + \frac{-a^4 - 8a^3bx^2}{4x^4}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**5,x)
```

output
```
6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**4/4 + (-a**4 - 8*a**3*b*x**2)
/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5}\,dx = \frac{1}{4}\,b^4x^4+2\,ab^3x^2+3\,a^2b^2\log\left(x^2\right)-\frac{8\,a^3bx^2+a^4}{4\,x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="maxima")
```

output
```
1/4*b^4*x^4 + 2*a*b^3*x^2 + 3*a^2*b^2*log(x^2) - 1/4*(8*a^3*b*x^2 + a^4)/x
^4
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5}\,dx = \frac{1}{4}\,b^4x^4+2\,ab^3x^2+3\,a^2b^2\log\left(x^2\right)-\frac{18\,a^2b^2x^4+8\,a^3bx^2+a^4}{4\,x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x, algorithm="giac")
```

output
```
1/4*b^4*x^4 + 2*a*b^3*x^2 + 3*a^2*b^2*log(x^2) - 1/4*(18*a^2*b^2*x^4 + 8*a
^3*b*x^2 + a^4)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5}\,dx = \frac{b^4\,x^4}{4}-\frac{\frac{a^4}{4}+2\,b\,a^3\,x^2}{x^4}+2\,a\,b^3\,x^2+6\,a^2\,b^2\,\ln\left(x\right)$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^5,x)
```

output
```
(b^4*x^4)/4 - (a^4/4 + 2*a^3*b*x^2)/x^4 + 2*a*b^3*x^2 + 6*a^2*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^5} \, dx = \frac{24 \log(x) \, a^2b^2x^4 - a^4 - 8a^3b \, x^2 + 8a \, b^3x^6 + b^4x^8}{4x^4}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x)
```

output
```
(24*log(x)*a**2*b**2*x**4 - a**4 - 8*a**3*b*x**2 + 8*a*b**3*x**6 + b**4*x*
*8)/(4*x**4)
```

3.309 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^7} dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^7} dx = -\frac{a^4}{6x^6} - \frac{a^3b}{x^4} - \frac{3a^2b^2}{x^2} + \frac{b^4x^2}{2} + 4ab^3\log(x)$$

output
```
-1/6*a^4/x^6-a^3*b/x^4-3*a^2*b^2/x^2+1/2*b^4*x^2+4*a*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^7} dx = -\frac{a^4}{6x^6} - \frac{a^3b}{x^4} - \frac{3a^2b^2}{x^2} + \frac{b^4x^2}{2} + 4ab^3\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]
```

output
```
-1/6*a^4/x^6 - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*b^3*Log[x
]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^7} \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^7}dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx^2)^4}{x^7}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^4}{x^8}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{a^4}{x^8}+\frac{4ba^3}{x^6}+\frac{6b^2a^2}{x^4}+\frac{4b^3a}{x^2}+b^4\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^4}{3x^6}-\frac{2a^3b}{x^4}-\frac{6a^2b^2}{x^2}+4ab^3\log\left(x^2\right)+b^4x^2\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]
```

output
```
(-1/3*a^4/x^6 - (2*a^3*b)/x^4 - (6*a^2*b^2)/x^2 + b^4*x^2 + 4*a*b^3*Log[x^
2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a^{4}}{6x^{6}}-\frac{a^{3}b}{x^{4}}-\frac{3a^{2}b^{2}}{x^{2}}+\frac{b^{4}x^{2}}{2}+4a\,b^{3}\ln\left(x\right)$	46
norman	$\frac{-\frac{1}{6}a^{4}+\frac{1}{2}b^{4}x^{8}-3a^{2}b^{2}x^{4}-a^{3}b\,x^{2}}{x^{6}}+4a\,b^{3}\ln\left(x\right)$	48
risch	$\frac{b^{4}x^{2}}{2}+\frac{-3a^{2}b^{2}x^{4}-a^{3}b\,x^{2}-\frac{1}{6}a^{4}}{x^{6}}+4a\,b^{3}\ln\left(x\right)$	48
parallelrisch	$\frac{3b^{4}x^{8}+24b^{3}a\ln\left(x\right)x^{6}-18a^{2}b^{2}x^{4}-6a^{3}b\,x^{2}-a^{4}}{6x^{6}}$	51

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^4/x^6-a^3*b/x^4-3*a^2*b^2/x^2+1/2*b^4*x^2+4*a*b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^7} \, dx = \frac{3 \, b^4x^8 + 24 \, ab^3x^6 \log(x) - 18 \, a^2b^2x^4 - 6 \, a^3bx^2 - a^4}{6 \, x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="fricas")
```

output
```
1/6*(3*b^4*x^8 + 24*a*b^3*x^6*log(x) - 18*a^2*b^2*x^4 - 6*a^3*b*x^2 - a^4)
/x^6
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^7} \, dx = 4ab^3 \log(x) + \frac{b^4x^2}{2} + \frac{-a^4 - 6a^3bx^2 - 18a^2b^2x^4}{6x^6}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**7,x)
```

output
```
4*a*b**3*log(x) + b**4*x**2/2 + (-a**4 - 6*a**3*b*x**2 - 18*a**2*b**2*x**4
)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^7}\,dx = \frac{1}{2}\,b^4x^2 + 2\,ab^3\log\left(x^2\right) - \frac{18\,a^2b^2x^4+6\,a^3bx^2+a^4}{6\,x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="maxima")
```

output
```
1/2*b^4*x^2 + 2*a*b^3*log(x^2) - 1/6*(18*a^2*b^2*x^4 + 6*a^3*b*x^2 + a^4)/
x^6
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^7}\,dx = \frac{1}{2}\,b^4x^2+2\,ab^3\log\left(x^2\right)-\frac{22\,ab^3x^6+18\,a^2b^2x^4+6\,a^3bx^2+a^4}{6\,x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x, algorithm="giac")
```

output
```
1/2*b^4*x^2 + 2*a*b^3*log(x^2) - 1/6*(22*a*b^3*x^6 + 18*a^2*b^2*x^4 + 6*a^
3*b*x^2 + a^4)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^7}\,dx = \frac{b^4\,x^2}{2} - \frac{\frac{a^4}{6}+a^3\,b\,x^2+3\,a^2\,b^2\,x^4}{x^6} + 4\,a\,b^3\,\ln\left(x\right)$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^7,x)
```

output
```
(b^4*x^2)/2 - (a^4/6 + a^3*b*x^2 + 3*a^2*b^2*x^4)/x^6 + 4*a*b^3*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^7}\,dx = \frac{24\log(x)\,a\,b^3x^6 - a^4 - 6a^3b\,x^2 - 18a^2b^2x^4 + 3b^4x^8}{6x^6}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x)
```

output
```
(24*log(x)*a*b**3*x**6 - a**4 - 6*a**3*b*x**2 - 18*a**2*b**2*x**4 + 3*b**4
*x**8)/(6*x**6)
```

3.310 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^9} dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^9} dx = -\frac{a^4}{8x^8} - \frac{2a^3b}{3x^6} - \frac{3a^2b^2}{2x^4} - \frac{2ab^3}{x^2} + b^4\log(x)$$

output
```
-1/8*a^4/x^8-2/3*a^3*b/x^6-3/2*a^2*b^2/x^4-2*a*b^3/x^2+b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^9} dx = -\frac{a^4}{8x^8} - \frac{2a^3b}{3x^6} - \frac{3a^2b^2}{2x^4} - \frac{2ab^3}{x^2} + b^4\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]
```

output
```
-1/8*a^4/x^8 - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3)/x^2 + b
^4*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^9}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^9}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{x^9}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^4}{x^{10}}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^4}{x^{10}}+\frac{4ba^3}{x^8}+\frac{6b^2a^2}{x^6}+\frac{4b^3a}{x^4}+\frac{b^4}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^4}{4x^8}-\frac{4a^3b}{3x^6}-\frac{3a^2b^2}{x^4}-\frac{4ab^3}{x^2}+b^4\log\left(x^2\right)\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]
```

output
```
(-1/4*a^4/x^8 - (4*a^3*b)/(3*x^6) - (3*a^2*b^2)/x^4 - (4*a*b^3)/x^2 + b^4*
Log[x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^{4}}{8x^{8}}-\frac{2a^{3}b}{3x^{6}}-\frac{3a^{2}b^{2}}{2x^{4}}-\frac{2a\,b^{3}}{x^{2}}+b^{4}\ln\left(x\right)$	45
norman	$\frac{-\frac{1}{8}a^{4}-2a\,b^{3}x^{6}-\frac{3}{2}a^{2}b^{2}x^{4}-\frac{2}{3}a^{3}b\,x^{2}}{x^{8}}+b^{4}\ln\left(x\right)$	47
risch	$\frac{-\frac{1}{8}a^{4}-2a\,b^{3}x^{6}-\frac{3}{2}a^{2}b^{2}x^{4}-\frac{2}{3}a^{3}b\,x^{2}}{x^{8}}+b^{4}\ln\left(x\right)$	47
parallelrisch	$\frac{24b^{4}\ln\left(x\right)x^{8}-48a\,b^{3}x^{6}-36a^{2}b^{2}x^{4}-16a^{3}b\,x^{2}-3a^{4}}{24x^{8}}$	51

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^4/x^8-2/3*a^3*b/x^6-3/2*a^2*b^2/x^4-2*a*b^3/x^2+b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^9}\,dx = \frac{24\,b^4x^8\log(x) - 48\,ab^3x^6 - 36\,a^2b^2x^4 - 16\,a^3bx^2 - 3\,a^4}{24\,x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="fricas")
```

output
```
1/24*(24*b^4*x^8*log(x) - 48*a*b^3*x^6 - 36*a^2*b^2*x^4 - 16*a^3*b*x^2 - 3
*a^4)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^9}\,dx = b^4\log(x) + \frac{-3a^4 - 16a^3bx^2 - 36a^2b^2x^4 - 48ab^3x^6}{24x^8}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**9,x)
```

output
```
b**4*log(x) + (-3*a**4 - 16*a**3*b*x**2 - 36*a**2*b**2*x**4 - 48*a*b**3*x*
*6)/(24*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^9}\,dx = \frac{1}{2}\,b^4\log\left(x^2\right) - \frac{48\,ab^3x^6+36\,a^2b^2x^4+16\,a^3bx^2+3\,a^4}{24\,x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="maxima")
```

output
```
1/2*b^4*log(x^2) - 1/24*(48*a*b^3*x^6 + 36*a^2*b^2*x^4 + 16*a^3*b*x^2 + 3*
a^4)/x^8
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^9}\,dx = \frac{1}{2}\,b^4\log\left(x^2\right) - \frac{25\,b^4x^8+48\,ab^3x^6+36\,a^2b^2x^4+16\,a^3bx^2+3\,a^4}{24\,x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x, algorithm="giac")
```

output
```
1/2*b^4*log(x^2) - 1/24*(25*b^4*x^8 + 48*a*b^3*x^6 + 36*a^2*b^2*x^4 + 16*a
^3*b*x^2 + 3*a^4)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^9}\,dx = b^4\,\ln\left(x\right) - \frac{\frac{a^4}{8}+\frac{2\,a^3\,b\,x^2}{3}+\frac{3\,a^2\,b^2\,x^4}{2}+2\,a\,b^3\,x^6}{x^8}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^9,x)
```

output
```
b^4*log(x) - (a^4/8 + (2*a^3*b*x^2)/3 + 2*a*b^3*x^6 + (3*a^2*b^2*x^4)/2)/x
^8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^9} dx = \frac{24 \log(x) b^4x^8 - 3a^4 - 16a^3b x^2 - 36a^2b^2x^4 - 48a b^3x^6}{24x^8}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x)
```

output
```
(24*log(x)*b**4*x**8 - 3*a**4 - 16*a**3*b*x**2 - 36*a**2*b**2*x**4 - 48*a*
b**3*x**6)/(24*x**8)
```

3.311 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}}\,dx = -\frac{(a+bx^2)^5}{10ax^{10}}$$

output
```
-1/10*(b*x^2+a)^5/a/x^10
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 52 vs. 2(19) = 38.

Time = 0.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.74

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}}\,dx = -\frac{a^4}{10x^{10}}-\frac{a^3b}{2x^8}-\frac{a^2b^2}{x^6}-\frac{ab^3}{x^4}-\frac{b^4}{2x^2}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]
```

output
```
-1/10*a^4/x^10 - (a^3*b)/(2*x^8) - (a^2*b^2)/x^6 - (a*b^3)/x^4 - b^4/(2*x^
2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1380, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}} \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^{11}}dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx^2)^4}{x^{11}}dx$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{(a+bx^2)^5}{10ax^{10}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]
```

output
```
-1/10*(a + b*x^2)^5/(a*x^10)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47

method	result	size
gosper	$-\frac{5b^4x^8+10a\,b^3x^6+10a^2b^2x^4+5a^3b\,x^2+a^4}{10x^{10}}$	47
default	$-\frac{b^3a}{x^4}-\frac{a^2b^2}{x^6}-\frac{b^4}{2x^2}-\frac{a^3b}{2x^8}-\frac{a^4}{10x^{10}}$	47
norman	$\frac{-\frac{1}{2}b^4x^8-a\,b^3x^6-a^2b^2x^4-\frac{1}{2}a^3b\,x^2-\frac{1}{10}a^4}{x^{10}}$	48
risch	$\frac{-\frac{1}{2}b^4x^8-a\,b^3x^6-a^2b^2x^4-\frac{1}{2}a^3b\,x^2-\frac{1}{10}a^4}{x^{10}}$	48
parallelrisch	$\frac{-5b^4x^8-10a\,b^3x^6-10a^2b^2x^4-5a^3b\,x^2-a^4}{10x^{10}}$	49
orering	$-\frac{(5b^4x^8+10a\,b^3x^6+10a^2b^2x^4+5a^3b\,x^2+a^4)(b^2x^4+2ab\,x^2+a^2)^2}{10x^{10}(b\,x^2+a)^4}$	76

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*(5*b^4*x^8+10*a*b^3*x^6+10*a^2*b^2*x^4+5*a^3*b*x^2+a^4)/x^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}}\,dx = -\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="fricas")
```

output
```
-1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(15) = 30.

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^{11}}\,dx = \frac{-a^4 - 5a^3bx^2 - 10a^2b^2x^4 - 10ab^3x^6 - 5b^4x^8}{10x^{10}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**11,x)
```

output
```
(-a**4 - 5*a**3*b*x**2 - 10*a**2*b**2*x**4 - 10*a*b**3*x**6 - 5*b**4*x**8)
/(10*x**10)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(17) = 34.

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^{11}}\,dx = -\frac{5\,b^4x^8 + 10\,ab^3x^6 + 10\,a^2b^2x^4 + 5\,a^3bx^2 + a^4}{10\,x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="maxima")
```

output
```
-1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}}\,dx = -\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x, algorithm="giac")
```

output
```
-1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/x^10
```

Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}}\,dx = -\frac{\frac{a^4}{10}+\frac{a^3\,b\,x^2}{2}+a^2\,b^2\,x^4+a\,b^3\,x^6+\frac{b^4\,x^8}{2}}{x^{10}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^11,x)
```

output
```
-(a^4/10 + (b^4*x^8)/2 + (a^3*b*x^2)/2 + a*b^3*x^6 + a^2*b^2*x^4)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}}\,dx = \frac{-5b^4x^8-10a\,b^3x^6-10a^2b^2x^4-5a^3b\,x^2-a^4}{10x^{10}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x)
```

output
```
( - a**4 - 5*a**3*b*x**2 - 10*a**2*b**2*x**4 - 10*a*b**3*x**6 - 5*b**4*x**
8)/(10*x**10)
```

3.312 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{13}} dx$

Optimal result

Integrand size = 24, antiderivative size = 40

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{13}} dx = -\frac{(a+bx^2)^5}{12ax^{12}} + \frac{b(a+bx^2)^5}{60a^2x^{10}}$$

output
```
-1/12*(b*x^2+a)^5/a/x^12+1/60*b*(b*x^2+a)^5/a^2/x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{13}} dx = -\frac{a^4}{12x^{12}} - \frac{2a^3b}{5x^{10}} - \frac{3a^2b^2}{4x^8} - \frac{2ab^3}{3x^6} - \frac{b^4}{4x^4}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
```

output
```
-1/12*a^4/x^12 - (2*a^3*b)/(5*x^10) - (3*a^2*b^2)/(4*x^8) - (2*a*b^3)/(3*x
^6) - b^4/(4*x^4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{13}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^{13}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{x^{13}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^4}{x^{14}}dx^2$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{b\int \frac{(bx^2+a)^4}{x^{12}}dx^2}{6a}-\frac{(a+bx^2)^5}{6ax^{12}}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2}\left(\frac{b(a+bx^2)^5}{30a^2x^{10}}-\frac{(a+bx^2)^5}{6ax^{12}}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
```

output
```
(-1/6*(a + b*x^2)^5/(a*x^12) + (b*(a + b*x^2)^5)/(30*a^2*x^10))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

method	result	size
default	$-\frac{a^4}{12x^{12}}-\frac{b^4}{4x^4}-\frac{2b^3a}{3x^6}-\frac{3a^2b^2}{4x^8}-\frac{2a^3b}{5x^{10}}$	47
norman	$\frac{-\frac{1}{4}b^4x^8-\frac{2}{3}a\,b^3x^6-\frac{3}{4}a^2b^2x^4-\frac{2}{5}a^3b\,x^2-\frac{1}{12}a^4}{x^{12}}$	48
risch	$\frac{-\frac{1}{4}b^4x^8-\frac{2}{3}a\,b^3x^6-\frac{3}{4}a^2b^2x^4-\frac{2}{5}a^3b\,x^2-\frac{1}{12}a^4}{x^{12}}$	48
gosper	$-\frac{15b^4x^8+40a\,b^3x^6+45a^2b^2x^4+24a^3b\,x^2+5a^4}{60x^{12}}$	49
parallelrisch	$\frac{-15b^4x^8-40a\,b^3x^6-45a^2b^2x^4-24a^3b\,x^2-5a^4}{60x^{12}}$	49
orering	$-\frac{\left(15b^4x^8+40a\,b^3x^6+45a^2b^2x^4+24a^3b\,x^2+5a^4\right)\left(b^2x^4+2ab\,x^2+a^2\right)^2}{60x^{12}\left(b\,x^2+a\right)^4}$	78

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*a^4/x^12-1/4*b^4/x^4-2/3*b^3*a/x^6-3/4*a^2*b^2/x^8-2/5*a^3*b/x^10
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{13}}\,dx=-\frac{15\,b^4x^8+40\,ab^3x^6+45\,a^2b^2x^4+24\,a^3bx^2+5\,a^4}{60\,x^{12}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="fricas")
```

output
```
-1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/
x^12
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{13}}\,dx = \frac{-5a^4-24a^3bx^2-45a^2b^2x^4-40ab^3x^6-15b^4x^8}{60x^{12}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**13,x)
```

output
```
(-5*a**4 - 24*a**3*b*x**2 - 45*a**2*b**2*x**4 - 40*a*b**3*x**6 - 15*b**4*x
**8)/(60*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{13}}\,dx = -\frac{15\,b^4x^8+40\,ab^3x^6+45\,a^2b^2x^4+24\,a^3bx^2+5\,a^4}{60\,x^{12}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="maxima")
```

output
```
-1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/
x^12
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{13}}\,dx = -\frac{15\,b^4x^8+40\,ab^3x^6+45\,a^2b^2x^4+24\,a^3bx^2+5\,a^4}{60\,x^{12}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x, algorithm="giac")
```

output
```
-1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2 + 5*a^4)/
x^12
```

Mupad [B] (verification not implemented)

Time = 18.65 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^{13}} \, dx = -\frac{\frac{a^4}{12} + \frac{2\,a^3\,b\,x^2}{5} + \frac{3\,a^2\,b^2\,x^4}{4} + \frac{2\,a\,b^3\,x^6}{3} + \frac{b^4\,x^8}{4}}{x^{12}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^13,x)
```

output
```
-(a^4/12 + (b^4*x^8)/4 + (2*a^3*b*x^2)/5 + (2*a*b^3*x^6)/3 + (3*a^2*b^2*x^
4)/4)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^{13}} \, dx = \frac{-15b^4x^8 - 40a\,b^3x^6 - 45a^2b^2x^4 - 24a^3b\,x^2 - 5a^4}{60x^{12}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x)
```

output
```
( - 5*a**4 - 24*a**3*b*x**2 - 45*a**2*b**2*x**4 - 40*a*b**3*x**6 - 15*b**4
*x**8)/(60*x**12)
```

3.313 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx = -\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

output
```
-1/14*a^4/x^14-1/3*a^3*b/x^12-3/5*a^2*b^2/x^10-1/2*a*b^3/x^8-1/6*b^4/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx = -\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

output
```
-1/14*a^4/x^14 - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3)/(2*x^8)
 - b^4/(6*x^6)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^{15}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{x^{15}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^4}{x^{16}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^4}{x^{16}}+\frac{4ba^3}{x^{14}}+\frac{6b^2a^2}{x^{12}}+\frac{4b^3a}{x^{10}}+\frac{b^4}{x^8}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^4}{7x^{14}}-\frac{2a^3b}{3x^{12}}-\frac{6a^2b^2}{5x^{10}}-\frac{ab^3}{x^8}-\frac{b^4}{3x^6}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

output
```
(-1/7*a^4/x^14 - (2*a^3*b)/(3*x^12) - (6*a^2*b^2)/(5*x^10) - (a*b^3)/x^8 -
 b^4/(3*x^6))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{4}}{14x^{14}}-\frac{a^{3}b}{3x^{12}}-\frac{3a^{2}b^{2}}{5x^{10}}-\frac{a\,b^{3}}{2x^{8}}-\frac{b^{4}}{6x^{6}}$	47
norman	$\frac{-\frac{1}{14}a^{4}-\frac{1}{3}a^{3}b\,x^{2}-\frac{3}{5}a^{2}b^{2}x^{4}-\frac{1}{2}a\,b^{3}x^{6}-\frac{1}{6}b^{4}x^{8}}{x^{14}}$	48
risch	$\frac{-\frac{1}{14}a^{4}-\frac{1}{3}a^{3}b\,x^{2}-\frac{3}{5}a^{2}b^{2}x^{4}-\frac{1}{2}a\,b^{3}x^{6}-\frac{1}{6}b^{4}x^{8}}{x^{14}}$	48
gosper	$-\frac{35b^{4}x^{8}+105a\,b^{3}x^{6}+126a^{2}b^{2}x^{4}+70a^{3}b\,x^{2}+15a^{4}}{210x^{14}}$	49
parallelrisch	$\frac{-35b^{4}x^{8}-105a\,b^{3}x^{6}-126a^{2}b^{2}x^{4}-70a^{3}b\,x^{2}-15a^{4}}{210x^{14}}$	49
orering	$-\frac{\left(35b^{4}x^{8}+105a\,b^{3}x^{6}+126a^{2}b^{2}x^{4}+70a^{3}b\,x^{2}+15a^{4}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}}{210x^{14}\left(b\,x^{2}+a\right)^{4}}$	78

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/14*a^4/x^14-1/3*a^3*b/x^12-3/5*a^2*b^2/x^10-1/2*a*b^3/x^8-1/6*b^4/x^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx = -\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="fricas")
```

output
```
-1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a
^4)/x^14
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx = \frac{-15a^4-70a^3bx^2-126a^2b^2x^4-105ab^3x^6-35b^4x^8}{210x^{14}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**15,x)
```

output
```
(-15*a**4 - 70*a**3*b*x**2 - 126*a**2*b**2*x**4 - 105*a*b**3*x**6 - 35*b**
4*x**8)/(210*x**14)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx = -\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="maxima")
```

output
```
-1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a
^4)/x^14
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx = -\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x, algorithm="giac")
```

output
```
-1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x^2 + 15*a
^4)/x^14
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx = -\frac{\frac{a^4}{14}+\frac{a^3\,b\,x^2}{3}+\frac{3\,a^2\,b^2\,x^4}{5}+\frac{a\,b^3\,x^6}{2}+\frac{b^4\,x^8}{6}}{x^{14}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^15,x)
```

output
```
-(a^4/14 + (b^4*x^8)/6 + (a^3*b*x^2)/3 + (a*b^3*x^6)/2 + (3*a^2*b^2*x^4)/5
)/x^14
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^{15}}\,dx = \frac{-35b^4x^8 - 105a\,b^3x^6 - 126a^2b^2x^4 - 70a^3b\,x^2 - 15a^4}{210x^{14}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x)
```

output
```
( - 15*a**4 - 70*a**3*b*x**2 - 126*a**2*b**2*x**4 - 105*a*b**3*x**6 - 35*b
**4*x**8)/(210*x**14)
```

3.314 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{17}} dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{17}} dx = -\frac{a^4}{16x^{16}} - \frac{2a^3b}{7x^{14}} - \frac{a^2b^2}{2x^{12}} - \frac{2ab^3}{5x^{10}} - \frac{b^4}{8x^8}$$

output
```
-1/16*a^4/x^16-2/7*a^3*b/x^14-1/2*a^2*b^2/x^12-2/5*a*b^3/x^10-1/8*b^4/x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{17}} dx = -\frac{a^4}{16x^{16}} - \frac{2a^3b}{7x^{14}} - \frac{a^2b^2}{2x^{12}} - \frac{2ab^3}{5x^{10}} - \frac{b^4}{8x^8}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^17,x]
```

output
```
-1/16*a^4/x^16 - (2*a^3*b)/(7*x^14) - (a^2*b^2)/(2*x^12) - (2*a*b^3)/(5*x^
10) - b^4/(8*x^8)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{17}}dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^{17}}dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx^2)^4}{x^{17}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^4}{x^{18}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{a^4}{x^{18}}+\frac{4ba^3}{x^{16}}+\frac{6b^2a^2}{x^{14}}+\frac{4b^3a}{x^{12}}+\frac{b^4}{x^{10}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^4}{8x^{16}}-\frac{4a^3b}{7x^{14}}-\frac{a^2b^2}{x^{12}}-\frac{4ab^3}{5x^{10}}-\frac{b^4}{4x^8}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^17,x]
```

output
```
(-1/8*a^4/x^16 - (4*a^3*b)/(7*x^14) - (a^2*b^2)/x^12 - (4*a*b^3)/(5*x^10)
- b^4/(4*x^8))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{4}}{16 x^{16}}-\frac{2 a^{3} b}{7 x^{14}}-\frac{a^{2} b^{2}}{2 x^{12}}-\frac{2 a \,b^{3}}{5 x^{10}}-\frac{b^{4}}{8 x^{8}}$	47
norman	$\frac{-\frac{1}{16} a^{4}-\frac{2}{7} a^{3} b \,x^{2}-\frac{1}{2} a^{2} b^{2} x^{4}-\frac{2}{5} a \,b^{3} x^{6}-\frac{1}{8} b^{4} x^{8}}{x^{16}}$	48
risch	$\frac{-\frac{1}{16} a^{4}-\frac{2}{7} a^{3} b \,x^{2}-\frac{1}{2} a^{2} b^{2} x^{4}-\frac{2}{5} a \,b^{3} x^{6}-\frac{1}{8} b^{4} x^{8}}{x^{16}}$	48
gosper	$-\frac{70 b^{4} x^{8}+224 a \,b^{3} x^{6}+280 a^{2} b^{2} x^{4}+160 a^{3} b \,x^{2}+35 a^{4}}{560 x^{16}}$	49
parallelrisch	$\frac{-70 b^{4} x^{8}-224 a \,b^{3} x^{6}-280 a^{2} b^{2} x^{4}-160 a^{3} b \,x^{2}-35 a^{4}}{560 x^{16}}$	49
orering	$-\frac{\left(70 b^{4} x^{8}+224 a \,b^{3} x^{6}+280 a^{2} b^{2} x^{4}+160 a^{3} b \,x^{2}+35 a^{4}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{2}}{560 x^{16} \left(b \,x^{2}+a \right)^{4}}$	78

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/16*a^4/x^16-2/7*a^3*b/x^14-1/2*a^2*b^2/x^12-2/5*a*b^3/x^10-1/8*b^4/x^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{17}}\,dx = -\frac{70\,b^4x^8+224\,ab^3x^6+280\,a^2b^2x^4+160\,a^3bx^2+35\,a^4}{560\,x^{16}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^17,x, algorithm="fricas")
```

output
```
-1/560*(70*b^4*x^8 + 224*a*b^3*x^6 + 280*a^2*b^2*x^4 + 160*a^3*b*x^2 + 35*
a^4)/x^16
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{17}}\,dx = \frac{-35a^4-160a^3bx^2-280a^2b^2x^4-224ab^3x^6-70b^4x^8}{560x^{16}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**17,x)
```

output
```
(-35*a**4 - 160*a**3*b*x**2 - 280*a**2*b**2*x**4 - 224*a*b**3*x**6 - 70*b*
*4*x**8)/(560*x**16)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{17}}\,dx = -\frac{70\,b^4x^8+224\,ab^3x^6+280\,a^2b^2x^4+160\,a^3bx^2+35\,a^4}{560\,x^{16}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^17,x, algorithm="maxima")
```

output
```
-1/560*(70*b^4*x^8 + 224*a*b^3*x^6 + 280*a^2*b^2*x^4 + 160*a^3*b*x^2 + 35*
a^4)/x^16
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{17}}\,dx = -\frac{70\,b^4x^8+224\,ab^3x^6+280\,a^2b^2x^4+160\,a^3bx^2+35\,a^4}{560\,x^{16}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^17,x, algorithm="giac")
```

output
```
-1/560*(70*b^4*x^8 + 224*a*b^3*x^6 + 280*a^2*b^2*x^4 + 160*a^3*b*x^2 + 35*
a^4)/x^16
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{17}}\,dx = -\frac{\frac{a^4}{16}+\frac{2\,a^3\,b\,x^2}{7}+\frac{a^2\,b^2\,x^4}{2}+\frac{2\,a\,b^3\,x^6}{5}+\frac{b^4\,x^8}{8}}{x^{16}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^17,x)
```

output
```
-(a^4/16 + (b^4*x^8)/8 + (2*a^3*b*x^2)/7 + (2*a*b^3*x^6)/5 + (a^2*b^2*x^4)
/2)/x^16
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^{17}} \, dx = \frac{-70b^4x^8 - 224a\,b^3x^6 - 280a^2b^2x^4 - 160a^3b\,x^2 - 35a^4}{560x^{16}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^17,x)
```

output
```
( - 35*a**4 - 160*a**3*b*x**2 - 280*a**2*b**2*x**4 - 224*a*b**3*x**6 - 70*
b**4*x**8)/(560*x**16)
```

3.315 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{19}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{19}}\,dx = -\frac{a^4}{18x^{18}}-\frac{a^3b}{4x^{16}}-\frac{3a^2b^2}{7x^{14}}-\frac{ab^3}{3x^{12}}-\frac{b^4}{10x^{10}}$$

output
```
-1/18*a^4/x^18-1/4*a^3*b/x^16-3/7*a^2*b^2/x^14-1/3*a*b^3/x^12-1/10*b^4/x^1
0
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{19}}\,dx = -\frac{a^4}{18x^{18}}-\frac{a^3b}{4x^{16}}-\frac{3a^2b^2}{7x^{14}}-\frac{ab^3}{3x^{12}}-\frac{b^4}{10x^{10}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^19,x]
```

output
```
-1/18*a^4/x^18 - (a^3*b)/(4*x^16) - (3*a^2*b^2)/(7*x^14) - (a*b^3)/(3*x^12
) - b^4/(10*x^10)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{19}}dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^{19}}dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx^2)^4}{x^{19}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^4}{x^{20}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{a^4}{x^{20}}+\frac{4ba^3}{x^{18}}+\frac{6b^2a^2}{x^{16}}+\frac{4b^3a}{x^{14}}+\frac{b^4}{x^{12}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^4}{9x^{18}}-\frac{a^3b}{2x^{16}}-\frac{6a^2b^2}{7x^{14}}-\frac{2ab^3}{3x^{12}}-\frac{b^4}{5x^{10}}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^19,x]
```

output
```
(-1/9*a^4/x^18 - (a^3*b)/(2*x^16) - (6*a^2*b^2)/(7*x^14) - (2*a*b^3)/(3*x^
12) - b^4/(5*x^10))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{4}}{18 x^{18}}-\frac{a^{3} b}{4 x^{16}}-\frac{3 a^{2} b^{2}}{7 x^{14}}-\frac{a \,b^{3}}{3 x^{12}}-\frac{b^{4}}{10 x^{10}}$	47
norman	$\frac{-\frac{1}{18} a^{4}-\frac{1}{4} a^{3} b \,x^{2}-\frac{3}{7} a^{2} b^{2} x^{4}-\frac{1}{3} a \,b^{3} x^{6}-\frac{1}{10} b^{4} x^{8}}{x^{18}}$	48
risch	$\frac{-\frac{1}{18} a^{4}-\frac{1}{4} a^{3} b \,x^{2}-\frac{3}{7} a^{2} b^{2} x^{4}-\frac{1}{3} a \,b^{3} x^{6}-\frac{1}{10} b^{4} x^{8}}{x^{18}}$	48
gosper	$-\frac{126 b^{4} x^{8}+420 a \,b^{3} x^{6}+540 a^{2} b^{2} x^{4}+315 a^{3} b \,x^{2}+70 a^{4}}{1260 x^{18}}$	49
parallelrisch	$\frac{-126 b^{4} x^{8}-420 a \,b^{3} x^{6}-540 a^{2} b^{2} x^{4}-315 a^{3} b \,x^{2}-70 a^{4}}{1260 x^{18}}$	49
orering	$-\frac{\left(126 b^{4} x^{8}+420 a \,b^{3} x^{6}+540 a^{2} b^{2} x^{4}+315 a^{3} b \,x^{2}+70 a^{4}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{2}}{1260 x^{18} \left(b \,x^{2}+a \right)^{4}}$	78

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/18*a^4/x^18-1/4*a^3*b/x^16-3/7*a^2*b^2/x^14-1/3*a*b^3/x^12-1/10*b^4/x^1
0
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^{19}}\,dx = -\frac{126\,b^4x^8 + 420\,ab^3x^6 + 540\,a^2b^2x^4 + 315\,a^3bx^2 + 70\,a^4}{1260\,x^{18}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^19,x, algorithm="fricas")
```

output
```
-1/1260*(126*b^4*x^8 + 420*a*b^3*x^6 + 540*a^2*b^2*x^4 + 315*a^3*b*x^2 + 7
0*a^4)/x^18
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^{19}}\,dx = \frac{-70a^4 - 315a^3bx^2 - 540a^2b^2x^4 - 420ab^3x^6 - 126b^4x^8}{1260x^{18}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**19,x)
```

output
```
(-70*a**4 - 315*a**3*b*x**2 - 540*a**2*b**2*x**4 - 420*a*b**3*x**6 - 126*b
**4*x**8)/(1260*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{19}}\,dx = -\frac{126\,b^4x^8+420\,ab^3x^6+540\,a^2b^2x^4+315\,a^3bx^2+70\,a^4}{1260\,x^{18}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^19,x, algorithm="maxima")
```

output
```
-1/1260*(126*b^4*x^8 + 420*a*b^3*x^6 + 540*a^2*b^2*x^4 + 315*a^3*b*x^2 + 7
0*a^4)/x^18
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{19}}\,dx = -\frac{126\,b^4x^8+420\,ab^3x^6+540\,a^2b^2x^4+315\,a^3bx^2+70\,a^4}{1260\,x^{18}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^19,x, algorithm="giac")
```

output
```
-1/1260*(126*b^4*x^8 + 420*a*b^3*x^6 + 540*a^2*b^2*x^4 + 315*a^3*b*x^2 + 7
0*a^4)/x^18
```

Mupad [B] (verification not implemented)

Time = 18.80 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{19}}\,dx = -\frac{\frac{a^4}{18}+\frac{a^3\,b\,x^2}{4}+\frac{3\,a^2\,b^2\,x^4}{7}+\frac{a\,b^3\,x^6}{3}+\frac{b^4\,x^8}{10}}{x^{18}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^19,x)
```

output
```
-(a^4/18 + (b^4*x^8)/10 + (a^3*b*x^2)/4 + (a*b^3*x^6)/3 + (3*a^2*b^2*x^4)/
7)/x^18
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^{19}} \, dx = \frac{-126b^4x^8 - 420a\,b^3x^6 - 540a^2b^2x^4 - 315a^3b\,x^2 - 70a^4}{1260x^{18}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^19,x)
```

output
```
( - 70*a**4 - 315*a**3*b*x**2 - 540*a**2*b**2*x**4 - 420*a*b**3*x**6 - 126
*b**4*x**8)/(1260*x**18)
```

3.316 $\int x^6(a^2+2abx^2+b^2x^4)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

output
```
1/7*a^4*x^7+4/9*a^3*b*x^9+6/11*a^2*b^2*x^11+4/13*a*b^3*x^13+1/15*b^4*x^15
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

input
```
Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^13)/13 +
(b^4*x^15)/15
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^2 dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^4 x^6 \left(bx^2+a\right)^4 dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int x^6 \left(a+bx^2\right)^4 dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^4x^6+4a^3bx^8+6a^2b^2x^{10}+4ab^3x^{12}+b^4x^{14}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

input
```
Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^13)/13 +
(b^4*x^15)/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{1}{7}a^4x^7+\frac{4}{9}a^3b\,x^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}a\,b^3x^{13}+\frac{1}{15}b^4x^{15}$	47
norman	$\frac{1}{7}a^4x^7+\frac{4}{9}a^3b\,x^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}a\,b^3x^{13}+\frac{1}{15}b^4x^{15}$	47
risch	$\frac{1}{7}a^4x^7+\frac{4}{9}a^3b\,x^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}a\,b^3x^{13}+\frac{1}{15}b^4x^{15}$	47
parallelrisch	$\frac{1}{7}a^4x^7+\frac{4}{9}a^3b\,x^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}a\,b^3x^{13}+\frac{1}{15}b^4x^{15}$	47
gosper	$\frac{x^7(3003b^4x^8+13860a\,b^3x^6+24570a^2b^2x^4+20020a^3b\,x^2+6435a^4)}{45045}$	49
orering	$\frac{x^7(3003b^4x^8+13860a\,b^3x^6+24570a^2b^2x^4+20020a^3b\,x^2+6435a^4)(b^2x^4+2ab\,x^2+a^2)^2}{45045(b\,x^2+a)^4}$	78

input
```
int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^4*x^7+4/9*a^3*b*x^9+6/11*a^2*b^2*x^11+4/13*a*b^3*x^13+1/15*b^4*x^15
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{1}{15}\,b^4x^{15}+\frac{4}{13}\,ab^3x^{13}+\frac{6}{11}\,a^2b^2x^{11}+\frac{4}{9}\,a^3bx^9+\frac{1}{7}\,a^4x^7$$

input
```
integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x^9 + 1/7*
a^4*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^7}{7}+\frac{4a^3bx^9}{9}+\frac{6a^2b^2x^{11}}{11}+\frac{4ab^3x^{13}}{13}+\frac{b^4x^{15}}{15}$$

input
```
integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
a**4*x**7/7 + 4*a**3*b*x**9/9 + 6*a**2*b**2*x**11/11 + 4*a*b**3*x**13/13 +
 b**4*x**15/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{1}{15}\,b^4x^{15}+\frac{4}{13}\,ab^3x^{13}+\frac{6}{11}\,a^2b^2x^{11}+\frac{4}{9}\,a^3bx^9+\frac{1}{7}\,a^4x^7$$

input
```
integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x^9 + 1/7*
a^4*x^7
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{1}{15}\,b^4x^{15}+\frac{4}{13}\,ab^3x^{13}+\frac{6}{11}\,a^2b^2x^{11}+\frac{4}{9}\,a^3bx^9+\frac{1}{7}\,a^4x^7$$

input
```
integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x^9 + 1/7*
a^4*x^7
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4\,x^7}{7}+\frac{4\,a^3\,b\,x^9}{9}+\frac{6\,a^2\,b^2\,x^{11}}{11}+\frac{4\,a\,b^3\,x^{13}}{13}+\frac{b^4\,x^{15}}{15}$$

input
```
int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(a^4*x^7)/7 + (b^4*x^15)/15 + (4*a^3*b*x^9)/9 + (4*a*b^3*x^13)/13 + (6*a^2
*b^2*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^2\,dx$$
$$=\frac{x^7(3003b^4x^8+13860a\,b^3x^6+24570a^2b^2x^4+20020a^3b\,x^2+6435a^4)}{45045}$$

input
```
int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(x**7*(6435*a**4 + 20020*a**3*b*x**2 + 24570*a**2*b**2*x**4 + 13860*a*b**3
*x**6 + 3003*b**4*x**8))/45045
```

3.317 $\int x^4(a^2+2abx^2+b^2x^4)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

output
```
1/5*a^4*x^5+4/7*a^3*b*x^7+2/3*a^2*b^2*x^9+4/11*a*b^3*x^11+1/13*b^4*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11)/11 + (b
^4*x^13)/13
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^2 dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^4x^4\left(bx^2+a\right)^4dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int x^4\left(a+bx^2\right)^4dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^4x^4+4a^3bx^6+6a^2b^2x^8+4ab^3x^{10}+b^4x^{12}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

input
```
Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11)/11 + (b
^4*x^13)/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{1}{5}a^4x^5+\frac{4}{7}a^3b\,x^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}a\,b^3x^{11}+\frac{1}{13}b^4x^{13}$	47
norman	$\frac{1}{5}a^4x^5+\frac{4}{7}a^3b\,x^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}a\,b^3x^{11}+\frac{1}{13}b^4x^{13}$	47
risch	$\frac{1}{5}a^4x^5+\frac{4}{7}a^3b\,x^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}a\,b^3x^{11}+\frac{1}{13}b^4x^{13}$	47
parallelrisch	$\frac{1}{5}a^4x^5+\frac{4}{7}a^3b\,x^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}a\,b^3x^{11}+\frac{1}{13}b^4x^{13}$	47
gosper	$\frac{x^5\left(1155b^4x^8+5460a\,b^3x^6+10010a^2b^2x^4+8580a^3b\,x^2+3003a^4\right)}{15015}$	49
orering	$\frac{x^5\left(1155b^4x^8+5460a\,b^3x^6+10010a^2b^2x^4+8580a^3b\,x^2+3003a^4\right)\left(b^2x^4+2ab\,x^2+a^2\right)^2}{15015\left(b\,x^2+a\right)^4}$	78

input
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^4*x^5+4/7*a^3*b*x^7+2/3*a^2*b^2*x^9+4/11*a*b^3*x^11+1/13*b^4*x^13
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^4(a^2+2abx^2+b^2x^4)^2\,dx=\frac{1}{13}\,b^4x^{13}+\frac{4}{11}\,ab^3x^{11}+\frac{2}{3}\,a^2b^2x^9+\frac{4}{7}\,a^3bx^7+\frac{1}{5}\,a^4x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7 + 1/5*a^
4*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int x^4(a^2+2abx^2+b^2x^4)^2\,dx=\frac{a^4x^5}{5}+\frac{4a^3bx^7}{7}+\frac{2a^2b^2x^9}{3}+\frac{4ab^3x^{11}}{11}+\frac{b^4x^{13}}{13}$$

input
```
integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
a**4*x**5/5 + 4*a**3*b*x**7/7 + 2*a**2*b**2*x**9/3 + 4*a*b**3*x**11/11 + b
**4*x**13/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^4(a^2+2abx^2+b^2x^4)^2\,dx=\frac{1}{13}\,b^4x^{13}+\frac{4}{11}\,ab^3x^{11}+\frac{2}{3}\,a^2b^2x^9+\frac{4}{7}\,a^3bx^7+\frac{1}{5}\,a^4x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7 + 1/5*a^
4*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{1}{13} b^4 x^{13} + \frac{4}{11} a b^3 x^{11} + \frac{2}{3} a^2 b^2 x^9 + \frac{4}{7} a^3 b x^7 + \frac{1}{5} a^4 x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7 + 1/5*a^
4*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{a^4\,x^5}{5} + \frac{4\,a^3\,b\,x^7}{7} + \frac{2\,a^2\,b^2\,x^9}{3} + \frac{4\,a\,b^3\,x^{11}}{11} + \frac{b^4\,x^{13}}{13}$$

input
```
int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(a^4*x^5)/5 + (b^4*x^13)/13 + (4*a^3*b*x^7)/7 + (4*a*b^3*x^11)/11 + (2*a^2
*b^2*x^9)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{x^5(1155b^4x^8 + 5460a\,b^3x^6 + 10010a^2b^2x^4 + 8580a^3b\,x^2 + 3003a^4)}{15015}$$

input
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(x**5*(3003*a**4 + 8580*a**3*b*x**2 + 10010*a**2*b**2*x**4 + 5460*a*b**3*x
**6 + 1155*b**4*x**8))/15015
```

3.318 $\int x^2(a^2+2abx^2+b^2x^4)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

output
```
1/3*a^4*x^3+4/5*a^3*b*x^5+6/7*a^2*b^2*x^7+4/9*a*b^3*x^9+1/11*b^4*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)/9 + (b^4
*x^11)/11
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^2 dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^4 x^2 \left(bx^2+a\right)^4 dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int x^2 \left(a+bx^2\right)^4 dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^4x^2+4a^3bx^4+6a^2b^2x^6+4ab^3x^8+b^4x^{10}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)/9 + (b^4
*x^11)/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{1}{3}a^4x^3+\frac{4}{5}a^3b\,x^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}a\,b^3x^9+\frac{1}{11}b^4x^{11}$	47
norman	$\frac{1}{3}a^4x^3+\frac{4}{5}a^3b\,x^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}a\,b^3x^9+\frac{1}{11}b^4x^{11}$	47
risch	$\frac{1}{3}a^4x^3+\frac{4}{5}a^3b\,x^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}a\,b^3x^9+\frac{1}{11}b^4x^{11}$	47
parallelrisch	$\frac{1}{3}a^4x^3+\frac{4}{5}a^3b\,x^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}a\,b^3x^9+\frac{1}{11}b^4x^{11}$	47
gosper	$\frac{x^3(315b^4x^8+1540a\,b^3x^6+2970a^2b^2x^4+2772a^3b\,x^2+1155a^4)}{3465}$	49
orering	$\frac{x^3(315b^4x^8+1540a\,b^3x^6+2970a^2b^2x^4+2772a^3b\,x^2+1155a^4)(b^2x^4+2ab\,x^2+a^2)^2}{3465(b\,x^2+a)^4}$	78

input
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^4*x^3+4/5*a^3*b*x^5+6/7*a^2*b^2*x^7+4/9*a*b^3*x^9+1/11*b^4*x^11
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{1}{11}\,b^4x^{11}+\frac{4}{9}\,ab^3x^9+\frac{6}{7}\,a^2b^2x^7+\frac{4}{5}\,a^3bx^5+\frac{1}{3}\,a^4x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 + 1/3*a^4*
x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{a^4x^3}{3}+\frac{4a^3bx^5}{5}+\frac{6a^2b^2x^7}{7}+\frac{4ab^3x^9}{9}+\frac{b^4x^{11}}{11}$$

input
```
integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
a**4*x**3/3 + 4*a**3*b*x**5/5 + 6*a**2*b**2*x**7/7 + 4*a*b**3*x**9/9 + b**
4*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{1}{11}\,b^4x^{11}+\frac{4}{9}\,ab^3x^9+\frac{6}{7}\,a^2b^2x^7+\frac{4}{5}\,a^3bx^5+\frac{1}{3}\,a^4x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 + 1/3*a^4*
x^3
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{1}{11}\,b^4x^{11}+\frac{4}{9}\,ab^3x^9+\frac{6}{7}\,a^2b^2x^7+\frac{4}{5}\,a^3bx^5+\frac{1}{3}\,a^4x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 + 1/3*a^4*
x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{a^4\,x^3}{3}+\frac{4\,a^3\,b\,x^5}{5}+\frac{6\,a^2\,b^2\,x^7}{7}+\frac{4\,a\,b^3\,x^9}{9}+\frac{b^4\,x^{11}}{11}$$

input
```
int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(a^4*x^3)/3 + (b^4*x^11)/11 + (4*a^3*b*x^5)/5 + (4*a*b^3*x^9)/9 + (6*a^2*b
^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{x^3(315b^4x^8+1540a\,b^3x^6+2970a^2b^2x^4+2772a^3b\,x^2+1155a^4)}{3465}$$

input
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(x**3*(1155*a**4 + 2772*a**3*b*x**2 + 2970*a**2*b**2*x**4 + 1540*a*b**3*x*
*6 + 315*b**4*x**8))/3465
```

3.319 $\int \left(a^2+2abx^2+b^2x^4\right)^2 dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \left(a^2+2abx^2+b^2x^4\right)^2 dx=a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

output
```
a^4*x+4/3*a^3*b*x^3+6/5*a^2*b^2*x^5+4/7*a*b^3*x^7+1/9*b^4*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \left(a^2+2abx^2+b^2x^4\right)^2 dx=a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (b^4*x^9)/
9
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1379, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a^2+2abx^2+b^2x^4)^2\,dx$$

$$\Big\downarrow \text{1379}$$

$$\frac{\int (b^2x^2+ab)^4\,dx}{b^4}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int (b^8x^8+4ab^7x^6+6a^2b^6x^4+4a^3b^5x^2+a^4b^4)\,dx}{b^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4b^4x+\frac{4}{3}a^3b^5x^3+\frac{6}{5}a^2b^6x^5+\frac{4}{7}ab^7x^7+\frac{b^8x^9}{9}}{b^4}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^4*b^4*x + (4*a^3*b^5*x^3)/3 + (6*a^2*b^6*x^5)/5 + (4*a*b^7*x^7)/7 + (b^
8*x^9)/9)/b^4
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result	size
default	$a^4x+\frac{4}{3}a^3b\,x^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}a\,b^3x^7+\frac{1}{9}b^4x^9$	44
norman	$a^4x+\frac{4}{3}a^3b\,x^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}a\,b^3x^7+\frac{1}{9}b^4x^9$	44
risch	$a^4x+\frac{4}{3}a^3b\,x^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}a\,b^3x^7+\frac{1}{9}b^4x^9$	44
parallelrisch	$a^4x+\frac{4}{3}a^3b\,x^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}a\,b^3x^7+\frac{1}{9}b^4x^9$	44
gosper	$\frac{x(35b^4x^8+180a\,b^3x^6+378a^2b^2x^4+420a^3b\,x^2+315a^4)}{315}$	47
orering	$\frac{x(35b^4x^8+180a\,b^3x^6+378a^2b^2x^4+420a^3b\,x^2+315a^4)(b^2x^4+2ab\,x^2+a^2)^2}{315(b\,x^2+a)^4}$	76

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a^4*x+4/3*a^3*b*x^3+6/5*a^2*b^2*x^5+4/7*a*b^3*x^7+1/9*b^4*x^9
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{1}{9}\,b^4x^9+\frac{4}{7}\,ab^3x^7+\frac{6}{5}\,a^2b^2x^5+\frac{4}{3}\,a^3bx^3+a^4x$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 + 4/3*a^3*b*x^3 + a^4*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \left(a^2+2abx^2+b^2x^4\right)^2\,dx = a^4x+\frac{4a^3bx^3}{3}+\frac{6a^2b^2x^5}{5}+\frac{4ab^3x^7}{7}+\frac{b^4x^9}{9}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
a**4*x + 4*a**3*b*x**3/3 + 6*a**2*b**2*x**5/5 + 4*a*b**3*x**7/7 + b**4*x**
9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \left(a^2+2abx^2+b^2x^4\right)^2\,dx = \frac{1}{9}\,b^4x^9+\frac{4}{7}\,ab^3x^7+\frac{4}{5}\,a^2b^2x^5+a^4x+\frac{2}{15}\,\left(3\,b^2x^5+10\,abx^3\right)a^2$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 4/5*a^2*b^2*x^5 + a^4*x + 2/15*(3*b^2*x^5 +
10*a*b*x^3)*a^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a^2+2abx^2+b^2x^4\right)^2\,dx = \frac{1}{9}\,b^4x^9+\frac{4}{7}\,ab^3x^7+\frac{6}{5}\,a^2b^2x^5+\frac{4}{3}\,a^3bx^3+a^4x$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 + 4/3*a^3*b*x^3 + a^4*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a^2+2abx^2+b^2x^4\right)^2\,dx=a^4\,x+\frac{4\,a^3\,b\,x^3}{3}+\frac{6\,a^2\,b^2\,x^5}{5}+\frac{4\,a\,b^3\,x^7}{7}+\frac{b^4\,x^9}{9}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
a^4*x + (b^4*x^9)/9 + (4*a^3*b*x^3)/3 + (4*a*b^3*x^7)/7 + (6*a^2*b^2*x^5)/
5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{x(35b^4x^8+180a\,b^3x^6+378a^2b^2x^4+420a^3b\,x^2+315a^4)}{315}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(x*(315*a**4 + 420*a**3*b*x**2 + 378*a**2*b**2*x**4 + 180*a*b**3*x**6 + 35
*b**4*x**8))/315
```

3.320 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^2}\,dx = -\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

output
```
-a^4/x+4*a^3*b*x+2*a^2*b^2*x^3+4/5*a*b^3*x^5+1/7*b^4*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^2}\,dx = -\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
```

output
```
-(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7)/7
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^2}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^2}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{x^2}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^4}{x^2}+4a^3b+6a^2b^2x^2+4ab^3x^4+b^4x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
```

output
```
-(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4a\,b^3x^5}{5}+\frac{b^4x^7}{7}$	45
risch	$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4a\,b^3x^5}{5}+\frac{b^4x^7}{7}$	45
norman	$\frac{\frac{1}{7}b^4x^8+\frac{4}{5}a\,b^3x^6+2a^2b^2x^4+4a^3b\,x^2-a^4}{x}$	48
gosper	$-\frac{-5b^4x^8-28a\,b^3x^6-70a^2b^2x^4-140a^3b\,x^2+35a^4}{35x}$	49
parallelrisch	$\frac{5b^4x^8+28a\,b^3x^6+70a^2b^2x^4+140a^3b\,x^2-35a^4}{35x}$	49
orering	$-\frac{(-5b^4x^8-28a\,b^3x^6-70a^2b^2x^4-140a^3b\,x^2+35a^4)(b^2x^4+2ab\,x^2+a^2)^2}{35x(b\,x^2+a)^4}$	78

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^4/x+4*a^3*b*x+2*a^2*b^2*x^3+4/5*a*b^3*x^5+1/7*b^4*x^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^2}\,dx = \frac{5\,b^4x^8+28\,ab^3x^6+70\,a^2b^2x^4+140\,a^3bx^2-35\,a^4}{35\,x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="fricas")
```

output
```
1/35*(5*b^4*x^8 + 28*a*b^3*x^6 + 70*a^2*b^2*x^4 + 140*a^3*b*x^2 - 35*a^4)/
x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^2}\,dx = -\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4ab^3x^5}{5}+\frac{b^4x^7}{7}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**2,x)
```

output
```
-a**4/x + 4*a**3*b*x + 2*a**2*b**2*x**3 + 4*a*b**3*x**5/5 + b**4*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^2}\,dx = \frac{1}{7}\,b^4x^7+\frac{4}{5}\,ab^3x^5+2\,a^2b^2x^3+4\,a^3bx-\frac{a^4}{x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="maxima")
```

output
```
1/7*b^4*x^7 + 4/5*a*b^3*x^5 + 2*a^2*b^2*x^3 + 4*a^3*b*x - a^4/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^2} dx = \frac{1}{7} b^4x^7 + \frac{4}{5} ab^3x^5 + 2 a^2b^2x^3 + 4 a^3bx - \frac{a^4}{x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x, algorithm="giac")
```

output
```
1/7*b^4*x^7 + 4/5*a*b^3*x^5 + 2*a^2*b^2*x^3 + 4*a^3*b*x - a^4/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^2} dx = \frac{b^4 x^7}{7} - \frac{a^4}{x} + \frac{4 a b^3 x^5}{5} + 2 a^2 b^2 x^3 + 4 a^3 b x$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^2,x)
```

output
```
(b^4*x^7)/7 - a^4/x + (4*a*b^3*x^5)/5 + 2*a^2*b^2*x^3 + 4*a^3*b*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^2} dx = \frac{5b^4x^8 + 28a b^3x^6 + 70a^2b^2x^4 + 140a^3b x^2 - 35a^4}{35x}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x)
```

output
```
( - 35*a**4 + 140*a**3*b*x**2 + 70*a**2*b**2*x**4 + 28*a*b**3*x**6 + 5*b**
4*x**8)/(35*x)
```

3.321 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^4} dx = -\frac{a^4}{3x^3} - \frac{4a^3b}{x} + 6a^2b^2x + \frac{4}{3}ab^3x^3 + \frac{b^4x^5}{5}$$

output
```
-1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^4} dx = -\frac{a^4}{3x^3} - \frac{4a^3b}{x} + 6a^2b^2x + \frac{4}{3}ab^3x^3 + \frac{b^4x^5}{5}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
```

output
```
-1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^4}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^4}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{x^4}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^4}{x^4}+\frac{4a^3b}{x^2}+6a^2b^2+4ab^3x^2+b^4x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
```

output
```
-1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4a\,b^3x^3}{3}+\frac{b^4x^5}{5}$	45
risch	$\frac{b^4x^5}{5}+\frac{4a\,b^3x^3}{3}+6a^2b^2x+\frac{-4a^3b\,x^2-\frac{1}{3}a^4}{x^3}$	47
norman	$\frac{\frac{1}{5}b^4x^8+\frac{4}{3}a\,b^3x^6+6a^2b^2x^4-4a^3b\,x^2-\frac{1}{3}a^4}{x^3}$	48
gosper	$-\frac{-3b^4x^8-20a\,b^3x^6-90a^2b^2x^4+60a^3b\,x^2+5a^4}{15x^3}$	49
parallelrisch	$\frac{3b^4x^8+20a\,b^3x^6+90a^2b^2x^4-60a^3b\,x^2-5a^4}{15x^3}$	49
orering	$-\frac{\left(-3b^4x^8-20a\,b^3x^6-90a^2b^2x^4+60a^3b\,x^2+5a^4\right)\left(b^2x^4+2ab\,x^2+a^2\right)^2}{15x^3\left(b\,x^2+a\right)^4}$	78

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^4} \, dx = \frac{3\,b^4x^8 + 20\,ab^3x^6 + 90\,a^2b^2x^4 - 60\,a^3bx^2 - 5\,a^4}{15\,x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="fricas")
```

output
```
1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^
3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^4} \, dx = 6a^2b^2x + \frac{4ab^3x^3}{3} + \frac{b^4x^5}{5} + \frac{-a^4 - 12a^3bx^2}{3x^3}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**4,x)
```

output
```
6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 + (-a**4 - 12*a**3*b*x**2)/(
3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^4} \, dx = \frac{1}{5}\,b^4x^5 + \frac{4}{3}\,ab^3x^3 + 6\,a^2b^2x - \frac{12\,a^3bx^2 + a^4}{3\,x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="maxima")
```

output
```
1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^4} \, dx = \frac{1}{5} b^4x^5 + \frac{4}{3} ab^3x^3 + 6 a^2b^2x - \frac{12 a^3bx^2 + a^4}{3 x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x, algorithm="giac")
```

output
```
1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^4} \, dx = \frac{b^4 x^5}{5} - \frac{\frac{a^4}{3} + 4 b a^3 x^2}{x^3} + 6 a^2 b^2 x + \frac{4 a b^3 x^3}{3}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^4,x)
```

output
```
(b^4*x^5)/5 - (a^4/3 + 4*a^3*b*x^2)/x^3 + 6*a^2*b^2*x + (4*a*b^3*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^4} \, dx = \frac{3b^4x^8 + 20a b^3x^6 + 90a^2b^2x^4 - 60a^3b x^2 - 5a^4}{15x^3}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x)
```

output
```
( - 5*a**4 - 60*a**3*b*x**2 + 90*a**2*b**2*x**4 + 20*a*b**3*x**6 + 3*b**4*
x**8)/(15*x**3)
```

3.322 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^6} dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6} dx = -\frac{a^4}{5x^5} - \frac{4a^3b}{3x^3} - \frac{6a^2b^2}{x} + 4ab^3x + \frac{b^4x^3}{3}$$

output
```
-1/5*a^4/x^5-4/3*a^3*b/x^3-6*a^2*b^2/x+4*a*b^3*x+1/3*b^4*x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6} dx = -\frac{a^4}{5x^5} - \frac{4a^3b}{3x^3} - \frac{6a^2b^2}{x} + 4ab^3x + \frac{b^4x^3}{3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
```

output
```
-1/5*a^4/x^5 - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b^4*x^3)/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^6}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{x^6}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^4}{x^6}+\frac{4a^3b}{x^4}+\frac{6a^2b^2}{x^2}+4ab^3+b^4x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
```

output
```
-1/5*a^4/x^5 - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b^4*x^3)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^{4}}{5x^{5}}-\frac{4a^{3}b}{3x^{3}}-\frac{6a^{2}b^{2}}{x}+4a\,b^{3}x+\frac{b^{4}x^{3}}{3}$	45
risch	$\frac{b^{4}x^{3}}{3}+4a\,b^{3}x+\frac{-6a^{2}b^{2}x^{4}-\frac{4}{3}a^{3}b\,x^{2}-\frac{1}{5}a^{4}}{x^{5}}$	47
norman	$\frac{\frac{1}{3}b^{4}x^{8}+4a\,b^{3}x^{6}-6a^{2}b^{2}x^{4}-\frac{4}{3}a^{3}b\,x^{2}-\frac{1}{5}a^{4}}{x^{5}}$	48
gosper	$-\frac{-5b^{4}x^{8}-60a\,b^{3}x^{6}+90a^{2}b^{2}x^{4}+20a^{3}b\,x^{2}+3a^{4}}{15x^{5}}$	49
parallelrisch	$\frac{5b^{4}x^{8}+60a\,b^{3}x^{6}-90a^{2}b^{2}x^{4}-20a^{3}b\,x^{2}-3a^{4}}{15x^{5}}$	49
orering	$-\frac{\left(-5b^{4}x^{8}-60a\,b^{3}x^{6}+90a^{2}b^{2}x^{4}+20a^{3}b\,x^{2}+3a^{4}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}}{15x^{5}\left(b\,x^{2}+a\right)^{4}}$	78

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^4/x^5-4/3*a^3*b/x^3-6*a^2*b^2/x+4*a*b^3*x+1/3*b^4*x^3
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6}\,dx = \frac{5\,b^4x^8+60\,ab^3x^6-90\,a^2b^2x^4-20\,a^3bx^2-3\,a^4}{15\,x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="fricas")
```

output
```
1/15*(5*b^4*x^8 + 60*a*b^3*x^6 - 90*a^2*b^2*x^4 - 20*a^3*b*x^2 - 3*a^4)/x^
5
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6}\,dx = 4ab^3x+\frac{b^4x^3}{3}+\frac{-3a^4-20a^3bx^2-90a^2b^2x^4}{15x^5}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**6,x)
```

output
```
4*a*b**3*x + b**4*x**3/3 + (-3*a**4 - 20*a**3*b*x**2 - 90*a**2*b**2*x**4)/
(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6}\,dx = \frac{1}{3}\,b^4x^3+4\,ab^3x-\frac{90\,a^2b^2x^4+20\,a^3bx^2+3\,a^4}{15\,x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="maxima")
```

output
```
1/3*b^4*x^3 + 4*a*b^3*x - 1/15*(90*a^2*b^2*x^4 + 20*a^3*b*x^2 + 3*a^4)/x^5
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6}\,dx = \frac{1}{3}\,b^4x^3+4\,ab^3x-\frac{90\,a^2b^2x^4+20\,a^3bx^2+3\,a^4}{15\,x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x, algorithm="giac")
```

output
```
1/3*b^4*x^3 + 4*a*b^3*x - 1/15*(90*a^2*b^2*x^4 + 20*a^3*b*x^2 + 3*a^4)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6}\,dx = \frac{b^4\,x^3}{3}-\frac{\frac{a^4}{5}+\frac{4\,a^3\,b\,x^2}{3}+6\,a^2\,b^2\,x^4}{x^5}+4\,a\,b^3\,x$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^6,x)
```

output
```
(b^4*x^3)/3 - (a^4/5 + (4*a^3*b*x^2)/3 + 6*a^2*b^2*x^4)/x^5 + 4*a*b^3*x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6}\,dx = \frac{5b^4x^8+60a\,b^3x^6-90a^2b^2x^4-20a^3b\,x^2-3a^4}{15x^5}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x)
```

output
```
( - 3*a**4 - 20*a**3*b*x**2 - 90*a**2*b**2*x**4 + 60*a*b**3*x**6 + 5*b**4*
x**8)/(15*x**5)
```

3.323 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^8}\,dx=-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

output
```
-1/7*a^4/x^7-4/5*a^3*b/x^5-2*a^2*b^2/x^3-4*a*b^3/x+b^4*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^8}\,dx=-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]
```

output
```
-1/7*a^4/x^7 - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x + b^4*x
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^8}\,dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^8}dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx^2)^4}{x^8}dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^4}{x^8}+\frac{4a^3b}{x^6}+\frac{6a^2b^2}{x^4}+\frac{4ab^3}{x^2}+b^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]
```

output
```
-1/7*a^4/x^7 - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x + b^4*x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a^{4}}{7 x^{7}}-\frac{4 a^{3} b}{5 x^{5}}-\frac{2 a^{2} b^{2}}{x^{3}}-\frac{4 a \,b^{3}}{x}+b^{4} x$	44
risch	$b^{4} x +\frac{-4 a \,b^{3} x^{6}-2 a^{2} b^{2} x^{4}-\frac{4}{5} a^{3} b \,x^{2}-\frac{1}{7} a^{4}}{x^{7}}$	46
norman	$\frac{b^{4} x^{8}-4 a \,b^{3} x^{6}-2 a^{2} b^{2} x^{4}-\frac{4}{5} a^{3} b \,x^{2}-\frac{1}{7} a^{4}}{x^{7}}$	47
gosper	$-\frac{-35 b^{4} x^{8}+140 a \,b^{3} x^{6}+70 a^{2} b^{2} x^{4}+28 a^{3} b \,x^{2}+5 a^{4}}{35 x^{7}}$	49
parallelrisch	$\frac{35 b^{4} x^{8}-140 a \,b^{3} x^{6}-70 a^{2} b^{2} x^{4}-28 a^{3} b \,x^{2}-5 a^{4}}{35 x^{7}}$	49
orering	$-\frac{\left(-35 b^{4} x^{8}+140 a \,b^{3} x^{6}+70 a^{2} b^{2} x^{4}+28 a^{3} b \,x^{2}+5 a^{4}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{2}}{35 x^{7} \left(b \,x^{2}+a \right)^{4}}$	78

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^4/x^7-4/5*a^3*b/x^5-2*a^2*b^2/x^3-4*a*b^3/x+b^4*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^8}\,dx = \frac{35\,b^4x^8 - 140\,ab^3x^6 - 70\,a^2b^2x^4 - 28\,a^3bx^2 - 5\,a^4}{35\,x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="fricas")
```

output
```
1/35*(35*b^4*x^8 - 140*a*b^3*x^6 - 70*a^2*b^2*x^4 - 28*a^3*b*x^2 - 5*a^4)/
x^7
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^8}\,dx = b^4x + \frac{-5a^4 - 28a^3bx^2 - 70a^2b^2x^4 - 140ab^3x^6}{35x^7}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**8,x)
```

output
```
b**4*x + (-5*a**4 - 28*a**3*b*x**2 - 70*a**2*b**2*x**4 - 140*a*b**3*x**6)/
(35*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^8}\,dx = b^4x - \frac{140\,ab^3x^6 + 70\,a^2b^2x^4 + 28\,a^3bx^2 + 5\,a^4}{35\,x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="maxima")
```

output
```
b^4*x - 1/35*(140*a*b^3*x^6 + 70*a^2*b^2*x^4 + 28*a^3*b*x^2 + 5*a^4)/x^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^8} \, dx = b^4x - \frac{140 \, ab^3x^6 + 70 \, a^2b^2x^4 + 28 \, a^3bx^2 + 5 \, a^4}{35 \, x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x, algorithm="giac")
```

output
```
b^4*x - 1/35*(140*a*b^3*x^6 + 70*a^2*b^2*x^4 + 28*a^3*b*x^2 + 5*a^4)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^8} \, dx = b^4 \, x - \frac{\frac{a^4}{7} + \frac{4 \, a^3 \, b \, x^2}{5} + 2 \, a^2 \, b^2 \, x^4 + 4 \, a \, b^3 \, x^6}{x^7}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^8,x)
```

output
```
b^4*x - (a^4/7 + (4*a^3*b*x^2)/5 + 4*a*b^3*x^6 + 2*a^2*b^2*x^4)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^8} \, dx = \frac{35b^4x^8 - 140a \, b^3x^6 - 70a^2b^2x^4 - 28a^3b \, x^2 - 5a^4}{35x^7}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x)
```

output
```
( - 5*a**4 - 28*a**3*b*x**2 - 70*a**2*b**2*x**4 - 140*a*b**3*x**6 + 35*b**
4*x**8)/(35*x**7)
```

3.324 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx = -\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

output
```
-1/9*a^4/x^9-4/7*a^3*b/x^7-6/5*a^2*b^2/x^5-4/3*a*b^3/x^3-b^4/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx = -\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

output
```
-1/9*a^4/x^9 - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3)/(3*x^3)
 - b^4/x
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^{10}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{x^{10}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^4}{x^{10}}+\frac{4a^3b}{x^8}+\frac{6a^2b^2}{x^6}+\frac{4ab^3}{x^4}+\frac{b^4}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

output
```
-1/9*a^4/x^9 - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3)/(3*x^3)
 - b^4/x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^{4}}{9 x^{9}}-\frac{4 a^{3} b}{7 x^{7}}-\frac{6 a^{2} b^{2}}{5 x^{5}}-\frac{4 a \,b^{3}}{3 x^{3}}-\frac{b^{4}}{x}$	47
norman	$\frac{-b^{4} x^{8}-\frac{4}{3} a \,b^{3} x^{6}-\frac{6}{5} a^{2} b^{2} x^{4}-\frac{4}{7} a^{3} b \,x^{2}-\frac{1}{9} a^{4}}{x^{9}}$	48
risch	$\frac{-b^{4} x^{8}-\frac{4}{3} a \,b^{3} x^{6}-\frac{6}{5} a^{2} b^{2} x^{4}-\frac{4}{7} a^{3} b \,x^{2}-\frac{1}{9} a^{4}}{x^{9}}$	48
gosper	$-\frac{315 b^{4} x^{8}+420 a \,b^{3} x^{6}+378 a^{2} b^{2} x^{4}+180 a^{3} b \,x^{2}+35 a^{4}}{315 x^{9}}$	49
parallelrisch	$\frac{-315 b^{4} x^{8}-420 a \,b^{3} x^{6}-378 a^{2} b^{2} x^{4}-180 a^{3} b \,x^{2}-35 a^{4}}{315 x^{9}}$	49
orering	$-\frac{\left(315 b^{4} x^{8}+420 a \,b^{3} x^{6}+378 a^{2} b^{2} x^{4}+180 a^{3} b \,x^{2}+35 a^{4}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{2}}{315 x^{9} \left(b \,x^{2}+a \right)^{4}}$	78

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^4/x^9-4/7*a^3*b/x^7-6/5*a^2*b^2/x^5-4/3*a*b^3/x^3-b^4/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx = -\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="fricas")
```

output
```
-1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35
*a^4)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx = \frac{-35a^4-180a^3bx^2-378a^2b^2x^4-420ab^3x^6-315b^4x^8}{315x^9}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**10,x)
```

output
```
(-35*a**4 - 180*a**3*b*x**2 - 378*a**2*b**2*x**4 - 420*a*b**3*x**6 - 315*b
**4*x**8)/(315*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx = -\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="maxima")
```

output
```
-1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35
*a^4)/x^9
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx = -\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x, algorithm="giac")
```

output
```
-1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b*x^2 + 35
*a^4)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx = -\frac{\frac{a^4}{9}+\frac{4\,a^3\,b\,x^2}{7}+\frac{6\,a^2\,b^2\,x^4}{5}+\frac{4\,a\,b^3\,x^6}{3}+b^4\,x^8}{x^9}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^10,x)
```

output
```
-(a^4/9 + b^4*x^8 + (4*a^3*b*x^2)/7 + (4*a*b^3*x^6)/3 + (6*a^2*b^2*x^4)/5)
/x^9
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx = \frac{-315b^4x^8-420a\,b^3x^6-378a^2b^2x^4-180a^3b\,x^2-35a^4}{315x^9}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x)
```

output
```
( - 35*a**4 - 180*a**3*b*x**2 - 378*a**2*b**2*x**4 - 420*a*b**3*x**6 - 315
*b**4*x**8)/(315*x**9)
```

3.325 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}} dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}} dx = -\frac{a^4}{11x^{11}} - \frac{4a^3b}{9x^9} - \frac{6a^2b^2}{7x^7} - \frac{4ab^3}{5x^5} - \frac{b^4}{3x^3}$$

output
```
-1/11*a^4/x^11-4/9*a^3*b/x^9-6/7*a^2*b^2/x^7-4/5*a*b^3/x^5-1/3*b^4/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}} dx = -\frac{a^4}{11x^{11}} - \frac{4a^3b}{9x^9} - \frac{6a^2b^2}{7x^7} - \frac{4ab^3}{5x^5} - \frac{b^4}{3x^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

output
```
-1/11*a^4/x^11 - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b^3)/(5*x^
5) - b^4/(3*x^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}}dx$$

$$\Big\downarrow \ 1380$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^{12}}dx}{b^4}$$

$$\Big\downarrow \ 27$$

$$\int \frac{(a+bx^2)^4}{x^{12}}dx$$

$$\Big\downarrow \ 244$$

$$\int \left(\frac{a^4}{x^{12}}+\frac{4a^3b}{x^{10}}+\frac{6a^2b^2}{x^8}+\frac{4ab^3}{x^6}+\frac{b^4}{x^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

output
```
-1/11*a^4/x^11 - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b^3)/(5*x^
5) - b^4/(3*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{4}}{11 x^{11}}-\frac{4 a^{3} b}{9 x^{9}}-\frac{6 a^{2} b^{2}}{7 x^{7}}-\frac{4 a\,b^{3}}{5 x^{5}}-\frac{b^{4}}{3 x^{3}}$	47
norman	$\frac{-\frac{1}{3} b^{4} x^{8}-\frac{4}{5} a\,b^{3} x^{6}-\frac{6}{7} a^{2} b^{2} x^{4}-\frac{4}{9} a^{3} b\,x^{2}-\frac{1}{11} a^{4}}{x^{11}}$	48
risch	$\frac{-\frac{1}{3} b^{4} x^{8}-\frac{4}{5} a\,b^{3} x^{6}-\frac{6}{7} a^{2} b^{2} x^{4}-\frac{4}{9} a^{3} b\,x^{2}-\frac{1}{11} a^{4}}{x^{11}}$	48
gosper	$-\frac{1155 b^{4} x^{8}+2772 a\,b^{3} x^{6}+2970 a^{2} b^{2} x^{4}+1540 a^{3} b\,x^{2}+315 a^{4}}{3465 x^{11}}$	49
parallelrisch	$\frac{-1155 b^{4} x^{8}-2772 a\,b^{3} x^{6}-2970 a^{2} b^{2} x^{4}-1540 a^{3} b\,x^{2}-315 a^{4}}{3465 x^{11}}$	49
orering	$-\frac{\left(1155 b^{4} x^{8}+2772 a\,b^{3} x^{6}+2970 a^{2} b^{2} x^{4}+1540 a^{3} b\,x^{2}+315 a^{4}\right) \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{2}}{3465 x^{11} \left(b\,x^{2}+a \right)^{4}}$	78

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*a^4/x^11-4/9*a^3*b/x^9-6/7*a^2*b^2/x^7-4/5*a*b^3/x^5-1/3*b^4/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{12}}\,dx = -\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="fricas")
```

output
```
-1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
 + 315*a^4)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^{12}}\,dx = \frac{-315a^4-1540a^3bx^2-2970a^2b^2x^4-2772ab^3x^6-1155b^4x^8}{3465x^{11}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**12,x)
```

output
```
(-315*a**4 - 1540*a**3*b*x**2 - 2970*a**2*b**2*x**4 - 2772*a*b**3*x**6 - 1
155*b**4*x**8)/(3465*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}}\,dx = -\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="maxima")
```

output
```
-1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
 + 315*a^4)/x^11
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}}\,dx = -\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x, algorithm="giac")
```

output
```
-1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*a^3*b*x^2
 + 315*a^4)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}}\,dx = -\frac{\frac{a^4}{11}+\frac{4\,a^3\,b\,x^2}{9}+\frac{6\,a^2\,b^2\,x^4}{7}+\frac{4\,a\,b^3\,x^6}{5}+\frac{b^4\,x^8}{3}}{x^{11}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^12,x)
```

output
```
-(a^4/11 + (b^4*x^8)/3 + (4*a^3*b*x^2)/9 + (4*a*b^3*x^6)/5 + (6*a^2*b^2*x^
4)/7)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}}\,dx = \frac{-1155b^4x^8-2772a\,b^3x^6-2970a^2b^2x^4-1540a^3b\,x^2-315a^4}{3465x^{11}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x)
```

output
```
( - 315*a**4 - 1540*a**3*b*x**2 - 2970*a**2*b**2*x**4 - 2772*a*b**3*x**6 -
 1155*b**4*x**8)/(3465*x**11)
```

3.326 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{14}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{14}}\,dx=-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

output
```
-1/13*a^4/x^13-4/11*a^3*b/x^11-2/3*a^2*b^2/x^9-4/7*a*b^3/x^7-1/5*b^4/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{14}}\,dx=-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
```

output
```
-1/13*a^4/x^13 - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a*b^3)/(7*
x^7) - b^4/(5*x^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{14}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{x^{14}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{x^{14}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^4}{x^{14}}+\frac{4a^3b}{x^{12}}+\frac{6a^2b^2}{x^{10}}+\frac{4ab^3}{x^8}+\frac{b^4}{x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
```

output
```
-1/13*a^4/x^13 - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a*b^3)/(7*
x^7) - b^4/(5*x^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{4}}{13x^{13}}-\frac{4a^{3}b}{11x^{11}}-\frac{2a^{2}b^{2}}{3x^{9}}-\frac{4a\,b^{3}}{7x^{7}}-\frac{b^{4}}{5x^{5}}$	47
norman	$\frac{-\frac{1}{5}b^{4}x^{8}-\frac{4}{7}a\,b^{3}x^{6}-\frac{2}{3}a^{2}b^{2}x^{4}-\frac{4}{11}a^{3}b\,x^{2}-\frac{1}{13}a^{4}}{x^{13}}$	48
risch	$\frac{-\frac{1}{5}b^{4}x^{8}-\frac{4}{7}a\,b^{3}x^{6}-\frac{2}{3}a^{2}b^{2}x^{4}-\frac{4}{11}a^{3}b\,x^{2}-\frac{1}{13}a^{4}}{x^{13}}$	48
gosper	$-\frac{3003b^{4}x^{8}+8580a\,b^{3}x^{6}+10010a^{2}b^{2}x^{4}+5460a^{3}b\,x^{2}+1155a^{4}}{15015x^{13}}$	49
parallelrisch	$\frac{-3003b^{4}x^{8}-8580a\,b^{3}x^{6}-10010a^{2}b^{2}x^{4}-5460a^{3}b\,x^{2}-1155a^{4}}{15015x^{13}}$	49
orering	$-\frac{\left(3003b^{4}x^{8}+8580a\,b^{3}x^{6}+10010a^{2}b^{2}x^{4}+5460a^{3}b\,x^{2}+1155a^{4}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}}{15015x^{13}\left(b\,x^{2}+a\right)^{4}}$	78

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/13*a^4/x^13-4/11*a^3*b/x^11-2/3*a^2*b^2/x^9-4/7*a*b^3/x^7-1/5*b^4/x^5
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^{14}}\,dx$$
$$= -\frac{3003\,b^4x^8 + 8580\,ab^3x^6 + 10010\,a^2b^2x^4 + 5460\,a^3bx^2 + 1155\,a^4}{15015\,x^{13}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="fricas")
```

output
```
-1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x
^2 + 1155*a^4)/x^13
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{x^{14}}\,dx$$
$$= \frac{-1155a^4 - 5460a^3bx^2 - 10010a^2b^2x^4 - 8580ab^3x^6 - 3003b^4x^8}{15015x^{13}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**14,x)
```

output
```
(-1155*a**4 - 5460*a**3*b*x**2 - 10010*a**2*b**2*x**4 - 8580*a*b**3*x**6 -
 3003*b**4*x**8)/(15015*x**13)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{14}}\,dx = -\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="maxima")
```

output
```
-1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x
^2 + 1155*a^4)/x^13
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{14}}\,dx = -\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x, algorithm="giac")
```

output
```
-1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 5460*a^3*b*x
^2 + 1155*a^4)/x^13
```

Mupad [B] (verification not implemented)

Time = 18.65 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{14}}\,dx = -\frac{\frac{a^4}{13}+\frac{4\,a^3\,b\,x^2}{11}+\frac{2\,a^2\,b^2\,x^4}{3}+\frac{4\,a\,b^3\,x^6}{7}+\frac{b^4\,x^8}{5}}{x^{13}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/x^14,x)
```

output
```
-(a^4/13 + (b^4*x^8)/5 + (4*a^3*b*x^2)/11 + (4*a*b^3*x^6)/7 + (2*a^2*b^2*x
^4)/3)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{14}}\,dx = \frac{-3003b^4x^8-8580a\,b^3x^6-10010a^2b^2x^4-5460a^3b\,x^2-1155a^4}{15015x^{13}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x)
```

output
```
( - 1155*a**4 - 5460*a**3*b*x**2 - 10010*a**2*b**2*x**4 - 8580*a*b**3*x**6
 - 3003*b**4*x**8)/(15015*x**13)
```

3.327 $\int x^{11}(a^2+2abx^2+b^2x^4)^3 dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^3 dx=\frac{a^6x^{12}}{12}+\frac{3}{7}a^5bx^{14}+\frac{15}{16}a^4b^2x^{16}+\frac{10}{9}a^3b^3x^{18}+\frac{3}{4}a^2b^4x^{20}+\frac{3}{11}ab^5x^{22}+\frac{b^6x^{24}}{24}$$

output
```
1/12*a^6*x^12+3/7*a^5*b*x^14+15/16*a^4*b^2*x^16+10/9*a^3*b^3*x^18+3/4*a^2*
b^4*x^20+3/11*a*b^5*x^22+1/24*b^6*x^24
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^3 dx=\frac{a^6x^{12}}{12}+\frac{3}{7}a^5bx^{14}+\frac{15}{16}a^4b^2x^{16}+\frac{10}{9}a^3b^3x^{18}+\frac{3}{4}a^2b^4x^{20}+\frac{3}{11}ab^5x^{22}+\frac{b^6x^{24}}{24}$$

input
```
Integrate[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^6*x^12)/12 + (3*a^5*b*x^14)/7 + (15*a^4*b^2*x^16)/16 + (10*a^3*b^3*x^18
)/9 + (3*a^2*b^4*x^20)/4 + (3*a*b^5*x^22)/11 + (b^6*x^24)/24
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^6x^{11}\left(bx^2+a\right)^6dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x^{11}\left(a+bx^2\right)^6dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^{10}\left(bx^2+a\right)^6dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(b^6x^{22}+6ab^5x^{20}+15a^2b^4x^{18}+20a^3b^3x^{16}+15a^4b^2x^{14}+6a^5bx^{12}+a^6x^{10}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^6x^{12}}{6}+\frac{6}{7}a^5bx^{14}+\frac{15}{8}a^4b^2x^{16}+\frac{20}{9}a^3b^3x^{18}+\frac{3}{2}a^2b^4x^{20}+\frac{6}{11}ab^5x^{22}+\frac{b^6x^{24}}{12}\right)$$

input
```
Int[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
((a^6*x^12)/6 + (6*a^5*b*x^14)/7 + (15*a^4*b^2*x^16)/8 + (20*a^3*b^3*x^18)
/9 + (3*a^2*b^4*x^20)/2 + (6*a*b^5*x^22)/11 + (b^6*x^24)/12)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{1}{12}a^6x^{12}+\frac{3}{7}a^5b\,x^{14}+\frac{15}{16}a^4b^2x^{16}+\frac{10}{9}a^3b^3x^{18}+\frac{3}{4}a^2b^4x^{20}+\frac{3}{11}a\,b^5x^{22}+\frac{1}{24}b^6x^{24}$	69
norman	$\frac{1}{12}a^6x^{12}+\frac{3}{7}a^5b\,x^{14}+\frac{15}{16}a^4b^2x^{16}+\frac{10}{9}a^3b^3x^{18}+\frac{3}{4}a^2b^4x^{20}+\frac{3}{11}a\,b^5x^{22}+\frac{1}{24}b^6x^{24}$	69
risch	$\frac{1}{12}a^6x^{12}+\frac{3}{7}a^5b\,x^{14}+\frac{15}{16}a^4b^2x^{16}+\frac{10}{9}a^3b^3x^{18}+\frac{3}{4}a^2b^4x^{20}+\frac{3}{11}a\,b^5x^{22}+\frac{1}{24}b^6x^{24}$	69
parallelrisch	$\frac{1}{12}a^6x^{12}+\frac{3}{7}a^5b\,x^{14}+\frac{15}{16}a^4b^2x^{16}+\frac{10}{9}a^3b^3x^{18}+\frac{3}{4}a^2b^4x^{20}+\frac{3}{11}a\,b^5x^{22}+\frac{1}{24}b^6x^{24}$	69
gosper	$\frac{x^{12}\left(462b^6x^{12}+3024a\,b^5x^{10}+8316a^2b^4x^8+12320a^3b^3x^6+10395a^4b^2x^4+4752a^5b\,x^2+924a^6\right)}{11088}$	71
orering	$\frac{x^{12}\left(462b^6x^{12}+3024a\,b^5x^{10}+8316a^2b^4x^8+12320a^3b^3x^6+10395a^4b^2x^4+4752a^5b\,x^2+924a^6\right)\left(b^2x^4+2ab\,x^2+a^2\right)^3}{11088\left(b\,x^2+a\right)^6}$	100

input
```
int(x^11*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^6*x^12+3/7*a^5*b*x^14+15/16*a^4*b^2*x^16+10/9*a^3*b^3*x^18+3/4*a^2*
b^4*x^20+3/11*a*b^5*x^22+1/24*b^6*x^24
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{24}\,b^6x^{24}+\frac{3}{11}\,ab^5x^{22}+\frac{3}{4}\,a^2b^4x^{20}+\frac{10}{9}\,a^3b^3x^{18}+\frac{15}{16}\,a^4b^2x^{16}+\frac{3}{7}\,a^5bx^{14}+\frac{1}{12}\,a^6x^{12}$$

input
```
integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/24*b^6*x^24 + 3/11*a*b^5*x^22 + 3/4*a^2*b^4*x^20 + 10/9*a^3*b^3*x^18 + 1
5/16*a^4*b^2*x^16 + 3/7*a^5*b*x^14 + 1/12*a^6*x^12
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^{12}}{12}+\frac{3a^5bx^{14}}{7}+\frac{15a^4b^2x^{16}}{16}+\frac{10a^3b^3x^{18}}{9}+\frac{3a^2b^4x^{20}}{4}+\frac{3ab^5x^{22}}{11}+\frac{b^6x^{24}}{24}$$

input
```
integrate(x**11*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
a**6*x**12/12 + 3*a**5*b*x**14/7 + 15*a**4*b**2*x**16/16 + 10*a**3*b**3*x*
*18/9 + 3*a**2*b**4*x**20/4 + 3*a*b**5*x**22/11 + b**6*x**24/24
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{24}\,b^6x^{24}+\frac{3}{11}\,ab^5x^{22}+\frac{3}{4}\,a^2b^4x^{20}+\frac{10}{9}\,a^3b^3x^{18}+\frac{15}{16}\,a^4b^2x^{16}+\frac{3}{7}\,a^5bx^{14}+\frac{1}{12}\,a^6x^{12}$$

input
```
integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/24*b^6*x^24 + 3/11*a*b^5*x^22 + 3/4*a^2*b^4*x^20 + 10/9*a^3*b^3*x^18 + 1
5/16*a^4*b^2*x^16 + 3/7*a^5*b*x^14 + 1/12*a^6*x^12
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{24}b^6x^{24}+\frac{3}{11}ab^5x^{22}+\frac{3}{4}a^2b^4x^{20}+\frac{10}{9}a^3b^3x^{18}+\frac{15}{16}a^4b^2x^{16}+\frac{3}{7}a^5bx^{14}+\frac{1}{12}a^6x^{12}$$

input
```
integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/24*b^6*x^24 + 3/11*a*b^5*x^22 + 3/4*a^2*b^4*x^20 + 10/9*a^3*b^3*x^18 + 1
5/16*a^4*b^2*x^16 + 3/7*a^5*b*x^14 + 1/12*a^6*x^12
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6\,x^{12}}{12}+\frac{3\,a^5\,b\,x^{14}}{7}+\frac{15\,a^4\,b^2\,x^{16}}{16}+\frac{10\,a^3\,b^3\,x^{18}}{9}+\frac{3\,a^2\,b^4\,x^{20}}{4}+\frac{3\,a\,b^5\,x^{22}}{11}+\frac{b^6\,x^{24}}{24}$$

input
```
int(x^11*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(a^6*x^12)/12 + (b^6*x^24)/24 + (3*a^5*b*x^14)/7 + (3*a*b^5*x^22)/11 + (15
*a^4*b^2*x^16)/16 + (10*a^3*b^3*x^18)/9 + (3*a^2*b^4*x^20)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$
$$=\frac{x^{12}(462b^6x^{12}+3024a\,b^5x^{10}+8316a^2b^4x^8+12320a^3b^3x^6+10395a^4b^2x^4+4752a^5b\,x^2+924a^6)}{11088}$$

input
```
int(x^11*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(x**12*(924*a**6 + 4752*a**5*b*x**2 + 10395*a**4*b**2*x**4 + 12320*a**3*b*
*3*x**6 + 8316*a**2*b**4*x**8 + 3024*a*b**5*x**10 + 462*b**6*x**12))/11088
```

3.328 $\int x^9(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int x^9\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^4(a+bx^2)^7}{14b^5}-\frac{a^3(a+bx^2)^8}{4b^5}+\frac{a^2(a+bx^2)^9}{3b^5}-\frac{a(a+bx^2)^{10}}{5b^5}+\frac{(a+bx^2)^{11}}{22b^5}$$

output
```
1/14*a^4*(b*x^2+a)^7/b^5-1/4*a^3*(b*x^2+a)^8/b^5+1/3*a^2*(b*x^2+a)^9/b^5-1
/5*a*(b*x^2+a)^10/b^5+1/22*(b*x^2+a)^11/b^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int x^9\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^{10}}{10}+\frac{1}{2}a^5bx^{12}+\frac{15}{14}a^4b^2x^{14}+\frac{5}{4}a^3b^3x^{16}+\frac{5}{6}a^2b^4x^{18}+\frac{3}{10}ab^5x^{20}+\frac{b^6x^{22}}{22}$$

input
```
Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^6*x^10)/10 + (a^5*b*x^12)/2 + (15*a^4*b^2*x^14)/14 + (5*a^3*b^3*x^16)/4
 + (5*a^2*b^4*x^18)/6 + (3*a*b^5*x^20)/10 + (b^6*x^22)/22
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^3 dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^6 x^9 \left(bx^2+a\right)^6 dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x^9 \left(a+bx^2\right)^6 dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^8 \left(bx^2+a\right)^6 dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{10}}{b^4}-\frac{4a\left(bx^2+a\right)^9}{b^4}+\frac{6a^2\left(bx^2+a\right)^8}{b^4}-\frac{4a^3\left(bx^2+a\right)^7}{b^4}+\frac{a^4\left(bx^2+a\right)^6}{b^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^4\left(a+bx^2\right)^7}{7b^5}-\frac{a^3\left(a+bx^2\right)^8}{2b^5}+\frac{2a^2\left(a+bx^2\right)^9}{3b^5}+\frac{\left(a+bx^2\right)^{11}}{11b^5}-\frac{2a\left(a+bx^2\right)^{10}}{5b^5}\right)$$

input
```
Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
((a^4*(a + b*x^2)^7)/(7*b^5) - (a^3*(a + b*x^2)^8)/(2*b^5) + (2*a^2*(a + b
*x^2)^9)/(3*b^5) - (2*a*(a + b*x^2)^10)/(5*b^5) + (a + b*x^2)^11/(11*b^5))
/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{1}{10}a^6x^{10}+\frac{1}{2}a^5b\,x^{12}+\frac{15}{14}a^4b^2x^{14}+\frac{5}{4}a^3b^3x^{16}+\frac{5}{6}a^2b^4x^{18}+\frac{3}{10}b^5a\,x^{20}+\frac{1}{22}b^6x^{22}$	69
norman	$\frac{1}{10}a^6x^{10}+\frac{1}{2}a^5b\,x^{12}+\frac{15}{14}a^4b^2x^{14}+\frac{5}{4}a^3b^3x^{16}+\frac{5}{6}a^2b^4x^{18}+\frac{3}{10}b^5a\,x^{20}+\frac{1}{22}b^6x^{22}$	69
risch	$\frac{1}{10}a^6x^{10}+\frac{1}{2}a^5b\,x^{12}+\frac{15}{14}a^4b^2x^{14}+\frac{5}{4}a^3b^3x^{16}+\frac{5}{6}a^2b^4x^{18}+\frac{3}{10}b^5a\,x^{20}+\frac{1}{22}b^6x^{22}$	69
parallelrisch	$\frac{1}{10}a^6x^{10}+\frac{1}{2}a^5b\,x^{12}+\frac{15}{14}a^4b^2x^{14}+\frac{5}{4}a^3b^3x^{16}+\frac{5}{6}a^2b^4x^{18}+\frac{3}{10}b^5a\,x^{20}+\frac{1}{22}b^6x^{22}$	69
gosper	$\frac{x^{10}\left(210b^6x^{12}+1386a\,b^5x^{10}+3850a^2b^4x^8+5775a^3b^3x^6+4950a^4b^2x^4+2310a^5b\,x^2+462a^6\right)}{4620}$	71
orering	$\frac{x^{10}\left(210b^6x^{12}+1386a\,b^5x^{10}+3850a^2b^4x^8+5775a^3b^3x^6+4950a^4b^2x^4+2310a^5b\,x^2+462a^6\right)\left(b^2x^4+2ab\,x^2+a^2\right)^3}{4620\left(b\,x^2+a\right)^6}$	100

input
```
int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*a^6*x^10+1/2*a^5*b*x^12+15/14*a^4*b^2*x^14+5/4*a^3*b^3*x^16+5/6*a^2*b
^4*x^18+3/10*b^5*a*x^20+1/22*b^6*x^22
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int x^9\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{22}\,b^6x^{22}+\frac{3}{10}\,ab^5x^{20}+\frac{5}{6}\,a^2b^4x^{18}+\frac{5}{4}\,a^3b^3x^{16}+\frac{15}{14}\,a^4b^2x^{14}+\frac{1}{2}\,a^5bx^{12}+\frac{1}{10}\,a^6x^{10}$$

input
```
integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/22*b^6*x^22 + 3/10*a*b^5*x^20 + 5/6*a^2*b^4*x^18 + 5/4*a^3*b^3*x^16 + 15
/14*a^4*b^2*x^14 + 1/2*a^5*b*x^12 + 1/10*a^6*x^10
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^{10}}{10}+\frac{a^5bx^{12}}{2}+\frac{15a^4b^2x^{14}}{14}+\frac{5a^3b^3x^{16}}{4}+\frac{5a^2b^4x^{18}}{6}+\frac{3ab^5x^{20}}{10}+\frac{b^6x^{22}}{22}$$

input
```
integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
a**6*x**10/10 + a**5*b*x**12/2 + 15*a**4*b**2*x**14/14 + 5*a**3*b**3*x**16
/4 + 5*a**2*b**4*x**18/6 + 3*a*b**5*x**20/10 + b**6*x**22/22
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{22}\,b^6x^{22}+\frac{3}{10}\,ab^5x^{20}+\frac{5}{6}\,a^2b^4x^{18}+\frac{5}{4}\,a^3b^3x^{16}+\frac{15}{14}\,a^4b^2x^{14}+\frac{1}{2}\,a^5bx^{12}+\frac{1}{10}\,a^6x^{10}$$

input
```
integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/22*b^6*x^22 + 3/10*a*b^5*x^20 + 5/6*a^2*b^4*x^18 + 5/4*a^3*b^3*x^16 + 15
/14*a^4*b^2*x^14 + 1/2*a^5*b*x^12 + 1/10*a^6*x^10
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{22}\,b^6x^{22}+\frac{3}{10}\,ab^5x^{20}+\frac{5}{6}\,a^2b^4x^{18}+\frac{5}{4}\,a^3b^3x^{16} +\frac{15}{14}\,a^4b^2x^{14}+\frac{1}{2}\,a^5bx^{12}+\frac{1}{10}\,a^6x^{10}$$

input
```
integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/22*b^6*x^22 + 3/10*a*b^5*x^20 + 5/6*a^2*b^4*x^18 + 5/4*a^3*b^3*x^16 + 15
/14*a^4*b^2*x^14 + 1/2*a^5*b*x^12 + 1/10*a^6*x^10
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{a^6\,x^{10}}{10}+\frac{a^5\,b\,x^{12}}{2}+\frac{15\,a^4\,b^2\,x^{14}}{14}+\frac{5\,a^3\,b^3\,x^{16}}{4} +\frac{5\,a^2\,b^4\,x^{18}}{6}+\frac{3\,a\,b^5\,x^{20}}{10}+\frac{b^6\,x^{22}}{22}$$

input
```
int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(a^6*x^10)/10 + (b^6*x^22)/22 + (a^5*b*x^12)/2 + (3*a*b^5*x^20)/10 + (15*a
^4*b^2*x^14)/14 + (5*a^3*b^3*x^16)/4 + (5*a^2*b^4*x^18)/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$$
$$= \frac{x^{10}(210b^6x^{12}+1386a\,b^5x^{10}+3850a^2b^4x^8+5775a^3b^3x^6+4950a^4b^2x^4+2310a^5b\,x^2+462a^6)}{4620}$$

input
```
int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(x**10*(462*a**6 + 2310*a**5*b*x**2 + 4950*a**4*b**2*x**4 + 5775*a**3*b**3
*x**6 + 3850*a**2*b**4*x**8 + 1386*a*b**5*x**10 + 210*b**6*x**12))/4620
```

3.329 $\int x^7(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^3\,dx=-\frac{a^3(a+bx^2)^7}{14b^4}+\frac{3a^2(a+bx^2)^8}{16b^4}-\frac{a(a+bx^2)^9}{6b^4}+\frac{(a+bx^2)^{10}}{20b^4}$$

output
```
-1/14*a^3*(b*x^2+a)^7/b^4+3/16*a^2*(b*x^2+a)^8/b^4-1/6*a*(b*x^2+a)^9/b^4+1
/20*(b*x^2+a)^10/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.14

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^8}{8}+\frac{3}{5}a^5bx^{10}+\frac{5}{4}a^4b^2x^{12}+\frac{10}{7}a^3b^3x^{14}+\frac{15}{16}a^2b^4x^{16}+\frac{1}{3}ab^5x^{18}+\frac{b^6x^{20}}{20}$$

input
```
Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^6*x^8)/8 + (3*a^5*b*x^10)/5 + (5*a^4*b^2*x^12)/4 + (10*a^3*b^3*x^14)/7
+ (15*a^2*b^4*x^16)/16 + (a*b^5*x^18)/3 + (b^6*x^20)/20
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3 dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^6x^7\left(bx^2+a\right)^6dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x^7\left(a+bx^2\right)^6dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^6\left(bx^2+a\right)^6dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^9}{b^3}-\frac{3a\left(bx^2+a\right)^8}{b^3}+\frac{3a^2\left(bx^2+a\right)^7}{b^3}-\frac{a^3\left(bx^2+a\right)^6}{b^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^3\left(a+bx^2\right)^7}{7b^4}+\frac{3a^2\left(a+bx^2\right)^8}{8b^4}+\frac{\left(a+bx^2\right)^{10}}{10b^4}-\frac{a\left(a+bx^2\right)^9}{3b^4}\right)$$

input
```
Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(-1/7*(a^3*(a + b*x^2)^7)/b^4 + (3*a^2*(a + b*x^2)^8)/(8*b^4) - (a*(a + b*
x^2)^9)/(3*b^4) + (a + b*x^2)^10/(10*b^4))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{1}{8}a^6x^8+\frac{3}{5}a^5b\,x^{10}+\frac{5}{4}a^4b^2x^{12}+\frac{10}{7}a^3b^3x^{14}+\frac{15}{16}a^2b^4x^{16}+\frac{1}{3}b^5a\,x^{18}+\frac{1}{20}b^6x^{20}$	69
norman	$\frac{1}{8}a^6x^8+\frac{3}{5}a^5b\,x^{10}+\frac{5}{4}a^4b^2x^{12}+\frac{10}{7}a^3b^3x^{14}+\frac{15}{16}a^2b^4x^{16}+\frac{1}{3}b^5a\,x^{18}+\frac{1}{20}b^6x^{20}$	69
risch	$\frac{1}{8}a^6x^8+\frac{3}{5}a^5b\,x^{10}+\frac{5}{4}a^4b^2x^{12}+\frac{10}{7}a^3b^3x^{14}+\frac{15}{16}a^2b^4x^{16}+\frac{1}{3}b^5a\,x^{18}+\frac{1}{20}b^6x^{20}$	69
parallelrisch	$\frac{1}{8}a^6x^8+\frac{3}{5}a^5b\,x^{10}+\frac{5}{4}a^4b^2x^{12}+\frac{10}{7}a^3b^3x^{14}+\frac{15}{16}a^2b^4x^{16}+\frac{1}{3}b^5a\,x^{18}+\frac{1}{20}b^6x^{20}$	69
gosper	$\frac{x^8\left(84b^6x^{12}+560a\,b^5x^{10}+1575a^2b^4x^8+2400a^3b^3x^6+2100a^4b^2x^4+1008a^5b\,x^2+210a^6\right)}{1680}$	71
orering	$\frac{x^8\left(84b^6x^{12}+560a\,b^5x^{10}+1575a^2b^4x^8+2400a^3b^3x^6+2100a^4b^2x^4+1008a^5b\,x^2+210a^6\right)\left(b^2x^4+2ab\,x^2+a^2\right)^3}{1680\left(b\,x^2+a\right)^6}$	100

input
```
int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^6*x^8+3/5*a^5*b*x^10+5/4*a^4*b^2*x^12+10/7*a^3*b^3*x^14+15/16*a^2*b^
4*x^16+1/3*b^5*a*x^18+1/20*b^6*x^20
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{20} \, b^6 x^{20} + \frac{1}{3} \, a b^5 x^{18} + \frac{15}{16} \, a^2 b^4 x^{16} + \frac{10}{7} \, a^3 b^3 x^{14} + \frac{5}{4} \, a^4 b^2 x^{12} + \frac{3}{5} \, a^5 b x^{10} + \frac{1}{8} \, a^6 x^8$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^3*x^14 +
5/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{a^6 x^8}{8} + \frac{3a^5 b x^{10}}{5} + \frac{5a^4 b^2 x^{12}}{4} + \frac{10a^3 b^3 x^{14}}{7} + \frac{15a^2 b^4 x^{16}}{16} + \frac{a b^5 x^{18}}{3} + \frac{b^6 x^{20}}{20}$$

input
```
integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
a**6*x**8/8 + 3*a**5*b*x**10/5 + 5*a**4*b**2*x**12/4 + 10*a**3*b**3*x**14/
7 + 15*a**2*b**4*x**16/16 + a*b**5*x**18/3 + b**6*x**20/20
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{20}\,b^6x^{20}+\frac{1}{3}\,ab^5x^{18}+\frac{15}{16}\,a^2b^4x^{16}+\frac{10}{7}\,a^3b^3x^{14}+\frac{5}{4}\,a^4b^2x^{12}+\frac{3}{5}\,a^5bx^{10}+\frac{1}{8}\,a^6x^8$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^3*x^14 +
5/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{20}\,b^6x^{20}+\frac{1}{3}\,ab^5x^{18}+\frac{15}{16}\,a^2b^4x^{16}+\frac{10}{7}\,a^3b^3x^{14}+\frac{5}{4}\,a^4b^2x^{12}+\frac{3}{5}\,a^5bx^{10}+\frac{1}{8}\,a^6x^8$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^3*x^14 +
5/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6\,x^8}{8}+\frac{3\,a^5\,b\,x^{10}}{5}+\frac{5\,a^4\,b^2\,x^{12}}{4}+\frac{10\,a^3\,b^3\,x^{14}}{7}+\frac{15\,a^2\,b^4\,x^{16}}{16}+\frac{a\,b^5\,x^{18}}{3}+\frac{b^6\,x^{20}}{20}$$

input
```
int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(a^6*x^8)/8 + (b^6*x^20)/20 + (3*a^5*b*x^10)/5 + (a*b^5*x^18)/3 + (5*a^4*b
^2*x^12)/4 + (10*a^3*b^3*x^14)/7 + (15*a^2*b^4*x^16)/16
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$
$$=\frac{x^8(84b^6x^{12}+560a\,b^5x^{10}+1575a^2b^4x^8+2400a^3b^3x^6+2100a^4b^2x^4+1008a^5b\,x^2+210a^6)}{1680}$$

input
```
int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(x**8*(210*a**6 + 1008*a**5*b*x**2 + 2100*a**4*b**2*x**4 + 2400*a**3*b**3*
x**6 + 1575*a**2*b**4*x**8 + 560*a*b**5*x**10 + 84*b**6*x**12))/1680
```

3.330 $\int x^5(a^2+2abx^2+b^2x^4)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 53

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{a^2(a+bx^2)^7}{14b^3} - \frac{a(a+bx^2)^8}{8b^3} + \frac{(a+bx^2)^9}{18b^3}$$

output
```
1/14*a^2*(b*x^2+a)^7/b^3-1/8*a*(b*x^2+a)^8/b^3+1/18*(b*x^2+a)^9/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.55

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{a^6x^6}{6} + \frac{3}{4}a^5bx^8 + \frac{3}{2}a^4b^2x^{10} + \frac{5}{3}a^3b^3x^{12} + \frac{15}{14}a^2b^4x^{14} + \frac{3}{8}ab^5x^{16} + \frac{b^6x^{18}}{18}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^6*x^6)/6 + (3*a^5*b*x^8)/4 + (3*a^4*b^2*x^10)/2 + (5*a^3*b^3*x^12)/3 +
(15*a^2*b^4*x^14)/14 + (3*a*b^5*x^16)/8 + (b^6*x^18)/18
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a^2 + 2abx^2 + b^2x^4)^3 \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int b^6 x^5 (bx^2 + a)^6 \, dx}{b^6}$$

$$\Big\downarrow \text{ 27}$$

$$\int x^5 (a + bx^2)^6 \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int x^4 (bx^2 + a)^6 \, dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2} \int \left(\frac{(bx^2 + a)^8}{b^2} - \frac{2a(bx^2 + a)^7}{b^2} + \frac{a^2 (bx^2 + a)^6}{b^2} \right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{a^2 (a + bx^2)^7}{7b^3} + \frac{(a + bx^2)^9}{9b^3} - \frac{a (a + bx^2)^8}{4b^3} \right)$$

input
```
Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
((a^2*(a + b*x^2)^7)/(7*b^3) - (a*(a + b*x^2)^8)/(4*b^3) + (a + b*x^2)^9/(
9*b^3))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{1}{6}a^6x^6+\frac{3}{4}a^5b\,x^8+\frac{3}{2}a^4b^2x^{10}+\frac{5}{3}a^3b^3x^{12}+\frac{15}{14}a^2b^4x^{14}+\frac{3}{8}b^5a\,x^{16}+\frac{1}{18}b^6x^{18}$	69
norman	$\frac{1}{6}a^6x^6+\frac{3}{4}a^5b\,x^8+\frac{3}{2}a^4b^2x^{10}+\frac{5}{3}a^3b^3x^{12}+\frac{15}{14}a^2b^4x^{14}+\frac{3}{8}b^5a\,x^{16}+\frac{1}{18}b^6x^{18}$	69
risch	$\frac{1}{6}a^6x^6+\frac{3}{4}a^5b\,x^8+\frac{3}{2}a^4b^2x^{10}+\frac{5}{3}a^3b^3x^{12}+\frac{15}{14}a^2b^4x^{14}+\frac{3}{8}b^5a\,x^{16}+\frac{1}{18}b^6x^{18}$	69
parallelrisch	$\frac{1}{6}a^6x^6+\frac{3}{4}a^5b\,x^8+\frac{3}{2}a^4b^2x^{10}+\frac{5}{3}a^3b^3x^{12}+\frac{15}{14}a^2b^4x^{14}+\frac{3}{8}b^5a\,x^{16}+\frac{1}{18}b^6x^{18}$	69
gosper	$\frac{x^6\left(28b^6x^{12}+189a\,b^5x^{10}+540a^2b^4x^8+840a^3b^3x^6+756a^4b^2x^4+378a^5b\,x^2+84a^6\right)}{504}$	71
orering	$\frac{x^6\left(28b^6x^{12}+189a\,b^5x^{10}+540a^2b^4x^8+840a^3b^3x^6+756a^4b^2x^4+378a^5b\,x^2+84a^6\right)\left(b^2x^4+2ab\,x^2+a^2\right)^3}{504\left(b\,x^2+a\right)^6}$	100

input
```
int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^6*x^6+3/4*a^5*b*x^8+3/2*a^4*b^2*x^10+5/3*a^3*b^3*x^12+15/14*a^2*b^4*
x^14+3/8*b^5*a*x^16+1/18*b^6*x^18
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{18} b^6 x^{18} + \frac{3}{8} a b^5 x^{16} + \frac{15}{14} a^2 b^4 x^{14} + \frac{5}{3} a^3 b^3 x^{12} + \frac{3}{2} a^4 b^2 x^{10} + \frac{3}{4} a^5 b x^8 + \frac{1}{6} a^6 x^6$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3*x^12 + 3
/2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{a^6 x^6}{6} + \frac{3a^5 b x^8}{4} + \frac{3a^4 b^2 x^{10}}{2} + \frac{5a^3 b^3 x^{12}}{3} + \frac{15a^2 b^4 x^{14}}{14} + \frac{3ab^5 x^{16}}{8} + \frac{b^6 x^{18}}{18}$$

input
```
integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
a**6*x**6/6 + 3*a**5*b*x**8/4 + 3*a**4*b**2*x**10/2 + 5*a**3*b**3*x**12/3
+ 15*a**2*b**4*x**14/14 + 3*a*b**5*x**16/8 + b**6*x**18/18
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{18}\,b^6x^{18}+\frac{3}{8}\,ab^5x^{16}+\frac{15}{14}\,a^2b^4x^{14}+\frac{5}{3}\,a^3b^3x^{12}+\frac{3}{2}\,a^4b^2x^{10}+\frac{3}{4}\,a^5bx^8+\frac{1}{6}\,a^6x^6$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3*x^12 + 3
/2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{18}\,b^6x^{18}+\frac{3}{8}\,ab^5x^{16}+\frac{15}{14}\,a^2b^4x^{14}+\frac{5}{3}\,a^3b^3x^{12}+\frac{3}{2}\,a^4b^2x^{10}+\frac{3}{4}\,a^5bx^8+\frac{1}{6}\,a^6x^6$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3*x^12 + 3
/2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6\,x^6}{6}+\frac{3\,a^5\,b\,x^8}{4}+\frac{3\,a^4\,b^2\,x^{10}}{2}+\frac{5\,a^3\,b^3\,x^{12}}{3}+\frac{15\,a^2\,b^4\,x^{14}}{14}+\frac{3\,a\,b^5\,x^{16}}{8}+\frac{b^6\,x^{18}}{18}$$

input
```
int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(a^6*x^6)/6 + (b^6*x^18)/18 + (3*a^5*b*x^8)/4 + (3*a*b^5*x^16)/8 + (3*a^4*
b^2*x^10)/2 + (5*a^3*b^3*x^12)/3 + (15*a^2*b^4*x^14)/14
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{x^6(28b^6x^{12}+189a\,b^5x^{10}+540a^2b^4x^8+840a^3b^3x^6+756a^4b^2x^4+378a^5b\,x^2+84a^6)}{504}$$

input
```
int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(x**6*(84*a**6 + 378*a**5*b*x**2 + 756*a**4*b**2*x**4 + 840*a**3*b**3*x**6
 + 540*a**2*b**4*x**8 + 189*a*b**5*x**10 + 28*b**6*x**12))/504
```

3.331 $\int x^3(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^3\,dx=-\frac{a(a+bx^2)^7}{14b^2}+\frac{(a+bx^2)^8}{16b^2}$$

output
```
-1/14*a*(b*x^2+a)^7/b^2+1/16*(b*x^2+a)^8/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 77 vs. 2(34) = 68.

Time = 0.00 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.26

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^4}{4}+a^5bx^6+\frac{15}{8}a^4b^2x^8+2a^3b^3x^{10}+\frac{5}{4}a^2b^4x^{12}+\frac{3}{7}ab^5x^{14}+\frac{b^6x^{16}}{16}$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^6*x^4)/4 + a^5*b*x^6 + (15*a^4*b^2*x^8)/8 + 2*a^3*b^3*x^10 + (5*a^2*b^4
*x^12)/4 + (3*a*b^5*x^14)/7 + (b^6*x^16)/16
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^3 dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int b^6x^3\left(bx^2+a\right)^6dx}{b^6}$$

$$\Big\downarrow \text{ 27}$$

$$\int x^3 \left(a+bx^2\right)^6dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int x^2\left(bx^2+a\right)^6dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^7}{b}-\frac{a\left(bx^2+a\right)^6}{b}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^8}{8b^2}-\frac{a\left(a+bx^2\right)^7}{7b^2}\right)$$

input
```
Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(-1/7*(a*(a + b*x^2)^7)/b^2 + (a + b*x^2)^8/(8*b^2))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

method	result	size
default	$\frac{1}{4}a^6x^4+a^5b\,x^6+\frac{15}{8}a^4b^2x^8+2a^3b^3x^{10}+\frac{5}{4}a^2b^4x^{12}+\frac{3}{7}b^5a\,x^{14}+\frac{1}{16}b^6x^{16}$	68
norman	$\frac{1}{4}a^6x^4+a^5b\,x^6+\frac{15}{8}a^4b^2x^8+2a^3b^3x^{10}+\frac{5}{4}a^2b^4x^{12}+\frac{3}{7}b^5a\,x^{14}+\frac{1}{16}b^6x^{16}$	68
risch	$\frac{1}{4}a^6x^4+a^5b\,x^6+\frac{15}{8}a^4b^2x^8+2a^3b^3x^{10}+\frac{5}{4}a^2b^4x^{12}+\frac{3}{7}b^5a\,x^{14}+\frac{1}{16}b^6x^{16}$	68
parallelrisch	$\frac{1}{4}a^6x^4+a^5b\,x^6+\frac{15}{8}a^4b^2x^8+2a^3b^3x^{10}+\frac{5}{4}a^2b^4x^{12}+\frac{3}{7}b^5a\,x^{14}+\frac{1}{16}b^6x^{16}$	68
gosper	$\frac{x^4(7b^6x^{12}+48a\,b^5x^{10}+140a^2b^4x^8+224a^3b^3x^6+210a^4b^2x^4+112a^5b\,x^2+28a^6)}{112}$	71
orering	$\frac{x^4(7b^6x^{12}+48a\,b^5x^{10}+140a^2b^4x^8+224a^3b^3x^6+210a^4b^2x^4+112a^5b\,x^2+28a^6)(b^2x^4+2ab\,x^2+a^2)^3}{112(b\,x^2+a)^6}$	100

input
```
int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^6*x^4+a^5*b*x^6+15/8*a^4*b^2*x^8+2*a^3*b^3*x^10+5/4*a^2*b^4*x^12+3/7
*b^5*a*x^14+1/16*b^6*x^16
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int x^3(a^2 + 2abx^2 + b^2x^4)^3 \, dx = \frac{1}{16} b^6x^{16} + \frac{3}{7} ab^5x^{14} + \frac{5}{4} a^2b^4x^{12} + 2 a^3b^3x^{10} + \frac{15}{8} a^4b^2x^8 + a^5bx^6 + \frac{1}{4} a^6x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^10 + 15/8*
a^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(27) = 54.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int x^3(a^2 + 2abx^2 + b^2x^4)^3 \, dx = \frac{a^6x^4}{4} + a^5bx^6 + \frac{15a^4b^2x^8}{8} + 2a^3b^3x^{10} + \frac{5a^2b^4x^{12}}{4} + \frac{3ab^5x^{14}}{7} + \frac{b^6x^{16}}{16}$$

input
```
integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
a**6*x**4/4 + a**5*b*x**6 + 15*a**4*b**2*x**8/8 + 2*a**3*b**3*x**10 + 5*a*
*2*b**4*x**12/4 + 3*a*b**5*x**14/7 + b**6*x**16/16
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{16}\,b^6x^{16}+\frac{3}{7}\,ab^5x^{14}+\frac{5}{4}\,a^2b^4x^{12}+2\,a^3b^3x^{10}+\frac{15}{8}\,a^4b^2x^8+a^5bx^6+\frac{1}{4}\,a^6x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^10 + 15/8*
a^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{16}\,b^6x^{16}+\frac{3}{7}\,ab^5x^{14}+\frac{5}{4}\,a^2b^4x^{12}+2\,a^3b^3x^{10}+\frac{15}{8}\,a^4b^2x^8+a^5bx^6+\frac{1}{4}\,a^6x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^10 + 15/8*
a^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{a^6 x^4}{4} + a^5 b x^6 + \frac{15 a^4 b^2 x^8}{8} + 2 a^3 b^3 x^{10} + \frac{5 a^2 b^4 x^{12}}{4} + \frac{3 a b^5 x^{14}}{7} + \frac{b^6 x^{16}}{16}$$

input
```
int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(a^6*x^4)/4 + (b^6*x^16)/16 + a^5*b*x^6 + (3*a*b^5*x^14)/7 + (15*a^4*b^2*x
^8)/8 + 2*a^3*b^3*x^10 + (5*a^2*b^4*x^12)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{x^4 (7b^6x^{12}+48a\,b^5x^{10}+140a^2b^4x^8+224a^3b^3x^6+210a^4b^2x^4+112a^5b\,x^2+28a^6)}{112}$$

input
```
int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(x**4*(28*a**6 + 112*a**5*b*x**2 + 210*a**4*b**2*x**4 + 224*a**3*b**3*x**6
 + 140*a**2*b**4*x**8 + 48*a*b**5*x**10 + 7*b**6*x**12))/112
```

3.332 $\int x(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{(a+bx^2)^7}{14b}$$

output
```
1/14*(b*x^2+a)^7/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{(a+bx^2)^7}{14b}$$

input
```
Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a + b*x^2)^7/(14*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1380, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx^2+b^2x^4\right)^3 dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^6x\left(bx^2+a\right)^6dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x\left(a+bx^2\right)^6dx$$

$$\Big\downarrow \text{241}$$

$$\frac{\left(a+bx^2\right)^7}{14b}$$

input
```
Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a + b*x^2)^7/(14*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(14) = 28.

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.31

method	result	size
default	$\frac{1}{2}a^6x^2+\frac{3}{2}a^5b\,x^4+\frac{5}{2}a^4b^2x^6+\frac{5}{2}a^3b^3x^8+\frac{3}{2}a^2b^4x^{10}+\frac{1}{2}b^5a\,x^{12}+\frac{1}{14}b^6x^{14}$	69
norman	$\frac{1}{2}a^6x^2+\frac{3}{2}a^5b\,x^4+\frac{5}{2}a^4b^2x^6+\frac{5}{2}a^3b^3x^8+\frac{3}{2}a^2b^4x^{10}+\frac{1}{2}b^5a\,x^{12}+\frac{1}{14}b^6x^{14}$	69
risch	$\frac{1}{2}a^6x^2+\frac{3}{2}a^5b\,x^4+\frac{5}{2}a^4b^2x^6+\frac{5}{2}a^3b^3x^8+\frac{3}{2}a^2b^4x^{10}+\frac{1}{2}b^5a\,x^{12}+\frac{1}{14}b^6x^{14}$	69
parallelrisch	$\frac{1}{2}a^6x^2+\frac{3}{2}a^5b\,x^4+\frac{5}{2}a^4b^2x^6+\frac{5}{2}a^3b^3x^8+\frac{3}{2}a^2b^4x^{10}+\frac{1}{2}b^5a\,x^{12}+\frac{1}{14}b^6x^{14}$	69
gosper	$\frac{x^2(b^6x^{12}+7a\,b^5x^{10}+21a^2b^4x^8+35a^3b^3x^6+35a^4b^2x^4+21a^5b\,x^2+7a^6)}{14}$	70
orering	$\frac{x^2(b^6x^{12}+7a\,b^5x^{10}+21a^2b^4x^8+35a^3b^3x^6+35a^4b^2x^4+21a^5b\,x^2+7a^6)(b^2x^4+2ab\,x^2+a^2)^3}{14(b\,x^2+a)^6}$	99

input
```
int(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^6*x^2+3/2*a^5*b*x^4+5/2*a^4*b^2*x^6+5/2*a^3*b^3*x^8+3/2*a^2*b^4*x^10
+1/2*b^5*a*x^12+1/14*b^6*x^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(14) = 28.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int x(a^2+2abx^2+b^2x^4)^3\,dx=\frac{1}{14}\,b^6x^{14}+\frac{1}{2}\,ab^5x^{12}+\frac{3}{2}\,a^2b^4x^{10}+\frac{5}{2}\,a^3b^3x^8+\frac{5}{2}\,a^4b^2x^6+\frac{3}{2}\,a^5bx^4+\frac{1}{2}\,a^6x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x^8 + 5/2*
a^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.88

$$\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^2}{2}+\frac{3a^5bx^4}{2}+\frac{5a^4b^2x^6}{2}+\frac{5a^3b^3x^8}{2}+\frac{3a^2b^4x^{10}}{2}+\frac{ab^5x^{12}}{2}+\frac{b^6x^{14}}{14}$$

input
```
integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
a**6*x**2/2 + 3*a**5*b*x**4/2 + 5*a**4*b**2*x**6/2 + 5*a**3*b**3*x**8/2 +
3*a**2*b**4*x**10/2 + a*b**5*x**12/2 + b**6*x**14/14
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{14}\,b^6x^{14}+\frac{1}{2}\,ab^5x^{12}+\frac{3}{2}\,a^2b^4x^{10}+\frac{5}{2}\,a^3b^3x^8+\frac{5}{2}\,a^4b^2x^6+\frac{3}{2}\,a^5bx^4+\frac{1}{2}\,a^6x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x^8 + 5/2*
a^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int x(a^2 + 2abx^2 + b^2x^4)^3 \, dx = \frac{1}{14} b^6x^{14} + \frac{1}{2} ab^5x^{12} + \frac{3}{2} a^2b^4x^{10} + \frac{5}{2} a^3b^3x^8 + \frac{5}{2} a^4b^2x^6 + \frac{3}{2} a^5bx^4 + \frac{1}{2} a^6x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x^8 + 5/2*
a^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.25

$$\int x(a^2 + 2abx^2 + b^2x^4)^3 \, dx = \frac{a^6 x^2}{2} + \frac{3 a^5 b x^4}{2} + \frac{5 a^4 b^2 x^6}{2} + \frac{5 a^3 b^3 x^8}{2} + \frac{3 a^2 b^4 x^{10}}{2} + \frac{a b^5 x^{12}}{2} + \frac{b^6 x^{14}}{14}$$

input
```
int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(a^6*x^2)/2 + (b^6*x^14)/14 + (3*a^5*b*x^4)/2 + (a*b^5*x^12)/2 + (5*a^4*b^
2*x^6)/2 + (5*a^3*b^3*x^8)/2 + (3*a^2*b^4*x^10)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.31

$$\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx = \frac{x^2(b^6x^{12}+7a\,b^5x^{10}+21a^2b^4x^8+35a^3b^3x^6+35a^4b^2x^4+21a^5b\,x^2+7a^6)}{14}$$

input
```
int(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(x**2*(7*a**6 + 21*a**5*b*x**2 + 35*a**4*b**2*x**4 + 35*a**3*b**3*x**6 + 2
1*a**2*b**4*x**8 + 7*a*b**5*x**10 + b**6*x**12))/14
```

3.333 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x} \, dx = 3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}+a^6\log(x)$$

output
```
3*a^5*b*x^2+15/4*a^4*b^2*x^4+10/3*a^3*b^3*x^6+15/8*a^2*b^4*x^8+3/5*a*b^5*x
^10+1/12*b^6*x^12+a^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x} \, dx = 3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}+a^6\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

output
```
3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
 + (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x}dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x}dx}{b^6}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx^2)^6}{x}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^6}{x^2}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(b^6x^{10}+6ab^5x^8+15a^2b^4x^6+20a^3b^3x^4+15a^4b^2x^2+6a^5b+\frac{a^6}{x^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(a^6\log\left(x^2\right)+6a^5bx^2+\frac{15}{2}a^4b^2x^4+\frac{20}{3}a^3b^3x^6+\frac{15}{4}a^2b^4x^8+\frac{6}{5}ab^5x^{10}+\frac{b^6x^{12}}{6}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

output
```
(6*a^5*b*x^2 + (15*a^4*b^2*x^4)/2 + (20*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/
4 + (6*a*b^5*x^10)/5 + (b^6*x^12)/6 + a^6*Log[x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

method	result	size
default	$3a^{5}b\,x^{2}+\frac{15a^{4}b^{2}x^{4}}{4}+\frac{10a^{3}b^{3}x^{6}}{3}+\frac{15a^{2}b^{4}x^{8}}{8}+\frac{3a\,b^{5}x^{10}}{5}+\frac{b^{6}x^{12}}{12}+a^{6}\ln\left(x\right)$	67
norman	$3a^{5}b\,x^{2}+\frac{15a^{4}b^{2}x^{4}}{4}+\frac{10a^{3}b^{3}x^{6}}{3}+\frac{15a^{2}b^{4}x^{8}}{8}+\frac{3a\,b^{5}x^{10}}{5}+\frac{b^{6}x^{12}}{12}+a^{6}\ln\left(x\right)$	67
parallelrisch	$3a^{5}b\,x^{2}+\frac{15a^{4}b^{2}x^{4}}{4}+\frac{10a^{3}b^{3}x^{6}}{3}+\frac{15a^{2}b^{4}x^{8}}{8}+\frac{3a\,b^{5}x^{10}}{5}+\frac{b^{6}x^{12}}{12}+a^{6}\ln\left(x\right)$	67
risch	$\frac{23a^{6}}{15}+a^{6}\ln\left(x\right)+\frac{b^{6}x^{12}}{12}+3a^{5}b\,x^{2}+\frac{15a^{4}b^{2}x^{4}}{4}+\frac{10a^{3}b^{3}x^{6}}{3}+\frac{15a^{2}b^{4}x^{8}}{8}+\frac{3a\,b^{5}x^{10}}{5}$	72

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x,x,method=_RETURNVERBOSE)
```

output
```
3*a^5*b*x^2+15/4*a^4*b^2*x^4+10/3*a^3*b^3*x^6+15/8*a^2*b^4*x^8+3/5*a*b^5*x
^10+1/12*b^6*x^12+a^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x}\,dx = \frac{1}{12}\,b^6x^{12} + \frac{3}{5}\,ab^5x^{10} + \frac{15}{8}\,a^2b^4x^8 + \frac{10}{3}\,a^3b^3x^6 + \frac{15}{4}\,a^4b^2x^4 + 3\,a^5bx^2 + a^6\log(x)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="fricas")
```

output
```
1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/
4*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x}\,dx = a^6\log(x) + 3a^5bx^2 + \frac{15a^4b^2x^4}{4} + \frac{10a^3b^3x^6}{3} + \frac{15a^2b^4x^8}{8} + \frac{3ab^5x^{10}}{5} + \frac{b^6x^{12}}{12}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x,x)
```

output
```
a**6*log(x) + 3*a**5*b*x**2 + 15*a**4*b**2*x**4/4 + 10*a**3*b**3*x**6/3 +
15*a**2*b**4*x**8/8 + 3*a*b**5*x**10/5 + b**6*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x} \, dx = \frac{1}{12}\, b^6 x^{12} + \frac{3}{5}\, ab^5 x^{10} + \frac{15}{8}\, a^2 b^4 x^8 + \frac{10}{3}\, a^3 b^3 x^6 + \frac{15}{4}\, a^4 b^2 x^4 + 3\, a^5 b x^2 + \frac{1}{2}\, a^6 \log\left(x^2\right)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="maxima")
```

output
```
1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/
4*a^4*b^2*x^4 + 3*a^5*b*x^2 + 1/2*a^6*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x} \, dx = \frac{1}{12}\, b^6 x^{12} + \frac{3}{5}\, ab^5 x^{10} + \frac{15}{8}\, a^2 b^4 x^8 + \frac{10}{3}\, a^3 b^3 x^6 + \frac{15}{4}\, a^4 b^2 x^4 + 3\, a^5 b x^2 + \frac{1}{2}\, a^6 \log\left(x^2\right)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x,x, algorithm="giac")
```

output
```
1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*x^6 + 15/
4*a^4*b^2*x^4 + 3*a^5*b*x^2 + 1/2*a^6*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x}\,dx = a^6\,\ln(x) + \frac{b^6\,x^{12}}{12} + 3\,a^5\,b\,x^2 + \frac{3\,a\,b^5\,x^{10}}{5} + \frac{15\,a^4\,b^2\,x^4}{4} + \frac{10\,a^3\,b^3\,x^6}{3} + \frac{15\,a^2\,b^4\,x^8}{8}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x,x)
```

output
```
a^6*log(x) + (b^6*x^12)/12 + 3*a^5*b*x^2 + (3*a*b^5*x^10)/5 + (15*a^4*b^2*
x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x}\,dx = \log(x)\,a^6 + 3a^5b\,x^2 + \frac{15a^4b^2x^4}{4} + \frac{10a^3b^3x^6}{3} + \frac{15a^2b^4x^8}{8} + \frac{3a\,b^5x^{10}}{5} + \frac{b^6x^{12}}{12}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x,x)
```

output
```
(120*log(x)*a**6 + 360*a**5*b*x**2 + 450*a**4*b**2*x**4 + 400*a**3*b**3*x*
*6 + 225*a**2*b**4*x**8 + 72*a*b**5*x**10 + 10*b**6*x**12)/120
```

3.334 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx = -\frac{a^6}{2x^2}+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}+6a^5b\log(x)$$

output
```
-1/2*a^6/x^2+15/2*a^4*b^2*x^2+5*a^3*b^3*x^4+5/2*a^2*b^4*x^6+3/4*a*b^5*x^8+
1/10*b^6*x^10+6*a^5*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx = -\frac{a^6}{2x^2}+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}+6a^5b\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

output
```
-1/2*a^6/x^2 + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2 + (3
*a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^3} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^3}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^6}{x^4}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(b^6x^8 + 6ab^5x^6 + 15a^2b^4x^4 + 20a^3b^3x^2 + 15a^4b^2 + \frac{6a^5b}{x^2} + \frac{a^6}{x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^6}{x^2} + 6a^5b\log(x^2) + 15a^4b^2x^2 + 10a^3b^3x^4 + 5a^2b^4x^6 + \frac{3}{2}ab^5x^8 + \frac{b^6x^{10}}{5}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

output
```
(-(a^6/x^2) + 15*a^4*b^2*x^2 + 10*a^3*b^3*x^4 + 5*a^2*b^4*x^6 + (3*a*b^5*x
^8)/2 + (b^6*x^10)/5 + 6*a^5*b*Log[x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{6}}{2 x^{2}}+\frac{15 a^{4} b^{2} x^{2}}{2}+5 a^{3} b^{3} x^{4}+\frac{5 a^{2} b^{4} x^{6}}{2}+\frac{3 a\, b^{5} x^{8}}{4}+\frac{b^{6} x^{10}}{10}+6 a^{5} b \ln \left(x \right)$	68
risch	$-\frac{a^{6}}{2 x^{2}}+\frac{15 a^{4} b^{2} x^{2}}{2}+5 a^{3} b^{3} x^{4}+\frac{5 a^{2} b^{4} x^{6}}{2}+\frac{3 a\, b^{5} x^{8}}{4}+\frac{b^{6} x^{10}}{10}+6 a^{5} b \ln \left(x \right)$	68
norman	$\frac{-\frac{1}{2} a^{6}+\frac{1}{10} b^{6} x^{12}+\frac{3}{4} a\, b^{5} x^{10}+\frac{5}{2} a^{2} b^{4} x^{8}+5 a^{3} b^{3} x^{6}+\frac{15}{2} a^{4} b^{2} x^{4}}{x^{2}}+6 a^{5} b \ln \left(x \right)$	70
parallelrisch	$\frac{2 b^{6} x^{12}+15 a\, b^{5} x^{10}+50 a^{2} b^{4} x^{8}+100 a^{3} b^{3} x^{6}+150 a^{4} b^{2} x^{4}+120 a^{5} b \ln \left(x \right) x^{2}-10 a^{6}}{20 x^{2}}$	73

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^6/x^2+15/2*a^4*b^2*x^2+5*a^3*b^3*x^4+5/2*a^2*b^4*x^6+3/4*a*b^5*x^8+
1/10*b^6*x^10+6*a^5*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^3}\,dx = \frac{2\,b^6x^{12} + 15\,ab^5x^{10} + 50\,a^2b^4x^8 + 100\,a^3b^3x^6 + 150\,a^4b^2x^4 + 120\,a^5bx^2\log(x) - 10\,a^6}{20\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="fricas")
```

output
```
1/20*(2*b^6*x^12 + 15*a*b^5*x^10 + 50*a^2*b^4*x^8 + 100*a^3*b^3*x^6 + 150*
a^4*b^2*x^4 + 120*a^5*b*x^2*log(x) - 10*a^6)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^3}\,dx = -\frac{a^6}{2x^2} + 6a^5b\log(x) + \frac{15a^4b^2x^2}{2} + 5a^3b^3x^4 + \frac{5a^2b^4x^6}{2} + \frac{3ab^5x^8}{4} + \frac{b^6x^{10}}{10}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**3,x)
```

output
```
-a**6/(2*x**2) + 6*a**5*b*log(x) + 15*a**4*b**2*x**2/2 + 5*a**3*b**3*x**4
+ 5*a**2*b**4*x**6/2 + 3*a*b**5*x**8/4 + b**6*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx = \frac{1}{10}\,b^6x^{10}+\frac{3}{4}\,ab^5x^8+\frac{5}{2}\,a^2b^4x^6+5\,a^3b^3x^4+\frac{15}{2}\,a^4b^2x^2+3\,a^5b\log\left(x^2\right)-\frac{a^6}{2\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="maxima")
```

output
```
1/10*b^6*x^10 + 3/4*a*b^5*x^8 + 5/2*a^2*b^4*x^6 + 5*a^3*b^3*x^4 + 15/2*a^4
*b^2*x^2 + 3*a^5*b*log(x^2) - 1/2*a^6/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx = \frac{1}{10}\,b^6x^{10}+\frac{3}{4}\,ab^5x^8+\frac{5}{2}\,a^2b^4x^6+5\,a^3b^3x^4+\frac{15}{2}\,a^4b^2x^2+3\,a^5b\log\left(x^2\right)-\frac{6\,a^5bx^2+a^6}{2\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x, algorithm="giac")
```

output
```
1/10*b^6*x^10 + 3/4*a*b^5*x^8 + 5/2*a^2*b^4*x^6 + 5*a^3*b^3*x^4 + 15/2*a^4
*b^2*x^2 + 3*a^5*b*log(x^2) - 1/2*(6*a^5*b*x^2 + a^6)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx=\frac{b^6\,x^{10}}{10}-\frac{a^6}{2\,x^2}+\frac{3\,a\,b^5\,x^8}{4}+6\,a^5\,b\,\ln(x)+\frac{15\,a^4\,b^2\,x^2}{2}+5\,a^3\,b^3\,x^4+\frac{5\,a^2\,b^4\,x^6}{2}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^3,x)
```

output
```
(b^6*x^10)/10 - a^6/(2*x^2) + (3*a*b^5*x^8)/4 + 6*a^5*b*log(x) + (15*a^4*b
^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx=\frac{120\log(x)\,a^5b\,x^2-10a^6+150a^4b^2x^4+100a^3b^3x^6+50a^2b^4x^8+15a\,b^5x^{10}+2b^6x^{12}}{20x^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x)
```

output
```
(120*log(x)*a**5*b*x**2 - 10*a**6 + 150*a**4*b**2*x**4 + 100*a**3*b**3*x**
6 + 50*a**2*b**4*x**8 + 15*a*b**5*x**10 + 2*b**6*x**12)/(20*x**2)
```

3.335 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^5}\,dx = -\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}+15a^4b^2\log(x)$$

output
```
-1/4*a^6/x^4-3*a^5*b/x^2+10*a^3*b^3*x^2+15/4*a^2*b^4*x^4+a*b^5*x^6+1/8*b^6
*x^8+15*a^4*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^5}\,dx = -\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}+15a^4b^2\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

output
```
-1/4*a^6/x^4 - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/4 + a*b^5
*x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^5}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^5}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^6}{x^6}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^6}{x^6}+\frac{6ba^5}{x^4}+\frac{15b^2a^4}{x^2}+20b^3a^3+15b^4x^2a^2+6b^5x^4a+b^6x^6\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^6}{2x^4}-\frac{6a^5b}{x^2}+15a^4b^2\log\left(x^2\right)+20a^3b^3x^2+\frac{15}{2}a^2b^4x^4+2ab^5x^6+\frac{b^6x^8}{4}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

output
```
(-1/2*a^6/x^4 - (6*a^5*b)/x^2 + 20*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/2 + 2*a*
b^5*x^6 + (b^6*x^8)/4 + 15*a^4*b^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a^{6}}{4x^{4}}-\frac{3a^{5}b}{x^{2}}+10a^{3}b^{3}x^{2}+\frac{15a^{2}b^{4}x^{4}}{4}+a\,b^{5}x^{6}+\frac{b^{6}x^{8}}{8}+15a^{4}b^{2}\ln\left(x\right)$	67
norman	$\frac{a\,b^{5}x^{10}-\frac{1}{4}a^{6}+\frac{1}{8}b^{6}x^{12}+\frac{15}{4}a^{2}b^{4}x^{8}+10a^{3}b^{3}x^{6}-3a^{5}b\,x^{2}}{x^{4}}+15a^{4}b^{2}\ln\left(x\right)$	69
risch	$\frac{b^{6}x^{8}}{8}+a\,b^{5}x^{6}+\frac{15a^{2}b^{4}x^{4}}{4}+10a^{3}b^{3}x^{2}+\frac{-3a^{5}b\,x^{2}-\frac{1}{4}a^{6}}{x^{4}}+15a^{4}b^{2}\ln\left(x\right)$	69
parallelrisch	$\frac{b^{6}x^{12}+8a\,b^{5}x^{10}+30a^{2}b^{4}x^{8}+80a^{3}b^{3}x^{6}+120a^{4}b^{2}\ln\left(x\right)x^{4}-24a^{5}b\,x^{2}-2a^{6}}{8x^{4}}$	72

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^6/x^4-3*a^5*b/x^2+10*a^3*b^3*x^2+15/4*a^2*b^4*x^4+a*b^5*x^6+1/8*b^6
*x^8+15*a^4*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^5}\,dx = \frac{b^6x^{12}+8\,ab^5x^{10}+30\,a^2b^4x^8+80\,a^3b^3x^6+120\,a^4b^2x^4\log(x)-24\,a^5bx^2-2\,a^6}{8\,x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="fricas")
```

output
```
1/8*(b^6*x^12 + 8*a*b^5*x^10 + 30*a^2*b^4*x^8 + 80*a^3*b^3*x^6 + 120*a^4*b
^2*x^4*log(x) - 24*a^5*b*x^2 - 2*a^6)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^5}\,dx = 15a^4b^2\log(x)+10a^3b^3x^2+\frac{15a^2b^4x^4}{4}+ab^5x^6+\frac{b^6x^8}{8}+\frac{-a^6-12a^5bx^2}{4x^4}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**5,x)
```

output
```
15*a**4*b**2*log(x) + 10*a**3*b**3*x**2 + 15*a**2*b**4*x**4/4 + a*b**5*x**
6 + b**6*x**8/8 + (-a**6 - 12*a**5*b*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^5} \, dx = \frac{1}{8} \, b^6x^8 + ab^5x^6 + \frac{15}{4} \, a^2b^4x^4 + 10 \, a^3b^3x^2 + \frac{15}{2} \, a^4b^2 \log\left(x^2\right) - \frac{12 \, a^5bx^2 + a^6}{4 \, x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="maxima")
```

output
```
1/8*b^6*x^8 + a*b^5*x^6 + 15/4*a^2*b^4*x^4 + 10*a^3*b^3*x^2 + 15/2*a^4*b^2
*log(x^2) - 1/4*(12*a^5*b*x^2 + a^6)/x^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^5} \, dx = \frac{1}{8} \, b^6x^8 + ab^5x^6 + \frac{15}{4} \, a^2b^4x^4 + 10 \, a^3b^3x^2 + \frac{15}{2} \, a^4b^2 \log\left(x^2\right) - \frac{45 \, a^4b^2x^4 + 12 \, a^5bx^2 + a^6}{4 \, x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x, algorithm="giac")
```

output
```
1/8*b^6*x^8 + a*b^5*x^6 + 15/4*a^2*b^4*x^4 + 10*a^3*b^3*x^2 + 15/2*a^4*b^2
*log(x^2) - 1/4*(45*a^4*b^2*x^4 + 12*a^5*b*x^2 + a^6)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^5}\,dx = \frac{b^6\,x^8}{8} - \frac{\frac{a^6}{4}+3\,b\,a^5\,x^2}{x^4} + a\,b^5\,x^6 + 10\,a^3\,b^3\,x^2 + \frac{15\,a^2\,b^4\,x^4}{4} + 15\,a^4\,b^2\,\ln(x)$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^5,x)
```

output
```
(b^6*x^8)/8 - (a^6/4 + 3*a^5*b*x^2)/x^4 + a*b^5*x^6 + 10*a^3*b^3*x^2 + (15
*a^2*b^4*x^4)/4 + 15*a^4*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^5}\,dx = \frac{120\log(x)\,a^4b^2x^4-2a^6-24a^5b\,x^2+80a^3b^3x^6+30a^2b^4x^8+8a\,b^5x^{10}+b^6x^{12}}{8x^4}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x)
```

output
```
(120*log(x)*a**4*b**2*x**4 - 2*a**6 - 24*a**5*b*x**2 + 80*a**3*b**3*x**6 +
 30*a**2*b**4*x**8 + 8*a*b**5*x**10 + b**6*x**12)/(8*x**4)
```

3.336 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7} dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7} dx = -\frac{a^6}{6x^6} - \frac{3a^5b}{2x^4} - \frac{15a^4b^2}{2x^2} + \frac{15}{2}a^2b^4x^2 + \frac{3}{2}ab^5x^4 + \frac{b^6x^6}{6} + 20a^3b^3\log(x)$$

output
```
-1/6*a^6/x^6-3/2*a^5*b/x^4-15/2*a^4*b^2/x^2+15/2*a^2*b^4*x^2+3/2*a*b^5*x^4
+1/6*b^6*x^6+20*a^3*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7} dx = -\frac{a^6}{6x^6} - \frac{3a^5b}{2x^4} - \frac{15a^4b^2}{2x^2} + \frac{15}{2}a^2b^4x^2 + \frac{3}{2}ab^5x^4 + \frac{b^6x^6}{6} + 20a^3b^3\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

output
```
-1/6*a^6/x^6 - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2*b^4*x^2)
/2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^7}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^7}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^6}{x^8}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^6}{x^8}+\frac{6ba^5}{x^6}+\frac{15b^2a^4}{x^4}+\frac{20b^3a^3}{x^2}+15b^4a^2+6b^5x^2a+b^6x^4\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^6}{3x^6}-\frac{3a^5b}{x^4}-\frac{15a^4b^2}{x^2}+20a^3b^3\log(x^2)+15a^2b^4x^2+3ab^5x^4+\frac{b^6x^6}{3}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

output
```
(-1/3*a^6/x^6 - (3*a^5*b)/x^4 - (15*a^4*b^2)/x^2 + 15*a^2*b^4*x^2 + 3*a*b^
5*x^4 + (b^6*x^6)/3 + 20*a^3*b^3*Log[x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^{6}}{6x^{6}}-\frac{3a^{5}b}{2x^{4}}-\frac{15a^{4}b^{2}}{2x^{2}}+\frac{15a^{2}b^{4}x^{2}}{2}+\frac{3a\,b^{5}x^{4}}{2}+\frac{b^{6}x^{6}}{6}+20a^{3}b^{3}\ln\left(x\right)$	68
norman	$\frac{-\frac{1}{6}a^{6}+\frac{1}{6}b^{6}x^{12}+\frac{3}{2}a\,b^{5}x^{10}+\frac{15}{2}a^{2}b^{4}x^{8}-\frac{15}{2}a^{4}b^{2}x^{4}-\frac{3}{2}a^{5}b\,x^{2}}{x^{6}}+20a^{3}b^{3}\ln\left(x\right)$	70
risch	$\frac{b^{6}x^{6}}{6}+\frac{3a\,b^{5}x^{4}}{2}+\frac{15a^{2}b^{4}x^{2}}{2}+\frac{-\frac{15}{2}a^{4}b^{2}x^{4}-\frac{3}{2}a^{5}b\,x^{2}-\frac{1}{6}a^{6}}{x^{6}}+20a^{3}b^{3}\ln\left(x\right)$	70
parallelrisch	$\frac{b^{6}x^{12}+9a\,b^{5}x^{10}+45a^{2}b^{4}x^{8}+120a^{3}b^{3}\ln\left(x\right)x^{6}-45a^{4}b^{2}x^{4}-9a^{5}b\,x^{2}-a^{6}}{6x^{6}}$	72

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^6/x^6-3/2*a^5*b/x^4-15/2*a^4*b^2/x^2+15/2*a^2*b^4*x^2+3/2*a*b^5*x^4
+1/6*b^6*x^6+20*a^3*b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7}\,dx = \frac{b^6x^{12}+9\,ab^5x^{10}+45\,a^2b^4x^8+120\,a^3b^3x^6\log{(x)}-45\,a^4b^2x^4-9\,a^5bx^2-a^6}{6\,x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="fricas")
```

output
```
1/6*(b^6*x^12 + 9*a*b^5*x^10 + 45*a^2*b^4*x^8 + 120*a^3*b^3*x^6*log(x) - 4
5*a^4*b^2*x^4 - 9*a^5*b*x^2 - a^6)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7}\,dx = 20a^3b^3\log{(x)}+\frac{15a^2b^4x^2}{2}+\frac{3ab^5x^4}{2}+\frac{b^6x^6}{6}+\frac{-a^6-9a^5bx^2-45a^4b^2x^4}{6x^6}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**7,x)
```

output
```
20*a**3*b**3*log(x) + 15*a**2*b**4*x**2/2 + 3*a*b**5*x**4/2 + b**6*x**6/6
+ (-a**6 - 9*a**5*b*x**2 - 45*a**4*b**2*x**4)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^7}\,dx = \frac{1}{6}\,b^6x^6 + \frac{3}{2}\,ab^5x^4 + \frac{15}{2}\,a^2b^4x^2 + 10\,a^3b^3\log\left(x^2\right) - \frac{45\,a^4b^2x^4 + 9\,a^5bx^2 + a^6}{6\,x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="maxima")
```

output
```
1/6*b^6*x^6 + 3/2*a*b^5*x^4 + 15/2*a^2*b^4*x^2 + 10*a^3*b^3*log(x^2) - 1/6
*(45*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/x^6
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^7}\,dx = \frac{1}{6}\,b^6x^6 + \frac{3}{2}\,ab^5x^4 + \frac{15}{2}\,a^2b^4x^2 + 10\,a^3b^3\log\left(x^2\right) - \frac{110\,a^3b^3x^6 + 45\,a^4b^2x^4 + 9\,a^5bx^2 + a^6}{6\,x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x, algorithm="giac")
```

output
```
1/6*b^6*x^6 + 3/2*a*b^5*x^4 + 15/2*a^2*b^4*x^2 + 10*a^3*b^3*log(x^2) - 1/6
*(110*a^3*b^3*x^6 + 45*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/x^6
```

Mupad [B] (verification not implemented)

Time = 18.70 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7}\,dx = \frac{b^6\,x^6}{6} - \frac{\frac{a^6}{6}+\frac{3\,a^5\,b\,x^2}{2}+\frac{15\,a^4\,b^2\,x^4}{2}}{x^6} + \frac{3\,a\,b^5\,x^4}{2} + \frac{15\,a^2\,b^4\,x^2}{2} + 20\,a^3\,b^3\,\ln(x)$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^7,x)
```

output
```
(b^6*x^6)/6 - (a^6/6 + (3*a^5*b*x^2)/2 + (15*a^4*b^2*x^4)/2)/x^6 + (3*a*b^
5*x^4)/2 + (15*a^2*b^4*x^2)/2 + 20*a^3*b^3*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7}\,dx = \frac{120\log(x)\,a^3b^3x^6-a^6-9a^5b\,x^2-45a^4b^2x^4+45a^2b^4x^8+9a\,b^5x^{10}+b^6x^{12}}{6x^6}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x)
```

output
```
(120*log(x)*a**3*b**3*x**6 - a**6 - 9*a**5*b*x**2 - 45*a**4*b**2*x**4 + 45
*a**2*b**4*x**8 + 9*a*b**5*x**10 + b**6*x**12)/(6*x**6)
```

3.337 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^9}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 73

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^9}\,dx = -\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+3ab^5x^2+\frac{b^6x^4}{4}+15a^2b^4\log(x)$$

output
```
-1/8*a^6/x^8-a^5*b/x^6-15/4*a^4*b^2/x^4-10*a^3*b^3/x^2+3*a*b^5*x^2+1/4*b^6
*x^4+15*a^2*b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^9}\,dx = -\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+3ab^5x^2+\frac{b^6x^4}{4}+15a^2b^4\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

output
```
-1/8*a^6/x^8 - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/x^2 + 3*a
*b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^9}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^9}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^9}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^6}{x^{10}}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^6}{x^{10}}+\frac{6ba^5}{x^8}+\frac{15b^2a^4}{x^6}+\frac{20b^3a^3}{x^4}+\frac{15b^4a^2}{x^2}+6b^5a+b^6x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^6}{4x^8}-\frac{2a^5b}{x^6}-\frac{15a^4b^2}{2x^4}-\frac{20a^3b^3}{x^2}+15a^2b^4\log(x^2)+6ab^5x^2+\frac{b^6x^4}{2}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

output
```
(-1/4*a^6/x^8 - (2*a^5*b)/x^6 - (15*a^4*b^2)/(2*x^4) - (20*a^3*b^3)/x^2 +
6*a*b^5*x^2 + (b^6*x^4)/2 + 15*a^2*b^4*Log[x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a^{6}}{8x^{8}}-\frac{a^{5}b}{x^{6}}-\frac{15a^{4}b^{2}}{4x^{4}}-\frac{10a^{3}b^{3}}{x^{2}}+3a\,b^{5}x^{2}+\frac{b^{6}x^{4}}{4}+15a^{2}b^{4}\ln\left(x\right)$	68
norman	$\frac{-\frac{1}{8}a^{6}+\frac{1}{4}b^{6}x^{12}+3a\,b^{5}x^{10}-10a^{3}b^{3}x^{6}-\frac{15}{4}a^{4}b^{2}x^{4}-a^{5}b\,x^{2}}{x^{8}}+15a^{2}b^{4}\ln\left(x\right)$	70
parallelrisch	$\frac{2b^{6}x^{12}+24a\,b^{5}x^{10}+120a^{2}b^{4}\ln\left(x\right)x^{8}-80a^{3}b^{3}x^{6}-30a^{4}b^{2}x^{4}-8a^{5}b\,x^{2}-a^{6}}{8x^{8}}$	73
risch	$\frac{b^{6}x^{4}}{4}+3a\,b^{5}x^{2}+9a^{2}b^{4}+\frac{-10a^{3}b^{3}x^{6}-\frac{15}{4}a^{4}b^{2}x^{4}-a^{5}b\,x^{2}-\frac{1}{8}a^{6}}{x^{8}}+15a^{2}b^{4}\ln\left(x\right)$	78

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^6/x^8-a^5*b/x^6-15/4*a^4*b^2/x^4-10*a^3*b^3/x^2+3*a*b^5*x^2+1/4*b^6
*x^4+15*a^2*b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^9}\,dx = \frac{2\,b^6x^{12} + 24\,ab^5x^{10} + 120\,a^2b^4x^8\log(x) - 80\,a^3b^3x^6 - 30\,a^4b^2x^4 - 8\,a^5bx^2 - a^6}{8\,x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="fricas")
```

output
```
1/8*(2*b^6*x^12 + 24*a*b^5*x^10 + 120*a^2*b^4*x^8*log(x) - 80*a^3*b^3*x^6
- 30*a^4*b^2*x^4 - 8*a^5*b*x^2 - a^6)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^9}\,dx = 15a^2b^4\log(x) + 3ab^5x^2 + \frac{b^6x^4}{4} + \frac{-a^6 - 8a^5bx^2 - 30a^4b^2x^4 - 80a^3b^3x^6}{8x^8}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**9,x)
```

output
```
15*a**2*b**4*log(x) + 3*a*b**5*x**2 + b**6*x**4/4 + (-a**6 - 8*a**5*b*x**2
 - 30*a**4*b**2*x**4 - 80*a**3*b**3*x**6)/(8*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^9} \, dx = \frac{1}{4} b^6 x^4 + 3 \, ab^5 x^2 + \frac{15}{2} \, a^2 b^4 \log \left(x^2\right) - \frac{80 \, a^3 b^3 x^6 + 30 \, a^4 b^2 x^4 + 8 \, a^5 b x^2 + a^6}{8 \, x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="maxima")
```

output
```
1/4*b^6*x^4 + 3*a*b^5*x^2 + 15/2*a^2*b^4*log(x^2) - 1/8*(80*a^3*b^3*x^6 +
30*a^4*b^2*x^4 + 8*a^5*b*x^2 + a^6)/x^8
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^9} \, dx = \frac{1}{4} b^6 x^4 + 3 \, ab^5 x^2 + \frac{15}{2} \, a^2 b^4 \log \left(x^2\right) - \frac{125 \, a^2 b^4 x^8 + 80 \, a^3 b^3 x^6 + 30 \, a^4 b^2 x^4 + 8 \, a^5 b x^2 + a^6}{8 \, x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x, algorithm="giac")
```

output
```
1/4*b^6*x^4 + 3*a*b^5*x^2 + 15/2*a^2*b^4*log(x^2) - 1/8*(125*a^2*b^4*x^8 +
 80*a^3*b^3*x^6 + 30*a^4*b^2*x^4 + 8*a^5*b*x^2 + a^6)/x^8
```

Mupad [B] (verification not implemented)

Time = 17.66 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^9}\,dx = \frac{b^6\,x^4}{4} - \frac{\frac{a^6}{8}+a^5\,b\,x^2+\frac{15\,a^4\,b^2\,x^4}{4}+10\,a^3\,b^3\,x^6}{x^8} + 3\,a\,b^5\,x^2 + 15\,a^2\,b^4\,\ln(x)$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^9,x)
```

output
```
(b^6*x^4)/4 - (a^6/8 + a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + 10*a^3*b^3*x^6)/x^
8 + 3*a*b^5*x^2 + 15*a^2*b^4*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^9}\,dx = \frac{120\log(x)\,a^2b^4x^8-a^6-8a^5b\,x^2-30a^4b^2x^4-80a^3b^3x^6+24a\,b^5x^{10}+2b^6x^{12}}{8x^8}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x)
```

output
```
(120*log(x)*a**2*b**4*x**8 - a**6 - 8*a**5*b*x**2 - 30*a**4*b**2*x**4 - 80
*a**3*b**3*x**6 + 24*a*b**5*x**10 + 2*b**6*x**12)/(8*x**8)
```

3.338 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{11}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{11}}\,dx=-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+\frac{b^6x^2}{2}+6ab^5\log(x)$$

output
```
-1/10*a^6/x^10-3/4*a^5*b/x^8-5/2*a^4*b^2/x^6-5*a^3*b^3/x^4-15/2*a^2*b^4/x^
2+1/2*b^6*x^2+6*a*b^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{11}}\,dx=-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+\frac{b^6x^2}{2}+6ab^5\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

output
```
-1/10*a^6/x^10 - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3*b^3)/x^4
 - (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{11}} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^{11}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^{11}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^6}{x^{12}}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^6}{x^{12}}+\frac{6ba^5}{x^{10}}+\frac{15b^2a^4}{x^8}+\frac{20b^3a^3}{x^6}+\frac{15b^4a^2}{x^4}+\frac{6b^5a}{x^2}+b^6\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^6}{5x^{10}}-\frac{3a^5b}{2x^8}-\frac{5a^4b^2}{x^6}-\frac{10a^3b^3}{x^4}-\frac{15a^2b^4}{x^2}+6ab^5\log\left(x^2\right)+b^6x^2\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

output
```
(-1/5*a^6/x^10 - (3*a^5*b)/(2*x^8) - (5*a^4*b^2)/x^6 - (10*a^3*b^3)/x^4 -
(15*a^2*b^4)/x^2 + b^6*x^2 + 6*a*b^5*Log[x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{6}}{10 x^{10}}-\frac{3 a^{5} b}{4 x^{8}}-\frac{5 a^{4} b^{2}}{2 x^{6}}-\frac{5 a^{3} b^{3}}{x^{4}}-\frac{15 a^{2} b^{4}}{2 x^{2}}+\frac{b^{6} x^{2}}{2}+6 a \,b^{5} \ln \left(x \right)$	68
norman	$\frac{-\frac{1}{10} a^{6}+\frac{1}{2} b^{6} x^{12}-\frac{15}{2} a^{2} b^{4} x^{8}-5 a^{3} b^{3} x^{6}-\frac{5}{2} a^{4} b^{2} x^{4}-\frac{3}{4} a^{5} b \,x^{2}}{x^{10}}+6 a \,b^{5} \ln \left(x \right)$	70
risch	$\frac{b^{6} x^{2}}{2}+\frac{-\frac{15}{2} a^{2} b^{4} x^{8}-5 a^{3} b^{3} x^{6}-\frac{5}{2} a^{4} b^{2} x^{4}-\frac{3}{4} a^{5} b \,x^{2}-\frac{1}{10} a^{6}}{x^{10}}+6 a \,b^{5} \ln \left(x \right)$	70
parallelrisch	$\frac{10 b^{6} x^{12}+120 b^{5} a \ln \left(x \right) x^{10}-150 a^{2} b^{4} x^{8}-100 a^{3} b^{3} x^{6}-50 a^{4} b^{2} x^{4}-15 a^{5} b \,x^{2}-2 a^{6}}{20 x^{10}}$	73

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^6/x^10-3/4*a^5*b/x^8-5/2*a^4*b^2/x^6-5*a^3*b^3/x^4-15/2*a^2*b^4/x^
2+1/2*b^6*x^2+6*a*b^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{11}}\,dx = \frac{10\,b^6x^{12}+120\,ab^5x^{10}\log(x)-150\,a^2b^4x^8-100\,a^3b^3x^6-50\,a^4b^2x^4-15\,a^5bx^2-2\,a^6}{20\,x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="fricas")
```

output
```
1/20*(10*b^6*x^12 + 120*a*b^5*x^10*log(x) - 150*a^2*b^4*x^8 - 100*a^3*b^3*
x^6 - 50*a^4*b^2*x^4 - 15*a^5*b*x^2 - 2*a^6)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{11}}\,dx = 6ab^5\log(x)+\frac{b^6x^2}{2}+\frac{-2a^6-15a^5bx^2-50a^4b^2x^4-100a^3b^3x^6-150a^2b^4x^8}{20x^{10}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**11,x)
```

output
```
6*a*b**5*log(x) + b**6*x**2/2 + (-2*a**6 - 15*a**5*b*x**2 - 50*a**4*b**2*x
**4 - 100*a**3*b**3*x**6 - 150*a**2*b**4*x**8)/(20*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{11}}\,dx = \frac{1}{2}\,b^6x^2+3\,ab^5\log\left(x^2\right) - \frac{150\,a^2b^4x^8+100\,a^3b^3x^6+50\,a^4b^2x^4+15\,a^5bx^2+2\,a^6}{20\,x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="maxima")
```

output
```
1/2*b^6*x^2 + 3*a*b^5*log(x^2) - 1/20*(150*a^2*b^4*x^8 + 100*a^3*b^3*x^6 +
 50*a^4*b^2*x^4 + 15*a^5*b*x^2 + 2*a^6)/x^10
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{11}}\,dx = \frac{1}{2}\,b^6x^2+3\,ab^5\log\left(x^2\right) - \frac{137\,ab^5x^{10}+150\,a^2b^4x^8+100\,a^3b^3x^6+50\,a^4b^2x^4+15\,a^5bx^2+2\,a^6}{20\,x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x, algorithm="giac")
```

output
```
1/2*b^6*x^2 + 3*a*b^5*log(x^2) - 1/20*(137*a*b^5*x^10 + 150*a^2*b^4*x^8 +
100*a^3*b^3*x^6 + 50*a^4*b^2*x^4 + 15*a^5*b*x^2 + 2*a^6)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{11}}\,dx = \frac{b^6\,x^2}{2} - \frac{\frac{a^6}{10}+\frac{3\,a^5\,b\,x^2}{4}+\frac{5\,a^4\,b^2\,x^4}{2}+5\,a^3\,b^3\,x^6+\frac{15\,a^2\,b^4\,x^8}{2}}{x^{10}} + 6\,a\,b^5\,\ln(x)$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^11,x)
```

output
```
(b^6*x^2)/2 - (a^6/10 + (3*a^5*b*x^2)/4 + (5*a^4*b^2*x^4)/2 + 5*a^3*b^3*x^
6 + (15*a^2*b^4*x^8)/2)/x^10 + 6*a*b^5*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{11}}\,dx = \frac{120\log(x)\,a\,b^5x^{10}-2a^6-15a^5b\,x^2-50a^4b^2x^4-100a^3b^3x^6-150a^2b^4x^8+10b^6x^{12}}{20x^{10}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x)
```

output
```
(120*log(x)*a*b**5*x**10 - 2*a**6 - 15*a**5*b*x**2 - 50*a**4*b**2*x**4 - 1
00*a**3*b**3*x**6 - 150*a**2*b**4*x**8 + 10*b**6*x**12)/(20*x**10)
```

3.339 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{13}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{13}}\,dx = -\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

output
```
-1/12*a^6/x^12-3/5*a^5*b/x^10-15/8*a^4*b^2/x^8-10/3*a^3*b^3/x^6-15/4*a^2*b
^4/x^4-3*a*b^5/x^2+b^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{13}}\,dx = -\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
```

output
```
-1/12*a^6/x^12 - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*a^3*b^3)/
(3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log[x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{13}} dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^{13}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a + bx^2)^6}{x^{13}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2 + a)^6}{x^{14}}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^6}{x^{14}} + \frac{6ba^5}{x^{12}} + \frac{15b^2a^4}{x^{10}} + \frac{20b^3a^3}{x^8} + \frac{15b^4a^2}{x^6} + \frac{6b^5a}{x^4} + \frac{b^6}{x^2}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^6}{6x^{12}} - \frac{6a^5b}{5x^{10}} - \frac{15a^4b^2}{4x^8} - \frac{20a^3b^3}{3x^6} - \frac{15a^2b^4}{2x^4} - \frac{6ab^5}{x^2} + b^6\log\left(x^2\right)\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
```

output
```
(-1/6*a^6/x^12 - (6*a^5*b)/(5*x^10) - (15*a^4*b^2)/(4*x^8) - (20*a^3*b^3)/
(3*x^6) - (15*a^2*b^4)/(2*x^4) - (6*a*b^5)/x^2 + b^6*Log[x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{6}}{12 x^{12}}-\frac{3 a^{5} b}{5 x^{10}}-\frac{15 a^{4} b^{2}}{8 x^{8}}-\frac{10 a^{3} b^{3}}{3 x^{6}}-\frac{15 a^{2} b^{4}}{4 x^{4}}-\frac{3 a \,b^{5}}{x^{2}}+b^{6} \ln \left(x \right)$	67
norman	$\frac{-\frac{1}{12} a^{6}-3 a \,b^{5} x^{10}-\frac{15}{4} a^{2} b^{4} x^{8}-\frac{10}{3} a^{3} b^{3} x^{6}-\frac{15}{8} a^{4} b^{2} x^{4}-\frac{3}{5} a^{5} b \,x^{2}}{x^{12}}+b^{6} \ln \left(x \right)$	69
risch	$\frac{-\frac{1}{12} a^{6}-3 a \,b^{5} x^{10}-\frac{15}{4} a^{2} b^{4} x^{8}-\frac{10}{3} a^{3} b^{3} x^{6}-\frac{15}{8} a^{4} b^{2} x^{4}-\frac{3}{5} a^{5} b \,x^{2}}{x^{12}}+b^{6} \ln \left(x \right)$	69
parallelrisch	$\frac{120 b^{6} \ln \left(x \right) x^{12}-360 a \,b^{5} x^{10}-450 a^{2} b^{4} x^{8}-400 a^{3} b^{3} x^{6}-225 a^{4} b^{2} x^{4}-72 a^{5} b \,x^{2}-10 a^{6}}{120 x^{12}}$	73

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*a^6/x^12-3/5*a^5*b/x^10-15/8*a^4*b^2/x^8-10/3*a^3*b^3/x^6-15/4*a^2*b
^4/x^4-3*a*b^5/x^2+b^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{13}} \, dx = \frac{120 \, b^6 x^{12} \log(x) - 360 \, ab^5 x^{10} - 450 \, a^2 b^4 x^8 - 400 \, a^3 b^3 x^6 - 225 \, a^4 b^2 x^4 - 72 \, a^5 b x^2 - 10 \, a^6}{120 \, x^{12}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="fricas")
```

output
```
1/120*(120*b^6*x^12*log(x) - 360*a*b^5*x^10 - 450*a^2*b^4*x^8 - 400*a^3*b^
3*x^6 - 225*a^4*b^2*x^4 - 72*a^5*b*x^2 - 10*a^6)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{13}} \, dx = b^6 \log(x) + \frac{-10a^6 - 72a^5bx^2 - 225a^4b^2x^4 - 400a^3b^3x^6 - 450a^2b^4x^8 - 360ab^5x^{10}}{120x^{12}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**13,x)
```

output
```
b**6*log(x) + (-10*a**6 - 72*a**5*b*x**2 - 225*a**4*b**2*x**4 - 400*a**3*b
**3*x**6 - 450*a**2*b**4*x**8 - 360*a*b**5*x**10)/(120*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{13}}\,dx = \frac{1}{2}\,b^6\log\left(x^2\right) - \frac{360\,ab^5x^{10}+450\,a^2b^4x^8+400\,a^3b^3x^6+225\,a^4b^2x^4+72\,a^5bx^2+10\,a^6}{120\,x^{12}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="maxima")
```

output
```
1/2*b^6*log(x^2) - 1/120*(360*a*b^5*x^10 + 450*a^2*b^4*x^8 + 400*a^3*b^3*x
^6 + 225*a^4*b^2*x^4 + 72*a^5*b*x^2 + 10*a^6)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{13}}\,dx = \frac{1}{2}\,b^6\log\left(x^2\right) - \frac{147\,b^6x^{12}+360\,ab^5x^{10}+450\,a^2b^4x^8+400\,a^3b^3x^6+225\,a^4b^2x^4+72\,a^5bx^2+10\,a^6}{120\,x^{12}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x, algorithm="giac")
```

output
```
1/2*b^6*log(x^2) - 1/120*(147*b^6*x^12 + 360*a*b^5*x^10 + 450*a^2*b^4*x^8
+ 400*a^3*b^3*x^6 + 225*a^4*b^2*x^4 + 72*a^5*b*x^2 + 10*a^6)/x^12
```

Mupad [B] (verification not implemented)

Time = 17.89 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{13}}\,dx = b^6 \ln(x) - \frac{\frac{a^6}{12} + \frac{3\,a^5\,b\,x^2}{5} + \frac{15\,a^4\,b^2\,x^4}{8} + \frac{10\,a^3\,b^3\,x^6}{3} + \frac{15\,a^2\,b^4\,x^8}{4} + 3\,a\,b^5\,x^{10}}{x^{12}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^13,x)
```

output
```
b^6*log(x) - (a^6/12 + (3*a^5*b*x^2)/5 + 3*a*b^5*x^10 + (15*a^4*b^2*x^4)/8
 + (10*a^3*b^3*x^6)/3 + (15*a^2*b^4*x^8)/4)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{13}}\,dx = \frac{120\log(x)\,b^6x^{12} - 10a^6 - 72a^5b\,x^2 - 225a^4b^2x^4 - 400a^3b^3x^6 - 450a^2b^4x^8 - 360a\,b^5x^{10}}{120x^{12}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x)
```

output
```
(120*log(x)*b**6*x**12 - 10*a**6 - 72*a**5*b*x**2 - 225*a**4*b**2*x**4 - 4
00*a**3*b**3*x**6 - 450*a**2*b**4*x**8 - 360*a*b**5*x**10)/(120*x**12)
```

3.340 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{15}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{15}}\,dx = -\frac{(a+bx^2)^7}{14ax^{14}}$$

output
```
-1/14*(b*x^2+a)^7/a/x^14
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(19) = 38.

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.32

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{15}}\,dx = -\frac{a^6}{14x^{14}}-\frac{a^5b}{2x^{12}}-\frac{3a^4b^2}{2x^{10}}-\frac{5a^3b^3}{2x^8}-\frac{5a^2b^4}{2x^6}-\frac{3ab^5}{2x^4}-\frac{b^6}{2x^2}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]
```

output
```
-1/14*a^6/x^14 - (a^5*b)/(2*x^12) - (3*a^4*b^2)/(2*x^10) - (5*a^3*b^3)/(2*
x^8) - (5*a^2*b^4)/(2*x^6) - (3*a*b^5)/(2*x^4) - b^6/(2*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1380, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{15}} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^{15}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^{15}}dx$$

$$\Big\downarrow \text{242}$$

$$-\frac{(a+bx^2)^7}{14ax^{14}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]
```

output
```
-1/14*(a + b*x^2)^7/(a*x^14)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.63

method	result	size
gosper	$-\frac{7b^6x^{12}+21a\,b^5x^{10}+35a^2b^4x^8+35a^3b^3x^6+21a^4b^2x^4+7a^5b\,x^2+a^6}{14x^{14}}$	69
default	$-\frac{a^5b}{2x^{12}}-\frac{3b^5a}{2x^4}-\frac{5a^2b^4}{2x^6}-\frac{b^6}{2x^2}-\frac{5a^3b^3}{2x^8}-\frac{3a^4b^2}{2x^{10}}-\frac{a^6}{14x^{14}}$	69
norman	$\frac{-\frac{1}{14}a^6-\frac{1}{2}a^5b\,x^2-\frac{3}{2}a^4b^2x^4-\frac{5}{2}a^3b^3x^6-\frac{5}{2}a^2b^4x^8-\frac{3}{2}a\,b^5x^{10}-\frac{1}{2}b^6x^{12}}{x^{14}}$	70
risch	$\frac{-\frac{1}{14}a^6-\frac{1}{2}a^5b\,x^2-\frac{3}{2}a^4b^2x^4-\frac{5}{2}a^3b^3x^6-\frac{5}{2}a^2b^4x^8-\frac{3}{2}a\,b^5x^{10}-\frac{1}{2}b^6x^{12}}{x^{14}}$	70
parallelrisch	$\frac{-7b^6x^{12}-21a\,b^5x^{10}-35a^2b^4x^8-35a^3b^3x^6-21a^4b^2x^4-7a^5b\,x^2-a^6}{14x^{14}}$	71
orering	$-\frac{(7b^6x^{12}+21a\,b^5x^{10}+35a^2b^4x^8+35a^3b^3x^6+21a^4b^2x^4+7a^5b\,x^2+a^6)(b^2x^4+2ab\,x^2+a^2)^3}{14x^{14}(b\,x^2+a)^6}$	98

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/14*(7*b^6*x^12+21*a*b^5*x^10+35*a^2*b^4*x^8+35*a^3*b^3*x^6+21*a^4*b^2*x
^4+7*a^5*b*x^2+a^6)/x^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.58

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{15}}\,dx = -\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="fricas")
```

output
```
-1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a
^4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.84

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{15}}\,dx = \frac{-a^6 - 7a^5bx^2 - 21a^4b^2x^4 - 35a^3b^3x^6 - 35a^2b^4x^8 - 21ab^5x^{10} - 7b^6x^{12}}{14x^{14}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**15,x)
```

output
```
(-a**6 - 7*a**5*b*x**2 - 21*a**4*b**2*x**4 - 35*a**3*b**3*x**6 - 35*a**2*b
**4*x**8 - 21*a*b**5*x**10 - 7*b**6*x**12)/(14*x**14)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.58

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{15}}\,dx = -\frac{7\,b^6x^{12} + 21\,ab^5x^{10} + 35\,a^2b^4x^8 + 35\,a^3b^3x^6 + 21\,a^4b^2x^4 + 7\,a^5bx^2 + a^6}{14\,x^{14}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="maxima")
```

output
```
-1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a
^4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.58

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{15}}\,dx = -\frac{7\,b^6x^{12} + 21\,ab^5x^{10} + 35\,a^2b^4x^8 + 35\,a^3b^3x^6 + 21\,a^4b^2x^4 + 7\,a^5bx^2 + a^6}{14\,x^{14}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x, algorithm="giac")
```

output
```
-1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x^6 + 21*a
^4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{15}}\,dx = -\frac{\frac{a^6}{14} + \frac{a^5\,b\,x^2}{2} + \frac{3\,a^4\,b^2\,x^4}{2} + \frac{5\,a^3\,b^3\,x^6}{2} + \frac{5\,a^2\,b^4\,x^8}{2} + \frac{3\,a\,b^5\,x^{10}}{2} + \frac{b^6\,x^{12}}{2}}{x^{14}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^15,x)
```

output
```
-(a^6/14 + (b^6*x^12)/2 + (a^5*b*x^2)/2 + (3*a*b^5*x^10)/2 + (3*a^4*b^2*x^
4)/2 + (5*a^3*b^3*x^6)/2 + (5*a^2*b^4*x^8)/2)/x^14
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{15}}\,dx = \frac{-7b^6x^{12}-21a\,b^5x^{10}-35a^2b^4x^8-35a^3b^3x^6-21a^4b^2x^4-7a^5b\,x^2-a^6}{14x^{14}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x)
```

output
```
( - a**6 - 7*a**5*b*x**2 - 21*a**4*b**2*x**4 - 35*a**3*b**3*x**6 - 35*a**2
*b**4*x**8 - 21*a*b**5*x**10 - 7*b**6*x**12)/(14*x**14)
```

3.341 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{17}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 40

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{17}}\,dx = -\frac{(a+bx^2)^7}{16ax^{16}}+\frac{b(a+bx^2)^7}{112a^2x^{14}}$$

output
```
-1/16*(b*x^2+a)^7/a/x^16+1/112*b*(b*x^2+a)^7/a^2/x^14
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.95

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{17}}\,dx = -\frac{a^6}{16x^{16}}-\frac{3a^5b}{7x^{14}}-\frac{5a^4b^2}{4x^{12}}-\frac{2a^3b^3}{x^{10}}-\frac{15a^2b^4}{8x^8}-\frac{ab^5}{x^6}-\frac{b^6}{4x^4}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

output
```
-1/16*a^6/x^16 - (3*a^5*b)/(7*x^14) - (5*a^4*b^2)/(4*x^12) - (2*a^3*b^3)/x
^10 - (15*a^2*b^4)/(8*x^8) - (a*b^5)/x^6 - b^6/(4*x^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{17}}\,dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^{17}}dx}{b^6}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx^2)^6}{x^{17}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^6}{x^{18}}dx^2$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{2}\left(-\frac{b\int \frac{(bx^2+a)^6}{x^{16}}dx^2}{8a}-\frac{(a+bx^2)^7}{8ax^{16}}\right)$$

$$\Big\downarrow \text{ 48}$$

$$\frac{1}{2}\left(\frac{b(a+bx^2)^7}{56a^2x^{14}}-\frac{(a+bx^2)^7}{8ax^{16}}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

output
```
(-1/8*(a + b*x^2)^7/(a*x^16) + (b*(a + b*x^2)^7)/(56*a^2*x^14))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

method	result	size
default	$-\frac{5a^4b^2}{4x^{12}}-\frac{b^6}{4x^4}-\frac{b^5a}{x^6}-\frac{a^6}{16x^{16}}-\frac{15a^2b^4}{8x^8}-\frac{2a^3b^3}{x^{10}}-\frac{3a^5b}{7x^{14}}$	69
norman	$\frac{-\frac{1}{16}a^6-\frac{3}{7}a^5b\,x^2-\frac{5}{4}a^4b^2x^4-2a^3b^3x^6-\frac{15}{8}a^2b^4x^8-a\,b^5x^{10}-\frac{1}{4}b^6x^{12}}{x^{16}}$	70
risch	$\frac{-\frac{1}{16}a^6-\frac{3}{7}a^5b\,x^2-\frac{5}{4}a^4b^2x^4-2a^3b^3x^6-\frac{15}{8}a^2b^4x^8-a\,b^5x^{10}-\frac{1}{4}b^6x^{12}}{x^{16}}$	70
gosper	$-\frac{28b^6x^{12}+112a\,b^5x^{10}+210a^2b^4x^8+224a^3b^3x^6+140a^4b^2x^4+48a^5b\,x^2+7a^6}{112x^{16}}$	71
parallelrisch	$\frac{-28b^6x^{12}-112a\,b^5x^{10}-210a^2b^4x^8-224a^3b^3x^6-140a^4b^2x^4-48a^5b\,x^2-7a^6}{112x^{16}}$	71
orering	$-\frac{\left(28b^6x^{12}+112a\,b^5x^{10}+210a^2b^4x^8+224a^3b^3x^6+140a^4b^2x^4+48a^5b\,x^2+7a^6\right)\left(b^2x^4+2ab\,x^2+a^2\right)^3}{112x^{16}\left(b\,x^2+a\right)^6}$	100

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x,method=_RETURNVERBOSE)
```

output
```
-5/4*a^4*b^2/x^12-1/4*b^6/x^4-b^5*a/x^6-1/16*a^6/x^16-15/8*a^2*b^4/x^8-2*a
^3*b^3/x^10-3/7*a^5*b/x^14
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.75

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{17}}\,dx = -\frac{28\,b^6x^{12}+112\,ab^5x^{10}+210\,a^2b^4x^8+224\,a^3b^3x^6+140\,a^4b^2x^4+48\,a^5bx^2+7\,a^6}{112\,x^{16}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="fricas")
```

output
```
-1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
 140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(32) = 64$.

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.88

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{17}}\,dx = \frac{-7a^6 - 48a^5bx^2 - 140a^4b^2x^4 - 224a^3b^3x^6 - 210a^2b^4x^8 - 112ab^5x^{10} - 28b^6x^{12}}{112x^{16}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**17,x)
```

output
```
(-7*a**6 - 48*a**5*b*x**2 - 140*a**4*b**2*x**4 - 224*a**3*b**3*x**6 - 210*
a**2*b**4*x**8 - 112*a*b**5*x**10 - 28*b**6*x**12)/(112*x**16)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.75

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{17}}\,dx = -\frac{28\,b^6x^{12} + 112\,ab^5x^{10} + 210\,a^2b^4x^8 + 224\,a^3b^3x^6 + 140\,a^4b^2x^4 + 48\,a^5bx^2 + 7\,a^6}{112\,x^{16}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="maxima")
```

output
```
-1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
 140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.75

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{17}}\,dx$$
$$= -\frac{28\,b^6x^{12} + 112\,ab^5x^{10} + 210\,a^2b^4x^8 + 224\,a^3b^3x^6 + 140\,a^4b^2x^4 + 48\,a^5bx^2 + 7\,a^6}{112\,x^{16}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x, algorithm="giac")
```

output
```
-1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*b^3*x^6 +
 140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

Mupad [B] (verification not implemented)

Time = 17.91 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{17}}\,dx$$
$$= -\frac{\frac{a^6}{16} + \frac{3\,a^5\,b\,x^2}{7} + \frac{5\,a^4\,b^2\,x^4}{4} + 2\,a^3\,b^3\,x^6 + \frac{15\,a^2\,b^4\,x^8}{8} + a\,b^5\,x^{10} + \frac{b^6\,x^{12}}{4}}{x^{16}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^17,x)
```

output
```
-(a^6/16 + (b^6*x^12)/4 + (3*a^5*b*x^2)/7 + a*b^5*x^10 + (5*a^4*b^2*x^4)/4
 + 2*a^3*b^3*x^6 + (15*a^2*b^4*x^8)/8)/x^16
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.75

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{17}}\,dx$$
$$= \frac{-28b^6x^{12} - 112a\,b^5x^{10} - 210a^2b^4x^8 - 224a^3b^3x^6 - 140a^4b^2x^4 - 48a^5b\,x^2 - 7a^6}{112x^{16}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x)
```

output
```
( - 7*a**6 - 48*a**5*b*x**2 - 140*a**4*b**2*x**4 - 224*a**3*b**3*x**6 - 21
0*a**2*b**4*x**8 - 112*a*b**5*x**10 - 28*b**6*x**12)/(112*x**16)
```

3.342 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{19}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{19}}\,dx = -\frac{(a+bx^2)^7}{18ax^{18}} + \frac{b(a+bx^2)^7}{72a^2x^{16}} - \frac{b^2(a+bx^2)^7}{504a^3x^{14}}$$

output
```
-1/18*(b*x^2+a)^7/a/x^18+1/72*b*(b*x^2+a)^7/a^2/x^16-1/504*b^2*(b*x^2+a)^7
/a^3/x^14
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{19}}\,dx = -\frac{a^6}{18x^{18}} - \frac{3a^5b}{8x^{16}} - \frac{15a^4b^2}{14x^{14}} - \frac{5a^3b^3}{3x^{12}} - \frac{3a^2b^4}{2x^{10}} - \frac{3ab^5}{4x^8} - \frac{b^6}{6x^6}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

output
```
-1/18*a^6/x^18 - (3*a^5*b)/(8*x^16) - (15*a^4*b^2)/(14*x^14) - (5*a^3*b^3)
/(3*x^12) - (3*a^2*b^4)/(2*x^10) - (3*a*b^5)/(4*x^8) - b^6/(6*x^6)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1380, 27, 243, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{19}}dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int \frac{b^6\left(bx^2+a\right)^6}{x^{19}}dx}{b^6}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{\left(a+bx^2\right)^6}{x^{19}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^6}{x^{20}}dx^2$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{2}\left(-\frac{2b\int \frac{\left(bx^2+a\right)^6}{x^{18}}dx^2}{9a}-\frac{\left(a+bx^2\right)^7}{9ax^{18}}\right)$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{2}\left(-\frac{2b\left(-\frac{b\int \frac{\left(bx^2+a\right)^6}{x^{16}}dx^2}{8a}-\frac{\left(a+bx^2\right)^7}{8ax^{16}}\right)}{9a}-\frac{\left(a+bx^2\right)^7}{9ax^{18}}\right)$$

$$\Big\downarrow \text{ 48}$$

$$\frac{1}{2}\left(-\frac{2b\left(\frac{b\left(a+bx^2\right)^7}{56a^2x^{14}}-\frac{\left(a+bx^2\right)^7}{8ax^{16}}\right)}{9a}-\frac{\left(a+bx^2\right)^7}{9ax^{18}}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

output
```
(-1/9*(a + b*x^2)^7/(a*x^18) - (2*b*(-1/8*(a + b*x^2)^7/(a*x^16) + (b*(a +
 b*x^2)^7)/(56*a^2*x^14)))/(9*a))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

method	result	size
default	$-\frac{5a^{3}b^{3}}{3x^{12}}-\frac{b^{6}}{6x^{6}}-\frac{3a^{5}b}{8x^{16}}-\frac{a^{6}}{18x^{18}}-\frac{3b^{5}a}{4x^{8}}-\frac{3a^{2}b^{4}}{2x^{10}}-\frac{15a^{4}b^{2}}{14x^{14}}$	69
norman	$\frac{-\frac{1}{18}a^{6}-\frac{3}{8}a^{5}b\,x^{2}-\frac{15}{14}a^{4}b^{2}x^{4}-\frac{5}{3}a^{3}b^{3}x^{6}-\frac{3}{2}a^{2}b^{4}x^{8}-\frac{3}{4}a\,b^{5}x^{10}-\frac{1}{6}b^{6}x^{12}}{x^{18}}$	70
risch	$\frac{-\frac{1}{18}a^{6}-\frac{3}{8}a^{5}b\,x^{2}-\frac{15}{14}a^{4}b^{2}x^{4}-\frac{5}{3}a^{3}b^{3}x^{6}-\frac{3}{2}a^{2}b^{4}x^{8}-\frac{3}{4}a\,b^{5}x^{10}-\frac{1}{6}b^{6}x^{12}}{x^{18}}$	70
gosper	$-\frac{84b^{6}x^{12}+378a\,b^{5}x^{10}+756a^{2}b^{4}x^{8}+840a^{3}b^{3}x^{6}+540a^{4}b^{2}x^{4}+189a^{5}b\,x^{2}+28a^{6}}{504x^{18}}$	71
parallelrisch	$\frac{-84b^{6}x^{12}-378a\,b^{5}x^{10}-756a^{2}b^{4}x^{8}-840a^{3}b^{3}x^{6}-540a^{4}b^{2}x^{4}-189a^{5}b\,x^{2}-28a^{6}}{504x^{18}}$	71
orering	$-\frac{\left(84b^{6}x^{12}+378a\,b^{5}x^{10}+756a^{2}b^{4}x^{8}+840a^{3}b^{3}x^{6}+540a^{4}b^{2}x^{4}+189a^{5}b\,x^{2}+28a^{6}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{3}}{504x^{18}\left(b\,x^{2}+a\right)^{6}}$	100

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x,method=_RETURNVERBOSE)
```

output
```
-5/3*a^3*b^3/x^12-1/6*b^6/x^6-3/8*a^5*b/x^16-1/18*a^6/x^18-3/4*b^5*a/x^8-3
/2*a^2*b^4/x^10-15/14*a^4*b^2/x^14
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{19}}\,dx = -\frac{84\,b^6x^{12}+378\,ab^5x^{10}+756\,a^2b^4x^8+840\,a^3b^3x^6+540\,a^4b^2x^4+189\,a^5bx^2+28\,a^6}{504\,x^{18}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="fricas")
```

output
```
-1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
 540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{19}}\,dx = \frac{-28a^6-189a^5bx^2-540a^4b^2x^4-840a^3b^3x^6-756a^2b^4x^8-378ab^5x^{10}-84b^6x^{12}}{504x^{18}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**19,x)
```

output
```
(-28*a**6 - 189*a**5*b*x**2 - 540*a**4*b**2*x**4 - 840*a**3*b**3*x**6 - 75
6*a**2*b**4*x**8 - 378*a*b**5*x**10 - 84*b**6*x**12)/(504*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{19}}\,dx = -\frac{84\,b^6x^{12}+378\,ab^5x^{10}+756\,a^2b^4x^8+840\,a^3b^3x^6+540\,a^4b^2x^4+189\,a^5bx^2+28\,a^6}{504\,x^{18}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="maxima")
```

output
```
-1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
 540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{19}}\,dx = -\frac{84\,b^6x^{12} + 378\,ab^5x^{10} + 756\,a^2b^4x^8 + 840\,a^3b^3x^6 + 540\,a^4b^2x^4 + 189\,a^5bx^2 + 28\,a^6}{504\,x^{18}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x, algorithm="giac")
```

output
```
-1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*b^3*x^6 +
 540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{19}}\,dx = -\frac{\frac{a^6}{18} + \frac{3\,a^5\,b\,x^2}{8} + \frac{15\,a^4\,b^2\,x^4}{14} + \frac{5\,a^3\,b^3\,x^6}{3} + \frac{3\,a^2\,b^4\,x^8}{2} + \frac{3\,a\,b^5\,x^{10}}{4} + \frac{b^6\,x^{12}}{6}}{x^{18}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^19,x)
```

output
```
-(a^6/18 + (b^6*x^12)/6 + (3*a^5*b*x^2)/8 + (3*a*b^5*x^10)/4 + (15*a^4*b^2
*x^4)/14 + (5*a^3*b^3*x^6)/3 + (3*a^2*b^4*x^8)/2)/x^18
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{19}}\,dx$$
$$= \frac{-84b^6x^{12}-378a\,b^5x^{10}-756a^2b^4x^8-840a^3b^3x^6-540a^4b^2x^4-189a^5b\,x^2-28a^6}{504x^{18}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x)
```

output
```
( - 28*a**6 - 189*a**5*b*x**2 - 540*a**4*b**2*x**4 - 840*a**3*b**3*x**6 -
756*a**2*b**4*x**8 - 378*a*b**5*x**10 - 84*b**6*x**12)/(504*x**18)
```

3.343 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{21}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 84

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{21}}\,dx=-\frac{\left(a+bx^2\right)^7}{20ax^{20}}+\frac{b\left(a+bx^2\right)^7}{60a^2x^{18}}-\frac{b^2\left(a+bx^2\right)^7}{240a^3x^{16}}+\frac{b^3\left(a+bx^2\right)^7}{1680a^4x^{14}}$$

output
```
-1/20*(b*x^2+a)^7/a/x^20+1/60*b*(b*x^2+a)^7/a^2/x^18-1/240*b^2*(b*x^2+a)^7
/a^3/x^16+1/1680*b^3*(b*x^2+a)^7/a^4/x^14
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{21}}\,dx=-\frac{a^6}{20x^{20}}-\frac{a^5b}{3x^{18}}-\frac{15a^4b^2}{16x^{16}}-\frac{10a^3b^3}{7x^{14}}-\frac{5a^2b^4}{4x^{12}}-\frac{3ab^5}{5x^{10}}-\frac{b^6}{8x^8}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

output
```
-1/20*a^6/x^20 - (a^5*b)/(3*x^18) - (15*a^4*b^2)/(16*x^16) - (10*a^3*b^3)/
(7*x^14) - (5*a^2*b^4)/(4*x^12) - (3*a*b^5)/(5*x^10) - b^6/(8*x^8)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1380, 27, 243, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{21}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^{21}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^{21}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^6}{x^{22}}dx^2$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{3b\int \frac{(bx^2+a)^6}{x^{20}}dx^2}{10a}-\frac{(a+bx^2)^7}{10ax^{20}}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{3b\left(-\frac{2b\int \frac{(bx^2+a)^6}{x^{18}}dx^2}{9a}-\frac{(a+bx^2)^7}{9ax^{18}}\right)}{10a}-\frac{(a+bx^2)^7}{10ax^{20}}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{3b\left(-\frac{2b\left(-\frac{b\int\frac{\left(bx^2+a\right)^6}{x^{16}}dx^2}{8a}-\frac{\left(a+bx^2\right)^7}{8ax^{16}}\right)}{9a}-\frac{\left(a+bx^2\right)^7}{9ax^{18}}\right)}{10a}-\frac{\left(a+bx^2\right)^7}{10ax^{20}}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2}\left(-\frac{3b\left(-\frac{2b\left(\frac{b\left(a+bx^2\right)^7}{56a^2x^{14}}-\frac{\left(a+bx^2\right)^7}{8ax^{16}}\right)}{9a}-\frac{\left(a+bx^2\right)^7}{9ax^{18}}\right)}{10a}-\frac{\left(a+bx^2\right)^7}{10ax^{20}}\right)$$

input

```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

output

```
(-1/10*(a + b*x^2)^7/(a*x^20) - (3*b*(-1/9*(a + b*x^2)^7/(a*x^18) - (2*b*(
-1/8*(a + b*x^2)^7/(a*x^16) + (b*(a + b*x^2)^7)/(56*a^2*x^14)))/(9*a)))/(1
0*a))/2
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{5a^2b^4}{4x^{12}}-\frac{15a^4b^2}{16x^{16}}-\frac{a^5b}{3x^{18}}-\frac{b^6}{8x^8}-\frac{3b^5a}{5x^{10}}-\frac{10a^3b^3}{7x^{14}}-\frac{a^6}{20x^{20}}$	69
norman	$\frac{-\frac{1}{20}a^6-\frac{1}{3}a^5b\,x^2-\frac{15}{16}a^4b^2x^4-\frac{10}{7}a^3b^3x^6-\frac{5}{4}a^2b^4x^8-\frac{3}{5}a\,b^5x^{10}-\frac{1}{8}b^6x^{12}}{x^{20}}$	70
risch	$\frac{-\frac{1}{20}a^6-\frac{1}{3}a^5b\,x^2-\frac{15}{16}a^4b^2x^4-\frac{10}{7}a^3b^3x^6-\frac{5}{4}a^2b^4x^8-\frac{3}{5}a\,b^5x^{10}-\frac{1}{8}b^6x^{12}}{x^{20}}$	70
gosper	$-\frac{210b^6x^{12}+1008a\,b^5x^{10}+2100a^2b^4x^8+2400a^3b^3x^6+1575a^4b^2x^4+560a^5b\,x^2+84a^6}{1680x^{20}}$	71
parallelrisch	$\frac{-210b^6x^{12}-1008a\,b^5x^{10}-2100a^2b^4x^8-2400a^3b^3x^6-1575a^4b^2x^4-560a^5b\,x^2-84a^6}{1680x^{20}}$	71
orering	$-\frac{\left(210b^6x^{12}+1008a\,b^5x^{10}+2100a^2b^4x^8+2400a^3b^3x^6+1575a^4b^2x^4+560a^5b\,x^2+84a^6\right)\left(b^2x^4+2ab\,x^2+a^2\right)^3}{1680x^{20}\left(b\,x^2+a\right)^6}$	100

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x,method=_RETURNVERBOSE)
```

output
```
-5/4*a^2*b^4/x^12-15/16*a^4*b^2/x^16-1/3*a^5*b/x^18-1/8*b^6/x^8-3/5*b^5*a/
x^10-10/7*a^3*b^3/x^14-1/20*a^6/x^20
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{21}}\,dx = -\frac{210\,b^6x^{12} + 1008\,ab^5x^{10} + 2100\,a^2b^4x^8 + 2400\,a^3b^3x^6 + 1575\,a^4b^2x^4 + 560\,a^5bx^2 + 84\,a^6}{1680\,x^{20}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="fricas")
```

output
```
-1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*
x^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{21}}\,dx = \frac{-84a^6 - 560a^5bx^2 - 1575a^4b^2x^4 - 2400a^3b^3x^6 - 2100a^2b^4x^8 - 1008ab^5x^{10} - 210b^6x^{12}}{1680x^{20}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**21,x)
```

output
```
(-84*a**6 - 560*a**5*b*x**2 - 1575*a**4*b**2*x**4 - 2400*a**3*b**3*x**6 -
2100*a**2*b**4*x**8 - 1008*a*b**5*x**10 - 210*b**6*x**12)/(1680*x**20)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{21}}\,dx = -\frac{210\,b^6x^{12}+1008\,ab^5x^{10}+2100\,a^2b^4x^8+2400\,a^3b^3x^6+1575\,a^4b^2x^4+560\,a^5bx^2+84\,a^6}{1680\,x^{20}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="maxima")
```

output
```
-1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*
x^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{21}}\,dx = -\frac{210\,b^6x^{12}+1008\,ab^5x^{10}+2100\,a^2b^4x^8+2400\,a^3b^3x^6+1575\,a^4b^2x^4+560\,a^5bx^2+84\,a^6}{1680\,x^{20}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x, algorithm="giac")
```

output
```
-1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400*a^3*b^3*
x^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

Mupad [B] (verification not implemented)

Time = 18.89 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{21}} \, dx = -\frac{\frac{a^6}{20} + \frac{a^5\, b\, x^2}{3} + \frac{15\, a^4\, b^2\, x^4}{16} + \frac{10\, a^3\, b^3\, x^6}{7} + \frac{5\, a^2\, b^4\, x^8}{4} + \frac{3\, a\, b^5\, x^{10}}{5} + \frac{b^6\, x^{12}}{8}}{x^{20}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^21,x)
```

output
```
-(a^6/20 + (b^6*x^12)/8 + (a^5*b*x^2)/3 + (3*a*b^5*x^10)/5 + (15*a^4*b^2*x
^4)/16 + (10*a^3*b^3*x^6)/7 + (5*a^2*b^4*x^8)/4)/x^20
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{21}} \, dx = \frac{-210b^6x^{12} - 1008a\, b^5x^{10} - 2100a^2b^4x^8 - 2400a^3b^3x^6 - 1575a^4b^2x^4 - 560a^5b\, x^2 - 84a^6}{1680x^{20}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x)
```

output
```
( - 84*a**6 - 560*a**5*b*x**2 - 1575*a**4*b**2*x**4 - 2400*a**3*b**3*x**6
- 2100*a**2*b**4*x**8 - 1008*a*b**5*x**10 - 210*b**6*x**12)/(1680*x**20)
```

3.344 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{23}} dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{23}} dx = -\frac{a^6}{22x^{22}} - \frac{3a^5b}{10x^{20}} - \frac{5a^4b^2}{6x^{18}} - \frac{5a^3b^3}{4x^{16}} - \frac{15a^2b^4}{14x^{14}} - \frac{ab^5}{2x^{12}} - \frac{b^6}{10x^{10}}$$

output
```
-1/22*a^6/x^22-3/10*a^5*b/x^20-5/6*a^4*b^2/x^18-5/4*a^3*b^3/x^16-15/14*a^2
*b^4/x^14-1/2*a*b^5/x^12-1/10*b^6/x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{23}} dx = -\frac{a^6}{22x^{22}} - \frac{3a^5b}{10x^{20}} - \frac{5a^4b^2}{6x^{18}} - \frac{5a^3b^3}{4x^{16}} - \frac{15a^2b^4}{14x^{14}} - \frac{ab^5}{2x^{12}} - \frac{b^6}{10x^{10}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^23,x]
```

output
```
-1/22*a^6/x^22 - (3*a^5*b)/(10*x^20) - (5*a^4*b^2)/(6*x^18) - (5*a^3*b^3)/
(4*x^16) - (15*a^2*b^4)/(14*x^14) - (a*b^5)/(2*x^12) - b^6/(10*x^10)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{23}}dx$$
$$\Big\downarrow \text{1380}$$
$$\frac{\int \frac{b^6(bx^2+a)^6}{x^{23}}dx}{b^6}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{(a+bx^2)^6}{x^{23}}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{(bx^2+a)^6}{x^{24}}dx^2$$
$$\Big\downarrow \text{53}$$
$$\frac{1}{2}\int \left(\frac{a^6}{x^{24}}+\frac{6ba^5}{x^{22}}+\frac{15b^2a^4}{x^{20}}+\frac{20b^3a^3}{x^{18}}+\frac{15b^4a^2}{x^{16}}+\frac{6b^5a}{x^{14}}+\frac{b^6}{x^{12}}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(-\frac{a^6}{11x^{22}}-\frac{3a^5b}{5x^{20}}-\frac{5a^4b^2}{3x^{18}}-\frac{5a^3b^3}{2x^{16}}-\frac{15a^2b^4}{7x^{14}}-\frac{ab^5}{x^{12}}-\frac{b^6}{5x^{10}}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^23,x]
```

output
```
(-1/11*a^6/x^22 - (3*a^5*b)/(5*x^20) - (5*a^4*b^2)/(3*x^18) - (5*a^3*b^3)/
(2*x^16) - (15*a^2*b^4)/(7*x^14) - (a*b^5)/x^12 - b^6/(5*x^10))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{6}}{22 x^{22}}-\frac{3 a^{5} b}{10 x^{20}}-\frac{5 a^{4} b^{2}}{6 x^{18}}-\frac{5 a^{3} b^{3}}{4 x^{16}}-\frac{15 a^{2} b^{4}}{14 x^{14}}-\frac{a \,b^{5}}{2 x^{12}}-\frac{b^{6}}{10 x^{10}}$	69
norman	$\frac{-\frac{1}{22} a^{6}-\frac{3}{10} a^{5} b \,x^{2}-\frac{5}{6} a^{4} b^{2} x^{4}-\frac{5}{4} a^{3} b^{3} x^{6}-\frac{15}{14} a^{2} b^{4} x^{8}-\frac{1}{2} a \,b^{5} x^{10}-\frac{1}{10} b^{6} x^{12}}{x^{22}}$	70
risch	$\frac{-\frac{1}{22} a^{6}-\frac{3}{10} a^{5} b \,x^{2}-\frac{5}{6} a^{4} b^{2} x^{4}-\frac{5}{4} a^{3} b^{3} x^{6}-\frac{15}{14} a^{2} b^{4} x^{8}-\frac{1}{2} a \,b^{5} x^{10}-\frac{1}{10} b^{6} x^{12}}{x^{22}}$	70
gosper	$-\frac{462 b^{6} x^{12}+2310 a \,b^{5} x^{10}+4950 a^{2} b^{4} x^{8}+5775 a^{3} b^{3} x^{6}+3850 a^{4} b^{2} x^{4}+1386 a^{5} b \,x^{2}+210 a^{6}}{4620 x^{22}}$	71
parallelrisch	$\frac{-462 b^{6} x^{12}-2310 a \,b^{5} x^{10}-4950 a^{2} b^{4} x^{8}-5775 a^{3} b^{3} x^{6}-3850 a^{4} b^{2} x^{4}-1386 a^{5} b \,x^{2}-210 a^{6}}{4620 x^{22}}$	71
orering	$-\frac{\left(462 b^{6} x^{12}+2310 a \,b^{5} x^{10}+4950 a^{2} b^{4} x^{8}+5775 a^{3} b^{3} x^{6}+3850 a^{4} b^{2} x^{4}+1386 a^{5} b \,x^{2}+210 a^{6}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{3}}{4620 x^{22} \left(b \,x^{2}+a \right)^{6}}$	100

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^23,x,method=_RETURNVERBOSE)
```

output
```
-1/22*a^6/x^22-3/10*a^5*b/x^20-5/6*a^4*b^2/x^18-5/4*a^3*b^3/x^16-15/14*a^2
*b^4/x^14-1/2*a*b^5/x^12-1/10*b^6/x^10
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{23}}\,dx = -\frac{462\,b^6x^{12}+2310\,ab^5x^{10}+4950\,a^2b^4x^8+5775\,a^3b^3x^6+3850\,a^4b^2x^4+1386\,a^5bx^2+210\,a^6}{4620\,x^{22}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^23,x, algorithm="fricas")
```

output
```
-1/4620*(462*b^6*x^12 + 2310*a*b^5*x^10 + 4950*a^2*b^4*x^8 + 5775*a^3*b^3*
x^6 + 3850*a^4*b^2*x^4 + 1386*a^5*b*x^2 + 210*a^6)/x^22
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{23}}\,dx = \frac{-210a^6-1386a^5bx^2-3850a^4b^2x^4-5775a^3b^3x^6-4950a^2b^4x^8-2310ab^5x^{10}-462b^6x^{12}}{4620x^{22}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**23,x)
```

output
```
(-210*a**6 - 1386*a**5*b*x**2 - 3850*a**4*b**2*x**4 - 5775*a**3*b**3*x**6
- 4950*a**2*b**4*x**8 - 2310*a*b**5*x**10 - 462*b**6*x**12)/(4620*x**22)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{23}} \, dx = -\frac{462 \, b^6x^{12} + 2310 \, ab^5x^{10} + 4950 \, a^2b^4x^8 + 5775 \, a^3b^3x^6 + 3850 \, a^4b^2x^4 + 1386 \, a^5bx^2 + 210 \, a^6}{4620 \, x^{22}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^23,x, algorithm="maxima")
```

output
```
-1/4620*(462*b^6*x^12 + 2310*a*b^5*x^10 + 4950*a^2*b^4*x^8 + 5775*a^3*b^3*
x^6 + 3850*a^4*b^2*x^4 + 1386*a^5*b*x^2 + 210*a^6)/x^22
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{23}} \, dx = -\frac{462 \, b^6x^{12} + 2310 \, ab^5x^{10} + 4950 \, a^2b^4x^8 + 5775 \, a^3b^3x^6 + 3850 \, a^4b^2x^4 + 1386 \, a^5bx^2 + 210 \, a^6}{4620 \, x^{22}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^23,x, algorithm="giac")
```

output
```
-1/4620*(462*b^6*x^12 + 2310*a*b^5*x^10 + 4950*a^2*b^4*x^8 + 5775*a^3*b^3*
x^6 + 3850*a^4*b^2*x^4 + 1386*a^5*b*x^2 + 210*a^6)/x^22
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{23}}\,dx = -\frac{\frac{a^6}{22}+\frac{3\,a^5\,b\,x^2}{10}+\frac{5\,a^4\,b^2\,x^4}{6}+\frac{5\,a^3\,b^3\,x^6}{4}+\frac{15\,a^2\,b^4\,x^8}{14}+\frac{a\,b^5\,x^{10}}{2}+\frac{b^6\,x^{12}}{10}}{x^{22}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^23,x)
```

output
```
-(a^6/22 + (b^6*x^12)/10 + (3*a^5*b*x^2)/10 + (a*b^5*x^10)/2 + (5*a^4*b^2*
x^4)/6 + (5*a^3*b^3*x^6)/4 + (15*a^2*b^4*x^8)/14)/x^22
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{23}}\,dx = \frac{-462b^6x^{12}-2310a\,b^5x^{10}-4950a^2b^4x^8-5775a^3b^3x^6-3850a^4b^2x^4-1386a^5b\,x^2-210a^6}{4620x^{22}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^23,x)
```

output
```
( - 210*a**6 - 1386*a**5*b*x**2 - 3850*a**4*b**2*x**4 - 5775*a**3*b**3*x**
6 - 4950*a**2*b**4*x**8 - 2310*a*b**5*x**10 - 462*b**6*x**12)/(4620*x**22)
```

3.345 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{25}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{25}}\,dx=-\frac{a^6}{24x^{24}}-\frac{3a^5b}{11x^{22}}-\frac{3a^4b^2}{4x^{20}}-\frac{10a^3b^3}{9x^{18}}-\frac{15a^2b^4}{16x^{16}}-\frac{3ab^5}{7x^{14}}-\frac{b^6}{12x^{12}}$$

output
```
-1/24*a^6/x^24-3/11*a^5*b/x^22-3/4*a^4*b^2/x^20-10/9*a^3*b^3/x^18-15/16*a^
2*b^4/x^16-3/7*a*b^5/x^14-1/12*b^6/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{25}}\,dx=-\frac{a^6}{24x^{24}}-\frac{3a^5b}{11x^{22}}-\frac{3a^4b^2}{4x^{20}}-\frac{10a^3b^3}{9x^{18}}-\frac{15a^2b^4}{16x^{16}}-\frac{3ab^5}{7x^{14}}-\frac{b^6}{12x^{12}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^25,x]
```

output
```
-1/24*a^6/x^24 - (3*a^5*b)/(11*x^22) - (3*a^4*b^2)/(4*x^20) - (10*a^3*b^3)
/(9*x^18) - (15*a^2*b^4)/(16*x^16) - (3*a*b^5)/(7*x^14) - b^6/(12*x^12)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{25}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^{25}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^{25}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^6}{x^{26}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^6}{x^{26}}+\frac{6ba^5}{x^{24}}+\frac{15b^2a^4}{x^{22}}+\frac{20b^3a^3}{x^{20}}+\frac{15b^4a^2}{x^{18}}+\frac{6b^5a}{x^{16}}+\frac{b^6}{x^{14}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^6}{12x^{24}}-\frac{6a^5b}{11x^{22}}-\frac{3a^4b^2}{2x^{20}}-\frac{20a^3b^3}{9x^{18}}-\frac{15a^2b^4}{8x^{16}}-\frac{6ab^5}{7x^{14}}-\frac{b^6}{6x^{12}}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^25,x]
```

output
```
(-1/12*a^6/x^24 - (6*a^5*b)/(11*x^22) - (3*a^4*b^2)/(2*x^20) - (20*a^3*b^3
)/(9*x^18) - (15*a^2*b^4)/(8*x^16) - (6*a*b^5)/(7*x^14) - b^6/(6*x^12))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{6}}{24 x^{24}}-\frac{3 a^{5} b}{11 x^{22}}-\frac{3 a^{4} b^{2}}{4 x^{20}}-\frac{10 a^{3} b^{3}}{9 x^{18}}-\frac{15 a^{2} b^{4}}{16 x^{16}}-\frac{3 a \,b^{5}}{7 x^{14}}-\frac{b^{6}}{12 x^{12}}$	69
norman	$\frac{-\frac{1}{24} a^{6}-\frac{15}{16} a^{2} b^{4} x^{8}-\frac{3}{7} a \,b^{5} x^{10}-\frac{1}{12} b^{6} x^{12}-\frac{3}{4} a^{4} b^{2} x^{4}-\frac{10}{9} a^{3} b^{3} x^{6}-\frac{3}{11} a^{5} b \,x^{2}}{x^{24}}$	70
risch	$\frac{-\frac{1}{24} a^{6}-\frac{15}{16} a^{2} b^{4} x^{8}-\frac{3}{7} a \,b^{5} x^{10}-\frac{1}{12} b^{6} x^{12}-\frac{3}{4} a^{4} b^{2} x^{4}-\frac{10}{9} a^{3} b^{3} x^{6}-\frac{3}{11} a^{5} b \,x^{2}}{x^{24}}$	70
gosper	$-\frac{924 b^{6} x^{12}+4752 a \,b^{5} x^{10}+10395 a^{2} b^{4} x^{8}+12320 a^{3} b^{3} x^{6}+8316 a^{4} b^{2} x^{4}+3024 a^{5} b \,x^{2}+462 a^{6}}{11088 x^{24}}$	71
parallelrisch	$\frac{-924 b^{6} x^{12}-4752 a \,b^{5} x^{10}-10395 a^{2} b^{4} x^{8}-12320 a^{3} b^{3} x^{6}-8316 a^{4} b^{2} x^{4}-3024 a^{5} b \,x^{2}-462 a^{6}}{11088 x^{24}}$	71
orering	$-\frac{\left(924 b^{6} x^{12}+4752 a \,b^{5} x^{10}+10395 a^{2} b^{4} x^{8}+12320 a^{3} b^{3} x^{6}+8316 a^{4} b^{2} x^{4}+3024 a^{5} b \,x^{2}+462 a^{6}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{3}}{11088 x^{24} \left(b \,x^{2}+a \right)^{6}}$	100

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^25,x,method=_RETURNVERBOSE)
```

output
```
-1/24*a^6/x^24-3/11*a^5*b/x^22-3/4*a^4*b^2/x^20-10/9*a^3*b^3/x^18-15/16*a^
2*b^4/x^16-3/7*a*b^5/x^14-1/12*b^6/x^12
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{25}} \, dx = -\frac{924 \, b^6x^{12} + 4752 \, ab^5x^{10} + 10395 \, a^2b^4x^8 + 12320 \, a^3b^3x^6 + 8316 \, a^4b^2x^4 + 3024 \, a^5bx^2 + 462 \, a^6}{11088 \, x^{24}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^25,x, algorithm="fricas")
```

output
```
-1/11088*(924*b^6*x^12 + 4752*a*b^5*x^10 + 10395*a^2*b^4*x^8 + 12320*a^3*b
^3*x^6 + 8316*a^4*b^2*x^4 + 3024*a^5*b*x^2 + 462*a^6)/x^24
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{25}} \, dx = \frac{-462a^6 - 3024a^5bx^2 - 8316a^4b^2x^4 - 12320a^3b^3x^6 - 10395a^2b^4x^8 - 4752ab^5x^{10} - 924b^6x^{12}}{11088x^{24}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**25,x)
```

output
```
(-462*a**6 - 3024*a**5*b*x**2 - 8316*a**4*b**2*x**4 - 12320*a**3*b**3*x**6
 - 10395*a**2*b**4*x**8 - 4752*a*b**5*x**10 - 924*b**6*x**12)/(11088*x**24
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{25}}\,dx = -\frac{924\,b^6x^{12} + 4752\,ab^5x^{10} + 10395\,a^2b^4x^8 + 12320\,a^3b^3x^6 + 8316\,a^4b^2x^4 + 3024\,a^5bx^2 + 462\,a^6}{11088\,x^{24}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^25,x, algorithm="maxima")
```

output
```
-1/11088*(924*b^6*x^12 + 4752*a*b^5*x^10 + 10395*a^2*b^4*x^8 + 12320*a^3*b
^3*x^6 + 8316*a^4*b^2*x^4 + 3024*a^5*b*x^2 + 462*a^6)/x^24
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{25}}\,dx = -\frac{924\,b^6x^{12} + 4752\,ab^5x^{10} + 10395\,a^2b^4x^8 + 12320\,a^3b^3x^6 + 8316\,a^4b^2x^4 + 3024\,a^5bx^2 + 462\,a^6}{11088\,x^{24}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^25,x, algorithm="giac")
```

output
```
-1/11088*(924*b^6*x^12 + 4752*a*b^5*x^10 + 10395*a^2*b^4*x^8 + 12320*a^3*b
^3*x^6 + 8316*a^4*b^2*x^4 + 3024*a^5*b*x^2 + 462*a^6)/x^24
```

Mupad [B] (verification not implemented)

Time = 19.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{25}}\,dx = -\frac{\frac{a^6}{24}+\frac{3\,a^5\,b\,x^2}{11}+\frac{3\,a^4\,b^2\,x^4}{4}+\frac{10\,a^3\,b^3\,x^6}{9}+\frac{15\,a^2\,b^4\,x^8}{16}+\frac{3\,a\,b^5\,x^{10}}{7}+\frac{b^6\,x^{12}}{12}}{x^{24}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^25,x)
```

output
```
-(a^6/24 + (b^6*x^12)/12 + (3*a^5*b*x^2)/11 + (3*a*b^5*x^10)/7 + (3*a^4*b^
2*x^4)/4 + (10*a^3*b^3*x^6)/9 + (15*a^2*b^4*x^8)/16)/x^24
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{25}}\,dx = \frac{-924b^6x^{12}-4752a\,b^5x^{10}-10395a^2b^4x^8-12320a^3b^3x^6-8316a^4b^2x^4-3024a^5b\,x^2-462a^6}{11088x^{24}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^25,x)
```

output
```
( - 462*a**6 - 3024*a**5*b*x**2 - 8316*a**4*b**2*x**4 - 12320*a**3*b**3*x*
*6 - 10395*a**2*b**4*x**8 - 4752*a*b**5*x**10 - 924*b**6*x**12)/(11088*x**
24)
```

3.346 $\int x^8(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int x^8\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

output
```
1/9*a^6*x^9+6/11*a^5*b*x^11+15/13*a^4*b^2*x^13+4/3*a^3*b^3*x^15+15/17*a^2*
b^4*x^17+6/19*a*b^5*x^19+1/21*b^6*x^21
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int x^8\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

input
```
Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/
3 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21)/21
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^6 x^8 \left(bx^2+a\right)^6 dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x^8 \left(a+bx^2\right)^6 dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^6x^8+6a^5bx^{10}+15a^4b^2x^{12}+20a^3b^3x^{14}+15a^2b^4x^{16}+6ab^5x^{18}+b^6x^{20}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

input
```
Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output

```
(a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/
3 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21)/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result
default	$\frac{1}{9}a^6x^9+\frac{6}{11}a^5b\,x^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}a\,b^5x^{19}+\frac{1}{21}b^6x^{21}$
norman	$\frac{1}{9}a^6x^9+\frac{6}{11}a^5b\,x^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}a\,b^5x^{19}+\frac{1}{21}b^6x^{21}$
risch	$\frac{1}{9}a^6x^9+\frac{6}{11}a^5b\,x^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}a\,b^5x^{19}+\frac{1}{21}b^6x^{21}$
parallelrisch	$\frac{1}{9}a^6x^9+\frac{6}{11}a^5b\,x^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}a\,b^5x^{19}+\frac{1}{21}b^6x^{21}$
gosper	$\frac{x^9\left(138567b^6x^{12}+918918a\,b^5x^{10}+2567565a^2b^4x^8+3879876a^3b^3x^6+3357585a^4b^2x^4+1587222a^5b\,x^2+323323a^6\right)}{2909907}$
orering	$\frac{x^9\left(138567b^6x^{12}+918918a\,b^5x^{10}+2567565a^2b^4x^8+3879876a^3b^3x^6+3357585a^4b^2x^4+1587222a^5b\,x^2+323323a^6\right)\left(b^2x^4+2ab\right.}{2909907\left(b\,x^2+a\right)^6}$

input
```
int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*a^6*x^9+6/11*a^5*b*x^11+15/13*a^4*b^2*x^13+4/3*a^3*b^3*x^15+15/17*a^2*
b^4*x^17+6/19*a*b^5*x^19+1/21*b^6*x^21
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{21}\,b^6x^{21}+\frac{6}{19}\,ab^5x^{19}+\frac{15}{17}\,a^2b^4x^{17}+\frac{4}{3}\,a^3b^3x^{15}+\frac{15}{13}\,a^4b^2x^{13}+\frac{6}{11}\,a^5bx^{11}+\frac{1}{9}\,a^6x^9$$

input
```
integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^3*x^15 +
15/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{a^6x^9}{9}+\frac{6a^5bx^{11}}{11}+\frac{15a^4b^2x^{13}}{13}+\frac{4a^3b^3x^{15}}{3}+\frac{15a^2b^4x^{17}}{17}+\frac{6ab^5x^{19}}{19}+\frac{b^6x^{21}}{21}$$

input
```
integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
a**6*x**9/9 + 6*a**5*b*x**11/11 + 15*a**4*b**2*x**13/13 + 4*a**3*b**3*x**1
5/3 + 15*a**2*b**4*x**17/17 + 6*a*b**5*x**19/19 + b**6*x**21/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{21}\, b^6 x^{21} + \frac{6}{19}\, a b^5 x^{19} + \frac{15}{17}\, a^2 b^4 x^{17} + \frac{4}{3}\, a^3 b^3 x^{15} + \frac{15}{13}\, a^4 b^2 x^{13} + \frac{6}{11}\, a^5 b x^{11} + \frac{1}{9}\, a^6 x^9$$

input
```
integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^3*x^15 +
15/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{21}\, b^6 x^{21} + \frac{6}{19}\, a b^5 x^{19} + \frac{15}{17}\, a^2 b^4 x^{17} + \frac{4}{3}\, a^3 b^3 x^{15} + \frac{15}{13}\, a^4 b^2 x^{13} + \frac{6}{11}\, a^5 b x^{11} + \frac{1}{9}\, a^6 x^9$$

input
```
integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^3*x^15 +
15/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{a^6\, x^9}{9} + \frac{6\, a^5\, b\, x^{11}}{11} + \frac{15\, a^4\, b^2\, x^{13}}{13} + \frac{4\, a^3\, b^3\, x^{15}}{3} + \frac{15\, a^2\, b^4\, x^{17}}{17} + \frac{6\, a\, b^5\, x^{19}}{19} + \frac{b^6\, x^{21}}{21}$$

input
```
int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(a^6*x^9)/9 + (b^6*x^21)/21 + (6*a^5*b*x^11)/11 + (6*a*b^5*x^19)/19 + (15*
a^4*b^2*x^13)/13 + (4*a^3*b^3*x^15)/3 + (15*a^2*b^4*x^17)/17
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{x^9(138567b^6x^{12}+918918a\,b^5x^{10}+2567565a^2b^4x^8+3879876a^3b^3x^6+3357585a^4b^2x^4+1587222a^5b\,x^2+}{2909907}$$

input
```
int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(x**9*(323323*a**6 + 1587222*a**5*b*x**2 + 3357585*a**4*b**2*x**4 + 387987
6*a**3*b**3*x**6 + 2567565*a**2*b**4*x**8 + 918918*a*b**5*x**10 + 138567*b
**6*x**12))/2909907
```

3.347 $\int x^6(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

output
```
1/7*a^6*x^7+2/3*a^5*b*x^9+15/11*a^4*b^2*x^11+20/13*a^3*b^3*x^13+a^2*b^4*x^
15+6/17*a*b^5*x^17+1/19*b^6*x^19
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

input
```
Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^3*x^13)/1
3 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$$
$$\Big\downarrow \text{1380}$$
$$\frac{\int b^6x^6 \left(bx^2+a\right)^6 dx}{b^6}$$
$$\Big\downarrow \text{27}$$
$$\int x^6 \left(a+bx^2\right)^6 dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(a^6x^6+6a^5bx^8+15a^4b^2x^{10}+20a^3b^3x^{12}+15a^2b^4x^{14}+6ab^5x^{16}+b^6x^{18}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

input
```
Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output

```
(a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^3*x^13)/1
3 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

method	result
default	$\frac{1}{7}a^6x^7+\frac{2}{3}a^5b\,x^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}a\,b^5x^{17}+\frac{1}{19}b^6x^{19}$
norman	$\frac{1}{7}a^6x^7+\frac{2}{3}a^5b\,x^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}a\,b^5x^{17}+\frac{1}{19}b^6x^{19}$
risch	$\frac{1}{7}a^6x^7+\frac{2}{3}a^5b\,x^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}a\,b^5x^{17}+\frac{1}{19}b^6x^{19}$
parallelrisch	$\frac{1}{7}a^6x^7+\frac{2}{3}a^5b\,x^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}a\,b^5x^{17}+\frac{1}{19}b^6x^{19}$
gosper	$\frac{x^7\left(51051b^6x^{12}+342342a\,b^5x^{10}+969969a^2b^4x^8+1492260a^3b^3x^6+1322685a^4b^2x^4+646646a^5b\,x^2+138567a^6\right)}{969969}$
orering	$\frac{x^7\left(51051b^6x^{12}+342342a\,b^5x^{10}+969969a^2b^4x^8+1492260a^3b^3x^6+1322685a^4b^2x^4+646646a^5b\,x^2+138567a^6\right)\left(b^2x^4+2ab\,x^2-\right.}{969969\left(b\,x^2+a\right)^6}$

input
```
int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^6*x^7+2/3*a^5*b*x^9+15/11*a^4*b^2*x^11+20/13*a^3*b^3*x^13+a^2*b^4*x^
15+6/17*a*b^5*x^17+1/19*b^6*x^19
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{19}\, b^6 x^{19} + \frac{6}{17}\, a b^5 x^{17} + a^2 b^4 x^{15} + \frac{20}{13}\, a^3 b^3 x^{13} + \frac{15}{11}\, a^4 b^2 x^{11} + \frac{2}{3}\, a^5 b x^9 + \frac{1}{7}\, a^6 x^7$$

input
```
integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^13 + 15/1
1*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{a^6 x^7}{7} + \frac{2a^5 b x^9}{3} + \frac{15a^4 b^2 x^{11}}{11} + \frac{20a^3 b^3 x^{13}}{13} + a^2 b^4 x^{15} + \frac{6ab^5 x^{17}}{17} + \frac{b^6 x^{19}}{19}$$

input
```
integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
a**6*x**7/7 + 2*a**5*b*x**9/3 + 15*a**4*b**2*x**11/11 + 20*a**3*b**3*x**13
/13 + a**2*b**4*x**15 + 6*a*b**5*x**17/17 + b**6*x**19/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int x^6 (a^2 + 2abx^2 + b^2x^4)^3 \, dx = \frac{1}{19} b^6 x^{19} + \frac{6}{17} ab^5 x^{17} + a^2 b^4 x^{15} + \frac{20}{13} a^3 b^3 x^{13} + \frac{15}{11} a^4 b^2 x^{11} + \frac{2}{3} a^5 b x^9 + \frac{1}{7} a^6 x^7$$

input
```
integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^13 + 15/1
1*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int x^6 (a^2 + 2abx^2 + b^2x^4)^3 \, dx = \frac{1}{19} b^6 x^{19} + \frac{6}{17} ab^5 x^{17} + a^2 b^4 x^{15} + \frac{20}{13} a^3 b^3 x^{13} + \frac{15}{11} a^4 b^2 x^{11} + \frac{2}{3} a^5 b x^9 + \frac{1}{7} a^6 x^7$$

input
```
integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^13 + 15/1
1*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int x^6 (a^2 + 2abx^2 + b^2x^4)^3 \, dx = \frac{a^6 x^7}{7} + \frac{2 a^5 b x^9}{3} + \frac{15 a^4 b^2 x^{11}}{11} + \frac{20 a^3 b^3 x^{13}}{13} + a^2 b^4 x^{15} + \frac{6 a b^5 x^{17}}{17} + \frac{b^6 x^{19}}{19}$$

input
```
int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(a^6*x^7)/7 + (b^6*x^19)/19 + (2*a^5*b*x^9)/3 + (6*a*b^5*x^17)/17 + (15*a^
4*b^2*x^11)/11 + (20*a^3*b^3*x^13)/13 + a^2*b^4*x^15
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int x^6(a^2+2abx^2+b^2x^4)^3\,dx$$
$$=\frac{x^7(51051b^6x^{12}+342342a\,b^5x^{10}+969969a^2b^4x^8+1492260a^3b^3x^6+1322685a^4b^2x^4+646646a^5b\,x^2+13}{969969}$$

input
```
int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(x**7*(138567*a**6 + 646646*a**5*b*x**2 + 1322685*a**4*b**2*x**4 + 1492260
*a**3*b**3*x**6 + 969969*a**2*b**4*x**8 + 342342*a*b**5*x**10 + 51051*b**6
*x**12))/969969
```

3.348 $\int x^4(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

output
```
1/5*a^6*x^5+6/7*a^5*b*x^7+5/3*a^4*b^2*x^9+20/11*a^3*b^3*x^11+15/13*a^2*b^4
*x^13+2/5*a*b^5*x^15+1/17*b^6*x^17
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x^11)/11 +
 (15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$$
$$\Big\downarrow \text{ 1380}$$
$$\frac{\int b^6 x^4 \left(bx^2+a\right)^6 dx}{b^6}$$
$$\Big\downarrow \text{ 27}$$
$$\int x^4 \left(a+bx^2\right)^6 dx$$
$$\Big\downarrow \text{ 244}$$
$$\int \left(a^6x^4+6a^5bx^6+15a^4b^2x^8+20a^3b^3x^{10}+15a^2b^4x^{12}+6ab^5x^{14}+b^6x^{16}\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a^6x^5}{5}+\frac{6}{7}a^5bx^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}ab^5x^{15}+\frac{b^6x^{17}}{17}$$

input
```
Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output

```
(a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x^11)/11 +
 (15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result
default	$\frac{1}{5}a^6x^5+\frac{6}{7}a^5b\,x^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}a\,b^5x^{15}+\frac{1}{17}b^6x^{17}$
norman	$\frac{1}{5}a^6x^5+\frac{6}{7}a^5b\,x^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}a\,b^5x^{15}+\frac{1}{17}b^6x^{17}$
risch	$\frac{1}{5}a^6x^5+\frac{6}{7}a^5b\,x^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}a\,b^5x^{15}+\frac{1}{17}b^6x^{17}$
parallelrisch	$\frac{1}{5}a^6x^5+\frac{6}{7}a^5b\,x^7+\frac{5}{3}a^4b^2x^9+\frac{20}{11}a^3b^3x^{11}+\frac{15}{13}a^2b^4x^{13}+\frac{2}{5}a\,b^5x^{15}+\frac{1}{17}b^6x^{17}$
gosper	$\frac{x^5(15015b^6x^{12}+102102a\,b^5x^{10}+294525a^2b^4x^8+464100a^3b^3x^6+425425a^4b^2x^4+218790a^5b\,x^2+51051a^6)}{255255}$
orering	$\frac{x^5(15015b^6x^{12}+102102a\,b^5x^{10}+294525a^2b^4x^8+464100a^3b^3x^6+425425a^4b^2x^4+218790a^5b\,x^2+51051a^6)(b^2x^4+2ab\,x^2+a^2}{255255(b\,x^2+a)^6}$

input
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^6*x^5+6/7*a^5*b*x^7+5/3*a^4*b^2*x^9+20/11*a^3*b^3*x^11+15/13*a^2*b^4
*x^13+2/5*a*b^5*x^15+1/17*b^6*x^17
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^4(a^2+2abx^2+b^2x^4)^3\,dx = \frac{1}{17}b^6x^{17}+\frac{2}{5}ab^5x^{15}+\frac{15}{13}a^2b^4x^{13}+\frac{20}{11}a^3b^3x^{11}+\frac{5}{3}a^4b^2x^9+\frac{6}{7}a^5bx^7+\frac{1}{5}a^6x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b^3*x^11 +
 5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int x^4(a^2+2abx^2+b^2x^4)^3\,dx = \frac{a^6x^5}{5}+\frac{6a^5bx^7}{7}+\frac{5a^4b^2x^9}{3}+\frac{20a^3b^3x^{11}}{11}+\frac{15a^2b^4x^{13}}{13}+\frac{2ab^5x^{15}}{5}+\frac{b^6x^{17}}{17}$$

input
```
integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
a**6*x**5/5 + 6*a**5*b*x**7/7 + 5*a**4*b**2*x**9/3 + 20*a**3*b**3*x**11/11
 + 15*a**2*b**4*x**13/13 + 2*a*b**5*x**15/5 + b**6*x**17/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{17}\,b^6x^{17} + \frac{2}{5}\,ab^5x^{15} + \frac{15}{13}\,a^2b^4x^{13} + \frac{20}{11}\,a^3b^3x^{11} + \frac{5}{3}\,a^4b^2x^9 + \frac{6}{7}\,a^5bx^7 + \frac{1}{5}\,a^6x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b^3*x^11 +
 5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{17}\,b^6x^{17} + \frac{2}{5}\,ab^5x^{15} + \frac{15}{13}\,a^2b^4x^{13} + \frac{20}{11}\,a^3b^3x^{11} + \frac{5}{3}\,a^4b^2x^9 + \frac{6}{7}\,a^5bx^7 + \frac{1}{5}\,a^6x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b^3*x^11 +
 5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{a^6\,x^5}{5} + \frac{6\,a^5\,b\,x^7}{7} + \frac{5\,a^4\,b^2\,x^9}{3} + \frac{20\,a^3\,b^3\,x^{11}}{11} + \frac{15\,a^2\,b^4\,x^{13}}{13} + \frac{2\,a\,b^5\,x^{15}}{5} + \frac{b^6\,x^{17}}{17}$$

input
```
int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(a^6*x^5)/5 + (b^6*x^17)/17 + (6*a^5*b*x^7)/7 + (2*a*b^5*x^15)/5 + (5*a^4*
b^2*x^9)/3 + (20*a^3*b^3*x^11)/11 + (15*a^2*b^4*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{x^5(15015b^6x^{12}+102102a\,b^5x^{10}+294525a^2b^4x^8+464100a^3b^3x^6+425425a^4b^2x^4+218790a^5b\,x^2+5105}{255255}$$

input
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(x**5*(51051*a**6 + 218790*a**5*b*x**2 + 425425*a**4*b**2*x**4 + 464100*a*
*3*b**3*x**6 + 294525*a**2*b**4*x**8 + 102102*a*b**5*x**10 + 15015*b**6*x*
*12))/255255
```

3.349 $\int x^2(a^2+2abx^2+b^2x^4)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

output
```
1/3*a^6*x^3+6/5*a^5*b*x^5+15/7*a^4*b^2*x^7+20/9*a^3*b^3*x^9+15/11*a^2*b^4*
x^11+6/13*a*b^5*x^13+1/15*b^6*x^15
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 +
(15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^3 dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^6x^2\left(bx^2+a\right)^6dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int x^2\left(a+bx^2\right)^6dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^6x^2+6a^5bx^4+15a^4b^2x^6+20a^3b^3x^8+15a^2b^4x^{10}+6ab^5x^{12}+b^6x^{14}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 +
(15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result
default	$\frac{1}{3}a^6x^3+\frac{6}{5}a^5b\,x^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}a\,b^5x^{13}+\frac{1}{15}b^6x^{15}$
norman	$\frac{1}{3}a^6x^3+\frac{6}{5}a^5b\,x^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}a\,b^5x^{13}+\frac{1}{15}b^6x^{15}$
risch	$\frac{1}{3}a^6x^3+\frac{6}{5}a^5b\,x^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}a\,b^5x^{13}+\frac{1}{15}b^6x^{15}$
parallelrisch	$\frac{1}{3}a^6x^3+\frac{6}{5}a^5b\,x^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}a\,b^5x^{13}+\frac{1}{15}b^6x^{15}$
gosper	$\frac{x^3(3003b^6x^{12}+20790a\,b^5x^{10}+61425a^2b^4x^8+100100a^3b^3x^6+96525a^4b^2x^4+54054a^5b\,x^2+15015a^6)}{45045}$
orering	$\frac{x^3(3003b^6x^{12}+20790a\,b^5x^{10}+61425a^2b^4x^8+100100a^3b^3x^6+96525a^4b^2x^4+54054a^5b\,x^2+15015a^6)(b^2x^4+2ab\,x^2+a^2)^3}{45045(b\,x^2+a)^6}$

input
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^6*x^3+6/5*a^5*b*x^5+15/7*a^4*b^2*x^7+20/9*a^3*b^3*x^9+15/11*a^2*b^4*
x^11+6/13*a*b^5*x^13+1/15*b^6*x^15
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{15}\,b^6x^{15}+\frac{6}{13}\,ab^5x^{13}+\frac{15}{11}\,a^2b^4x^{11}+\frac{20}{9}\,a^3b^3x^9+\frac{15}{7}\,a^4b^2x^7+\frac{6}{5}\,a^5bx^5+\frac{1}{3}\,a^6x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b^3*x^9 +
15/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{a^6x^3}{3}+\frac{6a^5bx^5}{5}+\frac{15a^4b^2x^7}{7}+\frac{20a^3b^3x^9}{9}+\frac{15a^2b^4x^{11}}{11}+\frac{6ab^5x^{13}}{13}+\frac{b^6x^{15}}{15}$$

input
```
integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
a**6*x**3/3 + 6*a**5*b*x**5/5 + 15*a**4*b**2*x**7/7 + 20*a**3*b**3*x**9/9
+ 15*a**2*b**4*x**11/11 + 6*a*b**5*x**13/13 + b**6*x**15/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{15}\, b^6x^{15} + \frac{6}{13}\, ab^5x^{13} + \frac{15}{11}\, a^2b^4x^{11} + \frac{20}{9}\, a^3b^3x^9 + \frac{15}{7}\, a^4b^2x^7 + \frac{6}{5}\, a^5bx^5 + \frac{1}{3}\, a^6x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b^3*x^9 +
15/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{15}\, b^6x^{15} + \frac{6}{13}\, ab^5x^{13} + \frac{15}{11}\, a^2b^4x^{11} + \frac{20}{9}\, a^3b^3x^9 + \frac{15}{7}\, a^4b^2x^7 + \frac{6}{5}\, a^5bx^5 + \frac{1}{3}\, a^6x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b^3*x^9 +
15/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{a^6\,x^3}{3} + \frac{6\,a^5\,b\,x^5}{5} + \frac{15\,a^4\,b^2\,x^7}{7} + \frac{20\,a^3\,b^3\,x^9}{9} + \frac{15\,a^2\,b^4\,x^{11}}{11} + \frac{6\,a\,b^5\,x^{13}}{13} + \frac{b^6\,x^{15}}{15}$$

input
```
int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(a^6*x^3)/3 + (b^6*x^15)/15 + (6*a^5*b*x^5)/5 + (6*a*b^5*x^13)/13 + (15*a^
4*b^2*x^7)/7 + (20*a^3*b^3*x^9)/9 + (15*a^2*b^4*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int x^2(a^2+2abx^2+b^2x^4)^3\,dx = \frac{x^3(3003b^6x^{12}+20790a\,b^5x^{10}+61425a^2b^4x^8+100100a^3b^3x^6+96525a^4b^2x^4+54054a^5b\,x^2+15015a^6)}{45045}$$

input
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(x**3*(15015*a**6 + 54054*a**5*b*x**2 + 96525*a**4*b**2*x**4 + 100100*a**3
*b**3*x**6 + 61425*a**2*b**4*x**8 + 20790*a*b**5*x**10 + 3003*b**6*x**12))
/45045
```

3.350 $\int (a^2+2abx^2+b^2x^4)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int (a^2+2abx^2+b^2x^4)^3\,dx = a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

output
```
a^6*x+2*a^5*b*x^3+3*a^4*b^2*x^5+20/7*a^3*b^3*x^7+5/3*a^2*b^4*x^9+6/11*a*b^
5*x^11+1/13*b^6*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int (a^2+2abx^2+b^2x^4)^3\,dx = a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)
/3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1379, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$$

$$\Big\downarrow \text{ 1379}$$

$$\frac{\int \left(b^2x^2+ab\right)^6dx}{b^6}$$

$$\Big\downarrow \text{ 210}$$

$$\frac{\int \left(b^{12}x^{12}+6ab^{11}x^{10}+15a^2b^{10}x^8+20a^3b^9x^6+15a^4b^8x^4+6a^5b^7x^2+a^6b^6\right)dx}{b^6}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^6b^6x+2a^5b^7x^3+3a^4b^8x^5+\frac{20}{7}a^3b^9x^7+\frac{5}{3}a^2b^{10}x^9+\frac{6}{11}ab^{11}x^{11}+\frac{b^{12}x^{13}}{13}}{b^6}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^6*b^6*x + 2*a^5*b^7*x^3 + 3*a^4*b^8*x^5 + (20*a^3*b^9*x^7)/7 + (5*a^2*b
^10*x^9)/3 + (6*a*b^11*x^11)/11 + (b^12*x^13)/13)/b^6
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

method	result	size
default	$a^6x+2a^5b\,x^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}a\,b^5x^{11}+\frac{1}{13}b^6x^{13}$	66
norman	$a^6x+2a^5b\,x^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}a\,b^5x^{11}+\frac{1}{13}b^6x^{13}$	66
risch	$a^6x+2a^5b\,x^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}a\,b^5x^{11}+\frac{1}{13}b^6x^{13}$	66
parallelrisch	$a^6x+2a^5b\,x^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}a\,b^5x^{11}+\frac{1}{13}b^6x^{13}$	66
gosper	$\frac{x(231b^6x^{12}+1638a\,b^5x^{10}+5005a^2b^4x^8+8580a^3b^3x^6+9009a^4b^2x^4+6006a^5b\,x^2+3003a^6)}{3003}$	69
orering	$\frac{x(231b^6x^{12}+1638a\,b^5x^{10}+5005a^2b^4x^8+8580a^3b^3x^6+9009a^4b^2x^4+6006a^5b\,x^2+3003a^6)(b^2x^4+2ab\,x^2+a^2)^3}{3003(b\,x^2+a)^6}$	98

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
a^6*x+2*a^5*b*x^3+3*a^4*b^2*x^5+20/7*a^3*b^3*x^7+5/3*a^2*b^4*x^9+6/11*a*b^
5*x^11+1/13*b^6*x^13
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{1}{13}\,b^6x^{13}+\frac{6}{11}\,ab^5x^{11}+\frac{5}{3}\,a^2b^4x^9 + \frac{20}{7}\,a^3b^3x^7+3\,a^4b^2x^5+2\,a^5bx^3+a^6x$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 5/3*a^2*b^4*x^9 + 20/7*a^3*b^3*x^7 + 3*a
^4*b^2*x^5 + 2*a^5*b*x^3 + a^6*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20a^3b^3x^7}{7} + \frac{5a^2b^4x^9}{3}+\frac{6ab^5x^{11}}{11}+\frac{b^6x^{13}}{13}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
a**6*x + 2*a**5*b*x**3 + 3*a**4*b**2*x**5 + 20*a**3*b**3*x**7/7 + 5*a**2*b
**4*x**9/3 + 6*a*b**5*x**11/11 + b**6*x**13/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.37

$$\int \left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{13}\,b^6x^{13}+\frac{6}{11}\,ab^5x^{11}+\frac{4}{3}\,a^2b^4x^9+\frac{8}{7}\,a^3b^3x^7+a^6x+\frac{1}{5}\,\left(3\,b^2x^5+10\,abx^3\right)a^4+\frac{1}{105}\,\left(35\,b^4x^9+180\,ab^3x^7+252\,a^2b^2x^5\right)a^2$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 4/3*a^2*b^4*x^9 + 8/7*a^3*b^3*x^7 + a^6*
x + 1/5*(3*b^2*x^5 + 10*a*b*x^3)*a^4 + 1/105*(35*b^4*x^9 + 180*a*b^3*x^7 +
 252*a^2*b^2*x^5)*a^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{1}{13}\,b^6x^{13}+\frac{6}{11}\,ab^5x^{11}+\frac{5}{3}\,a^2b^4x^9+\frac{20}{7}\,a^3b^3x^7+3\,a^4b^2x^5+2\,a^5bx^3+a^6x$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 5/3*a^2*b^4*x^9 + 20/7*a^3*b^3*x^7 + 3*a
^4*b^2*x^5 + 2*a^5*b*x^3 + a^6*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \left(a^2+2abx^2+b^2x^4\right)^3\,dx = a^6\,x+2\,a^5\,b\,x^3+3\,a^4\,b^2\,x^5+\frac{20\,a^3\,b^3\,x^7}{7}+\frac{5\,a^2\,b^4\,x^9}{3}+\frac{6\,a\,b^5\,x^{11}}{11}+\frac{b^6\,x^{13}}{13}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
a^6*x + (b^6*x^13)/13 + 2*a^5*b*x^3 + (6*a*b^5*x^11)/11 + 3*a^4*b^2*x^5 +
(20*a^3*b^3*x^7)/7 + (5*a^2*b^4*x^9)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \left(a^2+2abx^2+b^2x^4\right)^3\,dx = \frac{x(231b^6x^{12}+1638a\,b^5x^{10}+5005a^2b^4x^8+8580a^3b^3x^6+9009a^4b^2x^4+6006a^5b\,x^2+3003a^6)}{3003}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(x*(3003*a**6 + 6006*a**5*b*x**2 + 9009*a**4*b**2*x**4 + 8580*a**3*b**3*x*
*6 + 5005*a**2*b**4*x**8 + 1638*a*b**5*x**10 + 231*b**6*x**12))/3003
```

3.351 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2} dx = -\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

output
```
-a^6/x+6*a^5*b*x+5*a^4*b^2*x^3+4*a^3*b^3*x^5+15/7*a^2*b^4*x^7+2/3*a*b^5*x^
9+1/11*b^6*x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2} dx = -\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

output
```
-(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^4*x^7)/7
+ (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^2} dx$$

$$\Big\downarrow \ 1380$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^2} dx}{b^6}$$

$$\Big\downarrow \ 27$$

$$\int \frac{(a + bx^2)^6}{x^2} dx$$

$$\Big\downarrow \ 244$$

$$\int \left(\frac{a^6}{x^2} + 6a^5b + 15a^4b^2x^2 + 20a^3b^3x^4 + 15a^2b^4x^6 + 6ab^5x^8 + b^6x^{10}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^6}{x} + 6a^5bx + 5a^4b^2x^3 + 4a^3b^3x^5 + \frac{15}{7}a^2b^4x^7 + \frac{2}{3}ab^5x^9 + \frac{b^6x^{11}}{11}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

output

```
-(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^4*x^7)/7
+ (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a^{6}}{x}+6a^{5}bx+5a^{4}b^{2}x^{3}+4a^{3}b^{3}x^{5}+\frac{15a^{2}b^{4}x^{7}}{7}+\frac{2a\,b^{5}x^{9}}{3}+\frac{b^{6}x^{11}}{11}$	67
risch	$-\frac{a^{6}}{x}+6a^{5}bx+5a^{4}b^{2}x^{3}+4a^{3}b^{3}x^{5}+\frac{15a^{2}b^{4}x^{7}}{7}+\frac{2a\,b^{5}x^{9}}{3}+\frac{b^{6}x^{11}}{11}$	67
norman	$\frac{\frac{1}{11}b^{6}x^{12}+\frac{2}{3}a\,b^{5}x^{10}+\frac{15}{7}a^{2}b^{4}x^{8}+4a^{3}b^{3}x^{6}+5a^{4}b^{2}x^{4}+6a^{5}b\,x^{2}-a^{6}}{x}$	70
gosper	$-\frac{-21b^{6}x^{12}-154a\,b^{5}x^{10}-495a^{2}b^{4}x^{8}-924a^{3}b^{3}x^{6}-1155a^{4}b^{2}x^{4}-1386a^{5}b\,x^{2}+231a^{6}}{231x}$	71
parallelrisch	$\frac{21b^{6}x^{12}+154a\,b^{5}x^{10}+495a^{2}b^{4}x^{8}+924a^{3}b^{3}x^{6}+1155a^{4}b^{2}x^{4}+1386a^{5}b\,x^{2}-231a^{6}}{231x}$	71
orering	$-\frac{\left(-21b^{6}x^{12}-154a\,b^{5}x^{10}-495a^{2}b^{4}x^{8}-924a^{3}b^{3}x^{6}-1155a^{4}b^{2}x^{4}-1386a^{5}b\,x^{2}+231a^{6}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{3}}{231x\left(b\,x^{2}+a\right)^{6}}$	100

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^6/x+6*a^5*b*x+5*a^4*b^2*x^3+4*a^3*b^3*x^5+15/7*a^2*b^4*x^7+2/3*a*b^5*x^
9+1/11*b^6*x^11
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2}\,dx = \frac{21\,b^6x^{12}+154\,ab^5x^{10}+495\,a^2b^4x^8+924\,a^3b^3x^6+1155\,a^4b^2x^4+1386\,a^5bx^2-231\,a^6}{231\,x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="fricas")
```

output
```
1/231*(21*b^6*x^12 + 154*a*b^5*x^10 + 495*a^2*b^4*x^8 + 924*a^3*b^3*x^6 +
1155*a^4*b^2*x^4 + 1386*a^5*b*x^2 - 231*a^6)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2}\,dx = -\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15a^2b^4x^7}{7}+\frac{2ab^5x^9}{3}+\frac{b^6x^{11}}{11}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**2,x)
```

output
```
-a**6/x + 6*a**5*b*x + 5*a**4*b**2*x**3 + 4*a**3*b**3*x**5 + 15*a**2*b**4*
x**7/7 + 2*a*b**5*x**9/3 + b**6*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^2}\,dx = \frac{1}{11}\,b^6x^{11} + \frac{2}{3}\,ab^5x^9 + \frac{15}{7}\,a^2b^4x^7 + 4\,a^3b^3x^5 + 5\,a^4b^2x^3 + 6\,a^5bx - \frac{a^6}{x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="maxima")
```

output
```
1/11*b^6*x^11 + 2/3*a*b^5*x^9 + 15/7*a^2*b^4*x^7 + 4*a^3*b^3*x^5 + 5*a^4*b
^2*x^3 + 6*a^5*b*x - a^6/x
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^2}\,dx = \frac{1}{11}\,b^6x^{11} + \frac{2}{3}\,ab^5x^9 + \frac{15}{7}\,a^2b^4x^7 + 4\,a^3b^3x^5 + 5\,a^4b^2x^3 + 6\,a^5bx - \frac{a^6}{x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x, algorithm="giac")
```

output
```
1/11*b^6*x^11 + 2/3*a*b^5*x^9 + 15/7*a^2*b^4*x^7 + 4*a^3*b^3*x^5 + 5*a^4*b
^2*x^3 + 6*a^5*b*x - a^6/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2}\,dx = \frac{b^6\,x^{11}}{11}-\frac{a^6}{x}+\frac{2\,a\,b^5\,x^9}{3}+5\,a^4\,b^2\,x^3+4\,a^3\,b^3\,x^5+\frac{15\,a^2\,b^4\,x^7}{7}+6\,a^5\,b\,x$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^2,x)
```

output
```
(b^6*x^11)/11 - a^6/x + (2*a*b^5*x^9)/3 + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 +
(15*a^2*b^4*x^7)/7 + 6*a^5*b*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2}\,dx = \frac{21b^6x^{12}+154a\,b^5x^{10}+495a^2b^4x^8+924a^3b^3x^6+1155a^4b^2x^4+1386a^5b\,x^2-231a^6}{231x}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x)
```

output
```
( - 231*a**6 + 1386*a**5*b*x**2 + 1155*a**4*b**2*x**4 + 924*a**3*b**3*x**6
 + 495*a**2*b**4*x**8 + 154*a*b**5*x**10 + 21*b**6*x**12)/(231*x)
```

3.352 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^4}\,dx = -\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

output
```
-1/3*a^6/x^3-6*a^5*b/x+15*a^4*b^2*x+20/3*a^3*b^3*x^3+3*a^2*b^4*x^5+6/7*a*b
^5*x^7+1/9*b^6*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^4}\,dx = -\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

output
```
-1/3*a^6/x^3 - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 + 3*a^2*b^4
*x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^4} dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^4}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^4}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^6}{x^4} + \frac{6a^5b}{x^2} + 15a^4b^2 + 20a^3b^3x^2 + 15a^2b^4x^4 + 6ab^5x^6 + b^6x^8\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6}{3x^3} - \frac{6a^5b}{x} + 15a^4b^2x + \frac{20}{3}a^3b^3x^3 + 3a^2b^4x^5 + \frac{6}{7}ab^5x^7 + \frac{b^6x^9}{9}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

output

```
-1/3*a^6/x^3 - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 + 3*a^2*b^4
*x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{a^{6}}{3x^{3}}-\frac{6a^{5}b}{x}+15a^{4}b^{2}x+\frac{20a^{3}b^{3}x^{3}}{3}+3a^{2}b^{4}x^{5}+\frac{6a\,b^{5}x^{7}}{7}+\frac{b^{6}x^{9}}{9}$	67
risch	$\frac{b^{6}x^{9}}{9}+\frac{6a\,b^{5}x^{7}}{7}+3a^{2}b^{4}x^{5}+\frac{20a^{3}b^{3}x^{3}}{3}+15a^{4}b^{2}x+\frac{-6a^{5}b\,x^{2}-\frac{1}{3}a^{6}}{x^{3}}$	69
norman	$\frac{\frac{1}{9}b^{6}x^{12}+\frac{6}{7}a\,b^{5}x^{10}+3a^{2}b^{4}x^{8}+\frac{20}{3}a^{3}b^{3}x^{6}+15a^{4}b^{2}x^{4}-6a^{5}b\,x^{2}-\frac{1}{3}a^{6}}{x^{3}}$	70
gosper	$-\frac{-7b^{6}x^{12}-54a\,b^{5}x^{10}-189a^{2}b^{4}x^{8}-420a^{3}b^{3}x^{6}-945a^{4}b^{2}x^{4}+378a^{5}b\,x^{2}+21a^{6}}{63x^{3}}$	71
parallelrisch	$\frac{7b^{6}x^{12}+54a\,b^{5}x^{10}+189a^{2}b^{4}x^{8}+420a^{3}b^{3}x^{6}+945a^{4}b^{2}x^{4}-378a^{5}b\,x^{2}-21a^{6}}{63x^{3}}$	71
orering	$-\frac{\left(-7b^{6}x^{12}-54a\,b^{5}x^{10}-189a^{2}b^{4}x^{8}-420a^{3}b^{3}x^{6}-945a^{4}b^{2}x^{4}+378a^{5}b\,x^{2}+21a^{6}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{3}}{63x^{3}\left(b\,x^{2}+a\right)^{6}}$	100

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^6/x^3-6*a^5*b/x+15*a^4*b^2*x+20/3*a^3*b^3*x^3+3*a^2*b^4*x^5+6/7*a*b
^5*x^7+1/9*b^6*x^9
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^4}\,dx = \frac{7\,b^6x^{12}+54\,ab^5x^{10}+189\,a^2b^4x^8+420\,a^3b^3x^6+945\,a^4b^2x^4-378\,a^5bx^2-21\,a^6}{63\,x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="fricas")
```

output
```
1/63*(7*b^6*x^12 + 54*a*b^5*x^10 + 189*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 945
*a^4*b^2*x^4 - 378*a^5*b*x^2 - 21*a^6)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^4}\,dx = 15a^4b^2x+\frac{20a^3b^3x^3}{3}+3a^2b^4x^5+\frac{6ab^5x^7}{7}+\frac{b^6x^9}{9}+\frac{-a^6-18a^5bx^2}{3x^3}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**4,x)
```

output
```
15*a**4*b**2*x + 20*a**3*b**3*x**3/3 + 3*a**2*b**4*x**5 + 6*a*b**5*x**7/7
+ b**6*x**9/9 + (-a**6 - 18*a**5*b*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^4} \, dx = \frac{1}{9} b^6 x^9 + \frac{6}{7} ab^5 x^7 + 3 a^2 b^4 x^5 + \frac{20}{3} a^3 b^3 x^3 + 15 a^4 b^2 x - \frac{18 a^5 b x^2 + a^6}{3 x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="maxima")
```

output
```
1/9*b^6*x^9 + 6/7*a*b^5*x^7 + 3*a^2*b^4*x^5 + 20/3*a^3*b^3*x^3 + 15*a^4*b^
2*x - 1/3*(18*a^5*b*x^2 + a^6)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^4} \, dx = \frac{1}{9} b^6 x^9 + \frac{6}{7} ab^5 x^7 + 3 a^2 b^4 x^5 + \frac{20}{3} a^3 b^3 x^3 + 15 a^4 b^2 x - \frac{18 a^5 b x^2 + a^6}{3 x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x, algorithm="giac")
```

output
```
1/9*b^6*x^9 + 6/7*a*b^5*x^7 + 3*a^2*b^4*x^5 + 20/3*a^3*b^3*x^3 + 15*a^4*b^
2*x - 1/3*(18*a^5*b*x^2 + a^6)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^4}\,dx = \frac{b^6\,x^9}{9} - \frac{\frac{a^6}{3}+6\,b\,a^5\,x^2}{x^3} + 15\,a^4\,b^2\,x + \frac{6\,a\,b^5\,x^7}{7} + \frac{20\,a^3\,b^3\,x^3}{3} + 3\,a^2\,b^4\,x^5$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^4,x)
```

output
```
(b^6*x^9)/9 - (a^6/3 + 6*a^5*b*x^2)/x^3 + 15*a^4*b^2*x + (6*a*b^5*x^7)/7 +
 (20*a^3*b^3*x^3)/3 + 3*a^2*b^4*x^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^4}\,dx = \frac{7b^6x^{12}+54a\,b^5x^{10}+189a^2b^4x^8+420a^3b^3x^6+945a^4b^2x^4-378a^5b\,x^2-21a^6}{63x^3}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x)
```

output
```
( - 21*a**6 - 378*a**5*b*x**2 + 945*a**4*b**2*x**4 + 420*a**3*b**3*x**6 +
189*a**2*b**4*x**8 + 54*a*b**5*x**10 + 7*b**6*x**12)/(63*x**3)
```

3.353 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^6}\,dx = -\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

output
```
-1/5*a^6/x^5-2*a^5*b/x^3-15*a^4*b^2/x+20*a^3*b^3*x+5*a^2*b^4*x^3+6/5*a*b^5
*x^5+1/7*b^6*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^6}\,dx = -\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```

output
```
-1/5*a^6/x^5 - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*a^2*b^4*x
^3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^6}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^6}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^6}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^6}{x^6}+\frac{6a^5b}{x^4}+\frac{15a^4b^2}{x^2}+20a^3b^3+15a^2b^4x^2+6ab^5x^4+b^6x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```

output
```
-1/5*a^6/x^5 - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*a^2*b^4*x
^3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a^{6}}{5x^{5}}-\frac{2a^{5}b}{x^{3}}-\frac{15a^{4}b^{2}}{x}+20a^{3}b^{3}x+5a^{2}b^{4}x^{3}+\frac{6a\,b^{5}x^{5}}{5}+\frac{b^{6}x^{7}}{7}$	67
risch	$\frac{b^{6}x^{7}}{7}+\frac{6a\,b^{5}x^{5}}{5}+5a^{2}b^{4}x^{3}+20a^{3}b^{3}x+\frac{-15a^{4}b^{2}x^{4}-2a^{5}b\,x^{2}-\frac{1}{5}a^{6}}{x^{5}}$	69
norman	$\frac{\frac{1}{7}b^{6}x^{12}+\frac{6}{5}a\,b^{5}x^{10}+5a^{2}b^{4}x^{8}+20a^{3}b^{3}x^{6}-15a^{4}b^{2}x^{4}-2a^{5}b\,x^{2}-\frac{1}{5}a^{6}}{x^{5}}$	70
gosper	$-\frac{-5b^{6}x^{12}-42a\,b^{5}x^{10}-175a^{2}b^{4}x^{8}-700a^{3}b^{3}x^{6}+525a^{4}b^{2}x^{4}+70a^{5}b\,x^{2}+7a^{6}}{35x^{5}}$	71
parallelrisch	$\frac{5b^{6}x^{12}+42a\,b^{5}x^{10}+175a^{2}b^{4}x^{8}+700a^{3}b^{3}x^{6}-525a^{4}b^{2}x^{4}-70a^{5}b\,x^{2}-7a^{6}}{35x^{5}}$	71
orering	$-\frac{\left(-5b^{6}x^{12}-42a\,b^{5}x^{10}-175a^{2}b^{4}x^{8}-700a^{3}b^{3}x^{6}+525a^{4}b^{2}x^{4}+70a^{5}b\,x^{2}+7a^{6}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{3}}{35x^{5}\left(b\,x^{2}+a\right)^{6}}$	100

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^6/x^5-2*a^5*b/x^3-15*a^4*b^2/x+20*a^3*b^3*x+5*a^2*b^4*x^3+6/5*a*b^5
*x^5+1/7*b^6*x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^6}\,dx = \frac{5\,b^6x^{12}+42\,ab^5x^{10}+175\,a^2b^4x^8+700\,a^3b^3x^6-525\,a^4b^2x^4-70\,a^5bx^2-7\,a^6}{35\,x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="fricas")
```

output
```
1/35*(5*b^6*x^12 + 42*a*b^5*x^10 + 175*a^2*b^4*x^8 + 700*a^3*b^3*x^6 - 525
*a^4*b^2*x^4 - 70*a^5*b*x^2 - 7*a^6)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^6}\,dx = 20a^3b^3x+5a^2b^4x^3+\frac{6ab^5x^5}{5}+\frac{b^6x^7}{7} + \frac{-a^6-10a^5bx^2-75a^4b^2x^4}{5x^5}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**6,x)
```

output
```
20*a**3*b**3*x + 5*a**2*b**4*x**3 + 6*a*b**5*x**5/5 + b**6*x**7/7 + (-a**6
 - 10*a**5*b*x**2 - 75*a**4*b**2*x**4)/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^6} \, dx = \frac{1}{7} b^6x^7 + \frac{6}{5} ab^5x^5 + 5 a^2b^4x^3 + 20 a^3b^3x - \frac{75 a^4b^2x^4 + 10 a^5bx^2 + a^6}{5 x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="maxima")
```

output
```
1/7*b^6*x^7 + 6/5*a*b^5*x^5 + 5*a^2*b^4*x^3 + 20*a^3*b^3*x - 1/5*(75*a^4*b
^2*x^4 + 10*a^5*b*x^2 + a^6)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^6} \, dx = \frac{1}{7} b^6x^7 + \frac{6}{5} ab^5x^5 + 5 a^2b^4x^3 + 20 a^3b^3x - \frac{75 a^4b^2x^4 + 10 a^5bx^2 + a^6}{5 x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x, algorithm="giac")
```

output
```
1/7*b^6*x^7 + 6/5*a*b^5*x^5 + 5*a^2*b^4*x^3 + 20*a^3*b^3*x - 1/5*(75*a^4*b
^2*x^4 + 10*a^5*b*x^2 + a^6)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^6} \, dx = \frac{b^6 \, x^7}{7} - \frac{\frac{a^6}{5} + 2 \, a^5 \, b \, x^2 + 15 \, a^4 \, b^2 \, x^4}{x^5} + 20 \, a^3 \, b^3 \, x + \frac{6 \, a \, b^5 \, x^5}{5} + 5 \, a^2 \, b^4 \, x^3$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^6,x)
```

output
```
(b^6*x^7)/7 - (a^6/5 + 2*a^5*b*x^2 + 15*a^4*b^2*x^4)/x^5 + 20*a^3*b^3*x +
(6*a*b^5*x^5)/5 + 5*a^2*b^4*x^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^6} \, dx = \frac{5b^6x^{12} + 42a \, b^5x^{10} + 175a^2b^4x^8 + 700a^3b^3x^6 - 525a^4b^2x^4 - 70a^5b \, x^2 - 7a^6}{35x^5}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x)
```

output
```
( - 7*a**6 - 70*a**5*b*x**2 - 525*a**4*b**2*x**4 + 700*a**3*b**3*x**6 + 17
5*a**2*b**4*x**8 + 42*a*b**5*x**10 + 5*b**6*x**12)/(35*x**5)
```

3.354 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8}\,dx = -\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

output
```
-1/7*a^6/x^7-6/5*a^5*b/x^5-5*a^4*b^2/x^3-20*a^3*b^3/x+15*a^2*b^4*x+2*a*b^5
*x^3+1/5*b^6*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8}\,dx = -\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

output
```
-1/7*a^6/x^7 - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)/x + 15*a
^2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^8}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^8}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^6}{x^8}+\frac{6a^5b}{x^6}+\frac{15a^4b^2}{x^4}+\frac{20a^3b^3}{x^2}+15a^2b^4+6ab^5x^2+b^6x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

output
```
-1/7*a^6/x^7 - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)/x + 15*a
^2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2a\,b^5x^3+\frac{b^6x^5}{5}$	67
risch	$\frac{b^6x^5}{5}+2a\,b^5x^3+15a^2b^4x+\frac{-20a^3b^3x^6-5a^4b^2x^4-\frac{6}{5}a^5b\,x^2-\frac{1}{7}a^6}{x^7}$	69
norman	$\frac{\frac{1}{5}b^6x^{12}+2a\,b^5x^{10}+15a^2b^4x^8-20a^3b^3x^6-5a^4b^2x^4-\frac{6}{5}a^5b\,x^2-\frac{1}{7}a^6}{x^7}$	70
gosper	$-\frac{-7b^6x^{12}-70a\,b^5x^{10}-525a^2b^4x^8+700a^3b^3x^6+175a^4b^2x^4+42a^5b\,x^2+5a^6}{35x^7}$	71
parallelrisch	$\frac{7b^6x^{12}+70a\,b^5x^{10}+525a^2b^4x^8-700a^3b^3x^6-175a^4b^2x^4-42a^5b\,x^2-5a^6}{35x^7}$	71
orering	$-\frac{\left(-7b^6x^{12}-70a\,b^5x^{10}-525a^2b^4x^8+700a^3b^3x^6+175a^4b^2x^4+42a^5b\,x^2+5a^6\right)\left(b^2x^4+2ab\,x^2+a^2\right)^3}{35x^7\left(b\,x^2+a\right)^6}$	100

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^6/x^7-6/5*a^5*b/x^5-5*a^4*b^2/x^3-20*a^3*b^3/x+15*a^2*b^4*x+2*a*b^5
*x^3+1/5*b^6*x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8}\,dx = \frac{7\,b^6x^{12}+70\,ab^5x^{10}+525\,a^2b^4x^8-700\,a^3b^3x^6-175\,a^4b^2x^4-42\,a^5bx^2-5\,a^6}{35\,x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="fricas")
```

output
```
1/35*(7*b^6*x^12 + 70*a*b^5*x^10 + 525*a^2*b^4*x^8 - 700*a^3*b^3*x^6 - 175
*a^4*b^2*x^4 - 42*a^5*b*x^2 - 5*a^6)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8}\,dx = 15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5} + \frac{-5a^6-42a^5bx^2-175a^4b^2x^4-700a^3b^3x^6}{35x^7}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**8,x)
```

output
```
15*a**2*b**4*x + 2*a*b**5*x**3 + b**6*x**5/5 + (-5*a**6 - 42*a**5*b*x**2 -
 175*a**4*b**2*x**4 - 700*a**3*b**3*x**6)/(35*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^8} \, dx = \frac{1}{5} b^6x^5 + 2 ab^5x^3 + 15 a^2b^4x - \frac{700 a^3b^3x^6 + 175 a^4b^2x^4 + 42 a^5bx^2 + 5 a^6}{35 x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="maxima")
```

output
```
1/5*b^6*x^5 + 2*a*b^5*x^3 + 15*a^2*b^4*x - 1/35*(700*a^3*b^3*x^6 + 175*a^4
*b^2*x^4 + 42*a^5*b*x^2 + 5*a^6)/x^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^8} \, dx = \frac{1}{5} b^6x^5 + 2 ab^5x^3 + 15 a^2b^4x - \frac{700 a^3b^3x^6 + 175 a^4b^2x^4 + 42 a^5bx^2 + 5 a^6}{35 x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x, algorithm="giac")
```

output
```
1/5*b^6*x^5 + 2*a*b^5*x^3 + 15*a^2*b^4*x - 1/35*(700*a^3*b^3*x^6 + 175*a^4
*b^2*x^4 + 42*a^5*b*x^2 + 5*a^6)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^8} \, dx = \frac{b^6 \, x^5}{5} - \frac{\frac{a^6}{7} + \frac{6 \, a^5 \, b \, x^2}{5} + 5 \, a^4 \, b^2 \, x^4 + 20 \, a^3 \, b^3 \, x^6}{x^7} + 15 \, a^2 \, b^4 \, x + 2 \, a \, b^5 \, x^3$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^8,x)
```

output
```
(b^6*x^5)/5 - (a^6/7 + (6*a^5*b*x^2)/5 + 5*a^4*b^2*x^4 + 20*a^3*b^3*x^6)/x
^7 + 15*a^2*b^4*x + 2*a*b^5*x^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^8} \, dx = \frac{7b^6x^{12} + 70a \, b^5x^{10} + 525a^2b^4x^8 - 700a^3b^3x^6 - 175a^4b^2x^4 - 42a^5b \, x^2 - 5a^6}{35x^7}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x)
```

output
```
( - 5*a**6 - 42*a**5*b*x**2 - 175*a**4*b**2*x**4 - 700*a**3*b**3*x**6 + 52
5*a**2*b**4*x**8 + 70*a*b**5*x**10 + 7*b**6*x**12)/(35*x**7)
```

3.355 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{10}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{10}}\,dx=-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

output
```
-1/9*a^6/x^9-6/7*a^5*b/x^7-3*a^4*b^2/x^5-20/3*a^3*b^3/x^3-15*a^2*b^4/x+6*a
*b^5*x+1/3*b^6*x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{10}}\,dx=-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

output
```
-1/9*a^6/x^9 - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)/(3*x^3)
- (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{10}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^{10}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^{10}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^6}{x^{10}}+\frac{6a^5b}{x^8}+\frac{15a^4b^2}{x^6}+\frac{20a^3b^3}{x^4}+\frac{15a^2b^4}{x^2}+6ab^5+b^6x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

output
```
-1/9*a^6/x^9 - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)/(3*x^3)
- (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6a\,b^5x+\frac{b^6x^3}{3}$	67
risch	$\frac{b^6x^3}{3}+6a\,b^5x+\frac{-15a^2b^4x^8-\frac{20}{3}a^3b^3x^6-3a^4b^2x^4-\frac{6}{7}a^5b\,x^2-\frac{1}{9}a^6}{x^9}$	69
norman	$\frac{\frac{1}{3}b^6x^{12}+6a\,b^5x^{10}-15a^2b^4x^8-\frac{20}{3}a^3b^3x^6-3a^4b^2x^4-\frac{6}{7}a^5b\,x^2-\frac{1}{9}a^6}{x^9}$	70
gosper	$-\frac{-21b^6x^{12}-378a\,b^5x^{10}+945a^2b^4x^8+420a^3b^3x^6+189a^4b^2x^4+54a^5b\,x^2+7a^6}{63x^9}$	71
parallelrisch	$\frac{21b^6x^{12}+378a\,b^5x^{10}-945a^2b^4x^8-420a^3b^3x^6-189a^4b^2x^4-54a^5b\,x^2-7a^6}{63x^9}$	71
orering	$-\frac{\left(-21b^6x^{12}-378a\,b^5x^{10}+945a^2b^4x^8+420a^3b^3x^6+189a^4b^2x^4+54a^5b\,x^2+7a^6\right)\left(b^2x^4+2ab\,x^2+a^2\right)^3}{63x^9\left(b\,x^2+a\right)^6}$	100

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^6/x^9-6/7*a^5*b/x^7-3*a^4*b^2/x^5-20/3*a^3*b^3/x^3-15*a^2*b^4/x+6*a
*b^5*x+1/3*b^6*x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{10}} \, dx = \frac{21 \, b^6x^{12} + 378 \, ab^5x^{10} - 945 \, a^2b^4x^8 - 420 \, a^3b^3x^6 - 189 \, a^4b^2x^4 - 54 \, a^5bx^2 - 7 \, a^6}{63 \, x^9}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="fricas")
```

output
```
1/63*(21*b^6*x^12 + 378*a*b^5*x^10 - 945*a^2*b^4*x^8 - 420*a^3*b^3*x^6 - 1
89*a^4*b^2*x^4 - 54*a^5*b*x^2 - 7*a^6)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{10}} \, dx = 6ab^5x + \frac{b^6x^3}{3} + \frac{-7a^6 - 54a^5bx^2 - 189a^4b^2x^4 - 420a^3b^3x^6 - 945a^2b^4x^8}{63x^9}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**10,x)
```

output
```
6*a*b**5*x + b**6*x**3/3 + (-7*a**6 - 54*a**5*b*x**2 - 189*a**4*b**2*x**4
- 420*a**3*b**3*x**6 - 945*a**2*b**4*x**8)/(63*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{10}} \, dx = \frac{1}{3} b^6x^3 + 6 ab^5x - \frac{945 a^2b^4x^8 + 420 a^3b^3x^6 + 189 a^4b^2x^4 + 54 a^5bx^2 + 7 a^6}{63 x^9}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="maxima")
```

output
```
1/3*b^6*x^3 + 6*a*b^5*x - 1/63*(945*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 189*a^
4*b^2*x^4 + 54*a^5*b*x^2 + 7*a^6)/x^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{10}} \, dx = \frac{1}{3} b^6x^3 + 6 ab^5x - \frac{945 a^2b^4x^8 + 420 a^3b^3x^6 + 189 a^4b^2x^4 + 54 a^5bx^2 + 7 a^6}{63 x^9}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x, algorithm="giac")
```

output
```
1/3*b^6*x^3 + 6*a*b^5*x - 1/63*(945*a^2*b^4*x^8 + 420*a^3*b^3*x^6 + 189*a^
4*b^2*x^4 + 54*a^5*b*x^2 + 7*a^6)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{10}}\,dx = -\frac{\frac{a^6}{9}+\frac{6\,a^5\,b\,x^2}{7}+3\,a^4\,b^2\,x^4+\frac{20\,a^3\,b^3\,x^6}{3}+15\,a^2\,b^4\,x^8-6\,a\,b^5\,x^{10}-\frac{b^6\,x^{12}}{3}}{x^9}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^10,x)
```

output
```
-(a^6/9 - (b^6*x^12)/3 + (6*a^5*b*x^2)/7 - 6*a*b^5*x^10 + 3*a^4*b^2*x^4 +
(20*a^3*b^3*x^6)/3 + 15*a^2*b^4*x^8)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{10}}\,dx = \frac{21b^6x^{12}+378a\,b^5x^{10}-945a^2b^4x^8-420a^3b^3x^6-189a^4b^2x^4-54a^5b\,x^2-7a^6}{63x^9}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x)
```

output
```
( - 7*a**6 - 54*a**5*b*x**2 - 189*a**4*b**2*x**4 - 420*a**3*b**3*x**6 - 94
5*a**2*b**4*x**8 + 378*a*b**5*x**10 + 21*b**6*x**12)/(63*x**9)
```

3.356 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{12}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{12}}\,dx = -\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

output
```
-1/11*a^6/x^11-2/3*a^5*b/x^9-15/7*a^4*b^2/x^7-4*a^3*b^3/x^5-5*a^2*b^4/x^3-
6*a*b^5/x+b^6*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{12}}\,dx = -\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

output
```
-1/11*a^6/x^11 - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^3*b^3)/x^
5 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{12}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^{12}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^{12}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^6}{x^{12}}+\frac{6a^5b}{x^{10}}+\frac{15a^4b^2}{x^8}+\frac{20a^3b^3}{x^6}+\frac{15a^2b^4}{x^4}+\frac{6ab^5}{x^2}+b^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

output
```
-1/11*a^6/x^11 - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^3*b^3)/x^
5 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6a\,b^5}{x}+b^6x$	66
risch	$b^6x+\frac{-6a\,b^5x^{10}-5a^2b^4x^8-4a^3b^3x^6-\frac{15}{7}a^4b^2x^4-\frac{2}{3}a^5b\,x^2-\frac{1}{11}a^6}{x^{11}}$	68
norman	$\frac{b^6x^{12}-6a\,b^5x^{10}-5a^2b^4x^8-4a^3b^3x^6-\frac{15}{7}a^4b^2x^4-\frac{2}{3}a^5b\,x^2-\frac{1}{11}a^6}{x^{11}}$	69
gosper	$-\frac{-231b^6x^{12}+1386a\,b^5x^{10}+1155a^2b^4x^8+924a^3b^3x^6+495a^4b^2x^4+154a^5b\,x^2+21a^6}{231x^{11}}$	71
parallelrisch	$\frac{231b^6x^{12}-1386a\,b^5x^{10}-1155a^2b^4x^8-924a^3b^3x^6-495a^4b^2x^4-154a^5b\,x^2-21a^6}{231x^{11}}$	71
orering	$-\frac{\left(-231b^6x^{12}+1386a\,b^5x^{10}+1155a^2b^4x^8+924a^3b^3x^6+495a^4b^2x^4+154a^5b\,x^2+21a^6\right)\left(b^2x^4+2ab\,x^2+a^2\right)^3}{231x^{11}\left(b\,x^2+a\right)^6}$	100

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*a^6/x^11-2/3*a^5*b/x^9-15/7*a^4*b^2/x^7-4*a^3*b^3/x^5-5*a^2*b^4/x^3-
6*a*b^5/x+b^6*x
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{12}}\, dx = \frac{231\, b^6x^{12} - 1386\, ab^5x^{10} - 1155\, a^2b^4x^8 - 924\, a^3b^3x^6 - 495\, a^4b^2x^4 - 154\, a^5bx^2 - 21\, a^6}{231\, x^{11}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="fricas")
```

output
```
1/231*(231*b^6*x^12 - 1386*a*b^5*x^10 - 1155*a^2*b^4*x^8 - 924*a^3*b^3*x^6
 - 495*a^4*b^2*x^4 - 154*a^5*b*x^2 - 21*a^6)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{12}}\, dx = b^6x + \frac{-21a^6 - 154a^5bx^2 - 495a^4b^2x^4 - 924a^3b^3x^6 - 1155a^2b^4x^8 - 1386ab^5x^{10}}{231x^{11}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**12,x)
```

output
```
b**6*x + (-21*a**6 - 154*a**5*b*x**2 - 495*a**4*b**2*x**4 - 924*a**3*b**3*
x**6 - 1155*a**2*b**4*x**8 - 1386*a*b**5*x**10)/(231*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{12}} dx$$
$$= b^6x - \frac{1386\, ab^5x^{10} + 1155\, a^2b^4x^8 + 924\, a^3b^3x^6 + 495\, a^4b^2x^4 + 154\, a^5bx^2 + 21\, a^6}{231\, x^{11}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="maxima")
```

output
```
b^6*x - 1/231*(1386*a*b^5*x^10 + 1155*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 495*
a^4*b^2*x^4 + 154*a^5*b*x^2 + 21*a^6)/x^11
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{12}} dx$$
$$= b^6x - \frac{1386\, ab^5x^{10} + 1155\, a^2b^4x^8 + 924\, a^3b^3x^6 + 495\, a^4b^2x^4 + 154\, a^5bx^2 + 21\, a^6}{231\, x^{11}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x, algorithm="giac")
```

output
```
b^6*x - 1/231*(1386*a*b^5*x^10 + 1155*a^2*b^4*x^8 + 924*a^3*b^3*x^6 + 495*
a^4*b^2*x^4 + 154*a^5*b*x^2 + 21*a^6)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{12}}\,dx$$
$$= b^6\,x-\frac{\frac{a^6}{11}+\frac{2\,a^5\,b\,x^2}{3}+\frac{15\,a^4\,b^2\,x^4}{7}+4\,a^3\,b^3\,x^6+5\,a^2\,b^4\,x^8+6\,a\,b^5\,x^{10}}{x^{11}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^12,x)
```

output
```
b^6*x - (a^6/11 + (2*a^5*b*x^2)/3 + 6*a*b^5*x^10 + (15*a^4*b^2*x^4)/7 + 4*
a^3*b^3*x^6 + 5*a^2*b^4*x^8)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{12}}\,dx$$
$$= \frac{231b^6x^{12}-1386a\,b^5x^{10}-1155a^2b^4x^8-924a^3b^3x^6-495a^4b^2x^4-154a^5b\,x^2-21a^6}{231x^{11}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x)
```

output
```
( - 21*a**6 - 154*a**5*b*x**2 - 495*a**4*b**2*x**4 - 924*a**3*b**3*x**6 -
1155*a**2*b**4*x**8 - 1386*a*b**5*x**10 + 231*b**6*x**12)/(231*x**11)
```

3.357 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{14}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{14}}\,dx=-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

output
```
-1/13*a^6/x^13-6/11*a^5*b/x^11-5/3*a^4*b^2/x^9-20/7*a^3*b^3/x^7-3*a^2*b^4/
x^5-2*a*b^5/x^3-b^6/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{14}}\,dx=-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

output
```
-1/13*a^6/x^13 - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*a^3*b^3)/
(7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{14}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^{14}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^{14}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^6}{x^{14}}+\frac{6a^5b}{x^{12}}+\frac{15a^4b^2}{x^{10}}+\frac{20a^3b^3}{x^8}+\frac{15a^2b^4}{x^6}+\frac{6ab^5}{x^4}+\frac{b^6}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

output
```
-1/13*a^6/x^13 - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*a^3*b^3)/
(7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{a^{6}}{13 x^{13}}-\frac{6 a^{5} b}{11 x^{11}}-\frac{5 a^{4} b^{2}}{3 x^{9}}-\frac{20 a^{3} b^{3}}{7 x^{7}}-\frac{3 a^{2} b^{4}}{x^{5}}-\frac{2 a \,b^{5}}{x^{3}}-\frac{b^{6}}{x}$	69
norman	$\frac{-b^{6} x^{12}-2 a \,b^{5} x^{10}-3 a^{2} b^{4} x^{8}-\frac{20}{7} a^{3} b^{3} x^{6}-\frac{5}{3} a^{4} b^{2} x^{4}-\frac{6}{11} a^{5} b \,x^{2}-\frac{1}{13} a^{6}}{x^{13}}$	70
risch	$\frac{-b^{6} x^{12}-2 a \,b^{5} x^{10}-3 a^{2} b^{4} x^{8}-\frac{20}{7} a^{3} b^{3} x^{6}-\frac{5}{3} a^{4} b^{2} x^{4}-\frac{6}{11} a^{5} b \,x^{2}-\frac{1}{13} a^{6}}{x^{13}}$	70
gosper	$-\frac{3003 b^{6} x^{12}+6006 a \,b^{5} x^{10}+9009 a^{2} b^{4} x^{8}+8580 a^{3} b^{3} x^{6}+5005 a^{4} b^{2} x^{4}+1638 a^{5} b \,x^{2}+231 a^{6}}{3003 x^{13}}$	71
parallelrisch	$\frac{-3003 b^{6} x^{12}-6006 a \,b^{5} x^{10}-9009 a^{2} b^{4} x^{8}-8580 a^{3} b^{3} x^{6}-5005 a^{4} b^{2} x^{4}-1638 a^{5} b \,x^{2}-231 a^{6}}{3003 x^{13}}$	71
orering	$-\frac{\left(3003 b^{6} x^{12}+6006 a \,b^{5} x^{10}+9009 a^{2} b^{4} x^{8}+8580 a^{3} b^{3} x^{6}+5005 a^{4} b^{2} x^{4}+1638 a^{5} b \,x^{2}+231 a^{6}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{3}}{3003 x^{13} \left(b \,x^{2}+a \right)^{6}}$	100

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/13*a^6/x^13-6/11*a^5*b/x^11-5/3*a^4*b^2/x^9-20/7*a^3*b^3/x^7-3*a^2*b^4/
x^5-2*a*b^5/x^3-b^6/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{14}} dx = -\frac{3003 b^6x^{12} + 6006 ab^5x^{10} + 9009 a^2b^4x^8 + 8580 a^3b^3x^6 + 5005 a^4b^2x^4 + 1638 a^5bx^2 + 231 a^6}{3003 x^{13}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="fricas")
```

output
```
-1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3
*x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{x^{14}} dx = \frac{-231a^6 - 1638a^5bx^2 - 5005a^4b^2x^4 - 8580a^3b^3x^6 - 9009a^2b^4x^8 - 6006ab^5x^{10} - 3003b^6x^{12}}{3003x^{13}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**14,x)
```

output
```
(-231*a**6 - 1638*a**5*b*x**2 - 5005*a**4*b**2*x**4 - 8580*a**3*b**3*x**6
- 9009*a**2*b**4*x**8 - 6006*a*b**5*x**10 - 3003*b**6*x**12)/(3003*x**13)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{14}}\,dx = -\frac{3003\,b^6x^{12}+6006\,ab^5x^{10}+9009\,a^2b^4x^8+8580\,a^3b^3x^6+5005\,a^4b^2x^4+1638\,a^5bx^2+231\,a^6}{3003\,x^{13}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="maxima")
```

output
```
-1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3
*x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{14}}\,dx = -\frac{3003\,b^6x^{12}+6006\,ab^5x^{10}+9009\,a^2b^4x^8+8580\,a^3b^3x^6+5005\,a^4b^2x^4+1638\,a^5bx^2+231\,a^6}{3003\,x^{13}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x, algorithm="giac")
```

output
```
-1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 8580*a^3*b^3
*x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

Mupad [B] (verification not implemented)

Time = 18.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{14}}\,dx = -\frac{\frac{a^6}{13}+\frac{6\,a^5\,b\,x^2}{11}+\frac{5\,a^4\,b^2\,x^4}{3}+\frac{20\,a^3\,b^3\,x^6}{7}+3\,a^2\,b^4\,x^8+2\,a\,b^5\,x^{10}+b^6\,x^{12}}{x^{13}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^14,x)
```

output
```
-(a^6/13 + b^6*x^12 + (6*a^5*b*x^2)/11 + 2*a*b^5*x^10 + (5*a^4*b^2*x^4)/3
+ (20*a^3*b^3*x^6)/7 + 3*a^2*b^4*x^8)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{14}}\,dx = \frac{-3003b^6x^{12}-6006a\,b^5x^{10}-9009a^2b^4x^8-8580a^3b^3x^6-5005a^4b^2x^4-1638a^5b\,x^2-231a^6}{3003x^{13}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x)
```

output
```
( - 231*a**6 - 1638*a**5*b*x**2 - 5005*a**4*b**2*x**4 - 8580*a**3*b**3*x**
6 - 9009*a**2*b**4*x**8 - 6006*a*b**5*x**10 - 3003*b**6*x**12)/(3003*x**13
)
```

3.358 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx = -\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

output
```
-1/15*a^6/x^15-6/13*a^5*b/x^13-15/11*a^4*b^2/x^11-20/9*a^3*b^3/x^9-15/7*a^
2*b^4/x^7-6/5*a*b^5/x^5-1/3*b^6/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx = -\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

output
```
-1/15*a^6/x^15 - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (20*a^3*b^
3)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) - b^6/(3*x^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{x^{16}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{x^{16}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^6}{x^{16}}+\frac{6a^5b}{x^{14}}+\frac{15a^4b^2}{x^{12}}+\frac{20a^3b^3}{x^{10}}+\frac{15a^2b^4}{x^8}+\frac{6ab^5}{x^6}+\frac{b^6}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

output
```
-1/15*a^6/x^15 - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (20*a^3*b^
3)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) - b^6/(3*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result
default	$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6a\,b^5}{5x^5}-\frac{b^6}{3x^3}$
norman	$\frac{-\frac{1}{15}a^6-\frac{6}{13}a^5b\,x^2-\frac{15}{11}a^4b^2x^4-\frac{20}{9}a^3b^3x^6-\frac{15}{7}a^2b^4x^8-\frac{6}{5}a\,b^5x^{10}-\frac{1}{3}b^6x^{12}}{x^{15}}$
risch	$\frac{-\frac{1}{15}a^6-\frac{6}{13}a^5b\,x^2-\frac{15}{11}a^4b^2x^4-\frac{20}{9}a^3b^3x^6-\frac{15}{7}a^2b^4x^8-\frac{6}{5}a\,b^5x^{10}-\frac{1}{3}b^6x^{12}}{x^{15}}$
gosper	$-\frac{15015b^6x^{12}+54054a\,b^5x^{10}+96525a^2b^4x^8+100100a^3b^3x^6+61425a^4b^2x^4+20790a^5b\,x^2+3003a^6}{45045x^{15}}$
parallelrisch	$\frac{-15015b^6x^{12}-54054a\,b^5x^{10}-96525a^2b^4x^8-100100a^3b^3x^6-61425a^4b^2x^4-20790a^5b\,x^2-3003a^6}{45045x^{15}}$
orering	$-\frac{\left(15015b^6x^{12}+54054a\,b^5x^{10}+96525a^2b^4x^8+100100a^3b^3x^6+61425a^4b^2x^4+20790a^5b\,x^2+3003a^6\right)\left(b^2x^4+2ab\,x^2+a^2\right)^3}{45045x^{15}\left(b\,x^2+a\right)^6}$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*a^6/x^15-6/13*a^5*b/x^13-15/11*a^4*b^2/x^11-20/9*a^3*b^3/x^9-15/7*a^
2*b^4/x^7-6/5*a*b^5/x^5-1/3*b^6/x^3
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx = -\frac{15015\,b^6x^{12}+54054\,ab^5x^{10}+96525\,a^2b^4x^8+100100\,a^3b^3x^6+61425\,a^4b^2x^4+20790\,a^5bx^2+3003\,a^6}{45045\,x^{15}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="fricas")
```

output
```
-1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a
^3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx = \frac{-3003a^6-20790a^5bx^2-61425a^4b^2x^4-100100a^3b^3x^6-96525a^2b^4x^8-54054ab^5x^{10}-15015b^6x^{12}}{45045x^{15}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**16,x)
```

output
```
(-3003*a**6 - 20790*a**5*b*x**2 - 61425*a**4*b**2*x**4 - 100100*a**3*b**3*
x**6 - 96525*a**2*b**4*x**8 - 54054*a*b**5*x**10 - 15015*b**6*x**12)/(4504
5*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx = -\frac{15015\,b^6x^{12}+54054\,ab^5x^{10}+96525\,a^2b^4x^8+100100\,a^3b^3x^6+61425\,a^4b^2x^4+20790\,a^5bx^2+3003\,a^6}{45045\,x^{15}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="maxima")
```

output
```
-1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a
^3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx = -\frac{15015\,b^6x^{12}+54054\,ab^5x^{10}+96525\,a^2b^4x^8+100100\,a^3b^3x^6+61425\,a^4b^2x^4+20790\,a^5bx^2+3003\,a^6}{45045\,x^{15}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x, algorithm="giac")
```

output
```
-1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 + 100100*a
^3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*a^6)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx = -\frac{\frac{a^6}{15}+\frac{6\,a^5\,b\,x^2}{13}+\frac{15\,a^4\,b^2\,x^4}{11}+\frac{20\,a^3\,b^3\,x^6}{9}+\frac{15\,a^2\,b^4\,x^8}{7}+\frac{6\,a\,b^5\,x^{10}}{5}+\frac{b^6\,x^{12}}{3}}{x^{15}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/x^16,x)
```

output
```
-(a^6/15 + (b^6*x^12)/3 + (6*a^5*b*x^2)/13 + (6*a*b^5*x^10)/5 + (15*a^4*b^
2*x^4)/11 + (20*a^3*b^3*x^6)/9 + (15*a^2*b^4*x^8)/7)/x^15
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx = \frac{-15015b^6x^{12}-54054a\,b^5x^{10}-96525a^2b^4x^8-100100a^3b^3x^6-61425a^4b^2x^4-20790a^5b\,x^2-3003a^6}{45045x^{15}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x)
```

output
```
( - 3003*a**6 - 20790*a**5*b*x**2 - 61425*a**4*b**2*x**4 - 100100*a**3*b**
3*x**6 - 96525*a**2*b**4*x**8 - 54054*a*b**5*x**10 - 15015*b**6*x**12)/(45
045*x**15)
```

3.359 $\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx=-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6(a+bx^2)}+\frac{5a^4\log(a+bx^2)}{2b^6}$$

output
```
-2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^
2+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx = \frac{-48a^3bx^2+18a^2b^2x^4-8ab^3x^6+3b^4x^8+\frac{12a^5}{a+bx^2}+60a^4\log(a+bx^2)}{24b^6}$$

input
```
Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(-48*a^3*b*x^2 + 18*a^2*b^2*x^4 - 8*a*b^3*x^6 + 3*b^4*x^8 + (12*a^5)/(a +
b*x^2) + 60*a^4*Log[a + b*x^2])/(24*b^6)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^2\int \frac{x^{11}}{b^2\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^{11}}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^{10}}{\left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{x^6}{b^2}-\frac{2ax^4}{b^3}+\frac{3a^2x^2}{b^4}+\frac{5a^4}{b^5\left(bx^2+a\right)}-\frac{a^5}{b^5\left(bx^2+a\right)^2}-\frac{4a^3}{b^5}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{a^5}{b^6\left(a+bx^2\right)}+\frac{5a^4\log\left(a+bx^2\right)}{b^6}-\frac{4a^3x^2}{b^5}+\frac{3a^2x^4}{2b^4}-\frac{2ax^6}{3b^3}+\frac{x^8}{4b^2}\right)$$

input
```
Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
((-4*a^3*x^2)/b^5 + (3*a^2*x^4)/(2*b^4) - (2*a*x^6)/(3*b^3) + x^8/(4*b^2)
+ a^5/(b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/b^6)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{2a^{3}x^{2}}{b^{5}}+\frac{3a^{2}x^{4}}{4b^{4}}-\frac{a\,x^{6}}{3b^{3}}+\frac{x^{8}}{8b^{2}}+\frac{a^{5}}{2b^{6}\left(b\,x^{2}+a\right)}+\frac{5a^{4}\ln\left(b\,x^{2}+a\right)}{2b^{6}}$	74
default	$-\frac{-\frac{1}{4}b^{3}x^{8}+\frac{2}{3}b^{2}x^{6}a-\frac{3}{2}a^{2}b\,x^{4}+4a^{3}x^{2}}{2b^{5}}+\frac{5a^{4}\ln\left(b\,x^{2}+a\right)}{2b^{6}}+\frac{a^{5}}{2b^{6}\left(b\,x^{2}+a\right)}$	75
norman	$\frac{\frac{x^{10}}{8b}-\frac{5a\,x^{8}}{24b^{2}}+\frac{5a^{5}}{2b^{6}}+\frac{5a^{2}x^{6}}{12b^{3}}-\frac{5a^{3}x^{4}}{4b^{4}}}{b\,x^{2}+a}+\frac{5a^{4}\ln\left(b\,x^{2}+a\right)}{2b^{6}}$	76
parallelrisch	$\frac{3x^{10}b^{5}-5a\,x^{8}b^{4}+10a^{2}x^{6}b^{3}-30a^{3}x^{4}b^{2}+60\ln\left(b\,x^{2}+a\right)x^{2}a^{4}b+60\ln\left(b\,x^{2}+a\right)a^{5}+60a^{5}}{24b^{6}\left(b\,x^{2}+a\right)}$	90

input
```
int(x^11/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^
2+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx = \frac{3\,b^5x^{10}-5\,ab^4x^8+10\,a^2b^3x^6-30\,a^3b^2x^4-48\,a^4bx^2+12\,a^5+60\,(a^4bx^2+a^5)\log\left(bx^2+a\right)}{24\,(b^7x^2+ab^6)}$$

input
```
integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/24*(3*b^5*x^10 - 5*a*b^4*x^8 + 10*a^2*b^3*x^6 - 30*a^3*b^2*x^4 - 48*a^4*
b*x^2 + 12*a^5 + 60*(a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx = \frac{a^5}{2ab^6+2b^7x^2}+\frac{5a^4\log\left(a+bx^2\right)}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

input
```
integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
a**5/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*log(a + b*x**2)/(2*b**6) - 2*a**3*x
**2/b**5 + 3*a**2*x**4/(4*b**4) - a*x**6/(3*b**3) + x**8/(8*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx = \frac{a^5}{2\,(b^7x^2+ab^6)}+\frac{5\,a^4\log\left(bx^2+a\right)}{2\,b^6}+\frac{3\,b^3x^8-8\,ab^2x^6+18\,a^2bx^4-48\,a^3x^2}{24\,b^5}$$

input
```
integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2*a^5/(b^7*x^2 + a*b^6) + 5/2*a^4*log(b*x^2 + a)/b^6 + 1/24*(3*b^3*x^8 -
 8*a*b^2*x^6 + 18*a^2*b*x^4 - 48*a^3*x^2)/b^5
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx = \frac{5\,a^4\log\left(|bx^2+a|\right)}{2\,b^6}-\frac{5\,a^4bx^2+4\,a^5}{2\,(bx^2+a)b^6}+\frac{3\,b^6x^8-8\,ab^5x^6+18\,a^2b^4x^4-48\,a^3b^3x^2}{24\,b^8}$$

input
```
integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
5/2*a^4*log(abs(b*x^2 + a))/b^6 - 1/2*(5*a^4*b*x^2 + 4*a^5)/((b*x^2 + a)*b
^6) + 1/24*(3*b^6*x^8 - 8*a*b^5*x^6 + 18*a^2*b^4*x^4 - 48*a^3*b^3*x^2)/b^8
```

Mupad [B] (verification not implemented)

Time = 19.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx = \frac{x^8}{8\,b^2}+\frac{a^5}{2\,b\,\left(b^6\,x^2+a\,b^5\right)}-\frac{a\,x^6}{3\,b^3}+\frac{5\,a^4\,\ln\left(b\,x^2+a\right)}{2\,b^6}+\frac{3\,a^2\,x^4}{4\,b^4}-\frac{2\,a^3\,x^2}{b^5}$$

input
```
int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
x^8/(8*b^2) + a^5/(2*b*(a*b^5 + b^6*x^2)) - (a*x^6)/(3*b^3) + (5*a^4*log(a
 + b*x^2))/(2*b^6) + (3*a^2*x^4)/(4*b^4) - (2*a^3*x^2)/b^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{x^{11}}{a^2+2abx^2+b^2x^4}\,dx = \frac{60\log(b\,x^2+a)\,a^5+60\log(b\,x^2+a)\,a^4b\,x^2-60a^4b\,x^2-30a^3b^2x^4+10a^2b^3x^6-5a\,b^4x^8+3b^5x^{10}}{24b^6\,(b\,x^2+a)}$$

input
```
int(x^11/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(60*log(a + b*x**2)*a**5 + 60*log(a + b*x**2)*a**4*b*x**2 - 60*a**4*b*x**2
 - 30*a**3*b**2*x**4 + 10*a**2*b**3*x**6 - 5*a*b**4*x**8 + 3*b**5*x**10)/(
24*b**6*(a + b*x**2))
```

3.360 $\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx=\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}-\frac{a^4}{2b^5(a+bx^2)}-\frac{2a^3\log(a+bx^2)}{b^5}$$

output
```
3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b
*x^2+a)/b^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx=\frac{9a^2bx^2-3ab^2x^4+b^3x^6-\frac{3a^4}{a+bx^2}-12a^3\log(a+bx^2)}{6b^5}$$

input
```
Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(9*a^2*b*x^2 - 3*a*b^2*x^4 + b^3*x^6 - (3*a^4)/(a + b*x^2) - 12*a^3*Log[a
+ b*x^2])/(6*b^5)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{a^2+2abx^2+b^2x^4}dx$$

$$\Big\downarrow \text{1380}$$

$$b^2\int \frac{x^9}{b^2\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^9}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^8}{\left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^4}{b^4\left(bx^2+a\right)^2}-\frac{4a^3}{b^4\left(bx^2+a\right)}+\frac{3a^2}{b^4}-\frac{2x^2a}{b^3}+\frac{x^4}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^4}{b^5\left(a+bx^2\right)}-\frac{4a^3\log\left(a+bx^2\right)}{b^5}+\frac{3a^2x^2}{b^4}-\frac{ax^4}{b^3}+\frac{x^6}{3b^2}\right)$$

input
```
Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
((3*a^2*x^2)/b^4 - (a*x^4)/b^3 + x^6/(3*b^2) - a^4/(b^5*(a + b*x^2)) - (4*
a^3*Log[a + b*x^2])/b^5)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{3a^{2}x^{2}}{2b^{4}}-\frac{a\,x^{4}}{2b^{3}}+\frac{x^{6}}{6b^{2}}-\frac{a^{4}}{2b^{5}\left(b\,x^{2}+a\right)}-\frac{2a^{3}\ln\left(b\,x^{2}+a\right)}{b^{5}}$	63
default	$\frac{\frac{1}{3}b^{2}x^{6}-ab\,x^{4}+3a^{2}x^{2}}{2b^{4}}-\frac{2a^{3}\ln\left(b\,x^{2}+a\right)}{b^{5}}-\frac{a^{4}}{2b^{5}\left(b\,x^{2}+a\right)}$	64
norman	$\frac{\frac{a^{2}x^{4}}{b^{3}}+\frac{x^{8}}{6b}-\frac{a\,x^{6}}{3b^{2}}-\frac{2a^{4}}{b^{5}}}{b\,x^{2}+a}-\frac{2a^{3}\ln\left(b\,x^{2}+a\right)}{b^{5}}$	64
parallelrisch	$-\frac{-b^{4}x^{8}+2a\,b^{3}x^{6}-6a^{2}b^{2}x^{4}+12\ln\left(b\,x^{2}+a\right)x^{2}a^{3}b+12\ln\left(b\,x^{2}+a\right)a^{4}+12a^{4}}{6b^{5}\left(b\,x^{2}+a\right)}$	79

input
```
int(x^9/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b
*x^2+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx = \frac{b^4x^8-2\,ab^3x^6+6\,a^2b^2x^4+9\,a^3bx^2-3\,a^4-12\,(a^3bx^2+a^4)\log\left(bx^2+a\right)}{6\,(b^6x^2+ab^5)}$$

input
```
integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/6*(b^4*x^8 - 2*a*b^3*x^6 + 6*a^2*b^2*x^4 + 9*a^3*b*x^2 - 3*a^4 - 12*(a^3
*b*x^2 + a^4)*log(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx = -\frac{a^4}{2ab^5+2b^6x^2}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

input
```
integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
-a**4/(2*a*b**5 + 2*b**6*x**2) - 2*a**3*log(a + b*x**2)/b**5 + 3*a**2*x**2
/(2*b**4) - a*x**4/(2*b**3) + x**6/(6*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx=-\frac{a^4}{2\,(b^6x^2+ab^5)}-\frac{2\,a^3\log\left(bx^2+a\right)}{b^5}+\frac{b^2x^6-3\,abx^4+9\,a^2x^2}{6\,b^4}$$

input
```
integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/2*a^4/(b^6*x^2 + a*b^5) - 2*a^3*log(b*x^2 + a)/b^5 + 1/6*(b^2*x^6 - 3*a
*b*x^4 + 9*a^2*x^2)/b^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

$$\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx=-\frac{2\,a^3\log\left(|bx^2+a|\right)}{b^5}+\frac{b^4x^6-3\,ab^3x^4+9\,a^2b^2x^2}{6\,b^6}+\frac{4\,a^3bx^2+3\,a^4}{2\,(bx^2+a)b^5}$$

input
```
integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
-2*a^3*log(abs(b*x^2 + a))/b^5 + 1/6*(b^4*x^6 - 3*a*b^3*x^4 + 9*a^2*b^2*x^
2)/b^6 + 1/2*(4*a^3*b*x^2 + 3*a^4)/((b*x^2 + a)*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx=\frac{x^6}{6\,b^2}-\frac{a^4}{2\,b\,\left(b^5\,x^2+a\,b^4\right)}-\frac{a\,x^4}{2\,b^3}-\frac{2\,a^3\,\ln\left(b\,x^2+a\right)}{b^5}+\frac{3\,a^2\,x^2}{2\,b^4}$$

input
```
int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
x^6/(6*b^2) - a^4/(2*b*(a*b^4 + b^5*x^2)) - (a*x^4)/(2*b^3) - (2*a^3*log(a
 + b*x^2))/b^5 + (3*a^2*x^2)/(2*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{x^9}{a^2 + 2abx^2 + b^2x^4}\, dx = \frac{-12\log(b\,x^2+a)\,a^4 - 12\log(b\,x^2+a)\,a^3b\,x^2 + 12a^3b\,x^2 + 6a^2b^2x^4 - 2a\,b^3x^6 + b^4x^8}{6b^5\,(b\,x^2+a)}$$

input
```
int(x^9/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
( - 12*log(a + b*x**2)*a**4 - 12*log(a + b*x**2)*a**3*b*x**2 + 12*a**3*b*x
**2 + 6*a**2*b**2*x**4 - 2*a*b**3*x**6 + b**4*x**8)/(6*b**5*(a + b*x**2))
```

3.361 $\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 57

$$\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx=-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}+\frac{a^3}{2b^4(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{2b^4}$$

output
```
-a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx=\frac{-4abx^2+b^2x^4+\frac{2a^3}{a+bx^2}+6a^2\log(a+bx^2)}{4b^4}$$

input
```
Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(-4*a*b*x^2 + b^2*x^4 + (2*a^3)/(a + b*x^2) + 6*a^2*Log[a + b*x^2])/(4*b^4
)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{a^2+2abx^2+b^2x^4}dx$$
$$\Big\downarrow \text{1380}$$
$$b^2\int \frac{x^7}{b^2\left(bx^2+a\right)^2}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{x^7}{\left(a+bx^2\right)^2}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{x^6}{\left(bx^2+a\right)^2}dx^2$$
$$\Big\downarrow \text{49}$$
$$\frac{1}{2}\int \left(-\frac{a^3}{b^3\left(bx^2+a\right)^2}+\frac{3a^2}{b^3\left(bx^2+a\right)}-\frac{2a}{b^3}+\frac{x^2}{b^2}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{a^3}{b^4\left(a+bx^2\right)}+\frac{3a^2\log\left(a+bx^2\right)}{b^4}-\frac{2ax^2}{b^3}+\frac{x^4}{2b^2}\right)$$

input
```
Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
((-2*a*x^2)/b^3 + x^4/(2*b^2) + a^3/(b^4*(a + b*x^2)) + (3*a^2*Log[a + b*x
^2])/b^4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{-\frac{1}{2}b\,x^{4}+2a\,x^{2}}{2b^{3}}+\frac{3a^{2}\ln\left(b\,x^{2}+a\right)}{2b^{4}}+\frac{a^{3}}{2b^{4}\left(b\,x^{2}+a\right)}$	53
norman	$\frac{\frac{x^{6}}{4b}-\frac{3a\,x^{4}}{4b^{2}}+\frac{3a^{3}}{2b^{4}}}{b\,x^{2}+a}+\frac{3a^{2}\ln\left(b\,x^{2}+a\right)}{2b^{4}}$	54
risch	$\frac{x^{4}}{4b^{2}}-\frac{a\,x^{2}}{b^{3}}+\frac{a^{2}}{b^{4}}+\frac{a^{3}}{2b^{4}\left(b\,x^{2}+a\right)}+\frac{3a^{2}\ln\left(b\,x^{2}+a\right)}{2b^{4}}$	59
parallelrisch	$\frac{b^{3}x^{6}-3b^{2}x^{4}a+6\ln\left(b\,x^{2}+a\right)x^{2}a^{2}b+6\ln\left(b\,x^{2}+a\right)a^{3}+6a^{3}}{4b^{4}\left(b\,x^{2}+a\right)}$	67

input
```
int(x^7/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*(-1/2*b*x^4+2*a*x^2)+3/2*a^2*ln(b*x^2+a)/b^4+1/2*a^3/b^4/(b*x^2+a
)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx=\frac{b^3x^6-3\,ab^2x^4-4\,a^2bx^2+2\,a^3+6\,(a^2bx^2+a^3)\log\left(bx^2+a\right)}{4\,(b^5x^2+ab^4)}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/4*(b^3*x^6 - 3*a*b^2*x^4 - 4*a^2*b*x^2 + 2*a^3 + 6*(a^2*b*x^2 + a^3)*log
(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx=\frac{a^3}{2ab^4+2b^5x^2}+\frac{3a^2\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

input
```
integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
a**3/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*log(a + b*x**2)/(2*b**4) - a*x**2/b
**3 + x**4/(4*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{a^2 + 2abx^2 + b^2x^4} \, dx = \frac{a^3}{2\,(b^5x^2 + ab^4)} + \frac{3\,a^2 \log\left(bx^2 + a\right)}{2\,b^4} + \frac{bx^4 - 4\,ax^2}{4\,b^3}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2*a^3/(b^5*x^2 + a*b^4) + 3/2*a^2*log(b*x^2 + a)/b^4 + 1/4*(b*x^4 - 4*a*
x^2)/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{x^7}{a^2 + 2abx^2 + b^2x^4} \, dx = \frac{3\,a^2 \log\left(|bx^2 + a|\right)}{2\,b^4} + \frac{b^2x^4 - 4\,abx^2}{4\,b^4} - \frac{3\,a^2bx^2 + 2\,a^3}{2\,(bx^2 + a)b^4}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
3/2*a^2*log(abs(b*x^2 + a))/b^4 + 1/4*(b^2*x^4 - 4*a*b*x^2)/b^4 - 1/2*(3*a
^2*b*x^2 + 2*a^3)/((b*x^2 + a)*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{a^2 + 2abx^2 + b^2x^4} \, dx = \frac{x^4}{4\,b^2} + \frac{a^3}{2\,b\,\left(b^4\,x^2 + a\,b^3\right)} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,\ln\left(b\,x^2 + a\right)}{2\,b^4}$$

input
```
int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
x^4/(4*b^2) + a^3/(2*b*(a*b^3 + b^4*x^2)) - (a*x^2)/b^3 + (3*a^2*log(a + b
*x^2))/(2*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{x^7}{a^2+2abx^2+b^2x^4}\,dx = \frac{6\log(b\,x^2+a)\,a^3+6\log(b\,x^2+a)\,a^2b\,x^2-6a^2b\,x^2-3a\,b^2x^4+b^3x^6}{4b^4\,(b\,x^2+a)}$$

input
```
int(x^7/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(6*log(a + b*x**2)*a**3 + 6*log(a + b*x**2)*a**2*b*x**2 - 6*a**2*b*x**2 -
3*a*b**2*x**4 + b**3*x**6)/(4*b**4*(a + b*x**2))
```

3.362 $\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 44

$$\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx=\frac{x^2}{2b^2}-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}$$

output
```
1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx=\frac{bx^2-\frac{a^2}{a+bx^2}-2a\log\left(a+bx^2\right)}{2b^3}$$

input
```
Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(b*x^2 - a^2/(a + b*x^2) - 2*a*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a^2+2abx^2+b^2x^4}dx$$

$$\Big\downarrow \text{1380}$$

$$b^2\int \frac{x^5}{b^2(bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^5}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2(bx^2+a)^2}-\frac{2a}{b^2(bx^2+a)}+\frac{1}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{b^3(a+bx^2)}-\frac{2a\log(a+bx^2)}{b^3}+\frac{x^2}{b^2}\right)$$

input
```
Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(x^2/b^2 - a^2/(b^3*(a + b*x^2)) - (2*a*Log[a + b*x^2])/b^3)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x^{2}}{2b^{2}}-\frac{a^{2}}{2b^{3}\left(b\,x^{2}+a\right)}-\frac{a\ln\left(b\,x^{2}+a\right)}{b^{3}}$	41
risch	$\frac{x^{2}}{2b^{2}}-\frac{a^{2}}{2b^{3}\left(b\,x^{2}+a\right)}-\frac{a\ln\left(b\,x^{2}+a\right)}{b^{3}}$	41
norman	$\frac{\frac{x^{4}}{2b}-\frac{a^{2}}{b^{3}}}{b\,x^{2}+a}-\frac{a\ln\left(b\,x^{2}+a\right)}{b^{3}}$	43
parallelrisch	$-\frac{-b^{2}x^{4}+2\ln\left(b\,x^{2}+a\right)x^{2}ab+2\ln\left(b\,x^{2}+a\right)a^{2}+2a^{2}}{2b^{3}\left(b\,x^{2}+a\right)}$	57

input
```
int(x^5/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx = \frac{b^2x^4+abx^2-a^2-2\,(abx^2+a^2)\log\left(bx^2+a\right)}{2\,(b^4x^2+ab^3)}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^4 + a*b*x^2 - a^2 - 2*(a*b*x^2 + a^2)*log(b*x^2 + a))/(b^4*x^2
+ a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx = -\frac{a^2}{2ab^3+2b^4x^2} - \frac{a\log\left(a+bx^2\right)}{b^3} + \frac{x^2}{2b^2}$$

input
```
integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
-a**2/(2*a*b**3 + 2*b**4*x**2) - a*log(a + b*x**2)/b**3 + x**2/(2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx = -\frac{a^2}{2\,(b^4x^2+ab^3)} + \frac{x^2}{2\,b^2} - \frac{a\log\left(bx^2+a\right)}{b^3}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/2*a^2/(b^4*x^2 + a*b^3) + 1/2*x^2/b^2 - a*log(b*x^2 + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx=\frac{x^2}{2\,b^2}-\frac{a\log\left(|bx^2+a|\right)}{b^3}+\frac{2\,abx^2+a^2}{2\,(bx^2+a)b^3}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*x^2/b^2 - a*log(abs(b*x^2 + a))/b^3 + 1/2*(2*a*b*x^2 + a^2)/((b*x^2 +
a)*b^3)
```

Mupad [B] (verification not implemented)

Time = 19.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx=\frac{x^2}{2\,b^2}-\frac{a^2}{2\,\left(b^4\,x^2+a\,b^3\right)}-\frac{a\,\ln\left(b\,x^2+a\right)}{b^3}$$

input
```
int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
x^2/(2*b^2) - a^2/(2*(a*b^3 + b^4*x^2)) - (a*log(a + b*x^2))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx=\frac{-2\log(b\,x^2+a)\,a^2-2\log(b\,x^2+a)\,ab\,x^2+2ab\,x^2+b^2x^4}{2b^3\,(b\,x^2+a)}$$

input
```
int(x^5/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
( - 2*log(a + b*x**2)*a**2 - 2*log(a + b*x**2)*a*b*x**2 + 2*a*b*x**2 + b**
2*x**4)/(2*b**3*(a + b*x**2))
```

3.363 $\int \frac{x^3}{a^2+2abx^2+b^2x^4} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 33

$$\int \frac{x^3}{a^2+2abx^2+b^2x^4} \, dx = \frac{a}{2b^2(a+bx^2)} + \frac{\log(a+bx^2)}{2b^2}$$

output
```
1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{a^2+2abx^2+b^2x^4} \, dx = \frac{\frac{a}{a+bx^2} + \log(a+bx^2)}{2b^2}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(a/(a + b*x^2) + Log[a + b*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx$$
$$\Big\downarrow \text{1380}$$
$$b^2\int \frac{x^3}{b^2\left(bx^2+a\right)^2}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{x^3}{\left(a+bx^2\right)^2}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{x^2}{\left(bx^2+a\right)^2}dx^2$$
$$\Big\downarrow \text{49}$$
$$\frac{1}{2}\int \left(\frac{1}{b\left(bx^2+a\right)}-\frac{a}{b\left(bx^2+a\right)^2}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{a}{b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{b^2}\right)$$

input
```
Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(a/(b^2*(a + b*x^2)) + Log[a + b*x^2]/b^2)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{a}{2b^2(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^2}$	30
norman	$\frac{a}{2b^2(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^2}$	30
risch	$\frac{a}{2b^2(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^2}$	30
parallelrisch	$\frac{\ln(b\,x^2+a)x^2b+\ln(b\,x^2+a)a+a}{2b^2(b\,x^2+a)}$	40

input
```
int(x^3/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx=\frac{(bx^2+a)\log(bx^2+a)+a}{2(b^3x^2+ab^2)}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/2*((b*x^2 + a)*log(b*x^2 + a) + a)/(b^3*x^2 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx=\frac{a}{2ab^2+2b^3x^2}+\frac{\log(a+bx^2)}{2b^2}$$

input
```
integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx=\frac{a}{2(b^3x^2+ab^2)}+\frac{\log(bx^2+a)}{2b^2}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2*a/(b^3*x^2 + a*b^2) + 1/2*log(b*x^2 + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx=\frac{\log\left(|bx^2+a|\right)}{2\,b^2}+\frac{a}{2\,(bx^2+a)b^2}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*log(abs(b*x^2 + a))/b^2 + 1/2*a/((b*x^2 + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx=\frac{\ln\left(b\,x^2+a\right)}{2\,b^2}+\frac{a}{2\,b^2\,\left(b\,x^2+a\right)}$$

input
```
int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
log(a + b*x^2)/(2*b^2) + a/(2*b^2*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx=\frac{\log(b\,x^2+a)\,a+\log(b\,x^2+a)\,b\,x^2-b\,x^2}{2b^2\,(b\,x^2+a)}$$

input
```
int(x^3/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(log(a + b*x**2)*a + log(a + b*x**2)*b*x**2 - b*x**2)/(2*b**2*(a + b*x**2)
)
```

3.364 $\int \frac{x}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{x}{a^2+2abx^2+b^2x^4}\,dx = -\frac{1}{2b\,(a+bx^2)}$$

output
```
-1/2/b/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{a^2+2abx^2+b^2x^4}\,dx = -\frac{1}{2b\,(a+bx^2)}$$

input
```
Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*1/(b*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1380, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a^2+2abx^2+b^2x^4}dx$$

$$\Big\downarrow \text{1380}$$

$$b^2\int \frac{x}{b^2\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{241}$$

$$-\frac{1}{2b\left(a+bx^2\right)}$$

input
```
Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*1/(b*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{2b(b\,x^2+a)}$	15
default	$-\frac{1}{2b(b\,x^2+a)}$	15
norman	$-\frac{1}{2b(b\,x^2+a)}$	15
risch	$-\frac{1}{2b(b\,x^2+a)}$	15
parallelrisch	$-\frac{1}{2b(b\,x^2+a)}$	15
orering	$-\frac{b\,x^2+a}{2b(b^2x^4+2ab\,x^2+a^2)}$	33

input
```
int(x/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{a^2+2abx^2+b^2x^4}\,dx=-\frac{1}{2\left(b^2x^2+ab\right)}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/2/(b^2*x^2 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{a^2 + 2abx^2 + b^2x^4} \, dx = -\frac{1}{2ab + 2b^2x^2}$$

input
```
integrate(x/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
-1/(2*a*b + 2*b**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{a^2 + 2abx^2 + b^2x^4} \, dx = -\frac{1}{2\left(b^2x^2 + ab\right)}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/2/(b^2*x^2 + a*b)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{a^2 + 2abx^2 + b^2x^4} \, dx = -\frac{1}{2\left(bx^2 + a\right)b}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
-1/2/((b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{a^2 + 2abx^2 + b^2x^4}\,dx = -\frac{1}{2\,b\,\left(b\,x^2 + a\right)}$$

input
```
int(x/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
-1/(2*b*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x}{a^2 + 2abx^2 + b^2x^4}\,dx = \frac{x^2}{2a\,\left(b\,x^2 + a\right)}$$

input
```
int(x/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
x**2/(2*a*(a + b*x**2))
```

3.365 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{1}{x(a^2+2abx^2+b^2x^4)}\,dx = \frac{1}{2a(a+bx^2)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx^2)}{2a^2}$$

output
```
1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(a^2+2abx^2+b^2x^4)}\,dx = \frac{\frac{a}{a+bx^2}+2\log(x)-\log(a+bx^2)}{2a^2}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
(a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a^2 + 2abx^2 + b^2x^4)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$b^2 \int \frac{1}{b^2 x (bx^2 + a)^2} dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x (a + bx^2)^2} dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^2 (bx^2 + a)^2} dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(-\frac{b}{a^2 (bx^2 + a)} - \frac{b}{a (bx^2 + a)^2} + \frac{1}{a^2 x^2} \right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{\log (a + bx^2)}{a^2} + \frac{\log (x^2)}{a^2} + \frac{1}{a (a + bx^2)} \right)$$

input
```
Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
(1/(a*(a + b*x^2)) + Log[x^2]/a^2 - Log[a + b*x^2]/a^2)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{1}{2a(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	35
norman	$-\frac{b\,x^2}{2a^2(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	39
default	$-\frac{b\left(\frac{\ln\left(b\,x^2+a\right)}{b}-\frac{a}{b\left(b\,x^2+a\right)}\right)}{2a^2}+\frac{\ln(x)}{a^2}$	42
parallelrisch	$\frac{2\ln(x)x^2b-\ln(b\,x^2+a)x^2b-b\,x^2+2a\ln(x)-\ln(b\,x^2+a)a}{2a^2(b\,x^2+a)}$	60

input
```
int(1/x/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)}\,dx = -\frac{(bx^2+a)\log\left(bx^2+a\right)-2\left(bx^2+a\right)\log\left(x\right)-a}{2\left(a^2bx^2+a^3\right)}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 +
a^3)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)}\,dx = \frac{1}{2a^2+2abx^2}+\frac{\log\left(x\right)}{a^2}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

input
```
integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)}\,dx = \frac{1}{2\left(abx^2+a^2\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^2}+\frac{\log\left(x^2\right)}{2\,a^2}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)}\,dx = \frac{\log\left(x^2\right)}{2\,a^2} - \frac{\log\left(|bx^2+a|\right)}{2\,a^2} + \frac{bx^2+2\,a}{2\,(bx^2+a)a^2}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
 + a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)}\,dx = \frac{\ln\left(x\right)}{a^2} + \frac{1}{2\,a\,\left(b\,x^2+a\right)} - \frac{\ln\left(b\,x^2+a\right)}{2\,a^2}$$

input
```
int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

output
```
log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)}\,dx = \frac{-\log(b\,x^2+a)\,a-\log(b\,x^2+a)\,b\,x^2+2\log(x)\,a+2\log(x)\,b\,x^2-b\,x^2}{2a^2\left(b\,x^2+a\right)}$$

input
```
int(1/x/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
( - log(a + b*x**2)*a - log(a + b*x**2)*b*x**2 + 2*log(x)*a + 2*log(x)*b*x
**2 - b*x**2)/(2*a**2*(a + b*x**2))
```

3.366 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)}\,dx=-\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}$$

output
```
-1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)}\,dx=-\frac{a\left(\frac{1}{x^2}+\frac{b}{a+bx^2}\right)+4b\log(x)-2b\log\left(a+bx^2\right)}{2a^3}$$

input
```
Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
-1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^2+2abx^2+b^2x^4)} \, dx$$

$$\Big\downarrow \text{1380}$$

$$b^2 \int \frac{1}{b^2x^3 (bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^3 (a+bx^2)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(\frac{2b^2}{a^3 (bx^2+a)}+\frac{b^2}{a^2 (bx^2+a)^2}-\frac{2b}{a^3x^2}+\frac{1}{a^2x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{2b \log (x^2)}{a^3}+\frac{2b \log (a+bx^2)}{a^3}-\frac{b}{a^2 (a+bx^2)}-\frac{1}{a^2x^2}\right)$$

input

```
Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output

```
(-(1/(a^2*x^2)) - b/(a^2*(a + b*x^2)) - (2*b*Log[x^2])/a^3 + (2*b*Log[a +
b*x^2])/a^3)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

method	result	size
norman	$\frac{-\frac{b x^2}{a^2}-\frac{1}{2a}}{x^2(b x^2+a)}+\frac{b\ln\left(b x^2+a\right)}{a^3}-\frac{2b\ln\left(x\right)}{a^3}$	51
risch	$\frac{-\frac{b x^2}{a^2}-\frac{1}{2a}}{x^2(b x^2+a)}-\frac{2b\ln\left(x\right)}{a^3}+\frac{b\ln\left(-b x^2-a\right)}{a^3}$	54
default	$\frac{b^2\left(\frac{2\ln\left(b x^2+a\right)}{b}-\frac{a}{b\left(b x^2+a\right)}\right)}{2a^3}-\frac{1}{2a^2x^2}-\frac{2b\ln\left(x\right)}{a^3}$	55
parallelrisch	$-\frac{4\ln\left(x\right)x^4b^2-2\ln\left(b x^2+a\right)x^4b^2-2b^2x^4+4ab\ln\left(x\right)x^2-2\ln\left(b x^2+a\right)x^2ab+a^2}{2a^3x^2\left(b x^2+a\right)}$	80

input
```
int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
(-b/a^2*x^2-1/2/a)/x^2/(b*x^2+a)+b*ln(b*x^2+a)/a^3-2*b*ln(x)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)}\,dx = -\frac{2\,abx^2+a^2-2\,(b^2x^4+abx^2)\log(bx^2+a)+4\,(b^2x^4+abx^2)\log(x)}{2\,(a^3bx^4+a^4x^2)}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4
+ a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)}\,dx = \frac{-a-2bx^2}{2a^3x^2+2a^2bx^4} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

input
```
integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
(-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/
b + x**2)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)} dx = -\frac{2bx^2 + a}{2(a^2bx^4 + a^3x^2)} + \frac{b\log(bx^2 + a)}{a^3} - \frac{b\log(x^2)}{a^3}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - b*log(x^
2)/a^3
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)} dx = -\frac{b\log(x^2)}{a^3} + \frac{b\log(|bx^2 + a|)}{a^3} - \frac{2bx^2 + a}{2(bx^4 + ax^2)a^2}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
-b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 +
a*x^2)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)} dx = \frac{b\ln(b\,x^2 + a)}{a^3} - \frac{\frac{1}{2\,a} + \frac{b\,x^2}{a^2}}{b\,x^4 + a\,x^2} - \frac{2\,b\ln(x)}{a^3}$$

input
```
int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

output
```
(b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*lo
g(x))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.65

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)} \, dx = \frac{2 \log(b x^2 + a) ab x^2 + 2 \log(b x^2 + a) b^2 x^4 - 4 \log(x) ab x^2 - 4 \log(x) b^2 x^4 - a^2 + 2b^2 x^4}{2a^3 x^2 (b x^2 + a)}$$

input

```
int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output

```
(2*log(a + b*x**2)*a*b*x**2 + 2*log(a + b*x**2)*b**2*x**4 - 4*log(x)*a*b*x
**2 - 4*log(x)*b**2*x**4 - a**2 + 2*b**2*x**4)/(2*a**3*x**2*(a + b*x**2))
```

3.367 $\int \frac{1}{x^5(a^2+2abx^2+b^2x^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)}\,dx=-\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}$$

output
```
-1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*b^2*ln(b*
x^2+a)/a^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)}\,dx=\frac{a\left(-\frac{a}{x^4}+\frac{4b}{x^2}+\frac{2b^2}{a+bx^2}\right)+12b^2\log(x)-6b^2\log\left(a+bx^2\right)}{4a^4}$$

input
```
Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
(a*(-(a/x^4) + (4*b)/x^2 + (2*b^2)/(a + b*x^2)) + 12*b^2*Log[x] - 6*b^2*Lo
g[a + b*x^2])/(4*a^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a^2+2abx^2+b^2x^4)}dx$$

$$\Big\downarrow \text{1380}$$

$$b^2 \int \frac{1}{b^2x^5 (bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^5 (a+bx^2)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^6 (bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(-\frac{3b^3}{a^4 (bx^2+a)}-\frac{b^3}{a^3 (bx^2+a)^2}+\frac{3b^2}{a^4x^2}-\frac{2b}{a^3x^4}+\frac{1}{a^2x^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{3b^2 \log (x^2)}{a^4}-\frac{3b^2 \log (a+bx^2)}{a^4}+\frac{b^2}{a^3 (a+bx^2)}+\frac{2b}{a^3x^2}-\frac{1}{2a^2x^4}\right)$$

input
```
Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
(-1/2*1/(a^2*x^4) + (2*b)/(a^3*x^2) + b^2/(a^3*(a + b*x^2)) + (3*b^2*Log[x
^2])/a^4 - (3*b^2*Log[a + b*x^2])/a^4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{b^{3} \left(\frac{3 \ln \left(b \,x^{2}+a \right)}{b}-\frac{a}{b \left(b \,x^{2}+a \right)}\right)}{2 a^{4}}-\frac{1}{4 a^{2} x^{4}}+\frac{b}{a^{3} x^{2}}+\frac{3 b^{2} \ln \left(x \right)}{a^{4}}$	65
norman	$\frac{-\frac{1}{4 a}+\frac{3 b \,x^{2}}{4 a^{2}}-\frac{3 b^{3} x^{6}}{2 a^{4}}}{x^{4} \left(b \,x^{2}+a \right)}+\frac{3 b^{2} \ln \left(x \right)}{a^{4}}-\frac{3 b^{2} \ln \left(b \,x^{2}+a \right)}{2 a^{4}}$	67
risch	$\frac{\frac{3 b^{2} x^{4}}{2 a^{3}}+\frac{3 b \,x^{2}}{4 a^{2}}-\frac{1}{4 a}}{x^{4} \left(b \,x^{2}+a \right)}+\frac{3 b^{2} \ln \left(x \right)}{a^{4}}-\frac{3 b^{2} \ln \left(b \,x^{2}+a \right)}{2 a^{4}}$	67
parallelrisch	$\frac{12 \ln \left(x \right) x^{6} b^{3}-6 \ln \left(b \,x^{2}+a \right) x^{6} b^{3}-6 b^{3} x^{6}+12 \ln \left(x \right) x^{4} a \,b^{2}-6 \ln \left(b \,x^{2}+a \right) x^{4} a \,b^{2}+3 a^{2} b \,x^{2}-a^{3}}{4 a^{4} x^{4} \left(b \,x^{2}+a \right)}$	95

input
```
int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^3/a^4*(3/b*ln(b*x^2+a)-a/b/(b*x^2+a))-1/4/a^2/x^4+b/a^3/x^2+3*b^2*l
n(x)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)} dx = \frac{6 ab^2x^4 + 3 a^2bx^2 - a^3 - 6 (b^3x^6 + ab^2x^4) \log (bx^2 + a) + 12 (b^3x^6 + ab^2x^4) \log (x)}{4 (a^4bx^6 + a^5x^4)}$$

input
```
integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/4*(6*a*b^2*x^4 + 3*a^2*b*x^2 - a^3 - 6*(b^3*x^6 + a*b^2*x^4)*log(b*x^2 +
 a) + 12*(b^3*x^6 + a*b^2*x^4)*log(x))/(a^4*b*x^6 + a^5*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)} dx = \frac{-a^2 + 3abx^2 + 6b^2x^4}{4a^4x^4 + 4a^3bx^6} + \frac{3b^2 \log (x)}{a^4} - \frac{3b^2 \log \left(\frac{a}{b} + x^2\right)}{2a^4}$$

input
```
integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
(-a**2 + 3*a*b*x**2 + 6*b**2*x**4)/(4*a**4*x**4 + 4*a**3*b*x**6) + 3*b**2*
log(x)/a**4 - 3*b**2*log(a/b + x**2)/(2*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)} \, dx = \frac{6 \, b^2 x^4 + 3 \, abx^2 - a^2}{4 \, (a^3 bx^6 + a^4 x^4)} - \frac{3 \, b^2 \log \left(bx^2 + a\right)}{2 \, a^4} + \frac{3 \, b^2 \log \left(x^2\right)}{2 \, a^4}$$

input
```
integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/4*(6*b^2*x^4 + 3*a*b*x^2 - a^2)/(a^3*b*x^6 + a^4*x^4) - 3/2*b^2*log(b*x^
2 + a)/a^4 + 3/2*b^2*log(x^2)/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)} \, dx = \frac{3 \, b^2 \log \left(x^2\right)}{2 \, a^4} - \frac{3 \, b^2 \log \left(|bx^2 + a|\right)}{2 \, a^4} + \frac{3 \, b^3 x^2 + 4 \, ab^2}{2 \, (bx^2 + a)a^4} - \frac{9 \, b^2 x^4 - 4 \, abx^2 + a^2}{4 \, a^4 x^4}$$

input
```
integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
3/2*b^2*log(x^2)/a^4 - 3/2*b^2*log(abs(b*x^2 + a))/a^4 + 1/2*(3*b^3*x^2 +
4*a*b^2)/((b*x^2 + a)*a^4) - 1/4*(9*b^2*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)} \, dx = \frac{\frac{3 \, b \, x^2}{4 \, a^2} - \frac{1}{4 \, a} + \frac{3 \, b^2 \, x^4}{2 \, a^3}}{b \, x^6 + a \, x^4} - \frac{3 \, b^2 \, \ln \left(b \, x^2 + a\right)}{2 \, a^4} + \frac{3 \, b^2 \, \ln \left(x\right)}{a^4}$$

input
```
int(1/(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

output
```
((3*b*x^2)/(4*a^2) - 1/(4*a) + (3*b^2*x^4)/(2*a^3))/(a*x^4 + b*x^6) - (3*b
^2*log(a + b*x^2))/(2*a^4) + (3*b^2*log(x))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)} \, dx$$
$$= \frac{-6\log(b\,x^2 + a)\,a\,b^2x^4 - 6\log(b\,x^2 + a)\,b^3x^6 + 12\log(x)\,a\,b^2x^4 + 12\log(x)\,b^3x^6 - a^3 + 3a^2b\,x^2 - 6b^3x^6}{4a^4x^4\,(b\,x^2 + a)}$$

input
```
int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
( - 6*log(a + b*x**2)*a*b**2*x**4 - 6*log(a + b*x**2)*b**3*x**6 + 12*log(x
)*a*b**2*x**4 + 12*log(x)*b**3*x**6 - a**3 + 3*a**2*b*x**2 - 6*b**3*x**6)/
(4*a**4*x**4*(a + b*x**2))
```

3.368 $\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx=-\frac{4a^3x}{b^5}+\frac{a^2x^3}{b^4}-\frac{2ax^5}{5b^3}+\frac{x^7}{7b^2}-\frac{a^4x}{2b^5(a+bx^2)}+\frac{9a^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

output
```
-4*a^3*x/b^5+a^2*x^3/b^4-2/5*a*x^5/b^3+1/7*x^7/b^2-1/2*a^4*x/b^5/(b*x^2+a)
+9/2*a^(7/2)*arctan(b^(1/2)*x/a^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx=\frac{x\left(-280a^3+70a^2bx^2-28ab^2x^4+10b^3x^6-\frac{35a^4}{a+bx^2}\right)}{70b^5}+\frac{9a^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

input
```
Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(x*(-280*a^3 + 70*a^2*b*x^2 - 28*a*b^2*x^4 + 10*b^3*x^6 - (35*a^4)/(a + b*
x^2)))/(70*b^5) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}dx$$

$$\Big\downarrow \text{1380}$$

$$b^2\int \frac{x^{10}}{b^2\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{10}}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{9\int \frac{x^8}{bx^2+a}dx}{2b}-\frac{x^9}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{254}$$

$$\frac{9\int \left(\frac{x^6}{b}-\frac{ax^4}{b^2}+\frac{a^2x^2}{b^3}+\frac{a^4}{b^4\left(bx^2+a\right)}-\frac{a^3}{b^4}\right)dx}{2b}-\frac{x^9}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{9\left(\frac{a^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}\right)}{2b}-\frac{x^9}{2b\left(a+bx^2\right)}$$

input
```
Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*x^9/(b*(a + b*x^2)) + (9*(-((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5
)/(5*b^2) + x^7/(7*b) + (a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)))/(2
*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{-\frac{1}{7} b^{3} x^{7}+\frac{2}{5} b^{2} x^{5} a-a^{2} b\,x^{3}+4 a^{3} x}{b^{5}}+\frac{a^{4} \left(-\frac{x}{2 \left(b\,x^{2}+a \right)}+\frac{9 \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{b^{5}}$	76
risch	$\frac{x^{7}}{7 b^{2}}-\frac{2 a\,x^{5}}{5 b^{3}}+\frac{a^{2} x^{3}}{b^{4}}-\frac{4 a^{3} x}{b^{5}}-\frac{a^{4} x}{2 b^{5} \left(b\,x^{2}+a \right)}+\frac{9 \sqrt{-a b}\, a^{3} \ln \left(-\sqrt{-a b}\, x +a \right)}{4 b^{6}}-\frac{9 \sqrt{-a b}\, a^{3} \ln \left(\sqrt{-a b}\, x +a \right)}{4 b^{6}}$	107

input
```
int(x^10/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^5*(-1/7*b^3*x^7+2/5*b^2*x^5*a-a^2*b*x^3+4*a^3*x)+a^4/b^5*(-1/2*x/(b*x
^2+a)+9/2/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.41

$$\int \frac{x^{10}}{a^2 + 2abx^2 + b^2x^4} \, dx = \left[\frac{20\, b^4x^9 - 36\, ab^3x^7 + 84\, a^2b^2x^5 - 420\, a^3bx^3 - 630\, a^4x + 315\, (a^3bx^2 + a^4)\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 + 2\, bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right)}{140\, (b^6x^2 + ab^5)}, \; 1 \right.$$

input
```
integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
[1/140*(20*b^4*x^9 - 36*a*b^3*x^7 + 84*a^2*b^2*x^5 - 420*a^3*b*x^3 - 630*a
^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)))/(b^6*x^2 + a*b^5), 1/70*(10*b^4*x^9 - 18*a*b^3*x^7 + 42*a^2
*b^2*x^5 - 210*a^3*b*x^3 - 315*a^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(a/b)*arc
tan(b*x*sqrt(a/b)/a))/(b^6*x^2 + a*b^5)]
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{x^{10}}{a^2 + 2abx^2 + b^2x^4} \, dx = -\frac{a^4x}{2ab^5 + 2b^6x^2} - \frac{4a^3x}{b^5} + \frac{a^2x^3}{b^4} - \frac{2ax^5}{5b^3} - \frac{9\sqrt{-\frac{a^7}{b^{11}}} \log\left(x - \frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4} + \frac{9\sqrt{-\frac{a^7}{b^{11}}} \log\left(x + \frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4} + \frac{x^7}{7b^2}$$

input
```
integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
-a**4*x/(2*a*b**5 + 2*b**6*x**2) - 4*a**3*x/b**5 + a**2*x**3/b**4 - 2*a*x*
*5/(5*b**3) - 9*sqrt(-a**7/b**11)*log(x - b**5*sqrt(-a**7/b**11)/a**3)/4 +
 9*sqrt(-a**7/b**11)*log(x + b**5*sqrt(-a**7/b**11)/a**3)/4 + x**7/(7*b**2
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx = -\frac{a^4x}{2\,(b^6x^2+ab^5)}+\frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5} + \frac{5\,b^3x^7-14\,ab^2x^5+35\,a^2bx^3-140\,a^3x}{35\,b^5}$$

input
```
integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/2*a^4*x/(b^6*x^2 + a*b^5) + 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^
5) + 1/35*(5*b^3*x^7 - 14*a*b^2*x^5 + 35*a^2*b*x^3 - 140*a^3*x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx = \frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5}-\frac{a^4x}{2\,(bx^2+a)b^5} + \frac{5\,b^{12}x^7-14\,ab^{11}x^5+35\,a^2b^{10}x^3-140\,a^3b^9x}{35\,b^{14}}$$

input
```
integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*a^4*x/((b*x^2 + a)*b^5
) + 1/35*(5*b^12*x^7 - 14*a*b^11*x^5 + 35*a^2*b^10*x^3 - 140*a^3*b^9*x)/b^
14
```

Mupad [B] (verification not implemented)

Time = 19.47 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx = \frac{x^7}{7\,b^2}-\frac{2\,a\,x^5}{5\,b^3}-\frac{4\,a^3\,x}{b^5}+\frac{9\,a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{11/2}} + \frac{a^2\,x^3}{b^4}-\frac{a^4\,x}{2\,\left(b^6\,x^2+a\,b^5\right)}$$

input
```
int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
x^7/(7*b^2) - (2*a*x^5)/(5*b^3) - (4*a^3*x)/b^5 + (9*a^(7/2)*atan((b^(1/2)
*x)/a^(1/2)))/(2*b^(11/2)) + (a^2*x^3)/b^4 - (a^4*x)/(2*(a*b^5 + b^6*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

$$\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx = \frac{315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4+315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^2-315a^4bx-210a^3b^2x^3+42a^2b^3x^5-18a\,b^4 \cdots}{70b^6\left(b\,x^2+a\right)}$$

input
```
int(x^10/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4 + 315*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*x**2 - 315*a**4*b*x - 210*a**3*b*
*2*x**3 + 42*a**2*b**3*x**5 - 18*a*b**4*x**7 + 10*b**5*x**9)/(70*b**6*(a +
 b*x**2))
```

3.369 $\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx=\frac{3a^2x}{b^4}-\frac{2ax^3}{3b^3}+\frac{x^5}{5b^2}+\frac{a^3x}{2b^4(a+bx^2)}-\frac{7a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

output
```
3*a^2*x/b^4-2/3*a*x^3/b^3+1/5*x^5/b^2+1/2*a^3*x/b^4/(b*x^2+a)-7/2*a^(5/2)*
arctan(b^(1/2)*x/a^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx=\frac{x\left(90a^2-20abx^2+6b^2x^4+\frac{15a^3}{a+bx^2}\right)}{30b^4}-\frac{7a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

input
```
Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(x*(90*a^2 - 20*a*b*x^2 + 6*b^2*x^4 + (15*a^3)/(a + b*x^2)))/(30*b^4) - (7
*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{a^2+2abx^2+b^2x^4}dx$$
$$\Big\downarrow \text{ 1380}$$
$$b^2\int \frac{x^8}{b^2\left(bx^2+a\right)^2}dx$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{x^8}{\left(a+bx^2\right)^2}dx$$
$$\Big\downarrow \text{ 252}$$
$$\frac{7\int \frac{x^6}{bx^2+a}dx}{2b}-\frac{x^7}{2b\left(a+bx^2\right)}$$
$$\Big\downarrow \text{ 254}$$
$$\frac{7\int \left(\frac{x^4}{b}-\frac{ax^2}{b^2}-\frac{a^3}{b^3\left(bx^2+a\right)}+\frac{a^2}{b^3}\right)dx}{2b}-\frac{x^7}{2b\left(a+bx^2\right)}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{7\left(-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}\right)}{2b}-\frac{x^7}{2b\left(a+bx^2\right)}$$

input
```
Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*x^7/(b*(a + b*x^2)) + (7*((a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) -
 (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{1}{5}x^5b^2-\frac{2}{3}ab\,x^3+3a^2x}{b^4}-\frac{a^3\left(-\frac{x}{2\left(b\,x^2+a\right)}+\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^4}$	65
risch	$\frac{x^5}{5b^2}-\frac{2a\,x^3}{3b^3}+\frac{3a^2x}{b^4}+\frac{a^3x}{2b^4(b\,x^2+a)}+\frac{7\sqrt{-ab}\,a^2\ln\left(-\sqrt{-ab}\,x-a\right)}{4b^5}-\frac{7\sqrt{-ab}\,a^2\ln\left(\sqrt{-ab}\,x-a\right)}{4b^5}$	101

input
```
int(x^8/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/5*x^5*b^2-2/3*a*b*x^3+3*a^2*x)-a^3/b^4*(-1/2*x/(b*x^2+a)+7/2/(a*b
)^(1/2)*arctan(b/(a*b)^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.44

$$\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx = \left[\frac{12\,b^3x^7-28\,ab^2x^5+140\,a^2bx^3+210\,a^3x+105\,(a^2bx^2+a^3)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{60\,(b^5x^2+ab^4)}, \frac{6\,b^3x^7-14\,ab\right.$$

input
```
integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
[1/60*(12*b^3*x^7 - 28*a*b^2*x^5 + 140*a^2*b*x^3 + 210*a^3*x + 105*(a^2*b*
x^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^
5*x^2 + a*b^4), 1/30*(6*b^3*x^7 - 14*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*x
- 105*(a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^2 + a*b^
4)]
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.59

$$\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx = \frac{a^3x}{2ab^4+2b^5x^2}+\frac{3a^2x}{b^4}-\frac{2ax^3}{3b^3}+\frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x-\frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4} - \frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x+\frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4}+\frac{x^5}{5b^2}$$

input
```
integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
a**3*x/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*x/b**4 - 2*a*x**3/(3*b**3) + 7*sq
rt(-a**5/b**9)*log(x - b**4*sqrt(-a**5/b**9)/a**2)/4 - 7*sqrt(-a**5/b**9)*
log(x + b**4*sqrt(-a**5/b**9)/a**2)/4 + x**5/(5*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx = \frac{a^3x}{2\,(b^5x^2+ab^4)} - \frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4} + \frac{3\,b^2x^5-10\,abx^3+45\,a^2x}{15\,b^4}$$

input
```
integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2*a^3*x/(b^5*x^2 + a*b^4) - 7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4
) + 1/15*(3*b^2*x^5 - 10*a*b*x^3 + 45*a^2*x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx = -\frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4} + \frac{a^3x}{2\,(bx^2+a)b^4} + \frac{3\,b^8x^5-10\,ab^7x^3+45\,a^2b^6x}{15\,b^{10}}$$

input
```
integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
-7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*a^3*x/((b*x^2 + a)*b^
4) + 1/15*(3*b^8*x^5 - 10*a*b^7*x^3 + 45*a^2*b^6*x)/b^10
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx=\frac{x^5}{5\,b^2}-\frac{2\,a\,x^3}{3\,b^3}+\frac{3\,a^2\,x}{b^4}-\frac{7\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{9/2}}+\frac{a^3\,x}{2\,\left(b^5\,x^2+a\,b^4\right)}$$

input
```
int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
x^5/(5*b^2) - (2*a*x^3)/(3*b^3) + (3*a^2*x)/b^4 - (7*a^(5/2)*atan((b^(1/2)
*x)/a^(1/2)))/(2*b^(9/2)) + (a^3*x)/(2*(a*b^4 + b^5*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx$$
$$=\frac{-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^2+105a^3bx+70a^2b^2x^3-14a\,b^3x^5+6b^4x^7}{30b^5\,(b\,x^2+a)}$$

input
```
int(x^8/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
( - 105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3 - 105*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**2 + 105*a**3*b*x + 70*a**2*
b**2*x**3 - 14*a*b**3*x**5 + 6*b**4*x**7)/(30*b**5*(a + b*x**2))
```

3.370 $\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx=-\frac{2ax}{b^3}+\frac{x^3}{3b^2}-\frac{a^2x}{2b^3\left(a+bx^2\right)}+\frac{5a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

output
```
-2*a*x/b^3+1/3*x^3/b^2-1/2*a^2*x/b^3/(b*x^2+a)+5/2*a^(3/2)*arctan(b^(1/2)*
x/a^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx=\frac{x\left(-12a+2bx^2-\frac{3a^2}{a+bx^2}\right)}{6b^3}+\frac{5a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

input
```
Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(x*(-12*a + 2*b*x^2 - (3*a^2)/(a + b*x^2)))/(6*b^3) + (5*a^(3/2)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{a^2+2abx^2+b^2x^4}dx$$

$$\Big\downarrow \text{1380}$$

$$b^2\int \frac{x^6}{b^2\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^6}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5\int \frac{x^4}{bx^2+a}dx}{2b}-\frac{x^5}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{254}$$

$$\frac{5\int \left(\frac{a^2}{b^2(bx^2+a)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx}{2b}-\frac{x^5}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}\right)}{2b}-\frac{x^5}{2b\left(a+bx^2\right)}$$

input
```
Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*x^5/(b*(a + b*x^2)) + (5*(-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{-\frac{1}{3}b\,x^{3}+2ax}{b^{3}}+\frac{a^{2}\left(-\frac{x}{2\left(b\,x^{2}+a\right)}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	54
risch	$\frac{x^{3}}{3b^{2}}-\frac{2ax}{b^{3}}-\frac{a^{2}x}{2b^{3}\left(b\,x^{2}+a\right)}+\frac{5\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x+a\right)}{4b^{4}}-\frac{5\sqrt{-ab}\,a\ln\left(\sqrt{-ab}\,x+a\right)}{4b^{4}}$	82

input
```
int(x^6/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*(-1/3*b*x^3+2*a*x)+a^2/b^3*(-1/2*x/(b*x^2+a)+5/2/(a*b)^(1/2)*arctan
(b/(a*b)^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.52

$$\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx = \left[\frac{4\,b^2x^5-20\,abx^3-30\,a^2x+15\,(abx^2+a^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{12\,(b^4x^2+ab^3)}, \frac{2\,b^2x^5-10\,abx^3-15\,a^2x+15}{6\,(b^4x^2}\right.$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
[1/12*(4*b^2*x^5 - 20*a*b*x^3 - 30*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(-a/b)*l
og((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^4*x^2 + a*b^3), 1/6*(2*
b^2*x^5 - 10*a*b*x^3 - 15*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*
sqrt(a/b)/a))/(b^4*x^2 + a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx = -\frac{a^2x}{2ab^3+2b^4x^2} - \frac{2ax}{b^3} - \frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x-\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4} + \frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x+\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4} + \frac{x^3}{3b^2}$$

input
```
integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
-a**2*x/(2*a*b**3 + 2*b**4*x**2) - 2*a*x/b**3 - 5*sqrt(-a**3/b**7)*log(x -
 b**3*sqrt(-a**3/b**7)/a)/4 + 5*sqrt(-a**3/b**7)*log(x + b**3*sqrt(-a**3/b
**7)/a)/4 + x**3/(3*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx=-\frac{a^2x}{2\,(b^4x^2+ab^3)}+\frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3}+\frac{bx^3-6\,ax}{3\,b^3}$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/2*a^2*x/(b^4*x^2 + a*b^3) + 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^
3) + 1/3*(b*x^3 - 6*a*x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx=\frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3}-\frac{a^2x}{2\,(bx^2+a)b^3}+\frac{b^4x^3-6\,ab^3x}{3\,b^6}$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*a^2*x/((b*x^2 + a)*b^3
) + 1/3*(b^4*x^3 - 6*a*b^3*x)/b^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx = \frac{x^3}{3\,b^2}+\frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{7/2}}-\frac{a^2\,x}{2\,\left(b^4\,x^2+a\,b^3\right)}-\frac{2\,a\,x}{b^3}$$

input
```
int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
x^3/(3*b^2) + (5*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(7/2)) - (a^2*x)/
(2*(a*b^3 + b^4*x^2)) - (2*a*x)/b^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

$$\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2+15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2-15a^2bx-10a\,b^2x^3+2b^3x^5}{6b^4\left(b\,x^2+a\right)}$$

input
```
int(x^6/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 + 15*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 - 15*a**2*b*x - 10*a*b**2*x**3 +
2*b**3*x**5)/(6*b**4*(a + b*x**2))
```

3.371 $\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx=\frac{x}{b^2}+\frac{ax}{2b^2\left(a+bx^2\right)}-\frac{3\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}$$

output
```
x/b^2+1/2*a*x/b^2/(b*x^2+a)-3/2*a^(1/2)*arctan(b^(1/2)*x/a^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx=\frac{x}{b^2}+\frac{ax}{2b^2\left(a+bx^2\right)}-\frac{3\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}$$

input
```
Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
x/b^2 + (a*x)/(2*b^2*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^2\int \frac{x^4}{b^2\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^4}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{3\int \frac{x^2}{bx^2+a}dx}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{262}$$

$$\frac{3\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}\right)}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}$$

input
```
Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*x^3/(b*(a + b*x^2)) + (3*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/b^(3/2)))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x}{b^{2}}-\frac{a\left(-\frac{x}{2\left(b\,x^{2}+a\right)}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{2}}$	42
risch	$\frac{x}{b^{2}}+\frac{ax}{2b^{2}\left(b\,x^{2}+a\right)}+\frac{3\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{4b^{3}}-\frac{3\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{4b^{3}}$	72

input
```
int(x^4/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
x/b^2-a/b^2*(-1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.67

$$\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx = \left[\frac{4\,bx^3+3\,(bx^2+a)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,ax}{4\,(b^3x^2+ab^2)}, \frac{2\,bx^3-3\,(bx^2+a)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\,ax}{2\,(b^3x^2+ab^2)}\right]$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
[1/4*(4*b*x^3 + 3*(b*x^2 + a)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a
)/(b*x^2 + a)) + 6*a*x)/(b^3*x^2 + a*b^2), 1/2*(2*b*x^3 - 3*(b*x^2 + a)*sq
rt(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*a*x)/(b^3*x^2 + a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx = \frac{ax}{2ab^2+2b^3x^2} + \frac{3\sqrt{-\frac{a}{b^5}}\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4} - \frac{3\sqrt{-\frac{a}{b^5}}\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4} + \frac{x}{b^2}$$

input
```
integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
a*x/(2*a*b**2 + 2*b**3*x**2) + 3*sqrt(-a/b**5)*log(-b**2*sqrt(-a/b**5) + x
)/4 - 3*sqrt(-a/b**5)*log(b**2*sqrt(-a/b**5) + x)/4 + x/b**2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx = \frac{ax}{2\,(b^3x^2+ab^2)} - \frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{x}{b^2}$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2*a*x/(b^3*x^2 + a*b^2) - 3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) +
x/b^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx = -\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{ax}{2\,(bx^2+a)b^2} + \frac{x}{b^2}$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
-3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*a*x/((b*x^2 + a)*b^2) +
 x/b^2
```

Mupad [B] (verification not implemented)

Time = 18.86 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx = \frac{x}{b^2} + \frac{a\,x}{2\,\left(b^3\,x^2+a\,b^2\right)} - \frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}$$

input
```
int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
x/b^2 + (a*x)/(2*(a*b^2 + b^3*x^2)) - (3*a^(1/2)*atan((b^(1/2)*x)/a^(1/2))
)/(2*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int \frac{x^4}{a^2+2abx^2+b^2x^4}\,dx = \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^2+3abx+2b^2x^3}{2b^3\left(b\,x^2+a\right)}$$

input
```
int(x^4/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a - 3*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*b*x**2 + 3*a*b*x + 2*b**2*x**3)/(2*b**3*(a +
 b*x**2))
```

3.372 $\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 45

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx = -\frac{x}{2b\left(a+bx^2\right)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$

output
```
-1/2*x/b/(b*x^2+a)+1/2*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx = -\frac{x}{2b\left(a+bx^2\right)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}dx$$

$$\Big\downarrow \text{1380}$$

$$b^2\int \frac{x^2}{b^2(bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{\int \frac{1}{bx^2+a}dx}{2b}-\frac{x}{2b(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b(a+bx^2)}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{x}{2b\left(b\,x^{2}+a\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}$	36
risch	$-\frac{x}{2b\left(b\,x^{2}+a\right)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,b}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,b}$	62

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx = \left[-\frac{2\,abx+(bx^2+a)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\,(ab^3x^2+a^2b^2)},\; -\frac{abx-(bx^2+a)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,(ab^3x^2+a^2b^2)}\right]$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b*x + (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*x - (b*x^2 + a)*sqrt(a*b)*ar
ctan(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(36) = 72.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx = -\frac{x}{2ab+2b^2x^2} - \frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}$$

input
```
integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
-x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3)) +
x)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx = -\frac{x}{2\left(b^2x^2+ab\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b}$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/2*x/(b^2*x^2 + a*b) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b}-\frac{x}{2\left(bx^2+a\right)b}$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*x/((b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2\,b\,\left(b\,x^2+a\right)}$$

input
```
int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
atan((b^(1/2)*x)/a^(1/2))/(2*a^(1/2)*b^(3/2)) - x/(2*b*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^2-abx}{2a\,b^2\left(b\,x^2+a\right)}$$

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*b*x**2 - a*b*x)/(2*a*b**2*(a + b*x**2))
```

3.373 $\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx=\frac{x}{2a(a+bx^2)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

output
```
1/2*x/a/(b*x^2+a)+1/2*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx=\frac{x}{2a(a+bx^2)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
```

output
```
x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1379, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+2abx^2+b^2x^4}dx$$

$$\Big\downarrow \text{1379}$$

$$b^2\int \frac{1}{\left(b^2x^2+ab\right)^2}dx$$

$$\Big\downarrow \text{215}$$

$$b^2\left(\frac{\int \frac{1}{b^2x^2+ab}dx}{2ab}+\frac{x}{2ab^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{218}$$

$$b^2\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x}{2ab^2\left(a+bx^2\right)}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
```

output
```
b^2*(x/(2*a*b^2*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*b^(5
/2)))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x}{2a(b\,x^{2}+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	36
risch	$\frac{x}{2a(b\,x^{2}+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}$	62

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx = \left[\frac{2\,abx-(bx^2+a)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\,(a^2b^2x^2+a^3b)}, \frac{abx+(bx^2+a)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,(a^2b^2x^2+a^3b)}\right]$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(36) = 72.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx = \frac{x}{2a^2+2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

input
```
integrate(1/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) +
x)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{a^2 + 2abx^2 + b^2x^4}\,dx = \frac{x}{2\left(abx^2 + a^2\right)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{a^2 + 2abx^2 + b^2x^4}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{x}{2\left(bx^2 + a\right)a}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 18.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{1}{a^2 + 2abx^2 + b^2x^4}\,dx = \frac{x}{2\,a\,\left(b\,x^2 + a\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

input
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \frac{1}{a^2 + 2abx^2 + b^2x^4} \, dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b\,x^2 + abx}{2a^2b\,(b\,x^2 + a)}$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*b*x**2 + a*b*x)/(2*a**2*b*(a + b*x**2))
```

3.374 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)}\,dx = -\frac{1}{a^2x}-\frac{bx}{2a^2(a+bx^2)}-\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-1/a^2/x-1/2*b*x/a^2/(b*x^2+a)-3/2*b^(1/2)*arctan(b^(1/2)*x/a^(1/2))/a^(5/
2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)}\,dx = -\frac{1}{a^2x}-\frac{bx}{2a^2(a+bx^2)}-\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(5/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)} \, dx$$
$$\Big\downarrow \text{ 1380}$$
$$b^2 \int \frac{1}{b^2x^2 (bx^2 + a)^2} dx$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{1}{x^2 (a + bx^2)^2} dx$$
$$\Big\downarrow \text{ 253}$$
$$\frac{3 \int \frac{1}{x^2(bx^2+a)} dx}{2a} + \frac{1}{2ax (a + bx^2)}$$
$$\Big\downarrow \text{ 264}$$
$$\frac{3 \left(-\frac{b \int \frac{1}{bx^2+a} dx}{a} - \frac{1}{ax} \right)}{2a} + \frac{1}{2ax (a + bx^2)}$$
$$\Big\downarrow \text{ 218}$$
$$\frac{3 \left(-\frac{\sqrt{b} \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax} \right)}{2a} + \frac{1}{2ax (a + bx^2)}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
1/(2*a*x*(a + b*x^2)) + (3*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{b\left(\frac{x}{2b\,x^{2}+2a}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}-\frac{1}{a^{2}x}$	45
risch	$\frac{-\frac{3b\,x^{2}}{2a^{2}}-\frac{1}{a}}{x\left(b\,x^{2}+a\right)}+\frac{3\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{4a^{3}}-\frac{3\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{4a^{3}}$	78

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-b/a^2*(1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x))-1/a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.52

$$\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)}\,dx = \left[-\frac{6\,bx^2-3\,(bx^3+ax)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+4\,a}{4\,(a^2bx^3+a^3x)},\ -\frac{3\,bx^2+3\,(bx^3+ax)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,a}{2\,(a^2bx^3+a^3x)}\right]$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
[-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
- a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a
*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)}\,dx = \frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4} - \frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4} + \frac{-2a-3bx^2}{2a^3x+2a^2bx^3}$$

input
```
integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**
3*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)}\, dx = -\frac{3\, bx^2 + 2\, a}{2\, (a^2bx^3 + a^3x)} - \frac{3\, b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\, \sqrt{ab}a^2}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)}\, dx = -\frac{3\, b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\, \sqrt{ab}a^2} - \frac{3\, bx^2 + 2\, a}{2\, (bx^3 + ax)a^2}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
-3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
 + a*x)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)} \, dx = -\frac{\frac{1}{a} + \frac{3\,b\,x^2}{2\,a^2}}{b\,x^3 + a\,x} - \frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

input
```
int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

output
```
- (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^
(1/2)))/(2*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)} \, dx = \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) ax - 3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b\,x^3 - 2a^2 - 3ab\,x^2}{2a^3x\,(b\,x^2 + a)}$$

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*x - 3*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*x**3 - 2*a**2 - 3*a*b*x**2)/(2*a**3*x*(a
 + b*x**2))
```

3.375 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)}\,dx=-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}+\frac{b^2x}{2a^3\left(a+bx^2\right)}+\frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

output
```
-1/3/a^2/x^3+2*b/a^3/x+1/2*b^2*x/a^3/(b*x^2+a)+5/2*b^(3/2)*arctan(b^(1/2)*
x/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)}\,dx=-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}+\frac{b^2x}{2a^3\left(a+bx^2\right)}+\frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

input
```
Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
-1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1380, 27, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a^2+2abx^2+b^2x^4)}dx$$

$$\Big\downarrow \text{1380}$$

$$b^2 \int \frac{1}{b^2x^4 (bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^4 (a+bx^2)^2}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5\int \frac{1}{x^4(bx^2+a)}dx}{2a}+\frac{1}{2ax^3(a+bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{5\left(-\frac{b\int \frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3(a+bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{5\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{5\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

input
```
Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
1/(2*a*x^3*(a + b*x^2)) + (5*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{b^2\left(\frac{x}{2b\,x^2+2a}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}$	55
risch	$\frac{\frac{5b^2x^4}{2a^3}+\frac{5b\,x^2}{3a^2}-\frac{1}{3a}}{x^3(b\,x^2+a)}+\frac{5\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{4a^4}-\frac{5\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{4a^4}$	91

input
```
int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
b^2/a^3*(1/2*x/(b*x^2+a)+5/2/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x))-1/3/a^2/
x^3+2*b/a^3/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.57

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)}\,dx$$
$$=\left[\frac{30\,b^2x^4+20\,abx^2+15\left(b^2x^5+abx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-4\,a^2}{12\left(a^3bx^5+a^4x^3\right)},\frac{15\,b^2x^4+10\,abx^2+15\left(b^2x^5+\right.}{6\left(a^3bx^{\cdot}\right.}\right.$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
[1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1
/6*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sq
rt(b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)} dx = -\frac{5\sqrt{-\frac{b^3}{a^7}} \log \left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x\right)}{4} + \frac{5\sqrt{-\frac{b^3}{a^7}} \log \left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x\right)}{4} + \frac{-2a^2 + 10abx^2 + 15b^2x^4}{6a^4x^3 + 6a^3bx^5}$$

input
```
integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
-5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/
a**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*
b**2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)} dx = \frac{15 b^2x^4 + 10 abx^2 - 2 a^2}{6 (a^3bx^5 + a^4x^3)} + \frac{5 b^2 \arctan \left(\frac{bx}{\sqrt{ab}}\right)}{2 \sqrt{ab}a^3}$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arct
an(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)} dx = \frac{5 b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \sqrt{ab} a^3} + \frac{b^2x}{2 (bx^2 + a)a^3} + \frac{6 bx^2 - a}{3 a^3x^3}$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3
) + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 18.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)} dx = \frac{\frac{5 b x^2}{3 a^2} - \frac{1}{3 a} + \frac{5 b^2 x^4}{2 a^3}}{b x^5 + a x^3} + \frac{5 b^{3/2} \operatorname{atan}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{2 a^{7/2}}$$

input
```
int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

output
```
((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b
^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)} dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) ab\,x^3 + 15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b^2x^5 - 2a^3 + 10a^2b\,x^2 + 15a\,b^2x^4}{6a^4x^3\,(b\,x^2 + a)}$$

input
```
int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**3 + 15*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*x**5 - 2*a**3 + 10*a**2*b*x**2 +
15*a*b**2*x**4)/(6*a**4*x**3*(a + b*x**2))
```

3.376 $\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)}\,dx=-\frac{1}{5a^2x^5}+\frac{2b}{3a^3x^3}-\frac{3b^2}{a^4x}-\frac{b^3x}{2a^4\left(a+bx^2\right)}-\frac{7b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}$$

output
```
-1/5/a^2/x^5+2/3*b/a^3/x^3-3*b^2/a^4/x-1/2*b^3*x/a^4/(b*x^2+a)-7/2*b^(5/2)
*arctan(b^(1/2)*x/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)}\,dx=-\frac{1}{5a^2x^5}+\frac{2b}{3a^3x^3}-\frac{3b^2}{a^4x}-\frac{b^3x}{2a^4\left(a+bx^2\right)}-\frac{7b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}$$

input
```
Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
-1/5*1/(a^2*x^5) + (2*b)/(3*a^3*x^3) - (3*b^2)/(a^4*x) - (b^3*x)/(2*a^4*(a
 + b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1380, 27, 253, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)} dx$$

$$\Big\downarrow \text{1380}$$

$$b^2 \int \frac{1}{b^2x^6 (bx^2 + a)^2} dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^6 (a + bx^2)^2} dx$$

$$\Big\downarrow \text{253}$$

$$\frac{7 \int \frac{1}{x^6(bx^2+a)} dx}{2a} + \frac{1}{2ax^5 (a + bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{7 \left(-\frac{b \int \frac{1}{x^4 \left(bx^2+a\right)} dx}{a} - \frac{1}{5ax^5}\right)}{2a} + \frac{1}{2ax^5 (a + bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{7 \left(-\frac{b \left(-\frac{b \int \frac{1}{x^2 \left(bx^2+a\right)} dx}{a} - \frac{1}{3ax^3}\right)}{a} - \frac{1}{5ax^5}\right)}{2a} + \frac{1}{2ax^5 (a + bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

input
```
Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
1/(2*a*x^5*(a + b*x^2)) + (7*(-1/5*1/(a*x^5) - (b*(-1/3*1/(a*x^3) - (b*(-(
1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/a))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{b^3\left(\frac{x}{2b\,x^2+2a}+\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}-\frac{1}{5a^2x^5}-\frac{3b^2}{a^4x}+\frac{2b}{3a^3x^3}$	67
risch	$\frac{-\frac{7b^3x^6}{2a^4}-\frac{7b^2x^4}{3a^3}+\frac{7b\,x^2}{15a^2}-\frac{1}{5a}}{x^5(b\,x^2+a)}+\frac{7\sqrt{-ab}\,b^2\ln\left(-bx+\sqrt{-ab}\right)}{4a^5}-\frac{7\sqrt{-ab}\,b^2\ln\left(-bx-\sqrt{-ab}\right)}{4a^5}$	106

input
```
int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-b^3/a^4*(1/2*x/(b*x^2+a)+7/2/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x))-1/5/a^2
/x^5-3*b^2/a^4/x+2/3*b/a^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.48

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)}\,dx$$
$$= \left[-\frac{210\,b^3x^6 + 140\,ab^2x^4 - 28\,a^2bx^2 + 12\,a^3 - 105\,(b^3x^7 + ab^2x^5)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right)}{60\,(a^4bx^7 + a^5x^5)}, -\frac{105\,b^3x^6 + 70\,ab^2x^4 - 14\,a^2bx^2 + 6\,a^3 + 105\,(b^3x^7 + ab^2x^5)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{30\,(a^4bx^7 + a^5x^5)} \right]$$

input
```
integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
[-1/60*(210*b^3*x^6 + 140*a*b^2*x^4 - 28*a^2*b*x^2 + 12*a^3 - 105*(b^3*x^7
 + a*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/
(a^4*b*x^7 + a^5*x^5), -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 +
6*a^3 + 105*(b^3*x^7 + a*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b*x^
7 + a^5*x^5)]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)}\,dx = \frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3} + x\right)}{4} - \frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3} + x\right)}{4} + \frac{-6a^3 + 14a^2bx^2 - 70ab^2x^4 - 105b^3x^6}{30a^5x^5 + 30a^4bx^7}$$

input
```
integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
7*sqrt(-b**5/a**9)*log(-a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - 7*sqrt(-b**5/a
**9)*log(a**5*sqrt(-b**5/a**9)/b**3 + x)/4 + (-6*a**3 + 14*a**2*b*x**2 - 7
0*a*b**2*x**4 - 105*b**3*x**6)/(30*a**5*x**5 + 30*a**4*b*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)}\,dx = -\frac{105\,b^3x^6 + 70\,ab^2x^4 - 14\,a^2bx^2 + 6\,a^3}{30\,(a^4bx^7 + a^5x^5)} - \frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4}$$

input
```
integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a^3)/(a^4*b*x^7 + a^5
*x^5) - 7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)}\,dx = -\frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4} - \frac{b^3x}{2\,(bx^2 + a)a^4} - \frac{45\,b^2x^4 - 10\,abx^2 + 3\,a^2}{15\,a^4x^5}$$

input
```
integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
-7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/2*b^3*x/((b*x^2 + a)*a^
4) - 1/15*(45*b^2*x^4 - 10*a*b*x^2 + 3*a^2)/(a^4*x^5)
```

Mupad [B] (verification not implemented)

Time = 18.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^6\,(a^2+2abx^2+b^2x^4)}\,dx = -\frac{\frac{1}{5\,a}-\frac{7\,b\,x^2}{15\,a^2}+\frac{7\,b^2\,x^4}{3\,a^3}+\frac{7\,b^3\,x^6}{2\,a^4}}{b\,x^7+a\,x^5}-\frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{9/2}}$$

input
```
int(1/(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

output
```
- (1/(5*a) - (7*b*x^2)/(15*a^2) + (7*b^2*x^4)/(3*a^3) + (7*b^3*x^6)/(2*a^4
))/(a*x^5 + b*x^7) - (7*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^6\,(a^2+2abx^2+b^2x^4)}\,dx = \frac{-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^5-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^3x^7-6a^4+14a^3b\,x^2-70a^2b^2x^4-105a\,b^3x^6}{30a^5x^5\,(b\,x^2+a)}$$

input
```
int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
( - 105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*x**5 - 105*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*x**7 - 6*a**4 + 14*a**3*b
*x**2 - 70*a**2*b**2*x**4 - 105*a*b**3*x**6)/(30*a**5*x**5*(a + b*x**2))
```

3.377 $\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=-\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}+\frac{a^5}{6b^6\left(a+bx^2\right)^3}-\frac{5a^4}{4b^6\left(a+bx^2\right)^2}+\frac{5a^3}{b^6\left(a+bx^2\right)}+\frac{5a^2\log\left(a+bx^2\right)}{b^6}$$

output
```
-2*a*x^2/b^5+1/4*x^4/b^4+1/6*a^5/b^6/(b*x^2+a)^3-5/4*a^4/b^6/(b*x^2+a)^2+5
*a^3/b^6/(b*x^2+a)+5*a^2*ln(b*x^2+a)/b^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\frac{-24abx^2+3b^2x^4+\frac{2a^5}{\left(a+bx^2\right)^3}-\frac{15a^4}{\left(a+bx^2\right)^2}+\frac{60a^3}{a+bx^2}+60a^2\log\left(a+bx^2\right)}{12b^6}$$

input
```
Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(-24*a*b*x^2 + 3*b^2*x^4 + (2*a^5)/(a + b*x^2)^3 - (15*a^4)/(a + b*x^2)^2
+ (60*a^3)/(a + b*x^2) + 60*a^2*Log[a + b*x^2])/(12*b^6)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \text{1380}$$

$$b^4\int \frac{x^{11}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{11}}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^{10}}{(bx^2+a)^4}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(-\frac{a^5}{b^5(bx^2+a)^4}+\frac{5a^4}{b^5(bx^2+a)^3}-\frac{10a^3}{b^5(bx^2+a)^2}+\frac{10a^2}{b^5(bx^2+a)}-\frac{4a}{b^5}+\frac{x^2}{b^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^5}{3b^6(a+bx^2)^3}-\frac{5a^4}{2b^6(a+bx^2)^2}+\frac{10a^3}{b^6(a+bx^2)}+\frac{10a^2\log(a+bx^2)}{b^6}-\frac{4ax^2}{b^5}+\frac{x^4}{2b^4}\right)$$

input
```
Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
((-4*a*x^2)/b^5 + x^4/(2*b^4) + a^5/(3*b^6*(a + b*x^2)^3) - (5*a^4)/(2*b^6
*(a + b*x^2)^2) + (10*a^3)/(b^6*(a + b*x^2)) + (10*a^2*Log[a + b*x^2])/b^6
)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

method	result
norman	$\frac{\frac{x^{10}}{4b}-\frac{5a\,x^{8}}{4b^{2}}+\frac{55a^{5}}{6b^{6}}+\frac{15a^{3}x^{4}}{b^{4}}+\frac{45a^{4}x^{2}}{2b^{5}}}{\left(b\,x^{2}+a\right)^{3}}+\frac{5a^{2}\ln\left(b\,x^{2}+a\right)}{b^{6}}$
default	$\frac{\left(-b\,x^{2}+4a\right)^{2}}{4b^{6}}+\frac{a^{2}\left(\frac{10\ln\left(b\,x^{2}+a\right)}{b}+\frac{a^{3}}{3b\left(b\,x^{2}+a\right)^{3}}+\frac{10a}{b\left(b\,x^{2}+a\right)}-\frac{5a^{2}}{2b\left(b\,x^{2}+a\right)^{2}}\right)}{2b^{5}}$
risch	$\frac{x^{4}}{4b^{4}}-\frac{2a\,x^{2}}{b^{5}}+\frac{4a^{2}}{b^{6}}+\frac{5a^{3}b\,x^{4}+\frac{35a^{4}x^{2}}{4}+\frac{47a^{5}}{12b}}{b^{5}\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}+\frac{5a^{2}\ln\left(b\,x^{2}+a\right)}{b^{6}}$
parallelrisch	$\frac{3x^{10}b^{5}-15a\,x^{8}b^{4}+60\ln\left(b\,x^{2}+a\right)x^{6}a^{2}b^{3}+180\ln\left(b\,x^{2}+a\right)x^{4}a^{3}b^{2}+180a^{3}x^{4}b^{2}+180\ln\left(b\,x^{2}+a\right)x^{2}a^{4}b+270x^{2}a^{4}b+60\ln\left(b\,x^{2}+\right.}{12b^{6}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\left(b\,x^{2}+a\right)}$

input
```
int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/b*x^10-5/4*a/b^2*x^8+55/6*a^5/b^6+15*a^3/b^4*x^4+45/2*a^4/b^5*x^2)/(b
*x^2+a)^3+5*a^2*ln(b*x^2+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.51

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{3\,b^5x^{10}-15\,ab^4x^8-63\,a^2b^3x^6-9\,a^3b^2x^4+81\,a^4bx^2+47\,a^5+60\,(a^2b^3x^6+3\,a^3b^2x^4+3\,a^4bx^2+a^5)\log}{12\,(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)}$$

input
```
integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^5*x^10 - 15*a*b^4*x^8 - 63*a^2*b^3*x^6 - 9*a^3*b^2*x^4 + 81*a^4*
b*x^2 + 47*a^5 + 60*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*b*x^2 + a^5)*log(
b*x^2 + a))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{5a^2\log(a+bx^2)}{b^6} - \frac{2ax^2}{b^5} + \frac{47a^5+105a^4bx^2+60a^3b^2x^4}{12a^3b^6+36a^2b^7x^2+36ab^8x^4+12b^9x^6} + \frac{x^4}{4b^4}$$

input
```
integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
5*a**2*log(a + b*x**2)/b**6 - 2*a*x**2/b**5 + (47*a**5 + 105*a**4*b*x**2 +
 60*a**3*b**2*x**4)/(12*a**3*b**6 + 36*a**2*b**7*x**2 + 36*a*b**8*x**4 + 1
2*b**9*x**6) + x**4/(4*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{60\,a^3b^2x^4+105\,a^4bx^2+47\,a^5}{12\,(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)} + \frac{5\,a^2\log(bx^2+a)}{b^6} + \frac{bx^4-8\,ax^2}{4\,b^5}$$

input
```
integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/12*(60*a^3*b^2*x^4 + 105*a^4*b*x^2 + 47*a^5)/(b^9*x^6 + 3*a*b^8*x^4 + 3*
a^2*b^7*x^2 + a^3*b^6) + 5*a^2*log(b*x^2 + a)/b^6 + 1/4*(b*x^4 - 8*a*x^2)/
b^5
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{5\,a^2\log\left(|bx^2+a|\right)}{b^6} + \frac{b^4x^4-8\,ab^3x^2}{4\,b^8} - \frac{110\,a^2b^3x^6+270\,a^3b^2x^4+225\,a^4bx^2+63\,a^5}{12\,(bx^2+a)^3b^6}$$

input
```
integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
5*a^2*log(abs(b*x^2 + a))/b^6 + 1/4*(b^4*x^4 - 8*a*b^3*x^2)/b^8 - 1/12*(11
0*a^2*b^3*x^6 + 270*a^3*b^2*x^4 + 225*a^4*b*x^2 + 63*a^5)/((b*x^2 + a)^3*b
^6)
```

Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{47\,a^5}{12\,b}+\frac{35\,a^4\,x^2}{4}+5\,a^3\,b\,x^4}{a^3\,b^5+3\,a^2\,b^6\,x^2+3\,a\,b^7\,x^4+b^8\,x^6} + \frac{x^4}{4\,b^4} - \frac{2\,a\,x^2}{b^5} + \frac{5\,a^2\,\ln\left(b\,x^2+a\right)}{b^6}$$

input
```
int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
((47*a^5)/(12*b) + (35*a^4*x^2)/4 + 5*a^3*b*x^4)/(a^3*b^5 + b^8*x^6 + 3*a*
b^7*x^4 + 3*a^2*b^6*x^2) + x^4/(4*b^4) - (2*a*x^2)/b^5 + (5*a^2*log(a + b*
x^2))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.62

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^2}\,dx$$
$$= \frac{60\log(b\,x^2+a)\,a^5+180\log(b\,x^2+a)\,a^4b\,x^2+180\log(b\,x^2+a)\,a^3b^2x^4+60\log(b\,x^2+a)\,a^2b^3x^6+50a^5}{12b^6\,(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)}$$

input
```
int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(60*log(a + b*x**2)*a**5 + 180*log(a + b*x**2)*a**4*b*x**2 + 180*log(a + b
*x**2)*a**3*b**2*x**4 + 60*log(a + b*x**2)*a**2*b**3*x**6 + 50*a**5 + 90*a
**4*b*x**2 - 60*a**2*b**3*x**6 - 15*a*b**4*x**8 + 3*b**5*x**10)/(12*b**6*(
a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.378 $\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^2} \, dx = \frac{x^2}{2b^4} - \frac{a^4}{6b^5(a+bx^2)^3} + \frac{a^3}{b^5(a+bx^2)^2} - \frac{3a^2}{b^5(a+bx^2)} - \frac{2a\log(a+bx^2)}{b^5}$$

output
```
1/2*x^2/b^4-1/6*a^4/b^5/(b*x^2+a)^3+a^3/b^5/(b*x^2+a)^2-3*a^2/b^5/(b*x^2+a
)-2*a*ln(b*x^2+a)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^2} \, dx = -\frac{-3bx^2 + \frac{a^2(13a^2+30abx^2+18b^2x^4)}{(a+bx^2)^3} + 12a\log(a+bx^2)}{6b^5}$$

input
```
Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*(-3*b*x^2 + (a^2*(13*a^2 + 30*a*b*x^2 + 18*b^2*x^4))/(a + b*x^2)^3 +
12*a*Log[a + b*x^2])/b^5
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \text{1380}$$

$$b^4\int \frac{x^9}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^9}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^2+a)^4}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^4}{b^4(bx^2+a)^4}-\frac{4a^3}{b^4(bx^2+a)^3}+\frac{6a^2}{b^4(bx^2+a)^2}-\frac{4a}{b^4(bx^2+a)}+\frac{1}{b^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^4}{3b^5(a+bx^2)^3}+\frac{2a^3}{b^5(a+bx^2)^2}-\frac{6a^2}{b^5(a+bx^2)}-\frac{4a\log(a+bx^2)}{b^5}+\frac{x^2}{b^4}\right)$$

input
```
Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(x^2/b^4 - a^4/(3*b^5*(a + b*x^2)^3) + (2*a^3)/(b^5*(a + b*x^2)^2) - (6*a^
2)/(b^5*(a + b*x^2)) - (4*a*Log[a + b*x^2])/b^5)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

method	result
norman	$\frac{\frac{x^8}{2b}-\frac{11a^4}{3b^5}-\frac{6a^2x^4}{b^3}-\frac{9a^3x^2}{b^4}}{(b\,x^2+a)^3}-\frac{2a\ln\left(b\,x^2+a\right)}{b^5}$
default	$\frac{x^2}{2b^4}-\frac{a\left(\frac{4\ln\left(b\,x^2+a\right)}{b}+\frac{a^3}{3b\left(b\,x^2+a\right)^3}+\frac{6a}{b\left(b\,x^2+a\right)}-\frac{2a^2}{b\left(b\,x^2+a\right)^2}\right)}{2b^4}$
risch	$\frac{x^2}{2b^4}+\frac{-3a^2b\,x^4-5a^3x^2-\frac{13a^4}{6b}}{b^4(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}-\frac{2a\ln\left(b\,x^2+a\right)}{b^5}$
parallelrisch	$-\frac{-3b^4x^8+12\ln\left(b\,x^2+a\right)x^6a\,b^3+36\ln\left(b\,x^2+a\right)x^4a^2b^2+36a^2b^2x^4+36\ln\left(b\,x^2+a\right)x^2a^3b+54a^3b\,x^2+12\ln\left(b\,x^2+a\right)a^4+22a^4}{6b^5(b^2x^4+2ab\,x^2+a^2)(b\,x^2+a)}$

input
```
int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2/b*x^8-11/3*a^4/b^5-6*a^2/b^3*x^4-9*a^3/b^4*x^2)/(b*x^2+a)^3-2*a*ln(b*
x^2+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.61

$$\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{3\,b^4x^8+9\,ab^3x^6-9\,a^2b^2x^4-27\,a^3bx^2-13\,a^4-12\,(ab^3x^6+3\,a^2b^2x^4+3\,a^3bx^2+a^4)\log\left(bx^2+a\right)}{6\,(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5)}$$

input
```
integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/6*(3*b^4*x^8 + 9*a*b^3*x^6 - 9*a^2*b^2*x^4 - 27*a^3*b*x^2 - 13*a^4 - 12*
(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^8*x^6 +
 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{2a\log(a+bx^2)}{b^5} + \frac{-13a^4-30a^3bx^2-18a^2b^2x^4}{6a^3b^5+18a^2b^6x^2+18ab^7x^4+6b^8x^6} + \frac{x^2}{2b^4}$$

input
```
integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
-2*a*log(a + b*x**2)/b**5 + (-13*a**4 - 30*a**3*b*x**2 - 18*a**2*b**2*x**4
)/(6*a**3*b**5 + 18*a**2*b**6*x**2 + 18*a*b**7*x**4 + 6*b**8*x**6) + x**2/
(2*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{18\,a^2b^2x^4+30\,a^3bx^2+13\,a^4}{6\,(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5)} + \frac{x^2}{2\,b^4} - \frac{2\,a\log(bx^2+a)}{b^5}$$

input
```
integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(18*a^2*b^2*x^4 + 30*a^3*b*x^2 + 13*a^4)/(b^8*x^6 + 3*a*b^7*x^4 + 3*a
^2*b^6*x^2 + a^3*b^5) + 1/2*x^2/b^4 - 2*a*log(b*x^2 + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{x^2}{2\,b^4} - \frac{2\,a\log\left(|bx^2+a|\right)}{b^5} + \frac{22\,ab^3x^6+48\,a^2b^2x^4+36\,a^3bx^2+9\,a^4}{6\,(bx^2+a)^3b^5}$$

input
```
integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/2*x^2/b^4 - 2*a*log(abs(b*x^2 + a))/b^5 + 1/6*(22*a*b^3*x^6 + 48*a^2*b^2
*x^4 + 36*a^3*b*x^2 + 9*a^4)/((b*x^2 + a)^3*b^5)
```

Mupad [B] (verification not implemented)

Time = 17.66 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{x^2}{2\,b^4} - \frac{\frac{13\,a^4}{6\,b}+5\,a^3\,x^2+3\,a^2\,b\,x^4}{a^3\,b^4+3\,a^2\,b^5\,x^2+3\,a\,b^6\,x^4+b^7\,x^6} - \frac{2\,a\,\ln\left(b\,x^2+a\right)}{b^5}$$

input
```
int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
x^2/(2*b^4) - ((13*a^4)/(6*b) + 5*a^3*x^2 + 3*a^2*b*x^4)/(a^3*b^4 + b^7*x^
6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2) - (2*a*log(a + b*x^2))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{-12\log(b\,x^2+a)\,a^4-36\log(b\,x^2+a)\,a^3b\,x^2-36\log(b\,x^2+a)\,a^2b^2x^4-12\log(b\,x^2+a)\,a\,b^3x^6-10a^4 -}{6b^5\left(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3\right)}$$

input
```
int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
( - 12*log(a + b*x**2)*a**4 - 36*log(a + b*x**2)*a**3*b*x**2 - 36*log(a +
b*x**2)*a**2*b**2*x**4 - 12*log(a + b*x**2)*a*b**3*x**6 - 10*a**4 - 18*a**
3*b*x**2 + 12*a*b**3*x**6 + 3*b**4*x**8)/(6*b**5*(a**3 + 3*a**2*b*x**2 + 3
*a*b**2*x**4 + b**3*x**6))
```

3.379 $\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{a^3}{6b^4(a+bx^2)^3} - \frac{3a^2}{4b^4(a+bx^2)^2} + \frac{3a}{2b^4(a+bx^2)} + \frac{\log(a+bx^2)}{2b^4}$$

output
```
1/6*a^3/b^4/(b*x^2+a)^3-3/4*a^2/b^4/(b*x^2+a)^2+3/2*a/b^4/(b*x^2+a)+1/2*ln
(b*x^2+a)/b^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{a(11a^2+27abx^2+18b^2x^4)}{(a+bx^2)^3}+6\log(a+bx^2)}{12b^4}$$

input
```
Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
((a*(11*a^2 + 27*a*b*x^2 + 18*b^2*x^4))/(a + b*x^2)^3 + 6*Log[a + b*x^2])/
(12*b^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^4\int \frac{x^7}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^7}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^6}{(bx^2+a)^4}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(-\frac{a^3}{b^3(bx^2+a)^4}+\frac{3a^2}{b^3(bx^2+a)^3}-\frac{3a}{b^3(bx^2+a)^2}+\frac{1}{b^3(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{a^3}{3b^4(a+bx^2)^3}-\frac{3a^2}{2b^4(a+bx^2)^2}+\frac{3a}{b^4(a+bx^2)}+\frac{\log(a+bx^2)}{b^4}\right)$$

input
```
Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^3/(3*b^4*(a + b*x^2)^3) - (3*a^2)/(2*b^4*(a + b*x^2)^2) + (3*a)/(b^4*(a
 + b*x^2)) + Log[a + b*x^2]/b^4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{\frac{11a^3}{12b^4}+\frac{3a\,x^4}{2b^2}+\frac{9a^2x^2}{4b^3}}{(b\,x^2+a)^3}+\frac{\ln(b\,x^2+a)}{2b^4}$	54
default	$\frac{a^3}{6b^4(b\,x^2+a)^3}-\frac{3a^2}{4b^4(b\,x^2+a)^2}+\frac{3a}{2b^4(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^4}$	64
risch	$\frac{\frac{11a^3}{12b^4}+\frac{3a\,x^4}{2b^2}+\frac{9a^2x^2}{4b^3}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}+\frac{\ln(b\,x^2+a)}{2b^4}$	74
parallelrisch	$\frac{6\ln(b\,x^2+a)x^6b^3+18\ln(b\,x^2+a)x^4a\,b^2+18b^2x^4a+18\ln(b\,x^2+a)x^2a^2b+27a^2b\,x^2+6\ln(b\,x^2+a)a^3+11a^3}{12b^4(b^2x^4+2ab\,x^2+a^2)(b\,x^2+a)}$	122

input
```
int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(11/12*a^3/b^4+3/2*a/b^2*x^4+9/4*a^2/b^3*x^2)/(b*x^2+a)^3+1/2*ln(b*x^2+a)/
b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.44

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{18\,ab^2x^4+27\,a^2bx^2+11\,a^3+6\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\log(bx^2+a)}{12\,(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/12*(18*a*b^2*x^4 + 27*a^2*b*x^2 + 11*a^3 + 6*(b^3*x^6 + 3*a*b^2*x^4 + 3*
a^2*b*x^2 + a^3)*log(b*x^2 + a))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 +
a^3*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{11a^3+27a^2bx^2+18ab^2x^4}{12a^3b^4+36a^2b^5x^2+36ab^6x^4+12b^7x^6}+\frac{\log(a+bx^2)}{2b^4}$$

input
```
integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
(11*a**3 + 27*a**2*b*x**2 + 18*a*b**2*x**4)/(12*a**3*b**4 + 36*a**2*b**5*x
**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + log(a + b*x**2)/(2*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{18\,ab^2x^4+27\,a^2bx^2+11\,a^3}{12\,(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)}+\frac{\log(bx^2+a)}{2\,b^4}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/12*(18*a*b^2*x^4 + 27*a^2*b*x^2 + 11*a^3)/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2
*b^5*x^2 + a^3*b^4) + 1/2*log(b*x^2 + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\log(|bx^2+a|)}{2\,b^4}-\frac{11\,b^2x^6+15\,abx^4+6\,a^2x^2}{12\,(bx^2+a)^3b^3}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/2*log(abs(b*x^2 + a))/b^4 - 1/12*(11*b^2*x^6 + 15*a*b*x^4 + 6*a^2*x^2)/(
(b*x^2 + a)^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{11\,a^3}{12\,b^4}+\frac{3\,a\,x^4}{2\,b^2}+\frac{9\,a^2\,x^2}{4\,b^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}+\frac{\ln\left(b\,x^2+a\right)}{2\,b^4}$$

input
```
int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
((11*a^3)/(12*b^4) + (3*a*x^4)/(2*b^2) + (9*a^2*x^2)/(4*b^3))/(a^3 + b^3*x
^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4) + log(a + b*x^2)/(2*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.72

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^2}\,dx$$
$$= \frac{6\log(b\,x^2+a)\,a^3+18\log(b\,x^2+a)\,a^2b\,x^2+18\log(b\,x^2+a)\,a\,b^2x^4+6\log(b\,x^2+a)\,b^3x^6+5a^3+9a^2b\,x}{12b^4\left(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3\right)}$$

input
```
int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(6*log(a + b*x**2)*a**3 + 18*log(a + b*x**2)*a**2*b*x**2 + 18*log(a + b*x*
*2)*a*b**2*x**4 + 6*log(a + b*x**2)*b**3*x**6 + 5*a**3 + 9*a**2*b*x**2 - 6
*b**3*x**6)/(12*b**4*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.380 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^2} \, dx = \frac{x^6}{6a\left(a+bx^2\right)^3}$$

output
```
1/6*x^6/a/(b*x^2+a)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^2} \, dx = -\frac{a^2+3abx^2+3b^2x^4}{6b^3\left(a+bx^2\right)^3}$$

input
```
Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*(a^2 + 3*a*b*x^2 + 3*b^2*x^4)/(b^3*(a + b*x^2)^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1380, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^4\int \frac{x^5}{b^4\,(bx^2+a)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^5}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{ 242}$$

$$\frac{x^6}{6a\,(a+bx^2)^3}$$

input
```
Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
x^6/(6*a*(a + b*x^2)^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

method	result	size
norman	$\frac{-\frac{x^4}{2b}-\frac{a\,x^2}{2b^2}-\frac{a^2}{6b^3}}{\left(b\,x^2+a\right)^3}$	37
default	$-\frac{a^2}{6b^3\left(b\,x^2+a\right)^3}+\frac{a}{2b^3\left(b\,x^2+a\right)^2}-\frac{1}{2b^3\left(b\,x^2+a\right)}$	48
orering	$-\frac{\left(3b^2x^4+3ab\,x^2+a^2\right)\left(b\,x^2+a\right)}{6b^3\left(b^2x^4+2ab\,x^2+a^2\right)^2}$	52
gosper	$-\frac{3b^2x^4+3ab\,x^2+a^2}{6\left(b\,x^2+a\right)\left(b^2x^4+2ab\,x^2+a^2\right)b^3}$	54
parallelrisch	$\frac{-3b^2x^4-3ab\,x^2-a^2}{6b^3\left(b^2x^4+2ab\,x^2+a^2\right)\left(b\,x^2+a\right)}$	56
risch	$\frac{-\frac{x^4}{2b}-\frac{a\,x^2}{2b^2}-\frac{a^2}{6b^3}}{\left(b\,x^2+a\right)\left(b^2x^4+2ab\,x^2+a^2\right)}$	57

input
```
int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/b*x^4-1/2*a/b^2*x^2-1/6*a^2/b^3)/(b*x^2+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,\left(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3\right)}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2
+ a^3*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(14) = 28$.

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.16

$$\int \frac{x^5}{(a^2 + 2abx^2 + b^2x^4)^2}\,dx = \frac{-a^2 - 3abx^2 - 3b^2x^4}{6a^3b^3 + 18a^2b^4x^2 + 18ab^5x^4 + 6b^6x^6}$$

input
```
integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
(-a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*a**3*b**3 + 18*a**2*b**4*x**2 + 18*a
*b**5*x**4 + 6*b**6*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

$$\int \frac{x^5}{(a^2 + 2abx^2 + b^2x^4)^2}\,dx = -\frac{3\,b^2x^4 + 3\,abx^2 + a^2}{6\,(b^6x^6 + 3\,ab^5x^4 + 3\,a^2b^4x^2 + a^3b^3)}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2
+ a^3*b^3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{x^5}{(a^2 + 2abx^2 + b^2x^4)^2}\, dx = -\frac{3\, b^2x^4 + 3\, abx^2 + a^2}{6\, (bx^2 + a)^3 b^3}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/((b*x^2 + a)^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.16

$$\int \frac{x^5}{(a^2 + 2abx^2 + b^2x^4)^2}\, dx = -\frac{a^2 + 3\, a\, b\, x^2 + 3\, b^2\, x^4}{6\, a^3\, b^3 + 18\, a^2\, b^4\, x^2 + 18\, a\, b^5\, x^4 + 6\, b^6\, x^6}$$

input
```
int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
-(a^2 + 3*b^2*x^4 + 3*a*b*x^2)/(6*a^3*b^3 + 6*b^6*x^6 + 18*a*b^5*x^4 + 18*
a^2*b^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{x^5}{(a^2 + 2abx^2 + b^2x^4)^2}\, dx = \frac{x^6}{6a\, (b^3x^6 + 3a\, b^2x^4 + 3a^2b\, x^2 + a^3)}$$

input
```
int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
x**6/(6*a*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.381 $\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\frac{a}{6b^2\left(a+bx^2\right)^3}-\frac{1}{4b^2\left(a+bx^2\right)^2}$$

output
```
1/6*a/b^2/(b*x^2+a)^3-1/4/b^2/(b*x^2+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=-\frac{a+3bx^2}{12b^2\left(a+bx^2\right)^3}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/12*(a + 3*b*x^2)/(b^2*(a + b*x^2)^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \text{1380}$$

$$b^4\int \frac{x^3}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^3}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^4}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{1}{b(bx^2+a)^3}-\frac{a}{b(bx^2+a)^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a}{3b^2(a+bx^2)^3}-\frac{1}{2b^2(a+bx^2)^2}\right)$$

input
```
Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a/(3*b^2*(a + b*x^2)^3) - 1/(2*b^2*(a + b*x^2)^2))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{x^{2}}{4 b}-\frac{a}{12 b^{2}}}{\left(b \,x^{2}+a \right)^{3}}$	26
default	$\frac{a}{6 b^{2} \left(b \,x^{2}+a \right)^{3}}-\frac{1}{4 b^{2} \left(b \,x^{2}+a \right)^{2}}$	31
orering	$-\frac{\left(3 b \,x^{2}+a \right) \left(b \,x^{2}+a \right)}{12 b^{2} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{2}}$	41
gosper	$-\frac{3 b \,x^{2}+a}{12 \left(b \,x^{2}+a \right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right) b^{2}}$	43
risch	$\frac{-\frac{x^{2}}{4 b}-\frac{a}{12 b^{2}}}{\left(b \,x^{2}+a \right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}$	46
parallelrisch	$\frac{-3 b^{2} x^{2}-a b}{12 b^{3} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right) \left(b \,x^{2}+a \right)}$	48

input
```
int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/4/b*x^2-1/12*a/b^2)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{3\,bx^2+a}{12\,(b^5x^6+3\,ab^4x^4+3\,a^2b^3x^2+a^3b^2)}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/12*(3*b*x^2 + a)/(b^5*x^6 + 3*a*b^4*x^4 + 3*a^2*b^3*x^2 + a^3*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{-a-3bx^2}{12a^3b^2+36a^2b^3x^2+36ab^4x^4+12b^5x^6}$$

input
```
integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
(-a - 3*b*x**2)/(12*a**3*b**2 + 36*a**2*b**3*x**2 + 36*a*b**4*x**4 + 12*b*
*5*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{(a^2 + 2abx^2 + b^2x^4)^2} \, dx = -\frac{3\,bx^2 + a}{12\,(b^5x^6 + 3\,ab^4x^4 + 3\,a^2b^3x^2 + a^3b^2)}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/12*(3*b*x^2 + a)/(b^5*x^6 + 3*a*b^4*x^4 + 3*a^2*b^3*x^2 + a^3*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{x^3}{(a^2 + 2abx^2 + b^2x^4)^2} \, dx = -\frac{3\,bx^2 + a}{12\,(bx^2 + a)^3 b^2}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-1/12*(3*b*x^2 + a)/((b*x^2 + a)^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{x^3}{(a^2 + 2abx^2 + b^2x^4)^2} \, dx = -\frac{\frac{a}{12\,b^2} + \frac{x^2}{4\,b}}{a^3 + 3\,a^2\,b\,x^2 + 3\,a\,b^2\,x^4 + b^3\,x^6}$$

input
```
int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
-(a/(12*b^2) + x^2/(4*b))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{-3b\,x^2-a}{12b^2\left(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3\right)}$$

input
```
int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
( - a - 3*b*x**2)/(12*b**2*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x*
*6))
```

3.382 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=-\frac{1}{6b\left(a+bx^2\right)^3}$$

output
```
-1/6/b/(b*x^2+a)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=-\frac{1}{6b\left(a+bx^2\right)^3}$$

input
```
Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*1/(b*(a + b*x^2)^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1380, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^4\int \frac{x}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{ 241}$$

$$-\frac{1}{6b(a+bx^2)^3}$$

input
```
Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*1/(b*(a + b*x^2)^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{1}{6b\left(b\,x^{2}+a\right)^{3}}$	15
norman	$-\frac{1}{6b\left(b\,x^{2}+a\right)^{3}}$	15
orering	$-\frac{b\,x^{2}+a}{6b\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}}$	33
gosper	$-\frac{1}{6\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)b}$	35
risch	$-\frac{1}{6\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)b}$	35
parallelrisch	$-\frac{1}{6\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)b}$	35

input
```
int(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6/b/(b*x^2+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=-\frac{1}{6\left(b^4x^6+3\,ab^3x^4+3\,a^2b^2x^2+a^3b\right)}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/6/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 + a^3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{x}{(a^2 + 2abx^2 + b^2x^4)^2}\,dx = -\frac{1}{6a^3b + 18a^2b^2x^2 + 18ab^3x^4 + 6b^4x^6}$$

input
```
integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
-1/(6*a**3*b + 18*a**2*b**2*x**2 + 18*a*b**3*x**4 + 6*b**4*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \frac{x}{(a^2 + 2abx^2 + b^2x^4)^2}\,dx = -\frac{1}{6\,(b^4x^6 + 3\,ab^3x^4 + 3\,a^2b^2x^2 + a^3b)}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/6/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 + a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a^2 + 2abx^2 + b^2x^4)^2}\,dx = -\frac{1}{6\,(bx^2 + a)^3b}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-1/6/((b*x^2 + a)^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{x}{(a^2 + 2abx^2 + b^2x^4)^2} \, dx = -\frac{1}{6\,a^3\,b + 18\,a^2\,b^2\,x^2 + 18\,a\,b^3\,x^4 + 6\,b^4\,x^6}$$

input
```
int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
-1/(6*a^3*b + 6*b^4*x^6 + 18*a*b^3*x^4 + 18*a^2*b^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{x}{(a^2 + 2abx^2 + b^2x^4)^2} \, dx = -\frac{1}{6b\,(b^3x^6 + 3a\,b^2x^4 + 3a^2b\,x^2 + a^3)}$$

input
```
int(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
( - 1)/(6*b*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.383 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\frac{1}{6a\left(a+bx^2\right)^3}+\frac{1}{4a^2\left(a+bx^2\right)^2}+\frac{1}{2a^3\left(a+bx^2\right)}+\frac{\log(x)}{a^4}-\frac{\log\left(a+bx^2\right)}{2a^4}$$

output
```
1/6/a/(b*x^2+a)^3+1/4/a^2/(b*x^2+a)^2+1/2/a^3/(b*x^2+a)+ln(x)/a^4-1/2*ln(b
*x^2+a)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\frac{\frac{a\left(11a^2+15abx^2+6b^2x^4\right)}{\left(a+bx^2\right)^3}+12\log(x)-6\log\left(a+bx^2\right)}{12a^4}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
((a*(11*a^2 + 15*a*b*x^2 + 6*b^2*x^4))/(a + b*x^2)^3 + 12*Log[x] - 6*Log[a
 + b*x^2])/(12*a^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a^2 + 2abx^2 + b^2x^4)^2} \, dx$$
$$\Big\downarrow \text{1380}$$
$$b^4 \int \frac{1}{b^4 x (bx^2 + a)^4} dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1}{x (a + bx^2)^4} dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2} \int \frac{1}{x^2 (bx^2 + a)^4} dx^2$$
$$\Big\downarrow \text{54}$$
$$\frac{1}{2} \int \left(-\frac{b}{a^4 (bx^2 + a)} - \frac{b}{a^3 (bx^2 + a)^2} - \frac{b}{a^2 (bx^2 + a)^3} - \frac{b}{a (bx^2 + a)^4} + \frac{1}{a^4 x^2}\right) dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2} \left(-\frac{\log (a + bx^2)}{a^4} + \frac{\log (x^2)}{a^4} + \frac{1}{a^3 (a + bx^2)} + \frac{1}{2a^2 (a + bx^2)^2} + \frac{1}{3a (a + bx^2)^3}\right)$$

input
```
Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
(1/(3*a*(a + b*x^2)^3) + 1/(2*a^2*(a + b*x^2)^2) + 1/(a^3*(a + b*x^2)) + L
og[x^2]/a^4 - Log[a + b*x^2]/a^4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

method	result
norman	$\frac{-\frac{3b\,x^2}{2a^2}-\frac{9b^2x^4}{4a^3}-\frac{11b^3x^6}{12a^4}}{(b\,x^2+a)^3}+\frac{\ln(x)}{a^4}-\frac{\ln(b\,x^2+a)}{2a^4}$
default	$-\frac{b\left(\frac{\ln\left(b\,x^2+a\right)}{b}-\frac{a^3}{3b\left(b\,x^2+a\right)^3}-\frac{a^2}{2b\left(b\,x^2+a\right)^2}-\frac{a}{b\left(b\,x^2+a\right)}\right)}{2a^4}+\frac{\ln(x)}{a^4}$
risch	$\frac{\frac{b^2x^4}{2a^3}+\frac{5b\,x^2}{4a^2}+\frac{11}{12a}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}+\frac{\ln(x)}{a^4}-\frac{\ln(b\,x^2+a)}{2a^4}$
parallelrisch	$\frac{12\ln(x)x^6b^3-6\ln(b\,x^2+a)x^6b^3-11b^3x^6+36\ln(x)x^4a\,b^2-18\ln(b\,x^2+a)x^4a\,b^2-27b^2x^4a+36\ln(x)x^2a^2b-18\ln(b\,x^2+a)x^2a}{12a^4(b^2x^4+2ab\,x^2+a^2)(b\,x^2+a)}$

input
```
int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-3/2*b/a^2*x^2-9/4*b^2/a^3*x^4-11/12*b^3/a^4*x^6)/(b*x^2+a)^3+ln(x)/a^4-1
/2*ln(b*x^2+a)/a^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(62) = 124.

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.91

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$
$$=\frac{6\,ab^2x^4+15\,a^2bx^2+11\,a^3-6\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)+12\left(b^3x^6+3\,ab^2x^4+3\,a^2b\right.}{12\left(a^4b^3x^6+3\,a^5b^2x^4+3\,a^6bx^2+a^7\right)}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/12*(6*a*b^2*x^4 + 15*a^2*b*x^2 + 11*a^3 - 6*(b^3*x^6 + 3*a*b^2*x^4 + 3*a
^2*b*x^2 + a^3)*log(b*x^2 + a) + 12*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 +
 a^3)*log(x))/(a^4*b^3*x^6 + 3*a^5*b^2*x^4 + 3*a^6*b*x^2 + a^7)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{11a^2+15abx^2+6b^2x^4}{12a^6+36a^5bx^2+36a^4b^2x^4+12a^3b^3x^6} + \frac{\log\left(x\right)}{a^4} - \frac{\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

input
```
integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
(11*a**2 + 15*a*b*x**2 + 6*b**2*x**4)/(12*a**6 + 36*a**5*b*x**2 + 36*a**4*
b**2*x**4 + 12*a**3*b**3*x**6) + log(x)/a**4 - log(a/b + x**2)/(2*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{6\,b^2x^4+15\,abx^2+11\,a^2}{12\left(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6\right)} - \frac{\log\left(bx^2+a\right)}{2\,a^4} + \frac{\log\left(x^2\right)}{2\,a^4}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/12*(6*b^2*x^4 + 15*a*b*x^2 + 11*a^2)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^
5*b*x^2 + a^6) - 1/2*log(b*x^2 + a)/a^4 + 1/2*log(x^2)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\frac{\log\left(x^2\right)}{2\,a^4}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^4}+\frac{11\,b^3x^6+39\,ab^2x^4+48\,a^2bx^2+22\,a^3}{12\,\left(bx^2+a\right)^3a^4}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/2*log(x^2)/a^4 - 1/2*log(abs(b*x^2 + a))/a^4 + 1/12*(11*b^3*x^6 + 39*a*b
^2*x^4 + 48*a^2*b*x^2 + 22*a^3)/((b*x^2 + a)^3*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\frac{\ln\left(x\right)}{a^4}+\frac{\frac{11}{12\,a}+\frac{5\,b\,x^2}{4\,a^2}+\frac{b^2\,x^4}{2\,a^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}-\frac{\ln\left(b\,x^2+a\right)}{2\,a^4}$$

input
```
int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

output
```
log(x)/a^4 + (11/(12*a) + (5*b*x^2)/(4*a^2) + (b^2*x^4)/(2*a^3))/(a^3 + b^
3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4) - log(a + b*x^2)/(2*a^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.30

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

$$=\frac{-6\log(b\,x^2+a)\,a^3-18\log(b\,x^2+a)\,a^2b\,x^2-18\log(b\,x^2+a)\,a\,b^2x^4-6\log(b\,x^2+a)\,b^3x^6+12\log(x)\,a}{12a^4\left(b^3x^6+3a\,b^2x^4+3a^2b\,x^2\right.}$$

input
```
int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
( - 6*log(a + b*x**2)*a**3 - 18*log(a + b*x**2)*a**2*b*x**2 - 18*log(a + b
*x**2)*a*b**2*x**4 - 6*log(a + b*x**2)*b**3*x**6 + 12*log(x)*a**3 + 36*log
(x)*a**2*b*x**2 + 36*log(x)*a*b**2*x**4 + 12*log(x)*b**3*x**6 + 9*a**3 + 9
*a**2*b*x**2 - 2*b**3*x**6)/(12*a**4*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4
 + b**3*x**6))
```

3.384 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 84

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = -\frac{1}{2a^4x^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{4b\log(x)}{a^5}+\frac{2b\log\left(a+bx^2\right)}{a^5}$$

output
```
-1/2/a^4/x^2-1/6*b/a^2/(b*x^2+a)^3-1/2*b/a^3/(b*x^2+a)^2-3/2*b/a^4/(b*x^2+
a)-4*b*ln(x)/a^5+2*b*ln(b*x^2+a)/a^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = -\frac{\frac{a\left(3a^3+22a^2bx^2+30ab^2x^4+12b^3x^6\right)}{x^2\left(a+bx^2\right)^3}+24b\log(x)-12b\log\left(a+bx^2\right)}{6a^5}$$

input
```
Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
-1/6*((a*(3*a^3 + 22*a^2*b*x^2 + 30*a*b^2*x^4 + 12*b^3*x^6))/(x^2*(a + b*x
^2)^3) + 24*b*Log[x] - 12*b*Log[a + b*x^2])/a^5
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^2} dx$$

$$\Big\downarrow \text{1380}$$

$$b^4 \int \frac{1}{b^4 x^3 (bx^2 + a)^4} dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^3 (a + bx^2)^4} dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2 + a)^4} dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(\frac{4b^2}{a^5 (bx^2 + a)} + \frac{3b^2}{a^4 (bx^2 + a)^2} + \frac{2b^2}{a^3 (bx^2 + a)^3} + \frac{b^2}{a^2 (bx^2 + a)^4} - \frac{4b}{a^5 x^2} + \frac{1}{a^4 x^4} \right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{4b \log(x^2)}{a^5} + \frac{4b \log(a + bx^2)}{a^5} - \frac{3b}{a^4 (a + bx^2)} - \frac{1}{a^4 x^2} - \frac{b}{a^3 (a + bx^2)^2} - \frac{b}{3a^2 (a + bx^2)^3} \right)$$

input
```
Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
(-(1/(a^4*x^2)) - b/(3*a^2*(a + b*x^2)^3) - b/(a^3*(a + b*x^2)^2) - (3*b)/
(a^4*(a + b*x^2)) - (4*b*Log[x^2])/a^5 + (4*b*Log[a + b*x^2])/a^5)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

method	result
norman	$\frac{-\frac{1}{2a}+\frac{6b^{2}x^{4}}{a^{3}}+\frac{9b^{3}x^{6}}{a^{4}}+\frac{11b^{4}x^{8}}{3a^{5}}}{x^{2}\left(b\,x^{2}+a\right)^{3}}-\frac{4b\ln\left(x\right)}{a^{5}}+\frac{2b\ln\left(b\,x^{2}+a\right)}{a^{5}}$
default	$\frac{b^{2}\left(\frac{4\ln\left(b\,x^{2}+a\right)}{b}-\frac{a^{3}}{3b\left(b\,x^{2}+a\right)^{3}}-\frac{a^{2}}{b\left(b\,x^{2}+a\right)^{2}}-\frac{3a}{b\left(b\,x^{2}+a\right)}\right)}{2a^{5}}-\frac{1}{2a^{4}x^{2}}-\frac{4b\ln\left(x\right)}{a^{5}}$
risch	$\frac{-\frac{2b^{3}x^{6}}{a^{4}}-\frac{5b^{2}x^{4}}{a^{3}}-\frac{11b\,x^{2}}{3a^{2}}-\frac{1}{2a}}{x^{2}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\left(b\,x^{2}+a\right)}-\frac{4b\ln\left(x\right)}{a^{5}}+\frac{2b\ln\left(-b\,x^{2}-a\right)}{a^{5}}$
parallelrisch	$-\frac{24b^{4}\ln\left(x\right)x^{8}-12\ln\left(b\,x^{2}+a\right)x^{8}b^{4}-22b^{4}x^{8}+72b^{3}a\ln\left(x\right)x^{6}-36\ln\left(b\,x^{2}+a\right)x^{6}a\,b^{3}-54a\,b^{3}x^{6}+72a^{2}b^{2}\ln\left(x\right)x^{4}-36\ln\left(b\,x^{2}+a\right.}{6a^{5}x^{2}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\left(b\,x^{2}+a\right)}$

input
```
int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a+6*b^2/a^3*x^4+9*b^3/a^4*x^6+11/3*b^4/a^5*x^8)/x^2/(b*x^2+a)^3-4*b*
ln(x)/a^5+2*b*ln(b*x^2+a)/a^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(76) = 152.

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.94

$$\int \frac{1}{x^{3}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{2}}\,dx = -\frac{12\,ab^{3}x^{6}+30\,a^{2}b^{2}x^{4}+22\,a^{3}bx^{2}+3\,a^{4}-12\,\left(b^{4}x^{8}+3\,ab^{3}x^{6}+3\,a^{2}b^{2}x^{4}+a^{3}bx^{2}\right)\log\left(bx^{2}+a\right)+24\,(b^{4}}{6\,\left(a^{5}b^{3}x^{8}+3\,a^{6}b^{2}x^{6}+3\,a^{7}bx^{4}+a^{8}x^{2}\right)}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(12*a*b^3*x^6 + 30*a^2*b^2*x^4 + 22*a^3*b*x^2 + 3*a^4 - 12*(b^4*x^8 +
 3*a*b^3*x^6 + 3*a^2*b^2*x^4 + a^3*b*x^2)*log(b*x^2 + a) + 24*(b^4*x^8 + 3
*a*b^3*x^6 + 3*a^2*b^2*x^4 + a^3*b*x^2)*log(x))/(a^5*b^3*x^8 + 3*a^6*b^2*x
^6 + 3*a^7*b*x^4 + a^8*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^2} dx = \frac{-3a^3 - 22a^2bx^2 - 30ab^2x^4 - 12b^3x^6}{6a^7x^2 + 18a^6bx^4 + 18a^5b^2x^6 + 6a^4b^3x^8} - \frac{4b\log(x)}{a^5} + \frac{2b\log\left(\frac{a}{b} + x^2\right)}{a^5}$$

input
```
integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
(-3*a**3 - 22*a**2*b*x**2 - 30*a*b**2*x**4 - 12*b**3*x**6)/(6*a**7*x**2 +
18*a**6*b*x**4 + 18*a**5*b**2*x**6 + 6*a**4*b**3*x**8) - 4*b*log(x)/a**5 +
 2*b*log(a/b + x**2)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^2} dx = -\frac{12\,b^3x^6 + 30\,ab^2x^4 + 22\,a^2bx^2 + 3\,a^3}{6\,(a^4b^3x^8 + 3\,a^5b^2x^6 + 3\,a^6bx^4 + a^7x^2)} + \frac{2\,b\log(bx^2 + a)}{a^5} - \frac{2\,b\log(x^2)}{a^5}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/6*(12*b^3*x^6 + 30*a*b^2*x^4 + 22*a^2*b*x^2 + 3*a^3)/(a^4*b^3*x^8 + 3*a
^5*b^2*x^6 + 3*a^6*b*x^4 + a^7*x^2) + 2*b*log(b*x^2 + a)/a^5 - 2*b*log(x^2
)/a^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^2} dx = -\frac{2b\log(x^2)}{a^5} + \frac{2b\log(|bx^2+a|)}{a^5} + \frac{4bx^2-a}{2a^5x^2} - \frac{22b^4x^6 + 75ab^3x^4 + 87a^2b^2x^2 + 35a^3b}{6(bx^2+a)^3a^5}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-2*b*log(x^2)/a^5 + 2*b*log(abs(b*x^2 + a))/a^5 + 1/2*(4*b*x^2 - a)/(a^5*x
^2) - 1/6*(22*b^4*x^6 + 75*a*b^3*x^4 + 87*a^2*b^2*x^2 + 35*a^3*b)/((b*x^2
+ a)^3*a^5)
```

Mupad [B] (verification not implemented)

Time = 17.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^2} dx = \frac{2b\ln(bx^2+a)}{a^5} - \frac{\frac{1}{2a} + \frac{11bx^2}{3a^2} + \frac{5b^2x^4}{a^3} + \frac{2b^3x^6}{a^4}}{a^3x^2 + 3a^2bx^4 + 3ab^2x^6 + b^3x^8} - \frac{4b\ln(x)}{a^5}$$

input
```
int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

output
```
(2*b*log(a + b*x^2))/a^5 - (1/(2*a) + (11*b*x^2)/(3*a^2) + (5*b^2*x^4)/a^3
 + (2*b^3*x^6)/a^4)/(a^3*x^2 + b^3*x^8 + 3*a^2*b*x^4 + 3*a*b^2*x^6) - (4*b
*log(x))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.23

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^2} \, dx$$
$$= \frac{12 \log(b\,x^2 + a)\,a^3 b\,x^2 + 36 \log(b\,x^2 + a)\,a^2 b^2 x^4 + 36 \log(b\,x^2 + a)\,a\,b^3 x^6 + 12 \log(b\,x^2 + a)\,b^4 x^8 - 24 \log}{6a^5 x^2 (b^3 x^6 + 3a\,b^2 x}$$

input
```
int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(12*log(a + b*x**2)*a**3*b*x**2 + 36*log(a + b*x**2)*a**2*b**2*x**4 + 36*l
og(a + b*x**2)*a*b**3*x**6 + 12*log(a + b*x**2)*b**4*x**8 - 24*log(x)*a**3
*b*x**2 - 72*log(x)*a**2*b**2*x**4 - 72*log(x)*a*b**3*x**6 - 24*log(x)*b**
4*x**8 - 3*a**4 - 18*a**3*b*x**2 - 18*a**2*b**2*x**4 + 4*b**4*x**8)/(6*a**
5*x**2*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.385 $\int \frac{1}{x^5(a^2+2abx^2+b^2x^4)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \frac{1}{x^5 (a^2+2abx^2+b^2x^4)^2} dx = -\frac{1}{4a^4x^4} + \frac{2b}{a^5x^2} + \frac{b^2}{6a^3(a+bx^2)^3} + \frac{3b^2}{4a^4(a+bx^2)^2} + \frac{3b^2}{a^5(a+bx^2)} + \frac{10b^2\log(x)}{a^6} - \frac{5b^2\log(a+bx^2)}{a^6}$$

output
```
-1/4/a^4/x^4+2*b/a^5/x^2+1/6*b^2/a^3/(b*x^2+a)^3+3/4*b^2/a^4/(b*x^2+a)^2+3
*b^2/a^5/(b*x^2+a)+10*b^2*ln(x)/a^6-5*b^2*ln(b*x^2+a)/a^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^5 (a^2+2abx^2+b^2x^4)^2} dx = \frac{\frac{a(-3a^4+15a^3bx^2+110a^2b^2x^4+150ab^3x^6+60b^4x^8)}{x^4(a+bx^2)^3} + 120b^2\log(x) - 60b^2\log(a+bx^2)}{12a^6}$$

input
```
Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
((a*(-3*a^4 + 15*a^3*b*x^2 + 110*a^2*b^2*x^4 + 150*a*b^3*x^6 + 60*b^4*x^8)
)/(x^4*(a + b*x^2)^3) + 120*b^2*Log[x] - 60*b^2*Log[a + b*x^2])/(12*a^6)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a^2+2abx^2+b^2x^4)^2} \, dx$$

$$\Big\downarrow \text{1380}$$

$$b^4 \int \frac{1}{b^4x^5 (bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^5 (a+bx^2)^4}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^6 (bx^2+a)^4}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(-\frac{10b^3}{a^6(bx^2+a)}-\frac{6b^3}{a^5(bx^2+a)^2}-\frac{3b^3}{a^4(bx^2+a)^3}-\frac{b^3}{a^3(bx^2+a)^4}+\frac{10b^2}{a^6x^2}-\frac{4b}{a^5x^4}+\frac{1}{a^4x^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{10b^2\log(x^2)}{a^6}-\frac{10b^2\log(a+bx^2)}{a^6}+\frac{6b^2}{a^5(a+bx^2)}+\frac{4b}{a^5x^2}+\frac{3b^2}{2a^4(a+bx^2)^2}-\frac{1}{2a^4x^4}+\frac{b^2}{3a^3(a+bx^2)^3}\right)$$

input
```
Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
(-1/2*1/(a^4*x^4) + (4*b)/(a^5*x^2) + b^2/(3*a^3*(a + b*x^2)^3) + (3*b^2)/
(2*a^4*(a + b*x^2)^2) + (6*b^2)/(a^5*(a + b*x^2)) + (10*b^2*Log[x^2])/a^6
- (10*b^2*Log[a + b*x^2])/a^6)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

method	result
norman	$\frac{-\frac{1}{4a}+\frac{5b\,x^2}{4a^2}-\frac{15b^3x^6}{a^4}-\frac{45b^4x^8}{2a^5}-\frac{55b^5x^{10}}{6a^6}}{x^4(b\,x^2+a)^3}+\frac{10b^2\ln(x)}{a^6}-\frac{5b^2\ln(b\,x^2+a)}{a^6}$
default	$-\frac{b^3\left(-\frac{3a^2}{2b\left(b\,x^2+a\right)^2}+\frac{10\ln\left(b\,x^2+a\right)}{b}-\frac{a^3}{3b\left(b\,x^2+a\right)^3}-\frac{6a}{b\left(b\,x^2+a\right)}\right)}{2a^6}-\frac{1}{4a^4x^4}+\frac{2b}{a^5x^2}+\frac{10b^2\ln(x)}{a^6}$
risch	$\frac{\frac{5b^4x^8}{a^5}+\frac{25b^3x^6}{2a^4}+\frac{55b^2x^4}{6a^3}+\frac{5b\,x^2}{4a^2}-\frac{1}{4a}}{x^4(b^2x^4+2ab\,x^2+a^2)(b\,x^2+a)}+\frac{10b^2\ln(x)}{a^6}-\frac{5b^2\ln(b\,x^2+a)}{a^6}$
parallelrisch	$\frac{120\ln(x)x^{10}b^5-60\ln(b\,x^2+a)x^{10}b^5-110x^{10}b^5+360\ln(x)x^8a\,b^4-180\ln(b\,x^2+a)x^8a\,b^4-270a\,x^8b^4+360\ln(x)x^6a^2b^3-180\ln}{12a^6x^4(b^2x^4+2ab\,x^2+a^2)(b\,x^2+a)}$

input
```
int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/4/a+5/4*b/a^2*x^2-15*b^3/a^4*x^6-45/2*b^4/a^5*x^8-55/6*b^5/a^6*x^10)/x
^4/(b*x^2+a)^3+10*b^2*ln(x)/a^6-5*b^2*ln(b*x^2+a)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$
$$=\frac{60\,ab^4x^8+150\,a^2b^3x^6+110\,a^3b^2x^4+15\,a^4bx^2-3\,a^5-60\,(b^5x^{10}+3\,ab^4x^8+3\,a^2b^3x^6+a^3b^2x^4)\log(bx^2}{12\,(a^6b^3x^{10}+3\,a^7b^2x^8+3\,a^8bx^6+a^9x^4)}$$

input
```
integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/12*(60*a*b^4*x^8 + 150*a^2*b^3*x^6 + 110*a^3*b^2*x^4 + 15*a^4*b*x^2 - 3*
a^5 - 60*(b^5*x^10 + 3*a*b^4*x^8 + 3*a^2*b^3*x^6 + a^3*b^2*x^4)*log(b*x^2
+ a) + 120*(b^5*x^10 + 3*a*b^4*x^8 + 3*a^2*b^3*x^6 + a^3*b^2*x^4)*log(x))/
(a^6*b^3*x^10 + 3*a^7*b^2*x^8 + 3*a^8*b*x^6 + a^9*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)^2} \, dx = \frac{-3a^4 + 15a^3bx^2 + 110a^2b^2x^4 + 150ab^3x^6 + 60b^4x^8}{12a^8x^4 + 36a^7bx^6 + 36a^6b^2x^8 + 12a^5b^3x^{10}} + \frac{10b^2 \log(x)}{a^6} - \frac{5b^2 \log\left(\frac{a}{b} + x^2\right)}{a^6}$$

input
```
integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
(-3*a**4 + 15*a**3*b*x**2 + 110*a**2*b**2*x**4 + 150*a*b**3*x**6 + 60*b**4
*x**8)/(12*a**8*x**4 + 36*a**7*b*x**6 + 36*a**6*b**2*x**8 + 12*a**5*b**3*x
**10) + 10*b**2*log(x)/a**6 - 5*b**2*log(a/b + x**2)/a**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)^2} \, dx = \frac{60 b^4x^8 + 150 ab^3x^6 + 110 a^2b^2x^4 + 15 a^3bx^2 - 3 a^4}{12 (a^5b^3x^{10} + 3 a^6b^2x^8 + 3 a^7bx^6 + a^8x^4)} - \frac{5 b^2 \log(bx^2 + a)}{a^6} + \frac{5 b^2 \log(x^2)}{a^6}$$

input
```
integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/12*(60*b^4*x^8 + 150*a*b^3*x^6 + 110*a^2*b^2*x^4 + 15*a^3*b*x^2 - 3*a^4)
/(a^5*b^3*x^10 + 3*a^6*b^2*x^8 + 3*a^7*b*x^6 + a^8*x^4) - 5*b^2*log(b*x^2
+ a)/a^6 + 5*b^2*log(x^2)/a^6
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^5 \left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{5\,b^2\log\left(x^2\right)}{a^6} - \frac{5\,b^2\log\left(|bx^2+a|\right)}{a^6} + \frac{110\,b^5x^6+366\,ab^4x^4+411\,a^2b^3x^2+157\,a^3b^2}{12\,\left(bx^2+a\right)^3a^6} - \frac{30\,b^2x^4-8\,abx^2+a^2}{4\,a^6x^4}$$

input
```
integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
5*b^2*log(x^2)/a^6 - 5*b^2*log(abs(b*x^2 + a))/a^6 + 1/12*(110*b^5*x^6 + 3
66*a*b^4*x^4 + 411*a^2*b^3*x^2 + 157*a^3*b^2)/((b*x^2 + a)^3*a^6) - 1/4*(3
0*b^2*x^4 - 8*a*b*x^2 + a^2)/(a^6*x^4)
```

Mupad [B] (verification not implemented)

Time = 17.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^5 \left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{\frac{5\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{55\,b^2\,x^4}{6\,a^3}+\frac{25\,b^3\,x^6}{2\,a^4}+\frac{5\,b^4\,x^8}{a^5}}{a^3\,x^4+3\,a^2\,b\,x^6+3\,a\,b^2\,x^8+b^3\,x^{10}} - \frac{5\,b^2\,\ln\left(b\,x^2+a\right)}{a^6} + \frac{10\,b^2\,\ln\left(x\right)}{a^6}$$

input
```
int(1/(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

output
```
((5*b*x^2)/(4*a^2) - 1/(4*a) + (55*b^2*x^4)/(6*a^3) + (25*b^3*x^6)/(2*a^4)
 + (5*b^4*x^8)/a^5)/(a^3*x^4 + b^3*x^10 + 3*a^2*b*x^6 + 3*a*b^2*x^8) - (5*
b^2*log(a + b*x^2))/a^6 + (10*b^2*log(x))/a^6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)^2} dx$$
$$= \frac{-60 \log(b x^2 + a) a^3 b^2 x^4 - 180 \log(b x^2 + a) a^2 b^3 x^6 - 180 \log(b x^2 + a) a b^4 x^8 - 60 \log(b x^2 + a) b^5 x^{10} +}{12a^6 x^4 (}$$

input
```
int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
( - 60*log(a + b*x**2)*a**3*b**2*x**4 - 180*log(a + b*x**2)*a**2*b**3*x**6
 - 180*log(a + b*x**2)*a*b**4*x**8 - 60*log(a + b*x**2)*b**5*x**10 + 120*l
og(x)*a**3*b**2*x**4 + 360*log(x)*a**2*b**3*x**6 + 360*log(x)*a*b**4*x**8
+ 120*log(x)*b**5*x**10 - 3*a**5 + 15*a**4*b*x**2 + 90*a**3*b**2*x**4 + 90
*a**2*b**3*x**6 - 20*b**5*x**10)/(12*a**6*x**4*(a**3 + 3*a**2*b*x**2 + 3*a
*b**2*x**4 + b**3*x**6))
```

3.386 $\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{10a^2x}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^5}{5b^4}+\frac{a^5x}{6b^6(a+bx^2)^3}-\frac{31a^4x}{24b^6(a+bx^2)^2}+\frac{89a^3x}{16b^6(a+bx^2)}-\frac{231a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{13/2}}$$

output
```
10*a^2*x/b^6-4/3*a*x^3/b^5+1/5*x^5/b^4+1/6*a^5*x/b^6/(b*x^2+a)^3-31/24*a^4
*x/b^6/(b*x^2+a)^2+89/16*a^3*x/b^6/(b*x^2+a)-231/16*a^(5/2)*arctan(b^(1/2)
*x/a^(1/2))/b^(13/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{3465a^5x+9240a^4bx^3+7623a^3b^2x^5+1584a^2b^3x^7-176ab^4x^9+48b^5x^{11}}{240b^6(a+bx^2)^3}-\frac{231a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{13/2}}$$

input
```
Integrate[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(3465*a^5*x + 9240*a^4*b*x^3 + 7623*a^3*b^2*x^5 + 1584*a^2*b^3*x^7 - 176*a
*b^4*x^9 + 48*b^5*x^11)/(240*b^6*(a + b*x^2)^3) - (231*a^(5/2)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(16*b^(13/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1380, 27, 252, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^4\int \frac{x^{12}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^{12}}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{11\int \frac{x^{10}}{(bx^2+a)^3}dx}{6b}-\frac{x^{11}}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{11\left(\frac{9\int \frac{x^8}{(bx^2+a)^2}dx}{4b}-\frac{x^9}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^{11}}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{11\left(\frac{9\left(\frac{7\int \frac{x^6}{bx^2+a}dx}{2b}-\frac{x^7}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^9}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 254

$$\frac{11\left(\frac{9\left(\frac{7\int \left(\frac{x^4}{b}-\frac{ax^2}{b^2}-\frac{a^3}{b^3\left(bx^2+a\right)}+\frac{a^2}{b^3}\right)dx}{2b}-\frac{x^7}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^9}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 2009

$$\frac{11\left(\frac{9\left(\frac{7\left(-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}\right)}{2b}-\frac{x^7}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^9}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}$$

input
```
Int[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*x^11/(b*(a + b*x^2)^3) + (11*(-1/4*x^9/(b*(a + b*x^2)^2) + (9*(-1/2*x
^7/(b*(a + b*x^2)) + (7*((a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5
/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)))/(2*b)))/(4*b)))/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.72

method	result	s
default	$\frac{\frac{1}{5}x^5b^2-\frac{4}{3}ab\,x^3+10a^2x}{b^6}-\frac{a^3\left(\frac{-\frac{89}{16}x^5b^2-\frac{59}{6}ab\,x^3-\frac{71}{16}a^2x}{\left(b\,x^2+a\right)^3}+\frac{231\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{b^6}$	8
risch	$\frac{x^5}{5b^4}-\frac{4a\,x^3}{3b^5}+\frac{10a^2x}{b^6}+\frac{\frac{89}{16}a^3b^2x^5+\frac{59}{6}a^4b\,x^3+\frac{71}{16}x\,a^5}{b^6(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}+\frac{231\sqrt{-ab}\,a^2\ln\left(-\sqrt{-ab}\,x-a\right)}{32b^7}-\frac{231\sqrt{-ab}\,a^2\ln\left(\sqrt{-ab}\,x-a\right)}{32b^7}$	1

input
```
int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^6*(1/5*x^5*b^2-4/3*a*b*x^3+10*a^2*x)-1/b^6*a^3*((-89/16*x^5*b^2-59/6*a
*b*x^3-71/16*a^2*x)/(b*x^2+a)^3+231/16/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x)
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.73

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \left[\frac{96\,b^5x^{11}-352\,ab^4x^9+3168\,a^2b^3x^7+15246\,a^3b^2x^5+18480\,a^4bx^3+6930\,a^5x+3465\,(a^2b^3x^6+3\,a^3b^2x}{480\,(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)}\right.$$

input
```
integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
[1/480*(96*b^5*x^11 - 352*a*b^4*x^9 + 3168*a^2*b^3*x^7 + 15246*a^3*b^2*x^5
 + 18480*a^4*b*x^3 + 6930*a^5*x + 3465*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^
4*b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))
/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6), 1/240*(48*b^5*x^11 - 1
76*a*b^4*x^9 + 1584*a^2*b^3*x^7 + 7623*a^3*b^2*x^5 + 9240*a^4*b*x^3 + 3465
*a^5*x - 3465*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*b*x^2 + a^5)*sqrt(a/b)*
arctan(b*x*sqrt(a/b)/a))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)
]
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.46

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{10a^2x}{b^6} - \frac{4ax^3}{3b^5} + \frac{231\sqrt{-\frac{a^5}{b^{13}}}\log\left(x-\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}}{a^2}\right)}{32} - \frac{231\sqrt{-\frac{a^5}{b^{13}}}\log\left(x+\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}}{a^2}\right)}{32} + \frac{213a^5x+472a^4bx^3+267a^3b^2x^5}{48a^3b^6+144a^2b^7x^2+144ab^8x^4+48b^9x^6} + \frac{x^5}{5b^4}$$

input
```
integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
10*a**2*x/b**6 - 4*a*x**3/(3*b**5) + 231*sqrt(-a**5/b**13)*log(x - b**6*sq
rt(-a**5/b**13)/a**2)/32 - 231*sqrt(-a**5/b**13)*log(x + b**6*sqrt(-a**5/b
**13)/a**2)/32 + (213*a**5*x + 472*a**4*b*x**3 + 267*a**3*b**2*x**5)/(48*a
**3*b**6 + 144*a**2*b**7*x**2 + 144*a*b**8*x**4 + 48*b**9*x**6) + x**5/(5*
b**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{267\,a^3b^2x^5+472\,a^4bx^3+213\,a^5x}{48\,(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)} - \frac{231\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^6} + \frac{3\,b^2x^5-20\,abx^3+150\,a^2x}{15\,b^6}$$

input
```
integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/48*(267*a^3*b^2*x^5 + 472*a^4*b*x^3 + 213*a^5*x)/(b^9*x^6 + 3*a*b^8*x^4
+ 3*a^2*b^7*x^2 + a^3*b^6) - 231/16*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^6) + 1/15*(3*b^2*x^5 - 20*a*b*x^3 + 150*a^2*x)/b^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{231\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^6}+\frac{267\,a^3b^2x^5+472\,a^4bx^3+213\,a^5x}{48\,(bx^2+a)^3b^6}+\frac{3\,b^{16}x^5-20\,ab^{15}x^3+150\,a^2b^{14}x}{15\,b^{20}}$$

input
```
integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-231/16*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/48*(267*a^3*b^2*x^5
+ 472*a^4*b*x^3 + 213*a^5*x)/((b*x^2 + a)^3*b^6) + 1/15*(3*b^16*x^5 - 20*a
*b^15*x^3 + 150*a^2*b^14*x)/b^20
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{71\,a^5\,x}{16}+\frac{59\,a^4\,b\,x^3}{6}+\frac{89\,a^3\,b^2\,x^5}{16}}{a^3\,b^6+3\,a^2\,b^7\,x^2+3\,a\,b^8\,x^4+b^9\,x^6}+\frac{x^5}{5\,b^4}-\frac{4\,a\,x^3}{3\,b^5}+\frac{10\,a^2\,x}{b^6}-\frac{231\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,b^{13/2}}$$

input
```
int(x^12/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
((71*a^5*x)/16 + (59*a^4*b*x^3)/6 + (89*a^3*b^2*x^5)/16)/(a^3*b^6 + b^9*x^
6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2) + x^5/(5*b^4) - (4*a*x^3)/(3*b^5) + (10*a
^2*x)/b^6 - (231*a^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(16*b^(13/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.64

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^2}\,dx$$
$$= \frac{-3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5-10395\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4b\,x^2-10395\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b^2x^4-3}{240b^7\,(b^3x^6+3a}$$

input
```
int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
( - 3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5 - 10395*sqrt(b
)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*x**2 - 10395*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**4 - 3465*sqrt(b)*sqrt(a)*atan
((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*x**6 + 3465*a**5*b*x + 9240*a**4*b**2*
x**3 + 7623*a**3*b**3*x**5 + 1584*a**2*b**4*x**7 - 176*a*b**5*x**9 + 48*b*
*6*x**11)/(240*b**7*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.387 $\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^2} \, dx = -\frac{4ax}{b^5}+\frac{x^3}{3b^4}-\frac{a^4x}{6b^5(a+bx^2)^3}+\frac{25a^3x}{24b^5(a+bx^2)^2} - \frac{55a^2x}{16b^5(a+bx^2)}+\frac{105a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{11/2}}$$

output

```
-4*a*x/b^5+1/3*x^3/b^4-1/6*a^4*x/b^5/(b*x^2+a)^3+25/24*a^3*x/b^5/(b*x^2+a)
^2-55/16*a^2*x/b^5/(b*x^2+a)+105/16*a^(3/2)*arctan(b^(1/2)*x/a^(1/2))/b^(1
1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^2} \, dx = \frac{\frac{\sqrt{b}x(-315a^4-840a^3bx^2-693a^2b^2x^4-144ab^3x^6+16b^4x^8)}{(a+bx^2)^3}+315a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48b^{11/2}}$$

input
```
Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
((Sqrt[b]*x*(-315*a^4 - 840*a^3*b*x^2 - 693*a^2*b^2*x^4 - 144*a*b^3*x^6 +
16*b^4*x^8))/(a + b*x^2)^3 + 315*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(48*
b^(11/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1380, 27, 252, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^4\int \frac{x^{10}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^{10}}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{3\int \frac{x^8}{(bx^2+a)^3}dx}{2b}-\frac{x^9}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{3\left(\frac{7\int \frac{x^6}{(bx^2+a)^2}dx}{4b}-\frac{x^7}{4b(a+bx^2)^2}\right)}{2b}-\frac{x^9}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{3\left(\frac{7\left(\frac{5\int\frac{x^4}{bx^2+a}dx}{2b}-\frac{x^5}{2b(a+bx^2)}\right)}{4b}-\frac{x^7}{4b(a+bx^2)^2}\right)}{2b}-\frac{x^9}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \text{254}$$

$$\frac{3\left(\frac{7\left(\frac{5\int\left(\frac{a^2}{b^2(bx^2+a)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx}{2b}-\frac{x^5}{2b(a+bx^2)}\right)}{4b}-\frac{x^7}{4b(a+bx^2)^2}\right)}{2b}-\frac{x^9}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(\frac{7\left(\frac{5\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}\right)}{2b}-\frac{x^5}{2b(a+bx^2)}\right)}{4b}-\frac{x^7}{4b(a+bx^2)^2}\right)}{2b}-\frac{x^9}{6b\,(a+bx^2)^3}$$

input
```
Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*x^9/(b*(a + b*x^2)^3) + (3*(-1/4*x^7/(b*(a + b*x^2)^2) + (7*(-1/2*x^5
/(b*(a + b*x^2)) + (5*(-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/b^(5/2)))/(2*b)))/(4*b)))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{-\frac{1}{3} b\,x^{3}+4 a x}{b^{5}}+\frac{a^{2} \left(\frac{-\frac{55}{16} x^{5} b^{2}-\frac{35}{6} a b\,x^{3}-\frac{41}{16} a^{2} x}{\left(b\,x^{2}+a \right)^{3}}+\frac{105 \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{16 \sqrt{a b}}\right)}{b^{5}}$	74
risch	$\frac{x^{3}}{3 b^{4}}-\frac{4 a x}{b^{5}}+\frac{-\frac{55}{16} a^{2} b^{2} x^{5}-\frac{35}{6} a^{3} b\,x^{3}-\frac{41}{16} a^{4} x}{b^{5} \left(b\,x^{2}+a \right) \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)}+\frac{105 \sqrt{-a b}\, a \ln \left(-\sqrt{-a b}\, x+a \right)}{32 b^{6}}-\frac{105 \sqrt{-a b}\, a \ln \left(\sqrt{-a b}\, x+a \right)}{32 b^{6}}$	124

input
```
int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^5*(-1/3*b*x^3+4*a*x)+1/b^5*a^2*((-55/16*x^5*b^2-35/6*a*b*x^3-41/16*a^
2*x)/(b*x^2+a)^3+105/16/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.82

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^2}\,dx$$
$$= \left[\frac{32\,b^4x^9-288\,ab^3x^7-1386\,a^2b^2x^5-1680\,a^3bx^3-630\,a^4x+315\,(ab^3x^6+3\,a^2b^2x^4+3\,a^3bx^2+a^4)\sqrt{-}}{96\,(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5)}\right.$$

input
```
integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
[1/96*(32*b^4*x^9 - 288*a*b^3*x^7 - 1386*a^2*b^2*x^5 - 1680*a^3*b*x^3 - 63
0*a^4*x + 315*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 + a^4)*sqrt(-a/b)*l
og((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^8*x^6 + 3*a*b^7*x^4 + 3
*a^2*b^6*x^2 + a^3*b^5), 1/48*(16*b^4*x^9 - 144*a*b^3*x^7 - 693*a^2*b^2*x^
5 - 840*a^3*b*x^3 - 315*a^4*x + 315*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x
^2 + a^4)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^
2*b^6*x^2 + a^3*b^5)]
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.49

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{4ax}{b^5} - \frac{105\sqrt{-\frac{a^3}{b^{11}}}\log\left(x-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}}{a}\right)}{32} + \frac{105\sqrt{-\frac{a^3}{b^{11}}}\log\left(x+\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}}{a}\right)}{32} + \frac{-123a^4x-280a^3bx^3-165a^2b^2x^5}{48a^3b^5+144a^2b^6x^2+144ab^7x^4+48b^8x^6} + \frac{x^3}{3b^4}$$

input
```
integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
-4*a*x/b**5 - 105*sqrt(-a**3/b**11)*log(x - b**5*sqrt(-a**3/b**11)/a)/32 +
 105*sqrt(-a**3/b**11)*log(x + b**5*sqrt(-a**3/b**11)/a)/32 + (-123*a**4*x
 - 280*a**3*b*x**3 - 165*a**2*b**2*x**5)/(48*a**3*b**5 + 144*a**2*b**6*x**
2 + 144*a*b**7*x**4 + 48*b**8*x**6) + x**3/(3*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{165\,a^2b^2x^5+280\,a^3bx^3+123\,a^4x}{48\,(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5)} + \frac{105\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^5} + \frac{bx^3-12\,ax}{3\,b^5}$$

input
```
integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/48*(165*a^2*b^2*x^5 + 280*a^3*b*x^3 + 123*a^4*x)/(b^8*x^6 + 3*a*b^7*x^4
 + 3*a^2*b^6*x^2 + a^3*b^5) + 105/16*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^5) + 1/3*(b*x^3 - 12*a*x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.80

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{105\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^5} - \frac{165\,a^2b^2x^5+280\,a^3bx^3+123\,a^4x}{48\,(bx^2+a)^3b^5} + \frac{b^8x^3-12\,ab^7x}{3\,b^{12}}$$

input
```
integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
105/16*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/48*(165*a^2*b^2*x^5 +
 280*a^3*b*x^3 + 123*a^4*x)/((b*x^2 + a)^3*b^5) + 1/3*(b^8*x^3 - 12*a*b^7*
x)/b^12
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{x^3}{3\,b^4} - \frac{\frac{41\,a^4\,x}{16}+\frac{35\,a^3\,b\,x^3}{6}+\frac{55\,a^2\,b^2\,x^5}{16}}{a^3\,b^5+3\,a^2\,b^6\,x^2+3\,a\,b^7\,x^4+b^8\,x^6} + \frac{105\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,b^{11/2}} - \frac{4\,a\,x}{b^5}$$

input
```
int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
x^3/(3*b^4) - ((41*a^4*x)/16 + (35*a^3*b*x^3)/6 + (55*a^2*b^2*x^5)/16)/(a^
3*b^5 + b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2) + (105*a^(3/2)*atan((b^(1/2
)*x)/a^(1/2)))/(16*b^(11/2)) - (4*a*x)/b^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.72

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4+945\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^2+945\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2x^4+315\sqrt{b}\,\sqrt{}}{48b^6\left(b^3x^6+3a\,b^2x^4+3a^2b\,x^2\right.}$$

input
```
int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4 + 945*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*x**2 + 945*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**4 + 315*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a*b**3*x**6 - 315*a**4*b*x - 840*a**3*b**2*x**3 - 693*a**
2*b**3*x**5 - 144*a*b**4*x**7 + 16*b**5*x**9)/(48*b**6*(a**3 + 3*a**2*b*x*
*2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.388 $\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{x}{b^4}+\frac{a^3x}{6b^4(a+bx^2)^3}-\frac{19a^2x}{24b^4(a+bx^2)^2}+\frac{29ax}{16b^4(a+bx^2)}-\frac{35\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{9/2}}$$

output
```
x/b^4+1/6*a^3*x/b^4/(b*x^2+a)^3-19/24*a^2*x/b^4/(b*x^2+a)^2+29/16*a*x/b^4/
(b*x^2+a)-35/16*a^(1/2)*arctan(b^(1/2)*x/a^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{105a^3x+280a^2bx^3+231ab^2x^5+48b^3x^7}{48b^4(a+bx^2)^3}-\frac{35\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{9/2}}$$

input
```
Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(105*a^3*x + 280*a^2*b*x^3 + 231*a*b^2*x^5 + 48*b^3*x^7)/(48*b^4*(a + b*x^
2)^3) - (35*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(9/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1380, 27, 252, 252, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$\Big\downarrow \; 1380$$

$$b^4\int \frac{x^8}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{x^8}{(a+bx^2)^4}dx$$

$$\Big\downarrow \; 252$$

$$\frac{7\int \frac{x^6}{(bx^2+a)^3}dx}{6b}-\frac{x^7}{6b(a+bx^2)^3}$$

$$\Big\downarrow \; 252$$

$$\frac{7\left(\frac{5\int \frac{x^4}{(bx^2+a)^2}dx}{4b}-\frac{x^5}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^7}{6b(a+bx^2)^3}$$

$$\Big\downarrow \; 252$$

$$\frac{7\left(\frac{5\left(\frac{3\int \frac{x^2}{bx^2+a}dx}{2b}-\frac{x^3}{2b(a+bx^2)}\right)}{4b}-\frac{x^5}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^7}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{262}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{x}{b}-\frac{a\int\frac{1}{bx^2+a}dx}{b}\right)}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^5}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^5}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}$$

input
```
Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*x^7/(b*(a + b*x^2)^3) + (7*(-1/4*x^5/(b*(a + b*x^2)^2) + (5*(-1/2*x^3
/(b*(a + b*x^2)) + (3*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
))/(2*b)))/(4*b)))/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{x}{b^4}-\frac{a\left(\frac{-\frac{29}{16}x^5b^2-\frac{17}{6}ab\,x^3-\frac{19}{16}a^2x}{\left(b\,x^2+a\right)^3}+\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{b^4}$	62
risch	$\frac{x}{b^4}+\frac{\frac{29}{16}b^2x^5a+\frac{17}{6}a^2b\,x^3+\frac{19}{16}a^3x}{b^4(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}+\frac{35\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{32b^5}-\frac{35\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{32b^5}$	114

input
```
int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x/b^4-1/b^4*a*((-29/16*x^5*b^2-17/6*a*b*x^3-19/16*a^2*x)/(b*x^2+a)^3+35/16
/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.95

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx$$
$$= \left[\frac{96\,b^3x^7+462\,ab^2x^5+560\,a^2bx^3+210\,a^3x+105\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}}{bx^2+a}\right.}{96\,(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)}\right.$$

input
```
integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
[1/96*(96*b^3*x^7 + 462*a*b^2*x^5 + 560*a^2*b*x^3 + 210*a^3*x + 105*(b^3*x
^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-
a/b) - a)/(b*x^2 + a)))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4),
 1/48*(48*b^3*x^7 + 231*a*b^2*x^5 + 280*a^2*b*x^3 + 105*a^3*x - 105*(b^3*x
^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(
b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.44

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{35\sqrt{-\frac{a}{b^9}}\log\left(-b^4\sqrt{-\frac{a}{b^9}}+x\right)}{32} - \frac{35\sqrt{-\frac{a}{b^9}}\log\left(b^4\sqrt{-\frac{a}{b^9}}+x\right)}{32} + \frac{57a^3x+136a^2bx^3+87ab^2x^5}{48a^3b^4+144a^2b^5x^2+144ab^6x^4+48b^7x^6} + \frac{x}{b^4}$$

input
```
integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
35*sqrt(-a/b**9)*log(-b**4*sqrt(-a/b**9) + x)/32 - 35*sqrt(-a/b**9)*log(b*
*4*sqrt(-a/b**9) + x)/32 + (57*a**3*x + 136*a**2*b*x**3 + 87*a*b**2*x**5)/
(48*a**3*b**4 + 144*a**2*b**5*x**2 + 144*a*b**6*x**4 + 48*b**7*x**6) + x/b
**4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{87\,ab^2x^5+136\,a^2bx^3+57\,a^3x}{48\,(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)} - \frac{35\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^4} + \frac{x}{b^4}$$

input
```
integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/48*(87*a*b^2*x^5 + 136*a^2*b*x^3 + 57*a^3*x)/(b^7*x^6 + 3*a*b^6*x^4 + 3*
a^2*b^5*x^2 + a^3*b^4) - 35/16*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + x
/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{35\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^4} + \frac{x}{b^4} + \frac{87\,ab^2x^5+136\,a^2bx^3+57\,a^3x}{48\,(bx^2+a)^3b^4}$$

input
```
integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-35/16*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + x/b^4 + 1/48*(87*a*b^2*x^
5 + 136*a^2*b*x^3 + 57*a^3*x)/((b*x^2 + a)^3*b^4)
```

Mupad [B] (verification not implemented)

Time = 17.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{x}{b^4}+\frac{\frac{19\,a^3\,x}{16}+\frac{17\,a^2\,b\,x^3}{6}+\frac{29\,a\,b^2\,x^5}{16}}{a^3\,b^4+3\,a^2\,b^5\,x^2+3\,a\,b^6\,x^4+b^7\,x^6}-\frac{35\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,b^{9/2}}$$

input
```
int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
x/b^4 + ((19*a^3*x)/16 + (17*a^2*b*x^3)/6 + (29*a*b^2*x^5)/16)/(a^3*b^4 +
b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2) - (35*a^(1/2)*atan((b^(1/2)*x)/a^(1
/2)))/(16*b^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.84

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^2-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^4-105\sqrt{b}\,\cdots}{48b^5\left(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3\right)}$$

input
```
int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
( - 105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3 - 315*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**2 - 315*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*x**4 - 105*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*b**3*x**6 + 105*a**3*b*x + 280*a**2*b**2*x**3 + 231*a*b**
3*x**5 + 48*b**4*x**7)/(48*b**5*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b*
*3*x**6))
```

3.389 $\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^2} dx = -\frac{x^5}{6b(a+bx^2)^3} - \frac{5x^3}{24b^2(a+bx^2)^2} - \frac{5x}{16b^3(a+bx^2)} + \frac{5\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{a}b^{7/2}}$$

output
```
-1/6*x^5/b/(b*x^2+a)^3-5/24*x^3/b^2/(b*x^2+a)^2-5/16*x/b^3/(b*x^2+a)+5/16*
arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^2} dx = -\frac{x(15a^2+40abx^2+33b^2x^4)}{48b^3(a+bx^2)^3} + \frac{5\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{a}b^{7/2}}$$

input
```
Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/48*(x*(15*a^2 + 40*a*b*x^2 + 33*b^2*x^4))/(b^3*(a + b*x^2)^3) + (5*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(7/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1380, 27, 252, 252, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \text{1380}$$

$$b^4\int \frac{x^6}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^6}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5\int \frac{x^4}{(bx^2+a)^3}dx}{6b}-\frac{x^5}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{5\left(\frac{3\int \frac{x^2}{\left(bx^2+a\right)^2}dx}{4b}-\frac{x^3}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^5}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2b}-\frac{x}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^3}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^5}{6b(a+bx^2)^3}$$

$$\downarrow 218$$

$$\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^3}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^5}{6b\left(a+bx^2\right)^3}$$

input
```
Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*x^5/(b*(a + b*x^2)^3) + (5*(-1/4*x^3/(b*(a + b*x^2)^2) + (3*(-1/2*x/(
b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))))/(4*b)))
/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{-\frac{11x^5}{16b}-\frac{5a\,x^3}{6b^2}-\frac{5a^2x}{16b^3}}{(b\,x^2+a)^3}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16b^3\sqrt{ab}}$	58
risch	$\frac{-\frac{11x^5}{16b}-\frac{5a\,x^3}{6b^2}-\frac{5a^2x}{16b^3}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}-\frac{5\ln(bx+\sqrt{-ab})}{32\sqrt{-ab}\,b^3}+\frac{5\ln(-bx+\sqrt{-ab})}{32\sqrt{-ab}\,b^3}$	104

input
```
int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-11/16/b*x^5-5/6*a/b^2*x^3-5/16*a^2/b^3*x)/(b*x^2+a)^3+5/16/b^3/(a*b)^(1/
2)*arctan(b/(a*b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.06

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^2}\,dx = \left[-\frac{66\,ab^3x^5+80\,a^2b^2x^3+30\,a^3bx+15\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{96\,(ab^7x^6+3\,a^2b^6x^4+3\,a^3b^5x^2+a^4b^4)}, -\frac{33\,ab^3x^5+40\,a^2b^2x^3+15\,a^3bx-15\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{48\,(ab^7x^6+3\,a^2b^6x^4+3\,a^3b^5x^2+a^4b^4)}\right]$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
[-1/96*(66*a*b^3*x^5 + 80*a^2*b^2*x^3 + 30*a^3*b*x + 15*(b^3*x^6 + 3*a*b^2
*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x
^2 + a)))/(a*b^7*x^6 + 3*a^2*b^6*x^4 + 3*a^3*b^5*x^2 + a^4*b^4), -1/48*(33
*a*b^3*x^5 + 40*a^2*b^2*x^3 + 15*a^3*b*x - 15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a
^2*b*x^2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^7*x^6 + 3*a^2*b^6*x^
4 + 3*a^3*b^5*x^2 + a^4*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.61

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{5\sqrt{-\frac{1}{ab^7}}\log\left(-ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{32} + \frac{5\sqrt{-\frac{1}{ab^7}}\log\left(ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{32} + \frac{-15a^2x-40abx^3-33b^2x^5}{48a^3b^3+144a^2b^4x^2+144ab^5x^4+48b^6x^6}$$

input
```
integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
-5*sqrt(-1/(a*b**7))*log(-a*b**3*sqrt(-1/(a*b**7)) + x)/32 + 5*sqrt(-1/(a*
b**7))*log(a*b**3*sqrt(-1/(a*b**7)) + x)/32 + (-15*a**2*x - 40*a*b*x**3 -
33*b**2*x**5)/(48*a**3*b**3 + 144*a**2*b**4*x**2 + 144*a*b**5*x**4 + 48*b*
*6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{33\,b^2x^5+40\,abx^3+15\,a^2x}{48\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)} + \frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^3}$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/48*(33*b^2*x^5 + 40*a*b*x^3 + 15*a^2*x)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*
b^4*x^2 + a^3*b^3) + 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^3} - \frac{33\,b^2x^5+40\,abx^3+15\,a^2x}{48\,\left(bx^2+a\right)^3b^3}$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/48*(33*b^2*x^5 + 40*a*b*x^3
 + 15*a^2*x)/((b*x^2 + a)^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 17.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{5\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,\sqrt{a}\,b^{7/2}} - \frac{\frac{11\,x^5}{16\,b}+\frac{5\,a\,x^3}{6\,b^2}+\frac{5\,a^2\,x}{16\,b^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}$$

input
```
int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(5*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(1/2)*b^(7/2)) - ((11*x^5)/(16*b) + (5
*a*x^3)/(6*b^2) + (5*a^2*x)/(16*b^3))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b
^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.95

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$
$$= \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3+45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^2+45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^4+15\sqrt{b}\,\sqrt{a}\,ata}{48a\,b^4\left(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3\right)}$$

input
```
int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3 + 45*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**2 + 45*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a*b**2*x**4 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*b**3*x**6 - 15*a**3*b*x - 40*a**2*b**2*x**3 - 33*a*b**3*x**5)/(4
8*a*b**4*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.390 $\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 84

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{x^3}{6b(a+bx^2)^3} - \frac{x}{8b^2(a+bx^2)^2} + \frac{x}{16ab^2(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}$$

output
```
-1/6*x^3/b/(b*x^2+a)^3-1/8*x/b^2/(b*x^2+a)^2+1/16*x/a/b^2/(b*x^2+a)+1/16*a
rctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{-3a^2x-8abx^3+3b^2x^5}{48ab^2(a+bx^2)^3} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}$$

input
```
Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(-3*a^2*x - 8*a*b*x^3 + 3*b^2*x^5)/(48*a*b^2*(a + b*x^2)^3) + ArcTan[(Sqrt
[b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 27, 252, 252, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^4\int \frac{x^4}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^4}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{\int \frac{x^2}{(bx^2+a)^3}dx}{2b}-\frac{x^3}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\int \frac{1}{(bx^2+a)^2}dx}{4b}-\frac{x}{4b(a+bx^2)^2}}{2b}-\frac{x^3}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}}{4b}-\frac{x}{4b(a+bx^2)^2}}{2b}-\frac{x^3}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}}{4b}-\frac{x}{4b(a+bx^2)^2}}{2b}-\frac{x^3}{6b\left(a+bx^2\right)^3}$$

input
```
Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*x^3/(b*(a + b*x^2)^3) + (-1/4*x/(b*(a + b*x^2)^2) + (x/(2*a*(a + b*x^
2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]))/(4*b))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{x^5}{16a}-\frac{x^3}{6b}-\frac{ax}{16b^2}}{(b\,x^2+a)^3}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16b^2a\sqrt{ab}}$	58
risch	$\frac{\frac{x^5}{16a}-\frac{x^3}{6b}-\frac{ax}{16b^2}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)}-\frac{\ln(bx+\sqrt{-ab})}{32\sqrt{-ab}\,b^2a}+\frac{\ln(-bx+\sqrt{-ab})}{32\sqrt{-ab}\,b^2a}$	107

input
```
int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/16/a*x^5-1/6/b*x^3-1/16*a/b^2*x)/(b*x^2+a)^3+1/16/b^2/a/(a*b)^(1/2)*arc
tan(b/(a*b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.07

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx$$
$$=\left[\frac{6\,ab^3x^5-16\,a^2b^2x^3-6\,a^3bx-3\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{96\,(a^2b^6x^6+3\,a^3b^5x^4+3\,a^4b^4x^2+a^5b^3)},\frac{3\,ab^3x^5-8\right.$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
[1/96*(6*a*b^3*x^5 - 16*a^2*b^2*x^3 - 6*a^3*b*x - 3*(b^3*x^6 + 3*a*b^2*x^4
 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)))/(a^2*b^6*x^6 + 3*a^3*b^5*x^4 + 3*a^4*b^4*x^2 + a^5*b^3), 1/48*(3*a*b
^3*x^5 - 8*a^2*b^2*x^3 - 3*a^3*b*x + 3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^
2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^6*x^6 + 3*a^3*b^5*x^4 + 3
*a^4*b^4*x^2 + a^5*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(70) = 140$.

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.70

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^3b^5}}\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{a^3b^5}}\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{32} + \frac{-3a^2x-8abx^3+3b^2x^5}{48a^4b^2+144a^3b^3x^2+144a^2b^4x^4+48ab^5x^6}$$

input
```
integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
-sqrt(-1/(a**3*b**5))*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/32 + sqrt(-
1/(a**3*b**5))*log(a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/32 + (-3*a**2*x - 8
*a*b*x**3 + 3*b**2*x**5)/(48*a**4*b**2 + 144*a**3*b**3*x**2 + 144*a**2*b**
4*x**4 + 48*a*b**5*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{3\,b^2x^5-8\,abx^3-3\,a^2x}{48\,(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}ab^2}$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/48*(3*b^2*x^5 - 8*a*b*x^3 - 3*a^2*x)/(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*
b^3*x^2 + a^4*b^2) + 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}ab^2} + \frac{3\,b^2x^5 - 8\,abx^3 - 3\,a^2x}{48\,(bx^2+a)^3ab^2}$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/48*(3*b^2*x^5 - 8*a*b*x^3
 - 3*a^2*x)/((b*x^2 + a)^3*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 17.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{3/2}\,b^{5/2}} - \frac{\frac{x^3}{6\,b} - \frac{x^5}{16\,a} + \frac{a\,x}{16\,b^2}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6}$$

input
```
int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
atan((b^(1/2)*x)/a^(1/2))/(16*a^(3/2)*b^(5/2)) - (x^3/(6*b) - x^5/(16*a) +
 (a*x)/(16*b^2))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3*a*b^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.93

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3 + 9\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^2 + 9\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^4 + 3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b}{\sqrt{b}}\right.}{48a^2b^3\,(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)}$$

input
```
int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3 + 9*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**2 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(
sqrt(b)*sqrt(a)))*a*b**2*x**4 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt
(a)))*b**3*x**6 - 3*a**3*b*x - 8*a**2*b**2*x**3 + 3*a*b**3*x**5)/(48*a**2*
b**3*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.391 $\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{x}{6b(a+bx^2)^3}+\frac{x}{24ab(a+bx^2)^2}+\frac{x}{16a^2b(a+bx^2)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}$$

output
```
-1/6*x/b/(b*x^2+a)^3+1/24*x/a/b/(b*x^2+a)^2+1/16*x/a^2/b/(b*x^2+a)+1/16*ar
ctan(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{-3a^2x+8abx^3+3b^2x^5}{48a^2b(a+bx^2)^3}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(-3*a^2*x + 8*a*b*x^3 + 3*b^2*x^5)/(48*a^2*b*(a + b*x^2)^3) + ArcTan[(Sqrt
[b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 27, 252, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \text{1380}$$

$$b^4\int \frac{x^2}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{\int \frac{1}{(bx^2+a)^3}dx}{6b}-\frac{x}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\int \frac{1}{\left(bx^2+a\right)^2}dx}{4a}+\frac{x}{4a(a+bx^2)^2}}{6b}-\frac{x}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a(a+bx^2)^2}}{6b}-\frac{x}{6b(a+bx^2)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a(a+bx^2)^2}}{6b}-\frac{x}{6b\left(a+bx^2\right)^3}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*x/(b*(a + b*x^2)^3) + (x/(4*a*(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)
) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a))/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\frac{b\,x^{5}}{16a^{2}}+\frac{x^{3}}{6a}-\frac{x}{16b}}{\left(b\,x^{2}+a\right)^{3}}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16a^{2}b\sqrt{ab}}$	58
risch	$\frac{\frac{b\,x^{5}}{16a^{2}}+\frac{x^{3}}{6a}-\frac{x}{16b}}{\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{32\sqrt{-ab}\,b\,a^{2}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{32\sqrt{-ab}\,b\,a^{2}}$	107

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/16*b/a^2*x^5+1/6/a*x^3-1/16/b*x)/(b*x^2+a)^3+1/16/a^2/b/(a*b)^(1/2)*arc
tan(b/(a*b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.04

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \left[\frac{6\,ab^3x^5+16\,a^2b^2x^3-6\,a^3bx-3\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{96\,(a^3b^5x^6+3\,a^4b^4x^4+3\,a^5b^3x^2+a^6b^2)}, \frac{3\,ab^3x^5+8\right.$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
[1/96*(6*a*b^3*x^5 + 16*a^2*b^2*x^3 - 6*a^3*b*x - 3*(b^3*x^6 + 3*a*b^2*x^4
 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)))/(a^3*b^5*x^6 + 3*a^4*b^4*x^4 + 3*a^5*b^3*x^2 + a^6*b^2), 1/48*(3*a*b
^3*x^5 + 8*a^2*b^2*x^3 - 3*a^3*b*x + 3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^
2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^5*x^6 + 3*a^4*b^4*x^4 + 3
*a^5*b^3*x^2 + a^6*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(68) = 136$.

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.64

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^3}}\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{a^5b^3}}\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{32} + \frac{-3a^2x+8abx^3+3b^2x^5}{48a^5b+144a^4b^2x^2+144a^3b^3x^4+48a^2b^4x^6}$$

input
```
integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
-sqrt(-1/(a**5*b**3))*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/32 + sqrt(-1/(
a**5*b**3))*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/32 + (-3*a**2*x + 8*a*b*x
**3 + 3*b**2*x**5)/(48*a**5*b + 144*a**4*b**2*x**2 + 144*a**3*b**3*x**4 +
48*a**2*b**4*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{3\,b^2x^5+8\,abx^3-3\,a^2x}{48\,(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^2b}$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/48*(3*b^2*x^5 + 8*a*b*x^3 - 3*a^2*x)/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^
4*b^2*x^2 + a^5*b) + 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}a^2b} + \frac{3\,b^2x^5+8\,abx^3-3\,a^2x}{48\,(bx^2+a)^3a^2b}$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/48*(3*b^2*x^5 + 8*a*b*x^3
 - 3*a^2*x)/((b*x^2 + a)^3*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 17.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{x^3}{6\,a}-\frac{x}{16\,b}+\frac{b\,x^5}{16\,a^2}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{5/2}\,b^{3/2}}$$

input
```
int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(x^3/(6*a) - x/(16*b) + (b*x^5)/(16*a^2))/(a^3 + b^3*x^6 + 3*a^2*b*x^2 + 3
*a*b^2*x^4) + atan((b^(1/2)*x)/a^(1/2))/(16*a^(5/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.91

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^2}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3+9\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^2+9\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^4+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b}{\sqrt{b}}\right.}{48a^3b^2\,(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)}$$

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3 + 9*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**2 + 9*sqrt(b)*sqrt(a)*atan((b*x)/(
sqrt(b)*sqrt(a)))*a*b**2*x**4 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt
(a)))*b**3*x**6 - 3*a**3*b*x + 8*a**2*b**2*x**3 + 3*a*b**3*x**5)/(48*a**3*
b**2*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.392 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\frac{x}{6a\left(a+bx^2\right)^3}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}$$

output
```
1/6*x/a/(b*x^2+a)^3+5/24*x/a^2/(b*x^2+a)^2+5/16*x/a^3/(b*x^2+a)+5/16*arcta
n(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\frac{33a^2x+40abx^3+15b^2x^5}{48a^3\left(a+bx^2\right)^3}+\frac{5\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
```

output
```
(33*a^2*x + 40*a*b*x^3 + 15*b^2*x^5)/(48*a^3*(a + b*x^2)^3) + (5*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1379, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \text{1379}$$

$$b^4\int \frac{1}{(b^2x^2+ab)^4}dx$$

$$\Big\downarrow \text{215}$$

$$b^4\left(\frac{5\int \frac{1}{(b^2x^2+ab)^3}dx}{6ab}+\frac{x}{6ab^4(a+bx^2)^3}\right)$$

$$\Big\downarrow \text{215}$$

$$b^4\left(\frac{5\left(\frac{3\int \frac{1}{(b^2x^2+ab)^2}dx}{4ab}+\frac{x}{4ab^3(a+bx^2)^2}\right)}{6ab}+\frac{x}{6ab^4(a+bx^2)^3}\right)$$

$$\Big\downarrow \text{215}$$

$$b^4\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{b^2x^2+ab}dx}{2ab}+\frac{x}{2ab^2(a+bx^2)}\right)}{4ab}+\frac{x}{4ab^3(a+bx^2)^2}\right)}{6ab}+\frac{x}{6ab^4(a+bx^2)^3}\right)$$

$$\Big\downarrow \; 218$$

$$b^4\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x}{2ab^2(a+bx^2)}\right)}{4ab}+\frac{x}{4ab^3(a+bx^2)^2}\right)}{6ab}+\frac{x}{6ab^4(a+bx^2)^3}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
```

output
```
b^4*(x/(6*a*b^4*(a + b*x^2)^3) + (5*(x/(4*a*b^3*(a + b*x^2)^2) + (3*(x/(2*
a*b^2*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*b^(5/2))))/(4*
a*b)))/(6*a*b))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{x}{6a(b\,x^2+a)^3}+\frac{\frac{5x}{24a\left(b\,x^2+a\right)^2}+\frac{5\left(\frac{3x}{8a\left(b\,x^2+a\right)}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)}{6a}}{a}$	78
risch	$\frac{\frac{5b^2x^5}{16a^3}+\frac{5b\,x^3}{6a^2}+\frac{11x}{16a}}{(b^2x^4+2ab\,x^2+a^2)(b\,x^2+a)}-\frac{5\ln(bx+\sqrt{-ab})}{32\sqrt{-ab}\,a^3}+\frac{5\ln(-bx+\sqrt{-ab})}{32\sqrt{-ab}\,a^3}$	104

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x/a/(b*x^2+a)^3+5/6/a*(1/4*x/a/(b*x^2+a)^2+3/4/a*(1/2*x/a/(b*x^2+a)+1/
2/a/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.22

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^2}\,dx = \left[\frac{30\,ab^3x^5+80\,a^2b^2x^3+66\,a^3bx-15\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{96\,(a^4b^4x^6+3\,a^5b^3x^4+3\,a^6b^2x^2+a^7b)},\ \frac{15\,ab^3x\ldots}{}\right.$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
[1/96*(30*a*b^3*x^5 + 80*a^2*b^2*x^3 + 66*a^3*b*x - 15*(b^3*x^6 + 3*a*b^2*
x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^
2 + a)))/(a^4*b^4*x^6 + 3*a^5*b^3*x^4 + 3*a^6*b^2*x^2 + a^7*b), 1/48*(15*a
*b^3*x^5 + 40*a^2*b^2*x^3 + 33*a^3*b*x + 15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2
*b*x^2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^4*x^6 + 3*a^5*b^3*x^
4 + 3*a^6*b^2*x^2 + a^7*b)]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{5\sqrt{-\frac{1}{a^7b}}\log\left(-a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32} + \frac{5\sqrt{-\frac{1}{a^7b}}\log\left(a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32} + \frac{33a^2x+40abx^3+15b^2x^5}{48a^6+144a^5bx^2+144a^4b^2x^4+48a^3b^3x^6}$$

input
```
integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
-5*sqrt(-1/(a**7*b))*log(-a**4*sqrt(-1/(a**7*b)) + x)/32 + 5*sqrt(-1/(a**7
*b))*log(a**4*sqrt(-1/(a**7*b)) + x)/32 + (33*a**2*x + 40*a*b*x**3 + 15*b*
*2*x**5)/(48*a**6 + 144*a**5*b*x**2 + 144*a**4*b**2*x**4 + 48*a**3*b**3*x*
*6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{15\,b^2x^5+40\,abx^3+33\,a^2x}{48\,(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6)} + \frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/48*(15*b^2*x^5 + 40*a*b*x^3 + 33*a^2*x)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3
*a^5*b*x^2 + a^6) + 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}a^3} + \frac{15\,b^2x^5+40\,abx^3+33\,a^2x}{48\,(bx^2+a)^3a^3}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/48*(15*b^2*x^5 + 40*a*b*x^3
 + 33*a^2*x)/((b*x^2 + a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 17.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{11\,x}{16\,a}+\frac{5\,b\,x^3}{6\,a^2}+\frac{5\,b^2\,x^5}{16\,a^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6} + \frac{5\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{7/2}\,\sqrt{b}}$$

input
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
((11*x)/(16*a) + (5*b*x^3)/(6*a^2) + (5*b^2*x^5)/(16*a^3))/(a^3 + b^3*x^6
+ 3*a^2*b*x^2 + 3*a*b^2*x^4) + (5*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(7/2)*b
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.05

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^2}\,dx$$
$$= \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3+45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^2+45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^4+15\sqrt{b}\,\sqrt{a}\,ata}{48a^4b\,(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)}$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3 + 45*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**2 + 45*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a*b**2*x**4 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*b**3*x**6 + 33*a**3*b*x + 40*a**2*b**2*x**3 + 15*a*b**3*x**5)/(4
8*a**4*b*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.393 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = -\frac{1}{a^4x}-\frac{bx}{6a^2\left(a+bx^2\right)^3}-\frac{11bx}{24a^3\left(a+bx^2\right)^2}-\frac{19bx}{16a^4\left(a+bx^2\right)}-\frac{35\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{9/2}}$$

output
```
-1/a^4/x-1/6*b*x/a^2/(b*x^2+a)^3-11/24*b*x/a^3/(b*x^2+a)^2-19/16*b*x/a^4/(
b*x^2+a)-35/16*b^(1/2)*arctan(b^(1/2)*x/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = -\frac{48a^3+231a^2bx^2+280ab^2x^4+105b^3x^6}{48a^4x\left(a+bx^2\right)^3}-\frac{35\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{9/2}}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
-1/48*(48*a^3 + 231*a^2*b*x^2 + 280*a*b^2*x^4 + 105*b^3*x^6)/(a^4*x*(a + b
*x^2)^3) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(9/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1380, 27, 253, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a^2+2abx^2+b^2x^4\right)^2}dx$$

$$\Big\downarrow \text{1380}$$

$$b^4 \int \frac{1}{b^4x^2 \left(bx^2+a\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^2 \left(a+bx^2\right)^4}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{7 \int \frac{1}{x^2 \left(bx^2+a\right)^3}dx}{6a}+\frac{1}{6ax \left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{253}$$

$$\frac{7 \left(\frac{5 \int \frac{1}{x^2 \left(bx^2+a\right)^2}dx}{4a}+\frac{1}{4ax(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax \left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{253}$$

$$\frac{7 \left(\frac{5 \left(\frac{3 \int \frac{1}{x^2 \left(bx^2+a\right)}dx}{2a}+\frac{1}{2ax\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax \left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(-\frac{b\int\frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
1/(6*a*x*(a + b*x^2)^3) + (7*(1/(4*a*x*(a + b*x^2)^2) + (5*(1/(2*a*x*(a +
b*x^2)) + (3*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))
/(2*a)))/(4*a)))/(6*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{b\left(\frac{\frac{19}{16}x^{5}b^{2}+\frac{17}{6}ab\,x^{3}+\frac{29}{16}a^{2}x}{\left(b\,x^{2}+a\right)^{3}}+\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{a^{4}}-\frac{1}{a^{4}x}$	65
risch	$\frac{-\frac{35b^{3}x^{6}}{16a^{4}}-\frac{35b^{2}x^{4}}{6a^{3}}-\frac{77b\,x^{2}}{16a^{2}}-\frac{1}{a}}{x\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\left(b\,x^{2}+a\right)}+\frac{35\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{32a^{5}}-\frac{35\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{32a^{5}}$	120

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-b/a^4*((19/16*x^5*b^2+17/6*a*b*x^3+29/16*a^2*x)/(b*x^2+a)^3+35/16/(a*b)^(
1/2)*arctan(b/(a*b)^(1/2)*x))-1/a^4/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.98

$$\int \frac{1}{x^2 (a^2+2abx^2+b^2x^4)^2} dx$$
$$= \left[-\frac{210\,b^3x^6+560\,ab^2x^4+462\,a^2bx^2+96\,a^3-105\,(b^3x^7+3\,ab^2x^5+3\,a^2bx^3+a^3x)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{96\,(a^4b^3x^7+3\,a^5b^2x^5+3\,a^6bx^3+a^7x)}, \right.$$
$$\left. -\frac{105\,b^3x^6+280\,ab^2x^4+231\,a^2bx^2+48\,a^3+105\,(b^3x^7+3\,ab^2x^5+3\,a^2bx^3+a^3x)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{48\,(a^4b^3x^7+3\,a^5b^2x^5+3\,a^6bx^3+a^7x)} \right]$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
[-1/96*(210*b^3*x^6 + 560*a*b^2*x^4 + 462*a^2*b*x^2 + 96*a^3 - 105*(b^3*x^
7 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(
-b/a) - a)/(b*x^2 + a)))/(a^4*b^3*x^7 + 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*
x), -1/48*(105*b^3*x^6 + 280*a*b^2*x^4 + 231*a^2*b*x^2 + 48*a^3 + 105*(b^3
*x^7 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(
a^4*b^3*x^7 + 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x)]
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2 (a^2+2abx^2+b^2x^4)^2} dx = \frac{35\sqrt{-\frac{b}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}}{b}+x\right)}{32} - \frac{35\sqrt{-\frac{b}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}}{b}+x\right)}{32} + \frac{-48a^3-231a^2bx^2-280ab^2x^4-105b^3x^6}{48a^7x+144a^6bx^3+144a^5b^2x^5+48a^4b^3x^7}$$

input
```
integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
35*sqrt(-b/a**9)*log(-a**5*sqrt(-b/a**9)/b + x)/32 - 35*sqrt(-b/a**9)*log(
a**5*sqrt(-b/a**9)/b + x)/32 + (-48*a**3 - 231*a**2*b*x**2 - 280*a*b**2*x*
*4 - 105*b**3*x**6)/(48*a**7*x + 144*a**6*b*x**3 + 144*a**5*b**2*x**5 + 48
*a**4*b**3*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^2} \, dx = -\frac{105 \, b^3x^6 + 280 \, ab^2x^4 + 231 \, a^2bx^2 + 48 \, a^3}{48 \, (a^4b^3x^7 + 3 \, a^5b^2x^5 + 3 \, a^6bx^3 + a^7x)} - \frac{35 \, b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \, \sqrt{ab}a^4}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/48*(105*b^3*x^6 + 280*a*b^2*x^4 + 231*a^2*b*x^2 + 48*a^3)/(a^4*b^3*x^7
+ 3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x) - 35/16*b*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^2} \, dx = -\frac{35 \, b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \, \sqrt{ab}a^4} - \frac{1}{a^4x} - \frac{57 \, b^3x^5 + 136 \, ab^2x^3 + 87 \, a^2bx}{48 \, (bx^2 + a)^3a^4}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-35/16*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/(a^4*x) - 1/48*(57*b^3*
x^5 + 136*a*b^2*x^3 + 87*a^2*b*x)/((b*x^2 + a)^3*a^4)
```

Mupad [B] (verification not implemented)

Time = 17.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^2} \, dx = -\frac{\frac{1}{a} + \frac{77\,b\,x^2}{16\,a^2} + \frac{35\,b^2\,x^4}{6\,a^3} + \frac{35\,b^3\,x^6}{16\,a^4}}{a^3\,x + 3\,a^2\,b\,x^3 + 3\,a\,b^2\,x^5 + b^3\,x^7} - \frac{35\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{9/2}}$$

input
```
int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

output
```
- (1/a + (77*b*x^2)/(16*a^2) + (35*b^2*x^4)/(6*a^3) + (35*b^3*x^6)/(16*a^4
))/(a^3*x + b^3*x^7 + 3*a^2*b*x^3 + 3*a*b^2*x^5) - (35*b^(1/2)*atan((b^(1/
2)*x)/a^(1/2)))/(16*a^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^2} \, dx$$
$$= \frac{-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3x - 315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^3 - 315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^5 - 105\sqrt{b}}{48a^5x\,(b^3x^6 + 3a\,b^2x^4 + 3a^2b\,x^2 + a^3)}$$

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
( - 105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*x - 315*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**3 - 315*sqrt(b)*sqrt(a)*a
tan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*x**5 - 105*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*b**3*x**7 - 48*a**4 - 231*a**3*b*x**2 - 280*a**2*b**2*x
**4 - 105*a*b**3*x**6)/(48*a**5*x*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 +
b**3*x**6))
```

3.394 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{1}{3a^4x^3}+\frac{4b}{a^5x}+\frac{b^2x}{6a^3(a+bx^2)^3}+\frac{17b^2x}{24a^4(a+bx^2)^2}+\frac{41b^2x}{16a^5(a+bx^2)}+\frac{105b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{11/2}}$$

output
```
-1/3/a^4/x^3+4*b/a^5/x+1/6*b^2*x/a^3/(b*x^2+a)^3+17/24*b^2*x/a^4/(b*x^2+a)
^2+41/16*b^2*x/a^5/(b*x^2+a)+105/16*b^(3/2)*arctan(b^(1/2)*x/a^(1/2))/a^(1
1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{\sqrt{a}(-16a^4+144a^3bx^2+693a^2b^2x^4+840ab^3x^6+315b^4x^8)}{x^3(a+bx^2)^3}+315b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48a^{11/2}}$$

input
```
Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
((Sqrt[a]*(-16*a^4 + 144*a^3*b*x^2 + 693*a^2*b^2*x^4 + 840*a*b^3*x^6 + 315
*b^4*x^8))/(x^3*(a + b*x^2)^3) + 315*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(48*a^(11/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1380, 27, 253, 253, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a^2+2abx^2+b^2x^4\right)^2}dx$$

$$\Big\downarrow \text{1380}$$

$$b^4 \int \frac{1}{b^4x^4 \left(bx^2+a\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^4 \left(a+bx^2\right)^4}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{3 \int \frac{1}{x^4 \left(bx^2+a\right)^3}dx}{2a}+\frac{1}{6ax^3 \left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{253}$$

$$\frac{3 \left(\frac{7 \int \frac{1}{x^4 \left(bx^2+a\right)^2}dx}{4a}+\frac{1}{4ax^3 \left(a+bx^2\right)^2}\right)}{2a}+\frac{1}{6ax^3 \left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{253}$$

$$\frac{3\left(\frac{7\left(\frac{5\int \frac{1}{x^4\left(bx^2+a\right)}dx}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}$$

$\downarrow$ 264

$$\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\int \frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}$$

$\downarrow$ 264

$$\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}$$

$\downarrow$ 218

$$\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3(a+bx^2)}\right)}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{2a}+\frac{1}{6ax^3(a+bx^2)^3}$$

input
```
Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
1/(6*a*x^3*(a + b*x^2)^3) + (3*(1/(4*a*x^3*(a + b*x^2)^2) + (7*(1/(2*a*x^3
*(a + b*x^2)) + (5*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/a^(3/2)))/a))/(2*a)))/(4*a)))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{b^{2}\left(\frac{\frac{41}{16}x^{5}b^{2}+\frac{35}{6}ab\,x^{3}+\frac{55}{16}a^{2}x}{\left(b\,x^{2}+a\right)^{3}}+\frac{105\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{a^{5}}-\frac{1}{3a^{4}x^{3}}+\frac{4b}{a^{5}x}$	75
risch	$\frac{\frac{105b^{4}x^{8}}{16a^{5}}+\frac{35b^{3}x^{6}}{2a^{4}}+\frac{231b^{2}x^{4}}{16a^{3}}+\frac{3b\,x^{2}}{a^{2}}-\frac{1}{3a}}{x^{3}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\left(b\,x^{2}+a\right)}+\frac{105\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{32a^{6}}-\frac{105\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{32a^{6}}$	133

input
```
int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/a^5*((41/16*x^5*b^2+35/6*a*b*x^3+55/16*a^2*x)/(b*x^2+a)^3+105/16/(a*b)
^(1/2)*arctan(b/(a*b)^(1/2)*x))-1/3/a^4/x^3+4*b/a^5/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.84

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^2} dx$$
$$= \left[\frac{630 b^4x^8 + 1680 ab^3x^6 + 1386 a^2b^2x^4 + 288 a^3bx^2 - 32 a^4 + 315 (b^4x^9 + 3 ab^3x^7 + 3 a^2b^2x^5 + a^3bx^3) \sqrt{-}}{96 (a^5b^3x^9 + 3 a^6b^2x^7 + 3 a^7bx^5 + a^8x^3)} \right.$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
[1/96*(630*b^4*x^8 + 1680*a*b^3*x^6 + 1386*a^2*b^2*x^4 + 288*a^3*b*x^2 - 3
2*a^4 + 315*(b^4*x^9 + 3*a*b^3*x^7 + 3*a^2*b^2*x^5 + a^3*b*x^3)*sqrt(-b/a)
*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^3*x^9 + 3*a^6*b^2
*x^7 + 3*a^7*b*x^5 + a^8*x^3), 1/48*(315*b^4*x^8 + 840*a*b^3*x^6 + 693*a^2
*b^2*x^4 + 144*a^3*b*x^2 - 16*a^4 + 315*(b^4*x^9 + 3*a*b^3*x^7 + 3*a^2*b^2
*x^5 + a^3*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^5*b^3*x^9 + 3*a^6*b^2*
x^7 + 3*a^7*b*x^5 + a^8*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^2} dx = -\frac{105\sqrt{-\frac{b^3}{a^{11}}} \log \left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{b^2} + x\right)}{32} + \frac{105\sqrt{-\frac{b^3}{a^{11}}} \log \left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{b^2} + x\right)}{32} + \frac{-16a^4 + 144a^3bx^2 + 693a^2b^2x^4 + 840ab^3x^6 + 315b^4x^8}{48a^8x^3 + 144a^7bx^5 + 144a^6b^2x^7 + 48a^5b^3x^9}$$

input
```
integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
-105*sqrt(-b**3/a**11)*log(-a**6*sqrt(-b**3/a**11)/b**2 + x)/32 + 105*sqrt
(-b**3/a**11)*log(a**6*sqrt(-b**3/a**11)/b**2 + x)/32 + (-16*a**4 + 144*a*
*3*b*x**2 + 693*a**2*b**2*x**4 + 840*a*b**3*x**6 + 315*b**4*x**8)/(48*a**8
*x**3 + 144*a**7*b*x**5 + 144*a**6*b**2*x**7 + 48*a**5*b**3*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^2} dx = \frac{315 b^4x^8 + 840 ab^3x^6 + 693 a^2b^2x^4 + 144 a^3bx^2 - 16 a^4}{48 (a^5b^3x^9 + 3 a^6b^2x^7 + 3 a^7bx^5 + a^8x^3)} + \frac{105 b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \sqrt{ab}a^5}$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/48*(315*b^4*x^8 + 840*a*b^3*x^6 + 693*a^2*b^2*x^4 + 144*a^3*b*x^2 - 16*a
^4)/(a^5*b^3*x^9 + 3*a^6*b^2*x^7 + 3*a^7*b*x^5 + a^8*x^3) + 105/16*b^2*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^2} dx = \frac{105 b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \sqrt{ab}a^5} + \frac{315 b^4x^8 + 840 ab^3x^6 + 693 a^2b^2x^4 + 144 a^3bx^2 - 16 a^4}{48 (bx^3 + ax)^3 a^5}$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
105/16*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/48*(315*b^4*x^8 + 840
*a*b^3*x^6 + 693*a^2*b^2*x^4 + 144*a^3*b*x^2 - 16*a^4)/((b*x^3 + a*x)^3*a^
5)
```

Mupad [B] (verification not implemented)

Time = 17.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^2} \, dx = \frac{\frac{3\,b\,x^2}{a^2} - \frac{1}{3\,a} + \frac{231\,b^2\,x^4}{16\,a^3} + \frac{35\,b^3\,x^6}{2\,a^4} + \frac{105\,b^4\,x^8}{16\,a^5}}{a^3\,x^3 + 3\,a^2\,b\,x^5 + 3\,a\,b^2\,x^7 + b^3\,x^9} + \frac{105\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{11/2}}$$

input
```
int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

output
```
((3*b*x^2)/a^2 - 1/(3*a) + (231*b^2*x^4)/(16*a^3) + (35*b^3*x^6)/(2*a^4) +
 (105*b^4*x^8)/(16*a^5))/(a^3*x^3 + b^3*x^9 + 3*a^2*b*x^5 + 3*a*b^2*x^7) +
 (105*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.74

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^2} \, dx = \frac{315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^3 + 945\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2x^5 + 945\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^3x^7 + 315\sqrt{}}{48a^6x^3\,(b^3x^6 + 3a\,b^2x^4 + 3a^2b}$$

input
```
int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*x**3 + 945*sqrt(
b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**5 + 945*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*x**7 + 315*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*b**4*x**9 - 16*a**5 + 144*a**4*b*x**2 + 693*a**3*b
**2*x**4 + 840*a**2*b**3*x**6 + 315*a*b**4*x**8)/(48*a**6*x**3*(a**3 + 3*a
**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.395 $\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{1}{x^6 (a^2+2abx^2+b^2x^4)^2} \, dx = -\frac{1}{5a^4x^5}+\frac{4b}{3a^5x^3}-\frac{10b^2}{a^6x}-\frac{b^3x}{6a^4(a+bx^2)^3}-\frac{23b^3x}{24a^5(a+bx^2)^2}-\frac{71b^3x}{16a^6(a+bx^2)}-\frac{231b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{13/2}}$$

output
```
-1/5/a^4/x^5+4/3*b/a^5/x^3-10*b^2/a^6/x-1/6*b^3*x/a^4/(b*x^2+a)^3-23/24*b^
3*x/a^5/(b*x^2+a)^2-71/16*b^3*x/a^6/(b*x^2+a)-231/16*b^(5/2)*arctan(b^(1/2
)*x/a^(1/2))/a^(13/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^6 \left(a^2+2abx^2+b^2x^4\right)^2} \, dx = -\frac{48a^5-176a^4bx^2+1584a^3b^2x^4+7623a^2b^3x^6+9240ab^4x^8+3465b^5x^{10}}{240a^6x^5\left(a+bx^2\right)^3} - \frac{231b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{13/2}}$$

input
```
Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
-1/240*(48*a^5 - 176*a^4*b*x^2 + 1584*a^3*b^2*x^4 + 7623*a^2*b^3*x^6 + 924
0*a*b^4*x^8 + 3465*b^5*x^10)/(a^6*x^5*(a + b*x^2)^3) - (231*b^(5/2)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(13/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1380, 27, 253, 253, 253, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a^2+2abx^2+b^2x^4\right)^2} \, dx$$

$$\Big\downarrow \text{1380}$$

$$b^4 \int \frac{1}{b^4x^6\left(bx^2+a\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^6\left(a+bx^2\right)^4}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{11\int \frac{1}{x^6(bx^2+a)^3}dx}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

$$\Big\downarrow \textbf{253}$$

$$\frac{11\left(\frac{9\int \frac{1}{x^6\left(bx^2+a\right)^2}dx}{4a}+\frac{1}{4ax^5(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

$$\Big\downarrow \textbf{253}$$

$$\frac{11\left(\frac{9\left(\frac{7\int \frac{1}{x^6\left(bx^2+a\right)}dx}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^5(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

$$\Big\downarrow \textbf{264}$$

$$\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\int \frac{1}{x^4\left(bx^2+a\right)}dx}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^5(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

$$\Big\downarrow \textbf{264}$$

$$\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\int \frac{1}{x^2(bx^2+a)}dx}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5(a+bx^2)}\right)}{4a}+\frac{1}{4ax^5(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax^5(a+bx^2)^3}$$

$\downarrow$ 264

$$\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^5(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

$\downarrow$ 218

$$\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5(a+bx^2)}\right)}{4a}+\frac{1}{4ax^5(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax^5(a+bx^2)^3}$$

input
```
Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
1/(6*a*x^5*(a + b*x^2)^3) + (11*(1/(4*a*x^5*(a + b*x^2)^2) + (9*(1/(2*a*x^
5*(a + b*x^2)) + (7*(-1/5*1/(a*x^5) - (b*(-1/3*1/(a*x^3) - (b*(-(1/(a*x))
- (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/a))/(2*a)))/(4*a)))/
(6*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{b^{3}\left(\frac{\frac{71}{16}x^{5}b^{2}+\frac{59}{6}ab\,x^{3}+\frac{89}{16}a^{2}x}{\left(b\,x^{2}+a\right)^{3}}+\frac{231\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}}\right)}{a^{6}}-\frac{1}{5a^{4}x^{5}}-\frac{10b^{2}}{a^{6}x}+\frac{4b}{3a^{5}x^{3}}$	87
risch	$\frac{-\frac{231b^{5}x^{10}}{16a^{6}}-\frac{77b^{4}x^{8}}{2a^{5}}-\frac{2541b^{3}x^{6}}{80a^{4}}-\frac{33b^{2}x^{4}}{5a^{3}}+\frac{11b\,x^{2}}{15a^{2}}-\frac{1}{5a}}{x^{5}(b^{2}x^{4}+2ab\,x^{2}+a^{2})(b\,x^{2}+a)}+\frac{231\sqrt{-ab}\,b^{2}\ln\left(-bx+\sqrt{-ab}\right)}{32a^{7}}-\frac{231\sqrt{-ab}\,b^{2}\ln\left(-bx-\sqrt{-ab}\right)}{32a^{7}}$	148

input
```
int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^6*b^3*((71/16*x^5*b^2+59/6*a*b*x^3+89/16*a^2*x)/(b*x^2+a)^3+231/16/(a
*b)^(1/2)*arctan(b/(a*b)^(1/2)*x))-1/5/a^4/x^5-10*b^2/a^6/x+4/3*b/a^5/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$$

$$= \left[-\frac{6930\,b^5x^{10}+18480\,ab^4x^8+15246\,a^2b^3x^6+3168\,a^3b^2x^4-352\,a^4bx^2+96\,a^5-3465\,(b^5x^{11}+3\,ab^4x^9}{480\,(a^6b^3x^{11}+3\,a^7b^2x^9+3\,a^8bx^7+a^9x^5)}\right.$$

$$-\frac{3465\,b^5x^{10}+9240\,ab^4x^8+7623\,a^2b^3x^6+1584\,a^3b^2x^4-176\,a^4bx^2+48\,a^5+3465\,(b^5x^{11}+3\,ab^4x^9+3}{240\,(a^6b^3x^{11}+3\,a^7b^2x^9+3\,a^8bx^7+a^9x^5)}$$

input
```
integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
[-1/480*(6930*b^5*x^10 + 18480*a*b^4*x^8 + 15246*a^2*b^3*x^6 + 3168*a^3*b^
2*x^4 - 352*a^4*b*x^2 + 96*a^5 - 3465*(b^5*x^11 + 3*a*b^4*x^9 + 3*a^2*b^3*
x^7 + a^3*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
a)))/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 + a^9*x^5), -1/240*(3465*
b^5*x^10 + 9240*a*b^4*x^8 + 7623*a^2*b^3*x^6 + 1584*a^3*b^2*x^4 - 176*a^4*
b*x^2 + 48*a^5 + 3465*(b^5*x^11 + 3*a*b^4*x^9 + 3*a^2*b^3*x^7 + a^3*b^2*x^
5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*
x^7 + a^9*x^5)]
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)^2} \, dx = \frac{231\sqrt{-\frac{b^5}{a^{13}}} \log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}}{b^3} + x\right)}{32} - \frac{231\sqrt{-\frac{b^5}{a^{13}}} \log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}}{b^3} + x\right)}{32} + \frac{-48a^5 + 176a^4bx^2 - 1584a^3b^2x^4 - 7623a^2b^3x^6 - 9240ab^4x^8 - 3465b^5x^{10}}{240a^9x^5 + 720a^8bx^7 + 720a^7b^2x^9 + 240a^6b^3x^{11}}$$

input
```
integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
231*sqrt(-b**5/a**13)*log(-a**7*sqrt(-b**5/a**13)/b**3 + x)/32 - 231*sqrt(
-b**5/a**13)*log(a**7*sqrt(-b**5/a**13)/b**3 + x)/32 + (-48*a**5 + 176*a**
4*b*x**2 - 1584*a**3*b**2*x**4 - 7623*a**2*b**3*x**6 - 9240*a*b**4*x**8 -
3465*b**5*x**10)/(240*a**9*x**5 + 720*a**8*b*x**7 + 720*a**7*b**2*x**9 + 2
40*a**6*b**3*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)^2} \, dx = -\frac{3465 \, b^5 x^{10} + 9240 \, ab^4 x^8 + 7623 \, a^2 b^3 x^6 + 1584 \, a^3 b^2 x^4 - 176 \, a^4 b x^2 + 48 \, a^5}{240 \, (a^6 b^3 x^{11} + 3 \, a^7 b^2 x^9 + 3 \, a^8 b x^7 + a^9 x^5)} - \frac{231 \, b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \, \sqrt{ab} a^6}$$

input
```
integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/240*(3465*b^5*x^10 + 9240*a*b^4*x^8 + 7623*a^2*b^3*x^6 + 1584*a^3*b^2*x
^4 - 176*a^4*b*x^2 + 48*a^5)/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^8*b*x^7 +
 a^9*x^5) - 231/16*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)^2} \, dx = -\frac{231 \, b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16 \, \sqrt{ab} a^6} - \frac{213 \, b^5 x^5 + 472 \, ab^4 x^3 + 267 \, a^2 b^3 x}{48 \, (bx^2 + a)^3 a^6} - \frac{150 \, b^2 x^4 - 20 \, abx^2 + 3 \, a^2}{15 \, a^6 x^5}$$

input
```
integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-231/16*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/48*(213*b^5*x^5 + 47
2*a*b^4*x^3 + 267*a^2*b^3*x)/((b*x^2 + a)^3*a^6) - 1/15*(150*b^2*x^4 - 20*
a*b*x^2 + 3*a^2)/(a^6*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)^2} dx = -\frac{\frac{1}{5a} - \frac{11 b x^2}{15 a^2} + \frac{33 b^2 x^4}{5 a^3} + \frac{2541 b^3 x^6}{80 a^4} + \frac{77 b^4 x^8}{2 a^5} + \frac{231 b^5 x^{10}}{16 a^6}}{a^3 x^5 + 3 a^2 b x^7 + 3 a b^2 x^9 + b^3 x^{11}} - \frac{231 b^{5/2} \operatorname{atan}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{16 a^{13/2}}$$

input
```
int(1/(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

output
```
- (1/(5*a) - (11*b*x^2)/(15*a^2) + (33*b^2*x^4)/(5*a^3) + (2541*b^3*x^6)/(
80*a^4) + (77*b^4*x^8)/(2*a^5) + (231*b^5*x^10)/(16*a^6))/(a^3*x^5 + b^3*x
^11 + 3*a^2*b*x^7 + 3*a*b^2*x^9) - (231*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))
/(16*a^(13/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.66

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)^2} dx = \frac{-3465\sqrt{b}\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right) a^3 b^2 x^5 - 10395\sqrt{b}\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right) a^2 b^3 x^7 - 10395\sqrt{b}\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right) a\, b^4 x^9}{240 a^7 x^5 (b^3 x^6 +}$$

input
```
int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
( - 3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**5 - 10
395*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*x**7 - 10395*s
qrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*x**9 - 3465*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*x**11 - 48*a**6 + 176*a**5*b*x**2
 - 1584*a**4*b**2*x**4 - 7623*a**3*b**3*x**6 - 9240*a**2*b**4*x**8 - 3465*
a*b**5*x**10)/(240*a**7*x**5*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*
x**6))
```

3.396 $\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}+\frac{a^7}{10b^8(a+bx^2)^5}-\frac{7a^6}{8b^8(a+bx^2)^4}+\frac{7a^5}{2b^8(a+bx^2)^3}-\frac{35a^4}{4b^8(a+bx^2)^2}+\frac{35a^3}{2b^8(a+bx^2)}+\frac{21a^2\log(a+bx^2)}{2b^8}$$

output
```
-3*a*x^2/b^7+1/4*x^4/b^6+1/10*a^7/b^8/(b*x^2+a)^5-7/8*a^6/b^8/(b*x^2+a)^4+
7/2*a^5/b^8/(b*x^2+a)^3-35/4*a^4/b^8/(b*x^2+a)^2+35/2*a^3/b^8/(b*x^2+a)+21
/2*a^2*ln(b*x^2+a)/b^8
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{459a^7+1875a^6bx^2+2700a^5b^2x^4+1300a^4b^3x^6-400a^3b^4x^8-500a^2b^5x^{10}-70ab^6x^{12}+10b^7x^{14}+420a}{40b^8(a+bx^2)^5}$$

input
```
Integrate[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(459*a^7 + 1875*a^6*b*x^2 + 2700*a^5*b^2*x^4 + 1300*a^4*b^3*x^6 - 400*a^3*
b^4*x^8 - 500*a^2*b^5*x^10 - 70*a*b^6*x^12 + 10*b^7*x^14 + 420*a^2*(a + b*
x^2)^5*Log[a + b*x^2])/(40*b^8*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{x^{15}}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{15}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^{14}}{(bx^2+a)^6}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(-\frac{a^7}{b^7(bx^2+a)^6}+\frac{7a^6}{b^7(bx^2+a)^5}-\frac{21a^5}{b^7(bx^2+a)^4}+\frac{35a^4}{b^7(bx^2+a)^3}-\frac{35a^3}{b^7(bx^2+a)^2}+\frac{21a^2}{b^7(bx^2+a)}-\frac{6a}{b^7}+\frac{x^2}{b^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^7}{5b^8\left(a+bx^2\right)^5}-\frac{7a^6}{4b^8\left(a+bx^2\right)^4}+\frac{7a^5}{b^8\left(a+bx^2\right)^3}-\frac{35a^4}{2b^8\left(a+bx^2\right)^2}+\frac{35a^3}{b^8\left(a+bx^2\right)}+\frac{21a^2\log\left(a+bx^2\right)}{b^8}-\frac{6ax}{b^7}\right.$$

input
```
Int[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
((-6*a*x^2)/b^7 + x^4/(2*b^6) + a^7/(5*b^8*(a + b*x^2)^5) - (7*a^6)/(4*b^8
*(a + b*x^2)^4) + (7*a^5)/(b^8*(a + b*x^2)^3) - (35*a^4)/(2*b^8*(a + b*x^2
)^2) + (35*a^3)/(b^8*(a + b*x^2)) + (21*a^2*Log[a + b*x^2])/b^8)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.74

method	result
norman	$\frac{\frac{x^{14}}{4b}-\frac{7a\,x^{12}}{4b^{2}}+\frac{959a^{7}}{40b^{8}}+\frac{105a^{3}x^{8}}{2b^{4}}+\frac{315a^{4}x^{6}}{2b^{5}}+\frac{385a^{5}x^{4}}{2b^{6}}+\frac{875a^{6}x^{2}}{8b^{7}}}{\left(b\,x^{2}+a\right)^{5}}+\frac{21a^{2}\ln\left(b\,x^{2}+a\right)}{2b^{8}}$
default	$\frac{\left(-b\,x^{2}+6a\right)^{2}}{4b^{8}}+\frac{a^{2}\left(\frac{21\ln\left(b\,x^{2}+a\right)}{b}+\frac{7a^{3}}{b\left(b\,x^{2}+a\right)^{3}}+\frac{a^{5}}{5b\left(b\,x^{2}+a\right)^{5}}-\frac{35a^{2}}{2b\left(b\,x^{2}+a\right)^{2}}+\frac{35a}{b\left(b\,x^{2}+a\right)}-\frac{7a^{4}}{4b\left(b\,x^{2}+a\right)^{4}}\right)}{2b^{7}}$
risch	$\frac{x^{4}}{4b^{6}}-\frac{3a\,x^{2}}{b^{7}}+\frac{9a^{2}}{b^{8}}+\frac{\frac{35a^{3}b^{3}x^{8}}{2}+\frac{245a^{4}b^{2}x^{6}}{4}+\frac{329a^{5}b\,x^{4}}{4}+\frac{399a^{6}x^{2}}{8}+\frac{459a^{7}}{40b}}{b^{7}(b\,x^{2}+a)(b^{2}x^{4}+2ab\,x^{2}+a^{2})^{2}}+\frac{21a^{2}\ln\left(b\,x^{2}+a\right)}{2b^{8}}$
parallelrisch	$\frac{10b^{7}x^{14}-70a\,x^{12}b^{6}+420\ln(b\,x^{2}+a)x^{10}a^{2}b^{5}+2100\ln(b\,x^{2}+a)x^{8}a^{3}b^{4}+2100x^{8}a^{3}b^{4}+4200\ln(b\,x^{2}+a)x^{6}a^{4}b^{3}+6300x^{6}a^{4}b^{3}+}{40b^{8}(b^{2}x^{4}+2ab\,x^{2}+a^{2})^{2}(b\,x}$

input
```
int(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/4/b*x^14-7/4*a/b^2*x^12+959/40*a^7/b^8+105/2*a^3/b^4*x^8+315/2*a^4/b^5*
x^6+385/2*a^5/b^6*x^4+875/8*a^6/b^7*x^2)/(b*x^2+a)^5+21/2*a^2*ln(b*x^2+a)/
b^8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.53

$$\int \frac{x^{15}}{\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{3}}\,dx = \frac{10\,b^{7}x^{14}-70\,ab^{6}x^{12}-500\,a^{2}b^{5}x^{10}-400\,a^{3}b^{4}x^{8}+1300\,a^{4}b^{3}x^{6}+2700\,a^{5}b^{2}x^{4}+1875\,a^{6}bx^{2}+459\,a^{7}+42}{40\,(b^{13}x^{10}+5\,ab^{12}x^{8}+10\,a^{2}b^{11}x^{6}+10\,a^{3}b^{10}x^{4}}$$

input
```
integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/40*(10*b^7*x^14 - 70*a*b^6*x^12 - 500*a^2*b^5*x^10 - 400*a^3*b^4*x^8 + 1
300*a^4*b^3*x^6 + 2700*a^5*b^2*x^4 + 1875*a^6*b*x^2 + 459*a^7 + 420*(a^2*b
^5*x^10 + 5*a^3*b^4*x^8 + 10*a^4*b^3*x^6 + 10*a^5*b^2*x^4 + 5*a^6*b*x^2 +
a^7)*log(b*x^2 + a))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*
b^10*x^4 + 5*a^4*b^9*x^2 + a^5*b^8)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.13

$$\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$=\frac{21a^2\log(a+bx^2)}{2b^8}-\frac{3ax^2}{b^7}+\frac{459a^7+1995a^6bx^2+3290a^5b^2x^4+2450a^4b^3x^6+700a^3b^4x^8}{40a^5b^8+200a^4b^9x^2+400a^3b^{10}x^4+400a^2b^{11}x^6+200ab^{12}x^8+40b^{13}x^{10}}+\frac{x^4}{4b^6}$$

input
```
integrate(x**15/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
21*a**2*log(a + b*x**2)/(2*b**8) - 3*a*x**2/b**7 + (459*a**7 + 1995*a**6*b
*x**2 + 3290*a**5*b**2*x**4 + 2450*a**4*b**3*x**6 + 700*a**3*b**4*x**8)/(4
0*a**5*b**8 + 200*a**4*b**9*x**2 + 400*a**3*b**10*x**4 + 400*a**2*b**11*x*
*6 + 200*a*b**12*x**8 + 40*b**13*x**10) + x**4/(4*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$=\frac{700\,a^3b^4x^8+2450\,a^4b^3x^6+3290\,a^5b^2x^4+1995\,a^6bx^2+459\,a^7}{40\,(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a^5b^8)}+\frac{21\,a^2\log(bx^2+a)}{2\,b^8}+\frac{bx^4-12\,ax^2}{4\,b^7}$$

input
```
integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/40*(700*a^3*b^4*x^8 + 2450*a^4*b^3*x^6 + 3290*a^5*b^2*x^4 + 1995*a^6*b*x
^2 + 459*a^7)/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^
4 + 5*a^4*b^9*x^2 + a^5*b^8) + 21/2*a^2*log(b*x^2 + a)/b^8 + 1/4*(b*x^4 -
12*a*x^2)/b^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{21\,a^2\log\left(|bx^2+a|\right)}{2\,b^8} + \frac{b^6x^4-12\,ab^5x^2}{4\,b^{12}} - \frac{959\,a^2b^5x^{10}+4095\,a^3b^4x^8+7140\,a^4b^3x^6+6300\,a^5b^2x^4+2800\,a^6bx^2+500\,a^7}{40\,(bx^2+a)^5b^8}$$

input
```
integrate(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
21/2*a^2*log(abs(b*x^2 + a))/b^8 + 1/4*(b^6*x^4 - 12*a*b^5*x^2)/b^12 - 1/4
0*(959*a^2*b^5*x^10 + 4095*a^3*b^4*x^8 + 7140*a^4*b^3*x^6 + 6300*a^5*b^2*x
^4 + 2800*a^6*b*x^2 + 500*a^7)/((b*x^2 + a)^5*b^8)
```

Mupad [B] (verification not implemented)

Time = 17.75 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{\frac{459\,a^7}{40\,b}+\frac{399\,a^6\,x^2}{8}+\frac{329\,a^5\,b\,x^4}{4}+\frac{245\,a^4\,b^2\,x^6}{4}+\frac{35\,a^3\,b^3\,x^8}{2}}{a^5\,b^7+5\,a^4\,b^8\,x^2+10\,a^3\,b^9\,x^4+10\,a^2\,b^{10}\,x^6+5\,a\,b^{11}\,x^8+b^{12}\,x^{10}} + \frac{x^4}{4\,b^6} - \frac{3\,a\,x^2}{b^7} + \frac{21\,a^2\,\ln\left(b\,x^2+a\right)}{2\,b^8}$$

input
```
int(x^15/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((459*a^7)/(40*b) + (399*a^6*x^2)/8 + (329*a^5*b*x^4)/4 + (245*a^4*b^2*x^6
)/4 + (35*a^3*b^3*x^8)/2)/(a^5*b^7 + b^12*x^10 + 5*a*b^11*x^8 + 5*a^4*b^8*
x^2 + 10*a^3*b^9*x^4 + 10*a^2*b^10*x^6) + x^4/(4*b^6) - (3*a*x^2)/b^7 + (2
1*a^2*log(a + b*x^2))/(2*b^8)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.72

$$\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{420\log(b\,x^2+a)\,a^7+2100\log(b\,x^2+a)\,a^6b\,x^2+4200\log(b\,x^2+a)\,a^5b^2x^4+4200\log(b\,x^2+a)\,a^4b^3x^6+}{40b^8\,(b^5x^{10}-}$$

input
```
int(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(420*log(a + b*x**2)*a**7 + 2100*log(a + b*x**2)*a**6*b*x**2 + 4200*log(a
+ b*x**2)*a**5*b**2*x**4 + 4200*log(a + b*x**2)*a**4*b**3*x**6 + 2100*log(
a + b*x**2)*a**3*b**4*x**8 + 420*log(a + b*x**2)*a**2*b**5*x**10 + 539*a**
7 + 2275*a**6*b*x**2 + 3500*a**5*b**2*x**4 + 2100*a**4*b**3*x**6 - 420*a**
2*b**5*x**10 - 70*a*b**6*x**12 + 10*b**7*x**14)/(40*b**8*(a**5 + 5*a**4*b*
x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10)
)
```

3.397 $\int \frac{x^{13}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{x^{13}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{x^2}{2b^6}-\frac{a^6}{10b^7(a+bx^2)^5}+\frac{3a^5}{4b^7(a+bx^2)^4}-\frac{5a^4}{2b^7(a+bx^2)^3}+\frac{5a^3}{b^7(a+bx^2)^2}-\frac{15a^2}{2b^7(a+bx^2)}-\frac{3a\log(a+bx^2)}{b^7}$$

output
```
1/2*x^2/b^6-1/10*a^6/b^7/(b*x^2+a)^5+3/4*a^5/b^7/(b*x^2+a)^4-5/2*a^4/b^7/(
b*x^2+a)^3+5*a^3/b^7/(b*x^2+a)^2-15/2*a^2/b^7/(b*x^2+a)-3*a*ln(b*x^2+a)/b^
7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{x^{13}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{87a^6+375a^5bx^2+600a^4b^2x^4+400a^3b^3x^6+50a^2b^4x^8-50ab^5x^{10}-10b^6x^{12}+60a(a+bx^2)^5\log(a+}{20b^7(a+bx^2)^5}$$

input
```
Integrate[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/20*(87*a^6 + 375*a^5*b*x^2 + 600*a^4*b^2*x^4 + 400*a^3*b^3*x^6 + 50*a^2
*b^4*x^8 - 50*a*b^5*x^10 - 10*b^6*x^12 + 60*a*(a + b*x^2)^5*Log[a + b*x^2]
)/(b^7*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^6\int \frac{x^{13}}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x^{13}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^2+a)^6}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{a^6}{b^6(bx^2+a)^6}-\frac{6a^5}{b^6(bx^2+a)^5}+\frac{15a^4}{b^6(bx^2+a)^4}-\frac{20a^3}{b^6(bx^2+a)^3}+\frac{15a^2}{b^6(bx^2+a)^2}-\frac{6a}{b^6(bx^2+a)}+\frac{1}{b^6}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^6}{5b^7(a+bx^2)^5}+\frac{3a^5}{2b^7(a+bx^2)^4}-\frac{5a^4}{b^7(a+bx^2)^3}+\frac{10a^3}{b^7(a+bx^2)^2}-\frac{15a^2}{b^7(a+bx^2)}-\frac{6a\log(a+bx^2)}{b^7}+\frac{x^2}{b^6}\right)$$

input
```
Int[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(x^2/b^6 - a^6/(5*b^7*(a + b*x^2)^5) + (3*a^5)/(2*b^7*(a + b*x^2)^4) - (5*
a^4)/(b^7*(a + b*x^2)^3) + (10*a^3)/(b^7*(a + b*x^2)^2) - (15*a^2)/(b^7*(a
 + b*x^2)) - (6*a*Log[a + b*x^2])/b^7)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

method	result
norman	$\frac{\frac{x^{12}}{2b}-\frac{137a^6}{20b^7}-\frac{15a^2x^8}{b^3}-\frac{45a^3x^6}{b^4}-\frac{55a^4x^4}{b^5}-\frac{125a^5x^2}{4b^6}}{(b\,x^2+a)^5}-\frac{3a\ln\left(b\,x^2+a\right)}{b^7}$
risch	$\frac{x^2}{2b^6}+\frac{-\frac{15a^2b^3x^8}{2}-25a^3b^2x^6-\frac{65b\,a^4x^4}{2}-\frac{77a^5x^2}{4}-\frac{87a^6}{20b}}{b^6(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}-\frac{3a\ln\left(b\,x^2+a\right)}{b^7}$
default	$\frac{x^2}{2b^6}-\frac{a\left(\frac{6\ln\left(b\,x^2+a\right)}{b}-\frac{3a^4}{2b\left(b\,x^2+a\right)^4}-\frac{10a^2}{b\left(b\,x^2+a\right)^2}+\frac{5a^3}{b\left(b\,x^2+a\right)^3}+\frac{a^5}{5b\left(b\,x^2+a\right)^5}+\frac{15a}{b\left(b\,x^2+a\right)}\right)}{2b^6}$
parallelrisch	$-\frac{-10b^6x^{12}+60\ln\left(b\,x^2+a\right)x^{10}a\,b^5+300\ln\left(b\,x^2+a\right)x^8a^2b^4+300a^2b^4x^8+600\ln\left(b\,x^2+a\right)x^6a^3b^3+900a^3b^3x^6+600\ln\left(b\,x^2+a\right.}{20b^7(b^2x^4+2ab\,x^2+a^2)^2(b\,x^2+a)}$

input
```
int(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2/b*x^12-137/20*a^6/b^7-15*a^2/b^3*x^8-45*a^3/b^4*x^6-55*a^4/b^5*x^4-12
5/4*a^5/b^6*x^2)/(b*x^2+a)^5-3*a*ln(b*x^2+a)/b^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.61

$$\int \frac{x^{13}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{10\,b^6x^{12}+50\,ab^5x^{10}-50\,a^2b^4x^8-400\,a^3b^3x^6-600\,a^4b^2x^4-375\,a^5bx^2-87\,a^6-60\,(ab^5x^{10}+5\,a^2b^4x^8}{20\,(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2}$$

input
```
integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/20*(10*b^6*x^12 + 50*a*b^5*x^10 - 50*a^2*b^4*x^8 - 400*a^3*b^3*x^6 - 600
*a^4*b^2*x^4 - 375*a^5*b*x^2 - 87*a^6 - 60*(a*b^5*x^10 + 5*a^2*b^4*x^8 + 1
0*a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 + a^6)*log(b*x^2 + a))/(b^12*
x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a
^5*b^7)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.17

$$\int \frac{x^{13}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = -\frac{3a\log\left(a+bx^2\right)}{b^7} + \frac{-87a^6-385a^5bx^2-650a^4b^2x^4-500a^3b^3x^6-150a^2b^4x^8}{20a^5b^7+100a^4b^8x^2+200a^3b^9x^4+200a^2b^{10}x^6+100ab^{11}x^8+20b^{12}x^{10}} + \frac{x^2}{2b^6}$$

input
```
integrate(x**13/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
-3*a*log(a + b*x**2)/b**7 + (-87*a**6 - 385*a**5*b*x**2 - 650*a**4*b**2*x*
*4 - 500*a**3*b**3*x**6 - 150*a**2*b**4*x**8)/(20*a**5*b**7 + 100*a**4*b**
8*x**2 + 200*a**3*b**9*x**4 + 200*a**2*b**10*x**6 + 100*a*b**11*x**8 + 20*
b**12*x**10) + x**2/(2*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

$$\int \frac{x^{13}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = -\frac{150\,a^2b^4x^8+500\,a^3b^3x^6+650\,a^4b^2x^4+385\,a^5bx^2+87\,a^6}{20\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7\right)} + \frac{x^2}{2\,b^6} - \frac{3\,a\log\left(bx^2+a\right)}{b^7}$$

input
```
integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/20*(150*a^2*b^4*x^8 + 500*a^3*b^3*x^6 + 650*a^4*b^2*x^4 + 385*a^5*b*x^2
 + 87*a^6)/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 +
5*a^4*b^8*x^2 + a^5*b^7) + 1/2*x^2/b^6 - 3*a*log(b*x^2 + a)/b^7
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int \frac{x^{13}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{x^2}{2\,b^6}-\frac{3\,a\log\left(|bx^2+a|\right)}{b^7} + \frac{137\,ab^5x^{10}+535\,a^2b^4x^8+870\,a^3b^3x^6+720\,a^4b^2x^4+300\,a^5bx^2+50\,a^6}{20\,(bx^2+a)^5b^7}$$

input
```
integrate(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/2*x^2/b^6 - 3*a*log(abs(b*x^2 + a))/b^7 + 1/20*(137*a*b^5*x^10 + 535*a^2
*b^4*x^8 + 870*a^3*b^3*x^6 + 720*a^4*b^2*x^4 + 300*a^5*b*x^2 + 50*a^6)/((b
*x^2 + a)^5*b^7)
```

Mupad [B] (verification not implemented)

Time = 17.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

$$\int \frac{x^{13}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{x^2}{2\,b^6}-\frac{\frac{87\,a^6}{20\,b}+\frac{77\,a^5\,x^2}{4}+\frac{65\,a^4\,b\,x^4}{2}+25\,a^3\,b^2\,x^6+\frac{15\,a^2\,b^3\,x^8}{2}}{a^5\,b^6+5\,a^4\,b^7\,x^2+10\,a^3\,b^8\,x^4+10\,a^2\,b^9\,x^6+5\,a\,b^{10}\,x^8+b^{11}\,x^{10}} - \frac{3\,a\,\ln\left(b\,x^2+a\right)}{b^7}$$

input
```
int(x^13/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
x^2/(2*b^6) - ((87*a^6)/(20*b) + (77*a^5*x^2)/4 + (65*a^4*b*x^4)/2 + 25*a^
3*b^2*x^6 + (15*a^2*b^3*x^8)/2)/(a^5*b^6 + b^11*x^10 + 5*a*b^10*x^8 + 5*a^
4*b^7*x^2 + 10*a^3*b^8*x^4 + 10*a^2*b^9*x^6) - (3*a*log(a + b*x^2))/b^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.83

$$\int \frac{x^{13}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{-60\log(b\,x^2+a)\,a^6-300\log(b\,x^2+a)\,a^5b\,x^2-600\log(b\,x^2+a)\,a^4b^2x^4-600\log(b\,x^2+a)\,a^3b^3x^6-30}{20b^7\,(b^5x^{10}+5a\,b^4x^8+1}$$

input
```
int(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
( - 60*log(a + b*x**2)*a**6 - 300*log(a + b*x**2)*a**5*b*x**2 - 600*log(a
+ b*x**2)*a**4*b**2*x**4 - 600*log(a + b*x**2)*a**3*b**3*x**6 - 300*log(a
+ b*x**2)*a**2*b**4*x**8 - 60*log(a + b*x**2)*a*b**5*x**10 - 77*a**6 - 325
*a**5*b*x**2 - 500*a**4*b**2*x**4 - 300*a**3*b**3*x**6 + 60*a*b**5*x**10 +
 10*b**6*x**12)/(20*b**7*(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a*
*2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10))
```

3.398 $\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^3} dx = \frac{a^5}{10b^6(a+bx^2)^5} - \frac{5a^4}{8b^6(a+bx^2)^4} + \frac{5a^3}{3b^6(a+bx^2)^3} - \frac{5a^2}{2b^6(a+bx^2)^2} + \frac{5a}{2b^6(a+bx^2)} + \frac{\log(a+bx^2)}{2b^6}$$

output
```
1/10*a^5/b^6/(b*x^2+a)^5-5/8*a^4/b^6/(b*x^2+a)^4+5/3*a^3/b^6/(b*x^2+a)^3-5
/2*a^2/b^6/(b*x^2+a)^2+5/2*a/b^6/(b*x^2+a)+1/2*ln(b*x^2+a)/b^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.66

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^3} dx = \frac{\frac{a(137a^4+625a^3bx^2+1100a^2b^2x^4+900ab^3x^6+300b^4x^8)}{(a+bx^2)^5} + 60\log(a+bx^2)}{120b^6}$$

input
```
Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
((a*(137*a^4 + 625*a^3*b*x^2 + 1100*a^2*b^2*x^4 + 900*a*b^3*x^6 + 300*b^4*
x^8))/(a + b*x^2)^5 + 60*Log[a + b*x^2])/(120*b^6)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{x^{11}}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{11}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^{10}}{(bx^2+a)^6}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(-\frac{a^5}{b^5(bx^2+a)^6}+\frac{5a^4}{b^5(bx^2+a)^5}-\frac{10a^3}{b^5(bx^2+a)^4}+\frac{10a^2}{b^5(bx^2+a)^3}-\frac{5a}{b^5(bx^2+a)^2}+\frac{1}{b^5(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^5}{5b^6(a+bx^2)^5}-\frac{5a^4}{4b^6(a+bx^2)^4}+\frac{10a^3}{3b^6(a+bx^2)^3}-\frac{5a^2}{b^6(a+bx^2)^2}+\frac{5a}{b^6(a+bx^2)}+\frac{\log(a+bx^2)}{b^6}\right)$$

input
```
Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^5/(5*b^6*(a + b*x^2)^5) - (5*a^4)/(4*b^6*(a + b*x^2)^4) + (10*a^3)/(3*b
^6*(a + b*x^2)^3) - (5*a^2)/(b^6*(a + b*x^2)^2) + (5*a)/(b^6*(a + b*x^2))
+ Log[a + b*x^2]/b^6)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.70

method	result
norman	$\frac{\frac{137a^5}{120b^6}+\frac{5a\,x^8}{2b^2}+\frac{15a^2x^6}{2b^3}+\frac{55a^3x^4}{6b^4}+\frac{125a^4x^2}{24b^5}}{(b\,x^2+a)^5}+\frac{\ln(b\,x^2+a)}{2b^6}$
risch	$\frac{\frac{137a^5}{120b^6}+\frac{5a\,x^8}{2b^2}+\frac{15a^2x^6}{2b^3}+\frac{55a^3x^4}{6b^4}+\frac{125a^4x^2}{24b^5}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}+\frac{\ln(b\,x^2+a)}{2b^6}$
default	$\frac{a^5}{10b^6(b\,x^2+a)^5}-\frac{5a^4}{8b^6(b\,x^2+a)^4}+\frac{5a^3}{3b^6(b\,x^2+a)^3}-\frac{5a^2}{2b^6(b\,x^2+a)^2}+\frac{5a}{2b^6(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^6}$
parallelrisch	$\frac{60\ln(b\,x^2+a)x^{10}b^5+300\ln(b\,x^2+a)x^8a\,b^4+300a\,x^8b^4+600\ln(b\,x^2+a)x^6a^2b^3+900a^2x^6b^3+600\ln(b\,x^2+a)x^4a^3b^2+1100a^3}{120b^6(b^2x^4+2ab\,x^2+a^2)^2(b\,x^2+a)}$

input
```
int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(137/120*a^5/b^6+5/2*a/b^2*x^8+15/2*a^2/b^3*x^6+55/6*a^3/b^4*x^4+125/24*a^
4/b^5*x^2)/(b*x^2+a)^5+1/2*ln(b*x^2+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.54

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$=\frac{300\,ab^4x^8+900\,a^2b^3x^6+1100\,a^3b^2x^4+625\,a^4bx^2+137\,a^5+60\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x}{120\,(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)}$$

input
```
integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/120*(300*a*b^4*x^8 + 900*a^2*b^3*x^6 + 1100*a^3*b^2*x^4 + 625*a^4*b*x^2
+ 137*a^5 + 60*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 +
 5*a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9
*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \frac{x^{11}}{(a^2 + 2abx^2 + b^2x^4)^3}\, dx = \frac{137a^5 + 625a^4bx^2 + 1100a^3b^2x^4 + 900a^2b^3x^6 + 300ab^4x^8}{120a^5b^6 + 600a^4b^7x^2 + 1200a^3b^8x^4 + 1200a^2b^9x^6 + 600ab^{10}x^8 + 120b^{11}x^{10}} + \frac{\log(a + bx^2)}{2b^6}$$

input
```
integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
(137*a**5 + 625*a**4*b*x**2 + 1100*a**3*b**2*x**4 + 900*a**2*b**3*x**6 + 3
00*a*b**4*x**8)/(120*a**5*b**6 + 600*a**4*b**7*x**2 + 1200*a**3*b**8*x**4
+ 1200*a**2*b**9*x**6 + 600*a*b**10*x**8 + 120*b**11*x**10) + log(a + b*x*
*2)/(2*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11

$$\int \frac{x^{11}}{(a^2 + 2abx^2 + b^2x^4)^3}\, dx = \frac{300\, ab^4x^8 + 900\, a^2b^3x^6 + 1100\, a^3b^2x^4 + 625\, a^4bx^2 + 137\, a^5}{120\, (b^{11}x^{10} + 5\, ab^{10}x^8 + 10\, a^2b^9x^6 + 10\, a^3b^8x^4 + 5\, a^4b^7x^2 + a^5b^6)} + \frac{\log(bx^2 + a)}{2\, b^6}$$

input
```
integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/120*(300*a*b^4*x^8 + 900*a^2*b^3*x^6 + 1100*a^3*b^2*x^4 + 625*a^4*b*x^2
+ 137*a^5)/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5
*a^4*b^7*x^2 + a^5*b^6) + 1/2*log(b*x^2 + a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\log(|bx^2+a|)}{2\,b^6} - \frac{137\,b^4x^{10}+385\,ab^3x^8+470\,a^2b^2x^6+270\,a^3bx^4+60\,a^4x^2}{120\,(bx^2+a)^5b^5}$$

input
```
integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/2*log(abs(b*x^2 + a))/b^6 - 1/120*(137*b^4*x^10 + 385*a*b^3*x^8 + 470*a^
2*b^2*x^6 + 270*a^3*b*x^4 + 60*a^4*x^2)/((b*x^2 + a)^5*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{137\,a^5}{120\,b^6}+\frac{5\,a\,x^8}{2\,b^2}+\frac{15\,a^2\,x^6}{2\,b^3}+\frac{55\,a^3\,x^4}{6\,b^4}+\frac{125\,a^4\,x^2}{24\,b^5}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}} + \frac{\ln(b\,x^2+a)}{2\,b^6}$$

input
```
int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((137*a^5)/(120*b^6) + (5*a*x^8)/(2*b^2) + (15*a^2*x^6)/(2*b^3) + (55*a^3*
x^4)/(6*b^4) + (125*a^4*x^2)/(24*b^5))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a
*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6) + log(a + b*x^2)/(2*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.87

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{60\log(b\,x^2+a)\,a^5+300\log(b\,x^2+a)\,a^4b\,x^2+600\log(b\,x^2+a)\,a^3b^2x^4+600\log(b\,x^2+a)\,a^2b^3x^6+300}{120b^6\,(b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6}$$

input
```
int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(60*log(a + b*x**2)*a**5 + 300*log(a + b*x**2)*a**4*b*x**2 + 600*log(a + b
*x**2)*a**3*b**2*x**4 + 600*log(a + b*x**2)*a**2*b**3*x**6 + 300*log(a + b
*x**2)*a*b**4*x**8 + 60*log(a + b*x**2)*b**5*x**10 + 77*a**5 + 325*a**4*b*
x**2 + 500*a**3*b**2*x**4 + 300*a**2*b**3*x**6 - 60*b**5*x**10)/(120*b**6*
(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x
**8 + b**5*x**10))
```

3.399 $\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{x^{10}}{10a\,(a+bx^2)^5}$$

output
```
1/10*x^10/a/(b*x^2+a)^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(19) = 38.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.00

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{a^4+5a^3bx^2+10a^2b^2x^4+10ab^3x^6+5b^4x^8}{10b^5\,(a+bx^2)^5}$$

input
```
Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(a^4 + 5*a^3*b*x^2 + 10*a^2*b^2*x^4 + 10*a*b^3*x^6 + 5*b^4*x^8)/(b^5
*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1380, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{x^9}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^9}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{242}$$

$$\frac{x^{10}}{10a(a+bx^2)^5}$$

input
```
Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
x^10/(10*a*(a + b*x^2)^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(17) = 34.

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.11

method	result	size
norman	$\frac{-\frac{x^{8}}{2b}-\frac{a\,x^{6}}{b^{2}}-\frac{a^{2}x^{4}}{b^{3}}-\frac{a^{3}x^{2}}{2b^{4}}-\frac{a^{4}}{10b^{5}}}{\left(b\,x^{2}+a\right)^{5}}$	59
orering	$-\frac{\left(5b^{4}x^{8}+10a\,b^{3}x^{6}+10a^{2}b^{2}x^{4}+5a^{3}b\,x^{2}+a^{4}\right)\left(b\,x^{2}+a\right)}{10b^{5}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{3}}$	74
gosper	$-\frac{5b^{4}x^{8}+10a\,b^{3}x^{6}+10a^{2}b^{2}x^{4}+5a^{3}b\,x^{2}+a^{4}}{10\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}b^{5}}$	76
parallelrisch	$\frac{-5b^{4}x^{8}-10a\,b^{3}x^{6}-10a^{2}b^{2}x^{4}-5a^{3}b\,x^{2}-a^{4}}{10b^{5}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}\left(b\,x^{2}+a\right)}$	78
risch	$\frac{-\frac{x^{8}}{2b}-\frac{a\,x^{6}}{b^{2}}-\frac{a^{2}x^{4}}{b^{3}}-\frac{a^{3}x^{2}}{2b^{4}}-\frac{a^{4}}{10b^{5}}}{\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}}$	79
default	$-\frac{a^{2}}{b^{5}\left(b\,x^{2}+a\right)^{3}}-\frac{a^{4}}{10b^{5}\left(b\,x^{2}+a\right)^{5}}+\frac{a^{3}}{2b^{5}\left(b\,x^{2}+a\right)^{4}}+\frac{a}{b^{5}\left(b\,x^{2}+a\right)^{2}}-\frac{1}{2b^{5}\left(b\,x^{2}+a\right)}$	81

input
```
int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/b*x^8-a/b^2*x^6-a^2/b^3*x^4-1/2*a^3/b^4*x^2-1/10*a^4/b^5)/(b*x^2+a)^
5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(17) = 34.

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.37

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= -\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5)}$$

input
```
integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/(b^1
0*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a
^5*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(14) = 28$.

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 5.63

$$\int \frac{x^9}{(a^2 + 2abx^2 + b^2x^4)^3}\,dx = \frac{-a^4 - 5a^3bx^2 - 10a^2b^2x^4 - 10ab^3x^6 - 5b^4x^8}{10a^5b^5 + 50a^4b^6x^2 + 100a^3b^7x^4 + 100a^2b^8x^6 + 50ab^9x^8 + 10b^{10}x^{10}}$$

input
```
integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
(-a**4 - 5*a**3*b*x**2 - 10*a**2*b**2*x**4 - 10*a*b**3*x**6 - 5*b**4*x**8)
/(10*a**5*b**5 + 50*a**4*b**6*x**2 + 100*a**3*b**7*x**4 + 100*a**2*b**8*x*
*6 + 50*a*b**9*x**8 + 10*b**10*x**10)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.37

$$\int \frac{x^9}{(a^2 + 2abx^2 + b^2x^4)^3}\,dx = -\frac{5\,b^4x^8 + 10\,ab^3x^6 + 10\,a^2b^2x^4 + 5\,a^3bx^2 + a^4}{10\,(b^{10}x^{10} + 5\,ab^9x^8 + 10\,a^2b^8x^6 + 10\,a^3b^7x^4 + 5\,a^4b^6x^2 + a^5b^5)}$$

input
```
integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/(b^1
0*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a
^5*b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(17) = 34$.

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.89

$$\int \frac{x^9}{(a^2 + 2abx^2 + b^2x^4)^3}\,dx = -\frac{5\,b^4x^8 + 10\,ab^3x^6 + 10\,a^2b^2x^4 + 5\,a^3bx^2 + a^4}{10\,(bx^2 + a)^5 b^5}$$

input
```
integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 + a^4)/((b*
x^2 + a)^5*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 5.47

$$\int \frac{x^9}{(a^2 + 2abx^2 + b^2x^4)^3}\,dx = -\frac{a^4 + 5\,a^3\,b\,x^2 + 10\,a^2\,b^2\,x^4 + 10\,a\,b^3\,x^6 + 5\,b^4\,x^8}{10\,a^5\,b^5 + 50\,a^4\,b^6\,x^2 + 100\,a^3\,b^7\,x^4 + 100\,a^2\,b^8\,x^6 + 50\,a\,b^9\,x^8 + 10\,b^{10}\,x^{10}}$$

input
```
int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
-(a^4 + 5*b^4*x^8 + 5*a^3*b*x^2 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4)/(10*a^5*b
^5 + 10*b^10*x^10 + 50*a*b^9*x^8 + 50*a^4*b^6*x^2 + 100*a^3*b^7*x^4 + 100*
a^2*b^8*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$=\frac{x^{10}}{10a\,(b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6+10a^3b^2x^4+5a^4b\,x^2+a^5)}$$

input
```
int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
x**10/(10*a*(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6
+ 5*a*b**4*x**8 + b**5*x**10))
```

3.400 $\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx=\frac{x^8}{10a\left(a+bx^2\right)^5}+\frac{x^8}{40a^2\left(a+bx^2\right)^4}$$

output
```
1/10*x^8/a/(b*x^2+a)^5+1/40*x^8/a^2/(b*x^2+a)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx=-\frac{a^3+5a^2bx^2+10ab^2x^4+10b^3x^6}{40b^4\left(a+bx^2\right)^5}$$

input
```
Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/40*(a^3 + 5*a^2*b*x^2 + 10*a*b^2*x^4 + 10*b^3*x^6)/(b^4*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{x^7}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^7}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^6}{(bx^2+a)^6}dx^2$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(\frac{\int \frac{x^6}{(bx^2+a)^5}dx^2}{5a}+\frac{x^8}{5a(a+bx^2)^5}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2}\left(\frac{x^8}{20a^2(a+bx^2)^4}+\frac{x^8}{5a(a+bx^2)^5}\right)$$

input
```
Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(x^8/(5*a*(a + b*x^2)^5) + x^8/(20*a^2*(a + b*x^2)^4))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

method	result	size
norman	$\frac{-\frac{x^6}{4b}-\frac{a\,x^4}{4b^2}-\frac{a^2x^2}{8b^3}-\frac{a^3}{40b^4}}{(b\,x^2+a)^5}$	48
orering	$-\frac{(10b^3x^6+10b^2x^4a+5a^2b\,x^2+a^3)(b\,x^2+a)}{40b^4(b^2x^4+2ab\,x^2+a^2)^3}$	63
gosper	$-\frac{10b^3x^6+10b^2x^4a+5a^2b\,x^2+a^3}{40(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2b^4}$	65
default	$-\frac{1}{4b^4(b\,x^2+a)^2}+\frac{a}{2b^4(b\,x^2+a)^3}+\frac{a^3}{10b^4(b\,x^2+a)^5}-\frac{3a^2}{8b^4(b\,x^2+a)^4}$	65
risch	$\frac{-\frac{x^6}{4b}-\frac{a\,x^4}{4b^2}-\frac{a^2x^2}{8b^3}-\frac{a^3}{40b^4}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}$	68
parallelrisch	$\frac{-10b^4x^6-10a\,b^3x^4-5a^2b^2x^2-a^3b}{40b^5(b^2x^4+2ab\,x^2+a^2)^2(b\,x^2+a)}$	70

input
```
int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/4/b*x^6-1/4*a/b^2*x^4-1/8*a^2/b^3*x^2-1/40*a^3/b^4)/(b*x^2+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(35) = 70.

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.33

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{10\,b^3x^6+10\,ab^2x^4+5\,a^2bx^2+a^3}{40\,(b^9x^{10}+5\,ab^8x^8+10\,a^2b^7x^6+10\,a^3b^6x^4+5\,a^4b^5x^2+a^5b^4)}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/(b^9*x^10 + 5*a*b^8*
x^8 + 10*a^2*b^7*x^6 + 10*a^3*b^6*x^4 + 5*a^4*b^5*x^2 + a^5*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(31) = 62$.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \frac{x^7}{(a^2 + 2abx^2 + b^2x^4)^3} \, dx = \frac{-a^3 - 5a^2bx^2 - 10ab^2x^4 - 10b^3x^6}{40a^5b^4 + 200a^4b^5x^2 + 400a^3b^6x^4 + 400a^2b^7x^6 + 200ab^8x^8 + 40b^9x^{10}}$$

input
```
integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
(-a**3 - 5*a**2*b*x**2 - 10*a*b**2*x**4 - 10*b**3*x**6)/(40*a**5*b**4 + 20
0*a**4*b**5*x**2 + 400*a**3*b**6*x**4 + 400*a**2*b**7*x**6 + 200*a*b**8*x*
*8 + 40*b**9*x**10)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(35) = 70$.

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.33

$$\int \frac{x^7}{(a^2 + 2abx^2 + b^2x^4)^3} \, dx = -\frac{10\,b^3x^6 + 10\,ab^2x^4 + 5\,a^2bx^2 + a^3}{40\,(b^9x^{10} + 5\,ab^8x^8 + 10\,a^2b^7x^6 + 10\,a^3b^6x^4 + 5\,a^4b^5x^2 + a^5b^4)}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/(b^9*x^10 + 5*a*b^8*
x^8 + 10*a^2*b^7*x^6 + 10*a^3*b^6*x^4 + 5*a^4*b^5*x^2 + a^5*b^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{10\,b^3x^6+10\,ab^2x^4+5\,a^2bx^2+a^3}{40\,(bx^2+a)^5b^4}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/((b*x^2 + a)^5*b^4)
```

Mupad [B] (verification not implemented)

Time = 17.86 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.38

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{a^3+5\,a^2\,b\,x^2+10\,a\,b^2\,x^4+10\,b^3\,x^6}{40\,a^5\,b^4+200\,a^4\,b^5\,x^2+400\,a^3\,b^6\,x^4+400\,a^2\,b^7\,x^6+200\,a\,b^8\,x^8+40\,b^9\,x^{10}}$$

input
```
int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
-(a^3 + 10*b^3*x^6 + 5*a^2*b*x^2 + 10*a*b^2*x^4)/(40*a^5*b^4 + 40*b^9*x^10
 + 200*a*b^8*x^8 + 200*a^4*b^5*x^2 + 400*a^3*b^6*x^4 + 400*a^2*b^7*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.31

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{-10b^3x^6-10a\,b^2x^4-5a^2b\,x^2-a^3}{40b^4\left(b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6+10a^3b^2x^4+5a^4b\,x^2+a^5\right)}$$

input
```
int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
( - a**3 - 5*a**2*b*x**2 - 10*a*b**2*x**4 - 10*b**3*x**6)/(40*b**4*(a**5 +
 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x**8 + b
**5*x**10))
```

3.401 $\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 53

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{a^2}{10b^3(a+bx^2)^5}+\frac{a}{4b^3(a+bx^2)^4}-\frac{1}{6b^3(a+bx^2)^3}$$

output
```
-1/10*a^2/b^3/(b*x^2+a)^5+1/4*a/b^3/(b*x^2+a)^4-1/6/b^3/(b*x^2+a)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{a^2+5abx^2+10b^2x^4}{60b^3(a+bx^2)^5}$$

input
```
Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/60*(a^2 + 5*a*b*x^2 + 10*b^2*x^4)/(b^3*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3}dx$$
$$\Big\downarrow \text{1380}$$
$$b^6\int \frac{x^5}{b^6(bx^2+a)^6}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{x^5}{(a+bx^2)^6}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^6}dx^2$$
$$\Big\downarrow \text{53}$$
$$\frac{1}{2}\int \left(\frac{a^2}{b^2(bx^2+a)^6}-\frac{2a}{b^2(bx^2+a)^5}+\frac{1}{b^2(bx^2+a)^4}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(-\frac{a^2}{5b^3(a+bx^2)^5}+\frac{a}{2b^3(a+bx^2)^4}-\frac{1}{3b^3(a+bx^2)^3}\right)$$

input
```
Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(-1/5*a^2/(b^3*(a + b*x^2)^5) + a/(2*b^3*(a + b*x^2)^4) - 1/(3*b^3*(a + b*
x^2)^3))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

method	result	size
norman	$\frac{-\frac{x^{4}}{6b}-\frac{a\,x^{2}}{12b^{2}}-\frac{a^{2}}{60b^{3}}}{\left(b\,x^{2}+a\right)^{5}}$	37
default	$-\frac{a^{2}}{10b^{3}\left(b\,x^{2}+a\right)^{5}}+\frac{a}{4b^{3}\left(b\,x^{2}+a\right)^{4}}-\frac{1}{6b^{3}\left(b\,x^{2}+a\right)^{3}}$	48
orering	$-\frac{\left(10b^{2}x^{4}+5ab\,x^{2}+a^{2}\right)\left(b\,x^{2}+a\right)}{60b^{3}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{3}}$	52
gosper	$-\frac{10b^{2}x^{4}+5ab\,x^{2}+a^{2}}{60\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}b^{3}}$	54
risch	$\frac{-\frac{x^{4}}{6b}-\frac{a\,x^{2}}{12b^{2}}-\frac{a^{2}}{60b^{3}}}{\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}}$	57
parallelrisch	$\frac{-10b^{4}x^{4}-5a\,b^{3}x^{2}-a^{2}b^{2}}{60b^{5}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}\left(b\,x^{2}+a\right)}$	61

input
```
int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/6/b*x^4-1/12*a/b^2*x^2-1/60*a^2/b^3)/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{10\,b^2x^4+5\,abx^2+a^2}{60\,(b^8x^{10}+5\,ab^7x^8+10\,a^2b^6x^6+10\,a^3b^5x^4+5\,a^4b^4x^2+a^5b^3)}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/(b^8*x^10 + 5*a*b^7*x^8 + 10*a^2*b^6*
x^6 + 10*a^3*b^5*x^4 + 5*a^4*b^4*x^2 + a^5*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.57

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{-a^2-5abx^2-10b^2x^4}{60a^5b^3+300a^4b^4x^2+600a^3b^5x^4+600a^2b^6x^6+300ab^7x^8+60b^8x^{10}}$$

input
```
integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
(-a**2 - 5*a*b*x**2 - 10*b**2*x**4)/(60*a**5*b**3 + 300*a**4*b**4*x**2 + 6
00*a**3*b**5*x**4 + 600*a**2*b**6*x**6 + 300*a*b**7*x**8 + 60*b**8*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{10\,b^2x^4+5\,abx^2+a^2}{60\,(b^8x^{10}+5\,ab^7x^8+10\,a^2b^6x^6+10\,a^3b^5x^4+5\,a^4b^4x^2+a^5b^3)}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/(b^8*x^10 + 5*a*b^7*x^8 + 10*a^2*b^6*
x^6 + 10*a^3*b^5*x^4 + 5*a^4*b^4*x^2 + a^5*b^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{10\,b^2x^4+5\,abx^2+a^2}{60\,(bx^2+a)^5b^3}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/((b*x^2 + a)^5*b^3)
```

Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.53

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{\frac{a^2}{60\,b^3}+\frac{x^4}{6\,b}+\frac{a\,x^2}{12\,b^2}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

input
```
int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
-(a^2/(60*b^3) + x^4/(6*b) + (a*x^2)/(12*b^2))/(a^5 + b^5*x^10 + 5*a^4*b*x
^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.49

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{-10b^2x^4-5ab\,x^2-a^2}{60b^3\,(b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6+10a^3b^2x^4+5a^4b\,x^2+a^5)}$$

input
```
int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
( - a**2 - 5*a*b*x**2 - 10*b**2*x**4)/(60*b**3*(a**5 + 5*a**4*b*x**2 + 10*
a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10))
```

3.402 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{a}{10b^2(a+bx^2)^5} - \frac{1}{8b^2(a+bx^2)^4}$$

output
```
1/10*a/b^2/(b*x^2+a)^5-1/8/b^2/(b*x^2+a)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{a+5bx^2}{40b^2(a+bx^2)^5}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/40*(a + 5*b*x^2)/(b^2*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^3}dx$$
$$\Big\downarrow \text{ 1380}$$
$$b^6\int \frac{x^3}{b^6(bx^2+a)^6}dx$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{x^3}{(a+bx^2)^6}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^6}dx^2$$
$$\Big\downarrow \text{ 53}$$
$$\frac{1}{2}\int \left(\frac{1}{b(bx^2+a)^5}-\frac{a}{b(bx^2+a)^6}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(\frac{a}{5b^2(a+bx^2)^5}-\frac{1}{4b^2(a+bx^2)^4}\right)$$

input
```
Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a/(5*b^2*(a + b*x^2)^5) - 1/(4*b^2*(a + b*x^2)^4))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{x^{2}}{8 b}-\frac{a}{40 b^{2}}}{\left(b \,x^{2}+a \right)^{5}}$	26
default	$\frac{a}{10 b^{2} \left(b \,x^{2}+a \right)^{5}}-\frac{1}{8 b^{2} \left(b \,x^{2}+a \right)^{4}}$	31
orering	$-\frac{\left(5 b \,x^{2}+a \right) \left(b \,x^{2}+a \right)}{40 b^{2} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{3}}$	41
gosper	$-\frac{5 b \,x^{2}+a}{40 \left(b \,x^{2}+a \right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{2} b^{2}}$	43
risch	$\frac{-\frac{x^{2}}{8 b}-\frac{a}{40 b^{2}}}{\left(b \,x^{2}+a \right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{2}}$	46
parallelrisch	$\frac{-5 b^{4} x^{2}-b^{3} a}{40 b^{5} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{2} \left(b \,x^{2}+a \right)}$	50

input
```
int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8/b*x^2-1/40*a/b^2)/(b*x^2+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(30) = 60$.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \frac{x^3}{(a^2 + 2abx^2 + b^2x^4)^3} dx = -\frac{5\,bx^2 + a}{40\,(b^7x^{10} + 5\,ab^6x^8 + 10\,a^2b^5x^6 + 10\,a^3b^4x^4 + 5\,a^4b^3x^2 + a^5b^2)}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/40*(5*b*x^2 + a)/(b^7*x^10 + 5*a*b^6*x^8 + 10*a^2*b^5*x^6 + 10*a^3*b^4*
x^4 + 5*a^4*b^3*x^2 + a^5*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(29) = 58$.

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

$$\int \frac{x^3}{(a^2 + 2abx^2 + b^2x^4)^3} dx = \frac{-a - 5bx^2}{40a^5b^2 + 200a^4b^3x^2 + 400a^3b^4x^4 + 400a^2b^5x^6 + 200ab^6x^8 + 40b^7x^{10}}$$

input
```
integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
(-a - 5*b*x**2)/(40*a**5*b**2 + 200*a**4*b**3*x**2 + 400*a**3*b**4*x**4 +
400*a**2*b**5*x**6 + 200*a*b**6*x**8 + 40*b**7*x**10)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(30) = 60.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \frac{x^3}{(a^2 + 2abx^2 + b^2x^4)^3}\,dx$$
$$= -\frac{5\,bx^2 + a}{40\,(b^7x^{10} + 5\,ab^6x^8 + 10\,a^2b^5x^6 + 10\,a^3b^4x^4 + 5\,a^4b^3x^2 + a^5b^2)}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/40*(5*b*x^2 + a)/(b^7*x^10 + 5*a*b^6*x^8 + 10*a^2*b^5*x^6 + 10*a^3*b^4*
x^4 + 5*a^4*b^3*x^2 + a^5*b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{x^3}{(a^2 + 2abx^2 + b^2x^4)^3}\,dx = -\frac{5\,bx^2 + a}{40\,(bx^2 + a)^5b^2}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/40*(5*b*x^2 + a)/((b*x^2 + a)^5*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int \frac{x^3}{(a^2 + 2abx^2 + b^2x^4)^3}\,dx$$
$$= -\frac{\frac{a}{40\,b^2} + \frac{x^2}{8\,b}}{a^5 + 5\,a^4\,b\,x^2 + 10\,a^3\,b^2\,x^4 + 10\,a^2\,b^3\,x^6 + 5\,a\,b^4\,x^8 + b^5\,x^{10}}$$

input
```
int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
-(a/(40*b^2) + x^2/(8*b))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10
*a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{(a^2 + 2abx^2 + b^2x^4)^3}\,dx = \frac{-5b\,x^2 - a}{40b^2\,(b^5x^{10} + 5a\,b^4x^8 + 10a^2b^3x^6 + 10a^3b^2x^4 + 5a^4b\,x^2 + a^5)}$$

input
```
int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
( - a - 5*b*x**2)/(40*b**2*(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*
a**2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10))
```

3.403 $\int \frac{x}{(a^2+2abx^2+b^2x^4)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{x}{(a^2 + 2abx^2 + b^2x^4)^3} dx = -\frac{1}{10b(a + bx^2)^5}$$

output
```
-1/10/b/(b*x^2+a)^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a^2 + 2abx^2 + b^2x^4)^3} dx = -\frac{1}{10b(a + bx^2)^5}$$

input
```
Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*1/(b*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1380, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^6\int \frac{x}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{x}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{ 241}$$

$$-\frac{1}{10b(a+bx^2)^5}$$

input
```
Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*1/(b*(a + b*x^2)^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{1}{10b\left(b\,x^{2}+a\right)^{5}}$	15
norman	$-\frac{1}{10b\left(b\,x^{2}+a\right)^{5}}$	15
orering	$-\frac{b\,x^{2}+a}{10b\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{3}}$	33
gosper	$-\frac{1}{10\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}b}$	35
risch	$-\frac{1}{10\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}b}$	35
parallelrisch	$-\frac{1}{10\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}b}$	35

input
```
int(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/10/b/(b*x^2+a)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.69

$$\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = -\frac{1}{10\left(b^6x^{10}+5\,ab^5x^8+10\,a^2b^4x^6+10\,a^3b^3x^4+5\,a^4b^2x^2+a^5b\right)}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/10/(b^6*x^10 + 5*a*b^5*x^8 + 10*a^2*b^4*x^6 + 10*a^3*b^3*x^4 + 5*a^4*b^
2*x^2 + a^5*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(14) = 28$.

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.94

$$\int \frac{x}{(a^2 + 2abx^2 + b^2x^4)^3} \, dx = -\frac{1}{10a^5b + 50a^4b^2x^2 + 100a^3b^3x^4 + 100a^2b^4x^6 + 50ab^5x^8 + 10b^6x^{10}}$$

input
```
integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
-1/(10*a**5*b + 50*a**4*b**2*x**2 + 100*a**3*b**3*x**4 + 100*a**2*b**4*x**
6 + 50*a*b**5*x**8 + 10*b**6*x**10)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.69

$$\int \frac{x}{(a^2 + 2abx^2 + b^2x^4)^3} \, dx = -\frac{1}{10\,(b^6x^{10} + 5\,ab^5x^8 + 10\,a^2b^4x^6 + 10\,a^3b^3x^4 + 5\,a^4b^2x^2 + a^5b)}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/10/(b^6*x^10 + 5*a*b^5*x^8 + 10*a^2*b^4*x^6 + 10*a^3*b^3*x^4 + 5*a^4*b^
2*x^2 + a^5*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{1}{10\,(bx^2+a)^5b}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/10/((b*x^2 + a)^5*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.81

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{1}{10\,a^5\,b+50\,a^4\,b^2\,x^2+100\,a^3\,b^3\,x^4+100\,a^2\,b^4\,x^6+50\,a\,b^5\,x^8+10\,b^6\,x^{10}}$$

input
```
int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
-1/(10*a^5*b + 10*b^6*x^10 + 50*a*b^5*x^8 + 50*a^4*b^2*x^2 + 100*a^3*b^3*x
^4 + 100*a^2*b^4*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.62

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{1}{10b\,(b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6+10a^3b^2x^4+5a^4b\,x^2+a^5)}$$

input
```
int(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
( - 1)/(10*b*(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6
 + 5*a*b**4*x**8 + b**5*x**10))
```

3.404 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx=\frac{1}{10a\left(a+bx^2\right)^5}+\frac{1}{8a^2\left(a+bx^2\right)^4}+\frac{1}{6a^3\left(a+bx^2\right)^3}+\frac{1}{4a^4\left(a+bx^2\right)^2}+\frac{1}{2a^5\left(a+bx^2\right)}+\frac{\log(x)}{a^6}-\frac{\log\left(a+bx^2\right)}{2a^6}$$

output
```
1/10/a/(b*x^2+a)^5+1/8/a^2/(b*x^2+a)^4+1/6/a^3/(b*x^2+a)^3+1/4/a^4/(b*x^2+
a)^2+1/2/a^5/(b*x^2+a)+ln(x)/a^6-1/2*ln(b*x^2+a)/a^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx=\frac{\frac{a\left(137a^4+385a^3bx^2+470a^2b^2x^4+270ab^3x^6+60b^4x^8\right)}{\left(a+bx^2\right)^5}+120\log(x)-60\log\left(a+bx^2\right)}{120a^6}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
((a*(137*a^4 + 385*a^3*b*x^2 + 470*a^2*b^2*x^4 + 270*a*b^3*x^6 + 60*b^4*x^
8))/(a + b*x^2)^5 + 120*Log[x] - 60*Log[a + b*x^2])/(120*a^6)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^6\int \frac{1}{b^6x\left(bx^2+a\right)^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{x\left(a+bx^2\right)^6}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(bx^2+a\right)^6}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2}\int \left(-\frac{b}{a^6\left(bx^2+a\right)}-\frac{b}{a^5\left(bx^2+a\right)^2}-\frac{b}{a^4\left(bx^2+a\right)^3}-\frac{b}{a^3\left(bx^2+a\right)^4}-\frac{b}{a^2\left(bx^2+a\right)^5}-\frac{b}{a\left(bx^2+a\right)^6}+\frac{1}{a^6x^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{\log\left(a+bx^2\right)}{a^6}+\frac{\log\left(x^2\right)}{a^6}+\frac{1}{a^5\left(a+bx^2\right)}+\frac{1}{2a^4\left(a+bx^2\right)^2}+\frac{1}{3a^3\left(a+bx^2\right)^3}+\frac{1}{4a^2\left(a+bx^2\right)^4}+\frac{1}{5a\left(a+bx^2\right.}\right.$$

input
```
Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
(1/(5*a*(a + b*x^2)^5) + 1/(4*a^2*(a + b*x^2)^4) + 1/(3*a^3*(a + b*x^2)^3)
 + 1/(2*a^4*(a + b*x^2)^2) + 1/(a^5*(a + b*x^2)) + Log[x^2]/a^6 - Log[a +
b*x^2]/a^6)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

method	result
norman	$\frac{-\frac{5b\,x^{2}}{2a^{2}}-\frac{15b^{2}x^{4}}{2a^{3}}-\frac{55b^{3}x^{6}}{6a^{4}}-\frac{125b^{4}x^{8}}{24a^{5}}-\frac{137b^{5}x^{10}}{120a^{6}}}{\left(b\,x^{2}+a\right)^{5}}+\frac{\ln(x)}{a^{6}}-\frac{\ln\left(b\,x^{2}+a\right)}{2a^{6}}$
risch	$\frac{\frac{b^{4}x^{8}}{2a^{5}}+\frac{9b^{3}x^{6}}{4a^{4}}+\frac{47b^{2}x^{4}}{12a^{3}}+\frac{77b\,x^{2}}{24a^{2}}+\frac{137}{120a}}{\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}}+\frac{\ln(x)}{a^{6}}-\frac{\ln\left(b\,x^{2}+a\right)}{2a^{6}}$
default	$-\frac{b\left(\frac{\ln\left(b\,x^{2}+a\right)}{b}-\frac{a^{2}}{2b\left(b\,x^{2}+a\right)^{2}}-\frac{a^{4}}{4b\left(b\,x^{2}+a\right)^{4}}-\frac{a^{3}}{3b\left(b\,x^{2}+a\right)^{3}}-\frac{a^{5}}{5b\left(b\,x^{2}+a\right)^{5}}-\frac{a}{b\left(b\,x^{2}+a\right)}\right)}{2a^{6}}+\frac{\ln(x)}{a^{6}}$
parallelrisch	$120\ln(x)x^{10}b^{5}-60\ln\left(b\,x^{2}+a\right)x^{10}b^{5}-137x^{10}b^{5}+600\ln(x)x^{8}a\,b^{4}-300\ln\left(b\,x^{2}+a\right)x^{8}a\,b^{4}-625a\,x^{8}b^{4}+1200\ln(x)x^{6}a^{2}b^{3}-600$

input
```
int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-5/2*b/a^2*x^2-15/2*b^2/a^3*x^4-55/6*b^3/a^4*x^6-125/24*b^4/a^5*x^8-137/1
20*b^5/a^6*x^10)/(b*x^2+a)^5+ln(x)/a^6-1/2*ln(b*x^2+a)/a^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(90) = 180.

Time = 0.07 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.18

$$\int \frac{1}{x\,(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{60\,ab^4x^8+270\,a^2b^3x^6+470\,a^3b^2x^4+385\,a^4bx^2+137\,a^5-60\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4\,-}{120\,(a^6b^5x^{10}+5\,a^7b^4x^8+10\,a^8b^3x^6}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/120*(60*a*b^4*x^8 + 270*a^2*b^3*x^6 + 470*a^3*b^2*x^4 + 385*a^4*b*x^2 +
137*a^5 - 60*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5
*a^4*b*x^2 + a^5)*log(b*x^2 + a) + 120*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^
3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*log(x))/(a^6*b^5*x^10 + 5*a^7*
b^4*x^8 + 10*a^8*b^3*x^6 + 10*a^9*b^2*x^4 + 5*a^10*b*x^2 + a^11)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{137a^4+385a^3bx^2+470a^2b^2x^4+270ab^3x^6+60b^4x^8}{120a^{10}+600a^9bx^2+1200a^8b^2x^4+1200a^7b^3x^6+600a^6b^4x^8+120a^5b^5x^{10}} + \frac{\log\left(x\right)}{a^6} - \frac{\log\left(\frac{a}{b}+x^2\right)}{2a^6}$$

input
```
integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
(137*a**4 + 385*a**3*b*x**2 + 470*a**2*b**2*x**4 + 270*a*b**3*x**6 + 60*b*
*4*x**8)/(120*a**10 + 600*a**9*b*x**2 + 1200*a**8*b**2*x**4 + 1200*a**7*b*
*3*x**6 + 600*a**6*b**4*x**8 + 120*a**5*b**5*x**10) + log(x)/a**6 - log(a/
b + x**2)/(2*a**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{60\,b^4x^8+270\,ab^3x^6+470\,a^2b^2x^4+385\,a^3bx^2+137\,a^4}{120\left(a^5b^5x^{10}+5\,a^6b^4x^8+10\,a^7b^3x^6+10\,a^8b^2x^4+5\,a^9bx^2+a^{10}\right)} - \frac{\log\left(bx^2+a\right)}{2\,a^6} + \frac{\log\left(x^2\right)}{2\,a^6}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/120*(60*b^4*x^8 + 270*a*b^3*x^6 + 470*a^2*b^2*x^4 + 385*a^3*b*x^2 + 137*
a^4)/(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a
^9*b*x^2 + a^10) - 1/2*log(b*x^2 + a)/a^6 + 1/2*log(x^2)/a^6
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{\log\left(x^2\right)}{2\,a^6}-\frac{\log\left(|bx^2+a|\right)}{2\,a^6}+\frac{137\,b^5x^{10}+745\,ab^4x^8+1640\,a^2b^3x^6+1840\,a^3b^2x^4+1070\,a^4bx^2+274\,a^5}{120\,\left(bx^2+a\right)^5a^6}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/2*log(x^2)/a^6 - 1/2*log(abs(b*x^2 + a))/a^6 + 1/120*(137*b^5*x^10 + 745
*a*b^4*x^8 + 1640*a^2*b^3*x^6 + 1840*a^3*b^2*x^4 + 1070*a^4*b*x^2 + 274*a^
5)/((b*x^2 + a)^5*a^6)
```

Mupad [B] (verification not implemented)

Time = 18.65 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{\ln\left(x\right)}{a^6}-\frac{\ln\left(b\,x^2+a\right)}{2\,a^6}+\frac{\frac{137}{120\,a}+\frac{77\,b\,x^2}{24\,a^2}+\frac{47\,b^2\,x^4}{12\,a^3}+\frac{9\,b^3\,x^6}{4\,a^4}+\frac{b^4\,x^8}{2\,a^5}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

input
```
int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

output
```
log(x)/a^6 - log(a + b*x^2)/(2*a^6) + (137/(120*a) + (77*b*x^2)/(24*a^2) +
 (47*b^2*x^4)/(12*a^3) + (9*b^3*x^6)/(4*a^4) + (b^4*x^8)/(2*a^5))/(a^5 + b
^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.64

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{-60\log(b\,x^2+a)\,a^5-300\log(b\,x^2+a)\,a^4b\,x^2-600\log(b\,x^2+a)\,a^3b^2x^4-600\log(b\,x^2+a)\,a^2b^3x^6-30}{}$$

input
```
int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
( - 60*log(a + b*x**2)*a**5 - 300*log(a + b*x**2)*a**4*b*x**2 - 600*log(a
+ b*x**2)*a**3*b**2*x**4 - 600*log(a + b*x**2)*a**2*b**3*x**6 - 300*log(a
+ b*x**2)*a*b**4*x**8 - 60*log(a + b*x**2)*b**5*x**10 + 120*log(x)*a**5 +
600*log(x)*a**4*b*x**2 + 1200*log(x)*a**3*b**2*x**4 + 1200*log(x)*a**2*b**
3*x**6 + 600*log(x)*a*b**4*x**8 + 120*log(x)*b**5*x**10 + 125*a**5 + 325*a
**4*b*x**2 + 350*a**3*b**2*x**4 + 150*a**2*b**3*x**6 - 12*b**5*x**10)/(120
*a**6*(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*
b**4*x**8 + b**5*x**10))
```

3.405 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = -\frac{1}{2a^6x^2}-\frac{b}{10a^2\left(a+bx^2\right)^5}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{a^5\left(a+bx^2\right)^2}-\frac{5b}{2a^6\left(a+bx^2\right)}-\frac{6b\log(x)}{a^7}+\frac{3b\log\left(a+bx^2\right)}{a^7}$$

output
```
-1/2/a^6/x^2-1/10*b/a^2/(b*x^2+a)^5-1/4*b/a^3/(b*x^2+a)^4-1/2*b/a^4/(b*x^2
+a)^3-b/a^5/(b*x^2+a)^2-5/2*b/a^6/(b*x^2+a)-6*b*ln(x)/a^7+3*b*ln(b*x^2+a)/
a^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = -\frac{\frac{a\left(10a^5+137a^4bx^2+385a^3b^2x^4+470a^2b^3x^6+270ab^4x^8+60b^5x^{10}\right)}{x^2\left(a+bx^2\right)^5}+120b\log(x)-60b\log\left(a+bx^2\right)}{20a^7}$$

input
```
Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
-1/20*((a*(10*a^5 + 137*a^4*b*x^2 + 385*a^3*b^2*x^4 + 470*a^2*b^3*x^6 + 27
0*a*b^4*x^8 + 60*b^5*x^10))/(x^2*(a + b*x^2)^5) + 120*b*Log[x] - 60*b*Log[
a + b*x^2])/a^7
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1380, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^2+2abx^2+b^2x^4)^3} \, dx$$

$$\Big\downarrow \text{1380}$$

$$b^6 \int \frac{1}{b^6 x^3 (bx^2+a)^6} dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^3 (a+bx^2)^6} dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a)^6} dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(\frac{6b^2}{a^7 (bx^2+a)} + \frac{5b^2}{a^6 (bx^2+a)^2} + \frac{4b^2}{a^5 (bx^2+a)^3} + \frac{3b^2}{a^4 (bx^2+a)^4} + \frac{2b^2}{a^3 (bx^2+a)^5} + \frac{b^2}{a^2 (bx^2+a)^6} - \frac{6b}{a^7 x^2} + \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{6b \log (x^2)}{a^7} + \frac{6b \log (a+bx^2)}{a^7} - \frac{5b}{a^6 (a+bx^2)} - \frac{1}{a^6 x^2} - \frac{2b}{a^5 (a+bx^2)^2} - \frac{b}{a^4 (a+bx^2)^3} - \frac{b}{2a^3 (a+bx^2)^4} - \right.$$

input
```
Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
(-(1/(a^6*x^2)) - b/(5*a^2*(a + b*x^2)^5) - b/(2*a^3*(a + b*x^2)^4) - b/(a
^4*(a + b*x^2)^3) - (2*b)/(a^5*(a + b*x^2)^2) - (5*b)/(a^6*(a + b*x^2)) -
(6*b*Log[x^2])/a^7 + (6*b*Log[a + b*x^2])/a^7)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

method	result
norman	$\frac{-\frac{1}{2a}+\frac{15b^2x^4}{a^3}+\frac{45b^3x^6}{a^4}+\frac{55b^4x^8}{a^5}+\frac{125b^5x^{10}}{4a^6}+\frac{137b^6x^{12}}{20a^7}}{x^2(b\,x^2+a)^5}-\frac{6b\ln(x)}{a^7}+\frac{3b\ln(b\,x^2+a)}{a^7}$
risch	$\frac{-\frac{3b^5x^{10}}{a^6}-\frac{27b^4x^8}{2a^5}-\frac{47b^3x^6}{2a^4}-\frac{77b^2x^4}{4a^3}-\frac{137b\,x^2}{20a^2}-\frac{1}{2a}}{x^2(b^2x^4+2ab\,x^2+a^2)^2(b\,x^2+a)}-\frac{6b\ln(x)}{a^7}+\frac{3b\ln(-b\,x^2-a)}{a^7}$
default	$\frac{b^2\left(\frac{6\ln(b\,x^2+a)}{b}-\frac{a^4}{2b(b\,x^2+a)^4}-\frac{a^3}{b(b\,x^2+a)^3}-\frac{a^5}{5b(b\,x^2+a)^5}-\frac{5a}{b(b\,x^2+a)}-\frac{2a^2}{b(b\,x^2+a)^2}\right)}{2a^7}-\frac{1}{2a^6x^2}-\frac{6b\ln(x)}{a^7}$
parallelrisch	$-\frac{120b^6\ln(x)x^{12}-60\ln(b\,x^2+a)x^{12}b^6-137b^6x^{12}+600\ln(x)x^{10}a\,b^5-300\ln(b\,x^2+a)x^{10}a\,b^5-625a\,b^5x^{10}+1200a^2b^4\ln(x)x^8}{}$

input
```
int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a+15*b^2/a^3*x^4+45*b^3/a^4*x^6+55*b^4/a^5*x^8+125/4*b^5/a^6*x^10+13
7/20*b^6/a^7*x^12)/x^2/(b*x^2+a)^5-6*b*ln(x)/a^7+3*b*ln(b*x^2+a)/a^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(106) = 212.

Time = 0.07 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.16

$$\int \frac{1}{x^3\,(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{60\,ab^5x^{10}+270\,a^2b^4x^8+470\,a^3b^3x^6+385\,a^4b^2x^4+137\,a^5bx^2+10\,a^6-60\,(b^6x^{12}+5\,ab^5x^{10}+10\,a^2b^4}{20\,(a^7b^5x^{12}+5\,a^8b^4x^{10}}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/20*(60*a*b^5*x^10 + 270*a^2*b^4*x^8 + 470*a^3*b^3*x^6 + 385*a^4*b^2*x^4
 + 137*a^5*b*x^2 + 10*a^6 - 60*(b^6*x^12 + 5*a*b^5*x^10 + 10*a^2*b^4*x^8 +
 10*a^3*b^3*x^6 + 5*a^4*b^2*x^4 + a^5*b*x^2)*log(b*x^2 + a) + 120*(b^6*x^1
2 + 5*a*b^5*x^10 + 10*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 5*a^4*b^2*x^4 + a^5*b
*x^2)*log(x))/(a^7*b^5*x^12 + 5*a^8*b^4*x^10 + 10*a^9*b^3*x^8 + 10*a^10*b^
2*x^6 + 5*a^11*b*x^4 + a^12*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^3} \, dx = \frac{-10a^5 - 137a^4bx^2 - 385a^3b^2x^4 - 470a^2b^3x^6 - 270ab^4x^8 - 60b^5x^{10}}{20a^{11}x^2 + 100a^{10}bx^4 + 200a^9b^2x^6 + 200a^8b^3x^8 + 100a^7b^4x^{10} + 20a^6b^5x^{12}} - \frac{6b\log(x)}{a^7} + \frac{3b\log\left(\frac{a}{b} + x^2\right)}{a^7}$$

input
```
integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
(-10*a**5 - 137*a**4*b*x**2 - 385*a**3*b**2*x**4 - 470*a**2*b**3*x**6 - 27
0*a*b**4*x**8 - 60*b**5*x**10)/(20*a**11*x**2 + 100*a**10*b*x**4 + 200*a**
9*b**2*x**6 + 200*a**8*b**3*x**8 + 100*a**7*b**4*x**10 + 20*a**6*b**5*x**1
2) - 6*b*log(x)/a**7 + 3*b*log(a/b + x**2)/a**7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^3} \, dx = -\frac{60\, b^5x^{10} + 270\, ab^4x^8 + 470\, a^2b^3x^6 + 385\, a^3b^2x^4 + 137\, a^4bx^2 + 10\, a^5}{20\, (a^6b^5x^{12} + 5\, a^7b^4x^{10} + 10\, a^8b^3x^8 + 10\, a^9b^2x^6 + 5\, a^{10}bx^4 + a^{11}x^2)} + \frac{3\, b\log(bx^2 + a)}{a^7} - \frac{3\, b\log(x^2)}{a^7}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/20*(60*b^5*x^10 + 270*a*b^4*x^8 + 470*a^2*b^3*x^6 + 385*a^3*b^2*x^4 + 1
37*a^4*b*x^2 + 10*a^5)/(a^6*b^5*x^12 + 5*a^7*b^4*x^10 + 10*a^8*b^3*x^8 + 1
0*a^9*b^2*x^6 + 5*a^10*b*x^4 + a^11*x^2) + 3*b*log(b*x^2 + a)/a^7 - 3*b*lo
g(x^2)/a^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = -\frac{3\,b\log\left(x^2\right)}{a^7}+\frac{3\,b\log\left(|bx^2+a|\right)}{a^7}+\frac{6\,bx^2-a}{2\,a^7x^2} - \frac{137\,b^6x^{10}+735\,ab^5x^8+1590\,a^2b^4x^6+1740\,a^3b^3x^4+970\,a^4b^2x^2+224\,a^5b}{20\left(bx^2+a\right)^5a^7}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-3*b*log(x^2)/a^7 + 3*b*log(abs(b*x^2 + a))/a^7 + 1/2*(6*b*x^2 - a)/(a^7*x
^2) - 1/20*(137*b^6*x^10 + 735*a*b^5*x^8 + 1590*a^2*b^4*x^6 + 1740*a^3*b^3
*x^4 + 970*a^4*b^2*x^2 + 224*a^5*b)/((b*x^2 + a)^5*a^7)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{3\,b\,\ln\left(b\,x^2+a\right)}{a^7} - \frac{\frac{1}{2\,a}+\frac{137\,b\,x^2}{20\,a^2}+\frac{77\,b^2\,x^4}{4\,a^3}+\frac{47\,b^3\,x^6}{2\,a^4}+\frac{27\,b^4\,x^8}{2\,a^5}+\frac{3\,b^5\,x^{10}}{a^6}}{a^5\,x^2+5\,a^4\,b\,x^4+10\,a^3\,b^2\,x^6+10\,a^2\,b^3\,x^8+5\,a\,b^4\,x^{10}+b^5\,x^{12}} - \frac{6\,b\,\ln\left(x\right)}{a^7}$$

input
```
int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

output
```
(3*b*log(a + b*x^2))/a^7 - (1/(2*a) + (137*b*x^2)/(20*a^2) + (77*b^2*x^4)/
(4*a^3) + (47*b^3*x^6)/(2*a^4) + (27*b^4*x^8)/(2*a^5) + (3*b^5*x^10)/a^6)/
(a^5*x^2 + b^5*x^12 + 5*a^4*b*x^4 + 5*a*b^4*x^10 + 10*a^3*b^2*x^6 + 10*a^2
*b^3*x^8) - (6*b*log(x))/a^7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.54

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^3} \, dx$$
$$= \frac{60 \log(b\,x^2 + a)\,a^5 b\,x^2 + 300 \log(b\,x^2 + a)\,a^4 b^2 x^4 + 600 \log(b\,x^2 + a)\,a^3 b^3 x^6 + 600 \log(b\,x^2 + a)\,a^2 b^4 x^8 +}{}$$

input
```
int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(60*log(a + b*x**2)*a**5*b*x**2 + 300*log(a + b*x**2)*a**4*b**2*x**4 + 600
*log(a + b*x**2)*a**3*b**3*x**6 + 600*log(a + b*x**2)*a**2*b**4*x**8 + 300
*log(a + b*x**2)*a*b**5*x**10 + 60*log(a + b*x**2)*b**6*x**12 - 120*log(x)
*a**5*b*x**2 - 600*log(x)*a**4*b**2*x**4 - 1200*log(x)*a**3*b**3*x**6 - 12
00*log(x)*a**2*b**4*x**8 - 600*log(x)*a*b**5*x**10 - 120*log(x)*b**6*x**12
 - 10*a**6 - 125*a**5*b*x**2 - 325*a**4*b**2*x**4 - 350*a**3*b**3*x**6 - 1
50*a**2*b**4*x**8 + 12*b**6*x**12)/(20*a**7*x**2*(a**5 + 5*a**4*b*x**2 + 1
0*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10))
```

3.406 $\int \frac{1}{x^5(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 140

$$\int \frac{1}{x^5(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{1}{4a^6x^4}+\frac{3b}{a^7x^2}+\frac{b^2}{10a^3(a+bx^2)^5}+\frac{3b^2}{8a^4(a+bx^2)^4}+\frac{b^2}{a^5(a+bx^2)^3}+\frac{5b^2}{2a^6(a+bx^2)^2}+\frac{15b^2}{2a^7(a+bx^2)}+\frac{21b^2\log(x)}{a^8}-\frac{21b^2\log(a+bx^2)}{2a^8}$$

output
```
-1/4/a^6/x^4+3*b/a^7/x^2+1/10*b^2/a^3/(b*x^2+a)^5+3/8*b^2/a^4/(b*x^2+a)^4+
b^2/a^5/(b*x^2+a)^3+5/2*b^2/a^6/(b*x^2+a)^2+15/2*b^2/a^7/(b*x^2+a)+21*b^2*
ln(x)/a^8-21/2*b^2*ln(b*x^2+a)/a^8
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^5(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{a(-10a^6+70a^5bx^2+959a^4b^2x^4+2695a^3b^3x^6+3290a^2b^4x^8+1890ab^5x^{10}+420b^6x^{12})}{x^4(a+bx^2)^5}+840b^2\log(x)-420b^2\log(a+bx^2)}{40a^8}$$

input
```
Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
((a*(-10*a^6 + 70*a^5*b*x^2 + 959*a^4*b^2*x^4 + 2695*a^3*b^3*x^6 + 3290*a^
2*b^4*x^8 + 1890*a*b^5*x^10 + 420*b^6*x^12))/(x^4*(a + b*x^2)^5) + 840*b^2
*Log[x] - 420*b^2*Log[a + b*x^2])/(40*a^8)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1380, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)^3} \, dx$$

$$\Big\downarrow \text{1380}$$

$$b^6 \int \frac{1}{b^6 x^5 (bx^2 + a)^6} dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^5 (a + bx^2)^6} dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^6 (bx^2 + a)^6} dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(-\frac{21b^3}{a^8 (bx^2 + a)} - \frac{15b^3}{a^7 (bx^2 + a)^2} - \frac{10b^3}{a^6 (bx^2 + a)^3} - \frac{6b^3}{a^5 (bx^2 + a)^4} - \frac{3b^3}{a^4 (bx^2 + a)^5} - \frac{b^3}{a^3 (bx^2 + a)^6} + \frac{21b^2}{a^8 x^2} \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{21b^2 \log (x^2)}{a^8} - \frac{21b^2 \log (a + bx^2)}{a^8} + \frac{15b^2}{a^7 (a + bx^2)} + \frac{6b}{a^7 x^2} + \frac{5b^2}{a^6 (a + bx^2)^2} - \frac{1}{2a^6 x^4} + \frac{2b^2}{a^5 (a + bx^2)^3} + \frac{}{4a^4 (a} \right.$$

input
```
Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
(-1/2*1/(a^6*x^4) + (6*b)/(a^7*x^2) + b^2/(5*a^3*(a + b*x^2)^5) + (3*b^2)/
(4*a^4*(a + b*x^2)^4) + (2*b^2)/(a^5*(a + b*x^2)^3) + (5*b^2)/(a^6*(a + b*
x^2)^2) + (15*b^2)/(a^7*(a + b*x^2)) + (21*b^2*Log[x^2])/a^8 - (21*b^2*Log
[a + b*x^2])/a^8)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

method	result
norman	$\frac{-\frac{1}{4a}+\frac{7b\,x^{2}}{4a^{2}}-\frac{105b^{3}x^{6}}{2a^{4}}-\frac{315b^{4}x^{8}}{2a^{5}}-\frac{385b^{5}x^{10}}{2a^{6}}-\frac{875b^{6}x^{12}}{8a^{7}}-\frac{959b^{7}x^{14}}{40a^{8}}}{x^{4}(b\,x^{2}+a)^{5}}+\frac{21b^{2}\ln(x)}{a^{8}}-\frac{21b^{2}\ln(b\,x^{2}+a)}{2a^{8}}$
risch	$\frac{\frac{21b^{6}x^{12}}{2a^{7}}+\frac{189b^{5}x^{10}}{4a^{6}}+\frac{329b^{4}x^{8}}{4a^{5}}+\frac{539b^{3}x^{6}}{8a^{4}}+\frac{959b^{2}x^{4}}{40a^{3}}+\frac{7b\,x^{2}}{4a^{2}}-\frac{1}{4a}}{x^{4}(b^{2}x^{4}+2ab\,x^{2}+a^{2})^{2}(b\,x^{2}+a)}+\frac{21b^{2}\ln(x)}{a^{8}}-\frac{21b^{2}\ln(b\,x^{2}+a)}{2a^{8}}$
default	$-\frac{b^{3}\left(\frac{21\ln\left(b\,x^{2}+a\right)}{b}-\frac{3a^{4}}{4b\left(b\,x^{2}+a\right)^{4}}-\frac{2a^{3}}{b\left(b\,x^{2}+a\right)^{3}}-\frac{a^{5}}{5b\left(b\,x^{2}+a\right)^{5}}-\frac{5a^{2}}{b\left(b\,x^{2}+a\right)^{2}}-\frac{15a}{b\left(b\,x^{2}+a\right)}\right)}{2a^{8}}-\frac{1}{4a^{6}x^{4}}+\frac{3b}{a^{7}x^{2}}+\frac{21b^{2}\ln(x)}{a^{8}}$
parallelrisch	$-10a^{7}-4200\ln(b\,x^{2}+a)x^{10}a^{2}b^{5}-4200\ln(b\,x^{2}+a)x^{8}a^{3}b^{4}-2100\ln(b\,x^{2}+a)x^{6}a^{4}b^{3}-420\ln(b\,x^{2}+a)x^{4}a^{5}b^{2}-959b^{7}x^{14}+8400$

input
```
int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/4/a+7/4*b/a^2*x^2-105/2*b^3/a^4*x^6-315/2*b^4/a^5*x^8-385/2*b^5/a^6*x^
10-875/8*b^6/a^7*x^12-959/40*b^7/a^8*x^14)/x^4/(b*x^2+a)^5+21*b^2*ln(x)/a^
8-21/2*b^2*ln(b*x^2+a)/a^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(128) = 256$.

Time = 0.08 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.90

$$\int \frac{1}{x^5\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$
$$=\frac{420\,ab^6x^{12}+1890\,a^2b^5x^{10}+3290\,a^3b^4x^8+2695\,a^4b^3x^6+959\,a^5b^2x^4+70\,a^6bx^2-10\,a^7-420\,(b^7x^{14}+5}{40\,(a^8b^5x^{14}+5}$$

input
```
integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/40*(420*a*b^6*x^12 + 1890*a^2*b^5*x^10 + 3290*a^3*b^4*x^8 + 2695*a^4*b^3
*x^6 + 959*a^5*b^2*x^4 + 70*a^6*b*x^2 - 10*a^7 - 420*(b^7*x^14 + 5*a*b^6*x
^12 + 10*a^2*b^5*x^10 + 10*a^3*b^4*x^8 + 5*a^4*b^3*x^6 + a^5*b^2*x^4)*log(
b*x^2 + a) + 840*(b^7*x^14 + 5*a*b^6*x^12 + 10*a^2*b^5*x^10 + 10*a^3*b^4*x
^8 + 5*a^4*b^3*x^6 + a^5*b^2*x^4)*log(x))/(a^8*b^5*x^14 + 5*a^9*b^4*x^12 +
 10*a^10*b^3*x^10 + 10*a^11*b^2*x^8 + 5*a^12*b*x^6 + a^13*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)^3} \, dx = \frac{-10a^6 + 70a^5bx^2 + 959a^4b^2x^4 + 2695a^3b^3x^6 + 3290a^2b^4x^8 + 1890ab^5x^{10} + 420b^6x^{12}}{40a^{12}x^4 + 200a^{11}bx^6 + 400a^{10}b^2x^8 + 400a^9b^3x^{10} + 200a^8b^4x^{12} + 40a^7b^5x^{14}} + \frac{21b^2 \log(x)}{a^8} - \frac{21b^2 \log\left(\frac{a}{b} + x^2\right)}{2a^8}$$

input
```
integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
(-10*a**6 + 70*a**5*b*x**2 + 959*a**4*b**2*x**4 + 2695*a**3*b**3*x**6 + 32
90*a**2*b**4*x**8 + 1890*a*b**5*x**10 + 420*b**6*x**12)/(40*a**12*x**4 + 2
00*a**11*b*x**6 + 400*a**10*b**2*x**8 + 400*a**9*b**3*x**10 + 200*a**8*b**
4*x**12 + 40*a**7*b**5*x**14) + 21*b**2*log(x)/a**8 - 21*b**2*log(a/b + x*
*2)/(2*a**8)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)^3} \, dx = \frac{420\, b^6x^{12} + 1890\, ab^5x^{10} + 3290\, a^2b^4x^8 + 2695\, a^3b^3x^6 + 959\, a^4b^2x^4 + 70\, a^5bx^2 - 10\, a^6}{40\, (a^7b^5x^{14} + 5\, a^8b^4x^{12} + 10\, a^9b^3x^{10} + 10\, a^{10}b^2x^8 + 5\, a^{11}bx^6 + a^{12}x^4)} - \frac{21\, b^2 \log(bx^2 + a)}{2\, a^8} + \frac{21\, b^2 \log(x^2)}{2\, a^8}$$

input
```
integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/40*(420*b^6*x^12 + 1890*a*b^5*x^10 + 3290*a^2*b^4*x^8 + 2695*a^3*b^3*x^6
 + 959*a^4*b^2*x^4 + 70*a^5*b*x^2 - 10*a^6)/(a^7*b^5*x^14 + 5*a^8*b^4*x^12
 + 10*a^9*b^3*x^10 + 10*a^10*b^2*x^8 + 5*a^11*b*x^6 + a^12*x^4) - 21/2*b^2
*log(b*x^2 + a)/a^8 + 21/2*b^2*log(x^2)/a^8
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^5 (a^2+2abx^2+b^2x^4)^3}\,dx = \frac{21\,b^2\log\left(x^2\right)}{2\,a^8} - \frac{21\,b^2\log\left(|bx^2+a|\right)}{2\,a^8} - \frac{63\,b^2x^4-12\,abx^2+a^2}{4\,a^8x^4} + \frac{959\,b^7x^{10}+5095\,ab^6x^8+10890\,a^2b^5x^6+11730\,a^3b^4x^4+6390\,a^4b^3x^2+1418\,a^5b^2}{40\,(bx^2+a)^5a^8}$$

input
```
integrate(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
21/2*b^2*log(x^2)/a^8 - 21/2*b^2*log(abs(b*x^2 + a))/a^8 - 1/4*(63*b^2*x^4
 - 12*a*b*x^2 + a^2)/(a^8*x^4) + 1/40*(959*b^7*x^10 + 5095*a*b^6*x^8 + 108
90*a^2*b^5*x^6 + 11730*a^3*b^4*x^4 + 6390*a^4*b^3*x^2 + 1418*a^5*b^2)/((b*
x^2 + a)^5*a^8)
```

Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^5 (a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{7\,b\,x^2}{4\,a^2}-\frac{1}{4\,a}+\frac{959\,b^2\,x^4}{40\,a^3}+\frac{539\,b^3\,x^6}{8\,a^4}+\frac{329\,b^4\,x^8}{4\,a^5}+\frac{189\,b^5\,x^{10}}{4\,a^6}+\frac{21\,b^6\,x^{12}}{2\,a^7}}{a^5\,x^4+5\,a^4\,b\,x^6+10\,a^3\,b^2\,x^8+10\,a^2\,b^3\,x^{10}+5\,a\,b^4\,x^{12}+b^5\,x^{14}} - \frac{21\,b^2\,\ln\left(b\,x^2+a\right)}{2\,a^8} + \frac{21\,b^2\,\ln\left(x\right)}{a^8}$$

input
```
int(1/(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

output
```
((7*b*x^2)/(4*a^2) - 1/(4*a) + (959*b^2*x^4)/(40*a^3) + (539*b^3*x^6)/(8*a
^4) + (329*b^4*x^8)/(4*a^5) + (189*b^5*x^10)/(4*a^6) + (21*b^6*x^12)/(2*a^
7))/(a^5*x^4 + b^5*x^14 + 5*a^4*b*x^6 + 5*a*b^4*x^12 + 10*a^3*b^2*x^8 + 10
*a^2*b^3*x^10) - (21*b^2*log(a + b*x^2))/(2*a^8) + (21*b^2*log(x))/a^8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.21

$$\int \frac{1}{x^5 (a^2 + 2abx^2 + b^2x^4)^3} dx$$
$$= \frac{-420 \log(b x^2 + a) a^5 b^2 x^4 - 2100 \log(b x^2 + a) a^4 b^3 x^6 - 4200 \log(b x^2 + a) a^3 b^4 x^8 - 4200 \log(b x^2 + a) a^2}{}$$

input
```
int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
( - 420*log(a + b*x**2)*a**5*b**2*x**4 - 2100*log(a + b*x**2)*a**4*b**3*x*
*6 - 4200*log(a + b*x**2)*a**3*b**4*x**8 - 4200*log(a + b*x**2)*a**2*b**5*
x**10 - 2100*log(a + b*x**2)*a*b**6*x**12 - 420*log(a + b*x**2)*b**7*x**14
 + 840*log(x)*a**5*b**2*x**4 + 4200*log(x)*a**4*b**3*x**6 + 8400*log(x)*a*
*3*b**4*x**8 + 8400*log(x)*a**2*b**5*x**10 + 4200*log(x)*a*b**6*x**12 + 84
0*log(x)*b**7*x**14 - 10*a**7 + 70*a**6*b*x**2 + 875*a**5*b**2*x**4 + 2275
*a**4*b**3*x**6 + 2450*a**3*b**4*x**8 + 1050*a**2*b**5*x**10 - 84*b**7*x**
14)/(40*a**8*x**4*(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3
*x**6 + 5*a*b**4*x**8 + b**5*x**10))
```

3.407 $\int \frac{x^{16}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{x^{16}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{21a^2x}{b^8}-\frac{2ax^3}{b^7}+\frac{x^5}{5b^6}+\frac{a^7x}{10b^8(a+bx^2)^5} -\frac{71a^6x}{80b^8(a+bx^2)^4}+\frac{581a^5x}{160b^8(a+bx^2)^3}-\frac{1211a^4x}{128b^8(a+bx^2)^2} +\frac{5327a^3x}{256b^8(a+bx^2)}-\frac{9009a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{17/2}}$$

output
```
21*a^2*x/b^8-2*a*x^3/b^7+1/5*x^5/b^6+1/10*a^7*x/b^8/(b*x^2+a)^5-71/80*a^6*
x/b^8/(b*x^2+a)^4+581/160*a^5*x/b^8/(b*x^2+a)^3-1211/128*a^4*x/b^8/(b*x^2+
a)^2+5327/256*a^3*x/b^8/(b*x^2+a)-9009/256*a^(5/2)*arctan(b^(1/2)*x/a^(1/2
))/b^(17/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \frac{x^{16}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{\sqrt{b}x(45045a^7+210210a^6bx^2+384384a^5b^2x^4+338910a^4b^3x^6+137995a^3b^4x^8+16640a^2b^5x^{10}-1280ab^6x^{12}+256b^7x^{14})}{(a+bx^2)^5}-45045a^{5/2}\text{arcta}}{1280b^{17/2}}$$

input
```
Integrate[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
((Sqrt[b]*x*(45045*a^7 + 210210*a^6*b*x^2 + 384384*a^5*b^2*x^4 + 338910*a^
4*b^3*x^6 + 137995*a^3*b^4*x^8 + 16640*a^2*b^5*x^10 - 1280*a*b^6*x^12 + 25
6*b^7*x^14))/(a + b*x^2)^5 - 45045*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(1
280*b^(17/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1380, 27, 252, 252, 252, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{16}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$

$$\Big\downarrow \; 1380$$

$$b^6\int \frac{x^{16}}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{x^{16}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \; 252$$

$$\frac{3\int \frac{x^{14}}{(bx^2+a)^5}dx}{2b} - \frac{x^{15}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{3\left(\frac{13\int \frac{x^{12}}{\left(bx^2+a\right)^4}dx}{8b} - \frac{x^{13}}{8b(a+bx^2)^4}\right)}{2b} - \frac{x^{15}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{3\left(\frac{13\left(\frac{11\int \frac{x^{10}}{\left(bx^2+a\right)^3}dx}{6b} - \frac{x^{11}}{6b\left(a+bx^2\right)^3}\right)}{8b} - \frac{x^{13}}{8b(a+bx^2)^4}\right)}{2b} - \frac{x^{15}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\int \frac{x^{8}}{\left(bx^2+a\right)^2}dx}{4b} - \frac{x^{9}}{4b\left(a+bx^2\right)^2}\right)}{6b} - \frac{x^{11}}{6b\left(a+bx^2\right)^3}\right)}{8b} - \frac{x^{13}}{8b(a+bx^2)^4}\right)}{2b} - \frac{x^{15}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int\frac{x^6}{bx^2+a}dx}{2b}-\frac{x^7}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^9}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{13}}{8b(a+bx^2)^4}\right)}{2b}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int\left(\frac{x^4}{b}-\frac{ax^2}{b^2}-\frac{a^3}{b^3\left(bx^2+a\right)}+\frac{a^2}{b^3}\right)dx}{2b}-\frac{x^7}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^9}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{13}}{8b(a+bx^2)^4}\right)}{2b}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 2009$$

$$\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}\right)}{2b}-\frac{x^7}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^9}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{13}}{8b(a+bx^2)^4}\right)}{2b}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}$$

input
```
Int[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*x^15/(b*(a + b*x^2)^5) + (3*(-1/8*x^13/(b*(a + b*x^2)^4) + (13*(-1/6
*x^11/(b*(a + b*x^2)^3) + (11*(-1/4*x^9/(b*(a + b*x^2)^2) + (9*(-1/2*x^7/(
b*(a + b*x^2)) + (7*((a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)))/(2*b)))/(4*b)))/(6*b)))/(8*b)))/(2*
b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

method	result
default	$\frac{\frac{1}{5}x^5b^2-2ab\,x^3+21a^2x}{b^8}-\frac{a^3\left(\frac{-\frac{5327}{256}b^4x^9-\frac{9443}{128}a\,b^3x^7-\frac{1001}{10}a^2b^2x^5-\frac{7837}{128}a^3b\,x^3-\frac{3633}{256}a^4x}{\left(b\,x^2+a\right)^5}+\frac{9009\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}}\right)}{b^8}$
risch	$\frac{x^5}{5b^6}-\frac{2a\,x^3}{b^7}+\frac{21a^2x}{b^8}+\frac{\frac{5327}{256}a^3b^4x^9+\frac{9443}{128}a^4b^3x^7+\frac{1001}{10}a^5b^2x^5+\frac{7837}{128}a^6b\,x^3+\frac{3633}{256}a^7x}{b^8(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}+\frac{9009\sqrt{-ab}\,a^2\ln(-\sqrt{-ab}\,x-a)}{512b^9}-$

input
```
int(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^8*(1/5*x^5*b^2-2*a*b*x^3+21*a^2*x)-1/b^8*a^3*((-5327/256*b^4*x^9-9443/
128*a*b^3*x^7-1001/10*a^2*b^2*x^5-7837/128*a^3*b*x^3-3633/256*a^4*x)/(b*x^
2+a)^5+9009/256/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.91

$$\int \frac{x^{16}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$=\left[\frac{512\,b^7x^{15}-2560\,ab^6x^{13}+33280\,a^2b^5x^{11}+275990\,a^3b^4x^9+677820\,a^4b^3x^7+768768\,a^5b^2x^5+420420\,a}{2560\,(b^{13}x^{10}+5\,ab^{12}x^8+10}\right.$$

input
```
integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
[1/2560*(512*b^7*x^15 - 2560*a*b^6*x^13 + 33280*a^2*b^5*x^11 + 275990*a^3*
b^4*x^9 + 677820*a^4*b^3*x^7 + 768768*a^5*b^2*x^5 + 420420*a^6*b*x^3 + 900
90*a^7*x + 45045*(a^2*b^5*x^10 + 5*a^3*b^4*x^8 + 10*a^4*b^3*x^6 + 10*a^5*b
^2*x^4 + 5*a^6*b*x^2 + a^7)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/
(b*x^2 + a)))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^
4 + 5*a^4*b^9*x^2 + a^5*b^8), 1/1280*(256*b^7*x^15 - 1280*a*b^6*x^13 + 166
40*a^2*b^5*x^11 + 137995*a^3*b^4*x^9 + 338910*a^4*b^3*x^7 + 384384*a^5*b^2
*x^5 + 210210*a^6*b*x^3 + 45045*a^7*x - 45045*(a^2*b^5*x^10 + 5*a^3*b^4*x^
8 + 10*a^4*b^3*x^6 + 10*a^5*b^2*x^4 + 5*a^6*b*x^2 + a^7)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^1
0*x^4 + 5*a^4*b^9*x^2 + a^5*b^8)]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.40

$$\int \frac{x^{16}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{21a^2x}{b^8} - \frac{2ax^3}{b^7} + \frac{9009\sqrt{-\frac{a^5}{b^{17}}}\log\left(x - \frac{b^8\sqrt{-\frac{a^5}{b^{17}}}}{a^2}\right)}{512} - \frac{9009\sqrt{-\frac{a^5}{b^{17}}}\log\left(x + \frac{b^8\sqrt{-\frac{a^5}{b^{17}}}}{a^2}\right)}{512}$$
$$+ \frac{18165a^7x + 78370a^6bx^3 + 128128a^5b^2x^5 + 94430a^4b^3x^7 + 26635a^3b^4x^9}{1280a^5b^8 + 6400a^4b^9x^2 + 12800a^3b^{10}x^4 + 12800a^2b^{11}x^6 + 6400ab^{12}x^8 + 1280b^{13}x^{10}}$$
$$+ \frac{x^5}{5b^6}$$

input
```
integrate(x**16/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
21*a**2*x/b**8 - 2*a*x**3/b**7 + 9009*sqrt(-a**5/b**17)*log(x - b**8*sqrt(
-a**5/b**17)/a**2)/512 - 9009*sqrt(-a**5/b**17)*log(x + b**8*sqrt(-a**5/b*
*17)/a**2)/512 + (18165*a**7*x + 78370*a**6*b*x**3 + 128128*a**5*b**2*x**5
 + 94430*a**4*b**3*x**7 + 26635*a**3*b**4*x**9)/(1280*a**5*b**8 + 6400*a**
4*b**9*x**2 + 12800*a**3*b**10*x**4 + 12800*a**2*b**11*x**6 + 6400*a*b**12
*x**8 + 1280*b**13*x**10) + x**5/(5*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02

$$\int \frac{x^{16}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{26635\,a^3b^4x^9+94430\,a^4b^3x^7+128128\,a^5b^2x^5+78370\,a^6bx^3+18165\,a^7x}{1280\,(b^{13}x^{10}+5\,ab^{12}x^8+10\,a^2b^{11}x^6+10\,a^3b^{10}x^4+5\,a^4b^9x^2+a^5b^8)} - \frac{9009\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^8} + \frac{b^2x^5-10\,abx^3+105\,a^2x}{5\,b^8}$$

input
```
integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/1280*(26635*a^3*b^4*x^9 + 94430*a^4*b^3*x^7 + 128128*a^5*b^2*x^5 + 78370
*a^6*b*x^3 + 18165*a^7*x)/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^6 + 10
*a^3*b^10*x^4 + 5*a^4*b^9*x^2 + a^5*b^8) - 9009/256*a^3*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^8) + 1/5*(b^2*x^5 - 10*a*b*x^3 + 105*a^2*x)/b^8
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\int \frac{x^{16}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{9009\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^8} + \frac{26635\,a^3b^4x^9+94430\,a^4b^3x^7+128128\,a^5b^2x^5+78370\,a^6bx^3+18165\,a^7x}{1280\,(bx^2+a)^5b^8} + \frac{b^{24}x^5-10\,ab^{23}x^3+105\,a^2b^{22}x}{5\,b^{30}}$$

input
```
integrate(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-9009/256*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^8) + 1/1280*(26635*a^3*b^
4*x^9 + 94430*a^4*b^3*x^7 + 128128*a^5*b^2*x^5 + 78370*a^6*b*x^3 + 18165*a
^7*x)/((b*x^2 + a)^5*b^8) + 1/5*(b^24*x^5 - 10*a*b^23*x^3 + 105*a^2*b^22*x
)/b^30
```

Mupad [B] (verification not implemented)

Time = 17.53 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int \frac{x^{16}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{3633\,a^7\,x}{256}+\frac{7837\,a^6\,b\,x^3}{128}+\frac{1001\,a^5\,b^2\,x^5}{10}+\frac{9443\,a^4\,b^3\,x^7}{128}+\frac{5327\,a^3\,b^4\,x^9}{256}}{a^5\,b^8+5\,a^4\,b^9\,x^2+10\,a^3\,b^{10}\,x^4+10\,a^2\,b^{11}\,x^6+5\,a\,b^{12}\,x^8+b^{13}\,x^{10}} + \frac{x^5}{5\,b^6}-\frac{2\,a\,x^3}{b^7}+\frac{21\,a^2\,x}{b^8}-\frac{9009\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,b^{17/2}}$$

input
```
int(x^16/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((3633*a^7*x)/256 + (7837*a^6*b*x^3)/128 + (1001*a^5*b^2*x^5)/10 + (9443*a
^4*b^3*x^7)/128 + (5327*a^3*b^4*x^9)/256)/(a^5*b^8 + b^13*x^10 + 5*a*b^12*
x^8 + 5*a^4*b^9*x^2 + 10*a^3*b^10*x^4 + 10*a^2*b^11*x^6) + x^5/(5*b^6) - (
2*a*x^3)/b^7 + (21*a^2*x)/b^8 - (9009*a^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(
256*b^(17/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.87

$$\int \frac{x^{16}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{-45045\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^7-225225\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^6b\,x^2-450450\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5b^2x^4}{}$$

input
```
int(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
( - 45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7 - 225225*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b*x**2 - 450450*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**2*x**4 - 450450*sqrt(b)*sqrt(a)
*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**3*x**6 - 225225*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**4*x**8 - 45045*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**2*b**5*x**10 + 45045*a**7*b*x + 210210*a**6*b**2*
x**3 + 384384*a**5*b**3*x**5 + 338910*a**4*b**4*x**7 + 137995*a**3*b**5*x*
*9 + 16640*a**2*b**6*x**11 - 1280*a*b**7*x**13 + 256*b**8*x**15)/(1280*b**
9*(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4
*x**8 + b**5*x**10))
```

3.408 $\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\begin{aligned}\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx = &-\frac{6ax}{b^7}+\frac{x^3}{3b^6}-\frac{a^6x}{10b^7(a+bx^2)^5}+\frac{61a^5x}{80b^7(a+bx^2)^4}\\ &-\frac{1253a^4x}{480b^7(a+bx^2)^3}+\frac{2107a^3x}{384b^7(a+bx^2)^2}\\ &-\frac{2373a^2x}{256b^7(a+bx^2)}+\frac{3003a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{15/2}}\end{aligned}$$

output
```
-6*a*x/b^7+1/3*x^3/b^6-1/10*a^6*x/b^7/(b*x^2+a)^5+61/80*a^5*x/b^7/(b*x^2+a
)^4-1253/480*a^4*x/b^7/(b*x^2+a)^3+2107/384*a^3*x/b^7/(b*x^2+a)^2-2373/256
*a^2*x/b^7/(b*x^2+a)+3003/256*a^(3/2)*arctan(b^(1/2)*x/a^(1/2))/b^(15/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{\frac{\sqrt{b}x(-45045a^6-210210a^5bx^2-384384a^4b^2x^4-338910a^3b^3x^6-137995a^2b^4x^8-16640ab^5x^{10}+1280b^6x^{12})}{(a+bx^2)^5}+45045a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3840b^{15/2}}$$

input
```
Integrate[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
((Sqrt[b]*x*(-45045*a^6 - 210210*a^5*b*x^2 - 384384*a^4*b^2*x^4 - 338910*a
^3*b^3*x^6 - 137995*a^2*b^4*x^8 - 16640*a*b^5*x^10 + 1280*b^6*x^12))/(a +
b*x^2)^5 + 45045*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3840*b^(15/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1380, 27, 252, 252, 252, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$\Big\downarrow \text{1380}$$
$$b^6\int \frac{x^{14}}{b^6(bx^2+a)^6}dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{x^{14}}{(a+bx^2)^6}dx$$
$$\Big\downarrow \text{252}$$

$$\frac{13\int \frac{x^{12}}{(bx^2+a)^5}dx}{10b}-\frac{x^{13}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{13\left(\frac{11\int \frac{x^{10}}{\left(bx^2+a\right)^4}dx}{8b}-\frac{x^{11}}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^{13}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{13\left(\frac{11\left(\frac{3\int \frac{x^8}{\left(bx^2+a\right)^3}dx}{2b}-\frac{x^9}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{11}}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^{13}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\int \frac{x^6}{\left(bx^2+a\right)^2}dx}{4b}-\frac{x^7}{4b\left(a+bx^2\right)^2}\right)}{2b}-\frac{x^9}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{11}}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^{13}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{x^4}{bx^2+a}dx}{2b}-\frac{x^5}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^7}{4b\left(a+bx^2\right)^2}\right)}{2b}-\frac{x^9}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{11}}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 254

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\int\left(\frac{a^2}{b^2(bx^2+a)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx}{2b}-\frac{x^5}{2b(a+bx^2)}\right)}{4b}-\frac{x^7}{4b(a+bx^2)^2}\right)}{2b}-\frac{x^9}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^{11}}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^{13}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \; 2009$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}\right)}{2b}-\frac{x^5}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^7}{4b\left(a+bx^2\right)^2}\right)}{2b}-\frac{x^9}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{11}}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}$$

input
```
Int[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*x^13/(b*(a + b*x^2)^5) + (13*(-1/8*x^11/(b*(a + b*x^2)^4) + (11*(-1/
6*x^9/(b*(a + b*x^2)^3) + (3*(-1/4*x^7/(b*(a + b*x^2)^2) + (7*(-1/2*x^5/(b
*(a + b*x^2)) + (5*(-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/b^(5/2)))/(2*b)))/(4*b)))/(2*b)))/(8*b)))/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.66

method	result
default	$-\frac{-\frac{1}{3}b\,x^3+6ax}{b^7}+\frac{a^2\left(\frac{-\frac{2373}{256}b^4x^9-\frac{12131}{384}a\,b^3x^7-\frac{1253}{30}a^2b^2x^5-\frac{9629}{384}a^3b\,x^3-\frac{1467}{256}a^4x}{\left(b\,x^2+a\right)^5}+\frac{3003\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}}\right)}{b^7}$
risch	$\frac{x^3}{3b^6}-\frac{6ax}{b^7}+\frac{-\frac{2373}{256}a^2b^4x^9-\frac{12131}{384}a^3b^3x^7-\frac{1253}{30}a^4b^2x^5-\frac{9629}{384}a^5b\,x^3-\frac{1467}{256}a^6x}{b^7(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}+\frac{3003\sqrt{-ab}\,a\ln(-\sqrt{-ab}\,x+a)}{512b^8}-\frac{3003\sqrt{-a}}{}$

input
```
int(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^7*(-1/3*b*x^3+6*a*x)+1/b^7*a^2*((-2373/256*b^4*x^9-12131/384*a*b^3*x^
7-1253/30*a^2*b^2*x^5-9629/384*a^3*b*x^3-1467/256*a^4*x)/(b*x^2+a)^5+3003/
256/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.95

$$\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$

$$= \left[\frac{2560\,b^6x^{13}-33280\,ab^5x^{11}-275990\,a^2b^4x^9-677820\,a^3b^3x^7-768768\,a^4b^2x^5-420420\,a^5bx^3-90090\,}{7680\,(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6}\right.$$

input
```
integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
[1/7680*(2560*b^6*x^13 - 33280*a*b^5*x^11 - 275990*a^2*b^4*x^9 - 677820*a^
3*b^3*x^7 - 768768*a^4*b^2*x^5 - 420420*a^5*b*x^3 - 90090*a^6*x + 45045*(a
*b^5*x^10 + 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2
+ a^6)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^12*x
^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^
5*b^7), 1/3840*(1280*b^6*x^13 - 16640*a*b^5*x^11 - 137995*a^2*b^4*x^9 - 33
8910*a^3*b^3*x^7 - 384384*a^4*b^2*x^5 - 210210*a^5*b*x^3 - 45045*a^6*x + 4
5045*(a*b^5*x^10 + 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5
*b*x^2 + a^6)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^12*x^10 + 5*a*b^11*x^8
 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)]
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.41

$$\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{6ax}{b^7} - \frac{3003\sqrt{-\frac{a^3}{b^{15}}}\log\left(x-\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}}{a}\right)}{512} + \frac{3003\sqrt{-\frac{a^3}{b^{15}}}\log\left(x+\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}}{a}\right)}{512} + \frac{-22005a^6x-96290a^5bx^3-160384a^4b^2x^5-121310a^3b^3x^7-35595a^2b^4x^9}{3840a^5b^7+19200a^4b^8x^2+38400a^3b^9x^4+38400a^2b^{10}x^6+19200ab^{11}x^8+3840b^{12}x^{10}} + \frac{x^3}{3b^6}$$

input
```
integrate(x**14/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
-6*a*x/b**7 - 3003*sqrt(-a**3/b**15)*log(x - b**7*sqrt(-a**3/b**15)/a)/512
 + 3003*sqrt(-a**3/b**15)*log(x + b**7*sqrt(-a**3/b**15)/a)/512 + (-22005*
a**6*x - 96290*a**5*b*x**3 - 160384*a**4*b**2*x**5 - 121310*a**3*b**3*x**7
 - 35595*a**2*b**4*x**9)/(3840*a**5*b**7 + 19200*a**4*b**8*x**2 + 38400*a*
*3*b**9*x**4 + 38400*a**2*b**10*x**6 + 19200*a*b**11*x**8 + 3840*b**12*x**
10) + x**3/(3*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02

$$\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{35595\,a^2b^4x^9+121310\,a^3b^3x^7+160384\,a^4b^2x^5+96290\,a^5bx^3+22005\,a^6x}{3840\,(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7)} + \frac{3003\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^7} + \frac{bx^3-18\,ax}{3\,b^7}$$

input
```
integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/3840*(35595*a^2*b^4*x^9 + 121310*a^3*b^3*x^7 + 160384*a^4*b^2*x^5 + 962
90*a^5*b*x^3 + 22005*a^6*x)/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 +
10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7) + 3003/256*a^2*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*b^7) + 1/3*(b*x^3 - 18*a*x)/b^7
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.73

$$\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{3003\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^7} - \frac{35595\,a^2b^4x^9+121310\,a^3b^3x^7+160384\,a^4b^2x^5+96290\,a^5bx^3+22005\,a^6x}{3840\,(bx^2+a)^5b^7} + \frac{b^{12}x^3-18\,ab^{11}x}{3\,b^{18}}$$

input
```
integrate(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
3003/256*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) - 1/3840*(35595*a^2*b^4
*x^9 + 121310*a^3*b^3*x^7 + 160384*a^4*b^2*x^5 + 96290*a^5*b*x^3 + 22005*a
^6*x)/((b*x^2 + a)^5*b^7) + 1/3*(b^12*x^3 - 18*a*b^11*x)/b^18
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99

$$\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{x^3}{3\,b^6} - \frac{\frac{1467\,a^6\,x}{256} + \frac{9629\,a^5\,b\,x^3}{384} + \frac{1253\,a^4\,b^2\,x^5}{30} + \frac{12131\,a^3\,b^3\,x^7}{384} + \frac{2373\,a^2\,b^4\,x^9}{256}}{a^5\,b^7 + 5\,a^4\,b^8\,x^2 + 10\,a^3\,b^9\,x^4 + 10\,a^2\,b^{10}\,x^6 + 5\,a\,b^{11}\,x^8 + b^{12}\,x^{10}}$$
$$+ \frac{3003\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,b^{15/2}} - \frac{6\,a\,x}{b^7}$$

input
```
int(x^14/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
x^3/(3*b^6) - ((1467*a^6*x)/256 + (9629*a^5*b*x^3)/384 + (1253*a^4*b^2*x^5
)/30 + (12131*a^3*b^3*x^7)/384 + (2373*a^2*b^4*x^9)/256)/(a^5*b^7 + b^12*x
^10 + 5*a*b^11*x^8 + 5*a^4*b^8*x^2 + 10*a^3*b^9*x^4 + 10*a^2*b^10*x^6) + (
3003*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(256*b^(15/2)) - (6*a*x)/b^7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.92

$$\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{45045\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^6 + 225225\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5b\,x^2 + 450450\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4b^2x^4 +}{}$$

input
```
int(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**6 + 225225*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b*x**2 + 450450*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**2*x**4 + 450450*sqrt(b)*sqrt(a)*at
an((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**3*x**6 + 225225*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**2*b**4*x**8 + 45045*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*a*b**5*x**10 - 45045*a**6*b*x - 210210*a**5*b**2*x**3 -
 384384*a**4*b**3*x**5 - 338910*a**3*b**4*x**7 - 137995*a**2*b**5*x**9 - 1
6640*a*b**6*x**11 + 1280*b**7*x**13)/(3840*b**8*(a**5 + 5*a**4*b*x**2 + 10
*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10))
```

3.409 $\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{x}{b^6}+\frac{a^5x}{10b^6(a+bx^2)^5}-\frac{51a^4x}{80b^6(a+bx^2)^4}+\frac{281a^3x}{160b^6(a+bx^2)^3}-\frac{359a^2x}{128b^6(a+bx^2)^2}+\frac{843ax}{256b^6(a+bx^2)}-\frac{693\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{13/2}}$$

output
```
x/b^6+1/10*a^5*x/b^6/(b*x^2+a)^5-51/80*a^4*x/b^6/(b*x^2+a)^4+281/160*a^3*x
/b^6/(b*x^2+a)^3-359/128*a^2*x/b^6/(b*x^2+a)^2+843/256*a*x/b^6/(b*x^2+a)-6
93/256*a^(1/2)*arctan(b^(1/2)*x/a^(1/2))/b^(13/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{\sqrt{b}x(3465a^5+16170a^4bx^2+29568a^3b^2x^4+26070a^2b^3x^6+10615ab^4x^8+1280b^5x^{10})}{(a+bx^2)^5}-3465\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1280b^{13/2}}$$

input
```
Integrate[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
((Sqrt[b]*x*(3465*a^5 + 16170*a^4*b*x^2 + 29568*a^3*b^2*x^4 + 26070*a^2*b^
3*x^6 + 10615*a*b^4*x^8 + 1280*b^5*x^10))/(a + b*x^2)^5 - 3465*Sqrt[a]*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(1280*b^(13/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1380, 27, 252, 252, 252, 252, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{x^{12}}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{12}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{11\int \frac{x^{10}}{(bx^2+a)^5}dx}{10b}-\frac{x^{11}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \color{blue}{252}$$

$$\frac{11\left(\frac{9\int \frac{x^8}{\left(bx^2+a\right)^4}dx}{8b}-\frac{x^9}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^{11}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \color{blue}{252}$$

$$\frac{11\left(\frac{9\left(\frac{7\int \frac{x^6}{\left(bx^2+a\right)^3}dx}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^9}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^{11}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \color{blue}{252}$$

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\int \frac{x^4}{\left(bx^2+a\right)^2}dx}{4b}-\frac{x^5}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^9}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^{11}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \color{blue}{252}$$

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{x^2}{bx^2+a}dx}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^5}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^9}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{262}$$

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{x}{b}-\frac{a\int\frac{1}{bx^2+a}\,dx}{b}\right)}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^5}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^9}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 218$$

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^5}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^9}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}$$

input
```
Int[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*x^11/(b*(a + b*x^2)^5) + (11*(-1/8*x^9/(b*(a + b*x^2)^4) + (9*(-1/6*
x^7/(b*(a + b*x^2)^3) + (7*(-1/4*x^5/(b*(a + b*x^2)^2) + (5*(-1/2*x^3/(b*(
a + b*x^2)) + (3*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)))/(2
*b)))/(4*b)))/(6*b)))/(8*b)))/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

method	result
default	$\frac{x}{b^6}-\frac{a\left(\frac{-\frac{843}{256}b^4x^9-\frac{1327}{128}a\,b^3x^7-\frac{131}{10}a^2b^2x^5-\frac{977}{128}a^3b\,x^3-\frac{437}{256}a^4x}{\left(b\,x^2+a\right)^5}+\frac{693\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}}\right)}{b^6}$
risch	$\frac{x}{b^6}+\frac{\frac{843}{256}a\,b^4x^9+\frac{1327}{128}a^2b^3x^7+\frac{131}{10}a^3b^2x^5+\frac{977}{128}a^4b\,x^3+\frac{437}{256}x\,a^5}{b^6(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}+\frac{693\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{512b^7}-\frac{693\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{512b^7}$

input
```
int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
x/b^6-1/b^6*a*((-843/256*b^4*x^9-1327/128*a*b^3*x^7-131/10*a^2*b^2*x^5-977
/128*a^3*b*x^3-437/256*a^4*x)/(b*x^2+a)^5+693/256/(a*b)^(1/2)*arctan(b/(a*
b)^(1/2)*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.05

$$\int \frac{x^{12}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

$$= \left[\frac{2560\,b^5x^{11}+21230\,ab^4x^9+52140\,a^2b^3x^7+59136\,a^3b^2x^5+32340\,a^4bx^3+6930\,a^5x+3465\,(b^5x^{10}+5\,a}{2560\,(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4}\right.$$

input
```
integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
[1/2560*(2560*b^5*x^11 + 21230*a*b^4*x^9 + 52140*a^2*b^3*x^7 + 59136*a^3*b
^2*x^5 + 32340*a^4*b*x^3 + 6930*a^5*x + 3465*(b^5*x^10 + 5*a*b^4*x^8 + 10*
a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2 -
2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9
*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6), 1/1280*(1280*b^5*x^11 +
10615*a*b^4*x^9 + 26070*a^2*b^3*x^7 + 29568*a^3*b^2*x^5 + 16170*a^4*b*x^3
+ 3465*a^5*x - 3465*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*
x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^11*x^10 + 5
*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.36

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{693\sqrt{-\frac{a}{b^{13}}}\log\left(-b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{512} - \frac{693\sqrt{-\frac{a}{b^{13}}}\log\left(b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{512} + \frac{2185a^5x+9770a^4bx^3+16768a^3b^2x^5+13270a^2b^3x^7+4215ab^4x^9}{1280a^5b^6+6400a^4b^7x^2+12800a^3b^8x^4+12800a^2b^9x^6+6400ab^{10}x^8+1280b^{11}x^{10}} + \frac{x}{b^6}$$

input
```
integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
693*sqrt(-a/b**13)*log(-b**6*sqrt(-a/b**13) + x)/512 - 693*sqrt(-a/b**13)*
log(b**6*sqrt(-a/b**13) + x)/512 + (2185*a**5*x + 9770*a**4*b*x**3 + 16768
*a**3*b**2*x**5 + 13270*a**2*b**3*x**7 + 4215*a*b**4*x**9)/(1280*a**5*b**6
 + 6400*a**4*b**7*x**2 + 12800*a**3*b**8*x**4 + 12800*a**2*b**9*x**6 + 640
0*a*b**10*x**8 + 1280*b**11*x**10) + x/b**6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{4215\,ab^4x^9+13270\,a^2b^3x^7+16768\,a^3b^2x^5+9770\,a^4bx^3+2185\,a^5x}{1280\,(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)} - \frac{693\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^6}+\frac{x}{b^6}$$

input
```
integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/1280*(4215*a*b^4*x^9 + 13270*a^2*b^3*x^7 + 16768*a^3*b^2*x^5 + 9770*a^4*
b*x^3 + 2185*a^5*x)/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^
8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6) - 693/256*a*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^6) + x/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.66

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{693\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^6}+\frac{x}{b^6} + \frac{4215\,ab^4x^9+13270\,a^2b^3x^7+16768\,a^3b^2x^5+9770\,a^4bx^3+2185\,a^5x}{1280\,(bx^2+a)^5b^6}$$

input
```
integrate(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-693/256*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + x/b^6 + 1/1280*(4215*a*
b^4*x^9 + 13270*a^2*b^3*x^7 + 16768*a^3*b^2*x^5 + 9770*a^4*b*x^3 + 2185*a^
5*x)/((b*x^2 + a)^5*b^6)
```

Mupad [B] (verification not implemented)

Time = 17.91 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.99

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{437\,a^5\,x}{256}+\frac{977\,a^4\,b\,x^3}{128}+\frac{131\,a^3\,b^2\,x^5}{10}+\frac{1327\,a^2\,b^3\,x^7}{128}+\frac{843\,a\,b^4\,x^9}{256}}{a^5\,b^6+5\,a^4\,b^7\,x^2+10\,a^3\,b^8\,x^4+10\,a^2\,b^9\,x^6+5\,a\,b^{10}\,x^8+b^{11}\,x^{10}} + \frac{x}{b^6} - \frac{693\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,b^{13/2}}$$

input
```
int(x^12/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((437*a^5*x)/256 + (977*a^4*b*x^3)/128 + (843*a*b^4*x^9)/256 + (131*a^3*b^
2*x^5)/10 + (1327*a^2*b^3*x^7)/128)/(a^5*b^6 + b^11*x^10 + 5*a*b^10*x^8 +
5*a^4*b^7*x^2 + 10*a^3*b^8*x^4 + 10*a^2*b^9*x^6) + x/b^6 - (693*a^(1/2)*at
an((b^(1/2)*x)/a^(1/2)))/(256*b^(13/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.02

$$\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{-3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5 - 17325\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4b\,x^2 - 34650\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b^2x^4 - 3}{}$$

input
```
int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
( - 3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5 - 17325*sqrt(b
)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*x**2 - 34650*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**4 - 34650*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*x**6 - 17325*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a*b**4*x**8 - 3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*b**5*x**10 + 3465*a**5*b*x + 16170*a**4*b**2*x**3 + 29568*a**
3*b**3*x**5 + 26070*a**2*b**4*x**7 + 10615*a*b**5*x**9 + 1280*b**6*x**11)/
(1280*b**7*(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6 +
 5*a*b**4*x**8 + b**5*x**10))
```

3.410 $\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^3} \, dx = -\frac{x^9}{10b(a+bx^2)^5} - \frac{9x^7}{80b^2(a+bx^2)^4} - \frac{21x^5}{160b^3(a+bx^2)^3} - \frac{21x^3}{128b^4(a+bx^2)^2} - \frac{63x}{256b^5(a+bx^2)} + \frac{63\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{a}b^{11/2}}$$

output
```
-1/10*x^9/b/(b*x^2+a)^5-9/80*x^7/b^2/(b*x^2+a)^4-21/160*x^5/b^3/(b*x^2+a)^
3-21/128*x^3/b^4/(b*x^2+a)^2-63/256*x/b^5/(b*x^2+a)+63/256*arctan(b^(1/2)*
x/a^(1/2))/a^(1/2)/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^3} \, dx = -\frac{x(315a^4+1470a^3bx^2+2688a^2b^2x^4+2370ab^3x^6+965b^4x^8)}{1280b^5(a+bx^2)^5} + \frac{63\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{a}b^{11/2}}$$

input
```
Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/1280*(x*(315*a^4 + 1470*a^3*b*x^2 + 2688*a^2*b^2*x^4 + 2370*a*b^3*x^6 +
 965*b^4*x^8))/(b^5*(a + b*x^2)^5) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256
*Sqrt[a]*b^(11/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1380, 27, 252, 252, 252, 252, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a^2 + 2abx^2 + b^2x^4)^3} dx$$

$$\Big\downarrow \text{1380}$$

$$b^6 \int \frac{x^{10}}{b^6 (bx^2 + a)^6} dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{10}}{(a + bx^2)^6} dx$$

$$\Big\downarrow \text{252}$$

$$\frac{9 \int \frac{x^8}{(bx^2+a)^5} dx}{10b} - \frac{x^9}{10b (a + bx^2)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{9 \left(\frac{7 \int \frac{x^6}{(bx^2+a)^4} dx}{8b} - \frac{x^7}{8b(a+bx^2)^4} \right)}{10b} - \frac{x^9}{10b (a + bx^2)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{9\left(\frac{7\left(\frac{5\int \frac{x^4}{\left(bx^2+a\right)^3}dx}{6b}-\frac{x^5}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^7}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^9}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 252

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{x^2}{\left(bx^2+a\right)^2}dx}{4b}-\frac{x^3}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^5}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^7}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^9}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 252

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2b}-\frac{x}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^3}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^5}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^7}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^9}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 218

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b(a+bx^2)}\right)}{4b}-\frac{x^3}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^5}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^7}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^9}{10b\,(a+bx^2)^5}$$

input
```
Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*x^9/(b*(a + b*x^2)^5) + (9*(-1/8*x^7/(b*(a + b*x^2)^4) + (7*(-1/6*x^
5/(b*(a + b*x^2)^3) + (5*(-1/4*x^3/(b*(a + b*x^2)^2) + (3*(-1/2*x/(b*(a +
b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))))/(4*b)))/(6*b))
)/(8*b)))/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{-\frac{193x^9}{256b}-\frac{237a\,x^7}{128b^2}-\frac{21a^2x^5}{10b^3}-\frac{147a^3x^3}{128b^4}-\frac{63a^4x}{256b^5}}{(b\,x^2+a)^5}+\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256b^5\sqrt{ab}}$	80
risch	$\frac{-\frac{193x^9}{256b}-\frac{237a\,x^7}{128b^2}-\frac{21a^2x^5}{10b^3}-\frac{147a^3x^3}{128b^4}-\frac{63a^4x}{256b^5}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}-\frac{63\ln\left(bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^5}+\frac{63\ln\left(-bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^5}$	126

input
```
int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-193/256/b*x^9-237/128*a/b^2*x^7-21/10*a^2/b^3*x^5-147/128*a^3/b^4*x^3-63
/256*a^4/b^5*x)/(b*x^2+a)^5+63/256/b^5/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.19

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \left[-\frac{1930\,ab^5x^9+4740\,a^2b^4x^7+5376\,a^3b^3x^5+2940\,a^4b^2x^3+630\,a^5bx+315\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6}{2560\,(ab^{11}x^{10}+5\,a^2b^{10}x^8+10\,a^3b^9x^6+10\,a^4b^8x^4+5\,a^5b} \right.$$
$$-\frac{965\,ab^5x^9+2370\,a^2b^4x^7+2688\,a^3b^3x^5+1470\,a^4b^2x^3+315\,a^5bx-315\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+}{1280\,(ab^{11}x^{10}+5\,a^2b^{10}x^8+10\,a^3b^9x^6+10\,a^4b^8x^4+5\,a^5b^7x^2+}$$

input
```
integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
[-1/2560*(1930*a*b^5*x^9 + 4740*a^2*b^4*x^7 + 5376*a^3*b^3*x^5 + 2940*a^4*
b^2*x^3 + 630*a^5*b*x + 315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*
a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x -
a)/(b*x^2 + a)))/(a*b^11*x^10 + 5*a^2*b^10*x^8 + 10*a^3*b^9*x^6 + 10*a^4*b
^8*x^4 + 5*a^5*b^7*x^2 + a^6*b^6), -1/1280*(965*a*b^5*x^9 + 2370*a^2*b^4*x
^7 + 2688*a^3*b^3*x^5 + 1470*a^4*b^2*x^3 + 315*a^5*b*x - 315*(b^5*x^10 + 5
*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b
)*arctan(sqrt(a*b)*x/a))/(a*b^11*x^10 + 5*a^2*b^10*x^8 + 10*a^3*b^9*x^6 +
10*a^4*b^8*x^4 + 5*a^5*b^7*x^2 + a^6*b^6)]
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.50

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= -\frac{63\sqrt{-\frac{1}{ab^{11}}}\log\left(-ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{512}+\frac{63\sqrt{-\frac{1}{ab^{11}}}\log\left(ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{512}$$
$$+\frac{-315a^4x-1470a^3bx^3-2688a^2b^2x^5-2370ab^3x^7-965b^4x^9}{1280a^5b^5+6400a^4b^6x^2+12800a^3b^7x^4+12800a^2b^8x^6+6400ab^9x^8+1280b^{10}x^{10}}$$

input
```
integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
-63*sqrt(-1/(a*b**11))*log(-a*b**5*sqrt(-1/(a*b**11)) + x)/512 + 63*sqrt(-
1/(a*b**11))*log(a*b**5*sqrt(-1/(a*b**11)) + x)/512 + (-315*a**4*x - 1470*
a**3*b*x**3 - 2688*a**2*b**2*x**5 - 2370*a*b**3*x**7 - 965*b**4*x**9)/(128
0*a**5*b**5 + 6400*a**4*b**6*x**2 + 12800*a**3*b**7*x**4 + 12800*a**2*b**8
*x**6 + 6400*a*b**9*x**8 + 1280*b**10*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{965\,b^4x^9+2370\,ab^3x^7+2688\,a^2b^2x^5+1470\,a^3bx^3+315\,a^4x}{1280\,(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5)} + \frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^5}$$

input
```
integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/1280*(965*b^4*x^9 + 2370*a*b^3*x^7 + 2688*a^2*b^2*x^5 + 1470*a^3*b*x^3
+ 315*a^4*x)/(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 +
5*a^4*b^6*x^2 + a^5*b^5) + 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^5} - \frac{965\,b^4x^9+2370\,ab^3x^7+2688\,a^2b^2x^5+1470\,a^3bx^3+315\,a^4x}{1280\,(bx^2+a)^5b^5}$$

input
```
integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/1280*(965*b^4*x^9 + 2370*
a*b^3*x^7 + 2688*a^2*b^2*x^5 + 1470*a^3*b*x^3 + 315*a^4*x)/((b*x^2 + a)^5*
b^5)
```

Mupad [B] (verification not implemented)

Time = 18.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

$$\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{63\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,\sqrt{a}\,b^{11/2}}-\frac{\frac{193\,x^9}{256\,b}+\frac{237\,a\,x^7}{128\,b^2}+\frac{63\,a^4\,x}{256\,b^5}+\frac{21\,a^2\,x^5}{10\,b^3}+\frac{147\,a^3\,x^3}{128\,b^4}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

input
```
int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(63*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(1/2)*b^(11/2)) - ((193*x^9)/(256*b)
 + (237*a*x^7)/(128*b^2) + (63*a^4*x)/(256*b^5) + (21*a^2*x^5)/(10*b^3) +
(147*a^3*x^3)/(128*b^4))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*
a^3*b^2*x^4 + 10*a^2*b^3*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.15

$$\int \frac{x^{10}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5+1575\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4b\,x^2+3150\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b^2x^4+3150\sqrt{}}{1280a}$$

input
```
int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5 + 1575*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*x**2 + 3150*sqrt(b)*sqrt(a)*atan
((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**4 + 3150*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*a**2*b**3*x**6 + 1575*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt
(b)*sqrt(a)))*a*b**4*x**8 + 315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*b**5*x**10 - 315*a**5*b*x - 1470*a**4*b**2*x**3 - 2688*a**3*b**3*x**5
- 2370*a**2*b**4*x**7 - 965*a*b**5*x**9)/(1280*a*b**6*(a**5 + 5*a**4*b*x**
2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10))
```

3.411 $\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 122

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{x^7}{10b(a+bx^2)^5} - \frac{7x^5}{80b^2(a+bx^2)^4} - \frac{7x^3}{96b^3(a+bx^2)^3} - \frac{7x}{128b^4(a+bx^2)^2} + \frac{7x}{256ab^4(a+bx^2)} + \frac{7\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}$$

output
```
-1/10*x^7/b/(b*x^2+a)^5-7/80*x^5/b^2/(b*x^2+a)^4-7/96*x^3/b^3/(b*x^2+a)^3-
7/128*x/b^4/(b*x^2+a)^2+7/256*x/a/b^4/(b*x^2+a)+7/256*arctan(b^(1/2)*x/a^(
1/2))/a^(3/2)/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{x(105a^4+490a^3bx^2+896a^2b^2x^4+790ab^3x^6-105b^4x^8)}{3840ab^4(a+bx^2)^5} + \frac{7\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}$$

input
```
Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/3840*(x*(105*a^4 + 490*a^3*b*x^2 + 896*a^2*b^2*x^4 + 790*a*b^3*x^6 - 10
5*b^4*x^8))/(a*b^4*(a + b*x^2)^5) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a
^(3/2)*b^(9/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1380, 27, 252, 252, 252, 252, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{x^8}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^8}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{7\int \frac{x^6}{(bx^2+a)^5}dx}{10b}-\frac{x^7}{10b(a+bx^2)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{7\left(\frac{5\int \frac{x^4}{(bx^2+a)^4}dx}{8b}-\frac{x^5}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^7}{10b(a+bx^2)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{7\left(\frac{5\left(\frac{\int \frac{x^2}{\left(bx^2+a\right)^3}dx}{2b}-\frac{x^3}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^5}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^7}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \ 252$$

$$\frac{7\left(\frac{5\left(\frac{\frac{\int \frac{1}{\left(bx^2+a\right)^2}dx}{4b}-\frac{x}{4b\left(a+bx^2\right)^2}}{2b}-\frac{x^3}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^5}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^7}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \ 215$$

$$\frac{7\left(\frac{5\left(\frac{\frac{\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}}{4b}-\frac{x}{4b\left(a+bx^2\right)^2}}{2b}-\frac{x^3}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^5}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^7}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \ 218$$

$$\frac{7\left(\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}}{4b}-\frac{x}{4b\left(a+bx^2\right)^2}}{2b}-\frac{x^3}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^5}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^7}{10b\left(a+bx^2\right)^5}$$

input
```
Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*x^7/(b*(a + b*x^2)^5) + (7*(-1/8*x^5/(b*(a + b*x^2)^4) + (5*(-1/6*x^
3/(b*(a + b*x^2)^3) + (-1/4*x/(b*(a + b*x^2)^2) + (x/(2*a*(a + b*x^2)) + A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]))/(4*b))/(2*b)))/(8*b)))/(10
*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\frac{7x^9}{256a}-\frac{79x^7}{384b}-\frac{7a\,x^5}{30b^2}-\frac{49a^2x^3}{384b^3}-\frac{7a^3x}{256b^4}}{\left(b\,x^2+a\right)^5}+\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256b^4a\sqrt{ab}}$	80
risch	$\frac{\frac{7x^9}{256a}-\frac{79x^7}{384b}-\frac{7a\,x^5}{30b^2}-\frac{49a^2x^3}{384b^3}-\frac{7a^3x}{256b^4}}{\left(b\,x^2+a\right)\left(b^2x^4+2ab\,x^2+a^2\right)^2}-\frac{7\ln\left(bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^4a}+\frac{7\ln\left(-bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^4a}$	129

input
```
int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(7/256/a*x^9-79/384/b*x^7-7/30*a/b^2*x^5-49/384*a^2/b^3*x^3-7/256*a^3/b^4*
x)/(b*x^2+a)^5+7/256/b^4/a/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.20

$$\int \frac{x^8}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$
$$= \left[\frac{210\,ab^5x^9-1580\,a^2b^4x^7-1792\,a^3b^3x^5-980\,a^4b^2x^3-210\,a^5bx-105\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+1}{7680\,(a^2b^{10}x^{10}+5\,a^3b^9x^8+10\,a^4b^8x^6+10\,a^5b^7x^4+5\,a^6b^6x^2}\right.$$

input
```
integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
[1/7680*(210*a*b^5*x^9 - 1580*a^2*b^4*x^7 - 1792*a^3*b^3*x^5 - 980*a^4*b^2
*x^3 - 210*a^5*b*x - 105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3
*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)))/(a^2*b^10*x^10 + 5*a^3*b^9*x^8 + 10*a^4*b^8*x^6 + 10*a^5*b^7
*x^4 + 5*a^6*b^6*x^2 + a^7*b^5), 1/3840*(105*a*b^5*x^9 - 790*a^2*b^4*x^7 -
 896*a^3*b^3*x^5 - 490*a^4*b^2*x^3 - 105*a^5*b*x + 105*(b^5*x^10 + 5*a*b^4
*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a^2*b^10*x^10 + 5*a^3*b^9*x^8 + 10*a^4*b^8*x^6 + 10*a^
5*b^7*x^4 + 5*a^6*b^6*x^2 + a^7*b^5)]
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.59

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{7\sqrt{-\frac{1}{a^3b^9}}\log\left(-a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{512} + \frac{7\sqrt{-\frac{1}{a^3b^9}}\log\left(a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{512} + \frac{-105a^4x-490a^3bx^3-896a^2b^2x^5-790ab^3x^7+105b^4x^9}{3840a^6b^4+19200a^5b^5x^2+38400a^4b^6x^4+38400a^3b^7x^6+19200a^2b^8x^8+3840ab^9x^{10}}$$

input
```
integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
-7*sqrt(-1/(a**3*b**9))*log(-a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/512 + 7*s
qrt(-1/(a**3*b**9))*log(a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/512 + (-105*a*
*4*x - 490*a**3*b*x**3 - 896*a**2*b**2*x**5 - 790*a*b**3*x**7 + 105*b**4*x
**9)/(3840*a**6*b**4 + 19200*a**5*b**5*x**2 + 38400*a**4*b**6*x**4 + 38400
*a**3*b**7*x**6 + 19200*a**2*b**8*x**8 + 3840*a*b**9*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{105\,b^4x^9-790\,ab^3x^7-896\,a^2b^2x^5-490\,a^3bx^3-105\,a^4x}{3840\,(ab^9x^{10}+5\,a^2b^8x^8+10\,a^3b^7x^6+10\,a^4b^6x^4+5\,a^5b^5x^2+a^6b^4)} + \frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}ab^4}$$

input
```
integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/3840*(105*b^4*x^9 - 790*a*b^3*x^7 - 896*a^2*b^2*x^5 - 490*a^3*b*x^3 - 10
5*a^4*x)/(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5
*a^5*b^5*x^2 + a^6*b^4) + 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.69

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}ab^4} + \frac{105\,b^4x^9-790\,ab^3x^7-896\,a^2b^2x^5-490\,a^3bx^3-105\,a^4x}{3840\,(bx^2+a)^5ab^4}$$

input
```
integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/3840*(105*b^4*x^9 - 790*
a*b^3*x^7 - 896*a^2*b^2*x^5 - 490*a^3*b*x^3 - 105*a^4*x)/((b*x^2 + a)^5*a*
b^4)
```

Mupad [B] (verification not implemented)

Time = 18.81 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$=\frac{7\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{3/2}\,b^{9/2}}-\frac{\frac{79\,x^7}{384\,b}-\frac{7\,x^9}{256\,a}+\frac{7\,a\,x^5}{30\,b^2}+\frac{7\,a^3\,x}{256\,b^4}+\frac{49\,a^2\,x^3}{384\,b^3}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

input
```
int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(7*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(3/2)*b^(9/2)) - ((79*x^7)/(384*b) -
(7*x^9)/(256*a) + (7*a*x^5)/(30*b^2) + (7*a^3*x)/(256*b^4) + (49*a^2*x^3)/
(384*b^3))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 +
10*a^2*b^3*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.13

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$=\frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5+525\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4b\,x^2+1050\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b^2x^4+1050\sqrt{b}}{3840a^2b^5}$$

input
```
int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5 + 525*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*x**2 + 1050*sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**4 + 1050*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*a**2*b**3*x**6 + 525*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*a*b**4*x**8 + 105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))
)*b**5*x**10 - 105*a**5*b*x - 490*a**4*b**2*x**3 - 896*a**3*b**3*x**5 - 79
0*a**2*b**4*x**7 + 105*a*b**5*x**9)/(3840*a**2*b**5*(a**5 + 5*a**4*b*x**2
+ 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10))
```

3.412 $\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = -\frac{x^5}{10b\left(a+bx^2\right)^5}-\frac{x^3}{16b^2\left(a+bx^2\right)^4}-\frac{x}{32b^3\left(a+bx^2\right)^3}+\frac{x}{128ab^3\left(a+bx^2\right)^2}+\frac{3x}{256a^2b^3\left(a+bx^2\right)}+\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}$$

output
```
-1/10*x^5/b/(b*x^2+a)^5-1/16*x^3/b^2/(b*x^2+a)^4-1/32*x/b^3/(b*x^2+a)^3+1/
128*x/a/b^3/(b*x^2+a)^2+3/256*x/a^2/b^3/(b*x^2+a)+3/256*arctan(b^(1/2)*x/a
^(1/2))/a^(5/2)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.74

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{-15a^4x-70a^3bx^3-128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^2b^3\left(a+bx^2\right)^5}+\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}$$

input
```
Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(-15*a^4*x - 70*a^3*b*x^3 - 128*a^2*b^2*x^5 + 70*a*b^3*x^7 + 15*b^4*x^9)/(
1280*a^2*b^3*(a + b*x^2)^5) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(5/2)
*b^(7/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1380, 27, 252, 252, 252, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \; 1380$$

$$b^6 \int \frac{x^6}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{x^6}{(a+bx^2)^6}dx$$

$$\Big\downarrow \; 252$$

$$\frac{\int \frac{x^4}{(bx^2+a)^5}dx}{2b}-\frac{x^5}{10b(a+bx^2)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{\frac{3\int \frac{x^2}{(bx^2+a)^4}dx}{8b}-\frac{x^3}{8b(a+bx^2)^4}}{2b}-\frac{x^5}{10b(a+bx^2)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{\left(bx^2+a\right)^3}dx}{6b}-\frac{x}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^3}{8b(a+bx^2)^4}}{2b}-\frac{x^5}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 215

$$\frac{\frac{3\left(\frac{\frac{3\int \frac{1}{\left(bx^2+a\right)^2}dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}}{6b}-\frac{x}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^3}{8b(a+bx^2)^4}}{2b}-\frac{x^5}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 215

$$\frac{\frac{3\left(\frac{\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}}{6b}-\frac{x}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^3}{8b(a+bx^2)^4}}{2b}-\frac{x^5}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 218

$$\frac{\frac{3\left(\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}}{6b}-\frac{x}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^3}{8b(a+bx^2)^4}}{2b}-\frac{x^5}{10b\left(a+bx^2\right)^5}$$

input
```
Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*x^5/(b*(a + b*x^2)^5) + (-1/8*x^3/(b*(a + b*x^2)^4) + (3*(-1/6*x/(b*
(a + b*x^2)^3) + (x/(4*a*(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan
[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a))/(6*b)))/(8*b))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\frac{3b\,x^{9}}{256a^{2}}+\frac{7x^{7}}{128a}-\frac{x^{5}}{10b}-\frac{7a\,x^{3}}{128b^{2}}-\frac{3a^{2}x}{256b^{3}}}{\left(b\,x^{2}+a\right)^{5}}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256a^{2}b^{3}\sqrt{ab}}$	78
risch	$\frac{\frac{3b\,x^{9}}{256a^{2}}+\frac{7x^{7}}{128a}-\frac{x^{5}}{10b}-\frac{7a\,x^{3}}{128b^{2}}-\frac{3a^{2}x}{256b^{3}}}{\left(b\,x^{2}+a\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}}-\frac{3\ln\left(bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^{3}a^{2}}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^{3}a^{2}}$	127

input
```
int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/256*b/a^2*x^9+7/128/a*x^7-1/10/b*x^5-7/128*a/b^2*x^3-3/256*a^2/b^3*x)/(
b*x^2+a)^5+3/256/a^2/b^3/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.17

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \left[\frac{30\,ab^5x^9+140\,a^2b^4x^7-256\,a^3b^3x^5-140\,a^4b^2x^3-30\,a^5bx-15\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2}{2560\,(a^3b^9x^{10}+5\,a^4b^8x^8+10\,a^5b^7x^6+10\,a^6b^6x^4+5\,a^7b^5x^2+}\right.$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
[1/2560*(30*a*b^5*x^9 + 140*a^2*b^4*x^7 - 256*a^3*b^3*x^5 - 140*a^4*b^2*x^
3 - 30*a^5*b*x - 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*
x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^
2 + a)))/(a^3*b^9*x^10 + 5*a^4*b^8*x^8 + 10*a^5*b^7*x^6 + 10*a^6*b^6*x^4 +
 5*a^7*b^5*x^2 + a^8*b^4), 1/1280*(15*a*b^5*x^9 + 70*a^2*b^4*x^7 - 128*a^3
*b^3*x^5 - 70*a^4*b^2*x^3 - 15*a^5*b*x + 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a
^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt(a*b
)*x/a))/(a^3*b^9*x^10 + 5*a^4*b^8*x^8 + 10*a^5*b^7*x^6 + 10*a^6*b^6*x^4 +
5*a^7*b^5*x^2 + a^8*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.59

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= -\frac{3\sqrt{-\frac{1}{a^5b^7}}\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{512}+\frac{3\sqrt{-\frac{1}{a^5b^7}}\log\left(a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{512}$$
$$+\frac{-15a^4x-70a^3bx^3-128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^7b^3+6400a^6b^4x^2+12800a^5b^5x^4+12800a^4b^6x^6+6400a^3b^7x^8+1280a^2b^8x^{10}}$$

input
```
integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
-3*sqrt(-1/(a**5*b**7))*log(-a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/512 + 3*s
qrt(-1/(a**5*b**7))*log(a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/512 + (-15*a**
4*x - 70*a**3*b*x**3 - 128*a**2*b**2*x**5 + 70*a*b**3*x**7 + 15*b**4*x**9)
/(1280*a**7*b**3 + 6400*a**6*b**4*x**2 + 12800*a**5*b**5*x**4 + 12800*a**4
*b**6*x**6 + 6400*a**3*b**7*x**8 + 1280*a**2*b**8*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{15\,b^4x^9+70\,ab^3x^7-128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\,(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3)} + \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^2b^3}$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/1280*(15*b^4*x^9 + 70*a*b^3*x^7 - 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^
4*x)/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a
^6*b^4*x^2 + a^7*b^3) + 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^2b^3} + \frac{15\,b^4x^9+70\,ab^3x^7-128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\,(bx^2+a)^5a^2b^3}$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/1280*(15*b^4*x^9 + 70*
a*b^3*x^7 - 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x)/((b*x^2 + a)^5*a^2*
b^3)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{5/2}\,b^{7/2}} - \frac{\frac{x^5}{10\,b}-\frac{7\,x^7}{128\,a}+\frac{7\,a\,x^3}{128\,b^2}+\frac{3\,a^2\,x}{256\,b^3}-\frac{3\,b\,x^9}{256\,a^2}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}}$$

input
```
int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(3*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(5/2)*b^(7/2)) - (x^5/(10*b) - (7*x^7
)/(128*a) + (7*a*x^3)/(128*b^2) + (3*a^2*x)/(256*b^3) - (3*b*x^9)/(256*a^2
))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b
^3*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.11

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5+75\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4b\,x^2+150\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b^2x^4+150\sqrt{b}\,\sqrt{a}\,a}{1280a^3b^4\,(b^5x^{10}}$$

input
```
int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5 + 75*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*x**2 + 150*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**4 + 150*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*a**2*b**3*x**6 + 75*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqr
t(a)))*a*b**4*x**8 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5
*x**10 - 15*a**5*b*x - 70*a**4*b**2*x**3 - 128*a**3*b**3*x**5 + 70*a**2*b*
*4*x**7 + 15*a*b**5*x**9)/(1280*a**3*b**4*(a**5 + 5*a**4*b*x**2 + 10*a**3*
b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10))
```

3.413 $\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = -\frac{x^3}{10b\left(a+bx^2\right)^5}-\frac{3x}{80b^2\left(a+bx^2\right)^4}+\frac{x}{160ab^2\left(a+bx^2\right)^3}+\frac{x}{128a^2b^2\left(a+bx^2\right)^2}+\frac{3x}{256a^3b^2\left(a+bx^2\right)}+\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}$$

output
```
-1/10*x^3/b/(b*x^2+a)^5-3/80*x/b^2/(b*x^2+a)^4+1/160*x/a/b^2/(b*x^2+a)^3+1
/128*x/a^2/b^2/(b*x^2+a)^2+3/256*x/a^3/b^2/(b*x^2+a)+3/256*arctan(b^(1/2)*
x/a^(1/2))/a^(7/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{-15a^4x-70a^3bx^3+128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^3b^2\left(a+bx^2\right)^5}+\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}$$

input
```
Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(-15*a^4*x - 70*a^3*b*x^3 + 128*a^2*b^2*x^5 + 70*a*b^3*x^7 + 15*b^4*x^9)/(
1280*a^3*b^2*(a + b*x^2)^5) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(7/2)
*b^(5/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1380, 27, 252, 252, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{x^4}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^4}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{3\int \frac{x^2}{(bx^2+a)^5}dx}{10b}-\frac{x^3}{10b(a+bx^2)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{3\left(\frac{\int \frac{1}{(bx^2+a)^4}dx}{8b}-\frac{x}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^3}{10b(a+bx^2)^5}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\left(\frac{\frac{5\int \frac{1}{\left(bx^2+a\right)^3}dx}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}}{8b}-\frac{x}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^3}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 215

$$\frac{3\left(\frac{\frac{5\left(\frac{3\int \frac{1}{\left(bx^2+a\right)^2}dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}}{8b}-\frac{x}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^3}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 215

$$\frac{3\left(\frac{\frac{5\left(\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}}{8b}-\frac{x}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^3}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 218

$$\frac{3\left(\frac{\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}}{8b}-\frac{x}{8b(a+bx^2)^4}\right)}{10b}-\frac{x^3}{10b\left(a+bx^2\right)^5}$$

input
```
Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*x^3/(b*(a + b*x^2)^5) + (3*(-1/8*x/(b*(a + b*x^2)^4) + (x/(6*a*(a +
b*x^2)^3) + (5*(x/(4*a*(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a)))/(6*a))/(8*b)))/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\frac{3b^2x^9}{256a^3}+\frac{7b\,x^7}{128a^2}+\frac{x^5}{10a}-\frac{7x^3}{128b}-\frac{3ax}{256b^2}}{(b\,x^2+a)^5}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256a^3b^2\sqrt{ab}}$	78
risch	$\frac{\frac{3b^2x^9}{256a^3}+\frac{7b\,x^7}{128a^2}+\frac{x^5}{10a}-\frac{7x^3}{128b}-\frac{3ax}{256b^2}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}-\frac{3\ln\left(bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^2a^3}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b^2a^3}$	127

input
```
int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/256*b^2/a^3*x^9+7/128*b/a^2*x^7+1/10/a*x^5-7/128/b*x^3-3/256*a/b^2*x)/(
b*x^2+a)^5+3/256/a^3/b^2/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.15

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \left[\frac{30\,ab^5x^9+140\,a^2b^4x^7+256\,a^3b^3x^5-140\,a^4b^2x^3-30\,a^5bx-15\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2}{2560\,(a^4b^8x^{10}+5\,a^5b^7x^8+10\,a^6b^6x^6+10\,a^7b^5x^4+5\,a^8b^4x^2+}\right.$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
[1/2560*(30*a*b^5*x^9 + 140*a^2*b^4*x^7 + 256*a^3*b^3*x^5 - 140*a^4*b^2*x^
3 - 30*a^5*b*x - 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*
x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^
2 + a)))/(a^4*b^8*x^10 + 5*a^5*b^7*x^8 + 10*a^6*b^6*x^6 + 10*a^7*b^5*x^4 +
 5*a^8*b^4*x^2 + a^9*b^3), 1/1280*(15*a*b^5*x^9 + 70*a^2*b^4*x^7 + 128*a^3
*b^3*x^5 - 70*a^4*b^2*x^3 - 15*a^5*b*x + 15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a
^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arctan(sqrt(a*b
)*x/a))/(a^4*b^8*x^10 + 5*a^5*b^7*x^8 + 10*a^6*b^6*x^6 + 10*a^7*b^5*x^4 +
5*a^8*b^4*x^2 + a^9*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.58

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= -\frac{3\sqrt{-\frac{1}{a^7b^5}}\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{512} + \frac{3\sqrt{-\frac{1}{a^7b^5}}\log\left(a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{512}$$
$$+ \frac{-15a^4x-70a^3bx^3+128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^8b^2+6400a^7b^3x^2+12800a^6b^4x^4+12800a^5b^5x^6+6400a^4b^6x^8+1280a^3b^7x^{10}}$$

input
```
integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
-3*sqrt(-1/(a**7*b**5))*log(-a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/512 + 3*s
qrt(-1/(a**7*b**5))*log(a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/512 + (-15*a**
4*x - 70*a**3*b*x**3 + 128*a**2*b**2*x**5 + 70*a*b**3*x**7 + 15*b**4*x**9)
/(1280*a**8*b**2 + 6400*a**7*b**3*x**2 + 12800*a**6*b**4*x**4 + 12800*a**5
*b**5*x**6 + 6400*a**4*b**6*x**8 + 1280*a**3*b**7*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{15\,b^4x^9+70\,ab^3x^7+128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\,(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2)}$$
$$+ \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^3b^2}$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/1280*(15*b^4*x^9 + 70*a*b^3*x^7 + 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^
4*x)/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a
^7*b^3*x^2 + a^8*b^2) + 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}a^3b^2} + \frac{15\,b^4x^9+70\,ab^3x^7+128\,a^2b^2x^5-70\,a^3bx^3-15\,a^4x}{1280\,(bx^2+a)^5a^3b^2}$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2) + 1/1280*(15*b^4*x^9 + 70*
a*b^3*x^7 + 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x)/((b*x^2 + a)^5*a^3*
b^2)
```

Mupad [B] (verification not implemented)

Time = 18.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{x^5}{10\,a}-\frac{7\,x^3}{128\,b}+\frac{7\,b\,x^7}{128\,a^2}+\frac{3\,b^2\,x^9}{256\,a^3}-\frac{3\,a\,x}{256\,b^2}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{7/2}\,b^{5/2}}$$

input
```
int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(x^5/(10*a) - (7*x^3)/(128*b) + (7*b*x^7)/(128*a^2) + (3*b^2*x^9)/(256*a^3
) - (3*a*x)/(256*b^2))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8 + 10*a^
3*b^2*x^4 + 10*a^2*b^3*x^6) + (3*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(7/2)*b
^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.10

$$\int \frac{x^4}{(a^2 + 2abx^2 + b^2x^4)^3} \, dx$$
$$= \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^5 + 75\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^4 b\, x^2 + 150\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^3 b^2 x^4 + 150\sqrt{b}\,\sqrt{a}\, a}{1280a^4b^3\,(b^5x^{10}}$$

input
```
int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5 + 75*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*x**2 + 150*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**4 + 150*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*a**2*b**3*x**6 + 75*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqr
t(a)))*a*b**4*x**8 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5
*x**10 - 15*a**5*b*x - 70*a**4*b**2*x**3 + 128*a**3*b**3*x**5 + 70*a**2*b*
*4*x**7 + 15*a*b**5*x**9)/(1280*a**4*b**3*(a**5 + 5*a**4*b*x**2 + 10*a**3*
b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10))
```

3.414 $\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3} dx = -\frac{x}{10b(a+bx^2)^5}+\frac{x}{80ab(a+bx^2)^4}+\frac{7x}{480a^2b(a+bx^2)^3}+\frac{7x}{384a^3b(a+bx^2)^2}+\frac{7x}{256a^4b(a+bx^2)}+\frac{7\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}$$

output
```
-1/10*x/b/(b*x^2+a)^5+1/80*x/a/b/(b*x^2+a)^4+7/480*x/a^2/b/(b*x^2+a)^3+7/3
84*x/a^3/b/(b*x^2+a)^2+7/256*x/a^4/b/(b*x^2+a)+7/256*arctan(b^(1/2)*x/a^(1
/2))/a^(9/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3} dx = \frac{-105a^4x+790a^3bx^3+896a^2b^2x^5+490ab^3x^7+105b^4x^9}{3840a^4b(a+bx^2)^5}+\frac{7\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(-105*a^4*x + 790*a^3*b*x^3 + 896*a^2*b^2*x^5 + 490*a*b^3*x^7 + 105*b^4*x^
9)/(3840*a^4*b*(a + b*x^2)^5) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(9/
2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1380, 27, 252, 215, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \ 1380$$

$$b^6\int \frac{x^2}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{x^2}{(a+bx^2)^6}dx$$

$$\Big\downarrow \ 252$$

$$\frac{\int \frac{1}{(bx^2+a)^5}dx}{10b}-\frac{x}{10b(a+bx^2)^5}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{7\int \frac{1}{(bx^2+a)^4}dx}{8a}+\frac{x}{8a(a+bx^2)^4}}{10b}-\frac{x}{10b(a+bx^2)^5}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{7\left(\frac{5\int \frac{1}{\left(bx^2+a\right)^3}dx}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a(a+bx^2)^4}}{10b}-\frac{x}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 215

$$\frac{\frac{7\left(\frac{5\left(\frac{3\int \frac{1}{\left(bx^2+a\right)^2}dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a(a+bx^2)^4}}{10b}-\frac{x}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 215

$$\frac{\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a(a+bx^2)^4}}{10b}-\frac{x}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 218

$$\frac{\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a(a+bx^2)^4}}{10b}-\frac{x}{10b\left(a+bx^2\right)^5}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*x/(b*(a + b*x^2)^5) + (x/(8*a*(a + b*x^2)^4) + (7*(x/(6*a*(a + b*x^2
)^3) + (5*(x/(4*a*(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[
b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a)))/(6*a)))/(8*a))/(10*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\frac{7b^3x^9}{256a^4}+\frac{49b^2x^7}{384a^3}+\frac{7b\,x^5}{30a^2}+\frac{79x^3}{384a}-\frac{7x}{256b}}{(b\,x^2+a)^5}+\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256b\,a^4\sqrt{ab}}$	80
risch	$\frac{\frac{7b^3x^9}{256a^4}+\frac{49b^2x^7}{384a^3}+\frac{7b\,x^5}{30a^2}+\frac{79x^3}{384a}-\frac{7x}{256b}}{(b\,x^2+a)(b^2x^4+2ab\,x^2+a^2)^2}-\frac{7\ln\left(bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b\,a^4}+\frac{7\ln\left(-bx+\sqrt{-ab}\right)}{512\sqrt{-ab}\,b\,a^4}$	129

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(7/256*b^3/a^4*x^9+49/384*b^2/a^3*x^7+7/30*b/a^2*x^5+79/384/a*x^3-7/256/b*
x)/(b*x^2+a)^5+7/256/b/a^4/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.12

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$=\left[\frac{210\,ab^5x^9+980\,a^2b^4x^7+1792\,a^3b^3x^5+1580\,a^4b^2x^3-210\,a^5bx-105\,(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+1}{7680\,(a^5b^7x^{10}+5\,a^6b^6x^8+10\,a^7b^5x^6+10\,a^8b^4x^4+5\,a^9b^3x^2}\right.$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
[1/7680*(210*a*b^5*x^9 + 980*a^2*b^4*x^7 + 1792*a^3*b^3*x^5 + 1580*a^4*b^2
*x^3 - 210*a^5*b*x - 105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3
*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)))/(a^5*b^7*x^10 + 5*a^6*b^6*x^8 + 10*a^7*b^5*x^6 + 10*a^8*b^4*
x^4 + 5*a^9*b^3*x^2 + a^10*b^2), 1/3840*(105*a*b^5*x^9 + 490*a^2*b^4*x^7 +
 896*a^3*b^3*x^5 + 790*a^4*b^2*x^3 - 105*a^5*b*x + 105*(b^5*x^10 + 5*a*b^4
*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a^5*b^7*x^10 + 5*a^6*b^6*x^8 + 10*a^7*b^5*x^6 + 10*a^8
*b^4*x^4 + 5*a^9*b^3*x^2 + a^10*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= -\frac{7\sqrt{-\frac{1}{a^9b^3}}\log\left(-a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{512}+\frac{7\sqrt{-\frac{1}{a^9b^3}}\log\left(a^5b\sqrt{-\frac{1}{a^9b^3}}+x\right)}{512}$$
$$+\frac{-105a^4x+790a^3bx^3+896a^2b^2x^5+490ab^3x^7+105b^4x^9}{3840a^9b+19200a^8b^2x^2+38400a^7b^3x^4+38400a^6b^4x^6+19200a^5b^5x^8+3840a^4b^6x^{10}}$$

input
```
integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
-7*sqrt(-1/(a**9*b**3))*log(-a**5*b*sqrt(-1/(a**9*b**3)) + x)/512 + 7*sqrt
(-1/(a**9*b**3))*log(a**5*b*sqrt(-1/(a**9*b**3)) + x)/512 + (-105*a**4*x +
 790*a**3*b*x**3 + 896*a**2*b**2*x**5 + 490*a*b**3*x**7 + 105*b**4*x**9)/(
3840*a**9*b + 19200*a**8*b**2*x**2 + 38400*a**7*b**3*x**4 + 38400*a**6*b**
4*x**6 + 19200*a**5*b**5*x**8 + 3840*a**4*b**6*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$=\frac{105\,b^4x^9+490\,ab^3x^7+896\,a^2b^2x^5+790\,a^3bx^3-105\,a^4x}{3840\,(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b)}$$
$$+\frac{7\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^4b}$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/3840*(105*b^4*x^9 + 490*a*b^3*x^7 + 896*a^2*b^2*x^5 + 790*a^3*b*x^3 - 10
5*a^4*x)/(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 +
 5*a^8*b^2*x^2 + a^9*b) + 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}a^4b} + \frac{105\,b^4x^9+490\,ab^3x^7+896\,a^2b^2x^5+790\,a^3bx^3-105\,a^4x}{3840\,(bx^2+a)^5a^4b}$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b) + 1/3840*(105*b^4*x^9 + 490*
a*b^3*x^7 + 896*a^2*b^2*x^5 + 790*a^3*b*x^3 - 105*a^4*x)/((b*x^2 + a)^5*a^
4*b)
```

Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{79\,x^3}{384\,a}-\frac{7\,x}{256\,b}+\frac{7\,b\,x^5}{30\,a^2}+\frac{49\,b^2\,x^7}{384\,a^3}+\frac{7\,b^3\,x^9}{256\,a^4}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}} + \frac{7\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{9/2}\,b^{3/2}}$$

input
```
int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((79*x^3)/(384*a) - (7*x)/(256*b) + (7*b*x^5)/(30*a^2) + (49*b^2*x^7)/(384
*a^3) + (7*b^3*x^9)/(256*a^4))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 + 5*a*b^4*x^8
 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6) + (7*atan((b^(1/2)*x)/a^(1/2)))/(256*a
^(9/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.08

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5+525\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4b\,x^2+1050\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b^2x^4+1050\sqrt{b}}{3840a^5b^2}$$

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5 + 525*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*x**2 + 1050*sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**4 + 1050*sqrt(b)*sqrt(a)*atan((b*x)/
(sqrt(b)*sqrt(a)))*a**2*b**3*x**6 + 525*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*a*b**4*x**8 + 105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))
)*b**5*x**10 - 105*a**5*b*x + 790*a**4*b**2*x**3 + 896*a**3*b**3*x**5 + 49
0*a**2*b**4*x**7 + 105*a*b**5*x**9)/(3840*a**5*b**2*(a**5 + 5*a**4*b*x**2
+ 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10))
```

3.415 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{x}{10a(a+bx^2)^5}+\frac{9x}{80a^2(a+bx^2)^4}+\frac{21x}{160a^3(a+bx^2)^3}+\frac{21x}{128a^4(a+bx^2)^2}+\frac{63x}{256a^5(a+bx^2)}+\frac{63\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{11/2}\sqrt{b}}$$

output
```
1/10*x/a/(b*x^2+a)^5+9/80*x/a^2/(b*x^2+a)^4+21/160*x/a^3/(b*x^2+a)^3+21/12
8*x/a^4/(b*x^2+a)^2+63/256*x/a^5/(b*x^2+a)+63/256*arctan(b^(1/2)*x/a^(1/2)
)/a^(11/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{\sqrt{a}x(965a^4+2370a^3bx^2+2688a^2b^2x^4+1470ab^3x^6+315b^4x^8)}{(a+bx^2)^5}+\frac{315\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}{1280a^{11/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

output
```
((Sqrt[a]*x*(965*a^4 + 2370*a^3*b*x^2 + 2688*a^2*b^2*x^4 + 1470*a*b^3*x^6
+ 315*b^4*x^8))/(a + b*x^2)^5 + (315*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])
/(1280*a^(11/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.56, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1379, 215, 215, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \text{1379}$$

$$b^6\int \frac{1}{(b^2x^2+ab)^6}dx$$

$$\Big\downarrow \text{215}$$

$$b^6\left(\frac{9\int \frac{1}{(b^2x^2+ab)^5}dx}{10ab}+\frac{x}{10ab^6(a+bx^2)^5}\right)$$

$$\Big\downarrow \text{215}$$

$$b^6\left(\frac{9\left(\frac{7\int \frac{1}{(b^2x^2+ab)^4}dx}{8ab}+\frac{x}{8ab^5(a+bx^2)^4}\right)}{10ab}+\frac{x}{10ab^6(a+bx^2)^5}\right)$$

$$\Big\downarrow \text{215}$$

$$b^6\left(\frac{9\left(\frac{7\left(\frac{5\int \frac{1}{\left(b^2x^2+ab\right)^3}dx}{6ab}+\frac{x}{6ab^4\left(a+bx^2\right)^3}\right)}{8ab}+\frac{x}{8ab^5(a+bx^2)^4}\right)}{10ab}+\frac{x}{10ab^6\left(a+bx^2\right)^5}\right)$$

$$\Big\downarrow \text{215}$$

$$b^6\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{1}{\left(b^2x^2+ab\right)^2}dx}{4ab}+\frac{x}{4ab^3\left(a+bx^2\right)^2}\right)}{6ab}+\frac{x}{6ab^4\left(a+bx^2\right)^3}\right)}{8ab}+\frac{x}{8ab^5(a+bx^2)^4}\right)}{10ab}+\frac{x}{10ab^6\left(a+bx^2\right)^5}\right)$$

$$\Big\downarrow \text{215}$$

$$b^6\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int\frac{1}{b^2x^2+ab}dx}{2ab}+\frac{x}{2ab^2(a+bx^2)}\right)}{4ab}+\frac{x}{4ab^3(a+bx^2)^2}\right)}{6ab}+\frac{x}{6ab^4(a+bx^2)^3}\right)}{8ab}+\frac{x}{8ab^5(a+bx^2)^4}\right)}{10ab}+\frac{x}{10ab^6(a+bx^2)^5}\right)$$

$$\Big\downarrow \; 218$$

$$b^6\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x}{2ab^2\left(a+bx^2\right)}\right)}{4ab}+\frac{x}{4ab^3\left(a+bx^2\right)^2}\right)}{6ab}+\frac{x}{6ab^4\left(a+bx^2\right)^3}\right)}{8ab}+\frac{x}{8ab^5(a+bx^2)^4}\right)}{10ab}+\frac{x}{10ab^6\left(a+bx^2\right)^5}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

output
```
b^6*(x/(10*a*b^6*(a + b*x^2)^5) + (9*(x/(8*a*b^5*(a + b*x^2)^4) + (7*(x/(6
*a*b^4*(a + b*x^2)^3) + (5*(x/(4*a*b^3*(a + b*x^2)^2) + (3*(x/(2*a*b^2*(a
+ b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*b^(5/2))))/(4*a*b)))/(6
*a*b)))/(8*a*b)))/(10*a*b))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{x}{10a(b\,x^2+a)^5}+\frac{\frac{9x}{80a\left(b\,x^2+a\right)^4}+\frac{9\left(\frac{7x}{48a\left(b\,x^2+a\right)^3}+\frac{7\left(\frac{5x}{24a\left(b\,x^2+a\right)^2}+\frac{5\left(\frac{3x}{8a\left(b\,x^2+a\right)}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)}{6a}\right)}{8a}\right)}{10a}}{a}$	120
risch	$\frac{\frac{63b^4x^9}{256a^5}+\frac{147b^3x^7}{128a^4}+\frac{21b^2x^5}{10a^3}+\frac{237b\,x^3}{128a^2}+\frac{193x}{256a}}{(b^2x^4+2ab\,x^2+a^2)^2(b\,x^2+a)}-\frac{63\ln(bx+\sqrt{-ab})}{512\sqrt{-ab}\,a^5}+\frac{63\ln(-bx+\sqrt{-ab})}{512\sqrt{-ab}\,a^5}$	126

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*x/a/(b*x^2+a)^5+9/10/a*(1/8*x/a/(b*x^2+a)^4+7/8/a*(1/6*x/a/(b*x^2+a)^
3+5/6/a*(1/4*x/a/(b*x^2+a)^2+3/4/a*(1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*ar
ctan(b/(a*b)^(1/2)*x)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.42

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^3} \, dx$$
$$= \left[\frac{630\, ab^5x^9 + 2940\, a^2b^4x^7 + 5376\, a^3b^3x^5 + 4740\, a^4b^2x^3 + 1930\, a^5bx - 315\, (b^5x^{10} + 5\, ab^4x^8 + 10\, a^2b^3x^6 +}{2560\, (a^6b^6x^{10} + 5\, a^7b^5x^8 + 10\, a^8b^4x^6 + 10\, a^9b^3x^4 + 5\, a^{10}b^2x} \right.$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
[1/2560*(630*a*b^5*x^9 + 2940*a^2*b^4*x^7 + 5376*a^3*b^3*x^5 + 4740*a^4*b^
2*x^3 + 1930*a^5*b*x - 315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a
^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a
)/(b*x^2 + a)))/(a^6*b^6*x^10 + 5*a^7*b^5*x^8 + 10*a^8*b^4*x^6 + 10*a^9*b^
3*x^4 + 5*a^10*b^2*x^2 + a^11*b), 1/1280*(315*a*b^5*x^9 + 1470*a^2*b^4*x^7
 + 2688*a^3*b^3*x^5 + 2370*a^4*b^2*x^3 + 965*a^5*b*x + 315*(b^5*x^10 + 5*a
*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a))/(a^6*b^6*x^10 + 5*a^7*b^5*x^8 + 10*a^8*b^4*x^6 + 10
*a^9*b^3*x^4 + 5*a^10*b^2*x^2 + a^11*b)]
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.57

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= -\frac{63\sqrt{-\frac{1}{a^{11}b}}\log\left(-a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{512} + \frac{63\sqrt{-\frac{1}{a^{11}b}}\log\left(a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{512}$$
$$+ \frac{965a^4x+2370a^3bx^3+2688a^2b^2x^5+1470ab^3x^7+315b^4x^9}{1280a^{10}+6400a^9bx^2+12800a^8b^2x^4+12800a^7b^3x^6+6400a^6b^4x^8+1280a^5b^5x^{10}}$$

input
```
integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
-63*sqrt(-1/(a**11*b))*log(-a**6*sqrt(-1/(a**11*b)) + x)/512 + 63*sqrt(-1/
(a**11*b))*log(a**6*sqrt(-1/(a**11*b)) + x)/512 + (965*a**4*x + 2370*a**3*
b*x**3 + 2688*a**2*b**2*x**5 + 1470*a*b**3*x**7 + 315*b**4*x**9)/(1280*a**
10 + 6400*a**9*b*x**2 + 12800*a**8*b**2*x**4 + 12800*a**7*b**3*x**6 + 6400
*a**6*b**4*x**8 + 1280*a**5*b**5*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{315\,b^4x^9+1470\,ab^3x^7+2688\,a^2b^2x^5+2370\,a^3bx^3+965\,a^4x}{1280\,(a^5b^5x^{10}+5\,a^6b^4x^8+10\,a^7b^3x^6+10\,a^8b^2x^4+5\,a^9bx^2+a^{10})}$$
$$+ \frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^5}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/1280*(315*b^4*x^9 + 1470*a*b^3*x^7 + 2688*a^2*b^2*x^5 + 2370*a^3*b*x^3 +
 965*a^4*x)/(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^
4 + 5*a^9*b*x^2 + a^10) + 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}a^5} + \frac{315\,b^4x^9+1470\,ab^3x^7+2688\,a^2b^2x^5+2370\,a^3bx^3+965\,a^4x}{1280\,(bx^2+a)^5a^5}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/1280*(315*b^4*x^9 + 1470*
a*b^3*x^7 + 2688*a^2*b^2*x^5 + 2370*a^3*b*x^3 + 965*a^4*x)/((b*x^2 + a)^5*
a^5)
```

Mupad [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{193\,x}{256\,a}+\frac{237\,b\,x^3}{128\,a^2}+\frac{21\,b^2\,x^5}{10\,a^3}+\frac{147\,b^3\,x^7}{128\,a^4}+\frac{63\,b^4\,x^9}{256\,a^5}}{a^5+5\,a^4\,b\,x^2+10\,a^3\,b^2\,x^4+10\,a^2\,b^3\,x^6+5\,a\,b^4\,x^8+b^5\,x^{10}} + \frac{63\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{11/2}\,\sqrt{b}}$$

input
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((193*x)/(256*a) + (237*b*x^3)/(128*a^2) + (21*b^2*x^5)/(10*a^3) + (147*b^
3*x^7)/(128*a^4) + (63*b^4*x^9)/(256*a^5))/(a^5 + b^5*x^10 + 5*a^4*b*x^2 +
 5*a*b^4*x^8 + 10*a^3*b^2*x^4 + 10*a^2*b^3*x^6) + (63*atan((b^(1/2)*x)/a^(
1/2)))/(256*a^(11/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.30

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5+1575\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4b\,x^2+3150\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b^2x^4+3150\sqrt{}}{1280a}$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5 + 1575*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*x**2 + 3150*sqrt(b)*sqrt(a)*atan
((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**4 + 3150*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*a**2*b**3*x**6 + 1575*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt
(b)*sqrt(a)))*a*b**4*x**8 + 315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*b**5*x**10 + 965*a**5*b*x + 2370*a**4*b**2*x**3 + 2688*a**3*b**3*x**5
+ 1470*a**2*b**4*x**7 + 315*a*b**5*x**9)/(1280*a**6*b*(a**5 + 5*a**4*b*x**
2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10))
```

3.416 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = -\frac{1}{a^6x}-\frac{bx}{10a^2\left(a+bx^2\right)^5}-\frac{19bx}{80a^3\left(a+bx^2\right)^4}-\frac{71bx}{160a^4\left(a+bx^2\right)^3}-\frac{103bx}{128a^5\left(a+bx^2\right)^2}-\frac{437bx}{256a^6\left(a+bx^2\right)}-\frac{693\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{13/2}}$$

output
```
-1/a^6/x-1/10*b*x/a^2/(b*x^2+a)^5-19/80*b*x/a^3/(b*x^2+a)^4-71/160*b*x/a^4
/(b*x^2+a)^3-103/128*b*x/a^5/(b*x^2+a)^2-437/256*b*x/a^6/(b*x^2+a)-693/256
*b^(1/2)*arctan(b^(1/2)*x/a^(1/2))/a^(13/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^3} \, dx = -\frac{1280a^5 + 10615a^4bx^2 + 26070a^3b^2x^4 + 29568a^2b^3x^6 + 16170ab^4x^8 + 3465b^5x^{10}}{1280a^6x (a + bx^2)^5} - \frac{693\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{13/2}}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
-1/1280*(1280*a^5 + 10615*a^4*b*x^2 + 26070*a^3*b^2*x^4 + 29568*a^2*b^3*x^
6 + 16170*a*b^4*x^8 + 3465*b^5*x^10)/(a^6*x*(a + b*x^2)^5) - (693*Sqrt[b]*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(13/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.34, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1380, 27, 253, 253, 253, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^3} \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^6 \int \frac{1}{b^6x^2 (bx^2 + a)^6} dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{x^2 (a + bx^2)^6} dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{11\int \frac{1}{x^2(bx^2+a)^5}dx}{10a}+\frac{1}{10ax\,(a+bx^2)^5}$$

$$\Big\downarrow \textcolor{blue}{253}$$

$$\frac{11\left(\frac{9\int \frac{1}{x^2\left(bx^2+a\right)^4}dx}{8a}+\frac{1}{8ax(a+bx^2)^4}\right)}{10a}+\frac{1}{10ax\,(a+bx^2)^5}$$

$$\Big\downarrow \textcolor{blue}{253}$$

$$\frac{11\left(\frac{9\left(\frac{7\int \frac{1}{x^2\left(bx^2+a\right)^3}dx}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax(a+bx^2)^4}\right)}{10a}+\frac{1}{10ax\,(a+bx^2)^5}$$

$$\Big\downarrow \textcolor{blue}{253}$$

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\int \frac{1}{x^2\left(bx^2+a\right)^2}dx}{4a}+\frac{1}{4ax\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax(a+bx^2)^4}\right)}{10a}+\frac{1}{10ax\,(a+bx^2)^5}$$

$$\Big\downarrow \textcolor{blue}{253}$$

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{1}{x^2\left(bx^2+a\right)}dx}{2a}+\frac{1}{2ax\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax(a+bx^2)^4}\right)}{10a}+\frac{1}{10ax\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 264

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{b\int\frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax(a+bx^2)^4}\right)}{10a}+\frac{1}{10ax\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 218

$$\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a+bx^2)}\right)}{4a}+\frac{1}{4ax(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax(a+bx^2)^3}\right)}{8a}+\frac{1}{8ax(a+bx^2)^4}\right)}{10a}+\frac{1}{10ax\,(a+bx^2)^5}$$

input

```
Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
1/(10*a*x*(a + b*x^2)^5) + (11*(1/(8*a*x*(a + b*x^2)^4) + (9*(1/(6*a*x*(a
+ b*x^2)^3) + (7*(1/(4*a*x*(a + b*x^2)^2) + (5*(1/(2*a*x*(a + b*x^2)) + (3
*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*
a)))/(6*a)))/(8*a)))/(10*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

method	result
default	$-\frac{b\left(\frac{\frac{437}{256}b^4x^9+\frac{977}{128}a\,b^3x^7+\frac{131}{10}a^2b^2x^5+\frac{1327}{128}a^3b\,x^3+\frac{843}{256}a^4x}{\left(b\,x^2+a\right)^5}+\frac{693\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}}\right)}{a^6}-\frac{1}{a^6x}$
risch	$\frac{-\frac{693b^5x^{10}}{256a^6}-\frac{1617b^4x^8}{128a^5}-\frac{231b^3x^6}{10a^4}-\frac{2607b^2x^4}{128a^3}-\frac{2123b\,x^2}{256a^2}-\frac{1}{a}}{x(b^2x^4+2ab\,x^2+a^2)^2(b\,x^2+a)}+\frac{693\left(\sum_{\_R=\mathrm{RootOf}\left(a^{13}\_Z^2+b\right)}\_R\ln\left(\left(3\_R^2a^{13}+2b\right)x+a^7\_R\right)\right.}{512}$

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-b/a^6*((437/256*b^4*x^9+977/128*a*b^3*x^7+131/10*a^2*b^2*x^5+1327/128*a^3
*b*x^3+843/256*a^4*x)/(b*x^2+a)^5+693/256/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)
*x))-1/a^6/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.17

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

$$=\left[-\frac{6930\,b^5x^{10}+32340\,ab^4x^8+59136\,a^2b^3x^6+52140\,a^3b^2x^4+21230\,a^4bx^2+2560\,a^5-3465\,(b^5x^{11}+5\,a}{2560\,(a^6b^5x^{11}+5\,a^7b^4x^9+10\,a^8b^3x^7+10\,a^9b^2x}\right.$$

$$-\frac{3465\,b^5x^{10}+16170\,ab^4x^8+29568\,a^2b^3x^6+26070\,a^3b^2x^4+10615\,a^4bx^2+1280\,a^5+3465\,(b^5x^{11}+5\,a}{1280\,(a^6b^5x^{11}+5\,a^7b^4x^9+10\,a^8b^3x^7+10\,a^9b^2x^5+}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
[-1/2560*(6930*b^5*x^10 + 32340*a*b^4*x^8 + 59136*a^2*b^3*x^6 + 52140*a^3*
b^2*x^4 + 21230*a^4*b*x^2 + 2560*a^5 - 3465*(b^5*x^11 + 5*a*b^4*x^9 + 10*a
^2*b^3*x^7 + 10*a^3*b^2*x^5 + 5*a^4*b*x^3 + a^5*x)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^5*x^11 + 5*a^7*b^4*x^9 + 10*a^
8*b^3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x), -1/1280*(3465*b^5*x^1
0 + 16170*a*b^4*x^8 + 29568*a^2*b^3*x^6 + 26070*a^3*b^2*x^4 + 10615*a^4*b*
x^2 + 1280*a^5 + 3465*(b^5*x^11 + 5*a*b^4*x^9 + 10*a^2*b^3*x^7 + 10*a^3*b^
2*x^5 + 5*a^4*b*x^3 + a^5*x)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^5*x^11
+ 5*a^7*b^4*x^9 + 10*a^8*b^3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x)
]
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^3} dx = \frac{693\sqrt{-\frac{b}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b}{a^{13}}}}{b} + x\right)}{512} - \frac{693\sqrt{-\frac{b}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b}{a^{13}}}}{b} + x\right)}{512} + \frac{-1280a^5 - 10615a^4bx^2 - 26070a^3b^2x^4 - 29568a^2b^3x^6 - 16170ab^4x^8 - 3465b^5x^{10}}{1280a^{11}x + 6400a^{10}bx^3 + 12800a^9b^2x^5 + 12800a^8b^3x^7 + 6400a^7b^4x^9 + 1280a^6b^5x^{11}}$$

input
```
integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
693*sqrt(-b/a**13)*log(-a**7*sqrt(-b/a**13)/b + x)/512 - 693*sqrt(-b/a**13
)*log(a**7*sqrt(-b/a**13)/b + x)/512 + (-1280*a**5 - 10615*a**4*b*x**2 - 2
6070*a**3*b**2*x**4 - 29568*a**2*b**3*x**6 - 16170*a*b**4*x**8 - 3465*b**5
*x**10)/(1280*a**11*x + 6400*a**10*b*x**3 + 12800*a**9*b**2*x**5 + 12800*a
**8*b**3*x**7 + 6400*a**7*b**4*x**9 + 1280*a**6*b**5*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^3} \, dx = -\frac{3465 \, b^5 x^{10} + 16170 \, ab^4 x^8 + 29568 \, a^2 b^3 x^6 + 26070 \, a^3 b^2 x^4 + 10615 \, a^4 b x^2 + 1280 \, a^5}{1280 \, (a^6 b^5 x^{11} + 5 \, a^7 b^4 x^9 + 10 \, a^8 b^3 x^7 + 10 \, a^9 b^2 x^5 + 5 \, a^{10} b x^3 + a^{11} x)} - \frac{693 \, b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256 \, \sqrt{ab} a^6}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/1280*(3465*b^5*x^10 + 16170*a*b^4*x^8 + 29568*a^2*b^3*x^6 + 26070*a^3*b
^2*x^4 + 10615*a^4*b*x^2 + 1280*a^5)/(a^6*b^5*x^11 + 5*a^7*b^4*x^9 + 10*a^
8*b^3*x^7 + 10*a^9*b^2*x^5 + 5*a^10*b*x^3 + a^11*x) - 693/256*b*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*a^6)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^3} \, dx = -\frac{693 \, b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256 \, \sqrt{ab} a^6} - \frac{1}{a^6 x} - \frac{2185 \, b^5 x^9 + 9770 \, ab^4 x^7 + 16768 \, a^2 b^3 x^5 + 13270 \, a^3 b^2 x^3 + 4215 \, a^4 bx}{1280 \, (bx^2 + a)^5 a^6}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-693/256*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/(a^6*x) - 1/1280*(218
5*b^5*x^9 + 9770*a*b^4*x^7 + 16768*a^2*b^3*x^5 + 13270*a^3*b^2*x^3 + 4215*
a^4*b*x)/((b*x^2 + a)^5*a^6)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^3} \, dx$$
$$= -\frac{\frac{1}{a} + \frac{2123\,b\,x^2}{256\,a^2} + \frac{2607\,b^2\,x^4}{128\,a^3} + \frac{231\,b^3\,x^6}{10\,a^4} + \frac{1617\,b^4\,x^8}{128\,a^5} + \frac{693\,b^5\,x^{10}}{256\,a^6}}{a^5\,x + 5\,a^4\,b\,x^3 + 10\,a^3\,b^2\,x^5 + 10\,a^2\,b^3\,x^7 + 5\,a\,b^4\,x^9 + b^5\,x^{11}} - \frac{693\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{13/2}}$$

input
```
int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

output
```
- (1/a + (2123*b*x^2)/(256*a^2) + (2607*b^2*x^4)/(128*a^3) + (231*b^3*x^6)
/(10*a^4) + (1617*b^4*x^8)/(128*a^5) + (693*b^5*x^10)/(256*a^6))/(a^5*x +
b^5*x^11 + 5*a^4*b*x^3 + 5*a*b^4*x^9 + 10*a^3*b^2*x^5 + 10*a^2*b^3*x^7) -
(693*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(256*a^(13/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.13

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^3} \, dx$$
$$= \frac{-3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5x - 17325\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4b\,x^3 - 34650\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b^2x^5 -}{}$$

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
( - 3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*x - 17325*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*x**3 - 34650*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**5 - 34650*sqrt(b)*sqrt(a)*a
tan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*x**7 - 17325*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a*b**4*x**9 - 3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*b**5*x**11 - 1280*a**6 - 10615*a**5*b*x**2 - 26070*a**4*b**
2*x**4 - 29568*a**3*b**3*x**6 - 16170*a**2*b**4*x**8 - 3465*a*b**5*x**10)/
(1280*a**7*x*(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6
 + 5*a*b**4*x**8 + b**5*x**10))
```

3.417 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 147

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^3}\,dx=-\frac{1}{3a^6x^3}+\frac{6b}{a^7x}+\frac{b^2x}{10a^3\left(a+bx^2\right)^5}+\frac{29b^2x}{80a^4\left(a+bx^2\right)^4}+\frac{443b^2x}{480a^5\left(a+bx^2\right)^3}+\frac{827b^2x}{384a^6\left(a+bx^2\right)^2}+\frac{1467b^2x}{256a^7\left(a+bx^2\right)}+\frac{3003b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{15/2}}$$

output
```
-1/3/a^6/x^3+6*b/a^7/x+1/10*b^2*x/a^3/(b*x^2+a)^5+29/80*b^2*x/a^4/(b*x^2+a
)^4+443/480*b^2*x/a^5/(b*x^2+a)^3+827/384*b^2*x/a^6/(b*x^2+a)^2+1467/256*b
^2*x/a^7/(b*x^2+a)+3003/256*b^(3/2)*arctan(b^(1/2)*x/a^(1/2))/a^(15/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^3} dx = \frac{\frac{\sqrt{a}(-1280a^6+16640a^5bx^2+137995a^4b^2x^4+338910a^3b^3x^6+384384a^2b^4x^8+210210ab^5x^{10}+45045b^6x^{12})}{x^3(a+bx^2)^5} + 45045b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3840a^{15/2}}$$

input
```
Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
((Sqrt[a]*(-1280*a^6 + 16640*a^5*b*x^2 + 137995*a^4*b^2*x^4 + 338910*a^3*b
^3*x^6 + 384384*a^2*b^4*x^8 + 210210*a*b^5*x^10 + 45045*b^6*x^12))/(x^3*(a
 + b*x^2)^5) + 45045*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3840*a^(15/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1380, 27, 253, 253, 253, 253, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^3} dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^6 \int \frac{1}{b^6 x^4 (bx^2 + a)^6} dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{x^4 (a + bx^2)^6} dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{13\int \frac{1}{x^4(bx^2+a)^5}dx}{10a}+\frac{1}{10ax^3\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{13\left(\frac{11\int \frac{1}{x^4\left(bx^2+a\right)^4}dx}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right)}{10a}+\frac{1}{10ax^3\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{13\left(\frac{11\left(\frac{3\int \frac{1}{x^4\left(bx^2+a\right)^3}dx}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right)}{10a}+\frac{1}{10ax^3\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\int \frac{1}{x^4\left(bx^2+a\right)^2}dx}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right)}{10a}+\frac{1}{10ax^3\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{1}{x^4\left(bx^2+a\right)}dx}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right)}{10a}+\frac{1}{10ax^3\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 264$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\int\frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right)}{10a}+\frac{1}{10ax^3\left(a+bx^2\right)^5}$$

$\downarrow$ 264

$$\left(13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{b\int\frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3(a+bx^2)}\right)}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{2a}+\frac{1}{6ax^3(a+bx^2)^3}\right)}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right)$$

↓ 218

$$\left(13\,\frac{\left(11\,\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right.$$

input
```
Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
1/(10*a*x^3*(a + b*x^2)^5) + (13*(1/(8*a*x^3*(a + b*x^2)^4) + (11*(1/(6*a*
x^3*(a + b*x^2)^3) + (3*(1/(4*a*x^3*(a + b*x^2)^2) + (7*(1/(2*a*x^3*(a + b
*x^2)) + (5*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/a^(3/2)))/a))/(2*a)))/(4*a)))/(2*a)))/(8*a)))/(10*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

method	result
default	$\frac{b^{2}\left(\frac{\frac{1467}{256}b^{4}x^{9}+\frac{9629}{384}a\,b^{3}x^{7}+\frac{1253}{30}a^{2}b^{2}x^{5}+\frac{12131}{384}a^{3}b\,x^{3}+\frac{2373}{256}a^{4}x}{\left(b\,x^{2}+a\right)^{5}}+\frac{3003\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}}\right)}{a^{7}}-\frac{1}{3a^{6}x^{3}}+\frac{6b}{a^{7}x}$
risch	$\frac{\frac{3003b^{6}x^{12}}{256a^{7}}+\frac{7007b^{5}x^{10}}{128a^{6}}+\frac{1001b^{4}x^{8}}{10a^{5}}+\frac{11297b^{3}x^{6}}{128a^{4}}+\frac{27599b^{2}x^{4}}{768a^{3}}+\frac{13b\,x^{2}}{3a^{2}}-\frac{1}{3a}}{x^{3}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}\left(b\,x^{2}+a\right)}+\frac{3003\left(\sum_{\_R=\operatorname{RootOf}\left(a^{15}\_Z^{2}+b^{3}\right)}\_R\ln\left(\left(3\_R^{2}a^{15}+2\right.\right.\right.}{512}$

input
```
int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
b^2/a^7*((1467/256*b^4*x^9+9629/384*a*b^3*x^7+1253/30*a^2*b^2*x^5+12131/38
4*a^3*b*x^3+2373/256*a^4*x)/(b*x^2+a)^5+3003/256/(a*b)^(1/2)*arctan(b/(a*b
)^(1/2)*x))-1/3/a^6/x^3+6*b/a^7/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.97

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

$$=\left[\frac{90090\,b^6x^{12}+420420\,ab^5x^{10}+768768\,a^2b^4x^8+677820\,a^3b^3x^6+275990\,a^4b^2x^4+33280\,a^5bx^2-2560\,a}{7680\left(a^7b^5x^{13}+5\,a^8b^4x^{11}+10\,a^9b^3x\right.}\right.$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
[1/7680*(90090*b^6*x^12 + 420420*a*b^5*x^10 + 768768*a^2*b^4*x^8 + 677820*
a^3*b^3*x^6 + 275990*a^4*b^2*x^4 + 33280*a^5*b*x^2 - 2560*a^6 + 45045*(b^6
*x^13 + 5*a*b^5*x^11 + 10*a^2*b^4*x^9 + 10*a^3*b^3*x^7 + 5*a^4*b^2*x^5 + a
^5*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7
*b^5*x^13 + 5*a^8*b^4*x^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x
^5 + a^12*x^3), 1/3840*(45045*b^6*x^12 + 210210*a*b^5*x^10 + 384384*a^2*b^
4*x^8 + 338910*a^3*b^3*x^6 + 137995*a^4*b^2*x^4 + 16640*a^5*b*x^2 - 1280*a
^6 + 45045*(b^6*x^13 + 5*a*b^5*x^11 + 10*a^2*b^4*x^9 + 10*a^3*b^3*x^7 + 5*
a^4*b^2*x^5 + a^5*b*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^7*b^5*x^13 + 5*
a^8*b^4*x^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 + a^12*x^3)
]
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{3003\sqrt{-\frac{b^3}{a^{15}}}\log\left(-\frac{a^8\sqrt{-\frac{b^3}{a^{15}}}}{b^2}+x\right)}{512} + \frac{3003\sqrt{-\frac{b^3}{a^{15}}}\log\left(\frac{a^8\sqrt{-\frac{b^3}{a^{15}}}}{b^2}+x\right)}{512} + \frac{-1280a^6+16640a^5bx^2+137995a^4b^2x^4+338910a^3b^3x^6+384384a^2b^4x^8+210210ab^5x^{10}+45045b^6x^{12}}{3840a^{12}x^3+19200a^{11}bx^5+38400a^{10}b^2x^7+38400a^9b^3x^9+19200a^8b^4x^{11}+3840a^7b^5x^{13}}$$

input
```
integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
-3003*sqrt(-b**3/a**15)*log(-a**8*sqrt(-b**3/a**15)/b**2 + x)/512 + 3003*s
qrt(-b**3/a**15)*log(a**8*sqrt(-b**3/a**15)/b**2 + x)/512 + (-1280*a**6 +
16640*a**5*b*x**2 + 137995*a**4*b**2*x**4 + 338910*a**3*b**3*x**6 + 384384
*a**2*b**4*x**8 + 210210*a*b**5*x**10 + 45045*b**6*x**12)/(3840*a**12*x**3
 + 19200*a**11*b*x**5 + 38400*a**10*b**2*x**7 + 38400*a**9*b**3*x**9 + 192
00*a**8*b**4*x**11 + 3840*a**7*b**5*x**13)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4 (a^2+2abx^2+b^2x^4)^3} \, dx = \frac{45045\, b^6x^{12}+210210\, ab^5x^{10}+384384\, a^2b^4x^8+338910\, a^3b^3x^6+137995\, a^4b^2x^4+16640\, a^5bx^2-1280\, a^6}{3840\, (a^7b^5x^{13}+5\, a^8b^4x^{11}+10\, a^9b^3x^9+10\, a^{10}b^2x^7+5\, a^{11}bx^5+a^{12}x^3)} + \frac{3003\, b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\, \sqrt{ab}a^7}$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/3840*(45045*b^6*x^12 + 210210*a*b^5*x^10 + 384384*a^2*b^4*x^8 + 338910*a
^3*b^3*x^6 + 137995*a^4*b^2*x^4 + 16640*a^5*b*x^2 - 1280*a^6)/(a^7*b^5*x^1
3 + 5*a^8*b^4*x^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 + a^1
2*x^3) + 3003/256*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^4 (a^2+2abx^2+b^2x^4)^3} \, dx = \frac{3003\, b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\, \sqrt{ab}a^7} + \frac{18\, bx^2-a}{3\, a^7x^3} + \frac{22005\, b^6x^9+96290\, ab^5x^7+160384\, a^2b^4x^5+121310\, a^3b^3x^3+35595\, a^4b^2x}{3840\, (bx^2+a)^5a^7}$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
3003/256*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7) + 1/3*(18*b*x^2 - a)/(a
^7*x^3) + 1/3840*(22005*b^6*x^9 + 96290*a*b^5*x^7 + 160384*a^2*b^4*x^5 + 1
21310*a^3*b^3*x^3 + 35595*a^4*b^2*x)/((b*x^2 + a)^5*a^7)
```

Mupad [B] (verification not implemented)

Time = 18.64 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^3} \, dx = \frac{\frac{13\,b\,x^2}{3\,a^2} - \frac{1}{3\,a} + \frac{27599\,b^2\,x^4}{768\,a^3} + \frac{11297\,b^3\,x^6}{128\,a^4} + \frac{1001\,b^4\,x^8}{10\,a^5} + \frac{7007\,b^5\,x^{10}}{128\,a^6} + \frac{3003\,b^6\,x^{12}}{256\,a^7}}{a^5\,x^3 + 5\,a^4\,b\,x^5 + 10\,a^3\,b^2\,x^7 + 10\,a^2\,b^3\,x^9 + 5\,a\,b^4\,x^{11} + b^5\,x^{13}} + \frac{3003\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\,a^{15/2}}$$

input
```
int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

output
```
((13*b*x^2)/(3*a^2) - 1/(3*a) + (27599*b^2*x^4)/(768*a^3) + (11297*b^3*x^6
)/(128*a^4) + (1001*b^4*x^8)/(10*a^5) + (7007*b^5*x^10)/(128*a^6) + (3003*
b^6*x^12)/(256*a^7))/(a^5*x^3 + b^5*x^13 + 5*a^4*b*x^5 + 5*a*b^4*x^11 + 10
*a^3*b^2*x^7 + 10*a^2*b^3*x^9) + (3003*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/
(256*a^(15/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.93

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^3} \, dx = \frac{45045\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^5 b\,x^3 + 225225\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^4 b^2 x^5 + 450450\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^3 b^3 x}{}$$

input
```
int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b*x**3 + 225225*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**2*x**5 + 450450*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**3*x**7 + 450450*sqrt(b)*
sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**4*x**9 + 225225*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**5*x**11 + 45045*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*b**6*x**13 - 1280*a**7 + 16640*a**6*b*x**2 + 13
7995*a**5*b**2*x**4 + 338910*a**4*b**3*x**6 + 384384*a**3*b**4*x**8 + 2102
10*a**2*b**5*x**10 + 45045*a*b**6*x**12)/(3840*a**8*x**3*(a**5 + 5*a**4*b*
x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6 + 5*a*b**4*x**8 + b**5*x**10)
)
```

3.418 $\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\begin{aligned}\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)^3}\,dx = &-\frac{1}{5a^6x^5}+\frac{2b}{a^7x^3}-\frac{21b^2}{a^8x}-\frac{b^3x}{10a^4(a+bx^2)^5}\\ &-\frac{39b^3x}{80a^5(a+bx^2)^4}-\frac{251b^3x}{160a^6(a+bx^2)^3}\\ &-\frac{571b^3x}{128a^7(a+bx^2)^2}-\frac{3633b^3x}{256a^8(a+bx^2)}\\ &-\frac{9009b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{17/2}}\end{aligned}$$

output
```
-1/5/a^6/x^5+2*b/a^7/x^3-21*b^2/a^8/x-1/10*b^3*x/a^4/(b*x^2+a)^5-39/80*b^3
*x/a^5/(b*x^2+a)^4-251/160*b^3*x/a^6/(b*x^2+a)^3-571/128*b^3*x/a^7/(b*x^2+
a)^2-3633/256*b^3*x/a^8/(b*x^2+a)-9009/256*b^(5/2)*arctan(b^(1/2)*x/a^(1/2
))/a^(17/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)^3} dx = -\frac{256a^7 - 1280a^6bx^2 + 16640a^5b^2x^4 + 137995a^4b^3x^6 + 338910a^3b^4x^8 + 384384a^2b^5x^{10} + 210210ab^6x^{12}}{1280a^8x^5 (a + bx^2)^5} - \frac{9009b^{5/2} \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{17/2}}$$

input
```
Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
-1/1280*(256*a^7 - 1280*a^6*b*x^2 + 16640*a^5*b^2*x^4 + 137995*a^4*b^3*x^6
 + 338910*a^3*b^4*x^8 + 384384*a^2*b^5*x^10 + 210210*a*b^6*x^12 + 45045*b^
7*x^14)/(a^8*x^5*(a + b*x^2)^5) - (9009*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(256*a^(17/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.28, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1380, 27, 253, 253, 253, 253, 253, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)^3} dx$$

$$\Big\downarrow \text{1380}$$

$$b^6 \int \frac{1}{b^6x^6 (bx^2 + a)^6} dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^6 (a + bx^2)^6} dx$$

$$\Big\downarrow \ 253$$

$$\frac{3\int \frac{1}{x^6\left(bx^2+a\right)^5}dx}{2a}+\frac{1}{10ax^5\left(a+bx^2\right)^5}$$

$$\Big\downarrow \ 253$$

$$\frac{3\left(\frac{13\int \frac{1}{x^6\left(bx^2+a\right)^4}dx}{8a}+\frac{1}{8ax^5\left(a+bx^2\right)^4}\right)}{2a}+\frac{1}{10ax^5\left(a+bx^2\right)^5}$$

$$\Big\downarrow \ 253$$

$$\frac{3\left(\frac{13\left(\frac{11\int \frac{1}{x^6\left(bx^2+a\right)^3}dx}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^5\left(a+bx^2\right)^4}\right)}{2a}+\frac{1}{10ax^5\left(a+bx^2\right)^5}$$

$$\Big\downarrow \ 253$$

$$\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\int \frac{1}{x^6\left(bx^2+a\right)^2}dx}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^5\left(a+bx^2\right)^4}\right)}{2a}+\frac{1}{10ax^5\left(a+bx^2\right)^5}$$

$$\Big\downarrow \ 253$$

$$\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int \frac{1}{x^6\left(bx^2+a\right)}dx}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^5(a+bx^2)^4}\right)}{2a}+\frac{1}{10ax^5\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 264$$

$$\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\int\frac{1}{x^4\left(bx^2+a\right)}dx}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^5(a+bx^2)^4}\right)}{2a}+\frac{1}{10ax^5\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 264

$$3\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\int\frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^5(a+bx^2)^4}\right.$$

↓ 264

$$\left(\left(13\,\frac{\left(11\,\frac{\left(9\,\frac{\left(7\left(-\frac{b\left(-\frac{b\left(-\frac{b\int\frac{1}{bx^2+a}\,dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)\right)}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}\right.\right.$$

↓ 218

$$\Bigg(\Bigg(\frac{13 \left(\frac{11 \left(\frac{9 \left(\frac{7 \left(-\frac{b \left(-\frac{b \left(-\frac{\sqrt{b} \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax} \right)}{a} - \frac{1}{3ax^3} \right)}{a} - \frac{1}{5ax^5} \right)}{2a} + \frac{1}{2ax^5\left(a+bx^2\right)} \right)}{4a} + \frac{1}{4ax^5\left(a+bx^2\right)^2} \right)}{6a} + \frac{1}{6ax^5\left(a+bx^2\right)^3} \Bigg) \Bigg)$$

input
```
Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
1/(10*a*x^5*(a + b*x^2)^5) + (3*(1/(8*a*x^5*(a + b*x^2)^4) + (13*(1/(6*a*x
^5*(a + b*x^2)^3) + (11*(1/(4*a*x^5*(a + b*x^2)^2) + (9*(1/(2*a*x^5*(a + b
*x^2)) + (7*(-1/5*1/(a*x^5) - (b*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[
b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/a))/(2*a)))/(4*a)))/(6*a)))/
(8*a)))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

method	result
default	$-\frac{b^3\left(\frac{\frac{3633}{256}b^4x^9+\frac{7837}{128}a\,b^3x^7+\frac{1001}{10}a^2b^2x^5+\frac{9443}{128}a^3b\,x^3+\frac{5327}{256}a^4x}{\left(b\,x^2+a\right)^5}+\frac{9009\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{ab}}\right)}{a^8}-\frac{1}{5a^6x^5}-\frac{21b^2}{a^8x}+\frac{2b}{a^7x^3}$
risch	$\frac{-\frac{1}{5a}+\frac{b\,x^2}{a^2}-\frac{13b^2x^4}{a^3}-\frac{275999b^3x^6}{256a^4}-\frac{33891b^4x^8}{128a^5}-\frac{3003b^5x^{10}}{10a^6}-\frac{21021b^6x^{12}}{128a^7}-\frac{9009b^7x^{14}}{256a^8}}{x^5(b^2x^4+2ab\,x^2+a^2)^2(b\,x^2+a)}+\frac{9009\sqrt{-ab}\,b^2\ln(-bx+\sqrt{-ab})}{512a^9}-\frac{9009\sqrt{-al}}{}$

input
```
int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^8*b^3*((3633/256*b^4*x^9+7837/128*a*b^3*x^7+1001/10*a^2*b^2*x^5+9443/
128*a^3*b*x^3+5327/256*a^4*x)/(b*x^2+a)^5+9009/256/(a*b)^(1/2)*arctan(b/(a
*b)^(1/2)*x))-1/5/a^6/x^5-21*b^2/a^8/x+2*b/a^7/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.92

$$\int \frac{1}{x^6\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

$$= \left[-\frac{90090\,b^7x^{14}+420420\,ab^6x^{12}+768768\,a^2b^5x^{10}+677820\,a^3b^4x^8+275990\,a^4b^3x^6+33280\,a^5b^2x^4-25}{2560\left(a^8b^5x^{15}+5\,a^9b^4x^{13}+\right.}\right.$$

$$-\frac{45045\,b^7x^{14}+210210\,ab^6x^{12}+384384\,a^2b^5x^{10}+338910\,a^3b^4x^8+137995\,a^4b^3x^6+16640\,a^5b^2x^4-128}{1280\left(a^8b^5x^{15}+5\,a^9b^4x^{13}+10\,a\right.}$$

input
```
integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
[-1/2560*(90090*b^7*x^14 + 420420*a*b^6*x^12 + 768768*a^2*b^5*x^10 + 67782
0*a^3*b^4*x^8 + 275990*a^4*b^3*x^6 + 33280*a^5*b^2*x^4 - 2560*a^6*b*x^2 +
512*a^7 - 45045*(b^7*x^15 + 5*a*b^6*x^13 + 10*a^2*b^5*x^11 + 10*a^3*b^4*x^
9 + 5*a^4*b^3*x^7 + a^5*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
- a)/(b*x^2 + a)))/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10*
a^11*b^2*x^9 + 5*a^12*b*x^7 + a^13*x^5), -1/1280*(45045*b^7*x^14 + 210210*
a*b^6*x^12 + 384384*a^2*b^5*x^10 + 338910*a^3*b^4*x^8 + 137995*a^4*b^3*x^6
 + 16640*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256*a^7 + 45045*(b^7*x^15 + 5*a*b^
6*x^13 + 10*a^2*b^5*x^11 + 10*a^3*b^4*x^9 + 5*a^4*b^3*x^7 + a^5*b^2*x^5)*s
qrt(b/a)*arctan(x*sqrt(b/a)))/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3
*x^11 + 10*a^11*b^2*x^9 + 5*a^12*b*x^7 + a^13*x^5)]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)^3} \, dx = \frac{9009\sqrt{-\frac{b^5}{a^{17}}}\log\left(-\frac{a^9\sqrt{-\frac{b^5}{a^{17}}}}{b^3} + x\right)}{512} - \frac{9009\sqrt{-\frac{b^5}{a^{17}}}\log\left(\frac{a^9\sqrt{-\frac{b^5}{a^{17}}}}{b^3} + x\right)}{512} + \frac{-256a^7 + 1280a^6bx^2 - 16640a^5b^2x^4 - 137995a^4b^3x^6 - 338910a^3b^4x^8 - 384384a^2b^5x^{10} - 210210ab^6x^{1}}{1280a^{13}x^5 + 6400a^{12}bx^7 + 12800a^{11}b^2x^9 + 12800a^{10}b^3x^{11} + 6400a^9b^4x^{13} + 1280a^8b^5x^{15}}$$

input
```
integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
9009*sqrt(-b**5/a**17)*log(-a**9*sqrt(-b**5/a**17)/b**3 + x)/512 - 9009*sq
rt(-b**5/a**17)*log(a**9*sqrt(-b**5/a**17)/b**3 + x)/512 + (-256*a**7 + 12
80*a**6*b*x**2 - 16640*a**5*b**2*x**4 - 137995*a**4*b**3*x**6 - 338910*a**
3*b**4*x**8 - 384384*a**2*b**5*x**10 - 210210*a*b**6*x**12 - 45045*b**7*x*
*14)/(1280*a**13*x**5 + 6400*a**12*b*x**7 + 12800*a**11*b**2*x**9 + 12800*
a**10*b**3*x**11 + 6400*a**9*b**4*x**13 + 1280*a**8*b**5*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)^3} \, dx = -\frac{45045 \, b^7 x^{14} + 210210 \, ab^6 x^{12} + 384384 \, a^2 b^5 x^{10} + 338910 \, a^3 b^4 x^8 + 137995 \, a^4 b^3 x^6 + 16640 \, a^5 b^2 x^4 - 128}{1280 \, (a^8 b^5 x^{15} + 5 \, a^9 b^4 x^{13} + 10 \, a^{10} b^3 x^{11} + 10 \, a^{11} b^2 x^9 + 5 \, a^{12} b x^7 + a^{13} x^5)} - \frac{9009 \, b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256 \, \sqrt{ab} a^8}$$

input
```
integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/1280*(45045*b^7*x^14 + 210210*a*b^6*x^12 + 384384*a^2*b^5*x^10 + 338910
*a^3*b^4*x^8 + 137995*a^4*b^3*x^6 + 16640*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 2
56*a^7)/(a^8*b^5*x^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10*a^11*b^2*x^
9 + 5*a^12*b*x^7 + a^13*x^5) - 9009/256*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a^8)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)^3} \, dx = -\frac{9009 \, b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256 \, \sqrt{ab} a^8} - \frac{45045 \, b^7 x^{14} + 210210 \, ab^6 x^{12} + 384384 \, a^2 b^5 x^{10} + 338910 \, a^3 b^4 x^8 + 137995 \, a^4 b^3 x^6 + 16640 \, a^5 b^2 x^4 - 128}{1280 \, (bx^3 + ax)^5 a^8}$$

input
```
integrate(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-9009/256*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8) - 1/1280*(45045*b^7*x^
14 + 210210*a*b^6*x^12 + 384384*a^2*b^5*x^10 + 338910*a^3*b^4*x^8 + 137995
*a^4*b^3*x^6 + 16640*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256*a^7)/((b*x^3 + a*x
)^5*a^8)
```

Mupad [B] (verification not implemented)

Time = 18.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)^3} \, dx$$
$$= -\frac{\frac{1}{5a} - \frac{bx^2}{a^2} + \frac{13b^2x^4}{a^3} + \frac{27599b^3x^6}{256a^4} + \frac{33891b^4x^8}{128a^5} + \frac{3003b^5x^{10}}{10a^6} + \frac{21021b^6x^{12}}{128a^7} + \frac{9009b^7x^{14}}{256a^8}}{a^5x^5 + 5a^4bx^7 + 10a^3b^2x^9 + 10a^2b^3x^{11} + 5ab^4x^{13} + b^5x^{15}}$$
$$- \frac{9009 b^{5/2} \operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{17/2}}$$

input
```
int(1/(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

output
```
- (1/(5*a) - (b*x^2)/a^2 + (13*b^2*x^4)/a^3 + (27599*b^3*x^6)/(256*a^4) +
(33891*b^4*x^8)/(128*a^5) + (3003*b^5*x^10)/(10*a^6) + (21021*b^6*x^12)/(1
28*a^7) + (9009*b^7*x^14)/(256*a^8))/(a^5*x^5 + b^5*x^15 + 5*a^4*b*x^7 + 5
*a*b^4*x^13 + 10*a^3*b^2*x^9 + 10*a^2*b^3*x^11) - (9009*b^(5/2)*atan((b^(1
/2)*x)/a^(1/2)))/(256*a^(17/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.88

$$\int \frac{1}{x^6 (a^2 + 2abx^2 + b^2x^4)^3} \, dx$$
$$= \frac{-45045\sqrt{b}\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right) a^5b^2x^5 - 225225\sqrt{b}\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right) a^4b^3x^7 - 450450\sqrt{b}\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right) a^3}{}$$

input
```
int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
( - 45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**2*x**5 - 2
25225*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**3*x**7 - 45045
0*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**4*x**9 - 450450*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**5*x**11 - 225225*sqrt(
b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**6*x**13 - 45045*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**7*x**15 - 256*a**8 + 1280*a**7*b*x**2
 - 16640*a**6*b**2*x**4 - 137995*a**5*b**3*x**6 - 338910*a**4*b**4*x**8 -
384384*a**3*b**5*x**10 - 210210*a**2*b**6*x**12 - 45045*a*b**7*x**14)/(128
0*a**9*x**5*(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**6
+ 5*a*b**4*x**8 + b**5*x**10))
```

3.419 $\int \frac{1}{1+2x^2+x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{x}{2(1+x^2)}+\frac{\arctan(x)}{2}$$

output
```
x/(2*x^2+2)+1/2*arctan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{1}{2}\left(\frac{x}{1+x^2}+\arctan(x)\right)$$

input
```
Integrate[(1 + 2*x^2 + x^4)^(-1),x]
```

output
```
(x/(1 + x^2) + ArcTan[x])/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1379, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+2x^2+1}dx$$
$$\Big\downarrow \text{ 1379}$$
$$\int \frac{1}{(x^2+1)^2}dx$$
$$\Big\downarrow \text{ 215}$$
$$\frac{1}{2}\int \frac{1}{x^2+1}dx+\frac{x}{2(x^2+1)}$$
$$\Big\downarrow \text{ 216}$$
$$\frac{\arctan(x)}{2}+\frac{x}{2(x^2+1)}$$

input
```
Int[(1 + 2*x^2 + x^4)^(-1),x]
```

output
```
x/(2*(1 + x^2)) + ArcTan[x]/2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1379
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/
c^p   Int[(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n
2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p] && NeQ[p, 1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	16
risch	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+i\ln(x-i)-i\ln(x+i)-2x}{4(x^2+1)}$	52

input
```
int(1/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(x^2+1)+1/2*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{(x^2+1)\arctan(x)+x}{2\,(x^2+1)}$$

input
```
integrate(1/(x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{x}{2x^2+2} + \frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(1/(x**4+2*x**2+1),x)
```

output
```
x/(2*x**2 + 2) + atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan(x)$$

input
```
integrate(1/(x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan(x)$$

input
```
integrate(1/(x^4+2*x^2+1),x, algorithm="giac")
```

output
```
1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{\operatorname{atan}(x)}{2} + \frac{x}{2\,(x^2+1)}$$

input
```
int(1/(2*x^2 + x^4 + 1),x)
```

output
```
atan(x)/2 + x/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{1+2x^2+x^4}\,dx = \frac{atan(x)\,x^2 + atan(x) + x}{2x^2+2}$$

input
```
int(1/(x^4+2*x^2+1),x)
```

output
```
(atan(x)*x**2 + atan(x) + x)/(2*(x**2 + 1))
```

3.420 $\int \frac{x}{1+2x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 11

$$\int \frac{x}{1+2x^2+x^4} \, dx = -\frac{1}{2(1+x^2)}$$

output
```
-1/2/(x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+2x^2+x^4} \, dx = -\frac{1}{2(1+x^2)}$$

input
```
Integrate[x/(1 + 2*x^2 + x^4),x]
```

output
```
-1/2*1/(1 + x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1380, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^4+2x^2+1} \, dx$$
$$\Big\downarrow \text{ 1380}$$
$$\int \frac{x}{(x^2+1)^2}dx$$
$$\Big\downarrow \text{ 241}$$
$$-\frac{1}{2(x^2+1)}$$

input
```
Int[x/(1 + 2*x^2 + x^4),x]
```

output
```
-1/2*1/(1 + x^2)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gosper	$-\frac{1}{2(x^2+1)}$	10
default	$-\frac{1}{2(x^2+1)}$	10
norman	$-\frac{1}{2(x^2+1)}$	10
risch	$-\frac{1}{2(x^2+1)}$	10
parallelrisch	$-\frac{1}{2(x^2+1)}$	10
orering	$-\frac{x^2+1}{2(x^4+2x^2+1)}$	20

input
```
int(x/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+2x^2+x^4}\,dx=-\frac{1}{2\left(x^2+1\right)}$$

input
```
integrate(x/(x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
-1/2/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{x}{1+2x^2+x^4}\,dx = -\frac{1}{2x^2+2}$$

input
```
integrate(x/(x**4+2*x**2+1),x)
```

output
```
-1/(2*x**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+2x^2+x^4}\,dx = -\frac{1}{2\,(x^2+1)}$$

input
```
integrate(x/(x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
-1/2/(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{x}{1+2x^2+x^4}\,dx = -\frac{1}{2\,(x^2+1)}$$

input
```
integrate(x/(x^4+2*x^2+1),x, algorithm="giac")
```

output
```
-1/2/(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+2x^2+x^4}\,dx = -\frac{1}{2\left(x^2+1\right)}$$

input
```
int(x/(2*x^2 + x^4 + 1),x)
```

output
```
-1/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{x}{1+2x^2+x^4}\,dx = \frac{x^2}{2x^2+2}$$

input
```
int(x/(x^4+2*x^2+1),x)
```

output
```
x**2/(2*(x**2 + 1))
```

3.421 $\int \frac{x^2}{1+2x^2+x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{x^2}{1+2x^2+x^4}\,dx = -\frac{x}{2(1+x^2)}+\frac{\arctan(x)}{2}$$

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{1+2x^2+x^4}\,dx = -\frac{x}{2(1+x^2)}+\frac{\arctan(x)}{2}$$

input
```
Integrate[x^2/(1 + 2*x^2 + x^4),x]
```

output
```
-1/2*x/(1 + x^2) + ArcTan[x]/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1380, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^4+2x^2+1}\,dx$$

$$\Big\downarrow \ 1380$$

$$\int \frac{x^2}{(x^2+1)^2}dx$$

$$\Big\downarrow \ 252$$

$$\frac{1}{2}\int \frac{1}{x^2+1}dx-\frac{x}{2\,(x^2+1)}$$

$$\Big\downarrow \ 216$$

$$\frac{\arctan(x)}{2}-\frac{x}{2\,(x^2+1)}$$

input
```
Int[x^2/(1 + 2*x^2 + x^4),x]
```

output
```
-1/2*x/(1 + x^2) + ArcTan[x]/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
risch	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+i\ln(x-i)-i\ln(x+i)+2x}{4(x^2+1)}$	52

input
```
int(x^2/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{1+2x^2+x^4}\,dx=\frac{(x^2+1)\arctan(x)-x}{2(x^2+1)}$$

input
```
integrate(x^2/(x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{1+2x^2+x^4}\,dx = -\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(x**2/(x**4+2*x**2+1),x)
```

output
```
-x/(2*x**2 + 2) + atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{1+2x^2+x^4}\,dx = -\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan(x)$$

input
```
integrate(x^2/(x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{1+2x^2+x^4}\,dx = -\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan(x)$$

input
```
integrate(x^2/(x^4+2*x^2+1),x, algorithm="giac")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{1+2x^2+x^4}\,dx = \frac{\operatorname{atan}(x)}{2} - \frac{x}{2\,(x^2+1)}$$

input
```
int(x^2/(2*x^2 + x^4 + 1),x)
```

output
```
atan(x)/2 - x/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{1+2x^2+x^4}\,dx = \frac{atan(x)\,x^2 + atan(x) - x}{2x^2+2}$$

input
```
int(x^2/(x^4+2*x^2+1),x)
```

output
```
(atan(x)*x**2 + atan(x) - x)/(2*(x**2 + 1))
```

3.422 $\int \frac{x^3}{1+2x^2+x^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{x^3}{1+2x^2+x^4} \, dx = \frac{1}{2(1+x^2)} + \frac{1}{2}\log(1+x^2)$$

output
```
1/(2*x^2+2)+1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^2+x^4} \, dx = \frac{1}{2}\left(\frac{1}{1+x^2} + \log(1+x^2)\right)$$

input
```
Integrate[x^3/(1 + 2*x^2 + x^4),x]
```

output
```
((1 + x^2)^(-1) + Log[1 + x^2])/2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1380, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^4+2x^2+1}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^3}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^2}{(x^2+1)^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{1}{x^2+1}-\frac{1}{(x^2+1)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{x^2+1}+\log\left(x^2+1\right)\right)$$

input
```
Int[x^3/(1 + 2*x^2 + x^4),x]
```

output
```
((1 + x^2)^(-1) + Log[1 + x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{1}{2x^{2}+2}+\frac{\ln\left(x^{2}+1\right)}{2}$	19
norman	$\frac{1}{2x^{2}+2}+\frac{\ln\left(x^{2}+1\right)}{2}$	19
risch	$\frac{1}{2x^{2}+2}+\frac{\ln\left(x^{2}+1\right)}{2}$	19
parallelrisch	$\frac{\ln\left(x^{2}+1\right)x^{2}+1+\ln\left(x^{2}+1\right)}{2x^{2}+2}$	28

input
```
int(x^3/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+1)+1/2/(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{1+2x^2+x^4}\,dx = \frac{(x^2+1)\log(x^2+1)+1}{2(x^2+1)}$$

input
```
integrate(x^3/(x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*log(x^2 + 1) + 1)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{1+2x^2+x^4}\,dx = \frac{\log(x^2+1)}{2} + \frac{1}{2x^2+2}$$

input
```
integrate(x**3/(x**4+2*x**2+1),x)
```

output
```
log(x**2 + 1)/2 + 1/(2*x**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^2+x^4}\,dx = \frac{1}{2(x^2+1)} + \frac{1}{2}\log(x^2+1)$$

input
```
integrate(x^3/(x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
1/2/(x^2 + 1) + 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^2+x^4}\,dx = \frac{1}{2\,(x^2+1)} + \frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(x^3/(x^4+2*x^2+1),x, algorithm="giac")
```

output
```
1/2/(x^2 + 1) + 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 17.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1+2x^2+x^4}\,dx = \frac{\ln\left(x^2+1\right)}{2} + \frac{1}{2\,\left(x^2+1\right)}$$

input
```
int(x^3/(2*x^2 + x^4 + 1),x)
```

output
```
log(x^2 + 1)/2 + 1/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{x^3}{1+2x^2+x^4}\,dx = \frac{\log(x^2+1)\,x^2+\log(x^2+1)-x^2}{2x^2+2}$$

input
```
int(x^3/(x^4+2*x^2+1),x)
```

output
```
(log(x**2 + 1)*x**2 + log(x**2 + 1) - x**2)/(2*(x**2 + 1))
```

3.423 $\int \frac{x}{81-18x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{x}{81-18x^2+x^4} \, dx = \frac{1}{2(9-x^2)}$$

output
```
1/(-2*x^2+18)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{81-18x^2+x^4} \, dx = -\frac{1}{2(-9+x^2)}$$

input
```
Integrate[x/(81 - 18*x^2 + x^4),x]
```

output
```
-1/2*1/(-9 + x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1380, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^4-18x^2+81} \, dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x}{(9-x^2)^2}dx$$

$$\Big\downarrow \text{241}$$

$$\frac{1}{2(9-x^2)}$$

input
```
Int[x/(81 - 18*x^2 + x^4),x]
```

output
```
1/(2*(9 - x^2))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{1}{2(x^2-9)}$	10
default	$-\frac{1}{2(x^2-9)}$	10
norman	$-\frac{1}{2(x^2-9)}$	10
risch	$-\frac{1}{2(x^2-9)}$	10
parallelrisch	$-\frac{1}{2(x^2-9)}$	10
orering	$-\frac{(x-3)(x+3)}{2(x^4-18x^2+81)}$	21

input
```
int(x/(x^4-18*x^2+81),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x^2-9)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{81-18x^2+x^4}\,dx = -\frac{1}{2\,(x^2-9)}$$

input
```
integrate(x/(x^4-18*x^2+81),x, algorithm="fricas")
```

output
```
-1/2/(x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x}{81-18x^2+x^4}\,dx=-\frac{1}{2x^2-18}$$

input
```
integrate(x/(x**4-18*x**2+81),x)
```

output
```
-1/(2*x**2 - 18)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{81-18x^2+x^4}\,dx=-\frac{1}{2\left(x^2-9\right)}$$

input
```
integrate(x/(x^4-18*x^2+81),x, algorithm="maxima")
```

output
```
-1/2/(x^2 - 9)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{81-18x^2+x^4}\,dx=-\frac{1}{2\left(x^2-9\right)}$$

input
```
integrate(x/(x^4-18*x^2+81),x, algorithm="giac")
```

output
```
-1/2/(x^2 - 9)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{81-18x^2+x^4}\,dx = -\frac{1}{2\,(x^2-9)}$$

input
```
int(x/(x^4 - 18*x^2 + 81),x)
```

output
```
-1/(2*(x^2 - 9))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{x}{81-18x^2+x^4}\,dx = -\frac{x^2}{18x^2-162}$$

input
```
int(x/(x^4-18*x^2+81),x)
```

output
```
( - x**2)/(18*(x**2 - 9))
```

3.424 $\int \frac{x^3}{16-8x^2+x^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 24

$$\int \frac{x^3}{16-8x^2+x^4} \, dx = \frac{2}{4-x^2} + \frac{1}{2}\log\left(4-x^2\right)$$

output
```
2/(-x^2+4)+1/2*ln(-x^2+4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{16-8x^2+x^4} \, dx = -\frac{2}{-4+x^2} + \frac{1}{2}\log\left(-4+x^2\right)$$

input
```
Integrate[x^3/(16 - 8*x^2 + x^4),x]
```

output
```
-2/(-4 + x^2) + Log[-4 + x^2]/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1380, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^4-8x^2+16}\,dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{x^3}{(4-x^2)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^2}{(4-x^2)^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{1}{x^2-4}+\frac{4}{(x^2-4)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{4}{4-x^2}+\log\left(4-x^2\right)\right)$$

input
```
Int[x^3/(16 - 8*x^2 + x^4),x]
```

output
```
(4/(4 - x^2) + Log[4 - x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\ln(x^2-4)}{2}-\frac{2}{x^2-4}$	19
risch	$\frac{\ln(x^2-4)}{2}-\frac{2}{x^2-4}$	19
norman	$-\frac{2}{x^2-4}+\frac{\ln(x-2)}{2}+\frac{\ln(x+2)}{2}$	23
parallelrisch	$\frac{\ln(x-2)x^2+\ln(x+2)x^2-4-4\ln(x-2)-4\ln(x+2)}{2x^2-8}$	40

input
```
int(x^3/(x^4-8*x^2+16),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2-4)-2/(x^2-4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{16-8x^2+x^4}\,dx=\frac{(x^2-4)\log(x^2-4)-4}{2\,(x^2-4)}$$

input
```
integrate(x^3/(x^4-8*x^2+16),x, algorithm="fricas")
```

output
```
1/2*((x^2 - 4)*log(x^2 - 4) - 4)/(x^2 - 4)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{16-8x^2+x^4}\,dx=\frac{\log(x^2-4)}{2}-\frac{2}{x^2-4}$$

input
```
integrate(x**3/(x**4-8*x**2+16),x)
```

output
```
log(x**2 - 4)/2 - 2/(x**2 - 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{16-8x^2+x^4}\,dx=-\frac{2}{x^2-4}+\frac{1}{2}\log\left(x^2-4\right)$$

input
```
integrate(x^3/(x^4-8*x^2+16),x, algorithm="maxima")
```

output
```
-2/(x^2 - 4) + 1/2*log(x^2 - 4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{16-8x^2+x^4}\,dx = -\frac{2}{x^2-4} + \frac{1}{2}\log\left(\left|x^2-4\right|\right)$$

input
```
integrate(x^3/(x^4-8*x^2+16),x, algorithm="giac")
```

output
```
-2/(x^2 - 4) + 1/2*log(abs(x^2 - 4))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{16-8x^2+x^4}\,dx = \frac{\ln\left(x^2-4\right)}{2} - \frac{2}{x^2-4}$$

input
```
int(x^3/(x^4 - 8*x^2 + 16),x)
```

output
```
log(x^2 - 4)/2 - 2/(x^2 - 4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \frac{x^3}{16-8x^2+x^4}\,dx = \frac{\log(x-2)\,x^2 - 4\log(x-2) + \log(x+2)\,x^2 - 4\log(x+2) - x^2}{2x^2-8}$$

input
```
int(x^3/(x^4-8*x^2+16),x)
```

output
```
(log(x - 2)*x**2 - 4*log(x - 2) + log(x + 2)*x**2 - 4*log(x + 2) - x**2)/(
2*(x**2 - 4))
```

3.425 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 51

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right) \, dx = \frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}$$

output
```
2/7*a^2*(d*x)^(7/2)/d+4/11*a*b*(d*x)^(11/2)/d^3+2/15*b^2*(d*x)^(15/2)/d^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right) \, dx = \frac{2x(dx)^{5/2} \left(165a^2+210abx^2+77b^2x^4\right)}{1155}$$

input
```
Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(2*x*(d*x)^(5/2)*(165*a^2 + 210*a*b*x^2 + 77*b^2*x^4))/1155
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right) \, dx$$

$$\Big\downarrow \ 1380$$

$$\frac{\int b^2(dx)^{5/2} \left(bx^2+a\right)^2dx}{b^2}$$

$$\Big\downarrow \ 27$$

$$\int (dx)^{5/2} \left(a+bx^2\right)^2dx$$

$$\Big\downarrow \ 244$$

$$\int \left(a^2(dx)^{5/2}+\frac{2ab(dx)^{9/2}}{d^2}+\frac{b^2(dx)^{13/2}}{d^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2a^2(dx)^{7/2}}{7d}+\frac{4ab(dx)^{11/2}}{11d^3}+\frac{2b^2(dx)^{15/2}}{15d^5}$$

input
```
Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(2*a^2*(d*x)^(7/2))/(7*d) + (4*a*b*(d*x)^(11/2))/(11*d^3) + (2*b^2*(d*x)^(
15/2))/(15*d^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

method	result	size
gosper	$\frac{2x\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)\left(dx\right)^{\frac{5}{2}}}{1155}$	30
trager	$\frac{2d^{2}x^{3}\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)\sqrt{dx}}{1155}$	35
risch	$\frac{2d^{3}x^{4}\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)}{1155\sqrt{dx}}$	35
pseudoelliptic	$\frac{2d^{2}x^{3}\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)\sqrt{dx}}{1155}$	35
derivativedivides	$\frac{\frac{2b^{2}\left(dx\right)^{\frac{15}{2}}}{15}+\frac{4ab\,d^{2}\left(dx\right)^{\frac{11}{2}}}{11}+\frac{2a^{2}d^{4}\left(dx\right)^{\frac{7}{2}}}{7}}{d^{5}}$	42
default	$\frac{\frac{2b^{2}\left(dx\right)^{\frac{15}{2}}}{15}+\frac{4ab\,d^{2}\left(dx\right)^{\frac{11}{2}}}{11}+\frac{2a^{2}d^{4}\left(dx\right)^{\frac{7}{2}}}{7}}{d^{5}}$	42
orering	$\frac{2x\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)\left(dx\right)^{\frac{5}{2}}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}{1155\left(b\,x^{2}+a\right)^{2}}$	57

input
```
int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*x*(77*b^2*x^4+210*a*b*x^2+165*a^2)*(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right) dx = \frac{2}{1155} \left(77 b^2 d^2 x^7 + 210 abd^2 x^5 + 165 a^2 d^2 x^3\right) \sqrt{dx}$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
2/1155*(77*b^2*d^2*x^7 + 210*a*b*d^2*x^5 + 165*a^2*d^2*x^3)*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right) dx = \frac{2a^2x(dx)^{\frac{5}{2}}}{7} + \frac{4abx^3(dx)^{\frac{5}{2}}}{11} + \frac{2b^2x^5(dx)^{\frac{5}{2}}}{15}$$

input
```
integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
2*a**2*x*(d*x)**(5/2)/7 + 4*a*b*x**3*(d*x)**(5/2)/11 + 2*b**2*x**5*(d*x)**
(5/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right) dx = \frac{2 \left(77 (dx)^{\frac{15}{2}} b^2 + 210 (dx)^{\frac{11}{2}} abd^2 + 165 (dx)^{\frac{7}{2}} a^2 d^4\right)}{1155 d^5}$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
2/1155*(77*(d*x)^(15/2)*b^2 + 210*(d*x)^(11/2)*a*b*d^2 + 165*(d*x)^(7/2)*a
^2*d^4)/d^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int (dx)^{5/2} \left(a^2 + 2abx^2 + b^2x^4\right) \, dx = \frac{2}{15} \sqrt{dx} b^2 d^2 x^7 + \frac{4}{11} \sqrt{dx} a b d^2 x^5 + \frac{2}{7} \sqrt{dx} a^2 d^2 x^3$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
2/15*sqrt(d*x)*b^2*d^2*x^7 + 4/11*sqrt(d*x)*a*b*d^2*x^5 + 2/7*sqrt(d*x)*a^
2*d^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int (dx)^{5/2} \left(a^2 + 2abx^2 + b^2x^4\right) \, dx = \frac{\frac{2 b^2 (d x)^{15/2}}{15} + \frac{2 a^2 d^4 (d x)^{7/2}}{7} + \frac{4 a b d^2 (d x)^{11/2}}{11}}{d^5}$$

input
```
int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
((2*b^2*(d*x)^(15/2))/15 + (2*a^2*d^4*(d*x)^(7/2))/7 + (4*a*b*d^2*(d*x)^(1
1/2))/11)/d^5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{2\sqrt{x}\,\sqrt{d}\,d^2x^3(77b^2x^4+210ab\,x^2+165a^2)}{1155}$$

input
```
int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*d**2*x**3*(165*a**2 + 210*a*b*x**2 + 77*b**2*x**4))/115
5
```

3.426 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 51

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) \, dx = \frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}$$

output
```
2/5*a^2*(d*x)^(5/2)/d+4/9*a*b*(d*x)^(9/2)/d^3+2/13*b^2*(d*x)^(13/2)/d^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) \, dx = \frac{2}{585}x(dx)^{3/2} \left(117a^2+130abx^2+45b^2x^4\right)$$

input
```
Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(2*x*(d*x)^(3/2)*(117*a^2 + 130*a*b*x^2 + 45*b^2*x^4))/585
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) \, dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^2 (dx)^{3/2} \left(bx^2+a\right)^2dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int (dx)^{3/2} \left(a+bx^2\right)^2dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^2 (dx)^{3/2}+\frac{2ab(dx)^{7/2}}{d^2}+\frac{b^2 (dx)^{11/2}}{d^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2 (dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2 (dx)^{13/2}}{13d^5}$$

input
```
Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(2*a^2*(d*x)^(5/2))/(5*d) + (4*a*b*(d*x)^(9/2))/(9*d^3) + (2*b^2*(d*x)^(13
/2))/(13*d^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

method	result	size
gosper	$\frac{2x(45b^2x^4+130ab\,x^2+117a^2)(dx)^{\frac{3}{2}}}{585}$	30
pseudoelliptic	$\frac{2\left(\frac{5}{13}b^2x^4+\frac{10}{9}ab\,x^2+a^2\right)\sqrt{dx}\,d\,x^2}{5}$	31
trager	$\frac{2d\,x^2(45b^2x^4+130ab\,x^2+117a^2)\sqrt{dx}}{585}$	33
risch	$\frac{2d^2x^3(45b^2x^4+130ab\,x^2+117a^2)}{585\sqrt{dx}}$	35
derivativedivides	$\frac{\frac{2b^2(dx)^{\frac{13}{2}}}{13}+\frac{4ab\,d^2(dx)^{\frac{9}{2}}}{9}+\frac{2a^2d^4(dx)^{\frac{5}{2}}}{5}}{d^5}$	42
default	$\frac{\frac{2b^2(dx)^{\frac{13}{2}}}{13}+\frac{4ab\,d^2(dx)^{\frac{9}{2}}}{9}+\frac{2a^2d^4(dx)^{\frac{5}{2}}}{5}}{d^5}$	42
orering	$\frac{2x(45b^2x^4+130ab\,x^2+117a^2)(dx)^{\frac{3}{2}}(b^2x^4+2ab\,x^2+a^2)}{585(b\,x^2+a)^2}$	57

input
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
2/585*x*(45*b^2*x^4+130*a*b*x^2+117*a^2)*(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) dx = \frac{2}{585} \left(45 b^2 dx^6 + 130 abdx^4 + 117 a^2 dx^2\right) \sqrt{dx}$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
2/585*(45*b^2*d*x^6 + 130*a*b*d*x^4 + 117*a^2*d*x^2)*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) dx = \frac{2a^2x(dx)^{\frac{3}{2}}}{5} + \frac{4abx^3(dx)^{\frac{3}{2}}}{9} + \frac{2b^2x^5(dx)^{\frac{3}{2}}}{13}$$

input
```
integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
2*a**2*x*(d*x)**(3/2)/5 + 4*a*b*x**3*(d*x)**(3/2)/9 + 2*b**2*x**5*(d*x)**(
3/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) dx = \frac{2 \left(45 (dx)^{\frac{13}{2}} b^2 + 130 (dx)^{\frac{9}{2}} abd^2 + 117 (dx)^{\frac{5}{2}} a^2 d^4\right)}{585 d^5}$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
2/585*(45*(d*x)^(13/2)*b^2 + 130*(d*x)^(9/2)*a*b*d^2 + 117*(d*x)^(5/2)*a^2
*d^4)/d^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right) \, dx = \frac{2}{585} \left(45 \sqrt{dx} b^2 x^6 + 130 \sqrt{dx} a b x^4 + 117 \sqrt{dx} a^2 x^2\right) d$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
2/585*(45*sqrt(d*x)*b^2*x^6 + 130*sqrt(d*x)*a*b*x^4 + 117*sqrt(d*x)*a^2*x^
2)*d
```

Mupad [B] (verification not implemented)

Time = 17.62 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right) \, dx = \frac{90 \, b^2 \, (d \, x)^{13/2} + 234 \, a^2 \, d^4 \, (d \, x)^{5/2} + 260 \, a \, b \, d^2 \, (d \, x)^{9/2}}{585 \, d^5}$$

input
```
int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(90*b^2*(d*x)^(13/2) + 234*a^2*d^4*(d*x)^(5/2) + 260*a*b*d^2*(d*x)^(9/2))/
(585*d^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) \, dx = \frac{2\sqrt{x}\,\sqrt{d}\,d\,x^2(45b^2x^4+130ab\,x^2+117a^2)}{585}$$

input
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*d*x**2*(117*a**2 + 130*a*b*x**2 + 45*b**2*x**4))/585
```

3.427 $\int \sqrt{dx}(a^2 + 2abx^2 + b^2x^4)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 51

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}$$

output
```
2/3*a^2*(d*x)^(3/2)/d+4/7*a*b*(d*x)^(7/2)/d^3+2/11*b^2*(d*x)^(11/2)/d^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{2}{231}x\sqrt{dx}\left(77a^2+66abx^2+21b^2x^4\right)$$

input
```
Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(2*x*Sqrt[d*x]*(77*a^2 + 66*a*b*x^2 + 21*b^2*x^4))/231
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)\,dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int b^2\sqrt{dx}\left(bx^2+a\right)^2dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \sqrt{dx}\left(a+bx^2\right)^2dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^2\sqrt{dx}+\frac{2ab(dx)^{5/2}}{d^2}+\frac{b^2(dx)^{9/2}}{d^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}$$

input
```
Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(2*a^2*(d*x)^(3/2))/(3*d) + (4*a*b*(d*x)^(7/2))/(7*d^3) + (2*b^2*(d*x)^(11
/2))/(11*d^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

method	result	size
gosper	$\frac{2x\left(21b^2x^4+66ab\,x^2+77a^2\right)\sqrt{dx}}{231}$	30
trager	$\frac{2x\left(21b^2x^4+66ab\,x^2+77a^2\right)\sqrt{dx}}{231}$	30
pseudoelliptic	$\frac{2x\left(21b^2x^4+66ab\,x^2+77a^2\right)\sqrt{dx}}{231}$	30
risch	$\frac{2d\,x^2\left(21b^2x^4+66ab\,x^2+77a^2\right)}{231\sqrt{dx}}$	33
derivativedivides	$\frac{\frac{2b^2(dx)^{\frac{11}{2}}}{11}+\frac{4ab\,d^2(dx)^{\frac{7}{2}}}{7}+\frac{2a^2d^4(dx)^{\frac{3}{2}}}{3}}{d^5}$	42
default	$\frac{\frac{2b^2(dx)^{\frac{11}{2}}}{11}+\frac{4ab\,d^2(dx)^{\frac{7}{2}}}{7}+\frac{2a^2d^4(dx)^{\frac{3}{2}}}{3}}{d^5}$	42
orering	$\frac{2x\left(21b^2x^4+66ab\,x^2+77a^2\right)\sqrt{dx}\,\left(b^2x^4+2ab\,x^2+a^2\right)}{231\left(b\,x^2+a\right)^2}$	57

input
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
2/231*x*(21*b^2*x^4+66*a*b*x^2+77*a^2)*(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int \sqrt{dx}(a^2+2abx^2+b^2x^4)\,dx=\frac{2}{231}\left(21\,b^2x^5+66\,abx^3+77\,a^2x\right)\sqrt{dx}$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
2/231*(21*b^2*x^5 + 66*a*b*x^3 + 77*a^2*x)*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \sqrt{dx}(a^2+2abx^2+b^2x^4)\,dx=\frac{2a^2x\sqrt{dx}}{3}+\frac{4abx^3\sqrt{dx}}{7}+\frac{2b^2x^5\sqrt{dx}}{11}$$

input
```
integrate((d*x)**(1/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
2*a**2*x*sqrt(d*x)/3 + 4*a*b*x**3*sqrt(d*x)/7 + 2*b**2*x**5*sqrt(d*x)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \sqrt{dx}(a^2+2abx^2+b^2x^4)\,dx=\frac{2\left(21\,(dx)^{\frac{11}{2}}\,b^2+66\,(dx)^{\frac{7}{2}}\,abd^2+77\,(dx)^{\frac{3}{2}}\,a^2d^4\right)}{231\,d^5}$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
2/231*(21*(d*x)^(11/2)*b^2 + 66*(d*x)^(7/2)*a*b*d^2 + 77*(d*x)^(3/2)*a^2*d
^4)/d^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{2}{11}\sqrt{dx}b^2x^5+\frac{4}{7}\sqrt{dx}abx^3+\frac{2}{3}\sqrt{dx}a^2x$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
2/11*sqrt(d*x)*b^2*x^5 + 4/7*sqrt(d*x)*a*b*x^3 + 2/3*sqrt(d*x)*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{42\,b^2\,{\left(d\,x\right)}^{11/2}+154\,a^2\,d^4\,{\left(d\,x\right)}^{3/2}+132\,a\,b\,d^2\,{\left(d\,x\right)}^{7/2}}{231\,d^5}$$

input
```
int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(42*b^2*(d*x)^(11/2) + 154*a^2*d^4*(d*x)^(3/2) + 132*a*b*d^2*(d*x)^(7/2))/
(231*d^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.55

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{2\sqrt{x}\,\sqrt{d}\,x(21b^2x^4+66ab\,x^2+77a^2)}{231}$$

input
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x*(77*a**2 + 66*a*b*x**2 + 21*b**2*x**4))/231
```

3.428 $\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 49

$$\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx=\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}$$

output
```
2*a^2*(d*x)^(1/2)/d+4/5*a*b*(d*x)^(5/2)/d^3+2/9*b^2*(d*x)^(9/2)/d^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx=\frac{2x(45a^2+18abx^2+5b^2x^4)}{45\sqrt{dx}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
```

output
```
(2*x*(45*a^2 + 18*a*b*x^2 + 5*b^2*x^4))/(45*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \ 1380$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{\sqrt{dx}}dx}{b^2}$$

$$\Big\downarrow \ 27$$

$$\int \frac{(a+bx^2)^2}{\sqrt{dx}}dx$$

$$\Big\downarrow \ 244$$

$$\int \left(\frac{a^2}{\sqrt{dx}}+\frac{2ab(dx)^{3/2}}{d^2}+\frac{b^2(dx)^{7/2}}{d^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
```

output
```
(2*a^2*Sqrt[d*x])/d + (4*a*b*(d*x)^(5/2))/(5*d^3) + (2*b^2*(d*x)^(9/2))/(9
*d^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{2\left(5b^{2}x^{4}+18ab\,x^{2}+45a^{2}\right)x}{45\sqrt{dx}}$	30
risch	$\frac{2\left(5b^{2}x^{4}+18ab\,x^{2}+45a^{2}\right)x}{45\sqrt{dx}}$	30
trager	$\frac{\left(\frac{2}{9}b^{2}x^{4}+\frac{4}{5}ab\,x^{2}+2a^{2}\right)\sqrt{dx}}{d}$	31
pseudoelliptic	$\frac{2\sqrt{dx}\,\left(5b^{2}x^{4}+18ab\,x^{2}+45a^{2}\right)}{45d}$	32
derivativedivides	$\frac{\frac{2b^{2}(dx)^{\frac{9}{2}}}{9}+\frac{4ab\,d^{2}(dx)^{\frac{5}{2}}}{5}+2\sqrt{dx}\,a^{2}d^{4}}{d^{5}}$	41
default	$\frac{\frac{2b^{2}(dx)^{\frac{9}{2}}}{9}+\frac{4ab\,d^{2}(dx)^{\frac{5}{2}}}{5}+2\sqrt{dx}\,a^{2}d^{4}}{d^{5}}$	41
orering	$\frac{2\left(5b^{2}x^{4}+18ab\,x^{2}+45a^{2}\right)x\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}{45\left(b\,x^{2}+a\right)^{2}\sqrt{dx}}$	57

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*(5*b^2*x^4+18*a*b*x^2+45*a^2)*x/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{\sqrt{dx}}\,dx = \frac{2\,(5\,b^2x^4 + 18\,abx^2 + 45\,a^2)\sqrt{dx}}{45\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(5*b^2*x^4 + 18*a*b*x^2 + 45*a^2)*sqrt(d*x)/d
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{\sqrt{dx}}\,dx = \frac{2a^2x}{\sqrt{dx}} + \frac{4abx^3}{5\sqrt{dx}} + \frac{2b^2x^5}{9\sqrt{dx}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
```

output
```
2*a**2*x/sqrt(d*x) + 4*a*b*x**3/(5*sqrt(d*x)) + 2*b**2*x**5/(9*sqrt(d*x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{\sqrt{dx}}\,dx = \frac{2\left(45\,\sqrt{dx}a^2 + \frac{5\,(dx)^{\frac{9}{2}}b^2}{d^4} + \frac{18\,(dx)^{\frac{5}{2}}ab}{d^2}\right)}{45\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2/45*(45*sqrt(d*x)*a^2 + 5*(d*x)^(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a*b/d^2)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx = \frac{2\left(5\sqrt{dx}b^2x^4+18\sqrt{dx}abx^2+45\sqrt{dx}a^2\right)}{45\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
2/45*(5*sqrt(d*x)*b^2*x^4 + 18*sqrt(d*x)*a*b*x^2 + 45*sqrt(d*x)*a^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx = \frac{10\,b^2\,(d\,x)^{9/2}+90\,a^2\,d^4\,\sqrt{d\,x}+36\,a\,b\,d^2\,(d\,x)^{5/2}}{45\,d^5}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(1/2),x)
```

output
```
(10*b^2*(d*x)^(9/2) + 90*a^2*d^4*(d*x)^(1/2) + 36*a*b*d^2*(d*x)^(5/2))/(45
*d^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx=\frac{2\sqrt{x}\,\sqrt{d}\,(5b^2x^4+18ab\,x^2+45a^2)}{45d}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*(45*a**2 + 18*a*b*x**2 + 5*b**2*x**4))/(45*d)
```

3.429 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 49

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx = -\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}$$

output
```
-2*a^2/d/(d*x)^(1/2)+4/3*a*b*(d*x)^(3/2)/d^3+2/7*b^2*(d*x)^(7/2)/d^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx = -\frac{2x(21a^2-14abx^2-3b^2x^4)}{21(dx)^{3/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
```

output
```
(-2*x*(21*a^2 - 14*a*b*x^2 - 3*b^2*x^4))/(21*(d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{(dx)^{3/2}}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{(dx)^{3/2}}+\frac{2ab\sqrt{dx}}{d^2}+\frac{b^2(dx)^{5/2}}{d^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
```

output
```
(-2*a^2)/(d*Sqrt[d*x]) + (4*a*b*(d*x)^(3/2))/(3*d^3) + (2*b^2*(d*x)^(7/2))
/(7*d^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{2\left(-3 b^{2} x^{4}-14 a b \,x^{2}+21 a^{2}\right) x}{21 \left(d x \right)^{\frac{3}{2}}}$	30
risch	$-\frac{2\left(-3 b^{2} x^{4}-14 a b \,x^{2}+21 a^{2}\right)}{21 d \sqrt{d x}}$	32
pseudoelliptic	$\frac{6 b^{2} x^{4}+28 a b \,x^{2}-42 a^{2}}{21 d \sqrt{d x}}$	32
trager	$-\frac{2\left(-3 b^{2} x^{4}-14 a b \,x^{2}+21 a^{2}\right) \sqrt{d x}}{21 d^{2} x}$	35
derivativedivides	$\frac{\frac{2 b^{2} \left(d x \right)^{\frac{7}{2}}}{7}+\frac{4 a \,d^{2} b \left(d x \right)^{\frac{3}{2}}}{3}-\frac{2 a^{2} d^{4}}{\sqrt{d x}}}{d^{5}}$	42
default	$\frac{\frac{2 b^{2} \left(d x \right)^{\frac{7}{2}}}{7}+\frac{4 a \,d^{2} b \left(d x \right)^{\frac{3}{2}}}{3}-\frac{2 a^{2} d^{4}}{\sqrt{d x}}}{d^{5}}$	42
orering	$-\frac{2\left(-3 b^{2} x^{4}-14 a b \,x^{2}+21 a^{2}\right) x \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}{21 \left(b \,x^{2}+a \right)^{2} \left(d x \right)^{\frac{3}{2}}}$	57

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(-3*b^2*x^4-14*a*b*x^2+21*a^2)*x/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{(dx)^{3/2}}\,dx = \frac{2\,(3\,b^2x^4 + 14\,abx^2 - 21\,a^2)\sqrt{dx}}{21\,d^2x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/21*(3*b^2*x^4 + 14*a*b*x^2 - 21*a^2)*sqrt(d*x)/(d^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{(dx)^{3/2}}\,dx = -\frac{2a^2x}{(dx)^{\frac{3}{2}}} + \frac{4abx^3}{3\,(dx)^{\frac{3}{2}}} + \frac{2b^2x^5}{7\,(dx)^{\frac{3}{2}}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(3/2),x)
```

output
```
-2*a**2*x/(d*x)**(3/2) + 4*a*b*x**3/(3*(d*x)**(3/2)) + 2*b**2*x**5/(7*(d*x
)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{(dx)^{3/2}}\,dx = -\frac{2\left(\frac{21\,a^2}{\sqrt{dx}} - \frac{3\,(dx)^{\frac{7}{2}}b^2 + 14\,(dx)^{\frac{3}{2}}abd^2}{d^4}\right)}{21\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2/21*(21*a^2/sqrt(d*x) - (3*(d*x)^(7/2)*b^2 + 14*(d*x)^(3/2)*a*b*d^2)/d^4
)/d
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{(dx)^{3/2}} dx = -\frac{2\left(\frac{21a^2}{\sqrt{dx}} - \frac{3\sqrt{dx}b^2d^{27}x^3 + 14\sqrt{dx}abd^{27}x}{d^{28}}\right)}{21d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
-2/21*(21*a^2/sqrt(d*x) - (3*sqrt(d*x)*b^2*d^27*x^3 + 14*sqrt(d*x)*a*b*d^2
7*x)/d^28)/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{(dx)^{3/2}} dx = \frac{-42a^2 + 28abx^2 + 6b^2x^4}{21d\sqrt{dx}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(3/2),x)
```

output
```
(6*b^2*x^4 - 42*a^2 + 28*a*b*x^2)/(21*d*(d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx=\frac{2\sqrt{d}\,(3b^2x^4+14ab\,x^2-21a^2)}{21\sqrt{x}\,d^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x)
```

output
```
(2*sqrt(d)*( - 21*a**2 + 14*a*b*x**2 + 3*b**2*x**4))/(21*sqrt(x)*d**2)
```

3.430 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 49

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}}\,dx = -\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}$$

output
```
-2/3*a^2/d/(d*x)^(3/2)+4*a*b*(d*x)^(1/2)/d^3+2/5*b^2*(d*x)^(5/2)/d^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}}\,dx = -\frac{2x(5a^2-30abx^2-3b^2x^4)}{15(dx)^{5/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]
```

output
```
(-2*x*(5*a^2 - 30*a*b*x^2 - 3*b^2*x^4))/(15*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{(dx)^{5/2}}dx}{b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+bx^2)^2}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^2}{(dx)^{5/2}}+\frac{2ab}{d^2\sqrt{dx}}+\frac{b^2(dx)^{3/2}}{d^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]
```

output
```
(-2*a^2)/(3*d*(d*x)^(3/2)) + (4*a*b*Sqrt[d*x])/d^3 + (2*b^2*(d*x)^(5/2))/(
5*d^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{2\left(-3 b^{2} x^{4}-30 a b \,x^{2}+5 a^{2}\right) x}{15 \left(d x \right)^{\frac{5}{2}}}$	30
pseudoelliptic	$-\frac{2\left(-\frac{3}{5} b^{2} x^{4}-6 a b \,x^{2}+a^{2}\right)}{3 \sqrt{d x}\, d^{2} x}$	33
trager	$-\frac{2\left(-3 b^{2} x^{4}-30 a b \,x^{2}+5 a^{2}\right) \sqrt{d x}}{15 d^{3} x^{2}}$	35
risch	$-\frac{2\left(-3 b^{2} x^{4}-30 a b \,x^{2}+5 a^{2}\right)}{15 d^{2} x \sqrt{d x}}$	35
derivativedivides	$\frac{\frac{2 b^{2} \left(d x \right)^{\frac{5}{2}}}{5}+4 \sqrt{d x}\, a b \,d^{2}-\frac{2 a^{2} d^{4}}{3 \left(d x \right)^{\frac{3}{2}}}}{d^{5}}$	42
default	$\frac{\frac{2 b^{2} \left(d x \right)^{\frac{5}{2}}}{5}+4 \sqrt{d x}\, a b \,d^{2}-\frac{2 a^{2} d^{4}}{3 \left(d x \right)^{\frac{3}{2}}}}{d^{5}}$	42
orering	$-\frac{2\left(-3 b^{2} x^{4}-30 a b \,x^{2}+5 a^{2}\right) x \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}{15 \left(b \,x^{2}+a \right)^{2} \left(d x \right)^{\frac{5}{2}}}$	57

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*b^2*x^4-30*a*b*x^2+5*a^2)*x/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{(dx)^{5/2}}\,dx = \frac{2\,(3\,b^2x^4 + 30\,abx^2 - 5\,a^2)\sqrt{dx}}{15\,d^3x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^4 + 30*a*b*x^2 - 5*a^2)*sqrt(d*x)/(d^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{(dx)^{5/2}}\,dx = -\frac{2a^2x}{3\,(dx)^{\frac{5}{2}}} + \frac{4abx^3}{(dx)^{\frac{5}{2}}} + \frac{2b^2x^5}{5\,(dx)^{\frac{5}{2}}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(5/2),x)
```

output
```
-2*a**2*x/(3*(d*x)**(5/2)) + 4*a*b*x**3/(d*x)**(5/2) + 2*b**2*x**5/(5*(d*x
)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{(dx)^{5/2}}\,dx = -\frac{2\left(\frac{5\,a^2}{(dx)^{\frac{3}{2}}} - \frac{3\left((dx)^{\frac{5}{2}}b^2 + 10\,\sqrt{dx}abd^2\right)}{d^4}\right)}{15\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-2/15*(5*a^2/(d*x)^(3/2) - 3*((d*x)^(5/2)*b^2 + 10*sqrt(d*x)*a*b*d^2)/d^4)
/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}}\,dx=-\frac{2\left(\frac{5\,a^2}{\sqrt{dx}dx}-\frac{3\left(\sqrt{dx}b^2d^{18}x^2+10\,\sqrt{dx}abd^{18}\right)}{d^{20}}\right)}{15\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
-2/15*(5*a^2/(sqrt(d*x)*d*x) - 3*(sqrt(d*x)*b^2*d^18*x^2 + 10*sqrt(d*x)*a*
b*d^18)/d^20)/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}}\,dx=\frac{-10\,a^2+60\,a\,b\,x^2+6\,b^2\,x^4}{15\,d^2\,x\,\sqrt{d\,x}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(5/2),x)
```

output
```
(6*b^2*x^4 - 10*a^2 + 60*a*b*x^2)/(15*d^2*x*(d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{(dx)^{5/2}}\,dx = \frac{2\sqrt{d}\,(3b^2x^4 + 30ab\,x^2 - 5a^2)}{15\sqrt{x}\,d^3x}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x)
```

output
```
(2*sqrt(d)*( - 5*a**2 + 30*a*b*x**2 + 3*b**2*x**4))/(15*sqrt(x)*d**3*x)
```

3.431 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 49

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx=-\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}$$

output
```
-2/5*a^2/d/(d*x)^(5/2)-4*a*b/d^3/(d*x)^(1/2)+2/3*b^2*(d*x)^(3/2)/d^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx=\frac{2\sqrt{dx}(-3a^2-30abx^2+5b^2x^4)}{15d^4x^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
```

output
```
(2*Sqrt[d*x]*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*d^4*x^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{(dx)^{7/2}}dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{(dx)^{7/2}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{(dx)^{7/2}}+\frac{2ab}{d^2(dx)^{3/2}}+\frac{b^2\sqrt{dx}}{d^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
```

output
```
(-2*a^2)/(5*d*(d*x)^(5/2)) - (4*a*b)/(d^3*Sqrt[d*x]) + (2*b^2*(d*x)^(3/2))
/(3*d^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{2\left(-5 b^{2} x^{4}+30 a b \,x^{2}+3 a^{2}\right) x}{15 \left(d x \right)^{\frac{7}{2}}}$	30
pseudoelliptic	$-\frac{2\left(-\frac{5}{3} b^{2} x^{4}+10 a b \,x^{2}+a^{2}\right)}{5 \sqrt{d x}\, d^{3} x^{2}}$	33
trager	$-\frac{2\left(-5 b^{2} x^{4}+30 a b \,x^{2}+3 a^{2}\right) \sqrt{d x}}{15 d^{4} x^{3}}$	35
risch	$-\frac{2\left(-5 b^{2} x^{4}+30 a b \,x^{2}+3 a^{2}\right)}{15 d^{3} x^{2} \sqrt{d x}}$	35
derivativedivides	$\frac{\frac{2 \left(d x \right)^{\frac{3}{2}} b^{2}}{3}-\frac{4 d^{2} a b}{\sqrt{d x}}-\frac{2 a^{2} d^{4}}{5 \left(d x \right)^{\frac{5}{2}}}}{d^{5}}$	42
default	$\frac{\frac{2 \left(d x \right)^{\frac{3}{2}} b^{2}}{3}-\frac{4 d^{2} a b}{\sqrt{d x}}-\frac{2 a^{2} d^{4}}{5 \left(d x \right)^{\frac{5}{2}}}}{d^{5}}$	42
orering	$-\frac{2\left(-5 b^{2} x^{4}+30 a b \,x^{2}+3 a^{2}\right) x \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}{15 \left(b \,x^{2}+a \right)^{2} \left(d x \right)^{\frac{7}{2}}}$	57

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-5*b^2*x^4+30*a*b*x^2+3*a^2)*x/(d*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx=\frac{2\,(5\,b^2x^4-30\,abx^2-3\,a^2)\sqrt{dx}}{15\,d^4x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(5*b^2*x^4 - 30*a*b*x^2 - 3*a^2)*sqrt(d*x)/(d^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx=-\frac{2a^2x}{5\,(dx)^{\frac{7}{2}}}-\frac{4abx^3}{(dx)^{\frac{7}{2}}}+\frac{2b^2x^5}{3\,(dx)^{\frac{7}{2}}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(7/2),x)
```

output
```
-2*a**2*x/(5*(d*x)**(7/2)) - 4*a*b*x**3/(d*x)**(7/2) + 2*b**2*x**5/(3*(d*x
)**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx=\frac{2\left(\frac{5\,(dx)^{\frac{3}{2}}b^2}{d^4}-\frac{3\,(10\,abd^2x^2+a^2d^2)}{(dx)^{\frac{5}{2}}d^2}\right)}{15\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
2/15*(5*(d*x)^(3/2)*b^2/d^4 - 3*(10*a*b*d^2*x^2 + a^2*d^2)/((d*x)^(5/2)*d^
2))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx=\frac{2\left(5\sqrt{dx}b^2x-\frac{3\,(10\,abd^3x^2+a^2d^3)}{\sqrt{dx}d^2x^2}\right)}{15\,d^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x, algorithm="giac")
```

output
```
2/15*(5*sqrt(d*x)*b^2*x - 3*(10*a*b*d^3*x^2 + a^2*d^3)/(sqrt(d*x)*d^2*x^2)
)/d^4
```

Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx=-\frac{6\,a^2+60\,a\,b\,x^2-10\,b^2\,x^4}{15\,d^3\,x^2\,\sqrt{d\,x}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)/(d*x)^(7/2),x)
```

output
```
-(6*a^2 - 10*b^2*x^4 + 60*a*b*x^2)/(15*d^3*x^2*(d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx = \frac{2\sqrt{d}\,(5b^2x^4-30ab\,x^2-3a^2)}{15\sqrt{x}\,d^4x^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x)
```

output
```
(2*sqrt(d)*( - 3*a**2 - 30*a*b*x**2 + 5*b**2*x**4))/(15*sqrt(x)*d**4*x**2)
```

3.432 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 91

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{2a^4(dx)^{7/2}}{7d} + \frac{8a^3b(dx)^{11/2}}{11d^3} + \frac{4a^2b^2(dx)^{15/2}}{5d^5} + \frac{8ab^3(dx)^{19/2}}{19d^7} + \frac{2b^4(dx)^{23/2}}{23d^9}$$

output
```
2/7*a^4*(d*x)^(7/2)/d+8/11*a^3*b*(d*x)^(11/2)/d^3+4/5*a^2*b^2*(d*x)^(15/2)
/d^5+8/19*a*b^3*(d*x)^(19/2)/d^7+2/23*b^4*(d*x)^(23/2)/d^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{2x(dx)^{5/2}\left(24035a^4+61180a^3bx^2+67298a^2b^2x^4+35420ab^3x^6+7315b^4x^8\right)}{168245}$$

input
```
Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(2*x*(d*x)^(5/2)*(24035*a^4 + 61180*a^3*b*x^2 + 67298*a^2*b^2*x^4 + 35420*
a*b^3*x^6 + 7315*b^4*x^8))/168245
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^4 (dx)^{5/2} \left(bx^2+a\right)^4 dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int (dx)^{5/2} \left(a+bx^2\right)^4 dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^4 (dx)^{5/2}+\frac{4a^3b(dx)^{9/2}}{d^2}+\frac{6a^2b^2(dx)^{13/2}}{d^4}+\frac{4ab^3(dx)^{17/2}}{d^6}+\frac{b^4(dx)^{21/2}}{d^8}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4(dx)^{7/2}}{7d}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

input
```
Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(2*a^4*(d*x)^(7/2))/(7*d) + (8*a^3*b*(d*x)^(11/2))/(11*d^3) + (4*a^2*b^2*(
d*x)^(15/2))/(5*d^5) + (8*a*b^3*(d*x)^(19/2))/(19*d^7) + (2*b^4*(d*x)^(23/
2))/(23*d^9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{2x\left(7315b^{4}x^{8}+35420a\,b^{3}x^{6}+67298a^{2}b^{2}x^{4}+61180a^{3}b\,x^{2}+24035a^{4}\right)\left(dx\right)^{\frac{5}{2}}}{168245}$	52
pseudoelliptic	$\frac{2\sqrt{dx}\,d^{2}x^{3}\left(\frac{7}{23}b^{4}x^{8}+\frac{28}{19}a\,b^{3}x^{6}+\frac{14}{5}a^{2}b^{2}x^{4}+\frac{28}{11}a^{3}b\,x^{2}+a^{4}\right)}{7}$	55
trager	$\frac{2d^{2}x^{3}\left(7315b^{4}x^{8}+35420a\,b^{3}x^{6}+67298a^{2}b^{2}x^{4}+61180a^{3}b\,x^{2}+24035a^{4}\right)\sqrt{dx}}{168245}$	57
risch	$\frac{2d^{3}x^{4}\left(7315b^{4}x^{8}+35420a\,b^{3}x^{6}+67298a^{2}b^{2}x^{4}+61180a^{3}b\,x^{2}+24035a^{4}\right)}{168245\sqrt{dx}}$	57
derivativedivides	$\frac{\frac{2b^{4}\left(dx\right)^{\frac{23}{2}}}{23}+\frac{8a\,b^{3}d^{2}\left(dx\right)^{\frac{19}{2}}}{19}+\frac{4a^{2}d^{4}b^{2}\left(dx\right)^{\frac{15}{2}}}{5}+\frac{8a^{3}d^{6}b\left(dx\right)^{\frac{11}{2}}}{11}+\frac{2a^{4}d^{8}\left(dx\right)^{\frac{7}{2}}}{7}}{d^{9}}$	74
default	$\frac{\frac{2b^{4}\left(dx\right)^{\frac{23}{2}}}{23}+\frac{8a\,b^{3}d^{2}\left(dx\right)^{\frac{19}{2}}}{19}+\frac{4a^{2}d^{4}b^{2}\left(dx\right)^{\frac{15}{2}}}{5}+\frac{8a^{3}d^{6}b\left(dx\right)^{\frac{11}{2}}}{11}+\frac{2a^{4}d^{8}\left(dx\right)^{\frac{7}{2}}}{7}}{d^{9}}$	74
orering	$\frac{2x\left(7315b^{4}x^{8}+35420a\,b^{3}x^{6}+67298a^{2}b^{2}x^{4}+61180a^{3}b\,x^{2}+24035a^{4}\right)\left(dx\right)^{\frac{5}{2}}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}}{168245\left(b\,x^{2}+a\right)^{4}}$	81

input
```
int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/168245*x*(7315*b^4*x^8+35420*a*b^3*x^6+67298*a^2*b^2*x^4+61180*a^3*b*x^2
+24035*a^4)*(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int (dx)^{5/2} \left(a^2 + 2abx^2 + b^2x^4\right)^2 dx = \frac{2}{168245} \left(7315\, b^4d^2x^{11} + 35420\, ab^3d^2x^9 + 67298\, a^2b^2d^2x^7 + 61180\, a^3bd^2x^5 + 24035\, a^4d^2x^3\right)\sqrt{\cdot}$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
2/168245*(7315*b^4*d^2*x^11 + 35420*a*b^3*d^2*x^9 + 67298*a^2*b^2*d^2*x^7
+ 61180*a^3*b*d^2*x^5 + 24035*a^4*d^2*x^3)*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int (dx)^{5/2} \left(a^2 + 2abx^2 + b^2x^4\right)^2 dx = \frac{2a^4x(dx)^{\frac{5}{2}}}{7} + \frac{8a^3bx^3(dx)^{\frac{5}{2}}}{11} + \frac{4a^2b^2x^5(dx)^{\frac{5}{2}}}{5} + \frac{8ab^3x^7(dx)^{\frac{5}{2}}}{19} + \frac{2b^4x^9(dx)^{\frac{5}{2}}}{23}$$

input
```
integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
2*a**4*x*(d*x)**(5/2)/7 + 8*a**3*b*x**3*(d*x)**(5/2)/11 + 4*a**2*b**2*x**5
*(d*x)**(5/2)/5 + 8*a*b**3*x**7*(d*x)**(5/2)/19 + 2*b**4*x**9*(d*x)**(5/2)
/23
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{2 \left(7315 \, (dx)^{\frac{23}{2}} b^4 + 35420 \, (dx)^{\frac{19}{2}} ab^3 d^2 + 67298 \, (dx)^{\frac{15}{2}} a^2 b^2 d^4 + 61180 \, (dx)^{\frac{11}{2}} a^3 b d^6 + 24035 \, (\right.}{168245 \, d^9}$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
2/168245*(7315*(d*x)^(23/2)*b^4 + 35420*(d*x)^(19/2)*a*b^3*d^2 + 67298*(d*
x)^(15/2)*a^2*b^2*d^4 + 61180*(d*x)^(11/2)*a^3*b*d^6 + 24035*(d*x)^(7/2)*a
^4*d^8)/d^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{2}{23} \sqrt{dx} b^4 d^2 x^{11} + \frac{8}{19} \sqrt{dx} a b^3 d^2 x^9 + \frac{4}{5} \sqrt{dx} a^2 b^2 d^2 x^7 + \frac{8}{11} \sqrt{dx} a^3 b d^2 x^5 + \frac{2}{7} \sqrt{dx} a^4 d^2 x^3$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
2/23*sqrt(d*x)*b^4*d^2*x^11 + 8/19*sqrt(d*x)*a*b^3*d^2*x^9 + 4/5*sqrt(d*x)
*a^2*b^2*d^2*x^7 + 8/11*sqrt(d*x)*a^3*b*d^2*x^5 + 2/7*sqrt(d*x)*a^4*d^2*x^
3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.78

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^2\,dx = \frac{2\,a^4\,{\left(d\,x\right)}^{7/2}}{7\,d}+\frac{2\,b^4\,{\left(d\,x\right)}^{23/2}}{23\,d^9} + \frac{4\,a^2\,b^2\,{\left(d\,x\right)}^{15/2}}{5\,d^5}+\frac{8\,a^3\,b\,{\left(d\,x\right)}^{11/2}}{11\,d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{19/2}}{19\,d^7}$$

input
```
int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(2*a^4*(d*x)^(7/2))/(7*d) + (2*b^4*(d*x)^(23/2))/(23*d^9) + (4*a^2*b^2*(d*
x)^(15/2))/(5*d^5) + (8*a^3*b*(d*x)^(11/2))/(11*d^3) + (8*a*b^3*(d*x)^(19/
2))/(19*d^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int (dx)^{5/2} \left(a^2+2abx^2 +b^2x^4\right)^2\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,d^2x^3(7315b^4x^8+35420a\,b^3x^6+67298a^2b^2x^4+61180a^3b\,x^2+24035a^4)}{168245}$$

input
```
int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(2*sqrt(x)*sqrt(d)*d**2*x**3*(24035*a**4 + 61180*a**3*b*x**2 + 67298*a**2*
b**2*x**4 + 35420*a*b**3*x**6 + 7315*b**4*x**8))/168245
```

3.433 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 91

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{2a^4(dx)^{5/2}}{5d}+\frac{8a^3b(dx)^{9/2}}{9d^3} + \frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

output
```
2/5*a^4*(d*x)^(5/2)/d+8/9*a^3*b*(d*x)^(9/2)/d^3+12/13*a^2*b^2*(d*x)^(13/2)
/d^5+8/17*a*b^3*(d*x)^(17/2)/d^7+2/21*b^4*(d*x)^(21/2)/d^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int (dx)^{3/2} \left(a^2+2abx^2 +b^2x^4\right)^2 \, dx = \frac{2x(dx)^{3/2}\left(13923a^4+30940a^3bx^2+32130a^2b^2x^4+16380ab^3x^6+3315b^4x^8\right)}{69615}$$

input
```
Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(2*x*(d*x)^(3/2)*(13923*a^4 + 30940*a^3*b*x^2 + 32130*a^2*b^2*x^4 + 16380*
a*b^3*x^6 + 3315*b^4*x^8))/69615
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int b^4 (dx)^{3/2} \left(bx^2+a\right)^4 dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int (dx)^{3/2} \left(a+bx^2\right)^4 dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^4 (dx)^{3/2}+\frac{4a^3b(dx)^{7/2}}{d^2}+\frac{6a^2b^2(dx)^{11/2}}{d^4}+\frac{4ab^3(dx)^{15/2}}{d^6}+\frac{b^4(dx)^{19/2}}{d^8}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^4(dx)^{5/2}}{5d}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

input
```
Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(2*a^4*(d*x)^(5/2))/(5*d) + (8*a^3*b*(d*x)^(9/2))/(9*d^3) + (12*a^2*b^2*(d
*x)^(13/2))/(13*d^5) + (8*a*b^3*(d*x)^(17/2))/(17*d^7) + (2*b^4*(d*x)^(21/
2))/(21*d^9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{2x\left(3315b^{4}x^{8}+16380a\,b^{3}x^{6}+32130a^{2}b^{2}x^{4}+30940a^{3}b\,x^{2}+13923a^{4}\right)\left(dx\right)^{\frac{3}{2}}}{69615}$	52
pseudoelliptic	$\frac{2\sqrt{dx}\,d\left(\frac{5}{21}b^{4}x^{8}+\frac{20}{17}a\,b^{3}x^{6}+\frac{30}{13}a^{2}b^{2}x^{4}+\frac{20}{9}a^{3}b\,x^{2}+a^{4}\right)x^{2}}{5}$	53
trager	$\frac{2d\,x^{2}\left(3315b^{4}x^{8}+16380a\,b^{3}x^{6}+32130a^{2}b^{2}x^{4}+30940a^{3}b\,x^{2}+13923a^{4}\right)\sqrt{dx}}{69615}$	55
risch	$\frac{2d^{2}x^{3}\left(3315b^{4}x^{8}+16380a\,b^{3}x^{6}+32130a^{2}b^{2}x^{4}+30940a^{3}b\,x^{2}+13923a^{4}\right)}{69615\sqrt{dx}}$	57
derivativedivides	$\frac{\frac{2b^{4}\left(dx\right)^{\frac{21}{2}}}{21}+\frac{8a\,b^{3}d^{2}\left(dx\right)^{\frac{17}{2}}}{17}+\frac{12a^{2}d^{4}b^{2}\left(dx\right)^{\frac{13}{2}}}{13}+\frac{8a^{3}d^{6}b\left(dx\right)^{\frac{9}{2}}}{9}+\frac{2a^{4}d^{8}\left(dx\right)^{\frac{5}{2}}}{5}}{d^{9}}$	74
default	$\frac{\frac{2b^{4}\left(dx\right)^{\frac{21}{2}}}{21}+\frac{8a\,b^{3}d^{2}\left(dx\right)^{\frac{17}{2}}}{17}+\frac{12a^{2}d^{4}b^{2}\left(dx\right)^{\frac{13}{2}}}{13}+\frac{8a^{3}d^{6}b\left(dx\right)^{\frac{9}{2}}}{9}+\frac{2a^{4}d^{8}\left(dx\right)^{\frac{5}{2}}}{5}}{d^{9}}$	74
orering	$\frac{2x\left(3315b^{4}x^{8}+16380a\,b^{3}x^{6}+32130a^{2}b^{2}x^{4}+30940a^{3}b\,x^{2}+13923a^{4}\right)\left(dx\right)^{\frac{3}{2}}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}}{69615\left(b\,x^{2}+a\right)^{4}}$	81

input
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/69615*x*(3315*b^4*x^8+16380*a*b^3*x^6+32130*a^2*b^2*x^4+30940*a^3*b*x^2+
13923*a^4)*(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{2}{69615} \left(3315 \, b^4 dx^{10} + 16380 \, ab^3 dx^8 + 32130 \, a^2 b^2 dx^6 + 30940 \, a^3 b dx^4 + 13923 \, a^4 dx^2\right) \sqrt{dx}$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
2/69615*(3315*b^4*d*x^10 + 16380*a*b^3*d*x^8 + 32130*a^2*b^2*d*x^6 + 30940
*a^3*b*d*x^4 + 13923*a^4*d*x^2)*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{2a^4x(dx)^{\frac{3}{2}}}{5} + \frac{8a^3bx^3(dx)^{\frac{3}{2}}}{9} + \frac{12a^2b^2x^5(dx)^{\frac{3}{2}}}{13} + \frac{8ab^3x^7(dx)^{\frac{3}{2}}}{17} + \frac{2b^4x^9(dx)^{\frac{3}{2}}}{21}$$

input
```
integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
2*a**4*x*(d*x)**(3/2)/5 + 8*a**3*b*x**3*(d*x)**(3/2)/9 + 12*a**2*b**2*x**5
*(d*x)**(3/2)/13 + 8*a*b**3*x**7*(d*x)**(3/2)/17 + 2*b**4*x**9*(d*x)**(3/2
)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^2 \, dx = \frac{2 \left(3315 \, (dx)^{\frac{21}{2}} \, b^4 + 16380 \, (dx)^{\frac{17}{2}} \, ab^3d^2 + 32130 \, (dx)^{\frac{13}{2}} \, a^2b^2d^4 + 30940 \, (dx)^{\frac{9}{2}} \, a^3bd^6 + 13923 \, (dx)^{\frac{5}{2}} \, a^4d^8\right)}{69615 \, d^9}$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
2/69615*(3315*(d*x)^(21/2)*b^4 + 16380*(d*x)^(17/2)*a*b^3*d^2 + 32130*(d*x
)^(13/2)*a^2*b^2*d^4 + 30940*(d*x)^(9/2)*a^3*b*d^6 + 13923*(d*x)^(5/2)*a^4
*d^8)/d^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.81

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^2 \, dx = \frac{2}{69615} \left(3315 \sqrt{dx} b^4 x^{10} + 16380 \sqrt{dx} a b^3 x^8 + 32130 \sqrt{dx} a^2 b^2 x^6 + 30940 \sqrt{dx} a^3 b x^4 + 13923 \sqrt{dx} a^4 x^2\right) d$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
2/69615*(3315*sqrt(d*x)*b^4*x^10 + 16380*sqrt(d*x)*a*b^3*x^8 + 32130*sqrt(
d*x)*a^2*b^2*x^6 + 30940*sqrt(d*x)*a^3*b*x^4 + 13923*sqrt(d*x)*a^4*x^2)*d
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.78

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{2\,a^4\,{\left(d\,x\right)}^{5/2}}{5\,d}+\frac{2\,b^4\,{\left(d\,x\right)}^{21/2}}{21\,d^9}+\frac{12\,a^2\,b^2\,{\left(d\,x\right)}^{13/2}}{13\,d^5}+\frac{8\,a^3\,b\,{\left(d\,x\right)}^{9/2}}{9\,d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{17/2}}{17\,d^7}$$

input
```
int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(2*a^4*(d*x)^(5/2))/(5*d) + (2*b^4*(d*x)^(21/2))/(21*d^9) + (12*a^2*b^2*(d
*x)^(13/2))/(13*d^5) + (8*a^3*b*(d*x)^(9/2))/(9*d^3) + (8*a*b^3*(d*x)^(17/
2))/(17*d^7)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{2\sqrt{x}\,\sqrt{d}\,d\,x^2(3315b^4x^8+16380a\,b^3x^6+32130a^2b^2x^4+30940a^3b\,x^2+13923a^4)}{69615}$$

input
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(2*sqrt(x)*sqrt(d)*d*x**2*(13923*a**4 + 30940*a**3*b*x**2 + 32130*a**2*b**
2*x**4 + 16380*a*b**3*x**6 + 3315*b**4*x**8))/69615
```

3.434 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 91

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{2a^4(dx)^{3/2}}{3d}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

output
```
2/3*a^4*(d*x)^(3/2)/d+8/7*a^3*b*(d*x)^(7/2)/d^3+12/11*a^2*b^2*(d*x)^(11/2)
/d^5+8/15*a*b^3*(d*x)^(15/2)/d^7+2/19*b^4*(d*x)^(19/2)/d^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{2x\sqrt{dx}\left(7315a^4+12540a^3bx^2+11970a^2b^2x^4+5852ab^3x^6+1155b^4x^8\right)}{21945}$$

input
```
Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(2*x*Sqrt[d*x]*(7315*a^4 + 12540*a^3*b*x^2 + 11970*a^2*b^2*x^4 + 5852*a*b^
3*x^6 + 1155*b^4*x^8))/21945
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2 dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int b^4\sqrt{dx}\left(bx^2+a\right)^4dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int \sqrt{dx}\left(a+bx^2\right)^4dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^4\sqrt{dx}+\frac{4a^3b(dx)^{5/2}}{d^2}+\frac{6a^2b^2(dx)^{9/2}}{d^4}+\frac{4ab^3(dx)^{13/2}}{d^6}+\frac{b^4(dx)^{17/2}}{d^8}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^4(dx)^{3/2}}{3d}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

input
```
Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(2*a^4*(d*x)^(3/2))/(3*d) + (8*a^3*b*(d*x)^(7/2))/(7*d^3) + (12*a^2*b^2*(d
*x)^(11/2))/(11*d^5) + (8*a*b^3*(d*x)^(15/2))/(15*d^7) + (2*b^4*(d*x)^(19/
2))/(19*d^9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.55

method	result	size
pseudoelliptic	$\frac{2\sqrt{dx}\,\left(\frac{3}{19}b^{4}x^{8}+\frac{4}{5}a\,b^{3}x^{6}+\frac{18}{11}a^{2}b^{2}x^{4}+\frac{12}{7}a^{3}b\,x^{2}+a^{4}\right)x}{3}$	50
gosper	$\frac{2x\left(1155b^{4}x^{8}+5852a\,b^{3}x^{6}+11970a^{2}b^{2}x^{4}+12540a^{3}b\,x^{2}+7315a^{4}\right)\sqrt{dx}}{21945}$	52
trager	$\frac{2x\left(1155b^{4}x^{8}+5852a\,b^{3}x^{6}+11970a^{2}b^{2}x^{4}+12540a^{3}b\,x^{2}+7315a^{4}\right)\sqrt{dx}}{21945}$	52
risch	$\frac{2d\,x^{2}\left(1155b^{4}x^{8}+5852a\,b^{3}x^{6}+11970a^{2}b^{2}x^{4}+12540a^{3}b\,x^{2}+7315a^{4}\right)}{21945\sqrt{dx}}$	55
derivativedivides	$\frac{\frac{2b^{4}(dx)^{\frac{19}{2}}}{19}+\frac{8a\,b^{3}d^{2}(dx)^{\frac{15}{2}}}{15}+\frac{12a^{2}d^{4}b^{2}(dx)^{\frac{11}{2}}}{11}+\frac{8a^{3}b\,d^{6}(dx)^{\frac{7}{2}}}{7}+\frac{2a^{4}d^{8}(dx)^{\frac{3}{2}}}{3}}{d^{9}}$	74
default	$\frac{\frac{2b^{4}(dx)^{\frac{19}{2}}}{19}+\frac{8a\,b^{3}d^{2}(dx)^{\frac{15}{2}}}{15}+\frac{12a^{2}d^{4}b^{2}(dx)^{\frac{11}{2}}}{11}+\frac{8a^{3}b\,d^{6}(dx)^{\frac{7}{2}}}{7}+\frac{2a^{4}d^{8}(dx)^{\frac{3}{2}}}{3}}{d^{9}}$	74
orering	$\frac{2x\left(1155b^{4}x^{8}+5852a\,b^{3}x^{6}+11970a^{2}b^{2}x^{4}+12540a^{3}b\,x^{2}+7315a^{4}\right)\sqrt{dx}\,\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{2}}{21945\left(b\,x^{2}+a\right)^{4}}$	81

input
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x)^(1/2)*(3/19*b^4*x^8+4/5*a*b^3*x^6+18/11*a^2*b^2*x^4+12/7*a^3*b*x
^2+a^4)*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.56

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx = \frac{2}{21945}\left(1155\,b^4x^9+5852\,ab^3x^7+11970\,a^2b^2x^5+12540\,a^3bx^3+7315\,a^4x\right)\sqrt{dx}$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
2/21945*(1155*b^4*x^9 + 5852*a*b^3*x^7 + 11970*a^2*b^2*x^5 + 12540*a^3*b*x
^3 + 7315*a^4*x)*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx = \frac{2a^4x\sqrt{dx}}{3}+\frac{8a^3bx^3\sqrt{dx}}{7}+\frac{12a^2b^2x^5\sqrt{dx}}{11}+\frac{8ab^3x^7\sqrt{dx}}{15}+\frac{2b^4x^9\sqrt{dx}}{19}$$

input
```
integrate((d*x)**(1/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
2*a**4*x*sqrt(d*x)/3 + 8*a**3*b*x**3*sqrt(d*x)/7 + 12*a**2*b**2*x**5*sqrt(
d*x)/11 + 8*a*b**3*x**7*sqrt(d*x)/15 + 2*b**4*x**9*sqrt(d*x)/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx = \frac{2\left(1155\,(dx)^{\frac{19}{2}}\,b^4+5852\,(dx)^{\frac{15}{2}}\,ab^3d^2+11970\,(dx)^{\frac{11}{2}}\,a^2b^2d^4+12540\,(dx)^{\frac{7}{2}}\,a^3bd^6+7315\,(dx)^{\frac{3}{2}}\,a^4d^8\right)}{21945\,d^9}$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
2/21945*(1155*(d*x)^(19/2)*b^4 + 5852*(d*x)^(15/2)*a*b^3*d^2 + 11970*(d*x)
^(11/2)*a^2*b^2*d^4 + 12540*(d*x)^(7/2)*a^3*b*d^6 + 7315*(d*x)^(3/2)*a^4*d
^8)/d^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx = \frac{2}{19}\sqrt{dx}b^4x^9+\frac{8}{15}\sqrt{dx}ab^3x^7+\frac{12}{11}\sqrt{dx}a^2b^2x^5+\frac{8}{7}\sqrt{dx}a^3bx^3+\frac{2}{3}\sqrt{dx}a^4x$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
2/19*sqrt(d*x)*b^4*x^9 + 8/15*sqrt(d*x)*a*b^3*x^7 + 12/11*sqrt(d*x)*a^2*b^
2*x^5 + 8/7*sqrt(d*x)*a^3*b*x^3 + 2/3*sqrt(d*x)*a^4*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.78

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{2\,a^4\,{\left(d\,x\right)}^{3/2}}{3\,d}+\frac{2\,b^4\,{\left(d\,x\right)}^{19/2}}{19\,d^9}+\frac{12\,a^2\,b^2\,{\left(d\,x\right)}^{11/2}}{11\,d^5}+\frac{8\,a^3\,b\,{\left(d\,x\right)}^{7/2}}{7\,d^3}+\frac{8\,a\,b^3\,{\left(d\,x\right)}^{15/2}}{15\,d^7}$$

input
```
int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(2*a^4*(d*x)^(3/2))/(3*d) + (2*b^4*(d*x)^(19/2))/(19*d^9) + (12*a^2*b^2*(d
*x)^(11/2))/(11*d^5) + (8*a^3*b*(d*x)^(7/2))/(7*d^3) + (8*a*b^3*(d*x)^(15/
2))/(15*d^7)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.55

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx=\frac{2\sqrt{x}\,\sqrt{d}\,x(1155b^4x^8+5852a\,b^3x^6+11970a^2b^2x^4+12540a^3b\,x^2+7315a^4)}{21945}$$

input
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(2*sqrt(x)*sqrt(d)*x*(7315*a**4 + 12540*a**3*b*x**2 + 11970*a**2*b**2*x**4
 + 5852*a*b**3*x**6 + 1155*b**4*x**8))/21945
```

3.435 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 89

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}} \, dx = \frac{2a^4\sqrt{dx}}{d}+\frac{8a^3b(dx)^{5/2}}{5d^3}+\frac{4a^2b^2(dx)^{9/2}}{3d^5}+\frac{8ab^3(dx)^{13/2}}{13d^7}+\frac{2b^4(dx)^{17/2}}{17d^9}$$

output
```
2*a^4*(d*x)^(1/2)/d+8/5*a^3*b*(d*x)^(5/2)/d^3+4/3*a^2*b^2*(d*x)^(9/2)/d^5+
8/13*a*b^3*(d*x)^(13/2)/d^7+2/17*b^4*(d*x)^(17/2)/d^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}} \, dx = \frac{2x(3315a^4+2652a^3bx^2+2210a^2b^2x^4+1020ab^3x^6+195b^4x^8)}{3315\sqrt{dx}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

output
```
(2*x*(3315*a^4 + 2652*a^3*b*x^2 + 2210*a^2*b^2*x^4 + 1020*a*b^3*x^6 + 195*
b^4*x^8))/(3315*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{\sqrt{dx}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^4}{\sqrt{dx}}+\frac{4a^3b(dx)^{3/2}}{d^2}+\frac{6a^2b^2(dx)^{7/2}}{d^4}+\frac{4ab^3(dx)^{11/2}}{d^6}+\frac{b^4(dx)^{15/2}}{d^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4\sqrt{dx}}{d}+\frac{8a^3b(dx)^{5/2}}{5d^3}+\frac{4a^2b^2(dx)^{9/2}}{3d^5}+\frac{8ab^3(dx)^{13/2}}{13d^7}+\frac{2b^4(dx)^{17/2}}{17d^9}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

output
```
(2*a^4*Sqrt[d*x])/d + (8*a^3*b*(d*x)^(5/2))/(5*d^3) + (4*a^2*b^2*(d*x)^(9/
2))/(3*d^5) + (8*a*b^3*(d*x)^(13/2))/(13*d^7) + (2*b^4*(d*x)^(17/2))/(17*d
^9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{2\left(195 b^{4} x^{8}+1020 a\,b^{3} x^{6}+2210 a^{2} b^{2} x^{4}+2652 a^{3} b\,x^{2}+3315 a^{4}\right) x}{3315 \sqrt{d x}}$	52
risch	$\frac{2\left(195 b^{4} x^{8}+1020 a\,b^{3} x^{6}+2210 a^{2} b^{2} x^{4}+2652 a^{3} b\,x^{2}+3315 a^{4}\right) x}{3315 \sqrt{d x}}$	52
pseudoelliptic	$\frac{2 \sqrt{d x}\,\left(\frac{1}{17} b^{4} x^{8}+\frac{4}{13} a\,b^{3} x^{6}+\frac{2}{3} a^{2} b^{2} x^{4}+\frac{4}{5} a^{3} b\,x^{2}+a^{4}\right)}{d}$	52
trager	$\frac{\left(\frac{2}{17} b^{4} x^{8}+\frac{8}{13} a\,b^{3} x^{6}+\frac{4}{3} a^{2} b^{2} x^{4}+\frac{8}{5} a^{3} b\,x^{2}+2 a^{4}\right) \sqrt{d x}}{d}$	53
derivativedivides	$\frac{\frac{2 b^{4} \left(d x\right)^{\frac{17}{2}}}{17}+\frac{8 a\,b^{3} d^{2} \left(d x\right)^{\frac{13}{2}}}{13}+\frac{4 a^{2} d^{4} b^{2} \left(d x\right)^{\frac{9}{2}}}{3}+\frac{8 a^{3} d^{6} b \left(d x\right)^{\frac{5}{2}}}{5}+2 \sqrt{d x}\, a^{4} d^{8}}{d^{9}}$	73
default	$\frac{\frac{2 b^{4} \left(d x\right)^{\frac{17}{2}}}{17}+\frac{8 a\,b^{3} d^{2} \left(d x\right)^{\frac{13}{2}}}{13}+\frac{4 a^{2} d^{4} b^{2} \left(d x\right)^{\frac{9}{2}}}{3}+\frac{8 a^{3} d^{6} b \left(d x\right)^{\frac{5}{2}}}{5}+2 \sqrt{d x}\, a^{4} d^{8}}{d^{9}}$	73
orering	$\frac{2\left(195 b^{4} x^{8}+1020 a\,b^{3} x^{6}+2210 a^{2} b^{2} x^{4}+2652 a^{3} b\,x^{2}+3315 a^{4}\right) x \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{2}}{3315 \left(b\,x^{2}+a \right)^{4} \sqrt{d x}}$	81

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3315*(195*b^4*x^8+1020*a*b^3*x^6+2210*a^2*b^2*x^4+2652*a^3*b*x^2+3315*a^
4)*x/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}}\,dx = \frac{2\,(195\,b^4x^8+1020\,ab^3x^6+2210\,a^2b^2x^4+2652\,a^3bx^2+3315\,a^4)\sqrt{dx}}{3315\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/3315*(195*b^4*x^8 + 1020*a*b^3*x^6 + 2210*a^2*b^2*x^4 + 2652*a^3*b*x^2 +
 3315*a^4)*sqrt(d*x)/d
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}}\,dx = \frac{2a^4x}{\sqrt{dx}}+\frac{8a^3bx^3}{5\sqrt{dx}}+\frac{4a^2b^2x^5}{3\sqrt{dx}}+\frac{8ab^3x^7}{13\sqrt{dx}}+\frac{2b^4x^9}{17\sqrt{dx}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

output
```
2*a**4*x/sqrt(d*x) + 8*a**3*b*x**3/(5*sqrt(d*x)) + 4*a**2*b**2*x**5/(3*sqr
t(d*x)) + 8*a*b**3*x**7/(13*sqrt(d*x)) + 2*b**4*x**9/(17*sqrt(d*x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{\sqrt{dx}}\,dx = \frac{2\left(9945\sqrt{dx}a^4 + \frac{585\,(dx)^{\frac{17}{2}}b^4}{d^8} + \frac{3060\,(dx)^{\frac{13}{2}}ab^3}{d^6} + \frac{4420\,(dx)^{\frac{9}{2}}a^2b^2}{d^4} + 442\left(\frac{5\,(dx)^{\frac{9}{2}}b^2}{d^4} + \frac{18\,(dx)^{\frac{5}{2}}ab}{d^2}\right)a^2\right)}{9945\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2/9945*(9945*sqrt(d*x)*a^4 + 585*(d*x)^(17/2)*b^4/d^8 + 3060*(d*x)^(13/2)*
a*b^3/d^6 + 4420*(d*x)^(9/2)*a^2*b^2/d^4 + 442*(5*(d*x)^(9/2)*b^2/d^4 + 18
*(d*x)^(5/2)*a*b/d^2)*a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{\sqrt{dx}}\,dx = \frac{2\left(195\sqrt{dx}b^4x^8 + 1020\sqrt{dx}ab^3x^6 + 2210\sqrt{dx}a^2b^2x^4 + 2652\sqrt{dx}a^3bx^2 + 3315\sqrt{dx}a^4\right)}{3315\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x, algorithm="giac")
```

output
```
2/3315*(195*sqrt(d*x)*b^4*x^8 + 1020*sqrt(d*x)*a*b^3*x^6 + 2210*sqrt(d*x)*
a^2*b^2*x^4 + 2652*sqrt(d*x)*a^3*b*x^2 + 3315*sqrt(d*x)*a^4)/d
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}}\,dx = \frac{2\,a^4\,\sqrt{d\,x}}{d}+\frac{2\,b^4\,{(d\,x)}^{17/2}}{17\,d^9}+\frac{4\,a^2\,b^2\,{(d\,x)}^{9/2}}{3\,d^5}+\frac{8\,a^3\,b\,{(d\,x)}^{5/2}}{5\,d^3}+\frac{8\,a\,b^3\,{(d\,x)}^{13/2}}{13\,d^7}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(1/2),x)
```

output
```
(2*a^4*(d*x)^(1/2))/d + (2*b^4*(d*x)^(17/2))/(17*d^9) + (4*a^2*b^2*(d*x)^(
9/2))/(3*d^5) + (8*a^3*b*(d*x)^(5/2))/(5*d^3) + (8*a*b^3*(d*x)^(13/2))/(13
*d^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,(195b^4x^8+1020a\,b^3x^6+2210a^2b^2x^4+2652a^3b\,x^2+3315a^4)}{3315d}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*(3315*a**4 + 2652*a**3*b*x**2 + 2210*a**2*b**2*x**4 + 1
020*a*b**3*x**6 + 195*b**4*x**8))/(3315*d)
```

3.436 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 89

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx = -\frac{2a^4}{d\sqrt{dx}}+\frac{8a^3b(dx)^{3/2}}{3d^3}+\frac{12a^2b^2(dx)^{7/2}}{7d^5}+\frac{8ab^3(dx)^{11/2}}{11d^7}+\frac{2b^4(dx)^{15/2}}{15d^9}$$

output
```
-2*a^4/d/(d*x)^(1/2)+8/3*a^3*b*(d*x)^(3/2)/d^3+12/7*a^2*b^2*(d*x)^(7/2)/d^
5+8/11*a*b^3*(d*x)^(11/2)/d^7+2/15*b^4*(d*x)^(15/2)/d^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx = -\frac{2x(1155a^4-1540a^3bx^2-990a^2b^2x^4-420ab^3x^6-77b^4x^8)}{1155(dx)^{3/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

output
```
(-2*x*(1155*a^4 - 1540*a^3*b*x^2 - 990*a^2*b^2*x^4 - 420*a*b^3*x^6 - 77*b^
4*x^8))/(1155*(d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{(dx)^{3/2}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^4}{(dx)^{3/2}}+\frac{4a^3b\sqrt{dx}}{d^2}+\frac{6a^2b^2(dx)^{5/2}}{d^4}+\frac{4ab^3(dx)^{9/2}}{d^6}+\frac{b^4(dx)^{13/2}}{d^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^4}{d\sqrt{dx}}+\frac{8a^3b(dx)^{3/2}}{3d^3}+\frac{12a^2b^2(dx)^{7/2}}{7d^5}+\frac{8ab^3(dx)^{11/2}}{11d^7}+\frac{2b^4(dx)^{15/2}}{15d^9}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

output
```
(-2*a^4)/(d*Sqrt[d*x]) + (8*a^3*b*(d*x)^(3/2))/(3*d^3) + (12*a^2*b^2*(d*x)
^(7/2))/(7*d^5) + (8*a*b^3*(d*x)^(11/2))/(11*d^7) + (2*b^4*(d*x)^(15/2))/(
15*d^9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{2\left(-77 b^{4} x^{8}-420 a \,b^{3} x^{6}-990 a^{2} b^{2} x^{4}-1540 a^{3} b \,x^{2}+1155 a^{4}\right) x}{1155 \left(d x \right)^{\frac{3}{2}}}$	52
pseudoelliptic	$-\frac{2\left(-\frac{1}{15} b^{4} x^{8}-\frac{4}{11} a \,b^{3} x^{6}-\frac{6}{7} a^{2} b^{2} x^{4}-\frac{4}{3} a^{3} b \,x^{2}+a^{4}\right)}{\sqrt{d x}\, d}$	52
risch	$-\frac{2\left(-77 b^{4} x^{8}-420 a \,b^{3} x^{6}-990 a^{2} b^{2} x^{4}-1540 a^{3} b \,x^{2}+1155 a^{4}\right)}{1155 d \sqrt{d x}}$	54
trager	$-\frac{2\left(-77 b^{4} x^{8}-420 a \,b^{3} x^{6}-990 a^{2} b^{2} x^{4}-1540 a^{3} b \,x^{2}+1155 a^{4}\right) \sqrt{d x}}{1155 d^{2} x}$	57
derivativedivides	$\frac{\frac{2 b^{4} \left(d x \right)^{\frac{15}{2}}}{15}+\frac{8 a \,b^{3} d^{2} \left(d x \right)^{\frac{11}{2}}}{11}+\frac{12 a^{2} b^{2} d^{4} \left(d x \right)^{\frac{7}{2}}}{7}+\frac{8 a^{3} b \,d^{6} \left(d x \right)^{\frac{3}{2}}}{3}-\frac{2 a^{4} d^{8}}{\sqrt{d x}}}{d^{9}}$	74
default	$\frac{\frac{2 b^{4} \left(d x \right)^{\frac{15}{2}}}{15}+\frac{8 a \,b^{3} d^{2} \left(d x \right)^{\frac{11}{2}}}{11}+\frac{12 a^{2} b^{2} d^{4} \left(d x \right)^{\frac{7}{2}}}{7}+\frac{8 a^{3} b \,d^{6} \left(d x \right)^{\frac{3}{2}}}{3}-\frac{2 a^{4} d^{8}}{\sqrt{d x}}}{d^{9}}$	74
orering	$-\frac{2\left(-77 b^{4} x^{8}-420 a \,b^{3} x^{6}-990 a^{2} b^{2} x^{4}-1540 a^{3} b \,x^{2}+1155 a^{4}\right) x \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{2}}{1155 \left(b \,x^{2}+a \right)^{4} \left(d x \right)^{\frac{3}{2}}}$	81

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1155*(-77*b^4*x^8-420*a*b^3*x^6-990*a^2*b^2*x^4-1540*a^3*b*x^2+1155*a^4
)*x/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx = \frac{2\,(77\,b^4x^8+420\,ab^3x^6+990\,a^2b^2x^4+1540\,a^3bx^2-1155\,a^4)\sqrt{dx}}{1155\,d^2x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(77*b^4*x^8 + 420*a*b^3*x^6 + 990*a^2*b^2*x^4 + 1540*a^3*b*x^2 - 11
55*a^4)*sqrt(d*x)/(d^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx = -\frac{2a^4x}{(dx)^{\frac{3}{2}}}+\frac{8a^3bx^3}{3\,(dx)^{\frac{3}{2}}}+\frac{12a^2b^2x^5}{7\,(dx)^{\frac{3}{2}}}+\frac{8ab^3x^7}{11\,(dx)^{\frac{3}{2}}}+\frac{2b^4x^9}{15\,(dx)^{\frac{3}{2}}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(3/2),x)
```

output
```
-2*a**4*x/(d*x)**(3/2) + 8*a**3*b*x**3/(3*(d*x)**(3/2)) + 12*a**2*b**2*x**
5/(7*(d*x)**(3/2)) + 8*a*b**3*x**7/(11*(d*x)**(3/2)) + 2*b**4*x**9/(15*(d*
x)**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{(dx)^{3/2}}\,dx = -\frac{2\left(\frac{1155\,a^4}{\sqrt{dx}} - \frac{77\,(dx)^{\frac{15}{2}}b^4 + 420\,(dx)^{\frac{11}{2}}ab^3d^2 + 990\,(dx)^{\frac{7}{2}}a^2b^2d^4 + 1540\,(dx)^{\frac{3}{2}}a^3bd^6}{d^8}\right)}{1155\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2/1155*(1155*a^4/sqrt(d*x) - (77*(d*x)^(15/2)*b^4 + 420*(d*x)^(11/2)*a*b^
3*d^2 + 990*(d*x)^(7/2)*a^2*b^2*d^4 + 1540*(d*x)^(3/2)*a^3*b*d^6)/d^8)/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{(dx)^{3/2}}\,dx = -\frac{2\left(\frac{1155\,a^4}{\sqrt{dx}} - \frac{77\,\sqrt{dx}b^4d^{119}x^7 + 420\,\sqrt{dx}ab^3d^{119}x^5 + 990\,\sqrt{dx}a^2b^2d^{119}x^3 + 1540\,\sqrt{dx}a^3bd^{119}x}{d^{120}}\right)}{1155\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x, algorithm="giac")
```

output
```
-2/1155*(1155*a^4/sqrt(d*x) - (77*sqrt(d*x)*b^4*d^119*x^7 + 420*sqrt(d*x)*
a*b^3*d^119*x^5 + 990*sqrt(d*x)*a^2*b^2*d^119*x^3 + 1540*sqrt(d*x)*a^3*b*d
^119*x)/d^120)/d
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx = \frac{2\,b^4\,(d\,x)^{15/2}}{15\,d^9} - \frac{2\,a^4}{d\,\sqrt{d\,x}} + \frac{12\,a^2\,b^2\,(d\,x)^{7/2}}{7\,d^5} + \frac{8\,a^3\,b\,(d\,x)^{3/2}}{3\,d^3} + \frac{8\,a\,b^3\,(d\,x)^{11/2}}{11\,d^7}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(3/2),x)
```

output
```
(2*b^4*(d*x)^(15/2))/(15*d^9) - (2*a^4)/(d*(d*x)^(1/2)) + (12*a^2*b^2*(d*x
)^(7/2))/(7*d^5) + (8*a^3*b*(d*x)^(3/2))/(3*d^3) + (8*a*b^3*(d*x)^(11/2))/
(11*d^7)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx = \frac{2\sqrt{d}\,(77b^4x^8+420a\,b^3x^6+990a^2b^2x^4+1540a^3b\,x^2-1155a^4)}{1155\sqrt{x}\,d^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x)
```

output
```
(2*sqrt(d)*( - 1155*a**4 + 1540*a**3*b*x**2 + 990*a**2*b**2*x**4 + 420*a*b
**3*x**6 + 77*b**4*x**8))/(1155*sqrt(x)*d**2)
```

3.437 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 89

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{5/2}}\,dx = -\frac{2a^4}{3d(dx)^{3/2}}+\frac{8a^3b\sqrt{dx}}{d^3} + \frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

output
```
-2/3*a^4/d/(d*x)^(3/2)+8*a^3*b*(d*x)^(1/2)/d^3+12/5*a^2*b^2*(d*x)^(5/2)/d^
5+8/9*a*b^3*(d*x)^(9/2)/d^7+2/13*b^4*(d*x)^(13/2)/d^9
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{5/2}}\,dx = -\frac{2x(195a^4-2340a^3bx^2-702a^2b^2x^4-260ab^3x^6-45b^4x^8)}{585(dx)^{5/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

output
```
(-2*x*(195*a^4 - 2340*a^3*b*x^2 - 702*a^2*b^2*x^4 - 260*a*b^3*x^6 - 45*b^4
*x^8))/(585*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^4(bx^2+a)^4}{(dx)^{5/2}}dx}{b^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^4}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^4}{(dx)^{5/2}}+\frac{4a^3b}{d^2\sqrt{dx}}+\frac{6a^2b^2(dx)^{3/2}}{d^4}+\frac{4ab^3(dx)^{7/2}}{d^6}+\frac{b^4(dx)^{11/2}}{d^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^4}{3d(dx)^{3/2}}+\frac{8a^3b\sqrt{dx}}{d^3}+\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

output
```
(-2*a^4)/(3*d*(d*x)^(3/2)) + (8*a^3*b*Sqrt[d*x])/d^3 + (12*a^2*b^2*(d*x)^(
5/2))/(5*d^5) + (8*a*b^3*(d*x)^(9/2))/(9*d^7) + (2*b^4*(d*x)^(13/2))/(13*d
^9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{2\left(-45 b^{4} x^{8}-260 a \,b^{3} x^{6}-702 a^{2} b^{2} x^{4}-2340 a^{3} b \,x^{2}+195 a^{4}\right) x}{585 \left(d x \right)^{\frac{5}{2}}}$	52
pseudoelliptic	$-\frac{2\left(-\frac{3}{13} b^{4} x^{8}-\frac{4}{3} a \,b^{3} x^{6}-\frac{18}{5} a^{2} b^{2} x^{4}-12 a^{3} b \,x^{2}+a^{4}\right)}{3 \sqrt{d x}\, d^{2} x}$	55
trager	$-\frac{2\left(-45 b^{4} x^{8}-260 a \,b^{3} x^{6}-702 a^{2} b^{2} x^{4}-2340 a^{3} b \,x^{2}+195 a^{4}\right) \sqrt{d x}}{585 d^{3} x^{2}}$	57
risch	$-\frac{2\left(-45 b^{4} x^{8}-260 a \,b^{3} x^{6}-702 a^{2} b^{2} x^{4}-2340 a^{3} b \,x^{2}+195 a^{4}\right)}{585 d^{2} x \sqrt{d x}}$	57
derivativedivides	$\frac{\frac{2 b^{4} \left(d x \right)^{\frac{13}{2}}}{13}+\frac{8 a \,b^{3} d^{2} \left(d x \right)^{\frac{9}{2}}}{9}+\frac{12 a^{2} b^{2} d^{4} \left(d x \right)^{\frac{5}{2}}}{5}+8 \sqrt{d x}\, a^{3} b \,d^{6}-\frac{2 a^{4} d^{8}}{3 \left(d x \right)^{\frac{3}{2}}}}{d^{9}}$	74
default	$\frac{\frac{2 b^{4} \left(d x \right)^{\frac{13}{2}}}{13}+\frac{8 a \,b^{3} d^{2} \left(d x \right)^{\frac{9}{2}}}{9}+\frac{12 a^{2} b^{2} d^{4} \left(d x \right)^{\frac{5}{2}}}{5}+8 \sqrt{d x}\, a^{3} b \,d^{6}-\frac{2 a^{4} d^{8}}{3 \left(d x \right)^{\frac{3}{2}}}}{d^{9}}$	74
orering	$-\frac{2\left(-45 b^{4} x^{8}-260 a \,b^{3} x^{6}-702 a^{2} b^{2} x^{4}-2340 a^{3} b \,x^{2}+195 a^{4}\right) x \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{2}}{585 \left(b \,x^{2}+a \right)^{4} \left(d x \right)^{\frac{5}{2}}}$	81

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/585*(-45*b^4*x^8-260*a*b^3*x^6-702*a^2*b^2*x^4-2340*a^3*b*x^2+195*a^4)*
x/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{(dx)^{5/2}}\,dx = \frac{2\,(45\,b^4x^8 + 260\,ab^3x^6 + 702\,a^2b^2x^4 + 2340\,a^3bx^2 - 195\,a^4)\sqrt{dx}}{585\,d^3x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/585*(45*b^4*x^8 + 260*a*b^3*x^6 + 702*a^2*b^2*x^4 + 2340*a^3*b*x^2 - 195
*a^4)*sqrt(d*x)/(d^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{(dx)^{5/2}}\,dx = -\frac{2a^4x}{3\,(dx)^{\frac{5}{2}}} + \frac{8a^3bx^3}{(dx)^{\frac{5}{2}}} + \frac{12a^2b^2x^5}{5\,(dx)^{\frac{5}{2}}} + \frac{8ab^3x^7}{9\,(dx)^{\frac{5}{2}}} + \frac{2b^4x^9}{13\,(dx)^{\frac{5}{2}}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(5/2),x)
```

output
```
-2*a**4*x/(3*(d*x)**(5/2)) + 8*a**3*b*x**3/(d*x)**(5/2) + 12*a**2*b**2*x**
5/(5*(d*x)**(5/2)) + 8*a*b**3*x**7/(9*(d*x)**(5/2)) + 2*b**4*x**9/(13*(d*x
)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{(dx)^{5/2}}\,dx = -\frac{2\left(\frac{195\,a^4}{(dx)^{\frac{3}{2}}} - \frac{45\,(dx)^{\frac{13}{2}}b^4 + 260\,(dx)^{\frac{9}{2}}ab^3d^2 + 702\,(dx)^{\frac{5}{2}}a^2b^2d^4 + 2340\,\sqrt{dx}a^3bd^6}{d^8}\right)}{585\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-2/585*(195*a^4/(d*x)^(3/2) - (45*(d*x)^(13/2)*b^4 + 260*(d*x)^(9/2)*a*b^3
*d^2 + 702*(d*x)^(5/2)*a^2*b^2*d^4 + 2340*sqrt(d*x)*a^3*b*d^6)/d^8)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{(dx)^{5/2}}\,dx = -\frac{2\left(\frac{195\,a^4}{\sqrt{dx}dx} - \frac{45\,\sqrt{dx}b^4d^{102}x^6 + 260\,\sqrt{dx}ab^3d^{102}x^4 + 702\,\sqrt{dx}a^2b^2d^{102}x^2 + 2340\,\sqrt{dx}a^3bd^{102}}{d^{104}}\right)}{585\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x, algorithm="giac")
```

output
```
-2/585*(195*a^4/(sqrt(d*x)*d*x) - (45*sqrt(d*x)*b^4*d^102*x^6 + 260*sqrt(d
*x)*a*b^3*d^102*x^4 + 702*sqrt(d*x)*a^2*b^2*d^102*x^2 + 2340*sqrt(d*x)*a^3
*b*d^102)/d^104)/d
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{5/2}}\,dx = \frac{2\,b^4\,(d\,x)^{13/2}}{13\,d^9} - \frac{2\,a^4}{3\,d\,(d\,x)^{3/2}} + \frac{12\,a^2\,b^2\,(d\,x)^{5/2}}{5\,d^5} + \frac{8\,a^3\,b\,\sqrt{d\,x}}{d^3} + \frac{8\,a\,b^3\,(d\,x)^{9/2}}{9\,d^7}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(5/2),x)
```

output
```
(2*b^4*(d*x)^(13/2))/(13*d^9) - (2*a^4)/(3*d*(d*x)^(3/2)) + (12*a^2*b^2*(d
*x)^(5/2))/(5*d^5) + (8*a^3*b*(d*x)^(1/2))/d^3 + (8*a*b^3*(d*x)^(9/2))/(9*
d^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{5/2}}\,dx = \frac{2\sqrt{d}\,(45b^4x^8+260a\,b^3x^6+702a^2b^2x^4+2340a^3b\,x^2-195a^4)}{585\sqrt{x}\,d^3x}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x)
```

output
```
(2*sqrt(d)*( - 195*a**4 + 2340*a**3*b*x**2 + 702*a**2*b**2*x**4 + 260*a*b*
*3*x**6 + 45*b**4*x**8))/(585*sqrt(x)*d**3*x)
```

3.438 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 87

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{7/2}}\,dx = -\frac{2a^4}{5d(dx)^{5/2}}-\frac{8a^3b}{d^3\sqrt{dx}}+\frac{4a^2b^2(dx)^{3/2}}{d^5}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

output
```
-2/5*a^4/d/(d*x)^(5/2)-8*a^3*b/d^3/(d*x)^(1/2)+4*a^2*b^2*(d*x)^(3/2)/d^5+8
/7*a*b^3*(d*x)^(7/2)/d^7+2/11*b^4*(d*x)^(11/2)/d^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.69

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{7/2}}\,dx = -\frac{2\sqrt{dx}(77a^4+1540a^3bx^2-770a^2b^2x^4-220ab^3x^6-35b^4x^8)}{385d^4x^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```

output
```
(-2*Sqrt[d*x]*(77*a^4 + 1540*a^3*b*x^2 - 770*a^2*b^2*x^4 - 220*a*b^3*x^6 -
 35*b^4*x^8))/(385*d^4*x^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{7/2}}dx$$
$$\Big\downarrow \text{ 1380}$$
$$\frac{\int \frac{b^4(bx^2+a)^4}{(dx)^{7/2}}dx}{b^4}$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{(a+bx^2)^4}{(dx)^{7/2}}dx$$
$$\Big\downarrow \text{ 244}$$
$$\int \left(\frac{a^4}{(dx)^{7/2}}+\frac{4a^3b}{d^2(dx)^{3/2}}+\frac{6a^2b^2\sqrt{dx}}{d^4}+\frac{4ab^3(dx)^{5/2}}{d^6}+\frac{b^4(dx)^{9/2}}{d^8}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{2a^4}{5d(dx)^{5/2}}-\frac{8a^3b}{d^3\sqrt{dx}}+\frac{4a^2b^2(dx)^{3/2}}{d^5}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```

output
```
(-2*a^4)/(5*d*(d*x)^(5/2)) - (8*a^3*b)/(d^3*Sqrt[d*x]) + (4*a^2*b^2*(d*x)^
(3/2))/d^5 + (8*a*b^3*(d*x)^(7/2))/(7*d^7) + (2*b^4*(d*x)^(11/2))/(11*d^9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2\left(-35 b^{4} x^{8}-220 a \,b^{3} x^{6}-770 a^{2} b^{2} x^{4}+1540 a^{3} b \,x^{2}+77 a^{4}\right) x}{385 \left(d x \right)^{\frac{7}{2}}}$	52
pseudoelliptic	$-\frac{2\left(-\frac{5}{11} b^{4} x^{8}-\frac{20}{7} a \,b^{3} x^{6}-10 a^{2} b^{2} x^{4}+20 a^{3} b \,x^{2}+a^{4}\right)}{5 \sqrt{d x}\, d^{3} x^{2}}$	55
trager	$-\frac{2\left(-35 b^{4} x^{8}-220 a \,b^{3} x^{6}-770 a^{2} b^{2} x^{4}+1540 a^{3} b \,x^{2}+77 a^{4}\right) \sqrt{d x}}{385 d^{4} x^{3}}$	57
risch	$-\frac{2\left(-35 b^{4} x^{8}-220 a \,b^{3} x^{6}-770 a^{2} b^{2} x^{4}+1540 a^{3} b \,x^{2}+77 a^{4}\right)}{385 d^{3} x^{2} \sqrt{d x}}$	57
derivativedivides	$\frac{\frac{2 b^{4} \left(d x \right)^{\frac{11}{2}}}{11}+\frac{8 a \,b^{3} d^{2} \left(d x \right)^{\frac{7}{2}}}{7}+4 a^{2} b^{2} d^{4} \left(d x \right)^{\frac{3}{2}}-\frac{8 a^{3} b \,d^{6}}{\sqrt{d x}}-\frac{2 a^{4} d^{8}}{5 \left(d x \right)^{\frac{5}{2}}}}{d^{9}}$	74
default	$\frac{\frac{2 b^{4} \left(d x \right)^{\frac{11}{2}}}{11}+\frac{8 a \,b^{3} d^{2} \left(d x \right)^{\frac{7}{2}}}{7}+4 a^{2} b^{2} d^{4} \left(d x \right)^{\frac{3}{2}}-\frac{8 a^{3} b \,d^{6}}{\sqrt{d x}}-\frac{2 a^{4} d^{8}}{5 \left(d x \right)^{\frac{5}{2}}}}{d^{9}}$	74
orering	$-\frac{2\left(-35 b^{4} x^{8}-220 a \,b^{3} x^{6}-770 a^{2} b^{2} x^{4}+1540 a^{3} b \,x^{2}+77 a^{4}\right) x \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{2}}{385 \left(b \,x^{2}+a \right)^{4} \left(d x \right)^{\frac{7}{2}}}$	81

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/385*(-35*b^4*x^8-220*a*b^3*x^6-770*a^2*b^2*x^4+1540*a^3*b*x^2+77*a^4)*x
/(d*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{7/2}}\,dx = \frac{2\,(35\,b^4x^8+220\,ab^3x^6+770\,a^2b^2x^4-1540\,a^3bx^2-77\,a^4)\sqrt{dx}}{385\,d^4x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
2/385*(35*b^4*x^8 + 220*a*b^3*x^6 + 770*a^2*b^2*x^4 - 1540*a^3*b*x^2 - 77*
a^4)*sqrt(d*x)/(d^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{7/2}}\,dx = -\frac{2a^4x}{5\,(dx)^{\frac{7}{2}}}-\frac{8a^3bx^3}{(dx)^{\frac{7}{2}}}+\frac{4a^2b^2x^5}{(dx)^{\frac{7}{2}}}+\frac{8ab^3x^7}{7\,(dx)^{\frac{7}{2}}}+\frac{2b^4x^9}{11\,(dx)^{\frac{7}{2}}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(7/2),x)
```

output
```
-2*a**4*x/(5*(d*x)**(7/2)) - 8*a**3*b*x**3/(d*x)**(7/2) + 4*a**2*b**2*x**5
/(d*x)**(7/2) + 8*a*b**3*x**7/(7*(d*x)**(7/2)) + 2*b**4*x**9/(11*(d*x)**(7
/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{(dx)^{7/2}} dx = -\frac{2\left(\frac{77(20a^3bd^2x^2+a^4d^2)}{(dx)^{\frac{5}{2}}d^2} - \frac{5\left(7(dx)^{\frac{11}{2}}b^4+44(dx)^{\frac{7}{2}}ab^3d^2+154(dx)^{\frac{3}{2}}a^2b^2d^4\right)}{d^8}\right)}{385d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
-2/385*(77*(20*a^3*b*d^2*x^2 + a^4*d^2)/((d*x)^(5/2)*d^2) - 5*(7*(d*x)^(11
/2)*b^4 + 44*(d*x)^(7/2)*a*b^3*d^2 + 154*(d*x)^(3/2)*a^2*b^2*d^4)/d^8)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{(dx)^{7/2}} dx = -\frac{2\left(\frac{77(20a^3bd^3x^2+a^4d^3)}{\sqrt{dx}d^2x^2} - \frac{5\left(7\sqrt{dx}b^4d^{55}x^5+44\sqrt{dx}ab^3d^{55}x^3+154\sqrt{dx}a^2b^2d^{55}x\right)}{d^{55}}\right)}{385d^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x, algorithm="giac")
```

output
```
-2/385*(77*(20*a^3*b*d^3*x^2 + a^4*d^3)/(sqrt(d*x)*d^2*x^2) - 5*(7*sqrt(d*
x)*b^4*d^55*x^5 + 44*sqrt(d*x)*a*b^3*d^55*x^3 + 154*sqrt(d*x)*a^2*b^2*d^55
*x)/d^55)/d^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{(dx)^{7/2}}\,dx = \frac{2\,b^4\,(d\,x)^{11/2}}{11\,d^9} - \frac{\frac{2\,a^4\,d^2}{5} + 8\,b\,a^3\,d^2\,x^2}{d^3\,(d\,x)^{5/2}} + \frac{4\,a^2\,b^2\,(d\,x)^{3/2}}{d^5} + \frac{8\,a\,b^3\,(d\,x)^{7/2}}{7\,d^7}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^2/(d*x)^(7/2),x)
```

output
```
(2*b^4*(d*x)^(11/2))/(11*d^9) - ((2*a^4*d^2)/5 + 8*a^3*b*d^2*x^2)/(d^3*(d*
x)^(5/2)) + (4*a^2*b^2*(d*x)^(3/2))/d^5 + (8*a*b^3*(d*x)^(7/2))/(7*d^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^2}{(dx)^{7/2}}\,dx = \frac{2\sqrt{d}\,(35b^4x^8 + 220a\,b^3x^6 + 770a^2b^2x^4 - 1540a^3b\,x^2 - 77a^4)}{385\sqrt{x}\,d^4x^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x)
```

output
```
(2*sqrt(d)*( - 77*a**4 - 1540*a**3*b*x**2 + 770*a**2*b**2*x**4 + 220*a*b**
3*x**6 + 35*b**4*x**8))/(385*sqrt(x)*d**4*x**2)
```

3.439 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 129

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{2a^6(dx)^{7/2}}{7d}+\frac{12a^5b(dx)^{11/2}}{11d^3} +\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{4ab^5(dx)^{27/2}}{9d^{11}}+\frac{2b^6(dx)^{31/2}}{31d^{13}}$$

output
```
2/7*a^6*(d*x)^(7/2)/d+12/11*a^5*b*(d*x)^(11/2)/d^3+2*a^4*b^2*(d*x)^(15/2)/
d^5+40/19*a^3*b^3*(d*x)^(19/2)/d^7+30/23*a^2*b^4*(d*x)^(23/2)/d^9+4/9*a*b^
5*(d*x)^(27/2)/d^11+2/31*b^6*(d*x)^(31/2)/d^13
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.60

$$\int (dx)^{5/2} \left(a^2+2abx^2 +b^2x^4\right)^3 \, dx = \frac{2x(dx)^{5/2}\left(1341153a^6+5120766a^5bx^2+9388071a^4b^2x^4+9882180a^3b^3x^6+6122655a^2b^4x^8 \right.}{9388071}$$

input
```
Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(2*x*(d*x)^(5/2)*(1341153*a^6 + 5120766*a^5*b*x^2 + 9388071*a^4*b^2*x^4 +
9882180*a^3*b^3*x^6 + 6122655*a^2*b^4*x^8 + 2086238*a*b^5*x^10 + 302841*b^
6*x^12))/9388071
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int b^6(dx)^{5/2}\left(bx^2+a\right)^6dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int (dx)^{5/2}\left(a+bx^2\right)^6dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^6(dx)^{5/2}+\frac{6a^5b(dx)^{9/2}}{d^2}+\frac{15a^4b^2(dx)^{13/2}}{d^4}+\frac{20a^3b^3(dx)^{17/2}}{d^6}+\frac{15a^2b^4(dx)^{21/2}}{d^8}+\frac{6ab^5(dx)^{25/2}}{d^{10}}+\frac{b^6(dx)^{29/}}{d^{12}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^6(dx)^{7/2}}{7d}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{4ab^5(dx)^{27/2}}{9d^{11}}+\frac{2b^6(dx)^{31/2}}{31d^{13}}$$

input
```
Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(2*a^6*(d*x)^(7/2))/(7*d) + (12*a^5*b*(d*x)^(11/2))/(11*d^3) + (2*a^4*b^2*
(d*x)^(15/2))/d^5 + (40*a^3*b^3*(d*x)^(19/2))/(19*d^7) + (30*a^2*b^4*(d*x)
^(23/2))/(23*d^9) + (4*a*b^5*(d*x)^(27/2))/(9*d^11) + (2*b^6*(d*x)^(31/2))
/(31*d^13)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.57

method	result
gosper	$\frac{2x(302841b^6x^{12}+2086238a\,b^5x^{10}+6122655a^2b^4x^8+9882180a^3b^3x^6+9388071a^4b^2x^4+5120766a^5b\,x^2+1341153a^6)(d}{9388071}$
pseudoelliptic	$\frac{2\sqrt{dx}\,d^2x^3\left(\frac{7}{31}b^6x^{12}+\frac{14}{9}a\,b^5x^{10}+\frac{105}{23}a^2b^4x^8+\frac{140}{19}a^3b^3x^6+7a^4b^2x^4+\frac{42}{11}a^5b\,x^2+a^6\right)}{7}$
trager	$\frac{2d^2x^3(302841b^6x^{12}+2086238a\,b^5x^{10}+6122655a^2b^4x^8+9882180a^3b^3x^6+9388071a^4b^2x^4+5120766a^5b\,x^2+1341153a^6}{9388071}$
risch	$\frac{2d^3x^4(302841b^6x^{12}+2086238a\,b^5x^{10}+6122655a^2b^4x^8+9882180a^3b^3x^6+9388071a^4b^2x^4+5120766a^5b\,x^2+1341153a^6}{9388071\sqrt{dx}}$
orering	$\frac{2x(302841b^6x^{12}+2086238a\,b^5x^{10}+6122655a^2b^4x^8+9882180a^3b^3x^6+9388071a^4b^2x^4+5120766a^5b\,x^2+1341153a^6)(d}{9388071(b\,x^2+a)^6}$
derivativedivides	$\frac{\frac{2b^6(dx)^{\frac{31}{2}}}{31}+\frac{4a\,b^5d^2(dx)^{\frac{27}{2}}}{9}+\frac{30a^2d^4b^4(dx)^{\frac{23}{2}}}{23}+\frac{40a^3d^6b^3(dx)^{\frac{19}{2}}}{19}+2a^4d^8b^2(dx)^{\frac{15}{2}}+\frac{12a^5d^{10}b(dx)^{\frac{11}{2}}}{11}+\frac{2a^6d^{12}(dx)^{\frac{7}{2}}}{7}}{d^{13}}$
default	$\frac{\frac{2b^6(dx)^{\frac{31}{2}}}{31}+\frac{4a\,b^5d^2(dx)^{\frac{27}{2}}}{9}+\frac{30a^2d^4b^4(dx)^{\frac{23}{2}}}{23}+\frac{40a^3d^6b^3(dx)^{\frac{19}{2}}}{19}+2a^4d^8b^2(dx)^{\frac{15}{2}}+\frac{12a^5d^{10}b(dx)^{\frac{11}{2}}}{11}+\frac{2a^6d^{12}(dx)^{\frac{7}{2}}}{7}}{d^{13}}$

input
```
int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/9388071*x*(302841*b^6*x^12+2086238*a*b^5*x^10+6122655*a^2*b^4*x^8+988218
0*a^3*b^3*x^6+9388071*a^4*b^2*x^4+5120766*a^5*b*x^2+1341153*a^6)*(d*x)^(5/
2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.74

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{2}{9388071}\left(302841\,b^6d^2x^{15}+2086238\,ab^5d^2x^{13}+6122655\,a^2b^4d^2x^{11}+9882180\,a^3b^3d^2x^9+938\right.$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
2/9388071*(302841*b^6*d^2*x^15 + 2086238*a*b^5*d^2*x^13 + 6122655*a^2*b^4*
d^2*x^11 + 9882180*a^3*b^3*d^2*x^9 + 9388071*a^4*b^2*d^2*x^7 + 5120766*a^5
*b*d^2*x^5 + 1341153*a^6*d^2*x^3)*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{2a^6x(dx)^{\frac{5}{2}}}{7} + \frac{12a^5bx^3(dx)^{\frac{5}{2}}}{11} + 2a^4b^2x^5(dx)^{\frac{5}{2}} + \frac{40a^3b^3x^7(dx)^{\frac{5}{2}}}{19} + \frac{30a^2b^4x^9(dx)^{\frac{5}{2}}}{23} + \frac{4ab^5x^{11}(dx)^{\frac{5}{2}}}{9} + \frac{2b^6x^{13}(dx)^{\frac{5}{2}}}{31}$$

input
```
integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
2*a**6*x*(d*x)**(5/2)/7 + 12*a**5*b*x**3*(d*x)**(5/2)/11 + 2*a**4*b**2*x**
5*(d*x)**(5/2) + 40*a**3*b**3*x**7*(d*x)**(5/2)/19 + 30*a**2*b**4*x**9*(d*
x)**(5/2)/23 + 4*a*b**5*x**11*(d*x)**(5/2)/9 + 2*b**6*x**13*(d*x)**(5/2)/3
1
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{2\left(302841\,(dx)^{\frac{31}{2}}\,b^6+2086238\,(dx)^{\frac{27}{2}}\,ab^5d^2+6122655\,(dx)^{\frac{23}{2}}\,a^2b^4d^4+9882180\,(dx)^{\frac{19}{2}}\,a^3b^3d^{6}\right.}{9388071\,d^{13}}$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
2/9388071*(302841*(d*x)^(31/2)*b^6 + 2086238*(d*x)^(27/2)*a*b^5*d^2 + 6122
655*(d*x)^(23/2)*a^2*b^4*d^4 + 9882180*(d*x)^(19/2)*a^3*b^3*d^6 + 9388071*
(d*x)^(15/2)*a^4*b^2*d^8 + 5120766*(d*x)^(11/2)*a^5*b*d^10 + 1341153*(d*x)
^(7/2)*a^6*d^12)/d^13
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{2}{31}\sqrt{dx}b^6d^2x^{15}+\frac{4}{9}\sqrt{dx}ab^5d^2x^{13}+\frac{30}{23}\sqrt{dx}a^2b^4d^2x^{11}+\frac{40}{19}\sqrt{dx}a^3b^3d^2x^9+2\sqrt{dx}a^4b^2d^2x^7+\frac{12}{11}\sqrt{dx}a^5bd^2x^5+\frac{2}{7}\sqrt{dx}a^6d^2x^3$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
2/31*sqrt(d*x)*b^6*d^2*x^15 + 4/9*sqrt(d*x)*a*b^5*d^2*x^13 + 30/23*sqrt(d*
x)*a^2*b^4*d^2*x^11 + 40/19*sqrt(d*x)*a^3*b^3*d^2*x^9 + 2*sqrt(d*x)*a^4*b^
2*d^2*x^7 + 12/11*sqrt(d*x)*a^5*b*d^2*x^5 + 2/7*sqrt(d*x)*a^6*d^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{2\,a^6\,(d\,x)^{7/2}}{7\,d}+\frac{2\,b^6\,(d\,x)^{31/2}}{31\,d^{13}}+\frac{2\,a^4\,b^2\,(d\,x)^{15/2}}{d^5}+\frac{40\,a^3\,b^3\,(d\,x)^{19/2}}{19\,d^7}+\frac{30\,a^2\,b^4\,(d\,x)^{23/2}}{23\,d^9}+\frac{12\,a^5\,b\,(d\,x)^{11/2}}{11\,d^3}+\frac{4\,a\,b^5\,(d\,x)^{27/2}}{9\,d^{11}}$$

input
```
int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(2*a^6*(d*x)^(7/2))/(7*d) + (2*b^6*(d*x)^(31/2))/(31*d^13) + (2*a^4*b^2*(d
*x)^(15/2))/d^5 + (40*a^3*b^3*(d*x)^(19/2))/(19*d^7) + (30*a^2*b^4*(d*x)^(
23/2))/(23*d^9) + (12*a^5*b*(d*x)^(11/2))/(11*d^3) + (4*a*b^5*(d*x)^(27/2)
)/(9*d^11)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.60

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{2\sqrt{x}\,\sqrt{d}\,d^2x^3(302841b^6x^{12}+2086238a\,b^5x^{10}+6122655a^2b^4x^8+9882180a^3b^3x^6+9388071a^{4}\ldots}{9388071}$$

input
```
int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(2*sqrt(x)*sqrt(d)*d**2*x**3*(1341153*a**6 + 5120766*a**5*b*x**2 + 9388071
*a**4*b**2*x**4 + 9882180*a**3*b**3*x**6 + 6122655*a**2*b**4*x**8 + 208623
8*a*b**5*x**10 + 302841*b**6*x**12))/9388071
```

3.440 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 131

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{2a^6(dx)^{5/2}}{5d}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{10a^2b^4(dx)^{21/2}}{7d^9}+\frac{12ab^5(dx)^{25/2}}{25d^{11}}+\frac{2b^6(dx)^{29/2}}{29d^{13}}$$

output
```
2/5*a^6*(d*x)^(5/2)/d+4/3*a^5*b*(d*x)^(9/2)/d^3+30/13*a^4*b^2*(d*x)^(13/2)
/d^5+40/17*a^3*b^3*(d*x)^(17/2)/d^7+10/7*a^2*b^4*(d*x)^(21/2)/d^9+12/25*a*
b^5*(d*x)^(25/2)/d^11+2/29*b^6*(d*x)^(29/2)/d^13
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.59

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{2x(dx)^{3/2}\left(672945a^6+2243150a^5bx^2+3882375a^4b^2x^4+3958500a^3b^3x^6+2403375a^2b^4x^8+\right.}{3364725}$$

input
```
Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(2*x*(d*x)^(3/2)*(672945*a^6 + 2243150*a^5*b*x^2 + 3882375*a^4*b^2*x^4 + 3
958500*a^3*b^3*x^6 + 2403375*a^2*b^4*x^8 + 807534*a*b^5*x^10 + 116025*b^6*
x^12))/3364725
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$$

$$\Big\downarrow \textbf{1380}$$

$$\frac{\int b^6 (dx)^{3/2} \left(bx^2+a\right)^6 dx}{b^6}$$

$$\Big\downarrow \textbf{27}$$

$$\int (dx)^{3/2} \left(a+bx^2\right)^6 dx$$

$$\Big\downarrow \textbf{244}$$

$$\int \left(a^6 (dx)^{3/2}+\frac{6a^5b(dx)^{7/2}}{d^2}+\frac{15a^4b^2(dx)^{11/2}}{d^4}+\frac{20a^3b^3(dx)^{15/2}}{d^6}+\frac{15a^2b^4(dx)^{19/2}}{d^8}+\frac{6ab^5(dx)^{23/2}}{d^{10}}+\frac{b^6(dx)^{27/}}{d^{12}}\right.$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{2a^6(dx)^{5/2}}{5d}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{10a^2b^4(dx)^{21/2}}{7d^9}+\frac{12ab^5(dx)^{25/2}}{25d^{11}}+\frac{2b^6(dx)^{29/2}}{29d^{13}}$$

input
```
Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(2*a^6*(d*x)^(5/2))/(5*d) + (4*a^5*b*(d*x)^(9/2))/(3*d^3) + (30*a^4*b^2*(d
*x)^(13/2))/(13*d^5) + (40*a^3*b^3*(d*x)^(17/2))/(17*d^7) + (10*a^2*b^4*(d
*x)^(21/2))/(7*d^9) + (12*a*b^5*(d*x)^(25/2))/(25*d^11) + (2*b^6*(d*x)^(29
/2))/(29*d^13)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

method	result
gosper	$\frac{2x(116025b^6x^{12}+807534a\,b^5x^{10}+2403375a^2b^4x^8+3958500a^3b^3x^6+3882375a^4b^2x^4+2243150a^5b\,x^2+672945a^6)(dx)}{3364725}$
pseudoelliptic	$\frac{2\sqrt{dx}\,d(\frac{5}{29}b^6x^{12}+\frac{6}{5}a\,b^5x^{10}+\frac{25}{7}a^2b^4x^8+\frac{100}{17}a^3b^3x^6+\frac{75}{13}a^4b^2x^4+\frac{10}{3}a^5b\,x^2+a^6)x^2}{5}$
trager	$\frac{2d\,x^2(116025b^6x^{12}+807534a\,b^5x^{10}+2403375a^2b^4x^8+3958500a^3b^3x^6+3882375a^4b^2x^4+2243150a^5b\,x^2+672945a^6)\sqrt{}}{3364725}$
risch	$\frac{2d^2x^3(116025b^6x^{12}+807534a\,b^5x^{10}+2403375a^2b^4x^8+3958500a^3b^3x^6+3882375a^4b^2x^4+2243150a^5b\,x^2+672945a^6)}{3364725\sqrt{dx}}$
orering	$\frac{2x(116025b^6x^{12}+807534a\,b^5x^{10}+2403375a^2b^4x^8+3958500a^3b^3x^6+3882375a^4b^2x^4+2243150a^5b\,x^2+672945a^6)(dx)}{3364725(b\,x^2+a)^6}$
derivativedivides	$\frac{\frac{2b^6(dx)^{\frac{29}{2}}}{29}+\frac{12a\,b^5d^2(dx)^{\frac{25}{2}}}{25}+\frac{10a^2d^4b^4(dx)^{\frac{21}{2}}}{7}+\frac{40a^3d^6b^3(dx)^{\frac{17}{2}}}{17}+\frac{30a^4d^8b^2(dx)^{\frac{13}{2}}}{13}+\frac{4a^5d^{10}b(dx)^{\frac{9}{2}}}{3}+\frac{2a^6d^{12}(dx)^{\frac{5}{2}}}{5}}{d^{13}}$
default	$\frac{\frac{2b^6(dx)^{\frac{29}{2}}}{29}+\frac{12a\,b^5d^2(dx)^{\frac{25}{2}}}{25}+\frac{10a^2d^4b^4(dx)^{\frac{21}{2}}}{7}+\frac{40a^3d^6b^3(dx)^{\frac{17}{2}}}{17}+\frac{30a^4d^8b^2(dx)^{\frac{13}{2}}}{13}+\frac{4a^5d^{10}b(dx)^{\frac{9}{2}}}{3}+\frac{2a^6d^{12}(dx)^{\frac{5}{2}}}{5}}{d^{13}}$

input
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3364725*x*(116025*b^6*x^12+807534*a*b^5*x^10+2403375*a^2*b^4*x^8+3958500
*a^3*b^3*x^6+3882375*a^4*b^2*x^4+2243150*a^5*b*x^2+672945*a^6)*(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{2}{3364725}\left(116025\,b^6dx^{14}+807534\,ab^5dx^{12}+2403375\,a^2b^4dx^{10}+3958500\,a^3b^3dx^8+3882375\right.$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
2/3364725*(116025*b^6*d*x^14 + 807534*a*b^5*d*x^12 + 2403375*a^2*b^4*d*x^1
0 + 3958500*a^3*b^3*d*x^8 + 3882375*a^4*b^2*d*x^6 + 2243150*a^5*b*d*x^4 +
672945*a^6*d*x^2)*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{2a^6x(dx)^{\frac{3}{2}}}{5}+\frac{4a^5bx^3(dx)^{\frac{3}{2}}}{3}+\frac{30a^4b^2x^5(dx)^{\frac{3}{2}}}{13}+\frac{40a^3b^3x^7(dx)^{\frac{3}{2}}}{17}+\frac{10a^2b^4x^9(dx)^{\frac{3}{2}}}{7}+\frac{12ab^5x^{11}(dx)^{\frac{3}{2}}}{25}+\frac{2b^6x^{13}(dx)^{\frac{3}{2}}}{29}$$

input
```
integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
2*a**6*x*(d*x)**(3/2)/5 + 4*a**5*b*x**3*(d*x)**(3/2)/3 + 30*a**4*b**2*x**5
*(d*x)**(3/2)/13 + 40*a**3*b**3*x**7*(d*x)**(3/2)/17 + 10*a**2*b**4*x**9*(
d*x)**(3/2)/7 + 12*a*b**5*x**11*(d*x)**(3/2)/25 + 2*b**6*x**13*(d*x)**(3/2
)/29
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.80

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{2\left(116025\,(dx)^{\frac{29}{2}}b^6+807534\,(dx)^{\frac{25}{2}}ab^5d^2+2403375\,(dx)^{\frac{21}{2}}a^2b^4d^4+3958500\,(dx)^{\frac{17}{2}}a^3b^3d^6\right.}{3364725\,d^{13}}$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
2/3364725*(116025*(d*x)^(29/2)*b^6 + 807534*(d*x)^(25/2)*a*b^5*d^2 + 24033
75*(d*x)^(21/2)*a^2*b^4*d^4 + 3958500*(d*x)^(17/2)*a^3*b^3*d^6 + 3882375*(
d*x)^(13/2)*a^4*b^2*d^8 + 2243150*(d*x)^(9/2)*a^5*b*d^10 + 672945*(d*x)^(5
/2)*a^6*d^12)/d^13
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^3 \, dx = \frac{2}{3364725} \left(116025 \sqrt{dx} b^6 x^{14} + 807534 \sqrt{dx} a b^5 x^{12} + 2403375 \sqrt{dx} a^2 b^4 x^{10} + 3958500 \sqrt{dx} a^3 b^3\right.$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
2/3364725*(116025*sqrt(d*x)*b^6*x^14 + 807534*sqrt(d*x)*a*b^5*x^12 + 24033
75*sqrt(d*x)*a^2*b^4*x^10 + 3958500*sqrt(d*x)*a^3*b^3*x^8 + 3882375*sqrt(d
*x)*a^4*b^2*x^6 + 2243150*sqrt(d*x)*a^5*b*x^4 + 672945*sqrt(d*x)*a^6*x^2)*
d
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^3 \, dx = \frac{2\,a^6\,(d\,x)^{5/2}}{5\,d} + \frac{2\,b^6\,(d\,x)^{29/2}}{29\,d^{13}} + \frac{30\,a^4\,b^2\,(d\,x)^{13/2}}{13\,d^5} + \frac{40\,a^3\,b^3\,(d\,x)^{17/2}}{17\,d^7} + \frac{10\,a^2\,b^4\,(d\,x)^{21/2}}{7\,d^9} + \frac{4\,a^5\,b\,(d\,x)^{9/2}}{3\,d^3} + \frac{12\,a\,b^5\,(d\,x)^{25/2}}{25\,d^{11}}$$

input
```
int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(2*a^6*(d*x)^(5/2))/(5*d) + (2*b^6*(d*x)^(29/2))/(29*d^13) + (30*a^4*b^2*(
d*x)^(13/2))/(13*d^5) + (40*a^3*b^3*(d*x)^(17/2))/(17*d^7) + (10*a^2*b^4*(
d*x)^(21/2))/(7*d^9) + (4*a^5*b*(d*x)^(9/2))/(3*d^3) + (12*a*b^5*(d*x)^(25
/2))/(25*d^11)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.57

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{2\sqrt{x}\,\sqrt{d}\,d\,x^2(116025b^6x^{12}+807534a\,b^5x^{10}+2403375a^2b^4x^8+3958500a^3b^3x^6+3882375a^4b}{3364725}$$

input
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(2*sqrt(x)*sqrt(d)*d*x**2*(672945*a**6 + 2243150*a**5*b*x**2 + 3882375*a**
4*b**2*x**4 + 3958500*a**3*b**3*x**6 + 2403375*a**2*b**4*x**8 + 807534*a*b
**5*x**10 + 116025*b**6*x**12))/3364725
```

3.441 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 131

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{2a^6(dx)^{3/2}}{3d}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{12ab^5(dx)^{23/2}}{23d^{11}}+\frac{2b^6(dx)^{27/2}}{27d^{13}}$$

output
```
2/3*a^6*(d*x)^(3/2)/d+12/7*a^5*b*(d*x)^(7/2)/d^3+30/11*a^4*b^2*(d*x)^(11/2
)/d^5+8/3*a^3*b^3*(d*x)^(15/2)/d^7+30/19*a^2*b^4*(d*x)^(19/2)/d^9+12/23*a*
b^5*(d*x)^(23/2)/d^11+2/27*b^6*(d*x)^(27/2)/d^13
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.59

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$
$$=\frac{2x\sqrt{dx}(302841a^6+778734a^5bx^2+1238895a^4b^2x^4+1211364a^3b^3x^6+717255a^2b^4x^8+237006ab^5x^{10}+}{908523}$$

input
```
Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(2*x*Sqrt[d*x]*(302841*a^6 + 778734*a^5*b*x^2 + 1238895*a^4*b^2*x^4 + 1211
364*a^3*b^3*x^6 + 717255*a^2*b^4*x^8 + 237006*a*b^5*x^10 + 33649*b^6*x^12)
)/908523
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3 dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int b^6\sqrt{dx}\left(bx^2+a\right)^6 dx}{b^6}$$

$$\Big\downarrow \text{ 27}$$

$$\int \sqrt{dx}\left(a+bx^2\right)^6 dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^6\sqrt{dx}+\frac{6a^5b(dx)^{5/2}}{d^2}+\frac{15a^4b^2(dx)^{9/2}}{d^4}+\frac{20a^3b^3(dx)^{13/2}}{d^6}+\frac{15a^2b^4(dx)^{17/2}}{d^8}+\frac{6ab^5(dx)^{21/2}}{d^{10}}+\frac{b^6(dx)^{25/2}}{d^{12}}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^6(dx)^{3/2}}{3d}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{12ab^5(dx)^{23/2}}{23d^{11}}+\frac{2b^6(dx)^{27/2}}{27d^{13}}$$

input
```
Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(2*a^6*(d*x)^(3/2))/(3*d) + (12*a^5*b*(d*x)^(7/2))/(7*d^3) + (30*a^4*b^2*(
d*x)^(11/2))/(11*d^5) + (8*a^3*b^3*(d*x)^(15/2))/(3*d^7) + (30*a^2*b^4*(d*
x)^(19/2))/(19*d^9) + (12*a*b^5*(d*x)^(23/2))/(23*d^11) + (2*b^6*(d*x)^(27
/2))/(27*d^13)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$\frac{2\sqrt{dx}\,\left(\frac{1}{9}b^{6}x^{12}+\frac{18}{23}a\,b^{5}x^{10}+\frac{45}{19}a^{2}b^{4}x^{8}+4a^{3}b^{3}x^{6}+\frac{45}{11}a^{4}b^{2}x^{4}+\frac{18}{7}a^{5}b\,x^{2}+a^{6}\right)x}{3}$
gosper	$\frac{2x\left(33649b^{6}x^{12}+237006a\,b^{5}x^{10}+717255a^{2}b^{4}x^{8}+1211364a^{3}b^{3}x^{6}+1238895a^{4}b^{2}x^{4}+778734a^{5}b\,x^{2}+302841a^{6}\right)\sqrt{dx}}{908523}$
trager	$\frac{2x\left(33649b^{6}x^{12}+237006a\,b^{5}x^{10}+717255a^{2}b^{4}x^{8}+1211364a^{3}b^{3}x^{6}+1238895a^{4}b^{2}x^{4}+778734a^{5}b\,x^{2}+302841a^{6}\right)\sqrt{dx}}{908523}$
risch	$\frac{2d\,x^{2}\left(33649b^{6}x^{12}+237006a\,b^{5}x^{10}+717255a^{2}b^{4}x^{8}+1211364a^{3}b^{3}x^{6}+1238895a^{4}b^{2}x^{4}+778734a^{5}b\,x^{2}+302841a^{6}\right)}{908523\sqrt{dx}}$
orering	$\frac{2x\left(33649b^{6}x^{12}+237006a\,b^{5}x^{10}+717255a^{2}b^{4}x^{8}+1211364a^{3}b^{3}x^{6}+1238895a^{4}b^{2}x^{4}+778734a^{5}b\,x^{2}+302841a^{6}\right)\sqrt{dx}\,(b^{2}}{908523(b\,x^{2}+a)^{6}}$
derivativedivides	$\frac{\frac{2b^{6}(dx)^{\frac{27}{2}}}{27}+\frac{12a\,b^{5}d^{2}(dx)^{\frac{23}{2}}}{23}+\frac{30a^{2}d^{4}b^{4}(dx)^{\frac{19}{2}}}{19}+\frac{8a^{3}d^{6}b^{3}(dx)^{\frac{15}{2}}}{3}+\frac{30a^{4}d^{8}b^{2}(dx)^{\frac{11}{2}}}{11}+\frac{12a^{5}d^{10}b(dx)^{\frac{7}{2}}}{7}+\frac{2a^{6}d^{12}(dx)^{\frac{3}{2}}}{3}}{d^{13}}$
default	$\frac{\frac{2b^{6}(dx)^{\frac{27}{2}}}{27}+\frac{12a\,b^{5}d^{2}(dx)^{\frac{23}{2}}}{23}+\frac{30a^{2}d^{4}b^{4}(dx)^{\frac{19}{2}}}{19}+\frac{8a^{3}d^{6}b^{3}(dx)^{\frac{15}{2}}}{3}+\frac{30a^{4}d^{8}b^{2}(dx)^{\frac{11}{2}}}{11}+\frac{12a^{5}d^{10}b(dx)^{\frac{7}{2}}}{7}+\frac{2a^{6}d^{12}(dx)^{\frac{3}{2}}}{3}}{d^{13}}$

input
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x)^(1/2)*(1/9*b^6*x^12+18/23*a*b^5*x^10+45/19*a^2*b^4*x^8+4*a^3*b^3
*x^6+45/11*a^4*b^2*x^4+18/7*a^5*b*x^2+a^6)*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.56

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$
$$=\frac{2}{908523}\left(33649\,b^6x^{13}+237006\,ab^5x^{11}+717255\,a^2b^4x^9+1211364\,a^3b^3x^7+1238895\,a^4b^2x^5+778734\,a^5b\right.$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
2/908523*(33649*b^6*x^13 + 237006*a*b^5*x^11 + 717255*a^2*b^4*x^9 + 121136
4*a^3*b^3*x^7 + 1238895*a^4*b^2*x^5 + 778734*a^5*b*x^3 + 302841*a^6*x)*sqr
t(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{2a^6x\sqrt{dx}}{3}+\frac{12a^5bx^3\sqrt{dx}}{7}+\frac{30a^4b^2x^5\sqrt{dx}}{11}+\frac{8a^3b^3x^7\sqrt{dx}}{3}+\frac{30a^2b^4x^9\sqrt{dx}}{19}+\frac{12ab^5x^{11}\sqrt{dx}}{23}+\frac{2b^6x^{13}\sqrt{dx}}{27}$$

input
```
integrate((d*x)**(1/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
2*a**6*x*sqrt(d*x)/3 + 12*a**5*b*x**3*sqrt(d*x)/7 + 30*a**4*b**2*x**5*sqrt
(d*x)/11 + 8*a**3*b**3*x**7*sqrt(d*x)/3 + 30*a**2*b**4*x**9*sqrt(d*x)/19 +
 12*a*b**5*x**11*sqrt(d*x)/23 + 2*b**6*x**13*sqrt(d*x)/27
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.80

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$
$$=\frac{2\left(33649\,(dx)^{\frac{27}{2}}\,b^6+237006\,(dx)^{\frac{23}{2}}\,ab^5d^2+717255\,(dx)^{\frac{19}{2}}\,a^2b^4d^4+1211364\,(dx)^{\frac{15}{2}}\,a^3b^3d^6+1238895\,(dx\right.}{908523\,d^{13}}$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
2/908523*(33649*(d*x)^(27/2)*b^6 + 237006*(d*x)^(23/2)*a*b^5*d^2 + 717255*
(d*x)^(19/2)*a^2*b^4*d^4 + 1211364*(d*x)^(15/2)*a^3*b^3*d^6 + 1238895*(d*x
)^(11/2)*a^4*b^2*d^8 + 778734*(d*x)^(7/2)*a^5*b*d^10 + 302841*(d*x)^(3/2)*
a^6*d^12)/d^13
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.77

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{2}{27}\sqrt{dx}b^6x^{13}+\frac{12}{23}\sqrt{dx}ab^5x^{11}+\frac{30}{19}\sqrt{dx}a^2b^4x^9+\frac{8}{3}\sqrt{dx}a^3b^3x^7+\frac{30}{11}\sqrt{dx}a^4b^2x^5+\frac{12}{7}\sqrt{dx}a^5bx^3+\frac{2}{3}\sqrt{dx}a^6x$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
2/27*sqrt(d*x)*b^6*x^13 + 12/23*sqrt(d*x)*a*b^5*x^11 + 30/19*sqrt(d*x)*a^2
*b^4*x^9 + 8/3*sqrt(d*x)*a^3*b^3*x^7 + 30/11*sqrt(d*x)*a^4*b^2*x^5 + 12/7*
sqrt(d*x)*a^5*b*x^3 + 2/3*sqrt(d*x)*a^6*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx=\frac{2\,a^6\,(d\,x)^{3/2}}{3\,d}+\frac{2\,b^6\,(d\,x)^{27/2}}{27\,d^{13}}+\frac{30\,a^4\,b^2\,(d\,x)^{11/2}}{11\,d^5}+\frac{8\,a^3\,b^3\,(d\,x)^{15/2}}{3\,d^7}+\frac{30\,a^2\,b^4\,(d\,x)^{19/2}}{19\,d^9}+\frac{12\,a^5\,b\,(d\,x)^{7/2}}{7\,d^3}+\frac{12\,a\,b^5\,(d\,x)^{23/2}}{23\,d^{11}}$$

input
```
int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(2*a^6*(d*x)^(3/2))/(3*d) + (2*b^6*(d*x)^(27/2))/(27*d^13) + (30*a^4*b^2*(
d*x)^(11/2))/(11*d^5) + (8*a^3*b^3*(d*x)^(15/2))/(3*d^7) + (30*a^2*b^4*(d*
x)^(19/2))/(19*d^9) + (12*a^5*b*(d*x)^(7/2))/(7*d^3) + (12*a*b^5*(d*x)^(23
/2))/(23*d^11)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.55

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3\,dx$$
$$=\frac{2\sqrt{x}\,\sqrt{d}\,x(33649b^6x^{12}+237006a\,b^5x^{10}+717255a^2b^4x^8+1211364a^3b^3x^6+1238895a^4b^2x^4+778734a^5b}{908523}$$

input
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(2*sqrt(x)*sqrt(d)*x*(302841*a**6 + 778734*a**5*b*x**2 + 1238895*a**4*b**2
*x**4 + 1211364*a**3*b**3*x**6 + 717255*a**2*b**4*x**8 + 237006*a*b**5*x**
10 + 33649*b**6*x**12))/908523
```

3.442 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 129

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{\sqrt{dx}}\,dx = \frac{2a^6\sqrt{dx}}{d}+\frac{12a^5b(dx)^{5/2}}{5d^3}+\frac{10a^4b^2(dx)^{9/2}}{3d^5}+\frac{40a^3b^3(dx)^{13/2}}{13d^7}+\frac{30a^2b^4(dx)^{17/2}}{17d^9}+\frac{4ab^5(dx)^{21/2}}{7d^{11}}+\frac{2b^6(dx)^{25/2}}{25d^{13}}$$

output
```
2*a^6*(d*x)^(1/2)/d+12/5*a^5*b*(d*x)^(5/2)/d^3+10/3*a^4*b^2*(d*x)^(9/2)/d^
5+40/13*a^3*b^3*(d*x)^(13/2)/d^7+30/17*a^2*b^4*(d*x)^(17/2)/d^9+4/7*a*b^5*
(d*x)^(21/2)/d^11+2/25*b^6*(d*x)^(25/2)/d^13
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.60

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{\sqrt{dx}}\,dx = \frac{2x(116025a^6+139230a^5bx^2+193375a^4b^2x^4+178500a^3b^3x^6+102375a^2b^4x^8+33150ab^5x^{10}+4641b^6x}{116025\sqrt{dx}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```

output
```
(2*x*(116025*a^6 + 139230*a^5*b*x^2 + 193375*a^4*b^2*x^4 + 178500*a^3*b^3*
x^6 + 102375*a^2*b^4*x^8 + 33150*a*b^5*x^10 + 4641*b^6*x^12))/(116025*Sqrt
[d*x])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{\sqrt{dx}}dx$$
$$\Big\downarrow \text{1380}$$
$$\frac{\int \frac{b^6(bx^2+a)^6}{\sqrt{dx}}dx}{b^6}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{(a+bx^2)^6}{\sqrt{dx}}dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(\frac{a^6}{\sqrt{dx}}+\frac{6a^5b(dx)^{3/2}}{d^2}+\frac{15a^4b^2(dx)^{7/2}}{d^4}+\frac{20a^3b^3(dx)^{11/2}}{d^6}+\frac{15a^2b^4(dx)^{15/2}}{d^8}+\frac{6ab^5(dx)^{19/2}}{d^{10}}+\frac{b^6(dx)^{23/2}}{d^{12}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2a^6\sqrt{dx}}{d}+\frac{12a^5b(dx)^{5/2}}{5d^3}+\frac{10a^4b^2(dx)^{9/2}}{3d^5}+\frac{40a^3b^3(dx)^{13/2}}{13d^7}+\frac{30a^2b^4(dx)^{17/2}}{17d^9}+\frac{4ab^5(dx)^{21/2}}{7d^{11}}+\frac{2b^6(dx)^{25/2}}{25d^{13}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```

output
```
(2*a^6*Sqrt[d*x])/d + (12*a^5*b*(d*x)^(5/2))/(5*d^3) + (10*a^4*b^2*(d*x)^(
9/2))/(3*d^5) + (40*a^3*b^3*(d*x)^(13/2))/(13*d^7) + (30*a^2*b^4*(d*x)^(17
/2))/(17*d^9) + (4*a*b^5*(d*x)^(21/2))/(7*d^11) + (2*b^6*(d*x)^(25/2))/(25
*d^13)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.57

method	result
gosper	$\frac{2(4641b^6x^{12}+33150a\,b^5x^{10}+102375a^2b^4x^8+178500a^3b^3x^6+193375a^4b^2x^4+139230a^5b\,x^2+116025a^6)x}{116025\sqrt{dx}}$
risch	$\frac{2(4641b^6x^{12}+33150a\,b^5x^{10}+102375a^2b^4x^8+178500a^3b^3x^6+193375a^4b^2x^4+139230a^5b\,x^2+116025a^6)x}{116025\sqrt{dx}}$
pseudoelliptic	$\frac{2\sqrt{dx}\,(\frac{1}{25}b^6x^{12}+\frac{2}{7}a\,b^5x^{10}+\frac{15}{17}a^2b^4x^8+\frac{20}{13}a^3b^3x^6+\frac{5}{3}a^4b^2x^4+\frac{6}{5}a^5b\,x^2+a^6)}{d}$
trager	$\frac{(\frac{2}{25}b^6x^{12}+\frac{4}{7}a\,b^5x^{10}+\frac{30}{17}a^2b^4x^8+\frac{40}{13}a^3b^3x^6+\frac{10}{3}a^4b^2x^4+\frac{12}{5}a^5b\,x^2+2a^6)\sqrt{dx}}{d}$
orering	$\frac{2(4641b^6x^{12}+33150a\,b^5x^{10}+102375a^2b^4x^8+178500a^3b^3x^6+193375a^4b^2x^4+139230a^5b\,x^2+116025a^6)x(b^2x^4+2ab\,x}{116025(b\,x^2+a)^6\sqrt{dx}}$
derivativedivides	$\frac{\frac{2b^6(dx)^{\frac{25}{2}}}{25}+\frac{4a\,b^5d^2(dx)^{\frac{21}{2}}}{7}+\frac{30a^2d^4b^4(dx)^{\frac{17}{2}}}{17}+\frac{40a^3d^6b^3(dx)^{\frac{13}{2}}}{13}+\frac{10a^4d^8b^2(dx)^{\frac{9}{2}}}{3}+\frac{12a^5d^{10}b(dx)^{\frac{5}{2}}}{5}+2\sqrt{dx}\,a^6d^{12}}{d^{13}}$
default	$\frac{\frac{2b^6(dx)^{\frac{25}{2}}}{25}+\frac{4a\,b^5d^2(dx)^{\frac{21}{2}}}{7}+\frac{30a^2d^4b^4(dx)^{\frac{17}{2}}}{17}+\frac{40a^3d^6b^3(dx)^{\frac{13}{2}}}{13}+\frac{10a^4d^8b^2(dx)^{\frac{9}{2}}}{3}+\frac{12a^5d^{10}b(dx)^{\frac{5}{2}}}{5}+2\sqrt{dx}\,a^6d^{12}}{d^{13}}$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/116025*(4641*b^6*x^12+33150*a*b^5*x^10+102375*a^2*b^4*x^8+178500*a^3*b^3
*x^6+193375*a^4*b^2*x^4+139230*a^5*b*x^2+116025*a^6)*x/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{\sqrt{dx}}\,dx$$
$$= \frac{2\,(4641\,b^6x^{12}+33150\,ab^5x^{10}+102375\,a^2b^4x^8+178500\,a^3b^3x^6+193375\,a^4b^2x^4+139230\,a^5bx^2+11602}{116025\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/116025*(4641*b^6*x^12 + 33150*a*b^5*x^10 + 102375*a^2*b^4*x^8 + 178500*a
^3*b^3*x^6 + 193375*a^4*b^2*x^4 + 139230*a^5*b*x^2 + 116025*a^6)*sqrt(d*x)
/d
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{\sqrt{dx}}\,dx = \frac{2a^6x}{\sqrt{dx}} + \frac{12a^5bx^3}{5\sqrt{dx}} + \frac{10a^4b^2x^5}{3\sqrt{dx}} + \frac{40a^3b^3x^7}{13\sqrt{dx}} + \frac{30a^2b^4x^9}{17\sqrt{dx}} + \frac{4ab^5x^{11}}{7\sqrt{dx}} + \frac{2b^6x^{13}}{25\sqrt{dx}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
```

output
```
2*a**6*x/sqrt(d*x) + 12*a**5*b*x**3/(5*sqrt(d*x)) + 10*a**4*b**2*x**5/(3*s
qrt(d*x)) + 40*a**3*b**3*x**7/(13*sqrt(d*x)) + 30*a**2*b**4*x**9/(17*sqrt(
d*x)) + 4*a*b**5*x**11/(7*sqrt(d*x)) + 2*b**6*x**13/(25*sqrt(d*x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.20

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{\sqrt{dx}}\,dx = \frac{2\left(116025\sqrt{dx}a^6 + \frac{4641(dx)^{\frac{25}{2}}b^6}{d^{12}} + \frac{33150(dx)^{\frac{21}{2}}ab^5}{d^{10}} + \frac{81900(dx)^{\frac{17}{2}}a^2b^4}{d^8} + \frac{71400(dx)^{\frac{13}{2}}a^3b^3}{d^6} + 7735\left(\frac{5(dx)^{\frac{9}{2}}b^2}{d^4} + \frac{18(\right.\right.}{116025\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2/116025*(116025*sqrt(d*x)*a^6 + 4641*(d*x)^(25/2)*b^6/d^12 + 33150*(d*x)^
(21/2)*a*b^5/d^10 + 81900*(d*x)^(17/2)*a^2*b^4/d^8 + 71400*(d*x)^(13/2)*a^
3*b^3/d^6 + 7735*(5*(d*x)^(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a*b/d^2)*a^4 + 17
5*(117*(d*x)^(17/2)*b^4/d^8 + 612*(d*x)^(13/2)*a*b^3/d^6 + 884*(d*x)^(9/2)
*a^2*b^2/d^4)*a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{\sqrt{dx}}\,dx$$
$$= \frac{2\left(4641\sqrt{dx}b^6x^{12} + 33150\sqrt{dx}ab^5x^{10} + 102375\sqrt{dx}a^2b^4x^8 + 178500\sqrt{dx}a^3b^3x^6 + 193375\sqrt{dx}a^4b^2x^4 -\right.}{116025\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x, algorithm="giac")
```

output
```
2/116025*(4641*sqrt(d*x)*b^6*x^12 + 33150*sqrt(d*x)*a*b^5*x^10 + 102375*sq
rt(d*x)*a^2*b^4*x^8 + 178500*sqrt(d*x)*a^3*b^3*x^6 + 193375*sqrt(d*x)*a^4*
b^2*x^4 + 139230*sqrt(d*x)*a^5*b*x^2 + 116025*sqrt(d*x)*a^6)/d
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{\sqrt{dx}}\,dx = \frac{2\,a^6\,\sqrt{d\,x}}{d} + \frac{2\,b^6\,(d\,x)^{25/2}}{25\,d^{13}} + \frac{10\,a^4\,b^2\,(d\,x)^{9/2}}{3\,d^5} + \frac{40\,a^3\,b^3\,(d\,x)^{13/2}}{13\,d^7} + \frac{30\,a^2\,b^4\,(d\,x)^{17/2}}{17\,d^9} + \frac{12\,a^5\,b\,(d\,x)^{5/2}}{5\,d^3} + \frac{4\,a\,b^5\,(d\,x)^{21/2}}{7\,d^{11}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(1/2),x)
```

output
```
(2*a^6*(d*x)^(1/2))/d + (2*b^6*(d*x)^(25/2))/(25*d^13) + (10*a^4*b^2*(d*x)
^(9/2))/(3*d^5) + (40*a^3*b^3*(d*x)^(13/2))/(13*d^7) + (30*a^2*b^4*(d*x)^(
17/2))/(17*d^9) + (12*a^5*b*(d*x)^(5/2))/(5*d^3) + (4*a*b^5*(d*x)^(21/2))/
(7*d^11)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.57

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{\sqrt{dx}}\,dx$$
$$= \frac{2\sqrt{x}\,\sqrt{d}\,(4641b^6x^{12} + 33150a\,b^5x^{10} + 102375a^2b^4x^8 + 178500a^3b^3x^6 + 193375a^4b^2x^4 + 139230a^5b\,x^2 + 1}{116025d}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*(116025*a**6 + 139230*a**5*b*x**2 + 193375*a**4*b**2*x*
*4 + 178500*a**3*b**3*x**6 + 102375*a**2*b**4*x**8 + 33150*a*b**5*x**10 +
4641*b**6*x**12))/(116025*d)
```

3.443 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{3/2}}\,dx = -\frac{2a^6}{d\sqrt{dx}}+\frac{4a^5b(dx)^{3/2}}{d^3}+\frac{30a^4b^2(dx)^{7/2}}{7d^5} + \frac{40a^3b^3(dx)^{11/2}}{11d^7}+\frac{2a^2b^4(dx)^{15/2}}{d^9}+\frac{12ab^5(dx)^{19/2}}{19d^{11}}+\frac{2b^6(dx)^{23/2}}{23d^{13}}$$

output
```
-2*a^6/d/(d*x)^(1/2)+4*a^5*b*(d*x)^(3/2)/d^3+30/7*a^4*b^2*(d*x)^(7/2)/d^5+
40/11*a^3*b^3*(d*x)^(11/2)/d^7+2*a^2*b^4*(d*x)^(15/2)/d^9+12/19*a*b^5*(d*x
)^(19/2)/d^11+2/23*b^6*(d*x)^(23/2)/d^13
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{3/2}}\,dx = -\frac{2x(33649a^6-67298a^5bx^2-72105a^4b^2x^4-61180a^3b^3x^6-33649a^2b^4x^8-10626ab^5x^{10}-1463b^6x^{12})}{33649(dx)^{3/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

output
```
(-2*x*(33649*a^6 - 67298*a^5*b*x^2 - 72105*a^4*b^2*x^4 - 61180*a^3*b^3*x^6
 - 33649*a^2*b^4*x^8 - 10626*a*b^5*x^10 - 1463*b^6*x^12))/(33649*(d*x)^(3/
2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{(dx)^{3/2}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^6}{(dx)^{3/2}}+\frac{6a^5b\sqrt{dx}}{d^2}+\frac{15a^4b^2(dx)^{5/2}}{d^4}+\frac{20a^3b^3(dx)^{9/2}}{d^6}+\frac{15a^2b^4(dx)^{13/2}}{d^8}+\frac{6ab^5(dx)^{17/2}}{d^{10}}+\frac{b^6(dx)^{21/2}}{d^{12}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^6}{d\sqrt{dx}}+\frac{4a^5b(dx)^{3/2}}{d^3}+\frac{30a^4b^2(dx)^{7/2}}{7d^5}+\frac{40a^3b^3(dx)^{11/2}}{11d^7}+\frac{2a^2b^4(dx)^{15/2}}{d^9}+\frac{12ab^5(dx)^{19/2}}{19d^{11}}+\frac{2b^6(dx)^{23/2}}{23d^{13}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

output
```
(-2*a^6)/(d*Sqrt[d*x]) + (4*a^5*b*(d*x)^(3/2))/d^3 + (30*a^4*b^2*(d*x)^(7/
2))/(7*d^5) + (40*a^3*b^3*(d*x)^(11/2))/(11*d^7) + (2*a^2*b^4*(d*x)^(15/2)
)/d^9 + (12*a*b^5*(d*x)^(19/2))/(19*d^11) + (2*b^6*(d*x)^(23/2))/(23*d^13)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.59

method	result
gosper	$-\frac{2\left(-1463 b^{6} x^{12}-10626 a\,b^{5} x^{10}-33649 a^{2} b^{4} x^{8}-61180 a^{3} b^{3} x^{6}-72105 a^{4} b^{2} x^{4}-67298 a^{5} b\,x^{2}+33649 a^{6}\right) x}{33649 \left(d x \right)^{\frac{3}{2}}}$
pseudoelliptic	$-\frac{2\left(-\frac{1}{23} b^{6} x^{12}-\frac{6}{19} a\,b^{5} x^{10}-a^{2} b^{4} x^{8}-\frac{20}{11} a^{3} b^{3} x^{6}-\frac{15}{7} a^{4} b^{2} x^{4}-2 a^{5} b\,x^{2}+a^{6}\right)}{\sqrt{d x}\, d}$
risch	$-\frac{2\left(-1463 b^{6} x^{12}-10626 a\,b^{5} x^{10}-33649 a^{2} b^{4} x^{8}-61180 a^{3} b^{3} x^{6}-72105 a^{4} b^{2} x^{4}-67298 a^{5} b\,x^{2}+33649 a^{6}\right)}{33649 d \sqrt{d x}}$
trager	$-\frac{2\left(-1463 b^{6} x^{12}-10626 a\,b^{5} x^{10}-33649 a^{2} b^{4} x^{8}-61180 a^{3} b^{3} x^{6}-72105 a^{4} b^{2} x^{4}-67298 a^{5} b\,x^{2}+33649 a^{6}\right) \sqrt{d x}}{33649 d^{2} x}$
orering	$-\frac{2\left(-1463 b^{6} x^{12}-10626 a\,b^{5} x^{10}-33649 a^{2} b^{4} x^{8}-61180 a^{3} b^{3} x^{6}-72105 a^{4} b^{2} x^{4}-67298 a^{5} b\,x^{2}+33649 a^{6}\right) x \left(b^{2} x^{4}+2 a b\,x^{2}\right.}{33649 \left(b\,x^{2}+a \right)^{6} \left(d x \right)^{\frac{3}{2}}}$
derivativedivides	$\frac{\frac{2 b^{6} \left(d x \right)^{\frac{23}{2}}}{23}+\frac{12 a\,b^{5} d^{2} \left(d x \right)^{\frac{19}{2}}}{19}+2 a^{2} b^{4} d^{4} \left(d x \right)^{\frac{15}{2}}+\frac{40 a^{3} b^{3} d^{6} \left(d x \right)^{\frac{11}{2}}}{11}+\frac{30 a^{4} b^{2} d^{8} \left(d x \right)^{\frac{7}{2}}}{7}+4 a^{5} b\,d^{10} \left(d x \right)^{\frac{3}{2}}-\frac{2 a^{6} d^{12}}{\sqrt{d x}}}{d^{13}}$
default	$\frac{\frac{2 b^{6} \left(d x \right)^{\frac{23}{2}}}{23}+\frac{12 a\,b^{5} d^{2} \left(d x \right)^{\frac{19}{2}}}{19}+2 a^{2} b^{4} d^{4} \left(d x \right)^{\frac{15}{2}}+\frac{40 a^{3} b^{3} d^{6} \left(d x \right)^{\frac{11}{2}}}{11}+\frac{30 a^{4} b^{2} d^{8} \left(d x \right)^{\frac{7}{2}}}{7}+4 a^{5} b\,d^{10} \left(d x \right)^{\frac{3}{2}}-\frac{2 a^{6} d^{12}}{\sqrt{d x}}}{d^{13}}$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/33649*(-1463*b^6*x^12-10626*a*b^5*x^10-33649*a^2*b^4*x^8-61180*a^3*b^3*
x^6-72105*a^4*b^2*x^4-67298*a^5*b*x^2+33649*a^6)*x/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.62

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{3/2}}\,dx = \frac{2\,(1463\,b^6x^{12}+10626\,ab^5x^{10}+33649\,a^2b^4x^8+61180\,a^3b^3x^6+72105\,a^4b^2x^4}{33649\,d^2x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/33649*(1463*b^6*x^12 + 10626*a*b^5*x^10 + 33649*a^2*b^4*x^8 + 61180*a^3*
b^3*x^6 + 72105*a^4*b^2*x^4 + 67298*a^5*b*x^2 - 33649*a^6)*sqrt(d*x)/(d^2*
x)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{3/2}}\,dx = -\frac{2a^6x}{(dx)^{\frac{3}{2}}}+\frac{4a^5bx^3}{(dx)^{\frac{3}{2}}} + \frac{30a^4b^2x^5}{7\,(dx)^{\frac{3}{2}}}+\frac{40a^3b^3x^7}{11\,(dx)^{\frac{3}{2}}}+\frac{2a^2b^4x^9}{(dx)^{\frac{3}{2}}}+\frac{12ab^5x^{11}}{19\,(dx)^{\frac{3}{2}}}+\frac{2b^6x^{13}}{23\,(dx)^{\frac{3}{2}}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(3/2),x)
```

output
```
-2*a**6*x/(d*x)**(3/2) + 4*a**5*b*x**3/(d*x)**(3/2) + 30*a**4*b**2*x**5/(7
*(d*x)**(3/2)) + 40*a**3*b**3*x**7/(11*(d*x)**(3/2)) + 2*a**2*b**4*x**9/(d
*x)**(3/2) + 12*a*b**5*x**11/(19*(d*x)**(3/2)) + 2*b**6*x**13/(23*(d*x)**(
3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{3/2}}\,dx = -\frac{2\left(\frac{33649\,a^6}{\sqrt{dx}}-\frac{1463\,(dx)^{\frac{23}{2}}b^6+10626\,(dx)^{\frac{19}{2}}ab^5d^2+33649\,(dx)^{\frac{15}{2}}a^2b^4d^4+61180\,(dx)^{\frac{11}{2}}a^3b^3d^6+72105\,(dx)^{\frac{7}{2}}a^4b^2d^8+67298\,(dx)^{\frac{3}{2}}a^5bd^{10}}{d^{12}}\right)}{33649\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2/33649*(33649*a^6/sqrt(d*x) - (1463*(d*x)^(23/2)*b^6 + 10626*(d*x)^(19/2
)*a*b^5*d^2 + 33649*(d*x)^(15/2)*a^2*b^4*d^4 + 61180*(d*x)^(11/2)*a^3*b^3*
d^6 + 72105*(d*x)^(7/2)*a^4*b^2*d^8 + 67298*(d*x)^(3/2)*a^5*b*d^10)/d^12)/
d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{3/2}}\,dx =$$
$$-\frac{2\left(\frac{33649\,a^6}{\sqrt{dx}}-\frac{1463\,\sqrt{dx}b^6d^{275}x^{11}+10626\,\sqrt{dx}ab^5d^{275}x^9+33649\,\sqrt{dx}a^2b^4d^{275}x^7+61180\,\sqrt{dx}a^3b^3d^{275}x^5+72105\,\sqrt{dx}a^4b^2d^{275}x^3+67298\,\sqrt{dx}a^5bd^{275}x}{d^{276}}\right)}{33649\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x, algorithm="giac")
```

output
```
-2/33649*(33649*a^6/sqrt(d*x) - (1463*sqrt(d*x)*b^6*d^275*x^11 + 10626*sqr
t(d*x)*a*b^5*d^275*x^9 + 33649*sqrt(d*x)*a^2*b^4*d^275*x^7 + 61180*sqrt(d*
x)*a^3*b^3*d^275*x^5 + 72105*sqrt(d*x)*a^4*b^2*d^275*x^3 + 67298*sqrt(d*x)
*a^5*b*d^275*x)/d^276)/d
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.82

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{3/2}}\,dx = \frac{2\,b^6\,{(d\,x)}^{23/2}}{23\,d^{13}}-\frac{2\,a^6}{d\,\sqrt{d\,x}}+\frac{30\,a^4\,b^2\,{(d\,x)}^{7/2}}{7\,d^5}+\frac{40\,a^3\,b^3\,{(d\,x)}^{11/2}}{11\,d^7}+\frac{2\,a^2\,b^4\,{(d\,x)}^{15/2}}{d^9}+\frac{4\,a^5\,b\,{(d\,x)}^{3/2}}{d^3}+\frac{12\,a\,b^5\,{(d\,x)}^{19/2}}{19\,d^{11}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(3/2),x)
```

output
```
(2*b^6*(d*x)^(23/2))/(23*d^13) - (2*a^6)/(d*(d*x)^(1/2)) + (30*a^4*b^2*(d*
x)^(7/2))/(7*d^5) + (40*a^3*b^3*(d*x)^(11/2))/(11*d^7) + (2*a^2*b^4*(d*x)^
(15/2))/d^9 + (4*a^5*b*(d*x)^(3/2))/d^3 + (12*a*b^5*(d*x)^(19/2))/(19*d^11
)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{(dx)^{3/2}}\,dx = \frac{2\sqrt{d}\,(1463b^6x^{12} + 10626a\,b^5x^{10} + 33649a^2b^4x^8 + 61180a^3b^3x^6 + 72105a^4b^2x^4}{33649\sqrt{x}\,d^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x)
```

output
```
(2*sqrt(d)*( - 33649*a**6 + 67298*a**5*b*x**2 + 72105*a**4*b**2*x**4 + 611
80*a**3*b**3*x**6 + 33649*a**2*b**4*x**8 + 10626*a*b**5*x**10 + 1463*b**6*
x**12))/(33649*sqrt(x)*d**2)
```

3.444 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 127

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{5/2}}\,dx = -\frac{2a^6}{3d(dx)^{3/2}}+\frac{12a^5b\sqrt{dx}}{d^3}+\frac{6a^4b^2(dx)^{5/2}}{d^5}+\frac{40a^3b^3(dx)^{9/2}}{9d^7}+\frac{30a^2b^4(dx)^{13/2}}{13d^9}+\frac{12ab^5(dx)^{17/2}}{17d^{11}}+\frac{2b^6(dx)^{21/2}}{21d^{13}}$$

output
```
-2/3*a^6/d/(d*x)^(3/2)+12*a^5*b*(d*x)^(1/2)/d^3+6*a^4*b^2*(d*x)^(5/2)/d^5+
40/9*a^3*b^3*(d*x)^(9/2)/d^7+30/13*a^2*b^4*(d*x)^(13/2)/d^9+12/17*a*b^5*(d
*x)^(17/2)/d^11+2/21*b^6*(d*x)^(21/2)/d^13
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.61

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{5/2}}\,dx = -\frac{2x(4641a^6-83538a^5bx^2-41769a^4b^2x^4-30940a^3b^3x^6-16065a^2b^4x^8-4914ab^5x^{10}-663b^6x^{12})}{13923(dx)^{5/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```

output
```
(-2*x*(4641*a^6 - 83538*a^5*b*x^2 - 41769*a^4*b^2*x^4 - 30940*a^3*b^3*x^6
- 16065*a^2*b^4*x^8 - 4914*a*b^5*x^10 - 663*b^6*x^12))/(13923*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{(dx)^{5/2}}dx}{b^6}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^6}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^6}{(dx)^{5/2}}+\frac{6a^5b}{d^2\sqrt{dx}}+\frac{15a^4b^2(dx)^{3/2}}{d^4}+\frac{20a^3b^3(dx)^{7/2}}{d^6}+\frac{15a^2b^4(dx)^{11/2}}{d^8}+\frac{6ab^5(dx)^{15/2}}{d^{10}}+\frac{b^6(dx)^{19/2}}{d^{12}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^6}{3d(dx)^{3/2}}+\frac{12a^5b\sqrt{dx}}{d^3}+\frac{6a^4b^2(dx)^{5/2}}{d^5}+\frac{40a^3b^3(dx)^{9/2}}{9d^7}+\frac{30a^2b^4(dx)^{13/2}}{13d^9}+\frac{12ab^5(dx)^{17/2}}{17d^{11}}+\frac{2b^6(dx)^{21/2}}{21d^{13}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```

output
```
(-2*a^6)/(3*d*(d*x)^(3/2)) + (12*a^5*b*Sqrt[d*x])/d^3 + (6*a^4*b^2*(d*x)^(
5/2))/d^5 + (40*a^3*b^3*(d*x)^(9/2))/(9*d^7) + (30*a^2*b^4*(d*x)^(13/2))/(
13*d^9) + (12*a*b^5*(d*x)^(17/2))/(17*d^11) + (2*b^6*(d*x)^(21/2))/(21*d^1
3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.58

method	result
gosper	$-\frac{2\left(-663 b^{6} x^{12}-4914 a\,b^{5} x^{10}-16065 a^{2} b^{4} x^{8}-30940 a^{3} b^{3} x^{6}-41769 a^{4} b^{2} x^{4}-83538 a^{5} b\,x^{2}+4641 a^{6}\right) x}{13923 \left(d x \right)^{\frac{5}{2}}}$
pseudoelliptic	$-\frac{2\left(-\frac{1}{7} b^{6} x^{12}-\frac{18}{17} a\,b^{5} x^{10}-\frac{45}{13} a^{2} b^{4} x^{8}-\frac{20}{3} a^{3} b^{3} x^{6}-9 a^{4} b^{2} x^{4}-18 a^{5} b\,x^{2}+a^{6}\right)}{3 \sqrt{d x}\, d^{2} x}$
trager	$-\frac{2\left(-663 b^{6} x^{12}-4914 a\,b^{5} x^{10}-16065 a^{2} b^{4} x^{8}-30940 a^{3} b^{3} x^{6}-41769 a^{4} b^{2} x^{4}-83538 a^{5} b\,x^{2}+4641 a^{6}\right) \sqrt{d x}}{13923 d^{3} x^{2}}$
risch	$-\frac{2\left(-663 b^{6} x^{12}-4914 a\,b^{5} x^{10}-16065 a^{2} b^{4} x^{8}-30940 a^{3} b^{3} x^{6}-41769 a^{4} b^{2} x^{4}-83538 a^{5} b\,x^{2}+4641 a^{6}\right)}{13923 d^{2} x \sqrt{d x}}$
orering	$-\frac{2\left(-663 b^{6} x^{12}-4914 a\,b^{5} x^{10}-16065 a^{2} b^{4} x^{8}-30940 a^{3} b^{3} x^{6}-41769 a^{4} b^{2} x^{4}-83538 a^{5} b\,x^{2}+4641 a^{6}\right) x \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right.}{13923 \left(b\,x^{2}+a \right)^{6} \left(d x \right)^{\frac{5}{2}}}$
derivativedivides	$\frac{\frac{2 b^{6} \left(d x \right)^{\frac{21}{2}}}{21}+\frac{12 a\,b^{5} d^{2} \left(d x \right)^{\frac{17}{2}}}{17}+\frac{30 a^{2} b^{4} d^{4} \left(d x \right)^{\frac{13}{2}}}{13}+\frac{40 a^{3} b^{3} d^{6} \left(d x \right)^{\frac{9}{2}}}{9}+6 a^{4} b^{2} d^{8} \left(d x \right)^{\frac{5}{2}}+12 \sqrt{d x}\, a^{5} b\,d^{10}-\frac{2 a^{6} d^{12}}{3 \left(d x \right)^{\frac{3}{2}}}}{d^{13}}$
default	$\frac{\frac{2 b^{6} \left(d x \right)^{\frac{21}{2}}}{21}+\frac{12 a\,b^{5} d^{2} \left(d x \right)^{\frac{17}{2}}}{17}+\frac{30 a^{2} b^{4} d^{4} \left(d x \right)^{\frac{13}{2}}}{13}+\frac{40 a^{3} b^{3} d^{6} \left(d x \right)^{\frac{9}{2}}}{9}+6 a^{4} b^{2} d^{8} \left(d x \right)^{\frac{5}{2}}+12 \sqrt{d x}\, a^{5} b\,d^{10}-\frac{2 a^{6} d^{12}}{3 \left(d x \right)^{\frac{3}{2}}}}{d^{13}}$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/13923*(-663*b^6*x^12-4914*a*b^5*x^10-16065*a^2*b^4*x^8-30940*a^3*b^3*x^
6-41769*a^4*b^2*x^4-83538*a^5*b*x^2+4641*a^6)*x/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{5/2}}\,dx = \frac{2\left(663\,b^6x^{12}+4914\,ab^5x^{10}+16065\,a^2b^4x^8+30940\,a^3b^3x^6+41769\,a^4b^2x^4+8\right.}{13923\,d^3x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/13923*(663*b^6*x^12 + 4914*a*b^5*x^10 + 16065*a^2*b^4*x^8 + 30940*a^3*b^
3*x^6 + 41769*a^4*b^2*x^4 + 83538*a^5*b*x^2 - 4641*a^6)*sqrt(d*x)/(d^3*x^2
)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{(dx)^{5/2}}\,dx = -\frac{2a^6x}{3\,(dx)^{\frac{5}{2}}} + \frac{12a^5bx^3}{(dx)^{\frac{5}{2}}} + \frac{6a^4b^2x^5}{(dx)^{\frac{5}{2}}} + \frac{40a^3b^3x^7}{9\,(dx)^{\frac{5}{2}}} + \frac{30a^2b^4x^9}{13\,(dx)^{\frac{5}{2}}} + \frac{12ab^5x^{11}}{17\,(dx)^{\frac{5}{2}}} + \frac{2b^6x^{13}}{21\,(dx)^{\frac{5}{2}}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(5/2),x)
```

output
```
-2*a**6*x/(3*(d*x)**(5/2)) + 12*a**5*b*x**3/(d*x)**(5/2) + 6*a**4*b**2*x**
5/(d*x)**(5/2) + 40*a**3*b**3*x**7/(9*(d*x)**(5/2)) + 30*a**2*b**4*x**9/(1
3*(d*x)**(5/2)) + 12*a*b**5*x**11/(17*(d*x)**(5/2)) + 2*b**6*x**13/(21*(d*
x)**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.85

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^3}{(dx)^{5/2}}\,dx = -\frac{2\left(\frac{4641\,a^6}{(dx)^{\frac{3}{2}}} - \frac{663\,(dx)^{\frac{21}{2}}b^6 + 4914\,(dx)^{\frac{17}{2}}ab^5d^2 + 16065\,(dx)^{\frac{13}{2}}a^2b^4d^4 + 30940\,(dx)^{\frac{9}{2}}a^3b^3d^6 + 41769\,(dx)^{\frac{5}{2}}a^4b^2d^8 + 83538\,\sqrt{dx}a^5bd^{10}}{d^{12}}\right)}{13923\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-2/13923*(4641*a^6/(d*x)^(3/2) - (663*(d*x)^(21/2)*b^6 + 4914*(d*x)^(17/2)
*a*b^5*d^2 + 16065*(d*x)^(13/2)*a^2*b^4*d^4 + 30940*(d*x)^(9/2)*a^3*b^3*d^
6 + 41769*(d*x)^(5/2)*a^4*b^2*d^8 + 83538*sqrt(d*x)*a^5*b*d^10)/d^12)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.04

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{5/2}}\,dx =$$

$$-\frac{2\left(\frac{4641\,a^6}{\sqrt{dx}dx}-\frac{663\,\sqrt{dx}b^6d^{250}x^{10}+4914\,\sqrt{dx}ab^5d^{250}x^8+16065\,\sqrt{dx}a^2b^4d^{250}x^6+30940\,\sqrt{dx}a^3b^3d^{250}x^4+41769\,\sqrt{dx}a^4b^2d^{250}x^2+83538\,\sqrt{dx}a^5 \cdots}{d^{252}}\right.}{13923\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x, algorithm="giac")
```

output
```
-2/13923*(4641*a^6/(sqrt(d*x)*d*x) - (663*sqrt(d*x)*b^6*d^250*x^10 + 4914*
sqrt(d*x)*a*b^5*d^250*x^8 + 16065*sqrt(d*x)*a^2*b^4*d^250*x^6 + 30940*sqrt
(d*x)*a^3*b^3*d^250*x^4 + 41769*sqrt(d*x)*a^4*b^2*d^250*x^2 + 83538*sqrt(d
*x)*a^5*b*d^250)/d^252)/d
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.81

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{5/2}}\,dx = \frac{2\,b^6\,(d\,x)^{21/2}}{21\,d^{13}} - \frac{2\,a^6}{3\,d\,(d\,x)^{3/2}} + \frac{6\,a^4\,b^2\,(d\,x)^{5/2}}{d^5} + \frac{40\,a^3\,b^3\,(d\,x)^{9/2}}{9\,d^7} + \frac{30\,a^2\,b^4\,(d\,x)^{13/2}}{13\,d^9} + \frac{12\,a^5\,b\,\sqrt{d\,x}}{d^3} + \frac{12\,a\,b^5\,(d\,x)^{17/2}}{17\,d^{11}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(5/2),x)
```

output
```
(2*b^6*(d*x)^(21/2))/(21*d^13) - (2*a^6)/(3*d*(d*x)^(3/2)) + (6*a^4*b^2*(d
*x)^(5/2))/d^5 + (40*a^3*b^3*(d*x)^(9/2))/(9*d^7) + (30*a^2*b^4*(d*x)^(13/
2))/(13*d^9) + (12*a^5*b*(d*x)^(1/2))/d^3 + (12*a*b^5*(d*x)^(17/2))/(17*d^
11)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{5/2}}\,dx = \frac{2\sqrt{d}\,(663b^6x^{12}+4914a\,b^5x^{10}+16065a^2b^4x^8+30940a^3b^3x^6+41769a^4b^2x^4+}{13923\sqrt{x}\,d^3x}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x)
```

output
```
(2*sqrt(d)*( - 4641*a**6 + 83538*a**5*b*x**2 + 41769*a**4*b**2*x**4 + 3094
0*a**3*b**3*x**6 + 16065*a**2*b**4*x**8 + 4914*a*b**5*x**10 + 663*b**6*x**
12))/(13923*sqrt(x)*d**3*x)
```

3.445 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 127

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}\,dx = -\frac{2a^6}{5d(dx)^{5/2}}-\frac{12a^5b}{d^3\sqrt{dx}}+\frac{10a^4b^2(dx)^{3/2}}{d^5}+\frac{40a^3b^3(dx)^{7/2}}{7d^7}+\frac{30a^2b^4(dx)^{11/2}}{11d^9}+\frac{4ab^5(dx)^{15/2}}{5d^{11}}+\frac{2b^6(dx)^{19/2}}{19d^{13}}$$

output
```
-2/5*a^6/d/(d*x)^(5/2)-12*a^5*b/d^3/(d*x)^(1/2)+10*a^4*b^2*(d*x)^(3/2)/d^5
+40/7*a^3*b^3*(d*x)^(7/2)/d^7+30/11*a^2*b^4*(d*x)^(11/2)/d^9+4/5*a*b^5*(d*
x)^(15/2)/d^11+2/19*b^6*(d*x)^(19/2)/d^13
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}\,dx = -\frac{2\sqrt{dx}(1463a^6+43890a^5bx^2-36575a^4b^2x^4-20900a^3b^3x^6-9975a^2b^4x^8-2926ab^5x^{10}-385b^6x^{12})}{7315d^4x^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

output
```
(-2*Sqrt[d*x]*(1463*a^6 + 43890*a^5*b*x^2 - 36575*a^4*b^2*x^4 - 20900*a^3*
b^3*x^6 - 9975*a^2*b^4*x^8 - 2926*a*b^5*x^10 - 385*b^6*x^12))/(7315*d^4*x^
3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}dx$$

$$\Big\downarrow \ 1380$$

$$\frac{\int \frac{b^6(bx^2+a)^6}{(dx)^{7/2}}dx}{b^6}$$

$$\Big\downarrow \ 27$$

$$\int \frac{(a+bx^2)^6}{(dx)^{7/2}}dx$$

$$\Big\downarrow \ 244$$

$$\int \left(\frac{a^6}{(dx)^{7/2}}+\frac{6a^5b}{d^2(dx)^{3/2}}+\frac{15a^4b^2\sqrt{dx}}{d^4}+\frac{20a^3b^3(dx)^{5/2}}{d^6}+\frac{15a^2b^4(dx)^{9/2}}{d^8}+\frac{6ab^5(dx)^{13/2}}{d^{10}}+\frac{b^6(dx)^{17/2}}{d^{12}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^6}{5d(dx)^{5/2}}-\frac{12a^5b}{d^3\sqrt{dx}}+\frac{10a^4b^2(dx)^{3/2}}{d^5}+\frac{40a^3b^3(dx)^{7/2}}{7d^7}+\frac{30a^2b^4(dx)^{11/2}}{11d^9}+\frac{4ab^5(dx)^{15/2}}{5d^{11}}+\frac{2b^6(dx)^{19/2}}{19d^{13}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

output
```
(-2*a^6)/(5*d*(d*x)^(5/2)) - (12*a^5*b)/(d^3*Sqrt[d*x]) + (10*a^4*b^2*(d*x
)^(3/2))/d^5 + (40*a^3*b^3*(d*x)^(7/2))/(7*d^7) + (30*a^2*b^4*(d*x)^(11/2)
)/(11*d^9) + (4*a*b^5*(d*x)^(15/2))/(5*d^11) + (2*b^6*(d*x)^(19/2))/(19*d^
13)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.58

method	result
gosper	$-\frac{2(-385b^6x^{12}-2926a\,b^5x^{10}-9975a^2b^4x^8-20900a^3b^3x^6-36575a^4b^2x^4+43890a^5b\,x^2+1463a^6)x}{7315(dx)^{\frac{7}{2}}}$
pseudoelliptic	$-\frac{2(-\frac{5}{19}b^6x^{12}-2a\,b^5x^{10}-\frac{75}{11}a^2b^4x^8-\frac{100}{7}a^3b^3x^6-25a^4b^2x^4+30a^5b\,x^2+a^6)}{5\sqrt{dx}\,d^3x^2}$
trager	$-\frac{2(-385b^6x^{12}-2926a\,b^5x^{10}-9975a^2b^4x^8-20900a^3b^3x^6-36575a^4b^2x^4+43890a^5b\,x^2+1463a^6)\sqrt{dx}}{7315d^4x^3}$
risch	$-\frac{2(-385b^6x^{12}-2926a\,b^5x^{10}-9975a^2b^4x^8-20900a^3b^3x^6-36575a^4b^2x^4+43890a^5b\,x^2+1463a^6)}{7315d^3x^2\sqrt{dx}}$
orering	$-\frac{2(-385b^6x^{12}-2926a\,b^5x^{10}-9975a^2b^4x^8-20900a^3b^3x^6-36575a^4b^2x^4+43890a^5b\,x^2+1463a^6)x(b^2x^4+2ab\,x^2+a^2)}{7315(b\,x^2+a)^6(dx)^{\frac{7}{2}}}$
derivativedivides	$\frac{\frac{2b^6(dx)^{\frac{19}{2}}}{19}+\frac{4a\,b^5d^2(dx)^{\frac{15}{2}}}{5}+\frac{30a^2b^4d^4(dx)^{\frac{11}{2}}}{11}+\frac{40a^3b^3d^6(dx)^{\frac{7}{2}}}{7}+10a^4b^2d^8(dx)^{\frac{3}{2}}-\frac{12a^5b\,d^{10}}{\sqrt{dx}}-\frac{2a^6d^{12}}{5(dx)^{\frac{5}{2}}}}{d^{13}}$
default	$\frac{\frac{2b^6(dx)^{\frac{19}{2}}}{19}+\frac{4a\,b^5d^2(dx)^{\frac{15}{2}}}{5}+\frac{30a^2b^4d^4(dx)^{\frac{11}{2}}}{11}+\frac{40a^3b^3d^6(dx)^{\frac{7}{2}}}{7}+10a^4b^2d^8(dx)^{\frac{3}{2}}-\frac{12a^5b\,d^{10}}{\sqrt{dx}}-\frac{2a^6d^{12}}{5(dx)^{\frac{5}{2}}}}{d^{13}}$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7315*(-385*b^6*x^12-2926*a*b^5*x^10-9975*a^2*b^4*x^8-20900*a^3*b^3*x^6-
36575*a^4*b^2*x^4+43890*a^5*b*x^2+1463*a^6)*x/(d*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}\,dx = \frac{2\,(385\,b^6x^{12}+2926\,ab^5x^{10}+9975\,a^2b^4x^8+20900\,a^3b^3x^6+36575\,a^4b^2x^4-43}{7315\,d^4x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
2/7315*(385*b^6*x^12 + 2926*a*b^5*x^10 + 9975*a^2*b^4*x^8 + 20900*a^3*b^3*
x^6 + 36575*a^4*b^2*x^4 - 43890*a^5*b*x^2 - 1463*a^6)*sqrt(d*x)/(d^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}\,dx = -\frac{2a^6x}{5\,(dx)^{\frac{7}{2}}} - \frac{12a^5bx^3}{(dx)^{\frac{7}{2}}} + \frac{10a^4b^2x^5}{(dx)^{\frac{7}{2}}} + \frac{40a^3b^3x^7}{7\,(dx)^{\frac{7}{2}}} + \frac{30a^2b^4x^9}{11\,(dx)^{\frac{7}{2}}} + \frac{4ab^5x^{11}}{5\,(dx)^{\frac{7}{2}}} + \frac{2b^6x^{13}}{19\,(dx)^{\frac{7}{2}}}$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(7/2),x)
```

output
```
-2*a**6*x/(5*(d*x)**(7/2)) - 12*a**5*b*x**3/(d*x)**(7/2) + 10*a**4*b**2*x*
*5/(d*x)**(7/2) + 40*a**3*b**3*x**7/(7*(d*x)**(7/2)) + 30*a**2*b**4*x**9/(
11*(d*x)**(7/2)) + 4*a*b**5*x**11/(5*(d*x)**(7/2)) + 2*b**6*x**13/(19*(d*x
)**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}\,dx = -\frac{2\left(\frac{1463\,(30\,a^5bd^2x^2+a^6d^2)}{(dx)^{\frac{5}{2}}\,d^2} - \frac{385\,(dx)^{\frac{19}{2}}\,b^6+2926\,(dx)^{\frac{15}{2}}\,ab^5d^2+9975\,(dx)^{\frac{11}{2}}\,a^2b^4d^4+20900\,(dx)^{\frac{7}{2}}\,a^3b^3d^6+36575\,(dx)^{\frac{3}{2}}\,a^4b^2d^8}{d^{12}}\right)}{7315\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
-2/7315*(1463*(30*a^5*b*d^2*x^2 + a^6*d^2)/((d*x)^(5/2)*d^2) - (385*(d*x)^
(19/2)*b^6 + 2926*(d*x)^(15/2)*a*b^5*d^2 + 9975*(d*x)^(11/2)*a^2*b^4*d^4 +
 20900*(d*x)^(7/2)*a^3*b^3*d^6 + 36575*(d*x)^(3/2)*a^4*b^2*d^8)/d^12)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}\,dx = -\frac{2\left(\frac{1463\,(30\,a^5bd^3x^2+a^6d^3)}{\sqrt{dx}d^2x^2} - \frac{385\,\sqrt{dx}b^6d^{171}x^9+2926\,\sqrt{dx}ab^5d^{171}x^7+9975\,\sqrt{dx}a^2b^4d^{171}x^5+20900\,\sqrt{dx}a^3b^3d^{171}x^3+36575\,\sqrt{dx}a^4b^2d^{171}x}{d^{171}}\right.}{7315\,d^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x, algorithm="giac")
```

output
```
-2/7315*(1463*(30*a^5*b*d^3*x^2 + a^6*d^3)/(sqrt(d*x)*d^2*x^2) - (385*sqrt
(d*x)*b^6*d^171*x^9 + 2926*sqrt(d*x)*a*b^5*d^171*x^7 + 9975*sqrt(d*x)*a^2*
b^4*d^171*x^5 + 20900*sqrt(d*x)*a^3*b^3*d^171*x^3 + 36575*sqrt(d*x)*a^4*b^
2*d^171*x)/d^171)/d^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}\,dx = \frac{2\,b^6\,(d\,x)^{19/2}}{19\,d^{13}} - \frac{\frac{2\,a^6\,d^2}{5}+12\,b\,a^5\,d^2\,x^2}{d^3\,(d\,x)^{5/2}} + \frac{10\,a^4\,b^2\,(d\,x)^{3/2}}{d^5} + \frac{40\,a^3\,b^3\,(d\,x)^{7/2}}{7\,d^7} + \frac{30\,a^2\,b^4\,(d\,x)^{11/2}}{11\,d^9} + \frac{4\,a\,b^5\,(d\,x)^{15/2}}{5\,d^{11}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^3/(d*x)^(7/2),x)
```

output
```
(2*b^6*(d*x)^(19/2))/(19*d^13) - ((2*a^6*d^2)/5 + 12*a^5*b*d^2*x^2)/(d^3*(
d*x)^(5/2)) + (10*a^4*b^2*(d*x)^(3/2))/d^5 + (40*a^3*b^3*(d*x)^(7/2))/(7*d
^7) + (30*a^2*b^4*(d*x)^(11/2))/(11*d^9) + (4*a*b^5*(d*x)^(15/2))/(5*d^11)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

$$\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}\,dx = \frac{2\sqrt{d}\,(385b^6x^{12}+2926a\,b^5x^{10}+9975a^2b^4x^8+20900a^3b^3x^6+36575a^4b^2x^4-}{7315\sqrt{x}\,d^4x^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x)
```

output
```
(2*sqrt(d)*( - 1463*a**6 - 43890*a**5*b*x**2 + 36575*a**4*b**2*x**4 + 2090
0*a**3*b**3*x**6 + 9975*a**2*b**4*x**8 + 2926*a*b**5*x**10 + 385*b**6*x**1
2))/(7315*sqrt(x)*d**4*x**2)
```

3.446 $\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4} dx$

Optimal result

Integrand size = 28, antiderivative size = 239

$$\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4} dx = -\frac{9ad^5\sqrt{dx}}{2b^3}+\frac{9d^3(dx)^{5/2}}{10b^2} - \frac{d(dx)^{9/2}}{2b(a+bx^2)} - \frac{9a^{5/4}d^{11/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{13/4}} + \frac{9a^{5/4}d^{11/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{13/4}} + \frac{9a^{5/4}d^{11/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}b^{13/4}}$$

output

```
-9/2*a*d^5*(d*x)^(1/2)/b^3+9/10*d^3*(d*x)^(5/2)/b^2-1/2*d*(d*x)^(9/2)/b/(b
*x^2+a)-9/8*a^(5/4)*d^(11/2)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/
d^(1/2))*2^(1/2)/b^(13/4)+9/8*a^(5/4)*d^(11/2)*arctan(1+2^(1/2)*b^(1/4)*(d
*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(13/4)+9/8*a^(5/4)*d^(11/2)*arctanh(2
^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^
(13/4)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.74

$$\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{d^5\sqrt{dx}\left(4\sqrt[4]{b}\sqrt{x}(-45a^2-36abx^2+4b^2x^4)-45\sqrt{2}a^{5/4}(a+bx^2)\arctan\left(\frac{\sqrt{a}-}{\sqrt{2}\sqrt[4]{a}}\right.\right.}{40b^{13/4}\sqrt{x}\,(a+bx^2)}$$

input
```
Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(d^5*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(-45*a^2 - 36*a*b*x^2 + 4*b^2*x^4) - 45*
Sqrt[2]*a^(5/4)*(a + b*x^2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x])] + 45*Sqrt[2]*a^(5/4)*(a + b*x^2)*ArcTanh[(Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(40*b^(13/4)*Sqrt[x]*(a + b*x^
2))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.46, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {1380, 27, 252, 262, 262, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \ 1380$$

$$b^2\int \frac{(dx)^{11/2}}{b^2(bx^2+a)^2}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{(dx)^{11/2}}{(a+bx^2)^2}dx$$

$$\Big\downarrow \ 252$$

$$\frac{9d^2\int\frac{(dx)^{7/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 262

$$\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\int\frac{(dx)^{3/2}}{bx^2+a}dx}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 262

$$\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{ad^2\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 266

$$\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\int\frac{1}{bx^2+a}d\sqrt{dx}}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 755

$$\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}$$

$\downarrow$ 1476

$$\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}$$

$$\downarrow 1479$$

$$9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\cdots\right.}{\sqrt{2}\cdots}\right.}{\cdots}\right)}{b}\right)}{b}\right.$$

$$\overline{\qquad 4b \qquad}$$

↓ 25

$$9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right)}{\sqrt{2}\sqrt[4]{a}\cdots}\cdots\right.}{\cdots}\right)}{b}\cdots\right)}{b}\cdots\right)$$

$$\overline{\qquad\qquad\qquad 4b \qquad\qquad\qquad}$$

↓ 27

$$\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\cdots\right)}{2\sqrt{a}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4b}\cdots$$

$$\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\right.}{}\right.}{2\sqrt{a}}}{b}\right.}{b}\right.}{4b}$$

$$\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}$$

input
```
Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*(d*(d*x)^(9/2))/(b*(a + b*x^2)) + (9*d^2*((2*d*(d*x)^(5/2))/(5*b) - (
a*d^2*((2*d*Sqrt[d*x])/b - (2*a*d*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[
d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt
[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/b))/b))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{2(-b x^2+10a) x d^6}{5 b^3 \sqrt{dx}}+\frac{2a^2 d^7 \left(-\frac{\sqrt{dx}}{4\left(b d^2 x^2+a d^2\right)}+\frac{9\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cdots}}{\left(\frac{a d^2}{b}\cdots\right.}\right.\cdots\right)}{32 a d^2}\cdots\right)}{b^3}\cdots$
derivativedivides	$2d^3\left(-\frac{-\frac{b(dx)^{\frac{5}{2}}}{5}+2\sqrt{dx}\,a\,d^2}{b^3}+\frac{a^2 d^4\left(-\frac{\sqrt{dx}}{4\left(b d^2 x^2+a d^2\right)}+\frac{9\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+2\,\mathrm{arcta}\cdots\right)}{32 a d^2}\cdots\right)}{b^3}\cdots\right.$
default	$2d^3\left(-\frac{-\frac{b(dx)^{\frac{5}{2}}}{5}+2\sqrt{dx}\,a\,d^2}{b^3}+\frac{a^2 d^4\left(-\frac{\sqrt{dx}}{4\left(b d^2 x^2+a d^2\right)}+\frac{9\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+2\,\mathrm{arcta}\cdots\right)}{32 a d^2}\cdots\right)}{b^3}\cdots\right.$
pseudoelliptic	$\frac{d^5\left(-\frac{2\sqrt{dx}\left(-b x^2+10a\right)}{5}-\frac{\sqrt{dx}\,a^2}{2\left(b x^2+a\right)}+\frac{9\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}a\sqrt{2}}{16}+\frac{9\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}+1\right)\left(\frac{a d^2}{b}\cdots\right.}{8}\cdots\right)}{b^3}$

input

```
int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-b*x^2+10*a)*x/b^3/(d*x)^(1/2)*d^6+2*a^2/b^3*d^7*(-1/4*(d*x)^(1/2)/(
b*d^2*x^2+a*d^2)+9/32*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2
*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.23

$$\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{45\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}(b^4x^2+ab^3)\log\left(9\sqrt{dx}ad^5+9\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}b^3\right)-45\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}(\cdots}{\cdots}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/40*(45*(-a^5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3)*log(9*sqrt(d*x)*a*d^5 +
9*(-a^5*d^22/b^13)^(1/4)*b^3) - 45*(-a^5*d^22/b^13)^(1/4)*(-I*b^4*x^2 - I*
a*b^3)*log(9*sqrt(d*x)*a*d^5 + 9*I*(-a^5*d^22/b^13)^(1/4)*b^3) - 45*(-a^5*
d^22/b^13)^(1/4)*(I*b^4*x^2 + I*a*b^3)*log(9*sqrt(d*x)*a*d^5 - 9*I*(-a^5*d
^22/b^13)^(1/4)*b^3) - 45*(-a^5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3)*log(9*s
qrt(d*x)*a*d^5 - 9*(-a^5*d^22/b^13)^(1/4)*b^3) + 4*(4*b^2*d^5*x^4 - 36*a*b
*d^5*x^2 - 45*a^2*d^5)*sqrt(d*x))/(b^4*x^2 + a*b^3)
```

Sympy [F]

$$\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx = \int \frac{(dx)^{\frac{11}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
Integral((d*x)**(11/2)/(a + b*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.26

$$\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx = -\frac{\frac{40\sqrt{dx}a^2d^8}{b^4d^2x^2+ab^3d^2} - \frac{45\left(\frac{\sqrt{2}d^8\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}d^8\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{2\sqrt{2}d^7\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\right.}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right.}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}}{b^3}}{80\,d}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/80*(40*sqrt(d*x)*a^2*d^8/(b^4*d^2*x^2 + a*b^3*d^2) - 45*(sqrt(2)*d^8*lo
g(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d
^2)^(3/4)*b^(1/4)) - sqrt(2)*d^8*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*s
qrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^7*arct
an(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(
sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^7*arct
an(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt
(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a^2/b^3 - 32*((d*x
)^(5/2)*b*d^4 - 10*sqrt(d*x)*a*d^6)/b^3)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.28

$$\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx = -\frac{40\sqrt{dx}a^2d^8}{(bd^2x^2+ad^2)b^3} - \frac{90\sqrt{2}(ab^3d^2)^{\frac{1}{4}}ad^6\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4} - \frac{90\sqrt{2}(ab^3d^2)^{\frac{1}{4}}ad^6\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4} - \frac{4}{}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
-1/80*(40*sqrt(d*x)*a^2*d^8/((b*d^2*x^2 + a*d^2)*b^3) - 90*sqrt(2)*(a*b^3*
d^2)^(1/4)*a*d^6*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x)
)/(a*d^2/b)^(1/4))/b^4 - 90*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^6*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^4 - 45*sq
rt(2)*(a*b^3*d^2)^(1/4)*a*d^6*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
+ sqrt(a*d^2/b))/b^4 + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^6*log(d*x - sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^4 - 32*(sqrt(d*x)*b^8*d^6*x
^2 - 10*sqrt(d*x)*a*b^7*d^6)/b^10)/d
```

Mupad [B] (verification not implemented)

Time = 17.92 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.54

$$\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{2\,d^3\,(d\,x)^{5/2}}{5\,b^2} - \frac{9\,(-a)^{5/4}\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,b^{13/4}} - \frac{a^2\,d^7\,\sqrt{d\,x}}{2\,(b^4\,d^2\,x^2+a\,b^3\,d^2)} - \frac{4\,a\,d^5\,\sqrt{d\,x}}{b^3} + \frac{(-a)^{5/4}\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1\mathrm{i}}{(-a)^{1/4}\,\sqrt{d}}\right)\,9\mathrm{i}}{4\,b^{13/4}}$$

input
```
int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(2*d^3*(d*x)^(5/2))/(5*b^2) - (9*(-a)^(5/4)*d^(11/2)*atan((b^(1/4)*(d*x)^(
1/2))/((-a)^(1/4)*d^(1/2))))/(4*b^(13/4)) + ((-a)^(5/4)*d^(11/2)*atan((b^(
1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*9i)/(4*b^(13/4)) - (a^2*d^7*(d*
x)^(1/2))/(2*(a*b^3*d^2 + b^4*d^2*x^2)) - (4*a*d^5*(d*x)^(1/2))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{d}\,d^5\left(-90b^{\frac{3}{4}}a^{\frac{9}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-90b^{\frac{7}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)\right.}{}$$

input
```
int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(sqrt(d)*d**5*( - 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 90*b**(3/4)*
a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a*b*x**2 + 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a
**2 + 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqr
t(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*x**2 - 45*b**(3/4)*a**(1/4)
*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a
**2 - 45*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2
) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 45*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(
x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**2 + 45*b**(3/4)*a**
(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)
*a*b*x**2 - 360*sqrt(x)*a**2*b - 288*sqrt(x)*a*b**2*x**2 + 32*sqrt(x)*b**3
*x**4))/(80*b**4*(a + b*x**2))
```

3.447 $\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 221

$$\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4} \, dx = \frac{7d^3(dx)^{3/2}}{6b^2} - \frac{d(dx)^{7/2}}{2b(a+bx^2)} + \frac{7a^{3/4}d^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{11/4}} - \frac{7a^{3/4}d^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{11/4}} + \frac{7a^{3/4}d^{9/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}b^{11/4}}$$

output

```
7/6*d^3*(d*x)^(3/2)/b^2-1/2*d*(d*x)^(7/2)/b/(b*x^2+a)+7/8*a^(3/4)*d^(9/2)*
arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(11/4)-7/8
*a^(3/4)*d^(9/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(
1/2)/b^(11/4)+7/8*a^(3/4)*d^(9/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1
/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76

$$\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{d^4\sqrt{dx}\left(4b^{3/4}x^{3/2}(7a+4bx^2)+21\sqrt{2}a^{3/4}(a+bx^2)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+21\right.}{24b^{11/4}\sqrt{x}\,(a+bx^2)}$$

input
```
Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(d^4*Sqrt[d*x]*(4*b^(3/4)*x^(3/2)*(7*a + 4*b*x^2) + 21*Sqrt[2]*a^(3/4)*(a
+ b*x^2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] +
 21*Sqrt[2]*a^(3/4)*(a + b*x^2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/
(Sqrt[a] + Sqrt[b]*x)]))/(24*b^(11/4)*Sqrt[x]*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.48, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1380, 27, 252, 262, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^2\int \frac{(dx)^{9/2}}{b^2\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{9/2}}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{7d^2\int \frac{(dx)^{5/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{262}$$

$$\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{ad^2\int \frac{\sqrt{dx}}{bx^2+a}dx}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{826}$$

$$\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1479$$

$$7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\cdots\right.}}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{4b}\cdots$$

$$\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}$$

$\downarrow$ 25

$$7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\cdots}{\cdots}\right.}}{2\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{4b}\cdots$$

$$\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}$$

$\downarrow$ 27

$$\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{\cdots}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{4b}\cdots$$

$$\frac{d(dx)^{7/2}}{2b\,(a+bx^2)}$$

$\downarrow$ 1103

$$\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\cdots\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\cdots}}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{4b}\cdots$$

$$\frac{d(dx)^{7/2}}{2b\,(a+bx^2)}$$

input
```
Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*(d*(d*x)^(7/2))/(b*(a + b*x^2)) + (7*d^2*((2*d*(d*x)^(3/2))/(3*b) - (
2*a*d^3*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a
^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Lo
g[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sq
rt[b])))/b))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380

```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.85

method	result
derivativedivides	$2d^3\left(\frac{(dx)^{\frac{3}{2}}}{3b^2}-\frac{a d^2\left(-\frac{(dx)^{\frac{3}{2}}}{4\left(b d^2x^2+a d^2\right)}+\frac{7\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.}{32b\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\cdots\right)}{b^2}\cdots\right.$
default	$2d^3\left(\frac{(dx)^{\frac{3}{2}}}{3b^2}-\frac{a d^2\left(-\frac{(dx)^{\frac{3}{2}}}{4\left(b d^2x^2+a d^2\right)}+\frac{7\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.}{32b\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\cdots\right)}{b^2}\cdots\right.$
risch	$\frac{2x^2d^5}{3b^2\sqrt{dx}}-\frac{a\left(-\frac{(dx)^{\frac{3}{2}}}{2\left(b d^2x^2+a d^2\right)}+\frac{7\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\cdots\right.}{16b\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\cdots\right)}{b^2}\cdots$
pseudoelliptic	$-\frac{7\left(\frac{\sqrt{2}\,a\,d^2\left(b\,x^2+a\right)\ln\left(\frac{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)}{2}+\sqrt{2}\,a\,d^2\left(b\,x^2+a\right)\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}-1\right)+\sqrt{2}\,a\,d^2\left(b\,x^2+a\right)\cdots\right.}{8\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\left(b\,x^2+a\right)b^3}\cdots$

input
```
int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
2*d^3*(1/3*(d*x)^(3/2)/b^2-a*d^2/b^2*(-1/4*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)+7
/32/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2
/b)^(1/4)*(d*x)^(1/2)-1))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.31

$$\int \frac{(dx)^{9/2}}{a^2 + 2abx^2 + b^2x^4}\,dx =$$

$$-\frac{21\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}(b^3x^2+ab^2)\log\left(343\sqrt{dx}a^2d^{13}+343\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{3}{4}}b^8\right)+21\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}(-i\,b^3x^2-i\,ab^2)\log}{}$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/24*(21*(-a^3*d^18/b^11)^(1/4)*(b^3*x^2 + a*b^2)*log(343*sqrt(d*x)*a^2*d
^13 + 343*(-a^3*d^18/b^11)^(3/4)*b^8) + 21*(-a^3*d^18/b^11)^(1/4)*(-I*b^3*
x^2 - I*a*b^2)*log(343*sqrt(d*x)*a^2*d^13 + 343*I*(-a^3*d^18/b^11)^(3/4)*b
^8) + 21*(-a^3*d^18/b^11)^(1/4)*(I*b^3*x^2 + I*a*b^2)*log(343*sqrt(d*x)*a^
2*d^13 - 343*I*(-a^3*d^18/b^11)^(3/4)*b^8) - 21*(-a^3*d^18/b^11)^(1/4)*(b^
3*x^2 + a*b^2)*log(343*sqrt(d*x)*a^2*d^13 - 343*(-a^3*d^18/b^11)^(3/4)*b^8
) - 4*(4*b*d^4*x^3 + 7*a*d^4*x)*sqrt(d*x))/(b^3*x^2 + a*b^2)
```

Sympy [F]

$$\int \frac{(dx)^{9/2}}{a^2 + 2abx^2 + b^2x^4}\,dx = \int \frac{(dx)^{\frac{9}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
Integral((d*x)**(9/2)/(a + b*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.24

$$\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{\frac{24\,(dx)^{\frac{3}{2}}ad^6}{b^3d^2x^2+ab^2d^2} - \frac{21\,ad^6\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}\right.}{b}}{48\,d}$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/48*(24*(d*x)^(3/2)*a*d^6/(b^3*d^2*x^2 + a*b^2*d^2) - 21*a*d^6*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/
sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqr
t(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt
(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1
/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b
^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^2 + 32*(d*x)^(3/2)*d^4/b^2)
/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.25

$$\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{48}\left(\frac{24\,\sqrt{dx}ad^2x}{(bd^2x^2+ad^2)b^2} + \frac{32\,\sqrt{dx}x}{b^2} - \frac{42\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5d}\right.$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/48*(24*sqrt(d*x)*a*d^2*x/((b*d^2*x^2 + a*d^2)*b^2) + 32*sqrt(d*x)*x/b^2
- 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
 + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*d) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))
/(b^5*d) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*
sqrt(d*x) + sqrt(a*d^2/b))/(b^5*d) - 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x
- sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^5*d))*d^4
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.51

$$\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{2\,d^3\,(d\,x)^{3/2}}{3\,b^2} + \frac{7\,(-a)^{3/4}\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,b^{11/4}} + \frac{a\,d^5\,(d\,x)^{3/2}}{2\,(b^3\,d^2\,x^2+a\,b^2\,d^2)} + \frac{(-a)^{3/4}\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1\mathrm{i}}{(-a)^{1/4}\,\sqrt{d}}\right)\,7\mathrm{i}}{4\,b^{11/4}}$$

input
```
int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(2*d^3*(d*x)^(3/2))/(3*b^2) + (7*(-a)^(3/4)*d^(9/2)*atan((b^(1/4)*(d*x)^(1
/2))/((-a)^(1/4)*d^(1/2))))/(4*b^(11/4)) + ((-a)^(3/4)*d^(9/2)*atan((b^(1/
4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*7i)/(4*b^(11/4)) + (a*d^5*(d*x)^(
3/2))/(2*(a*b^2*d^2 + b^3*d^2*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.44

$$\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{d}\,d^4\left(42b^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+42b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2}{}$$

input

```
int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output

```
(sqrt(d)*d**4*(42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 42*b**(1/4)*a**(3/
4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*
a**(1/4)*sqrt(2)))*b*x**2 - 42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a - 42*b*
*(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b
))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**2 - 21*b**(1/4)*a**(3/4)*sqrt(2)*log(
 - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a - 21*b**(1/4
)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sq
rt(b)*x)*b*x**2 + 21*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/
4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a + 21*b**(1/4)*a**(3/4)*sqrt(2)*log(sqr
t(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 + 56*sqrt(x)*
a*b*x + 32*sqrt(x)*b**2*x**3))/(48*b**3*(a + b*x**2))
```

3.448 $\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 221

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{5d^3\sqrt{dx}}{2b^2}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5\sqrt[4]{a}d^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}d^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}d^{7/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}b^{9/4}}$$

output
```
5/2*d^3*(d*x)^(1/2)/b^2-1/2*d*(d*x)^(5/2)/b/(b*x^2+a)+5/8*a^(1/4)*d^(7/2)*
arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(9/4)-5/8*
a^(1/4)*d^(7/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1
/2)/b^(9/4)-5/8*a^(1/4)*d^(7/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2
)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{d^3\sqrt{dx}\left(4\sqrt[4]{b}\sqrt{x}(5a+4bx^2)+5\sqrt{2}\sqrt[4]{a}(a+bx^2)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-5\sqrt{2}\sqrt[4]{}\right.}{8b^{9/4}\sqrt{x}\,(a+bx^2)}$$

input
```
Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(d^3*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(5*a + 4*b*x^2) + 5*Sqrt[2]*a^(1/4)*(a +
 b*x^2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] -
5*Sqrt[2]*a^(1/4)*(a + b*x^2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(S
qrt[a] + Sqrt[b]*x)]))/(8*b^(9/4)*Sqrt[x]*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.47, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1380, 27, 252, 262, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^2\int \frac{(dx)^{7/2}}{b^2\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(dx)^{7/2}}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{5d^2\int \frac{(dx)^{3/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 262$$

$$\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{ad^2\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 266$$

$$\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\int \frac{1}{bx^2+a}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 755$$

$$\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 1476$$

$$\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b(a+bx^2)}$$

$$\Big\downarrow \; 1082$$

$$\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b(a+bx^2)}$$

$$\Big\downarrow \; 217$$

$$\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{2\ldots}\right.}{b}\right.}{4b}$$

$$\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\text{arc}}{}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}$$

$$\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{a}\sqrt{\cdots}}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\cdots}}\right.}{2\sqrt{a}}\right.}{b}\right.}{4b}$$

$$\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{d\cdots}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}\cdots\right)}{2\sqrt{a}}\cdots\right)}{b}\cdots\right)}{4b}$$

$$\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}$$

input
```
Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*(d*(d*x)^(5/2))/(b*(a + b*x^2)) + (5*d^2*((2*d*Sqrt[d*x])/b - (2*a*d*
((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4
)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log
[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sq
rt[a])))/b))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.86

method	result
derivativedivides	$2d^3\left(\frac{\sqrt{dx}}{b^2}-\frac{a\,d^2\left(-\frac{\sqrt{dx}}{4\left(b\,d^2x^2+a\,d^2\right)}+\frac{5\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2 \right.}{32a\,d^2}}{b^2}\right.$
default	$2d^3\left(\frac{\sqrt{dx}}{b^2}-\frac{a\,d^2\left(-\frac{\sqrt{dx}}{4\left(b\,d^2x^2+a\,d^2\right)}+\frac{5\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2 \right.}{32a\,d^2}}{b^2}\right.$
risch	$\frac{2x\,d^4}{b^2\sqrt{dx}}-\frac{2a\,d^5\left(-\frac{\sqrt{dx}}{4\left(b\,d^2x^2+a\,d^2\right)}+\frac{5\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arcta} \right.}{32a\,d^2}}{b^2}$
pseudoelliptic	$\frac{d^3\left(2\sqrt{dx}+\frac{\sqrt{dx}\,a}{2b\,x^2+2a}-\frac{5\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}}{16}-\frac{5\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}}{8}-\frac{5\arctan\left(\right.}{} \right.}{b^2}$

input
```
int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
2*d^3*((d*x)^(1/2)/b^2-1/b^2*a*d^2*(-1/4*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)+5/3
2*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d
^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.18

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx =$$
$$-\frac{5\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}(b^3x^2+ab^2)\log\left(5\sqrt{dx}d^3+5\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}b^2\right)+5\left(-\frac{ad^{14}}{b^9}\right)^{\frac{1}{4}}(i\,b^3x^2+i\,ab^2)\log\left(5\sqrt{dx}d^3+5\right.}{}$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/8*(5*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*sqrt(d*x)*d^3 + 5*(-a*
d^14/b^9)^(1/4)*b^2) + 5*(-a*d^14/b^9)^(1/4)*(I*b^3*x^2 + I*a*b^2)*log(5*s
qrt(d*x)*d^3 + 5*I*(-a*d^14/b^9)^(1/4)*b^2) + 5*(-a*d^14/b^9)^(1/4)*(-I*b^
3*x^2 - I*a*b^2)*log(5*sqrt(d*x)*d^3 - 5*I*(-a*d^14/b^9)^(1/4)*b^2) - 5*(-
a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*sqrt(d*x)*d^3 - 5*(-a*d^14/b^9)^
(1/4)*b^2) - 4*(4*b*d^3*x^2 + 5*a*d^3)*sqrt(d*x))/(b^3*x^2 + a*b^2)
```

Sympy [F]

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx = \int \frac{(dx)^{\frac{7}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
Integral((d*x)**(7/2)/(a + b*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.28

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{\frac{8\sqrt{dx}ad^6}{b^3d^2x^2+ab^2d^2}+\frac{32\sqrt{dx}d^4}{b^2}-\frac{5\left(\frac{\sqrt{2}d^6\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^6\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{d\ldots}\right.}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}\ldots}{\ldots}}{16\ldots}$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/16*(8*sqrt(d*x)*a*d^6/(b^3*d^2*x^2 + a*b^2*d^2) + 32*sqrt(d*x)*d^4/b^2 -
 5*(sqrt(2)*d^6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sqrt(
2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) +
2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) +
2*sqrt(2)*d^5*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(
d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*
a/b^2)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.26

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{\frac{8\sqrt{dx}ad^6}{(bd^2x^2+ad^2)b^2}-\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d^4\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}-\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d^4\arctan\left(-\ldots\right.}{b^3}\ldots}{\ldots}$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/16*(8*sqrt(d*x)*a*d^6/((b*d^2*x^2 + a*d^2)*b^2) - 10*sqrt(2)*(a*b^3*d^2)
^(1/4)*d^4*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/b^3 - 10*sqrt(2)*(a*b^3*d^2)^(1/4)*d^4*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 - 5*sqrt(2)*(a*
b^3*d^2)^(1/4)*d^4*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^
2/b))/b^3 + 5*sqrt(2)*(a*b^3*d^2)^(1/4)*d^4*log(d*x - sqrt(2)*(a*d^2/b)^(1
/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 + 32*sqrt(d*x)*d^4/b^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.51

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{2\,d^3\,\sqrt{d\,x}}{b^2} - \frac{5\,{(-a)}^{1/4}\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{4\,b^{9/4}} + \frac{a\,d^5\,\sqrt{d\,x}}{2\,\left(b^3\,d^2\,x^2+a\,b^2\,d^2\right)} + \frac{{(-a)}^{1/4}\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{(-a)}^{1/4}\,\sqrt{d}}\right)\,5{}\mathrm{i}}{4\,b^{9/4}}$$

input
```
int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(2*d^3*(d*x)^(1/2))/b^2 - (5*(-a)^(1/4)*d^(7/2)*atan((b^(1/4)*(d*x)^(1/2))
/((-a)^(1/4)*d^(1/2))))/(4*b^(9/4)) + ((-a)^(1/4)*d^(7/2)*atan((b^(1/4)*(d
*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*5i)/(4*b^(9/4)) + (a*d^5*(d*x)^(1/2))/
(2*(a*b^2*d^2 + b^3*d^2*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.44

$$\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{d}\,d^3\left(10b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+10b^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2}{}$$

input
```
int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output

```
(sqrt(d)*d**3*(10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 10*b**(3/4)*a**(1/
4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*
a**(1/4)*sqrt(2)))*b*x**2 - 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a - 10*b*
*(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b
))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**2 + 5*b**(3/4)*a**(1/4)*sqrt(2)*log(
- sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a + 5*b**(3/4)*
a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt
(b)*x)*b*x**2 - 5*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*
sqrt(2) + sqrt(a) + sqrt(b)*x)*a - 5*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)
*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 + 40*sqrt(x)*a*b
+ 32*sqrt(x)*b**2*x**2))/(16*b**3*(a + b*x**2))
```

3.449 $\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx=-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}-\frac{3d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{3d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3d^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

output
```
-1/2*d*(d*x)^(3/2)/b/(b*x^2+a)-3/8*d^(5/2)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^
(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(1/4)/b^(7/4)+3/8*d^(5/2)*arctan(1+2^(1/2
)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(1/4)/b^(7/4)-3/8*d^(5/2)
*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*
2^(1/2)/a^(1/4)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.75

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx = -\frac{(dx)^{5/2}\left(4\sqrt[4]{a}b^{3/4}x^{3/2}+3\sqrt{2}(a+bx^2)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+3\sqrt{2}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)\right)}{8\sqrt[4]{a}b^{7/4}x^{5/2}\left(a+bx^2\right)}$$

input
```
Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/8*((d*x)^(5/2)*(4*a^(1/4)*b^(3/4)*x^(3/2) + 3*Sqrt[2]*(a + b*x^2)*ArcTa
n[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(a
+ b*x^2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])
)/(a^(1/4)*b^(7/4)*x^(5/2)*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1380, 27, 252, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}dx$$

$$\Big\downarrow \text{1380}$$

$$b^2\int \frac{(dx)^{5/2}}{b^2\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{5/2}}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{3d^2\int \frac{\sqrt{dx}}{bx^2+a}dx}{4b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 266

$$\frac{3d\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{3d^3\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 826

$$\frac{3d^3\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{3d^3\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{3d^3\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 217

$$\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 1479

$$\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 25

$$\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

input
```
Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*(d*(d*x)^(3/2))/(b*(a + b*x^2)) + (3*d^3*((-(ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log
[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.84

method	result
derivativedivides	$2d^3\left(-\frac{(dx)^{\frac{3}{2}}}{4b(b\,d^2x^2+a\,d^2)}+\frac{3\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{}}{\left(\frac{a\,d^2}{b}\right.}\right.\right.}{32b^2\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$
default	$2d^3\left(-\frac{(dx)^{\frac{3}{2}}}{4b(b\,d^2x^2+a\,d^2)}+\frac{3\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{}}{\left(\frac{a\,d^2}{b}\right.}\right.\right.}{32b^2\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$
pseudoelliptic	$\frac{\left(-8x\sqrt{dx}\,b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}+3d\sqrt{2}\,(b\,x^2+a)\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\right.\right.}{16\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}(b\,x^2+a)b^2}$

input
```
int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
2*d^3*(-1/4/b*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)+3/32/b^2/(a*d^2/b)^(1/4)*2^(1/
2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/
b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.27

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx = -\frac{4\sqrt{dx}d^2x - 3\left(b^2x^2+ab\right)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\log\left(27\sqrt{dx}d^7+27\left(-\frac{d^{10}}{ab^7}\right)^{\frac{3}{4}}ab^5\right)+3\left(i\,b^2x^2+i\,ab\right)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\log\left(27\right.}{}$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/8*(4*sqrt(d*x)*d^2*x - 3*(b^2*x^2 + a*b)*(-d^10/(a*b^7))^(1/4)*log(27*s
qrt(d*x)*d^7 + 27*(-d^10/(a*b^7))^(3/4)*a*b^5) + 3*(I*b^2*x^2 + I*a*b)*(-d
^10/(a*b^7))^(1/4)*log(27*sqrt(d*x)*d^7 + 27*I*(-d^10/(a*b^7))^(3/4)*a*b^5
) + 3*(-I*b^2*x^2 - I*a*b)*(-d^10/(a*b^7))^(1/4)*log(27*sqrt(d*x)*d^7 - 27
*I*(-d^10/(a*b^7))^(3/4)*a*b^5) + 3*(b^2*x^2 + a*b)*(-d^10/(a*b^7))^(1/4)*
log(27*sqrt(d*x)*d^7 - 27*(-d^10/(a*b^7))^(3/4)*a*b^5))/(b^2*x^2 + a*b)
```

Sympy [F]

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
Integral((d*x)**(5/2)/(a + b*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.25

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx = -\frac{\frac{8(dx)^{\frac{3}{2}}d^4}{b^2d^2x^2+abd^2} - \frac{3d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}(ad^2)^{\frac{1}{4}}\sqrt{dx}\cdots\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}\cdots\right)}{b}}{16d}$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/16*(8*(d*x)^(3/2)*d^4/(b^2*d^2*x^2 + a*b*d^2) - 3*d^4*(2*sqrt(2)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sq
rt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(
a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x
 + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(
3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
+ sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(136) = 272.

Time = 0.12 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.36

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx = -\frac{1}{16}\left(\frac{8\sqrt{dx}d^2x}{(bd^2x^2+ad^2)b} - \frac{6\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4d} - \frac{6\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\cdots\right.\right.}{2\left(\cdots\right.}\cdots\right)}{ab^4d}\cdots\right)$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
-1/16*(8*sqrt(d*x)*d^2*x/((b*d^2*x^2 + a*d^2)*b) - 6*sqrt(2)*(a*b^3*d^2)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^
(1/4))/(a*b^4*d) - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^4*d) + 3*sqrt(2)*(a
*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a*b^4*d) - 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4*d))*d^2
```

Mupad [B] (verification not implemented)

Time = 18.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.45

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{3\,d^{5/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{1/4}\,b^{7/4}} - \frac{3\,d^{5/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{1/4}\,b^{7/4}} - \frac{d^3\,(d\,x)^{3/2}}{2\,b\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

input
```
int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(3*d^(5/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(1/4)
*b^(7/4)) - (3*d^(5/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/
(4*(-a)^(1/4)*b^(7/4)) - (d^3*(d*x)^(3/2))/(2*b*(a*d^2 + b*d^2*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.53

$$\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{d}\,d^2\left(-6b^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-6b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2}{}$$

input
```
int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(sqrt(d)*d**2*( - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a - 6*b**(1/4)*a**(3
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*b*x**2 + 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 6*b**
(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**2 + 3*b**(1/4)*a**(3/4)*sqrt(2)*log( -
 sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a + 3*b**(1/4)*a
**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(
b)*x)*b*x**2 - 3*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*s
qrt(2) + sqrt(a) + sqrt(b)*x)*a - 3*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*
b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 - 8*sqrt(x)*a*b*x)
)/(16*a*b**2*(a + b*x**2))
```

3.450 $\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx = -\frac{d\sqrt{dx}}{2b\,(a+bx^2)} - \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

output
```
-1/2*d*(d*x)^(1/2)/b/(b*x^2+a)-1/8*d^(3/2)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^
(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(3/4)/b^(5/4)+1/8*d^(3/2)*arctan(1+2^(1/2
)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(3/4)/b^(5/4)+1/8*d^(3/2)
*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*
2^(1/2)/a^(3/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.75

$$\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{(dx)^{3/2}\left(-4a^{3/4}\sqrt[4]{b}\sqrt{x}-\sqrt{2}(a+bx^2)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\sqrt{2}(a+bx^2)\text{ar}\ldots\right.}{8a^{3/4}b^{5/4}x^{3/2}(a+bx^2)}$$

input
```
Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
((d*x)^(3/2)*(-4*a^(3/4)*b^(1/4)*Sqrt[x] - Sqrt[2]*(a + b*x^2)*ArcTan[(Sqr
t[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]*(a + b*x^2)
*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(8*a^(
3/4)*b^(5/4)*x^(3/2)*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1380, 27, 252, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}dx$$

$$\Big\downarrow \text{1380}$$

$$b^2\int \frac{(dx)^{3/2}}{b^2(bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{3/2}}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{d^2 \int \frac{1}{\sqrt{dx}(bx^2+a)}dx}{4b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 266$$

$$\frac{d \int \frac{1}{bx^2+a}d\sqrt{dx}}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 755$$

$$\frac{d\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{d\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1476$$

$$\frac{d\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{d\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{d\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

$\downarrow$ 1479

$$\frac{d\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}$$

$$\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

$\downarrow$ 25

$$\frac{d\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}$$

$$\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{d\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}-$$

$$\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{d\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}$$

$$\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

input
```
Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/2*(d*Sqrt[d*x])/(b*(a + b*x^2)) + (d*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Lo
g[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.87

method	result
derivativedivides	$2d^3\left(-\frac{\sqrt{dx}}{4b(b\,d^2x^2+a\,d^2)}+\frac{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{32b\,d^2a}\right.$
default	$2d^3\left(-\frac{\sqrt{dx}}{4b(b\,d^2x^2+a\,d^2)}+\frac{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{32b\,d^2a}\right.$
pseudoelliptic	$\frac{d\left(\frac{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(b\,x^2+a\right)\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)}{2}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(b\,x^2+a\right)\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\right.}{8b(b\,x^2+a)a}$

input
```
int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
2*d^3*(-1/4/b*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)+1/32/b*(a*d^2/b)^(1/4)/d^2/a*2
^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*
d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1
)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.15

$$\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{(b^2x^2+ab)\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}+\sqrt{dx}d\right)-(-i\,b^2x^2-i\,ab)\left(-\frac{d^6}{a^3b^5}\right)^{\frac{1}{4}}\,\mathrm{l}}{}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/8*((b^2*x^2 + a*b)*(-d^6/(a^3*b^5))^(1/4)*log(a*b*(-d^6/(a^3*b^5))^(1/4)
 + sqrt(d*x)*d) - (-I*b^2*x^2 - I*a*b)*(-d^6/(a^3*b^5))^(1/4)*log(I*a*b*(-
d^6/(a^3*b^5))^(1/4) + sqrt(d*x)*d) - (I*b^2*x^2 + I*a*b)*(-d^6/(a^3*b^5))
^(1/4)*log(-I*a*b*(-d^6/(a^3*b^5))^(1/4) + sqrt(d*x)*d) - (b^2*x^2 + a*b)*
(-d^6/(a^3*b^5))^(1/4)*log(-a*b*(-d^6/(a^3*b^5))^(1/4) + sqrt(d*x)*d) - 4*
sqrt(d*x)*d)/(b^2*x^2 + a*b)
```

Sympy [F]

$$\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
Integral((d*x)**(3/2)/(a + b*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.30

$$\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx = -\frac{\frac{8\sqrt{dx}d^4}{b^2d^2x^2+abd^2}-\frac{\frac{\sqrt{2}d^4\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^4\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}d^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}}{b}}{16\,d}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/16*(8*sqrt(d*x)*d^4/(b^2*d^2*x^2 + a*b*d^2) - (sqrt(2)*d^4*log(sqrt(b)*
d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*
b^(1/4)) - sqrt(2)*d^4*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b
^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^3*arctan(1/2*sqr
t(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sq
rt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^3*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*s
qrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/b)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(136) = 272.

Time = 0.12 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.35

$$\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx = -\frac{\frac{8\sqrt{dx}d^4}{(bd^2x^2+ad^2)b}-\frac{2\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^2}-\frac{2\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^2}-\sqrt{2}(ab^3}{16\,d}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
-1/16*(8*sqrt(d*x)*d^4/((b*d^2*x^2 + a*d^2)*b) - 2*sqrt(2)*(a*b^3*d^2)^(1/
4)*d^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/(a*b^2) - 2*sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^2) - sqrt(2)*(a
*b^3*d^2)^(1/4)*d^2*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d
^2/b))/(a*b^2) + sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*log(d*x - sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 17.80 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.45

$$\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx = -\frac{d^{3/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{3/4}\,b^{5/4}} - \frac{d^{3/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{3/4}\,b^{5/4}} - \frac{d^3\,\sqrt{d\,x}}{2\,b\,(b\,d^2\,x^2+a\,d^2)}$$

input
```
int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
- (d^(3/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(3/4)
*b^(5/4)) - (d^(3/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4
*(-a)^(3/4)*b^(5/4)) - (d^3*(d*x)^(1/2))/(2*b*(a*d^2 + b*d^2*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.50

$$\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{d}\,d\left(-2b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2b^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2}{}$$

input
```
int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(sqrt(d)*d*( - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a - 2*b**(3/4)*a**(1/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*b*x**2 + 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1
/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 2*b**(3/
4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*b*x**2 - b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt
(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a - b**(3/4)*a**(1/4)
*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b
*x**2 + b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) +
sqrt(a) + sqrt(b)*x)*a + b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a*
*(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 - 8*sqrt(x)*a*b))/(16*a*b**2*
(a + b*x**2))
```

3.451 $\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 206

$$\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx = \frac{(dx)^{3/2}}{2ad\,(a+bx^2)} - \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}} - \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}$$

output
```
1/2*(d*x)^(3/2)/a/d/(b*x^2+a)-1/8*d^(1/2)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(
1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(5/4)/b^(3/4)+1/8*d^(1/2)*arctan(1+2^(1/2)
*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(5/4)/b^(3/4)-1/8*d^(1/2)*
arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2
^(1/2)/a^(5/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx = -\frac{\sqrt{dx}\left(-4\sqrt[4]{a}b^{3/4}x^{3/2}+\sqrt{2}(a+bx^2)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\sqrt{2}(a+bx^2)\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)\right)}{8a^{5/4}b^{3/4}\sqrt{x}\,(a+bx^2)}$$

input
```
Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
-1/8*(Sqrt[d*x]*(-4*a^(1/4)*b^(3/4)*x^(3/2) + Sqrt[2]*(a + b*x^2)*ArcTan[(
Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]*(a + b*x
^2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(a^
(5/4)*b^(3/4)*Sqrt[x]*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.46, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1380, 27, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}dx$$

$$\Big\downarrow \ 1380$$

$$b^2\int \frac{\sqrt{dx}}{b^2\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{\sqrt{dx}}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \ 253$$

$$\frac{\int \frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 266

$$\frac{\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{d\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 826

$$\frac{d\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{d\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{d\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 217

$$\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 1479

$$\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right.}{2a}$$

$$\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 25

$$\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}$$

$$\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}+$$

$$\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

input
```
Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(d*x)^(3/2)/(2*a*d*(a + b*x^2)) + (d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[
d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*
d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.87

method	result
derivativedivides	$2d^3\left(\frac{(dx)^{\frac{3}{2}}}{4a\,d^2(b\,d^2x^2+a\,d^2)}+\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d}\cdots}{\left(\frac{a\,d^2}{b}\right)\cdots}\right.\cdots}{32a\,d^2b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\cdots$
default	$2d^3\left(\frac{(dx)^{\frac{3}{2}}}{4a\,d^2(b\,d^2x^2+a\,d^2)}+\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d}\cdots}{\left(\frac{a\,d^2}{b}\right)\cdots}\right.\cdots}{32a\,d^2b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\cdots$
pseudoelliptic	$\frac{8x\sqrt{dx}\,b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}+d\sqrt{2}\,(b\,x^2+a)\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right.\cdots}{16\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b(b\,x^2+a)a}$

input
```
int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
2*d^3*(1/4*(d*x)^(3/2)/a/d^2/(b*d^2*x^2+a*d^2)+1/32/a/d^2/b/(a*d^2/b)^(1/4
)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d
*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/
(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2
)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx = \frac{(abx^2+a^2)\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}\log\left(a^4b^2\left(-\frac{d^2}{a^5b^3}\right)^{\frac{3}{4}}+\sqrt{dx}d\right)-(i\,abx^2+i\,a^2)\left(-\frac{d^2}{a^5b^3}\right)^{\frac{1}{4}}\log\left(i\,a^4b^2\left(-\frac{d^2}{a^5b^3}\right)^{\frac{3}{4}}+\sqrt{}\right.}{}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/8*((a*b*x^2 + a^2)*(-d^2/(a^5*b^3))^(1/4)*log(a^4*b^2*(-d^2/(a^5*b^3))^(
3/4) + sqrt(d*x)*d) - (I*a*b*x^2 + I*a^2)*(-d^2/(a^5*b^3))^(1/4)*log(I*a^4
*b^2*(-d^2/(a^5*b^3))^(3/4) + sqrt(d*x)*d) - (-I*a*b*x^2 - I*a^2)*(-d^2/(a
^5*b^3))^(1/4)*log(-I*a^4*b^2*(-d^2/(a^5*b^3))^(3/4) + sqrt(d*x)*d) - (a*b
*x^2 + a^2)*(-d^2/(a^5*b^3))^(1/4)*log(-a^4*b^2*(-d^2/(a^5*b^3))^(3/4) + s
qrt(d*x)*d) + 4*sqrt(d*x)*x)/(a*b*x^2 + a^2)
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx = \int \frac{\sqrt{dx}}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
Integral(sqrt(d*x)/(a + b*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{dx}}{a^2 + 2abx^2 + b^2x^4}\,dx = \frac{\frac{8\,(dx)^{\frac{3}{2}}d^2}{abd^2x^2+a^2d^2} + \frac{d^2\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b\right.}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{a}}{16\,d}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/16*(8*(d*x)^(3/2)*d^2/(a*b*d^2*x^2 + a^2*d^2) + d^2*(2*sqrt(2)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(
a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*
sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x +
sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4
)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + s
qrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{dx}}{a^2 + 2abx^2 + b^2x^4}\,dx = \frac{\frac{8\,\sqrt{dx}d^3x}{(bd^2x^2+ad^2)a} + \frac{2\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3} + \frac{2\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3} - \frac{\sqrt{2}(ab^3d^2)}{}}{16\,d}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/16*(8*sqrt(d*x)*d^3*x/((b*d^2*x^2 + a*d^2)*a) + 2*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^2*b^3) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^3) - sqrt(2)*(a*b^
3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(a^2*b^3) + sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(a^2*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{d}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{5/4}\,b^{3/4}} - \frac{\sqrt{d}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{4\,(-a)^{5/4}\,b^{3/4}} + \frac{d\,(d\,x)^{3/2}}{2\,a\,\left(b\,d^2\,x^2+a\,d^2\right)}$$

input
```
int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(d^(1/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(5/4)*
b^(3/4)) - (d^(1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(
-a)^(5/4)*b^(3/4)) + (d*(d*x)^(3/2))/(2*a*(a*d^2 + b*d^2*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{d}\left(-2b^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2+2b^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+\cdots}{b^{\frac{1}{4}}a^{\frac{1}{4}}\cdots}\right.\right.}{}$$

input
```
int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(sqrt(d)*( - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a - 2*b**(1/4)*a**(3/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*b*x**2 + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 2*b**(1/4)
*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*b*x**2 + b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x
)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a + b**(1/4)*a**(3/4)*s
qrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x
**2 - b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sq
rt(a) + sqrt(b)*x)*a - b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(
1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 + 8*sqrt(x)*a*b*x))/(16*a**2*b*
(a + b*x**2))
```

3.452 $\int \frac{1}{\sqrt{dx}(a^2+2abx^2+b^2x^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 206

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)}\,dx=\frac{\sqrt{dx}}{2ad\,(a+bx^2)}-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}$$

output
```
1/2*(d*x)^(1/2)/a/d/(b*x^2+a)-3/8*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(
1/4)/d^(1/2))*2^(1/2)/a^(7/4)/b^(1/4)/d^(1/2)+3/8*arctan(1+2^(1/2)*b^(1/4)
*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(7/4)/b^(1/4)/d^(1/2)+3/8*arctanh(
2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a
^(7/4)/b^(1/4)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)}\,dx = \frac{\sqrt{x}\left(4a^{3/4}\sqrt[4]{b}\sqrt{x}-3\sqrt{2}(a+bx^2)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+3\sqrt{2}(a+bx^2)\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)\right)}{8a^{7/4}\sqrt[4]{b}\sqrt{dx}\,(a+bx^2)}$$

input
```
Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
(Sqrt[x]*(4*a^(3/4)*b^(1/4)*Sqrt[x] - 3*Sqrt[2]*(a + b*x^2)*ArcTan[(Sqrt[a
] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(a + b*x^2)*
ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(8*a^(7
/4)*b^(1/4)*Sqrt[d*x]*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1380, 27, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^2\int \frac{1}{b^2\sqrt{dx}\,(bx^2+a)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\sqrt{dx}\,(a+bx^2)^2}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{3\int \frac{1}{\sqrt{dx}(bx^2+a)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 266

$$\frac{3\int \frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 755

$$\frac{3\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 217

$$\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}$$

$$\Big\downarrow \ 1479$$

$$\frac{3\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{a}\sqrt{d}}\cdots\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right.}{2\sqrt{a}}\cdots\right.}{2ad}\cdots$$

$$\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+$$

$$\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}$$

$$\frac{\sqrt{dx}}{2ad\,(a+bx^2)}$$

input
```
Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
Sqrt[d*x]/(2*a*d*(a + b*x^2)) + (3*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqr
t[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380

```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86

method	result
derivativedivides	$2d^3\left(\frac{\sqrt{dx}}{4d^2a\left(b\,d^2x^2+a\,d^2\right)}+\frac{3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\cdots\right.}{32d^4a^2}\right.$
default	$2d^3\left(\frac{\sqrt{dx}}{4d^2a\left(b\,d^2x^2+a\,d^2\right)}+\frac{3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\cdots\right.}{32d^4a^2}\right.$
pseudoelliptic	$\frac{8\sqrt{dx}\,a+3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(b\,x^2+a\right)\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\cdots\right.}{16d\,a^2\left(b\,x^2+a\right)}$

input
```
int(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
2*d^3*(1/4*(d*x)^(1/2)/d^2/a/(b*d^2*x^2+a*d^2)+3/32/d^4/a^2*(a*d^2/b)^(1/4
)*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d
*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/
(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2
)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)}\,dx$$
$$= \frac{3\,(abdx^2+a^2d)\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}\log\left(a^2d\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-3\,(-i\,abdx^2-i\,a^2d)\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}\log\left(i\,a^2d\left(-\frac{1}{a^7bd^2}\right.\right.}{}$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/8*(3*(a*b*d*x^2 + a^2*d)*(-1/(a^7*b*d^2))^(1/4)*log(a^2*d*(-1/(a^7*b*d^2
))^(1/4) + sqrt(d*x)) - 3*(-I*a*b*d*x^2 - I*a^2*d)*(-1/(a^7*b*d^2))^(1/4)*
log(I*a^2*d*(-1/(a^7*b*d^2))^(1/4) + sqrt(d*x)) - 3*(I*a*b*d*x^2 + I*a^2*d
)*(-1/(a^7*b*d^2))^(1/4)*log(-I*a^2*d*(-1/(a^7*b*d^2))^(1/4) + sqrt(d*x))
- 3*(a*b*d*x^2 + a^2*d)*(-1/(a^7*b*d^2))^(1/4)*log(-a^2*d*(-1/(a^7*b*d^2))
^(1/4) + sqrt(d*x)) + 4*sqrt(d*x))/(a*b*d*x^2 + a^2*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)}\,dx = \int \frac{1}{\sqrt{dx}\,(a+bx^2)^2}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
Integral(1/(sqrt(d*x)*(a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)}\,dx$$

$$= \frac{\frac{8\sqrt{dx}d^2}{abd^2x^2+a^2d^2} + \frac{3\left(\frac{\sqrt{2}d^2\log\left(\sqrt{b}dx+\sqrt{2}(ad^2)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}d^2\log\left(\sqrt{b}dx-\sqrt{2}(ad^2)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{d\cdots}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}\cdots\right)}{a}}{16\,d}$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/16*(8*sqrt(d*x)*d^2/(a*b*d^2*x^2 + a^2*d^2) + 3*(sqrt(2)*d^2*log(sqrt(b)
*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)
*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*
b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d*arctan(1/2*sqrt
(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqr
t(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d*arctan(-1/2*sqrt(
2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt
(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/a)/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)}\,dx = \frac{\sqrt{dx}d}{2\,(bd^2x^2+ad^2)a} + \frac{3\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2bd} + \frac{3\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2bd} + \frac{3\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^2bd} - \frac{3\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^2bd}$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*sqrt(d*x)*d/((b*d^2*x^2 + a*d^2)*a) + 3/8*sqrt(2)*(a*b^3*d^2)^(1/4)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/
(a^2*b*d) + 3/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b*d) + 3/16*sqrt(2)*(a*b
^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a^2*b*d) - 3/16*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1
/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b*d)
```

Mupad [B] (verification not implemented)

Time = 17.91 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)}\,dx = \frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{d\,x}}{(-a)^{1/4}\sqrt{d}}\right)}{4\,(-a)^{7/4}\,b^{1/4}\sqrt{d}} + \frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\sqrt{d\,x}}{(-a)^{1/4}\sqrt{d}}\right)}{4\,(-a)^{7/4}\,b^{1/4}\sqrt{d}} + \frac{d\sqrt{d\,x}}{2\,a\,(b\,d^2\,x^2+a\,d^2)}$$

input
```
int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

output
```
(3*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^(7/4)*b^(1/4)
*d^(1/2)) + (3*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(4*(-a)^
(7/4)*b^(1/4)*d^(1/2)) + (d*(d*x)^(1/2))/(2*a*(a*d^2 + b*d^2*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)}\,dx = \frac{\sqrt{d}\left(-6b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-6b^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2+6b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right.}{}$$

input
```
int(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(sqrt(d)*( - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a - 6*b**(3/4)*a**(1/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*b*x**2 + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 6*b**(3/4)
*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*b*x**2 - 3*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt
(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a - 3*b**(3/4)*a**(1/
4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)
*b*x**2 + 3*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2
) + sqrt(a) + sqrt(b)*x)*a + 3*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1
/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 + 8*sqrt(x)*a*b))/(16*a
**2*b*d*(a + b*x**2))
```

3.453 $\int \frac{1}{(dx)^{3/2}(a^2+2abx^2+b^2x^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 223

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = -\frac{5}{2a^2d\sqrt{dx}} + \frac{1}{2ad\sqrt{dx}\,(a+bx^2)} + \frac{5\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{9/4}d^{3/2}} - \frac{5\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{9/4}d^{3/2}} + \frac{5\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}a^{9/4}d^{3/2}}$$

output
```
-5/2/a^2/d/(d*x)^(1/2)+1/2/a/d/(d*x)^(1/2)/(b*x^2+a)+5/8*b^(1/4)*arctan(1-
2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(9/4)/d^(3/2)-5/8*b
^(1/4)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(9/
4)/d^(3/2)+5/8*b^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)
/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(9/4)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.74

$$\int \frac{1}{(dx)^{3/2} (a^2 + 2abx^2 + b^2x^4)} dx = \frac{x\left(-4\sqrt[4]{a}(4a + 5bx^2) + 5\sqrt{2}\sqrt[4]{b}\sqrt{x}(a + bx^2)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + \right.}{8a^{9/4}(dx)^{3/2}(a + bx^2)}$$

input
```
Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
(x*(-4*a^(1/4)*(4*a + 5*b*x^2) + 5*Sqrt[2]*b^(1/4)*Sqrt[x]*(a + b*x^2)*Arc
Tan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*Sqrt[2]*b
^(1/4)*Sqrt[x]*(a + b*x^2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt
[a] + Sqrt[b]*x)]))/(8*a^(9/4)*(d*x)^(3/2)*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.46, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1380, 27, 253, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2} (a^2 + 2abx^2 + b^2x^4)} dx$$

$$\Big\downarrow \; 1380$$

$$b^2 \int \frac{1}{b^2(dx)^{3/2} (bx^2 + a)^2} dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{1}{(dx)^{3/2} (a + bx^2)^2} dx$$

$$\Big\downarrow \; 253$$

$$\frac{5\int \frac{1}{(dx)^{3/2}(bx^2+a)} dx}{4a} + \frac{1}{2ad\sqrt{dx}\,(a + bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{5\left(-\frac{b\int \frac{\sqrt{dx}}{bx^2+a}dx}{ad^2}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\,(a+bx^2)}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(-\frac{2b\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{ad^3}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(-\frac{2b\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\,(a+bx^2)}$$

$$\Big\downarrow \text{826}$$

$$\frac{5\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\,(a+bx^2)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\,(a+bx^2)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\,(a+bx^2)}$$

$\Big\downarrow$ 217

$$\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\,(a+bx^2)}$$

$\Big\downarrow$ 1479

$$5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}\cdots\right)}{4a}$$

$$\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}$$

$\downarrow$ 25

$$5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}\cdots\right)}{4a}$$

$$\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}$$

$\downarrow$ 27

$$\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}$$

$$\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}-\dots\right)}{4a}$$

$$\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}$$

input
```
Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
1/(2*a*d*Sqrt[d*x]*(a + b*x^2)) + (5*(-2/(a*d*Sqrt[d*x]) - (2*b*((-(ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt
[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(a*d)))/(
4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{2}{a^2 d\sqrt{dx}}-\frac{b\left(\frac{(dx)^{\frac{3}{2}}}{2b\,d^2x^2+2a\,d^2}+\frac{5\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\cdots\right.\right)}{16b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2 d}$
derivativedivides	$2d^3\left(-\frac{1}{a^2d^4\sqrt{dx}}-\frac{b\left(\frac{(dx)^{\frac{3}{2}}}{4b\,d^2x^2+4a\,d^2}+\frac{5\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.\right)}{32b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^4}\cdots\right.$
default	$2d^3\left(-\frac{1}{a^2d^4\sqrt{dx}}-\frac{b\left(\frac{(dx)^{\frac{3}{2}}}{4b\,d^2x^2+4a\,d^2}+\frac{5\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.\right)}{32b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^4}\cdots\right.$
pseudoelliptic	$-\frac{5\left(\frac{\sqrt{2}\left(b\,x^2+a\right)\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)\right)\sqrt{\cdots}}{2}\cdots\right)}{8\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,d\,a^2(b\,x^2+a)}$

input
```
int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-2/a^2/d/(d*x)^(1/2)-1/a^2*b*(1/2*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)+5/16/b/(a*
d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arct
an(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)
*(d*x)^(1/2)-1)))/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.32

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx =$$
$$-\frac{5\left(a^2bd^2x^3+a^3d^2x\right)\left(-\frac{b}{a^9d^6}\right)^{\frac{1}{4}}\log\left(125\,a^7d^5\left(-\frac{b}{a^9d^6}\right)^{\frac{3}{4}}+125\,\sqrt{dx}b\right)+5\left(-i\,a^2bd^2x^3-i\,a^3d^2x\right)\left(-\frac{b}{a^9d^6}\right)^{\frac{1}{4}}\mathrm{lo}}{}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/8*(5*(a^2*b*d^2*x^3 + a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*log(125*a^7*d^5*(
-b/(a^9*d^6))^(3/4) + 125*sqrt(d*x)*b) + 5*(-I*a^2*b*d^2*x^3 - I*a^3*d^2*x
)*(-b/(a^9*d^6))^(1/4)*log(125*I*a^7*d^5*(-b/(a^9*d^6))^(3/4) + 125*sqrt(d
*x)*b) + 5*(I*a^2*b*d^2*x^3 + I*a^3*d^2*x)*(-b/(a^9*d^6))^(1/4)*log(-125*I
*a^7*d^5*(-b/(a^9*d^6))^(3/4) + 125*sqrt(d*x)*b) - 5*(a^2*b*d^2*x^3 + a^3*
d^2*x)*(-b/(a^9*d^6))^(1/4)*log(-125*a^7*d^5*(-b/(a^9*d^6))^(3/4) + 125*sq
rt(d*x)*b) + 4*(5*b*x^2 + 4*a)*sqrt(d*x))/(a^2*b*d^2*x^3 + a^3*d^2*x)
```

Sympy [F]

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\left(a+bx^2\right)^2}\,dx$$

input
```
integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
Integral(1/((d*x)**(3/2)*(a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.20

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = -\frac{\frac{8\,(5\,bd^2x^2+4\,ad^2)}{(dx)^{\frac{5}{2}}a^2b+\sqrt{dx}a^3d^2}+\frac{5\,b\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)\right.}{\left(ad^2\right)^{\frac{1}{4}}b}\right.}{a^2}}{16\,d}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/16*(8*(5*b*d^2*x^2 + 4*a*d^2)/((d*x)^(5/2)*a^2*b + sqrt(d*x)*a^3*d^2) +
 5*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt
(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b))
+ 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*
x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - s
qrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)
*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1
/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.32

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = -\frac{\frac{8\,(5\,bd^2x^2+4\,ad^2)}{\left(\sqrt{dx}bd^2x^2+\sqrt{dx}ad^2\right)a^2}+\frac{10\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2d^2}+\frac{10\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2d^2}-}{16\,d}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
-1/16*(8*(5*b*d^2*x^2 + 4*a*d^2)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)*
a^2) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2*d^2) + 10*sqrt(2)*(a*b^3*d^
2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^
2/b)^(1/4))/(a^3*b^2*d^2) - 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2*d^2) + 5*sqrt(2)*(a*b^
3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(a^3*b^2*d^2))/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.46

$$\int \frac{1}{(dx)^{3/2}\,(a^2+2abx^2+b^2x^4)}\,dx = \frac{5\,{(-b)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{9/4}\,d^{3/2}} - \frac{5\,{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{9/4}\,d^{3/2}} - \frac{\frac{2\,d}{a}+\frac{5\,b\,d\,x^2}{2\,a^2}}{b\,{(d\,x)}^{5/2}+a\,d^2\,\sqrt{d\,x}}$$

input
```
int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

output
```
(5*(-b)^(1/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(4*a^(9/4
)*d^(3/2)) - (5*(-b)^(1/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))
))/(4*a^(9/4)*d^(3/2)) - ((2*d)/a + (5*b*d*x^2)/(2*a^2))/(b*(d*x)^(5/2) +
a*d^2*(d*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.50

$$\int \frac{1}{(dx)^{3/2}\,(a^2+2abx^2+b^2x^4)}\,dx = \frac{\sqrt{d}\left(10\sqrt{x}\,b^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+10\sqrt{x}\,b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a}{}\right.\right.}{}$$

input
```
int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(sqrt(d)*(10*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 10*sqrt(x)*b**(
1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**2 - 10*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*
sqrt(2)))*a - 10*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**2 - 5*sqrt
(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + s
qrt(a) + sqrt(b)*x)*a - 5*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)
*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 + 5*sqrt(x)*b**(1
/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqr
t(b)*x)*a + 5*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1
/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 - 32*a**2 - 40*a*b*x**2))/(16*sq
rt(x)*a**3*d**2*(a + b*x**2))
```

3.454 $\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 223

$$\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)}\,dx = -\frac{7}{6a^2d(dx)^{3/2}} + \frac{1}{2ad(dx)^{3/2}(a+bx^2)} + \frac{7b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{11/4}d^{5/2}} - \frac{7b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{11/4}d^{5/2}} - \frac{7b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}a^{11/4}d^{5/2}}$$

output
```
-7/6/a^2/d/(d*x)^(3/2)+1/2/a/d/(d*x)^(3/2)/(b*x^2+a)+7/8*b^(3/4)*arctan(1-
2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(11/4)/d^(5/2)-7/8*
b^(3/4)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(1
1/4)/d^(5/2)-7/8*b^(3/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/
2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(11/4)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.74

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx=\frac{x\left(-4a^{3/4}(4a+7bx^2)+21\sqrt{2}b^{3/4}x^{3/2}(a+bx^2)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)\right.}{24a^{11/4}(dx)^{5/2}\left(a+b\right.}$$

input
```
Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
(x*(-4*a^(3/4)*(4*a + 7*b*x^2) + 21*Sqrt[2]*b^(3/4)*x^(3/2)*(a + b*x^2)*Ar
cTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 21*Sqrt[2]
*b^(3/4)*x^(3/2)*(a + b*x^2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sq
rt[a] + Sqrt[b]*x)]))/(24*a^(11/4)*(d*x)^(5/2)*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.48, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1380, 27, 253, 264, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)}dx$$

$$\Big\downarrow \; 1380$$

$$b^2\int \frac{1}{b^2(dx)^{5/2}\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{1}{(dx)^{5/2}\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \; 253$$

$$\frac{7\int \frac{1}{(dx)^{5/2}(bx^2+a)}dx}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(-\frac{b\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{ad^2}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{7\left(-\frac{2b\int \frac{1}{bx^2+a}d\sqrt{dx}}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{7\left(-\frac{2b\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{7\left(-\frac{2b\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{7\left(-\frac{2b\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

$\big\downarrow$ 1082

$$\frac{7\left(-\frac{2b\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

$\big\downarrow$ 217

$$\frac{7\left(-\frac{2b\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1479$$

$$7\left(-\frac{2b\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\ldots\right.}{\sqrt{2}\sqrt[4]{a}\ldots}\right.}{2\sqrt{a}}\ldots\right.}{ad^3}\ldots\right.}{4a}$$

$$\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 25$$

$$7\left(-\frac{2b\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a}\cdots}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\right.}{2\sqrt{a}}\cdots\right)}{ad^3}\cdots\right)}{4a}$$

$$\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

$$\downarrow 27$$

$$\frac{7\left(-\frac{2b\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\right.}{4a}$$

$$\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1103$$

$$7\left(-\frac{2b\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}\right)$$
$$\overline{\qquad 4a \qquad}$$

$$\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}$$

input
```
Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
1/(2*a*d*(d*x)^(3/2)*(a + b*x^2)) + (7*(-2/(3*a*d*(d*x)^(3/2)) - (2*b*((d*
(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqr
t[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a
])))/(a*d^3)))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{2}{3a^2 x\sqrt{dx}\,d^2}-\frac{2b\left(\frac{\sqrt{dx}}{4b\,d^2x^2+4a\,d^2}+\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\,\text{arc}\right.}{32a\,d^2}\right.}{a^2d}$
derivativedivides	$2d^3\left(-\frac{1}{3a^2d^4(dx)^{\frac{3}{2}}}-\frac{b\left(\frac{\sqrt{dx}}{4b\,d^2x^2+4a\,d^2}+\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right.\right.}{32a\,d^2}\right.}{a^2d^4}\right.$
default	$2d^3\left(-\frac{1}{3a^2d^4(dx)^{\frac{3}{2}}}-\frac{b\left(\frac{\sqrt{dx}}{4b\,d^2x^2+4a\,d^2}+\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right.\right.}{32a\,d^2}\right.}{a^2d^4}\right.$
pseudoelliptic	$d^3\left(-\frac{2}{3a^2d^4(dx)^{\frac{3}{2}}}-\frac{\sqrt{dx}\,b}{2d^6(b\,x^2+a)a^2}-\frac{7\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b}{16a^3d^6}-\frac{7\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)}\right.}{8a}\right.$

input
```
int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a^2/x/(d*x)^(1/2)/d^2-2/a^2*b/d*(1/4*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)+7/
32*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1
/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*
d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.43

$$\int \frac{1}{(dx)^{5/2}\,(a^2+2abx^2+b^2x^4)}\,dx =$$

$$-\frac{21\,(a^2bd^3x^4+a^3d^3x^2)\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}\log\left(7\,a^3d^3\left(-\frac{b^3}{a^{11}d^{10}}\right)^{\frac{1}{4}}+7\,\sqrt{dx}b\right)+21\,(i\,a^2bd^3x^4+i\,a^3d^3x^2)\left(-\frac{b^3}{a^{11}d^{10}}\right.}{}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/24*(21*(a^2*b*d^3*x^4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10))^(1/4)*log(7*a^3
*d^3*(-b^3/(a^11*d^10))^(1/4) + 7*sqrt(d*x)*b) + 21*(I*a^2*b*d^3*x^4 + I*a
^3*d^3*x^2)*(-b^3/(a^11*d^10))^(1/4)*log(7*I*a^3*d^3*(-b^3/(a^11*d^10))^(1
/4) + 7*sqrt(d*x)*b) + 21*(-I*a^2*b*d^3*x^4 - I*a^3*d^3*x^2)*(-b^3/(a^11*d
^10))^(1/4)*log(-7*I*a^3*d^3*(-b^3/(a^11*d^10))^(1/4) + 7*sqrt(d*x)*b) - 2
1*(a^2*b*d^3*x^4 + a^3*d^3*x^2)*(-b^3/(a^11*d^10))^(1/4)*log(-7*a^3*d^3*(-
b^3/(a^11*d^10))^(1/4) + 7*sqrt(d*x)*b) + 4*(7*b*x^2 + 4*a)*sqrt(d*x))/(a^
2*b*d^3*x^4 + a^3*d^3*x^2)
```

Sympy [F]

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx=\int \frac{1}{(dx)^{\frac{5}{2}}\left(a+bx^2\right)^2}\,dx$$

input
```
integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
Integral(1/((d*x)**(5/2)*(a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.23

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = -\frac{\frac{8\left(7\,bd^2x^2+4\,ad^2\right)}{(dx)^{\frac{7}{2}}a^2b+(dx)^{\frac{3}{2}}a^3d^2}+\frac{21\left(\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}}+\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)\ldots}{2\sqrt{\ldots}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{\ldots}}\ldots\right)}{a^2}}{48\,d}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/48*(8*(7*b*d^2*x^2 + 4*a*d^2)/((d*x)^(7/2)*a^2*b + (d*x)^(3/2)*a^3*d^2)
 + 21*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b
^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*d*x - sqrt
(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) + 2*sqrt(2)
*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b)
)/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d) + 2*sqrt(2)
*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b
))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d))/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.24

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = -\frac{\sqrt{dx}b}{2\left(bd^2x^2+ad^2\right)a^2d} - \frac{7\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3d^3} - \frac{7\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3d^3} - \frac{7\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^3d^3} + \frac{7\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^3d^3} - \frac{2}{3\sqrt{dx}a^2d^2x}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(d*x)*b/((b*d^2*x^2 + a*d^2)*a^2*d) - 7/8*sqrt(2)*(a*b^3*d^2)^(1/
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1
/4))/(a^3*d^3) - 7/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*d^3) - 7/16*sqrt(2)
*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^
2/b))/(a^3*d^3) + 7/16*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/
b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*d^3) - 2/3/(sqrt(d*x)*a^2*d^2*x)
```

Mupad [B] (verification not implemented)

Time = 17.75 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.46

$$\int \frac{1}{(dx)^{5/2} (a^2 + 2abx^2 + b^2x^4)} \, dx = \frac{7 \, (-b)^{3/4} \operatorname{atan}\left(\frac{(-b)^{1/4} \sqrt{d \, x}}{a^{1/4} \sqrt{d}}\right)}{4 \, a^{11/4} \, d^{5/2}} - \frac{\frac{2 \, d}{3 \, a} + \frac{7 \, b \, d \, x^2}{6 \, a^2}}{b \, (d \, x)^{7/2} + a \, d^2 \, (d \, x)^{3/2}} + \frac{7 \, (-b)^{3/4} \operatorname{atanh}\left(\frac{(-b)^{1/4} \sqrt{d \, x}}{a^{1/4} \sqrt{d}}\right)}{4 \, a^{11/4} \, d^{5/2}}$$

input
```
int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

output
```
(7*(-b)^(3/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(4*a^(11/4
)*d^(5/2)) - ((2*d)/(3*a) + (7*b*d*x^2)/(6*a^2))/(b*(d*x)^(7/2) + a*d^2*(d
*x)^(3/2)) + (7*(-b)^(3/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2)
)))/(4*a^(11/4)*d^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.53

$$\int \frac{1}{(dx)^{5/2} (a^2 + 2abx^2 + b^2x^4)} \, dx = \frac{\sqrt{d} \left(42 \sqrt{x} \, b^{\frac{3}{4}} a^{\frac{5}{4}} \sqrt{2} \, atan\left(\frac{b^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2} - 2\sqrt{x} \sqrt{b}}{b^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) x + 42 \sqrt{x} \, b^{\frac{7}{4}} a^{\frac{1}{4}} \sqrt{2} \, atan\left(\frac{b^{\frac{1}{4}}}{}\right.\right.}{}$$

input
```
int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(sqrt(d)*(42*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*x + 42*sqrt(x)*b*
*(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b
))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**3 - 42*sqrt(x)*b**(3/4)*a**(1/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4
)*sqrt(2)))*a*x - 42*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**3 + 21
*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2
) + sqrt(a) + sqrt(b)*x)*a*x + 21*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log( -
 sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**3 - 21*sqrt
(x)*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt
(a) + sqrt(b)*x)*a*x - 21*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b*
*(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**3 - 32*a**2 - 56*a*b*x
**2))/(48*sqrt(x)*a**3*d**3*x*(a + b*x**2))
```

3.455 $\int \frac{1}{(dx)^{7/2}(a^2+2abx^2+b^2x^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 241

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = -\frac{9}{10a^2d(dx)^{5/2}}+\frac{9b}{2a^3d^3\sqrt{dx}} + \frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}-\frac{9b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{13/4}d^{7/2}} + \frac{9b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{13/4}d^{7/2}}-\frac{9b^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{4\sqrt{2}a^{13/4}d^{7/2}}$$

output
```
-9/10/a^2/d/(d*x)^(5/2)+9/2*b/a^3/d^3/(d*x)^(1/2)+1/2/a/d/(d*x)^(5/2)/(b*x
^2+a)-9/8*b^(5/4)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^
(1/2)/a^(13/4)/d^(7/2)+9/8*b^(5/4)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^
(1/4)/d^(1/2))*2^(1/2)/a^(13/4)/d^(7/2)-9/8*b^(5/4)*arctanh(2^(1/2)*a^(1/4
)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(13/4)/d^(7/2
)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.75

$$\int \frac{1}{(dx)^{7/2} (a^2 + 2abx^2 + b^2x^4)} dx =$$
$$-\frac{\sqrt{dx}\left(4\sqrt[4]{a}(4a^2 - 36abx^2 - 45b^2x^4) + 45\sqrt{2}b^{5/4}x^{5/2}(a + bx^2)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 45\sqrt{2}b^{5/4}x^{5/2}(a +\right.}{40a^{13/4}d^4x^3(a + bx^2)}$$

input
```
Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
-1/40*(Sqrt[d*x]*(4*a^(1/4)*(4*a^2 - 36*a*b*x^2 - 45*b^2*x^4) + 45*Sqrt[2]
*b^(5/4)*x^(5/2)*(a + b*x^2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x])] + 45*Sqrt[2]*b^(5/4)*x^(5/2)*(a + b*x^2)*ArcTanh[(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(a^(13/4)*d^4*x^3*(a
+ b*x^2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.46, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {1380, 27, 253, 264, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{7/2} (a^2 + 2abx^2 + b^2x^4)} dx$$
$$\Big\downarrow \text{1380}$$
$$b^2 \int \frac{1}{b^2(dx)^{7/2} (bx^2 + a)^2} dx$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1}{(dx)^{7/2} (a + bx^2)^2} dx$$

$$\Big\downarrow \text{253}$$

$$\frac{9\int \frac{1}{(dx)^{7/2}(bx^2+a)}dx}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{9\left(-\frac{b\int \frac{1}{(dx)^{3/2}\left(bx^2+a\right)}dx}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{9\left(-\frac{b\left(-\frac{b\int \frac{\sqrt{dx}}{bx^2+a}dx}{ad^2}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{ad^3}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{826}$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\,(a+bx^2)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+$$

$$\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1479$$

$$9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{}}{\sqrt[4]{b}}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right.}{ad}\right.}{ad^2}\right.$$

$$\overline{4a}$$

$$\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow 25$$

$$9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}\cdots\right)}{ad^2}\cdots\right)}{4a\cdots}$$

$$\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\cdots\right)}{ad^2}\cdots\right)}{4a}$$

$$\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1103$$

$$9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}}{ad^2}\right.$$

$$4a$$

$$\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

input
```
Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

output
```
1/(2*a*d*(d*x)^(5/2)*(a + b*x^2)) + (9*(-2/(5*a*d*(d*x)^(5/2)) - (b*(-2/(a
*d*Sqrt[d*x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b
]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*S
qrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d]))/(2*Sqrt[b])))/(a*d)))/(a*d^2)))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{2(-10b\,x^2+a)}{5a^3\sqrt{dx}\,x^2d^3}+\frac{b^2\left(\frac{(dx)^{\frac{3}{2}}}{2b\,d^2x^2+2a\,d^2}+\frac{9\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{16b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}}{a^3d^3}$
derivativedivides	$2d^3\left(-\frac{1}{5a^2d^4(dx)^{\frac{5}{2}}}+\frac{2b}{a^3d^6\sqrt{dx}}+\frac{b^2\left(\frac{(dx)^{\frac{3}{2}}}{4b\,d^2x^2+4a\,d^2}+\frac{9\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d}}{\left(\frac{a\,d^2}{b}\right.}\right.\right.}{32b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}}{a^3d^6}\right.$
default	$2d^3\left(-\frac{1}{5a^2d^4(dx)^{\frac{5}{2}}}+\frac{2b}{a^3d^6\sqrt{dx}}+\frac{b^2\left(\frac{(dx)^{\frac{3}{2}}}{4b\,d^2x^2+4a\,d^2}+\frac{9\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d}}{\left(\frac{a\,d^2}{b}\right.}\right.\right.}{32b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}}{a^3d^6}\right.$
pseudoelliptic	$-\frac{2\left(-\frac{45b\sqrt{2}\left(b\,x^2+a\right)\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)\right.}{32}\right.}{5(dx)^{\frac{5}{2}}\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}d^3a^3(b\,x^2+a)}$

input
```
int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-10*b*x^2+a)/a^3/(d*x)^(1/2)/x^2/d^3+1/a^3*b^2*(1/2*(d*x)^(3/2)/(b*d
^2*x^2+a*d^2)+9/16/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan
(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))/d^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.41

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = \frac{45\left(a^3bd^4x^5+a^4d^4x^3\right)\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}\log\left(729\,a^{10}d^{11}\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{3}{4}}+729\,\sqrt{}\right.}{}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
1/40*(45*(a^3*b*d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*log(729*a^
10*d^11*(-b^5/(a^13*d^14))^(3/4) + 729*sqrt(d*x)*b^4) - 45*(I*a^3*b*d^4*x^
5 + I*a^4*d^4*x^3)*(-b^5/(a^13*d^14))^(1/4)*log(729*I*a^10*d^11*(-b^5/(a^1
3*d^14))^(3/4) + 729*sqrt(d*x)*b^4) - 45*(-I*a^3*b*d^4*x^5 - I*a^4*d^4*x^3
)*(-b^5/(a^13*d^14))^(1/4)*log(-729*I*a^10*d^11*(-b^5/(a^13*d^14))^(3/4) +
 729*sqrt(d*x)*b^4) - 45*(a^3*b*d^4*x^5 + a^4*d^4*x^3)*(-b^5/(a^13*d^14))^
(1/4)*log(-729*a^10*d^11*(-b^5/(a^13*d^14))^(3/4) + 729*sqrt(d*x)*b^4) + 4
*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)*sqrt(d*x))/(a^3*b*d^4*x^5 + a^4*d^4*x^3
)
```

Sympy [F]

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = \int \frac{1}{(dx)^{\frac{7}{2}}\left(a+bx^2\right)^2}\,dx$$

input
```
integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
Integral(1/((d*x)**(7/2)*(a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.20

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = \frac{\frac{8\left(45\,b^2d^4x^4+36\,abd^4x^2-4\,a^2d^4\right)}{(dx)^{\frac{9}{2}}a^3bd^2+(dx)^{\frac{5}{2}}a^4d^4}+\frac{45\,b^2\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\,\sqrt{2}\,\mathrm{ar}\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
1/80*(8*(45*b^2*d^4*x^4 + 36*a*b*d^4*x^2 - 4*a^2*d^4)/((d*x)^(9/2)*a^3*b*d
^2 + (d*x)^(5/2)*a^4*d^4) + 45*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqr
t(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*
d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(s
qrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/
4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sq
rt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^
(1/4)*b^(3/4)))/(a^3*d^2))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.27

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)}\,dx = \frac{\sqrt{dx}b^2x}{2\,(bd^2x^2+ad^2)a^3d^2} + \frac{9\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4bd^5} + \frac{9\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4bd^5} - \frac{9\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^4bd^5} + \frac{9\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^4bd^5} + \frac{2\,(10\,bd^2x^2-ad^2)}{5\,\sqrt{dx}a^3d^5x^2}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
1/2*sqrt(d*x)*b^2*x/((b*d^2*x^2 + a*d^2)*a^3*d^2) + 9/8*sqrt(2)*(a*b^3*d^2
)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/(a^4*b*d^5) + 9/8*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d^5) - 9/1
6*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
sqrt(a*d^2/b))/(a^4*b*d^5) + 9/16*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt
(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*d^5) + 2/5*(10*b*d^2
*x^2 - a*d^2)/(sqrt(d*x)*a^3*d^5*x^2)
```

Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.47

$$\int \frac{1}{(dx)^{7/2}\,(a^2+2abx^2+b^2x^4)}\,dx = \frac{\frac{9\,b^2\,d\,x^4}{2\,a^3}-\frac{2\,d}{5\,a}+\frac{18\,b\,d\,x^2}{5\,a^2}}{b\,{(d\,x)}^{9/2}+a\,d^2\,{(d\,x)}^{5/2}} - \frac{9\,{(-b)}^{5/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{13/4}\,d^{7/2}} + \frac{9\,{(-b)}^{5/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{4\,a^{13/4}\,d^{7/2}}$$

input
```
int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)),x)
```

output
```
((9*b^2*d*x^4)/(2*a^3) - (2*d)/(5*a) + (18*b*d*x^2)/(5*a^2))/(b*(d*x)^(9/2
) + a*d^2*(d*x)^(5/2)) - (9*(-b)^(5/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1
/4)*d^(1/2))))/(4*a^(13/4)*d^(7/2)) + (9*(-b)^(5/4)*atanh(((-b)^(1/4)*(d*x
)^(1/2))/(a^(1/4)*d^(1/2))))/(4*a^(13/4)*d^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.49

$$\int \frac{1}{(dx)^{7/2}\,(a^2+2abx^2+b^2x^4)}\,dx = \frac{\sqrt{d}\left(-90\sqrt{x}\,b^{\frac{5}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2-90\sqrt{x}\,b^{\frac{9}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\right.\right.}{}$$

input
```
int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(sqrt(d)*( - 90*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*x**2 - 90*sq
rt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 + 90*sqrt(x)*b**(1/4)*a*
*(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1
/4)*a**(1/4)*sqrt(2)))*a*b*x**2 + 90*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(
2)))*b**2*x**4 + 45*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1
/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 45*sqrt(x)*b**(1/4)
*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqr
t(b)*x)*b**2*x**4 - 45*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1
/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 45*sqrt(x)*b**(1/4)
*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b
)*x)*b**2*x**4 - 32*a**3 + 288*a**2*b*x**2 + 360*a*b**2*x**4))/(80*sqrt(x)
*a**4*d**4*x**2*(a + b*x**2))
```

3.456 $\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 291

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{663ad^9\sqrt{dx}}{64b^5}+\frac{663d^7(dx)^{5/2}}{320b^4}-\frac{d(dx)^{17/2}}{6b\,(a+bx^2)^3}-\frac{17d^3(dx)^{13/2}}{48b^2\,(a+bx^2)^2}-\frac{221d^5(dx)^{9/2}}{192b^3\,(a+bx^2)}-\frac{663a^{5/4}d^{19/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}b^{21/4}}+\frac{663a^{5/4}d^{19/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}b^{21/4}}+\frac{663a^{5/4}d^{19/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}b^{21/4}}$$

output

```
-663/64*a*d^9*(d*x)^(1/2)/b^5+663/320*d^7*(d*x)^(5/2)/b^4-1/6*d*(d*x)^(17/
2)/b/(b*x^2+a)^3-17/48*d^3*(d*x)^(13/2)/b^2/(b*x^2+a)^2-221/192*d^5*(d*x)^
(9/2)/b^3/(b*x^2+a)-663/256*a^(5/4)*d^(19/2)*arctan(1-2^(1/2)*b^(1/4)*(d*x
)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(21/4)+663/256*a^(5/4)*d^(19/2)*arctan(
1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(21/4)+663/256*a^
(5/4)*d^(19/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2
)+b^(1/2)*x))*2^(1/2)/b^(21/4)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.70

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{d^9\sqrt{dx}\left(4\sqrt[4]{b}\sqrt{x}(-9945a^4-27846a^3bx^2-24973a^2b^2x^4-6528ab^3x^6+384b\right.}{}$$

input
```
Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(d^9*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(-9945*a^4 - 27846*a^3*b*x^2 - 24973*a^2
*b^2*x^4 - 6528*a*b^3*x^6 + 384*b^4*x^8) + 9945*Sqrt[2]*a^(5/4)*(a + b*x^2
)^3*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 994
5*Sqrt[2]*a^(5/4)*(a + b*x^2)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/
(Sqrt[a] + Sqrt[b]*x)]))/(3840*b^(21/4)*Sqrt[x]*(a + b*x^2)^3)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.44, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 252, 252, 252, 262, 262, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^4\int \frac{(dx)^{19/2}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{19/2}}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{17d^2\int\frac{(dx)^{15/2}}{(bx^2+a)^3}dx}{12b}-\frac{d(dx)^{17/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{17d^2\left(\frac{13d^2\int\frac{(dx)^{11/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{13/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\int\frac{(dx)^{7/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \text{262}$$

$$\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\int\frac{(dx)^{3/2}}{bx^2+a}dx}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \text{262}$$

$$\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{ad^2\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 266

$$\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\int\frac{1}{bx^2+a}d\sqrt{dx}}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 755

$$\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{ 1476}$$

$$\left(13d^2 \frac{\left(9d^2 \left(\frac{2d(dx)^{5/2}}{5b} - \frac{\left(ad^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{\left(2ad \left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}} + \frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}} + \frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}} \right)}{2\sqrt{a}} \right. \right.}{b} \right. \right.}{b} \right. \right.}{4b} \right.$$

↓ 1082

$$13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4b}-\ldots\right.\right.$$

↓ 217

$$\left(13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right.\right.$$

↓ 1479

$$
13d^2 \left(\frac{9d^2 \left(\frac{2d(dx)^{5/2}}{5b} - \frac{ad^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{2ad \left(\frac{d \left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + \cdots \right.}{b} \cdots}{b} \cdots}{4b} \cdots
$$

↓ 25

$$13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\text{ar}\cdots\right.}{\cdots}\right.}{b}\right.}{b}\right.}{4b}\right.$$

$$\Big\downarrow \; 27$$

$$13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{a}\sqrt{d}\cdots}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\right.}{\cdots}\right.}{b}\right.}{b}\right.}{4b}\right.$$

↓ 1103

$$13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{}\right)}{b}\right)}{b}\right)}{4b}\right.$$

input
```
Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*(d*(d*x)^(17/2))/(b*(a + b*x^2)^3) + (17*d^2*(-1/4*(d*(d*x)^(13/2))/(
b*(a + b*x^2)^2) + (13*d^2*(-1/2*(d*(d*x)^(9/2))/(b*(a + b*x^2)) + (9*d^2*
((2*d*(d*x)^(5/2))/(5*b) - (a*d^2*((2*d*Sqrt[d*x])/b - (2*a*d*((d*(-(ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d +
 Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/b))
/b))/(4*b)))/(8*b)))/(12*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{2(-b\,x^2+20a)\,x\,d^{10}}{5b^5\sqrt{dx}}+\frac{2a^2d^{11}\left(\frac{-\frac{617b^2(dx)^{\frac{9}{2}}}{384}-\frac{173ab\,d^2(dx)^{\frac{5}{2}}}{64}-\frac{151\sqrt{dx}\,a^2d^4}{128}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{d}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{d}}\right.\right.}{}\right.}{b^5}$
derivativedivides	$2d^7\left(-\frac{-\frac{b(dx)^{\frac{5}{2}}}{5}+4\sqrt{dx}\,a\,d^2}{b^5}+\frac{a^2d^4\left(\frac{-\frac{617b^2(dx)^{\frac{9}{2}}}{384}-\frac{173ab\,d^2(dx)^{\frac{5}{2}}}{64}-\frac{151\sqrt{dx}\,a^2d^4}{128}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right.}{dx-\left(\frac{a\,d^2}{b}\right.}\right.\right.}{}\right.}{b^5}\right.$
default	$2d^7\left(-\frac{-\frac{b(dx)^{\frac{5}{2}}}{5}+4\sqrt{dx}\,a\,d^2}{b^5}+\frac{a^2d^4\left(\frac{-\frac{617b^2(dx)^{\frac{9}{2}}}{384}-\frac{173ab\,d^2(dx)^{\frac{5}{2}}}{64}-\frac{151\sqrt{dx}\,a^2d^4}{128}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right.}{dx-\left(\frac{a\,d^2}{b}\right.}\right.\right.}{}\right.}{b^5}\right.$
pseudoelliptic	$\frac{663d^5\left(\frac{\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a\,d^4\sqrt{2}\left(b\,x^2+a\right)^3}{2}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^4\left(b\,x^2+a\right)^3\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-\right.\right.}{}$

input
```
int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-b*x^2+20*a)*x/b^5/(d*x)^(1/2)*d^10+2*a^2/b^5*d^11*((-617/384*b^2*(d
*x)^(9/2)-173/64*a*b*d^2*(d*x)^(5/2)-151/128*(d*x)^(1/2)*a^2*d^4)/(b*d^2*x
^2+a*d^2)^3+663/1024*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*
arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.49

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{9945\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5)\log\left(663\sqrt{dx}ad^9+663\,\left(\right.\right.}{}$$

input
```
integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/3840*(9945*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2
 + a^3*b^5)*log(663*sqrt(d*x)*a*d^9 + 663*(-a^5*d^38/b^21)^(1/4)*b^5) - 99
45*(-a^5*d^38/b^21)^(1/4)*(-I*b^8*x^6 - 3*I*a*b^7*x^4 - 3*I*a^2*b^6*x^2 -
I*a^3*b^5)*log(663*sqrt(d*x)*a*d^9 + 663*I*(-a^5*d^38/b^21)^(1/4)*b^5) - 9
945*(-a^5*d^38/b^21)^(1/4)*(I*b^8*x^6 + 3*I*a*b^7*x^4 + 3*I*a^2*b^6*x^2 +
I*a^3*b^5)*log(663*sqrt(d*x)*a*d^9 - 663*I*(-a^5*d^38/b^21)^(1/4)*b^5) - 9
945*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^
5)*log(663*sqrt(d*x)*a*d^9 - 663*(-a^5*d^38/b^21)^(1/4)*b^5) + 4*(384*b^4*
d^9*x^8 - 6528*a*b^3*d^9*x^6 - 24973*a^2*b^2*d^9*x^4 - 27846*a^3*b*d^9*x^2
 - 9945*a^4*d^9)*sqrt(d*x))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b
^5)
```

Sympy [F]

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx=\int \frac{(dx)^{\frac{19}{2}}}{(a+bx^2)^4}\,dx$$

input
```
integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral((d*x)**(19/2)/(a + b*x**2)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.24

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx=$$

$$-\frac{40\left(617\,(dx)^{\frac{9}{2}}a^2b^2d^{12}+1038\,(dx)^{\frac{5}{2}}a^3bd^{14}+453\,\sqrt{dx}a^4d^{16}\right)}{b^8d^6x^6+3\,ab^7d^6x^4+3\,a^2b^6d^6x^2+a^3b^5d^6}-\frac{9945\left(\frac{\sqrt{2}d^{12}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^{12}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{\cdots}\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}\cdots\right.}{7\cdots}$$

input
```
integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/7680*(40*(617*(d*x)^(9/2)*a^2*b^2*d^12 + 1038*(d*x)^(5/2)*a^3*b*d^14 +
453*sqrt(d*x)*a^4*d^16)/(b^8*d^6*x^6 + 3*a*b^7*d^6*x^4 + 3*a^2*b^6*d^6*x^2
 + a^3*b^5*d^6) - 9945*(sqrt(2)*d^12*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/
4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^12*l
og(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*
d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^11*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(
1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)
*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^11*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)
^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(
a)*sqrt(b)*d)*sqrt(a)))*a^2/b^5 - 3072*((d*x)^(5/2)*b*d^8 - 20*sqrt(d*x)*a
*d^10)/b^5)/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.19

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{19890\sqrt{2}(ab^3d^2)^{\frac{1}{4}}ad^{10}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6}+\frac{19890\sqrt{2}(ab^3d^2)^{\frac{1}{4}}ad^{10}\arctan\left(-\frac{\sqrt{2}(\sqrt{}}{}\right.}{b^6}}{}$$

input
```
integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/7680*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^10*arctan(1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^6 + 19890*sqrt(2)*(a*b^
3*d^2)^(1/4)*a*d^10*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(
d*x))/(a*d^2/b)^(1/4))/b^6 + 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^10*log(d*x
 + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^6 - 9945*sqrt(2)*(
a*b^3*d^2)^(1/4)*a*d^10*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/b^6 - 40*(617*sqrt(d*x)*a^2*b^2*d^16*x^4 + 1038*sqrt(d*x)*a^3*b
*d^16*x^2 + 453*sqrt(d*x)*a^4*d^16)/((b*d^2*x^2 + a*d^2)^3*b^5) + 3072*(sq
rt(d*x)*b^16*d^10*x^2 - 20*sqrt(d*x)*a*b^15*d^10)/b^20)/d
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.65

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{2\,d^7\,(d\,x)^{5/2}}{5\,b^4} - \frac{\frac{151\,a^4\,d^{15}\,\sqrt{d\,x}}{64}+\frac{617\,a^2\,b^2\,d^{11}\,(d\,x)^{9/2}}{192}+\frac{173\,a^3\,b\,d^{13}\,(d\,x)^{5/2}}{32}}{a^3\,b^5\,d^6+3\,a^2\,b^6\,d^6\,x^2+3\,a\,b^7\,d^6\,x^4+b^8\,d^6\,x^6} - \frac{663\,(-a)^{5/4}\,d^{19/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,b^{21/4}} - \frac{8\,a\,d^9\,\sqrt{d\,x}}{b^5} + \frac{(-a)^{5/4}\,d^{19/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1\mathrm{i}}{(-a)^{1/4}\,\sqrt{d}}\right)\,663\mathrm{i}}{128\,b^{21/4}}$$

input
```
int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(2*d^7*(d*x)^(5/2))/(5*b^4) - ((151*a^4*d^15*(d*x)^(1/2))/64 + (617*a^2*b^
2*d^11*(d*x)^(9/2))/192 + (173*a^3*b*d^13*(d*x)^(5/2))/32)/(a^3*b^5*d^6 +
b^8*d^6*x^6 + 3*a*b^7*d^6*x^4 + 3*a^2*b^6*d^6*x^2) - (663*(-a)^(5/4)*d^(19
/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*b^(21/4)) + ((-
a)^(5/4)*d^(19/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*663i
)/(128*b^(21/4)) - (8*a*d^9*(d*x)^(1/2))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.31

$$\int \frac{(dx)^{19/2}}{(a^2 + 2abx^2 + b^2x^4)^2}\, dx = \text{Too large to display}$$

input
```
int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(sqrt(d)*d**9*( - 19890*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 - 59670*b**
(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 - 59670*b**(3/4)*a**(1/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*a**2*b**2*x**4 - 19890*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3
*x**6 + 19890*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 + 59670*b**(3/4)*a**(
1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4
)*a**(1/4)*sqrt(2)))*a**3*b*x**2 + 59670*b**(3/4)*a**(1/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
*a**2*b**2*x**4 + 19890*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 - 99
45*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sq
rt(a) + sqrt(b)*x)*a**4 - 29835*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b
**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3*b*x**2 - 29835*b**(3/
4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + s
qrt(b)*x)*a**2*b**2*x**4 - 9945*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b
**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**3*x**6 + 9945*b**(...
```

3.457 $\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 273

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2} dx = \frac{385d^7(dx)^{3/2}}{192b^4} - \frac{d(dx)^{15/2}}{6b(a+bx^2)^3} - \frac{5d^3(dx)^{11/2}}{16b^2(a+bx^2)^2} - \frac{55d^5(dx)^{7/2}}{64b^3(a+bx^2)} + \frac{385a^{3/4}d^{17/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}b^{19/4}} - \frac{385a^{3/4}d^{17/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}b^{19/4}} + \frac{385a^{3/4}d^{17/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}(\sqrt{a}+\sqrt{b}x)}\right)}{128\sqrt{2}b^{19/4}}$$

output
```
385/192*d^7*(d*x)^(3/2)/b^4-1/6*d*(d*x)^(15/2)/b/(b*x^2+a)^3-5/16*d^3*(d*x
)^(11/2)/b^2/(b*x^2+a)^2-55/64*d^5*(d*x)^(7/2)/b^3/(b*x^2+a)+385/256*a^(3/
4)*d^(17/2)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/
b^(19/4)-385/256*a^(3/4)*d^(17/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(
1/4)/d^(1/2))*2^(1/2)/b^(19/4)+385/256*a^(3/4)*d^(17/2)*arctanh(2^(1/2)*a^
(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(19/4)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.71

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{d^8\sqrt{dx}\Big(4b^{3/4}x^{3/2}(385a^3+990a^2bx^2+765ab^2x^4+128b^3x^6)-1155\sqrt{2}a^{3/4}(a}{768b^{19/4}}$$

input
```
Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(d^8*Sqrt[d*x]*(4*b^(3/4)*x^(3/2)*(385*a^3 + 990*a^2*b*x^2 + 765*a*b^2*x^4
 + 128*b^3*x^6) - 1155*Sqrt[2]*a^(3/4)*(a + b*x^2)^3*ArcTan[(-Sqrt[a] + Sq
rt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 1155*Sqrt[2]*a^(3/4)*(a + b*
x^2)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/
(768*b^(19/4)*Sqrt[x]*(a + b*x^2)^3)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.45, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1380, 27, 252, 252, 252, 262, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$\Big\downarrow \; 1380$$

$$b^4\int \frac{(dx)^{17/2}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{(dx)^{17/2}}{(a+bx^2)^4}dx$$

$$\Big\downarrow \; 252$$

$$\frac{5d^2\int \frac{(dx)^{13/2}}{(bx^2+a)^3}dx}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 252

$$\frac{5d^2\left(\frac{11d^2\int \frac{(dx)^{9/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{11/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 252

$$\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\int \frac{(dx)^{5/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 262

$$\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{ad^2\int \frac{\sqrt{dx}}{bx^2+a}dx}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 266

$$\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 27

$$\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 826

$$\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 1476

$$\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-}{4b}$$

$$\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}$$

↓ 1082

$$\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)}{4b(a+}}{4b}$$

$$\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}$$

↓ 217

$$\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}$$

$$\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}$$

↓ 1479

$$5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}(\ldots}{\sqrt[4]{b}(x\ldots}\ldots}{\ldots}}{2\sqrt{b}}\ldots\right)}{b}\ldots\right)}{4b}\ldots\right)}{8b}\ldots\right.$$

↓ 25

$$5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\cdots\right.}{\sqrt[4]{b}\left(xd+\cdots\right.}\cdots}{2\cdots}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8b}\cdots\right.$$

$$\Big\downarrow \text{27}$$

$$5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{\cdots}}{\sqrt[4]{b}}}\cdots}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}\cdots}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8b}\cdots$$

↓ 1103

$$5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots\right.}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right.}{b}\cdots\right.}{4b}\cdots\right.}{8b}\cdots\right.}{4b}\cdots$$

$$\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}$$

input
```
Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*(d*(d*x)^(15/2))/(b*(a + b*x^2)^3) + (5*d^2*(-1/4*(d*(d*x)^(11/2))/(b
*(a + b*x^2)^2) + (11*d^2*(-1/2*(d*(d*x)^(7/2))/(b*(a + b*x^2)) + (7*d^2*(
(2*d*(d*x)^(3/2))/(3*b) - (2*a*d^3*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*
x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d
+ Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/b))/(4*b)))/(8*b)))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.80

method	result
derivativedivides	$2d^7\left(\frac{(dx)^{\frac{3}{2}}}{3b^4}-\frac{ad^2\left(\frac{-\frac{127b^2(dx)^{\frac{11}{2}}}{128}-\frac{101ab\,d^2(dx)^{\frac{7}{2}}}{64}-\frac{257a^2d^4(dx)^{\frac{3}{2}}}{384}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{385\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\,a\right.}{1024b\left(\right.}\right.}{b^4}\right.$
default	$2d^7\left(\frac{(dx)^{\frac{3}{2}}}{3b^4}-\frac{ad^2\left(\frac{-\frac{127b^2(dx)^{\frac{11}{2}}}{128}-\frac{101ab\,d^2(dx)^{\frac{7}{2}}}{64}-\frac{257a^2d^4(dx)^{\frac{3}{2}}}{384}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{385\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\,a\right.}{1024b\left(\right.}\right.}{b^4}\right.$
risch	$\frac{2x^2d^9}{3b^4\sqrt{dx}}-\frac{a\left(\frac{-\frac{127b^2(dx)^{\frac{11}{2}}}{64}-\frac{101ab\,d^2(dx)^{\frac{7}{2}}}{32}-\frac{257a^2d^4(dx)^{\frac{3}{2}}}{192}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{385\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\right.\right.}{512b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.}{b^4}$
pseudoelliptic	$-\frac{385d^8\left(-8\sqrt{dx}\left(\frac{128}{385}b^3x^6+\frac{153}{77}b^2x^4a+\frac{18}{7}a^2b\,x^2+a^3\right)bx\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}+3\sqrt{2}\,ad\left(b\,x^2+a\right)^3\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{a}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{a}}\right.\right.\right.}{1536\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^5\left(b\,x^2+a\right)^3}$

input

```
int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*d^7*(1/3*(d*x)^(3/2)/b^4-a*d^2/b^4*((-127/128*b^2*(d*x)^(11/2)-101/64*a*
b*d^2*(d*x)^(7/2)-257/384*a^2*d^4*(d*x)^(3/2))/(b*d^2*x^2+a*d^2)^3+385/102
4/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))
+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b
)^(1/4)*(d*x)^(1/2)-1))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.57

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx =$$

$$-\frac{1155\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{1}{4}}(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)\log\left(57066625\,\sqrt{dx}a^2d^{25}+57066625\left(-\frac{a^3d^{34}}{b^{19}}\right)^{\frac{3}{4}}b^{14}\right)}{}$$

input
```
integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/768*(1155*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2
 + a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^25 + 57066625*(-a^3*d^34/b^19)^(3
/4)*b^14) + 1155*(-a^3*d^34/b^19)^(1/4)*(-I*b^7*x^6 - 3*I*a*b^6*x^4 - 3*I*
a^2*b^5*x^2 - I*a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^25 + 57066625*I*(-a^
3*d^34/b^19)^(3/4)*b^14) + 1155*(-a^3*d^34/b^19)^(1/4)*(I*b^7*x^6 + 3*I*a*
b^6*x^4 + 3*I*a^2*b^5*x^2 + I*a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^25 - 5
7066625*I*(-a^3*d^34/b^19)^(3/4)*b^14) - 1155*(-a^3*d^34/b^19)^(1/4)*(b^7*
x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^
25 - 57066625*(-a^3*d^34/b^19)^(3/4)*b^14) - 4*(128*b^3*d^8*x^7 + 765*a*b^
2*d^8*x^5 + 990*a^2*b*d^8*x^3 + 385*a^3*d^8*x)*sqrt(d*x))/(b^7*x^6 + 3*a*b
^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)
```

Sympy [F]

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \int \frac{(dx)^{\frac{17}{2}}}{(a+bx^2)^4}\,dx$$

input
```
integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral((d*x)**(17/2)/(a + b*x**2)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.22

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{\frac{1155\,ad^{10}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\cdots}{b^4}}{1536\,d}$$

input
```
integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/1536*(1155*a*d^10*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*
b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt
(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(
1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)
*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b
^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sq
rt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4))
)/b^4 - 1024*(d*x)^(3/2)*d^8/b^4 - 8*(381*(d*x)^(11/2)*a*b^2*d^10 + 606*(d
*x)^(7/2)*a^2*b*d^12 + 257*(d*x)^(3/2)*a^3*d^14)/(b^7*d^6*x^6 + 3*a*b^6*d^
6*x^4 + 3*a^2*b^5*d^6*x^2 + a^3*b^4*d^6))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.16

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{1}{1536}d^8\left(\frac{1024\sqrt{dx}x}{b^4} - \frac{2310\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7d} - \right.$$

input
```
integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/1536*d^8*(1024*sqrt(d*x)*x/b^4 - 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1
/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*d
) - 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*d) + 1155*sqrt(2)*(a*b^3*d^2)^(
3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^7*d)
- 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*
x) + sqrt(a*d^2/b))/(b^7*d) + 8*(381*sqrt(d*x)*a*b^2*d^6*x^5 + 606*sqrt(d*
x)*a^2*b*d^6*x^3 + 257*sqrt(d*x)*a^3*d^6*x)/((b*d^2*x^2 + a*d^2)^3*b^4))
```

Mupad [B] (verification not implemented)

Time = 18.37 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.63

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{257\,a^3\,d^{13}\,(d\,x)^{3/2}}{192}+\frac{101\,a^2\,b\,d^{11}\,(d\,x)^{7/2}}{32}+\frac{127\,a\,b^2\,d^9\,(d\,x)^{11/2}}{64}}{a^3\,b^4\,d^6+3\,a^2\,b^5\,d^6\,x^2+3\,a\,b^6\,d^6\,x^4+b^7\,d^6\,x^6} + \frac{2\,d^7\,(d\,x)^{3/2}}{3\,b^4} + \frac{385\,(-a)^{3/4}\,d^{17/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,b^{19/4}} + \frac{(-a)^{3/4}\,d^{17/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1\mathrm{i}}{(-a)^{1/4}\,\sqrt{d}}\right)\,385\mathrm{i}}{128\,b^{19/4}}$$

input
```
int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
((257*a^3*d^13*(d*x)^(3/2))/192 + (101*a^2*b*d^11*(d*x)^(7/2))/32 + (127*a
*b^2*d^9*(d*x)^(11/2))/64)/(a^3*b^4*d^6 + b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 +
3*a^2*b^5*d^6*x^2) + (2*d^7*(d*x)^(3/2))/(3*b^4) + (385*(-a)^(3/4)*d^(17/2
)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*b^(19/4)) + ((-a)
^(3/4)*d^(17/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*385i)/
(128*b^(19/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.42

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(sqrt(d)*d**8*(2310*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 6930*b**(1/4)
*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 + 6930*b**(1/4)*a**(3/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(
2)))*a*b**2*x**4 + 2310*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 - 2310
*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqr
t(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 - 6930*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**2*b*x**2 - 6930*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 -
2310*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)
*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 - 1155*b**(1/4)*a**(3/4)*
sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*
*3 - 3465*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(
2) + sqrt(a) + sqrt(b)*x)*a**2*b*x**2 - 3465*b**(1/4)*a**(3/4)*sqrt(2)*log
( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 -
 1155*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) +
 sqrt(a) + sqrt(b)*x)*b**3*x**6 + 1155*b**(1/4)*a**(3/4)*sqrt(2)*log(sq...
```

3.458 $\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 273

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{195d^7\sqrt{dx}}{64b^4}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}-\frac{13d^3(dx)^{9/2}}{48b^2\left(a+bx^2\right)^2}-\frac{39d^5(dx)^{5/2}}{64b^3\left(a+bx^2\right)}+\frac{195\sqrt[4]{a}d^{15/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}b^{17/4}}-\frac{195\sqrt[4]{a}d^{15/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}b^{17/4}}-\frac{195\sqrt[4]{a}d^{15/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}b^{17/4}}$$

output
```
195/64*d^7*(d*x)^(1/2)/b^4-1/6*d*(d*x)^(13/2)/b/(b*x^2+a)^3-13/48*d^3*(d*x
)^(9/2)/b^2/(b*x^2+a)^2-39/64*d^5*(d*x)^(5/2)/b^3/(b*x^2+a)+195/256*a^(1/4
)*d^(15/2)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b
^(17/4)-195/256*a^(1/4)*d^(15/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1
/4)/d^(1/2))*2^(1/2)/b^(17/4)-195/256*a^(1/4)*d^(15/2)*arctanh(2^(1/2)*a^(
1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(17/4)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.64

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{d^7\sqrt{dx}\left(\frac{4\sqrt[4]{b}\sqrt{x}(585a^3+1638a^2bx^2+1469ab^2x^4+384b^3x^6)}{(a+bx^2)^3}+585\sqrt{2}\sqrt[4]{a}\arctan\left(\frac{\sqrt{a}-\sqrt{}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{}}\right.\right.}{768b^{17/4}\sqrt{x}}$$

input
```
Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(d^7*Sqrt[d*x]*((4*b^(1/4)*Sqrt[x]*(585*a^3 + 1638*a^2*b*x^2 + 1469*a*b^2*
x^4 + 384*b^3*x^6))/(a + b*x^2)^3 + 585*Sqrt[2]*a^(1/4)*ArcTan[(Sqrt[a] -
Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 585*Sqrt[2]*a^(1/4)*ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(768*b^(17/4)
*Sqrt[x])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.44, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1380, 27, 252, 252, 252, 262, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \ 1380$$

$$b^4\int \frac{(dx)^{15/2}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{(dx)^{15/2}}{(a+bx^2)^4}dx$$

$$\Big\downarrow \ 252$$

$$\frac{13d^2\int\frac{(dx)^{11/2}}{(bx^2+a)^3}dx}{12b}-\frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \ 252$$

$$\frac{13d^2\left(\frac{9d^2\int\frac{(dx)^{7/2}}{(bx^2+a)^2}dx}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \ 252$$

$$\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\int\frac{(dx)^{3/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \ 262$$

$$\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{ad^2\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \ 266$$

$$\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\int\frac{1}{bx^2+a}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \ 755$$

$$\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{ 1476}$$

$$13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}\right.$$

↓ 1082

$$13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(\cdots}{4b(a\cdots}$$

↓ 217

$$13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}$$

↓ 1479

$$13d^2 \left(\frac{9d^2 \left(\frac{5d^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{2ad \left(\frac{d \left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d \left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}} \cdots \right.}{\cdots} \right.}{b} \cdots \right.}{4b} \cdots \right.}{8b} \cdots \right.$$

↓ 25

$$13d^2 \left(\frac{9d^2 \left(\frac{5d^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{2ad \left(\frac{d \left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d} - 2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd + \frac{\sqrt{a}d}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)} d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} + \frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d} + \sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd + \frac{\sqrt{a}d}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)} d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d \left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}} + \cdots \right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \cdots \right.}{\cdots} \cdots}{b} \cdots \right)}{4b} \cdots \right)}{8b} \cdots$$

$$\Big\downarrow \; 27$$

$$13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\cdots\right.}{\sqrt{2}\sqrt[4]{\cdots}}\cdots\right.}{2\sqrt{a}}\cdots\right.}{b}\cdots\right.}{4b}\cdots\right.}{8b}\cdots\right.$$

↓ 1103

$$13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}\right.}{2\sqrt{2}\cdots}\right.}{2\sqrt{a}}}{b}}{4b}}{8b}$$

input
```
Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*(d*(d*x)^(13/2))/(b*(a + b*x^2)^3) + (13*d^2*(-1/4*(d*(d*x)^(9/2))/(b
*(a + b*x^2)^2) + (9*d^2*(-1/2*(d*(d*x)^(5/2))/(b*(a + b*x^2)) + (5*d^2*((
2*d*Sqrt[d*x])/b - (2*a*d*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a
^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/
(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d +
Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/b))/(4*b)))/(8*b)))/(12*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.81

method	result
derivativedivides	$2d^7\left(\frac{\sqrt{dx}}{b^4}-\frac{a\,d^2\left(\frac{-\frac{317b^2(dx)^{\frac{9}{2}}}{384}-\frac{81ab\,d^2(dx)^{\frac{5}{2}}}{64}-\frac{67\sqrt{dx}\,a^2d^4}{128}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{195\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\cdots\right.}{1024a\cdots}\right)}{b^4}\cdots\right.$
default	$2d^7\left(\frac{\sqrt{dx}}{b^4}-\frac{a\,d^2\left(\frac{-\frac{317b^2(dx)^{\frac{9}{2}}}{384}-\frac{81ab\,d^2(dx)^{\frac{5}{2}}}{64}-\frac{67\sqrt{dx}\,a^2d^4}{128}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{195\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\cdots\right.}{1024a\cdots}\right)}{b^4}\cdots\right.$
risch	$\frac{2x\,d^8}{b^4\sqrt{dx}}-\frac{2a\,d^9\left(\frac{-\frac{317b^2(dx)^{\frac{9}{2}}}{384}-\frac{81ab\,d^2(dx)^{\frac{5}{2}}}{64}-\frac{67\sqrt{dx}\,a^2d^4}{128}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{195\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\,\mathrm{ar}\cdots\right.}{1024a\,d^2\cdots}\right)}{b^4}\cdots$
pseudoelliptic	$-\frac{d^7\left(\left(-3072b^3x^6-11752b^2x^4a-13104a^2b\,x^2-4680a^3\right)\sqrt{dx}+585\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(b\,x^2+a\right)^3\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\cdots}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\cdots}\right)\cdots\right.\right.}{1536\left(b\,x^2+a\right)^3b^4}$

input
```
int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*d^7*((d*x)^(1/2)/b^4-1/b^4*a*d^2*((-317/384*b^2*(d*x)^(9/2)-81/64*a*b*d^
2*(d*x)^(5/2)-67/128*(d*x)^(1/2)*a^2*d^4)/(b*d^2*x^2+a*d^2)^3+195/1024*(a*
d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+
2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)
^(1/4)*(d*x)^(1/2)-1))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.46

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx =$$

$$-\frac{585\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)\log\left(195\sqrt{dx}d^7+195\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}b^4\right)+585\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}}(\cdots}{\cdots}$$

input
```
integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/768*(585*(-a*d^30/b^17)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 +
a^3*b^4)*log(195*sqrt(d*x)*d^7 + 195*(-a*d^30/b^17)^(1/4)*b^4) + 585*(-a*d
^30/b^17)^(1/4)*(I*b^7*x^6 + 3*I*a*b^6*x^4 + 3*I*a^2*b^5*x^2 + I*a^3*b^4)*
log(195*sqrt(d*x)*d^7 + 195*I*(-a*d^30/b^17)^(1/4)*b^4) + 585*(-a*d^30/b^1
7)^(1/4)*(-I*b^7*x^6 - 3*I*a*b^6*x^4 - 3*I*a^2*b^5*x^2 - I*a^3*b^4)*log(19
5*sqrt(d*x)*d^7 - 195*I*(-a*d^30/b^17)^(1/4)*b^4) - 585*(-a*d^30/b^17)^(1/
4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(195*sqrt(d*x)*d^7
 - 195*(-a*d^30/b^17)^(1/4)*b^4) - 4*(384*b^3*d^7*x^6 + 1469*a*b^2*d^7*x^4
 + 1638*a^2*b*d^7*x^2 + 585*a^3*d^7)*sqrt(d*x))/(b^7*x^6 + 3*a*b^6*x^4 + 3
*a^2*b^5*x^2 + a^3*b^4)
```

Sympy [F]

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \int \frac{(dx)^{\frac{15}{2}}}{(a+bx^2)^4}\,dx$$

input
```
integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral((d*x)**(15/2)/(a + b*x**2)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.26

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{3072\sqrt{dx}d^8}{b^4} + \frac{8\left(317(dx)^{\frac{9}{2}}ab^2d^{10}+486(dx)^{\frac{5}{2}}a^2bd^{12}+201\sqrt{dx}a^3d^{14}\right)}{b^7d^6x^6+3ab^6d^6x^4+3a^2b^5d^6x^2+a^3b^4d^6} - \frac{585\left(\frac{\sqrt{2}d^{10}\log\left(\sqrt{b}dx+\sqrt{2}(\right.}{(ad^2)}\right.}{}}{}$$

input
```
integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/1536*(3072*sqrt(d*x)*d^8/b^4 + 8*(317*(d*x)^(9/2)*a*b^2*d^10 + 486*(d*x)
^(5/2)*a^2*b*d^12 + 201*sqrt(d*x)*a^3*d^14)/(b^7*d^6*x^6 + 3*a*b^6*d^6*x^4
 + 3*a^2*b^5*d^6*x^2 + a^3*b^4*d^6) - 585*(sqrt(2)*d^10*log(sqrt(b)*d*x +
sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4
)) - sqrt(2)*d^10*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4
) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^9*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)
*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^9*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b
)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a/b^4)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.16

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{\frac{1170\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^8\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5}+\frac{1170\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^8\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5}+\frac{585\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^8}{}}{}$$

input
```
integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-1/1536*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*d^8*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^5 + 1170*sqrt(2)*(a*b^3*d^
2)^(1/4)*d^8*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/b^5 + 585*sqrt(2)*(a*b^3*d^2)^(1/4)*d^8*log(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5 - 585*sqrt(2)*(a*b^3*d^2)^(
1/4)*d^8*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5
- 3072*sqrt(d*x)*d^8/b^4 - 8*(317*sqrt(d*x)*a*b^2*d^14*x^4 + 486*sqrt(d*x)
*a^2*b*d^14*x^2 + 201*sqrt(d*x)*a^3*d^14)/((b*d^2*x^2 + a*d^2)^3*b^4))/d
```

Mupad [B] (verification not implemented)

Time = 17.81 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.63

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{67\,a^3\,d^{13}\,\sqrt{d\,x}}{64}+\frac{81\,a^2\,b\,d^{11}\,{(d\,x)}^{5/2}}{32}+\frac{317\,a\,b^2\,d^9\,{(d\,x)}^{9/2}}{192}}{a^3\,b^4\,d^6+3\,a^2\,b^5\,d^6\,x^2+3\,a\,b^6\,d^6\,x^4+b^7\,d^6\,x^6} + \frac{2\,d^7\,\sqrt{d\,x}}{b^4} - \frac{195\,{(-a)}^{1/4}\,d^{15/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{128\,b^{17/4}} + \frac{{(-a)}^{1/4}\,d^{15/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1{}\mathrm{i}}{{(-a)}^{1/4}\,\sqrt{d}}\right)\,195{}\mathrm{i}}{128\,b^{17/4}}$$

input
```
int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
((67*a^3*d^13*(d*x)^(1/2))/64 + (81*a^2*b*d^11*(d*x)^(5/2))/32 + (317*a*b^
2*d^9*(d*x)^(9/2))/192)/(a^3*b^4*d^6 + b^7*d^6*x^6 + 3*a*b^6*d^6*x^4 + 3*a
^2*b^5*d^6*x^2) + (2*d^7*(d*x)^(1/2))/b^4 - (195*(-a)^(1/4)*d^(15/2)*atan(
(b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*b^(17/4)) + ((-a)^(1/4)*
d^(15/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*195i)/(128*b^
(17/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 660, normalized size of antiderivative = 2.42

$$\int \frac{(dx)^{15/2}}{(a^2 + 2abx^2 + b^2x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(sqrt(d)*d**7*(1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 3510*b**(3/4)
*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 + 3510*b**(3/4)*a**(1/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(
2)))*a*b**2*x**4 + 1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 - 1170
*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqr
t(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 - 3510*b**(3/4)*a**(1/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**2*b*x**2 - 3510*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 -
1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)
*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 585*b**(3/4)*a**(1/4)*s
qrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**
3 + 1755*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2
) + sqrt(a) + sqrt(b)*x)*a**2*b*x**2 + 1755*b**(3/4)*a**(1/4)*sqrt(2)*log(
 - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 +
585*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + s
qrt(a) + sqrt(b)*x)*b**3*x**6 - 585*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(...
```

3.459 $\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 256

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{d(dx)^{11/2}}{6b(a+bx^2)^3} - \frac{11d^3(dx)^{7/2}}{48b^2(a+bx^2)^2} - \frac{77d^5(dx)^{3/2}}{192b^3(a+bx^2)} - \frac{77d^{13/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\sqrt[4]{a}b^{15/4}} + \frac{77d^{13/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\sqrt[4]{a}b^{15/4}} - \frac{77d^{13/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}\sqrt[4]{a}b^{15/4}}$$

output

```
-1/6*d*(d*x)^(11/2)/b/(b*x^2+a)^3-11/48*d^3*(d*x)^(7/2)/b^2/(b*x^2+a)^2-77
/192*d^5*(d*x)^(3/2)/b^3/(b*x^2+a)-77/256*d^(13/2)*arctan(1-2^(1/2)*b^(1/4
)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(1/4)/b^(15/4)+77/256*d^(13/2)*ar
ctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(1/4)/b^(15/
4)-77/256*d^(13/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^
(1/2)+b^(1/2)*x))*2^(1/2)/a^(1/4)/b^(15/4)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.64

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{d^6\sqrt{dx}\left(-\frac{4b^{3/4}x^{3/2}(77a^2+198abx^2+153b^2x^4)}{(a+bx^2)^3}-\frac{231\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}}-\frac{231\sqrt{2}\mathrm{ar}\cdots}{\cdots}\right)}{768b^{15/4}\sqrt{x}}$$

input
```
Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(d^6*Sqrt[d*x]*((-4*b^(3/4)*x^(3/2)*(77*a^2 + 198*a*b*x^2 + 153*b^2*x^4))/
(a + b*x^2)^3 - (231*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x])])/a^(1/4) - (231*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4)))/(768*b^(15/4)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.45, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {1380, 27, 252, 252, 252, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^4\int \frac{(dx)^{13/2}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(dx)^{13/2}}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{11d^2\int \frac{(dx)^{9/2}}{(bx^2+a)^3}dx}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 252$$

$$\frac{11d^2\left(\frac{7d^2\int \frac{(dx)^{5/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 252$$

$$\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\int \frac{\sqrt{dx}}{bx^2+a}dx}{4b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 266$$

$$\frac{11d^2\left(\frac{7d^2\left(\frac{3d\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 826$$

$$\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{1476}$$

$$\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{1082}$$

$$\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 217

$$\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

$\big\downarrow$ 1479

$$11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\sqrt[4]{\cdots}}\right.}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\cdots}}}{2\sqrt{b}}\cdots\right)}{2b}\cdots\right)}{8b}\cdots\right)}{12b}\cdots$$

$$\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{}\cdots}{\sqrt[4]{b}}\right.}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\cdots\right)}{2b}\cdots\right)}{8b}\cdots\right)}{12b}\cdots$$

$$\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

$\big\downarrow$ 27

$$\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2b}-\frac{d}{2b}\cdots}{8b}\cdots}{12b}\cdots$$

$$\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 1103

$$\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right.}{2b}\right.}{8b}\right.}{12b}$$

$$\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}$$

input
```
Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*(d*(d*x)^(11/2))/(b*(a + b*x^2)^3) + (11*d^2*(-1/4*(d*(d*x)^(7/2))/(b
*(a + b*x^2)^2) + (7*d^2*(-1/2*(d*(d*x)^(3/2))/(b*(a + b*x^2)) + (3*d^3*((
-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*
d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(
2*b)))/(8*b)))/(12*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.79

method	result
derivativedivides	$2d^7\left(\frac{-\frac{51(dx)^{\frac{11}{2}}}{128b}-\frac{33d^2a(dx)^{\frac{7}{2}}}{64b^2}-\frac{77d^4a^2(dx)^{\frac{3}{2}}}{384b^3}}{(b\,d^2x^2+a\,d^2)^3}+\frac{77\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+\right.\right.}{1024b^4\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$
default	$2d^7\left(\frac{-\frac{51(dx)^{\frac{11}{2}}}{128b}-\frac{33d^2a(dx)^{\frac{7}{2}}}{64b^2}-\frac{77d^4a^2(dx)^{\frac{3}{2}}}{384b^3}}{(b\,d^2x^2+a\,d^2)^3}+\frac{77\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+\right.\right.}{1024b^4\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$
pseudoelliptic	$\frac{77d^6\left(-8\left(\frac{153}{77}b^2x^4+\frac{18}{7}ab\,x^2+a^2\right)\sqrt{dx}\,bx\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}+3d\sqrt{2}\,(b\,x^2+a)^3\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\right.\right.\right.}{1536\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}(b\,x^2+a)^3b^4}$

input
```
int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*d^7*((-51/128/b*(d*x)^(11/2)-33/64/b^2*d^2*a*(d*x)^(7/2)-77/384/b^3*d^4*
a^2*(d*x)^(3/2))/(b*d^2*x^2+a*d^2)^3+77/1024/b^4/(a*d^2/b)^(1/4)*2^(1/2)*(
ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(
1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.59

$$\int\frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\frac{231\left(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3\right)\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\log\left(456533\sqrt{dx}d^{19}+4565\right.}{}$$

input
```
integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/768*(231*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^1
5))^(1/4)*log(456533*sqrt(d*x)*d^19 + 456533*(-d^26/(a*b^15))^(3/4)*a*b^11
) - 231*(I*b^6*x^6 + 3*I*a*b^5*x^4 + 3*I*a^2*b^4*x^2 + I*a^3*b^3)*(-d^26/(
a*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 + 456533*I*(-d^26/(a*b^15))^(3/4)
*a*b^11) - 231*(-I*b^6*x^6 - 3*I*a*b^5*x^4 - 3*I*a^2*b^4*x^2 - I*a^3*b^3)*
(-d^26/(a*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 - 456533*I*(-d^26/(a*b^15
))^(3/4)*a*b^11) - 231*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(
-d^26/(a*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 - 456533*(-d^26/(a*b^15))^
(3/4)*a*b^11) - 4*(153*b^2*d^6*x^5 + 198*a*b*d^6*x^3 + 77*a^2*d^6*x)*sqrt(
d*x))/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)
```

Sympy [F]

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx=\int \frac{(dx)^{\frac{13}{2}}}{(a+bx^2)^4}\,dx$$

input
```
integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral((d*x)**(13/2)/(a + b*x**2)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.24

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx=\frac{231\,d^8\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\ldots}{\ldots}\right.}{b^3}$$

input
```
integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/1536*(231*d^8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/
4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d
)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
- 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*s
qrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4
) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)
*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b^3
 - 8*(153*(d*x)^(11/2)*b^2*d^8 + 198*(d*x)^(7/2)*a*b*d^10 + 77*(d*x)^(3/2)
*a^2*d^12)/(b^6*d^6*x^6 + 3*a*b^5*d^6*x^4 + 3*a^2*b^4*d^6*x^2 + a^3*b^3*d^
6))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.23

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx =$$

$$-\frac{1}{1536}d^6\left(\frac{8\left(153\sqrt{dx}b^2d^6x^5+198\sqrt{dx}abd^6x^3+77\sqrt{dx}a^2d^6x\right)}{(bd^2x^2+ad^2)^3b^3}-\frac{462\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\right.}{2\left(\frac{ad^2}{b}\right.}\right.}{ab^6d}\right.$$

input
```
integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-1/1536*d^6*(8*(153*sqrt(d*x)*b^2*d^6*x^5 + 198*sqrt(d*x)*a*b*d^6*x^3 + 77
*sqrt(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*b^3) - 462*sqrt(2)*(a*b^3*d^2
)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/(a*b^6*d) - 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6*d) + 231*sqr
t(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a*b^6*d) - 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6*d))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.60

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{77\,d^{13/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{1/4}\,b^{15/4}} - \frac{\frac{51\,d^7\,(d\,x)^{11/2}}{64\,b}+\frac{77\,a^2\,d^{11}\,(d\,x)^{3/2}}{192\,b^3}+\frac{33\,a\,d^9\,(d\,x)^{7/2}}{32\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6} - \frac{77\,d^{13/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{1/4}\,b^{15/4}}$$

input

```
int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output

```
(77*d^(13/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(
1/4)*b^(15/4)) - ((51*d^7*(d*x)^(11/2))/(64*b) + (77*a^2*d^11*(d*x)^(3/2))
/(192*b^3) + (33*a*d^9*(d*x)^(7/2))/(32*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a
^2*b*d^6*x^2 + 3*a*b^2*d^6*x^4) - (77*d^(13/2)*atanh((b^(1/4)*(d*x)^(1/2))
/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(1/4)*b^(15/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.55

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \text{Too large to display}$$

input

```
int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output

```
(sqrt(d)*d**6*( - 462*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 - 1386*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 - 1386*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a*b**2*x**4 - 462*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 462
*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqr
t(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 1386*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**2*b*x**2 + 1386*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 +
462*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*
sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 231*b**(1/4)*a**(3/4)*sq
rt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3
 + 693*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2)
+ sqrt(a) + sqrt(b)*x)*a**2*b*x**2 + 693*b**(1/4)*a**(3/4)*sqrt(2)*log( -
sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 + 231
*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt
(a) + sqrt(b)*x)*b**3*x**6 - 231*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*...
```

3.460 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 256

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = -\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3} - \frac{3d^3(dx)^{5/2}}{16b^2\left(a+bx^2\right)^2} - \frac{15d^5\sqrt{dx}}{64b^3\left(a+bx^2\right)} - \frac{15d^{11/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{3/4}b^{13/4}} + \frac{15d^{11/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{3/4}b^{13/4}} + \frac{15d^{11/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}a^{3/4}b^{13/4}}$$

output

```
-1/6*d*(d*x)^(9/2)/b/(b*x^2+a)^3-3/16*d^3*(d*x)^(5/2)/b^2/(b*x^2+a)^2-15/6
4*d^5*(d*x)^(1/2)/b^3/(b*x^2+a)-15/256*d^(11/2)*arctan(1-2^(1/2)*b^(1/4)*(
d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(3/4)/b^(13/4)+15/256*d^(11/2)*arcta
n(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(3/4)/b^(13/4)+
15/256*d^(11/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/
2)+b^(1/2)*x))*2^(1/2)/a^(3/4)/b^(13/4)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.64

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{d^5\sqrt{dx}\left(-\frac{4\sqrt[4]{b}(45a^2+126abx^2+113b^2x^4)}{(a+bx^2)^3}-\frac{45\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}\sqrt{x}}+\frac{45\sqrt{2}\text{arctanh}}{a}\right.}{768b^{13/4}}$$

input
```
Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(d^5*Sqrt[d*x]*((-4*b^(1/4)*(45*a^2 + 126*a*b*x^2 + 113*b^2*x^4))/(a + b*x
^2)^3 - (45*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x])])/(a^(3/4)*Sqrt[x]) + (45*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(3/4)*Sqrt[x])))/(768*b^(13/4))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.45, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {1380, 27, 252, 252, 252, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^4\int \frac{(dx)^{11/2}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{11/2}}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{3d^2\int \frac{(dx)^{7/2}}{(bx^2+a)^3}dx}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 252

$$\frac{3d^2\left(\frac{5d^2\int \frac{(dx)^{3/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{5/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 252

$$\frac{3d^2\left(\frac{5d^2\left(\frac{d^2\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 266

$$\frac{3d^2\left(\frac{5d^2\left(\frac{d\int \frac{1}{bx^2+a}d\sqrt{dx}}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 755

$$\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 27

$$\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b(a+bx^2)^2}\right)}{4b}$$

$$\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

$\big\downarrow$ 1082

$$\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

$$\downarrow \text{217}$$

$$\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 1479

$$3d^2\frac{\left(5d^2\frac{\left(d\frac{\left(d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(\cdots\right.}{\sqrt{2}\cdots}\right.}{2\sqrt{a}}\cdots\right.}{2b}\cdots\right.}{8b}\cdots$$

$$4b$$

↓ 25

$$3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\ldots\right.}{\sqrt{2}\sqrt[4]{a}\ldots}\right.}{2\sqrt{a}}\right.}{2b}\right.}{8b}\right.}{4b}$$

↓ 27

$$\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right.}{2b}\right.}{8b}\right.}{4b}$$

$$\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 1103

$$\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{a}}}{2b}}{8b}}{4b}$$

$$\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}$$

input
```
Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*(d*(d*x)^(9/2))/(b*(a + b*x^2)^3) + (3*d^2*(-1/4*(d*(d*x)^(5/2))/(b*(
a + b*x^2)^2) + (5*d^2*(-1/2*(d*Sqrt[d*x])/(b*(a + b*x^2)) + (d*((d*(-(Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d
 + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(
2*b)))/(8*b)))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.82

method	result
derivativedivides	$2d^7\left(\frac{-\frac{113(dx)^{\frac{9}{2}}}{384b}-\frac{21a\,d^2(dx)^{\frac{5}{2}}}{64b^2}-\frac{15a^2d^4\sqrt{dx}}{128b^3}}{(b\,d^2x^2+a\,d^2)^3}+\frac{15\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\ldots\right.\right.}{1024b^3a\,d^2}\right.$
default	$2d^7\left(\frac{-\frac{113(dx)^{\frac{9}{2}}}{384b}-\frac{21a\,d^2(dx)^{\frac{5}{2}}}{64b^2}-\frac{15a^2d^4\sqrt{dx}}{128b^3}}{(b\,d^2x^2+a\,d^2)^3}+\frac{15\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\ldots\right.\right.}{1024b^3a\,d^2}\right.$
pseudoelliptic	$\frac{15d^5\left(\frac{\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(b\,x^2+a\right)^3}{2}+\sqrt{2}\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\left(b\,x^2+a\right)^3\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)+\sqrt{2}\left(\ldots\right.\right.}{256b^3\left(b\,x^2+a\right)^3a}$

input

```
int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output

```
2*d^7*((-113/384/b*(d*x)^(9/2)-21/64*a*d^2/b^2*(d*x)^(5/2)-15/128*a^2*d^4/
b^3*(d*x)^(1/2))/(b*d^2*x^2+a*d^2)^3+15/1024/b^3*(a*d^2/b)^(1/4)/a/d^2*2^(
1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^
2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1))
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.58

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{45\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}\log\left(15\,\sqrt{dx}d^5+15\left(-\frac{d^{22}}{a^3b^{13}}\right.\right.}{}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/768*(45*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b^
13))^(1/4)*log(15*sqrt(d*x)*d^5 + 15*(-d^22/(a^3*b^13))^(1/4)*a*b^3) - 45*
(-I*b^6*x^6 - 3*I*a*b^5*x^4 - 3*I*a^2*b^4*x^2 - I*a^3*b^3)*(-d^22/(a^3*b^1
3))^(1/4)*log(15*sqrt(d*x)*d^5 + 15*I*(-d^22/(a^3*b^13))^(1/4)*a*b^3) - 45
*(I*b^6*x^6 + 3*I*a*b^5*x^4 + 3*I*a^2*b^4*x^2 + I*a^3*b^3)*(-d^22/(a^3*b^1
3))^(1/4)*log(15*sqrt(d*x)*d^5 - 15*I*(-d^22/(a^3*b^13))^(1/4)*a*b^3) - 45
*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b^13))^(1/4
)*log(15*sqrt(d*x)*d^5 - 15*(-d^22/(a^3*b^13))^(1/4)*a*b^3) - 4*(113*b^2*d
^5*x^4 + 126*a*b*d^5*x^2 + 45*a^2*d^5)*sqrt(d*x))/(b^6*x^6 + 3*a*b^5*x^4 +
 3*a^2*b^4*x^2 + a^3*b^3)
```

Sympy [F]

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \int \frac{(dx)^{\frac{11}{2}}}{(a+bx^2)^4}\,dx$$

input
```
integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral((d*x)**(11/2)/(a + b*x**2)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.27

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx =$$

$$-\frac{\frac{8\left(113\,(dx)^{\frac{9}{2}}b^2d^8+126\,(dx)^{\frac{5}{2}}abd^{10}+45\,\sqrt{dx}a^2d^{12}\right)}{b^6d^6x^6+3\,ab^5d^6x^4+3\,a^2b^4d^6x^2+a^3b^3d^6}-\frac{45\left(\frac{\sqrt{2}d^8\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^8\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{}}{1536\,d}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/1536*(8*(113*(d*x)^(9/2)*b^2*d^8 + 126*(d*x)^(5/2)*a*b*d^10 + 45*sqrt(d
*x)*a^2*d^12)/(b^6*d^6*x^6 + 3*a*b^5*d^6*x^4 + 3*a^2*b^4*d^6*x^2 + a^3*b^3
*d^6) - 45*(sqrt(2)*d^8*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*
b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^8*log(sqrt(b)*d*x
 - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(
1/4)) + 2*sqrt(2)*d^7*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) +
2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqr
t(a)) + 2*sqrt(2)*d^7*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) -
 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sq
rt(a)))/b^3)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.22

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{90\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^6\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}+\frac{90\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^6\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-\right.}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{ab^4}}{}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/1536*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*d^6*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^4) + 90*sqrt(2)*(a*b^3*d^2
)^(1/4)*d^6*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a
*d^2/b)^(1/4))/(a*b^4) + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*d^6*log(d*x + sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4) - 45*sqrt(2)*(a*b^3*d
^2)^(1/4)*d^6*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a*b^4) - 8*(113*sqrt(d*x)*b^2*d^12*x^4 + 126*sqrt(d*x)*a*b*d^12*x^2 + 45
*sqrt(d*x)*a^2*d^12)/((b*d^2*x^2 + a*d^2)^3*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 18.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.60

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{\frac{113\,d^7\,(d\,x)^{9/2}}{192\,b}+\frac{15\,a^2\,d^{11}\,\sqrt{d\,x}}{64\,b^3}+\frac{21\,a\,d^9\,(d\,x)^{5/2}}{32\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6} - \frac{15\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{3/4}\,b^{13/4}} - \frac{15\,d^{11/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{3/4}\,b^{13/4}}$$

input
```
int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
- ((113*d^7*(d*x)^(9/2))/(192*b) + (15*a^2*d^11*(d*x)^(1/2))/(64*b^3) + (2
1*a*d^9*(d*x)^(5/2))/(32*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 +
3*a*b^2*d^6*x^4) - (15*d^(11/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(
1/2))))/(128*(-a)^(3/4)*b^(13/4)) - (15*d^(11/2)*atanh((b^(1/4)*(d*x)^(1/2
))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(3/4)*b^(13/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 653, normalized size of antiderivative = 2.55

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(sqrt(d)*d**5*( - 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 - 270*b**(3/4)
*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 - 270*b**(3/4)*a**(1/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*a*b**2*x**4 - 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 90*b**(
3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 270*b**(3/4)*a**(1/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
*a**2*b*x**2 + 270*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 + 90*b**(
3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 - 45*b**(3/4)*a**(1/4)*sqrt(2)*log
( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3 - 135*b*
*(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a)
 + sqrt(b)*x)*a**2*b*x**2 - 135*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b
**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 - 45*b**(3/4)*
a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt
(b)*x)*b**3*x**6 + 45*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a*...
```

3.461 $\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 259

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{d(dx)^{7/2}}{6b(a+bx^2)^3} - \frac{7d^3(dx)^{3/2}}{48b^2(a+bx^2)^2} + \frac{7d^3(dx)^{3/2}}{64ab^2(a+bx^2)} - \frac{7d^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{5/4}b^{11/4}} + \frac{7d^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{5/4}b^{11/4}} - \frac{7d^{9/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}a^{5/4}b^{11/4}}$$

output
```
-1/6*d*(d*x)^(7/2)/b/(b*x^2+a)^3-7/48*d^3*(d*x)^(3/2)/b^2/(b*x^2+a)^2+7/64
*d^3*(d*x)^(3/2)/a/b^2/(b*x^2+a)-7/256*d^(9/2)*arctan(1-2^(1/2)*b^(1/4)*(d
*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(5/4)/b^(11/4)+7/256*d^(9/2)*arctan(1
+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(5/4)/b^(11/4)-7/2
56*d^(9/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^
(1/2)*x))*2^(1/2)/a^(5/4)/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.63

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{d^4\sqrt{dx}\left(\frac{4\sqrt[4]{a}b^{3/4}x^{3/2}(-7a^2-18abx^2+21b^2x^4)}{(a+bx^2)^3}-21\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-21\sqrt{}\right.}{768a^{5/4}b^{11/4}\sqrt{x}}$$

input
```
Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(d^4*Sqrt[d*x]*((4*a^(1/4)*b^(3/4)*x^(3/2)*(-7*a^2 - 18*a*b*x^2 + 21*b^2*x
^4))/(a + b*x^2)^3 - 21*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x])] - 21*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
])/(Sqrt[a] + Sqrt[b]*x)]))/(768*a^(5/4)*b^(11/4)*Sqrt[x])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.43, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {1380, 27, 252, 252, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^4\int \frac{(dx)^{9/2}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{9/2}}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{7d^2\int\frac{(dx)^{5/2}}{(bx^2+a)^3}dx}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 252

$$\frac{7d^2\left(\frac{3d^2\int\frac{\sqrt{dx}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 253

$$\frac{7d^2\left(\frac{3d^2\left(\frac{\int\frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 266

$$\frac{7d^2\left(\frac{3d^2\left(\frac{\int\frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 27

$$\frac{7d^2\left(\frac{3d^2\left(\frac{d\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 826

$$\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{1476}$$

$$\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{1082}$$

$$\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 217

$$\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 1479

$$\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}\right)}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}\right)}{8b}\right)}{12b}$$

$$\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 25$$

$$7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right.\cdots}{2a}\cdots}{8b}\cdots}{12b}\cdots$$

$$\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 27$$

$$7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)\cdots}{2ad\left(a\cdots\right.}\cdots\right)}{8b}\cdots\right)}{12b}\cdots$$

$$\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 1103

$$\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}\right)}{8b}\right)}{12b}$$

$$\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}$$

input
```
Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*(d*(d*x)^(7/2))/(b*(a + b*x^2)^3) + (7*d^2*(-1/4*(d*(d*x)^(3/2))/(b*(
a + b*x^2)^2) + (3*d^2*((d*x)^(3/2)/(2*a*d*(a + b*x^2)) + (d*((-(ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]
*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*
Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*a)))/(8*b
)))/(12*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.80

method	result
derivativedivides	$2d^7\left(\frac{\frac{7(dx)^{\frac{11}{2}}}{128d^2a}-\frac{3(dx)^{\frac{7}{2}}}{64b}-\frac{7d^2a(dx)^{\frac{3}{2}}}{384b^2}}{(b\,d^2x^2+a\,d^2)^3}+\frac{7\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arct}\right.}{1024d^2a\,b^3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$
default	$2d^7\left(\frac{\frac{7(dx)^{\frac{11}{2}}}{128d^2a}-\frac{3(dx)^{\frac{7}{2}}}{64b}-\frac{7d^2a(dx)^{\frac{3}{2}}}{384b^2}}{(b\,d^2x^2+a\,d^2)^3}+\frac{7\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arct}\right.}{1024d^2a\,b^3\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$
pseudoelliptic	$\frac{7d^4\left(-8\sqrt{dx}\,(-3b^2x^4+\frac{18}{7}ab\,x^2+a^2)bx\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}+3d\sqrt{2}\,(b\,x^2+a)^3\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\right.\right.\right.}{1536\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^3(b\,x^2+a)^3a}$

input
```
int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*d^7*((7/128/d^2/a*(d*x)^(11/2)-3/64/b*(d*x)^(7/2)-7/384*d^2*a/b^2*(d*x)^
(3/2))/(b*d^2*x^2+a*d^2)^3+7/1024/d^2/a/b^3/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d
*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*
(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.66

$$\int\frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx=\frac{21\left(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\log\left(343\,\sqrt{dx}d^{13}+343\,\left(-\right.\right.}{}$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/768*(21*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^18/(a^
5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 + 343*(-d^18/(a^5*b^11))^(3/4)*a^4*b
^8) - 21*(I*a*b^5*x^6 + 3*I*a^2*b^4*x^4 + 3*I*a^3*b^3*x^2 + I*a^4*b^2)*(-d
^18/(a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 + 343*I*(-d^18/(a^5*b^11))^(3
/4)*a^4*b^8) - 21*(-I*a*b^5*x^6 - 3*I*a^2*b^4*x^4 - 3*I*a^3*b^3*x^2 - I*a^
4*b^2)*(-d^18/(a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 - 343*I*(-d^18/(a^5
*b^11))^(3/4)*a^4*b^8) - 21*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a
^4*b^2)*(-d^18/(a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 - 343*(-d^18/(a^5*
b^11))^(3/4)*a^4*b^8) + 4*(21*b^2*d^4*x^5 - 18*a*b*d^4*x^3 - 7*a^2*d^4*x)*
sqrt(d*x))/(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [F]

$$\int \frac{(dx)^{9/2}}{(a^2 + 2abx^2 + b^2x^4)^2}\, dx = \int \frac{(dx)^{\frac{9}{2}}}{(a + bx^2)^4}\, dx$$

input
```
integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral((d*x)**(9/2)/(a + b*x**2)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.25

$$\int \frac{(dx)^{9/2}}{(a^2 + 2abx^2 + b^2x^4)^2}\, dx = \frac{21\, d^6 \left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}} + 2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}} - 2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} - \frac{\sqrt{2}\log\left(\cdots\right.}{\cdots} \cdots \right.}{ab^2 \cdots}$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/1536*(21*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)
*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) -
 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sq
rt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a*b
^2) + 8*(21*(d*x)^(11/2)*b^2*d^6 - 18*(d*x)^(7/2)*a*b*d^8 - 7*(d*x)^(3/2)*
a^2*d^10)/(a*b^5*d^6*x^6 + 3*a^2*b^4*d^6*x^4 + 3*a^3*b^3*d^6*x^2 + a^4*b^2
*d^6))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.22

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{1}{1536}\,d^4\left(\frac{8\left(21\sqrt{dx}b^2d^6x^5-18\sqrt{dx}abd^6x^3-7\sqrt{dx}a^2d^6x\right)}{\left(bd^2x^2+ad^2\right)^3ab^2}+\frac{42\sqrt{2}(ab^3d^2)}{}\right.$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/1536*d^4*(8*(21*sqrt(d*x)*b^2*d^6*x^5 - 18*sqrt(d*x)*a*b*d^6*x^3 - 7*sqr
t(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*a*b^2) + 42*sqrt(2)*(a*b^3*d^2)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^
(1/4))/(a^2*b^5*d) + 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5*d) - 21*sqrt
(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^2*b^5*d) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^5*d))
```

Mupad [B] (verification not implemented)

Time = 18.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.58

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{7\,d^{9/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{5/4}\,b^{11/4}} - \frac{7\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{5/4}\,b^{11/4}} - \frac{\frac{3\,d^7\,(d\,x)^{7/2}}{32\,b}-\frac{7\,d^5\,(d\,x)^{11/2}}{64\,a}+\frac{7\,a\,d^9\,(d\,x)^{3/2}}{192\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}$$

input

```
int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output

```
(7*d^(9/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(5
/4)*b^(11/4)) - (7*d^(9/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))
))/(128*(-a)^(5/4)*b^(11/4)) - ((3*d^7*(d*x)^(7/2))/(32*b) - (7*d^5*(d*x)^
(11/2))/(64*a) + (7*a*d^9*(d*x)^(3/2))/(192*b^2))/(a^3*d^6 + b^3*d^6*x^6 +
 3*a^2*b*d^6*x^2 + 3*a*b^2*d^6*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.53

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \text{Too large to display}$$

input

```
int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(sqrt(d)*d**4*( - 42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 - 126*b**(1/4)
*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 - 126*b**(1/4)*a**(3/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*a*b**2*x**4 - 42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 42*b**(
1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 126*b**(1/4)*a**(3/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
*a**2*b*x**2 + 126*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 + 42*b**(
1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 21*b**(1/4)*a**(3/4)*sqrt(2)*log
( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3 + 63*b**
(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a)
+ sqrt(b)*x)*a**2*b*x**2 + 63*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**
(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 + 21*b**(1/4)*a*
*(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b
)*x)*b**3*x**6 - 21*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(...
```

3.462 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 259

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = -\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}-\frac{5d^3\sqrt{dx}}{48b^2\left(a+bx^2\right)^2}+\frac{5d^3\sqrt{dx}}{192ab^2\left(a+bx^2\right)}-\frac{5d^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{7/4}b^{9/4}}+\frac{5d^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{7/4}b^{9/4}}+\frac{5d^{7/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}a^{7/4}b^{9/4}}$$

output
```
-1/6*d*(d*x)^(5/2)/b/(b*x^2+a)^3-5/48*d^3*(d*x)^(1/2)/b^2/(b*x^2+a)^2+5/19
2*d^3*(d*x)^(1/2)/a/b^2/(b*x^2+a)-5/256*d^(7/2)*arctan(1-2^(1/2)*b^(1/4)*(
d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(7/4)/b^(9/4)+5/256*d^(7/2)*arctan(1
+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(7/4)/b^(9/4)+5/25
6*d^(7/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(
1/2)*x))*2^(1/2)/a^(7/4)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.63

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{d^3\sqrt{dx}\left(\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}(-15a^2-42abx^2+5b^2x^4)}{(a+bx^2)^3}-15\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+15\sqrt{2}\right.}{768a^{7/4}b^{9/4}\sqrt{x}}$$

input
```
Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(d^3*Sqrt[d*x]*((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-15*a^2 - 42*a*b*x^2 + 5*b^2*x
^4))/(a + b*x^2)^3 - 15*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x])] + 15*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
])/(Sqrt[a] + Sqrt[b]*x)]))/(768*a^(7/4)*b^(9/4)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.45, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {1380, 27, 252, 252, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^4\int \frac{(dx)^{7/2}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(dx)^{7/2}}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{5d^2\int\frac{(dx)^{3/2}}{(bx^2+a)^3}dx}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 252

$$\frac{5d^2\left(\frac{d^2\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)^2}dx}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 253

$$\frac{5d^2\left(\frac{d^2\left(\frac{3\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 266

$$\frac{5d^2\left(\frac{d^2\left(\frac{3\int\frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 755

$$\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 27

$$\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

$$\downarrow 1476$$

$$\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{12b}$$

$$\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 1082

$$\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

$\big\downarrow$ 217

$$\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 1479

$$5d^2 \left(\frac{d^2 \left(\frac{3 \left(\frac{d \left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d \left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1 \cdots\right.}{\sqrt{2}\cdots} \cdots \right.}{2\sqrt{a}} \cdots \right.}{2ad} \cdots \right.}{8b} \cdots \right.$$

$$\overline{12b}$$

$\Big\downarrow$ 25

$$5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\cdots}\cdots\right)}{2\sqrt{a}}\cdots\right)}{2ad}\cdots\right)}{8b}\cdots\right)}{12b}\cdots$$

↓ 27

$$\frac{5d^2 \left(\frac{d^2 \left(\frac{3 \left(\frac{d \left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}} + \frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d \left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} \right)}{2ad} \cdots \right)}{8b} \cdots \right)}{12b} \cdots$$

$$\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 1103

$$\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{\cdots}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right)}{2\sqrt{a}}\cdots\right)}{2ad}\cdots\right)}{8b}\cdots\right)}{12b}$$

$$\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}$$

input

```
Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*(d*(d*x)^(5/2))/(b*(a + b*x^2)^3) + (5*d^2*(-1/4*(d*Sqrt[d*x])/(b*(a
+ b*x^2)^2) + (d^2*(Sqrt[d*x]/(2*a*d*(a + b*x^2)) + (3*((d*(-(ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b
]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]
*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*a*d)))/
(8*b)))/(12*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.80

method	result
derivativedivides	$2d^7\left(\frac{\frac{5(dx)^{\frac{9}{2}}}{384a\,d^2}-\frac{7(dx)^{\frac{5}{2}}}{64b}-\frac{5a\,d^2\sqrt{dx}}{128b^2}}{(b\,d^2x^2+a\,d^2)^3}+\frac{5\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)\right.}{1024a^2d^4b^2}\right.$
default	$2d^7\left(\frac{\frac{5(dx)^{\frac{9}{2}}}{384a\,d^2}-\frac{7(dx)^{\frac{5}{2}}}{64b}-\frac{5a\,d^2\sqrt{dx}}{128b^2}}{(b\,d^2x^2+a\,d^2)^3}+\frac{5\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)\right.}{1024a^2d^4b^2}\right.$
pseudoelliptic	$\frac{5\left(\frac{\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(b\,x^2+a\right)^3}{2}+\sqrt{2}\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}(b\,x^2+a)^3\arctan\left(\frac{\sqrt{2}\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)+\sqrt{2}\left(\frac{a\,d}{b}\right.\right.}{256b^2a^2(b\,x^2+a)^3}$

input
```
int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*d^7*((5/384/a/d^2*(d*x)^(9/2)-7/64/b*(d*x)^(5/2)-5/128*a*d^2/b^2*(d*x)^(
1/2))/(b*d^2*x^2+a*d^2)^3+5/1024/a^2/d^4/b^2*(a*d^2/b)^(1/4)*2^(1/2)*(ln((
d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)
*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.65

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{15\,(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2)\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}\log\left(5\,a^2b^2\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}+5\,\sqrt{\right.}$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/768*(15*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^
7*b^9))^(1/4)*log(5*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)*d^3) - 1
5*(-I*a*b^5*x^6 - 3*I*a^2*b^4*x^4 - 3*I*a^3*b^3*x^2 - I*a^4*b^2)*(-d^14/(a
^7*b^9))^(1/4)*log(5*I*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)*d^3)
- 15*(I*a*b^5*x^6 + 3*I*a^2*b^4*x^4 + 3*I*a^3*b^3*x^2 + I*a^4*b^2)*(-d^14/
(a^7*b^9))^(1/4)*log(-5*I*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)*d^
3) - 15*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^7*
b^9))^(1/4)*log(-5*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)*d^3) + 4*
(5*b^2*d^3*x^4 - 42*a*b*d^3*x^2 - 15*a^2*d^3)*sqrt(d*x))/(a*b^5*x^6 + 3*a^
2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [F]

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \int \frac{(dx)^{\frac{7}{2}}}{(a+bx^2)^4}\,dx$$

input
```
integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral((d*x)**(7/2)/(a + b*x**2)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.28

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{8\left(5\,(dx)^{\frac{9}{2}}b^2d^6-42\,(dx)^{\frac{5}{2}}abd^8-15\,\sqrt{dx}a^2d^{10}\right)}{ab^5d^6x^6+3\,a^2b^4d^6x^4+3\,a^3b^3d^6x^2+a^4b^2d^6} + \frac{15\left(\frac{\sqrt{2}d^6\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}d^6\,\mathrm{l}}{}\right.}{}}{}$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/1536*(8*(5*(d*x)^(9/2)*b^2*d^6 - 42*(d*x)^(5/2)*a*b*d^8 - 15*sqrt(d*x)*a
^2*d^10)/(a*b^5*d^6*x^6 + 3*a^2*b^4*d^6*x^4 + 3*a^3*b^3*d^6*x^2 + a^4*b^2*
d^6) + 15*(sqrt(2)*d^6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b
^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x
- sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1
/4)) + 2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2
*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt
(a)) + 2*sqrt(2)*d^5*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) -
2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqr
t(a)))/(a*b^2))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.22

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{30\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^4\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3} + \frac{30\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^4\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-\right.}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{a^2b^3}}{}$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/1536*(30*sqrt(2)*(a*b^3*d^2)^(1/4)*d^4*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^3) + 30*sqrt(2)*(a*b^3*d
^2)^(1/4)*d^4*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/(a^2*b^3) + 15*sqrt(2)*(a*b^3*d^2)^(1/4)*d^4*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^3) - 15*sqrt(2)*(a
*b^3*d^2)^(1/4)*d^4*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d
^2/b))/(a^2*b^3) + 8*(5*sqrt(d*x)*b^2*d^10*x^4 - 42*sqrt(d*x)*a*b*d^10*x^2
 - 15*sqrt(d*x)*a^2*d^10)/((b*d^2*x^2 + a*d^2)^3*a*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 18.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.58

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{5\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{7/4}\,b^{9/4}} - \frac{\frac{7\,d^7\,(d\,x)^{5/2}}{32\,b}-\frac{5\,d^5\,(d\,x)^{9/2}}{192\,a}+\frac{5\,a\,d^9\,\sqrt{d\,x}}{64\,b^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6} + \frac{5\,d^{7/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{7/4}\,b^{9/4}}$$

input
```
int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(5*d^(7/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-a)^(7/
4)*b^(9/4)) - ((7*d^7*(d*x)^(5/2))/(32*b) - (5*d^5*(d*x)^(9/2))/(192*a) +
(5*a*d^9*(d*x)^(1/2))/(64*b^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 +
 3*a*b^2*d^6*x^4) + (5*d^(7/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(
1/2))))/(128*(-a)^(7/4)*b^(9/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 653, normalized size of antiderivative = 2.52

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(sqrt(d)*d**3*( - 30*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 - 90*b**(3/4)*
a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 - 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))
)*a*b**2*x**4 - 30*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 30*b**(3/
4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(
1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*
*2*b*x**2 + 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 + 30*b**(3/4)
*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 - 15*b**(3/4)*a**(1/4)*sqrt(2)*log( -
sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3 - 45*b**(3/4
)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sq
rt(b)*x)*a**2*b*x**2 - 45*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4
)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 - 15*b**(3/4)*a**(1/
4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)
*b**3*x**6 + 15*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)...
```

3.463 $\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 258

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = -\frac{d(dx)^{3/2}}{6b(a+bx^2)^3}+\frac{d(dx)^{3/2}}{16ab(a+bx^2)^2}+\frac{5d(dx)^{3/2}}{64a^2b(a+bx^2)}-\frac{5d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{9/4}b^{7/4}}+\frac{5d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{9/4}b^{7/4}}-\frac{5d^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}a^{9/4}b^{7/4}}$$

output
```
-1/6*d*(d*x)^(3/2)/b/(b*x^2+a)^3+1/16*d*(d*x)^(3/2)/a/b/(b*x^2+a)^2+5/64*d
*(d*x)^(3/2)/a^2/b/(b*x^2+a)-5/256*d^(5/2)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^
(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(9/4)/b^(7/4)+5/256*d^(5/2)*arctan(1+2^(1
/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(9/4)/b^(7/4)-5/256*d^(
5/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*
x))*2^(1/2)/a^(9/4)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.62

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{(dx)^{5/2}\left(\frac{4\sqrt[4]{a}b^{3/4}x^{3/2}(-5a^2+42abx^2+15b^2x^4)}{(a+bx^2)^3} - 15\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - 15\cdots\right)}{768a^{9/4}b^{7/4}x^{5/2}}$$

input
```
Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
((d*x)^(5/2)*((4*a^(1/4)*b^(3/4)*x^(3/2)*(-5*a^2 + 42*a*b*x^2 + 15*b^2*x^4
))/(a + b*x^2)^3 - 15*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x])] - 15*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
/(Sqrt[a] + Sqrt[b]*x)]))/(768*a^(9/4)*b^(7/4)*x^(5/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.43, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {1380, 27, 252, 253, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^4\int \frac{(dx)^{5/2}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{5/2}}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{d^2\int \frac{\sqrt{dx}}{(bx^2+a)^3}dx}{4b}-\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{d^2\left(\frac{5\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^2}dx}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{d^2\left(\frac{5\left(\frac{\int \frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{d^2\left(\frac{5\left(\frac{\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d^2\left(\frac{5\left(\frac{d\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3}$$

$$\Big\downarrow \text{1476}$$

$$\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{1082}$$

$$\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}$$

$\big\downarrow$ 1479

$$d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}}{8a}}{4b}$$

$$\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 25

$$d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}}{8a}\right)}{4b}$$

$$\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 27$$

$$d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2 \cdots\right.}\right)}{8a} \cdots\right)}{4b}$$

$$\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 1103

$$\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}+\dots\right)}{8a}\right)}{4b}$$

$$\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}$$

input
```
Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*(d*(d*x)^(3/2))/(b*(a + b*x^2)^3) + (d^2*((d*x)^(3/2)/(4*a*d*(a + b*x
^2)^2) + (5*((d*x)^(3/2)/(2*a*d*(a + b*x^2)) + (d*((-(ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) +
 ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) +
 Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/
(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*a)))/(8*a)))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.80

method	result
derivativedivides	$2d^7\left(\frac{\frac{5b(dx)^{\frac{11}{2}}}{128a^2d^4}+\frac{7(dx)^{\frac{7}{2}}}{64a\,d^2}-\frac{5(dx)^{\frac{3}{2}}}{384b}}{(b\,d^2x^2+a\,d^2)^3}+\frac{5\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{1024a^2d^4b^2\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$
default	$2d^7\left(\frac{\frac{5b(dx)^{\frac{11}{2}}}{128a^2d^4}+\frac{7(dx)^{\frac{7}{2}}}{64a\,d^2}-\frac{5(dx)^{\frac{3}{2}}}{384b}}{(b\,d^2x^2+a\,d^2)^3}+\frac{5\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{1024a^2d^4b^2\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$
pseudoelliptic	$\frac{5d^2\left(-8\sqrt{dx}\,bx\left(-3b^2x^4-\frac{42}{5}ab\,x^2+a^2\right)\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}+3d\sqrt{2}\,(b\,x^2+a)^3\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\right.\right.\right.}{1536\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^2a^2(b\,x^2+a)^3}$

input
```
int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*d^7*((5/128/a^2/d^4*b*(d*x)^(11/2)+7/64/a/d^2*(d*x)^(7/2)-5/384/b*(d*x)^
(3/2))/(b*d^2*x^2+a*d^2)^3+5/1024/a^2/d^4/b^2/(a*d^2/b)^(1/4)*2^(1/2)*(ln(
(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4
)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.66

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{15\left(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b\right)\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{1}{4}}\log\left(125\,a^7b^5\left(-\frac{d^{10}}{a^9b^7}\right)^{\frac{3}{4}}+1\right.}{}$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/768*(15*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^10/(a^
9*b^7))^(1/4)*log(125*a^7*b^5*(-d^10/(a^9*b^7))^(3/4) + 125*sqrt(d*x)*d^7)
 - 15*(I*a^2*b^4*x^6 + 3*I*a^3*b^3*x^4 + 3*I*a^4*b^2*x^2 + I*a^5*b)*(-d^10
/(a^9*b^7))^(1/4)*log(125*I*a^7*b^5*(-d^10/(a^9*b^7))^(3/4) + 125*sqrt(d*x
)*d^7) - 15*(-I*a^2*b^4*x^6 - 3*I*a^3*b^3*x^4 - 3*I*a^4*b^2*x^2 - I*a^5*b)
*(-d^10/(a^9*b^7))^(1/4)*log(-125*I*a^7*b^5*(-d^10/(a^9*b^7))^(3/4) + 125*
sqrt(d*x)*d^7) - 15*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*
(-d^10/(a^9*b^7))^(1/4)*log(-125*a^7*b^5*(-d^10/(a^9*b^7))^(3/4) + 125*sqr
t(d*x)*d^7) + 4*(15*b^2*d^2*x^5 + 42*a*b*d^2*x^3 - 5*a^2*d^2*x)*sqrt(d*x))
/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)
```

Sympy [F]

$$\int \frac{(dx)^{5/2}}{(a^2 + 2abx^2 + b^2x^4)^2}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{(a + bx^2)^4}\,dx$$

input
```
integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral((d*x)**(5/2)/(a + b*x**2)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.25

$$\int \frac{(dx)^{5/2}}{(a^2 + 2abx^2 + b^2x^4)^2}\,dx = \frac{15\,d^4\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} - \frac{\sqrt{2}\log\left(\cdots\right.}{\cdots}\cdots\right)}{a^2b\cdots}$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/1536*(15*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)
*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) -
 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sq
rt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^2
*b) + 8*(15*(d*x)^(11/2)*b^2*d^4 + 42*(d*x)^(7/2)*a*b*d^6 - 5*(d*x)^(3/2)*
a^2*d^8)/(a^2*b^4*d^6*x^6 + 3*a^3*b^3*d^6*x^4 + 3*a^4*b^2*d^6*x^2 + a^5*b*
d^6))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.23

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{1}{1536}\,d^2\left(\frac{8\left(15\sqrt{dx}b^2d^6x^5+42\sqrt{dx}abd^6x^3-5\sqrt{dx}a^2d^6x\right)}{\left(bd^2x^2+ad^2\right)^3a^2b}+\frac{30\sqrt{2}(ab^3d^2)}{}\right.$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/1536*d^2*(8*(15*sqrt(d*x)*b^2*d^6*x^5 + 42*sqrt(d*x)*a*b*d^6*x^3 - 5*sqr
t(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*a^2*b) + 30*sqrt(2)*(a*b^3*d^2)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^
(1/4))/(a^3*b^4*d) + 30*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4*d) - 15*sqrt
(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^3*b^4*d) + 15*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4*d))
```

Mupad [B] (verification not implemented)

Time = 18.73 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.58

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\frac{\frac{7\,d^5\,{\left(d\,x\right)}^{7/2}}{32\,a}-\frac{5\,d^7\,{\left(d\,x\right)}^{3/2}}{192\,b}+\frac{5\,b\,d^3\,{\left(d\,x\right)}^{11/2}}{64\,a^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6}+\frac{5\,d^{5/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{9/4}\,b^{7/4}}-\frac{5\,d^{5/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{\left(-a\right)}^{1/4}\,\sqrt{d}}\right)}{128\,{\left(-a\right)}^{9/4}\,b^{7/4}}$$

input
```
int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
((7*d^5*(d*x)^(7/2))/(32*a) - (5*d^7*(d*x)^(3/2))/(192*b) + (5*b*d^3*(d*x)
^(11/2))/(64*a^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*d^6*
x^4) + (5*d^(5/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(
-a)^(9/4)*b^(7/4)) - (5*d^(5/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^
(1/2))))/(128*(-a)^(9/4)*b^(7/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.53

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\text{Too large to display}$$

input
```
int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(sqrt(d)*d**2*( - 30*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 - 90*b**(1/4)*
a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 - 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))
)*a*b**2*x**4 - 30*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 30*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(
1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*
*2*b*x**2 + 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 + 30*b**(1/4)
*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 15*b**(1/4)*a**(3/4)*sqrt(2)*log( -
sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3 + 45*b**(1/4
)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sq
rt(b)*x)*a**2*b*x**2 + 45*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4
)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 + 15*b**(1/4)*a**(3/
4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)
*b**3*x**6 - 15*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)...
```

3.464 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 258

$$\begin{aligned}\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx &= -\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}+\frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2} \\ &+\frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)}-\frac{7d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{11/4}b^{5/4}} \\ &+\frac{7d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{7d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}a^{11/4}b^{5/4}}\end{aligned}$$

output

```
-1/6*d*(d*x)^(1/2)/b/(b*x^2+a)^3+1/48*d*(d*x)^(1/2)/a/b/(b*x^2+a)^2+7/192*
d*(d*x)^(1/2)/a^2/b/(b*x^2+a)-7/256*d^(3/2)*arctan(1-2^(1/2)*b^(1/4)*(d*x)
^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(11/4)/b^(5/4)+7/256*d^(3/2)*arctan(1+2^
(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(11/4)/b^(5/4)+7/256*
d^(3/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/
2)*x))*2^(1/2)/a^(11/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.62

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{(dx)^{3/2}\left(\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}(-21a^2+18abx^2+7b^2x^4)}{(a+bx^2)^3} - 21\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 21\sqrt{}\right.}{768a^{11/4}b^{5/4}x^{3/2}}$$

input
```
Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
((d*x)^(3/2)*((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-21*a^2 + 18*a*b*x^2 + 7*b^2*x^4
))/(a + b*x^2)^3 - 21*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x])] + 21*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
/(Sqrt[a] + Sqrt[b]*x)]))/(768*a^(11/4)*b^(5/4)*x^(3/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.45, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {1380, 27, 252, 253, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$\Big\downarrow \; 1380$$

$$b^4\int \frac{(dx)^{3/2}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{(dx)^{3/2}}{(a+bx^2)^4}dx$$

$$\Big\downarrow \; 252$$

$$\frac{d^2\int \frac{1}{\sqrt{dx}(bx^2+a)^3}dx}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \ 253$$

$$\frac{d^2\left(\frac{7\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^2}dx}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \ 253$$

$$\frac{d^2\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \ 266$$

$$\frac{d^2\left(\frac{7\left(\frac{3\int \frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \ 755$$

$$\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{ad}}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{ad}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 1476

$$\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 1082

$$\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 217

$$\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

$\big\downarrow$ 1479

$$\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\sqrt[4]{\cdots}}\cdots\right)}{2\sqrt{a}}\cdots\right)}{2ad}\cdots\right)}{8a}\cdots\right)}{12b}$$

↓ 25

$$d^2\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\cdots\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}\cdots\right)}{2\sqrt{a}}\cdots\right)}{2ad}\cdots\right)}{8a}\cdots\right)}{12b}\cdots$$

$$\Big\downarrow \; 27$$

$$\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\cdots}{2\cdots}\cdots\right)}{8a}\cdots\right)}{12b\cdots}$$

$$\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

$\downarrow$ 1103

$$d^2\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{2\sqrt{a}}\right.}{2ad}\right.}{8a}\right.}{12b}\right.$$

$$\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

input
```
Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
-1/6*(d*Sqrt[d*x])/(b*(a + b*x^2)^3) + (d^2*(Sqrt[d*x]/(4*a*d*(a + b*x^2)^
2) + (7*(Sqrt[d*x]/(2*a*d*(a + b*x^2)) + (3*((d*(-(ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])
+ Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]
/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*a*d)))/(8*a)))/(12
*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.80

method	result
derivativedivides	$2d^7\left(\frac{\frac{7b(dx)^{\frac{9}{2}}}{384a^2d^4}+\frac{3(dx)^{\frac{5}{2}}}{64a\,d^2}-\frac{7\sqrt{dx}}{128b}}{(b\,d^2x^2+a\,d^2)^3}+\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\right.}{1024a^3d^6b}\right.$
default	$2d^7\left(\frac{\frac{7b(dx)^{\frac{9}{2}}}{384a^2d^4}+\frac{3(dx)^{\frac{5}{2}}}{64a\,d^2}-\frac{7\sqrt{dx}}{128b}}{(b\,d^2x^2+a\,d^2)^3}+\frac{7\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\right.}{1024a^3d^6b}\right.$
pseudoelliptic	$\frac{7d\left(\frac{\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(b\,x^2+a\right)^3}{2}+\sqrt{2}\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}(b\,x^2+a)^3\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)+\sqrt{2}\left(\frac{a}{}\right.\right.}{256a^3b(b\,x^2+a)^3}$

input

```
int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output

```
2*d^7*((7/384/a^2/d^4*b*(d*x)^(9/2)+3/64/a/d^2*(d*x)^(5/2)-7/128/b*(d*x)^(
1/2))/(b*d^2*x^2+a*d^2)^3+7/1024/a^3/d^6/b*(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*
x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(
d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.57

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{21\,(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b)\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}\log\left(7\,a^3b\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}+7\,\sqrt{}\right.}{}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/768*(21*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^1
1*b^5))^(1/4)*log(7*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) - 21*(-
I*a^2*b^4*x^6 - 3*I*a^3*b^3*x^4 - 3*I*a^4*b^2*x^2 - I*a^5*b)*(-d^6/(a^11*b
^5))^(1/4)*log(7*I*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) - 21*(I*
a^2*b^4*x^6 + 3*I*a^3*b^3*x^4 + 3*I*a^4*b^2*x^2 + I*a^5*b)*(-d^6/(a^11*b^5
))^(1/4)*log(-7*I*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) - 21*(a^2
*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11*b^5))^(1/4)*
log(-7*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) + 4*(7*b^2*d*x^4 + 1
8*a*b*d*x^2 - 21*a^2*d)*sqrt(d*x))/(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^
2*x^2 + a^5*b)
```

Sympy [F]

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(a+bx^2)^4}\,dx$$

input
```
integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral((d*x)**(3/2)/(a + b*x**2)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.29

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{\frac{8\left(7\,(dx)^{\frac{9}{2}}b^2d^4+18\,(dx)^{\frac{5}{2}}abd^6-21\,\sqrt{dx}a^2d^8\right)}{a^2b^4d^6x^6+3\,a^3b^3d^6x^4+3\,a^4b^2d^6x^2+a^5bd^6}+\frac{21\left(\frac{\sqrt{2}d^4\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^4\log\ldots}{\ldots}\right.}{\ldots}}{\ldots}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/1536*(8*(7*(d*x)^(9/2)*b^2*d^4 + 18*(d*x)^(5/2)*a*b*d^6 - 21*sqrt(d*x)*a
^2*d^8)/(a^2*b^4*d^6*x^6 + 3*a^3*b^3*d^6*x^4 + 3*a^4*b^2*d^6*x^2 + a^5*b*d
^6) + 21*(sqrt(2)*d^4*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^
(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^4*log(sqrt(b)*d*x -
 sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/
4)) + 2*sqrt(2)*d^3*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(
a)) + 2*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2
*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt
(a)))/(a^2*b))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.22

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2}+\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-\ldots\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2}\ldots}{\ldots}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/1536*(42*sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2) + 42*sqrt(2)*(a*b^3*d
^2)^(1/4)*d^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/(a^3*b^2) + 21*sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*log(d*x + sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2) - 21*sqrt(2)*(a
*b^3*d^2)^(1/4)*d^2*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d
^2/b))/(a^3*b^2) + 8*(7*sqrt(d*x)*b^2*d^8*x^4 + 18*sqrt(d*x)*a*b*d^8*x^2 -
 21*sqrt(d*x)*a^2*d^8)/((b*d^2*x^2 + a*d^2)^3*a^2*b))/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.58

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{3\,d^5\,(d\,x)^{5/2}}{32\,a}-\frac{7\,d^7\,\sqrt{d\,x}}{64\,b}+\frac{7\,b\,d^3\,(d\,x)^{9/2}}{192\,a^2}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6} - \frac{7\,d^{3/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{11/4}\,b^{5/4}} - \frac{7\,d^{3/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{11/4}\,b^{5/4}}$$

input
```
int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
((3*d^5*(d*x)^(5/2))/(32*a) - (7*d^7*(d*x)^(1/2))/(64*b) + (7*b*d^3*(d*x)^
(9/2))/(192*a^2))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^2*d^6*x
^4) - (7*d^(3/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-
a)^(11/4)*b^(5/4)) - (7*d^(3/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^
(1/2))))/(128*(-a)^(11/4)*b^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.52

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(sqrt(d)*d*( - 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 - 126*b**(3/4)*a*
*(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1
/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 - 126*b**(3/4)*a**(1/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
*a*b**2*x**4 - 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 42*b**(3/4
)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 126*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(
1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*
*2*b*x**2 + 126*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
+ 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 + 42*b**(3/4
)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 - 21*b**(3/4)*a**(1/4)*sqrt(2)*log( -
 sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3 - 63*b**(3/
4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + s
qrt(b)*x)*a**2*b*x**2 - 63*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/
4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 - 21*b**(3/4)*a**(1
/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x
)*b**3*x**6 + 21*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4...
```

3.465 $\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 258

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{(dx)^{3/2}}{6ad\,(a+bx^2)^3} + \frac{3(dx)^{3/2}}{16a^2d\,(a+bx^2)^2} + \frac{15(dx)^{3/2}}{64a^3d\,(a+bx^2)} - \frac{15\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{13/4}b^{3/4}} + \frac{15\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{13/4}b^{3/4}} - \frac{15\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}a^{13/4}b^{3/4}}$$

output
```
1/6*(d*x)^(3/2)/a/d/(b*x^2+a)^3+3/16*(d*x)^(3/2)/a^2/d/(b*x^2+a)^2+15/64*(
d*x)^(3/2)/a^3/d/(b*x^2+a)-15/256*d^(1/2)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(
1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(13/4)/b^(3/4)+15/256*d^(1/2)*arctan(1+2^(
1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(13/4)/b^(3/4)-15/256*
d^(1/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/
2)*x))*2^(1/2)/a^(13/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\sqrt{dx}\left(\frac{4\sqrt[4]{a}x^{3/2}(113a^2+126abx^2+45b^2x^4)}{(a+bx^2)^3}-\frac{45\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}}-\frac{45\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{b^{3/4}}\right)}{768a^{13/4}\sqrt{x}}$$

input
```
Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(Sqrt[d*x]*((4*a^(1/4)*x^(3/2)*(113*a^2 + 126*a*b*x^2 + 45*b^2*x^4))/(a +
b*x^2)^3 - (45*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])])/b^(3/4) - (45*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x])/(Sqrt[a] + Sqrt[b]*x)])/b^(3/4)))/(768*a^(13/4)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.43, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {1380, 27, 253, 253, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^4\int \frac{\sqrt{dx}}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{dx}}{(a+bx^2)^4}dx$$

$\Big\downarrow$ 253

$$\frac{3\int \frac{\sqrt{dx}}{(bx^2+a)^3}dx}{4a}+\frac{(dx)^{3/2}}{6ad(a+bx^2)^3}$$

$\Big\downarrow$ 253

$$\frac{3\left(\frac{5\int \frac{\sqrt{dx}}{(bx^2+a)^2}dx}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad(a+bx^2)^3}$$

$\Big\downarrow$ 253

$$\frac{3\left(\frac{5\left(\frac{\int \frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad(a+bx^2)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad(a+bx^2)^3}$$

$\Big\downarrow$ 266

$$\frac{3\left(\frac{5\left(\frac{\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad(a+bx^2)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad(a+bx^2)^3}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{5\left(\frac{d\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad(a+bx^2)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad(a+bx^2)^3}$$

$\Big\downarrow$ 826

$$\frac{3\left(\frac{5\left(\frac{d\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}$$

$$\Big\downarrow 217$$

$$\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{1479}$$

$$3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\ldots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\ldots}{2a}\ldots}{8a}\ldots}{4a}\ldots$$

$$\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 25

$$3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right.}{2a}}{8a}\right.}{4a}$$

$$\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}$$

$\downarrow$ 27

$$\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}}{8a}}{4a}$$

$$\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}$$

$\downarrow$ 1103

$$\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}+\frac{}{2a}\right)}{8a}\right)}{4a}$$

$$\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}$$

input
```
Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(d*x)^(3/2)/(6*a*d*(a + b*x^2)^3) + (3*((d*x)^(3/2)/(4*a*d*(a + b*x^2)^2)
+ (5*((d*x)^(3/2)/(2*a*d*(a + b*x^2)) + (d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sq
rt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*a)))/(8*a)))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.82

method	result
derivativedivides	$2d^7\left(\frac{\frac{15b^2(dx)^{\frac{11}{2}}}{128a^3d^6}+\frac{21b(dx)^{\frac{7}{2}}}{64a^2d^4}+\frac{113(dx)^{\frac{3}{2}}}{384a\,d^2}}{(b\,d^2x^2+a\,d^2)^3}+\frac{15\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+\right.}{1024a^3d^6b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$
default	$2d^7\left(\frac{\frac{15b^2(dx)^{\frac{11}{2}}}{128a^3d^6}+\frac{21b(dx)^{\frac{7}{2}}}{64a^2d^4}+\frac{113(dx)^{\frac{3}{2}}}{384a\,d^2}}{(b\,d^2x^2+a\,d^2)^3}+\frac{15\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+\right.}{1024a^3d^6b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.$
pseudoelliptic	$\frac{904\left(\frac{45}{113}b^2x^4+\frac{126}{113}ab\,x^2+a^2\right)\sqrt{dx}\,bx\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}+45d\sqrt{2}\,(b\,x^2+a)^3\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{}\right.\right.}{1536\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}a^3b(b\,x^2+a)^3}$

input
```
int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*d^7*((15/128/a^3/d^6*b^2*(d*x)^(11/2)+21/64/a^2*b/d^4*(d*x)^(7/2)+113/38
4/a/d^2*(d*x)^(3/2))/(b*d^2*x^2+a*d^2)^3+15/1024/a^3/d^6/b/(a*d^2/b)^(1/4)
*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*
x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(
a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)
-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$= \frac{45\,(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6)\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}\log\left(3375\,a^{10}b^2\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{3}{4}}+3375\,\sqrt{dx}d\right)-45\,(i\,a^3b^3x^6}{}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/768*(45*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)*(-d^2/(a^13*b^
3))^(1/4)*log(3375*a^10*b^2*(-d^2/(a^13*b^3))^(3/4) + 3375*sqrt(d*x)*d) -
45*(I*a^3*b^3*x^6 + 3*I*a^4*b^2*x^4 + 3*I*a^5*b*x^2 + I*a^6)*(-d^2/(a^13*b
^3))^(1/4)*log(3375*I*a^10*b^2*(-d^2/(a^13*b^3))^(3/4) + 3375*sqrt(d*x)*d)
 - 45*(-I*a^3*b^3*x^6 - 3*I*a^4*b^2*x^4 - 3*I*a^5*b*x^2 - I*a^6)*(-d^2/(a^
13*b^3))^(1/4)*log(-3375*I*a^10*b^2*(-d^2/(a^13*b^3))^(3/4) + 3375*sqrt(d*
x)*d) - 45*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)*(-d^2/(a^13*b
^3))^(1/4)*log(-3375*a^10*b^2*(-d^2/(a^13*b^3))^(3/4) + 3375*sqrt(d*x)*d)
+ 4*(45*b^2*x^5 + 126*a*b*x^3 + 113*a^2*x)*sqrt(d*x))/(a^3*b^3*x^6 + 3*a^4
*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{(a^2 + 2abx^2 + b^2x^4)^2} \, dx = \int \frac{\sqrt{dx}}{(a + bx^2)^4} \, dx$$

input
```
integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral(sqrt(d*x)/(a + b*x**2)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{dx}}{(a^2 + 2abx^2 + b^2x^4)^2} \, dx$$

$$= \frac{\frac{8\left(45\,(dx)^{\frac{11}{2}}b^2d^2 + 126\,(dx)^{\frac{7}{2}}abd^4 + 113\,(dx)^{\frac{3}{2}}a^2d^6\right)}{a^3b^3d^6x^6 + 3\,a^4b^2d^6x^4 + 3\,a^5bd^6x^2 + a^6d^6} + \frac{45\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}} + 2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}} - 2\right.}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right.}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}\right.}{}}{1536\,d}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/1536*(8*(45*(d*x)^(11/2)*b^2*d^2 + 126*(d*x)^(7/2)*a*b*d^4 + 113*(d*x)^(
3/2)*a^2*d^6)/(a^3*b^3*d^6*x^6 + 3*a^4*b^2*d^6*x^4 + 3*a^5*b*d^6*x^2 + a^6
*d^6) + 45*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)
*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) -
 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sq
rt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^3)
/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{90\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^3}+\frac{90\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^3}-\frac{45\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\right.}{a^4b^3}}{1536}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/1536*(90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3) + 90*sqrt(2)*(a*b^3*d^2)^
(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/(a^4*b^3) - 45*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2
/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) + 45*sqrt(2)*(a*b^3*d^2)^(3
/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3)
 + 8*(45*sqrt(d*x)*b^2*d^7*x^5 + 126*sqrt(d*x)*a*b*d^7*x^3 + 113*sqrt(d*x)
*a^2*d^7*x)/((b*d^2*x^2 + a*d^2)^3*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{113\,d^5\,(d\,x)^{3/2}}{192\,a}+\frac{21\,b\,d^3\,(d\,x)^{7/2}}{32\,a^2}+\frac{15\,b^2\,d\,(d\,x)^{11/2}}{64\,a^3}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6} - \frac{15\,\sqrt{d}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{13/4}\,b^{3/4}} + \frac{15\,\sqrt{d}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{128\,(-a)^{13/4}\,b^{3/4}}$$

input
```
int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
((113*d^5*(d*x)^(3/2))/(192*a) + (21*b*d^3*(d*x)^(7/2))/(32*a^2) + (15*b^2
*d*(d*x)^(11/2))/(64*a^3))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*
b^2*d^6*x^4) - (15*d^(1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))
))/(128*(-a)^(13/4)*b^(3/4)) + (15*d^(1/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-
a)^(1/4)*d^(1/2))))/(128*(-a)^(13/4)*b^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.52

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(sqrt(d)*( - 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
- 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 - 270*b**(1/4)*a**(
3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4
)*a**(1/4)*sqrt(2)))*a**2*b*x**2 - 270*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a
*b**2*x**4 - 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
- 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 90*b**(1/4)*
a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a**3 + 270*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/
4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2
*b*x**2 + 270*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 + 90*b**(1/4)*
a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 45*b**(1/4)*a**(3/4)*sqrt(2)*log( - s
qrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3 + 135*b**(1/4
)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sq
rt(b)*x)*a**2*b*x**2 + 135*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/
4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 + 45*b**(1/4)*a**(3
/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x
)*b**3*x**6 - 45*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4...
```

3.466 $\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 258

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\sqrt{dx}}{6ad\,(a+bx^2)^3} + \frac{11\sqrt{dx}}{48a^2d\,(a+bx^2)^2} + \frac{77\sqrt{dx}}{192a^3d\,(a+bx^2)} - \frac{77\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}} + \frac{77\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}} + \frac{77\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}$$

output
```
1/6*(d*x)^(1/2)/a/d/(b*x^2+a)^3+11/48*(d*x)^(1/2)/a^2/d/(b*x^2+a)^2+77/192
*(d*x)^(1/2)/a^3/d/(b*x^2+a)-77/256*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a
^(1/4)/d^(1/2))*2^(1/2)/a^(15/4)/b^(1/4)/d^(1/2)+77/256*arctan(1+2^(1/2)*b
^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(15/4)/b^(1/4)/d^(1/2)+77/25
6*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))
*2^(1/2)/a^(15/4)/b^(1/4)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\sqrt{x}\left(\frac{4a^{3/4}\sqrt{x}(153a^2+198abx^2+77b^2x^4)}{(a+bx^2)^3}-\frac{231\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}}+\frac{231\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{b}}\right)}{768a^{15/4}\sqrt{dx}}$$

input
```
Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
(Sqrt[x]*((4*a^(3/4)*Sqrt[x]*(153*a^2 + 198*a*b*x^2 + 77*b^2*x^4))/(a + b*
x^2)^3 - (231*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])])/b^(1/4) + (231*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x])/(Sqrt[a] + Sqrt[b]*x)])/b^(1/4)))/(768*a^(15/4)*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.45, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {1380, 27, 253, 253, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^4\int \frac{1}{b^4\sqrt{dx}\,(bx^2+a)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\sqrt{dx}\,(a+bx^2)^4}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{11\int \frac{1}{\sqrt{dx}(bx^2+a)^3}dx}{12a}+\frac{\sqrt{dx}}{6ad(a+bx^2)^3}$$

$$\Big\downarrow \text{253}$$

$$\frac{11\left(\frac{7\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^2}dx}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad(a+bx^2)^3}$$

$$\Big\downarrow \text{253}$$

$$\frac{11\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad(a+bx^2)^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{11\left(\frac{7\left(\frac{3\int \frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad(a+bx^2)^3}$$

$$\Big\downarrow \text{755}$$

$$\frac{11\left(\frac{7\left(\frac{3\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad(a+bx^2)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 1476$$

$$\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}$$

$\big\downarrow$ 1082

$$\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}$$

$\downarrow$ 217

$$\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 1479

$$11\left(\frac{7\left(\frac{3\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\cdots\right.}{\sqrt{2}\sqrt[4]{\cdots}}\right.}{2\sqrt{a}}\cdots\right.}{2ad}\cdots\right.}{8a}\cdots\right.}{12a}$$

↓ 25

$$11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right.}{2\sqrt{a}}\right.}{2ad}\right.}{8a}\right.}{12a}$$

↓ 27

$$\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\cdots\right)}{8a}\cdots\right)}{12a}$$

$$\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}$$

$\downarrow$ 1103

$$\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{a}}}{2ad}}{8a}}{12a}$$

$$\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}$$

input

```
Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
Sqrt[d*x]/(6*a*d*(a + b*x^2)^3) + (11*(Sqrt[d*x]/(4*a*d*(a + b*x^2)^2) + (
7*(Sqrt[d*x]/(2*a*d*(a + b*x^2)) + (3*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[
Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*a*d)))/(8*a)))/(12*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.81

method	result
derivativedivides	$2d^7\left(\frac{\frac{77b^2(dx)^{\frac{9}{2}}}{384a^3d^6}+\frac{33b(dx)^{\frac{5}{2}}}{64a^2d^4}+\frac{51\sqrt{dx}}{128a\,d^2}}{(b\,d^2x^2+a\,d^2)^3}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+\right.\right.}{1024a^4d^8}\right.$
default	$2d^7\left(\frac{\frac{77b^2(dx)^{\frac{9}{2}}}{384a^3d^6}+\frac{33b(dx)^{\frac{5}{2}}}{64a^2d^4}+\frac{51\sqrt{dx}}{128a\,d^2}}{(b\,d^2x^2+a\,d^2)^3}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+\right.\right.}{1024a^4d^8}\right.$
pseudoelliptic	$\frac{77\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,(b\,x^2+a)^3+154\sqrt{2}\,\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}(b\,x^2+a)^3\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)+154}{512d\,a^4(b\,x^2+a)^3}$

input
```
int(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*d^7*((77/384/a^3/d^6*b^2*(d*x)^(9/2)+33/64/a^2/d^4*b*(d*x)^(5/2)+51/128/
a/d^2*(d*x)^(1/2))/(b*d^2*x^2+a*d^2)^3+77/1024/a^4/d^8*(a*d^2/b)^(1/4)*2^(
1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^
2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1))
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^2}\,dx$$
$$= \frac{231\,(a^3b^3dx^6+3\,a^4b^2dx^4+3\,a^5bdx^2+a^6d)\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}\log\left(a^4d\left(-\frac{1}{a^{15}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-231\,(-i\,a^3b^3dx^6-\ldots}{\ldots}$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/768*(231*(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(
a^15*b*d^2))^(1/4)*log(a^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) - 231*(-
I*a^3*b^3*d*x^6 - 3*I*a^4*b^2*d*x^4 - 3*I*a^5*b*d*x^2 - I*a^6*d)*(-1/(a^15
*b*d^2))^(1/4)*log(I*a^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) - 231*(I*a
^3*b^3*d*x^6 + 3*I*a^4*b^2*d*x^4 + 3*I*a^5*b*d*x^2 + I*a^6*d)*(-1/(a^15*b*
d^2))^(1/4)*log(-I*a^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) - 231*(a^3*b
^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a^15*b*d^2))^(1/4
)*log(-a^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) + 4*(77*b^2*x^4 + 198*a*
b*x^2 + 153*a^2)*sqrt(d*x))/(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x
^2 + a^6*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^2}\,dx = \int \frac{1}{\sqrt{dx}\,(a+bx^2)^4}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral(1/(sqrt(d*x)*(a + b*x**2)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^2}\,dx$$

$$= \frac{\frac{8\left(77\,(dx)^{\frac{9}{2}}b^2d^2+198\,(dx)^{\frac{5}{2}}abd^4+153\,\sqrt{dx}a^2d^6\right)}{a^3b^3d^6x^6+3\,a^4b^2d^6x^4+3\,a^5bd^6x^2+a^6d^6}+\frac{231\left(\frac{\sqrt{2}d^2\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^2\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{}}{1536\,d}$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/1536*(8*(77*(d*x)^(9/2)*b^2*d^2 + 198*(d*x)^(5/2)*a*b*d^4 + 153*sqrt(d*x
)*a^2*d^6)/(a^3*b^3*d^6*x^6 + 3*a^4*b^2*d^6*x^4 + 3*a^5*b*d^6*x^2 + a^6*d^
6) + 231*(sqrt(2)*d^2*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^
(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x -
 sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/
4)) + 2*sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sq
rt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)
) + 2*sqrt(2)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqr
t(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a))
)/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{77\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4bd} + \frac{77\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4bd} + \frac{77\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^4bd} - \frac{77\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^4bd} + \frac{77\,\sqrt{dx}b^2d^5x^4+198\,\sqrt{dx}abd^5x^2+153\,\sqrt{dx}a^2d^5}{192\,(bd^2x^2+ad^2)^3a^3}$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
77/256*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d) + 77/256*sqrt(2)*(a*b^3*d^2)^
(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/(a^4*b*d) + 77/512*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a
*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*d) - 77/512*sqrt(2)*(a*b^3
*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a^4*b*d) + 1/192*(77*sqrt(d*x)*b^2*d^5*x^4 + 198*sqrt(d*x)*a*b*d^5*x^2 + 1
53*sqrt(d*x)*a^2*d^5)/((b*d^2*x^2 + a*d^2)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 18.61 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{\frac{51\,d^5\,\sqrt{d\,x}}{64\,a}+\frac{33\,b\,d^3\,{(d\,x)}^{5/2}}{32\,a^2}+\frac{77\,b^2\,d\,{(d\,x)}^{9/2}}{192\,a^3}}{a^3\,d^6+3\,a^2\,b\,d^6\,x^2+3\,a\,b^2\,d^6\,x^4+b^3\,d^6\,x^6} + \frac{77\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{128\,{(-a)}^{15/4}\,b^{1/4}\,\sqrt{d}} + \frac{77\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{128\,{(-a)}^{15/4}\,b^{1/4}\,\sqrt{d}}$$

input
```
int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

output
```
((51*d^5*(d*x)^(1/2))/(64*a) + (33*b*d^3*(d*x)^(5/2))/(32*a^2) + (77*b^2*d
*(d*x)^(9/2))/(192*a^3))/(a^3*d^6 + b^3*d^6*x^6 + 3*a^2*b*d^6*x^2 + 3*a*b^
2*d^6*x^4) + (77*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(128*(-
a)^(15/4)*b^(1/4)*d^(1/2)) + (77*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d
^(1/2))))/(128*(-a)^(15/4)*b^(1/4)*d^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 653, normalized size of antiderivative = 2.53

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output

```
(sqrt(d)*( - 462*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 - 1386*b**(3/4)*a*
*(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1
/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 - 1386*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))
)*a*b**2*x**4 - 462*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 462*b**(
3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 1386*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))
)*a**2*b*x**2 + 1386*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 + 462*b
**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(
b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 - 231*b**(3/4)*a**(1/4)*sqrt(2)
*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3 - 69
3*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqr
t(a) + sqrt(b)*x)*a**2*b*x**2 - 693*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(
x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 - 231*b**(
3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) +
 sqrt(b)*x)*b**3*x**6 + 231*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1...
```

3.467 $\int \frac{1}{(dx)^{3/2}(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 275

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = -\frac{195}{64a^4d\sqrt{dx}}+\frac{1}{6ad\sqrt{dx}\,(a+bx^2)^3}+\frac{13}{48a^2d\sqrt{dx}\,(a+bx^2)^2}+\frac{39}{64a^3d\sqrt{dx}\,(a+bx^2)}+\frac{195\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{17/4}d^{3/2}}-\frac{195\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}a^{17/4}d^{3/2}}$$

output

```
-195/64/a^4/d/(d*x)^(1/2)+1/6/a/d/(d*x)^(1/2)/(b*x^2+a)^3+13/48/a^2/d/(d*x
)^(1/2)/(b*x^2+a)^2+39/64/a^3/d/(d*x)^(1/2)/(b*x^2+a)+195/256*b^(1/4)*arct
an(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(17/4)/d^(3/2)
-195/256*b^(1/4)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(
1/2)/a^(17/4)/d^(3/2)+195/256*b^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x
)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(17/4)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.63

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{x\left(-\frac{4\sqrt[4]{a}(384a^3+1469a^2bx^2+1638ab^2x^4+585b^3x^6)}{(a+bx^2)^3}+585\sqrt{2}\sqrt[4]{b}\sqrt{x}\arctan\left(\frac{}{\sqrt{2}\cdots}\right.\right.}{768a^{17/4}(dx)^{3/2}\cdots}$$

input
```
Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
(x*((-4*a^(1/4)*(384*a^3 + 1469*a^2*b*x^2 + 1638*a*b^2*x^4 + 585*b^3*x^6))
/(a + b*x^2)^3 + 585*Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 585*Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTanh[(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(768*a^(17/4)*(d
*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.43, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1380, 27, 253, 253, 253, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^4\int \frac{1}{b^4(dx)^{3/2}\left(bx^2+a\right)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^2\right)^4}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{13\int \frac{1}{(dx)^{3/2}(bx^2+a)^3}dx}{12a}+\frac{1}{6ad\sqrt{dx}\,(a+bx^2)^3}$$

$\Big\downarrow$ 253

$$\frac{13\left(\frac{9\int \frac{1}{(dx)^{3/2}\left(bx^2+a\right)^2}dx}{8a}+\frac{1}{4ad\sqrt{dx}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\,(a+bx^2)^3}$$

$\Big\downarrow$ 253

$$\frac{13\left(\frac{9\left(\frac{5\int \frac{1}{(dx)^{3/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\,(a+bx^2)^3}$$

$\Big\downarrow$ 264

$$\frac{13\left(\frac{9\left(\frac{5\left(-\frac{b\int \frac{\sqrt{dx}}{bx^2+a}dx}{ad^2}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\,(a+bx^2)^3}$$

$\Big\downarrow$ 266

$$\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{ad^3}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\,(a+bx^2)^3}$$

$\Big\downarrow$ 27

$$\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 826

$$\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 1476

$$\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{\cdots}{4a\cdots}\right)}{12a}$$

$$\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}$$

↓ 1082

$$\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-dx-1}\,d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}\,d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}(a+\cdots}\right)}{12a}$$

$$\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}$$

↓ 217

$$\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}(a+bx^2)^2}\right)}{12a}$$

$$\frac{1}{6ad\sqrt{dx}\,(a+bx^2)^3}$$

↓ 1479

$$13\left(\frac{9\left(\frac{5-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{d}\cdots\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}\cdots}{\sqrt[4]{b}}\cdots\right)}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots}{2\sqrt{b}}\cdots\right)}{ad}\cdots}{4a}\cdots\right)}{8a}\cdots\right)\cdots$$

$$\Big\downarrow \text{25}$$

$$13\left(\frac{9\left(\frac{5-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right.}{ad}}{4a}\right.}{8a}\right.$$

$$\Big\downarrow 27$$

$$13\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}\right)}{4a}\right)}{8a}\right)$$

↓ 1103

$$13\left(9\left(5-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}}{ad}\right.\right.\right.$$

$$\frac{\cdots}{4a} \qquad \frac{\cdots}{8a} \qquad \frac{\cdots}{12a}$$

$$\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}$$

input
```
Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
1/(6*a*d*Sqrt[d*x]*(a + b*x^2)^3) + (13*(1/(4*a*d*Sqrt[d*x]*(a + b*x^2)^2)
 + (9*(1/(2*a*d*Sqrt[d*x]*(a + b*x^2)) + (5*(-2/(a*d*Sqrt[d*x]) - (2*b*((-
(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d
 + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(a
*d)))/(4*a)))/(8*a)))/(12*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{2}{a^{4} d \sqrt{dx}}-\frac{b\left(\frac{\frac{67 b^{2}(dx)^{\frac{11}{2}}}{64}+\frac{81 a b\, d^{2}(dx)^{\frac{7}{2}}}{32}+\frac{317 a^{2} d^{4}(dx)^{\frac{3}{2}}}{192}}{\left(b\, d^{2} x^{2}+a\, d^{2}\right)^{3}}+\frac{195\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\, d^{2}}{b}}}{dx+\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\, d^{2}}{b}}}\right)+2\arctan\left(\cdots\right.\right.}{512 b\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}}\cdots\right)}{a^{4} d}$ …
derivativedivides	$2 d^{7}\left(-\frac{1}{a^{4} d^{8}\sqrt{dx}}-\frac{b\left(\frac{\frac{67 b^{2}(dx)^{\frac{11}{2}}}{128}+\frac{81 a b\, d^{2}(dx)^{\frac{7}{2}}}{64}+\frac{317 a^{2} d^{4}(dx)^{\frac{3}{2}}}{384}}{\left(b\, d^{2} x^{2}+a\, d^{2}\right)^{3}}+\frac{195\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\, d^{2}}{b}}}{dx+\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\, d^{2}}{b}}}\right)+2\cdots\right.}{1024 b\left(\cdots\right.}\cdots\right)}{a^{4} d^{8}}\cdots\right.$ …
default	$2 d^{7}\left(-\frac{1}{a^{4} d^{8}\sqrt{dx}}-\frac{b\left(\frac{\frac{67 b^{2}(dx)^{\frac{11}{2}}}{128}+\frac{81 a b\, d^{2}(dx)^{\frac{7}{2}}}{64}+\frac{317 a^{2} d^{4}(dx)^{\frac{3}{2}}}{384}}{\left(b\, d^{2} x^{2}+a\, d^{2}\right)^{3}}+\frac{195\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\, d^{2}}{b}}}{dx+\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\, d^{2}}{b}}}\right)+2\cdots\right.}{1024 b\left(\cdots\right.}\cdots\right)}{a^{4} d^{8}}\cdots\right.$ …
pseudoelliptic	$-\frac{2\left(\frac{195\sqrt{2}\left(b\, x^{2}+a\right)^{3}\left(\ln\left(\frac{dx-\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\, d^{2}}{b}}}{dx+\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\, d^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}}\right)\cdots\right.}{1024}\cdots\right)}{\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\, d\, a^{4}\left(b\, x^{2}+a\right)^{3}}$ …

input
```
int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^4/d/(d*x)^(1/2)-b/a^4*(2*(67/128*b^2*(d*x)^(11/2)+81/64*a*b*d^2*(d*x)
^(7/2)+317/384*a^2*d^4*(d*x)^(3/2))/(b*d^2*x^2+a*d^2)^3+195/512/b/(a*d^2/b
)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^
(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x
)^(1/2)-1)))/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.66

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx =$$

$$-\frac{585\left(a^4b^3d^2x^7+3\,a^5b^2d^2x^5+3\,a^6bd^2x^3+a^7d^2x\right)\left(-\frac{b}{a^{17}d^6}\right)^{\frac{1}{4}}\log\left(7414875\,a^{13}d^5\left(-\frac{b}{a^{17}d^6}\right)^{\frac{3}{4}}+7414875\,\sqrt{dx}\right.}{}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/768*(585*(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d
^2*x)*(-b/(a^17*d^6))^(1/4)*log(7414875*a^13*d^5*(-b/(a^17*d^6))^(3/4) + 7
414875*sqrt(d*x)*b) + 585*(-I*a^4*b^3*d^2*x^7 - 3*I*a^5*b^2*d^2*x^5 - 3*I*
a^6*b*d^2*x^3 - I*a^7*d^2*x)*(-b/(a^17*d^6))^(1/4)*log(7414875*I*a^13*d^5*
(-b/(a^17*d^6))^(3/4) + 7414875*sqrt(d*x)*b) + 585*(I*a^4*b^3*d^2*x^7 + 3*
I*a^5*b^2*d^2*x^5 + 3*I*a^6*b*d^2*x^3 + I*a^7*d^2*x)*(-b/(a^17*d^6))^(1/4)
*log(-7414875*I*a^13*d^5*(-b/(a^17*d^6))^(3/4) + 7414875*sqrt(d*x)*b) - 58
5*(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)*(-b/
(a^17*d^6))^(1/4)*log(-7414875*a^13*d^5*(-b/(a^17*d^6))^(3/4) + 7414875*sq
rt(d*x)*b) + 4*(585*b^3*x^6 + 1638*a*b^2*x^4 + 1469*a^2*b*x^2 + 384*a^3)*s
qrt(d*x))/(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2
*x)
```

Sympy [F]

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+bx^2\right)^4}\,dx$$

input
```
integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral(1/((d*x)**(3/2)*(a + b*x**2)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.19

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=$$

$$-\frac{\frac{8\left(585\,b^3d^6x^6+1638\,ab^2d^6x^4+1469\,a^2bd^6x^2+384\,a^3d^6\right)}{(dx)^{\frac{13}{2}}a^4b^3+3\,(dx)^{\frac{9}{2}}a^5b^2d^2+3\,(dx)^{\frac{5}{2}}a^6bd^4+\sqrt{dx}a^7d^6}+\frac{585\,b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}\right.}{2\sqrt{\sqrt{a}\sqrt{}}}\right.}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}\right.}{}}{1536\,d}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/1536*(8*(585*b^3*d^6*x^6 + 1638*a*b^2*d^6*x^4 + 1469*a^2*b*d^6*x^2 + 38
4*a^3*d^6)/((d*x)^(13/2)*a^4*b^3 + 3*(d*x)^(9/2)*a^5*b^2*d^2 + 3*(d*x)^(5/
2)*a^6*b*d^4 + sqrt(d*x)*a^7*d^6) + 585*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d
))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/
(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d
^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)
*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((
a*d^2)^(1/4)*b^(3/4)))/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.19

$$\int \frac{1}{(dx)^{3/2} (a^2 + 2abx^2 + b^2x^4)^2} dx =$$

$$-\frac{\frac{3072}{\sqrt{dx}a^4} + \frac{8\left(201\sqrt{dx}b^3d^5x^5 + 486\sqrt{dx}ab^2d^5x^3 + 317\sqrt{dx}a^2bd^5x\right)}{(bd^2x^2+ad^2)^3a^4} + \frac{1170\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2d^2} + \frac{1170\sqrt{2}(ab^3 \cdots}{\cdots}}{\cdots}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-1/1536*(3072/(sqrt(d*x)*a^4) + 8*(201*sqrt(d*x)*b^3*d^5*x^5 + 486*sqrt(d*
x)*a*b^2*d^5*x^3 + 317*sqrt(d*x)*a^2*b*d^5*x)/((b*d^2*x^2 + a*d^2)^3*a^4)
+ 1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2*d^2) + 1170*sqrt(2)*(a*b^3*d^2
)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2
/b)^(1/4))/(a^5*b^2*d^2) - 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)
*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^2*d^2) + 585*sqrt(2)*(a
*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^5*b^2*d^2))/d
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.60

$$\int \frac{1}{(dx)^{3/2} (a^2 + 2abx^2 + b^2x^4)^2} dx = \frac{195\,(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{d\,x}}{a^{1/4}\sqrt{d}}\right)}{128\,a^{17/4}\,d^{3/2}} - \frac{195\,(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{d\,x}}{a^{1/4}\sqrt{d}}\right)}{128\,a^{17/4}\,d^{3/2}} - \frac{\frac{2\,d^5}{a} + \frac{1469\,b\,d^5\,x^2}{192\,a^2} + \frac{273\,b^2\,d^5\,x^4}{32\,a^3} + \frac{195\,b^3\,d^5\,x^6}{64\,a^4}}{b^3\,(d\,x)^{13/2} + a^3\,d^6\,\sqrt{d\,x} + 3\,a^2\,b\,d^4\,(d\,x)^{5/2} + 3\,a\,b^2\,d^2\,(d\,x)^{9/2}}$$

input
```
int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

output
```
(195*(-b)^(1/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*a^
(17/4)*d^(3/2)) - (195*(-b)^(1/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d
^(1/2))))/(128*a^(17/4)*d^(3/2)) - ((2*d^5)/a + (1469*b*d^5*x^2)/(192*a^2)
 + (273*b^2*d^5*x^4)/(32*a^3) + (195*b^3*d^5*x^6)/(64*a^4))/(b^3*(d*x)^(13
/2) + a^3*d^6*(d*x)^(1/2) + 3*a^2*b*d^4*(d*x)^(5/2) + 3*a*b^2*d^2*(d*x)^(9
/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 688, normalized size of antiderivative = 2.50

$$\int \frac{1}{(dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(sqrt(d)*(1170*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 3510*sqrt(
x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*s
qrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 + 3510*sqrt(x)*b**(1/4)*a
**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 + 1170*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*b**3*x**6 - 1170*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/
4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3
 - 3510*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
+ 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 - 3510*sqrt(
x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*s
qrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 - 1170*sqrt(x)*b**(1/4)*a
**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 - 585*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*
log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3 - 175
5*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(
2) + sqrt(a) + sqrt(b)*x)*a**2*b*x**2 - 1755*sqrt(x)*b**(1/4)*a**(3/4)*sqr
t(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**
2*x**4 - 585*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*...
```

3.468 $\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 275

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = -\frac{385}{192a^4d(dx)^{3/2}} + \frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{5}{16a^2d(dx)^{3/2}\left(a+bx^2\right)^2} + \frac{55}{64a^3d(dx)^{3/2}\left(a+bx^2\right)} + \frac{385b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{19/4}d^{5/2}} - \frac{385b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{19/4}d^{5/2}} - \frac{385b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}a^{19/4}d^{5/2}}$$

output
```
-385/192/a^4/d/(d*x)^(3/2)+1/6/a/d/(d*x)^(3/2)/(b*x^2+a)^3+5/16/a^2/d/(d*x
)^(3/2)/(b*x^2+a)^2+55/64/a^3/d/(d*x)^(3/2)/(b*x^2+a)+385/256*b^(3/4)*arct
an(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(19/4)/d^(5/2)
-385/256*b^(3/4)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(
1/2)/a^(19/4)/d^(5/2)-385/256*b^(3/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x
)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(19/4)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.63

$$\int \frac{1}{(dx)^{5/2} (a^2 + 2abx^2 + b^2x^4)^2} \, dx = \frac{x\left(-\frac{4a^{3/4}(128a^3+765a^2bx^2+990ab^2x^4+385b^3x^6)}{(a+bx^2)^3} + 1155\sqrt{2}b^{3/4}x^{3/2} \arctan\left(\right.\right.}{768a^{19/4}(dx)^{5/}}$$

input
```
Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
(x*((-4*a^(3/4)*(128*a^3 + 765*a^2*b*x^2 + 990*a*b^2*x^4 + 385*b^3*x^6))/(
a + b*x^2)^3 + 1155*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 1155*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTanh[(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(768*a^(19/4)*(d
*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.44, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1380, 27, 253, 253, 253, 264, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{5/2} (a^2 + 2abx^2 + b^2x^4)^2} \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^4 \int \frac{1}{b^4 (dx)^{5/2} (bx^2 + a)^4} dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{(dx)^{5/2} (a + bx^2)^4} dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{5\int \frac{1}{(dx)^{5/2}(bx^2+a)^3}dx}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 253

$$\frac{5\left(\frac{11\int \frac{1}{(dx)^{5/2}\left(bx^2+a\right)^2}dx}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 253

$$\frac{5\left(\frac{11\left(\frac{7\int \frac{1}{(dx)^{5/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 264

$$\frac{5\left(\frac{11\left(\frac{7\left(-\frac{b\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{ad^2}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 266

$$\frac{5\left(\frac{11\left(\frac{7\left(-\frac{2b\int \frac{1}{bx^2+a}d\sqrt{dx}}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 755$$

$$\frac{5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 1476

$$5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx\right.}\cdots\right)}{8a}\cdots\right.$$

↓ 1082

$$5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\right.$$

$\downarrow$ 217

$$5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{\cdots}{4ad(dx\cdots}\right.$$

↓ 1479

$$5\left(11\left(7\left(-\frac{2b\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan \cdots}{\cdots}\right.}{2\sqrt{a}}\right.}{ad^3}\right.}{4a}\right.}{8a}$$

↓ 25

$$5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\right.}{\sqrt{2}\sqrt[4]{\ }}\right.}{2\sqrt{a}}}{ad^3}\right.}{4a}\right.}{8a}\right.$$

$\downarrow$ 27

$$5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}\right)}{4a}\right)}{8a}\right)$$

↓ 1103

$$5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d-\cdots\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right)}{2\sqrt{a}}\cdots\right)}{ad^{3}}\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right.$$

input
```
Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
1/(6*a*d*(d*x)^(3/2)*(a + b*x^2)^3) + (5*(1/(4*a*d*(d*x)^(3/2)*(a + b*x^2)
^2) + (11*(1/(2*a*d*(d*x)^(3/2)*(a + b*x^2)) + (7*(-2/(3*a*d*(d*x)^(3/2))
- (2*b*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-
1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x
]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))
)/(2*Sqrt[a])))/(a*d^3)))/(4*a)))/(8*a)))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{2}{3a^4 x\sqrt{dx}\, d^2}-\frac{2b\left(\frac{\frac{257b^2(dx)^{\frac{9}{2}}}{384}+\frac{101ab\,d^2(dx)^{\frac{5}{2}}}{64}+\frac{127\sqrt{dx}\,a^2d^4}{128}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{385\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)\right.}{102}\right.}{a^4d}$
derivativedivides	$2d^7\left(-\frac{1}{3a^4d^8(dx)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{257b^2(dx)^{\frac{9}{2}}}{384}+\frac{101ab\,d^2(dx)^{\frac{5}{2}}}{64}+\frac{127\sqrt{dx}\,a^2d^4}{128}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{385\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}\right.\right.}{}\right.}{a^4d^8}\right.$
default	$2d^7\left(-\frac{1}{3a^4d^8(dx)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{257b^2(dx)^{\frac{9}{2}}}{384}+\frac{101ab\,d^2(dx)^{\frac{5}{2}}}{64}+\frac{127\sqrt{dx}\,a^2d^4}{128}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{385\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}\right.\right.}{}\right.}{a^4d^8}\right.$
pseudoelliptic	$-\frac{385\left(\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b\sqrt{2}\left(b\,x^2+a\right)^3\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{}\right.\right.\right.}{512(dx)^{\frac{3}{2}}d^3a^5\left(b\,x^2+a\right)^3}$

input
```
int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/a^4/x/(d*x)^(1/2)/d^2-2*b/a^4/d*((257/384*b^2*(d*x)^(9/2)+101/64*a*b*
d^2*(d*x)^(5/2)+127/128*(d*x)^(1/2)*a^2*d^4)/(b*d^2*x^2+a*d^2)^3+385/1024*
(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2
/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.75

$$\int \frac{1}{(dx)^{5/2} (a^2 + 2abx^2 + b^2x^4)^2} \, dx =$$

$$-\frac{1155 \left(a^4b^3d^3x^8 + 3\,a^5b^2d^3x^6 + 3\,a^6bd^3x^4 + a^7d^3x^2\right)\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}} \log\left(385\,a^5d^3\left(-\frac{b^3}{a^{19}d^{10}}\right)^{\frac{1}{4}} + 385\,\sqrt{dx}b\right) +}{}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/768*(1155*(a^4*b^3*d^3*x^8 + 3*a^5*b^2*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*
d^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*log(385*a^5*d^3*(-b^3/(a^19*d^10))^(1/4)
 + 385*sqrt(d*x)*b) + 1155*(I*a^4*b^3*d^3*x^8 + 3*I*a^5*b^2*d^3*x^6 + 3*I*
a^6*b*d^3*x^4 + I*a^7*d^3*x^2)*(-b^3/(a^19*d^10))^(1/4)*log(385*I*a^5*d^3*
(-b^3/(a^19*d^10))^(1/4) + 385*sqrt(d*x)*b) + 1155*(-I*a^4*b^3*d^3*x^8 - 3
*I*a^5*b^2*d^3*x^6 - 3*I*a^6*b*d^3*x^4 - I*a^7*d^3*x^2)*(-b^3/(a^19*d^10))
^(1/4)*log(-385*I*a^5*d^3*(-b^3/(a^19*d^10))^(1/4) + 385*sqrt(d*x)*b) - 11
55*(a^4*b^3*d^3*x^8 + 3*a^5*b^2*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d^3*x^2)*(
-b^3/(a^19*d^10))^(1/4)*log(-385*a^5*d^3*(-b^3/(a^19*d^10))^(1/4) + 385*sq
rt(d*x)*b) + 4*(385*b^3*x^6 + 990*a*b^2*x^4 + 765*a^2*b*x^2 + 128*a^3)*sqr
t(d*x))/(a^4*b^3*d^3*x^8 + 3*a^5*b^2*d^3*x^6 + 3*a^6*b*d^3*x^4 + a^7*d^3*x
^2)
```

Sympy [F]

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\int \frac{1}{(dx)^{\frac{5}{2}}\left(a+bx^2\right)^4}\,dx$$

input
```
integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral(1/((d*x)**(5/2)*(a + b*x**2)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.22

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=$$

$$-\frac{\frac{8\left(385\,b^3d^6x^6+990\,ab^2d^6x^4+765\,a^2bd^6x^2+128\,a^3d^6\right)}{(dx)^{\frac{15}{2}}a^4b^3+3\,(dx)^{\frac{11}{2}}a^5b^2d^2+3\,(dx)^{\frac{7}{2}}a^6bd^4+(dx)^{\frac{3}{2}}a^7d^6}+\frac{1155\left(\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}\right.}{\left(ad^2\right)^{\frac{3}{4}}}\right.}{}}{1536\,d}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/1536*(8*(385*b^3*d^6*x^6 + 990*a*b^2*d^6*x^4 + 765*a^2*b*d^6*x^2 + 128*
a^3*d^6)/((d*x)^(15/2)*a^4*b^3 + 3*(d*x)^(11/2)*a^5*b^2*d^2 + 3*(d*x)^(7/2
)*a^6*b*d^4 + (d*x)^(3/2)*a^7*d^6) + 1155*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x
 + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sq
rt(2)*b^(3/4)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) +
sqrt(a)*d)/(a*d^2)^(3/4) + 2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)
^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(
a)*sqrt(b)*d)*sqrt(a)*d) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2
)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt
(a)*sqrt(b)*d)*sqrt(a)*d))/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.12

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = -\frac{385\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^5d^3} - \frac{385\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^5d^3} - \frac{385\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^5d^3} + \frac{385\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^5d^3} - \frac{385\,b^3d^6x^6+990\,ab^2d^6x^4+765\,a^2bd^6x^2+128\,a^3d^6}{192\left(\sqrt{dx}bd^2x^2+\sqrt{dx}ad^2\right)^3a^4d}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-385/256*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*d^3) - 385/256*sqrt(2)*(a*b^3*d^
2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^
2/b)^(1/4))/(a^5*d^3) - 385/512*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*d^3) + 385/512*sqrt(2)*(
a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a^5*d^3) - 1/192*(385*b^3*d^6*x^6 + 990*a*b^2*d^6*x^4 + 765*a^2*b*d^6
*x^2 + 128*a^3*d^6)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)^3*a^4*d)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.60

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{385\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{19/4}\,d^{5/2}} - \frac{\frac{2\,d^5}{3\,a}+\frac{255\,b\,d^5\,x^2}{64\,a^2}+\frac{165\,b^2\,d^5\,x^4}{32\,a^3}+\frac{385\,b^3\,d^5\,x^6}{192\,a^4}}{b^3\,{\left(d\,x\right)}^{15/2}+a^3\,d^6\,{\left(d\,x\right)}^{3/2}+3\,a^2\,b\,d^4\,{\left(d\,x\right)}^{7/2}+3\,a\,b^2\,d^2\,{\left(d\,x\right)}^{11/2}} + \frac{385\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{19/4}\,d^{5/2}}$$

input
```
int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

output
```
(385*(-b)^(3/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*a^(
19/4)*d^(5/2)) - ((2*d^5)/(3*a) + (255*b*d^5*x^2)/(64*a^2) + (165*b^2*d^5*
x^4)/(32*a^3) + (385*b^3*d^5*x^6)/(192*a^4))/(b^3*(d*x)^(15/2) + a^3*d^6*(
d*x)^(3/2) + 3*a^2*b*d^4*(d*x)^(7/2) + 3*a*b^2*d^2*(d*x)^(11/2)) + (385*(-
b)^(3/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(128*a^(19/4)*
d^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 695, normalized size of antiderivative = 2.53

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output

```
(sqrt(d)*(2310*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*x + 6930*sqr
t(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)
*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**3 + 6930*sqrt(x)*b**(3/4)
*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**5 + 2310*sqrt(x)*b**(3/4)*a**(1/4)*sqr
t(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/
4)*sqrt(2)))*b**3*x**7 - 2310*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(
1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*
*3*x - 6930*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**3 - 6930*s
qrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(
x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**5 - 2310*sqrt(x)*b**(3/
4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**7 + 1155*sqrt(x)*b**(3/4)*a**(1/4)*sqr
t(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3*
x + 3465*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4
)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**2*b*x**3 + 3465*sqrt(x)*b**(3/4)*a**(1
/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x
)*a*b**2*x**5 + 1155*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b...
```

3.469 $\int \frac{1}{(dx)^{7/2}(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 293

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = -\frac{663}{320a^4d(dx)^{5/2}} + \frac{663b}{64a^5d^3\sqrt{dx}} + \frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{17}{48a^2d(dx)^{5/2}\left(a+bx^2\right)^2} + \frac{221}{192a^3d(dx)^{5/2}\left(a+bx^2\right)} - \frac{663b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{21/4}d^{7/2}} + \frac{663b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{21/4}d^{7/2}} - \frac{663b^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{128\sqrt{2}a^{21/4}d^{7/2}}$$

output

```
-663/320/a^4/d/(d*x)^(5/2)+663/64*b/a^5/d^3/(d*x)^(1/2)+1/6/a/d/(d*x)^(5/2
)/(b*x^2+a)^3+17/48/a^2/d/(d*x)^(5/2)/(b*x^2+a)^2+221/192/a^3/d/(d*x)^(5/2
)/(b*x^2+a)-663/256*b^(5/4)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d
^(1/2))*2^(1/2)/a^(21/4)/d^(7/2)+663/256*b^(5/4)*arctan(1+2^(1/2)*b^(1/4)*
(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(21/4)/d^(7/2)-663/256*b^(5/4)*arct
anh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/
2)/a^(21/4)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.65

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{\sqrt{dx}\left(\frac{4\sqrt[4]{a}(-384a^4+6528a^3bx^2+24973a^2b^2x^4+27846ab^3x^6+9945b^4x^8)}{(a+bx^2)^3}-9945\sqrt{2}b^5\right.}{3840c}$$

input
```
Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
(Sqrt[d*x]*((4*a^(1/4)*(-384*a^4 + 6528*a^3*b*x^2 + 24973*a^2*b^2*x^4 + 27
846*a*b^3*x^6 + 9945*b^4*x^8))/(a + b*x^2)^3 - 9945*Sqrt[2]*b^(5/4)*x^(5/2
)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 9945*S
qrt[2]*b^(5/4)*x^(5/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a]
+ Sqrt[b]*x)]))/(3840*a^(21/4)*d^4*x^3)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.43, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 253, 253, 253, 264, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^4\int \frac{1}{b^4(dx)^{7/2}\left(bx^2+a\right)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{(dx)^{7/2}\left(a+bx^2\right)^4}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{17\int \frac{1}{(dx)^{7/2}(bx^2+a)^3}dx}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}$$

$$\Big\downarrow \ 253$$

$$\frac{17\left(\frac{13\int \frac{1}{(dx)^{7/2}\left(bx^2+a\right)^2}dx}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}$$

$$\Big\downarrow \ 253$$

$$\frac{17\left(\frac{13\left(\frac{9\int \frac{1}{(dx)^{7/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}$$

$$\Big\downarrow \ 264$$

$$\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\int \frac{1}{(dx)^{3/2}\left(bx^2+a\right)}dx}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}$$

$$\Big\downarrow \ 264$$

$$\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{b\int\frac{\sqrt{dx}}{bx^2+a}dx}{ad^2}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}$$

$\big\downarrow$ 266

$$\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\int\frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{ad^3}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}$$

$\big\downarrow$ 27

$$\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}$$

$\downarrow$ 826

$$\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}$$

$\Big\downarrow$ 1476

$$17\left(\frac{13\left(\frac{9\left(\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{}{2ad(d}\right)}{8a}\right.$$

↓ 1082

$$\left(\frac{17\left(\frac{13\left(\frac{9\left(b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+\right.}}{8a}\right.$$

↓ 217

$$17\left(\frac{13\left(\frac{9\left(b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}\right)-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}$$

↓ 1479

$$\left(13\left(\frac{9-\frac{b-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\right.}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right)}{ad}\cdots}{ad^2}\cdots}{4a}\cdots\right)\right.\right.$$

$$\Big\downarrow \text{25}$$

$$\left(13\;\frac{\left(9-\frac{\left(b-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{\cdots}}{\sqrt[4]{b}\cdots}\right.}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\cdots}}{2\sqrt{b}}\cdots\right.}{ad}\cdots\right.}{ad^2}\cdots\right.}{4a}\cdots\right.$$

$\Big\downarrow$ 27

$$13\left(\frac{9-\frac{b-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}\cdots}{ad^2}\cdots}{4a}\cdots\right.$$

↓ 1103

$$\left(17\left(13\left(9-\frac{\left(b-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}\right)}{ad^2}\right)}{4a}\right)}{8a}\right.$$

input
```
Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

output
```
1/(6*a*d*(d*x)^(5/2)*(a + b*x^2)^3) + (17*(1/(4*a*d*(d*x)^(5/2)*(a + b*x^2
)^2) + (13*(1/(2*a*d*(d*x)^(5/2)*(a + b*x^2)) + (9*(-2/(5*a*d*(d*x)^(5/2))
 - (b*(-2/(a*d*Sqrt[d*x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x]
)/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]
))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d +
Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(a*d)))/(a*d^2)))/(4*a)))/(8*a)))/(12*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{2(-20b\,x^2+a)}{5a^5\sqrt{dx}\,x^2d^3}+\frac{b^2\left(\frac{\frac{151b^2(dx)^{\frac{11}{2}}}{64}+\frac{173ab\,d^2(dx)^{\frac{7}{2}}}{32}+\frac{617a^2d^4(dx)^{\frac{3}{2}}}{192}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{663\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\,a\right.}{512b\left(\right.}\right)}{a^5d^3}$
derivativedivides	$2d^7\left(-\frac{1}{5d^8a^4(dx)^{\frac{5}{2}}}+\frac{4b}{d^{10}a^5\sqrt{dx}}+\frac{b^2\left(\frac{\frac{151b^2(dx)^{\frac{11}{2}}}{128}+\frac{173ab\,d^2(dx)^{\frac{7}{2}}}{64}+\frac{617a^2d^4(dx)^{\frac{3}{2}}}{384}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{663\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{}\right)}{d^{10}a^5}\right.$
default	$2d^7\left(-\frac{1}{5d^8a^4(dx)^{\frac{5}{2}}}+\frac{4b}{d^{10}a^5\sqrt{dx}}+\frac{b^2\left(\frac{\frac{151b^2(dx)^{\frac{11}{2}}}{128}+\frac{173ab\,d^2(dx)^{\frac{7}{2}}}{64}+\frac{617a^2d^4(dx)^{\frac{3}{2}}}{384}}{\left(b\,d^2x^2+a\,d^2\right)^3}+\frac{663\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{}\right)}{d^{10}a^5}\right.$
pseudoelliptic	$-\frac{2\left(-\frac{3315b\,x^2\sqrt{2}\left(b\,x^2+a\right)^3\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right.}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{1024}\right.}{5\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,d^3a^5x^2(b\,x^2+a)^3}$

input
```
int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-20*b*x^2+a)/a^5/(d*x)^(1/2)/x^2/d^3+b^2/a^5*(2*(151/128*b^2*(d*x)^(
11/2)+173/64*a*b*d^2*(d*x)^(7/2)+617/384*a^2*d^4*(d*x)^(3/2))/(b*d^2*x^2+a
*d^2)^3+663/512/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^
(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))/d^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.71

$$\int \frac{1}{(dx)^{7/2}\,(a^2+2abx^2+b^2x^4)^2}\,dx = \frac{9945\,(a^5b^3d^4x^9+3\,a^6b^2d^4x^7+3\,a^7bd^4x^5+a^8d^4x^3)\left(-\frac{b^5}{a^{21}d^{14}}\right)^{\frac{1}{4}}\log\left(2\right.}{}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/3840*(9945*(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5 + a^8*
d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*log(291434247*a^16*d^11*(-b^5/(a^21*d^14
))^(3/4) + 291434247*sqrt(d*x)*b^4) - 9945*(I*a^5*b^3*d^4*x^9 + 3*I*a^6*b^
2*d^4*x^7 + 3*I*a^7*b*d^4*x^5 + I*a^8*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*lo
g(291434247*I*a^16*d^11*(-b^5/(a^21*d^14))^(3/4) + 291434247*sqrt(d*x)*b^4
) - 9945*(-I*a^5*b^3*d^4*x^9 - 3*I*a^6*b^2*d^4*x^7 - 3*I*a^7*b*d^4*x^5 - I
*a^8*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*log(-291434247*I*a^16*d^11*(-b^5/(a
^21*d^14))^(3/4) + 291434247*sqrt(d*x)*b^4) - 9945*(a^5*b^3*d^4*x^9 + 3*a^
6*b^2*d^4*x^7 + 3*a^7*b*d^4*x^5 + a^8*d^4*x^3)*(-b^5/(a^21*d^14))^(1/4)*lo
g(-291434247*a^16*d^11*(-b^5/(a^21*d^14))^(3/4) + 291434247*sqrt(d*x)*b^4)
 + 4*(9945*b^4*x^8 + 27846*a*b^3*x^6 + 24973*a^2*b^2*x^4 + 6528*a^3*b*x^2
- 384*a^4)*sqrt(d*x))/(a^5*b^3*d^4*x^9 + 3*a^6*b^2*d^4*x^7 + 3*a^7*b*d^4*x
^5 + a^8*d^4*x^3)
```

Sympy [F]

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\int \frac{1}{(dx)^{\frac{7}{2}}\left(a+bx^2\right)^4}\,dx$$

input
```
integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral(1/((d*x)**(7/2)*(a + b*x**2)**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.19

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx=\frac{\frac{8\left(9945\,b^4d^8x^8+27846\,ab^3d^8x^6+24973\,a^2b^2d^8x^4+6528\,a^3bd^8x^2-384\,a^4d^8\right)}{(dx)^{\frac{17}{2}}a^5b^3d^2+3\,(dx)^{\frac{13}{2}}a^6b^2d^4+3\,(dx)^{\frac{9}{2}}a^7bd^6+(dx)^{\frac{5}{2}}a^8d^8}+\frac{9945\,b^2\left(\frac{2\sqrt{2}\ldots}{\ldots}\right.}{\ldots}}{\ldots}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/7680*(8*(9945*b^4*d^8*x^8 + 27846*a*b^3*d^8*x^6 + 24973*a^2*b^2*d^8*x^4
+ 6528*a^3*b*d^8*x^2 - 384*a^4*d^8)/((d*x)^(17/2)*a^5*b^3*d^2 + 3*(d*x)^(1
3/2)*a^6*b^2*d^4 + 3*(d*x)^(9/2)*a^7*b*d^6 + (d*x)^(5/2)*a^8*d^8) + 9945*b
^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)
*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqr
t(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d
)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4
)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^5*d^2))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.19

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^2}\,dx = \frac{663\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^6bd^5} + \frac{663\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^6bd^5} - \frac{663\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^6bd^5} + \frac{663\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^6bd^5} + \frac{453\sqrt{dx}b^4d^5x^5+1038\sqrt{dx}ab^3d^5x^3+617\sqrt{dx}a^2b^2d^5x}{192\left(bd^2x^2+ad^2\right)^3a^5d^3} + \frac{2\left(20\,bd^2x^2-ad^2\right)}{5\sqrt{dx}a^5d^5x^2}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
663/256*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d^5) + 663/256*sqrt(2)*(a*b^3*d
^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/(a^6*b*d^5) - 663/512*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqr
t(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b*d^5) + 663/512*sqrt
(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^6*b*d^5) + 1/192*(453*sqrt(d*x)*b^4*d^5*x^5 + 1038*sqrt(d*x)*a
*b^3*d^5*x^3 + 617*sqrt(d*x)*a^2*b^2*d^5*x)/((b*d^2*x^2 + a*d^2)^3*a^5*d^3
) + 2/5*(20*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^5*d^5*x^2)
```

Mupad [B] (verification not implemented)

Time = 18.67 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.61

$$\int \frac{1}{(dx)^{7/2} (a^2 + 2abx^2 + b^2x^4)^2} dx = \frac{\frac{34\,b\,d^5\,x^2}{5\,a^2} - \frac{2\,d^5}{5\,a} + \frac{24973\,b^2\,d^5\,x^4}{960\,a^3} + \frac{4641\,b^3\,d^5\,x^6}{160\,a^4} + \frac{663\,b^4\,d^5\,x^8}{64\,a^5}}{b^3\,(d\,x)^{17/2} + a^3\,d^6\,(d\,x)^{5/2} + 3\,a^2\,b\,d^4\,(d\,x)^{9/2} + 3\,a\,b^2\,d^2\,(d\,x)^{13/2}} - \frac{663\,(-b)^{5/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{21/4}\,d^{7/2}} + \frac{663\,(-b)^{5/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{128\,a^{21/4}\,d^{7/2}}$$

input
```
int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^2),x)
```

output
```
((34*b*d^5*x^2)/(5*a^2) - (2*d^5)/(5*a) + (24973*b^2*d^5*x^4)/(960*a^3) +
(4641*b^3*d^5*x^6)/(160*a^4) + (663*b^4*d^5*x^8)/(64*a^5))/(b^3*(d*x)^(17/
2) + a^3*d^6*(d*x)^(5/2) + 3*a^2*b*d^4*(d*x)^(9/2) + 3*a*b^2*d^2*(d*x)^(13
/2)) - (663*(-b)^(5/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(
128*a^(21/4)*d^(7/2)) + (663*(-b)^(5/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^
(1/4)*d^(1/2))))/(128*a^(21/4)*d^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.44

$$\int \frac{1}{(dx)^{7/2} (a^2 + 2abx^2 + b^2x^4)^2} dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(sqrt(d)*( - 19890*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 -
 59670*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 - 59670*sq
rt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 - 19890*sqrt(x)*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 + 19890*sqrt(x)*b**(1/4)*a**(3/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1
/4)*sqrt(2)))*a**3*b*x**2 + 59670*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))
)*a**2*b**2*x**4 + 59670*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*
x**6 + 19890*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 + 9945*sq
rt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) +
 sqrt(a) + sqrt(b)*x)*a**3*b*x**2 + 29835*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2
)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**2*b**
2*x**4 + 29835*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a
**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**3*x**6 + 9945*sqrt(x)*b**(1...
```

3.470 $\int \frac{(dx)^{27/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 343

$$\begin{aligned}\int \frac{(dx)^{27/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx &= -\frac{69615ad^{13}\sqrt{dx}}{4096b^7}+\frac{13923d^{11}(dx)^{5/2}}{4096b^6} \\ &\quad -\frac{d(dx)^{25/2}}{10b(a+bx^2)^5}-\frac{5d^3(dx)^{21/2}}{32b^2(a+bx^2)^4}-\frac{35d^5(dx)^{17/2}}{128b^3(a+bx^2)^3}-\frac{595d^7(dx)^{13/2}}{1024b^4(a+bx^2)^2} \\ &\quad -\frac{7735d^9(dx)^{9/2}}{4096b^5(a+bx^2)}-\frac{69615a^{5/4}d^{27/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}b^{29/4}} \\ &\quad +\frac{69615a^{5/4}d^{27/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}b^{29/4}} \\ &\quad +\frac{69615a^{5/4}d^{27/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}b^{29/4}}\end{aligned}$$

output
```
-69615/4096*a*d^13*(d*x)^(1/2)/b^7+13923/4096*d^11*(d*x)^(5/2)/b^6-1/10*d*
(d*x)^(25/2)/b/(b*x^2+a)^5-5/32*d^3*(d*x)^(21/2)/b^2/(b*x^2+a)^4-35/128*d^
5*(d*x)^(17/2)/b^3/(b*x^2+a)^3-595/1024*d^7*(d*x)^(13/2)/b^4/(b*x^2+a)^2-7
735/4096*d^9*(d*x)^(9/2)/b^5/(b*x^2+a)-69615/16384*a^(5/4)*d^(27/2)*arctan
(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(29/4)+69615/163
84*a^(5/4)*d^(27/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*
2^(1/2)/b^(29/4)+69615/16384*a^(5/4)*d^(27/2)*arctanh(2^(1/2)*a^(1/4)*b^(1
/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(29/4)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.66

$$\int \frac{(dx)^{27/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{d^{13}\sqrt{dx}\Big(4\sqrt[4]{b}\sqrt{x}(-348075a^6-1670760a^5bx^2-3171350a^4b^2x^4-2951200a^3}{}$$

input
```
Integrate[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(d^13*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(-348075*a^6 - 1670760*a^5*b*x^2 - 3171
350*a^4*b^2*x^4 - 2951200*a^3*b^3*x^6 - 1317575*a^2*b^4*x^8 - 204800*a*b^5
*x^10 + 8192*b^6*x^12) + 348075*Sqrt[2]*a^(5/4)*(a + b*x^2)^5*ArcTan[(-Sqr
t[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 348075*Sqrt[2]*a^(5
/4)*(a + b*x^2)^5*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqr
t[b]*x)]))/(81920*b^(29/4)*Sqrt[x]*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.43, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.679$, Rules used = {1380, 27, 252, 252, 252, 252, 252, 262, 262, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{27/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{(dx)^{27/2}}{b^6\left(bx^2+a\right)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{27/2}}{\left(a+bx^2\right)^6}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5d^2\int \frac{(dx)^{23/2}}{\left(bx^2+a\right)^5}dx}{4b}-\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{5d^2\left(\frac{21d^2\int \frac{(dx)^{19/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^4}\right)}{4b}-\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{5d^2\left(\frac{21d^2\left(\frac{17d^2\int \frac{(dx)^{15/2}}{\left(bx^2+a\right)^3}dx}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^4}\right)}{4b}-\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{5d^2\left(\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\int\frac{(dx)^{11/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{21/2}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 252

$$\frac{5d^2\left(\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\int\frac{(dx)^{7/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{21/2}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 262

$$\frac{5d^2\left(\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\int\frac{(dx)^{3/2}}{bx^2+a}dx}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{21/2}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{25/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 262

$$5d^2 \left(\frac{21d^2 \left(\frac{17d^2 \left(\frac{13d^2 \left(\frac{9d^2 \left(\frac{2d(dx)^{5/2}}{5b} - \frac{ad^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{ad^2 \int \frac{1}{\sqrt{dx}\left(bx^2+a\right)} dx}{b} \right)}{b} \right)}{4b} - \frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)} \right)}{8b} - \frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2} \right)}{12b} - \frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3} \right)}{16b} - \frac{d(dx)^{21/2}}{8b(a+bx^2)^4} \right.$$

↓ 266

$$5d^2\left(\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\int\frac{1}{bx^2+a}d\sqrt{dx}}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{21/2}}{8b(a+bx^2)^4}\right.$$

↓ 755

$$\left(21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}\right)$$

↓ 27

$$\left(21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(}{6b(}\right.\right.$$

↓ 1476

$$
\left(\left(\left(\left(13d^2\,\frac{\left(9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{\left(ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{\left(2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}\,d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\cdots}{\sqrt[4]{b}}}\cdots}{2\sqrt{b}}\cdots\right)}{2\sqrt{a}}\cdots\right)\cdots\right)}{b}\cdots\right)\cdots\right)}{b}\cdots\right)\cdots\right)}{4b}\cdots\right)\cdots\right)\cdots\right)\cdots\right)
$$

↓ 1082

$$2ad\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{2\sqrt{a}}\right.$$

$$13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\cdots\right.}{b}\right.}{b}\right.}{4b}\right.$$

↓ 217

$$\left(\left(\left(13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4b}\right) -\right.\right.\right.$$

↓ 1479

$$d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.$$

$$2ad\left(\frac{\cdots}{2\sqrt{a}}\right.$$

$$ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{\cdots}{b}\right.$$

$$9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{\cdots}{b}\right.$$

$$13d^2\,\frac{\cdots}{4b}$$

↓ 25

$$13d^2\,\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{\cdots}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right)\cdots}{2\sqrt{a}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4b}\cdots$$

$\downarrow$ 27

$$d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)$$

$$2ad\left(\frac{\cdots}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\cdots}{\sqrt{\cdots}}\right)}{\cdots}\right)$$

$$ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{\cdots}{b}\right)$$

$$9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{\cdots}{b}\right)$$

$$13d^2\,\frac{\cdots}{4b}$$

↓ 1103

$$13d^2 \left(\frac{9d^2 \left(\frac{2d(dx)^{5/2}}{5b} - \frac{ad^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{2ad \left(\frac{d \left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}} + 1 \right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}} \right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d \left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx} + \right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} \right.}{} \right.}{b} \right.}{b} \right.}{4b} \right.$$

input
```
Int[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*(d*x)^(25/2))/(b*(a + b*x^2)^5) + (5*d^2*(-1/8*(d*(d*x)^(21/2))/(
b*(a + b*x^2)^4) + (21*d^2*(-1/6*(d*(d*x)^(17/2))/(b*(a + b*x^2)^3) + (17*
d^2*(-1/4*(d*(d*x)^(13/2))/(b*(a + b*x^2)^2) + (13*d^2*(-1/2*(d*(d*x)^(9/2
))/(b*(a + b*x^2)) + (9*d^2*((2*d*(d*x)^(5/2))/(5*b) - (a*d^2*((2*d*Sqrt[d
*x])/b - (2*a*d*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]
) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]
*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*
x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d])))/(2*Sqrt[a])))/b))/b))/(4*b)))/(8*b)))/(12*b)))/(16*b)))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 14.83 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{d^{13}\left(\left(65536b^6x^{12}-1638400a\,b^5x^{10}-10540600a^2b^4x^8-23609600a^3b^3x^6-25370800a^4b^2x^4-13366080a^5b\,x^2-278460\right.\right.}{}$
risch	$-\frac{2\left(-b\,x^2+30a\right)x\,d^{14}}{5b^7\sqrt{dx}}+\frac{2a^2d^{15}\left(\frac{-\frac{20463a^4d^8\sqrt{dx}}{8192}-\frac{56269a^3d^6b(dx)^{\frac{5}{2}}}{5120}-\frac{75471a^2d^4b^2(dx)^{\frac{9}{2}}}{4096}-\frac{3597a\,d^2b^3(dx)^{\frac{13}{2}}}{256}-\frac{341139b}{81}}{\left(b\,d^2x^2+a\,d^2\right)^5}\right.}{}$
derivativedivides	$2d^{11}\left(-\frac{-\frac{b(dx)^{\frac{5}{2}}}{5}+6\sqrt{dx}\,a\,d^2}{b^7}+\frac{a^2d^4\left(\frac{-\frac{20463a^4d^8\sqrt{dx}}{8192}-\frac{56269a^3d^6b(dx)^{\frac{5}{2}}}{5120}-\frac{75471a^2d^4b^2(dx)^{\frac{9}{2}}}{4096}-\frac{3597a\,d^2b^3(dx)^{\frac{13}{2}}}{256}}{\left(b\,d^2x^2+a\,d^2\right)^5}\right.}{}\right.$
default	$2d^{11}\left(-\frac{-\frac{b(dx)^{\frac{5}{2}}}{5}+6\sqrt{dx}\,a\,d^2}{b^7}+\frac{a^2d^4\left(\frac{-\frac{20463a^4d^8\sqrt{dx}}{8192}-\frac{56269a^3d^6b(dx)^{\frac{5}{2}}}{5120}-\frac{75471a^2d^4b^2(dx)^{\frac{9}{2}}}{4096}-\frac{3597a\,d^2b^3(dx)^{\frac{13}{2}}}{256}}{\left(b\,d^2x^2+a\,d^2\right)^5}\right.}{}\right.$

input
```
int((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/163840*d^13*((65536*b^6*x^12-1638400*a*b^5*x^10-10540600*a^2*b^4*x^8-236
09600*a^3*b^3*x^6-25370800*a^4*b^2*x^4-13366080*a^5*b*x^2-2784600*a^6)*(d*
x)^(1/2)+348075*(a*d^2/b)^(1/4)*a*2^(1/2)*(b*x^2+a)^5*(ln((d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2)))-2*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)
^(1/4))))/b^7/(b*x^2+a)^5
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.66

$$\int \frac{(dx)^{27/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{348075\left(-\frac{a^5d^{54}}{b^{29}}\right)^{\frac{1}{4}}(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a}{}$$

input
```
integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/81920*(348075*(-a^5*d^54/b^29)^(1/4)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*
b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)*log(69615*sqrt(d*x)*a
*d^13 + 69615*(-a^5*d^54/b^29)^(1/4)*b^7) - 348075*(-a^5*d^54/b^29)^(1/4)*
(-I*b^12*x^10 - 5*I*a*b^11*x^8 - 10*I*a^2*b^10*x^6 - 10*I*a^3*b^9*x^4 - 5*
I*a^4*b^8*x^2 - I*a^5*b^7)*log(69615*sqrt(d*x)*a*d^13 + 69615*I*(-a^5*d^54
/b^29)^(1/4)*b^7) - 348075*(-a^5*d^54/b^29)^(1/4)*(I*b^12*x^10 + 5*I*a*b^1
1*x^8 + 10*I*a^2*b^10*x^6 + 10*I*a^3*b^9*x^4 + 5*I*a^4*b^8*x^2 + I*a^5*b^7
)*log(69615*sqrt(d*x)*a*d^13 - 69615*I*(-a^5*d^54/b^29)^(1/4)*b^7) - 34807
5*(-a^5*d^54/b^29)^(1/4)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*
a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)*log(69615*sqrt(d*x)*a*d^13 - 69615*
(-a^5*d^54/b^29)^(1/4)*b^7) + 4*(8192*b^6*d^13*x^12 - 204800*a*b^5*d^13*x^
10 - 1317575*a^2*b^4*d^13*x^8 - 2951200*a^3*b^3*d^13*x^6 - 3171350*a^4*b^2
*d^13*x^4 - 1670760*a^5*b*d^13*x^2 - 348075*a^6*d^13)*sqrt(d*x))/(b^12*x^1
0 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*
b^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{27/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(27/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.23

$$\int \frac{(dx)^{27/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx =$$

$$-\frac{\frac{8\left(170695\,(dx)^{\frac{17}{2}}a^2b^4d^{16}+575520\,(dx)^{\frac{13}{2}}a^3b^3d^{18}+754710\,(dx)^{\frac{9}{2}}a^4b^2d^{20}+450152\,(dx)^{\frac{5}{2}}a^5bd^{22}+102315\,\sqrt{dx}a^6d^{24}\right)}{b^{12}d^{10}x^{10}+5\,ab^{11}d^{10}x^8+10\,a^2b^{10}d^{10}x^6+10\,a^3b^9d^{10}x^4+5\,a^4b^8d^{10}x^2+a^5b^7d^{10}}-\frac{348075\left(\frac{\sqrt{2}d^{16}\log\left(\sqrt{\cdots}\right.}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

input
```
integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/163840*(8*(170695*(d*x)^(17/2)*a^2*b^4*d^16 + 575520*(d*x)^(13/2)*a^3*b
^3*d^18 + 754710*(d*x)^(9/2)*a^4*b^2*d^20 + 450152*(d*x)^(5/2)*a^5*b*d^22
+ 102315*sqrt(d*x)*a^6*d^24)/(b^12*d^10*x^10 + 5*a*b^11*d^10*x^8 + 10*a^2*
b^10*d^10*x^6 + 10*a^3*b^9*d^10*x^4 + 5*a^4*b^8*d^10*x^2 + a^5*b^7*d^10) -
 348075*(sqrt(2)*d^16*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^
(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^16*log(sqrt(b)*d*x
- sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1
/4)) + 2*sqrt(2)*d^15*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) +
2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqr
t(a)) + 2*sqrt(2)*d^15*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4)
- 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*s
qrt(a)))*a^2/b^7 - 65536*((d*x)^(5/2)*b*d^12 - 30*sqrt(d*x)*a*d^14)/b^7)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^{27/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{\frac{696150\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad^{14}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8}+\frac{696150\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad^{14}\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right)}{b^8}\cdots}{\cdots}$$

input
```
integrate((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/163840*(696150*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^14*arctan(1/2*sqrt(2)*(sqrt
(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^8 + 696150*sqrt(2)*(
a*b^3*d^2)^(1/4)*a*d^14*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*s
qrt(d*x))/(a*d^2/b)^(1/4))/b^8 + 348075*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^14*l
og(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^8 - 348075*s
qrt(2)*(a*b^3*d^2)^(1/4)*a*d^14*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/b^8 - 8*(170695*sqrt(d*x)*a^2*b^4*d^24*x^8 + 575520*sqr
t(d*x)*a^3*b^3*d^24*x^6 + 754710*sqrt(d*x)*a^4*b^2*d^24*x^4 + 450152*sqrt(
d*x)*a^5*b*d^24*x^2 + 102315*sqrt(d*x)*a^6*d^24)/((b*d^2*x^2 + a*d^2)^5*b^
7) + 65536*(sqrt(d*x)*b^24*d^14*x^2 - 30*sqrt(d*x)*a*b^23*d^14)/b^30)/d
```

Mupad [B] (verification not implemented)

Time = 18.67 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.72

$$\int \frac{(dx)^{27/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{2\,d^{11}\,(d\,x)^{5/2}}{5\,b^6} - \frac{\frac{20463\,a^6\,d^{23}\,\sqrt{d\,x}}{4096}+\frac{75471\,a^4\,b^2\,d^{19}\,(d\,x)^{9/2}}{2048}+\frac{3597\,a^3\,b^3\,d^{17}\,(d\,x)^{13/2}}{128}+\frac{34139\,a^2\,b^4\,d^{15}\,(d\,x)^{17/2}}{4096}+\frac{56269\,a^5\,b\,d^{21}\,(d\,x)^{5/2}}{2560}}{a^5\,b^7\,d^{10}+5\,a^4\,b^8\,d^{10}\,x^2+10\,a^3\,b^9\,d^{10}\,x^4+10\,a^2\,b^{10}\,d^{10}\,x^6+5\,a\,b^{11}\,d^{10}\,x^8+b^{12}\,d^{10}\,x^{10}} - \frac{69615\,(-a)^{5/4}\,d^{27/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,b^{29/4}} - \frac{12\,a\,d^{13}\,\sqrt{d\,x}}{b^7} + \frac{(-a)^{5/4}\,d^{27/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1\mathrm{i}}{(-a)^{1/4}\,\sqrt{d}}\right)\,69615\mathrm{i}}{8192\,b^{29/4}}$$

input
```
int((d*x)^(27/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(2*d^11*(d*x)^(5/2))/(5*b^6) - ((20463*a^6*d^23*(d*x)^(1/2))/4096 + (75471
*a^4*b^2*d^19*(d*x)^(9/2))/2048 + (3597*a^3*b^3*d^17*(d*x)^(13/2))/128 + (
34139*a^2*b^4*d^15*(d*x)^(17/2))/4096 + (56269*a^5*b*d^21*(d*x)^(5/2))/256
0)/(a^5*b^7*d^10 + b^12*d^10*x^10 + 5*a*b^11*d^10*x^8 + 5*a^4*b^8*d^10*x^2
 + 10*a^3*b^9*d^10*x^4 + 10*a^2*b^10*d^10*x^6) - (69615*(-a)^(5/4)*d^(27/2
)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*b^(29/4)) + ((-a
)^(5/4)*d^(27/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*69615
i)/(8192*b^(29/4)) - (12*a*d^13*(d*x)^(1/2))/b^7
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1015, normalized size of antiderivative = 2.96

$$\int \frac{(dx)^{27/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*d**13*( - 696150*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**6 - 3480750
*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqr
t(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5*b*x**2 - 6961500*b**(3/4)*a**(1/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*a**4*b**2*x**4 - 6961500*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))
)*a**3*b**3*x**6 - 3480750*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**4*x**
8 - 696150*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*s
qrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**5*x**10 + 696150*b**(3/4
)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*a**6 + 3480750*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))
)*a**5*b*x**2 + 6961500*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b**2*x**4 +
 6961500*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqr
t(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**3*x**6 + 3480750*b**(3/
4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**4*x**8 + 696150*b**(3/4)*a**(1/4)*s...
```

3.471 $\int \frac{(dx)^{25/2}}{(a^2+2abx^2+b^2x^4)^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 325

$$\begin{aligned}\int \frac{(dx)^{25/2}}{(a^2+2abx^2+b^2x^4)^3} dx &= \frac{33649d^{11}(dx)^{3/2}}{12288b^6} - \frac{d(dx)^{23/2}}{10b(a+bx^2)^5} \\ &- \frac{23d^3(dx)^{19/2}}{160b^2(a+bx^2)^4} - \frac{437d^5(dx)^{15/2}}{1920b^3(a+bx^2)^3} - \frac{437d^7(dx)^{11/2}}{1024b^4(a+bx^2)^2} \\ &- \frac{4807d^9(dx)^{7/2}}{4096b^5(a+bx^2)} + \frac{33649a^{3/4}d^{25/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}b^{27/4}} \\ &- \frac{33649a^{3/4}d^{25/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}b^{27/4}} \\ &+ \frac{33649a^{3/4}d^{25/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}b^{27/4}}\end{aligned}$$

output
```
33649/12288*d^11*(d*x)^(3/2)/b^6-1/10*d*(d*x)^(23/2)/b/(b*x^2+a)^5-23/160*
d^3*(d*x)^(19/2)/b^2/(b*x^2+a)^4-437/1920*d^5*(d*x)^(15/2)/b^3/(b*x^2+a)^3
-437/1024*d^7*(d*x)^(11/2)/b^4/(b*x^2+a)^2-4807/4096*d^9*(d*x)^(7/2)/b^5/(
b*x^2+a)+33649/16384*a^(3/4)*d^(25/2)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)
/a^(1/4)/d^(1/2))*2^(1/2)/b^(27/4)-33649/16384*a^(3/4)*d^(25/2)*arctan(1+2
^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(27/4)+33649/16384*a
^(3/4)*d^(25/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/
2)+b^(1/2)*x))*2^(1/2)/b^(27/4)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.66

$$\int \frac{(dx)^{25/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{d^{12}\sqrt{dx}\left(4b^{3/4}x^{3/2}(168245a^5+769120a^4bx^2+1367810a^3b^2x^4+1157176a^2b^3\right.}{}$$

input
```
Integrate[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(d^12*Sqrt[d*x]*(4*b^(3/4)*x^(3/2)*(168245*a^5 + 769120*a^4*b*x^2 + 136781
0*a^3*b^2*x^4 + 1157176*a^2*b^3*x^6 + 437345*a*b^4*x^8 + 40960*b^5*x^10) -
 504735*Sqrt[2]*a^(3/4)*(a + b*x^2)^5*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 504735*Sqrt[2]*a^(3/4)*(a + b*x^2)^5*ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(245760*b^(27
/4)*Sqrt[x]*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.43, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1380, 27, 252, 252, 252, 252, 252, 262, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{25/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$

$$\Big\downarrow \; 1380$$

$$b^6\int \frac{(dx)^{25/2}}{b^6\,(bx^2+a)^6}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{(dx)^{25/2}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \; 252$$

$$\frac{23d^2\int \frac{(dx)^{21/2}}{(bx^2+a)^5}dx}{20b}-\frac{d(dx)^{23/2}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{23d^2\left(\frac{19d^2\int \frac{(dx)^{17/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{23/2}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{23d^2\left(\frac{19d^2\left(\frac{5d^2\int \frac{(dx)^{13/2}}{\left(bx^2+a\right)^3}dx}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{23/2}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{23d^2\left(\frac{19d^2\left(\frac{5d^2\left(\frac{11d^2\int\frac{(dx)^{9/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 252

$$\frac{23d^2\left(\frac{19d^2\left(\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\int\frac{(dx)^{5/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 262

$$\frac{23d^2\left(\frac{19d^2\left(\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{ad^2\int\frac{\sqrt{dx}}{bx^2+a}dx}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 266

$$\frac{23d^2\left(\frac{19d^2\left(\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \ 27$$

$$\frac{23d^2\left(\frac{19d^2\left(\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{23/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 826

$$23d^2 \left(\frac{19d^2 \left(\frac{5d^2 \left(\frac{11d^2 \left(\frac{7d^2 \left(\frac{2d(dx)^{3/2}}{3b} - \frac{2ad^3 \left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}} - \frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}} \right)}{b} \right)}{4b} - \frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)} \right)}{8b} - \frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2} \right)}{4b} - \frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3} \right)}{16b} - \frac{d\cdots}{8b\cdots}$$

↓ 1476

$$19d^2\left(5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{\cdots}{2b}\cdots}{8b}\cdots\right)}{4b}\cdots\right)\cdots\right.$$

↓ 1082

$$\left(19d^2\frac{\left(5d^2\frac{\left(11d^2\frac{\left(7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right.}{8b}\right.}{4b}\right.$$

↓ 217

$$\left(19d^2\left(\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(d}{4b\left(a\right.}\right.}{4b}\right.\right.$$

↓ 1479

$$5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\cdots}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8b}\cdots\right.$$

↓ 25

$$\left(5d^2 \frac{\left(11d^2 \frac{\left(7d^2 \left(\frac{2d(dx)^{3/2}}{3b} - \frac{2ad^3 \left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} - \frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} + \cdots}{2\sqrt{b}} \cdots \right)}{b} \cdots \right) \cdots \right)}{4b} \cdots \right)}{8b} \cdots \right.$$

$$\Big\downarrow 27$$

$$5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\cdots}{xd+\frac{\sqrt{a}\cdots}{\sqrt{b}\cdots}}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8b}\cdots\right)\cdots$$

↓ 1103

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}$$

$$2\sqrt{b}$$

$$\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(\right.}{}$$

$$2\sqrt{b}$$

$$2ad^3$$

$$7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{}{b}\right.$$

$$11d^2 \quad \frac{}{4b}$$

$$5d^2 \quad \frac{}{8b}$$

$$19d^2 \quad \frac{}{4b}$$

input
```
Int[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*(d*x)^(23/2))/(b*(a + b*x^2)^5) + (23*d^2*(-1/8*(d*(d*x)^(19/2))/
(b*(a + b*x^2)^4) + (19*d^2*(-1/6*(d*(d*x)^(15/2))/(b*(a + b*x^2)^3) + (5*
d^2*(-1/4*(d*(d*x)^(11/2))/(b*(a + b*x^2)^2) + (11*d^2*(-1/2*(d*(d*x)^(7/2
))/(b*(a + b*x^2)) + (7*d^2*((2*d*(d*x)^(3/2))/(3*b) - (2*a*d^3*((-(ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt
[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/b))/(4*b)
))/(8*b)))/(4*b)))/(16*b)))/(20*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 14.73 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.77

method	result
derivativedivides	$2d^{11}\left(\frac{(dx)^{\frac{3}{2}}}{3b^6}-\frac{a\,d^2\left(\frac{-\frac{25457a^4d^8(dx)^{\frac{3}{2}}}{24576}-\frac{3527a^3d^6b(dx)^{\frac{7}{2}}}{768}-\frac{95821a^2d^4b^2(dx)^{\frac{11}{2}}}{12288}-\frac{31149a\,d^2b^3(dx)^{\frac{15}{2}}}{5120}-\frac{15503b^4(dx)^{\frac{19}{2}}}{8192}}{\left(b\,d^2x^2+a\,d^2\right)^5}\right.}{b}\right.$
default	$2d^{11}\left(\frac{(dx)^{\frac{3}{2}}}{3b^6}-\frac{a\,d^2\left(\frac{-\frac{25457a^4d^8(dx)^{\frac{3}{2}}}{24576}-\frac{3527a^3d^6b(dx)^{\frac{7}{2}}}{768}-\frac{95821a^2d^4b^2(dx)^{\frac{11}{2}}}{12288}-\frac{31149a\,d^2b^3(dx)^{\frac{15}{2}}}{5120}-\frac{15503b^4(dx)^{\frac{19}{2}}}{8192}}{\left(b\,d^2x^2+a\,d^2\right)^5}\right.}{b}\right.$
risch	$\frac{2x^2d^{13}}{3b^6\sqrt{dx}}-\frac{a\left(\frac{-\frac{25457a^4d^8(dx)^{\frac{3}{2}}}{12288}-\frac{3527a^3d^6b(dx)^{\frac{7}{2}}}{384}-\frac{95821a^2d^4b^2(dx)^{\frac{11}{2}}}{6144}-\frac{31149a\,d^2b^3(dx)^{\frac{15}{2}}}{2560}-\frac{15503b^4(dx)^{\frac{19}{2}}}{4096}}{\left(b\,d^2x^2+a\,d^2\right)^5}+\frac{33649\sqrt{}}{}\right.}{b^6}$
pseudoelliptic	$\frac{33649\left(8\left(\frac{8192}{33649}x^{10}b^5+\frac{3803}{1463}a\,x^8b^4+\frac{2648}{385}a^2x^6b^3+\frac{626}{77}a^3x^4b^2+\frac{32}{7}x^2a^4b+a^5\right)\sqrt{dx}\,bx\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}-3\sqrt{2}\,ad\left(b\,x^2+a\right)^5\left(\ln\left(\right.\right.\right.}{98304\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^7(b\,x}$

input
```
int((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^11*(1/3*(d*x)^(3/2)/b^6-a*d^2/b^6*((-25457/24576*a^4*d^8*(d*x)^(3/2)-3
527/768*a^3*d^6*b*(d*x)^(7/2)-95821/12288*a^2*d^4*b^2*(d*x)^(11/2)-31149/5
120*a*d^2*b^3*(d*x)^(15/2)-15503/8192*b^4*(d*x)^(19/2))/(b*d^2*x^2+a*d^2)^
5+33649/65536/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^
2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1
/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.74

$$\int \frac{(dx)^{25/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx =$$

$$-\frac{504735\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{1}{4}}(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)\log\left(38099255258449\sqrt{\right.}}{}$$

input
```
integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/245760*(504735*(-a^3*d^50/b^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^
2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*log(38099255258449*s
qrt(d*x)*a^2*d^37 + 38099255258449*(-a^3*d^50/b^27)^(3/4)*b^20) + 504735*(
-a^3*d^50/b^27)^(1/4)*(-I*b^11*x^10 - 5*I*a*b^10*x^8 - 10*I*a^2*b^9*x^6 -
10*I*a^3*b^8*x^4 - 5*I*a^4*b^7*x^2 - I*a^5*b^6)*log(38099255258449*sqrt(d*
x)*a^2*d^37 + 38099255258449*I*(-a^3*d^50/b^27)^(3/4)*b^20) + 504735*(-a^3
*d^50/b^27)^(1/4)*(I*b^11*x^10 + 5*I*a*b^10*x^8 + 10*I*a^2*b^9*x^6 + 10*I*
a^3*b^8*x^4 + 5*I*a^4*b^7*x^2 + I*a^5*b^6)*log(38099255258449*sqrt(d*x)*a^
2*d^37 - 38099255258449*I*(-a^3*d^50/b^27)^(3/4)*b^20) - 504735*(-a^3*d^50
/b^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 +
 5*a^4*b^7*x^2 + a^5*b^6)*log(38099255258449*sqrt(d*x)*a^2*d^37 - 38099255
258449*(-a^3*d^50/b^27)^(3/4)*b^20) - 4*(40960*b^5*d^12*x^11 + 437345*a*b^
4*d^12*x^9 + 1157176*a^2*b^3*d^12*x^7 + 1367810*a^3*b^2*d^12*x^5 + 769120*
a^4*b*d^12*x^3 + 168245*a^5*d^12*x)*sqrt(d*x))/(b^11*x^10 + 5*a*b^10*x^8 +
 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{25/2}}{(a^2 + 2abx^2 + b^2x^4)^3}\, dx = \text{Timed out}$$

input
```
integrate((d*x)**(25/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.21

$$\int \frac{(dx)^{25/2}}{(a^2 + 2abx^2 + b^2x^4)^3}\, dx = -\frac{\frac{504735\, ad^{14} \left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}} + 2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}} b^{\frac{1}{4}} - 2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}dx + \sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}} + \sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}} b^{\frac{3}{4}}} \right.}{b^6}}{}$$

input
```
integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/491520*(504735*a*d^14*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1
/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*
sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)
*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqr
t(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*
x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x
- sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3
/4)))/b^6 - 327680*(d*x)^(3/2)*d^12/b^6 - 8*(232545*(d*x)^(19/2)*a*b^4*d^1
4 + 747576*(d*x)^(15/2)*a^2*b^3*d^16 + 958210*(d*x)^(11/2)*a^3*b^2*d^18 +
564320*(d*x)^(7/2)*a^4*b*d^20 + 127285*(d*x)^(3/2)*a^5*d^22)/(b^11*d^10*x^
10 + 5*a*b^10*d^10*x^8 + 10*a^2*b^9*d^10*x^6 + 10*a^3*b^8*d^10*x^4 + 5*a^4
*b^7*d^10*x^2 + a^5*b^6*d^10))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.09

$$\int \frac{(dx)^{25/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{1}{491520}\,d^{12}\left(\frac{327680\,\sqrt{dx}x}{b^6} - \frac{1009470\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{\right.}}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{b^9d}\right.$$

input
```
integrate((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/491520*d^12*(327680*sqrt(d*x)*x/b^6 - 1009470*sqrt(2)*(a*b^3*d^2)^(3/4)*
arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4)
)/(b^9*d) - 1009470*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^9*d) + 504735*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(b^9*d) - 504735*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b
)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^9*d) + 8*(232545*sqrt(d*x)*a*b^4*d^1
0*x^9 + 747576*sqrt(d*x)*a^2*b^3*d^10*x^7 + 958210*sqrt(d*x)*a^3*b^2*d^10*
x^5 + 564320*sqrt(d*x)*a^4*b*d^10*x^3 + 127285*sqrt(d*x)*a^5*d^10*x)/((b*d
^2*x^2 + a*d^2)^5*b^6))
```

Mupad [B] (verification not implemented)

Time = 18.63 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.71

$$\int \frac{(dx)^{25/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{25457\,a^5\,d^{21}\,(d\,x)^{3/2}}{12288}+\frac{95821\,a^3\,b^2\,d^{17}\,(d\,x)^{11/2}}{6144}+\frac{31149\,a^2\,b^3\,d^{15}\,(d\,x)^{15/2}}{2560}+\frac{3527\,a^4\,b\,d^{19}\,(d\,x)^{7/2}}{384}}{a^5\,b^6\,d^{10}+5\,a^4\,b^7\,d^{10}\,x^2+10\,a^3\,b^8\,d^{10}\,x^4+10\,a^2\,b^9\,d^{10}\,x^6+5\,a\,b^{10}\,d^{10}\,x}$$

$$+\frac{2\,d^{11}\,(d\,x)^{3/2}}{3\,b^6}+\frac{33649\,(-a)^{3/4}\,d^{25/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,b^{27/4}}$$

$$+\frac{(-a)^{3/4}\,d^{25/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1\mathrm{i}}{(-a)^{1/4}\,\sqrt{d}}\right)\,33649\mathrm{i}}{8192\,b^{27/4}}$$

input
```
int((d*x)^(25/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((25457*a^5*d^21*(d*x)^(3/2))/12288 + (95821*a^3*b^2*d^17*(d*x)^(11/2))/61
44 + (31149*a^2*b^3*d^15*(d*x)^(15/2))/2560 + (3527*a^4*b*d^19*(d*x)^(7/2)
)/384 + (15503*a*b^4*d^13*(d*x)^(19/2))/4096)/(a^5*b^6*d^10 + b^11*d^10*x^
10 + 5*a*b^10*d^10*x^8 + 5*a^4*b^7*d^10*x^2 + 10*a^3*b^8*d^10*x^4 + 10*a^2
*b^9*d^10*x^6) + (2*d^11*(d*x)^(3/2))/(3*b^6) + (33649*(-a)^(3/4)*d^(25/2)
*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*b^(27/4)) + ((-a)
^(3/4)*d^(25/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*33649i
)/(8192*b^(27/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1003, normalized size of antiderivative = 3.09

$$\int \frac{(dx)^{25/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*d**12*(1009470*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 + 5047350*b
**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(
b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 10094700*b**(1/4)*a**(3/4)*
sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**
(1/4)*sqrt(2)))*a**3*b**2*x**4 + 10094700*b**(1/4)*a**(3/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))
)*a**2*b**3*x**6 + 5047350*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 +
 1009470*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 - 1009470*b**(1/4)*a
**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*a**5 - 5047350*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a
**4*b*x**2 - 10094700*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 - 1
0094700*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt
(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*x**6 - 5047350*b**(1/4
)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 1009470*b**(1/4)*a**(3/4)*sqrt...
```

3.472 $\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 325

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{13923d^{11}\sqrt{dx}}{4096b^6} - \frac{d(dx)^{21/2}}{10b(a+bx^2)^5} - \frac{21d^3(dx)^{17/2}}{160b^2(a+bx^2)^4} - \frac{119d^5(dx)^{13/2}}{640b^3(a+bx^2)^3} - \frac{1547d^7(dx)^{9/2}}{5120b^4(a+bx^2)^2} - \frac{13923d^9(dx)^{5/2}}{20480b^5(a+bx^2)} + \frac{13923\sqrt[4]{a}d^{23/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}b^{25/4}} - \frac{13923\sqrt[4]{a}d^{23/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}b^{25/4}} - \frac{13923\sqrt[4]{a}d^{23/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}b^{25/4}}$$

output
```
13923/4096*d^11*(d*x)^(1/2)/b^6-1/10*d*(d*x)^(21/2)/b/(b*x^2+a)^5-21/160*d
^3*(d*x)^(17/2)/b^2/(b*x^2+a)^4-119/640*d^5*(d*x)^(13/2)/b^3/(b*x^2+a)^3-1
547/5120*d^7*(d*x)^(9/2)/b^4/(b*x^2+a)^2-13923/20480*d^9*(d*x)^(5/2)/b^5/(
b*x^2+a)+13923/16384*a^(1/4)*d^(23/2)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)
/a^(1/4)/d^(1/2))*2^(1/2)/b^(25/4)-13923/16384*a^(1/4)*d^(23/2)*arctan(1+2
^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(25/4)-13923/16384*a
^(1/4)*d^(23/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/
2)+b^(1/2)*x))*2^(1/2)/b^(25/4)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.66

$$\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{d^{11}\sqrt{dx}\left(4\sqrt[4]{b}\sqrt{x}(69615a^5+334152a^4bx^2+634270a^3b^2x^4+590240a^2b^3x^6+\right.}{}$$

input
```
Integrate[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(d^11*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(69615*a^5 + 334152*a^4*b*x^2 + 634270*
a^3*b^2*x^4 + 590240*a^2*b^3*x^6 + 263515*a*b^4*x^8 + 40960*b^5*x^10) - 69
615*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])] - 69615*Sqrt[2]*a^(1/4)*(a + b*x^2)^5*ArcTanh[(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(81920*b^(25/4)*Sq
rt[x]*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.43, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1380, 27, 252, 252, 252, 252, 252, 262, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^6\int \frac{(dx)^{23/2}}{b^6\left(bx^2+a\right)^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(dx)^{23/2}}{\left(a+bx^2\right)^6}dx$$

$$\Big\downarrow \; 252$$

$$\frac{21d^2\int \frac{(dx)^{19/2}}{(bx^2+a)^5}dx}{20b}-\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{21d^2\left(\frac{17d^2\int \frac{(dx)^{15/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{17/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\int \frac{(dx)^{11/2}}{\left(bx^2+a\right)^3}dx}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{17/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\int \frac{(dx)^{7/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{17/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\int\frac{(dx)^{3/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{17/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \ 262$$

$$\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{ad^2\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{17/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5}$$

$\big\downarrow$ 266

$$\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\int \frac{1}{bx^2+a}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{17/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 755

$$21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\right.$$

$$\Big\downarrow 27$$

$$\left(21d^2\,\frac{\left(17d^2\,\frac{\left(13d^2\,\frac{\left(9d^2\,\frac{\left(5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{a}}\right)}{b}\right)\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(\cdots}{8b(\cdots}\right.$$

↓ 1476

$$\left(\quad\left(\quad 13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}-\right.}{8b}\right.\right.\right.$$

↓ 1082

$$\left(\quad 13d^2 \left(\frac{9d^2 \left(\frac{5d^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{2ad \left(\frac{d \int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2} d\sqrt{dx}}{2\sqrt{a}} + \frac{d \left(\frac{\int \frac{1}{-dx-1} d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\int \frac{1}{-dx-1} d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} \right)}{b} \right)}{4b} - \frac{d(dx)^{5/2}}{2b\left(a+bx^2\right.}}{8b} \right.\right.$$

↓ 217

$$\left(\left(13d^2\left(9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d}{4b(}\right.\right.\right.$$

↓ 1479

$$\left(\vphantom{\frac{1}{1}}\right.\left(\vphantom{\frac{1}{1}}\right.\left(\vphantom{\frac{1}{1}}\right. 13d^2 \, \frac{\left(9d^2 \, \frac{\left(5d^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{2ad \left(\frac{d \left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d\left(\frac{\text{arc}\cdots}{\cdots}\right.}{\cdots} \right)}{b} \cdots \right.}{4b} \cdots \right.}{8b} \cdots$$

↓ 25

$$\frac{2d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\cdots\right.}{\sqrt{2}\cdots}\right.}{\cdots}$$

$$5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\cdots\right)}{b}\cdots\right.$$

$$9d^2\left(\frac{\cdots}{4b}\cdots\right.$$

$$13d^2\left(\frac{\cdots}{8b}\cdots\right.$$

↓ 27

$$13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\ldots}{b}}{4b}}{8b}\right.$$

$$d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.$$

↓ 1103

$$\left(\quad \left(\; 13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log \cdots}{\cdots}\right.}{2\sqrt{a}\cdots}\right.}{b\cdots}\right.}{4b\cdots}\right.}{8b\cdots}\right.\right.\right.$$

input
```
Int[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*(d*x)^(21/2))/(b*(a + b*x^2)^5) + (21*d^2*(-1/8*(d*(d*x)^(17/2))/
(b*(a + b*x^2)^4) + (17*d^2*(-1/6*(d*(d*x)^(13/2))/(b*(a + b*x^2)^3) + (13
*d^2*(-1/4*(d*(d*x)^(9/2))/(b*(a + b*x^2)^2) + (9*d^2*(-1/2*(d*(d*x)^(5/2)
)/(b*(a + b*x^2)) + (5*d^2*((2*d*Sqrt[d*x])/b - (2*a*d*((d*(-(ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b
]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]
*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/b))/(4*b))
)/(8*b)))/(12*b)))/(16*b)))/(20*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 14.77 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{d^{11}\left(\left(-327680x^{10}b^5-2108120a\,x^8b^4-4721920a^2x^6b^3-5074160a^3x^4b^2-2673216x^2a^4b-556920a^5\right)\sqrt{dx}+69615\left(\right.\right.}{1638}$
derivativedivides	$2d^{11}\left(\frac{\sqrt{dx}}{b^6}-\frac{a\,d^2\left(\frac{-\frac{5731a^4d^8\sqrt{dx}}{8192}-\frac{16169a^3d^6b(dx)^{\frac{5}{2}}}{5120}-\frac{22467a^2d^4b^2(dx)^{\frac{9}{2}}}{4096}-\frac{1129a\,d^2b^3(dx)^{\frac{13}{2}}}{256}-\frac{11743b^4(dx)^{\frac{17}{2}}}{8192}}{\left(b\,d^2x^2+a\,d^2\right)^5}+\frac{139}{}\right.}{}\right.$
default	$2d^{11}\left(\frac{\sqrt{dx}}{b^6}-\frac{a\,d^2\left(\frac{-\frac{5731a^4d^8\sqrt{dx}}{8192}-\frac{16169a^3d^6b(dx)^{\frac{5}{2}}}{5120}-\frac{22467a^2d^4b^2(dx)^{\frac{9}{2}}}{4096}-\frac{1129a\,d^2b^3(dx)^{\frac{13}{2}}}{256}-\frac{11743b^4(dx)^{\frac{17}{2}}}{8192}}{\left(b\,d^2x^2+a\,d^2\right)^5}+\frac{139}{}\right.}{}\right.$
risch	$\frac{2x\,d^{12}}{b^6\sqrt{dx}}-\frac{2a\,d^{13}\left(\frac{-\frac{5731a^4d^8\sqrt{dx}}{8192}-\frac{16169a^3d^6b(dx)^{\frac{5}{2}}}{5120}-\frac{22467a^2d^4b^2(dx)^{\frac{9}{2}}}{4096}-\frac{1129a\,d^2b^3(dx)^{\frac{13}{2}}}{256}-\frac{11743b^4(dx)^{\frac{17}{2}}}{8192}}{\left(b\,d^2x^2+a\,d^2\right)^5}+\frac{13923\left(\right.}{}\right.}{b^6}$

input
```
int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/163840*d^11*((-327680*b^5*x^10-2108120*a*b^4*x^8-4721920*a^2*b^3*x^6-50
74160*a^3*b^2*x^4-2673216*a^4*b*x^2-556920*a^5)*(d*x)^(1/2)+69615*(a*d^2/b
)^(1/4)*2^(1/2)*(b*x^2+a)^5*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))
+2*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))+2*arctan(
(2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))))/b^6/(b*x^2+a)^5
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.65

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx =$$

$$-\frac{69615\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}}(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)\log\left(13923\,\sqrt{dx}d^{11}+1392\right.}{}$$

input
```
integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/81920*(69615*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^
9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*log(13923*sqrt(d*x)*d^11
 + 13923*(-a*d^46/b^25)^(1/4)*b^6) + 69615*(-a*d^46/b^25)^(1/4)*(I*b^11*x^
10 + 5*I*a*b^10*x^8 + 10*I*a^2*b^9*x^6 + 10*I*a^3*b^8*x^4 + 5*I*a^4*b^7*x^
2 + I*a^5*b^6)*log(13923*sqrt(d*x)*d^11 + 13923*I*(-a*d^46/b^25)^(1/4)*b^6
) + 69615*(-a*d^46/b^25)^(1/4)*(-I*b^11*x^10 - 5*I*a*b^10*x^8 - 10*I*a^2*b
^9*x^6 - 10*I*a^3*b^8*x^4 - 5*I*a^4*b^7*x^2 - I*a^5*b^6)*log(13923*sqrt(d*
x)*d^11 - 13923*I*(-a*d^46/b^25)^(1/4)*b^6) - 69615*(-a*d^46/b^25)^(1/4)*(
b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2
 + a^5*b^6)*log(13923*sqrt(d*x)*d^11 - 13923*(-a*d^46/b^25)^(1/4)*b^6) - 4
*(40960*b^5*d^11*x^10 + 263515*a*b^4*d^11*x^8 + 590240*a^2*b^3*d^11*x^6 +
634270*a^3*b^2*d^11*x^4 + 334152*a^4*b*d^11*x^2 + 69615*a^5*d^11)*sqrt(d*x
))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7
*x^2 + a^5*b^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.24

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{327680\sqrt{dx}d^{12}}{b^6} + \frac{8\left(58715\,(dx)^{\frac{17}{2}}ab^4d^{14}+180640\,(dx)^{\frac{13}{2}}a^2b^3d^{16}+224670\,(dx)^{\frac{9}{2}}a^3b^2d^{18}+129352\,(dx)\right.}{b^{11}d^{10}x^{10}+5\,ab^{10}d^{10}x^8+10\,a^2b^9d^{10}x^6+10\,a^3b^8d^{10}x^4+5\,a^4b^7d^{10}x}}{}$$

input
```
integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/163840*(327680*sqrt(d*x)*d^12/b^6 + 8*(58715*(d*x)^(17/2)*a*b^4*d^14 + 1
80640*(d*x)^(13/2)*a^2*b^3*d^16 + 224670*(d*x)^(9/2)*a^3*b^2*d^18 + 129352
*(d*x)^(5/2)*a^4*b*d^20 + 28655*sqrt(d*x)*a^5*d^22)/(b^11*d^10*x^10 + 5*a*
b^10*d^10*x^8 + 10*a^2*b^9*d^10*x^6 + 10*a^3*b^8*d^10*x^4 + 5*a^4*b^7*d^10
*x^2 + a^5*b^6*d^10) - 69615*(sqrt(2)*d^14*log(sqrt(b)*d*x + sqrt(2)*(a*d^
2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*
d^14*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d
)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^13*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(s
qrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^13*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt
(sqrt(a)*sqrt(b)*d)*sqrt(a)))*a/b^6)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.09

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx =$$

$$-\frac{\frac{139230\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^{12}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}+\frac{139230\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^{12}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}+\frac{69615\sqrt{2}(ab}{}$$

input
```
integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/163840*(139230*sqrt(2)*(a*b^3*d^2)^(1/4)*d^12*arctan(1/2*sqrt(2)*(sqrt(
2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^7 + 139230*sqrt(2)*(a
*b^3*d^2)^(1/4)*d^12*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt
(d*x))/(a*d^2/b)^(1/4))/b^7 + 69615*sqrt(2)*(a*b^3*d^2)^(1/4)*d^12*log(d*x
 + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7 - 69615*sqrt(2)*
(a*b^3*d^2)^(1/4)*d^12*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/b^7 - 327680*sqrt(d*x)*d^12/b^6 - 8*(58715*sqrt(d*x)*a*b^4*d^22*
x^8 + 180640*sqrt(d*x)*a^2*b^3*d^22*x^6 + 224670*sqrt(d*x)*a^3*b^2*d^22*x^
4 + 129352*sqrt(d*x)*a^4*b*d^22*x^2 + 28655*sqrt(d*x)*a^5*d^22)/((b*d^2*x^
2 + a*d^2)^5*b^6))/d
```

Mupad [B] (verification not implemented)

Time = 18.49 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.71

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{5731\,a^5\,d^{21}\,\sqrt{d\,x}}{4096}+\frac{22467\,a^3\,b^2\,d^{17}\,(d\,x)^{9/2}}{2048}+\frac{1129\,a^2\,b^3\,d^{15}\,(d\,x)^{13/2}}{128}+\frac{16169\,a^4\,b\,d^{19}\,(d\,x)^{5/2}}{2560}+}{a^5\,b^6\,d^{10}+5\,a^4\,b^7\,d^{10}\,x^2+10\,a^3\,b^8\,d^{10}\,x^4+10\,a^2\,b^9\,d^{10}\,x^6+5\,a\,b^{10}\,d^{10}\,x^8}$$

$$+\frac{2\,d^{11}\,\sqrt{d\,x}}{b^6}-\frac{13923\,(-a)^{1/4}\,d^{23/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,b^{25/4}}$$

$$+\frac{(-a)^{1/4}\,d^{23/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}\,1\mathrm{i}}{(-a)^{1/4}\,\sqrt{d}}\right)\,13923\mathrm{i}}{8192\,b^{25/4}}$$

input
```
int((d*x)^(23/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((5731*a^5*d^21*(d*x)^(1/2))/4096 + (22467*a^3*b^2*d^17*(d*x)^(9/2))/2048
+ (1129*a^2*b^3*d^15*(d*x)^(13/2))/128 + (16169*a^4*b*d^19*(d*x)^(5/2))/25
60 + (11743*a*b^4*d^13*(d*x)^(17/2))/4096)/(a^5*b^6*d^10 + b^11*d^10*x^10
+ 5*a*b^10*d^10*x^8 + 5*a^4*b^7*d^10*x^2 + 10*a^3*b^8*d^10*x^4 + 10*a^2*b^
9*d^10*x^6) + (2*d^11*(d*x)^(1/2))/b^6 - (13923*(-a)^(1/4)*d^(23/2)*atan((
b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*b^(25/4)) + ((-a)^(1/4)*
d^(23/2)*atan((b^(1/4)*(d*x)^(1/2)*1i)/((-a)^(1/4)*d^(1/2)))*13923i)/(8192
*b^(25/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1002, normalized size of antiderivative = 3.08

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*d**11*(139230*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 + 696150*b**
(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 1392300*b**(3/4)*a**(1/4)*sqr
t(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/
4)*sqrt(2)))*a**3*b**2*x**4 + 1392300*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(
1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*
*2*b**3*x**6 + 696150*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 + 1392
30*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*s
qrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 - 139230*b**(3/4)*a**(1/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*a**5 - 696150*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x*
*2 - 1392300*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 - 1392300*b*
*(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b
))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*x**6 - 696150*b**(3/4)*a**(1/4)*
sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**
(1/4)*sqrt(2)))*a*b**4*x**8 - 139230*b**(3/4)*a**(1/4)*sqrt(2)*atan((b*...
```

3.473 $\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 308

$$\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = -\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5} - \frac{19d^3(dx)^{15/2}}{160b^2\left(a+bx^2\right)^4} - \frac{19d^5(dx)^{11/2}}{128b^3\left(a+bx^2\right)^3} - \frac{209d^7(dx)^{7/2}}{1024b^4\left(a+bx^2\right)^2} - \frac{1463d^9(dx)^{3/2}}{4096b^5\left(a+bx^2\right)} - \frac{4389d^{21/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\sqrt[4]{a}b^{23/4}} + \frac{4389d^{21/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\sqrt[4]{a}b^{23/4}} - \frac{4389d^{21/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}\sqrt[4]{a}b^{23/4}}$$

output
```
-1/10*d*(d*x)^(19/2)/b/(b*x^2+a)^5-19/160*d^3*(d*x)^(15/2)/b^2/(b*x^2+a)^4
-19/128*d^5*(d*x)^(11/2)/b^3/(b*x^2+a)^3-209/1024*d^7*(d*x)^(7/2)/b^4/(b*x
^2+a)^2-1463/4096*d^9*(d*x)^(3/2)/b^5/(b*x^2+a)-4389/16384*d^(21/2)*arctan
(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(1/4)/b^(23/4)+4
389/16384*d^(21/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2
^(1/2)/a^(1/4)/b^(23/4)-4389/16384*d^(21/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4
)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(1/4)/b^(23/4)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.67

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{d^9(dx)^{3/2}\left(-4\sqrt[4]{a}b^{3/4}x^{3/2}(7315a^4+33440a^3bx^2+59470a^2b^2x^4+50312ab^3x\right.}{}$$

input
```
Integrate[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(d^9*(d*x)^(3/2)*(-4*a^(1/4)*b^(3/4)*x^(3/2)*(7315*a^4 + 33440*a^3*b*x^2 +
 59470*a^2*b^2*x^4 + 50312*a*b^3*x^6 + 19015*b^4*x^8) + 21945*Sqrt[2]*(a +
 b*x^2)^5*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])]
 - 21945*Sqrt[2]*(a + b*x^2)^5*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(
Sqrt[a] + Sqrt[b]*x)]))/(81920*a^(1/4)*b^(23/4)*x^(3/2)*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.43, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 252, 252, 252, 252, 252, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{(dx)^{21/2}}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{21/2}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{19d^2\int\frac{(dx)^{17/2}}{(bx^2+a)^5}dx}{20b}-\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{19d^2\left(\frac{15d^2\int\frac{(dx)^{13/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{19d^2\left(\frac{15d^2\left(\frac{11d^2\int\frac{(dx)^{9/2}}{\left(bx^2+a\right)^3}dx}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{19d^2\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\int\frac{(dx)^{5/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{19d^2\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\int\frac{\sqrt{dx}}{bx^2+a}dx}{4b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \; 266$$

$$\frac{19d^2\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d\int\frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{19/2}}{10b\,(a+bx^2)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{19d^2\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{826}$$

$$\frac{19d^2\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{19/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 1476

$$\left(\frac{19d^2\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)}{4b\left(a+b\right.}\right.}{12b}\right.}{16b}\right.$$

↓ 1082

$$\left(\frac{19d^2\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\right.}{16b}\right.$$

↓ 217

$$19d^2 \frac{\left(15d^2 \frac{\left(11d^2 \frac{\left(7d^2 \frac{\left(3d^3 \left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{11}}{6b\left(a+bx\right.}\right.}{16b}$$

↓ 1479

$$15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\cdots\right.}{\sqrt[4]{b}\left(xd+\cdots\right.}}{2\sqrt{\cdots}}}{2\sqrt{b}}\cdots\right)}{2b}\cdots\right)}{8b}\cdots\right)}{12b}\cdots\right)$$

$$\Big\downarrow \text{25}$$

$$\Bigg(15d^2\Bigg(\frac{11d^2\Bigg(\frac{7d^2\Bigg(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\cdots\right.}}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right.}{2b}\cdots}{8b}\cdots}{12b}\cdots$$

↓ 27

$$15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}}{2b}}{8b}}{12b}$$

↓ 1103

$$19d^2\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}}{2\sqrt{b}}\right.\cdots}{2b}\cdots}{8b}\cdots}{12b}\cdots}{16b}\cdots\right.$$

input
```
Int[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*(d*x)^(19/2))/(b*(a + b*x^2)^5) + (19*d^2*(-1/8*(d*(d*x)^(15/2))/
(b*(a + b*x^2)^4) + (15*d^2*(-1/6*(d*(d*x)^(11/2))/(b*(a + b*x^2)^3) + (11
*d^2*(-1/4*(d*(d*x)^(7/2))/(b*(a + b*x^2)^2) + (7*d^2*(-1/2*(d*(d*x)^(3/2)
)/(b*(a + b*x^2)) + (3*d^3*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(
1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*
Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b
]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*b)))/(8*b)))/(12*b)))/(16*b)))/(20*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 14.74 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.76

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{1463 d^{8} a^{4} (dx)^{\frac{3}{2}}}{8192 b^{5}}-\frac{209 d^{6} a^{3} (dx)^{\frac{7}{2}}}{256 b^{4}}-\frac{5947 a^{2} d^{4} (dx)^{\frac{11}{2}}}{4096 b^{3}}-\frac{6289 a\, d^{2} (dx)^{\frac{15}{2}}}{5120 b^{2}}-\frac{3803 (dx)^{\frac{19}{2}}}{8192 b}}{(b\, d^{2} x^{2}+a\, d^{2})^{5}}+\frac{4389\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\, d^{2}}{b}\right.}{dx+\left(\frac{a\, d^{2}}{b}\right.}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{-\frac{1463 d^{8} a^{4} (dx)^{\frac{3}{2}}}{8192 b^{5}}-\frac{209 d^{6} a^{3} (dx)^{\frac{7}{2}}}{256 b^{4}}-\frac{5947 a^{2} d^{4} (dx)^{\frac{11}{2}}}{4096 b^{3}}-\frac{6289 a\, d^{2} (dx)^{\frac{15}{2}}}{5120 b^{2}}-\frac{3803 (dx)^{\frac{19}{2}}}{8192 b}}{(b\, d^{2} x^{2}+a\, d^{2})^{5}}+\frac{4389\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\, d^{2}}{b}\right.}{dx+\left(\frac{a\, d^{2}}{b}\right.}\right.\right.}{}\right.$
pseudoelliptic	$\frac{1463 d^{10}\left(-8\sqrt{dx}\, b x\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}}\left(\frac{3803}{1463} b^{4} x^{8}+\frac{2648}{385} a\, b^{3} x^{6}+\frac{626}{77} a^{2} b^{2} x^{4}+\frac{32}{7} a^{3} b\, x^{2}+a^{4}\right)+3 d\sqrt{2}\,(b\, x^{2}+a)^{5}\left(\ln\left(\frac{dx-\left(\frac{a\, d^{2}}{b}\right)}{dx+\left(\frac{a\, d^{2}}{b}\right)}\right.\right.\right.}{32768\left(\frac{a\, d^{2}}{b}\right)^{\frac{1}{4}} b^{6} (b\, x^{2}+a)^{5}}$

input
```
int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^11*((-1463/8192/b^5*d^8*a^4*(d*x)^(3/2)-209/256/b^4*d^6*a^3*(d*x)^(7/2
)-5947/4096*a^2*d^4/b^3*(d*x)^(11/2)-6289/5120*a*d^2/b^2*(d*x)^(15/2)-3803
/8192/b*(d*x)^(19/2))/(b*d^2*x^2+a*d^2)^5+4389/65536/b^6/(a*d^2/b)^(1/4)*2
^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*
d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1
)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.77

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{21945\,(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5)\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{1}{4}}}{}$$

input
```
integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/81920*(21945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4
+ 5*a^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4)*log(84546715869*sqrt(d*x
)*d^31 + 84546715869*(-d^42/(a*b^23))^(3/4)*a*b^17) - 21945*(I*b^10*x^10 +
 5*I*a*b^9*x^8 + 10*I*a^2*b^8*x^6 + 10*I*a^3*b^7*x^4 + 5*I*a^4*b^6*x^2 + I
*a^5*b^5)*(-d^42/(a*b^23))^(1/4)*log(84546715869*sqrt(d*x)*d^31 + 84546715
869*I*(-d^42/(a*b^23))^(3/4)*a*b^17) - 21945*(-I*b^10*x^10 - 5*I*a*b^9*x^8
 - 10*I*a^2*b^8*x^6 - 10*I*a^3*b^7*x^4 - 5*I*a^4*b^6*x^2 - I*a^5*b^5)*(-d^
42/(a*b^23))^(1/4)*log(84546715869*sqrt(d*x)*d^31 - 84546715869*I*(-d^42/(
a*b^23))^(3/4)*a*b^17) - 21945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 +
 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4)*log(8454
6715869*sqrt(d*x)*d^31 - 84546715869*(-d^42/(a*b^23))^(3/4)*a*b^17) - 4*(1
9015*b^4*d^10*x^9 + 50312*a*b^3*d^10*x^7 + 59470*a^2*b^2*d^10*x^5 + 33440*
a^3*b*d^10*x^3 + 7315*a^4*d^10*x)*sqrt(d*x))/(b^10*x^10 + 5*a*b^9*x^8 + 10
*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.22

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{21945\,d^{12}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac14}b^{\frac14}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac14}b^{\frac14}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\cdots\right.}{b^5}$$

input
```
integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/163840*(21945*d^12*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*
b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt
(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(
1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)
*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b
^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sq
rt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4))
)/b^5 - 8*(19015*(d*x)^(19/2)*b^4*d^12 + 50312*(d*x)^(15/2)*a*b^3*d^14 + 5
9470*(d*x)^(11/2)*a^2*b^2*d^16 + 33440*(d*x)^(7/2)*a^3*b*d^18 + 7315*(d*x)
^(3/2)*a^4*d^20)/(b^10*d^10*x^10 + 5*a*b^9*d^10*x^8 + 10*a^2*b^8*d^10*x^6
+ 10*a^3*b^7*d^10*x^4 + 5*a^4*b^6*d^10*x^2 + a^5*b^5*d^10))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.14

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{1}{163840}\,d^{10}\left(\frac{43890\sqrt{2}(ab^3d^2)^{\frac34}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac14}}\right)}{ab^8d}+\frac{43890\sqrt{2}(\cdots}{\cdots}\right.$$

input
```
integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/163840*d^10*(43890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^8*d) + 43890*sqrt(2)
*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d
*x))/(a*d^2/b)^(1/4))/(a*b^8*d) - 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x
+ sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^8*d) + 21945*sqr
t(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a*b^8*d) - 8*(19015*sqrt(d*x)*b^4*d^10*x^9 + 50312*sqrt(d*x)*a*
b^3*d^10*x^7 + 59470*sqrt(d*x)*a^2*b^2*d^10*x^5 + 33440*sqrt(d*x)*a^3*b*d^
10*x^3 + 7315*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*b^5))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.69

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{4389\,d^{21/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{1/4}\,b^{23/4}} - \frac{\frac{3803\,d^{11}\,(d\,x)^{19/2}}{4096\,b}+\frac{5947\,a^2\,d^{15}\,(d\,x)^{11/2}}{2048\,b^3}+\frac{209\,a^3\,d^{17}\,(d\,x)^{7/2}}{128\,b^4}+\frac{1463\,a^4\,d^{19}\,(d\,x)^{3/2}}{4096\,b^5}+\frac{6289\,a\,d^{13}\,(d\,x)^{15/2}}{2560\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}} - \frac{4389\,d^{21/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{1/4}\,b^{23/4}}$$

input
```
int((d*x)^(21/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(4389*d^(21/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a
)^(1/4)*b^(23/4)) - ((3803*d^11*(d*x)^(19/2))/(4096*b) + (5947*a^2*d^15*(d
*x)^(11/2))/(2048*b^3) + (209*a^3*d^17*(d*x)^(7/2))/(128*b^4) + (1463*a^4*
d^19*(d*x)^(3/2))/(4096*b^5) + (6289*a*d^13*(d*x)^(15/2))/(2560*b^2))/(a^5
*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d
^10*x^4 + 10*a^2*b^3*d^10*x^6) - (4389*d^(21/2)*atanh((b^(1/4)*(d*x)^(1/2)
)/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(1/4)*b^(23/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 996, normalized size of antiderivative = 3.23

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Too large to display}$$

input

```
int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output

```
(sqrt(d)*d**10*( - 43890*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 219450*b
**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(
b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 438900*b**(1/4)*a**(3/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1
/4)*sqrt(2)))*a**3*b**2*x**4 - 438900*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(
1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*
*2*b**3*x**6 - 219450*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 4389
0*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sq
rt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 43890*b**(1/4)*a**(3/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*a**5 + 219450*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2
 + 438900*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sq
rt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 438900*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*x**6 + 219450*b**(1/4)*a**(3/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4
)*sqrt(2)))*a*b**4*x**8 + 43890*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4...
```

3.474 $\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 308

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{d(dx)^{17/2}}{10b(a+bx^2)^5} - \frac{17d^3(dx)^{13/2}}{160b^2(a+bx^2)^4} - \frac{221d^5(dx)^{9/2}}{1920b^3(a+bx^2)^3} - \frac{663d^7(dx)^{5/2}}{5120b^4(a+bx^2)^2} - \frac{663d^9\sqrt{dx}}{4096b^5(a+bx^2)} - \frac{663d^{19/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{3/4}b^{21/4}} + \frac{663d^{19/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{3/4}b^{21/4}} + \frac{663d^{19/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}a^{3/4}b^{21/4}}$$

output
```
-1/10*d*(d*x)^(17/2)/b/(b*x^2+a)^5-17/160*d^3*(d*x)^(13/2)/b^2/(b*x^2+a)^4
-221/1920*d^5*(d*x)^(9/2)/b^3/(b*x^2+a)^3-663/5120*d^7*(d*x)^(5/2)/b^4/(b*
x^2+a)^2-663/4096*d^9*(d*x)^(1/2)/b^5/(b*x^2+a)-663/16384*d^(19/2)*arctan(
1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(3/4)/b^(21/4)+66
3/16384*d^(19/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(
1/2)/a^(3/4)/b^(21/4)+663/16384*d^(19/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(
d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(3/4)/b^(21/4)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.67

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{d^9\sqrt{dx}\Big(-4a^{3/4}\sqrt[4]{b}\sqrt{x}(9945a^4+47736a^3bx^2+90610a^2b^2x^4+84320ab^3x^6+}{24}$$

input
```
Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(d^9*Sqrt[d*x]*(-4*a^(3/4)*b^(1/4)*Sqrt[x]*(9945*a^4 + 47736*a^3*b*x^2 + 9
0610*a^2*b^2*x^4 + 84320*a*b^3*x^6 + 37645*b^4*x^8) + 9945*Sqrt[2]*(a + b*
x^2)^5*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] +
9945*Sqrt[2]*(a + b*x^2)^5*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt
[a] + Sqrt[b]*x)]))/(245760*a^(3/4)*b^(21/4)*Sqrt[x]*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.43, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 252, 252, 252, 252, 252, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{(dx)^{19/2}}{b^6\,(bx^2+a)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{19/2}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{17d^2\int \frac{(dx)^{15/2}}{(bx^2+a)^5}dx}{20b}-\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{17d^2\left(\frac{13d^2\int \frac{(dx)^{11/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{13/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{17d^2\left(\frac{13d^2\left(\frac{3d^2\int \frac{(dx)^{7/2}}{\left(bx^2+a\right)^3}dx}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{13/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{17d^2\left(\frac{13d^2\left(\frac{3d^2\left(\frac{5d^2\int \frac{(dx)^{3/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{13/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{252}$$

$$\frac{17d^2\left(\frac{13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d^2\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{13/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 266

$$\frac{17d^2\left(\frac{13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\int\frac{1}{bx^2+a}d\sqrt{dx}}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{13/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 755

$$\frac{17d^2\left(\frac{13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{13/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{17d^2\left(\frac{13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{13/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{17/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 1476$$

$$13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(a}{4b\left(a\right.}}{4b}\right.$$

↓ 1082

$$\left(13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}\right.\right.$$

↓ 217

$$13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx}{6b\left(a+\right.}\right.$$

↓ 1479

$$13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{}}{2b}\right.}{8b}\right.}{4b}\right.$$

↓ 25

$$\left(13d^2 \frac{\left(3d^2 \frac{\left(5d^2 \frac{\left(d \frac{\left(d \left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} + \frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d \left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \cdots \right.}{2\sqrt{\cdots}} \cdots \right.}{2b} \cdots \right.}{8b} \cdots \right.}{4b} \cdots \right.$$

$\downarrow$ 27

$$\left(13d^2 \frac{\left(3d^2 \frac{\left(5d^2 \frac{\left(d \frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}} + \frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}} \cdots \right.}{2\sqrt{a}} \cdots \right.}{2b} \cdots \right.}{8b} \cdots \right.}{4b} \cdots \right.$$

↓ 1103

$$\Bigg(13d^2 \Bigg(\frac{3d^2 \left(\frac{5d^2 \left(\frac{d \left(\frac{d \left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d \left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots\right)}{2\sqrt{2}\sqrt[4]{\cdots}} \cdots \right)}{2\sqrt{a}} \cdots \right)}{2b} \cdots \right)}{8b} \cdots \right)}{4b} \cdots$$

input
```
Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*(d*x)^(17/2))/(b*(a + b*x^2)^5) + (17*d^2*(-1/8*(d*(d*x)^(13/2))/
(b*(a + b*x^2)^4) + (13*d^2*(-1/6*(d*(d*x)^(9/2))/(b*(a + b*x^2)^3) + (3*d
^2*(-1/4*(d*(d*x)^(5/2))/(b*(a + b*x^2)^2) + (5*d^2*(-1/2*(d*Sqrt[d*x])/(b
*(a + b*x^2)) + (d*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*
Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt
[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]
*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d])))/(2*Sqrt[a])))/(2*b)))/(8*b)))/(4*b)))/(16*b)))/(20*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 14.63 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\left(\left(-301160a\,x^8b^4-674560a^2x^6b^3-724880a^3x^4b^2-381888x^2a^4b-79560a^5\right)\sqrt{dx}+9945\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,(b\,x^2+a)^5\left(\ln\left(\right.\right.\right.}{491520b^5(b\,x^2+a)^5}$
derivativedivides	$2d^{11}\left(\frac{-\frac{663a^4d^8\sqrt{dx}}{8192b^5}-\frac{1989a^3d^6(dx)^{\frac{5}{2}}}{5120b^4}-\frac{9061a^2d^4(dx)^{\frac{9}{2}}}{12288b^3}-\frac{527a\,d^2(dx)^{\frac{13}{2}}}{768b^2}-\frac{7529(dx)^{\frac{17}{2}}}{24576b}}{(b\,d^2x^2+a\,d^2)^5}+\frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+(}{dx-(}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{-\frac{663a^4d^8\sqrt{dx}}{8192b^5}-\frac{1989a^3d^6(dx)^{\frac{5}{2}}}{5120b^4}-\frac{9061a^2d^4(dx)^{\frac{9}{2}}}{12288b^3}-\frac{527a\,d^2(dx)^{\frac{13}{2}}}{768b^2}-\frac{7529(dx)^{\frac{17}{2}}}{24576b}}{(b\,d^2x^2+a\,d^2)^5}+\frac{663\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+(}{dx-(}\right.\right.}{}\right.$

input
```
int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/491520*((-301160*a*b^4*x^8-674560*a^2*b^3*x^6-724880*a^3*b^2*x^4-381888*
a^4*b*x^2-79560*a^5)*(d*x)^(1/2)+9945*(a*d^2/b)^(1/4)*2^(1/2)*(b*x^2+a)^5*
(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan((2^(1/2)*(d*x)^(1/
2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))))*d^9/b^5/(b*x^2+a)^5/a
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.76

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{9945\left(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5\right)\left(-\frac{d^{38}}{a^3b^{21}}\right)^{\frac{1}{4}}}{}$$

input
```
integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/245760*(9945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4
+ 5*a^4*b^6*x^2 + a^5*b^5)*(-d^38/(a^3*b^21))^(1/4)*log(663*sqrt(d*x)*d^9
+ 663*(-d^38/(a^3*b^21))^(1/4)*a*b^5) - 9945*(-I*b^10*x^10 - 5*I*a*b^9*x^8
 - 10*I*a^2*b^8*x^6 - 10*I*a^3*b^7*x^4 - 5*I*a^4*b^6*x^2 - I*a^5*b^5)*(-d^
38/(a^3*b^21))^(1/4)*log(663*sqrt(d*x)*d^9 + 663*I*(-d^38/(a^3*b^21))^(1/4
)*a*b^5) - 9945*(I*b^10*x^10 + 5*I*a*b^9*x^8 + 10*I*a^2*b^8*x^6 + 10*I*a^3
*b^7*x^4 + 5*I*a^4*b^6*x^2 + I*a^5*b^5)*(-d^38/(a^3*b^21))^(1/4)*log(663*s
qrt(d*x)*d^9 - 663*I*(-d^38/(a^3*b^21))^(1/4)*a*b^5) - 9945*(b^10*x^10 + 5
*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-
d^38/(a^3*b^21))^(1/4)*log(663*sqrt(d*x)*d^9 - 663*(-d^38/(a^3*b^21))^(1/4
)*a*b^5) - 4*(37645*b^4*d^9*x^8 + 84320*a*b^3*d^9*x^6 + 90610*a^2*b^2*d^9*
x^4 + 47736*a^3*b*d^9*x^2 + 9945*a^4*d^9)*sqrt(d*x))/(b^10*x^10 + 5*a*b^9*
x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx=\text{Timed out}$$

input
```
integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.25

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx =$$

$$-\frac{8\left(37645\,(dx)^{\frac{17}{2}}b^4d^{12}+84320\,(dx)^{\frac{13}{2}}ab^3d^{14}+90610\,(dx)^{\frac{9}{2}}a^2b^2d^{16}+47736\,(dx)^{\frac{5}{2}}a^3bd^{18}+9945\,\sqrt{dx}a^4d^{20}\right)}{b^{10}d^{10}x^{10}+5\,ab^9d^{10}x^8+10\,a^2b^8d^{10}x^6+10\,a^3b^7d^{10}x^4+5\,a^4b^6d^{10}x^2+a^5b^5d^{10}} - \frac{9945\left(\frac{\sqrt{2}d^{12}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)\right.}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{}$$

input
```
integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/491520*(8*(37645*(d*x)^(17/2)*b^4*d^12 + 84320*(d*x)^(13/2)*a*b^3*d^14
+ 90610*(d*x)^(9/2)*a^2*b^2*d^16 + 47736*(d*x)^(5/2)*a^3*b*d^18 + 9945*sqr
t(d*x)*a^4*d^20)/(b^10*d^10*x^10 + 5*a*b^9*d^10*x^8 + 10*a^2*b^8*d^10*x^6
+ 10*a^3*b^7*d^10*x^4 + 5*a^4*b^6*d^10*x^2 + a^5*b^5*d^10) - 9945*(sqrt(2)
*d^12*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*
d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^12*log(sqrt(b)*d*x - sqrt(2)*(a*d^2
)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)
*d^11*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt
(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2
)*d^11*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sq
rt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/b^5)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.14

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{19890\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^{10}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6} + \frac{19890\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^{10}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{2}\right.}{ab^6}}{}$$

input
```
integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/491520*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*d^10*arctan(1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6) + 19890*sqrt(2)*(
a*b^3*d^2)^(1/4)*d^10*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqr
t(d*x))/(a*d^2/b)^(1/4))/(a*b^6) + 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*d^10*log
(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6) - 9945*s
qrt(2)*(a*b^3*d^2)^(1/4)*d^10*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
+ sqrt(a*d^2/b))/(a*b^6) - 8*(37645*sqrt(d*x)*b^4*d^20*x^8 + 84320*sqrt(d*
x)*a*b^3*d^20*x^6 + 90610*sqrt(d*x)*a^2*b^2*d^20*x^4 + 47736*sqrt(d*x)*a^3
*b*d^20*x^2 + 9945*sqrt(d*x)*a^4*d^20)/((b*d^2*x^2 + a*d^2)^5*b^5))/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.69

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{\frac{7529\,d^{11}\,(d\,x)^{17/2}}{12288\,b}+\frac{9061\,a^2\,d^{15}\,(d\,x)^{9/2}}{6144\,b^3}+\frac{1989\,a^3\,d^{17}\,(d\,x)^{5/2}}{2560\,b^4}+\frac{663\,a^4\,d^{19}\,\sqrt{d\,x}}{4096\,b^5}+\frac{527\,a\,d^{13}\,(d\,x)^{13/2}}{384\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}} - \frac{663\,d^{19/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{3/4}\,b^{21/4}} - \frac{663\,d^{19/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{3/4}\,b^{21/4}}$$

input
```
int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
- ((7529*d^11*(d*x)^(17/2))/(12288*b) + (9061*a^2*d^15*(d*x)^(9/2))/(6144*
b^3) + (1989*a^3*d^17*(d*x)^(5/2))/(2560*b^4) + (663*a^4*d^19*(d*x)^(1/2))
/(4096*b^5) + (527*a*d^13*(d*x)^(13/2))/(384*b^2))/(a^5*d^10 + b^5*d^10*x^
10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^
3*d^10*x^6) - (663*d^(19/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2)
)))/(8192*(-a)^(3/4)*b^(21/4)) - (663*d^(19/2)*atanh((b^(1/4)*(d*x)^(1/2))
/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(3/4)*b^(21/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 995, normalized size of antiderivative = 3.23

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*d**9*( - 19890*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 99450*b**
(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 198900*b**(3/4)*a**(1/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4
)*sqrt(2)))*a**3*b**2*x**4 - 198900*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/
4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2
*b**3*x**6 - 99450*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 19890*b
**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(
b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 19890*b**(3/4)*a**(1/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4
)*sqrt(2)))*a**5 + 99450*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 1
98900*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x
)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 198900*b**(3/4)*a
**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*x**6 + 99450*b**(3/4)*a**(1/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a*b**4*x**8 + 19890*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**...
```

3.475 $\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 311

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{d(dx)^{15/2}}{10b(a+bx^2)^5} - \frac{3d^3(dx)^{11/2}}{32b^2(a+bx^2)^4} - \frac{11d^5(dx)^{7/2}}{128b^3(a+bx^2)^3} - \frac{77d^7(dx)^{3/2}}{1024b^4(a+bx^2)^2} + \frac{231d^7(dx)^{3/2}}{4096ab^4(a+bx^2)} - \frac{231d^{17/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{5/4}b^{19/4}} + \frac{231d^{17/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{5/4}b^{19/4}} - \frac{231d^{17/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}a^{5/4}b^{19/4}}$$

output
```
-1/10*d*(d*x)^(15/2)/b/(b*x^2+a)^5-3/32*d^3*(d*x)^(11/2)/b^2/(b*x^2+a)^4-1
1/128*d^5*(d*x)^(7/2)/b^3/(b*x^2+a)^3-77/1024*d^7*(d*x)^(3/2)/b^4/(b*x^2+a
)^2+231/4096*d^7*(d*x)^(3/2)/a/b^4/(b*x^2+a)-231/16384*d^(17/2)*arctan(1-2
^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(5/4)/b^(19/4)+231/1
6384*d^(17/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2
)/a^(5/4)/b^(19/4)-231/16384*d^(17/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x
)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(5/4)/b^(19/4)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.66

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{d^8\sqrt{dx}\Big(4\sqrt[4]{a}b^{3/4}x^{3/2}(-385a^4-1760a^3bx^2-3130a^2b^2x^4-2648ab^3x^6+115}{819}$$

input
```
Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(d^8*Sqrt[d*x]*(4*a^(1/4)*b^(3/4)*x^(3/2)*(-385*a^4 - 1760*a^3*b*x^2 - 313
0*a^2*b^2*x^4 - 2648*a*b^3*x^6 + 1155*b^4*x^8) + 1155*Sqrt[2]*(a + b*x^2)^
5*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 1155*
Sqrt[2]*(a + b*x^2)^5*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
 Sqrt[b]*x)]))/(81920*a^(5/4)*b^(19/4)*Sqrt[x]*(a + b*x^2)^5)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.42, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 252, 252, 252, 252, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \ 1380$$

$$b^6\int \frac{(dx)^{17/2}}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{(dx)^{17/2}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \ 252$$

$$\frac{3d^2\int \frac{(dx)^{13/2}}{(bx^2+a)^5}dx}{4b}-\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 252

$$\frac{3d^2\left(\frac{11d^2\int \frac{(dx)^{9/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 252

$$\frac{3d^2\left(\frac{11d^2\left(\frac{7d^2\int \frac{(dx)^{5/2}}{\left(bx^2+a\right)^3}dx}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 252

$$\frac{3d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 253

$$\frac{3d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{\int \frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{3d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 826

$$\frac{3d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{15/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 1476

$$3d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)}\right)}{12b}\right.\right.}{16b}\right.$$

↓ 1082

$$3d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(\ldots}{6b(\ldots}\ldots\right)}{16b}\ldots$$

↓ 217

$$3d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}$$

↓ 1479

$$\left(11d^2 \left(7d^2 \left(3d^2 \left(\frac{d \left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} - \frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\cdots\right.}}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{b}} \cdots \right)}{2a} \cdots \right)}{8b} \cdots \right)}{12b} \cdots \right. \right.$$

↓ 25

$$11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}}{2\sqrt{b}}\right.}{2a}\right.}{8b}\right.}{12b}\right.$$

$$\Big\downarrow \mathbf{27}$$

$$\left(11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{d}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}}{2a}}{8b}}{12b}$$

$$\Big\downarrow$$ 1103

$$3d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots}{2\sqrt{b}}\cdots\right.}{2a}\cdots\right.}{8b}\cdots\right.}{12b}\cdots\right.}{16b}\cdots\right.$$

input
```
Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*(d*x)^(15/2))/(b*(a + b*x^2)^5) + (3*d^2*(-1/8*(d*(d*x)^(11/2))/(
b*(a + b*x^2)^4) + (11*d^2*(-1/6*(d*(d*x)^(7/2))/(b*(a + b*x^2)^3) + (7*d^
2*(-1/4*(d*(d*x)^(3/2))/(b*(a + b*x^2)^2) + (3*d^2*((d*x)^(3/2)/(2*a*d*(a
+ b*x^2)) + (d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (
-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*
x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])
)/(2*Sqrt[b])))/(2*a)))/(8*b)))/(12*b)))/(16*b)))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 14.70 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.77

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{77a^3d^6(dx)^{\frac{3}{2}}}{8192b^4}-\frac{11a^2d^4(dx)^{\frac{7}{2}}}{256b^3}-\frac{313a\,d^2(dx)^{\frac{11}{2}}}{4096b^2}-\frac{331(dx)^{\frac{15}{2}}}{5120b}+\frac{231(dx)^{\frac{19}{2}}}{8192a\,d^2}}{(b\,d^2x^2+a\,d^2)^5}+\frac{231\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{-\frac{77a^3d^6(dx)^{\frac{3}{2}}}{8192b^4}-\frac{11a^2d^4(dx)^{\frac{7}{2}}}{256b^3}-\frac{313a\,d^2(dx)^{\frac{11}{2}}}{4096b^2}-\frac{331(dx)^{\frac{15}{2}}}{5120b}+\frac{231(dx)^{\frac{19}{2}}}{8192a\,d^2}}{(b\,d^2x^2+a\,d^2)^5}+\frac{231\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-}\right.\right.}{}\right.$
pseudoelliptic	$\frac{77d^8\left(-8\left(-3b^4x^8+\frac{2648}{385}a\,b^3x^6+\frac{626}{77}a^2b^2x^4+\frac{32}{7}a^3b\,x^2+a^4\right)\sqrt{dx}\,bx\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}+3d\sqrt{2}\,(b\,x^2+a)^5\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}}\right.\right.\right.}{32768\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^5(b\,x^2+a)^5a}$

input
```
int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^11*((-77/8192*a^3*d^6/b^4*(d*x)^(3/2)-11/256/b^3*a^2*d^4*(d*x)^(7/2)-3
13/4096/b^2*a*d^2*(d*x)^(11/2)-331/5120/b*(d*x)^(15/2)+231/8192/a/d^2*(d*x
)^(19/2))/(b*d^2*x^2+a*d^2)^5+231/65536/a/d^2/b^5/(a*d^2/b)^(1/4)*2^(1/2)*
(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^
(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.82

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{1155\left(ab^9x^{10}+5\,a^2b^8x^8+10\,a^3b^7x^6+10\,a^4b^6x^4+5\,a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{34}}{a^5b^{19}}\right)}{}$$

input
```
integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/81920*(1155*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^
4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*log(12326391*sqrt(d*
x)*d^25 + 12326391*(-d^34/(a^5*b^19))^(3/4)*a^4*b^14) - 1155*(I*a*b^9*x^10
 + 5*I*a^2*b^8*x^8 + 10*I*a^3*b^7*x^6 + 10*I*a^4*b^6*x^4 + 5*I*a^5*b^5*x^2
 + I*a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*log(12326391*sqrt(d*x)*d^25 + 12326
391*I*(-d^34/(a^5*b^19))^(3/4)*a^4*b^14) - 1155*(-I*a*b^9*x^10 - 5*I*a^2*b
^8*x^8 - 10*I*a^3*b^7*x^6 - 10*I*a^4*b^6*x^4 - 5*I*a^5*b^5*x^2 - I*a^6*b^4
)*(-d^34/(a^5*b^19))^(1/4)*log(12326391*sqrt(d*x)*d^25 - 12326391*I*(-d^34
/(a^5*b^19))^(3/4)*a^4*b^14) - 1155*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b
^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^34/(a^5*b^19))^(1/4
)*log(12326391*sqrt(d*x)*d^25 - 12326391*(-d^34/(a^5*b^19))^(3/4)*a^4*b^14
) + 4*(1155*b^4*d^8*x^9 - 2648*a*b^3*d^8*x^7 - 3130*a^2*b^2*d^8*x^5 - 1760
*a^3*b*d^8*x^3 - 385*a^4*d^8*x)*sqrt(d*x))/(a*b^9*x^10 + 5*a^2*b^8*x^8 + 1
0*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.23

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{1155\,d^{10}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\cdots}{\cdots}\right.}{ab^4 \cdots}$$

input
```
integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/163840*(1155*d^10*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b
^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(
b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1
/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*
d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^
(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqr
t(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))
/(a*b^4) + 8*(1155*(d*x)^(19/2)*b^4*d^10 - 2648*(d*x)^(15/2)*a*b^3*d^12 -
3130*(d*x)^(11/2)*a^2*b^2*d^14 - 1760*(d*x)^(7/2)*a^3*b*d^16 - 385*(d*x)^(
3/2)*a^4*d^18)/(a*b^9*d^10*x^10 + 5*a^2*b^8*d^10*x^8 + 10*a^3*b^7*d^10*x^6
 + 10*a^4*b^6*d^10*x^4 + 5*a^5*b^5*d^10*x^2 + a^6*b^4*d^10))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.14

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{1}{163840}\,d^8\left(\frac{2310\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^7d}+\frac{2310\sqrt{2}(ab^3\cdots}{\cdots}\right.$$

input
```
integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/163840*d^8*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^7*d) + 2310*sqrt(2)*
(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*
x))/(a*d^2/b)^(1/4))/(a^2*b^7*d) - 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x
+ sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^7*d) + 1155*sq
rt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/(a^2*b^7*d) + 8*(1155*sqrt(d*x)*b^4*d^10*x^9 - 2648*sqrt(d*x)*a
*b^3*d^10*x^7 - 3130*sqrt(d*x)*a^2*b^2*d^10*x^5 - 1760*sqrt(d*x)*a^3*b*d^1
0*x^3 - 385*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a*b^4))
```

Mupad [B] (verification not implemented)

Time = 17.73 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.68

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{231\,d^{17/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{5/4}\,b^{19/4}} - \frac{231\,d^{17/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{5/4}\,b^{19/4}} - \frac{\frac{331\,d^{11}\,(d\,x)^{15/2}}{2560\,b}-\frac{231\,d^9\,(d\,x)^{19/2}}{4096\,a}+\frac{11\,a^2\,d^{15}\,(d\,x)^{7/2}}{128\,b^3}+\frac{77\,a^3\,d^{17}\,(d\,x)^{3/2}}{4096\,b^4}+\frac{313\,a\,d^{13}\,(d\,x)^{11/2}}{2048\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}}$$

input
```
int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(231*d^(17/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a
)^(5/4)*b^(19/4)) - (231*d^(17/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d
^(1/2))))/(8192*(-a)^(5/4)*b^(19/4)) - ((331*d^11*(d*x)^(15/2))/(2560*b) -
 (231*d^9*(d*x)^(19/2))/(4096*a) + (11*a^2*d^15*(d*x)^(7/2))/(128*b^3) + (
77*a^3*d^17*(d*x)^(3/2))/(4096*b^4) + (313*a*d^13*(d*x)^(11/2))/(2048*b^2)
)/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3
*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 996, normalized size of antiderivative = 3.20

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*d**8*( - 2310*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 11550*b**(
1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 23100*b**(1/4)*a**(3/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*
sqrt(2)))*a**3*b**2*x**4 - 23100*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*
*3*x**6 - 11550*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
- 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 2310*b**(1
/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/
(b**(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 2310*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**5 + 11550*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 23100*
b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 23100*b**(1/4)*a**(3/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*a**2*b**3*x**6 + 11550*b**(1/4)*a**(3/4)*sqrt(2)*atan((b*
*(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*
a*b**4*x**8 + 2310*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq...
```

3.476 $\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 311

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{d(dx)^{13/2}}{10b(a+bx^2)^5} - \frac{13d^3(dx)^{9/2}}{160b^2(a+bx^2)^4} - \frac{39d^5(dx)^{5/2}}{640b^3(a+bx^2)^3} - \frac{39d^7\sqrt{dx}}{1024b^4(a+bx^2)^2} + \frac{39d^7\sqrt{dx}}{4096ab^4(a+bx^2)} - \frac{117d^{15/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{7/4}b^{17/4}} + \frac{117d^{15/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{7/4}b^{17/4}} + \frac{117d^{15/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}a^{7/4}b^{17/4}}$$

output
```
-1/10*d*(d*x)^(13/2)/b/(b*x^2+a)^5-13/160*d^3*(d*x)^(9/2)/b^2/(b*x^2+a)^4-
39/640*d^5*(d*x)^(5/2)/b^3/(b*x^2+a)^3-39/1024*d^7*(d*x)^(1/2)/b^4/(b*x^2+
a)^2+39/4096*d^7*(d*x)^(1/2)/a/b^4/(b*x^2+a)-117/16384*d^(15/2)*arctan(1-2
^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(7/4)/b^(17/4)+117/1
6384*d^(15/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2
)/a^(7/4)/b^(17/4)+117/16384*d^(15/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x
)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(7/4)/b^(17/4)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.60

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{d^7\sqrt{dx}\left(-\frac{4a^{3/4}\sqrt[4]{b}\left(585a^4+2808a^3bx^2+5330a^2b^2x^4+4960ab^3x^6-195b^4x^8\right)}{(a+bx^2)^5}-\frac{585\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{x}}+\cdots\right)}{81920a^{7/4}b^{17/4}}$$

input
```
Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(d^7*Sqrt[d*x]*((-4*a^(3/4)*b^(1/4)*(585*a^4 + 2808*a^3*b*x^2 + 5330*a^2*b
^2*x^4 + 4960*a*b^3*x^6 - 195*b^4*x^8))/(a + b*x^2)^5 - (585*Sqrt[2]*ArcTa
n[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/Sqrt[x] + (585
*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])
/Sqrt[x]))/(81920*a^(7/4)*b^(17/4))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.43, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 252, 252, 252, 252, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{(dx)^{15/2}}{b^6\left(bx^2+a\right)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{15/2}}{(a+bx^2)^6}dx$$

$\Big\downarrow$ 252

$$\frac{13d^2\int \frac{(dx)^{11/2}}{(bx^2+a)^5}dx}{20b}-\frac{d(dx)^{13/2}}{10b(a+bx^2)^5}$$

$\Big\downarrow$ 252

$$\frac{13d^2\left(\frac{9d^2\int \frac{(dx)^{7/2}}{(bx^2+a)^4}dx}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{13/2}}{10b(a+bx^2)^5}$$

$\Big\downarrow$ 252

$$\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\int \frac{(dx)^{3/2}}{(bx^2+a)^3}dx}{12b}-\frac{d(dx)^{5/2}}{6b(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{13/2}}{10b(a+bx^2)^5}$$

$\Big\downarrow$ 252

$$\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\int \frac{1}{\sqrt{dx}(bx^2+a)^2}dx}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{13/2}}{10b(a+bx^2)^5}$$

$\Big\downarrow$ 253

$$\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5}$$

$\big\downarrow$ 266

$$\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\int\frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5}$$

$\big\downarrow$ 755

$$\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{13/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 1476$$

$$9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{\cdots}}{4b\left(a+\cdots\right.}\cdots}{12b}\cdots\right.$$

↓ 1082

$$\left(9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{12b}\right.\right.$$

↓ 217

$$\left(9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)}{6b\left(a+b\right.}\right.\right.$$

↓ 1479

$$\Bigg(9d^2\Bigg(5d^2\Bigg(d^2\Bigg(3\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1 \cdots\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right.}{\cdots}\cdots\right)}{2ad}\cdots\Bigg)}{8b}\cdots\Bigg)}{12b}\cdots$$

$$\Big\downarrow 25$$

$$9d^2 \left(5d^2 \left(\frac{d^2 \left(\frac{3 \left(\frac{d \left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} + \frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \cdots}{2\sqrt{a}} \cdots}{2ad} \cdots}{8b} \cdots}{12b} \cdots$$

$$\Big\downarrow 27$$

$$9d^2 \left(\frac{5d^2 \left(\frac{d^2 \left(\frac{3 \left(\frac{d \left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}} + \frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a}\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\cdots\right)}{2\sqrt{a}} \cdots \right)}{2ad}\cdots \right)}{8b}\cdots \right)}{12b}\cdots \right.$$

↓ 1103

$$9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\cdots\right.}{2\sqrt{2}\sqrt[4]{a}\cdots}\right.}{2\sqrt{a}}\cdots}{2ad}\cdots\right.}{8b}\cdots\right.}{12b}\cdots\right.$$

input
```
Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*(d*x)^(13/2))/(b*(a + b*x^2)^5) + (13*d^2*(-1/8*(d*(d*x)^(9/2))/(
b*(a + b*x^2)^4) + (9*d^2*(-1/6*(d*(d*x)^(5/2))/(b*(a + b*x^2)^3) + (5*d^2
*(-1/4*(d*Sqrt[d*x])/(b*(a + b*x^2)^2) + (d^2*(Sqrt[d*x]/(2*a*d*(a + b*x^2
)) + (3*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(
-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*
x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])
))/(2*Sqrt[a])))/(2*a*d)))/(8*b)))/(12*b)))/(16*b)))/(20*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 14.64 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.77

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{117a^3d^6\sqrt{dx}}{8192b^4}-\frac{351a^2d^4(dx)^{\frac{5}{2}}}{5120b^3}-\frac{533a\,d^2(dx)^{\frac{9}{2}}}{4096b^2}-\frac{31(dx)^{\frac{13}{2}}}{256b}+\frac{39(dx)^{\frac{17}{2}}}{8192a\,d^2}}{(b\,d^2x^2+a\,d^2)^5}+\frac{117\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{\ldots}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{\ldots}}\right.\right.\ldots}{\ldots}\right.$
default	$2d^{11}\left(\frac{-\frac{117a^3d^6\sqrt{dx}}{8192b^4}-\frac{351a^2d^4(dx)^{\frac{5}{2}}}{5120b^3}-\frac{533a\,d^2(dx)^{\frac{9}{2}}}{4096b^2}-\frac{31(dx)^{\frac{13}{2}}}{256b}+\frac{39(dx)^{\frac{17}{2}}}{8192a\,d^2}}{(b\,d^2x^2+a\,d^2)^5}+\frac{117\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{\ldots}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{\ldots}}\right.\right.\ldots}{\ldots}\right.$
pseudoelliptic	$-\frac{\left(\left(-1560a\,x^8b^4+39680a^2x^6b^3+42640a^3x^4b^2+22464x^2a^4b+4680a^5\right)\sqrt{dx}+585\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,(b\,x^2+a)^5\left(2\arctan\left(\ldots\right.\right.\right.}{163840b^4a^2(b\,x^2+a)\ldots}$

input
```
int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^11*((-117/8192*a^3*d^6/b^4*(d*x)^(1/2)-351/5120*a^2*d^4/b^3*(d*x)^(5/2
)-533/4096/b^2*a*d^2*(d*x)^(9/2)-31/256/b*(d*x)^(13/2)+39/8192/a/d^2*(d*x)
^(17/2))/(b*d^2*x^2+a*d^2)^5+117/65536/a^2/d^4/b^4*(a*d^2/b)^(1/4)*2^(1/2)
*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)
^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.82

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{585\left(ab^9x^{10}+5\,a^2b^8x^8+10\,a^3b^7x^6+10\,a^4b^6x^4+5\,a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{1}{4}}}{}$$

input
```
integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/81920*(585*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4
 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30/(a^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7
 + 117*(-d^30/(a^7*b^17))^(1/4)*a^2*b^4) - 585*(-I*a*b^9*x^10 - 5*I*a^2*b^
8*x^8 - 10*I*a^3*b^7*x^6 - 10*I*a^4*b^6*x^4 - 5*I*a^5*b^5*x^2 - I*a^6*b^4)
*(-d^30/(a^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7 + 117*I*(-d^30/(a^7*b^17))
^(1/4)*a^2*b^4) - 585*(I*a*b^9*x^10 + 5*I*a^2*b^8*x^8 + 10*I*a^3*b^7*x^6 +
 10*I*a^4*b^6*x^4 + 5*I*a^5*b^5*x^2 + I*a^6*b^4)*(-d^30/(a^7*b^17))^(1/4)*
log(117*sqrt(d*x)*d^7 - 117*I*(-d^30/(a^7*b^17))^(1/4)*a^2*b^4) - 585*(a*b
^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2
+ a^6*b^4)*(-d^30/(a^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7 - 117*(-d^30/(a^
7*b^17))^(1/4)*a^2*b^4) + 4*(195*b^4*d^7*x^8 - 4960*a*b^3*d^7*x^6 - 5330*a
^2*b^2*d^7*x^4 - 2808*a^3*b*d^7*x^2 - 585*a^4*d^7)*sqrt(d*x))/(a*b^9*x^10
+ 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^
4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.26

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{8\left(195\,(dx)^{\frac{17}{2}}b^4d^{10}-4960\,(dx)^{\frac{13}{2}}ab^3d^{12}-5330\,(dx)^{\frac{9}{2}}a^2b^2d^{14}-2808\,(dx)^{\frac{5}{2}}a^3bd^{16}-585\,\sqrt{dx}a^4d^{18}\right)}{ab^9d^{10}x^{10}+5\,a^2b^8d^{10}x^8+10\,a^3b^7d^{10}x^6+10\,a^4b^6d^{10}x^4+5\,a^5b^5d^{10}x^2+a^6b^4d^{10}} + \cdots$$

input
```
integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/163840*(8*(195*(d*x)^(17/2)*b^4*d^10 - 4960*(d*x)^(13/2)*a*b^3*d^12 - 53
30*(d*x)^(9/2)*a^2*b^2*d^14 - 2808*(d*x)^(5/2)*a^3*b*d^16 - 585*sqrt(d*x)*
a^4*d^18)/(a*b^9*d^10*x^10 + 5*a^2*b^8*d^10*x^8 + 10*a^3*b^7*d^10*x^6 + 10
*a^4*b^6*d^10*x^4 + 5*a^5*b^5*d^10*x^2 + a^6*b^4*d^10) + 585*(sqrt(2)*d^10
*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((
a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^10*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/
4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^9*
arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/s
qrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^9*
arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/
sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a*b^4))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.14

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{\frac{1170\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d^8\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5} + \frac{1170\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d^8\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b} \cdots\right.\right.}{2\left(\frac{ad}{b} \cdots\right.}\right)}{a^2b^5} \cdots}{\cdots}$$

input
```
integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/163840*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*d^8*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5) + 1170*sqrt(2)*(a
*b^3*d^2)^(1/4)*d^8*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(
d*x))/(a*d^2/b)^(1/4))/(a^2*b^5) + 585*sqrt(2)*(a*b^3*d^2)^(1/4)*d^8*log(d
*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^5) - 585*sq
rt(2)*(a*b^3*d^2)^(1/4)*d^8*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
sqrt(a*d^2/b))/(a^2*b^5) + 8*(195*sqrt(d*x)*b^4*d^18*x^8 - 4960*sqrt(d*x)*
a*b^3*d^18*x^6 - 5330*sqrt(d*x)*a^2*b^2*d^18*x^4 - 2808*sqrt(d*x)*a^3*b*d^
18*x^2 - 585*sqrt(d*x)*a^4*d^18)/((b*d^2*x^2 + a*d^2)^5*a*b^4))/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.68

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{117\,d^{15/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{7/4}\,b^{17/4}} - \frac{\frac{31\,d^{11}\,(d\,x)^{13/2}}{128\,b}-\frac{39\,d^9\,(d\,x)^{17/2}}{4096\,a}+\frac{351\,a^2\,d^{15}\,(d\,x)^{5/2}}{2560\,b^3}+\frac{117\,a^3\,d^{17}\,\sqrt{d\,x}}{4096\,b^4}+\frac{533\,a\,d^{13}\,(d\,x)^{9/2}}{2048\,b^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}} + \frac{117\,d^{15/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{7/4}\,b^{17/4}}$$

input
```
int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(117*d^(15/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)
^(7/4)*b^(17/4)) - ((31*d^11*(d*x)^(13/2))/(128*b) - (39*d^9*(d*x)^(17/2))
/(4096*a) + (351*a^2*d^15*(d*x)^(5/2))/(2560*b^3) + (117*a^3*d^17*(d*x)^(1
/2))/(4096*b^4) + (533*a*d^13*(d*x)^(9/2))/(2048*b^2))/(a^5*d^10 + b^5*d^1
0*x^10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^
2*b^3*d^10*x^6) + (117*d^(15/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^
(1/2))))/(8192*(-a)^(7/4)*b^(17/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 995, normalized size of antiderivative = 3.20

$$\int \frac{(dx)^{15/2}}{(a^2 + 2abx^2 + b^2x^4)^3} \, dx = \text{Too large to display}$$

input
```
int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*d**7*( - 1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 5850*b**(3
/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/
(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 11700*b**(3/4)*a**(1/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*s
qrt(2)))*a**3*b**2*x**4 - 11700*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**
3*x**6 - 5850*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 1170*b**(3/4
)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 1170*b**(3/4)*a**(1/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(
2)))*a**5 + 5850*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 11700*b**
(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 11700*b**(3/4)*a**(1/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1
/4)*sqrt(2)))*a**2*b**3*x**6 + 5850*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/
4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*
*4*x**8 + 1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2...
```

3.477 $\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 314

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{d(dx)^{11/2}}{10b(a+bx^2)^5} - \frac{11d^3(dx)^{7/2}}{160b^2(a+bx^2)^4} - \frac{77d^5(dx)^{3/2}}{1920b^3(a+bx^2)^3} + \frac{77d^5(dx)^{3/2}}{5120ab^3(a+bx^2)^2} + \frac{77d^5(dx)^{3/2}}{4096a^2b^3(a+bx^2)} - \frac{77d^{13/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{9/4}b^{15/4}} + \frac{77d^{13/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{9/4}b^{15/4}} - \frac{77d^{13/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}(\sqrt{a}+\sqrt{b}x)}\right)}{8192\sqrt{2}a^{9/4}b^{15/4}}$$

output
```
-1/10*d*(d*x)^(11/2)/b/(b*x^2+a)^5-11/160*d^3*(d*x)^(7/2)/b^2/(b*x^2+a)^4-
77/1920*d^5*(d*x)^(3/2)/b^3/(b*x^2+a)^3+77/5120*d^5*(d*x)^(3/2)/a/b^3/(b*x
^2+a)^2+77/4096*d^5*(d*x)^(3/2)/a^2/b^3/(b*x^2+a)-77/16384*d^(13/2)*arctan
(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(9/4)/b^(15/4)+7
7/16384*d^(13/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(
1/2)/a^(9/4)/b^(15/4)-77/16384*d^(13/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d
*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(9/4)/b^(15/4)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.59

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{d^6\sqrt{dx}\left(\frac{4\sqrt[4]{a}b^{3/4}x^{3/2}(-385a^4-1760a^3bx^2-3130a^2b^2x^4+5544ab^3x^6+1155b^4x^8)}{(a+bx^2)^5}-1155\sqrt{2}\,\text{ar}\right.}{245760a^{9/4}b^{15/4}\sqrt{x}}$$

input
```
Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(d^6*Sqrt[d*x]*((4*a^(1/4)*b^(3/4)*x^(3/2)*(-385*a^4 - 1760*a^3*b*x^2 - 31
30*a^2*b^2*x^4 + 5544*a*b^3*x^6 + 1155*b^4*x^8))/(a + b*x^2)^5 - 1155*Sqrt
[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 1155
*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])
)/(245760*a^(9/4)*b^(15/4)*Sqrt[x])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.40, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 252, 252, 252, 253, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$

$$\Big\downarrow \; 1380$$

$$b^6\int \frac{(dx)^{13/2}}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{(dx)^{13/2}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \; 252$$

$$\frac{11d^2\int \frac{(dx)^{9/2}}{(bx^2+a)^5}dx}{20b}-\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 252

$$\frac{11d^2\left(\frac{7d^2\int \frac{(dx)^{5/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 252

$$\frac{11d^2\left(\frac{7d^2\left(\frac{d^2\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^3}dx}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 253

$$\frac{11d^2\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^2}dx}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 253

$$\frac{11d^2\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{\int\frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 266

$$\frac{11d^2\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{\int\frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 27

$$\frac{11d^2\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 826$$

$$\frac{11d^2\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{11/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 1476

$$11d^2\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}}{4b}}{16b}$$

↓ 1082

$$11d^2 \left(\frac{7d^2 \left(\frac{d^2 \left(\frac{5 \left(\frac{d \left(\frac{\frac{\int \frac{1}{-dx-1} d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\int \frac{1}{-dx-1} d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} - \frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2} d\sqrt{dx}}{2\sqrt{b}} \right)}{2a} + \frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} \right)}{8a} + \frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2} \right)}{4b} - \frac{d(dx}{6b\left(a-\right.} \right.}{16b}$$

↓ 217

$$11d^2\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}$$

↓ 1479

$$7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\cdots}{\cdots}\right.}\cdots}{2\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{b}}\cdots\right.}{2a}\cdots\right.}{8a}\cdots\right.}{4b}\cdots\right.$$

$$\Big\downarrow 25$$

$$\left(7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\sqrt[4]{\cdots}}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}}{2\sqrt{b}}\cdots\right.}{2a}\cdots\right.}{8a}\cdots\right.}{4b}\cdots\right.\right.$$

$$\Big\downarrow \text{27}$$

$$7d^2\left(\frac{d^2\left(\frac{\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}\right.}{8a}\right.}{4b}\right.$$

↓ 1103

$$11d^2 \left(\frac{7d^2 \left(\frac{d^2 \left(\frac{5 \left(\frac{d \left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} - \frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d\cdots\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} \cdots \right.}{2a} \cdots \right.}{8a} \cdots \right.}{4b} \cdots \right.}{16b} \cdots \right.$$

input
```
Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*(d*x)^(11/2))/(b*(a + b*x^2)^5) + (11*d^2*(-1/8*(d*(d*x)^(7/2))/(
b*(a + b*x^2)^4) + (7*d^2*(-1/6*(d*(d*x)^(3/2))/(b*(a + b*x^2)^3) + (d^2*(
(d*x)^(3/2)/(4*a*d*(a + b*x^2)^2) + (5*((d*x)^(3/2)/(2*a*d*(a + b*x^2)) +
(d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4
)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqr
t[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]
)))/(2*a)))/(8*a)))/(4*b)))/(16*b)))/(20*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 14.66 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.75

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{77d^4a^2(dx)^{\frac{3}{2}}}{24576b^3}-\frac{11d^2a(dx)^{\frac{7}{2}}}{768b^2}-\frac{313(dx)^{\frac{11}{2}}}{12288b}+\frac{231(dx)^{\frac{15}{2}}}{5120a\,d^2}+\frac{77b(dx)^{\frac{19}{2}}}{8192a^2d^4}}{(b\,d^2x^2+a\,d^2)^5}+\frac{77\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right.\right.}{6}\right.$
default	$2d^{11}\left(\frac{-\frac{77d^4a^2(dx)^{\frac{3}{2}}}{24576b^3}-\frac{11d^2a(dx)^{\frac{7}{2}}}{768b^2}-\frac{313(dx)^{\frac{11}{2}}}{12288b}+\frac{231(dx)^{\frac{15}{2}}}{5120a\,d^2}+\frac{77b(dx)^{\frac{19}{2}}}{8192a^2d^4}}{(b\,d^2x^2+a\,d^2)^5}+\frac{77\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right.\right.}{6}\right.$
pseudoelliptic	$\frac{77d^6\left(-8\left(-3b^4x^8-\frac{72}{5}a\,b^3x^6+\frac{626}{77}a^2b^2x^4+\frac{32}{7}a^3b\,x^2+a^4\right)\sqrt{dx}\,bx\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}+3d\sqrt{2}\,(b\,x^2+a)^5\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}}\right.\right.\right.}{98304\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}b^4a^2(b\,x^2+a)^5}$

input
```
int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^11*((-77/24576/b^3*d^4*a^2*(d*x)^(3/2)-11/768/b^2*d^2*a*(d*x)^(7/2)-31
3/12288/b*(d*x)^(11/2)+231/5120/a/d^2*(d*x)^(15/2)+77/8192/a^2/d^4*b*(d*x)
^(19/2))/(b*d^2*x^2+a*d^2)^5+77/65536/a^2/d^4/b^4/(a*d^2/b)^(1/4)*2^(1/2)*
(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^
(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.84

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{1155\,(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3)\left(-\frac{d^{26}}{a^9b^{15}}\right)}{}$$

input
```
integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/245760*(1155*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5
*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*log(456533*sqrt(d
*x)*d^19 + 456533*(-d^26/(a^9*b^15))^(3/4)*a^7*b^11) - 1155*(I*a^2*b^8*x^1
0 + 5*I*a^3*b^7*x^8 + 10*I*a^4*b^6*x^6 + 10*I*a^5*b^5*x^4 + 5*I*a^6*b^4*x^
2 + I*a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 + 456533
*I*(-d^26/(a^9*b^15))^(3/4)*a^7*b^11) - 1155*(-I*a^2*b^8*x^10 - 5*I*a^3*b^
7*x^8 - 10*I*a^4*b^6*x^6 - 10*I*a^5*b^5*x^4 - 5*I*a^6*b^4*x^2 - I*a^7*b^3)
*(-d^26/(a^9*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 - 456533*I*(-d^26/(a^9
*b^15))^(3/4)*a^7*b^11) - 1155*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*
x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*l
og(456533*sqrt(d*x)*d^19 - 456533*(-d^26/(a^9*b^15))^(3/4)*a^7*b^11) + 4*(
1155*b^4*d^6*x^9 + 5544*a*b^3*d^6*x^7 - 3130*a^2*b^2*d^6*x^5 - 1760*a^3*b*
d^6*x^3 - 385*a^4*d^6*x)*sqrt(d*x))/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4
*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)
```

Sympy [F]

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \int \frac{(dx)^{\frac{13}{2}}}{(a+bx^2)^6}\,dx$$

input
```
integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Integral((d*x)**(13/2)/(a + b*x**2)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.23

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{1155\,d^8\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\,\mathrm{lo}}{}\right.}{a^2b^3}$$

input
```
integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/491520*(1155*d^8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^
(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b
)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/
4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d
)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(
1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt
(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/
(a^2*b^3) + 8*(1155*(d*x)^(19/2)*b^4*d^8 + 5544*(d*x)^(15/2)*a*b^3*d^10 -
3130*(d*x)^(11/2)*a^2*b^2*d^12 - 1760*(d*x)^(7/2)*a^3*b*d^14 - 385*(d*x)^(
3/2)*a^4*d^16)/(a^2*b^8*d^10*x^10 + 5*a^3*b^7*d^10*x^8 + 10*a^4*b^6*d^10*x
^6 + 10*a^5*b^5*d^10*x^4 + 5*a^6*b^4*d^10*x^2 + a^7*b^3*d^10))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.13

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{1}{491520}\,d^6\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^6d}+\frac{2310\sqrt{2}\left(ab^3\right.}{}\right.$$

input
```
integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/491520*d^6*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^6*d) + 2310*sqrt(2)*
(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*
x))/(a*d^2/b)^(1/4))/(a^3*b^6*d) - 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x
+ sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^6*d) + 1155*sq
rt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/(a^3*b^6*d) + 8*(1155*sqrt(d*x)*b^4*d^10*x^9 + 5544*sqrt(d*x)*a
*b^3*d^10*x^7 - 3130*sqrt(d*x)*a^2*b^2*d^10*x^5 - 1760*sqrt(d*x)*a^3*b*d^1
0*x^3 - 385*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^2*b^3))
```

Mupad [B] (verification not implemented)

Time = 17.81 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.66

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{77\,d^{13/2}\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{d\,x}}{(-a)^{1/4}\sqrt{d}}\right)}{8192\,(-a)^{9/4}\,b^{15/4}} - \frac{\frac{313\,d^{11}\,(d\,x)^{11/2}}{6144\,b}-\frac{231\,d^9\,(d\,x)^{15/2}}{2560\,a}+\frac{77\,a^2\,d^{15}\,(d\,x)^{3/2}}{12288\,b^3}+\frac{11\,a\,d^{13}\,(d\,x)^{7/2}}{384\,b^2}-\frac{77\,b\,d^7\,(d\,x)^{19/2}}{4096\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}} - \frac{77\,d^{13/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\sqrt{d\,x}}{(-a)^{1/4}\sqrt{d}}\right)}{8192\,(-a)^{9/4}\,b^{15/4}}$$

input
```
int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(77*d^(13/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^
(9/4)*b^(15/4)) - ((313*d^11*(d*x)^(11/2))/(6144*b) - (231*d^9*(d*x)^(15/2
))/(2560*a) + (77*a^2*d^15*(d*x)^(3/2))/(12288*b^3) + (11*a*d^13*(d*x)^(7/
2))/(384*b^2) - (77*b*d^7*(d*x)^(19/2))/(4096*a^2))/(a^5*d^10 + b^5*d^10*x
^10 + 5*a^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b
^3*d^10*x^6) - (77*d^(13/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2
))))/(8192*(-a)^(9/4)*b^(15/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 996, normalized size of antiderivative = 3.17

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*d**6*( - 2310*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 11550*b**(
1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 23100*b**(1/4)*a**(3/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*
sqrt(2)))*a**3*b**2*x**4 - 23100*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*
*3*x**6 - 11550*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
- 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 2310*b**(1
/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/
(b**(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 2310*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**5 + 11550*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 23100*
b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 23100*b**(1/4)*a**(3/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*a**2*b**3*x**6 + 11550*b**(1/4)*a**(3/4)*sqrt(2)*atan((b*
*(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*
a*b**4*x**8 + 2310*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq...
```

3.478 $\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 314

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = -\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5} - \frac{9d^3(dx)^{5/2}}{160b^2\left(a+bx^2\right)^4} - \frac{3d^5\sqrt{dx}}{128b^3\left(a+bx^2\right)^3} + \frac{3d^5\sqrt{dx}}{1024ab^3\left(a+bx^2\right)^2} + \frac{21d^5\sqrt{dx}}{4096a^2b^3\left(a+bx^2\right)} - \frac{63d^{11/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{11/4}b^{13/4}} + \frac{63d^{11/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{11/4}b^{13/4}} + \frac{63d^{11/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}a^{11/4}b^{13/4}}$$

output
```
-1/10*d*(d*x)^(9/2)/b/(b*x^2+a)^5-9/160*d^3*(d*x)^(5/2)/b^2/(b*x^2+a)^4-3/
128*d^5*(d*x)^(1/2)/b^3/(b*x^2+a)^3+3/1024*d^5*(d*x)^(1/2)/a/b^3/(b*x^2+a)
^2+21/4096*d^5*(d*x)^(1/2)/a^2/b^3/(b*x^2+a)-63/16384*d^(11/2)*arctan(1-2^
(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(11/4)/b^(13/4)+63/16
384*d^(11/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)
/a^(11/4)/b^(13/4)+63/16384*d^(11/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)
^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(11/4)/b^(13/4)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.59

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{d^5\sqrt{dx}\left(\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}(-315a^4-1512a^3bx^2-2870a^2b^2x^4+480ab^3x^6+105b^4x^8)}{(a+bx^2)^5}-315\sqrt{2}\,\text{arctan}\right.}{81920a^{11/4}b^{13/4}\sqrt{x}}$$

input
```
Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(d^5*Sqrt[d*x]*((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-315*a^4 - 1512*a^3*b*x^2 - 28
70*a^2*b^2*x^4 + 480*a*b^3*x^6 + 105*b^4*x^8))/(a + b*x^2)^5 - 315*Sqrt[2]
*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 315*Sqr
t[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(8
1920*a^(11/4)*b^(13/4)*Sqrt[x])
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.41, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 252, 252, 252, 253, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^6\int \frac{(dx)^{11/2}}{b^6\,(bx^2+a)^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(dx)^{11/2}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{9d^2\int\frac{(dx)^{7/2}}{(bx^2+a)^5}dx}{20b}-\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 252

$$\frac{9d^2\left(\frac{5d^2\int\frac{(dx)^{3/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 252

$$\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)^3}dx}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 253

$$\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{7\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)^2}dx}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 253

$$\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{266}$$

$$\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\int\frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{755}$$

$$\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{9/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 1476$$

$$\left(5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx\right.}\right.}{12b}\right.\right.$$

↓ 1082

$$\left(5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12b}-\cdots\right.\right.$$

↓ 217

$$\left(5d^2 \left(\frac{d^2 \left(\frac{7 \left(\frac{3 \left(\frac{d \int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2} d\sqrt{dx}}{2\sqrt{a}} + \frac{d \left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} \right)}{2ad} + \frac{\sqrt{dx}}{2ad\left(a+bx^2\right)} \right)}{8a} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2} \right)}{12b} - \frac{d\sqrt{dx}}{6b\left(a+bx^2\right.}$$

↓ 1479

$$5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\cdots\right)}{2\sqrt{\cdots}}\cdots\right)}{2ad}\cdots\right)}{8a}\cdots\right)}{12b}\cdots\right)$$

$$\Big\downarrow \, 25$$

$$\Bigg(5d^2 \Bigg(\frac{d^2 \left(\frac{7 \left(\frac{3 \left(\frac{d \left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} + \frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d \left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\text{arcta}\cdots}{\cdots} \right.}{2\sqrt{a}} \cdots}{2ad} \cdots}{8a} \cdots}{12b} \cdots$$

$$\Big\downarrow \, 27$$

$$\Bigg(5d^2\Bigg(d^2\Bigg(7\Bigg(3\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{d\ldots}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\ldots\right)}{2\sqrt{a}}\ldots\right)\bigg/(2ad)\ldots\Bigg)\bigg/(8a)\ldots\Bigg)\bigg/(12b)\ldots$$

↓ 1103

$$5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}\cdots\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\cdots\right.}{2\sqrt{a}}\cdots\right)}{2ad}\cdots\right)}{8a}\cdots\right)}{12b}\cdots\right.$$

input
```
Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*(d*x)^(9/2))/(b*(a + b*x^2)^5) + (9*d^2*(-1/8*(d*(d*x)^(5/2))/(b*
(a + b*x^2)^4) + (5*d^2*(-1/6*(d*Sqrt[d*x])/(b*(a + b*x^2)^3) + (d^2*(Sqrt
[d*x]/(4*a*d*(a + b*x^2)^2) + (7*(Sqrt[d*x]/(2*a*d*(a + b*x^2)) + (3*((d*(
-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt
[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]
)))/(2*a*d)))/(8*a)))/(12*b)))/(16*b)))/(20*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 14.66 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.75

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{63a^2d^4\sqrt{dx}}{8192b^3}-\frac{189a\,d^2(dx)^{\frac{5}{2}}}{5120b^2}-\frac{287(dx)^{\frac{9}{2}}}{4096b}+\frac{3(dx)^{\frac{13}{2}}}{256a\,d^2}+\frac{21b(dx)^{\frac{17}{2}}}{8192a^2d^4}}{(b\,d^2x^2+a\,d^2)^5}+\frac{63\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{-\frac{63a^2d^4\sqrt{dx}}{8192b^3}-\frac{189a\,d^2(dx)^{\frac{5}{2}}}{5120b^2}-\frac{287(dx)^{\frac{9}{2}}}{4096b}+\frac{3(dx)^{\frac{13}{2}}}{256a\,d^2}+\frac{21b(dx)^{\frac{17}{2}}}{8192a^2d^4}}{(b\,d^2x^2+a\,d^2)^5}+\frac{63\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}\right.\right.}{}\right.$
pseudoelliptic	$\frac{d^5\left(\left(840a\,x^8b^4+3840a^2x^6b^3-22960a^3x^4b^2-12096x^2a^4b-2520a^5\right)\sqrt{dx}+315\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,(b\,x^2+a)^5\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)}{dx-\left(\frac{a\,d^2}{b}\right)}\right.\right.\right.}{163840a^3b^3(b\,x^2+a)^5}$

input
```
int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^11*((-63/8192*a^2*d^4/b^3*(d*x)^(1/2)-189/5120*a*d^2/b^2*(d*x)^(5/2)-2
87/4096/b*(d*x)^(9/2)+3/256/a/d^2*(d*x)^(13/2)+21/8192/a^2/d^4*b*(d*x)^(17
/2))/(b*d^2*x^2+a*d^2)^5+63/65536/a^3/d^6/b^3*(a*d^2/b)^(1/4)*2^(1/2)*(ln(
(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4
)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.83

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{315\,(a^2b^8x^{10}+5\,a^3b^7x^8+10\,a^4b^6x^6+10\,a^5b^5x^4+5\,a^6b^4x^2+a^7b^3)\left(-\frac{d^{22}}{a^{11}b^{13}}\right)}{}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/81920*(315*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x
^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^22/(a^11*b^13))^(1/4)*log(63*a^3*b^3*(-d
^22/(a^11*b^13))^(1/4) + 63*sqrt(d*x)*d^5) - 315*(-I*a^2*b^8*x^10 - 5*I*a^
3*b^7*x^8 - 10*I*a^4*b^6*x^6 - 10*I*a^5*b^5*x^4 - 5*I*a^6*b^4*x^2 - I*a^7*
b^3)*(-d^22/(a^11*b^13))^(1/4)*log(63*I*a^3*b^3*(-d^22/(a^11*b^13))^(1/4)
 + 63*sqrt(d*x)*d^5) - 315*(I*a^2*b^8*x^10 + 5*I*a^3*b^7*x^8 + 10*I*a^4*b^6
*x^6 + 10*I*a^5*b^5*x^4 + 5*I*a^6*b^4*x^2 + I*a^7*b^3)*(-d^22/(a^11*b^13))
^(1/4)*log(-63*I*a^3*b^3*(-d^22/(a^11*b^13))^(1/4) + 63*sqrt(d*x)*d^5) - 3
15*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6
*b^4*x^2 + a^7*b^3)*(-d^22/(a^11*b^13))^(1/4)*log(-63*a^3*b^3*(-d^22/(a^11
*b^13))^(1/4) + 63*sqrt(d*x)*d^5) + 4*(105*b^4*d^5*x^8 + 480*a*b^3*d^5*x^6
 - 2870*a^2*b^2*d^5*x^4 - 1512*a^3*b*d^5*x^2 - 315*a^4*d^5)*sqrt(d*x))/(a^
2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x
^2 + a^7*b^3)
```

Sympy [F]

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \int \frac{(dx)^{\frac{11}{2}}}{(a+bx^2)^6}\,dx$$

input
```
integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Integral((d*x)**(11/2)/(a + b*x**2)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.25

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{8\left(105\,(dx)^{\frac{17}{2}}b^4d^8+480\,(dx)^{\frac{13}{2}}ab^3d^{10}-2870\,(dx)^{\frac{9}{2}}a^2b^2d^{12}-1512\,(dx)^{\frac{5}{2}}a^3bd^{14}-315\sqrt{dx}a^4d^{16}\right)}{a^2b^8d^{10}x^{10}+5\,a^3b^7d^{10}x^8+10\,a^4b^6d^{10}x^6+10\,a^5b^5d^{10}x^4+5\,a^6b^4d^{10}x^2+a^7b^3d^{10}} + \frac{315 \ldots}{\ldots}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/163840*(8*(105*(d*x)^(17/2)*b^4*d^8 + 480*(d*x)^(13/2)*a*b^3*d^10 - 2870
*(d*x)^(9/2)*a^2*b^2*d^12 - 1512*(d*x)^(5/2)*a^3*b*d^14 - 315*sqrt(d*x)*a^
4*d^16)/(a^2*b^8*d^10*x^10 + 5*a^3*b^7*d^10*x^8 + 10*a^4*b^6*d^10*x^6 + 10
*a^5*b^5*d^10*x^4 + 5*a^6*b^4*d^10*x^2 + a^7*b^3*d^10) + 315*(sqrt(2)*d^8*
log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a
*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^8*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)
*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^7*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqr
t(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^7*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sq
rt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^2*b^3))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.13

$$\int \frac{(dx)^{11/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{\frac{630\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d^6\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^4} + \frac{630\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d^6\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}} \ldots\right)}{2\left(\frac{ad^2}{b}\right) \ldots}\right)}{a^3b^4} \ldots}{\ldots}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/163840*(630*sqrt(2)*(a*b^3*d^2)^(1/4)*d^6*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4) + 630*sqrt(2)*(a*b
^3*d^2)^(1/4)*d^6*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*
x))/(a*d^2/b)^(1/4))/(a^3*b^4) + 315*sqrt(2)*(a*b^3*d^2)^(1/4)*d^6*log(d*x
 + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4) - 315*sqrt
(2)*(a*b^3*d^2)^(1/4)*d^6*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sq
rt(a*d^2/b))/(a^3*b^4) + 8*(105*sqrt(d*x)*b^4*d^16*x^8 + 480*sqrt(d*x)*a*b
^3*d^16*x^6 - 2870*sqrt(d*x)*a^2*b^2*d^16*x^4 - 1512*sqrt(d*x)*a^3*b*d^16*
x^2 - 315*sqrt(d*x)*a^4*d^16)/((b*d^2*x^2 + a*d^2)^5*a^2*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.66

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{\frac{287\,d^{11}\,(d\,x)^{9/2}}{2048\,b}-\frac{3\,d^9\,(d\,x)^{13/2}}{128\,a}+\frac{63\,a^2\,d^{15}\,\sqrt{d\,x}}{4096\,b^3}+\frac{189\,a\,d^{13}\,(d\,x)^{5/2}}{2560\,b^2}-\frac{21\,b\,d^7\,(d\,x)^{17/2}}{4096\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}} - \frac{63\,d^{11/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{11/4}\,b^{13/4}} - \frac{63\,d^{11/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{11/4}\,b^{13/4}}$$

input
```
int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
- ((287*d^11*(d*x)^(9/2))/(2048*b) - (3*d^9*(d*x)^(13/2))/(128*a) + (63*a^
2*d^15*(d*x)^(1/2))/(4096*b^3) + (189*a*d^13*(d*x)^(5/2))/(2560*b^2) - (21
*b*d^7*(d*x)^(17/2))/(4096*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*
x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) - (63*
d^(11/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(11/
4)*b^(13/4)) - (63*d^(11/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2
))))/(8192*(-a)^(11/4)*b^(13/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 995, normalized size of antiderivative = 3.17

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Too large to display}$$

input

```
int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output

```
(sqrt(d)*d**5*( - 630*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 3150*b**(3/
4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 6300*b**(3/4)*a**(1/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**3*b**2*x**4 - 6300*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*x
**6 - 3150*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*s
qrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 630*b**(3/4)*a*
*(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1
/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 630*b**(3/4)*a**(1/4)*sqrt(2)*atan((b*
*(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*
a**5 + 3150*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 6300*b**(3/4)*
a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 6300*b**(3/4)*a**(1/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**2*b**3*x**6 + 3150*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8
 + 630*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sq...
```

3.479 $\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 317

$$\begin{aligned}\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx &= -\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5} \\ &\quad -\frac{7d^3(dx)^{3/2}}{160b^2\,(a+bx^2)^4}+\frac{7d^3(dx)^{3/2}}{640ab^2\,(a+bx^2)^3}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2\,(a+bx^2)^2} \\ &\quad +\frac{63d^3(dx)^{3/2}}{4096a^3b^2\,(a+bx^2)}-\frac{63d^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{13/4}b^{11/4}} \\ &\quad +\frac{63d^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{13/4}b^{11/4}}-\frac{63d^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}a^{13/4}b^{11/4}}\end{aligned}$$

output

```
-1/10*d*(d*x)^(7/2)/b/(b*x^2+a)^5-7/160*d^3*(d*x)^(3/2)/b^2/(b*x^2+a)^4+7/
640*d^3*(d*x)^(3/2)/a/b^2/(b*x^2+a)^3+63/5120*d^3*(d*x)^(3/2)/a^2/b^2/(b*x
^2+a)^2+63/4096*d^3*(d*x)^(3/2)/a^3/b^2/(b*x^2+a)-63/16384*d^(9/2)*arctan(
1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(13/4)/b^(11/4)+6
3/16384*d^(9/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1
/2)/a^(13/4)/b^(11/4)-63/16384*d^(9/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*
x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(13/4)/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.59

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{d^4\sqrt{dx}\left(\frac{4\sqrt[4]{a}b^{3/4}x^{3/2}(-105a^4-480a^3bx^2+2870a^2b^2x^4+1512ab^3x^6+315b^4x^8)}{(a+bx^2)^5}-315\sqrt{2}\arcta\right.}{81920a^{13/4}b^{11/4}\sqrt{x}}$$

input
```
Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(d^4*Sqrt[d*x]*((4*a^(1/4)*b^(3/4)*x^(3/2)*(-105*a^4 - 480*a^3*b*x^2 + 287
0*a^2*b^2*x^4 + 1512*a*b^3*x^6 + 315*b^4*x^8))/(a + b*x^2)^5 - 315*Sqrt[2]
*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 315*Sqr
t[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(8
1920*a^(13/4)*b^(11/4)*Sqrt[x])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.38, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 252, 252, 253, 253, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{(dx)^{9/2}}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{9/2}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{7d^2\int\frac{(dx)^{5/2}}{(bx^2+a)^5}dx}{20b}-\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 252

$$\frac{7d^2\left(\frac{3d^2\int\frac{\sqrt{dx}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{3/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 253

$$\frac{7d^2\left(\frac{3d^2\left(\frac{3\int\frac{\sqrt{dx}}{\left(bx^2+a\right)^3}dx}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{3/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 253

$$\frac{7d^2\left(\frac{3d^2\left(\frac{3\left(\frac{5\int\frac{\sqrt{dx}}{\left(bx^2+a\right)^2}dx}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{3/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 253

$$\frac{7d^2\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{\int\frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{3/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}$$

$\Big\downarrow$ 266

$$\frac{7d^2\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{\int\frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{3/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{7/2}}{10b\,(a+bx^2)^5}$$

$\Big\downarrow$ 27

$$\frac{7d^2\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{d\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{3/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 826

$$\frac{7d^2\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{3/2}}{8b(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{7/2}}{10b\left(a+bx^2\right)^5}$$

$$\downarrow \; 1476$$

$$7d^2 \left(\frac{3d^2 \left(\frac{3 \left(\frac{5 \left(\frac{d \left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}} \right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)} \right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2} \right)}{4a}+ \cdots \right)}{16b} \cdots$$

↓ 1082

$$7d^2\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{-dx-1}\,d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}\,d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3}}{6ad\left(a+b\right.}\right.}{16b}\right.$$

↓ 217

$$7d^2\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16b}$$

↓ 1479

$$3d^2\Bigg(\frac{3\Bigg(\frac{5\Bigg(\frac{d\Bigg(\frac{\frac{\arctan\Big(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\Big)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\Big(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\Big)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\Big(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\Big)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\Big(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{b}\Big(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}}{2\sqrt{b}}\cdots}{2a}\cdots}{8a}\cdots}{4a}\cdots$$

↓ 25

$$3d^2\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\cdots}{\sqrt[4]{b}}\cdots\right.}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots}{2\sqrt{b}}\cdots\right)}{2a}\cdots\right)}{8a}\cdots\right)}{4a}\cdots\right.$$

$$\Big\downarrow \; 27$$

$$3d^2\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}+\cdots}{8a}\cdots}{4a}\cdots$$

↓ 1103

$$7d^2 \left(\frac{3d^2 \left(\frac{3 \left(\frac{5 \left(\frac{d \left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} - \frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} \cdots}{2a}}{8a}}{4a}}{16b}$$

input
```
Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*(d*x)^(7/2))/(b*(a + b*x^2)^5) + (7*d^2*(-1/8*(d*(d*x)^(3/2))/(b*
(a + b*x^2)^4) + (3*d^2*((d*x)^(3/2)/(6*a*d*(a + b*x^2)^3) + (3*((d*x)^(3/
2)/(4*a*d*(a + b*x^2)^2) + (5*((d*x)^(3/2)/(2*a*d*(a + b*x^2)) + (d*((-(Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d +
Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*a)
))/(8*a)))/(4*a)))/(16*b)))/(20*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 14.63 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.74

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{21 d^2 a (dx)^{\frac{3}{2}}}{8192 b^2}-\frac{3 (dx)^{\frac{7}{2}}}{256 b}+\frac{287 (dx)^{\frac{11}{2}}}{4096 d^2 a}+\frac{189 b (dx)^{\frac{15}{2}}}{5120 a^2 d^4}+\frac{63 b^2 (dx)^{\frac{19}{2}}}{8192 a^3 d^6}}{(b d^2 x^2+a d^2)^5}+\frac{63\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)\right.}{6553}\right.$
default	$2d^{11}\left(\frac{-\frac{21 d^2 a (dx)^{\frac{3}{2}}}{8192 b^2}-\frac{3 (dx)^{\frac{7}{2}}}{256 b}+\frac{287 (dx)^{\frac{11}{2}}}{4096 d^2 a}+\frac{189 b (dx)^{\frac{15}{2}}}{5120 a^2 d^4}+\frac{63 b^2 (dx)^{\frac{19}{2}}}{8192 a^3 d^6}}{(b d^2 x^2+a d^2)^5}+\frac{63\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)\right.}{6553}\right.$
pseudoelliptic	$-\frac{21 d^4\left(8\sqrt{dx}\, b\left(-3b^4x^8-\frac{72}{5}a b^3 x^6-\frac{82}{3}a^2 b^2 x^4+\frac{32}{7}a^3 b x^2+a^4\right)x\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}-3d\sqrt{2}\,(b x^2+a)^5\left(\ln\left(\frac{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{}}\right.\right.\right.}{32768\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} a^3 b^3 (b x^2+a)^5}$

input
```
int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^11*((-21/8192*d^2*a/b^2*(d*x)^(3/2)-3/256/b*(d*x)^(7/2)+287/4096/d^2/a
*(d*x)^(11/2)+189/5120/a^2*b/d^4*(d*x)^(15/2)+63/8192/a^3/d^6*b^2*(d*x)^(1
9/2))/(b*d^2*x^2+a*d^2)^5+63/65536/a^3/d^6/b^3/(a*d^2/b)^(1/4)*2^(1/2)*(ln
((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/
4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.82

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{315\,(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2)\left(-\frac{d^{18}}{a^{13}b^{11}}\right)}{}$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/81920*(315*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x
^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*log(250047*a^10*b^
8*(-d^18/(a^13*b^11))^(3/4) + 250047*sqrt(d*x)*d^13) - 315*(I*a^3*b^7*x^10
 + 5*I*a^4*b^6*x^8 + 10*I*a^5*b^5*x^6 + 10*I*a^6*b^4*x^4 + 5*I*a^7*b^3*x^2
 + I*a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)*log(250047*I*a^10*b^8*(-d^18/(a^13
*b^11))^(3/4) + 250047*sqrt(d*x)*d^13) - 315*(-I*a^3*b^7*x^10 - 5*I*a^4*b^
6*x^8 - 10*I*a^5*b^5*x^6 - 10*I*a^6*b^4*x^4 - 5*I*a^7*b^3*x^2 - I*a^8*b^2)
*(-d^18/(a^13*b^11))^(1/4)*log(-250047*I*a^10*b^8*(-d^18/(a^13*b^11))^(3/4
) + 250047*sqrt(d*x)*d^13) - 315*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^
5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4
)*log(-250047*a^10*b^8*(-d^18/(a^13*b^11))^(3/4) + 250047*sqrt(d*x)*d^13)
+ 4*(315*b^4*d^4*x^9 + 1512*a*b^3*d^4*x^7 + 2870*a^2*b^2*d^4*x^5 - 480*a^3
*b*d^4*x^3 - 105*a^4*d^4*x)*sqrt(d*x))/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*
a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)
```

Sympy [F]

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \int \frac{(dx)^{\frac{9}{2}}}{(a+bx^2)^6}\,dx$$

input
```
integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Integral((d*x)**(9/2)/(a + b*x**2)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.21

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{315\,d^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\cdots}{\cdots}\right.}{a^3b^2}$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/163840*(315*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(
1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)
*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)
*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1
/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(
2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(
a^3*b^2) + 8*(315*(d*x)^(19/2)*b^4*d^6 + 1512*(d*x)^(15/2)*a*b^3*d^8 + 287
0*(d*x)^(11/2)*a^2*b^2*d^10 - 480*(d*x)^(7/2)*a^3*b*d^12 - 105*(d*x)^(3/2)
*a^4*d^14)/(a^3*b^7*d^10*x^10 + 5*a^4*b^6*d^10*x^8 + 10*a^5*b^5*d^10*x^6 +
 10*a^6*b^4*d^10*x^4 + 5*a^7*b^3*d^10*x^2 + a^8*b^2*d^10))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{1}{163840}\,d^4\left(\frac{630\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^5d}+\frac{630\sqrt{2}(ab^3d^2\cdots}{\cdots}\right.$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/163840*d^4*(630*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^5*d) + 630*sqrt(2)*(a
*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x)
)/(a*d^2/b)^(1/4))/(a^4*b^5*d) - 315*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^5*d) + 315*sqrt(2
)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d
^2/b))/(a^4*b^5*d) + 8*(315*sqrt(d*x)*b^4*d^10*x^9 + 1512*sqrt(d*x)*a*b^3*
d^10*x^7 + 2870*sqrt(d*x)*a^2*b^2*d^10*x^5 - 480*sqrt(d*x)*a^3*b*d^10*x^3
- 105*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^3*b^2))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.65

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{287\,d^9\,(d\,x)^{11/2}}{2048\,a}-\frac{3\,d^{11}\,(d\,x)^{7/2}}{128\,b}+\frac{63\,b^2\,d^5\,(d\,x)^{19/2}}{4096\,a^3}-\frac{21\,a\,d^{13}\,(d\,x)^{3/2}}{4096\,b^2}+\frac{189\,b\,d^7\,(d\,x)^{15/2}}{2560\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}} - \frac{63\,d^{9/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{13/4}\,b^{11/4}}+\frac{63\,d^{9/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{13/4}\,b^{11/4}}$$

input
```
int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((287*d^9*(d*x)^(11/2))/(2048*a) - (3*d^11*(d*x)^(7/2))/(128*b) + (63*b^2*
d^5*(d*x)^(19/2))/(4096*a^3) - (21*a*d^13*(d*x)^(3/2))/(4096*b^2) + (189*b
*d^7*(d*x)^(15/2))/(2560*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x^
2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) - (63*d^
(9/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(13/4)*
b^(11/4)) + (63*d^(9/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))
/(8192*(-a)^(13/4)*b^(11/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 996, normalized size of antiderivative = 3.14

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \text{Too large to display}$$

input

```
int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output

```
(sqrt(d)*d**4*( - 630*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 3150*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 6300*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**3*b**2*x**4 - 6300*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*x
**6 - 3150*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*s
qrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 630*b**(1/4)*a*
*(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1
/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 630*b**(1/4)*a**(3/4)*sqrt(2)*atan((b*
*(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*
a**5 + 3150*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 6300*b**(1/4)*
a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 6300*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**2*b**3*x**6 + 3150*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8
 + 630*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sq...
```

3.480 $\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 317

$$\begin{aligned}\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx &= -\frac{d(dx)^{5/2}}{10b(a+bx^2)^5} \\ &\quad -\frac{d^3\sqrt{dx}}{32b^2(a+bx^2)^4}+\frac{d^3\sqrt{dx}}{384ab^2(a+bx^2)^3}+\frac{11d^3\sqrt{dx}}{3072a^2b^2(a+bx^2)^2} \\ &\quad +\frac{77d^3\sqrt{dx}}{12288a^3b^2(a+bx^2)}-\frac{77d^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{15/4}b^{9/4}} \\ &\quad +\frac{77d^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^{7/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}a^{15/4}b^{9/4}}\end{aligned}$$

output
```
-1/10*d*(d*x)^(5/2)/b/(b*x^2+a)^5-1/32*d^3*(d*x)^(1/2)/b^2/(b*x^2+a)^4+1/3
84*d^3*(d*x)^(1/2)/a/b^2/(b*x^2+a)^3+11/3072*d^3*(d*x)^(1/2)/a^2/b^2/(b*x^
2+a)^2+77/12288*d^3*(d*x)^(1/2)/a^3/b^2/(b*x^2+a)-77/16384*d^(7/2)*arctan(
1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(15/4)/b^(9/4)+77
/16384*d^(7/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/
2)/a^(15/4)/b^(9/4)+77/16384*d^(7/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)
^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(15/4)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.59

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{d^3\sqrt{dx}\left(\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}(-1155a^4-5544a^3bx^2+3130a^2b^2x^4+1760ab^3x^6+385b^4x^8)}{(a+bx^2)^5}-1155\sqrt{2}\,\text{arc}\right.}{245760a^{15/4}b^{9/4}\sqrt{x}}$$

input
```
Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(d^3*Sqrt[d*x]*((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-1155*a^4 - 5544*a^3*b*x^2 + 3
130*a^2*b^2*x^4 + 1760*a*b^3*x^6 + 385*b^4*x^8))/(a + b*x^2)^5 - 1155*Sqrt
[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 1155
*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])
)/(245760*a^(15/4)*b^(9/4)*Sqrt[x])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.40, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 252, 252, 253, 253, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{(dx)^{7/2}}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(dx)^{7/2}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{d^2\int \frac{(dx)^{3/2}}{\left(bx^2+a\right)^5}dx}{4b}-\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 252$$

$$\frac{d^2\left(\frac{d^2\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^4}dx}{16b}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^4}\right)}{4b}-\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 253$$

$$\frac{d^2\left(\frac{d^2\left(\frac{11\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^3}dx}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^4}\right)}{4b}-\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 253$$

$$\frac{d^2\left(\frac{d^2\left(\frac{11\left(\frac{7\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^2}dx}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^4}\right)}{4b}-\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 253$$

$$\frac{d^2\left(\frac{d^2\left(\frac{11\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d\sqrt{dx}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 266

$$\frac{d^2\left(\frac{d^2\left(\frac{11\left(\frac{7\left(\frac{3\int \frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d\sqrt{dx}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 755

$$\frac{d^2\left(\frac{d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d\sqrt{dx}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow 27$$

$$\frac{d^2\left(\frac{d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d\sqrt{dx}}{8b(a+bx^2)^4}\right)}{4b}-\frac{d(dx)^{5/2}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow 1476$$

$$d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}\right.$$

↓ 1082

$$d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\quad}{6ad}\right.\right.$$

↓ 217

$$d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad(a+bx^2)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad(a+bx^2)^3}\right.$$

↓ 1479

$$d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\text{arc}\cdots}{\cdots}\right)}{2\sqrt{a}}\cdots\right)}{2ad}\cdots\right)}{8a}\cdots\right)}{12a}\cdots\right.$$

↓ 25

$$\left(d^2 \left(\frac{11 \left(\frac{7 \left(\frac{3 \left(\frac{d \left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} + \frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d \left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(\cdots \right.}{\sqrt{2} \cdots} \cdots \right.}{2\sqrt{a}} \cdots \right.}{2ad} \cdots \right.}{8a} \cdots \right.}{12a} \cdots \right. \right.$$

↓ 27

$$d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots}{2\sqrt{a}}\cdots}{2ad}\cdots}{8a}\cdots}{12a}\cdots$$

↓ 1103

$$d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right.}{2\sqrt{a}}\cdots\right.}{2ad}\cdots\right.}{8a}\cdots\right.}{12a}\cdots\right.$$

input
```
Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*(d*x)^(5/2))/(b*(a + b*x^2)^5) + (d^2*(-1/8*(d*Sqrt[d*x])/(b*(a +
 b*x^2)^4) + (d^2*(Sqrt[d*x]/(6*a*d*(a + b*x^2)^3) + (11*(Sqrt[d*x]/(4*a*d
*(a + b*x^2)^2) + (7*(Sqrt[d*x]/(2*a*d*(a + b*x^2)) + (3*((d*(-(ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt
[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*a*d))
)/(8*a)))/(12*a)))/(16*b)))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 14.57 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.74

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{77a\,d^2\sqrt{dx}}{8192b^2}-\frac{231(dx)^{\frac{5}{2}}}{5120b}+\frac{313(dx)^{\frac{9}{2}}}{12288a\,d^2}+\frac{11b(dx)^{\frac{13}{2}}}{768a^2d^4}+\frac{77b^2(dx)^{\frac{17}{2}}}{24576a^3d^6}}{(b\,d^2x^2+a\,d^2)^5}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\cdots}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\cdots}\right)\cdots\right.}{\cdots}\cdots\right.$
default	$2d^{11}\left(\frac{-\frac{77a\,d^2\sqrt{dx}}{8192b^2}-\frac{231(dx)^{\frac{5}{2}}}{5120b}+\frac{313(dx)^{\frac{9}{2}}}{12288a\,d^2}+\frac{11b(dx)^{\frac{13}{2}}}{768a^2d^4}+\frac{77b^2(dx)^{\frac{17}{2}}}{24576a^3d^6}}{(b\,d^2x^2+a\,d^2)^5}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\cdots}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\cdots}\right)\cdots\right.}{\cdots}\cdots\right.$
pseudoelliptic	$\frac{d^3\left(\left(3080a\,x^8b^4+14080a^2x^6b^3+25040a^3x^4b^2-44352x^2a^4b-9240a^5\right)\sqrt{dx}+1155\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,(b\,x^2+a)^5\left(\ln\left(\frac{dx+\left(\cdots\right.}{dx-\left(\cdots\right.}\cdots\right)\cdots\right)\cdots\right)}{491520a^4b^2(b\,x^2+a)^5}$

input
```
int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^11*((-77/8192*a*d^2/b^2*(d*x)^(1/2)-231/5120/b*(d*x)^(5/2)+313/12288/a
/d^2*(d*x)^(9/2)+11/768/a^2/d^4*b*(d*x)^(13/2)+77/24576/a^3/d^6*b^2*(d*x)^
(17/2))/(b*d^2*x^2+a*d^2)^5+77/65536/a^4/d^8/b^2*(a*d^2/b)^(1/4)*2^(1/2)*(
ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(
1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.82

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{1155\,(a^3b^7x^{10}+5\,a^4b^6x^8+10\,a^5b^5x^6+10\,a^6b^4x^4+5\,a^7b^3x^2+a^8b^2)\left(-\frac{d^{14}}{a^{15}b^9}\right)}{}$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/245760*(1155*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4
*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^14/(a^15*b^9))^(1/4)*log(77*a^4*b^2*(-
d^14/(a^15*b^9))^(1/4) + 77*sqrt(d*x)*d^3) - 1155*(-I*a^3*b^7*x^10 - 5*I*a
^4*b^6*x^8 - 10*I*a^5*b^5*x^6 - 10*I*a^6*b^4*x^4 - 5*I*a^7*b^3*x^2 - I*a^8
*b^2)*(-d^14/(a^15*b^9))^(1/4)*log(77*I*a^4*b^2*(-d^14/(a^15*b^9))^(1/4) +
 77*sqrt(d*x)*d^3) - 1155*(I*a^3*b^7*x^10 + 5*I*a^4*b^6*x^8 + 10*I*a^5*b^5
*x^6 + 10*I*a^6*b^4*x^4 + 5*I*a^7*b^3*x^2 + I*a^8*b^2)*(-d^14/(a^15*b^9))^
(1/4)*log(-77*I*a^4*b^2*(-d^14/(a^15*b^9))^(1/4) + 77*sqrt(d*x)*d^3) - 115
5*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*
b^3*x^2 + a^8*b^2)*(-d^14/(a^15*b^9))^(1/4)*log(-77*a^4*b^2*(-d^14/(a^15*b
^9))^(1/4) + 77*sqrt(d*x)*d^3) + 4*(385*b^4*d^3*x^8 + 1760*a*b^3*d^3*x^6 +
 3130*a^2*b^2*d^3*x^4 - 5544*a^3*b*d^3*x^2 - 1155*a^4*d^3)*sqrt(d*x))/(a^3
*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^
2 + a^8*b^2)
```

Sympy [F]

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \int \frac{(dx)^{\frac{7}{2}}}{(a+bx^2)^6}\,dx$$

input
```
integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Integral((d*x)**(7/2)/(a + b*x**2)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.24

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{8\left(385\,(dx)^{\frac{17}{2}}b^4d^6+1760\,(dx)^{\frac{13}{2}}ab^3d^8+3130\,(dx)^{\frac{9}{2}}a^2b^2d^{10}-5544\,(dx)^{\frac{5}{2}}a^3bd^{12}-1155\,\sqrt{dx}a^4d^{14}\right)}{a^3b^7d^{10}x^{10}+5\,a^4b^6d^{10}x^8+10\,a^5b^5d^{10}x^6+10\,a^6b^4d^{10}x^4+5\,a^7b^3d^{10}x^2+a^8b^2d^{10}} + -$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/491520*(8*(385*(d*x)^(17/2)*b^4*d^6 + 1760*(d*x)^(13/2)*a*b^3*d^8 + 3130
*(d*x)^(9/2)*a^2*b^2*d^10 - 5544*(d*x)^(5/2)*a^3*b*d^12 - 1155*sqrt(d*x)*a
^4*d^14)/(a^3*b^7*d^10*x^10 + 5*a^4*b^6*d^10*x^8 + 10*a^5*b^5*d^10*x^6 + 1
0*a^6*b^4*d^10*x^4 + 5*a^7*b^3*d^10*x^2 + a^8*b^2*d^10) + 1155*(sqrt(2)*d^
6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(
(a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/
4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^5*
arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/s
qrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^5*
arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/
sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^3*b^2))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{\frac{2310\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d^4\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^3} + \frac{2310\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d^4\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right.\right.}{2\left(\frac{ad}{b}\right.}\right.}{a^4b^3}}{}$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/491520*(2310*sqrt(2)*(a*b^3*d^2)^(1/4)*d^4*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3) + 2310*sqrt(2)*(a
*b^3*d^2)^(1/4)*d^4*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(
d*x))/(a*d^2/b)^(1/4))/(a^4*b^3) + 1155*sqrt(2)*(a*b^3*d^2)^(1/4)*d^4*log(
d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) - 1155*
sqrt(2)*(a*b^3*d^2)^(1/4)*d^4*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
+ sqrt(a*d^2/b))/(a^4*b^3) + 8*(385*sqrt(d*x)*b^4*d^14*x^8 + 1760*sqrt(d*x
)*a*b^3*d^14*x^6 + 3130*sqrt(d*x)*a^2*b^2*d^14*x^4 - 5544*sqrt(d*x)*a^3*b*
d^14*x^2 - 1155*sqrt(d*x)*a^4*d^14)/((b*d^2*x^2 + a*d^2)^5*a^3*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.65

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{313\,d^9\,(d\,x)^{9/2}}{6144\,a}-\frac{231\,d^{11}\,(d\,x)^{5/2}}{2560\,b}+\frac{77\,b^2\,d^5\,(d\,x)^{17/2}}{12288\,a^3}-\frac{77\,a\,d^{13}\,\sqrt{d\,x}}{4096\,b^2}+\frac{11\,b\,d^7\,(d\,x)^{13/2}}{384\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}} + \frac{77\,d^{7/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{15/4}\,b^{9/4}} + \frac{77\,d^{7/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{15/4}\,b^{9/4}}$$

input
```
int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((313*d^9*(d*x)^(9/2))/(6144*a) - (231*d^11*(d*x)^(5/2))/(2560*b) + (77*b^
2*d^5*(d*x)^(17/2))/(12288*a^3) - (77*a*d^13*(d*x)^(1/2))/(4096*b^2) + (11
*b*d^7*(d*x)^(13/2))/(384*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*x
^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) + (77*d
^(7/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(15/4)
*b^(9/4)) + (77*d^(7/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))
/(8192*(-a)^(15/4)*b^(9/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 995, normalized size of antiderivative = 3.14

$$\int \frac{(dx)^{7/2}}{(a^2 + 2abx^2 + b^2x^4)^3} \, dx = \text{Too large to display}$$

input
```
int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*d**3*( - 2310*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 11550*b**(
3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 23100*b**(3/4)*a**(1/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*
sqrt(2)))*a**3*b**2*x**4 - 23100*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*
*3*x**6 - 11550*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
- 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 2310*b**(3
/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/
(b**(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 2310*b**(3/4)*a**(1/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**5 + 11550*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 23100*
b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 23100*b**(3/4)*a**(1/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*a**2*b**3*x**6 + 11550*b**(3/4)*a**(1/4)*sqrt(2)*atan((b*
*(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*
a*b**4*x**8 + 2310*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq...
```

3.481 $\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 312

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^3} dx = -\frac{d(dx)^{3/2}}{10b(a+bx^2)^5} + \frac{3d(dx)^{3/2}}{160ab(a+bx^2)^4} + \frac{13d(dx)^{3/2}}{640a^2b(a+bx^2)^3} + \frac{117d(dx)^{3/2}}{5120a^3b(a+bx^2)^2} + \frac{117d(dx)^{3/2}}{4096a^4b(a+bx^2)} - \frac{117d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{17/4}b^{7/4}} + \frac{117d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{17/4}b^{7/4}} - \frac{117d^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}(\sqrt{a}+\sqrt{b}x)}\right)}{8192\sqrt{2}a^{17/4}b^{7/4}}$$

output

```
-1/10*d*(d*x)^(3/2)/b/(b*x^2+a)^5+3/160*d*(d*x)^(3/2)/a/b/(b*x^2+a)^4+13/6
40*d*(d*x)^(3/2)/a^2/b/(b*x^2+a)^3+117/5120*d*(d*x)^(3/2)/a^3/b/(b*x^2+a)^
2+117/4096*d*(d*x)^(3/2)/a^4/b/(b*x^2+a)-117/16384*d^(5/2)*arctan(1-2^(1/2
)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(17/4)/b^(7/4)+117/16384*
d^(5/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(1
7/4)/b^(7/4)-117/16384*d^(5/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)
/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(17/4)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.59

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{(dx)^{5/2}\left(\frac{4\sqrt[4]{a}b^{3/4}x^{3/2}(-195a^4+4960a^3bx^2+5330a^2b^2x^4+2808ab^3x^6+585b^4x^8)}{(a+bx^2)^5}-585\sqrt{2}\,\mathrm{arc}\right.}{81920a^{17/4}b^{7/4}x^{5/2}}$$

input
```
Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
((d*x)^(5/2)*((4*a^(1/4)*b^(3/4)*x^(3/2)*(-195*a^4 + 4960*a^3*b*x^2 + 5330
*a^2*b^2*x^4 + 2808*a*b^3*x^6 + 585*b^4*x^8))/(a + b*x^2)^5 - 585*Sqrt[2]*
ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 585*Sqrt
[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(81
920*a^(17/4)*b^(7/4)*x^(5/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.40, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 252, 253, 253, 253, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$

$$\Big\downarrow \; 1380$$

$$b^6\int \frac{(dx)^{5/2}}{b^6(bx^2+a)^6}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{(dx)^{5/2}}{(a+bx^2)^6}dx$$

$$\Big\downarrow \; 252$$

$$\frac{3d^2\int \frac{\sqrt{dx}}{(bx^2+a)^5}dx}{20b}-\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5}$$

$\Big\downarrow$ 253

$$\frac{3d^2\left(\frac{13\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^4}dx}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5}$$

$\Big\downarrow$ 253

$$\frac{3d^2\left(\frac{13\left(\frac{3\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^3}dx}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5}$$

$\Big\downarrow$ 253

$$\frac{3d^2\left(\frac{13\left(\frac{3\left(\frac{5\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^2}dx}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5}$$

$\Big\downarrow$ 253

$$\frac{3d^2\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{\int\frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 266

$$\frac{3d^2\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{\int\frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 27

$$\frac{3d^2\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 826

$$\frac{3d^2\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20b}-\frac{d(dx)^{3/2}}{10b\left(a+bx^2\right)^5}$$

$\downarrow$ 1476

$$3d^2\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\right.}{16a}\right.$$

↓ 1082

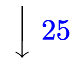
$$3d^2\left(\frac{13\left(\frac{3\left(\frac{5d\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/}}{6ad\left(a+bx\right.}}{16a}\right.$$

↓ 217

$$3d^2\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+$$

↓ 1479

$$\left(\frac{13\left(\frac{3\left(\frac{5\left(d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\cdots}\right.}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\cdots}{2\sqrt{b}}\cdots\right.\right.}{2a}\cdots\right.}{8a}\cdots\right.}{4a}\cdots\right.$$


25

$$\left(13\frac{\left(3\frac{\left(5\frac{\left(d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}\right)}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)\right)}{2a}\right)}{8a}\right)}{4a}$$

↓ 27

$$13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}+\frac{\cdots}{2\cdots}\right)}{8a}\cdots\right)}{4a}\cdots\right)\cdots$$

↓ 1103

$$3d^2\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right.}{2a}\right.}{8a}\right.}{4a}\right.}{16a}\right.$$

input
```
Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*(d*x)^(3/2))/(b*(a + b*x^2)^5) + (3*d^2*((d*x)^(3/2)/(8*a*d*(a +
b*x^2)^4) + (13*((d*x)^(3/2)/(6*a*d*(a + b*x^2)^3) + (3*((d*x)^(3/2)/(4*a*
d*(a + b*x^2)^2) + (5*((d*x)^(3/2)/(2*a*d*(a + b*x^2)) + (d*((-(ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*
d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*S
qrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*a)))/(8*a)
))/(4*a)))/(16*a)))/(20*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 14.54 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.76

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{39(dx)^{\frac{3}{2}}}{8192b}+\frac{31(dx)^{\frac{7}{2}}}{256a\,d^{2}}+\frac{533b(dx)^{\frac{11}{2}}}{4096a^{2}d^{4}}+\frac{351b^{2}(dx)^{\frac{15}{2}}}{5120a^{3}d^{6}}+\frac{117b^{3}(dx)^{\frac{19}{2}}}{8192a^{4}d^{8}}}{(b\,d^{2}x^{2}+a\,d^{2})^{5}}+\frac{117\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right.\right.}{655}\right.$
default	$2d^{11}\left(\frac{-\frac{39(dx)^{\frac{3}{2}}}{8192b}+\frac{31(dx)^{\frac{7}{2}}}{256a\,d^{2}}+\frac{533b(dx)^{\frac{11}{2}}}{4096a^{2}d^{4}}+\frac{351b^{2}(dx)^{\frac{15}{2}}}{5120a^{3}d^{6}}+\frac{117b^{3}(dx)^{\frac{19}{2}}}{8192a^{4}d^{8}}}{(b\,d^{2}x^{2}+a\,d^{2})^{5}}+\frac{117\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right.\right.}{655}\right.$
pseudoelliptic	$\frac{39d^{2}\left(-8\sqrt{dx}\left(-3b^{4}x^{8}-\frac{72}{5}a\,b^{3}x^{6}-\frac{82}{3}a^{2}b^{2}x^{4}-\frac{992}{39}a^{3}b\,x^{2}+a^{4}\right)bx\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}+3d\sqrt{2}\,(b\,x^{2}+a)^{5}\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}}\right.\right.\right.}{32768\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}a^{4}b^{2}(b\,x^{2}+a)^{5}}$

input
```
int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^11*((-39/8192/b*(d*x)^(3/2)+31/256/a/d^2*(d*x)^(7/2)+533/4096/a^2/d^4*
b*(d*x)^(11/2)+351/5120/a^3*b^2/d^6*(d*x)^(15/2)+117/8192/a^4/d^8*b^3*(d*x
)^(19/2))/(b*d^2*x^2+a*d^2)^5+117/65536/a^4/d^8/b^2/(a*d^2/b)^(1/4)*2^(1/2
)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b
)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.82

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{585\,(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b)\left(-\frac{d^{10}}{a^{17}b^7}\right)^{\frac{1}{4}}}{}$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/81920*(585*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x
^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*log(1601613*a^13*b^5*
(-d^10/(a^17*b^7))^(3/4) + 1601613*sqrt(d*x)*d^7) - 585*(I*a^4*b^6*x^10 +
5*I*a^5*b^5*x^8 + 10*I*a^6*b^4*x^6 + 10*I*a^7*b^3*x^4 + 5*I*a^8*b^2*x^2 +
I*a^9*b)*(-d^10/(a^17*b^7))^(1/4)*log(1601613*I*a^13*b^5*(-d^10/(a^17*b^7)
)^(3/4) + 1601613*sqrt(d*x)*d^7) - 585*(-I*a^4*b^6*x^10 - 5*I*a^5*b^5*x^8
- 10*I*a^6*b^4*x^6 - 10*I*a^7*b^3*x^4 - 5*I*a^8*b^2*x^2 - I*a^9*b)*(-d^10/
(a^17*b^7))^(1/4)*log(-1601613*I*a^13*b^5*(-d^10/(a^17*b^7))^(3/4) + 16016
13*sqrt(d*x)*d^7) - 585*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 1
0*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*log(-16016
13*a^13*b^5*(-d^10/(a^17*b^7))^(3/4) + 1601613*sqrt(d*x)*d^7) + 4*(585*b^4
*d^2*x^9 + 2808*a*b^3*d^2*x^7 + 5330*a^2*b^2*d^2*x^5 + 4960*a^3*b*d^2*x^3
- 195*a^4*d^2*x)*sqrt(d*x))/(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6
 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)
```

Sympy [F]

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{(a+bx^2)^6}\,dx$$

input
```
integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Integral((d*x)**(5/2)/(a + b*x**2)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.23

5

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{8\left(585\,(dx)^{\frac{19}{2}}b^4d^4+2808\,(dx)^{\frac{15}{2}}ab^3d^6+5330\,(dx)^{\frac{11}{2}}a^2b^2d^8+4960\,(dx)^{\frac{7}{2}}a^3bd^{10}-195\,(dx)^{\frac{3}{2}}a^4d^{12}\right)}{a^4b^6d^{10}x^{10}+5\,a^5b^5d^{10}x^8+10\,a^6b^4d^{10}x^6+10\,a^7b^3d^{10}x^4+5\,a^8b^2d^{10}x^2+a^9bd^{10}} + -$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/163840*(8*(585*(d*x)^(19/2)*b^4*d^4 + 2808*(d*x)^(15/2)*a*b^3*d^6 + 5330
*(d*x)^(11/2)*a^2*b^2*d^8 + 4960*(d*x)^(7/2)*a^3*b*d^10 - 195*(d*x)^(3/2)*
a^4*d^12)/(a^4*b^6*d^10*x^10 + 5*a^5*b^5*d^10*x^8 + 10*a^6*b^4*d^10*x^6 +
10*a^7*b^3*d^10*x^4 + 5*a^8*b^2*d^10*x^2 + a^9*b*d^10) + 585*d^4*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))
/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sq
rt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(sqr
t(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(
1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*
b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^4*b))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.14

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{1}{163840}\,d^2\left(\frac{1170\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^4d} + \frac{1170\,\sqrt{2}(ab^3\right.$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/163840*d^2*(1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^4*d) + 1170*sqrt(2)*
(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*
x))/(a*d^2/b)^(1/4))/(a^5*b^4*d) - 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x +
 sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^4*d) + 585*sqrt
(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^5*b^4*d) + 8*(585*sqrt(d*x)*b^4*d^10*x^9 + 2808*sqrt(d*x)*a*b^
3*d^10*x^7 + 5330*sqrt(d*x)*a^2*b^2*d^10*x^5 + 4960*sqrt(d*x)*a^3*b*d^10*x
^3 - 195*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x^2 + a*d^2)^5*a^4*b))
```

Mupad [B] (verification not implemented)

Time = 17.75 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.67

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{31\,d^9\,(d\,x)^{7/2}}{128\,a}-\frac{39\,d^{11}\,(d\,x)^{3/2}}{4096\,b}+\frac{351\,b^2\,d^5\,(d\,x)^{15/2}}{2560\,a^3}+\frac{117\,b^3\,d^3\,(d\,x)^{19/2}}{4096\,a^4}+\frac{533\,b\,d^7\,(d\,x)^{11/2}}{2048\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}} + \frac{117\,d^{5/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{17/4}\,b^{7/4}} - \frac{117\,d^{5/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{17/4}\,b^{7/4}}$$

input
```
int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((31*d^9*(d*x)^(7/2))/(128*a) - (39*d^11*(d*x)^(3/2))/(4096*b) + (351*b^2*
d^5*(d*x)^(15/2))/(2560*a^3) + (117*b^3*d^3*(d*x)^(19/2))/(4096*a^4) + (53
3*b*d^7*(d*x)^(11/2))/(2048*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10
*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) + (11
7*d^(5/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(17
/4)*b^(7/4)) - (117*d^(5/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2
))))/(8192*(-a)^(17/4)*b^(7/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 996, normalized size of antiderivative = 3.19

$$\int \frac{(dx)^{5/2}}{(a^2 + 2abx^2 + b^2x^4)^3} \, dx = \text{Too large to display}$$

input
```
int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*d**2*( - 1170*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 5850*b**(1
/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/
(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 11700*b**(1/4)*a**(3/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*s
qrt(2)))*a**3*b**2*x**4 - 11700*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**
3*x**6 - 5850*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 1170*b**(1/4
)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 1170*b**(1/4)*a**(3/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(
2)))*a**5 + 5850*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 11700*b**
(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 11700*b**(1/4)*a**(3/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1
/4)*sqrt(2)))*a**2*b**3*x**6 + 5850*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/
4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*
*4*x**8 + 1170*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2...
```

3.482 $\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 312

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = -\frac{d\sqrt{dx}}{10b(a+bx^2)^5} + \frac{d\sqrt{dx}}{160ab(a+bx^2)^4} + \frac{d\sqrt{dx}}{128a^2b(a+bx^2)^3} + \frac{11d\sqrt{dx}}{1024a^3b(a+bx^2)^2} + \frac{77d\sqrt{dx}}{4096a^4b(a+bx^2)} - \frac{231d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{19/4}b^{5/4}} + \frac{231d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{19/4}b^{5/4}} + \frac{231d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}a^{19/4}b^{5/4}}$$

output
```
-1/10*d*(d*x)^(1/2)/b/(b*x^2+a)^5+1/160*d*(d*x)^(1/2)/a/b/(b*x^2+a)^4+1/12
8*d*(d*x)^(1/2)/a^2/b/(b*x^2+a)^3+11/1024*d*(d*x)^(1/2)/a^3/b/(b*x^2+a)^2+
77/4096*d*(d*x)^(1/2)/a^4/b/(b*x^2+a)-231/16384*d^(3/2)*arctan(1-2^(1/2)*b
^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(19/4)/b^(5/4)+231/16384*d^(
3/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(19/4
)/b^(5/4)+231/16384*d^(3/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^
(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(19/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.59

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx=\frac{(dx)^{3/2}\left(\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}\left(-1155a^4+2648a^3bx^2+3130a^2b^2x^4+1760ab^3x^6+385b^4x^8\right)}{(a+bx^2)^5}-1155\sqrt{2}\,\mathrm{ar}\right.}{81920a^{19/4}b^{5/4}x^{3/2}}$$

input
```
Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
((d*x)^(3/2)*((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-1155*a^4 + 2648*a^3*b*x^2 + 313
0*a^2*b^2*x^4 + 1760*a*b^3*x^6 + 385*b^4*x^8))/(a + b*x^2)^5 - 1155*Sqrt[2
]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 1155*S
qrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/
(81920*a^(19/4)*b^(5/4)*x^(3/2))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.42, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.607, Rules used = {1380, 27, 252, 253, 253, 253, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}dx$$

$$\Big\downarrow \ 1380$$

$$b^6\int \frac{(dx)^{3/2}}{b^6\left(bx^2+a\right)^6}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{(dx)^{3/2}}{\left(a+bx^2\right)^6}dx$$

$$\Big\downarrow \ 252$$

$$\frac{d^2 \int \frac{1}{\sqrt{dx}(bx^2+a)^5}dx}{20b} - \frac{d\sqrt{dx}}{10b\,(a+bx^2)^5}$$

$\big\downarrow$ 253

$$\frac{d^2\left(\frac{15\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^4}dx}{16a} + \frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20b} - \frac{d\sqrt{dx}}{10b\,(a+bx^2)^5}$$

$\big\downarrow$ 253

$$\frac{d^2\left(\frac{15\left(\frac{11\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^3}dx}{12a} + \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a} + \frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20b} - \frac{d\sqrt{dx}}{10b\,(a+bx^2)^5}$$

$\big\downarrow$ 253

$$\frac{d^2\left(\frac{15\left(\frac{11\left(\frac{7\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^2}dx}{8a} + \frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a} + \frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a} + \frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20b} - \frac{d\sqrt{dx}}{10b\,(a+bx^2)^5}$$

$\big\downarrow$ 253

$$\frac{d^2\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20b}-\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 266

$$\frac{d^2\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\int \frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20b}-\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 755

$$\frac{d^2\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20b}-\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{d^2\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20b}-\frac{d\sqrt{dx}}{10b\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 1476$$

$$\left(15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}\,d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}\right.\right.$$

↓ 1082

$$\left(15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{}{6ad\cdots}\right.\right.$$

↓ 217

$$\left(15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right.\right.$$

↓ 1479

$$\left(15\,\frac{\left(11\,\frac{\left(7\,\frac{\left(3\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\text{arc}\ldots}{\ldots}\right.}{2\sqrt{a}}\ldots\right.}{2ad}\ldots\right.}{8a}\ldots\right.}{12a}\ldots\right.$$

$$\Big\downarrow 25$$

$$15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(\cdots\right.}{\sqrt{2}\cdots}\cdots\right.}{2\sqrt{a}}\cdots\right.}{2ad}\cdots\right.}{8a}\cdots\right.}{12a}\cdots\right.$$

$$\Big\downarrow \, 27$$

$$\left(15\frac{\left(11\frac{\left(7\frac{\left(3\frac{\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{2\sqrt{a}}\right.}{2ad}\right.}{8a}\right.}{12a}\right.$$

↓ 1103

$$15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{\cdots}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\cdots}\cdots\right)}{2\sqrt{a}\cdots}\cdots}{2ad\cdots}\cdots}{8a\cdots}\cdots}{12a\cdots}\cdots$$

input
```
Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
-1/10*(d*Sqrt[d*x])/(b*(a + b*x^2)^5) + (d^2*(Sqrt[d*x]/(8*a*d*(a + b*x^2)
^4) + (15*(Sqrt[d*x]/(6*a*d*(a + b*x^2)^3) + (11*(Sqrt[d*x]/(4*a*d*(a + b*
x^2)^2) + (7*(Sqrt[d*x]/(2*a*d*(a + b*x^2)) + (3*((d*(-(ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))
 + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[
d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*a*d)))/(8*a))
)/(12*a)))/(16*a)))/(20*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.76

method	result
derivativedivides	$2d^{11}\left(\frac{-\frac{231\sqrt{dx}}{8192b}+\frac{331(dx)^{\frac{5}{2}}}{5120a\,d^2}+\frac{313b(dx)^{\frac{9}{2}}}{4096a^2d^4}+\frac{11b^2(dx)^{\frac{13}{2}}}{256a^3d^6}+\frac{77b^3(dx)^{\frac{17}{2}}}{8192a^4d^8}}{(b\,d^2x^2+a\,d^2)^5}+\frac{231\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{-\frac{231\sqrt{dx}}{8192b}+\frac{331(dx)^{\frac{5}{2}}}{5120a\,d^2}+\frac{313b(dx)^{\frac{9}{2}}}{4096a^2d^4}+\frac{11b^2(dx)^{\frac{13}{2}}}{256a^3d^6}+\frac{77b^3(dx)^{\frac{17}{2}}}{8192a^4d^8}}{(b\,d^2x^2+a\,d^2)^5}+\frac{231\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+}\right.\right.}{}\right.$
pseudoelliptic	$\frac{d\left(\left(3080a\,x^8b^4+14080a^2x^6b^3+25040a^3x^4b^2+21184x^2a^4b-9240a^5\right)\sqrt{dx}+1155\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,(b\,x^2+a)^5\left(\ln\left(\frac{dx+\left(\frac{a\,d}{b}\right.}{dx-\left(\frac{a\,d}{b}\right.}\right.\right.\right.}{163840b\,a^5(b\,x^2+a)^5}$

input
```
int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^11*((-231/8192/b*(d*x)^(1/2)+331/5120/a/d^2*(d*x)^(5/2)+313/4096/a^2/d
^4*b*(d*x)^(9/2)+11/256/a^3/d^6*b^2*(d*x)^(13/2)+77/8192/a^4/d^8*b^3*(d*x)
^(17/2))/(b*d^2*x^2+a*d^2)^5+231/65536/a^5/d^10/b*(a*d^2/b)^(1/4)*2^(1/2)*
(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^
(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.73

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{1155\,(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b)\left(-\frac{d^6}{a^{19}b^5}\right)}{}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/81920*(1155*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*
x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^6/(a^19*b^5))^(1/4)*log(231*a^5*b*(-d^6/(
a^19*b^5))^(1/4) + 231*sqrt(d*x)*d) - 1155*(-I*a^4*b^6*x^10 - 5*I*a^5*b^5*
x^8 - 10*I*a^6*b^4*x^6 - 10*I*a^7*b^3*x^4 - 5*I*a^8*b^2*x^2 - I*a^9*b)*(-d
^6/(a^19*b^5))^(1/4)*log(231*I*a^5*b*(-d^6/(a^19*b^5))^(1/4) + 231*sqrt(d*
x)*d) - 1155*(I*a^4*b^6*x^10 + 5*I*a^5*b^5*x^8 + 10*I*a^6*b^4*x^6 + 10*I*a
^7*b^3*x^4 + 5*I*a^8*b^2*x^2 + I*a^9*b)*(-d^6/(a^19*b^5))^(1/4)*log(-231*I
*a^5*b*(-d^6/(a^19*b^5))^(1/4) + 231*sqrt(d*x)*d) - 1155*(a^4*b^6*x^10 + 5
*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-
d^6/(a^19*b^5))^(1/4)*log(-231*a^5*b*(-d^6/(a^19*b^5))^(1/4) + 231*sqrt(d*
x)*d) + 4*(385*b^4*d*x^8 + 1760*a*b^3*d*x^6 + 3130*a^2*b^2*d*x^4 + 2648*a^
3*b*d*x^2 - 1155*a^4*d)*sqrt(d*x))/(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*
b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)
```

Sympy [F]

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(a+bx^2)^6}\,dx$$

input
```
integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Integral((d*x)**(3/2)/(a + b*x**2)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.26

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{8\left(385\,(dx)^{\frac{17}{2}}b^4d^4+1760\,(dx)^{\frac{13}{2}}ab^3d^6+3130\,(dx)^{\frac{9}{2}}a^2b^2d^8+2648\,(dx)^{\frac{5}{2}}a^3bd^{10}-1155\,\sqrt{dx}a^4d^{12}\right)}{a^4b^6d^{10}x^{10}+5\,a^5b^5d^{10}x^8+10\,a^6b^4d^{10}x^6+10\,a^7b^3d^{10}x^4+5\,a^8b^2d^{10}x^2+a^9bd^{10}} + \ldots$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/163840*(8*(385*(d*x)^(17/2)*b^4*d^4 + 1760*(d*x)^(13/2)*a*b^3*d^6 + 3130
*(d*x)^(9/2)*a^2*b^2*d^8 + 2648*(d*x)^(5/2)*a^3*b*d^10 - 1155*sqrt(d*x)*a^
4*d^12)/(a^4*b^6*d^10*x^10 + 5*a^5*b^5*d^10*x^8 + 10*a^6*b^4*d^10*x^6 + 10
*a^7*b^3*d^10*x^4 + 5*a^8*b^2*d^10*x^2 + a^9*b*d^10) + 1155*(sqrt(2)*d^4*l
og(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*
d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^4*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*
sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^3*arc
tan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt
(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^3*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqr
t(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/(a^4*b))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.13

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{2310\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2} + \frac{2310\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\ldots\right.\right.}{2\left(\ldots\right.}\ldots\right)}{a^5b^2}\ldots}{\ldots}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/163840*(2310*sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2) + 2310*sqrt(2)*(a
*b^3*d^2)^(1/4)*d^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(
d*x))/(a*d^2/b)^(1/4))/(a^5*b^2) + 1155*sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*log(
d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^2) - 1155*
sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
+ sqrt(a*d^2/b))/(a^5*b^2) + 8*(385*sqrt(d*x)*b^4*d^12*x^8 + 1760*sqrt(d*x
)*a*b^3*d^12*x^6 + 3130*sqrt(d*x)*a^2*b^2*d^12*x^4 + 2648*sqrt(d*x)*a^3*b*
d^12*x^2 - 1155*sqrt(d*x)*a^4*d^12)/((b*d^2*x^2 + a*d^2)^5*a^4*b))/d
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.67

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{331\,d^9\,(d\,x)^{5/2}}{2560\,a}-\frac{231\,d^{11}\,\sqrt{d\,x}}{4096\,b}+\frac{11\,b^2\,d^5\,(d\,x)^{13/2}}{128\,a^3}+\frac{77\,b^3\,d^3\,(d\,x)^{17/2}}{4096\,a^4}+\frac{313\,b\,d^7\,(d\,x)^{9/2}}{2048\,a^2}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x}$$
$$-\frac{231\,d^{3/2}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{19/4}\,b^{5/4}}-\frac{231\,d^{3/2}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{19/4}\,b^{5/4}}$$

input
```
int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((331*d^9*(d*x)^(5/2))/(2560*a) - (231*d^11*(d*x)^(1/2))/(4096*b) + (11*b^
2*d^5*(d*x)^(13/2))/(128*a^3) + (77*b^3*d^3*(d*x)^(17/2))/(4096*a^4) + (31
3*b*d^7*(d*x)^(9/2))/(2048*a^2))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^4*b*d^10*
x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^6) - (231
*d^(3/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(19/
4)*b^(5/4)) - (231*d^(3/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2)
)))/(8192*(-a)^(19/4)*b^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 993, normalized size of antiderivative = 3.18

$$\int \frac{(dx)^{3/2}}{(a^2 + 2abx^2 + b^2x^4)^3} \, dx = \text{Too large to display}$$

input
```
int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*d*( - 2310*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 11550*b**(3/4
)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 23100*b**(3/4)*a**(1/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**3*b**2*x**4 - 23100*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**
(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*
x**6 - 11550*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 2310*b**(3/4)
*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 2310*b**(3/4)*a**(1/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*a**5 + 11550*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 23100*b**
(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 23100*b**(3/4)*a**(1/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1
/4)*sqrt(2)))*a**2*b**3*x**6 + 11550*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1
/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b
**4*x**8 + 2310*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(...
```

3.483 $\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 310

$$\begin{aligned}\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^3}\,dx &= \frac{(dx)^{3/2}}{10ad\,(a+bx^2)^5}+\frac{17(dx)^{3/2}}{160a^2d\,(a+bx^2)^4}\\
&+\frac{221(dx)^{3/2}}{1920a^3d\,(a+bx^2)^3}+\frac{663(dx)^{3/2}}{5120a^4d\,(a+bx^2)^2}\\
&+\frac{663(dx)^{3/2}}{4096a^5d\,(a+bx^2)}-\frac{663\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{21/4}b^{3/4}}\\
&+\frac{663\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{21/4}b^{3/4}}\\
&-\frac{663\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}a^{21/4}b^{3/4}}\end{aligned}$$

output
```
1/10*(d*x)^(3/2)/a/d/(b*x^2+a)^5+17/160*(d*x)^(3/2)/a^2/d/(b*x^2+a)^4+221/
1920*(d*x)^(3/2)/a^3/d/(b*x^2+a)^3+663/5120*(d*x)^(3/2)/a^4/d/(b*x^2+a)^2+
663/4096*(d*x)^(3/2)/a^5/d/(b*x^2+a)-663/16384*d^(1/2)*arctan(1-2^(1/2)*b^
(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(21/4)/b^(3/4)+663/16384*d^(1
/2)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(21/4)
/b^(3/4)-663/16384*d^(1/2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(
1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(21/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\sqrt{dx}\left(\frac{4\sqrt[4]{a}x^{3/2}(37645a^4+84320a^3bx^2+90610a^2b^2x^4+47736ab^3x^6+9945b^4x^8)}{(a+bx^2)^5}-\frac{9945\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}}-\frac{9945\sqrt{2}\text{arcta}}{}\right.}{245760a^{21/4}\sqrt{x}}$$

input
```
Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(Sqrt[d*x]*((4*a^(1/4)*x^(3/2)*(37645*a^4 + 84320*a^3*b*x^2 + 90610*a^2*b^
2*x^4 + 47736*a*b^3*x^6 + 9945*b^4*x^8))/(a + b*x^2)^5 - (9945*Sqrt[2]*Arc
Tan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) - (9
945*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x
)])/b^(3/4)))/(245760*a^(21/4)*Sqrt[x])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.41, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 253, 253, 253, 253, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed

below.

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{\sqrt{dx}}{b^6\left(bx^2+a\right)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{dx}}{\left(a+bx^2\right)^6}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{17\int \frac{\sqrt{dx}}{(bx^2+a)^5}dx}{20a}+\frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{17\left(\frac{13\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^4}dx}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{17\left(\frac{13\left(\frac{3\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^3}dx}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{17\left(\frac{13\left(\frac{3\left(\frac{5\int\frac{\sqrt{dx}}{\left(bx^2+a\right)^2}dx}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5}$$

$\downarrow$ 253

$$\frac{17\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{\int\frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5}$$

$\downarrow$ 266

$$\frac{17\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{\int \frac{d^3 x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 27

$$\frac{17\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 826

$$\frac{17\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5}$$

$\downarrow$ 1476

$$17\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{}{6a\cdots}\right)}{16a}\right.$$

↓ 1082

$$17\,\frac{\left(13\,\frac{\left(3\,\frac{\left(5\,\frac{d\left(\frac{\frac{\int \frac{1}{-dx-1}\,d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}\,d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right.}$$

$$\overline{\qquad\qquad 16a \qquad\qquad}$$

$\downarrow$ 217

$$\left(17\,\frac{\left(13\,\frac{\left(3\,\frac{\left(5\,\frac{\left(d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\right.$$

↓ 1479

$$13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\sqrt[4]{b}\cdots}\right.}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\cdots}}{2\sqrt{b}}\cdots\right.}{2a}\cdots\right.}{8a}\cdots\right.}{4a}\cdots\right.$$

↓ 25

$$\left(\frac{13\left(\frac{3\left(\frac{5\left(d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d} \cdots}{\sqrt[4]{b}}\cdots\right.}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\cdots}}{2\sqrt{b}}\cdots\right.\right.}{2a}\cdots\right.}{8a}\cdots\right.}{4a}\cdots\right.$$

$$\Big\downarrow 27$$

$$13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}\right)+\frac{\cdots}{2a\cdots}}{8a}\cdots\right)}{4a}\cdots\right)\cdots$$

↓ 1103

$$\frac{17\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right.}{2a}\right.}{8a}\right.}{4a}\right.}{16a}\right.}$$

input
```
Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(d*x)^(3/2)/(10*a*d*(a + b*x^2)^5) + (17*((d*x)^(3/2)/(8*a*d*(a + b*x^2)^4
) + (13*((d*x)^(3/2)/(6*a*d*(a + b*x^2)^3) + (3*((d*x)^(3/2)/(4*a*d*(a + b
*x^2)^2) + (5*((d*x)^(3/2)/(2*a*d*(a + b*x^2)) + (d*((-(ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))
 + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])
 + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]
]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*a)))/(8*a)))/(4*a)
))/(16*a)))/(20*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.79

method	result
derivativedivides	$2d^{11}\left(\frac{\frac{7529(dx)^{\frac{3}{2}}}{24576a\,d^{2}}+\frac{527b(dx)^{\frac{7}{2}}}{768a^{2}d^{4}}+\frac{9061b^{2}(dx)^{\frac{11}{2}}}{12288a^{3}d^{6}}+\frac{1989b^{3}(dx)^{\frac{15}{2}}}{5120a^{4}d^{8}}+\frac{663b^{4}(dx)^{\frac{19}{2}}}{8192d^{10}a^{5}}}{(b\,d^{2}x^{2}+a\,d^{2})^{5}}+\frac{663\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{}}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{\frac{7529(dx)^{\frac{3}{2}}}{24576a\,d^{2}}+\frac{527b(dx)^{\frac{7}{2}}}{768a^{2}d^{4}}+\frac{9061b^{2}(dx)^{\frac{11}{2}}}{12288a^{3}d^{6}}+\frac{1989b^{3}(dx)^{\frac{15}{2}}}{5120a^{4}d^{8}}+\frac{663b^{4}(dx)^{\frac{19}{2}}}{8192d^{10}a^{5}}}{(b\,d^{2}x^{2}+a\,d^{2})^{5}}+\frac{663\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{}}\right.\right.}{}\right.$
pseudoelliptic	$\frac{60232\sqrt{dx}\,\left(\frac{1989}{7529}b^{4}x^{8}+\frac{47736}{37645}a\,b^{3}x^{6}+\frac{18122}{7529}a^{2}b^{2}x^{4}+\frac{16864}{7529}a^{3}b\,x^{2}+a^{4}\right)bx\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}+1989d\sqrt{2}\,(b\,x^{2}+a)^{5}\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right.}{dx+\left(\frac{a\,d^{2}}{b}\right.}\right.\right.}{98304\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}b\,a^{5}(b\,x^{2}+a)^{5}}$

input
```
int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^11*((7529/24576/a/d^2*(d*x)^(3/2)+527/768/a^2*b/d^4*(d*x)^(7/2)+9061/1
2288/a^3/d^6*b^2*(d*x)^(11/2)+1989/5120/a^4*b^3/d^8*(d*x)^(15/2)+663/8192/
d^10/a^5*b^4*(d*x)^(19/2))/(b*d^2*x^2+a*d^2)^5+663/65536/d^10/a^5/b/(a*d^2
/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(
2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d
*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^3}\,dx$$
$$= \frac{9945\,(a^5b^5x^{10}+5\,a^6b^4x^8+10\,a^7b^3x^6+10\,a^8b^2x^4+5\,a^9bx^2+a^{10})\left(-\frac{d^2}{a^{21}b^3}\right)^{\frac{1}{4}}\log\left(291434247\,a^{16}b^2\left(-\frac{d^2}{a^{21}\ldots}\right.\right.}{\ldots}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/245760*(9945*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2
*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^21*b^3))^(1/4)*log(291434247*a^16*b^2*
(-d^2/(a^21*b^3))^(3/4) + 291434247*sqrt(d*x)*d) - 9945*(I*a^5*b^5*x^10 +
5*I*a^6*b^4*x^8 + 10*I*a^7*b^3*x^6 + 10*I*a^8*b^2*x^4 + 5*I*a^9*b*x^2 + I*
a^10)*(-d^2/(a^21*b^3))^(1/4)*log(291434247*I*a^16*b^2*(-d^2/(a^21*b^3))^(
3/4) + 291434247*sqrt(d*x)*d) - 9945*(-I*a^5*b^5*x^10 - 5*I*a^6*b^4*x^8 -
10*I*a^7*b^3*x^6 - 10*I*a^8*b^2*x^4 - 5*I*a^9*b*x^2 - I*a^10)*(-d^2/(a^21*
b^3))^(1/4)*log(-291434247*I*a^16*b^2*(-d^2/(a^21*b^3))^(3/4) + 291434247*
sqrt(d*x)*d) - 9945*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^
8*b^2*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^21*b^3))^(1/4)*log(-291434247*a^1
6*b^2*(-d^2/(a^21*b^3))^(3/4) + 291434247*sqrt(d*x)*d) + 4*(9945*b^4*x^9 +
 47736*a*b^3*x^7 + 90610*a^2*b^2*x^5 + 84320*a^3*b*x^3 + 37645*a^4*x)*sqrt
(d*x))/(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5
*a^9*b*x^2 + a^10)
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \int \frac{\sqrt{dx}}{(a+bx^2)^6}\,dx$$

input
```
integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Integral(sqrt(d*x)/(a + b*x**2)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{dx}}{(a^2 + 2abx^2 + b^2x^4)^3}\,dx$$

$$= \frac{\frac{8\left(9945\,(dx)^{\frac{19}{2}}b^4d^2+47736\,(dx)^{\frac{15}{2}}ab^3d^4+90610\,(dx)^{\frac{11}{2}}a^2b^2d^6+84320\,(dx)^{\frac{7}{2}}a^3bd^8+37645\,(dx)^{\frac{3}{2}}a^4d^{10}\right)}{a^5b^5d^{10}x^{10}+5\,a^6b^4d^{10}x^8+10\,a^7b^3d^{10}x^6+10\,a^8b^2d^{10}x^4+5\,a^9bd^{10}x^2+a^{10}d^{10}} + \frac{9945\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right.\right.}{2\,\sqrt{}}\right.}{\sqrt{\sqrt{a}\sqrt{b}}}\right.}{}}{491}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/491520*(8*(9945*(d*x)^(19/2)*b^4*d^2 + 47736*(d*x)^(15/2)*a*b^3*d^4 + 90
610*(d*x)^(11/2)*a^2*b^2*d^6 + 84320*(d*x)^(7/2)*a^3*b*d^8 + 37645*(d*x)^(
3/2)*a^4*d^10)/(a^5*b^5*d^10*x^10 + 5*a^6*b^4*d^10*x^8 + 10*a^7*b^3*d^10*x
^6 + 10*a^8*b^2*d^10*x^4 + 5*a^9*b*d^10*x^2 + a^10*d^10) + 9945*d^2*(2*sqr
t(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(
b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))
/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(
sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*
x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a^5)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{19890\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^3}+\frac{19890\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^3}-\frac{9945\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\log}{}}{}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/491520*(19890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b^3) + 19890*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/(a^6*b^3) - 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt
(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b^3) + 9945*sqrt(2)*(a
*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^6*b^3) + 8*(9945*sqrt(d*x)*b^4*d^11*x^9 + 47736*sqrt(d*x)*a*b^3*d^11
*x^7 + 90610*sqrt(d*x)*a^2*b^2*d^11*x^5 + 84320*sqrt(d*x)*a^3*b*d^11*x^3 +
 37645*sqrt(d*x)*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*a^5))/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{7529\,d^9\,(d\,x)^{3/2}}{12288\,a}+\frac{9061\,b^2\,d^5\,(d\,x)^{11/2}}{6144\,a^3}+\frac{1989\,b^3\,d^3\,(d\,x)^{15/2}}{2560\,a^4}+\frac{527\,b\,d^7\,(d\,x)^{7/2}}{384\,a^2}+\frac{663\,b^4\,d\,(d\,x)^{19/2}}{4096\,a^5}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}} - \frac{663\,\sqrt{d}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{21/4}\,b^{3/4}} + \frac{663\,\sqrt{d}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{(-a)^{1/4}\,\sqrt{d}}\right)}{8192\,(-a)^{21/4}\,b^{3/4}}$$

input
```
int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
((7529*d^9*(d*x)^(3/2))/(12288*a) + (9061*b^2*d^5*(d*x)^(11/2))/(6144*a^3)
 + (1989*b^3*d^3*(d*x)^(15/2))/(2560*a^4) + (527*b*d^7*(d*x)^(7/2))/(384*a
^2) + (663*b^4*d*(d*x)^(19/2))/(4096*a^5))/(a^5*d^10 + b^5*d^10*x^10 + 5*a
^4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x
^6) - (663*d^(1/2)*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192
*(-a)^(21/4)*b^(3/4)) + (663*d^(1/2)*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/
4)*d^(1/2))))/(8192*(-a)^(21/4)*b^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 993, normalized size of antiderivative = 3.20

$$\int \frac{\sqrt{dx}}{(a^2 + 2abx^2 + b^2x^4)^3}\, dx = \text{Too large to display}$$

input
```
int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*( - 19890*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 99450*b**(1/4)
*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 198900*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**3*b**2*x**4 - 198900*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3
*x**6 - 99450*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 19890*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 19890*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**5 + 99450*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 198900
*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqr
t(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 198900*b**(1/4)*a**(3/
4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*
a**(1/4)*sqrt(2)))*a**2*b**3*x**6 + 99450*b**(1/4)*a**(3/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))
)*a*b**4*x**8 + 19890*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)...
```

3.484 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 310

$$\begin{aligned}\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^3}\,dx &= \frac{\sqrt{dx}}{10ad\,(a+bx^2)^5}+\frac{19\sqrt{dx}}{160a^2d\,(a+bx^2)^4}\\ &\quad+\frac{19\sqrt{dx}}{128a^3d\,(a+bx^2)^3}+\frac{209\sqrt{dx}}{1024a^4d\,(a+bx^2)^2}\\ &\quad+\frac{1463\sqrt{dx}}{4096a^5d\,(a+bx^2)}-\frac{4389\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}\\ &\quad+\frac{4389\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}\\ &\quad+\frac{4389\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}\end{aligned}$$

output
```
1/10*(d*x)^(1/2)/a/d/(b*x^2+a)^5+19/160*(d*x)^(1/2)/a^2/d/(b*x^2+a)^4+19/1
28*(d*x)^(1/2)/a^3/d/(b*x^2+a)^3+209/1024*(d*x)^(1/2)/a^4/d/(b*x^2+a)^2+14
63/4096*(d*x)^(1/2)/a^5/d/(b*x^2+a)-4389/16384*arctan(1-2^(1/2)*b^(1/4)*(d
*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(23/4)/b^(1/4)/d^(1/2)+4389/16384*arc
tan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(23/4)/b^(1/4
)/d^(1/2)+4389/16384*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(
a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(23/4)/b^(1/4)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

$$= \frac{\sqrt{x}\left(\frac{4a^{3/4}\sqrt{x}(19015a^4+50312a^3bx^2+59470a^2b^2x^4+33440ab^3x^6+7315b^4x^8)}{(a+bx^2)^5} - \frac{21945\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}} + \frac{21945\sqrt{2}\mathrm{arctan}}{}\right.}{81920a^{23/4}\sqrt{dx}}$$

input
```
Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
(Sqrt[x]*((4*a^(3/4)*Sqrt[x]*(19015*a^4 + 50312*a^3*b*x^2 + 59470*a^2*b^2*
x^4 + 33440*a*b^3*x^6 + 7315*b^4*x^8))/(a + b*x^2)^5 - (21945*Sqrt[2]*ArcT
an[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (21
945*Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x
)])/b^(1/4)))/(81920*a^(23/4)*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.42, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$, Rules used = {1380, 27, 253, 253, 253, 253, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$$

$$\Big\downarrow \; 1380$$

$$b^6\int \frac{1}{b^6\sqrt{dx}\left(bx^2+a\right)^6}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{1}{\sqrt{dx}\left(a+bx^2\right)^6}dx$$

$$\Big\downarrow \; 253$$

$$\frac{19\int \frac{1}{\sqrt{dx}(bx^2+a)^5}dx}{20a}+\frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 253$$

$$\frac{19\left(\frac{15\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^4}dx}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 253$$

$$\frac{19\left(\frac{15\left(\frac{11\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^3}dx}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 253$$

$$\frac{19\left(\frac{15\left(\frac{11\left(\frac{7\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^2}dx}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5}$$

$\downarrow$ 253

$$\frac{19\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5}$$

$\downarrow$ 266

$$\frac{19\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\int\frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5}$$

$\downarrow$ 755

$$\frac{19\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5}$$

$$\Big\downarrow 27$$

$$\frac{19\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{20a}+\frac{\sqrt{dx}}{10ad\left(a+bx^2\right)^5}$$

$$\Big\downarrow 1476$$

$$\left(15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}\right.\right.$$

↓ 1082

$$15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{}{6ad}$$

↓ 217

$$\left(15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right.\right.$$

↓ 1479

$$15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\cdots\right)}{2\sqrt{a}}\cdots\right)}{2ad}\cdots\right)}{8a}\cdots\right)}{12a}\cdots\right)$$

↓ 25

$$\left(15\frac{\left(11\frac{\left(7\frac{\left(3\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(\cdots\right.}{\sqrt{2}\cdots}\right.}{2\sqrt{a}}\cdots}{2ad}\cdots}{8a}\cdots}{12a}\cdots\right.$$

↓ 27

$$\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{2\sqrt{a}}\right.}{2ad}\right.}{8a}\right.}{12a}\right.\right.$$

↓ 1103

$$15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{2\sqrt{a}}\right.}{2ad}\right.}{8a}\right.}{12a}\right.$$

input
```
Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
Sqrt[d*x]/(10*a*d*(a + b*x^2)^5) + (19*(Sqrt[d*x]/(8*a*d*(a + b*x^2)^4) +
(15*(Sqrt[d*x]/(6*a*d*(a + b*x^2)^3) + (11*(Sqrt[d*x]/(4*a*d*(a + b*x^2)^2
) + (7*(Sqrt[d*x]/(2*a*d*(a + b*x^2)) + (3*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) +
 Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/
(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*a*d)))/(8*a)))/(12*
a)))/(16*a)))/(20*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\left(58520a\,x^{8}b^{4}+267520a^{2}x^{6}b^{3}+475760a^{3}x^{4}b^{2}+402496x^{2}a^{4}b+152120a^{5}\right)\sqrt{dx}+21945\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(b\,x^{2}+a\right)^{5}\left(\ln\left(\frac{dx+}{dx-}\right.\right.}{163840d\,a^{6}\left(b\,x^{2}+a\right)^{5}}$
derivativedivides	$2d^{11}\left(\frac{\frac{3803\sqrt{dx}}{8192a\,d^{2}}+\frac{6289b\left(dx\right)^{\frac{5}{2}}}{5120a^{2}d^{4}}+\frac{5947b^{2}\left(dx\right)^{\frac{9}{2}}}{4096a^{3}d^{6}}+\frac{209b^{3}\left(dx\right)^{\frac{13}{2}}}{256a^{4}d^{8}}+\frac{1463b^{4}\left(dx\right)^{\frac{17}{2}}}{8192a^{5}d^{10}}}{\left(b\,d^{2}x^{2}+a\,d^{2}\right)^{5}}+\frac{4389\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{}}\right.\right.}{}\right.$
default	$2d^{11}\left(\frac{\frac{3803\sqrt{dx}}{8192a\,d^{2}}+\frac{6289b\left(dx\right)^{\frac{5}{2}}}{5120a^{2}d^{4}}+\frac{5947b^{2}\left(dx\right)^{\frac{9}{2}}}{4096a^{3}d^{6}}+\frac{209b^{3}\left(dx\right)^{\frac{13}{2}}}{256a^{4}d^{8}}+\frac{1463b^{4}\left(dx\right)^{\frac{17}{2}}}{8192a^{5}d^{10}}}{\left(b\,d^{2}x^{2}+a\,d^{2}\right)^{5}}+\frac{4389\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{}}\right.\right.}{}\right.$

input
```
int(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/163840*((58520*a*b^4*x^8+267520*a^2*b^3*x^6+475760*a^3*b^2*x^4+402496*a^
4*b*x^2+152120*a^5)*(d*x)^(1/2)+21945*(a*d^2/b)^(1/4)*2^(1/2)*(b*x^2+a)^5*
(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan((2^(1/2)*(d*x)^(1/
2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))))/d/a^6/(b*x^2+a)^5
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{21945\,(a^5b^5dx^{10}+5\,a^6b^4dx^8+10\,a^7b^3dx^6+10\,a^8b^2dx^4+5\,a^9bdx^2+a^{10}d)\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}\log\left(a^6d\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}\right.}{}$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/81920*(21945*(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a
^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)*(-1/(a^23*b*d^2))^(1/4)*log(a^6*d*(
-1/(a^23*b*d^2))^(1/4) + sqrt(d*x)) - 21945*(-I*a^5*b^5*d*x^10 - 5*I*a^6*b
^4*d*x^8 - 10*I*a^7*b^3*d*x^6 - 10*I*a^8*b^2*d*x^4 - 5*I*a^9*b*d*x^2 - I*a
^10*d)*(-1/(a^23*b*d^2))^(1/4)*log(I*a^6*d*(-1/(a^23*b*d^2))^(1/4) + sqrt(
d*x)) - 21945*(I*a^5*b^5*d*x^10 + 5*I*a^6*b^4*d*x^8 + 10*I*a^7*b^3*d*x^6 +
 10*I*a^8*b^2*d*x^4 + 5*I*a^9*b*d*x^2 + I*a^10*d)*(-1/(a^23*b*d^2))^(1/4)*
log(-I*a^6*d*(-1/(a^23*b*d^2))^(1/4) + sqrt(d*x)) - 21945*(a^5*b^5*d*x^10
+ 5*a^6*b^4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 +
a^10*d)*(-1/(a^23*b*d^2))^(1/4)*log(-a^6*d*(-1/(a^23*b*d^2))^(1/4) + sqrt(
d*x)) + 4*(7315*b^4*x^8 + 33440*a*b^3*x^6 + 59470*a^2*b^2*x^4 + 50312*a^3*
b*x^2 + 19015*a^4)*sqrt(d*x))/(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x^8 + 10*a^7*b
^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^3}\,dx = \int \frac{1}{\sqrt{dx}\,(a+bx^2)^6}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Integral(1/(sqrt(d*x)*(a + b*x**2)**6), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^3}\,dx$$

$$= \frac{8\left(7315\,(dx)^{\frac{17}{2}}b^4d^2+33440\,(dx)^{\frac{13}{2}}ab^3d^4+59470\,(dx)^{\frac{9}{2}}a^2b^2d^6+50312\,(dx)^{\frac{5}{2}}a^3bd^8+19015\,\sqrt{dx}a^4d^{10}\right)}{a^5b^5d^{10}x^{10}+5\,a^6b^4d^{10}x^8+10\,a^7b^3d^{10}x^6+10\,a^8b^2d^{10}x^4+5\,a^9bd^{10}x^2+a^{10}d^{10}} + \frac{21945\left(\frac{\sqrt{2}d^2\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{}\right.}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{16}$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/163840*(8*(7315*(d*x)^(17/2)*b^4*d^2 + 33440*(d*x)^(13/2)*a*b^3*d^4 + 59
470*(d*x)^(9/2)*a^2*b^2*d^6 + 50312*(d*x)^(5/2)*a^3*b*d^8 + 19015*sqrt(d*x
)*a^4*d^10)/(a^5*b^5*d^10*x^10 + 5*a^6*b^4*d^10*x^8 + 10*a^7*b^3*d^10*x^6
+ 10*a^8*b^2*d^10*x^4 + 5*a^9*b*d^10*x^2 + a^10*d^10) + 21945*(sqrt(2)*d^2
*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((
a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4
)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d*arc
tan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt
(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(
sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/a^5)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{4389\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^6bd} + \frac{4389\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^6bd} + \frac{4389\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^6bd} - \frac{4389\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^6bd} + \frac{7315\,\sqrt{dx}b^4d^9x^8+33440\,\sqrt{dx}ab^3d^9x^6+59470\,\sqrt{dx}a^2b^2d^9x^4+50312\,\sqrt{dx}a^3bd^9x^2+19015\,\sqrt{dx}a^4d^9}{20480\,(bd^2x^2+ad^2)^5a^5}$$

input

```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output

```
4389/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d) + 4389/16384*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/(a^6*b*d) + 4389/32768*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x
+ sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b*d) - 4389/3276
8*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
sqrt(a*d^2/b))/(a^6*b*d) + 1/20480*(7315*sqrt(d*x)*b^4*d^9*x^8 + 33440*sqr
t(d*x)*a*b^3*d^9*x^6 + 59470*sqrt(d*x)*a^2*b^2*d^9*x^4 + 50312*sqrt(d*x)*a
^3*b*d^9*x^2 + 19015*sqrt(d*x)*a^4*d^9)/((b*d^2*x^2 + a*d^2)^5*a^5)
```

Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^3}\,dx = \frac{\frac{3803\,d^9\,\sqrt{d\,x}}{4096\,a}+\frac{5947\,b^2\,d^5\,{(d\,x)}^{9/2}}{2048\,a^3}+\frac{209\,b^3\,d^3\,{(d\,x)}^{13/2}}{128\,a^4}+\frac{6289\,b\,d^7\,{(d\,x)}^{5/2}}{2560\,a^2}+\frac{1463\,b^4\,d\,{(d\,x)}^{17/2}}{4096\,a^5}}{a^5\,d^{10}+5\,a^4\,b\,d^{10}\,x^2+10\,a^3\,b^2\,d^{10}\,x^4+10\,a^2\,b^3\,d^{10}\,x^6+5\,a\,b^4\,d^{10}\,x^8+b^5\,d^{10}\,x^{10}} + \frac{4389\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{8192\,{(-a)}^{23/4}\,b^{1/4}\,\sqrt{d}}+\frac{4389\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{d\,x}}{{(-a)}^{1/4}\,\sqrt{d}}\right)}{8192\,{(-a)}^{23/4}\,b^{1/4}\,\sqrt{d}}$$

input
```
int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

output
```
((3803*d^9*(d*x)^(1/2))/(4096*a) + (5947*b^2*d^5*(d*x)^(9/2))/(2048*a^3) +
 (209*b^3*d^3*(d*x)^(13/2))/(128*a^4) + (6289*b*d^7*(d*x)^(5/2))/(2560*a^2
) + (1463*b^4*d*(d*x)^(17/2))/(4096*a^5))/(a^5*d^10 + b^5*d^10*x^10 + 5*a^
4*b*d^10*x^2 + 5*a*b^4*d^10*x^8 + 10*a^3*b^2*d^10*x^4 + 10*a^2*b^3*d^10*x^
6) + (4389*atan((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1/2))))/(8192*(-a)^(2
3/4)*b^(1/4)*d^(1/2)) + (4389*atanh((b^(1/4)*(d*x)^(1/2))/((-a)^(1/4)*d^(1
/2))))/(8192*(-a)^(23/4)*b^(1/4)*d^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 995, normalized size of antiderivative = 3.21

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*( - 43890*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 219450*b**(3/4
)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 438900*b**(3/4)*a**(1/4)*sqrt(2)*
atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sq
rt(2)))*a**3*b**2*x**4 - 438900*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**
3*x**6 - 219450*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
- 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 43890*b**(
3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 43890*b**(3/4)*a**(1/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*s
qrt(2)))*a**5 + 219450*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 438
900*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*
sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 438900*b**(3/4)*a**
(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/
4)*a**(1/4)*sqrt(2)))*a**2*b**3*x**6 + 219450*b**(3/4)*a**(1/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt
(2)))*a*b**4*x**8 + 43890*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(...
```

3.485 $\int \frac{1}{(dx)^{3/2}(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 327

$$\begin{aligned}\int \frac{1}{(dx)^{3/2}(a^2+2abx^2+b^2x^4)^3}\,dx &= -\frac{13923}{4096a^6d\sqrt{dx}}+\frac{1}{10ad\sqrt{dx}\,(a+bx^2)^5}\\&+\frac{21}{160a^2d\sqrt{dx}\,(a+bx^2)^4}+\frac{119}{640a^3d\sqrt{dx}\,(a+bx^2)^3}+\frac{1547}{5120a^4d\sqrt{dx}\,(a+bx^2)^2}\\&+\frac{13923}{20480a^5d\sqrt{dx}\,(a+bx^2)}+\frac{13923\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{25/4}d^{3/2}}\\&-\frac{13923\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}(\sqrt{a}+\sqrt{b}x)}\right)}{8192\sqrt{2}a^{25/4}d^{3/2}}\end{aligned}$$

output
```
-13923/4096/a^6/d/(d*x)^(1/2)+1/10/a/d/(d*x)^(1/2)/(b*x^2+a)^5+21/160/a^2/
d/(d*x)^(1/2)/(b*x^2+a)^4+119/640/a^3/d/(d*x)^(1/2)/(b*x^2+a)^3+1547/5120/
a^4/d/(d*x)^(1/2)/(b*x^2+a)^2+13923/20480/a^5/d/(d*x)^(1/2)/(b*x^2+a)+1392
3/16384*b^(1/4)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1
/2)/a^(25/4)/d^(3/2)-13923/16384*b^(1/4)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1
/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(25/4)/d^(3/2)+13923/16384*b^(1/4)*arctanh(
2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a
^(25/4)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.60

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx=\frac{x\left(-\frac{4\sqrt[4]{a}\left(40960a^5+263515a^4bx^2+590240a^3b^2x^4+634270a^2b^3x^6+334152ab^4x^8+69615b^5x\right.}{(a+bx^2)^5}\right.}{}$$

input
```
Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
(x*((-4*a^(1/4)*(40960*a^5 + 263515*a^4*b*x^2 + 590240*a^3*b^2*x^4 + 63427
0*a^2*b^3*x^6 + 334152*a*b^4*x^8 + 69615*b^5*x^10))/(a + b*x^2)^5 + 69615*
Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x])] + 69615*Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(81920*a^(25/4)*(d*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.41, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {1380, 27, 253, 253, 253, 253, 253, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{1}{b^6(dx)^{3/2}\left(bx^2+a\right)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^2\right)^6}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{21\int \frac{1}{(dx)^{3/2}(bx^2+a)^5}dx}{20a}+\frac{1}{10ad\sqrt{dx}\,(a+bx^2)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{21\left(\frac{17\int \frac{1}{(dx)^{3/2}\left(bx^2+a\right)^4}dx}{16a}+\frac{1}{8ad\sqrt{dx}(a+bx^2)^4}\right)}{20a}+\frac{1}{10ad\sqrt{dx}\,(a+bx^2)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{21\left(\frac{17\left(\frac{13\int \frac{1}{(dx)^{3/2}\left(bx^2+a\right)^3}dx}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad\sqrt{dx}(a+bx^2)^4}\right)}{20a}+\frac{1}{10ad\sqrt{dx}\,(a+bx^2)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\int \frac{1}{(dx)^{3/2}\left(bx^2+a\right)^2}dx}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad\sqrt{dx}(a+bx^2)^4}\right)}{20a}+\frac{1}{10ad\sqrt{dx}\,(a+bx^2)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\left(\frac{5\int\frac{1}{(dx)^{3/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad\sqrt{dx}(a+bx^2)^4}\right)}{20a}+\frac{1}{10ad\sqrt{dx}\,(a+bx^2)^5}$$

$$\Big\downarrow \ 264$$

$$\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\left(\frac{5\left(-\frac{b\int \frac{\sqrt{dx}}{bx^2+a}dx}{ad^2}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad\sqrt{dx}(a+bx^2)^4}\right)}{20a}+\frac{1}{10ad\sqrt{dx}\,(a+bx^2)^5}$$

$\Big\downarrow$ 266

$$\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{ad^3}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad\sqrt{dx}(a+bx^2)^4}\right)}{20a}+\frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 27

$$\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad\sqrt{dx}(a+bx^2)^4}\right)}{20a}+\frac{1}{10ad\sqrt{dx}\left(a+bx^2\right)^5}$$

$$\Big\downarrow \; 826$$

$$21\left(\frac{17\left(\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}\right)}{16a}+\frac{}{8a\cdots}\right.$$

↓ 1476

17

13

9

5

$$-\frac{2b\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}$$

$$\frac{\left(\ldots\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}$$

$$\frac{\left(\ldots\right)}{8a}$$

$$\frac{\left(\ldots\right)}{12a}$$

$$\Big\downarrow$$ 1082

$$\left(\left(17\frac{\left(13\frac{\left(9\frac{\left(5\left(-\frac{2b\left(\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{\quad}{4ad\sqrt{dx}\cdots}\right)}{12a}\cdots$$

↓ 217

$$17\left(\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)}\right)}{12a}\right.$$

↓ 1479

$$\left(13-\frac{\left(9-\frac{\left(5-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\sqrt{\cdots}\right.}\cdots}{2\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{b}}\cdots\right)}{ad}\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right.$$

↓ 25

$$\Bigg(\Bigg(\Bigg(13\,\frac{\Bigg(9\,\frac{\Bigg(5-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\cdots}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}}{2\sqrt{b}}\cdots}{ad}\cdots}{4a}\cdots}{8a}\cdots$$

↓ 27

$$\left(\left(\left(\left(13\frac{\left(9\frac{\left(5-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}}{ad}}{4a}}{8a}$$

↓ 1103

$$17\left(13\left(9\left(5-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}\cdots\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots}{2\sqrt{b}}\cdots\right.}{ad}\cdots\right.}{4a}\cdots\right.}{8a}\cdots\right.}{12a}\cdots$$

input
```
Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
1/(10*a*d*Sqrt[d*x]*(a + b*x^2)^5) + (21*(1/(8*a*d*Sqrt[d*x]*(a + b*x^2)^4
) + (17*(1/(6*a*d*Sqrt[d*x]*(a + b*x^2)^3) + (13*(1/(4*a*d*Sqrt[d*x]*(a +
b*x^2)^2) + (9*(1/(2*a*d*Sqrt[d*x]*(a + b*x^2)) + (5*(-2/(a*d*Sqrt[d*x]) -
 (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(
1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[
Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt
[b])))/(a*d)))/(4*a)))/(8*a)))/(12*a)))/(16*a)))/(20*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{2}{a^6 d\sqrt{dx}}-\frac{b\left(\frac{\frac{11743a^4d^8(dx)^{\frac{3}{2}}}{4096}+\frac{1129a^3d^6b(dx)^{\frac{7}{2}}}{128}+\frac{22467a^2d^4b^2(dx)^{\frac{11}{2}}}{2048}+\frac{16169a\,d^2b^3(dx)^{\frac{15}{2}}}{2560}+\frac{5731b^4(dx)^{\frac{19}{2}}}{4096}}{\left(b\,d^2x^2+a\,d^2\right)^5}+\frac{13923\sqrt{2}}{}\right.}{a^6 d}$
derivativedivides	$2d^{11}\left(-\frac{b\left(\frac{\frac{11743a^4d^8(dx)^{\frac{3}{2}}}{8192}+\frac{1129a^3d^6b(dx)^{\frac{7}{2}}}{256}+\frac{22467a^2d^4b^2(dx)^{\frac{11}{2}}}{4096}+\frac{16169a\,d^2b^3(dx)^{\frac{15}{2}}}{5120}+\frac{5731b^4(dx)^{\frac{19}{2}}}{8192}}{\left(b\,d^2x^2+a\,d^2\right)^5}+\frac{13923\sqrt{2}\left(\ln\right.}{}\right.}{a^6d^{12}}\right.$
default	$2d^{11}\left(-\frac{b\left(\frac{\frac{11743a^4d^8(dx)^{\frac{3}{2}}}{8192}+\frac{1129a^3d^6b(dx)^{\frac{7}{2}}}{256}+\frac{22467a^2d^4b^2(dx)^{\frac{11}{2}}}{4096}+\frac{16169a\,d^2b^3(dx)^{\frac{15}{2}}}{5120}+\frac{5731b^4(dx)^{\frac{19}{2}}}{8192}}{\left(b\,d^2x^2+a\,d^2\right)^5}+\frac{13923\sqrt{2}\left(\ln\right.}{}\right.}{a^6d^{12}}\right.$
pseudoelliptic	$-\frac{13923\left(\frac{\sqrt{2}\left(b\,x^2+a\right)^5\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{2}\right.}{16384\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,d\,a^6(b}$

input
```
int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/a^6/d/(d*x)^(1/2)-1/a^6*b*(2*(11743/8192*a^4*d^8*(d*x)^(3/2)+1129/256*a
^3*d^6*b*(d*x)^(7/2)+22467/4096*a^2*d^4*b^2*(d*x)^(11/2)+16169/5120*a*d^2*
b^3*(d*x)^(15/2)+5731/8192*b^4*(d*x)^(19/2))/(b*d^2*x^2+a*d^2)^5+13923/327
68/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))
)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/
b)^(1/4)*(d*x)^(1/2)-1)))/d
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.89

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx =$$

$$-\frac{69615\left(a^6b^5d^2x^{11}+5\,a^7b^4d^2x^9+10\,a^8b^3d^2x^7+10\,a^9b^2d^2x^5+5\,a^{10}bd^2x^3+a^{11}d^2x\right)\left(-\frac{b}{a^{25}d^6}\right)^{\frac{1}{4}}\log\Big(269897}{}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/81920*(69615*(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7
 + 10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)*(-b/(a^25*d^6))^(1/
4)*log(2698972561467*a^19*d^5*(-b/(a^25*d^6))^(3/4) + 2698972561467*sqrt(d
*x)*b) + 69615*(-I*a^6*b^5*d^2*x^11 - 5*I*a^7*b^4*d^2*x^9 - 10*I*a^8*b^3*d
^2*x^7 - 10*I*a^9*b^2*d^2*x^5 - 5*I*a^10*b*d^2*x^3 - I*a^11*d^2*x)*(-b/(a^
25*d^6))^(1/4)*log(2698972561467*I*a^19*d^5*(-b/(a^25*d^6))^(3/4) + 269897
2561467*sqrt(d*x)*b) + 69615*(I*a^6*b^5*d^2*x^11 + 5*I*a^7*b^4*d^2*x^9 + 1
0*I*a^8*b^3*d^2*x^7 + 10*I*a^9*b^2*d^2*x^5 + 5*I*a^10*b*d^2*x^3 + I*a^11*d
^2*x)*(-b/(a^25*d^6))^(1/4)*log(-2698972561467*I*a^19*d^5*(-b/(a^25*d^6))^
(3/4) + 2698972561467*sqrt(d*x)*b) - 69615*(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d
^2*x^9 + 10*a^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11
*d^2*x)*(-b/(a^25*d^6))^(1/4)*log(-2698972561467*a^19*d^5*(-b/(a^25*d^6))^
(3/4) + 2698972561467*sqrt(d*x)*b) + 4*(69615*b^5*x^10 + 334152*a*b^4*x^8
+ 634270*a^2*b^3*x^6 + 590240*a^3*b^2*x^4 + 263515*a^4*b*x^2 + 40960*a^5)*
sqrt(d*x))/(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7 + 10
*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)
```

Sympy [F]

$$\int \frac{1}{(dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^3} \, dx = \int \frac{1}{(dx)^{\frac{3}{2}} \left(a + bx^2\right)^6} \, dx$$

input
```
integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Integral(1/((d*x)**(3/2)*(a + b*x**2)**6), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.19

$$\int \frac{1}{(dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^3} \, dx =$$

$$-\frac{\frac{8\left(69615\,b^5d^{10}x^{10}+334152\,ab^4d^{10}x^8+634270\,a^2b^3d^{10}x^6+590240\,a^3b^2d^{10}x^4+263515\,a^4bd^{10}x^2+40960\,a^5d^{10}\right)}{(dx)^{\frac{21}{2}}a^6b^5+5\,(dx)^{\frac{17}{2}}a^7b^4d^2+10\,(dx)^{\frac{13}{2}}a^8b^3d^4+10\,(dx)^{\frac{9}{2}}a^9b^2d^6+5\,(dx)^{\frac{5}{2}}a^{10}bd^8+\sqrt{dx}a^{11}d^{10}} + \frac{69615\,b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\cdots\right.}{\cdots}\right.}{\sqrt{\sqrt{\cdots}}}\cdots\right.}{\cdots}}{10\cdots}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/163840*(8*(69615*b^5*d^10*x^10 + 334152*a*b^4*d^10*x^8 + 634270*a^2*b^3
*d^10*x^6 + 590240*a^3*b^2*d^10*x^4 + 263515*a^4*b*d^10*x^2 + 40960*a^5*d^
10)/((d*x)^(21/2)*a^6*b^5 + 5*(d*x)^(17/2)*a^7*b^4*d^2 + 10*(d*x)^(13/2)*a
^8*b^3*d^4 + 10*(d*x)^(9/2)*a^9*b^2*d^6 + 5*(d*x)^(5/2)*a^10*b*d^8 + sqrt(
d*x)*a^11*d^10) + 69615*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(
1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)
*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4
)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sq
rt(b)*d)*sqrt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d
*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x
 - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(
3/4)))/a^6)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.12

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx =$$

$$-\frac{\frac{327680}{\sqrt{dx}a^6}+\frac{139230\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^7b^2d^2}+\frac{139230\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^7b^2d^2}-\frac{69615\sqrt{}\cdots}{\cdots}}{\cdots}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-1/163840*(327680/(sqrt(d*x)*a^6) + 139230*sqrt(2)*(a*b^3*d^2)^(3/4)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^
7*b^2*d^2) + 139230*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b^2*d^2) - 69615*sqr
t(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a^7*b^2*d^2) + 69615*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*b^2*d^2) + 8*(28655*sqrt
(d*x)*b^5*d^9*x^9 + 129352*sqrt(d*x)*a*b^4*d^9*x^7 + 224670*sqrt(d*x)*a^2*
b^3*d^9*x^5 + 180640*sqrt(d*x)*a^3*b^2*d^9*x^3 + 58715*sqrt(d*x)*a^4*b*d^9
*x)/((b*d^2*x^2 + a*d^2)^5*a^6))/d
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.69

$$\int \frac{1}{(dx)^{3/2} (a^2 + 2abx^2 + b^2x^4)^3} \, dx = \frac{13923 \, (-b)^{1/4} \, \mathrm{atanh}\left(\frac{(-b)^{1/4} \sqrt{d\,x}}{a^{1/4} \sqrt{d}}\right)}{8192 \, a^{25/4} \, d^{3/2}} - \frac{13923 \, (-b)^{1/4} \, \mathrm{atan}\left(\frac{(-b)^{1/4} \sqrt{d\,x}}{a^{1/4} \sqrt{d}}\right)}{8192 \, a^{25/4} \, d^{3/2}} - \frac{\frac{2\,d^9}{a} + \frac{52703\,b\,d^9\,x^2}{4096\,a^2} + \frac{3689\,b^2\,d^9\,x^4}{128\,a^3} + \frac{63427\,b^3\,d^9\,x^6}{2048\,a^4} + \frac{41769\,b^4\,d^9\,x^8}{2560\,a^5} + \frac{13923\,b^5\,d^9\,x^{10}}{4096\,a^6}}{b^5\,(d\,x)^{21/2} + a^5\,d^{10}\,\sqrt{d\,x} + 10\,a^3\,b^2\,d^6\,(d\,x)^{9/2} + 10\,a^2\,b^3\,d^4\,(d\,x)^{13/2} + 5\,a^4\,b\,d^8\,(d\,x)^{5/2} + 5\,a\,b^4\,d^2\,(d\,x)^{17/}}$$

input
```
int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

output
```
(13923*(-b)^(1/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192
*a^(25/4)*d^(3/2)) - (13923*(-b)^(1/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1
/4)*d^(1/2))))/(8192*a^(25/4)*d^(3/2)) - ((2*d^9)/a + (52703*b*d^9*x^2)/(4
096*a^2) + (3689*b^2*d^9*x^4)/(128*a^3) + (63427*b^3*d^9*x^6)/(2048*a^4) +
 (41769*b^4*d^9*x^8)/(2560*a^5) + (13923*b^5*d^9*x^10)/(4096*a^6))/(b^5*(d
*x)^(21/2) + a^5*d^10*(d*x)^(1/2) + 10*a^3*b^2*d^6*(d*x)^(9/2) + 10*a^2*b^
3*d^4*(d*x)^(13/2) + 5*a^4*b*d^8*(d*x)^(5/2) + 5*a*b^4*d^2*(d*x)^(17/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1042, normalized size of antiderivative = 3.19

$$\int \frac{1}{(dx)^{3/2} (a^2 + 2abx^2 + b^2x^4)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*(139230*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 + 696150*s
qrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 + 1392300*sqrt(x)*b**
(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 + 1392300*sqrt(x)*b**(1/4)*a
**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*x**6 + 696150*sqrt(x)*b**(1/4)*a**(3/4)*
sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**
(1/4)*sqrt(2)))*a*b**4*x**8 + 139230*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(
2)))*b**5*x**10 - 139230*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 -
696150*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 1392300*sqr
t(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)
*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 - 1392300*sqrt(x)*b*
*(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b
))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*x**6 - 696150*sqrt(x)*b**(1/4)*a
**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b...
```

3.486 $\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 327

$$
\begin{aligned}
\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= -\frac{33649}{12288a^6d(dx)^{3/2}} \\
&+ \frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5} + \frac{23}{160a^2d(dx)^{3/2}\left(a+bx^2\right)^4} \\
&+ \frac{437}{1920a^3d(dx)^{3/2}\left(a+bx^2\right)^3} + \frac{437}{1024a^4d(dx)^{3/2}\left(a+bx^2\right)^2} \\
&+ \frac{4807}{4096a^5d(dx)^{3/2}\left(a+bx^2\right)} + \frac{33649b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{27/4}d^{5/2}} \\
&- \frac{33649b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{27/4}d^{5/2}} - \frac{33649b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}a^{27/4}d^{5/2}}
\end{aligned}
$$

output
```
-33649/12288/a^6/d/(d*x)^(3/2)+1/10/a/d/(d*x)^(3/2)/(b*x^2+a)^5+23/160/a^2
/d/(d*x)^(3/2)/(b*x^2+a)^4+437/1920/a^3/d/(d*x)^(3/2)/(b*x^2+a)^3+437/1024
/a^4/d/(d*x)^(3/2)/(b*x^2+a)^2+4807/4096/a^5/d/(d*x)^(3/2)/(b*x^2+a)+33649
/16384*b^(3/4)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/
2)/a^(27/4)/d^(5/2)-33649/16384*b^(3/4)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/
2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(27/4)/d^(5/2)-33649/16384*b^(3/4)*arctanh(2
^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^
(27/4)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.60

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{x\left(-\frac{4a^{3/4}(40960a^5+437345a^4bx^2+1157176a^3b^2x^4+1367810a^2b^3x^6+769120ab^4x^8+168245}{(a+bx^2)^5}\right.}{}$$

input
```
Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
(x*((-4*a^(3/4)*(40960*a^5 + 437345*a^4*b*x^2 + 1157176*a^3*b^2*x^4 + 1367
810*a^2*b^3*x^6 + 769120*a*b^4*x^8 + 168245*b^5*x^10))/(a + b*x^2)^5 + 504
735*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x])] - 504735*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTanh[(Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(245760*a^(27/4)*(d*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.42, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1380, 27, 253, 253, 253, 253, 253, 264, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^3}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^6 \int \frac{1}{b^6 (dx)^{5/2} \left(bx^2+a\right)^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{(dx)^{5/2} \left(a+bx^2\right)^6}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{23 \int \frac{1}{(dx)^{5/2} \left(bx^2+a\right)^5}dx}{20 a}+\frac{1}{10 a d (dx)^{3/2} \left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{23 \left(\frac{19 \int \frac{1}{(dx)^{5/2} \left(bx^2+a\right)^4}dx}{16 a}+\frac{1}{8 a d (dx)^{3/2} \left(a+bx^2\right)^4}\right)}{20 a}+\frac{1}{10 a d (dx)^{3/2} \left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{23 \left(\frac{19 \left(\frac{5 \int \frac{1}{(dx)^{5/2} \left(bx^2+a\right)^3}dx}{4 a}+\frac{1}{6 a d (dx)^{3/2} \left(a+bx^2\right)^3}\right)}{16 a}+\frac{1}{8 a d (dx)^{3/2} \left(a+bx^2\right)^4}\right)}{20 a}+\frac{1}{10 a d (dx)^{3/2} \left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{23 \left(\frac{19 \left(\frac{5 \left(\frac{11 \int \frac{1}{(dx)^{5/2} \left(bx^2+a\right)^2}dx}{8 a}+\frac{1}{4 a d (dx)^{3/2} \left(a+bx^2\right)^2}\right)}{4 a}+\frac{1}{6 a d (dx)^{3/2} \left(a+bx^2\right)^3}\right)}{16 a}+\frac{1}{8 a d (dx)^{3/2} \left(a+bx^2\right)^4}\right)}{20 a}+\frac{1}{10 a d (dx)^{3/2} \left(a+bx^2\right)^5}$$

$$\downarrow \text{253}$$

$$\frac{23\left(\frac{19\left(\frac{5\left(\frac{11\left(\frac{7\int \frac{1}{(dx)^{5/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad(dx)^{3/2}(a+bx^2)^4}\right)}{20a}+\frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5}$$

$$\downarrow \text{264}$$

$$\frac{23\left(\frac{19\left(\frac{5\left(\frac{11\left(\frac{7\left(-\frac{b\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{ad^2}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad(dx)^{3/2}(a\cdots}}{20a}$$

$$\frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{266}$$

$$\frac{23\left(\frac{19\left(\frac{5\left(\frac{11\left(\frac{7\left(-\frac{2b\int\frac{1}{bx^2+a}d\sqrt{dx}}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad(dx)^{3/2}(a+b\cdots}\right.}{20a}$$

$$\frac{1}{10ad(dx)^{3/2}\left(a+bx^2\right)^5}$$

$$\downarrow 755$$

$$23\left(\frac{19\left(\frac{5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)^2}\right)}{4a}+\frac{}{6ad(dx}\cdots}{16a}\right.$$

$$\Big\downarrow 27$$

$$23\,\frac{19\,\frac{5\,\frac{11\,\frac{7\left(-\frac{2b\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}}{8a}+\frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)^2}}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a \cdots\right)}}{16a}$$

↓ 1476

$$\left(\left(\left(5\frac{\left(11\frac{\left(7\left(-\frac{2b\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)\right)}{4a}+\frac{}{2ad(dx)^{\cdots}}\right)}{8a}\right.\right.\right.$$

↓ 1082

$$\left(\left(\left(\frac{5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx\right.}\right.}{8a}\right.\right.\right.\right.$$

↓ 217

$$\left(\left(\frac{5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{\cdots}{4a\cdots}\right.\right.$$

↓ 1479

$$\left(\quad \left(\left(5 \frac{\left(11 \frac{\left(7 - \frac{2b \left(\frac{d \left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}} + \frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{\cdots}\right.}{ad^3}\right.}{4a}\right.}{8a}\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$2b\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\mathrm{ar}\cdots}{\cdots}\right)}{2\sqrt{a}}\cdots\right.$$

$$7-\frac{\cdots}{ad^3}$$

$$11\frac{\cdots}{4a}$$

$$5\frac{\cdots}{8a}$$

$\Big\downarrow$ 27

$$\left(\left(\left(5\,\frac{\left(11\,\frac{\left(7-\frac{2b\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\cdots\right)}{2\sqrt{a}}\cdots\right)}{ad^{3}}\cdots\right)}{4a}\cdots\right)}{8a}\cdots$$

↓ 1103

$$\left(\left(\left(5\,\frac{\left(11\,\frac{\left(7-\frac{2b\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}\cdots\right.}{2\sqrt{a}}\cdots\right)}{ad^3}\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right.\right.\right.$$

input
```
Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
1/(10*a*d*(d*x)^(3/2)*(a + b*x^2)^5) + (23*(1/(8*a*d*(d*x)^(3/2)*(a + b*x^
2)^4) + (19*(1/(6*a*d*(d*x)^(3/2)*(a + b*x^2)^3) + (5*(1/(4*a*d*(d*x)^(3/2
)*(a + b*x^2)^2) + (11*(1/(2*a*d*(d*x)^(3/2)*(a + b*x^2)) + (7*(-2/(3*a*d*
(d*x)^(3/2)) - (2*b*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqr
t[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b
]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d])))/(2*Sqrt[a])))/(a*d^3)))/(4*a)))/(8*a)))/(4*a)))/(16*a)))/(2
0*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{2}{3a^6 x\sqrt{dx}\,d^2}-\frac{2b\left(\frac{\frac{15503a^4d^8\sqrt{dx}}{8192}+\frac{31149a^3d^6b(dx)^{\frac{5}{2}}}{5120}+\frac{95821a^2d^4b^2(dx)^{\frac{9}{2}}}{12288}+\frac{3527a\,d^2b^3(dx)^{\frac{13}{2}}}{768}+\frac{25457b^4(dx)^{\frac{17}{2}}}{24576}}{\left(b\,d^2x^2+a\,d^2\right)^5}+\frac{3364}{}\right.}{}$
pseudoelliptic	$-\frac{33649\left(\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}bx\sqrt{2}\,(b\,x^2+a)^5\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\right.\right.\right.}{32768\sqrt{dx}\,d^3a^7}$
derivativedivides	$2d^{11}\left(-\frac{b\left(\frac{\frac{15503a^4d^8\sqrt{dx}}{8192}+\frac{31149a^3d^6b(dx)^{\frac{5}{2}}}{5120}+\frac{95821a^2d^4b^2(dx)^{\frac{9}{2}}}{12288}+\frac{3527a\,d^2b^3(dx)^{\frac{13}{2}}}{768}+\frac{25457b^4(dx)^{\frac{17}{2}}}{24576}}{\left(b\,d^2x^2+a\,d^2\right)^5}+\frac{33649\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{}\right.}{a^6d^{12}}\right.$
default	$2d^{11}\left(-\frac{b\left(\frac{\frac{15503a^4d^8\sqrt{dx}}{8192}+\frac{31149a^3d^6b(dx)^{\frac{5}{2}}}{5120}+\frac{95821a^2d^4b^2(dx)^{\frac{9}{2}}}{12288}+\frac{3527a\,d^2b^3(dx)^{\frac{13}{2}}}{768}+\frac{25457b^4(dx)^{\frac{17}{2}}}{24576}}{\left(b\,d^2x^2+a\,d^2\right)^5}+\frac{33649\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{}\right.}{a^6d^{12}}\right.$

input
```
int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3/a^6/x/(d*x)^(1/2)/d^2-2/a^6*b/d*((15503/8192*a^4*d^8*(d*x)^(1/2)+3114
9/5120*a^3*d^6*b*(d*x)^(5/2)+95821/12288*a^2*d^4*b^2*(d*x)^(9/2)+3527/768*
a*d^2*b^3*(d*x)^(13/2)+25457/24576*b^4*(d*x)^(17/2))/(b*d^2*x^2+a*d^2)^5+3
3649/65536*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(
1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.97

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx =$$

$$-\frac{504735\left(a^6b^5d^3x^{12}+5\,a^7b^4d^3x^{10}+10\,a^8b^3d^3x^8+10\,a^9b^2d^3x^6+5\,a^{10}bd^3x^4+a^{11}d^3x^2\right)\left(-\frac{b^3}{a^{27}d^{10}}\right)^{\frac{1}{4}}\log\left(33\right.}{}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/245760*(504735*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*
x^8 + 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a^27*d^
10))^(1/4)*log(33649*a^7*d^3*(-b^3/(a^27*d^10))^(1/4) + 33649*sqrt(d*x)*b)
 + 504735*(I*a^6*b^5*d^3*x^12 + 5*I*a^7*b^4*d^3*x^10 + 10*I*a^8*b^3*d^3*x^
8 + 10*I*a^9*b^2*d^3*x^6 + 5*I*a^10*b*d^3*x^4 + I*a^11*d^3*x^2)*(-b^3/(a^2
7*d^10))^(1/4)*log(33649*I*a^7*d^3*(-b^3/(a^27*d^10))^(1/4) + 33649*sqrt(d
*x)*b) + 504735*(-I*a^6*b^5*d^3*x^12 - 5*I*a^7*b^4*d^3*x^10 - 10*I*a^8*b^3
*d^3*x^8 - 10*I*a^9*b^2*d^3*x^6 - 5*I*a^10*b*d^3*x^4 - I*a^11*d^3*x^2)*(-b
^3/(a^27*d^10))^(1/4)*log(-33649*I*a^7*d^3*(-b^3/(a^27*d^10))^(1/4) + 3364
9*sqrt(d*x)*b) - 504735*(a^6*b^5*d^3*x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^
3*d^3*x^8 + 10*a^9*b^2*d^3*x^6 + 5*a^10*b*d^3*x^4 + a^11*d^3*x^2)*(-b^3/(a
^27*d^10))^(1/4)*log(-33649*a^7*d^3*(-b^3/(a^27*d^10))^(1/4) + 33649*sqrt(
d*x)*b) + 4*(168245*b^5*x^10 + 769120*a*b^4*x^8 + 1367810*a^2*b^3*x^6 + 11
57176*a^3*b^2*x^4 + 437345*a^4*b*x^2 + 40960*a^5)*sqrt(d*x))/(a^6*b^5*d^3*
x^12 + 5*a^7*b^4*d^3*x^10 + 10*a^8*b^3*d^3*x^8 + 10*a^9*b^2*d^3*x^6 + 5*a^
10*b*d^3*x^4 + a^11*d^3*x^2)
```

Sympy [F]

$$\int \frac{1}{(dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \int \frac{1}{(dx)^{\frac{5}{2}} \left(a+bx^2\right)^6}\,dx$$

input
```
integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Integral(1/((d*x)**(5/2)*(a + b*x**2)**6), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.21

$$\int \frac{1}{(dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^3}\,dx =$$

$$-\frac{\frac{8\,(168245\,b^5d^{10}x^{10}+769120\,ab^4d^{10}x^8+1367810\,a^2b^3d^{10}x^6+1157176\,a^3b^2d^{10}x^4+437345\,a^4bd^{10}x^2+40960\,a^5d^{10})}{(dx)^{\frac{23}{2}}a^6b^5+5\,(dx)^{\frac{19}{2}}a^7b^4d^2+10\,(dx)^{\frac{15}{2}}a^8b^3d^4+10\,(dx)^{\frac{11}{2}}a^9b^2d^6+5\,(dx)^{\frac{7}{2}}a^{10}bd^8+(dx)^{\frac{3}{2}}a^{11}d^{10}} + \frac{504735\left(\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}dx-\right.}{}\right.}{}}{}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/491520*(8*(168245*b^5*d^10*x^10 + 769120*a*b^4*d^10*x^8 + 1367810*a^2*b
^3*d^10*x^6 + 1157176*a^3*b^2*d^10*x^4 + 437345*a^4*b*d^10*x^2 + 40960*a^5
*d^10)/((d*x)^(23/2)*a^6*b^5 + 5*(d*x)^(19/2)*a^7*b^4*d^2 + 10*(d*x)^(15/2
)*a^8*b^3*d^4 + 10*(d*x)^(11/2)*a^9*b^2*d^6 + 5*(d*x)^(7/2)*a^10*b*d^8 + (
d*x)^(3/2)*a^11*d^10) + 504735*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x + sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2)*b^(3/
4)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/
(a*d^2)^(3/4) + 2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1
/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*
d)*sqrt(a)*d) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(
1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)
*d)*sqrt(a)*d))/a^6)/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.09

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx =$$

$$-\frac{33649\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^7d^3}$$

$$-\frac{33649\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^7d^3}$$

$$-\frac{33649\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^7d^3}$$

$$+\frac{33649\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^7d^3}-\frac{2}{3\,\sqrt{dx}a^6d^2x}$$

$$-\frac{127285\,\sqrt{dx}b^5d^8x^8+564320\,\sqrt{dx}ab^4d^8x^6+958210\,\sqrt{dx}a^2b^3d^8x^4+747576\,\sqrt{dx}a^3b^2d^8x^2+232545\,\sqrt{dx}a}{61440\left(bd^2x^2+ad^2\right)^5a^6d}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-33649/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/
b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*d^3) - 33649/16384*sqrt(2)*(
a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x
))/(a*d^2/b)^(1/4))/(a^7*d^3) - 33649/32768*sqrt(2)*(a*b^3*d^2)^(1/4)*log(
d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*d^3) + 33649
/32768*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*
x) + sqrt(a*d^2/b))/(a^7*d^3) - 2/3/(sqrt(d*x)*a^6*d^2*x) - 1/61440*(12728
5*sqrt(d*x)*b^5*d^8*x^8 + 564320*sqrt(d*x)*a*b^4*d^8*x^6 + 958210*sqrt(d*x
)*a^2*b^3*d^8*x^4 + 747576*sqrt(d*x)*a^3*b^2*d^8*x^2 + 232545*sqrt(d*x)*a^
4*b*d^8)/((b*d^2*x^2 + a*d^2)^5*a^6*d)
```

Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.69

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{33649\,{\left(-b\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{27/4}\,d^{5/2}} - \frac{\frac{2\,d^9}{3\,a}+\frac{87469\,b\,d^9\,x^2}{12288\,a^2}+\frac{144647\,b^2\,d^9\,x^4}{7680\,a^3}+\frac{136781\,b^3\,d^9\,x^6}{6144\,a^4}+\frac{4807\,b^4\,d^9\,x^8}{384\,a^5}+\frac{33649\,b^5\,d^9\,x^{10}}{12288\,a^6}}{b^5\,{\left(d\,x\right)}^{23/2}+a^5\,d^{10}\,{\left(d\,x\right)}^{3/2}+10\,a^3\,b^2\,d^6\,{\left(d\,x\right)}^{11/2}+10\,a^2\,b^3\,d^4\,{\left(d\,x\right)}^{15/2}+5\,a^4\,b\,d^8\,{\left(d\,x\right)}^{7/2}+5\,a\,b^4\,d^2\,{\left(d\,x\right.}} + \frac{33649\,{\left(-b\right)}^{3/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{27/4}\,d^{5/2}}$$

input
```
int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

output
```
(33649*(-b)^(3/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*
a^(27/4)*d^(5/2)) - ((2*d^9)/(3*a) + (87469*b*d^9*x^2)/(12288*a^2) + (1446
47*b^2*d^9*x^4)/(7680*a^3) + (136781*b^3*d^9*x^6)/(6144*a^4) + (4807*b^4*d
^9*x^8)/(384*a^5) + (33649*b^5*d^9*x^10)/(12288*a^6))/(b^5*(d*x)^(23/2) +
a^5*d^10*(d*x)^(3/2) + 10*a^3*b^2*d^6*(d*x)^(11/2) + 10*a^2*b^3*d^4*(d*x)^
(15/2) + 5*a^4*b*d^8*(d*x)^(7/2) + 5*a*b^4*d^2*(d*x)^(19/2)) + (33649*(-b)
^(3/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*a^(27/4)*d
^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1049, normalized size of antiderivative = 3.21

$$\int \frac{1}{(dx)^{5/2} (a^2 + 2abx^2 + b^2x^4)^3} dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(sqrt(d)*(1009470*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5*x + 50473
50*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*s
qrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**3 + 10094700*sqrt(x
)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sq
rt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**5 + 10094700*sqrt(x)*b**(
3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*x**7 + 5047350*sqrt(x)*b**(3/4)*a*
*(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1
/4)*a**(1/4)*sqrt(2)))*a*b**4*x**9 + 1009470*sqrt(x)*b**(3/4)*a**(1/4)*sqr
t(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/
4)*sqrt(2)))*b**5*x**11 - 1009470*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))
)*a**5*x - 5047350*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**3 -
 10094700*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**5 - 10094
700*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*x**7 - 5047350*sqr
t(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt...
```

3.487 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 345

$$
\begin{aligned}
\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx &= -\frac{13923}{4096a^6d(dx)^{5/2}} \\
&+ \frac{69615b}{4096a^7d^3\sqrt{dx}} + \frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5} + \frac{5}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^4} \\
&+ \frac{35}{128a^3d(dx)^{5/2}\left(a+bx^2\right)^3} + \frac{595}{1024a^4d(dx)^{5/2}\left(a+bx^2\right)^2} \\
&+ \frac{7735}{4096a^5d(dx)^{5/2}\left(a+bx^2\right)} - \frac{69615b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{29/4}d^{7/2}} \\
&+ \frac{69615b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{29/4}d^{7/2}} - \frac{69615b^{5/4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{8192\sqrt{2}a^{29/4}d^{7/2}}
\end{aligned}
$$

output
```
-13923/4096/a^6/d/(d*x)^(5/2)+69615/4096*b/a^7/d^3/(d*x)^(1/2)+1/10/a/d/(d
*x)^(5/2)/(b*x^2+a)^5+5/32/a^2/d/(d*x)^(5/2)/(b*x^2+a)^4+35/128/a^3/d/(d*x
)^(5/2)/(b*x^2+a)^3+595/1024/a^4/d/(d*x)^(5/2)/(b*x^2+a)^2+7735/4096/a^5/d
/(d*x)^(5/2)/(b*x^2+a)-69615/16384*b^(5/4)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^
(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(29/4)/d^(7/2)+69615/16384*b^(5/4)*arctan
(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(29/4)/d^(7/2)-6
9615/16384*b^(5/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^
(1/2)+b^(1/2)*x))*2^(1/2)/a^(29/4)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.61

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx=\frac{\sqrt{dx}\left(\frac{4\sqrt[4]{a}(-8192a^6+204800a^5bx^2+1317575a^4b^2x^4+2951200a^3b^3x^6+3171350a^2b^4x^8+1\cdots}{(a+bx^2)^5}\right.}{\cdots}$$

input
```
Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
(Sqrt[d*x]*((4*a^(1/4)*(-8192*a^6 + 204800*a^5*b*x^2 + 1317575*a^4*b^2*x^4
 + 2951200*a^3*b^3*x^6 + 3171350*a^2*b^4*x^8 + 1670760*a*b^5*x^10 + 348075
*b^6*x^12))/(a + b*x^2)^5 - 348075*Sqrt[2]*b^(5/4)*x^(5/2)*ArcTan[(Sqrt[a]
 - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 348075*Sqrt[2]*b^(5/4)*
x^(5/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))
/(81920*a^(29/4)*d^4*x^3)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.41, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.679, Rules used = {1380, 27, 253, 253, 253, 253, 253, 264, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}dx$$

$$\Big\downarrow \text{1380}$$

$$b^6\int \frac{1}{b^6(dx)^{7/2}\left(bx^2+a\right)^6}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{(dx)^{7/2}\left(a+bx^2\right)^6}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5\int \frac{1}{(dx)^{7/2}(bx^2+a)^5}dx}{4a}+\frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{5\left(\frac{21\int \frac{1}{(dx)^{7/2}\left(bx^2+a\right)^4}dx}{16a}+\frac{1}{8ad(dx)^{5/2}(a+bx^2)^4}\right)}{4a}+\frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{5\left(\frac{21\left(\frac{17\int \frac{1}{(dx)^{7/2}\left(bx^2+a\right)^3}dx}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad(dx)^{5/2}(a+bx^2)^4}\right)}{4a}+\frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5}$$

$$\Big\downarrow \text{253}$$

$$\frac{5\left(\frac{21\left(\frac{17\left(\frac{13\int \frac{1}{(dx)^{7/2}\left(bx^2+a\right)^2}dx}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad(dx)^{5/2}(a+bx^2)^4}\right)}{4a}+\frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5}$$

$$\Big\downarrow \ 253$$

$$\frac{5\left(\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\int \frac{1}{(dx)^{7/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad(dx)^{5/2}(a+bx^2)^4}\right)}{4a}+\frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5}$$

$$\Big\downarrow \ 264$$

$$\frac{5\left(\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\int \frac{1}{(dx)^{3/2}\left(bx^2+a\right)}dx}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad(dx)^{5/2}}\right.}{4a}$$

$$\frac{1}{10ad(dx)^{5/2}\left(a+bx^2\right)^5}$$

$\Big\downarrow$ 264

$$5\left(\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{b\int\frac{\sqrt{dx}}{bx^2+a}\,dx}{ad^2}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{}{8ad(d}\right.$$

↓ 266

$$5\left(\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\int\frac{d^3x}{bx^2d^2+ad^2}\,d\sqrt{dx}}{ad^3}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}\right)}{16a}-\right.$$

↓ 27

$$5\left(\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}\right)}{16a} -$$

↓ 826

$$\left(21\left(\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\cdots$$

↓ 1476

$$\left(\left(\left(\frac{17\left(\frac{13\left(9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a} - \cdots}{8a}$$

↓ 1082

$$17\left(13\left(9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)\Big/4a+\frac{\ldots}{2ad(dx)^{5}}\ldots\right)\Big/8a \ldots$$

↓ 217

$$\left(\left(\left(17\frac{\left(13\frac{\left(9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx\right.}\right)}{8a}\right.\right.\right.$$

↓ 1479

$$13\left(9\left(b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}\cdots\right.}}{2\sqrt{2}\cdots}}{2\sqrt{b}}\cdots\right)}{ad}\cdots\right)}{ad^2}\cdots\right)}{4a}\cdots$$

↓ 25

$$2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\cdots}{}\right.}}{2\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{b}}\cdots\right.$$

$$b-\frac{\cdots}{ad}$$

$$9-\frac{\cdots}{ad^2}$$

$$13\ \frac{\cdots}{4a}$$

$\Big\downarrow$ 27

$$13 - \frac{9 - \frac{b - \frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} - \frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}} + \frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{\cdots}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}} \cdots}{ad}}{ad^2}}{4a}$$

↓ 1103

$$\Bigg(\Bigg(17\frac{\Bigg(13\frac{\Bigg(9-\frac{\Bigg(b-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}\cdots\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\cdots}{2\sqrt{b}}\cdots\right.}{ad}\cdots}{ad^2}\cdots}{4a}\cdots}{8a}\cdots$$

input
```
Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

output
```
1/(10*a*d*(d*x)^(5/2)*(a + b*x^2)^5) + (5*(1/(8*a*d*(d*x)^(5/2)*(a + b*x^2
)^4) + (21*(1/(6*a*d*(d*x)^(5/2)*(a + b*x^2)^3) + (17*(1/(4*a*d*(d*x)^(5/2
)*(a + b*x^2)^2) + (13*(1/(2*a*d*(d*x)^(5/2)*(a + b*x^2)) + (9*(-2/(5*a*d*
(d*x)^(5/2)) - (b*(-2/(a*d*Sqrt[d*x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[
Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(a*d)))/(a*d^2)))/(4*a)))/(8
*a)))/(12*a)))/(16*a)))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{2(-30b\,x^2+a)}{5a^7\sqrt{dx}\,x^2d^3}+\frac{b^2\left(\frac{\frac{34139a^4d^8(dx)^{\frac{3}{2}}}{4096}+\frac{3597a^3d^6b(dx)^{\frac{7}{2}}}{128}+\frac{75471a^2d^4b^2(dx)^{\frac{11}{2}}}{2048}+\frac{56269a\,d^2b^3(dx)^{\frac{15}{2}}}{2560}+\frac{20463b^4(dx)^{\frac{19}{2}}}{4096}}{\left(b\,d^2x^2+a\,d^2\right)^5} \dots\right)}{a^7 \dots}$
derivativedivides	$2d^{11}\left(-\frac{1}{5a^6d^{12}(dx)^{\frac{5}{2}}}+\frac{6b}{a^7d^{14}\sqrt{dx}}+\frac{b^2\left(\frac{\frac{34139a^4d^8(dx)^{\frac{3}{2}}}{8192}+\frac{3597a^3d^6b(dx)^{\frac{7}{2}}}{256}+\frac{75471a^2d^4b^2(dx)^{\frac{11}{2}}}{4096}+\frac{56269a\,d^2b^3(d \dots}{5120} \dots}{\left(b\,d^2x^2+a\,d^2\right)^5} \dots\right)}{\dots} \dots\right.$
default	$2d^{11}\left(-\frac{1}{5a^6d^{12}(dx)^{\frac{5}{2}}}+\frac{6b}{a^7d^{14}\sqrt{dx}}+\frac{b^2\left(\frac{\frac{34139a^4d^8(dx)^{\frac{3}{2}}}{8192}+\frac{3597a^3d^6b(dx)^{\frac{7}{2}}}{256}+\frac{75471a^2d^4b^2(dx)^{\frac{11}{2}}}{4096}+\frac{56269a\,d^2b^3(d \dots}{5120} \dots}{\left(b\,d^2x^2+a\,d^2\right)^5} \dots\right)}{\dots} \dots\right.$
pseudoelliptic	$\frac{\frac{69615b\,x^2\sqrt{2}\left(b\,x^2+a\right)^5\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right) \dots\right.}{32768} \dots}{d^3a^7x^2\sqrt{dx}\,(b\,x^2+a \dots}$

input
```
int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-30*b*x^2+a)/a^7/(d*x)^(1/2)/x^2/d^3+b^2/a^7*(2*(34139/8192*a^4*d^8*
(d*x)^(3/2)+3597/256*a^3*d^6*b*(d*x)^(7/2)+75471/4096*a^2*d^4*b^2*(d*x)^(1
1/2)+56269/5120*a*d^2*b^3*(d*x)^(15/2)+20463/8192*b^4*(d*x)^(19/2))/(b*d^2
*x^2+a*d^2)^5+69615/32768/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2
*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))/d^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.92

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/81920*(348075*(a^7*b^5*d^4*x^13 + 5*a^8*b^4*d^4*x^11 + 10*a^9*b^3*d^4*x^
9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4*x^5 + a^12*d^4*x^3)*(-b^5/(a^29*d^1
4))^(1/4)*log(337371570183375*a^22*d^11*(-b^5/(a^29*d^14))^(3/4) + 3373715
70183375*sqrt(d*x)*b^4) - 348075*(I*a^7*b^5*d^4*x^13 + 5*I*a^8*b^4*d^4*x^1
1 + 10*I*a^9*b^3*d^4*x^9 + 10*I*a^10*b^2*d^4*x^7 + 5*I*a^11*b*d^4*x^5 + I*
a^12*d^4*x^3)*(-b^5/(a^29*d^14))^(1/4)*log(337371570183375*I*a^22*d^11*(-b
^5/(a^29*d^14))^(3/4) + 337371570183375*sqrt(d*x)*b^4) - 348075*(-I*a^7*b^
5*d^4*x^13 - 5*I*a^8*b^4*d^4*x^11 - 10*I*a^9*b^3*d^4*x^9 - 10*I*a^10*b^2*d
^4*x^7 - 5*I*a^11*b*d^4*x^5 - I*a^12*d^4*x^3)*(-b^5/(a^29*d^14))^(1/4)*log
(-337371570183375*I*a^22*d^11*(-b^5/(a^29*d^14))^(3/4) + 337371570183375*s
qrt(d*x)*b^4) - 348075*(a^7*b^5*d^4*x^13 + 5*a^8*b^4*d^4*x^11 + 10*a^9*b^3
*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4*x^5 + a^12*d^4*x^3)*(-b^5/(a
^29*d^14))^(1/4)*log(-337371570183375*a^22*d^11*(-b^5/(a^29*d^14))^(3/4) +
 337371570183375*sqrt(d*x)*b^4) + 4*(348075*b^6*x^12 + 1670760*a*b^5*x^10
+ 3171350*a^2*b^4*x^8 + 2951200*a^3*b^3*x^6 + 1317575*a^4*b^2*x^4 + 204800
*a^5*b*x^2 - 8192*a^6)*sqrt(d*x))/(a^7*b^5*d^4*x^13 + 5*a^8*b^4*d^4*x^11 +
 10*a^9*b^3*d^4*x^9 + 10*a^10*b^2*d^4*x^7 + 5*a^11*b*d^4*x^5 + a^12*d^4*x^
3)
```

Sympy [F]

$$\int \frac{1}{(dx)^{7/2} (a^2 + 2abx^2 + b^2x^4)^3} \, dx = \int \frac{1}{(dx)^{\frac{7}{2}} (a + bx^2)^6} \, dx$$

input
```
integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Integral(1/((d*x)**(7/2)*(a + b*x**2)**6), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.19

$$\int \frac{1}{(dx)^{7/2} (a^2 + 2abx^2 + b^2x^4)^3} \, dx = \frac{8 \left(348075 \, b^6 d^{12} x^{12} + 1670760 \, ab^5 d^{12} x^{10} + 3171350 \, a^2 b^4 d^{12} x^8 + 2951200 \, a^3 b^3 d^{12} x^6 + 1317575 \, a^4 \right.}{(dx)^{\frac{25}{2}} a^7 b^5 d^2 + 5 \, (dx)^{\frac{21}{2}} a^8 b^4 d^4 + 10 \, (dx)^{\frac{17}{2}} a^9 b^3 d^6 + 10 \, (dx)^{\frac{13}{2}} a^{10} b^2 d^8 + 5 \, (dx)}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/163840*(8*(348075*b^6*d^12*x^12 + 1670760*a*b^5*d^12*x^10 + 3171350*a^2*
b^4*d^12*x^8 + 2951200*a^3*b^3*d^12*x^6 + 1317575*a^4*b^2*d^12*x^4 + 20480
0*a^5*b*d^12*x^2 - 8192*a^6*d^12)/((d*x)^(25/2)*a^7*b^5*d^2 + 5*(d*x)^(21/
2)*a^8*b^4*d^4 + 10*(d*x)^(17/2)*a^9*b^3*d^6 + 10*(d*x)^(13/2)*a^10*b^2*d^
8 + 5*(d*x)^(9/2)*a^11*b*d^10 + (d*x)^(5/2)*a^12*d^12) + 348075*b^2*(2*sqr
t(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(
b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))
/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) - sqrt(2)*log(
sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2
)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*
x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^7*d^2))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.05

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{69615\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^8bd^5}$$
$$+\frac{69615\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^8bd^5}$$
$$-\frac{69615\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^8bd^5}$$
$$+\frac{69615\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^8bd^5}$$
$$+\frac{348075\,b^6d^{12}x^{12}+1670760\,ab^5d^{12}x^{10}+3171350\,a^2b^4d^{12}x^8+2951200\,a^3b^3d^{12}x^6+1317575\,a^4b^2d^{12}x^4+20}{20480\left(\sqrt{dx}bd^2x^2+\sqrt{dx}ad^2\right)^5a^7d^3}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
69615/16384*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^8*b*d^5) + 69615/16384*sqrt(2)*
(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*
x))/(a*d^2/b)^(1/4))/(a^8*b*d^5) - 69615/32768*sqrt(2)*(a*b^3*d^2)^(3/4)*l
og(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^8*b*d^5) +
69615/32768*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(a^8*b*d^5) + 1/20480*(348075*b^6*d^12*x^12 + 167
0760*a*b^5*d^12*x^10 + 3171350*a^2*b^4*d^12*x^8 + 2951200*a^3*b^3*d^12*x^6
 + 1317575*a^4*b^2*d^12*x^4 + 204800*a^5*b*d^12*x^2 - 8192*a^6*d^12)/((sqr
t(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)^5*a^7*d^3)
```

Mupad [B] (verification not implemented)

Time = 17.90 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.69

$$\int \frac{1}{(dx)^{7/2} \left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \frac{\frac{10\,b\,d^9\,x^2}{a^2}-\frac{2\,d^9}{5\,a}+\frac{263515\,b^2\,d^9\,x^4}{4096\,a^3}+\frac{18445\,b^3\,d^9\,x^6}{128\,a^4}+\frac{317135\,b^4\,d^9\,x^8}{2048\,a^5}+\cdots}{b^5\,{\left(d\,x\right)}^{25/2}+a^5\,d^{10}\,{\left(d\,x\right)}^{5/2}+10\,a^3\,b^2\,d^6\,{\left(d\,x\right)}^{13/2}+10\,a^2\,b^3\,d^4\,{\left(d\,x\right)}^{17/\cdots}} - \frac{69615\,{\left(-b\right)}^{5/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{29/4}\,d^{7/2}}+\frac{69615\,{\left(-b\right)}^{5/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{d\,x}}{a^{1/4}\,\sqrt{d}}\right)}{8192\,a^{29/4}\,d^{7/2}}$$

input
```
int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^3),x)
```

output
```
((10*b*d^9*x^2)/a^2 - (2*d^9)/(5*a) + (263515*b^2*d^9*x^4)/(4096*a^3) + (1
8445*b^3*d^9*x^6)/(128*a^4) + (317135*b^4*d^9*x^8)/(2048*a^5) + (41769*b^5
*d^9*x^10)/(512*a^6) + (69615*b^6*d^9*x^12)/(4096*a^7))/(b^5*(d*x)^(25/2)
+ a^5*d^10*(d*x)^(5/2) + 10*a^3*b^2*d^6*(d*x)^(13/2) + 10*a^2*b^3*d^4*(d*x
)^(17/2) + 5*a^4*b*d^8*(d*x)^(9/2) + 5*a*b^4*d^2*(d*x)^(21/2)) - (69615*(-
b)^(5/4)*atan(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/2))))/(8192*a^(29/4)*
d^(7/2)) + (69615*(-b)^(5/4)*atanh(((-b)^(1/4)*(d*x)^(1/2))/(a^(1/4)*d^(1/
2))))/(8192*a^(29/4)*d^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1068, normalized size of antiderivative = 3.10

$$\int \frac{1}{(dx)^{7/2} \left(a^2+2abx^2+b^2x^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output

```
(sqrt(d)*( - 696150*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1
/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5*b*x**2
- 3480750*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b**2*x**4 - 69615
00*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*s
qrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**3*x**6 - 6961500*sqrt
(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*
sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**4*x**8 - 3480750*sqrt(x)*b**
(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**5*x**10 - 696150*sqrt(x)*b**(1/4)*a**(
3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4
)*a**(1/4)*sqrt(2)))*b**6*x**12 + 696150*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*s
qrt(2)))*a**5*b*x**2 + 3480750*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a
**4*b**2*x**4 + 6961500*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**
3*x**6 + 6961500*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**4*x**8
+ 3480750*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr...
```

3.488 $\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}+\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}$$

output
```
a*x^6*((b*x^2+a)^2)^(1/2)/(6*b*x^2+6*a)+b*x^8*((b*x^2+a)^2)^(1/2)/(8*b*x^2
+8*a)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{x^6(4a+3bx^2)\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(a+bx^2)^2}\right)\right)}{24\left(-a^2-abx^2+\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}$$

input
```
Integrate[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(x^6*(4*a + 3*b*x^2)*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2]
)))/(24*(-a^2 - a*b*x^2 + Sqrt[a^2]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt{a^2+2abx^2+b^2x^4} \, dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int bx^5 \left(bx^2+a\right) dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^5 \left(bx^2+a\right) dx}{a+bx^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(bx^7+ax^5\right) dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \left(\frac{ax^6}{6}+\frac{bx^8}{8}\right)}{a+bx^2}$$

input
```
Int[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a*x^6)/6 + (b*x^8)/8))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$\frac{x^{6} \left(3 b \,x^{2}+4 a \right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{24}$	24
gosper	$\frac{x^{6} \left(3 b \,x^{2}+4 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{24 b \,x^{2}+24 a}$	36
default	$\frac{x^{6} \left(3 b \,x^{2}+4 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{24 b \,x^{2}+24 a}$	36
orering	$\frac{x^{6} \left(3 b \,x^{2}+4 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{24 b \,x^{2}+24 a}$	36
risch	$\frac{a \,x^{6} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{6 b \,x^{2}+6 a}+\frac{b \,x^{8} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{8 b \,x^{2}+8 a}$	54

input
```
int(x^5*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*x^6*(3*b*x^2+4*a)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{8}bx^8+\frac{1}{6}ax^6$$

input
```
integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*b*x^8 + 1/6*a*x^6
```

Sympy [F]

$$\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x^5\sqrt{(a+bx^2)^2}\,dx$$

input
```
integrate(x**5*((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x**5*sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{8}bx^8+\frac{1}{6}ax^6$$

input
```
integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/8*b*x^8 + 1/6*a*x^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{8}\,bx^8\text{sgn}\left(bx^2+a\right)+\frac{1}{6}\,ax^6\text{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^5*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*b*x^8*sgn(b*x^2 + a) + 1/6*a*x^6*sgn(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 18.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(a^3-4\,a^2\,b\,x^2-5\,a\,b^2\,x^4+3\,b\,x^2\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)\right)}{24\,b^3}$$

input
```
int(x^5*((a + b*x^2)^2)^(1/2),x)
```

output
```
((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a^3 - 4*a^2*b*x^2 - 5*a*b^2*x^4 + 3*b*
x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)))/(24*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{x^6(3b\,x^2+4a)}{24}$$

input
```
int(x^5*((b*x^2+a)^2)^(1/2),x)
```

output
```
(x**6*(4*a + 3*b*x**2))/24
```

3.489 $\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx=-\frac{a(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}+\frac{(a^2+2abx^2+b^2x^4)^{3/2}}{6b^2}$$

output
```
-1/4*a*(b*x^2+a)*((b*x^2+a)^2)^(1/2)/b^2+1/6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)
/b^2
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{\sqrt{(a+bx^2)^2}(3ax^4+2bx^6)}{12\,(a+bx^2)}$$

input
```
Integrate[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(3*a*x^4 + 2*b*x^6))/(12*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1434, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \ 1434$$

$$\frac{1}{2}\int x^2\sqrt{b^2x^4+2abx^2+a^2}dx^2$$

$$\Big\downarrow \ 1100$$

$$\frac{1}{2}\left(\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{3b^2}-\frac{a\int \sqrt{b^2x^4+2abx^2+a^2}dx^2}{b}\right)$$

$$\Big\downarrow \ 1079$$

$$\frac{1}{2}\left(\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{3b^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^2x^2+ab\right)dx^2}{b^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \ 17$$

$$\frac{1}{2}\left(\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{3b^2}-\frac{a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}\right)$$

input
```
Int[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(-1/2*(a*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^2 + (a^2 + 2*a*b*x
^2 + b^2*x^4)^(3/2)/(3*b^2))/2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.36

method	result	size
pseudoelliptic	$\frac{x^{4} \left(2 b \,x^{2}+3 a \right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{12}$	24
gosper	$\frac{x^{4} \left(2 b \,x^{2}+3 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{12 b \,x^{2}+12 a}$	36
default	$\frac{x^{4} \left(2 b \,x^{2}+3 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{12 b \,x^{2}+12 a}$	36
orering	$\frac{x^{4} \left(2 b \,x^{2}+3 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{12 b \,x^{2}+12 a}$	36
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, b \,x^{6}}{6 b \,x^{2}+6 a}+\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, a \,x^{4}}{4 b \,x^{2}+4 a}$	54

input
```
int(x^3*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^4*(2*b*x^2+3*a)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.19

$$\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*b*x^6 + 1/4*a*x^4
```

Sympy [F]

$$\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x^3\sqrt{(a+bx^2)^2}\,dx$$

input
```
integrate(x**3*((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x**3*sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.19

$$\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/6*b*x^6 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.34

$$\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{12}\left(2\,bx^6+3\,ax^4\right)\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^3*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/12*(2*b*x^6 + 3*a*x^4)*sgn(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 18.72 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(8\,b^2\,\left(a^2+b^2\,x^4\right)-12\,a^2\,b^2+4\,a\,b^3\,x^2\right)}{48\,b^4}$$

input
```
int(x^3*((a + b*x^2)^2)^(1/2),x)
```

output
```
((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(8*b^2*(a^2 + b^2*x^4) - 12*a^2*b^2 + 4
*a*b^3*x^2))/(48*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.22

$$\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{x^4(2b\,x^2+3a)}{12}$$

input
```
int(x^3*((b*x^2+a)^2)^(1/2),x)
```

output
```
(x**4*(3*a + 2*b*x**2))/12
```

3.490 $\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 36

$$\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}$$

output
```
1/4*(b*x^2+a)*((b*x^2+a)^2)^(1/2)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 88 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.44

$$\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{x^2(2a+bx^2)\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(a+bx^2)^2}\right)\right)}{-4a^2-4abx^2+4\sqrt{a^2}\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(x^2*(2*a + b*x^2)*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2]))
)/(-4*a^2 - 4*a*b*x^2 + 4*Sqrt[a^2]*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1432, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \sqrt{b^2x^4+2abx^2+a^2}dx^2$$

$$\Big\downarrow \text{1079}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (b^2x^2+ab)\,dx^2}{2b\,(a+bx^2)}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}$$

input
```
Int[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{x^{2}\left(b\,x^{2}+2a\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{4}$	23
default	$\frac{\left(b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{4b}$	24
risch	$\frac{\left(b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{4b}$	24
gosper	$\frac{x^{2}\left(b\,x^{2}+2a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{4b\,x^{2}+4a}$	35
orering	$\frac{x^{2}\left(b\,x^{2}+2a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{4b\,x^{2}+4a}$	35

input
```
int(x*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(b*x^2+2*a)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.36

$$\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

input
```
integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/2*a*x^2
```

Sympy [F]

$$\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x\sqrt{\left(a+bx^2\right)^2}\,dx$$

input
```
integrate(x*((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x*sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.39

$$\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{\left(bx^2+a\right)^2}{4\,b}$$

input
```
integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^2/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{4}\left(bx^4+2\,ax^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(b*x^4 + 2*a*x^2)*sgn(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx=\left(\frac{a}{4\,b}+\frac{x^2}{4}\right)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

input
```
int(x*((a + b*x^2)^2)^(1/2),x)
```

output
```
(a/(4*b) + x^2/4)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.39

$$\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{x^2(b\,x^2+2a)}{4}$$

input
```
int(x*((b*x^2+a)^2)^(1/2),x)
```

output
```
(x**2*(2*a + b*x**2))/4
```

3.491 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x} \, dx = \frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)} + \frac{a\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

output
```
b*x^2*((b*x^2+a)^2)^(1/2)/(2*b*x^2+2*a)+a*((b*x^2+a)^2)^(1/2)*ln(x)/(b*x^2
+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 454 vs. 2(75) = 150.

Time = 0.56 (sec) , antiderivative size = 454, normalized size of antiderivative = 6.05

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x} \, dx$$
$$= \frac{-2a\sqrt{a^2}bx^2 - 2\sqrt{a^2}b^2x^4 + 2abx^2\sqrt{\left(a+bx^2\right)^2} - 2a\left(a^2+abx^2-\sqrt{a^2}\sqrt{\left(a+bx^2\right)^2}\right)\operatorname{arctanh}\left(\frac{bx^2}{\sqrt{a^2}-\sqrt{(a+}}\right.}{}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
```

output
```
(-2*a*Sqrt[a^2]*b*x^2 - 2*Sqrt[a^2]*b^2*x^4 + 2*a*b*x^2*Sqrt[(a + b*x^2)^2
] - 2*a*(a^2 + a*b*x^2 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2])*ArcTanh[(b*x^2)/(S
qrt[a^2] - Sqrt[(a + b*x^2)^2])] - 2*((a^2)^(3/2) + a*Sqrt[a^2]*b*x^2 - a^
2*Sqrt[(a + b*x^2)^2])*Log[x^2] + (a^2)^(3/2)*Log[Sqrt[a^2] - b*x^2 - Sqrt
[(a + b*x^2)^2]] + a*Sqrt[a^2]*b*x^2*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x
^2)^2]] - a^2*Sqrt[(a + b*x^2)^2]*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)
^2]] + (a^2)^(3/2)*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]] + a*Sqrt[a
^2]*b*x^2*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]] - a^2*Sqrt[(a + b*x
^2)^2]*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]])/(4*(a^2 + a*b*x^2 - S
qrt[a^2]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{x}dx}{b(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{x}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a}{x}+bx\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a\log(x)+\frac{bx^2}{2}\right)}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((b*x^2)/2 + a*Log[x]))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(b\,x^{2}+a+a\ln\left(b\,x^{2}\right)\right)}{2}$	26
default	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(b\,x^{2}+2a\ln\left(x\right)\right)}{2b\,x^{2}+2a}$	34
risch	$\frac{b\,x^{2}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{2b\,x^{2}+2a}+\frac{a\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(x\right)}{b\,x^{2}+a}$	52

input
```
int(((b*x^2+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*csgn(b*x^2+a)*(b*x^2+a+a*ln(b*x^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx=\frac{1}{2}\,bx^2+a\log\left(x\right)$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/2*b*x^2 + a*log(x)
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx=\int \frac{\sqrt{\left(a+bx^2\right)^2}}{x}\,dx$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/x,x)
```

output
```
Integral(sqrt((a + b*x**2)**2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x}\,dx = \frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/2*b*x^2 + 1/2*a*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x}\,dx = \frac{1}{2}\,bx^2\mathrm{sgn}\left(bx^2 + a\right) + \frac{1}{2}\,a\log\left(x^2\right)\mathrm{sgn}\left(bx^2 + a\right)$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/2*b*x^2*sgn(b*x^2 + a) + 1/2*a*log(x^2)*sgn(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 17.86 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x}\,dx = \frac{\sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{2} - \frac{\ln\left(a\,b + \frac{a^2}{x^2} + \frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^2}\right)\sqrt{a^2}}{2} + \frac{a\,b\,\ln\left(a\,b + \sqrt{\left(b\,x^2 + a\right)^2}\,\sqrt{b^2} + b^2\,x^2\right)}{2\,\sqrt{b^2}}$$

input
```
int(((a + b*x^2)^2)^(1/2)/x,x)
```

output
```
(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/2 - (log(a*b + a^2/x^2 + ((a^2)^(1/2)*(a
^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/x^2)*(a^2)^(1/2))/2 + (a*b*log(a*b + ((a
+ b*x^2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx=\log(x)\,a+\frac{b\,x^2}{2}$$

input
```
int(((b*x^2+a)^2)^(1/2)/x,x)
```

output
```
(2*log(x)*a + b*x**2)/2
```

3.492 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3} \, dx = -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)} + \frac{b\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

output
```
-1/2*a*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+b*((b*x^2+a)^2)^(1/2)*ln(x)/(b*x^
2+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 178 vs. 2(75) = 150.

Time = 0.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.37

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3} \, dx$$
$$= \frac{a\sqrt{a^2} - a\sqrt{(a+bx^2)^2} - 2abx^2\text{arctanh}\left(\frac{bx^2}{\sqrt{a^2}-\sqrt{(a+bx^2)^2}}\right) - 2\sqrt{a^2}bx^2\log(x^2) + \sqrt{a^2}bx^2\log\Big(a\Big(\sqrt{a^2} - bx \cdots}{4ax^2}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
```

output
```
(a*Sqrt[a^2] - a*Sqrt[(a + b*x^2)^2] - 2*a*b*x^2*ArcTanh[(b*x^2)/(Sqrt[a^2
] - Sqrt[(a + b*x^2)^2])] - 2*Sqrt[a^2]*b*x^2*Log[x^2] + Sqrt[a^2]*b*x^2*L
og[a*(Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2])] + Sqrt[a^2]*b*x^2*Log[a*(S
qrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2])])/(4*a*x^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{x^3}dx}{b(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{x^3}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a}{x^3}+\frac{b}{x}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(b\log(x)-\frac{a}{2x^2}\right)}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/2*a/x^2 + b*Log[x]))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.37

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a \right) \left(-\ln \left(b\,x^{2}\right) b\,x^{2}+a \right)}{2 x^{2}}$	28
default	$\frac{\sqrt{\left(b\,x^{2}+a \right)^{2}}\, \left(2 \ln \left(x \right) x^{2} b -a \right)}{2 x^{2} \left(b\,x^{2}+a \right)}$	38
risch	$-\frac{a \sqrt{\left(b\,x^{2}+a \right)^{2}}}{2 x^{2} \left(b\,x^{2}+a \right)}+\frac{b \sqrt{\left(b\,x^{2}+a \right)^{2}}\, \ln \left(x \right)}{b\,x^{2}+a}$	52

input
```
int(((b*x^2+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*csgn(b*x^2+a)*(-ln(b*x^2)*b*x^2+a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^3} \, dx = \frac{2\,bx^2\log{(x)} - a}{2\,x^2}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*b*x^2*log(x) - a)/x^2
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^3} \, dx = \int \frac{\sqrt{(a + bx^2)^2}}{x^3} \, dx$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt((a + b*x**2)**2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^3} \, dx = \frac{1}{2}\,b\log{(x^2)} - \frac{a}{2\,x^2}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
1/2*b*log(x^2) - 1/2*a/x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{1}{2}\,b\log\left(x^2\right)\operatorname{sgn}\left(bx^2+a\right) - \frac{bx^2\operatorname{sgn}(bx^2+a)+a\operatorname{sgn}(bx^2+a)}{2\,x^2}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/2*b*log(x^2)*sgn(b*x^2 + a) - 1/2*(b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 +
a))/x^2
```

Mupad [B] (verification not implemented)

Time = 17.88 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{\ln\left(a\,b+\sqrt{(b\,x^2+a)^2}\,\sqrt{b^2}+b^2\,x^2\right)\,\sqrt{b^2}}{2} - \frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^2} - \frac{a\,b\,\ln\left(a\,b+\frac{a^2}{x^2}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^2}\right)}{2\,\sqrt{a^2}}$$

input
```
int(((a + b*x^2)^2)^(1/2)/x^3,x)
```

output
```
(log(a*b + ((a + b*x^2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2)*(b^2)^(1/2))/2 - (
a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(2*x^2) - (a*b*log(a*b + a^2/x^2 + ((a^2)
^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/x^2))/(2*(a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^3}\,dx = \frac{2\log(x)\,b\,x^2 - a}{2x^2}$$

input

```
int(((b*x^2+a)^2)^(1/2)/x^3,x)
```

output

```
(2*log(x)*b*x**2 - a)/(2*x**2)
```

3.493 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 39

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx=-\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

output
```
-1/4*(b*x^2+a)*((b*x^2+a)^2)^(1/2)/a/x^4
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx=-\frac{\sqrt{(a+bx^2)^2}(a+2bx^2)}{4x^4(a+bx^2)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]
```

output
```
-1/4*(Sqrt[(a + b*x^2)^2]*(a + 2*b*x^2))/(x^4*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1434, 1102, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{\sqrt{b^2x^4+2abx^2+a^2}}{x^6}dx^2$$

$$\Big\downarrow \text{1102}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{x^6}dx^2}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{x^6}dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]
```

output
```
-1/4*((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a*x^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 1102
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*F
racPart[p]))   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c,
 d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$-\frac{\left(2 b \,x^{2}+a \right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{4 x^{4}}$	22
gosper	$-\frac{\left(2 b \,x^{2}+a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{4 x^{4} \left(b \,x^{2}+a \right)}$	34
default	$-\frac{\left(2 b \,x^{2}+a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{4 x^{4} \left(b \,x^{2}+a \right)}$	34
orering	$-\frac{\left(2 b \,x^{2}+a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{4 x^{4} \left(b \,x^{2}+a \right)}$	34
risch	$\frac{\left(-\frac{b \,x^{2}}{2}-\frac{a}{4}\right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{x^{4} \left(b \,x^{2}+a \right)}$	35

input
```
int(((b*x^2+a)^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*b*x^2+a)*csgn(b*x^2+a)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx = -\frac{2\,bx^2+a}{4\,x^4}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/4*(2*b*x^2 + a)/x^4
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx = \int \frac{\sqrt{(a+bx^2)^2}}{x^5}\,dx$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/x**5,x)
```

output
```
Integral(sqrt((a + b*x**2)**2)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx = -\frac{2\,bx^2+a}{4\,x^4}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x^2 + a)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx = -\frac{2\,bx^2\mathrm{sgn}(bx^2+a)+a\mathrm{sgn}(bx^2+a)}{4\,x^4}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/4*(2*b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a))/x^4
```

Mupad [B] (verification not implemented)

Time = 17.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx = -\frac{\left(2\,b\,x^2+a\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{4\,x^4\,\left(b\,x^2+a\right)}$$

input
```
int(((a + b*x^2)^2)^(1/2)/x^5,x)
```

output
```
-((a + 2*b*x^2)*((a + b*x^2)^2)^(1/2))/(4*x^4*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^5} \, dx = \frac{-2b\, x^2 - a}{4x^4}$$

input
```
int(((b*x^2+a)^2)^(1/2)/x^5,x)
```

output
```
( - a - 2*b*x**2)/(4*x**4)
```

3.494 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx = -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)}$$

output
```
-1/6*a*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)-1/4*b*((b*x^2+a)^2)^(1/2)/x^4/(b*
x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx = -\frac{\sqrt{(a+bx^2)^2}(2a+3bx^2)}{12x^6(a+bx^2)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]
```

output
```
-1/12*(Sqrt[(a + b*x^2)^2]*(2*a + 3*b*x^2))/(x^6*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{x^7}dx}{b(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{x^7}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a}{x^7}+\frac{b}{x^5}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{6x^6}-\frac{b}{4x^4}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]
```

output
```
((-1/6*a/x^6 - b/(4*x^4))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$-\frac{(3b\,x^2+2a)\,\mathrm{csgn}(b\,x^2+a)}{12x^6}$	24
risch	$\frac{\left(-\frac{b\,x^2}{4}-\frac{a}{6}\right)\sqrt{(b\,x^2+a)^2}}{x^6(b\,x^2+a)}$	35
gosper	$-\frac{(3b\,x^2+2a)\sqrt{(b\,x^2+a)^2}}{12x^6(b\,x^2+a)}$	36
default	$-\frac{(3b\,x^2+2a)\sqrt{(b\,x^2+a)^2}}{12x^6(b\,x^2+a)}$	36
orering	$-\frac{(3b\,x^2+2a)\sqrt{(b\,x^2+a)^2}}{12x^6(b\,x^2+a)}$	36

input
```
int(((b*x^2+a)^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3*b*x^2+2*a)*csgn(b*x^2+a)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^7} \, dx = -\frac{3 \, bx^2 + 2 \, a}{12 \, x^6}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
-1/12*(3*b*x^2 + 2*a)/x^6
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^7} \, dx = \int \frac{\sqrt{(a + bx^2)^2}}{x^7} \, dx$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/x**7,x)
```

output
```
Integral(sqrt((a + b*x**2)**2)/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^7} \, dx = -\frac{3 \, bx^2 + 2 \, a}{12 \, x^6}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/12*(3*b*x^2 + 2*a)/x^6
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx = -\frac{3\,bx^2\mathrm{sgn}(bx^2+a)+2\,a\mathrm{sgn}(bx^2+a)}{12\,x^6}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^7,x, algorithm="giac")
```

output
```
-1/12*(3*b*x^2*sgn(b*x^2 + a) + 2*a*sgn(b*x^2 + a))/x^6
```

Mupad [B] (verification not implemented)

Time = 17.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx = -\frac{(3\,b\,x^2+2\,a)\,\sqrt{(b\,x^2+a)^2}}{12\,x^6\,(b\,x^2+a)}$$

input
```
int(((a + b*x^2)^2)^(1/2)/x^7,x)
```

output
```
-((2*a + 3*b*x^2)*((a + b*x^2)^2)^(1/2))/(12*x^6*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx = \frac{-3b\,x^2-2a}{12x^6}$$

input
```
int(((b*x^2+a)^2)^(1/2)/x^7,x)
```

output
```
( - 2*a - 3*b*x**2)/(12*x**6)
```

3.495 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9} dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9} dx = -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}$$

output
```
-1/8*a*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)-1/6*b*((b*x^2+a)^2)^(1/2)/x^6/(b*
x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9} dx = -\frac{\sqrt{(a+bx^2)^2}(3a+4bx^2)}{24x^8(a+bx^2)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
```

output
```
-1/24*(Sqrt[(a + b*x^2)^2]*(3*a + 4*b*x^2))/(x^8*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{x^9}dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{x^9}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a}{x^9}+\frac{b}{x^7}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a}{8x^8}-\frac{b}{6x^6}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
```

output
```
((-1/8*a/x^8 - b/(6*x^6))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$-\frac{\left(4 b \,x^{2}+3 a \right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{24 x^{8}}$	24
risch	$\frac{\left(-\frac{b \,x^{2}}{6}-\frac{a}{8}\right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{x^{8} \left(b \,x^{2}+a \right)}$	35
gosper	$-\frac{\left(4 b \,x^{2}+3 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{24 x^{8} \left(b \,x^{2}+a \right)}$	36
default	$-\frac{\left(4 b \,x^{2}+3 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{24 x^{8} \left(b \,x^{2}+a \right)}$	36
orering	$-\frac{\left(4 b \,x^{2}+3 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{24 x^{8} \left(b \,x^{2}+a \right)}$	36

input
```
int(((b*x^2+a)^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(4*b*x^2+3*a)*csgn(b*x^2+a)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx=-\frac{4\,bx^2+3\,a}{24\,x^8}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
-1/24*(4*b*x^2 + 3*a)/x^8
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx=\int \frac{\sqrt{(a+bx^2)^2}}{x^9}\,dx$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/x**9,x)
```

output
```
Integral(sqrt((a + b*x**2)**2)/x**9, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx=-\frac{4\,bx^2+3\,a}{24\,x^8}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-1/24*(4*b*x^2 + 3*a)/x^8
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx = -\frac{4\,bx^2\mathrm{sgn}(bx^2+a)+3\,a\mathrm{sgn}(bx^2+a)}{24\,x^8}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^9,x, algorithm="giac")
```

output
```
-1/24*(4*b*x^2*sgn(b*x^2 + a) + 3*a*sgn(b*x^2 + a))/x^8
```

Mupad [B] (verification not implemented)

Time = 17.55 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx = -\frac{(4\,b\,x^2+3\,a)\,\sqrt{(b\,x^2+a)^2}}{24\,x^8\,(b\,x^2+a)}$$

input
```
int(((a + b*x^2)^2)^(1/2)/x^9,x)
```

output
```
-((3*a + 4*b*x^2)*((a + b*x^2)^2)^(1/2))/(24*x^8*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx = \frac{-4b\,x^2-3a}{24x^8}$$

input
```
int(((b*x^2+a)^2)^(1/2)/x^9,x)
```

output
```
( - 3*a - 4*b*x**2)/(24*x**8)
```

3.496 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx=-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\,(a+bx^2)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\,(a+bx^2)}$$

output
```
-1/10*a*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)-1/8*b*((b*x^2+a)^2)^(1/2)/x^8/(
b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx=-\frac{\sqrt{(a+bx^2)^2}(4a+5bx^2)}{40x^{10}\,(a+bx^2)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
```

output
```
-1/40*(Sqrt[(a + b*x^2)^2]*(4*a + 5*b*x^2))/(x^10*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{x^{11}}dx}{b(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{x^{11}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a}{x^{11}}+\frac{b}{x^9}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{10x^{10}}-\frac{b}{8x^8}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
```

output
```
((-1/10*a/x^10 - b/(8*x^8))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$-\frac{\left(5 b \,x^{2}+4 a \right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{40 x^{10}}$	24
risch	$\frac{\left(-\frac{b \,x^{2}}{8}-\frac{a}{10}\right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{x^{10} \left(b \,x^{2}+a \right)}$	35
gosper	$-\frac{\left(5 b \,x^{2}+4 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{40 x^{10} \left(b \,x^{2}+a \right)}$	36
default	$-\frac{\left(5 b \,x^{2}+4 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{40 x^{10} \left(b \,x^{2}+a \right)}$	36
orering	$-\frac{\left(5 b \,x^{2}+4 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{40 x^{10} \left(b \,x^{2}+a \right)}$	36

input
```
int(((b*x^2+a)^2)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(5*b*x^2+4*a)*csgn(b*x^2+a)/x^10
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx=-\frac{5\,bx^2+4\,a}{40\,x^{10}}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="fricas")
```

output
```
-1/40*(5*b*x^2 + 4*a)/x^10
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx=\text{Timed out}$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/x**11,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx=-\frac{5\,bx^2+4\,a}{40\,x^{10}}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="maxima")
```

output
```
-1/40*(5*b*x^2 + 4*a)/x^10
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx = -\frac{5\,bx^2\mathrm{sgn}(bx^2+a)+4\,a\mathrm{sgn}(bx^2+a)}{40\,x^{10}}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^11,x, algorithm="giac")
```

output
```
-1/40*(5*b*x^2*sgn(b*x^2 + a) + 4*a*sgn(b*x^2 + a))/x^10
```

Mupad [B] (verification not implemented)

Time = 17.82 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx = -\frac{(5\,b\,x^2+4\,a)\,\sqrt{(b\,x^2+a)^2}}{40\,x^{10}\,(b\,x^2+a)}$$

input
```
int(((a + b*x^2)^2)^(1/2)/x^11,x)
```

output
```
-((4*a + 5*b*x^2)*((a + b*x^2)^2)^(1/2))/(40*x^10*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx = \frac{-5b\,x^2-4a}{40x^{10}}$$

input
```
int(((b*x^2+a)^2)^(1/2)/x^11,x)
```

output
```
( - 4*a - 5*b*x**2)/(40*x**10)
```

3.497 $\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5\,(a+bx^2)}+\frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}$$

output
```
a*x^5*((b*x^2+a)^2)^(1/2)/(5*b*x^2+5*a)+b*x^7*((b*x^2+a)^2)^(1/2)/(7*b*x^2
+7*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{\sqrt{(a+bx^2)^2}(7ax^5+5bx^7)}{35\,(a+bx^2)}$$

input
```
Integrate[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(7*a*x^5 + 5*b*x^7))/(35*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int bx^4\left(bx^2+a\right)dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^4\left(bx^2+a\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(bx^6+ax^4\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{ax^5}{5}+\frac{bx^7}{7}\right)}{a+bx^2}$$

input
```
Int[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a*x^5)/5 + (b*x^7)/7))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x^{5}\left(5b\,x^{2}+7a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{35b\,x^{2}+35a}$	36
default	$\frac{x^{5}\left(5b\,x^{2}+7a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{35b\,x^{2}+35a}$	36
orering	$\frac{x^{5}\left(5b\,x^{2}+7a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{35b\,x^{2}+35a}$	36
risch	$\frac{a\,x^{5}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{5b\,x^{2}+5a}+\frac{b\,x^{7}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{7b\,x^{2}+7a}$	54

input
```
int(x^4*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*x^5*(5*b*x^2+7*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

input
```
integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/7*b*x^7 + 1/5*a*x^5
```

Sympy [F]

$$\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x^4\sqrt{(a+bx^2)^2}\,dx$$

input
```
integrate(x**4*((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x**4*sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

input
```
integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/7*b*x^7 + 1/5*a*x^5
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{7}\,bx^7\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{5}\,ax^5\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^4*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/7*b*x^7*sgn(b*x^2 + a) + 1/5*a*x^5*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x^4\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int(x^4*((a + b*x^2)^2)^(1/2),x)
```

output
```
int(x^4*((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{x^5(5b\,x^2+7a)}{35}$$

input
```
int(x^4*((b*x^2+a)^2)^(1/2),x)
```

output
```
(x**5*(7*a + 5*b*x**2))/35
```

3.498 $\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}$$

output
```
a*x^3*((b*x^2+a)^2)^(1/2)/(3*b*x^2+3*a)+b*x^5*((b*x^2+a)^2)^(1/2)/(5*b*x^2
+5*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{\sqrt{(a+bx^2)^2}(5ax^3+3bx^5)}{15(a+bx^2)}$$

input
```
Integrate[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(5*a*x^3 + 3*b*x^5))/(15*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int bx^2\left(bx^2+a\right)dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2\left(bx^2+a\right)dx}{a+bx^2}$$

$$\Big\downarrow \ 244$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(bx^4+ax^2\right)dx}{a+bx^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{ax^3}{3}+\frac{bx^5}{5}\right)}{a+bx^2}$$

input
```
Int[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a*x^3)/3 + (b*x^5)/5))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{x^{3}\left(3 b \,x^{2}+5 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{15 b \,x^{2}+15 a}$	36
default	$\frac{x^{3}\left(3 b \,x^{2}+5 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{15 b \,x^{2}+15 a}$	36
orering	$\frac{x^{3}\left(3 b \,x^{2}+5 a \right) \sqrt{\left(b \,x^{2}+a \right)^{2}}}{15 b \,x^{2}+15 a}$	36
risch	$\frac{a \,x^{3} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{3 b \,x^{2}+3 a}+\frac{b \,x^{5} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{5 b \,x^{2}+5 a}$	54

input
```
int(x^2*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x^3*(3*b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/5*b*x^5 + 1/3*a*x^3
```

Sympy [F]

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x^2\sqrt{(a+bx^2)^2}\,dx$$

input
```
integrate(x**2*((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.16

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*b*x^5 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{5}\,bx^5\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,ax^3\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^2*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/5*b*x^5*sgn(b*x^2 + a) + 1/3*a*x^3*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x^2\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int(x^2*((a + b*x^2)^2)^(1/2),x)
```

output
```
int(x^2*((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{x^3(3b\,x^2+5a)}{15}$$

input
```
int(x^2*((b*x^2+a)^2)^(1/2),x)
```

output
```
(x**3*(5*a + 3*b*x**2))/15
```

3.499 $\int \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

output
```
a*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+b*x^3*((b*x^2+a)^2)^(1/2)/(3*b*x^2+3*a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

$$\int \sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{\sqrt{\left(a+bx^2\right)^2}\left(3ax+bx^3\right)}{3\left(a+bx^2\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(3*a*x + b*x^3))/(3*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1384, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (b^2x^2+ab)\,dx}{b\,(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(abx+\frac{b^2x^3}{3}\right)\sqrt{a^2+2abx^2+b^2x^4}}{b\,(a+bx^2)}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a*b*x + (b^2*x^3)/3)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(b*(a + b*x^2))
```

Defintions of rubi rules used

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

method	result	size
gosper	$\frac{x(b\,x^2+3a)\sqrt{(b\,x^2+a)^2}}{3b\,x^2+3a}$	33
default	$\frac{x(b\,x^2+3a)\sqrt{(b\,x^2+a)^2}}{3b\,x^2+3a}$	33
orering	$\frac{x(b\,x^2+3a)\sqrt{(b\,x^2+a)^2}}{3b\,x^2+3a}$	33
risch	$\frac{ax\sqrt{(b\,x^2+a)^2}}{b\,x^2+a}+\frac{b\,x^3\sqrt{(b\,x^2+a)^2}}{3b\,x^2+3a}$	51

input
```
int(((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.14

$$\int \sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{1}{3}\,bx^3+ax$$

input
```
integrate(((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*b*x^3 + a*x
```

Sympy [F]

$$\int \sqrt{a^2 + 2abx^2 + b^2x^4}\,dx = \int \sqrt{(a + bx^2)^2}\,dx$$

input
```
integrate(((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.14

$$\int \sqrt{a^2 + 2abx^2 + b^2x^4}\,dx = \frac{1}{3}\,bx^3 + ax$$

input
```
integrate(((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*b*x^3 + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.27

$$\int \sqrt{a^2 + 2abx^2 + b^2x^4}\,dx = \frac{1}{3}\,(bx^3 + 3\,ax)\mathrm{sgn}\left(bx^2 + a\right)$$

input
```
integrate(((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(b*x^3 + 3*a*x)*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2 + 2abx^2 + b^2x^4}\, dx = \int \sqrt{(b\,x^2 + a)^2}\, dx$$

input
```
int(((a + b*x^2)^2)^(1/2),x)
```

output
```
int(((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.16

$$\int \sqrt{a^2 + 2abx^2 + b^2x^4}\, dx = \frac{x(b\,x^2 + 3a)}{3}$$

input
```
int(((b*x^2+a)^2)^(1/2),x)
```

output
```
(x*(3*a + b*x**2))/3
```

3.500 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 72

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2} dx = -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)} + \frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

output
```
-a*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+b*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2} dx = \frac{(-a+bx^2)\sqrt{(a+bx^2)^2}}{x(a+bx^2)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
```

output
```
((-a + b*x^2)*Sqrt[(a + b*x^2)^2])/(x*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{x^2}dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{x^2}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a}{x^2}+b\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(bx-\frac{a}{x}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
```

output
```
((-(a/x) + b*x)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{\left(-b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x\left(b\,x^{2}+a\right)}$	34
default	$-\frac{\left(-b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x\left(b\,x^{2}+a\right)}$	34
orering	$-\frac{\left(-b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x\left(b\,x^{2}+a\right)}$	34
risch	$-\frac{a\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x\left(b\,x^{2}+a\right)}+\frac{bx\sqrt{\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}$	51

input
```
int(((b*x^2+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(-b*x^2+a)*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{bx^2-a}{x}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(b*x^2 - a)/x
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \int \frac{\sqrt{(a+bx^2)^2}}{x^2}\,dx$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a + b*x**2)**2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = bx - \frac{a}{x}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
b*x - a/x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^2} \, dx = bx\text{sgn}\left(bx^2 + a\right) - \frac{a\text{sgn}(bx^2 + a)}{x}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
b*x*sgn(b*x^2 + a) - a*sgn(b*x^2 + a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^2} \, dx = \int \frac{\sqrt{\left(b\,x^2 + a\right)^2}}{x^2} \, dx$$

input
```
int(((a + b*x^2)^2)^(1/2)/x^2,x)
```

output
```
int(((a + b*x^2)^2)^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^2} \, dx = \frac{b\,x^2 - a}{x}$$

input
```
int(((b*x^2+a)^2)^(1/2)/x^2,x)
```

output
```
( - a + b*x**2)/x
```

3.501 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 77

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}$$

output
```
-1/3*a*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-b*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = -\frac{\sqrt{(a+bx^2)^2}(a+3bx^2)}{3x^3(a+bx^2)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]
```

output
```
-1/3*(Sqrt[(a + b*x^2)^2]*(a + 3*b*x^2))/(x^3*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{x^4}dx}{b(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{x^4}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a}{x^4}+\frac{b}{x^2}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{3x^3}-\frac{b}{x}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]
```

output
```
((-1/3*a/x^3 - b/x)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{(3b\,x^2+a)\sqrt{(b\,x^2+a)^2}}{3x^3(b\,x^2+a)}$	34
default	$-\frac{(3b\,x^2+a)\sqrt{(b\,x^2+a)^2}}{3x^3(b\,x^2+a)}$	34
orering	$-\frac{(3b\,x^2+a)\sqrt{(b\,x^2+a)^2}}{3x^3(b\,x^2+a)}$	34
risch	$\frac{\left(-b\,x^2-\frac{a}{3}\right)\sqrt{(b\,x^2+a)^2}}{x^3(b\,x^2+a)}$	35

input
```
int(((b*x^2+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*b*x^2+a)*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^4}\,dx = -\frac{3\,bx^2 + a}{3\,x^3}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(3*b*x^2 + a)/x^3
```

Sympy [F]

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^4}\,dx = \int \frac{\sqrt{(a + bx^2)^2}}{x^4}\,dx$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/x**4,x)
```

output
```
Integral(sqrt((a + b*x**2)**2)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^4}\,dx = -\frac{3\,bx^2 + a}{3\,x^3}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(3*b*x^2 + a)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = -\frac{3\,bx^2\mathrm{sgn}(bx^2+a)+a\mathrm{sgn}(bx^2+a)}{3\,x^3}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/3*(3*b*x^2*sgn(b*x^2 + a) + a*sgn(b*x^2 + a))/x^3
```

Mupad [B] (verification not implemented)

Time = 17.71 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = -\frac{(3\,b\,x^2+a)\,\sqrt{(b\,x^2+a)^2}}{3\,x^3\,(b\,x^2+a)}$$

input
```
int(((a + b*x^2)^2)^(1/2)/x^4,x)
```

output
```
-((a + 3*b*x^2)*((a + b*x^2)^2)^(1/2))/(3*x^3*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = \frac{-3b\,x^2-a}{3x^3}$$

input
```
int(((b*x^2+a)^2)^(1/2)/x^4,x)
```

output
```
( - a - 3*b*x**2)/(3*x**3)
```

3.502 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx=-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}$$

output
```
-1/5*a*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-1/3*b*((b*x^2+a)^2)^(1/2)/x^3/(b*
x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx=-\frac{\sqrt{(a+bx^2)^2}(3a+5bx^2)}{15x^5(a+bx^2)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]
```

output
```
-1/15*(Sqrt[(a + b*x^2)^2]*(3*a + 5*b*x^2))/(x^5*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \frac{b(bx^2+a)}{x^6}dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \frac{bx^2+a}{x^6}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(\frac{a}{x^6}+\frac{b}{x^4}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{5x^5}-\frac{b}{3x^3}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]
```

output
```
((-1/5*a/x^5 - b/(3*x^3))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^{2}}{3}-\frac{a}{5}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x^{5}\left(b\,x^{2}+a\right)}$	35
gosper	$-\frac{\left(5b\,x^{2}+3a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{15x^{5}\left(b\,x^{2}+a\right)}$	36
default	$-\frac{\left(5b\,x^{2}+3a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{15x^{5}\left(b\,x^{2}+a\right)}$	36
orering	$-\frac{\left(5b\,x^{2}+3a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{15x^{5}\left(b\,x^{2}+a\right)}$	36

input
```
int(((b*x^2+a)^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/x^5*(-1/3*b*x^2-1/5*a)/(b*x^2+a)*((b*x^2+a)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = -\frac{5\,bx^2+3\,a}{15\,x^5}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/15*(5*b*x^2 + 3*a)/x^5
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = \int \frac{\sqrt{(a+bx^2)^2}}{x^6}\,dx$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/x**6,x)
```

output
```
Integral(sqrt((a + b*x**2)**2)/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = -\frac{5\,bx^2+3\,a}{15\,x^5}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
-1/15*(5*b*x^2 + 3*a)/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = -\frac{5\,bx^2\mathrm{sgn}(bx^2+a)+3\,a\mathrm{sgn}(bx^2+a)}{15\,x^5}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^6,x, algorithm="giac")
```

output
```
-1/15*(5*b*x^2*sgn(b*x^2 + a) + 3*a*sgn(b*x^2 + a))/x^5
```

Mupad [B] (verification not implemented)

Time = 18.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = -\frac{(5\,b\,x^2+3\,a)\,\sqrt{(b\,x^2+a)^2}}{15\,x^5\,(b\,x^2+a)}$$

input
```
int(((a + b*x^2)^2)^(1/2)/x^6,x)
```

output
```
-((3*a + 5*b*x^2)*((a + b*x^2)^2)^(1/2))/(15*x^5*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx = \frac{-5b\,x^2-3a}{15x^5}$$

input
```
int(((b*x^2+a)^2)^(1/2)/x^6,x)
```

output
```
( - 3*a - 5*b*x**2)/(15*x**5)
```

3.503 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx = -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\,(a+bx^2)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\,(a+bx^2)}$$

output
```
-1/7*a*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-1/5*b*((b*x^2+a)^2)^(1/2)/x^5/(b*
x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx = -\frac{\sqrt{(a+bx^2)^2}(5a+7bx^2)}{35x^7\,(a+bx^2)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]
```

output
```
-1/35*(Sqrt[(a + b*x^2)^2]*(5*a + 7*b*x^2))/(x^7*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{x^8}dx}{b(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{x^8}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a}{x^8}+\frac{b}{x^6}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{7x^7}-\frac{b}{5x^5}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]
```

output
```
((-1/7*a/x^7 - b/(5*x^5))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^{2}}{5}-\frac{a}{7}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x^{7}\left(b\,x^{2}+a\right)}$	35
gosper	$-\frac{\left(7b\,x^{2}+5a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{35x^{7}\left(b\,x^{2}+a\right)}$	36
default	$-\frac{\left(7b\,x^{2}+5a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{35x^{7}\left(b\,x^{2}+a\right)}$	36
orering	$-\frac{\left(7b\,x^{2}+5a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{35x^{7}\left(b\,x^{2}+a\right)}$	36

input
```
int(((b*x^2+a)^2)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/x^7*(-1/5*b*x^2-1/7*a)/(b*x^2+a)*((b*x^2+a)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx = -\frac{7\,bx^2+5\,a}{35\,x^7}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-1/35*(7*b*x^2 + 5*a)/x^7
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx = \int \frac{\sqrt{(a+bx^2)^2}}{x^8}\,dx$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/x**8,x)
```

output
```
Integral(sqrt((a + b*x**2)**2)/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx = -\frac{7\,bx^2+5\,a}{35\,x^7}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
-1/35*(7*b*x^2 + 5*a)/x^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx = -\frac{7\,bx^2\mathrm{sgn}(bx^2+a)+5\,a\mathrm{sgn}(bx^2+a)}{35\,x^7}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^8,x, algorithm="giac")
```

output
```
-1/35*(7*b*x^2*sgn(b*x^2 + a) + 5*a*sgn(b*x^2 + a))/x^7
```

Mupad [B] (verification not implemented)

Time = 18.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx = -\frac{(7\,b\,x^2+5\,a)\,\sqrt{(b\,x^2+a)^2}}{35\,x^7\,(b\,x^2+a)}$$

input
```
int(((a + b*x^2)^2)^(1/2)/x^8,x)
```

output
```
-((5*a + 7*b*x^2)*((a + b*x^2)^2)^(1/2))/(35*x^7*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8}\,dx = \frac{-7b\,x^2-5a}{35x^7}$$

input
```
int(((b*x^2+a)^2)^(1/2)/x^8,x)
```

output
```
( - 5*a - 7*b*x**2)/(35*x**7)
```

3.504 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}} dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}} dx = -\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)} - \frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}$$

output
```
-1/9*a*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-1/7*b*((b*x^2+a)^2)^(1/2)/x^7/(b*
x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}} dx = -\frac{\sqrt{(a+bx^2)^2}(7a+9bx^2)}{63x^9(a+bx^2)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]
```

output
```
-1/63*(Sqrt[(a + b*x^2)^2]*(7*a + 9*b*x^2))/(x^9*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \frac{b(bx^2+a)}{x^{10}}dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \frac{bx^2+a}{x^{10}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(\frac{a}{x^{10}}+\frac{b}{x^8}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a}{9x^9}-\frac{b}{7x^7}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]
```

output
```
((-1/9*a/x^9 - b/(7*x^7))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{\left(-\frac{b\,x^{2}}{7}-\frac{a}{9}\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x^{9}\left(b\,x^{2}+a\right)}$	35
gosper	$-\frac{\left(9b\,x^{2}+7a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{63x^{9}\left(b\,x^{2}+a\right)}$	36
default	$-\frac{\left(9b\,x^{2}+7a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{63x^{9}\left(b\,x^{2}+a\right)}$	36
orering	$-\frac{\left(9b\,x^{2}+7a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{63x^{9}\left(b\,x^{2}+a\right)}$	36

input
```
int(((b*x^2+a)^2)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/x^9*(-1/7*b*x^2-1/9*a)/(b*x^2+a)*((b*x^2+a)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^{10}}\,dx = -\frac{9\,bx^2 + 7\,a}{63\,x^9}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="fricas")
```

output
```
-1/63*(9*b*x^2 + 7*a)/x^9
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^{10}}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/x**10,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{x^{10}}\,dx = -\frac{9\,bx^2 + 7\,a}{63\,x^9}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="maxima")
```

output
```
-1/63*(9*b*x^2 + 7*a)/x^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}}\,dx = -\frac{9\,bx^2\mathrm{sgn}(bx^2+a)+7\,a\mathrm{sgn}(bx^2+a)}{63\,x^9}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/x^10,x, algorithm="giac")
```

output
```
-1/63*(9*b*x^2*sgn(b*x^2 + a) + 7*a*sgn(b*x^2 + a))/x^9
```

Mupad [B] (verification not implemented)

Time = 18.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}}\,dx = -\frac{(9\,b\,x^2+7\,a)\,\sqrt{(b\,x^2+a)^2}}{63\,x^9\,(b\,x^2+a)}$$

input
```
int(((a + b*x^2)^2)^(1/2)/x^10,x)
```

output
```
-((7*a + 9*b*x^2)*((a + b*x^2)^2)^(1/2))/(63*x^9*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}}\,dx = \frac{-9b\,x^2-7a}{63x^9}$$

input
```
int(((b*x^2+a)^2)^(1/2)/x^10,x)
```

output
```
( - 7*a - 9*b*x**2)/(63*x**9)
```

3.505 $\int x^9(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^9\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\,(a+bx^2)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\,(a+bx^2)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\,(a+bx^2)}+\frac{b^3x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\,(a+bx^2)}$$

output
```
a^3*x^10*((b*x^2+a)^2)^(1/2)/(10*b*x^2+10*a)+a^2*b*x^12*((b*x^2+a)^2)^(1/2
)/(4*b*x^2+4*a)+3*a*b^2*x^14*((b*x^2+a)^2)^(1/2)/(14*b*x^2+14*a)+b^3*x^16*
((b*x^2+a)^2)^(1/2)/(16*b*x^2+16*a)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int x^9\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{x^{10}(56a^3+140a^2bx^2+120ab^2x^4+35b^3x^6)\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(a+bx^2)^2}\right)\right)}{560\left(-a^2-abx^2+\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}$$

input
```
Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(x^10*(56*a^3 + 140*a^2*b*x^2 + 120*a*b^2*x^4 + 35*b^3*x^6)*(Sqrt[a^2]*b*x
^2 + a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])))/(560*(-a^2 - a*b*x^2 + Sqrt[a^2
]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$$
$$\Big\downarrow \text{ 1384}$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int b^3 x^9 \left(bx^2+a\right)^3 dx}{b^3 \left(a+bx^2\right)}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^9 \left(bx^2+a\right)^3 dx}{a+bx^2}$$
$$\Big\downarrow \text{ 243}$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^8 \left(bx^2+a\right)^3 dx^2}{2 \left(a+bx^2\right)}$$
$$\Big\downarrow \text{ 49}$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(b^3x^{14}+3ab^2x^{12}+3a^2bx^{10}+a^3x^8\right) dx^2}{2 \left(a+bx^2\right)}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \left(\frac{a^3x^{10}}{5}+\frac{1}{2}a^2bx^{12}+\frac{3}{7}ab^2x^{14}+\frac{b^3x^{16}}{8}\right)}{2 \left(a+bx^2\right)}$$

input
```
Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^3*x^10)/5 + (a^2*b*x^12)/2 + (3*a*b^2
*x^14)/7 + (b^3*x^16)/8))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.28

method	result	size
pseudoelliptic	$\frac{x^{10}\left(35b^3x^6+120b^2x^4a+140a^2b\,x^2+56a^3\right)\operatorname{csgn}\left(b\,x^2+a\right)}{560}$	46
gosper	$\frac{x^{10}\left(35b^3x^6+120b^2x^4a+140a^2b\,x^2+56a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{560\left(b\,x^2+a\right)^3}$	58
default	$\frac{x^{10}\left(35b^3x^6+120b^2x^4a+140a^2b\,x^2+56a^3\right)\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{560\left(b\,x^2+a\right)^3}$	58
orering	$\frac{x^{10}\left(35b^3x^6+120b^2x^4a+140a^2b\,x^2+56a^3\right)\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{3}{2}}}{560\left(b\,x^2+a\right)^3}$	67
risch	$\frac{a^3x^{10}\sqrt{\left(b\,x^2+a\right)^2}}{10b\,x^2+10a}+\frac{a^2b\,x^{12}\sqrt{\left(b\,x^2+a\right)^2}}{4b\,x^2+4a}+\frac{3\sqrt{\left(b\,x^2+a\right)^2}\,b^2a\,x^{14}}{14\left(b\,x^2+a\right)}+\frac{b^3x^{16}\sqrt{\left(b\,x^2+a\right)^2}}{16b\,x^2+16a}$	116

input
```
int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/560*x^10*(35*b^3*x^6+120*a*b^2*x^4+140*a^2*b*x^2+56*a^3)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^9\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

input
```
integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Sympy [F]

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \int x^9 \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**9*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{1}{16}\, b^3x^{16} + \frac{3}{14}\, ab^2x^{14} + \frac{1}{4}\, a^2bx^{12} + \frac{1}{10}\, a^3x^{10}$$

input
```
integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{1}{16}\, b^3x^{16}\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{14}\, ab^2x^{14}\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{4}\, a^2bx^{12}\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{10}\, a^3x^{10}\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/16*b^3*x^16*sgn(b*x^2 + a) + 3/14*a*b^2*x^14*sgn(b*x^2 + a) + 1/4*a^2*b*
x^12*sgn(b*x^2 + a) + 1/10*a^3*x^10*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \int x^9 \, \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2} \, dx$$

input
```
int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{x^{10}(35b^3x^6+120a\,b^2x^4+140a^2b\,x^2+56a^3)}{560}$$

input
```
int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(x**10*(56*a**3 + 140*a**2*b*x**2 + 120*a*b**2*x**4 + 35*b**3*x**6))/560
```

3.506 $\int x^7(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{ab^2x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}$$

output
```
a^3*x^8*((b*x^2+a)^2)^(1/2)/(8*b*x^2+8*a)+3*a^2*b*x^10*((b*x^2+a)^2)^(1/2)
/(10*b*x^2+10*a)+a*b^2*x^12*((b*x^2+a)^2)^(1/2)/(4*b*x^2+4*a)+b^3*x^14*((b
*x^2+a)^2)^(1/2)/(14*b*x^2+14*a)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{x^8(35a^3+84a^2bx^2+70ab^2x^4+20b^3x^6)\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(a+bx^2)^2}\right)\right)}{280\left(-a^2-abx^2+\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}$$

input
```
Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(x^8*(35*a^3 + 84*a^2*b*x^2 + 70*a*b^2*x^4 + 20*b^3*x^6)*(Sqrt[a^2]*b*x^2
+ a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])))/(280*(-a^2 - a*b*x^2 + Sqrt[a^2]*S
qrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int b^3x^7 \left(bx^2+a\right)^3dx}{b^3 \left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^7 \left(bx^2+a\right)^3dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^6 \left(bx^2+a\right)^3dx^2}{2 \left(a+bx^2\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(b^3x^{12}+3ab^2x^{10}+3a^2bx^8+a^3x^6\right)dx^2}{2 \left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \left(\frac{a^3x^8}{4}+\frac{3}{5}a^2bx^{10}+\frac{1}{2}ab^2x^{12}+\frac{b^3x^{14}}{7}\right)}{2 \left(a+bx^2\right)}$$

input
```
Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^3*x^8)/4 + (3*a^2*b*x^10)/5 + (a*b^2*
x^12)/2 + (b^3*x^14)/7))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.28

method	result	size
pseudoelliptic	$\frac{x^{8} \left(20 b^{3} x^{6}+70 b^{2} x^{4} a +84 a^{2} b \,x^{2}+35 a^{3}\right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{280}$	46
gosper	$\frac{x^{8} \left(20 b^{3} x^{6}+70 b^{2} x^{4} a +84 a^{2} b \,x^{2}+35 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{280 \left(b \,x^{2}+a \right)^{3}}$	58
default	$\frac{x^{8} \left(20 b^{3} x^{6}+70 b^{2} x^{4} a +84 a^{2} b \,x^{2}+35 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{280 \left(b \,x^{2}+a \right)^{3}}$	58
orering	$\frac{x^{8} \left(20 b^{3} x^{6}+70 b^{2} x^{4} a +84 a^{2} b \,x^{2}+35 a^{3}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{3}{2}}}{280 \left(b \,x^{2}+a \right)^{3}}$	67
risch	$\frac{a^{3} x^{8} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{8 b \,x^{2}+8 a}+\frac{3 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, a^{2} b \,x^{10}}{10 \left(b \,x^{2}+a \right)}+\frac{a \,b^{2} x^{12} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{4 b \,x^{2}+4 a}+\frac{b^{3} x^{14} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{14 b \,x^{2}+14 a}$	116

input
```
int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/280*x^8*(20*b^3*x^6+70*a*b^2*x^4+84*a^2*b*x^2+35*a^3)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx = \frac{1}{14}\,b^3x^{14}+\frac{1}{4}\,ab^2x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{8}\,a^3x^8$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Sympy [F]

$$\int x^7 (a^2 + 2abx^2 + b^2x^4)^{3/2} \, dx = \int x^7 \left((a + bx^2)^2 \right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**7*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^7 (a^2 + 2abx^2 + b^2x^4)^{3/2} \, dx = \frac{1}{14} b^3x^{14} + \frac{1}{4} ab^2x^{12} + \frac{3}{10} a^2bx^{10} + \frac{1}{8} a^3x^8$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^7 (a^2 + 2abx^2 + b^2x^4)^{3/2} \, dx = \frac{1}{14} b^3x^{14} \mathrm{sgn}(bx^2 + a) + \frac{1}{4} ab^2x^{12} \mathrm{sgn}(bx^2 + a) + \frac{3}{10} a^2bx^{10} \mathrm{sgn}(bx^2 + a) + \frac{1}{8} a^3x^8 \mathrm{sgn}(bx^2 + a)$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/14*b^3*x^14*sgn(b*x^2 + a) + 1/4*a*b^2*x^12*sgn(b*x^2 + a) + 3/10*a^2*b*
x^10*sgn(b*x^2 + a) + 1/8*a^3*x^8*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx=\int x^7\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2} \,d x$$

input
```
int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx=\frac{x^8 (20b^3 x^6+70a\,b^2 x^4+84a^2 b\,x^2+35a^3)}{280}$$

input
```
int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(x**8*(35*a**3 + 84*a**2*b*x**2 + 70*a*b**2*x**4 + 20*b**3*x**6))/280
```

3.507 $\int x^5(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 106

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{a^2(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^3}-\frac{a(a^2+2abx^2+b^2x^4)^{5/2}}{5b^3}+\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^3}$$

output
```
1/8*a^2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/b^3-1/5*a*(b^2*x^4+2*a*b*x
^2+a^2)^(5/2)/b^3+1/12*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/b^3
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{x^6(20a^3+45a^2bx^2+36ab^2x^4+10b^3x^6)\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(a+bx^2)^2}\right)\right)}{120\left(-a^2-abx^2+\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(x^6*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3*x^6)*(Sqrt[a^2]*b*x^2
+ a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])))/(120*(-a^2 - a*b*x^2 + Sqrt[a^2]*S
qrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int b^3x^5 \left(bx^2+a\right)^3 dx}{b^3 \left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^5 \left(bx^2+a\right)^3 dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^4 \left(bx^2+a\right)^3 dx^2}{2 \left(a+bx^2\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(b^3x^{10}+3ab^2x^8+3a^2bx^6+a^3x^4\right) dx^2}{2 \left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \left(\frac{a^3x^6}{3}+\frac{3}{4}a^2bx^8+\frac{3}{5}ab^2x^{10}+\frac{b^3x^{12}}{6}\right)}{2 \left(a+bx^2\right)}$$

input
```
Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^3*x^6)/3 + (3*a^2*b*x^8)/4 + (3*a*b^2
*x^10)/5 + (b^3*x^12)/6))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.43

method	result	size
pseudoelliptic	$\frac{x^{6}\left(10 b^{3} x^{6}+36 b^{2} x^{4} a+45 a^{2} b\,x^{2}+20 a^{3}\right) \operatorname{csgn}\left(b\,x^{2}+a\right)}{120}$	46
gosper	$\frac{x^{6}\left(10 b^{3} x^{6}+36 b^{2} x^{4} a+45 a^{2} b\,x^{2}+20 a^{3}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{120 \left(b\,x^{2}+a\right)^{3}}$	58
default	$\frac{x^{6}\left(10 b^{3} x^{6}+36 b^{2} x^{4} a+45 a^{2} b\,x^{2}+20 a^{3}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{120 \left(b\,x^{2}+a\right)^{3}}$	58
orering	$\frac{x^{6}\left(10 b^{3} x^{6}+36 b^{2} x^{4} a+45 a^{2} b\,x^{2}+20 a^{3}\right) \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{120 \left(b\,x^{2}+a\right)^{3}}$	67
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, b^{3} x^{12}}{12 b\,x^{2}+12 a}+\frac{3 \sqrt{\left(b\,x^{2}+a\right)^{2}}\, a\,b^{2} x^{10}}{10 \left(b\,x^{2}+a\right)}+\frac{3 \sqrt{\left(b\,x^{2}+a\right)^{2}}\, a^{2} b\,x^{8}}{8 \left(b\,x^{2}+a\right)}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, a^{3} x^{6}}{6 b\,x^{2}+6 a}$	116

input
```
int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.33

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{1}{12}\, b^{3} x^{12} + \frac{3}{10}\, a b^{2} x^{10} + \frac{3}{8}\, a^{2} b x^{8} + \frac{1}{6}\, a^{3} x^{6}$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Sympy [F]

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \int x^5 \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**5*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.33

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{12}b^3x^{12} + \frac{3}{10}ab^2x^{10} + \frac{3}{8}a^2bx^8 + \frac{1}{6}a^3x^6$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{12}b^3x^{12}\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{10}ab^2x^{10}\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{8}a^2bx^8\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{6}a^3x^6\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/12*b^3*x^12*sgn(b*x^2 + a) + 3/10*a*b^2*x^10*sgn(b*x^2 + a) + 3/8*a^2*b*
x^8*sgn(b*x^2 + a) + 1/6*a^3*x^6*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\int x^5\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

input
```
int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{x^6(10b^3x^6+36a\,b^2x^4+45a^2b\,x^2+20a^3)}{120}$$

input
```
int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(x**6*(20*a**3 + 45*a**2*b*x**2 + 36*a*b**2*x**4 + 10*b**3*x**6))/120
```

3.508 $\int x^3(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx = -\frac{a(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}{8b^2}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}$$

output
```
-1/8*a*(b*x^2+a)^3*((b*x^2+a)^2)^(1/2)/b^2+1/10*(b^2*x^4+2*a*b*x^2+a^2)^(5
/2)/b^2
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx = \frac{x^4(10a^3+20a^2bx^2+15ab^2x^4+4b^3x^6)\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(a+bx^2)^2}\right)\right)}{40\left(-a^2-abx^2+\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(x^4*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*x^6)*(Sqrt[a^2]*b*x^2 +
 a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])))/(40*(-a^2 - a*b*x^2 + Sqrt[a^2]*Sqr
t[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1434, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int x^2 \left(b^2x^4+2abx^2+a^2\right)^{3/2} dx^2$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{2}\left(\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{5b^2}-\frac{a\int \left(b^2x^4+2abx^2+a^2\right)^{3/2} dx^2}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{2}\left(\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{5b^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^2x^2+ab\right)^3 dx^2}{b^4\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{2}\left(\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{5b^2}-\frac{a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}\right)$$

input
```
Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(-1/4*(a*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^2 + (a^2 + 2*a*b
*x^2 + b^2*x^4)^(5/2)/(5*b^2))/2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{x^{4} \left(4 b^{3} x^{6}+15 b^{2} x^{4} a+20 a^{2} b \,x^{2}+10 a^{3}\right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{40}$	46
gosper	$\frac{x^{4} \left(4 b^{3} x^{6}+15 b^{2} x^{4} a+20 a^{2} b \,x^{2}+10 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{40 \left(b \,x^{2}+a \right)^{3}}$	58
default	$\frac{x^{4} \left(4 b^{3} x^{6}+15 b^{2} x^{4} a+20 a^{2} b \,x^{2}+10 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{40 \left(b \,x^{2}+a \right)^{3}}$	58
orering	$\frac{x^{4} \left(4 b^{3} x^{6}+15 b^{2} x^{4} a+20 a^{2} b \,x^{2}+10 a^{3}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{3}{2}}}{40 \left(b \,x^{2}+a \right)^{3}}$	67
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, b^{3} x^{10}}{10 b \,x^{2}+10 a}+\frac{3 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, a \,b^{2} x^{8}}{8 \left(b \,x^{2}+a \right)}+\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, a^{2} b \,x^{6}}{2 b \,x^{2}+2 a}+\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, a^{3} x^{4}}{4 b \,x^{2}+4 a}$	116

input
```
int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*x^4*(4*b^3*x^6+15*a*b^2*x^4+20*a^2*b*x^2+10*a^3)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int x^3 \left(a^2+2 a b x^2+b^2 x^4\right)^{3/2} \, dx=\frac{1}{10} b^3 x^{10}+\frac{3}{8} a b^2 x^8+\frac{1}{2} a^2 b x^6+\frac{1}{4} a^3 x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Sympy [F]

$$\int x^3 \left(a^2+2 a b x^2+b^2 x^4\right)^{3/2} \, dx=\int x^{3} \left(\left(a + b x^{2}\right)^{2}\right)^{\frac{3}{2}}\, dx$$

input
```
integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**3*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{1}{10}\,b^3x^{10} + \frac{3}{8}\,ab^2x^8 + \frac{1}{2}\,a^2bx^6 + \frac{1}{4}\,a^3x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{1}{40}\left(4\,b^3x^{10} + 15\,ab^2x^8 + 20\,a^2bx^6 + 10\,a^3x^4\right)\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/40*(4*b^3*x^10 + 15*a*b^2*x^8 + 20*a^2*b*x^6 + 10*a^3*x^4)*sgn(b*x^2 + a
)
```

Mupad [B] (verification not implemented)

Time = 17.68 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\left(-a^2+3\,a\,b\,x^2+4\,b^2\,x^4\right)}{40\,b^2}$$

input
```
int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)*(4*b^2*x^4 - a^2 + 3*a*b*x^2))/(40*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{x^4(4b^3x^6+15a\,b^2x^4+20a^2b\,x^2+10a^3)}{40}$$

input
```
int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(x**4*(10*a**3 + 20*a**2*b*x**2 + 15*a*b**2*x**4 + 4*b**3*x**6))/40
```

3.509 $\int x(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}{8b}$$

output
```
1/8*(b*x^2+a)^3*((b*x^2+a)^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{(a+bx^2)\left((a+bx^2)^2\right)^{3/2}}{8b}$$

input
```
Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*((a + b*x^2)^2)^(3/2))/(8*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1432, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{3/2}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \left(b^2x^4+2abx^2+a^2\right)^{3/2}dx^2$$

$$\Big\downarrow \text{1079}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^2x^2+ab\right)^3dx^2}{2b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8b}$$

input
```
Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}} \left(b\,x^{2}+a\right)}{8b}$	24
risch	$\frac{\left(b\,x^{2}+a\right)^{3} \sqrt{\left(b\,x^{2}+a\right)^{2}}}{8b}$	26
pseudoelliptic	$\frac{x^{2} \left(b^{3} x^{6}+4 b^{2} x^{4} a+6 a^{2} b\,x^{2}+4 a^{3}\right) \operatorname{csgn}\left(b\,x^{2}+a\right)}{8}$	45
gosper	$\frac{x^{2} \left(b^{3} x^{6}+4 b^{2} x^{4} a+6 a^{2} b\,x^{2}+4 a^{3}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{8 \left(b\,x^{2}+a\right)^{3}}$	57
orering	$\frac{x^{2} \left(b^{3} x^{6}+4 b^{2} x^{4} a+6 a^{2} b\,x^{2}+4 a^{3}\right) \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{8 \left(b\,x^{2}+a\right)^{3}}$	66

input
```
int(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*((b*x^2+a)^2)^(3/2)*(b*x^2+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{1}{8}\,b^3x^8+\frac{1}{2}\,ab^2x^6+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/8*b^3*x^8 + 1/2*a*b^2*x^6 + 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

Sympy [F]

$$\int x(a^2+2abx^2+b^2x^4)^{3/2}\,dx=\int x\left((a+bx^2)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int x(a^2+2abx^2+b^2x^4)^{3/2}\,dx=\frac{1}{8}\,b^3x^8+\frac{1}{2}\,ab^2x^6+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*b^3*x^8 + 1/2*a*b^2*x^6 + 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int x(a^2+2abx^2+b^2x^4)^{3/2}\,dx=\frac{1}{8}\left(2\left(bx^4+2\,ax^2\right)a^2+\left(bx^4+2\,ax^2\right)^2b\right)\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(2*(b*x^4 + 2*a*x^2)*a^2 + (b*x^4 + 2*a*x^2)^2*b)*sgn(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 17.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{\left(b^2\,x^2+a\,b\right)\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}{8\,b^2}$$

input
```
int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
((a*b + b^2*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2))/(8*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{x^2(b^3x^6+4a\,b^2x^4+6a^2b\,x^2+4a^3)}{8}$$

input
```
int(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(x**2*(4*a**3 + 6*a**2*b*x**2 + 4*a*b**2*x**4 + b**3*x**6))/8
```

3.510 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x} dx$

Optimal result

Integrand size = 26, antiderivative size = 163

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x} dx = \frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)} + \frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)} + \frac{b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)} + \frac{a^3\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

output
```
3*a^2*b*x^2*((b*x^2+a)^2)^(1/2)/(2*b*x^2+2*a)+3*a*b^2*x^4*((b*x^2+a)^2)^(1
/2)/(4*b*x^2+4*a)+b^3*x^6*((b*x^2+a)^2)^(1/2)/(6*b*x^2+6*a)+a^3*((b*x^2+a)
^2)^(1/2)*ln(x)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x} dx = \frac{\sqrt{(a+bx^2)^2}(bx^2(18a^2+9abx^2+2b^2x^4)+12a^3\log(x))}{12(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(b*x^2*(18*a^2 + 9*a*b*x^2 + 2*b^2*x^4) + 12*a^3*Log[
x]))/(12*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3(bx^2+a)^3}{x}dx}{b^3(a+bx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{x}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{x^2}dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^3x^4+3ab^2x^2+3a^2b+\frac{a^3}{x^2}\right)dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a^3\log(x^2)+3a^2bx^2+\frac{3}{2}ab^2x^4+\frac{b^3x^6}{3}\right)}{2(a+bx^2)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(3*a^2*b*x^2 + (3*a*b^2*x^4)/2 + (b^3*x^6
)/3 + a^3*Log[x^2]))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.29

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}(b\,x^2+a)\,(2b^3x^6+9b^2x^4a+6a^3\ln(x^2)+18a^2b\,x^2)}{12}$	47
default	$\frac{\left((b\,x^2+a)^2\right)^{\frac{3}{2}}(2b^3x^6+9b^2x^4a+18a^2b\,x^2+12a^3\ln(x))}{12(b\,x^2+a)^3}$	57
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,b(\frac{1}{6}b^2x^6+\frac{3}{4}ab\,x^4+\frac{3}{2}a^2x^2)}{b\,x^2+a}+\frac{a^3\sqrt{(b\,x^2+a)^2}\,\ln(x)}{b\,x^2+a}$	74

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/12*csgn(b*x^2+a)*(2*b^3*x^6+9*b^2*x^4*a+6*a^3*ln(x^2)+18*a^2*b*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.20

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x}\,dx=\frac{1}{6}\,b^3x^6+\frac{3}{4}\,ab^2x^4+\frac{3}{2}\,a^2bx^2+a^3\log\left(x\right)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x}\,dx=\int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.20

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x}\,dx = \frac{1}{6}\,b^3x^6 + \frac{3}{4}\,ab^2x^4 + \frac{3}{2}\,a^2bx^2 + a^3\log\left(x\right)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x}\,dx = \frac{1}{6}\,b^3x^6\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{4}\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{2}\,a^3\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/6*b^3*x^6*sgn(b*x^2 + a) + 3/4*a*b^2*x^4*sgn(b*x^2 + a) + 3/2*a^2*b*x^2*
sgn(b*x^2 + a) + 1/2*a^3*log(x^2)*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x}\,dx = \int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}{x}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.20

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x}\,dx = \log(x)\,a^3 + \frac{3a^2b\,x^2}{2} + \frac{3a\,b^2x^4}{4} + \frac{b^3x^6}{6}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x)
```

output
```
(12*log(x)*a**3 + 18*a**2*b*x**2 + 9*a*b**2*x**4 + 2*b**3*x**6)/12
```

3.511 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^3} dx = -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)} + \frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)} + \frac{b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)} + \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

output
```
-1/2*a^3*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+3*a*b^2*x^2*((b*x^2+a)^2)^(1/2)
/(2*b*x^2+2*a)+b^3*x^4*((b*x^2+a)^2)^(1/2)/(4*b*x^2+4*a)+3*a^2*b*((b*x^2+a
)^2)^(1/2)*ln(x)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.38

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^3} dx = \frac{\sqrt{(a+bx^2)^2}(-2a^3+6ab^2x^4+b^3x^6+12a^2bx^2\log(x))}{4x^2(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(-2*a^3 + 6*a*b^2*x^4 + b^3*x^6 + 12*a^2*b*x^2*Log[x]
))/(4*x^2*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3\left(bx^2+a\right)^3}{x^3}dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^3}dx}{a+bx^2}$$

$$\Big\downarrow \ 243$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^4}dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \ 49$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{x^4}+\frac{3ba^2}{x^2}+3b^2a+b^3x^2\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^3}{x^2}+3a^2b\log\left(x^2\right)+3ab^2x^2+\frac{b^3x^4}{2}\right)}{2\left(a+bx^2\right)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-(a^3/x^2) + 3*a*b^2*x^2 + (b^3*x^4)/2 +
 3*a^2*b*Log[x^2]))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.29

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(-\frac{b^{3} x^{6}}{2}-3 b^{2} x^{4} a-3 a^{2} b \ln\left(x^{2}\right) x^{2}+a^{3}\right)}{2 x^{2}}$	48
default	$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}} \left(b^{3} x^{6}+6 b^{2} x^{4} a+12 \ln\left(x\right) x^{2} a^{2} b-2 a^{3}\right)}{4 x^{2} \left(b\,x^{2}+a\right)^{3}}$	59
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, b \left(b\,x^{2}+3 a\right)^{2}}{4 b\,x^{2}+4 a}-\frac{a^{3} \sqrt{\left(b\,x^{2}+a\right)^{2}}}{2 x^{2} \left(b\,x^{2}+a\right)}+\frac{3 a^{2} b \sqrt{\left(b\,x^{2}+a\right)^{2}}\, \ln\left(x\right)}{b\,x^{2}+a}$	92

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*csgn(b*x^2+a)*(-1/2*b^3*x^6-3*b^2*x^4*a-3*a^2*b*ln(x^2)*x^2+a^3)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^3}\,dx=\frac{b^3x^6+6\,ab^2x^4+12\,a^2bx^2\log\left(x\right)-2\,a^3}{4\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^3}\,dx=\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**3,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.21

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^3}\,dx = \frac{1}{4}\,b^3x^4 + \frac{3}{2}\,ab^2x^2 + 3\,a^2b\log\left(x\right) - \frac{a^3}{2\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3*a^2*b*log(x) - 1/2*a^3/x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^3}\,dx = \frac{1}{4}\,b^3x^4\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{2}\,ab^2x^2\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{2}\,a^2b\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right) - \frac{3\,a^2bx^2\mathrm{sgn}(bx^2+a) + a^3\mathrm{sgn}(bx^2+a)}{2\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/4*b^3*x^4*sgn(b*x^2 + a) + 3/2*a*b^2*x^2*sgn(b*x^2 + a) + 3/2*a^2*b*log(
x^2)*sgn(b*x^2 + a) - 1/2*(3*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a)
)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^3}\,dx = \int \frac{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}{x^3}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^3,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^3}\,dx=\frac{12\log(x)\,a^2b\,x^2-2a^3+6a\,b^2x^4+b^3x^6}{4x^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x)
```

output
```
(12*log(x)*a**2*b*x**2 - 2*a**3 + 6*a*b**2*x**4 + b**3*x**6)/(4*x**2)
```

3.512 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^5} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^5} \, dx = -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)} + \frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)} + \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

output
```
-1/4*a^3*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)-3/2*a^2*b*((b*x^2+a)^2)^(1/2)/x
^2/(b*x^2+a)+b^3*x^2*((b*x^2+a)^2)^(1/2)/(2*b*x^2+2*a)+3*a*b^2*((b*x^2+a)^
2)^(1/2)*ln(x)/(b*x^2+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 612 vs. 2(164) = 328.

Time = 0.83 (sec) , antiderivative size = 612, normalized size of antiderivative = 3.73

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^5} \, dx = \frac{4a^4\sqrt{a^2}+28a^3\sqrt{a^2}bx^2+35(a^2)^{3/2}b^2x^4+3a\sqrt{a^2}b^3x^6-8\sqrt{a^2}b^4x^8-4a^4\sqrt{}}{}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
```

output
```
(4*a^4*Sqrt[a^2] + 28*a^3*Sqrt[a^2]*b*x^2 + 35*(a^2)^(3/2)*b^2*x^4 + 3*a*S
qrt[a^2]*b^3*x^6 - 8*Sqrt[a^2]*b^4*x^8 - 4*a^4*Sqrt[(a + b*x^2)^2] - 24*a^
3*b*x^2*Sqrt[(a + b*x^2)^2] - 11*a^2*b^2*x^4*Sqrt[(a + b*x^2)^2] + 8*a*b^3
*x^6*Sqrt[(a + b*x^2)^2] - 24*a*b^2*x^4*(a^2 + a*b*x^2 - Sqrt[a^2]*Sqrt[(a
 + b*x^2)^2])*ArcTanh[(b*x^2)/(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])] - 24*b^2*
x^4*((a^2)^(3/2) + a*Sqrt[a^2]*b*x^2 - a^2*Sqrt[(a + b*x^2)^2])*Log[x^2] +
 12*(a^2)^(3/2)*b^2*x^4*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] + 12*
a*Sqrt[a^2]*b^3*x^6*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] - 12*a^2*
b^2*x^4*Sqrt[(a + b*x^2)^2]*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] +
 12*(a^2)^(3/2)*b^2*x^4*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]] + 12*
a*Sqrt[a^2]*b^3*x^6*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]] - 12*a^2*
b^2*x^4*Sqrt[(a + b*x^2)^2]*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]])/
(16*x^4*(a^2 + a*b*x^2 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.44, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^5} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \frac{b^3(bx^2+a)^3}{x^5} dx}{b^3(a + bx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \frac{(bx^2+a)^3}{x^5} dx}{a + bx^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{x^6}dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{x^6}+\frac{3ba^2}{x^4}+\frac{3b^2a}{x^2}+b^3\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^3}{2x^4}-\frac{3a^2b}{x^2}+3ab^2\log\left(x^2\right)+b^3x^2\right)}{2\left(a+bx^2\right)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/2*a^3/x^4 - (3*a^2*b)/x^2 + b^3*x^2 +
 3*a*b^2*Log[x^2]))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.29

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(-6 b^{2} a \ln\left(x^{2}\right) x^{4}-2 b^{3} x^{6}+6 a^{2} b\,x^{2}+a^{3}\right)}{4 x^{4}}$	48
default	$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(2 b^{3} x^{6}+12 \ln\left(x\right) x^{4} a\,b^{2}-6 a^{2} b\,x^{2}-a^{3}\right)}{4 x^{4}\left(b\,x^{2}+a\right)^{3}}$	60
risch	$\frac{b^{3} x^{2}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{2 b\,x^{2}+2 a}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{3}{2} a^{2} b\,x^{2}-\frac{1}{4} a^{3}\right)}{\left(b\,x^{2}+a\right) x^{4}}+\frac{3 a\,b^{2}\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(x\right)}{b\,x^{2}+a}$	97

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*csgn(b*x^2+a)*(-6*b^2*a*ln(x^2)*x^4-2*b^3*x^6+6*a^2*b*x^2+a^3)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^5}\,dx=\frac{2\,b^3x^6+12\,ab^2x^4\log\left(x\right)-6\,a^2bx^2-a^3}{4\,x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
1/4*(2*b^3*x^6 + 12*a*b^2*x^4*log(x) - 6*a^2*b*x^2 - a^3)/x^4
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^5}\, dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{3}{2}}}{x^5}\, dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**5,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.21

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^5}\, dx = \frac{1}{2}\, b^3x^2 + 3\, ab^2 \log{(x)} - \frac{3\, a^2b}{2\, x^2} - \frac{a^3}{4\, x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
1/2*b^3*x^2 + 3*a*b^2*log(x) - 3/2*a^2*b/x^2 - 1/4*a^3/x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^5}\, dx = \frac{1}{2}\, b^3x^2\mathrm{sgn}\left(bx^2 + a\right) + \frac{3}{2}\, ab^2 \log{\left(x^2\right)} \,\mathrm{sgn}\left(bx^2 + a\right) - \frac{9\, ab^2x^4\mathrm{sgn}(bx^2 + a) + 6\, a^2bx^2\mathrm{sgn}(bx^2 + a) + a^3\mathrm{sgn}(bx^2 + a)}{4\, x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
1/2*b^3*x^2*sgn(b*x^2 + a) + 3/2*a*b^2*log(x^2)*sgn(b*x^2 + a) - 1/4*(9*a*
b^2*x^4*sgn(b*x^2 + a) + 6*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/
x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^5}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{3/2}}{x^5}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^5,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^5}\,dx = \frac{12\log(x)\,a\,b^2x^4 - a^3 - 6a^2b\,x^2 + 2b^3x^6}{4x^4}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x)
```

output
```
(12*log(x)*a*b**2*x**4 - a**3 - 6*a**2*b*x**2 + 2*b**3*x**6)/(4*x**4)
```

3.513 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 163

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^7}\,dx = -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)} -\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

output
```
-1/6*a^3*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)-3/4*a^2*b*((b*x^2+a)^2)^(1/2)/x
^4/(b*x^2+a)-3/2*a*b^2*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+b^3*((b*x^2+a)^2)
^(1/2)*ln(x)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.63

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^7}\,dx = \frac{2a^3\sqrt{a^2}+9(a^2)^{3/2}bx^2+18a\sqrt{a^2}b^2x^4-2a^3\sqrt{(a+bx^2)^2}-7a^2bx^2\sqrt{(a+b\cdots}}{\cdots}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```

output
```
(2*a^3*Sqrt[a^2] + 9*(a^2)^(3/2)*b*x^2 + 18*a*Sqrt[a^2]*b^2*x^4 - 2*a^3*Sq
rt[(a + b*x^2)^2] - 7*a^2*b*x^2*Sqrt[(a + b*x^2)^2] - 11*a*b^2*x^4*Sqrt[(a
 + b*x^2)^2] - 12*a*b^3*x^6*ArcTanh[(b*x^2)/(Sqrt[a^2] - Sqrt[(a + b*x^2)^
2])] - 12*Sqrt[a^2]*b^3*x^6*Log[x^2] + 6*Sqrt[a^2]*b^3*x^6*Log[a*(Sqrt[a^2
] - b*x^2 - Sqrt[(a + b*x^2)^2])] + 6*Sqrt[a^2]*b^3*x^6*Log[a*(Sqrt[a^2] +
 b*x^2 - Sqrt[(a + b*x^2)^2])])/(24*a*x^6)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.45, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3\left(bx^2+a\right)^3}{x^7}dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^7}dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^8}dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{x^8}+\frac{3ba^2}{x^6}+\frac{3b^2a}{x^4}+\frac{b^3}{x^2}\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^3}{3x^6}-\frac{3a^2b}{2x^4}-\frac{3ab^2}{x^2}+b^3\log\left(x^2\right)\right)}{2\left(a+bx^2\right)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/3*a^3/x^6 - (3*a^2*b)/(2*x^4) - (3*a*
b^2)/x^2 + b^3*Log[x^2]))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
 - 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.31

method	result	size
pseudoelliptic	$\frac{\mathrm{csgn}\left(b\,x^{2}+a\right)\left(6b^{3}\ln\left(x^{2}\right)x^{6}-18b^{2}x^{4}a-9a^{2}b\,x^{2}-2a^{3}\right)}{12x^{6}}$	50
default	$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(12\ln\left(x\right)x^{6}b^{3}-18b^{2}x^{4}a-9a^{2}b\,x^{2}-2a^{3}\right)}{12x^{6}\left(b\,x^{2}+a\right)^{3}}$	60
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{3}{2}b^{2}x^{4}a-\frac{3}{4}a^{2}b\,x^{2}-\frac{1}{6}a^{3}\right)}{\left(b\,x^{2}+a\right)x^{6}}+\frac{b^{3}\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(x\right)}{b\,x^{2}+a}$	76

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/12*csgn(b*x^2+a)*(6*b^3*ln(x^2)*x^6-18*b^2*x^4*a-9*a^2*b*x^2-2*a^3)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{3/2}}{x^{7}}\,dx=\frac{12\,b^{3}x^{6}\log\left(x\right)-18\,ab^{2}x^{4}-9\,a^{2}bx^{2}-2\,a^{3}}{12\,x^{6}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
1/12*(12*b^3*x^6*log(x) - 18*a*b^2*x^4 - 9*a^2*b*x^2 - 2*a^3)/x^6
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^7}\, dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{3}{2}}}{x^7}\, dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**7,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.20

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^7}\, dx = b^3 \log{(x)} - \frac{3\, ab^2}{2\, x^2} - \frac{3\, a^2 b}{4\, x^4} - \frac{a^3}{6\, x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
b^3*log(x) - 3/2*a*b^2/x^2 - 3/4*a^2*b/x^4 - 1/6*a^3/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^7}\, dx = \frac{1}{2}\, b^3 \log{\left(x^2\right)} \operatorname{sgn}\left(bx^2 + a\right) - \frac{11\, b^3 x^6 \operatorname{sgn}(bx^2 + a) + 18\, ab^2 x^4 \operatorname{sgn}(bx^2 + a) + 9\, a^2 b x^2 \operatorname{sgn}(bx^2 + a) + 2\, a^3 \operatorname{sgn}(bx^2 + a)}{12\, x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
1/2*b^3*log(x^2)*sgn(b*x^2 + a) - 1/12*(11*b^3*x^6*sgn(b*x^2 + a) + 18*a*b
^2*x^4*sgn(b*x^2 + a) + 9*a^2*b*x^2*sgn(b*x^2 + a) + 2*a^3*sgn(b*x^2 + a))
/x^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^7} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{3/2}}{x^7} \, dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^7,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^7} \, dx = \frac{12\log(x)\,b^3x^6 - 2a^3 - 9a^2b\,x^2 - 18a\,b^2x^4}{12x^6}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x)
```

output
```
(12*log(x)*b**3*x**6 - 2*a**3 - 9*a**2*b*x**2 - 18*a*b**2*x**4)/(12*x**6)
```

3.514 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^9} dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^9} dx = -\frac{(a+bx^2)^3 \sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

output
```
-1/8*(b*x^2+a)^3*((b*x^2+a)^2)^(1/2)/a/x^8
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^9} dx = -\frac{\sqrt{(a+bx^2)^2}(a^3+4a^2bx^2+6ab^2x^4+4b^3x^6)}{8x^8(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]
```

output
```
-1/8*(Sqrt[(a + b*x^2)^2]*(a^3 + 4*a^2*b*x^2 + 6*a*b^2*x^4 + 4*b^3*x^6))/(
x^8*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1384, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^9}dx$$

$$\Big\downarrow \textbf{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3\left(bx^2+a\right)^3}{x^9}dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^9}dx}{a+bx^2}$$

$$\Big\downarrow \textbf{242}$$

$$-\frac{\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]
```

output
```
-1/8*((a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a*x^8)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$-\frac{(2b\,x^2+a)\,(2b^2x^4+2ab\,x^2+a^2)\,\mathrm{csgn}(b\,x^2+a)}{8x^8}$	41
gosper	$-\frac{(4b^3x^6+6b^2x^4a+4a^2b\,x^2+a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{8x^8(b\,x^2+a)^3}$	56
default	$-\frac{(4b^3x^6+6b^2x^4a+4a^2b\,x^2+a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{8x^8(b\,x^2+a)^3}$	56
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{2}b^3x^6-\frac{3}{4}b^2x^4a-\frac{1}{2}a^2b\,x^2-\frac{1}{8}a^3\right)}{(b\,x^2+a)x^8}$	57
orering	$-\frac{(4b^3x^6+6b^2x^4a+4a^2b\,x^2+a^3)\,(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{2}}}{8x^8(b\,x^2+a)^3}$	65

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(2*b*x^2+a)*(2*b^2*x^4+2*a*b*x^2+a^2)*csgn(b*x^2+a)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^9}\,dx=-\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
-1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^9}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**9,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**9, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^9}\,dx = -\frac{b^3}{2\,x^2} - \frac{3\,ab^2}{4\,x^4} - \frac{a^2b}{2\,x^6} - \frac{a^3}{8\,x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
-1/2*b^3/x^2 - 3/4*a*b^2/x^4 - 1/2*a^2*b/x^6 - 1/8*a^3/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(28) = 56.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^9}\,dx = -\frac{4\,b^3x^6\mathrm{sgn}(bx^2+a) + 6\,ab^2x^4\mathrm{sgn}(bx^2+a) + 4\,a^2bx^2\mathrm{sgn}(bx^2+a) + a^3\mathrm{sgn}(bx^2+a)}{8\,x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
-1/8*(4*b^3*x^6*sgn(b*x^2 + a) + 6*a*b^2*x^4*sgn(b*x^2 + a) + 4*a^2*b*x^2*
sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^8
```

Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.68

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^9}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^2\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\,\left(b\,x^2+a\right)}-\frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^6\,\left(b\,x^2+a\right)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^9,x)
```

output
```
- (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (b^3*(a^2
+ b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*x^2*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (a^2*b*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(2*x^6*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^9}\,dx = \frac{-4b^3x^6-6a\,b^2x^4-4a^2b\,x^2-a^3}{8x^8}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x)
```

output
```
( - a**3 - 4*a**2*b*x**2 - 6*a*b**2*x**4 - 4*b**3*x**6)/(8*x**8)
```

3.515 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{11}}\,dx = -\frac{(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}{10ax^{10}} + \frac{b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}{40a^2x^8}$$

output
```
-1/10*(b*x^2+a)^3*((b*x^2+a)^2)^(1/2)/a/x^10+1/40*b*(b*x^2+a)^3*((b*x^2+a)
^2)^(1/2)/a^2/x^8
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{11}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(4a^3+15a^2bx^2+20ab^2x^4+10b^3x^6)}{40x^{10}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

output
```
-1/40*(Sqrt[(a + b*x^2)^2]*(4*a^3 + 15*a^2*b*x^2 + 20*a*b^2*x^4 + 10*b^3*x
^6))/(x^10*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{11}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3\left(bx^2+a\right)^3}{x^{11}}dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^{11}}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^{12}}dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{b\int \frac{\left(bx^2+a\right)^3}{x^{10}}dx^2}{5a}-\frac{\left(a+bx^2\right)^4}{5ax^{10}}\right)}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{b\left(a+bx^2\right)^4}{20a^2x^8}-\frac{\left(a+bx^2\right)^4}{5ax^{10}}\right)}{2\left(a+bx^2\right)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/5*(a + b*x^2)^4/(a*x^10) + (b*(a + b*
x^2)^4)/(20*a^2*x^8)))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}(b\,x^2+a)\left(\frac{5}{2}b^3x^6+5b^2x^4a+\frac{15}{4}a^2b\,x^2+a^3\right)}{10x^{10}}$	44
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{4}b^3x^6-\frac{1}{2}b^2x^4a-\frac{3}{8}a^2b\,x^2-\frac{1}{10}a^3\right)}{(b\,x^2+a)x^{10}}$	57
gosper	$-\frac{(10b^3x^6+20b^2x^4a+15a^2b\,x^2+4a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{40x^{10}(b\,x^2+a)^3}$	58
default	$-\frac{(10b^3x^6+20b^2x^4a+15a^2b\,x^2+4a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{40x^{10}(b\,x^2+a)^3}$	58
orering	$-\frac{(10b^3x^6+20b^2x^4a+15a^2b\,x^2+4a^3)\,(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{2}}}{40x^{10}(b\,x^2+a)^3}$	67

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*csgn(b*x^2+a)*(5/2*b^3*x^6+5*b^2*x^4*a+15/4*a^2*b*x^2+a^3)/x^10
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{11}}\,dx=-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="fricas")
```

output
```
-1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{11}}\,dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**11,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**11, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{11}}\,dx = -\frac{b^3}{4\,x^4} - \frac{ab^2}{2\,x^6} - \frac{3\,a^2b}{8\,x^8} - \frac{a^3}{10\,x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="maxima")
```

output
```
-1/4*b^3/x^4 - 1/2*a*b^2/x^6 - 3/8*a^2*b/x^8 - 1/10*a^3/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{11}}\,dx = -\frac{10\,b^3x^6\mathrm{sgn}(bx^2 + a) + 20\,ab^2x^4\mathrm{sgn}(bx^2 + a) + 15\,a^2bx^2\mathrm{sgn}(bx^2 + a) + 4\,a^3\mathrm{sgn}(bx^2 + a)}{40\,x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x, algorithm="giac")
```

output
```
-1/40*(10*b^3*x^6*sgn(b*x^2 + a) + 20*a*b^2*x^4*sgn(b*x^2 + a) + 15*a^2*b*
x^2*sgn(b*x^2 + a) + 4*a^3*sgn(b*x^2 + a))/x^10
```

Mupad [B] (verification not implemented)

Time = 17.81 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.80

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{11}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,(b\,x^2+a)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\,(b\,x^2+a)} - \frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^6\,(b\,x^2+a)} - \frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,(b\,x^2+a)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^11,x)
```

output
```
- (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (b^3*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(2*x^6*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(8*x^8*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{11}}\,dx = \frac{-10b^3x^6-20a\,b^2x^4-15a^2b\,x^2-4a^3}{40x^{10}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x)
```

output
```
( - 4*a**3 - 15*a**2*b*x**2 - 20*a*b**2*x**4 - 10*b**3*x**6)/(40*x**10)
```

3.516 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{13}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{13}}\,dx = -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)} - \frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}$$

output
```
-1/12*a^3*((b*x^2+a)^2)^(1/2)/x^12/(b*x^2+a)-3/10*a^2*b*((b*x^2+a)^2)^(1/2
)/x^10/(b*x^2+a)-3/8*a*b^2*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)-1/6*b^3*((b*x
^2+a)^2)^(1/2)/x^6/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{13}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(10a^3+36a^2bx^2+45ab^2x^4+20b^3x^6)}{120x^{12}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
```

output
```
-1/120*(Sqrt[(a + b*x^2)^2]*(10*a^3 + 36*a^2*b*x^2 + 45*a*b^2*x^4 + 20*b^3
*x^6))/(x^12*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{13}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3(bx^2+a)^3}{x^{13}}dx}{b^3(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{x^{13}}dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{x^{14}}dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{x^{14}}+\frac{3ba^2}{x^{12}}+\frac{3b^2a}{x^{10}}+\frac{b^3}{x^8}\right)dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{6x^{12}}-\frac{3a^2b}{5x^{10}}-\frac{3ab^2}{4x^8}-\frac{b^3}{3x^6}\right)\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
```

output
```
((-1/6*a^3/x^12 - (3*a^2*b)/(5*x^10) - (3*a*b^2)/(4*x^8) - b^3/(3*x^6))*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.26

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(2 b^{3} x^{6}+\frac{9}{2} b^{2} x^{4} a+\frac{18}{5} a^{2} b\,x^{2}+a^{3}\right)}{12 x^{12}}$	44
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{1}{6} b^{3} x^{6}-\frac{3}{8} b^{2} x^{4} a-\frac{3}{10} a^{2} b\,x^{2}-\frac{1}{12} a^{3}\right)}{\left(b\,x^{2}+a\right) x^{12}}$	57
gosper	$-\frac{\left(20 b^{3} x^{6}+45 b^{2} x^{4} a+36 a^{2} b\,x^{2}+10 a^{3}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{120 x^{12} \left(b\,x^{2}+a\right)^{3}}$	58
default	$-\frac{\left(20 b^{3} x^{6}+45 b^{2} x^{4} a+36 a^{2} b\,x^{2}+10 a^{3}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{120 x^{12} \left(b\,x^{2}+a\right)^{3}}$	58
orering	$-\frac{\left(20 b^{3} x^{6}+45 b^{2} x^{4} a+36 a^{2} b\,x^{2}+10 a^{3}\right) \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{120 x^{12} \left(b\,x^{2}+a\right)^{3}}$	67

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*csgn(b*x^2+a)*(2*b^3*x^6+9/2*b^2*x^4*a+18/5*a^2*b*x^2+a^3)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{13}}\,dx=-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="fricas")
```

output
```
-1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{13}}\,dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**13,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**13, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{13}}\,dx = -\frac{b^3}{6\,x^6} - \frac{3\,ab^2}{8\,x^8} - \frac{3\,a^2b}{10\,x^{10}} - \frac{a^3}{12\,x^{12}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="maxima")
```

output
```
-1/6*b^3/x^6 - 3/8*a*b^2/x^8 - 3/10*a^2*b/x^10 - 1/12*a^3/x^12
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{13}}\,dx = -\frac{20\,b^3x^6\mathrm{sgn}(bx^2 + a) + 45\,ab^2x^4\mathrm{sgn}(bx^2 + a) + 36\,a^2bx^2\mathrm{sgn}(bx^2 + a) + 10\,a^3\mathrm{sgn}(bx^2 + a)}{120\,x^{12}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x, algorithm="giac")
```

output
```
-1/120*(20*b^3*x^6*sgn(b*x^2 + a) + 45*a*b^2*x^4*sgn(b*x^2 + a) + 36*a^2*b
*x^2*sgn(b*x^2 + a) + 10*a^3*sgn(b*x^2 + a))/x^12
```

Mupad [B] (verification not implemented)

Time = 17.74 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{13}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^6\,\left(b\,x^2+a\right)}-\frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,\left(b\,x^2+a\right)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^13,x)
```

output
```
- (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (b^3*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(6*x^6*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*
a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{13}}\,dx = \frac{-20b^3x^6-45a\,b^2x^4-36a^2b\,x^2-10a^3}{120x^{12}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x)
```

output
```
( - 10*a**3 - 36*a**2*b*x**2 - 45*a*b**2*x**4 - 20*b**3*x**6)/(120*x**12)
```

3.517 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{15}} dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{15}} dx = -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)} - \frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)} - \frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}$$

output
```
-1/14*a^3*((b*x^2+a)^2)^(1/2)/x^14/(b*x^2+a)-1/4*a^2*b*((b*x^2+a)^2)^(1/2)
/x^12/(b*x^2+a)-3/10*a*b^2*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)-1/8*b^3*((b*
x^2+a)^2)^(1/2)/x^8/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{15}} dx = -\frac{\sqrt{(a+bx^2)^2}(20a^3+70a^2bx^2+84ab^2x^4+35b^3x^6)}{280x^{14}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
```

output
```
-1/280*(Sqrt[(a + b*x^2)^2]*(20*a^3 + 70*a^2*b*x^2 + 84*a*b^2*x^4 + 35*b^3
*x^6))/(x^14*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{15}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3\left(bx^2+a\right)^3}{x^{15}}dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^{15}}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^{16}}dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{x^{16}}+\frac{3ba^2}{x^{14}}+\frac{3b^2a}{x^{12}}+\frac{b^3}{x^{10}}\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a^3}{7x^{14}}-\frac{a^2b}{2x^{12}}-\frac{3ab^2}{5x^{10}}-\frac{b^3}{4x^8}\right)\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
```

output
```
((-1/7*a^3/x^14 - (a^2*b)/(2*x^12) - (3*a*b^2)/(5*x^10) - b^3/(4*x^8))*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.28

method	result	size
pseudoelliptic	$-\frac{(35b^3x^6+84b^2x^4a+70a^2b\,x^2+20a^3)\,\mathrm{csgn}(b\,x^2+a)}{280x^{14}}$	46
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{14}a^3-\frac{1}{4}a^2b\,x^2-\frac{3}{10}b^2x^4a-\frac{1}{8}b^3x^6\right)}{(b\,x^2+a)x^{14}}$	57
gosper	$-\frac{(35b^3x^6+84b^2x^4a+70a^2b\,x^2+20a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{280x^{14}(b\,x^2+a)^3}$	58
default	$-\frac{(35b^3x^6+84b^2x^4a+70a^2b\,x^2+20a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{280x^{14}(b\,x^2+a)^3}$	58
orering	$-\frac{(35b^3x^6+84b^2x^4a+70a^2b\,x^2+20a^3)\,(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{2}}}{280x^{14}(b\,x^2+a)^3}$	67

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/280*(35*b^3*x^6+84*a*b^2*x^4+70*a^2*b*x^2+20*a^3)*csgn(b*x^2+a)/x^14
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{15}}\,dx = -\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="fricas")
```

output
```
-1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{15}}\,dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**15,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**15, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{15}}\,dx = -\frac{b^3}{8\,x^8} - \frac{3\,ab^2}{10\,x^{10}} - \frac{a^2b}{4\,x^{12}} - \frac{a^3}{14\,x^{14}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="maxima")
```

output
```
-1/8*b^3/x^8 - 3/10*a*b^2/x^10 - 1/4*a^2*b/x^12 - 1/14*a^3/x^14
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{15}}\,dx = -\frac{35\,b^3x^6\mathrm{sgn}(bx^2 + a) + 84\,ab^2x^4\mathrm{sgn}(bx^2 + a) + 70\,a^2bx^2\mathrm{sgn}(bx^2 + a) + 20\,a^3\mathrm{sgn}(bx^2 + a)}{280\,x^{14}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x, algorithm="giac")
```

output
```
-1/280*(35*b^3*x^6*sgn(b*x^2 + a) + 84*a*b^2*x^4*sgn(b*x^2 + a) + 70*a^2*b
*x^2*sgn(b*x^2 + a) + 20*a^3*sgn(b*x^2 + a))/x^14
```

Mupad [B] (verification not implemented)

Time = 17.79 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{15}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,(b\,x^2+a)} - \frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,(b\,x^2+a)} - \frac{3\,a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,(b\,x^2+a)} - \frac{a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^{12}\,(b\,x^2+a)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^15,x)
```

output
```
- (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (b^3*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (a^2*b*(a^2 + b^2*x^4 + 2*
a*b*x^2)^(1/2))/(4*x^12*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{15}}\,dx = \frac{-35b^3x^6 - 84a\,b^2x^4 - 70a^2b\,x^2 - 20a^3}{280x^{14}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x)
```

output
```
( - 20*a**3 - 70*a**2*b*x**2 - 84*a*b**2*x**4 - 35*b**3*x**6)/(280*x**14)
```

3.518 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{17}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{17}}\,dx = -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}(a+bx^2)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)} - \frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)} - \frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}$$

output
```
-1/16*a^3*((b*x^2+a)^2)^(1/2)/x^16/(b*x^2+a)-3/14*a^2*b*((b*x^2+a)^2)^(1/2
)/x^14/(b*x^2+a)-1/4*a*b^2*((b*x^2+a)^2)^(1/2)/x^12/(b*x^2+a)-1/10*b^3*((b
*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{17}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(35a^3+120a^2bx^2+140ab^2x^4+56b^3x^6)}{560x^{16}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
```

output
```
-1/560*(Sqrt[(a + b*x^2)^2]*(35*a^3 + 120*a^2*b*x^2 + 140*a*b^2*x^4 + 56*b
^3*x^6))/(x^16*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.47, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{17}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3(bx^2+a)^3}{x^{17}}dx}{b^3(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{x^{17}}dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{x^{18}}dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{x^{18}}+\frac{3ba^2}{x^{16}}+\frac{3b^2a}{x^{14}}+\frac{b^3}{x^{12}}\right)dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{8x^{16}}-\frac{3a^2b}{7x^{14}}-\frac{ab^2}{2x^{12}}-\frac{b^3}{5x^{10}}\right)\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
```

output
```
((-1/8*a^3/x^16 - (3*a^2*b)/(7*x^14) - (a*b^2)/(2*x^12) - b^3/(5*x^10))*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.26

method	result	size
pseudoelliptic	$-\frac{\left(\frac{8}{5}b^{3}x^{6}+4b^{2}x^{4}a+\frac{24}{7}a^{2}b\,x^{2}+a^{3}\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{16x^{16}}$	44
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{1}{16}a^{3}-\frac{3}{14}a^{2}b\,x^{2}-\frac{1}{4}b^{2}x^{4}a-\frac{1}{10}b^{3}x^{6}\right)}{\left(b\,x^{2}+a\right)x^{16}}$	57
gosper	$-\frac{\left(56b^{3}x^{6}+140b^{2}x^{4}a+120a^{2}b\,x^{2}+35a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{560x^{16}\left(b\,x^{2}+a\right)^{3}}$	58
default	$-\frac{\left(56b^{3}x^{6}+140b^{2}x^{4}a+120a^{2}b\,x^{2}+35a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{560x^{16}\left(b\,x^{2}+a\right)^{3}}$	58
orering	$-\frac{\left(56b^{3}x^{6}+140b^{2}x^{4}a+120a^{2}b\,x^{2}+35a^{3}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{560x^{16}\left(b\,x^{2}+a\right)^{3}}$	67

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(8/5*b^3*x^6+4*b^2*x^4*a+24/7*a^2*b*x^2+a^3)*csgn(b*x^2+a)/x^16
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{3/2}}{x^{17}}\,dx = -\frac{56\,b^{3}x^{6}+140\,ab^{2}x^{4}+120\,a^{2}bx^{2}+35\,a^{3}}{560\,x^{16}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="fricas")
```

output
```
-1/560*(56*b^3*x^6 + 140*a*b^2*x^4 + 120*a^2*b*x^2 + 35*a^3)/x^16
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{17}}\,dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**17,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**17, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{17}}\,dx = -\frac{b^3}{10\,x^{10}} - \frac{ab^2}{4\,x^{12}} - \frac{3\,a^2b}{14\,x^{14}} - \frac{a^3}{16\,x^{16}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="maxima")
```

output
```
-1/10*b^3/x^10 - 1/4*a*b^2/x^12 - 3/14*a^2*b/x^14 - 1/16*a^3/x^16
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{17}}\,dx = -\frac{56\,b^3x^6\mathrm{sgn}(bx^2 + a) + 140\,ab^2x^4\mathrm{sgn}(bx^2 + a) + 120\,a^2bx^2\mathrm{sgn}(bx^2 + a) + 35\,a^3\mathrm{sgn}(bx^2 + a)}{560\,x^{16}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x, algorithm="giac")
```

output
```
-1/560*(56*b^3*x^6*sgn(b*x^2 + a) + 140*a*b^2*x^4*sgn(b*x^2 + a) + 120*a^2
*b*x^2*sgn(b*x^2 + a) + 35*a^3*sgn(b*x^2 + a))/x^16
```

Mupad [B] (verification not implemented)

Time = 17.62 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{17}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\,(b\,x^2+a)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,(b\,x^2+a)} -\frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^{12}\,(b\,x^2+a)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,(b\,x^2+a)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^17,x)
```

output
```
- (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (b^3*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (a*b^2*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(4*x^12*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2
*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{17}}\,dx = \frac{-56b^3x^6-140a\,b^2x^4-120a^2b\,x^2-35a^3}{560x^{16}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x)
```

output
```
( - 35*a**3 - 120*a**2*b*x**2 - 140*a*b**2*x**4 - 56*b**3*x**6)/(560*x**16
)
```

3.519 $\int x^8(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^8\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}$$

output
```
a^3*x^9*((b*x^2+a)^2)^(1/2)/(9*b*x^2+9*a)+3*a^2*b*x^11*((b*x^2+a)^2)^(1/2)
/(11*b*x^2+11*a)+3*a*b^2*x^13*((b*x^2+a)^2)^(1/2)/(13*b*x^2+13*a)+b^3*x^15
*((b*x^2+a)^2)^(1/2)/(15*b*x^2+15*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^8\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{x^9\sqrt{\left(a+bx^2\right)^2}(715a^3+1755a^2bx^2+1485ab^2x^4+429b^3x^6)}{6435\left(a+bx^2\right)}$$

input
```
Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(x^9*Sqrt[(a + b*x^2)^2]*(715*a^3 + 1755*a^2*b*x^2 + 1485*a*b^2*x^4 + 429*
b^3*x^6))/(6435*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int b^3x^8 \left(bx^2+a\right)^3 dx}{b^3 (a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^8 \left(bx^2+a\right)^3 dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(b^3x^{14}+3ab^2x^{12}+3a^2bx^{10}+a^3x^8\right) dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \left(\frac{a^3x^9}{9}+\frac{3}{11}a^2bx^{11}+\frac{3}{13}ab^2x^{13}+\frac{b^3x^{15}}{15}\right)}{a+bx^2}$$

input
```
Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^3*x^9)/9 + (3*a^2*b*x^11)/11 + (3*a*b
^2*x^13)/13 + (b^3*x^15)/15))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{9} \left(429 b^{3} x^{6}+1485 b^{2} x^{4} a +1755 a^{2} b \,x^{2}+715 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{6435 \left(b \,x^{2}+a \right)^{3}}$	58
default	$\frac{x^{9} \left(429 b^{3} x^{6}+1485 b^{2} x^{4} a +1755 a^{2} b \,x^{2}+715 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{6435 \left(b \,x^{2}+a \right)^{3}}$	58
orering	$\frac{x^{9} \left(429 b^{3} x^{6}+1485 b^{2} x^{4} a +1755 a^{2} b \,x^{2}+715 a^{3}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{3}{2}}}{6435 \left(b \,x^{2}+a \right)^{3}}$	67
risch	$\frac{a^{3} x^{9} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{9 b \,x^{2}+9 a}+\frac{3 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, a^{2} b \,x^{11}}{11 \left(b \,x^{2}+a \right)}+\frac{3 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, b^{2} a \,x^{13}}{13 \left(b \,x^{2}+a \right)}+\frac{b^{3} x^{15} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{15 b \,x^{2}+15 a}$	116

input
```
int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6435*x^9*(429*b^3*x^6+1485*a*b^2*x^4+1755*a^2*b*x^2+715*a^3)*((b*x^2+a)^
2)^(3/2)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{15} b^3x^{15} + \frac{3}{13} ab^2x^{13} + \frac{3}{11} a^2bx^{11} + \frac{1}{9} a^3x^9$$

input
```
integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15*b^3*x^15 + 3/13*a*b^2*x^13 + 3/11*a^2*b*x^11 + 1/9*a^3*x^9
```

Sympy [F]

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \int x^8 \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**8*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{15}\, b^3x^{15} + \frac{3}{13}\, ab^2x^{13} + \frac{3}{11}\, a^2bx^{11} + \frac{1}{9}\, a^3x^9$$

input
```
integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/15*b^3*x^15 + 3/13*a*b^2*x^13 + 3/11*a^2*b*x^11 + 1/9*a^3*x^9
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{15}\, b^3x^{15}\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{13}\, ab^2x^{13}\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{11}\, a^2bx^{11}\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{9}\, a^3x^9\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/15*b^3*x^15*sgn(b*x^2 + a) + 3/13*a*b^2*x^13*sgn(b*x^2 + a) + 3/11*a^2*b
*x^11*sgn(b*x^2 + a) + 1/9*a^3*x^9*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \int x^8\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2} \,d x$$

input
```
int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{x^9(429b^3x^6+1485a\,b^2x^4+1755a^2b\,x^2+715a^3)}{6435}$$

input
```
int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(x**9*(715*a**3 + 1755*a**2*b*x**2 + 1485*a*b**2*x**4 + 429*b**3*x**6))/64
35
```

3.520 $\int x^6(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}$$

output
```
a^3*x^7*((b*x^2+a)^2)^(1/2)/(7*b*x^2+7*a)+a^2*b*x^9*((b*x^2+a)^2)^(1/2)/(3
*b*x^2+3*a)+3*a*b^2*x^11*((b*x^2+a)^2)^(1/2)/(11*b*x^2+11*a)+b^3*x^13*((b*
x^2+a)^2)^(1/2)/(13*b*x^2+13*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{x^7\sqrt{\left(a+bx^2\right)^2}\left(429a^3+1001a^2bx^2+819ab^2x^4+231b^3x^6\right)}{3003\left(a+bx^2\right)}$$

input
```
Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(x^7*Sqrt[(a + b*x^2)^2]*(429*a^3 + 1001*a^2*b*x^2 + 819*a*b^2*x^4 + 231*b
^3*x^6))/(3003*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int b^3x^6 \left(bx^2+a\right)^3 dx}{b^3 \left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^6 \left(bx^2+a\right)^3 dx}{a+bx^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(b^3x^{12}+3ab^2x^{10}+3a^2bx^8+a^3x^6\right) dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \left(\frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}\right)}{a+bx^2}$$

input
```
Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^3*x^7)/7 + (a^2*b*x^9)/3 + (3*a*b^2*x
^11)/11 + (b^3*x^13)/13))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{7}\left(231 b^{3} x^{6}+819 b^{2} x^{4} a+1001 a^{2} b\,x^{2}+429 a^{3}\right) \left(\left(b\,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{3003 \left(b\,x^{2}+a \right)^{3}}$	58
default	$\frac{x^{7}\left(231 b^{3} x^{6}+819 b^{2} x^{4} a+1001 a^{2} b\,x^{2}+429 a^{3}\right) \left(\left(b\,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{3003 \left(b\,x^{2}+a \right)^{3}}$	58
orering	$\frac{x^{7}\left(231 b^{3} x^{6}+819 b^{2} x^{4} a+1001 a^{2} b\,x^{2}+429 a^{3}\right) \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{3003 \left(b\,x^{2}+a \right)^{3}}$	67
risch	$\frac{b^{3} x^{13} \sqrt{\left(b\,x^{2}+a \right)^{2}}}{13 b\,x^{2}+13 a}+\frac{3 \sqrt{\left(b\,x^{2}+a \right)^{2}}\, a\,b^{2} x^{11}}{11 \left(b\,x^{2}+a \right)}+\frac{a^{2} b\,x^{9} \sqrt{\left(b\,x^{2}+a \right)^{2}}}{3 b\,x^{2}+3 a}+\frac{a^{3} x^{7} \sqrt{\left(b\,x^{2}+a \right)^{2}}}{7 b\,x^{2}+7 a}$	116

input
```
int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3003*x^7*(231*b^3*x^6+819*a*b^2*x^4+1001*a^2*b*x^2+429*a^3)*((b*x^2+a)^2
)^(3/2)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^6 \left(a^2 + 2abx^2 + b^2x^4\right)^{3/2} dx = \frac{1}{13} b^3 x^{13} + \frac{3}{11} ab^2 x^{11} + \frac{1}{3} a^2 b x^9 + \frac{1}{7} a^3 x^7$$

input
```
integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Sympy [F]

$$\int x^6 \left(a^2 + 2abx^2 + b^2x^4\right)^{3/2} dx = \int x^6 \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**6*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{13}b^3x^{13} + \frac{3}{11}ab^2x^{11} + \frac{1}{3}a^2bx^9 + \frac{1}{7}a^3x^7$$

input
```
integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{13}b^3x^{13}\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{11}ab^2x^{11}\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{3}a^2bx^9\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{7}a^3x^7\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/13*b^3*x^13*sgn(b*x^2 + a) + 3/11*a*b^2*x^11*sgn(b*x^2 + a) + 1/3*a^2*b*
x^9*sgn(b*x^2 + a) + 1/7*a^3*x^7*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \int x^6 \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2} dx$$

input
```
int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int x^6 (a^2 + 2abx^2 + b^2x^4)^{3/2} \, dx = \frac{x^7(231b^3x^6 + 819a\,b^2x^4 + 1001a^2b\,x^2 + 429a^3)}{3003}$$

input
```
int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(x**7*(429*a**3 + 1001*a**2*b*x**2 + 819*a*b**2*x**4 + 231*b**3*x**6))/300
3
```

3.521 $\int x^4(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{ab^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}$$

output
```
a^3*x^5*((b*x^2+a)^2)^(1/2)/(5*b*x^2+5*a)+3*a^2*b*x^7*((b*x^2+a)^2)^(1/2)/
(7*b*x^2+7*a)+a*b^2*x^9*((b*x^2+a)^2)^(1/2)/(3*b*x^2+3*a)+b^3*x^11*((b*x^2
+a)^2)^(1/2)/(11*b*x^2+11*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{x^5\sqrt{\left(a+bx^2\right)^2}\left(231a^3+495a^2bx^2+385ab^2x^4+105b^3x^6\right)}{1155\left(a+bx^2\right)}$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(x^5*Sqrt[(a + b*x^2)^2]*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^
3*x^6))/(1155*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int b^3x^4 \left(bx^2+a\right)^3 dx}{b^3 \left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int x^4 \left(bx^2+a\right)^3 dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(b^3x^{10}+3ab^2x^8+3a^2bx^6+a^3x^4\right) dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \left(\frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}\right)}{a+bx^2}$$

input
```
Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x
^9)/3 + (b^3*x^11)/11))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{5} \left(105 b^{3} x^{6}+385 b^{2} x^{4} a +495 a^{2} b \,x^{2}+231 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{1155 \left(b \,x^{2}+a \right)^{3}}$	58
default	$\frac{x^{5} \left(105 b^{3} x^{6}+385 b^{2} x^{4} a +495 a^{2} b \,x^{2}+231 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{1155 \left(b \,x^{2}+a \right)^{3}}$	58
orering	$\frac{x^{5} \left(105 b^{3} x^{6}+385 b^{2} x^{4} a +495 a^{2} b \,x^{2}+231 a^{3}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{3}{2}}}{1155 \left(b \,x^{2}+a \right)^{3}}$	67
risch	$\frac{b^{3} x^{11} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{11 b \,x^{2}+11 a}+\frac{a \,b^{2} x^{9} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{3 b \,x^{2}+3 a}+\frac{3 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, a^{2} b \,x^{7}}{7 \left(b \,x^{2}+a \right)}+\frac{a^{3} x^{5} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{5 b \,x^{2}+5 a}$	116

input
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*((b*x^2+a)^2)
^(3/2)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^4(a^2+2abx^2+b^2x^4)^{3/2}\,dx=\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Sympy [F]

$$\int x^4(a^2+2abx^2+b^2x^4)^{3/2}\,dx=\int x^4\left((a+bx^2)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**4*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^4(a^2+2abx^2+b^2x^4)^{3/2}\,dx=\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^4(a^2+2abx^2+b^2x^4)^{3/2}\,dx=\frac{1}{11}b^3x^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}ab^2x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{3}{7}a^2bx^7\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{5}a^3x^5\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/11*b^3*x^11*sgn(b*x^2 + a) + 1/3*a*b^2*x^9*sgn(b*x^2 + a) + 3/7*a^2*b*x^
7*sgn(b*x^2 + a) + 1/5*a^3*x^5*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a^2+2abx^2+b^2x^4)^{3/2}\,dx=\int x^4\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

input
```
int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int x^4(a^2+2abx^2+b^2x^4)^{3/2}\,dx=\frac{x^5(105b^3x^6+385a\,b^2x^4+495a^2b\,x^2+231a^3)}{1155}$$

input
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(x**5*(231*a**3 + 495*a**2*b*x**2 + 385*a*b**2*x**4 + 105*b**3*x**6))/1155
```

3.522 $\int x^2(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\,(a+bx^2)}+\frac{3ab^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}+\frac{b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}$$

output
```
a^3*x^3*((b*x^2+a)^2)^(1/2)/(3*b*x^2+3*a)+3*a^2*b*x^5*((b*x^2+a)^2)^(1/2)/
(5*b*x^2+5*a)+3*a*b^2*x^7*((b*x^2+a)^2)^(1/2)/(7*b*x^2+7*a)+b^3*x^9*((b*x^
2+a)^2)^(1/2)/(9*b*x^2+9*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{\sqrt{(a+bx^2)^2}\,(105a^3x^3+189a^2bx^5+135ab^2x^7+35b^3x^9)}{315\,(a+bx^2)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(105*a^3*x^3 + 189*a^2*b*x^5 + 135*a*b^2*x^7 + 35*b^3
*x^9))/(315*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^3x^2\left(bx^2+a\right)^3dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2\left(bx^2+a\right)^3dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^3x^8+3ab^2x^6+3a^2bx^4+a^3x^2\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}\right)}{a+bx^2}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2
*x^7)/7 + (b^3*x^9)/9))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x^{3}\left(35 b^{3} x^{6}+135 b^{2} x^{4} a+189 a^{2} b \,x^{2}+105 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{315 \left(b \,x^{2}+a \right)^{3}}$	58
default	$\frac{x^{3}\left(35 b^{3} x^{6}+135 b^{2} x^{4} a+189 a^{2} b \,x^{2}+105 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{315 \left(b \,x^{2}+a \right)^{3}}$	58
orering	$\frac{x^{3}\left(35 b^{3} x^{6}+135 b^{2} x^{4} a+189 a^{2} b \,x^{2}+105 a^{3}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{3}{2}}}{315 \left(b \,x^{2}+a \right)^{3}}$	67
risch	$\frac{b^{3} x^{9} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{9 b \,x^{2}+9 a}+\frac{3 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, a \,b^{2} x^{7}}{7 \left(b \,x^{2}+a \right)}+\frac{3 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, a^{2} b \,x^{5}}{5 \left(b \,x^{2}+a \right)}+\frac{a^{3} x^{3} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{3 b \,x^{2}+3 a}$	116

input
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*((b*x^2+a)^2)^(
3/2)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Sympy [F]

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \int x^2 \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**2*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{1}{9} b^3 x^9 \mathrm{sgn}\left(bx^2+a\right) + \frac{3}{7} ab^2 x^7 \mathrm{sgn}\left(bx^2+a\right) + \frac{3}{5} a^2 b x^5 \mathrm{sgn}\left(bx^2+a\right) + \frac{1}{3} a^3 x^3 \mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/9*b^3*x^9*sgn(b*x^2 + a) + 3/7*a*b^2*x^7*sgn(b*x^2 + a) + 3/5*a^2*b*x^5*
sgn(b*x^2 + a) + 1/3*a^3*x^3*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \int x^2 \, {\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2} \,d x$$

input
```
int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{x^3 (35b^3x^6+135a\,b^2x^4+189a^2b\,x^2+105a^3)}{315}$$

input
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(x**3*(105*a**3 + 189*a**2*b*x**2 + 135*a*b**2*x**4 + 35*b**3*x**6))/315
```

3.523 $\int (a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 159

$$\int (a^2+2abx^2+b^2x^4)^{3/2}\,dx=\frac{a^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{a^2bx^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{3ab^2x^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}+\frac{b^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}$$

output
```
a^3*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^2*b*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a
)+3*a*b^2*x^5*((b*x^2+a)^2)^(1/2)/(5*b*x^2+5*a)+b^3*x^7*((b*x^2+a)^2)^(1/2
)/(7*b*x^2+7*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.37

$$\int (a^2+2abx^2+b^2x^4)^{3/2}\,dx=\frac{\sqrt{(a+bx^2)^2}(35a^3x+35a^2bx^3+21ab^2x^5+5b^3x^7)}{35(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(35*a^3*x + 35*a^2*b*x^3 + 21*a*b^2*x^5 + 5*b^3*x^7))
/(35*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.47, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1384, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^2x^2+ab\right)^3dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 210}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^6x^6+3ab^5x^4+3a^2b^4x^2+a^3b^3\right)dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a^3b^3x+a^2b^4x^3+\frac{3}{5}ab^5x^5+\frac{b^6x^7}{7}\right)}{b^3\left(a+bx^2\right)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(a^3*b^3*x + a^2*b^4*x^3 + (3*a*b^5*x^5)/
5 + (b^6*x^7)/7))/(b^3*(a + b*x^2))
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.35

method	result	size
gosper	$\frac{x\left(5b^{3}x^{6}+21b^{2}x^{4}a+35a^{2}b\,x^{2}+35a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{35\left(b\,x^{2}+a\right)^{3}}$	56
default	$\frac{x\left(5b^{3}x^{6}+21b^{2}x^{4}a+35a^{2}b\,x^{2}+35a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{35\left(b\,x^{2}+a\right)^{3}}$	56
orering	$\frac{x\left(5b^{3}x^{6}+21b^{2}x^{4}a+35a^{2}b\,x^{2}+35a^{3}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{35\left(b\,x^{2}+a\right)^{3}}$	65
risch	$\frac{b^{3}x^{7}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{7b\,x^{2}+7a}+\frac{3\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{2}x^{5}a}{5\left(b\,x^{2}+a\right)}+\frac{a^{2}b\,x^{3}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}+\frac{a^{3}x\sqrt{\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}$	112

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)^(3/2)/(b
*x^2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.19

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{7}b^3x^7+\frac{3}{5}ab^2x^5+a^2bx^3+a^3x$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Sympy [F]

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \int \left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}} dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.19

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{7}b^3x^7+\frac{3}{5}ab^2x^5+a^2bx^3+a^3x$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.40

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{1}{7}b^3x^7\mathrm{sgn}\left(bx^2+a\right) + \frac{3}{5}ab^2x^5\mathrm{sgn}\left(bx^2+a\right) + a^2bx^3\mathrm{sgn}\left(bx^2+a\right) + a^3x\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/7*b^3*x^7*sgn(b*x^2 + a) + 3/5*a*b^2*x^5*sgn(b*x^2 + a) + a^2*b*x^3*sgn(
b*x^2 + a) + a^3*x*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \int \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2} dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.22

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{x(5b^3x^6+21a\,b^2x^4+35a^2b\,x^2+35a^3)}{35}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(x*(35*a**3 + 35*a**2*b*x**2 + 21*a*b**2*x**4 + 5*b**3*x**6))/35
```

3.524 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 158

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^2}\,dx = -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)} + \frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}$$

output
```
-a^3*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+3*a^2*b*x*((b*x^2+a)^2)^(1/2)/(b*x^2+
a)+a*b^2*x^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+b^3*x^5*((b*x^2+a)^2)^(1/2)/(5*
b*x^2+5*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.38

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^2}\,dx = \frac{\sqrt{(a+bx^2)^2}(-5a^3+15a^2bx^2+5ab^2x^4+b^3x^6)}{5x(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(5*x
*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3(bx^2+a)^3}{x^2}dx}{b^3(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{x^2}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^3x^4+3ab^2x^2+3a^2b+\frac{a^3}{x^2}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}\right)}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*
x^5)/5))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.37

method	result	size
gosper	$-\frac{\left(-b^{3} x^{6}-5 b^{2} x^{4} a-15 a^{2} b \,x^{2}+5 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{5 x \left(b \,x^{2}+a \right)^{3}}$	58
default	$-\frac{\left(-b^{3} x^{6}-5 b^{2} x^{4} a-15 a^{2} b \,x^{2}+5 a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{5 x \left(b \,x^{2}+a \right)^{3}}$	58
orering	$-\frac{\left(-b^{3} x^{6}-5 b^{2} x^{4} a-15 a^{2} b \,x^{2}+5 a^{3}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{3}{2}}}{5 x \left(b \,x^{2}+a \right)^{3}}$	67
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, b \left(\frac{1}{5} x^{5} b^{2}+a b \,x^{3}+3 a^{2} x \right)}{b \,x^{2}+a}-\frac{a^{3} \sqrt{\left(b \,x^{2}+a \right)^{2}}}{x \left(b \,x^{2}+a \right)}$	73

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*((b*x^2+a)^2)^(3/2)/x/(b*x^
2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^2}\,dx = \frac{b^3x^6+5\,ab^2x^4+15\,a^2bx^2-5\,a^3}{5\,x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^2}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**2,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^2}\,dx = \frac{1}{5}\,b^3x^5+ab^2x^3+3\,a^2bx-\frac{a^3}{x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^2} dx = \frac{1}{5} b^3x^5 \text{sgn}(bx^2 + a) + ab^2x^3 \text{sgn}(bx^2 + a) + 3 a^2bx \text{sgn}(bx^2 + a) - \frac{a^3 \text{sgn}(bx^2 + a)}{x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/5*b^3*x^5*sgn(b*x^2 + a) + a*b^2*x^3*sgn(b*x^2 + a) + 3*a^2*b*x*sgn(b*x^
2 + a) - a^3*sgn(b*x^2 + a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^2} dx = \int \frac{(a^2 + 2\, a\, b\, x^2 + b^2\, x^4)^{3/2}}{x^2} \,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^2,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^2}\,dx = \frac{b^3x^6 + 5a\,b^2x^4 + 15a^2b\,x^2 - 5a^3}{5x}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x)
```

output
```
( - 5*a**3 + 15*a**2*b*x**2 + 5*a*b**2*x**4 + b**3*x**6)/(5*x)
```

3.525 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 161

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}\,dx = -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\,(a+bx^2)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x\,(a+bx^2)} + \frac{3ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}$$

output
```
-1/3*a^3*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-3*a^2*b*((b*x^2+a)^2)^(1/2)/x/(
b*x^2+a)+3*a*b^2*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+b^3*x^3*((b*x^2+a)^2)^(1/
2)/(3*b*x^2+3*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.37

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}\,dx = -\frac{\sqrt{(a+bx^2)^2}\,(a^3+9a^2bx^2-9ab^2x^4-b^3x^6)}{3x^3\,(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

output
```
-1/3*(Sqrt[(a + b*x^2)^2]*(a^3 + 9*a^2*b*x^2 - 9*a*b^2*x^4 - b^3*x^6))/(x^
3*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3\left(bx^2+a\right)^3}{x^4}dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^4}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{x^4}+\frac{3ba^2}{x^2}+3b^2a+b^3x^2\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3ab^2x+\frac{b^3x^3}{3}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

output
```
((-1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.35

method	result	size
gosper	$-\frac{\left(-b^{3} x^{6}-9 b^{2} x^{4} a+9 a^{2} b \,x^{2}+a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{3 x^{3} \left(b \,x^{2}+a \right)^{3}}$	56
default	$-\frac{\left(-b^{3} x^{6}-9 b^{2} x^{4} a+9 a^{2} b \,x^{2}+a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{3 x^{3} \left(b \,x^{2}+a \right)^{3}}$	56
orering	$-\frac{\left(-b^{3} x^{6}-9 b^{2} x^{4} a+9 a^{2} b \,x^{2}+a^{3}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{3}{2}}}{3 x^{3} \left(b \,x^{2}+a \right)^{3}}$	65
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, b^{2} \left(\frac{1}{3} b \,x^{3}+3 a x \right)}{b \,x^{2}+a}+\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(-3 a^{2} b \,x^{2}-\frac{1}{3} a^{3}\right)}{\left(b \,x^{2}+a \right) x^{3}}$	76

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-b^3*x^6-9*a*b^2*x^4+9*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^3/(b*x^2
+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}\,dx=\frac{b^3x^6+9\,ab^2x^4-9\,a^2bx^2-a^3}{3\,x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}\,dx=\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**4,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.20

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}\,dx=\frac{1}{3}\,b^3x^3+3\,ab^2x-\frac{3\,a^2b}{x}-\frac{a^3}{3\,x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3 + 3*a*b^2*x - 3*a^2*b/x - 1/3*a^3/x^3
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^4}\,dx = \frac{1}{3}\,b^3x^3\mathrm{sgn}\left(bx^2+a\right) + 3\,ab^2x\mathrm{sgn}\left(bx^2+a\right) - \frac{9\,a^2bx^2\mathrm{sgn}(bx^2+a) + a^3\mathrm{sgn}(bx^2+a)}{3\,x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
1/3*b^3*x^3*sgn(b*x^2 + a) + 3*a*b^2*x*sgn(b*x^2 + a) - 1/3*(9*a^2*b*x^2*s
gn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^4}\,dx = \int \frac{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}{x^4}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^4,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^4} \, dx = \frac{b^3x^6 + 9a\,b^2x^4 - 9a^2b\,x^2 - a^3}{3x^3}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x)
```

output
```
( - a**3 - 9*a**2*b*x**2 + 9*a*b**2*x**4 + b**3*x**6)/(3*x**3)
```

3.526 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 158

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^6}\,dx = -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)} - \frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3(a+bx^2)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)} + \frac{b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

output
```
-1/5*a^3*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-a^2*b*((b*x^2+a)^2)^(1/2)/x^3/(
b*x^2+a)-3*a*b^2*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+b^3*x*((b*x^2+a)^2)^(1/2)
/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^6}\,dx = -\frac{\sqrt{(a+bx^2)^2}(a^3+5a^2bx^2+15ab^2x^4-5b^3x^6)}{5x^5(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
```

output
```
-1/5*(Sqrt[(a + b*x^2)^2]*(a^3 + 5*a^2*b*x^2 + 15*a*b^2*x^4 - 5*b^3*x^6))/
(x^5*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3\left(bx^2+a\right)^3}{x^6}dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^6}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{x^6}+\frac{3ba^2}{x^4}+\frac{3b^2a}{x^2}+b^3\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
```

output
```
((-1/5*a^3/x^5 - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.35

method	result	size
gosper	$-\frac{\left(-5 b^{3} x^{6}+15 b^{2} x^{4} a+5 a^{2} b \,x^{2}+a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{5 x^{5} \left(b \,x^{2}+a \right)^{3}}$	56
default	$-\frac{\left(-5 b^{3} x^{6}+15 b^{2} x^{4} a+5 a^{2} b \,x^{2}+a^{3}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{5 x^{5} \left(b \,x^{2}+a \right)^{3}}$	56
orering	$-\frac{\left(-5 b^{3} x^{6}+15 b^{2} x^{4} a+5 a^{2} b \,x^{2}+a^{3}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{3}{2}}}{5 x^{5} \left(b \,x^{2}+a \right)^{3}}$	65
risch	$\frac{b^{3} x \sqrt{\left(b \,x^{2}+a \right)^{2}}}{b \,x^{2}+a}+\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(-3 b^{2} x^{4} a -a^{2} b \,x^{2}-\frac{1}{5} a^{3}\right)}{\left(b \,x^{2}+a \right) x^{5}}$	75

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-5*b^3*x^6+15*a*b^2*x^4+5*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x^5/(b*
x^2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^6} dx = \frac{5 b^3x^6 - 15 ab^2x^4 - 5 a^2bx^2 - a^3}{5 x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^6} dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{3}{2}}}{x^6} dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**6,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^6} dx = b^3x - \frac{3 ab^2}{x} - \frac{a^2b}{x^3} - \frac{a^3}{5 x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
b^3*x - 3*a*b^2/x - a^2*b/x^3 - 1/5*a^3/x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^6} \, dx = b^3 x \operatorname{sgn}(bx^2 + a) - \frac{15\, ab^2x^4 \operatorname{sgn}(bx^2 + a) + 5\, a^2bx^2 \operatorname{sgn}(bx^2 + a) + a^3 \operatorname{sgn}(bx^2 + a)}{5\, x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
b^3*x*sgn(b*x^2 + a) - 1/5*(15*a*b^2*x^4*sgn(b*x^2 + a) + 5*a^2*b*x^2*sgn(
b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^6} \, dx = \int \frac{(a^2 + 2\, a\, b\, x^2 + b^2\, x^4)^{3/2}}{x^6} \, dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^6,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^6}\, dx = \frac{5b^3x^6 - 15a\, b^2x^4 - 5a^2b\, x^2 - a^3}{5x^5}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x)
```

output
```
( - a**3 - 5*a**2*b*x**2 - 15*a*b**2*x**4 + 5*b**3*x**6)/(5*x**5)
```

3.527 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 163

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^8}\,dx = -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)} - \frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3(a+bx^2)} - \frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}$$

output
```
-1/7*a^3*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-3/5*a^2*b*((b*x^2+a)^2)^(1/2)/x
^5/(b*x^2+a)-a*b^2*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-b^3*((b*x^2+a)^2)^(1/
2)/x/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^8}\,dx = -\frac{\sqrt{(a+bx^2)^2}(5a^3+21a^2bx^2+35ab^2x^4+35b^3x^6)}{35x^7(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

output
```
-1/35*(Sqrt[(a + b*x^2)^2]*(5*a^3 + 21*a^2*b*x^2 + 35*a*b^2*x^4 + 35*b^3*x
^6))/(x^7*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3\left(bx^2+a\right)^3}{x^8}dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^8}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{x^8}+\frac{3ba^2}{x^6}+\frac{3b^2a}{x^4}+\frac{b^3}{x^2}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

output
```
((-1/7*a^3/x^7 - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.35

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-b^{3} x^{6}-b^{2} x^{4} a-\frac{3}{5} a^{2} b\,x^{2}-\frac{1}{7} a^{3}\right)}{\left(b\,x^{2}+a\right) x^{7}}$	57
gosper	$-\frac{\left(35 b^{3} x^{6}+35 b^{2} x^{4} a+21 a^{2} b\,x^{2}+5 a^{3}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{35 x^{7} \left(b\,x^{2}+a\right)^{3}}$	58
default	$-\frac{\left(35 b^{3} x^{6}+35 b^{2} x^{4} a+21 a^{2} b\,x^{2}+5 a^{3}\right) \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{35 x^{7} \left(b\,x^{2}+a\right)^{3}}$	58
orering	$-\frac{\left(35 b^{3} x^{6}+35 b^{2} x^{4} a+21 a^{2} b\,x^{2}+5 a^{3}\right) \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{35 x^{7} \left(b\,x^{2}+a\right)^{3}}$	67

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*(-b^3*x^6-b^2*x^4*a-3/5*a^2*b*x^2-1/7*a^3)/x
^7
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^8}\,dx = -\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
-1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^8}\,dx = \int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**8,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^8}\,dx = -\frac{b^3}{x}-\frac{ab^2}{x^3}-\frac{3\,a^2b}{5\,x^5}-\frac{a^3}{7\,x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
-b^3/x - a*b^2/x^3 - 3/5*a^2*b/x^5 - 1/7*a^3/x^7
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^8} dx = -\frac{35 b^3x^6\text{sgn}(bx^2 + a) + 35 ab^2x^4\text{sgn}(bx^2 + a) + 21 a^2bx^2\text{sgn}(bx^2 + a) + 5 a^3\text{sgn}(bx^2 + a)}{35 x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
-1/35*(35*b^3*x^6*sgn(b*x^2 + a) + 35*a*b^2*x^4*sgn(b*x^2 + a) + 21*a^2*b*
x^2*sgn(b*x^2 + a) + 5*a^3*sgn(b*x^2 + a))/x^7
```

Mupad [B] (verification not implemented)

Time = 18.87 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^8} dx = -\frac{a^3 \sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{7\,x^7\,(b\,x^2 + a)} - \frac{b^3 \sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{x\,(b\,x^2 + a)} - \frac{a\,b^2 \sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{x^3\,(b\,x^2 + a)} - \frac{3\,a^2\,b \sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{5\,x^5\,(b\,x^2 + a)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^8,x)
```

output
```
- (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (b^3*(a^2
+ b^2*x^4 + 2*a*b*x^2)^(1/2))/(x*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^4 + 2*
a*b*x^2)^(1/2))/(x^3*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(5*x^5*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^8}\,dx = \frac{-35b^3x^6-35a\,b^2x^4-21a^2b\,x^2-5a^3}{35x^7}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x)
```

output
```
( - 5*a**3 - 21*a**2*b*x**2 - 35*a*b**2*x**4 - 35*b**3*x**6)/(35*x**7)
```

3.528 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{10}}\,dx=-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

output
```
-1/9*a^3*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-3/7*a^2*b*((b*x^2+a)^2)^(1/2)/x
^7/(b*x^2+a)-3/5*a*b^2*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-1/3*b^3*((b*x^2+a
)^2)^(1/2)/x^3/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{10}}\,dx=-\frac{\sqrt{\left(a+bx^2\right)^2}\left(35a^3+135a^2bx^2+189ab^2x^4+105b^3x^6\right)}{315x^9\left(a+bx^2\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

output
```
-1/315*(Sqrt[(a + b*x^2)^2]*(35*a^3 + 135*a^2*b*x^2 + 189*a*b^2*x^4 + 105*
b^3*x^6))/(x^9*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{10}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3(bx^2+a)^3}{x^{10}}dx}{b^3(a+bx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{x^{10}}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{x^{10}}+\frac{3ba^2}{x^8}+\frac{3b^2a}{x^6}+\frac{b^3}{x^4}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

output
```
((-1/9*a^3/x^9 - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(5*x^5) - b^3/(3*x^3))*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{1}{3}b^{3}x^{6}-\frac{3}{5}b^{2}x^{4}a-\frac{3}{7}a^{2}b\,x^{2}-\frac{1}{9}a^{3}\right)}{\left(b\,x^{2}+a\right)x^{9}}$	57
gosper	$-\frac{\left(105b^{3}x^{6}+189b^{2}x^{4}a+135a^{2}b\,x^{2}+35a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{315x^{9}\left(b\,x^{2}+a\right)^{3}}$	58
default	$-\frac{\left(105b^{3}x^{6}+189b^{2}x^{4}a+135a^{2}b\,x^{2}+35a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{315x^{9}\left(b\,x^{2}+a\right)^{3}}$	58
orering	$-\frac{\left(105b^{3}x^{6}+189b^{2}x^{4}a+135a^{2}b\,x^{2}+35a^{3}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{315x^{9}\left(b\,x^{2}+a\right)^{3}}$	67

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*(-1/3*b^3*x^6-3/5*b^2*x^4*a-3/7*a^2*b*x^2-1/
9*a^3)/x^9
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{10}}\,dx = -\frac{105\,b^3x^6+189\,ab^2x^4+135\,a^2bx^2+35\,a^3}{315\,x^9}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
-1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{10}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**10,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**10, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{10}}\,dx = -\frac{b^3}{3\,x^3}-\frac{3\,ab^2}{5\,x^5}-\frac{3\,a^2b}{7\,x^7}-\frac{a^3}{9\,x^9}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
-1/3*b^3/x^3 - 3/5*a*b^2/x^5 - 3/7*a^2*b/x^7 - 1/9*a^3/x^9
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{10}} \, dx = -\frac{105 \, b^3 x^6 \mathrm{sgn}(bx^2 + a) + 189 \, ab^2 x^4 \mathrm{sgn}(bx^2 + a) + 135 \, a^2 b x^2 \mathrm{sgn}(bx^2 + a) + 35 \, a^3 \mathrm{sgn}(bx^2 + a)}{315 \, x^9}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x, algorithm="giac")
```

output
```
-1/315*(105*b^3*x^6*sgn(b*x^2 + a) + 189*a*b^2*x^4*sgn(b*x^2 + a) + 135*a^
2*b*x^2*sgn(b*x^2 + a) + 35*a^3*sgn(b*x^2 + a))/x^9
```

Mupad [B] (verification not implemented)

Time = 18.42 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{10}} \, dx = -\frac{a^3 \sqrt{a^2 + 2 \, a \, b \, x^2 + b^2 \, x^4}}{9 \, x^9 \, (b \, x^2 + a)} - \frac{b^3 \sqrt{a^2 + 2 \, a \, b \, x^2 + b^2 \, x^4}}{3 \, x^3 \, (b \, x^2 + a)} - \frac{3 \, a \, b^2 \sqrt{a^2 + 2 \, a \, b \, x^2 + b^2 \, x^4}}{5 \, x^5 \, (b \, x^2 + a)} - \frac{3 \, a^2 \, b \sqrt{a^2 + 2 \, a \, b \, x^2 + b^2 \, x^4}}{7 \, x^7 \, (b \, x^2 + a)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^10,x)
```

output
```
- (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (b^3*(a^2
+ b^2*x^4 + 2*a*b*x^2)^(1/2))/(3*x^3*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(5*x^5*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(7*x^7*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{10}}\,dx = \frac{-105b^3x^6 - 189a\,b^2x^4 - 135a^2b\,x^2 - 35a^3}{315x^9}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x)
```

output
```
( - 35*a**3 - 135*a**2*b*x**2 - 189*a*b**2*x**4 - 105*b**3*x**6)/(315*x**9
)
```

3.529 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{12}}\,dx = -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)} - \frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9(a+bx^2)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)} - \frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}$$

output
```
-1/11*a^3*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-1/3*a^2*b*((b*x^2+a)^2)^(1/2)
/x^9/(b*x^2+a)-3/7*a*b^2*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-1/5*b^3*((b*x^2
+a)^2)^(1/2)/x^5/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{12}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(105a^3+385a^2bx^2+495ab^2x^4+231b^3x^6)}{1155x^{11}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

output
```
-1/1155*(Sqrt[(a + b*x^2)^2]*(105*a^3 + 385*a^2*b*x^2 + 495*a*b^2*x^4 + 23
1*b^3*x^6))/(x^11*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{12}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3\left(bx^2+a\right)^3}{x^{12}}dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^3}{x^{12}}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{x^{12}}+\frac{3ba^2}{x^{10}}+\frac{3b^2a}{x^8}+\frac{b^3}{x^6}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

output
```
((-1/11*a^3/x^11 - (a^2*b)/(3*x^9) - (3*a*b^2)/(7*x^7) - b^3/(5*x^5))*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{1}{5}b^{3}x^{6}-\frac{3}{7}b^{2}x^{4}a-\frac{1}{3}a^{2}b\,x^{2}-\frac{1}{11}a^{3}\right)}{\left(b\,x^{2}+a\right)x^{11}}$	57
gosper	$-\frac{\left(231b^{3}x^{6}+495b^{2}x^{4}a+385a^{2}b\,x^{2}+105a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{1155x^{11}\left(b\,x^{2}+a\right)^{3}}$	58
default	$-\frac{\left(231b^{3}x^{6}+495b^{2}x^{4}a+385a^{2}b\,x^{2}+105a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{1155x^{11}\left(b\,x^{2}+a\right)^{3}}$	58
orering	$-\frac{\left(231b^{3}x^{6}+495b^{2}x^{4}a+385a^{2}b\,x^{2}+105a^{3}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{1155x^{11}\left(b\,x^{2}+a\right)^{3}}$	67

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*(-1/5*b^3*x^6-3/7*b^2*x^4*a-1/3*a^2*b*x^2-1/
11*a^3)/x^11
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{12}}\,dx = -\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="fricas")
```

output
```
-1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{12}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**12,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**12, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{12}}\,dx = -\frac{b^3}{5\,x^5}-\frac{3\,ab^2}{7\,x^7}-\frac{a^2b}{3\,x^9}-\frac{a^3}{11\,x^{11}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="maxima")
```

output
```
-1/5*b^3/x^5 - 3/7*a*b^2/x^7 - 1/3*a^2*b/x^9 - 1/11*a^3/x^11
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{12}}\, dx = -\frac{231\, b^3x^6\mathrm{sgn}(bx^2 + a) + 495\, ab^2x^4\mathrm{sgn}(bx^2 + a) + 385\, a^2bx^2\mathrm{sgn}(bx^2 + a) + 105\, a^3\mathrm{sgn}(bx^2 + a)}{1155\, x^{11}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x, algorithm="giac")
```

output
```
-1/1155*(231*b^3*x^6*sgn(b*x^2 + a) + 495*a*b^2*x^4*sgn(b*x^2 + a) + 385*a
^2*b*x^2*sgn(b*x^2 + a) + 105*a^3*sgn(b*x^2 + a))/x^11
```

Mupad [B] (verification not implemented)

Time = 17.74 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{12}}\, dx = -\frac{a^3\sqrt{a^2 + 2\, a\, b\, x^2 + b^2\, x^4}}{11\, x^{11}\, (b\, x^2 + a)} - \frac{b^3\sqrt{a^2 + 2\, a\, b\, x^2 + b^2\, x^4}}{5\, x^5\, (b\, x^2 + a)} - \frac{3\, a\, b^2\sqrt{a^2 + 2\, a\, b\, x^2 + b^2\, x^4}}{7\, x^7\, (b\, x^2 + a)} - \frac{a^2\, b\sqrt{a^2 + 2\, a\, b\, x^2 + b^2\, x^4}}{3\, x^9\, (b\, x^2 + a)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^12,x)
```

output
```
- (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (b^3*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(5*x^5*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (a^2*b*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(3*x^9*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{12}}\,dx=\frac{-231b^3x^6-495a\,b^2x^4-385a^2b\,x^2-105a^3}{1155x^{11}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x)
```

output
```
( - 105*a**3 - 385*a**2*b*x**2 - 495*a*b**2*x**4 - 231*b**3*x**6)/(1155*x*
*11)
```

3.530 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{14}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{14}}\,dx=-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}$$

output
```
-1/13*a^3*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)-3/11*a^2*b*((b*x^2+a)^2)^(1/2
)/x^11/(b*x^2+a)-1/3*a*b^2*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-1/7*b^3*((b*x
^2+a)^2)^(1/2)/x^7/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{14}}\,dx=-\frac{\sqrt{\left(a+bx^2\right)^2}\left(231a^3+819a^2bx^2+1001ab^2x^4+429b^3x^6\right)}{3003x^{13}\left(a+bx^2\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

output
```
-1/3003*(Sqrt[(a + b*x^2)^2]*(231*a^3 + 819*a^2*b*x^2 + 1001*a*b^2*x^4 + 4
29*b^3*x^6))/(x^13*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{14}} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \frac{b^3(bx^2+a)^3}{x^{14}}dx}{b^3(a + bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \frac{(bx^2+a)^3}{x^{14}}dx}{a + bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \left(\frac{a^3}{x^{14}} + \frac{3ba^2}{x^{12}} + \frac{3b^2a}{x^{10}} + \frac{b^3}{x^8}\right)dx}{a + bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{13x^{13}} - \frac{3a^2b}{11x^{11}} - \frac{ab^2}{3x^9} - \frac{b^3}{7x^7}\right)\sqrt{a^2 + 2abx^2 + b^2x^4}}{a + bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

output
```
((-1/13*a^3/x^13 - (3*a^2*b)/(11*x^11) - (a*b^2)/(3*x^9) - b^3/(7*x^7))*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 7.83 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{1}{7}b^{3}x^{6}-\frac{1}{3}b^{2}x^{4}a-\frac{3}{11}a^{2}b\,x^{2}-\frac{1}{13}a^{3}\right)}{\left(b\,x^{2}+a\right)x^{13}}$	57
gosper	$-\frac{\left(429b^{3}x^{6}+1001b^{2}x^{4}a+819a^{2}b\,x^{2}+231a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{3003x^{13}\left(b\,x^{2}+a\right)^{3}}$	58
default	$-\frac{\left(429b^{3}x^{6}+1001b^{2}x^{4}a+819a^{2}b\,x^{2}+231a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{3003x^{13}\left(b\,x^{2}+a\right)^{3}}$	58
orering	$-\frac{\left(429b^{3}x^{6}+1001b^{2}x^{4}a+819a^{2}b\,x^{2}+231a^{3}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{3003x^{13}\left(b\,x^{2}+a\right)^{3}}$	67

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*(-1/7*b^3*x^6-1/3*b^2*x^4*a-3/11*a^2*b*x^2-1
/13*a^3)/x^13
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{14}}\,dx = -\frac{429\,b^3x^6+1001\,ab^2x^4+819\,a^2bx^2+231\,a^3}{3003\,x^{13}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="fricas")
```

output
```
-1/3003*(429*b^3*x^6 + 1001*a*b^2*x^4 + 819*a^2*b*x^2 + 231*a^3)/x^13
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{14}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**14,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**14, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{14}}\,dx = -\frac{b^3}{7\,x^7}-\frac{ab^2}{3\,x^9}-\frac{3\,a^2b}{11\,x^{11}}-\frac{a^3}{13\,x^{13}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="maxima")
```

output
```
-1/7*b^3/x^7 - 1/3*a*b^2/x^9 - 3/11*a^2*b/x^11 - 1/13*a^3/x^13
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{14}}\,dx = -\frac{429\,b^3x^6\mathrm{sgn}(bx^2+a)+1001\,ab^2x^4\mathrm{sgn}(bx^2+a)+819\,a^2bx^2\mathrm{sgn}(bx^2+a)+231\,a^3\mathrm{sgn}(bx^2+a)}{3003\,x^{13}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x, algorithm="giac")
```

output
```
-1/3003*(429*b^3*x^6*sgn(b*x^2 + a) + 1001*a*b^2*x^4*sgn(b*x^2 + a) + 819*
a^2*b*x^2*sgn(b*x^2 + a) + 231*a^3*sgn(b*x^2 + a))/x^13
```

Mupad [B] (verification not implemented)

Time = 17.52 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{14}}\,dx = -\frac{a^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}-\frac{b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)} -\frac{a\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^9\,\left(b\,x^2+a\right)}-\frac{3\,a^2\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^14,x)
```

output
```
- (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (b^3*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (a*b^2*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(3*x^9*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(11*x^11*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{14}}\,dx = \frac{-429b^3x^6-1001a\,b^2x^4-819a^2b\,x^2-231a^3}{3003x^{13}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x)
```

output
```
( - 231*a**3 - 819*a**2*b*x**2 - 1001*a*b**2*x**4 - 429*b**3*x**6)/(3003*x
**13)
```

3.531 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{16}} dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{16}} dx = -\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}(a+bx^2)} - \frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)} - \frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)} - \frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}$$

output
```
-1/15*a^3*((b*x^2+a)^2)^(1/2)/x^15/(b*x^2+a)-3/13*a^2*b*((b*x^2+a)^2)^(1/2
)/x^13/(b*x^2+a)-3/11*a*b^2*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-1/9*b^3*((b
*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{16}} dx = -\frac{\sqrt{(a+bx^2)^2}(429a^3+1485a^2bx^2+1755ab^2x^4+715b^3x^6)}{6435x^{15}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
```

output
```
-1/6435*(Sqrt[(a + b*x^2)^2]*(429*a^3 + 1485*a^2*b*x^2 + 1755*a*b^2*x^4 +
715*b^3*x^6))/(x^15*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{16}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3(bx^2+a)^3}{x^{16}}dx}{b^3(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{x^{16}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{x^{16}}+\frac{3ba^2}{x^{14}}+\frac{3b^2a}{x^{12}}+\frac{b^3}{x^{10}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^3}{15x^{15}}-\frac{3a^2b}{13x^{13}}-\frac{3ab^2}{11x^{11}}-\frac{b^3}{9x^9}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
```

output
```
((-1/15*a^3/x^15 - (3*a^2*b)/(13*x^13) - (3*a*b^2)/(11*x^11) - b^3/(9*x^9)
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 10.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{1}{15}a^{3}-\frac{3}{13}a^{2}b\,x^{2}-\frac{3}{11}b^{2}x^{4}a-\frac{1}{9}b^{3}x^{6}\right)}{\left(b\,x^{2}+a\right)x^{15}}$	57
gosper	$-\frac{\left(715b^{3}x^{6}+1755b^{2}x^{4}a+1485a^{2}b\,x^{2}+429a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{6435x^{15}\left(b\,x^{2}+a\right)^{3}}$	58
default	$-\frac{\left(715b^{3}x^{6}+1755b^{2}x^{4}a+1485a^{2}b\,x^{2}+429a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{6435x^{15}\left(b\,x^{2}+a\right)^{3}}$	58
orering	$-\frac{\left(715b^{3}x^{6}+1755b^{2}x^{4}a+1485a^{2}b\,x^{2}+429a^{3}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{6435x^{15}\left(b\,x^{2}+a\right)^{3}}$	67

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*(-1/15*a^3-3/13*a^2*b*x^2-3/11*b^2*x^4*a-1/9
*b^3*x^6)/x^15
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{16}}\,dx = -\frac{715\,b^3x^6+1755\,ab^2x^4+1485\,a^2bx^2+429\,a^3}{6435\,x^{15}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="fricas")
```

output
```
-1/6435*(715*b^3*x^6 + 1755*a*b^2*x^4 + 1485*a^2*b*x^2 + 429*a^3)/x^15
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{16}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**16,x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/x**16, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.21

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{16}}\,dx = -\frac{b^3}{9\,x^9}-\frac{3\,ab^2}{11\,x^{11}}-\frac{3\,a^2b}{13\,x^{13}}-\frac{a^3}{15\,x^{15}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="maxima")
```

output
```
-1/9*b^3/x^9 - 3/11*a*b^2/x^11 - 3/13*a^2*b/x^13 - 1/15*a^3/x^15
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{16}}\,dx = -\frac{715\,b^3x^6\mathrm{sgn}(bx^2 + a) + 1755\,ab^2x^4\mathrm{sgn}(bx^2 + a) + 1485\,a^2bx^2\mathrm{sgn}(bx^2 + a) + 429\,a^3\mathrm{sgn}(bx^2 + a)}{6435\,x^{15}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x, algorithm="giac")
```

output
```
-1/6435*(715*b^3*x^6*sgn(b*x^2 + a) + 1755*a*b^2*x^4*sgn(b*x^2 + a) + 1485
*a^2*b*x^2*sgn(b*x^2 + a) + 429*a^3*sgn(b*x^2 + a))/x^15
```

Mupad [B] (verification not implemented)

Time = 17.78 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{16}}\,dx = -\frac{a^3\sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{15\,x^{15}\,(b\,x^2 + a)} - \frac{b^3\sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{9\,x^9\,(b\,x^2 + a)} - \frac{3\,a\,b^2\sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{11\,x^{11}\,(b\,x^2 + a)} - \frac{3\,a^2\,b\sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{13\,x^{13}\,(b\,x^2 + a)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/x^16,x)
```

output
```
- (a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(15*x^15*(a + b*x^2)) - (b^3*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (3*a*b^2*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (3*a^2*b*(a^2 + b^2*x^4 +
2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{x^{16}}\, dx = \frac{-715b^3x^6 - 1755a\, b^2x^4 - 1485a^2b\, x^2 - 429a^3}{6435x^{15}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x)
```

output
```
( - 429*a**3 - 1485*a**2*b*x**2 - 1755*a*b**2*x**4 - 715*b**3*x**6)/(6435*
x**15)
```

3.532 $\int x^{13}(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\,(a+bx^2)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\,(a+bx^2)}+\frac{5a^3b^2x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{2\,(a+bx^2)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\,(a+bx^2)}+\frac{b^5x^{24}\sqrt{a^2+2abx^2+b^2x^4}}{24\,(a+bx^2)}$$

output
```
a^5*x^14*((b*x^2+a)^2)^(1/2)/(14*b*x^2+14*a)+5*a^4*b*x^16*((b*x^2+a)^2)^(1
/2)/(16*b*x^2+16*a)+5*a^3*b^2*x^18*((b*x^2+a)^2)^(1/2)/(9*b*x^2+9*a)+a^2*b
^3*x^20*((b*x^2+a)^2)^(1/2)/(2*b*x^2+2*a)+5*a*b^4*x^22*((b*x^2+a)^2)^(1/2)
/(22*b*x^2+22*a)+b^5*x^24*((b*x^2+a)^2)^(1/2)/(24*b*x^2+24*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^{14}\sqrt{(a+bx^2)^2}(792a^5+3465a^4bx^2+6160a^3b^2x^4+5544a^2b^3x^6+2520ab^4x^8+462b^5x^{10})}{11088\,(a+bx^2)}$$

input
```
Integrate[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^14*Sqrt[(a + b*x^2)^2]*(792*a^5 + 3465*a^4*b*x^2 + 6160*a^3*b^2*x^4 + 5
544*a^2*b^3*x^6 + 2520*a*b^4*x^8 + 462*b^5*x^10))/(11088*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5x^{13}\left(bx^2+a\right)^5dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^{13}\left(bx^2+a\right)^5dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^{12}\left(bx^2+a\right)^5dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(b^5x^{22}+5ab^4x^{20}+10a^2b^3x^{18}+10a^3b^2x^{16}+5a^4bx^{14}+a^5x^{12}\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^5x^{14}}{7}+\frac{5}{8}a^4bx^{16}+\frac{10}{9}a^3b^2x^{18}+a^2b^3x^{20}+\frac{5}{11}ab^4x^{22}+\frac{b^5x^{24}}{12}\right)}{2\left(a+bx^2\right)}$$

input
```
Int[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^5*x^14)/7 + (5*a^4*b*x^16)/8 + (10*a^
3*b^2*x^18)/9 + a^2*b^3*x^20 + (5*a*b^4*x^22)/11 + (b^5*x^24)/12))/(2*(a +
 b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.43 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26

method	result
pseudoelliptic	$\frac{x^{14}\left(\frac{7}{12}x^{10}b^{5}+\frac{35}{11}a\,x^{8}b^{4}+7a^{2}x^{6}b^{3}+\frac{70}{9}a^{3}x^{4}b^{2}+\frac{35}{8}x^{2}a^{4}b+a^{5}\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{14}$
gosper	$\frac{x^{14}\left(462x^{10}b^{5}+2520a\,x^{8}b^{4}+5544a^{2}x^{6}b^{3}+6160a^{3}x^{4}b^{2}+3465x^{2}a^{4}b+792a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{11088\left(b\,x^{2}+a\right)^{5}}$
default	$\frac{x^{14}\left(462x^{10}b^{5}+2520a\,x^{8}b^{4}+5544a^{2}x^{6}b^{3}+6160a^{3}x^{4}b^{2}+3465x^{2}a^{4}b+792a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{11088\left(b\,x^{2}+a\right)^{5}}$
orering	$\frac{x^{14}\left(462x^{10}b^{5}+2520a\,x^{8}b^{4}+5544a^{2}x^{6}b^{3}+6160a^{3}x^{4}b^{2}+3465x^{2}a^{4}b+792a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{11088\left(b\,x^{2}+a\right)^{5}}$
risch	$\frac{b^{5}x^{24}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{24b\,x^{2}+24a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{3}b^{2}x^{18}}{9\left(b\,x^{2}+a\right)}+\frac{a^{5}x^{14}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{14b\,x^{2}+14a}+\frac{a^{2}b^{3}x^{20}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{2b\,x^{2}+2a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b\,a^{4}x^{16}}{16\left(b\,x^{2}+a\right)}+$

input
```
int(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/14*x^14*(7/12*x^10*b^5+35/11*a*x^8*b^4+7*a^2*x^6*b^3+70/9*a^3*x^4*b^2+35
/8*x^2*a^4*b+a^5)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{13}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{5/2}\,dx=\frac{1}{24}\,b^{5}x^{24}+\frac{5}{22}\,ab^{4}x^{22}+\frac{1}{2}\,a^{2}b^{3}x^{20}+\frac{5}{9}\,a^{3}b^{2}x^{18}+\frac{5}{16}\,a^{4}bx^{16}+\frac{1}{14}\,a^{5}x^{14}$$

input
```
integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/
16*a^4*b*x^16 + 1/14*a^5*x^14
```

Sympy [F]

$$\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^{13}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**13*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**13*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{24}\,b^5x^{24}+\frac{5}{22}\,ab^4x^{22}+\frac{1}{2}\,a^2b^3x^{20}+\frac{5}{9}\,a^3b^2x^{18}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{14}\,a^5x^{14}$$

input
```
integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/
16*a^4*b*x^16 + 1/14*a^5*x^14
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{24}\,b^5x^{24}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{22}\,ab^4x^{22}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{2}\,a^2b^3x^{20}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,a^3b^2x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{16}\,a^4bx^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{14}\,a^5x^{14}\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/24*b^5*x^24*sgn(b*x^2 + a) + 5/22*a*b^4*x^22*sgn(b*x^2 + a) + 1/2*a^2*b^
3*x^20*sgn(b*x^2 + a) + 5/9*a^3*b^2*x^18*sgn(b*x^2 + a) + 5/16*a^4*b*x^16*
sgn(b*x^2 + a) + 1/14*a^5*x^14*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^{13}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

input
```
int(x^13*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^13*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^{14}\left(462b^5x^{10}+2520a\,b^4x^8+5544a^2b^3x^6+6160a^3b^2x^4+3465a^4b\,x^2+792a^5\right)}{11088}$$

input
```
int(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**14*(792*a**5 + 3465*a**4*b*x**2 + 6160*a**3*b**2*x**4 + 5544*a**2*b**3
*x**6 + 2520*a*b**4*x**8 + 462*b**5*x**10))/11088
```

3.533 $\int x^{11}(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{12\,(a+bx^2)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\,(a+bx^2)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{8\,(a+bx^2)}+\frac{5a^2b^3x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{4\,(a+bx^2)}+\frac{b^5x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22\,(a+bx^2)}$$

output
```
a^5*x^12*((b*x^2+a)^2)^(1/2)/(12*b*x^2+12*a)+5*a^4*b*x^14*((b*x^2+a)^2)^(1
/2)/(14*b*x^2+14*a)+5*a^3*b^2*x^16*((b*x^2+a)^2)^(1/2)/(8*b*x^2+8*a)+5*a^2
*b^3*x^18*((b*x^2+a)^2)^(1/2)/(9*b*x^2+9*a)+a*b^4*x^20*((b*x^2+a)^2)^(1/2)
/(4*b*x^2+4*a)+b^5*x^22*((b*x^2+a)^2)^(1/2)/(22*b*x^2+22*a)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.53

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^{12}(462a^5+1980a^4bx^2+3465a^3b^2x^4+3080a^2b^3x^6+1386ab^4x^8+252b^5x^{10})\left(\sqrt{a^2}bx^2+a\right.}{5544\left(-a^2-abx^2+\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}$$

input
```
Integrate[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^12*(462*a^5 + 1980*a^4*b*x^2 + 3465*a^3*b^2*x^4 + 3080*a^2*b^3*x^6 + 13
86*a*b^4*x^8 + 252*b^5*x^10)*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b
*x^2)^2])))/(5544*(-a^2 - a*b*x^2 + Sqrt[a^2]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5x^{11}\left(bx^2+a\right)^5dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^{11}\left(bx^2+a\right)^5dx}{a+bx^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^{10}\left(bx^2+a\right)^5dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^5x^{20}+5ab^4x^{18}+10a^2b^3x^{16}+10a^3b^2x^{14}+5a^4bx^{12}+a^5x^{10}\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^5x^{12}}{6}+\frac{5}{7}a^4bx^{14}+\frac{5}{4}a^3b^2x^{16}+\frac{10}{9}a^2b^3x^{18}+\frac{1}{2}ab^4x^{20}+\frac{b^5x^{22}}{11}\right)}{2\left(a+bx^2\right)}$$

input
```
Int[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^5*x^12)/6 + (5*a^4*b*x^14)/7 + (5*a^3
*b^2*x^16)/4 + (10*a^2*b^3*x^18)/9 + (a*b^4*x^20)/2 + (b^5*x^22)/11))/(2*(
a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26

method	result
pseudoelliptic	$\frac{x^{12}\left(\frac{6}{11}x^{10}b^{5}+3a\,x^{8}b^{4}+\frac{20}{3}a^{2}x^{6}b^{3}+\frac{15}{2}a^{3}x^{4}b^{2}+\frac{30}{7}x^{2}a^{4}b+a^{5}\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{12}$
gosper	$\frac{x^{12}\left(252x^{10}b^{5}+1386a\,x^{8}b^{4}+3080a^{2}x^{6}b^{3}+3465a^{3}x^{4}b^{2}+1980x^{2}a^{4}b+462a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{5544\left(b\,x^{2}+a\right)^{5}}$
default	$\frac{x^{12}\left(252x^{10}b^{5}+1386a\,x^{8}b^{4}+3080a^{2}x^{6}b^{3}+3465a^{3}x^{4}b^{2}+1980x^{2}a^{4}b+462a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{5544\left(b\,x^{2}+a\right)^{5}}$
orering	$\frac{x^{12}\left(252x^{10}b^{5}+1386a\,x^{8}b^{4}+3080a^{2}x^{6}b^{3}+3465a^{3}x^{4}b^{2}+1980x^{2}a^{4}b+462a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{5544\left(b\,x^{2}+a\right)^{5}}$
risch	$\frac{a^{5}x^{12}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{12b\,x^{2}+12a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b\,a^{4}x^{14}}{14\left(b\,x^{2}+a\right)}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{3}b^{2}x^{16}}{8\left(b\,x^{2}+a\right)}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}b^{3}x^{18}}{9\left(b\,x^{2}+a\right)}+\frac{a\,b^{4}x^{20}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{4b\,x^{2}+4a}$

input
```
int(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^12*(6/11*x^10*b^5+3*a*x^8*b^4+20/3*a^2*x^6*b^3+15/2*a^3*x^4*b^2+30/
7*x^2*a^4*b+a^5)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

input
```
integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/1
4*a^4*b*x^14 + 1/12*a^5*x^12
```

Sympy [F]

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^{11}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**11*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**11*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

input
```
integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/1
4*a^4*b*x^14 + 1/12*a^5*x^12
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{22}\,b^5x^{22}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{4}\,ab^4x^{20}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{9}\,a^2b^3x^{18}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{8}\,a^3b^2x^{16}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{14}\,a^4bx^{14}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{12}\,a^5x^{12}\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/22*b^5*x^22*sgn(b*x^2 + a) + 1/4*a*b^4*x^20*sgn(b*x^2 + a) + 5/9*a^2*b^3
*x^18*sgn(b*x^2 + a) + 5/8*a^3*b^2*x^16*sgn(b*x^2 + a) + 5/14*a^4*b*x^14*s
gn(b*x^2 + a) + 1/12*a^5*x^12*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^{11}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

input
```
int(x^11*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^11*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^{12}(252b^5x^{10}+1386a\,b^4x^8+3080a^2b^3x^6+3465a^3b^2x^4+1980a^4b\,x^2+462a^5)}{5544}$$

input
```
int(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**12*(462*a**5 + 1980*a**4*b*x**2 + 3465*a**3*b**2*x**4 + 3080*a**2*b**3
*x**6 + 1386*a*b**4*x**8 + 252*b**5*x**10))/5544
```

3.534 $\int x^9(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^9\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{a^5x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{5a^4bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{12\left(a+bx^2\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5ab^4x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{18\left(a+bx^2\right)}+\frac{b^5x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{20\left(a+bx^2\right)}$$

output
```
a^5*x^10*((b*x^2+a)^2)^(1/2)/(10*b*x^2+10*a)+5*a^4*b*x^12*((b*x^2+a)^2)^(1
/2)/(12*b*x^2+12*a)+5*a^3*b^2*x^14*((b*x^2+a)^2)^(1/2)/(7*b*x^2+7*a)+5*a^2
*b^3*x^16*((b*x^2+a)^2)^(1/2)/(8*b*x^2+8*a)+5*a*b^4*x^18*((b*x^2+a)^2)^(1/
2)/(18*b*x^2+18*a)+b^5*x^20*((b*x^2+a)^2)^(1/2)/(20*b*x^2+20*a)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.53

$$\int x^9 (a^2 + 2abx^2 + b^2x^4)^{5/2} dx = \frac{x^{10}(252a^5 + 1050a^4bx^2 + 1800a^3b^2x^4 + 1575a^2b^3x^6 + 700ab^4x^8 + 126b^5x^{10}) \left(\sqrt{a^2}bx^2 + a\left(\right.\right.}{2520 \left(-a^2 - abx^2 + \sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}$$

input
```
Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^10*(252*a^5 + 1050*a^4*b*x^2 + 1800*a^3*b^2*x^4 + 1575*a^2*b^3*x^6 + 70
0*a*b^4*x^8 + 126*b^5*x^10)*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b*
x^2)^2])))/(2520*(-a^2 - a*b*x^2 + Sqrt[a^2]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 (a^2 + 2abx^2 + b^2x^4)^{5/2} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int b^5x^9 (bx^2 + a)^5 dx}{b^5 (a + bx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int x^9 (bx^2 + a)^5 dx}{a + bx^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^8\left(bx^2+a\right)^5dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^5x^{18}+5ab^4x^{16}+10a^2b^3x^{14}+10a^3b^2x^{12}+5a^4bx^{10}+a^5x^8\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^5x^{10}}{5}+\frac{5}{6}a^4bx^{12}+\frac{10}{7}a^3b^2x^{14}+\frac{5}{4}a^2b^3x^{16}+\frac{5}{9}ab^4x^{18}+\frac{b^5x^{20}}{10}\right)}{2\left(a+bx^2\right)}$$

input
```
Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^5*x^10)/5 + (5*a^4*b*x^12)/6 + (10*a^
3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/4 + (5*a*b^4*x^18)/9 + (b^5*x^20)/10))/(2
*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.27

method	result
pseudoelliptic	$\frac{x^{10}\left(126x^{10}b^{5}+700a\,x^{8}b^{4}+1575a^{2}x^{6}b^{3}+1800a^{3}x^{4}b^{2}+1050x^{2}a^{4}b+252a^{5}\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{2520}$
gosper	$\frac{x^{10}\left(126x^{10}b^{5}+700a\,x^{8}b^{4}+1575a^{2}x^{6}b^{3}+1800a^{3}x^{4}b^{2}+1050x^{2}a^{4}b+252a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{2520\left(b\,x^{2}+a\right)^{5}}$
default	$\frac{x^{10}\left(126x^{10}b^{5}+700a\,x^{8}b^{4}+1575a^{2}x^{6}b^{3}+1800a^{3}x^{4}b^{2}+1050x^{2}a^{4}b+252a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{2520\left(b\,x^{2}+a\right)^{5}}$
orering	$\frac{x^{10}\left(126x^{10}b^{5}+700a\,x^{8}b^{4}+1575a^{2}x^{6}b^{3}+1800a^{3}x^{4}b^{2}+1050x^{2}a^{4}b+252a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{2520\left(b\,x^{2}+a\right)^{5}}$
risch	$\frac{a^{5}x^{10}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{10b\,x^{2}+10a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b\,a^{4}x^{12}}{12\left(b\,x^{2}+a\right)}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{3}b^{2}x^{14}}{7\left(b\,x^{2}+a\right)}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}b^{3}x^{16}}{8\left(b\,x^{2}+a\right)}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{4}a\,x}{18\left(b\,x^{2}+a\right)}$

input
```
int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2520*x^10*(126*b^5*x^10+700*a*b^4*x^8+1575*a^2*b^3*x^6+1800*a^3*b^2*x^4+
1050*a^4*b*x^2+252*a^5)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{20} b^5 x^{20} + \frac{5}{18} a b^4 x^{18} + \frac{5}{8} a^2 b^3 x^{16} + \frac{5}{7} a^3 b^2 x^{14} + \frac{5}{12} a^4 b x^{12} + \frac{1}{10} a^5 x^{10}$$

input
```
integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/
12*a^4*b*x^12 + 1/10*a^5*x^10
```

Sympy [F]

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \int x^9 \left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}} dx$$

input
```
integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**9*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{20} b^5 x^{20} + \frac{5}{18} a b^4 x^{18} + \frac{5}{8} a^2 b^3 x^{16} + \frac{5}{7} a^3 b^2 x^{14} + \frac{5}{12} a^4 b x^{12} + \frac{1}{10} a^5 x^{10}$$

input
```
integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/
12*a^4*b*x^12 + 1/10*a^5*x^10
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{20} b^5 x^{20} \text{sgn}\left(bx^2+a\right) + \frac{5}{18} ab^4 x^{18} \text{sgn}\left(bx^2+a\right) + \frac{5}{8} a^2 b^3 x^{16} \text{sgn}\left(bx^2+a\right) + \frac{5}{7} a^3 b^2 x^{14} \text{sgn}\left(bx^2+a\right) + \frac{5}{12} a^4 b x^{12} \text{sgn}\left(bx^2+a\right) + \frac{1}{10} a^5 x^{10} \text{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/20*b^5*x^20*sgn(b*x^2 + a) + 5/18*a*b^4*x^18*sgn(b*x^2 + a) + 5/8*a^2*b^
3*x^16*sgn(b*x^2 + a) + 5/7*a^3*b^2*x^14*sgn(b*x^2 + a) + 5/12*a^4*b*x^12*
sgn(b*x^2 + a) + 1/10*a^5*x^10*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \int x^9 \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2} dx$$

input
```
int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^9*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^{10}(126b^5x^{10}+700a\,b^4x^8+1575a^2b^3x^6+1800a^3b^2x^4+1050a^4b\,x^2+252a^5)}{2520}$$

input
```
int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**10*(252*a**5 + 1050*a**4*b*x**2 + 1800*a**3*b**2*x**4 + 1575*a**2*b**3
*x**6 + 700*a*b**4*x**8 + 126*b**5*x**10))/2520
```

3.535 $\int x^7(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 160

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=-\frac{a^3(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^4}+\frac{3a^2(a+bx^2)^6\sqrt{a^2+2abx^2+b^2x^4}}{14b^4}-\frac{3a(a+bx^2)^7\sqrt{a^2+2abx^2+b^2x^4}}{16b^4}+\frac{(a+bx^2)^8\sqrt{a^2+2abx^2+b^2x^4}}{18b^4}$$

output
```
-1/12*a^3*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/b^4+3/14*a^2*(b*x^2+a)^6*((b*x^2
+a)^2)^(1/2)/b^4-3/16*a*(b*x^2+a)^7*((b*x^2+a)^2)^(1/2)/b^4+1/18*(b*x^2+a)
^8*((b*x^2+a)^2)^(1/2)/b^4
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^8(126a^5+504a^4bx^2+840a^3b^2x^4+720a^2b^3x^6+315ab^4x^8+56b^5x^{10})\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}\right.\right.}{1008\left(-a^2-abx^2+\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}$$

input
```
Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^8*(126*a^5 + 504*a^4*b*x^2 + 840*a^3*b^2*x^4 + 720*a^2*b^3*x^6 + 315*a*
b^4*x^8 + 56*b^5*x^10)*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^
2])))/(1008*(-a^2 - a*b*x^2 + Sqrt[a^2]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5x^7\left(bx^2+a\right)^5dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^7\left(bx^2+a\right)^5dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^6\left(bx^2+a\right)^5dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{\left(bx^2+a\right)^8}{b^3}-\frac{3a\left(bx^2+a\right)^7}{b^3}+\frac{3a^2\left(bx^2+a\right)^6}{b^3}-\frac{a^3\left(bx^2+a\right)^5}{b^3}\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^3(a+bx^2)^6}{6b^4}+\frac{3a^2(a+bx^2)^7}{7b^4}+\frac{(a+bx^2)^9}{9b^4}-\frac{3a(a+bx^2)^8}{8b^4}\right)}{2(a+bx^2)}$$

input
```
Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/6*(a^3*(a + b*x^2)^6)/b^4 + (3*a^2*(a
 + b*x^2)^7)/(7*b^4) - (3*a*(a + b*x^2)^8)/(8*b^4) + (a + b*x^2)^9/(9*b^4)
))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.41

method	result
pseudoelliptic	$\frac{x^{8}\left(\frac{4}{9}x^{10}b^{5}+\frac{5}{2}a\,x^{8}b^{4}+\frac{40}{7}a^{2}x^{6}b^{3}+\frac{20}{3}a^{3}x^{4}b^{2}+4x^{2}a^{4}b+a^{5}\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{8}$
gosper	$\frac{x^{8}\left(56x^{10}b^{5}+315a\,x^{8}b^{4}+720a^{2}x^{6}b^{3}+840a^{3}x^{4}b^{2}+504x^{2}a^{4}b+126a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{1008\left(b\,x^{2}+a\right)^{5}}$
default	$\frac{x^{8}\left(56x^{10}b^{5}+315a\,x^{8}b^{4}+720a^{2}x^{6}b^{3}+840a^{3}x^{4}b^{2}+504x^{2}a^{4}b+126a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{1008\left(b\,x^{2}+a\right)^{5}}$
orering	$\frac{x^{8}\left(56x^{10}b^{5}+315a\,x^{8}b^{4}+720a^{2}x^{6}b^{3}+840a^{3}x^{4}b^{2}+504x^{2}a^{4}b+126a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{1008\left(b\,x^{2}+a\right)^{5}}$
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{5}x^{8}}{8b\,x^{2}+8a}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b\,a^{4}x^{10}}{2b\,x^{2}+2a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{3}b^{2}x^{12}}{6\left(b\,x^{2}+a\right)}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}b^{3}x^{14}}{7\left(b\,x^{2}+a\right)}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{4}a\,x^{16}}{16\left(b\,x^{2}+a\right)}$

input
```
int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^8*(4/9*x^10*b^5+5/2*a*x^8*b^4+40/7*a^2*x^6*b^3+20/3*a^3*x^4*b^2+4*x^
2*a^4*b+a^5)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{18}\,b^5x^{18}+\frac{5}{16}\,ab^4x^{16}+\frac{5}{7}\,a^2b^3x^{14}+\frac{5}{6}\,a^3b^2x^{12}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{8}\,a^5x^8$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/
2*a^4*b*x^10 + 1/8*a^5*x^8
```

Sympy [F]

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx = \int x^7 \left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}} \, dx$$

input
```
integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**7*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx = \frac{1}{18}\,b^5x^{18} + \frac{5}{16}\,ab^4x^{16} + \frac{5}{7}\,a^2b^3x^{14} + \frac{5}{6}\,a^3b^2x^{12} + \frac{1}{2}\,a^4bx^{10} + \frac{1}{8}\,a^5x^8$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/
2*a^4*b*x^10 + 1/8*a^5*x^8
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx = \frac{1}{18}\,b^5x^{18}\mathrm{sgn}\left(bx^2+a\right) + \frac{5}{16}\,ab^4x^{16}\mathrm{sgn}\left(bx^2+a\right) + \frac{5}{7}\,a^2b^3x^{14}\mathrm{sgn}\left(bx^2+a\right) + \frac{5}{6}\,a^3b^2x^{12}\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{2}\,a^4bx^{10}\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{8}\,a^5x^8\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/18*b^5*x^18*sgn(b*x^2 + a) + 5/16*a*b^4*x^16*sgn(b*x^2 + a) + 5/7*a^2*b^
3*x^14*sgn(b*x^2 + a) + 5/6*a^3*b^2*x^12*sgn(b*x^2 + a) + 1/2*a^4*b*x^10*s
gn(b*x^2 + a) + 1/8*a^5*x^8*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^7\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

input
```
int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.37

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^8(56b^5x^{10}+315a\,b^4x^8+720a^2b^3x^6+840a^3b^2x^4+504a^4b\,x^2+126a^5)}{1008}$$

input
```
int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**8*(126*a**5 + 504*a**4*b*x**2 + 840*a**3*b**2*x**4 + 720*a**2*b**3*x**
6 + 315*a*b**4*x**8 + 56*b**5*x**10))/1008
```

3.536 $\int x^5(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{a^2(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^3}-\frac{a(a+bx^2)^6\sqrt{a^2+2abx^2+b^2x^4}}{7b^3}+\frac{(a+bx^2)^7\sqrt{a^2+2abx^2+b^2x^4}}{16b^3}$$

output
```
1/12*a^2*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/b^3-1/7*a*(b*x^2+a)^6*((b*x^2+a)^
2)^(1/2)/b^3+1/16*(b*x^2+a)^7*((b*x^2+a)^2)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.13

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^6(56a^5+210a^4bx^2+336a^3b^2x^4+280a^2b^3x^6+120ab^4x^8+21b^5x^{10})\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\right.\right.}{336\left(-a^2-abx^2+\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^6*(56*a^5 + 210*a^4*b*x^2 + 336*a^3*b^2*x^4 + 280*a^2*b^3*x^6 + 120*a*b
^4*x^8 + 21*b^5*x^10)*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2
])))/(336*(-a^2 - a*b*x^2 + Sqrt[a^2]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^{5/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5x^5\left(bx^2+a\right)^5dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^5\left(bx^2+a\right)^5dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^4\left(bx^2+a\right)^5dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{\left(bx^2+a\right)^7}{b^2}-\frac{2a\left(bx^2+a\right)^6}{b^2}+\frac{a^2\left(bx^2+a\right)^5}{b^2}\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^2\left(a+bx^2\right)^6}{6b^3}+\frac{\left(a+bx^2\right)^8}{8b^3}-\frac{2a\left(a+bx^2\right)^7}{7b^3}\right)}{2\left(a+bx^2\right)}$$

input
```
Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^2*(a + b*x^2)^6)/(6*b^3) - (2*a*(a +
b*x^2)^7)/(7*b^3) + (a + b*x^2)^8/(8*b^3)))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$\frac{x^{6}\left(\frac{3}{8}x^{10}b^{5}+\frac{15}{7}a\,x^{8}b^{4}+5a^{2}x^{6}b^{3}+6a^{3}x^{4}b^{2}+\frac{15}{4}x^{2}a^{4}b+a^{5}\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{6}$
gosper	$\frac{x^{6}\left(21x^{10}b^{5}+120a\,x^{8}b^{4}+280a^{2}x^{6}b^{3}+336a^{3}x^{4}b^{2}+210x^{2}a^{4}b+56a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{336\left(b\,x^{2}+a\right)^{5}}$
default	$\frac{x^{6}\left(21x^{10}b^{5}+120a\,x^{8}b^{4}+280a^{2}x^{6}b^{3}+336a^{3}x^{4}b^{2}+210x^{2}a^{4}b+56a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{336\left(b\,x^{2}+a\right)^{5}}$
orering	$\frac{x^{6}\left(21x^{10}b^{5}+120a\,x^{8}b^{4}+280a^{2}x^{6}b^{3}+336a^{3}x^{4}b^{2}+210x^{2}a^{4}b+56a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{336\left(b\,x^{2}+a\right)^{5}}$
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{5}x^{6}}{6b\,x^{2}+6a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b\,a^{4}x^{8}}{8\left(b\,x^{2}+a\right)}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{3}b^{2}x^{10}}{b\,x^{2}+a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}b^{3}x^{12}}{6\left(b\,x^{2}+a\right)}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{4}a\,x^{14}}{14\left(b\,x^{2}+a\right)}$

input
```
int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*(3/8*x^10*b^5+15/7*a*x^8*b^4+5*a^2*x^6*b^3+6*a^3*x^4*b^2+15/4*x^2*
a^4*b+a^5)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.47

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^
4*b*x^8 + 1/6*a^5*x^6
```

Sympy [F]

$$\int x^5 (a^2 + 2abx^2 + b^2x^4)^{5/2} \, dx = \int x^5 \left(\left(a + bx^2\right)^2 \right)^{\frac{5}{2}} \, dx$$

input
```
integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**5*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.47

$$\int x^5 (a^2 + 2abx^2 + b^2x^4)^{5/2} \, dx = \frac{1}{16} b^5x^{16} + \frac{5}{14} ab^4x^{14} + \frac{5}{6} a^2b^3x^{12} + a^3b^2x^{10} + \frac{5}{8} a^4bx^8 + \frac{1}{6} a^5x^6$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^
4*b*x^8 + 1/6*a^5*x^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int x^5 (a^2 + 2abx^2 + b^2x^4)^{5/2} \, dx = \frac{1}{16} b^5x^{16}\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{14} ab^4x^{14}\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{6} a^2b^3x^{12}\mathrm{sgn}\left(bx^2 + a\right) + a^3b^2x^{10}\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{8} a^4bx^8\mathrm{sgn}\left(bx^2 + a\right) + \frac{1}{6} a^5x^6\mathrm{sgn}\left(bx^2 + a\right)$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/16*b^5*x^16*sgn(b*x^2 + a) + 5/14*a*b^4*x^14*sgn(b*x^2 + a) + 5/6*a^2*b^
3*x^12*sgn(b*x^2 + a) + a^3*b^2*x^10*sgn(b*x^2 + a) + 5/8*a^4*b*x^8*sgn(b*
x^2 + a) + 1/6*a^5*x^6*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^5\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

input
```
int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.50

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^6(21b^5x^{10}+120a\,b^4x^8+280a^2b^3x^6+336a^3b^2x^4+210a^4b\,x^2+56a^5)}{336}$$

input
```
int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**6*(56*a**5 + 210*a**4*b*x**2 + 336*a**3*b**2*x**4 + 280*a**2*b**3*x**6
 + 120*a*b**4*x**8 + 21*b**5*x**10))/336
```

3.537 $\int x^3(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = -\frac{a(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b^2}+\frac{(a^2+2abx^2+b^2x^4)^{7/2}}{14b^2}$$

output
```
-1/12*a*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/b^2+1/14*(b^2*x^4+2*a*b*x^2+a^2)^(
7/2)/b^2
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.96

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{x^4(21a^5+70a^4bx^2+105a^3b^2x^4+84a^2b^3x^6+35ab^4x^8+6b^5x^{10})\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(\right.\right.}{84\left(-a^2-abx^2+\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^4*(21*a^5 + 70*a^4*b*x^2 + 105*a^3*b^2*x^4 + 84*a^2*b^3*x^6 + 35*a*b^4*
x^8 + 6*b^5*x^10)*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])))
/(84*(-a^2 - a*b*x^2 + Sqrt[a^2]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1434, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int x^2 \left(b^2x^4+2abx^2+a^2\right)^{5/2} dx^2$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{2} \left(\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{7b^2}-\frac{a\int \left(b^2x^4+2abx^2+a^2\right)^{5/2} dx^2}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{2} \left(\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{7b^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^2x^2+ab\right)^5 dx^2}{b^6\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{2} \left(\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{7b^2}-\frac{a\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{6b^2}\right)$$

input
```
Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(-1/6*(a*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/b^2 + (a^2 + 2*a*b
*x^2 + b^2*x^4)^(7/2)/(7*b^2))/2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{x^{4} \left(\frac{2}{7} x^{10} b^{5}+\frac{5}{3} a\,x^{8} b^{4}+4 a^{2} x^{6} b^{3}+5 a^{3} x^{4} b^{2}+\frac{10}{3} x^{2} a^{4} b +a^{5}\right) \operatorname{csgn}\left(b\,x^{2}+a \right)}{4}$
gosper	$\frac{x^{4} \left(6 x^{10} b^{5}+35 a\,x^{8} b^{4}+84 a^{2} x^{6} b^{3}+105 a^{3} x^{4} b^{2}+70 x^{2} a^{4} b +21 a^{5}\right) \left(\left(b\,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}{84 \left(b\,x^{2}+a \right)^{5}}$
default	$\frac{x^{4} \left(6 x^{10} b^{5}+35 a\,x^{8} b^{4}+84 a^{2} x^{6} b^{3}+105 a^{3} x^{4} b^{2}+70 x^{2} a^{4} b +21 a^{5}\right) \left(\left(b\,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}{84 \left(b\,x^{2}+a \right)^{5}}$
orering	$\frac{x^{4} \left(6 x^{10} b^{5}+35 a\,x^{8} b^{4}+84 a^{2} x^{6} b^{3}+105 a^{3} x^{4} b^{2}+70 x^{2} a^{4} b +21 a^{5}\right) \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{84 \left(b\,x^{2}+a \right)^{5}}$
risch	$\frac{\sqrt{\left(b\,x^{2}+a \right)^{2}}\, a^{5} x^{4}}{4 b\,x^{2}+4 a}+\frac{5 \sqrt{\left(b\,x^{2}+a \right)^{2}}\, b\,a^{4} x^{6}}{6 \left(b\,x^{2}+a \right)}+\frac{5 \sqrt{\left(b\,x^{2}+a \right)^{2}}\, a^{3} b^{2} x^{8}}{4 \left(b\,x^{2}+a \right)}+\frac{\sqrt{\left(b\,x^{2}+a \right)^{2}}\, a^{2} b^{3} x^{10}}{b\,x^{2}+a}+\frac{5 \sqrt{\left(b\,x^{2}+a \right)^{2}}\, b^{4} a\,x^{12}}{12 \left(b\,x^{2}+a \right)}$

input
```
int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*(2/7*x^10*b^5+5/3*a*x^8*b^4+4*a^2*x^6*b^3+5*a^3*x^4*b^2+10/3*x^2*a
^4*b+a^5)*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{14}\,b^5x^{14}+\frac{5}{12}\,ab^4x^{12}+a^2b^3x^{10}+\frac{5}{4}\,a^3b^2x^8+\frac{5}{6}\,a^4bx^6+\frac{1}{4}\,a^5x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4
*b*x^6 + 1/4*a^5*x^4
```

Sympy [F]

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^3\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**3*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{14}\,b^5x^{14}+\frac{5}{12}\,ab^4x^{12}+a^2b^3x^{10}+\frac{5}{4}\,a^3b^2x^8+\frac{5}{6}\,a^4bx^6+\frac{1}{4}\,a^5x^4$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4
*b*x^6 + 1/4*a^5*x^4
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{84}\left(6\,b^5x^{14}+35\,ab^4x^{12}+84\,a^2b^3x^{10}+105\,a^3b^2x^8+70\,a^4bx^6+21\,a^5x^4\right)\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/84*(6*b^5*x^14 + 35*a*b^4*x^12 + 84*a^2*b^3*x^10 + 105*a^3*b^2*x^8 + 70*
a^4*b*x^6 + 21*a^5*x^4)*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^3\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

input
```
int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^4(6b^5x^{10}+35a\,b^4x^8+84a^2b^3x^6+105a^3b^2x^4+70a^4b\,x^2+21a^5)}{84}$$

input
```
int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**4*(21*a**5 + 70*a**4*b*x**2 + 105*a**3*b**2*x**4 + 84*a**2*b**3*x**6 +
 35*a*b**4*x**8 + 6*b**5*x**10))/84
```

3.538 $\int x(a^2 + 2abx^2 + b^2x^4)^{5/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b}$$

output
```
1/12*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{(a+bx^2)\left((a+bx^2)^2\right)^{5/2}}{12b}$$

input
```
Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*((a + b*x^2)^2)^(5/2))/(12*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1432, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{5/2}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \left(b^2x^4+2abx^2+a^2\right)^{5/2}dx^2$$

$$\Big\downarrow \text{1079}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^2x^2+ab\right)^5dx^2}{2b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12b}$$

input
```
Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}\left(b\,x^{2}+a\right)}{12b}$	24
risch	$\frac{\left(b\,x^{2}+a\right)^{5}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{12b}$	26
pseudoelliptic	$\frac{x^{2}\left(x^{10}b^{5}+6a\,x^{8}b^{4}+15a^{2}x^{6}b^{3}+20a^{3}x^{4}b^{2}+15x^{2}a^{4}b+6a^{5}\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{12}$	67
gosper	$\frac{x^{2}\left(x^{10}b^{5}+6a\,x^{8}b^{4}+15a^{2}x^{6}b^{3}+20a^{3}x^{4}b^{2}+15x^{2}a^{4}b+6a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{12\left(b\,x^{2}+a\right)^{5}}$	79
orering	$\frac{x^{2}\left(x^{10}b^{5}+6a\,x^{8}b^{4}+15a^{2}x^{6}b^{3}+20a^{3}x^{4}b^{2}+15x^{2}a^{4}b+6a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{12\left(b\,x^{2}+a\right)^{5}}$	88

input
```
int(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*((b*x^2+a)^2)^(5/2)*(b*x^2+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{12}\,b^5x^{12}+\frac{1}{2}\,ab^4x^{10}+\frac{5}{4}\,a^2b^3x^8+\frac{5}{3}\,a^3b^2x^6+\frac{5}{4}\,a^4bx^4+\frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12*b^5*x^12 + 1/2*a*b^4*x^10 + 5/4*a^2*b^3*x^8 + 5/3*a^3*b^2*x^6 + 5/4*a
^4*b*x^4 + 1/2*a^5*x^2
```

Sympy [F]

$$\int x(a^2+2abx^2+b^2x^4)^{5/2}\,dx=\int x\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x*((a + b*x**2)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int x(a^2+2abx^2+b^2x^4)^{5/2}\,dx=\frac{1}{12}\,b^5x^{12}+\frac{1}{2}\,ab^4x^{10}+\frac{5}{4}\,a^2b^3x^8+\frac{5}{3}\,a^3b^2x^6+\frac{5}{4}\,a^4bx^4+\frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/12*b^5*x^12 + 1/2*a*b^4*x^10 + 5/4*a^2*b^3*x^8 + 5/3*a^3*b^2*x^6 + 5/4*a
^4*b*x^4 + 1/2*a^5*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{12}\left(3\left(bx^4+2\,ax^2\right)a^4+3\left(bx^4+2\,ax^2\right)^2a^2b+\left(bx^4+2\,ax^2\right)^3b^2\right)\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/12*(3*(b*x^4 + 2*a*x^2)*a^4 + 3*(b*x^4 + 2*a*x^2)^2*a^2*b + (b*x^4 + 2*a
*x^2)^3*b^2)*sgn(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 17.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\left(b^2\,x^2+a\,b\right)\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{12\,b^2}$$

input
```
int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
((a*b + b^2*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2))/(12*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int x\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^2(b^5x^{10}+6a\,b^4x^8+15a^2b^3x^6+20a^3b^2x^4+15a^4b\,x^2+6a^5)}{12}$$

input
```
int(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**2*(6*a**5 + 15*a**4*b*x**2 + 20*a**3*b**2*x**4 + 15*a**2*b**3*x**6 + 6
*a*b**4*x**8 + b**5*x**10))/12
```

3.539 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x}\,dx=\frac{5a^4bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{5ab^4x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{b^5x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

output
```
5*a^4*b*x^2*((b*x^2+a)^2)^(1/2)/(2*b*x^2+2*a)+5*a^3*b^2*x^4*((b*x^2+a)^2)^
(1/2)/(2*b*x^2+2*a)+5*a^2*b^3*x^6*((b*x^2+a)^2)^(1/2)/(3*b*x^2+3*a)+5*a*b^
4*x^8*((b*x^2+a)^2)^(1/2)/(8*b*x^2+8*a)+b^5*x^10*((b*x^2+a)^2)^(1/2)/(10*b
*x^2+10*a)+a^5*((b*x^2+a)^2)^(1/2)*ln(x)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x}\,dx = \frac{\sqrt{(a+bx^2)^2}\,(bx^2(300a^4+300a^3bx^2+200a^2b^2x^4+75ab^3x^6+12b^4x^8)+12}{120\,(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(b*x^2*(300*a^4 + 300*a^3*b*x^2 + 200*a^2*b^2*x^4 + 7
5*a*b^3*x^6 + 12*b^4*x^8) + 120*a^5*Log[x]))/(120*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x}dx}{a+bx^2}$$

$$\Big\downarrow \; 243$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^2}dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \; 49$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(b^5x^8+5ab^4x^6+10a^2b^3x^4+10a^3b^2x^2+5a^4b+\frac{a^5}{x^2}\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a^5\log\left(x^2\right)+5a^4bx^2+5a^3b^2x^4+\frac{10}{3}a^2b^3x^6+\frac{5}{4}ab^4x^8+\frac{b^5x^{10}}{5}\right)}{2\left(a+bx^2\right)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(5*a^4*b*x^2 + 5*a^3*b^2*x^4 + (10*a^2*b^
3*x^6)/3 + (5*a*b^4*x^8)/4 + (b^5*x^10)/5 + a^5*Log[x^2]))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
 - 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.27

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(12 x^{10} b^{5}+75 a\,x^{8} b^{4}+200 a^{2} x^{6} b^{3}+300 a^{3} x^{4} b^{2}+60 a^{5} \ln \left(x^{2}\right)+300 x^{2} a^{4} b\right)}{120}$	69
default	$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}} \left(12 x^{10} b^{5}+75 a\,x^{8} b^{4}+200 a^{2} x^{6} b^{3}+300 a^{3} x^{4} b^{2}+300 x^{2} a^{4} b+120 a^{5} \ln \left(x\right)\right)}{120 \left(b\,x^{2}+a\right)^{5}}$	79
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, b\left(\frac{1}{10} b^{4} x^{10}+\frac{5}{8} a\,b^{3} x^{8}+\frac{5}{3} a^{2} b^{2} x^{6}+\frac{5}{2} a^{3} b\,x^{4}+\frac{5}{2} a^{4} x^{2}\right)}{b\,x^{2}+a}+\frac{a^{5} \sqrt{\left(b\,x^{2}+a\right)^{2}}\, \ln \left(x\right)}{b\,x^{2}+a}$	96

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/120*csgn(b*x^2+a)*(12*x^10*b^5+75*a*x^8*b^4+200*a^2*x^6*b^3+300*a^3*x^4*
b^2+60*a^5*ln(x^2)+300*x^2*a^4*b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x}\,dx = \frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8 + \frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+a^5\log\left(x\right)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="fricas")
```

output
```
1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^
4*b*x^2 + a^5*log(x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x} \, dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{5}{2}}}{x} \, dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x} \, dx = \frac{1}{10} \, b^5x^{10} + \frac{5}{8} \, ab^4x^8 + \frac{5}{3} \, a^2b^3x^6 + \frac{5}{2} \, a^3b^2x^4 + \frac{5}{2} \, a^4bx^2 + a^5 \log(x)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="maxima")
```

output
```
1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^
4*b*x^2 + a^5*log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x} \, dx = \frac{1}{10} \, b^5x^{10}\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{8} \, ab^4x^8\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{3} \, a^2b^3x^6\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{2} \, a^3b^2x^4\mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{2} \, a^4bx^2\mathrm{sgn}\left(bx^2 + a\right) + \frac{1}{2} \, a^5 \log\left(x^2\right) \mathrm{sgn}\left(bx^2 + a\right)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x, algorithm="giac")
```

output
```
1/10*b^5*x^10*sgn(b*x^2 + a) + 5/8*a*b^4*x^8*sgn(b*x^2 + a) + 5/3*a^2*b^3*
x^6*sgn(b*x^2 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^2 + a) + 5/2*a^4*b*x^2*sgn(b*
x^2 + a) + 1/2*a^5*log(x^2)*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{5/2}}{x} \, dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x} \, dx = \log(x)\,a^5 + \frac{5a^4b\,x^2}{2} + \frac{5a^3b^2x^4}{2} + \frac{5a^2b^3x^6}{3} + \frac{5a\,b^4x^8}{8} + \frac{b^5x^{10}}{10}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x)
```

output
```
(120*log(x)*a**5 + 300*a**4*b*x**2 + 300*a**3*b**2*x**4 + 200*a**2*b**3*x*
*6 + 75*a*b**4*x**8 + 12*b**5*x**10)/120
```

3.540 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 250

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^3}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)} + \frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{5a^2b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)} + \frac{5ab^4x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)} + \frac{b^5x^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)} + \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

output
```
-1/2*a^5*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+5*a^3*b^2*x^2*((b*x^2+a)^2)^(1/
2)/(b*x^2+a)+5*a^2*b^3*x^4*((b*x^2+a)^2)^(1/2)/(2*b*x^2+2*a)+5*a*b^4*x^6*(
(b*x^2+a)^2)^(1/2)/(6*b*x^2+6*a)+b^5*x^8*((b*x^2+a)^2)^(1/2)/(8*b*x^2+8*a)
+5*a^4*b*((b*x^2+a)^2)^(1/2)*ln(x)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^3}\,dx = \frac{\sqrt{(a+bx^2)^2}(-12a^5+120a^3b^2x^4+60a^2b^3x^6+20ab^4x^8+3b^5x^{10}+120a^4b}{24x^2\,(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(-12*a^5 + 120*a^3*b^2*x^4 + 60*a^2*b^3*x^6 + 20*a*b^
4*x^8 + 3*b^5*x^10 + 120*a^4*b*x^2*Log[x]))/(24*x^2*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^3}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5\left(bx^2+a\right)^5}{x^3}dx}{b^5\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^5}{x^3}dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^5}{x^4}dx^2}{2\,(a+bx^2)}$$

$$\Big\downarrow \ 49$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^5x^6+5ab^4x^4+10a^2b^3x^2+10a^3b^2+\frac{5a^4b}{x^2}+\frac{a^5}{x^4}\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^5}{x^2}+5a^4b\log\left(x^2\right)+10a^3b^2x^2+5a^2b^3x^4+\frac{5}{3}ab^4x^6+\frac{b^5x^8}{4}\right)}{2\left(a+bx^2\right)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-(a^5/x^2) + 10*a^3*b^2*x^2 + 5*a^2*b^3*
x^4 + (5*a*b^4*x^6)/3 + (b^5*x^8)/4 + 5*a^4*b*Log[x^2]))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
 - 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.28

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(-\frac{x^{10} b^{5}}{4}-\frac{5 a\,x^{8} b^{4}}{3}-5 a^{2} x^{6} b^{3}-10 a^{3} x^{4} b^{2}-5 b\,a^{4} \ln\left(x^{2}\right) x^{2}+a^{5}\right)}{2 x^{2}}$	70
default	$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}} \left(3 x^{10} b^{5}+20 a\,x^{8} b^{4}+60 a^{2} x^{6} b^{3}+120 a^{3} x^{4} b^{2}+120 b\,a^{4} \ln\left(x\right) x^{2}-12 a^{5}\right)}{24 x^{2} \left(b\,x^{2}+a\right)^{5}}$	82
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, b^{2} \left(\frac{1}{8} b^{3} x^{8}+\frac{5}{6} b^{2} x^{6} a+\frac{5}{2} a^{2} b\,x^{4}+5 a^{3} x^{2}\right)}{b\,x^{2}+a}-\frac{a^{5} \sqrt{\left(b\,x^{2}+a\right)^{2}}}{2 x^{2} \left(b\,x^{2}+a\right)}+\frac{5 a^{4} b \sqrt{\left(b\,x^{2}+a\right)^{2}}\, \ln\left(x\right)}{b\,x^{2}+a}$	117

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*csgn(b*x^2+a)*(-1/4*x^10*b^5-5/3*a*x^8*b^4-5*a^2*x^6*b^3-10*a^3*x^4*b
^2-5*b*a^4*ln(x^2)*x^2+a^5)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^3}\,dx=\frac{3\,b^5x^{10}+20\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4+120\,a^4bx^2\log\left(x\right)-12\,a^5}{24\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
1/24*(3*b^5*x^10 + 20*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4 + 120*a
^4*b*x^2*log(x) - 12*a^5)/x^2
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^3}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**3,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^3}\,dx = \frac{1}{8}\,b^5x^8 + \frac{5}{6}\,ab^4x^6 + \frac{5}{2}\,a^2b^3x^4 + 5\,a^3b^2x^2 + 5\,a^4b\log(x) - \frac{a^5}{2\,x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5*a^4*b*lo
g(x) - 1/2*a^5/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.50

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^3}\,dx &= \frac{1}{8}\,b^5x^8\mathrm{sgn}\left(bx^2+a\right) \\ &+ \frac{5}{6}\,ab^4x^6\mathrm{sgn}\left(bx^2+a\right) + \frac{5}{2}\,a^2b^3x^4\mathrm{sgn}\left(bx^2+a\right) + 5\,a^3b^2x^2\mathrm{sgn}\left(bx^2+a\right) \\ &+ \frac{5}{2}\,a^4b\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right) - \frac{5\,a^4bx^2\mathrm{sgn}(bx^2+a) + a^5\mathrm{sgn}(bx^2+a)}{2\,x^2}\end{aligned}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x, algorithm="giac")
```

output
```
1/8*b^5*x^8*sgn(b*x^2 + a) + 5/6*a*b^4*x^6*sgn(b*x^2 + a) + 5/2*a^2*b^3*x^
4*sgn(b*x^2 + a) + 5*a^3*b^2*x^2*sgn(b*x^2 + a) + 5/2*a^4*b*log(x^2)*sgn(b
*x^2 + a) - 1/2*(5*a^4*b*x^2*sgn(b*x^2 + a) + a^5*sgn(b*x^2 + a))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^3} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{5/2}}{x^3} \, dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^3,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^3} \, dx = \frac{120 \log(x)\, a^4 b\, x^2 - 12a^5 + 120a^3b^2x^4 + 60a^2b^3x^6 + 20a\, b^4x^8 + 3b^5x^{10}}{24x^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x)
```

output
```
(120*log(x)*a**4*b*x**2 - 12*a**5 + 120*a**3*b**2*x**4 + 60*a**2*b**3*x**6
 + 20*a*b**4*x**8 + 3*b**5*x**10)/(24*x**2)
```

3.541 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 250

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^5}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)} + \frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{5ab^4x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)} + \frac{b^5x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)} + \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

output
```
-1/4*a^5*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)-5/2*a^4*b*((b*x^2+a)^2)^(1/2)/x
^2/(b*x^2+a)+5*a^2*b^3*x^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5*a*b^4*x^4*((b*x
^2+a)^2)^(1/2)/(4*b*x^2+4*a)+b^5*x^6*((b*x^2+a)^2)^(1/2)/(6*b*x^2+6*a)+10*
a^3*b^2*((b*x^2+a)^2)^(1/2)*ln(x)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.10

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^5}\,dx = \frac{(12a^5+120a^4bx^2+57a^3b^2x^4-240a^2b^3x^6-60ab^4x^8-8b^5x^{10})\left(\sqrt{a^2}bx^2+\right.}{48x^4\left(a^2+abx^2-\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}$$
$$-5a^3b^2\text{arctanh}\left(\frac{bx^2}{\sqrt{a^2}-\sqrt{(a+bx^2)^2}}\right)-5(a^2)^{3/2}b^2\log(x^2)$$
$$+\frac{5}{2}(a^2)^{3/2}b^2\log\left(\sqrt{a^2}-bx^2-\sqrt{(a+bx^2)^2}\right)+\frac{5}{2}(a^2)^{3/2}b^2\log\left(\sqrt{a^2}+bx^2-\sqrt{(a+bx^2)^2}\right)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
```

output
```
((12*a^5 + 120*a^4*b*x^2 + 57*a^3*b^2*x^4 - 240*a^2*b^3*x^6 - 60*a*b^4*x^8
 - 8*b^5*x^10)*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])))/(4
8*x^4*(a^2 + a*b*x^2 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2])) - 5*a^3*b^2*ArcTanh
[(b*x^2)/(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])] - 5*(a^2)^(3/2)*b^2*Log[x^2] +
 (5*(a^2)^(3/2)*b^2*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]])/2 + (5*(
a^2)^(3/2)*b^2*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]])/2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^5}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^5}dx}{b^5(a+bx^2)}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^5}dx}{a+bx^2}$$

$\Big\downarrow$ 243

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^6}dx^2}{2(a+bx^2)}$$

$\Big\downarrow$ 49

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5}{x^6}+\frac{5ba^4}{x^4}+\frac{10b^2a^3}{x^2}+10b^3a^2+5b^4x^2a+b^5x^4\right)dx^2}{2(a+bx^2)}$$

$\Big\downarrow$ 2009

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^5}{2x^4}-\frac{5a^4b}{x^2}+10a^3b^2\log(x^2)+10a^2b^3x^2+\frac{5}{2}ab^4x^4+\frac{b^5x^6}{3}\right)}{2(a+bx^2)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/2*a^5/x^4 - (5*a^4*b)/x^2 + 10*a^2*b^
3*x^2 + (5*a*b^4*x^4)/2 + (b^5*x^6)/3 + 10*a^3*b^2*Log[x^2]))/(2*(a + b*x^
2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.28

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(-\frac{2 x^{10} b^{5}}{3}-5 a\,x^{8} b^{4}-20 a^{2} x^{6} b^{3}-20 a^{3} b^{2} \ln\left(x^{2}\right) x^{4}+10 x^{2} a^{4} b+a^{5}\right)}{4 x^{4}}$	70
default	$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}} \left(2 x^{10} b^{5}+15 a\,x^{8} b^{4}+60 a^{2} x^{6} b^{3}+120 \ln\left(x\right) x^{4} a^{3} b^{2}-30 x^{2} a^{4} b-3 a^{5}\right)}{12 x^{4} \left(b\,x^{2}+a\right)^{5}}$	82
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, b^{3} \left(\frac{1}{6} b^{2} x^{6}+\frac{5}{4} a b\,x^{4}+5 a^{2} x^{2}\right)}{b\,x^{2}+a}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, \left(-\frac{5}{2} x^{2} a^{4} b-\frac{1}{4} a^{5}\right)}{\left(b\,x^{2}+a\right) x^{4}}+\frac{10 a^{3} b^{2} \sqrt{\left(b\,x^{2}+a\right)^{2}}\, \ln\left(x\right)}{b\,x^{2}+a}$	119

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*csgn(b*x^2+a)*(-2/3*x^10*b^5-5*a*x^8*b^4-20*a^2*x^6*b^3-20*a^3*b^2*ln
(x^2)*x^4+10*x^2*a^4*b+a^5)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^5}\,dx = \frac{2\,b^5x^{10} + 15\,ab^4x^8 + 60\,a^2b^3x^6 + 120\,a^3b^2x^4\log(x) - 30\,a^4bx^2 - 3\,a^5}{12\,x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
1/12*(2*b^5*x^10 + 15*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4*log(x)
- 30*a^4*b*x^2 - 3*a^5)/x^4
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^5}\,dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**5,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^5}\,dx = \frac{1}{6}\,b^5x^6 + \frac{5}{4}\,ab^4x^4 + 5\,a^2b^3x^2 + 10\,a^3b^2\log(x) - \frac{5\,a^4b}{2\,x^2} - \frac{a^5}{4\,x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 10*a^3*b^2*log(x) - 5/2*a^4*
b/x^2 - 1/4*a^5/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.51

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^5}\,dx = \frac{1}{6}\,b^5x^6\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{4}\,ab^4x^4\mathrm{sgn}\left(bx^2+a\right) + 5\,a^2b^3x^2\mathrm{sgn}\left(bx^2+a\right)+5\,a^3b^2\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right) - \frac{30\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+10\,a^4bx^2\mathrm{sgn}(bx^2+a)+a^5\mathrm{sgn}(bx^2+a)}{4\,x^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x, algorithm="giac")
```

output
```
1/6*b^5*x^6*sgn(b*x^2 + a) + 5/4*a*b^4*x^4*sgn(b*x^2 + a) + 5*a^2*b^3*x^2*
sgn(b*x^2 + a) + 5*a^3*b^2*log(x^2)*sgn(b*x^2 + a) - 1/4*(30*a^3*b^2*x^4*s
gn(b*x^2 + a) + 10*a^4*b*x^2*sgn(b*x^2 + a) + a^5*sgn(b*x^2 + a))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^5}\,dx = \int \frac{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}{x^5}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^5,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^5}\,dx = \frac{120\log(x)\,a^3b^2x^4-3a^5-30a^4b\,x^2+60a^2b^3x^6+15a\,b^4x^8+2b^5x^{10}}{12x^4}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x)
```

output
```
(120*log(x)*a**3*b**2*x**4 - 3*a**5 - 30*a**4*b*x**2 + 60*a**2*b**3*x**6 +
 15*a*b**4*x**8 + 2*b**5*x**10)/(12*x**4)
```

3.542 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^7} dx$

Optimal result

Integrand size = 26, antiderivative size = 250

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^7} dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^2(a+bx^2)} + \frac{5ab^4x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)} + \frac{b^5x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)} + \frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

output
```
-1/6*a^5*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)-5/4*a^4*b*((b*x^2+a)^2)^(1/2)/x
^4/(b*x^2+a)-5*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+5*a*b^4*x^2*((b*x
^2+a)^2)^(1/2)/(2*b*x^2+2*a)+b^5*x^4*((b*x^2+a)^2)^(1/2)/(4*b*x^2+4*a)+10*
a^2*b^3*((b*x^2+a)^2)^(1/2)*ln(x)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.12

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^7}\,dx = \frac{(4a^5+30a^4bx^2+120a^3b^2x^4+53a^2b^3x^6-60ab^4x^8-6b^5x^{10})\left(\sqrt{a^2}bx^2+a\right.}{24x^6\left(a^2+abx^2-\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}$$
$$-5a^2b^3\text{arctanh}\left(\frac{bx^2}{\sqrt{a^2}-\sqrt{(a+bx^2)^2}}\right)-5a\sqrt{a^2}b^3\log(x^2)$$
$$+\frac{5}{2}a\sqrt{a^2}b^3\log\left(\sqrt{a^2}-bx^2-\sqrt{(a+bx^2)^2}\right)+\frac{5}{2}a\sqrt{a^2}b^3\log\left(\sqrt{a^2}+bx^2-\sqrt{(a+bx^2)^2}\right)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```

output
```
((4*a^5 + 30*a^4*b*x^2 + 120*a^3*b^2*x^4 + 53*a^2*b^3*x^6 - 60*a*b^4*x^8 -
 6*b^5*x^10)*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])))/(24*
x^6*(a^2 + a*b*x^2 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2])) - 5*a^2*b^3*ArcTanh[(
b*x^2)/(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])] - 5*a*Sqrt[a^2]*b^3*Log[x^2] + (
5*a*Sqrt[a^2]*b^3*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]])/2 + (5*a*S
qrt[a^2]*b^3*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]])/2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^7}\,dx$$
$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^7}dx}{b^5(a+bx^2)}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^7}dx}{a+bx^2}$$

$\Big\downarrow$ 243

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^8}dx^2}{2(a+bx^2)}$$

$\Big\downarrow$ 49

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5}{x^8}+\frac{5ba^4}{x^6}+\frac{10b^2a^3}{x^4}+\frac{10b^3a^2}{x^2}+5b^4a+b^5x^2\right)dx^2}{2(a+bx^2)}$$

$\Big\downarrow$ 2009

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^5}{3x^6}-\frac{5a^4b}{2x^4}-\frac{10a^3b^2}{x^2}+10a^2b^3\log\left(x^2\right)+5ab^4x^2+\frac{b^5x^4}{2}\right)}{2(a+bx^2)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/3*a^5/x^6 - (5*a^4*b)/(2*x^4) - (10*a
^3*b^2)/x^2 + 5*a*b^4*x^2 + (b^5*x^4)/2 + 10*a^2*b^3*Log[x^2]))/(2*(a + b*
x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.28

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(-\frac{3 x^{10} b^{5}}{2}-15 a\,x^{8} b^{4}-30 a^{2} b^{3} \ln\left(x^{2}\right) x^{6}+30 a^{3} x^{4} b^{2}+\frac{15 x^{2} a^{4} b}{2}+a^{5}\right)}{6 x^{6}}$	70
default	$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}} \left(3 x^{10} b^{5}+30 a\,x^{8} b^{4}+120 \ln\left(x\right) x^{6} a^{2} b^{3}-60 a^{3} x^{4} b^{2}-15 x^{2} a^{4} b-2 a^{5}\right)}{12 x^{6} \left(b\,x^{2}+a\right)^{5}}$	82
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, b^{3} \left(b\,x^{2}+5 a\right)^{2}}{4 b\,x^{2}+4 a}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, \left(-5 a^{3} x^{4} b^{2}-\frac{5}{4} x^{2} a^{4} b-\frac{1}{6} a^{5}\right)}{\left(b\,x^{2}+a\right) x^{6}}+\frac{10 a^{2} b^{3} \sqrt{\left(b\,x^{2}+a\right)^{2}}\, \ln\left(x\right)}{b\,x^{2}+a}$	118

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*csgn(b*x^2+a)*(-3/2*x^10*b^5-15*a*x^8*b^4-30*a^2*b^3*ln(x^2)*x^6+30*a
^3*x^4*b^2+15/2*x^2*a^4*b+a^5)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^7}\,dx = \frac{3\,b^5x^{10}+30\,ab^4x^8+120\,a^2b^3x^6\log\left(x\right)-60\,a^3b^2x^4-15\,a^4bx^2-2\,a^5}{12\,x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
1/12*(3*b^5*x^10 + 30*a*b^4*x^8 + 120*a^2*b^3*x^6*log(x) - 60*a^3*b^2*x^4
- 15*a^4*b*x^2 - 2*a^5)/x^6
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^7}\,dx = \int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**7,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^7}\,dx = \frac{1}{4}\,b^5x^4+\frac{5}{2}\,ab^4x^2+10\,a^2b^3\log\left(x\right)-\frac{5\,a^3b^2}{x^2}-\frac{5\,a^4b}{4\,x^4}-\frac{a^5}{6\,x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 10*a^2*b^3*log(x) - 5*a^3*b^2/x^2 - 5/4*a^4*
b/x^4 - 1/6*a^5/x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.51

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^7} dx = \frac{1}{4} b^5 x^4 \text{sgn}\left(bx^2 + a\right) + \frac{5}{2} ab^4 x^2 \text{sgn}\left(bx^2 + a\right) + 5 a^2 b^3 \log\left(x^2\right) \text{sgn}\left(bx^2 + a\right) - \frac{110 a^2 b^3 x^6 \text{sgn}(bx^2 + a) + 60 a^3 b^2 x^4 \text{sgn}(bx^2 + a) + 15 a^4 b x^2 \text{sgn}(bx^2 + a) + 2 a^5 \text{sgn}(bx^2 + a)}{12 x^6}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x, algorithm="giac")
```

output
```
1/4*b^5*x^4*sgn(b*x^2 + a) + 5/2*a*b^4*x^2*sgn(b*x^2 + a) + 5*a^2*b^3*log(
x^2)*sgn(b*x^2 + a) - 1/12*(110*a^2*b^3*x^6*sgn(b*x^2 + a) + 60*a^3*b^2*x^
4*sgn(b*x^2 + a) + 15*a^4*b*x^2*sgn(b*x^2 + a) + 2*a^5*sgn(b*x^2 + a))/x^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^7} dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{5/2}}{x^7} \,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^7,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^7} \, dx = \frac{120 \log(x) \, a^2b^3x^6 - 2a^5 - 15a^4b \, x^2 - 60a^3b^2x^4 + 30a \, b^4x^8 + 3b^5x^{10}}{12x^6}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x)
```

output
```
(120*log(x)*a**2*b**3*x**6 - 2*a**5 - 15*a**4*b*x**2 - 60*a**3*b**2*x**4 +
 30*a*b**4*x**8 + 3*b**5*x**10)/(12*x**6)
```

3.543 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^9}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 250

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^9}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^4(a+bx^2)} - \frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^2(a+bx^2)} + \frac{b^5x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)} + \frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

output
```
-1/8*a^5*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)-5/6*a^4*b*((b*x^2+a)^2)^(1/2)/x
^6/(b*x^2+a)-5/2*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)-5*a^2*b^3*((b*x
^2+a)^2)^(1/2)/x^2/(b*x^2+a)+b^5*x^2*((b*x^2+a)^2)^(1/2)/(2*b*x^2+2*a)+5*a
*b^4*((b*x^2+a)^2)^(1/2)*ln(x)/(b*x^2+a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 696 vs. $2(250) = 500$.

Time = 1.02 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.78

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^9}\,dx = \frac{48a^6\sqrt{a^2}+368a^5\sqrt{a^2}bx^2+1280a^4\sqrt{a^2}b^2x^4+2880a^3\sqrt{a^2}b^3x^6+2677(a^2)^{3/}}{}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

output
```
(48*a^6*Sqrt[a^2] + 368*a^5*Sqrt[a^2]*b*x^2 + 1280*a^4*Sqrt[a^2]*b^2*x^4 +
 2880*a^3*Sqrt[a^2]*b^3*x^6 + 2677*(a^2)^(3/2)*b^4*x^8 + 565*a*Sqrt[a^2]*b
^5*x^10 - 192*Sqrt[a^2]*b^6*x^12 - 48*a^6*Sqrt[(a + b*x^2)^2] - 320*a^5*b*
x^2*Sqrt[(a + b*x^2)^2] - 960*a^4*b^2*x^4*Sqrt[(a + b*x^2)^2] - 1920*a^3*b
^3*x^6*Sqrt[(a + b*x^2)^2] - 757*a^2*b^4*x^8*Sqrt[(a + b*x^2)^2] + 192*a*b
^5*x^10*Sqrt[(a + b*x^2)^2] - 960*a*b^4*x^8*(a^2 + a*b*x^2 - Sqrt[a^2]*Sqr
t[(a + b*x^2)^2])*ArcTanh[(b*x^2)/(Sqrt[a^2] - Sqrt[(a + b*x^2)^2])] - 960
*b^4*x^8*((a^2)^(3/2) + a*Sqrt[a^2]*b*x^2 - a^2*Sqrt[(a + b*x^2)^2])*Log[x
^2] + 480*(a^2)^(3/2)*b^4*x^8*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]]
 + 480*a*Sqrt[a^2]*b^5*x^10*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] -
 480*a^2*b^4*x^8*Sqrt[(a + b*x^2)^2]*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x
^2)^2]] + 480*(a^2)^(3/2)*b^4*x^8*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)
^2]] + 480*a*Sqrt[a^2]*b^5*x^10*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2
]] - 480*a^2*b^4*x^8*Sqrt[(a + b*x^2)^2]*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a +
 b*x^2)^2]])/(384*x^8*(a^2 + a*b*x^2 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.38, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^9}\,dx$$

$\Big\downarrow$ 1384

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5\left(bx^2+a\right)^5}{x^9}dx}{b^5\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^5}{x^9}dx}{a+bx^2}$$

$\Big\downarrow$ 243

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^5}{x^{10}}dx^2}{2\left(a+bx^2\right)}$$

$\Big\downarrow$ 49

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5}{x^{10}}+\frac{5ba^4}{x^8}+\frac{10b^2a^3}{x^6}+\frac{10b^3a^2}{x^4}+\frac{5b^4a}{x^2}+b^5\right)dx^2}{2\left(a+bx^2\right)}$$

$\Big\downarrow$ 2009

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^5}{4x^8}-\frac{5a^4b}{3x^6}-\frac{5a^3b^2}{x^4}-\frac{10a^2b^3}{x^2}+5ab^4\log\left(x^2\right)+b^5x^2\right)}{2\left(a+bx^2\right)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/4*a^5/x^8 - (5*a^4*b)/(3*x^6) - (5*a^
3*b^2)/x^4 - (10*a^2*b^3)/x^2 + b^5*x^2 + 5*a*b^4*Log[x^2]))/(2*(a + b*x^2
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.28

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(-4x^{10}b^{5}-20b^{4}a\ln\left(x^{2}\right)x^{8}+40a^{2}x^{6}b^{3}+20a^{3}x^{4}b^{2}+\frac{20x^{2}a^{4}b}{3}+a^{5}\right)}{8x^{8}}$	70
default	$\frac{\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}\left(12x^{10}b^{5}+120\ln\left(x\right)x^{8}a\,b^{4}-120a^{2}x^{6}b^{3}-60a^{3}x^{4}b^{2}-20x^{2}a^{4}b-3a^{5}\right)}{24x^{8}\left(b\,x^{2}+a\right)^{5}}$	82
risch	$\frac{b^{5}x^{2}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{2b\,x^{2}+2a}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-5a^{2}x^{6}b^{3}-\frac{5}{2}a^{3}x^{4}b^{2}-\frac{5}{6}x^{2}a^{4}b-\frac{1}{8}a^{5}\right)}{\left(b\,x^{2}+a\right)x^{8}}+\frac{5a\,b^{4}\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(x\right)}{b\,x^{2}+a}$	119

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*csgn(b*x^2+a)*(-4*x^10*b^5-20*b^4*a*ln(x^2)*x^8+40*a^2*x^6*b^3+20*a^3
*x^4*b^2+20/3*x^2*a^4*b+a^5)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^9}\,dx = \frac{12\,b^5x^{10}+120\,ab^4x^8\log\left(x\right)-120\,a^2b^3x^6-60\,a^3b^2x^4-20\,a^4bx^2-3\,a^5}{24\,x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

output
```
1/24*(12*b^5*x^10 + 120*a*b^4*x^8*log(x) - 120*a^2*b^3*x^6 - 60*a^3*b^2*x^
4 - 20*a^4*b*x^2 - 3*a^5)/x^8
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^9}\,dx = \int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**9,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**9, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^9}\,dx = \frac{1}{2}\,b^5x^2+5\,ab^4\log\left(x\right)-\frac{5\,a^2b^3}{x^2}-\frac{5\,a^3b^2}{2\,x^4}-\frac{5\,a^4b}{6\,x^6}-\frac{a^5}{8\,x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

output
```
1/2*b^5*x^2 + 5*a*b^4*log(x) - 5*a^2*b^3/x^2 - 5/2*a^3*b^2/x^4 - 5/6*a^4*b
/x^6 - 1/8*a^5/x^8
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.50

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^9}\,dx = \frac{1}{2}\,b^5x^2\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{2}\,ab^4\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right) - \frac{125\,ab^4x^8\mathrm{sgn}(bx^2+a)+120\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+60\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+20\,a^4bx^2\mathrm{sgn}(bx^2+a)+3\,a^5\mathrm{s}}{24\,x^8}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x, algorithm="giac")
```

output
```
1/2*b^5*x^2*sgn(b*x^2 + a) + 5/2*a*b^4*log(x^2)*sgn(b*x^2 + a) - 1/24*(125
*a*b^4*x^8*sgn(b*x^2 + a) + 120*a^2*b^3*x^6*sgn(b*x^2 + a) + 60*a^3*b^2*x^
4*sgn(b*x^2 + a) + 20*a^4*b*x^2*sgn(b*x^2 + a) + 3*a^5*sgn(b*x^2 + a))/x^8
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^9}\,dx = \int \frac{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}{x^9}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^9,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^9, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^9} dx = \frac{120 \log(x) a b^4 x^8 - 3a^5 - 20a^4 b x^2 - 60a^3 b^2 x^4 - 120a^2 b^3 x^6 + 12b^5 x^{10}}{24x^8}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x)
```

output
```
(120*log(x)*a*b**4*x**8 - 3*a**5 - 20*a**4*b*x**2 - 60*a**3*b**2*x**4 - 12
0*a**2*b**3*x**6 + 12*b**5*x**10)/(24*x**8)
```

3.544 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{11}}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)} -\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^6(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^4(a+bx^2)} -\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}$$

output
```
-1/10*a^5*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)-5/8*a^4*b*((b*x^2+a)^2)^(1/2)
/x^8/(b*x^2+a)-5/3*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)-5/2*a^2*b^3*(
(b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)-5/2*a*b^4*((b*x^2+a)^2)^(1/2)/x^2/(b*x^2+
a)+b^5*((b*x^2+a)^2)^(1/2)*ln(x)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{11}}\,dx = \frac{1}{240}\left(-\frac{\sqrt{(a+bx^2)^2}(12a^4+63a^3bx^2+137a^2b^2x^4+163ab^3x^6+137b^4x^8)}{x^{10}} + \frac{\sqrt{a^2}(12a^4+75a^3bx^2+200a^2b^2x^4+300ab^3x^6+300b^4x^8)}{x^{10}} - 120b^5\operatorname{arctanh}\left(\frac{bx^2}{\sqrt{a^2}-\sqrt{(a+bx^2)^2}}\right) - \frac{120\sqrt{a^2}b^5\log(x^2)}{a} + \frac{60\sqrt{a^2}b^5\log\left(a\left(\sqrt{a^2}-bx^2-\sqrt{(a+bx^2)^2}\right)\right)}{a} + \frac{60\sqrt{a^2}b^5\log\left(a\left(\sqrt{a^2}+bx^2-\sqrt{(a+bx^2)^2}\right)\right)}{a}\right)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

output
```
(-((Sqrt[(a + b*x^2)^2]*(12*a^4 + 63*a^3*b*x^2 + 137*a^2*b^2*x^4 + 163*a*b
^3*x^6 + 137*b^4*x^8))/x^10) + (Sqrt[a^2]*(12*a^4 + 75*a^3*b*x^2 + 200*a^2
*b^2*x^4 + 300*a*b^3*x^6 + 300*b^4*x^8))/x^10 - 120*b^5*ArcTanh[(b*x^2)/(S
qrt[a^2] - Sqrt[(a + b*x^2)^2])] - (120*Sqrt[a^2]*b^5*Log[x^2])/a + (60*Sq
rt[a^2]*b^5*Log[a*(Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2])])/a + (60*Sqrt
[a^2]*b^5*Log[a*(Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2])])/a)/240
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{11}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^{11}}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{11}}dx}{a+bx^2}$$

$$\Big\downarrow \ 243$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{12}}dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \ 49$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5}{x^{12}}+\frac{5ba^4}{x^{10}}+\frac{10b^2a^3}{x^8}+\frac{10b^3a^2}{x^6}+\frac{5b^4a}{x^4}+\frac{b^5}{x^2}\right)dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^5}{5x^{10}}-\frac{5a^4b}{4x^8}-\frac{10a^3b^2}{3x^6}-\frac{5a^2b^3}{x^4}-\frac{5ab^4}{x^2}+b^5\log(x^2)\right)}{2(a+bx^2)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/5*a^5/x^10 - (5*a^4*b)/(4*x^8) - (10*
a^3*b^2)/(3*x^6) - (5*a^2*b^3)/x^4 - (5*a*b^4)/x^2 + b^5*Log[x^2]))/(2*(a
+ b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.28

method	result	size
pseudoelliptic	$-\frac{\left(-5 b^{5} \ln \left(x^{2}\right) x^{10}+a \left(25 b^{4} x^{8}+25 a \,b^{3} x^{6}+\frac{50}{3} a^{2} b^{2} x^{4}+\frac{25}{4} a^{3} b \,x^{2}+a^{4}\right)\right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{10 x^{10}}$	70
default	$\frac{\left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}} \left(120 \ln \left(x \right) x^{10} b^{5}-300 a \,x^{8} b^{4}-300 a^{2} x^{6} b^{3}-200 a^{3} x^{4} b^{2}-75 x^{2} a^{4} b -12 a^{5}\right)}{120 x^{10} \left(b \,x^{2}+a \right)^{5}}$	82
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(-\frac{5}{2} a \,x^{8} b^{4}-\frac{5}{2} a^{2} x^{6} b^{3}-\frac{5}{3} a^{3} x^{4} b^{2}-\frac{5}{8} x^{2} a^{4} b -\frac{1}{10} a^{5}\right)}{\left(b \,x^{2}+a \right) x^{10}}+\frac{b^{5} \sqrt{\left(b \,x^{2}+a \right)^{2}}\, \ln \left(x \right)}{b \,x^{2}+a}$	98

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*(-5*b^5*ln(x^2)*x^10+a*(25*b^4*x^8+25*a*b^3*x^6+50/3*a^2*b^2*x^4+25/
4*a^3*b*x^2+a^4))*csgn(b*x^2+a)/x^10
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{11}}\,dx = \frac{120\,b^5x^{10}\log{(x)} - 300\,ab^4x^8 - 300\,a^2b^3x^6 - 200\,a^3b^2x^4 - 75\,a^4bx^2 - 12\,a^5}{120\,x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

output
```
1/120*(120*b^5*x^10*log(x) - 300*a*b^4*x^8 - 300*a^2*b^3*x^6 - 200*a^3*b^2
*x^4 - 75*a^4*b*x^2 - 12*a^5)/x^10
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{11}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^{11}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**11,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**11, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{11}}\,dx = b^5\log(x) - \frac{5\,ab^4}{2\,x^2} - \frac{5\,a^2b^3}{2\,x^4} - \frac{5\,a^3b^2}{3\,x^6} - \frac{5\,a^4b}{8\,x^8} - \frac{a^5}{10\,x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

output
```
b^5*log(x) - 5/2*a*b^4/x^2 - 5/2*a^2*b^3/x^4 - 5/3*a^3*b^2/x^6 - 5/8*a^4*b
/x^8 - 1/10*a^5/x^10
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.50

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{11}}\,dx = \frac{1}{2}\,b^5\log\left(x^2\right)\operatorname{sgn}\left(bx^2+a\right) - \frac{137\,b^5x^{10}\operatorname{sgn}(bx^2+a) + 300\,ab^4x^8\operatorname{sgn}(bx^2+a) + 300\,a^2b^3x^6\operatorname{sgn}(bx^2+a) + 200\,a^3b^2x^4\operatorname{sgn}(bx^2+a) + 75}{120\,x^{10}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x, algorithm="giac")
```

output
```
1/2*b^5*log(x^2)*sgn(b*x^2 + a) - 1/120*(137*b^5*x^10*sgn(b*x^2 + a) + 300
*a*b^4*x^8*sgn(b*x^2 + a) + 300*a^2*b^3*x^6*sgn(b*x^2 + a) + 200*a^3*b^2*x
^4*sgn(b*x^2 + a) + 75*a^4*b*x^2*sgn(b*x^2 + a) + 12*a^5*sgn(b*x^2 + a))/x
^10
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{11}}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{5/2}}{x^{11}}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^11,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^11, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{11}}\,dx = \frac{120\log(x)\,b^5x^{10} - 12a^5 - 75a^4b\,x^2 - 200a^3b^2x^4 - 300a^2b^3x^6 - 300a\,b^4x^8}{120x^{10}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x)
```

output
```
(120*log(x)*b**5*x**10 - 12*a**5 - 75*a**4*b*x**2 - 200*a**3*b**2*x**4 - 3
00*a**2*b**3*x**6 - 300*a*b**4*x**8)/(120*x**10)
```

3.545 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{13}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{13}}\,dx=-\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

output
```
-1/12*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/a/x^12
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{13}}\,dx = -\frac{\sqrt{\left(a+bx^2\right)^2}\left(a^5+6a^4bx^2+15a^3b^2x^4+20a^2b^3x^6+15ab^4x^8+6b^5x^{10}\right)}{12x^{12}\left(a+bx^2\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

output
```
-1/12*(Sqrt[(a + b*x^2)^2]*(a^5 + 6*a^4*b*x^2 + 15*a^3*b^2*x^4 + 20*a^2*b^
3*x^6 + 15*a*b^4*x^8 + 6*b^5*x^10))/(x^12*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1384, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{13}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5\left(bx^2+a\right)^5}{x^{13}}dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^5}{x^{13}}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

output
```
-1/12*((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a*x^12)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

method	result	size
pseudoelliptic	$-\frac{(2b\,x^2+a)\,(3b^2x^4+3ab\,x^2+a^2)\,(b^2x^4+ab\,x^2+a^2)\,\operatorname{csgn}(b\,x^2+a)}{12x^{12}}$	58
gosper	$-\frac{(6x^{10}b^5+15a\,x^8b^4+20a^2x^6b^3+15a^3x^4b^2+6x^2a^4b+a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{12x^{12}(b\,x^2+a)^5}$	78
default	$-\frac{(6x^{10}b^5+15a\,x^8b^4+20a^2x^6b^3+15a^3x^4b^2+6x^2a^4b+a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{12x^{12}(b\,x^2+a)^5}$	78
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{2}x^{10}b^5-\frac{5}{4}a\,x^8b^4-\frac{5}{3}a^2x^6b^3-\frac{5}{4}a^3x^4b^2-\frac{1}{2}x^2a^4b-\frac{1}{12}a^5\right)}{(b\,x^2+a)x^{12}}$	79
orering	$-\frac{(6x^{10}b^5+15a\,x^8b^4+20a^2x^6b^3+15a^3x^4b^2+6x^2a^4b+a^5)\,(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{2}}}{12x^{12}(b\,x^2+a)^5}$	87

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(2*b*x^2+a)*(3*b^2*x^4+3*a*b*x^2+a^2)*(b^2*x^4+a*b*x^2+a^2)*csgn(b*x
^2+a)/x^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(28) = 56$.

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{13}} \, dx = -\frac{6\,b^5x^{10} + 15\,ab^4x^8 + 20\,a^2b^3x^6 + 15\,a^3b^2x^4 + 6\,a^4bx^2 + a^5}{12\,x^{12}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="fricas")
```

output
```
-1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4
*b*x^2 + a^5)/x^12
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{13}} \, dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{5}{2}}}{x^{13}} \, dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**13,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**13, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(28) = 56$.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{13}} \, dx = -\frac{b^5}{2\,x^2} - \frac{5\,ab^4}{4\,x^4} - \frac{5\,a^2b^3}{3\,x^6} - \frac{5\,a^3b^2}{4\,x^8} - \frac{a^4b}{2\,x^{10}} - \frac{a^5}{12\,x^{12}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="maxima")
```

output
```
-1/2*b^5/x^2 - 5/4*a*b^4/x^4 - 5/3*a^2*b^3/x^6 - 5/4*a^3*b^2/x^8 - 1/2*a^4
*b/x^10 - 1/12*a^5/x^12
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(28) = 56$.

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.59

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{13}}\,dx = -\frac{6\,b^5x^{10}\mathrm{sgn}(bx^2+a)+15\,ab^4x^8\mathrm{sgn}(bx^2+a)+20\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+15\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+6\,a^4bx^2\mathrm{s}}{12\,x^{12}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x, algorithm="giac")
```

output
```
-1/12*(6*b^5*x^10*sgn(b*x^2 + a) + 15*a*b^4*x^8*sgn(b*x^2 + a) + 20*a^2*b^
3*x^6*sgn(b*x^2 + a) + 15*a^3*b^2*x^4*sgn(b*x^2 + a) + 6*a^4*b*x^2*sgn(b*x
^2 + a) + a^5*sgn(b*x^2 + a))/x^12
```

Mupad [B] (verification not implemented)

Time = 18.49 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.63

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{13}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,\left(b\,x^2+a\right)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^2\,\left(b\,x^2+a\right)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\,\left(b\,x^2+a\right)} - \frac{a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^{10}\,\left(b\,x^2+a\right)} - \frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^6\,\left(b\,x^2+a\right)} - \frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^8\,\left(b\,x^2+a\right)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^13,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*x^2*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (a^4*b*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(2*x^10*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(3*x^6*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2
))/(4*x^8*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{13}}\, dx = \frac{-6b^5x^{10} - 15a\,b^4x^8 - 20a^2b^3x^6 - 15a^3b^2x^4 - 6a^4b\,x^2 - a^5}{12x^{12}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x)
```

output
```
( - a**5 - 6*a**4*b*x**2 - 15*a**3*b**2*x**4 - 20*a**2*b**3*x**6 - 15*a*b*
*4*x**8 - 6*b**5*x**10)/(12*x**12)
```

3.546 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{15}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{15}}\,dx=-\frac{\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{14ax^{14}}+\frac{b\left(a+bx^2\right)^5\sqrt{a^2+2abx^2+b^2x^4}}{84a^2x^{12}}$$

output
```
-1/14*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/a/x^14+1/84*b*(b*x^2+a)^5*((b*x^2+a)
^2)^(1/2)/a^2/x^12
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{15}}\,dx=-\frac{\sqrt{\left(a+bx^2\right)^2}\left(6a^5+35a^4bx^2+84a^3b^2x^4+105a^2b^3x^6+70ab^4x^8+21b^5x^{10}\right)}{84x^{14}\left(a+bx^2\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

output
```
-1/84*(Sqrt[(a + b*x^2)^2]*(6*a^5 + 35*a^4*b*x^2 + 84*a^3*b^2*x^4 + 105*a^
2*b^3*x^6 + 70*a*b^4*x^8 + 21*b^5*x^10))/(x^14*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{15}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^{15}}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{15}}dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{16}}dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{55}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{b\int \frac{(bx^2+a)^5}{x^{14}}dx^2}{7a}-\frac{(a+bx^2)^6}{7ax^{14}}\right)}{2(a+bx^2)}$$

$$\Big\downarrow \text{48}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{b(a+bx^2)^6}{42a^2x^{12}}-\frac{(a+bx^2)^6}{7ax^{14}}\right)}{2(a+bx^2)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/7*(a + b*x^2)^6/(a*x^14) + (b*(a + b*
x^2)^6)/(42*a^2*x^12)))/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.52 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{\left(21 x^{10} b^{5}+70 a \,x^{8} b^{4}+105 a^{2} x^{6} b^{3}+84 a^{3} x^{4} b^{2}+35 x^{2} a^{4} b +6 a^{5}\right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{84 x^{14}}$	68
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(-\frac{1}{14} a^{5}-\frac{5}{12} x^{2} a^{4} b -a^{3} x^{4} b^{2}-\frac{5}{4} a^{2} x^{6} b^{3}-\frac{5}{6} a \,x^{8} b^{4}-\frac{1}{4} x^{10} b^{5}\right)}{\left(b \,x^{2}+a \right) x^{14}}$	79
gosper	$-\frac{\left(21 x^{10} b^{5}+70 a \,x^{8} b^{4}+105 a^{2} x^{6} b^{3}+84 a^{3} x^{4} b^{2}+35 x^{2} a^{4} b +6 a^{5}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}{84 x^{14} \left(b \,x^{2}+a \right)^{5}}$	80
default	$-\frac{\left(21 x^{10} b^{5}+70 a \,x^{8} b^{4}+105 a^{2} x^{6} b^{3}+84 a^{3} x^{4} b^{2}+35 x^{2} a^{4} b +6 a^{5}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}{84 x^{14} \left(b \,x^{2}+a \right)^{5}}$	80
orering	$-\frac{\left(21 x^{10} b^{5}+70 a \,x^{8} b^{4}+105 a^{2} x^{6} b^{3}+84 a^{3} x^{4} b^{2}+35 x^{2} a^{4} b +6 a^{5}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{5}{2}}}{84 x^{14} \left(b \,x^{2}+a \right)^{5}}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/84*(21*b^5*x^10+70*a*b^4*x^8+105*a^2*b^3*x^6+84*a^3*b^2*x^4+35*a^4*b*x^
2+6*a^5)*csgn(b*x^2+a)/x^14
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{15}}\,dx = -\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="fricas")
```

output
```
-1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*
a^4*b*x^2 + 6*a^5)/x^14
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{15}}\,dx=\int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^{15}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**15,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**15, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{15}}\,dx=-\frac{b^5}{4\,x^4}-\frac{5\,ab^4}{6\,x^6}-\frac{5\,a^2b^3}{4\,x^8}-\frac{a^3b^2}{x^{10}}-\frac{5\,a^4b}{12\,x^{12}}-\frac{a^5}{14\,x^{14}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="maxima")
```

output
```
-1/4*b^5/x^4 - 5/6*a*b^4/x^6 - 5/4*a^2*b^3/x^8 - a^3*b^2/x^10 - 5/12*a^4*b
/x^12 - 1/14*a^5/x^14
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{15}}\,dx=$$
$$-\frac{21\,b^5x^{10}\mathrm{sgn}(bx^2+a)+70\,ab^4x^8\mathrm{sgn}(bx^2+a)+105\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+84\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+35\,a^4b}{84\,x^{14}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x, algorithm="giac")
```

output
```
-1/84*(21*b^5*x^10*sgn(b*x^2 + a) + 70*a*b^4*x^8*sgn(b*x^2 + a) + 105*a^2*
b^3*x^6*sgn(b*x^2 + a) + 84*a^3*b^2*x^4*sgn(b*x^2 + a) + 35*a^4*b*x^2*sgn(
b*x^2 + a) + 6*a^5*sgn(b*x^2 + a))/x^14
```

Mupad [B] (verification not implemented)

Time = 17.60 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.75

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{15}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,(b\,x^2+a)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^4\,(b\,x^2+a)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^6\,(b\,x^2+a)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,(b\,x^2+a)} - \frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^8\,(b\,x^2+a)} - \frac{a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^{10}\,(b\,x^2+a)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^15,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*x^4*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(6*x^6*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*
a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(4*x^8*(a + b*x^2)) - (a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/
2))/(x^10*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{15}}\,dx = \frac{-21b^5x^{10}-70a\,b^4x^8-105a^2b^3x^6-84a^3b^2x^4-35a^4b\,x^2-6a^5}{84x^{14}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x)
```

output
```
( - 6*a**5 - 35*a**4*b*x**2 - 84*a**3*b**2*x**4 - 105*a**2*b**3*x**6 - 70*
a*b**4*x**8 - 21*b**5*x**10)/(84*x**14)
```

3.547 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{17}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 128

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{17}}\,dx = -\frac{(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{16ax^{16}} + \frac{b(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{56a^2x^{14}} - \frac{b^2(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{336a^3x^{12}}$$

output
```
-1/16*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/a/x^16+1/56*b*(b*x^2+a)^5*((b*x^2+a)
^2)^(1/2)/a^2/x^14-1/336*b^2*(b*x^2+a)^5*((b*x^2+a)^2)^(1/2)/a^3/x^12
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{17}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(21a^5+120a^4bx^2+280a^3b^2x^4+336a^2b^3x^6+210ab^4x^8+56b^5x^{10})}{336x^{16}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

output
```
-1/336*(Sqrt[(a + b*x^2)^2]*(21*a^5 + 120*a^4*b*x^2 + 280*a^3*b^2*x^4 + 33
6*a^2*b^3*x^6 + 210*a*b^4*x^8 + 56*b^5*x^10))/(x^16*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1384, 27, 243, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{17}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5\left(bx^2+a\right)^5}{x^{17}}dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^5}{x^{17}}dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^5}{x^{18}}dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{55}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{b\int \frac{\left(bx^2+a\right)^5}{x^{16}}dx^2}{4a}-\frac{\left(a+bx^2\right)^6}{8ax^{16}}\right)}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{55}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{b\left(-\frac{b\int\frac{\left(bx^2+a\right)^5}{x^{14}}dx^2}{7a}-\frac{\left(a+bx^2\right)^6}{7ax^{14}}\right)}{4a}-\frac{\left(a+bx^2\right)^6}{8ax^{16}}\right)}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{48}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{b\left(\frac{b\left(a+bx^2\right)^6}{42a^2x^{12}}-\frac{\left(a+bx^2\right)^6}{7ax^{14}}\right)}{4a}-\frac{\left(a+bx^2\right)^6}{8ax^{16}}\right)}{2\left(a+bx^2\right)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/8*(a + b*x^2)^6/(a*x^16) - (b*(-1/7*(
a + b*x^2)^6/(a*x^14) + (b*(a + b*x^2)^6)/(42*a^2*x^12)))/(4*a)))/(2*(a +
b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.77 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(\frac{8}{3}x^{10}b^{5}+10a\,x^{8}b^{4}+16a^{2}x^{6}b^{3}+\frac{40}{3}a^{3}x^{4}b^{2}+\frac{40}{7}x^{2}a^{4}b+a^{5}\right)}{16x^{16}}$	66
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{1}{16}a^{5}-\frac{5}{14}x^{2}a^{4}b-\frac{5}{6}a^{3}x^{4}b^{2}-a^{2}x^{6}b^{3}-\frac{5}{8}a\,x^{8}b^{4}-\frac{1}{6}x^{10}b^{5}\right)}{\left(b\,x^{2}+a\right)x^{16}}$	79
gosper	$-\frac{\left(56x^{10}b^{5}+210a\,x^{8}b^{4}+336a^{2}x^{6}b^{3}+280a^{3}x^{4}b^{2}+120x^{2}a^{4}b+21a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{336x^{16}\left(b\,x^{2}+a\right)^{5}}$	80
default	$-\frac{\left(56x^{10}b^{5}+210a\,x^{8}b^{4}+336a^{2}x^{6}b^{3}+280a^{3}x^{4}b^{2}+120x^{2}a^{4}b+21a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{336x^{16}\left(b\,x^{2}+a\right)^{5}}$	80
orering	$-\frac{\left(56x^{10}b^{5}+210a\,x^{8}b^{4}+336a^{2}x^{6}b^{3}+280a^{3}x^{4}b^{2}+120x^{2}a^{4}b+21a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{336x^{16}\left(b\,x^{2}+a\right)^{5}}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/16*csgn(b*x^2+a)*(8/3*x^10*b^5+10*a*x^8*b^4+16*a^2*x^6*b^3+40/3*a^3*x^4
*b^2+40/7*x^2*a^4*b+a^5)/x^16
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{17}} dx = -\frac{56\,b^5x^{10} + 210\,ab^4x^8 + 336\,a^2b^3x^6 + 280\,a^3b^2x^4 + 120\,a^4bx^2 + 21\,a^5}{336\,x^{16}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="fricas")
```

output
```
-1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 +
120*a^4*b*x^2 + 21*a^5)/x^16
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{17}} dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{5}{2}}}{x^{17}}\, dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**17,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**17, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.45

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{17}}\,dx = -\frac{b^5}{6\,x^6} - \frac{5\,ab^4}{8\,x^8} - \frac{a^2b^3}{x^{10}} - \frac{5\,a^3b^2}{6\,x^{12}} - \frac{5\,a^4b}{14\,x^{14}} - \frac{a^5}{16\,x^{16}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="maxima")
```

output
```
-1/6*b^5/x^6 - 5/8*a*b^4/x^8 - a^2*b^3/x^10 - 5/6*a^3*b^2/x^12 - 5/14*a^4*
b/x^14 - 1/16*a^5/x^16
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{17}}\,dx =$$
$$-\frac{56\,b^5x^{10}\mathrm{sgn}(bx^2+a) + 210\,ab^4x^8\mathrm{sgn}(bx^2+a) + 336\,a^2b^3x^6\mathrm{sgn}(bx^2+a) + 280\,a^3b^2x^4\mathrm{sgn}(bx^2+a) + 120}{336\,x^{16}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x, algorithm="giac")
```

output
```
-1/336*(56*b^5*x^10*sgn(b*x^2 + a) + 210*a*b^4*x^8*sgn(b*x^2 + a) + 336*a^
2*b^3*x^6*sgn(b*x^2 + a) + 280*a^3*b^2*x^4*sgn(b*x^2 + a) + 120*a^4*b*x^2*
sgn(b*x^2 + a) + 21*a^5*sgn(b*x^2 + a))/x^16
```

Mupad [B] (verification not implemented)

Time = 17.91 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.80

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{17}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\,(b\,x^2+a)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^6\,(b\,x^2+a)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,(b\,x^2+a)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,(b\,x^2+a)} - \frac{a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^{10}\,(b\,x^2+a)} - \frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^{12}\,(b\,x^2+a)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^17,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(6*x^6*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*
a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^
2)^(1/2))/(x^10*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2
))/(6*x^12*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{17}}\,dx = \frac{-56b^5x^{10}-210a\,b^4x^8-336a^2b^3x^6-280a^3b^2x^4-120a^4b\,x^2-21a^5}{336x^{16}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x)
```

output
```
( - 21*a**5 - 120*a**4*b*x**2 - 280*a**3*b**2*x**4 - 336*a**2*b**3*x**6 -
210*a*b**4*x**8 - 56*b**5*x**10)/(336*x**16)
```

3.548 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{19}} dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{19}} dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}(a+bx^2)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}(a+bx^2)} - \frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^{12}(a+bx^2)} - \frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^{10}(a+bx^2)} - \frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}$$

output
```
-1/18*a^5*((b*x^2+a)^2)^(1/2)/x^18/(b*x^2+a)-5/16*a^4*b*((b*x^2+a)^2)^(1/2
)/x^16/(b*x^2+a)-5/7*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^14/(b*x^2+a)-5/6*a^2*b^
3*((b*x^2+a)^2)^(1/2)/x^12/(b*x^2+a)-1/2*a*b^4*((b*x^2+a)^2)^(1/2)/x^10/(b
*x^2+a)-1/8*b^5*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{19}} \, dx = -\frac{\sqrt{(a+bx^2)^2}(56a^5 + 315a^4bx^2 + 720a^3b^2x^4 + 840a^2b^3x^6 + 504ab^4x^8 + 126b^5x^{10})}{1008x^{18}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
```

output
```
-1/1008*(Sqrt[(a + b*x^2)^2]*(56*a^5 + 315*a^4*b*x^2 + 720*a^3*b^2*x^4 + 8
40*a^2*b^3*x^6 + 504*a*b^4*x^8 + 126*b^5*x^10))/(x^18*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{19}} \, dx$$

$$\Big\downarrow \; 1384$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \frac{b^5(bx^2+a)^5}{x^{19}} dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \frac{(bx^2+a)^5}{x^{19}} dx}{a+bx^2}$$

$$\Big\downarrow \; 243$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{20}}dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5}{x^{20}}+\frac{5ba^4}{x^{18}}+\frac{10b^2a^3}{x^{16}}+\frac{10b^3a^2}{x^{14}}+\frac{5b^4a}{x^{12}}+\frac{b^5}{x^{10}}\right)dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{9x^{18}}-\frac{5a^4b}{8x^{16}}-\frac{10a^3b^2}{7x^{14}}-\frac{5a^2b^3}{3x^{12}}-\frac{ab^4}{x^{10}}-\frac{b^5}{4x^8}\right)\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
```

output
```
((-1/9*a^5/x^18 - (5*a^4*b)/(8*x^16) - (10*a^3*b^2)/(7*x^14) - (5*a^2*b^3)
/(3*x^12) - (a*b^4)/x^10 - b^5/(4*x^8))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26

method	result	size
pseudoelliptic	$-\frac{\left(\frac{9}{4}x^{10}b^{5}+9a\,x^{8}b^{4}+15a^{2}x^{6}b^{3}+\frac{90}{7}a^{3}x^{4}b^{2}+\frac{45}{8}x^{2}a^{4}b+a^{5}\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{18x^{18}}$	66
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{1}{18}a^{5}-\frac{5}{16}x^{2}a^{4}b-\frac{5}{7}a^{3}x^{4}b^{2}-\frac{5}{6}a^{2}x^{6}b^{3}-\frac{1}{2}a\,x^{8}b^{4}-\frac{1}{8}x^{10}b^{5}\right)}{\left(b\,x^{2}+a\right)x^{18}}$	79
gosper	$-\frac{\left(126x^{10}b^{5}+504a\,x^{8}b^{4}+840a^{2}x^{6}b^{3}+720a^{3}x^{4}b^{2}+315x^{2}a^{4}b+56a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{1008x^{18}\left(b\,x^{2}+a\right)^{5}}$	80
default	$-\frac{\left(126x^{10}b^{5}+504a\,x^{8}b^{4}+840a^{2}x^{6}b^{3}+720a^{3}x^{4}b^{2}+315x^{2}a^{4}b+56a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{1008x^{18}\left(b\,x^{2}+a\right)^{5}}$	80
orering	$-\frac{\left(126x^{10}b^{5}+504a\,x^{8}b^{4}+840a^{2}x^{6}b^{3}+720a^{3}x^{4}b^{2}+315x^{2}a^{4}b+56a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{1008x^{18}\left(b\,x^{2}+a\right)^{5}}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/18*(9/4*x^10*b^5+9*a*x^8*b^4+15*a^2*x^6*b^3+90/7*a^3*x^4*b^2+45/8*x^2*a
^4*b+a^5)*csgn(b*x^2+a)/x^18
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{19}} \, dx = -\frac{126 \, b^5x^{10} + 504 \, ab^4x^8 + 840 \, a^2b^3x^6 + 720 \, a^3b^2x^4 + 315 \, a^4bx^2 + 56 \, a^5}{1008 \, x^{18}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="fricas")
```

output
```
-1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4
+ 315*a^4*b*x^2 + 56*a^5)/x^18
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{19}} \, dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{5}{2}}}{x^{19}} \, dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**19,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**19, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{19}} \, dx = -\frac{b^5}{8 \, x^8} - \frac{ab^4}{2 \, x^{10}} - \frac{5 \, a^2b^3}{6 \, x^{12}} - \frac{5 \, a^3b^2}{7 \, x^{14}} - \frac{5 \, a^4b}{16 \, x^{16}} - \frac{a^5}{18 \, x^{18}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="maxima")
```

output
```
-1/8*b^5/x^8 - 1/2*a*b^4/x^10 - 5/6*a^2*b^3/x^12 - 5/7*a^3*b^2/x^14 - 5/16
*a^4*b/x^16 - 1/18*a^5/x^18
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{19}}\,dx = -\frac{126\,b^5x^{10}\mathrm{sgn}(bx^2 + a) + 504\,ab^4x^8\mathrm{sgn}(bx^2 + a) + 840\,a^2b^3x^6\mathrm{sgn}(bx^2 + a) + 720\,a^3b^2x^4\mathrm{sgn}(bx^2 + a) + 31}{1008\,x^{18}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x, algorithm="giac")
```

output
```
-1/1008*(126*b^5*x^10*sgn(b*x^2 + a) + 504*a*b^4*x^8*sgn(b*x^2 + a) + 840*
a^2*b^3*x^6*sgn(b*x^2 + a) + 720*a^3*b^2*x^4*sgn(b*x^2 + a) + 315*a^4*b*x^
2*sgn(b*x^2 + a) + 56*a^5*sgn(b*x^2 + a))/x^18
```

Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{19}}\,dx = {} & -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{18\,x^{18}\,\left(b\,x^2+a\right)} \\ & -\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^8\,\left(b\,x^2+a\right)}-\frac{a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^{10}\,\left(b\,x^2+a\right)} \\ & -\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\,\left(b\,x^2+a\right)}-\frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{6\,x^{12}\,\left(b\,x^2+a\right)} \\ & -\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^{14}\,\left(b\,x^2+a\right)}\end{aligned}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^19,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(18*x^18*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*x^8*(a + b*x^2)) - (a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(2*x^10*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(6*x^12*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(7*x^14*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{19}} \, dx = \frac{-126b^5x^{10} - 504a\,b^4x^8 - 840a^2b^3x^6 - 720a^3b^2x^4 - 315a^4b\,x^2 - 56a^5}{1008x^{18}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x)
```

output
```
( - 56*a**5 - 315*a**4*b*x**2 - 720*a**3*b**2*x**4 - 840*a**2*b**3*x**6 -
504*a*b**4*x**8 - 126*b**5*x**10)/(1008*x**18)
```

3.549 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{21}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{21}}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}(a+bx^2)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}(a+bx^2)} - \frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}(a+bx^2)} - \frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)} - \frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}$$

output

```
-1/20*a^5*((b*x^2+a)^2)^(1/2)/x^20/(b*x^2+a)-5/18*a^4*b*((b*x^2+a)^2)^(1/2
)/x^18/(b*x^2+a)-5/8*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^16/(b*x^2+a)-5/7*a^2*b^
3*((b*x^2+a)^2)^(1/2)/x^14/(b*x^2+a)-5/12*a*b^4*((b*x^2+a)^2)^(1/2)/x^12/(
b*x^2+a)-1/10*b^5*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{21}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(126a^5+700a^4bx^2+1575a^3b^2x^4+1800a^2b^3x^6+1050ab^4x^8+252b^5x^{10})}{2520x^{20}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
```

output
```
-1/2520*(Sqrt[(a + b*x^2)^2]*(126*a^5 + 700*a^4*b*x^2 + 1575*a^3*b^2*x^4 +
 1800*a^2*b^3*x^6 + 1050*a*b^4*x^8 + 252*b^5*x^10))/(x^20*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{21}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^{21}}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{21}}dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{22}}dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5}{x^{22}}+\frac{5ba^4}{x^{20}}+\frac{10b^2a^3}{x^{18}}+\frac{10b^3a^2}{x^{16}}+\frac{5b^4a}{x^{14}}+\frac{b^5}{x^{12}}\right)dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{10x^{20}}-\frac{5a^4b}{9x^{18}}-\frac{5a^3b^2}{4x^{16}}-\frac{10a^2b^3}{7x^{14}}-\frac{5ab^4}{6x^{12}}-\frac{b^5}{5x^{10}}\right)\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
```

output
```
((-1/10*a^5/x^20 - (5*a^4*b)/(9*x^18) - (5*a^3*b^2)/(4*x^16) - (10*a^2*b^3
)/(7*x^14) - (5*a*b^4)/(6*x^12) - b^5/(5*x^10))*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.84 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26

method	result	size
pseudoelliptic	$-\frac{\left(2x^{10}b^{5}+\frac{25}{3}a\,x^{8}b^{4}+\frac{100}{7}a^{2}x^{6}b^{3}+\frac{25}{2}a^{3}x^{4}b^{2}+\frac{50}{9}x^{2}a^{4}b+a^{5}\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{20x^{20}}$	66
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{1}{20}a^{5}-\frac{5}{18}x^{2}a^{4}b-\frac{5}{8}a^{3}x^{4}b^{2}-\frac{5}{7}a^{2}x^{6}b^{3}-\frac{5}{12}a\,x^{8}b^{4}-\frac{1}{10}x^{10}b^{5}\right)}{\left(b\,x^{2}+a\right)x^{20}}$	79
gosper	$-\frac{\left(252x^{10}b^{5}+1050a\,x^{8}b^{4}+1800a^{2}x^{6}b^{3}+1575a^{3}x^{4}b^{2}+700x^{2}a^{4}b+126a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{2520x^{20}\left(b\,x^{2}+a\right)^{5}}$	80
default	$-\frac{\left(252x^{10}b^{5}+1050a\,x^{8}b^{4}+1800a^{2}x^{6}b^{3}+1575a^{3}x^{4}b^{2}+700x^{2}a^{4}b+126a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{2520x^{20}\left(b\,x^{2}+a\right)^{5}}$	80
orering	$-\frac{\left(252x^{10}b^{5}+1050a\,x^{8}b^{4}+1800a^{2}x^{6}b^{3}+1575a^{3}x^{4}b^{2}+700x^{2}a^{4}b+126a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{2520x^{20}\left(b\,x^{2}+a\right)^{5}}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(2*x^10*b^5+25/3*a*x^8*b^4+100/7*a^2*x^6*b^3+25/2*a^3*x^4*b^2+50/9*x
^2*a^4*b+a^5)*csgn(b*x^2+a)/x^20
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{21}}\,dx = -\frac{252\,b^5x^{10}+1050\,ab^4x^8+1800\,a^2b^3x^6+1575\,a^3b^2x^4+700\,a^4bx^2+126\,a^5}{2520\,x^{20}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="fricas")
```

output
```
-1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x
^4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{21}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^{21}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**21,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**21, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{21}}\,dx = -\frac{b^5}{10\,x^{10}}-\frac{5\,ab^4}{12\,x^{12}}-\frac{5\,a^2b^3}{7\,x^{14}}-\frac{5\,a^3b^2}{8\,x^{16}}-\frac{5\,a^4b}{18\,x^{18}}-\frac{a^5}{20\,x^{20}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="maxima")
```

output
```
-1/10*b^5/x^10 - 5/12*a*b^4/x^12 - 5/7*a^2*b^3/x^14 - 5/8*a^3*b^2/x^16 - 5
/18*a^4*b/x^18 - 1/20*a^5/x^20
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{21}}\,dx = -\frac{252\,b^5x^{10}\mathrm{sgn}(bx^2+a)+1050\,ab^4x^8\mathrm{sgn}(bx^2+a)+1800\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+1575\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+}{2520\,x^{20}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x, algorithm="giac")
```

output
```
-1/2520*(252*b^5*x^10*sgn(b*x^2 + a) + 1050*a*b^4*x^8*sgn(b*x^2 + a) + 180
0*a^2*b^3*x^6*sgn(b*x^2 + a) + 1575*a^3*b^2*x^4*sgn(b*x^2 + a) + 700*a^4*b
*x^2*sgn(b*x^2 + a) + 126*a^5*sgn(b*x^2 + a))/x^20
```

Mupad [B] (verification not implemented)

Time = 17.89 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{21}}\,dx &= -\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{20\,x^{20}\,(b\,x^2+a)} \\ &- \frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{10\,x^{10}\,(b\,x^2+a)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,(b\,x^2+a)} \\ &- \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{18\,x^{18}\,(b\,x^2+a)} - \frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^{14}\,(b\,x^2+a)} \\ &- \frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^{16}\,(b\,x^2+a)}\end{aligned}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^21,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(20*x^20*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(10*x^10*(a + b*x^2)) - (5*a*b^4*(a^2 + b^
2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(18*x^18*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*
a*b*x^2)^(1/2))/(7*x^14*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(8*x^16*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{21}}\,dx = \frac{-252b^5x^{10} - 1050a\,b^4x^8 - 1800a^2b^3x^6 - 1575a^3b^2x^4 - 700a^4b\,x^2 - 126a^5}{2520x^{20}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x)
```

output
```
( - 126*a**5 - 700*a**4*b*x**2 - 1575*a**3*b**2*x**4 - 1800*a**2*b**3*x**6
 - 1050*a*b**4*x**8 - 252*b**5*x**10)/(2520*x**20)
```

3.550 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{23}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{23}}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}(a+bx^2)} - \frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}(a+bx^2)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}(a+bx^2)} - \frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}(a+bx^2)} - \frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)} - \frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)}$$

output

```
-1/22*a^5*((b*x^2+a)^2)^(1/2)/x^22/(b*x^2+a)-1/4*a^4*b*((b*x^2+a)^2)^(1/2)
/x^20/(b*x^2+a)-5/9*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^18/(b*x^2+a)-5/8*a^2*b^3
*((b*x^2+a)^2)^(1/2)/x^16/(b*x^2+a)-5/14*a*b^4*((b*x^2+a)^2)^(1/2)/x^14/(b
*x^2+a)-1/12*b^5*((b*x^2+a)^2)^(1/2)/x^12/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{23}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(252a^5+1386a^4bx^2+3080a^3b^2x^4+3465a^2b^3x^6+1980ab^4x^8+462b^5x^{10})}{5544x^{22}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
```

output
```
-1/5544*(Sqrt[(a + b*x^2)^2]*(252*a^5 + 1386*a^4*b*x^2 + 3080*a^3*b^2*x^4
+ 3465*a^2*b^3*x^6 + 1980*a*b^4*x^8 + 462*b^5*x^10))/(x^22*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{23}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^{23}}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{23}}dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{24}}dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5}{x^{24}}+\frac{5ba^4}{x^{22}}+\frac{10b^2a^3}{x^{20}}+\frac{10b^3a^2}{x^{18}}+\frac{5b^4a}{x^{16}}+\frac{b^5}{x^{14}}\right)dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{11x^{22}}-\frac{a^4b}{2x^{20}}-\frac{10a^3b^2}{9x^{18}}-\frac{5a^2b^3}{4x^{16}}-\frac{5ab^4}{7x^{14}}-\frac{b^5}{6x^{12}}\right)\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
```

output
```
((-1/11*a^5/x^22 - (a^4*b)/(2*x^20) - (10*a^3*b^2)/(9*x^18) - (5*a^2*b^3)/
(4*x^16) - (5*a*b^4)/(7*x^14) - b^5/(6*x^12))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 2.71 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26

method	result	size
pseudoelliptic	$-\frac{\left(\frac{11}{6} x^{10} b^{5}+\frac{55}{7} a \,x^{8} b^{4}+\frac{55}{4} a^{2} x^{6} b^{3}+\frac{110}{9} a^{3} x^{4} b^{2}+\frac{11}{2} x^{2} a^{4} b +a^{5}\right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{22 x^{22}}$	66
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(-\frac{1}{22} a^{5}-\frac{1}{4} x^{2} a^{4} b -\frac{5}{9} a^{3} x^{4} b^{2}-\frac{5}{8} a^{2} x^{6} b^{3}-\frac{5}{14} a \,x^{8} b^{4}-\frac{1}{12} x^{10} b^{5}\right)}{\left(b \,x^{2}+a \right) x^{22}}$	79
gosper	$-\frac{\left(462 x^{10} b^{5}+1980 a \,x^{8} b^{4}+3465 a^{2} x^{6} b^{3}+3080 a^{3} x^{4} b^{2}+1386 x^{2} a^{4} b +252 a^{5}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}{5544 x^{22} \left(b \,x^{2}+a \right)^{5}}$	80
default	$-\frac{\left(462 x^{10} b^{5}+1980 a \,x^{8} b^{4}+3465 a^{2} x^{6} b^{3}+3080 a^{3} x^{4} b^{2}+1386 x^{2} a^{4} b +252 a^{5}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}{5544 x^{22} \left(b \,x^{2}+a \right)^{5}}$	80
orering	$-\frac{\left(462 x^{10} b^{5}+1980 a \,x^{8} b^{4}+3465 a^{2} x^{6} b^{3}+3080 a^{3} x^{4} b^{2}+1386 x^{2} a^{4} b +252 a^{5}\right) \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{5}{2}}}{5544 x^{22} \left(b \,x^{2}+a \right)^{5}}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x,method=_RETURNVERBOSE)
```

output
```
-1/22*(11/6*x^10*b^5+55/7*a*x^8*b^4+55/4*a^2*x^6*b^3+110/9*a^3*x^4*b^2+11/
2*x^2*a^4*b+a^5)*csgn(b*x^2+a)/x^22
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{23}}\,dx = -\frac{462\,b^5x^{10}+1980\,ab^4x^8+3465\,a^2b^3x^6+3080\,a^3b^2x^4+1386\,a^4bx^2+252\,a^5}{5544\,x^{22}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="fricas")
```

output
```
-1/5544*(462*b^5*x^10 + 1980*a*b^4*x^8 + 3465*a^2*b^3*x^6 + 3080*a^3*b^2*x
^4 + 1386*a^4*b*x^2 + 252*a^5)/x^22
```

Sympy [F]

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{23}}\,dx = \int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{23}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**23,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**23, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{23}}\,dx = -\frac{b^5}{12\,x^{12}}-\frac{5\,ab^4}{14\,x^{14}}-\frac{5\,a^2b^3}{8\,x^{16}}-\frac{5\,a^3b^2}{9\,x^{18}}-\frac{a^4b}{4\,x^{20}}-\frac{a^5}{22\,x^{22}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="maxima")
```

output
```
-1/12*b^5/x^12 - 5/14*a*b^4/x^14 - 5/8*a^2*b^3/x^16 - 5/9*a^3*b^2/x^18 - 1
/4*a^4*b/x^20 - 1/22*a^5/x^22
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{23}}\,dx = -\frac{462\,b^5x^{10}\mathrm{sgn}(bx^2+a)+1980\,ab^4x^8\mathrm{sgn}(bx^2+a)+3465\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+3080\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+}{5544\,x^{22}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x, algorithm="giac")
```

output
```
-1/5544*(462*b^5*x^10*sgn(b*x^2 + a) + 1980*a*b^4*x^8*sgn(b*x^2 + a) + 346
5*a^2*b^3*x^6*sgn(b*x^2 + a) + 3080*a^3*b^2*x^4*sgn(b*x^2 + a) + 1386*a^4*
b*x^2*sgn(b*x^2 + a) + 252*a^5*sgn(b*x^2 + a))/x^22
```

Mupad [B] (verification not implemented)

Time = 18.06 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{23}}\,dx &= -\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{22\,x^{22}\,(b\,x^2+a)} \\ &- \frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{12\,x^{12}\,(b\,x^2+a)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,(b\,x^2+a)} \\ &- \frac{a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,x^{20}\,(b\,x^2+a)} - \frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,x^{16}\,(b\,x^2+a)} \\ &- \frac{5\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^{18}\,(b\,x^2+a)}\end{aligned}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^23,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(22*x^22*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(12*x^12*(a + b*x^2)) - (5*a*b^4*(a^2 + b^
2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (a^4*b*(a^2 + b^2*x^4 +
2*a*b*x^2)^(1/2))/(4*x^20*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(8*x^16*(a + b*x^2)) - (5*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)
^(1/2))/(9*x^18*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{23}}\,dx = \frac{-462b^5x^{10}-1980a\,b^4x^8-3465a^2b^3x^6-3080a^3b^2x^4-1386a^4b\,x^2-252a^5}{5544x^{22}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x)
```

output
```
( - 252*a**5 - 1386*a**4*b*x**2 - 3080*a**3*b**2*x**4 - 3465*a**2*b**3*x**
6 - 1980*a*b**4*x**8 - 462*b**5*x**10)/(5544*x**22)
```

3.551 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{25}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{25}}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}(a+bx^2)} - \frac{a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^{20}(a+bx^2)} - \frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}(a+bx^2)} - \frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}(a+bx^2)} - \frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}$$

output
```
-1/24*a^5*((b*x^2+a)^2)^(1/2)/x^24/(b*x^2+a)-5/22*a^4*b*((b*x^2+a)^2)^(1/2
)/x^22/(b*x^2+a)-1/2*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^20/(b*x^2+a)-5/9*a^2*b^
3*((b*x^2+a)^2)^(1/2)/x^18/(b*x^2+a)-5/16*a*b^4*((b*x^2+a)^2)^(1/2)/x^16/(
b*x^2+a)-1/14*b^5*((b*x^2+a)^2)^(1/2)/x^14/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{25}}\,dx = -\frac{\sqrt{(a+bx^2)^2}\,(462a^5+2520a^4bx^2+5544a^3b^2x^4+6160a^2b^3x^6+3465ab^4x^8+792b^5x^{10})}{11088x^{24}\,(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

output
```
-1/11088*(Sqrt[(a + b*x^2)^2]*(462*a^5 + 2520*a^4*b*x^2 + 5544*a^3*b^2*x^4
 + 6160*a^2*b^3*x^6 + 3465*a*b^4*x^8 + 792*b^5*x^10))/(x^24*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{25}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^{25}}dx}{b^5\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{25}}dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\frac{(bx^2+a)^5}{x^{26}}dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{a^5}{x^{26}}+\frac{5ba^4}{x^{24}}+\frac{10b^2a^3}{x^{22}}+\frac{10b^3a^2}{x^{20}}+\frac{5b^4a}{x^{18}}+\frac{b^5}{x^{16}}\right)dx^2}{2(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{12x^{24}}-\frac{5a^4b}{11x^{22}}-\frac{a^3b^2}{x^{20}}-\frac{10a^2b^3}{9x^{18}}-\frac{5ab^4}{8x^{16}}-\frac{b^5}{7x^{14}}\right)\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

output
```
((-1/12*a^5/x^24 - (5*a^4*b)/(11*x^22) - (a^3*b^2)/x^20 - (10*a^2*b^3)/(9*
x^18) - (5*a*b^4)/(8*x^16) - b^5/(7*x^14))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(2*(a + b*x^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 4.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26

method	result	size
pseudoelliptic	$-\frac{\left(\frac{12}{7}x^{10}b^{5}+\frac{15}{2}a\,x^{8}b^{4}+\frac{40}{3}a^{2}x^{6}b^{3}+12a^{3}x^{4}b^{2}+\frac{60}{11}x^{2}a^{4}b+a^{5}\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{24x^{24}}$	66
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{5}{22}x^{2}a^{4}b-\frac{1}{24}a^{5}-\frac{1}{2}a^{3}x^{4}b^{2}-\frac{5}{16}a\,x^{8}b^{4}-\frac{5}{9}a^{2}x^{6}b^{3}-\frac{1}{14}x^{10}b^{5}\right)}{\left(b\,x^{2}+a\right)x^{24}}$	79
gosper	$-\frac{\left(792x^{10}b^{5}+3465a\,x^{8}b^{4}+6160a^{2}x^{6}b^{3}+5544a^{3}x^{4}b^{2}+2520x^{2}a^{4}b+462a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{11088x^{24}\left(b\,x^{2}+a\right)^{5}}$	80
default	$-\frac{\left(792x^{10}b^{5}+3465a\,x^{8}b^{4}+6160a^{2}x^{6}b^{3}+5544a^{3}x^{4}b^{2}+2520x^{2}a^{4}b+462a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{11088x^{24}\left(b\,x^{2}+a\right)^{5}}$	80
orering	$-\frac{\left(792x^{10}b^{5}+3465a\,x^{8}b^{4}+6160a^{2}x^{6}b^{3}+5544a^{3}x^{4}b^{2}+2520x^{2}a^{4}b+462a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{11088x^{24}\left(b\,x^{2}+a\right)^{5}}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(12/7*x^10*b^5+15/2*a*x^8*b^4+40/3*a^2*x^6*b^3+12*a^3*x^4*b^2+60/11*
x^2*a^4*b+a^5)*csgn(b*x^2+a)/x^24
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{25}}\,dx = -\frac{792\,b^5x^{10}+3465\,ab^4x^8+6160\,a^2b^3x^6+5544\,a^3b^2x^4+2520\,a^4bx^2+462\,a^5}{11088\,x^{24}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="fricas")
```

output
```
-1/11088*(792*b^5*x^10 + 3465*a*b^4*x^8 + 6160*a^2*b^3*x^6 + 5544*a^3*b^2*
x^4 + 2520*a^4*b*x^2 + 462*a^5)/x^24
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{25}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^{25}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**25,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**25, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{25}}\,dx = -\frac{b^5}{14\,x^{14}}-\frac{5\,ab^4}{16\,x^{16}}-\frac{5\,a^2b^3}{9\,x^{18}}-\frac{a^3b^2}{2\,x^{20}}-\frac{5\,a^4b}{22\,x^{22}}-\frac{a^5}{24\,x^{24}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="maxima")
```

output
```
-1/14*b^5/x^14 - 5/16*a*b^4/x^16 - 5/9*a^2*b^3/x^18 - 1/2*a^3*b^2/x^20 - 5
/22*a^4*b/x^22 - 1/24*a^5/x^24
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{25}}\,dx = -\frac{792\,b^5x^{10}\mathrm{sgn}(bx^2+a)+3465\,ab^4x^8\mathrm{sgn}(bx^2+a)+6160\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+5544\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+}{11088\,x^{24}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x, algorithm="giac")
```

output
```
-1/11088*(792*b^5*x^10*sgn(b*x^2 + a) + 3465*a*b^4*x^8*sgn(b*x^2 + a) + 61
60*a^2*b^3*x^6*sgn(b*x^2 + a) + 5544*a^3*b^2*x^4*sgn(b*x^2 + a) + 2520*a^4
*b*x^2*sgn(b*x^2 + a) + 462*a^5*sgn(b*x^2 + a))/x^24
```

Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{25}}\,dx &= -\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{24\,x^{24}\,\left(b\,x^2+a\right)} \\ &- \frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{14\,x^{14}\,\left(b\,x^2+a\right)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{16\,x^{16}\,\left(b\,x^2+a\right)} \\ &- \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{22\,x^{22}\,\left(b\,x^2+a\right)} \\ &- \frac{5\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^{18}\,\left(b\,x^2+a\right)} - \frac{a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,x^{20}\,\left(b\,x^2+a\right)}\end{aligned}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^25,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(24*x^24*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(14*x^14*(a + b*x^2)) - (5*a*b^4*(a^2 + b^
2*x^4 + 2*a*b*x^2)^(1/2))/(16*x^16*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(22*x^22*(a + b*x^2)) - (5*a^2*b^3*(a^2 + b^2*x^4 + 2*
a*b*x^2)^(1/2))/(9*x^18*(a + b*x^2)) - (a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(2*x^20*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{25}}\,dx = \frac{-792b^5x^{10} - 3465a\,b^4x^8 - 6160a^2b^3x^6 - 5544a^3b^2x^4 - 2520a^4b\,x^2 - 462a^5}{11088x^{24}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x)
```

output
```
( - 462*a**5 - 2520*a**4*b*x**2 - 5544*a**3*b**2*x**4 - 6160*a**2*b**3*x**
6 - 3465*a*b**4*x**8 - 792*b**5*x**10)/(11088*x**24)
```

3.552 $\int x^{12}(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^{12}(a^2+2abx^2+b^2x^4)^{5/2}\,dx = \frac{a^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13(a+bx^2)} + \frac{a^4bx^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)} + \frac{10a^3b^2x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17(a+bx^2)} + \frac{10a^2b^3x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19(a+bx^2)} + \frac{5ab^4x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21(a+bx^2)} + \frac{b^5x^{23}\sqrt{a^2+2abx^2+b^2x^4}}{23(a+bx^2)}$$

output
```
a^5*x^13*((b*x^2+a)^2)^(1/2)/(13*b*x^2+13*a)+a^4*b*x^15*((b*x^2+a)^2)^(1/2
)/(3*b*x^2+3*a)+10*a^3*b^2*x^17*((b*x^2+a)^2)^(1/2)/(17*b*x^2+17*a)+10*a^2
*b^3*x^19*((b*x^2+a)^2)^(1/2)/(19*b*x^2+19*a)+5*a*b^4*x^21*((b*x^2+a)^2)^(
1/2)/(21*b*x^2+21*a)+b^5*x^23*((b*x^2+a)^2)^(1/2)/(23*b*x^2+23*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^{12}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^{13}\sqrt{(a+bx^2)^2}(156009a^5+676039a^4bx^2+1193010a^3b^2x^4+1067430a^2b^3x^6+482885ab^4}{2028117\,(a+bx^2)}$$

input
```
Integrate[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^13*Sqrt[(a + b*x^2)^2]*(156009*a^5 + 676039*a^4*b*x^2 + 1193010*a^3*b^2
*x^4 + 1067430*a^2*b^3*x^6 + 482885*a*b^4*x^8 + 88179*b^5*x^10))/(2028117*
(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5x^{12}\left(bx^2+a\right)^5dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^{12}\left(bx^2+a\right)^5dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(b^5x^{22}+5ab^4x^{20}+10a^2b^3x^{18}+10a^3b^2x^{16}+5a^4bx^{14}+a^5x^{12}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^5x^{13}}{13}+\frac{1}{3}a^4bx^{15}+\frac{10}{17}a^3b^2x^{17}+\frac{10}{19}a^2b^3x^{19}+\frac{5}{21}ab^4x^{21}+\frac{b^5x^{23}}{23}\right)}{a+bx^2}$$

input
```
Int[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^5*x^13)/13 + (a^4*b*x^15)/3 + (10*a^3
*b^2*x^17)/17 + (10*a^2*b^3*x^19)/19 + (5*a*b^4*x^21)/21 + (b^5*x^23)/23))
/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
 - 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 5.99 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{13}\left(88179x^{10}b^{5}+482885a\,x^{8}b^{4}+1067430a^{2}x^{6}b^{3}+1193010a^{3}x^{4}b^{2}+676039x^{2}a^{4}b+156009a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{2028117\left(b\,x^{2}+a\right)^{5}}$
default	$\frac{x^{13}\left(88179x^{10}b^{5}+482885a\,x^{8}b^{4}+1067430a^{2}x^{6}b^{3}+1193010a^{3}x^{4}b^{2}+676039x^{2}a^{4}b+156009a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{2028117\left(b\,x^{2}+a\right)^{5}}$
orering	$\frac{x^{13}\left(88179x^{10}b^{5}+482885a\,x^{8}b^{4}+1067430a^{2}x^{6}b^{3}+1193010a^{3}x^{4}b^{2}+676039x^{2}a^{4}b+156009a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{2028117\left(b\,x^{2}+a\right)^{5}}$
risch	$\frac{a^{5}x^{13}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{13b\,x^{2}+13a}+\frac{a^{4}b\,x^{15}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{3b\,x^{2}+3a}+\frac{10\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{3}b^{2}x^{17}}{17\left(b\,x^{2}+a\right)}+\frac{10\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}b^{3}x^{19}}{19\left(b\,x^{2}+a\right)}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{4}a\,x^{21}}{21\left(b\,x^{2}+a\right)}+$

input
```
int(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2028117*x^13*(88179*b^5*x^10+482885*a*b^4*x^8+1067430*a^2*b^3*x^6+119301
0*a^3*b^2*x^4+676039*a^4*b*x^2+156009*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{12}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{5/2}\,dx=\frac{1}{23}\,b^{5}x^{23}+\frac{5}{21}\,ab^{4}x^{21}+\frac{10}{19}\,a^{2}b^{3}x^{19}+\frac{10}{17}\,a^{3}b^{2}x^{17}+\frac{1}{3}\,a^{4}bx^{15}+\frac{1}{13}\,a^{5}x^{13}$$

input
```
integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/23*b^5*x^23 + 5/21*a*b^4*x^21 + 10/19*a^2*b^3*x^19 + 10/17*a^3*b^2*x^17
+ 1/3*a^4*b*x^15 + 1/13*a^5*x^13
```

Sympy [F]

$$\int x^{12}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^{12}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**12*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**12*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{12}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{23}\,b^5x^{23}+\frac{5}{21}\,ab^4x^{21}+\frac{10}{19}\,a^2b^3x^{19}+\frac{10}{17}\,a^3b^2x^{17}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{13}\,a^5x^{13}$$

input
```
integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/23*b^5*x^23 + 5/21*a*b^4*x^21 + 10/19*a^2*b^3*x^19 + 10/17*a^3*b^2*x^17
+ 1/3*a^4*b*x^15 + 1/13*a^5*x^13
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^{12}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{23}\,b^5x^{23}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{21}\,ab^4x^{21}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{19}\,a^2b^3x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{17}\,a^3b^2x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{3}\,a^4bx^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{13}\,a^5x^{13}\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/23*b^5*x^23*sgn(b*x^2 + a) + 5/21*a*b^4*x^21*sgn(b*x^2 + a) + 10/19*a^2*
b^3*x^19*sgn(b*x^2 + a) + 10/17*a^3*b^2*x^17*sgn(b*x^2 + a) + 1/3*a^4*b*x^
15*sgn(b*x^2 + a) + 1/13*a^5*x^13*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{12}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^{12}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

input
```
int(x^12*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^12*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int x^{12}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^{13}(88179b^5x^{10}+482885a\,b^4x^8+1067430a^2b^3x^6+1193010a^3b^2x^4+676039a^4b\,x^2+15600}{2028117}$$

input
```
int(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**13*(156009*a**5 + 676039*a**4*b*x**2 + 1193010*a**3*b**2*x**4 + 106743
0*a**2*b**3*x**6 + 482885*a*b**4*x**8 + 88179*b**5*x**10))/2028117
```

3.553 $\int x^{10}(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{a^5x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\,(a+bx^2)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{2a^3b^2x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\,(a+bx^2)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\,(a+bx^2)}+\frac{b^5x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21\,(a+bx^2)}$$

output
```
a^5*x^11*((b*x^2+a)^2)^(1/2)/(11*b*x^2+11*a)+5*a^4*b*x^13*((b*x^2+a)^2)^(1
/2)/(13*b*x^2+13*a)+2*a^3*b^2*x^15*((b*x^2+a)^2)^(1/2)/(3*b*x^2+3*a)+10*a^
2*b^3*x^17*((b*x^2+a)^2)^(1/2)/(17*b*x^2+17*a)+5*a*b^4*x^19*((b*x^2+a)^2)^
(1/2)/(19*b*x^2+19*a)+b^5*x^21*((b*x^2+a)^2)^(1/2)/(21*b*x^2+21*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^{11}\sqrt{(a+bx^2)^2}(88179a^5+373065a^4bx^2+646646a^3b^2x^4+570570a^2b^3x^6+255255ab^4x^8+\cdots}{969969\,(a+bx^2)}$$

input
```
Integrate[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^11*Sqrt[(a + b*x^2)^2]*(88179*a^5 + 373065*a^4*b*x^2 + 646646*a^3*b^2*x
^4 + 570570*a^2*b^3*x^6 + 255255*a*b^4*x^8 + 46189*b^5*x^10))/(969969*(a +
 b*x^2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}dx$$

$$\Big\downarrow \; 1384$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5x^{10}\left(bx^2+a\right)^5dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^{10}\left(bx^2+a\right)^5dx}{a+bx^2}$$

$$\Big\downarrow \; 244$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(b^5x^{20}+5ab^4x^{18}+10a^2b^3x^{16}+10a^3b^2x^{14}+5a^4bx^{12}+a^5x^{10}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^5x^{11}}{11}+\frac{5}{13}a^4bx^{13}+\frac{2}{3}a^3b^2x^{15}+\frac{10}{17}a^2b^3x^{17}+\frac{5}{19}ab^4x^{19}+\frac{b^5x^{21}}{21}\right)}{a+bx^2}$$

input
```
Int[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^5*x^11)/11 + (5*a^4*b*x^13)/13 + (2*a
^3*b^2*x^15)/3 + (10*a^2*b^3*x^17)/17 + (5*a*b^4*x^19)/19 + (b^5*x^21)/21)
)/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{11}\left(46189x^{10}b^{5}+255255a\,x^{8}b^{4}+570570a^{2}x^{6}b^{3}+646646a^{3}x^{4}b^{2}+373065x^{2}a^{4}b+88179a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{969969\left(b\,x^{2}+a\right)^{5}}$
default	$\frac{x^{11}\left(46189x^{10}b^{5}+255255a\,x^{8}b^{4}+570570a^{2}x^{6}b^{3}+646646a^{3}x^{4}b^{2}+373065x^{2}a^{4}b+88179a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{969969\left(b\,x^{2}+a\right)^{5}}$
orering	$\frac{x^{11}\left(46189x^{10}b^{5}+255255a\,x^{8}b^{4}+570570a^{2}x^{6}b^{3}+646646a^{3}x^{4}b^{2}+373065x^{2}a^{4}b+88179a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{969969\left(b\,x^{2}+a\right)^{5}}$
risch	$\frac{a^{5}x^{11}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{11b\,x^{2}+11a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b\,a^{4}x^{13}}{13\left(b\,x^{2}+a\right)}+\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{3}b^{2}x^{15}}{3\left(b\,x^{2}+a\right)}+\frac{10\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}b^{3}x^{17}}{17\left(b\,x^{2}+a\right)}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{4}a\,x^{19}}{19\left(b\,x^{2}+a\right)}+$

input
```
int(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/969969*x^11*(46189*b^5*x^10+255255*a*b^4*x^8+570570*a^2*b^3*x^6+646646*a
^3*b^2*x^4+373065*a^4*b*x^2+88179*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{10}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{5/2}\,dx=\frac{1}{21}\,b^{5}x^{21}+\frac{5}{19}\,ab^{4}x^{19}+\frac{10}{17}\,a^{2}b^{3}x^{17}+\frac{2}{3}\,a^{3}b^{2}x^{15}+\frac{5}{13}\,a^{4}bx^{13}+\frac{1}{11}\,a^{5}x^{11}$$

input
```
integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/21*b^5*x^21 + 5/19*a*b^4*x^19 + 10/17*a^2*b^3*x^17 + 2/3*a^3*b^2*x^15 +
5/13*a^4*b*x^13 + 1/11*a^5*x^11
```

Sympy [F]

$$\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^{10}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**10*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**10*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{21}\,b^5x^{21}+\frac{5}{19}\,ab^4x^{19}+\frac{10}{17}\,a^2b^3x^{17}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{11}\,a^5x^{11}$$

input
```
integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/21*b^5*x^21 + 5/19*a*b^4*x^19 + 10/17*a^2*b^3*x^17 + 2/3*a^3*b^2*x^15 +
5/13*a^4*b*x^13 + 1/11*a^5*x^11
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{21}\,b^5x^{21}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{19}\,ab^4x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{17}\,a^2b^3x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\,a^3b^2x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{13}\,a^4bx^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{11}\,a^5x^{11}\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/21*b^5*x^21*sgn(b*x^2 + a) + 5/19*a*b^4*x^19*sgn(b*x^2 + a) + 10/17*a^2*
b^3*x^17*sgn(b*x^2 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^2 + a) + 5/13*a^4*b*x^1
3*sgn(b*x^2 + a) + 1/11*a^5*x^11*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^{10}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

input
```
int(x^10*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^10*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int x^{10}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^{11}\left(46189b^5x^{10}+255255a\,b^4x^8+570570a^2b^3x^6+646646a^3b^2x^4+373065a^4b\,x^2+88179a^5\right)}{969969}$$

input
```
int(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**11*(88179*a**5 + 373065*a**4*b*x**2 + 646646*a**3*b**2*x**4 + 570570*a
**2*b**3*x**6 + 255255*a*b**4*x**8 + 46189*b**5*x**10))/969969
```

3.554 $\int x^8(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\begin{aligned}\int x^8\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx &= \frac{a^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)} \\ &+ \frac{5a^4bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)} \\ &+ \frac{2a^2b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)} \\ &+ \frac{5ab^4x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}+\frac{b^5x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19\left(a+bx^2\right)}\end{aligned}$$

output
```
a^5*x^9*((b*x^2+a)^2)^(1/2)/(9*b*x^2+9*a)+5*a^4*b*x^11*((b*x^2+a)^2)^(1/2)
/(11*b*x^2+11*a)+10*a^3*b^2*x^13*((b*x^2+a)^2)^(1/2)/(13*b*x^2+13*a)+2*a^2
*b^3*x^15*((b*x^2+a)^2)^(1/2)/(3*b*x^2+3*a)+5*a*b^4*x^17*((b*x^2+a)^2)^(1/
2)/(17*b*x^2+17*a)+b^5*x^19*((b*x^2+a)^2)^(1/2)/(19*b*x^2+19*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^8 (a^2 + 2abx^2 + b^2x^4)^{5/2}\, dx = \frac{x^9\sqrt{(a+bx^2)^2}(46189a^5 + 188955a^4bx^2 + 319770a^3b^2x^4 + 277134a^2b^3x^6 + 122265ab^4x^8 +}{415701\,(a+bx^2)}$$

input
```
Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^9*Sqrt[(a + b*x^2)^2]*(46189*a^5 + 188955*a^4*b*x^2 + 319770*a^3*b^2*x^
4 + 277134*a^2*b^3*x^6 + 122265*a*b^4*x^8 + 21879*b^5*x^10))/(415701*(a +
b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 (a^2 + 2abx^2 + b^2x^4)^{5/2}\, dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5x^8(bx^2+a)^5dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^8(bx^2+a)^5dx}{a+bx^2}$$

$$\Big\downarrow \ 244$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(b^5x^{18}+5ab^4x^{16}+10a^2b^3x^{14}+10a^3b^2x^{12}+5a^4bx^{10}+a^5x^8\right)dx}{a+bx^2}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}\right)}{a+bx^2}$$

input
```
Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^5*x^9)/9 + (5*a^4*b*x^11)/11 + (10*a^
3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3 + (5*a*b^4*x^17)/17 + (b^5*x^19)/19))/
(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{9}\left(21879x^{10}b^{5}+122265a\,x^{8}b^{4}+277134a^{2}x^{6}b^{3}+319770a^{3}x^{4}b^{2}+188955x^{2}a^{4}b+46189a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{415701\left(b\,x^{2}+a\right)^{5}}$
default	$\frac{x^{9}\left(21879x^{10}b^{5}+122265a\,x^{8}b^{4}+277134a^{2}x^{6}b^{3}+319770a^{3}x^{4}b^{2}+188955x^{2}a^{4}b+46189a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{415701\left(b\,x^{2}+a\right)^{5}}$
orering	$\frac{x^{9}\left(21879x^{10}b^{5}+122265a\,x^{8}b^{4}+277134a^{2}x^{6}b^{3}+319770a^{3}x^{4}b^{2}+188955x^{2}a^{4}b+46189a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{415701\left(b\,x^{2}+a\right)^{5}}$
risch	$\frac{a^{5}x^{9}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{9b\,x^{2}+9a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b\,a^{4}x^{11}}{11\left(b\,x^{2}+a\right)}+\frac{10\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{3}b^{2}x^{13}}{13\left(b\,x^{2}+a\right)}+\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}b^{3}x^{15}}{3\left(b\,x^{2}+a\right)}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{4}a\,x^{17}}{17\left(b\,x^{2}+a\right)}+\frac{b^{5}}{}$

input
```
int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/415701*x^9*(21879*b^5*x^10+122265*a*b^4*x^8+277134*a^2*b^3*x^6+319770*a^
3*b^2*x^4+188955*a^4*b*x^2+46189*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{8}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{5/2}\,dx=\frac{1}{19}\,b^{5}x^{19}+\frac{5}{17}\,ab^{4}x^{17}+\frac{2}{3}\,a^{2}b^{3}x^{15}+\frac{10}{13}\,a^{3}b^{2}x^{13}+\frac{5}{11}\,a^{4}bx^{11}+\frac{1}{9}\,a^{5}x^{9}$$

input
```
integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 +
5/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Sympy [F]

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^8 \left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**8*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

input
```
integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 +
5/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{19}\,b^5x^{19}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{17}\,ab^4x^{17}\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\,a^2b^3x^{15}\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{13}\,a^3b^2x^{13}\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{11}\,a^4bx^{11}\mathrm{sgn}\left(bx^2+a\right)+\frac{1}{9}\,a^5x^9\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/19*b^5*x^19*sgn(b*x^2 + a) + 5/17*a*b^4*x^17*sgn(b*x^2 + a) + 2/3*a^2*b^
3*x^15*sgn(b*x^2 + a) + 10/13*a^3*b^2*x^13*sgn(b*x^2 + a) + 5/11*a^4*b*x^1
1*sgn(b*x^2 + a) + 1/9*a^5*x^9*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx = \int x^8 \, \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2} \, dx$$

input
```
int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^8*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx = \frac{x^9(21879b^5x^{10}+122265a\,b^4x^8+277134a^2b^3x^6+319770a^3b^2x^4+188955a^4b\,x^2+46189a^5)}{415701}$$

input
```
int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**9*(46189*a**5 + 188955*a**4*b*x**2 + 319770*a**3*b**2*x**4 + 277134*a*
*2*b**3*x**6 + 122265*a*b**4*x**8 + 21879*b**5*x**10))/415701
```

3.555 $\int x^6(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^6\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{a^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{5a^4bx^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{ab^4x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{b^5x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\left(a+bx^2\right)}$$

output
```
a^5*x^7*((b*x^2+a)^2)^(1/2)/(7*b*x^2+7*a)+5*a^4*b*x^9*((b*x^2+a)^2)^(1/2)/
(9*b*x^2+9*a)+10*a^3*b^2*x^11*((b*x^2+a)^2)^(1/2)/(11*b*x^2+11*a)+10*a^2*b
^3*x^13*((b*x^2+a)^2)^(1/2)/(13*b*x^2+13*a)+a*b^4*x^15*((b*x^2+a)^2)^(1/2)
/(3*b*x^2+3*a)+b^5*x^17*((b*x^2+a)^2)^(1/2)/(17*b*x^2+17*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^6(a^2+2abx^2+b^2x^4)^{5/2}\,dx=\frac{x^7\sqrt{(a+bx^2)^2}(21879a^5+85085a^4bx^2+139230a^3b^2x^4+117810a^2b^3x^6+51051ab^4x^8+90$$

$$153153\,(a+bx^2)$$

input
```
Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^7*Sqrt[(a + b*x^2)^2]*(21879*a^5 + 85085*a^4*b*x^2 + 139230*a^3*b^2*x^4
 + 117810*a^2*b^3*x^6 + 51051*a*b^4*x^8 + 9009*b^5*x^10))/(153153*(a + b*x
^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6(a^2+2abx^2+b^2x^4)^{5/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5x^6(bx^2+a)^5dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^6(bx^2+a)^5dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(b^5x^{16}+5ab^4x^{14}+10a^2b^3x^{12}+10a^3b^2x^{10}+5a^4bx^8+a^5x^6\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}\right)}{a+bx^2}$$

input
```
Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^5*x^7)/7 + (5*a^4*b*x^9)/9 + (10*a^3*
b^2*x^11)/11 + (10*a^2*b^3*x^13)/13 + (a*b^4*x^15)/3 + (b^5*x^17)/17))/(a
+ b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{7}\left(9009x^{10}b^{5}+51051a\,x^{8}b^{4}+117810a^{2}x^{6}b^{3}+139230a^{3}x^{4}b^{2}+85085x^{2}a^{4}b+21879a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{153153\left(b\,x^{2}+a\right)^{5}}$
default	$\frac{x^{7}\left(9009x^{10}b^{5}+51051a\,x^{8}b^{4}+117810a^{2}x^{6}b^{3}+139230a^{3}x^{4}b^{2}+85085x^{2}a^{4}b+21879a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{153153\left(b\,x^{2}+a\right)^{5}}$
orering	$\frac{x^{7}\left(9009x^{10}b^{5}+51051a\,x^{8}b^{4}+117810a^{2}x^{6}b^{3}+139230a^{3}x^{4}b^{2}+85085x^{2}a^{4}b+21879a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{153153\left(b\,x^{2}+a\right)^{5}}$
risch	$\frac{a^{5}x^{7}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{7b\,x^{2}+7a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b\,a^{4}x^{9}}{9\left(b\,x^{2}+a\right)}+\frac{10\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{3}b^{2}x^{11}}{11\left(b\,x^{2}+a\right)}+\frac{10\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}b^{3}x^{13}}{13\left(b\,x^{2}+a\right)}+\frac{a\,b^{4}x^{15}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{3b\,x^{2}+3a}+\frac{b^{5}x}{}$

input
```
int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/153153*x^7*(9009*b^5*x^10+51051*a*b^4*x^8+117810*a^2*b^3*x^6+139230*a^3*
b^2*x^4+85085*a^4*b*x^2+21879*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^{6}\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{5/2}\,dx=\frac{1}{17}\,b^{5}x^{17}+\frac{1}{3}\,ab^{4}x^{15}+\frac{10}{13}\,a^{2}b^{3}x^{13}+\frac{10}{11}\,a^{3}b^{2}x^{11}+\frac{5}{9}\,a^{4}bx^{9}+\frac{1}{7}\,a^{5}x^{7}$$

input
```
integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
 5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Sympy [F]

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \int x^6 \left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}} dx$$

input
```
integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**6*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{17}b^5x^{17} + \frac{1}{3}ab^4x^{15} + \frac{10}{13}a^2b^3x^{13} + \frac{10}{11}a^3b^2x^{11} + \frac{5}{9}a^4bx^9 + \frac{1}{7}a^5x^7$$

input
```
integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
 5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{17}b^5x^{17}\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{3}ab^4x^{15}\mathrm{sgn}\left(bx^2+a\right) + \frac{10}{13}a^2b^3x^{13}\mathrm{sgn}\left(bx^2+a\right) + \frac{10}{11}a^3b^2x^{11}\mathrm{sgn}\left(bx^2+a\right) + \frac{5}{9}a^4bx^9\mathrm{sgn}\left(bx^2+a\right) + \frac{1}{7}a^5x^7\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/17*b^5*x^17*sgn(b*x^2 + a) + 1/3*a*b^4*x^15*sgn(b*x^2 + a) + 10/13*a^2*b
^3*x^13*sgn(b*x^2 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^2 + a) + 5/9*a^4*b*x^9
*sgn(b*x^2 + a) + 1/7*a^5*x^7*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int x^6\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}\,dx$$

input
```
int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^6*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x^7(9009b^5x^{10}+51051a\,b^4x^8+117810a^2b^3x^6+139230a^3b^2x^4+85085a^4b\,x^2+21879a^5)}{153153}$$

input
```
int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**7*(21879*a**5 + 85085*a**4*b*x**2 + 139230*a**3*b**2*x**4 + 117810*a**
2*b**3*x**6 + 51051*a*b**4*x**8 + 9009*b**5*x**10))/153153
```

3.556 $\int x^4(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{a^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{10a^3b^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{5ab^4x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{b^5x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}$$

output
```
a^5*x^5*((b*x^2+a)^2)^(1/2)/(5*b*x^2+5*a)+5*a^4*b*x^7*((b*x^2+a)^2)^(1/2)/
(7*b*x^2+7*a)+10*a^3*b^2*x^9*((b*x^2+a)^2)^(1/2)/(9*b*x^2+9*a)+10*a^2*b^3*
x^11*((b*x^2+a)^2)^(1/2)/(11*b*x^2+11*a)+5*a*b^4*x^13*((b*x^2+a)^2)^(1/2)/
(13*b*x^2+13*a)+b^5*x^15*((b*x^2+a)^2)^(1/2)/(15*b*x^2+15*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{x^5\sqrt{(a+bx^2)^2}(9009a^5+32175a^4bx^2+50050a^3b^2x^4+40950a^2b^3x^6+17325ab^4x^8+3003b}{45045(a+bx^2)}$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^5*Sqrt[(a + b*x^2)^2]*(9009*a^5 + 32175*a^4*b*x^2 + 50050*a^3*b^2*x^4 +
 40950*a^2*b^3*x^6 + 17325*a*b^4*x^8 + 3003*b^5*x^10))/(45045*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5x^4\left(bx^2+a\right)^5dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^4\left(bx^2+a\right)^5dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^5x^{14}+5ab^4x^{12}+10a^2b^3x^{10}+10a^3b^2x^8+5a^4bx^6+a^5x^4\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}\right)}{a+bx^2}$$

input
```
Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^5*x^5)/5 + (5*a^4*b*x^7)/7 + (10*a^3*
b^2*x^9)/9 + (10*a^2*b^3*x^11)/11 + (5*a*b^4*x^13)/13 + (b^5*x^15)/15))/(a
 + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x^{5}\left(3003x^{10}b^{5}+17325a\,x^{8}b^{4}+40950a^{2}x^{6}b^{3}+50050a^{3}x^{4}b^{2}+32175x^{2}a^{4}b+9009a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{45045\left(b\,x^{2}+a\right)^{5}}$
default	$\frac{x^{5}\left(3003x^{10}b^{5}+17325a\,x^{8}b^{4}+40950a^{2}x^{6}b^{3}+50050a^{3}x^{4}b^{2}+32175x^{2}a^{4}b+9009a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{45045\left(b\,x^{2}+a\right)^{5}}$
orering	$\frac{x^{5}\left(3003x^{10}b^{5}+17325a\,x^{8}b^{4}+40950a^{2}x^{6}b^{3}+50050a^{3}x^{4}b^{2}+32175x^{2}a^{4}b+9009a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{45045\left(b\,x^{2}+a\right)^{5}}$
risch	$\frac{a^{5}x^{5}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{5b\,x^{2}+5a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b\,a^{4}x^{7}}{7\left(b\,x^{2}+a\right)}+\frac{10\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{3}b^{2}x^{9}}{9\left(b\,x^{2}+a\right)}+\frac{10\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}b^{3}x^{11}}{11\left(b\,x^{2}+a\right)}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{4}a\,x^{13}}{13\left(b\,x^{2}+a\right)}+\frac{b^{5}x}{}$

input
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/45045*x^5*(3003*b^5*x^10+17325*a*b^4*x^8+40950*a^2*b^3*x^6+50050*a^3*b^2
*x^4+32175*a^4*b*x^2+9009*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 +
5/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Sympy [F]

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \int x^4 \left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}} dx$$

input
```
integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**4*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{15} b^5 x^{15} + \frac{5}{13} a b^4 x^{13} + \frac{10}{11} a^2 b^3 x^{11} + \frac{10}{9} a^3 b^2 x^9 + \frac{5}{7} a^4 b x^7 + \frac{1}{5} a^5 x^5$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 +
5/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.41

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{15} b^5 x^{15} \operatorname{sgn}\left(bx^2+a\right) + \frac{5}{13} a b^4 x^{13} \operatorname{sgn}\left(bx^2+a\right) + \frac{10}{11} a^2 b^3 x^{11} \operatorname{sgn}\left(bx^2+a\right) + \frac{10}{9} a^3 b^2 x^9 \operatorname{sgn}\left(bx^2+a\right) + \frac{5}{7} a^4 b x^7 \operatorname{sgn}\left(bx^2+a\right) + \frac{1}{5} a^5 x^5 \operatorname{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/15*b^5*x^15*sgn(b*x^2 + a) + 5/13*a*b^4*x^13*sgn(b*x^2 + a) + 10/11*a^2*
b^3*x^11*sgn(b*x^2 + a) + 10/9*a^3*b^2*x^9*sgn(b*x^2 + a) + 5/7*a^4*b*x^7*
sgn(b*x^2 + a) + 1/5*a^5*x^5*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx = \int x^4 \,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2} \,dx$$

input
```
int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx = \frac{x^5(3003b^5x^{10}+17325a\,b^4x^8+40950a^2b^3x^6+50050a^3b^2x^4+32175a^4b\,x^2+9009a^5)}{45045}$$

input
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**5*(9009*a**5 + 32175*a**4*b*x**2 + 50050*a**3*b**2*x**4 + 40950*a**2*b
**3*x**6 + 17325*a*b**4*x**8 + 3003*b**5*x**10))/45045
```

3.557 $\int x^2(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{a^4bx^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{b^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}$$

output

```
a^5*x^3*((b*x^2+a)^2)^(1/2)/(3*b*x^2+3*a)+a^4*b*x^5*((b*x^2+a)^2)^(1/2)/(b
*x^2+a)+10*a^3*b^2*x^7*((b*x^2+a)^2)^(1/2)/(7*b*x^2+7*a)+10*a^2*b^3*x^9*((
b*x^2+a)^2)^(1/2)/(9*b*x^2+9*a)+5*a*b^4*x^11*((b*x^2+a)^2)^(1/2)/(11*b*x^2
+11*a)+b^5*x^13*((b*x^2+a)^2)^(1/2)/(13*b*x^2+13*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int x^2(a^2+2abx^2+b^2x^4)^{5/2}\,dx=\frac{x^3\sqrt{(a+bx^2)^2}(3003a^5+9009a^4bx^2+12870a^3b^2x^4+10010a^2b^3x^6+4095ab^4x^8+693b^5x^{10})}{9009\,(a+bx^2)}$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^3*Sqrt[(a + b*x^2)^2]*(3003*a^5 + 9009*a^4*b*x^2 + 12870*a^3*b^2*x^4 +
10010*a^2*b^3*x^6 + 4095*a*b^4*x^8 + 693*b^5*x^10))/(9009*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a^2+2abx^2+b^2x^4)^{5/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5x^2(bx^2+a)^5dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2(bx^2+a)^5dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (b^5x^{12}+5ab^4x^{10}+10a^2b^3x^8+10a^3b^2x^6+5a^4bx^4+a^5x^2)\,dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}\right)}{a+bx^2}$$

input
```
Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^5*x^3)/3 + a^4*b*x^5 + (10*a^3*b^2*x^
7)/7 + (10*a^2*b^3*x^9)/9 + (5*a*b^4*x^11)/11 + (b^5*x^13)/13))/(a + b*x^2
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result
gosper	$\frac{x^{3}\left(693x^{10}b^{5}+4095a\,x^{8}b^{4}+10010a^{2}x^{6}b^{3}+12870a^{3}x^{4}b^{2}+9009x^{2}a^{4}b+3003a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{9009\left(b\,x^{2}+a\right)^{5}}$
default	$\frac{x^{3}\left(693x^{10}b^{5}+4095a\,x^{8}b^{4}+10010a^{2}x^{6}b^{3}+12870a^{3}x^{4}b^{2}+9009x^{2}a^{4}b+3003a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{9009\left(b\,x^{2}+a\right)^{5}}$
orering	$\frac{x^{3}\left(693x^{10}b^{5}+4095a\,x^{8}b^{4}+10010a^{2}x^{6}b^{3}+12870a^{3}x^{4}b^{2}+9009x^{2}a^{4}b+3003a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{9009\left(b\,x^{2}+a\right)^{5}}$
risch	$\frac{b^{5}x^{13}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{13b\,x^{2}+13a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a\,b^{4}x^{11}}{11\left(b\,x^{2}+a\right)}+\frac{10\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}b^{3}x^{9}}{9\left(b\,x^{2}+a\right)}+\frac{10\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{3}b^{2}x^{7}}{7\left(b\,x^{2}+a\right)}+\frac{a^{4}b\,x^{5}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}+\frac{a^{5}x^{3}}{3}$

input
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/9009*x^3*(693*b^5*x^10+4095*a*b^4*x^8+10010*a^2*b^3*x^6+12870*a^3*b^2*x^
4+9009*a^4*b*x^2+3003*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^
4*b*x^5 + 1/3*a^5*x^3
```

Sympy [F]

$$\int x^2 \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \int x^2 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}} dx$$

input
```
integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**2*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int x^2 \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \frac{1}{13} b^5 x^{13} + \frac{5}{11} ab^4 x^{11} + \frac{10}{9} a^2 b^3 x^9 + \frac{10}{7} a^3 b^2 x^7 + a^4 b x^5 + \frac{1}{3} a^5 x^3$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^
4*b*x^5 + 1/3*a^5*x^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41

$$\int x^2 \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \frac{1}{13} b^5 x^{13} \mathrm{sgn}\left(bx^2 + a\right) + \frac{5}{11} ab^4 x^{11} \mathrm{sgn}\left(bx^2 + a\right) + \frac{10}{9} a^2 b^3 x^9 \mathrm{sgn}\left(bx^2 + a\right) + \frac{10}{7} a^3 b^2 x^7 \mathrm{sgn}\left(bx^2 + a\right) + a^4 b x^5 \mathrm{sgn}\left(bx^2 + a\right) + \frac{1}{3} a^5 x^3 \mathrm{sgn}\left(bx^2 + a\right)$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/13*b^5*x^13*sgn(b*x^2 + a) + 5/11*a*b^4*x^11*sgn(b*x^2 + a) + 10/9*a^2*b
^3*x^9*sgn(b*x^2 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^2 + a) + a^4*b*x^5*sgn(b*
x^2 + a) + 1/3*a^5*x^3*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a^2+2abx^2+b^2x^4)^{5/2}\,dx=\int x^2\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

input
```
int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int x^2(a^2+2abx^2+b^2x^4)^{5/2}\,dx=\frac{x^3(693b^5x^{10}+4095a\,b^4x^8+10010a^2b^3x^6+12870a^3b^2x^4+9009a^4b\,x^2+3003a^5)}{9009}$$

input
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**3*(3003*a**5 + 9009*a**4*b*x**2 + 12870*a**3*b**2*x**4 + 10010*a**2*b*
*3*x**6 + 4095*a*b**4*x**8 + 693*b**5*x**10))/9009
```

3.558 $\int \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 248

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{a^5x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{5a^4bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)} + \frac{2a^3b^2x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{10a^2b^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)} + \frac{5ab^4x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)} + \frac{b^5x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}$$

output
```
a^5*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+5*a^4*b*x^3*((b*x^2+a)^2)^(1/2)/(3*b*x
^2+3*a)+2*a^3*b^2*x^5*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10*a^2*b^3*x^7*((b*x^2
+a)^2)^(1/2)/(7*b*x^2+7*a)+5*a*b^4*x^9*((b*x^2+a)^2)^(1/2)/(9*b*x^2+9*a)+b
^5*x^11*((b*x^2+a)^2)^(1/2)/(11*b*x^2+11*a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.33

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{\sqrt{(a+bx^2)^2}\left(693a^5x+1155a^4bx^3+1386a^3b^2x^5+990a^2b^3x^7+385ab^4x^9+63b^5x^{11}\right)}{693\,(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(693*a^5*x + 1155*a^4*b*x^3 + 1386*a^3*b^2*x^5 + 990*
a^2*b^3*x^7 + 385*a*b^4*x^9 + 63*b^5*x^11))/(693*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.41, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1384, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^2x^2+ab\right)^5dx}{b^5\,(a+bx^2)}$$

$$\Big\downarrow \text{210}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^{10}x^{10}+5ab^9x^8+10a^2b^8x^6+10a^3b^7x^4+5a^4b^6x^2+a^5b^5\right)dx}{b^5\,(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a^5b^5x+\frac{5}{3}a^4b^6x^3+2a^3b^7x^5+\frac{10}{7}a^2b^8x^7+\frac{5}{9}ab^9x^9+\frac{b^{10}x^{11}}{11}\right)}{b^5\,(a+bx^2)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(a^5*b^5*x + (5*a^4*b^6*x^3)/3 + 2*a^3*b^
7*x^5 + (10*a^2*b^8*x^7)/7 + (5*a*b^9*x^9)/9 + (b^10*x^11)/11))/(b^5*(a +
b*x^2))
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.31

method	result
gosper	$\frac{x\left(63x^{10}b^{5}+385a\,x^{8}b^{4}+990a^{2}x^{6}b^{3}+1386a^{3}x^{4}b^{2}+1155x^{2}a^{4}b+693a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{693\left(b\,x^{2}+a\right)^{5}}$
default	$\frac{x\left(63x^{10}b^{5}+385a\,x^{8}b^{4}+990a^{2}x^{6}b^{3}+1386a^{3}x^{4}b^{2}+1155x^{2}a^{4}b+693a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{693\left(b\,x^{2}+a\right)^{5}}$
orering	$\frac{x\left(63x^{10}b^{5}+385a\,x^{8}b^{4}+990a^{2}x^{6}b^{3}+1386a^{3}x^{4}b^{2}+1155x^{2}a^{4}b+693a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{693\left(b\,x^{2}+a\right)^{5}}$
risch	$\frac{b^{5}x^{11}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{11b\,x^{2}+11a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a\,b^{4}x^{9}}{9\left(b\,x^{2}+a\right)}+\frac{10\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{2}b^{3}x^{7}}{7\left(b\,x^{2}+a\right)}+\frac{2a^{3}b^{2}x^{5}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a^{4}b\,x^{3}}{3\left(b\,x^{2}+a\right)}+\frac{a^{5}x\sqrt{}}{b}$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/693*x*(63*b^5*x^10+385*a*b^4*x^8+990*a^2*b^3*x^6+1386*a^3*b^2*x^4+1155*a
^4*b*x^2+693*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.22

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4
*b*x^3 + a^5*x
```

Sympy [F]

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int \left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.22

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{11}b^5x^{11}+\frac{5}{9}ab^4x^9+\frac{10}{7}a^2b^3x^7+2a^3b^2x^5+\frac{5}{3}a^4bx^3+a^5x$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4
*b*x^3 + a^5*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.41

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{1}{11}b^5x^{11}\mathrm{sgn}\left(bx^2+a\right) + \frac{5}{9}ab^4x^9\mathrm{sgn}\left(bx^2+a\right)+\frac{10}{7}a^2b^3x^7\mathrm{sgn}\left(bx^2+a\right) + 2a^3b^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{5}{3}a^4bx^3\mathrm{sgn}\left(bx^2+a\right)+a^5x\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/11*b^5*x^11*sgn(b*x^2 + a) + 5/9*a*b^4*x^9*sgn(b*x^2 + a) + 10/7*a^2*b^3
*x^7*sgn(b*x^2 + a) + 2*a^3*b^2*x^5*sgn(b*x^2 + a) + 5/3*a^4*b*x^3*sgn(b*x
^2 + a) + a^5*x*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x(63b^5x^{10}+385a\,b^4x^8+990a^2b^3x^6+1386a^3b^2x^4+1155a^4b\,x^2+693a^5)}{693}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x*(693*a**5 + 1155*a**4*b*x**2 + 1386*a**3*b**2*x**4 + 990*a**2*b**3*x**6
 + 385*a*b**4*x**8 + 63*b**5*x**10))/693
```

3.559 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 247

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^2}\,dx = &-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)} \\ &+\frac{5a^4bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)} \\ &+\frac{2a^2b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} \\ &+\frac{5ab^4x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{b^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}\end{aligned}$$

output
```
-a^5*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+5*a^4*b*x*((b*x^2+a)^2)^(1/2)/(b*x^2+
a)+10*a^3*b^2*x^3*((b*x^2+a)^2)^(1/2)/(3*b*x^2+3*a)+2*a^2*b^3*x^5*((b*x^2+
a)^2)^(1/2)/(b*x^2+a)+5*a*b^4*x^7*((b*x^2+a)^2)^(1/2)/(7*b*x^2+7*a)+b^5*x^
9*((b*x^2+a)^2)^(1/2)/(9*b*x^2+9*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.34

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^2} \, dx = \frac{\sqrt{(a + bx^2)^2}(-63a^5 + 315a^4bx^2 + 210a^3b^2x^4 + 126a^2b^3x^6 + 45ab^4x^8 + 7b^5}{63x\,(a + bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(-63*a^5 + 315*a^4*b*x^2 + 210*a^3*b^2*x^4 + 126*a^2*
b^3*x^6 + 45*a*b^4*x^8 + 7*b^5*x^10))/(63*x*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^2} \, dx$$

$$\Big\downarrow \; 1384$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \frac{b^5(bx^2+a)^5}{x^2} dx}{b^5\,(a + bx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \frac{(bx^2+a)^5}{x^2} dx}{a + bx^2}$$

$$\Big\downarrow \; 244$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int \left(b^5x^8 + 5ab^4x^6 + 10a^2b^3x^4 + 10a^3b^2x^2 + 5a^4b + \frac{a^5}{x^2}\right) dx}{a + bx^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}\right)}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/
3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/7 + (b^5*x^9)/9))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-7x^{10}b^{5}-45a\,x^{8}b^{4}-126a^{2}x^{6}b^{3}-210a^{3}x^{4}b^{2}-315x^{2}a^{4}b+63a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{63x\left(b\,x^{2}+a\right)^{5}}$	80
default	$-\frac{\left(-7x^{10}b^{5}-45a\,x^{8}b^{4}-126a^{2}x^{6}b^{3}-210a^{3}x^{4}b^{2}-315x^{2}a^{4}b+63a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{63x\left(b\,x^{2}+a\right)^{5}}$	80
orering	$-\frac{\left(-7x^{10}b^{5}-45a\,x^{8}b^{4}-126a^{2}x^{6}b^{3}-210a^{3}x^{4}b^{2}-315x^{2}a^{4}b+63a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{63x\left(b\,x^{2}+a\right)^{5}}$	89
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b\left(\frac{1}{9}b^{4}x^{9}+\frac{5}{7}a\,b^{3}x^{7}+2a^{2}b^{2}x^{5}+\frac{10}{3}a^{3}b\,x^{3}+5a^{4}x\right)}{b\,x^{2}+a}-\frac{a^{5}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{x\left(b\,x^{2}+a\right)}$	96

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/63*(-7*b^5*x^10-45*a*b^4*x^8-126*a^2*b^3*x^6-210*a^3*b^2*x^4-315*a^4*b*
x^2+63*a^5)*((b*x^2+a)^2)^(5/2)/x/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^2}\,dx=\frac{7\,b^5x^{10}+45\,ab^4x^8+126\,a^2b^3x^6+210\,a^3b^2x^4+315\,a^4bx^2-63\,a^5}{63\,x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x^4 + 315*
a^4*b*x^2 - 63*a^5)/x
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^2} \, dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{5}{2}}}{x^2} \, dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**2,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^2} \, dx = \frac{1}{9} b^5x^9 + \frac{5}{7} ab^4x^7 + 2 a^2b^3x^5 + \frac{10}{3} a^3b^2x^3 + 5 a^4bx - \frac{a^5}{x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
 - a^5/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.42

$$\begin{aligned}\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^2} \, dx &= \frac{1}{9} b^5x^9 \mathrm{sgn}\left(bx^2 + a\right) \\ &+ \frac{5}{7} ab^4x^7 \mathrm{sgn}\left(bx^2 + a\right) + 2 a^2b^3x^5 \mathrm{sgn}\left(bx^2 + a\right) \\ &+ \frac{10}{3} a^3b^2x^3 \mathrm{sgn}\left(bx^2 + a\right) + 5 a^4bx \mathrm{sgn}\left(bx^2 + a\right) - \frac{a^5 \mathrm{sgn}(bx^2 + a)}{x}\end{aligned}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x, algorithm="giac")
```

output
```
1/9*b^5*x^9*sgn(b*x^2 + a) + 5/7*a*b^4*x^7*sgn(b*x^2 + a) + 2*a^2*b^3*x^5*
sgn(b*x^2 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^2 + a) + 5*a^4*b*x*sgn(b*x^2 + a
) - a^5*sgn(b*x^2 + a)/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^2} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{5/2}}{x^2} \, dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^2,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^2} \, dx = \frac{7b^5x^{10} + 45a\,b^4x^8 + 126a^2b^3x^6 + 210a^3b^2x^4 + 315a^4b\,x^2 - 63a^5}{63x}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x)
```

output
```
( - 63*a**5 + 315*a**4*b*x**2 + 210*a**3*b**2*x**4 + 126*a**2*b**3*x**6 +
45*a*b**4*x**8 + 7*b**5*x**10)/(63*x)
```

3.560 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 246

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^4}\,dx=-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{10a^3b^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{ab^4x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

output

```
-1/3*a^5*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-5*a^4*b*((b*x^2+a)^2)^(1/2)/x/(
b*x^2+a)+10*a^3*b^2*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+10*a^2*b^3*x^3*((b*x^2
+a)^2)^(1/2)/(3*b*x^2+3*a)+a*b^4*x^5*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+b^5*x^7
*((b*x^2+a)^2)^(1/2)/(7*b*x^2+7*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^4}\,dx = \frac{\sqrt{(a+bx^2)^2}(-7a^5-105a^4bx^2+210a^3b^2x^4+70a^2b^3x^6+21ab^4x^8+3b^5x^1}{21x^3(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(-7*a^5 - 105*a^4*b*x^2 + 210*a^3*b^2*x^4 + 70*a^2*b^
3*x^6 + 21*a*b^4*x^8 + 3*b^5*x^10))/(21*x^3*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^4}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^4}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^4}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(b^5x^6+5ab^4x^4+10a^2b^3x^2+10a^3b^2+\frac{5a^4b}{x^2}+\frac{a^5}{x^4}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^5}{3x^3}-\frac{5a^4b}{x}+10a^3b^2x+\frac{10}{3}a^2b^3x^3+ab^4x^5+\frac{b^5x^7}{7}\right)}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/3*a^5/x^3 - (5*a^4*b)/x + 10*a^3*b^2*
x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5 + (b^5*x^7)/7))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.33

method	result	size
gosper	$-\frac{\left(-3x^{10}b^{5}-21a\,x^{8}b^{4}-70a^{2}x^{6}b^{3}-210a^{3}x^{4}b^{2}+105x^{2}a^{4}b+7a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{21x^{3}\left(b\,x^{2}+a\right)^{5}}$	80
default	$-\frac{\left(-3x^{10}b^{5}-21a\,x^{8}b^{4}-70a^{2}x^{6}b^{3}-210a^{3}x^{4}b^{2}+105x^{2}a^{4}b+7a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{21x^{3}\left(b\,x^{2}+a\right)^{5}}$	80
orering	$-\frac{\left(-3x^{10}b^{5}-21a\,x^{8}b^{4}-70a^{2}x^{6}b^{3}-210a^{3}x^{4}b^{2}+105x^{2}a^{4}b+7a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{21x^{3}\left(b\,x^{2}+a\right)^{5}}$	89
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{2}\left(\frac{1}{7}b^{3}x^{7}+b^{2}x^{5}a+\frac{10}{3}a^{2}b\,x^{3}+10a^{3}x\right)}{b\,x^{2}+a}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\left(-5x^{2}a^{4}b-\frac{1}{3}a^{5}\right)}{\left(b\,x^{2}+a\right)x^{3}}$	97

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/21*(-3*b^5*x^10-21*a*b^4*x^8-70*a^2*b^3*x^6-210*a^3*b^2*x^4+105*a^4*b*x
^2+7*a^5)*((b*x^2+a)^2)^(5/2)/x^3/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^4}\,dx=\frac{3\,b^5x^{10}+21\,ab^4x^8+70\,a^2b^3x^6+210\,a^3b^2x^4-105\,a^4bx^2-7\,a^5}{21\,x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^4 - 105*a
^4*b*x^2 - 7*a^5)/x^3
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^4} \, dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{5}{2}}}{x^4} \, dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**4,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^4} \, dx = \frac{1}{7} b^5x^7 + ab^4x^5 + \frac{10}{3} a^2b^3x^3 + 10 a^3b^2x - \frac{5 a^4b}{x} - \frac{a^5}{3 x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 5*a^4*b/x - 1/
3*a^5/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^4} \, dx = \frac{1}{7} b^5x^7 \text{sgn}(bx^2 + a) + ab^4x^5 \text{sgn}(bx^2 + a) + \frac{10}{3} a^2b^3x^3 \text{sgn}(bx^2 + a) + 10 a^3b^2x \text{sgn}(bx^2 + a) - \frac{15 a^4bx^2 \text{sgn}(bx^2 + a) + a^5 \text{sgn}(bx^2 + a)}{3 x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x, algorithm="giac")
```

output
```
1/7*b^5*x^7*sgn(b*x^2 + a) + a*b^4*x^5*sgn(b*x^2 + a) + 10/3*a^2*b^3*x^3*s
gn(b*x^2 + a) + 10*a^3*b^2*x*sgn(b*x^2 + a) - 1/3*(15*a^4*b*x^2*sgn(b*x^2
+ a) + a^5*sgn(b*x^2 + a))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^4} dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{5/2}}{x^4} dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^4,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^4} dx = \frac{3b^5x^{10} + 21a\,b^4x^8 + 70a^2b^3x^6 + 210a^3b^2x^4 - 105a^4b\,x^2 - 7a^5}{21x^3}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x)
```

output
```
( - 7*a**5 - 105*a**4*b*x**2 + 210*a**3*b**2*x**4 + 70*a**2*b**3*x**6 + 21
*a*b**4*x**8 + 3*b**5*x**10)/(21*x**3)
```

3.561 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 249

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^6}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)} + \frac{10a^2b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{5ab^4x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)} + \frac{b^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}$$

output

```
-1/5*a^5*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-5/3*a^4*b*((b*x^2+a)^2)^(1/2)/x
^3/(b*x^2+a)-10*a^3*b^2*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+10*a^2*b^3*x*((b*x
^2+a)^2)^(1/2)/(b*x^2+a)+5*a*b^4*x^3*((b*x^2+a)^2)^(1/2)/(3*b*x^2+3*a)+b^5
*x^5*((b*x^2+a)^2)^(1/2)/(5*b*x^2+5*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^6}\,dx = \frac{\sqrt{(a+bx^2)^2}(-3a^5-25a^4bx^2-150a^3b^2x^4+150a^2b^3x^6+25ab^4x^8+3b^5x^{10})}{15x^5(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```

output
```
(Sqrt[(a + b*x^2)^2]*(-3*a^5 - 25*a^4*b*x^2 - 150*a^3*b^2*x^4 + 150*a^2*b^
3*x^6 + 25*a*b^4*x^8 + 3*b^5*x^10))/(15*x^5*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^6}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^6}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^6}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^5}{x^6}+\frac{5ba^4}{x^4}+\frac{10b^2a^3}{x^2}+10b^3a^2+5b^4x^2a+b^5x^4\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}\right)}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*(-1/5*a^5/x^5 - (5*a^4*b)/(3*x^3) - (10*a
^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^4*x^3)/3 + (b^5*x^5)/5))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-3x^{10}b^{5}-25a\,x^{8}b^{4}-150a^{2}x^{6}b^{3}+150a^{3}x^{4}b^{2}+25x^{2}a^{4}b+3a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{15x^{5}\left(b\,x^{2}+a\right)^{5}}$	80
default	$-\frac{\left(-3x^{10}b^{5}-25a\,x^{8}b^{4}-150a^{2}x^{6}b^{3}+150a^{3}x^{4}b^{2}+25x^{2}a^{4}b+3a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{15x^{5}\left(b\,x^{2}+a\right)^{5}}$	80
orering	$-\frac{\left(-3x^{10}b^{5}-25a\,x^{8}b^{4}-150a^{2}x^{6}b^{3}+150a^{3}x^{4}b^{2}+25x^{2}a^{4}b+3a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{15x^{5}\left(b\,x^{2}+a\right)^{5}}$	89
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{3}\left(\frac{1}{5}x^{5}b^{2}+\frac{5}{3}ab\,x^{3}+10a^{2}x\right)}{b\,x^{2}+a}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\left(-10a^{3}x^{4}b^{2}-\frac{5}{3}x^{2}a^{4}b-\frac{1}{5}a^{5}\right)}{\left(b\,x^{2}+a\right)x^{5}}$	98

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-3*b^5*x^10-25*a*b^4*x^8-150*a^2*b^3*x^6+150*a^3*b^2*x^4+25*a^4*b*x
^2+3*a^5)*((b*x^2+a)^2)^(5/2)/x^5/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^6}\,dx=\frac{3\,b^5x^{10}+25\,ab^4x^8+150\,a^2b^3x^6-150\,a^3b^2x^4-25\,a^4bx^2-3\,a^5}{15\,x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x^4 - 25*a
^4*b*x^2 - 3*a^5)/x^5
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^6}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**6,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^6}\,dx = \frac{1}{5}\,b^5x^5 + \frac{5}{3}\,ab^4x^3 + 10\,a^2b^3x - \frac{10\,a^3b^2}{x} - \frac{5\,a^4b}{3\,x^3} - \frac{a^5}{5\,x^5}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 10*a^3*b^2/x - 5/3*a^4*b/x^3
- 1/5*a^5/x^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.43

$$\begin{aligned}\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^6}\,dx &= \frac{1}{5}\,b^5x^5\mathrm{sgn}\left(bx^2+a\right) \\ &+ \frac{5}{3}\,ab^4x^3\mathrm{sgn}\left(bx^2+a\right) + 10\,a^2b^3x\mathrm{sgn}\left(bx^2+a\right) \\ &- \frac{150\,a^3b^2x^4\mathrm{sgn}(bx^2+a) + 25\,a^4bx^2\mathrm{sgn}(bx^2+a) + 3\,a^5\mathrm{sgn}(bx^2+a)}{15\,x^5}\end{aligned}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x, algorithm="giac")
```

output
```
1/5*b^5*x^5*sgn(b*x^2 + a) + 5/3*a*b^4*x^3*sgn(b*x^2 + a) + 10*a^2*b^3*x*s
gn(b*x^2 + a) - 1/15*(150*a^3*b^2*x^4*sgn(b*x^2 + a) + 25*a^4*b*x^2*sgn(b*
x^2 + a) + 3*a^5*sgn(b*x^2 + a))/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^6}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{5/2}}{x^6}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^6,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^6}\,dx = \frac{3b^5x^{10} + 25a\,b^4x^8 + 150a^2b^3x^6 - 150a^3b^2x^4 - 25a^4b\,x^2 - 3a^5}{15x^5}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x)
```

output
```
( - 3*a**5 - 25*a**4*b*x**2 - 150*a**3*b**2*x**4 + 150*a**2*b**3*x**6 + 25
*a*b**4*x**8 + 3*b**5*x**10)/(15*x**5)
```

3.562 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^8} dx$

Optimal result

Integrand size = 26, antiderivative size = 247

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^8} dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)} - \frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)} - \frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)} + \frac{5ab^4x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}$$

output
```
-1/7*a^5*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-a^4*b*((b*x^2+a)^2)^(1/2)/x^5/(
b*x^2+a)-10/3*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-10*a^2*b^3*((b*x^2
+a)^2)^(1/2)/x/(b*x^2+a)+5*a*b^4*x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+b^5*x^3*(
(b*x^2+a)^2)^(1/2)/(3*b*x^2+3*a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^8}\,dx = -\frac{\sqrt{(a+bx^2)^2}(3a^5+21a^4bx^2+70a^3b^2x^4+210a^2b^3x^6-105ab^4x^8-7b^5x^{10})}{21x^7(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

output
```
-1/21*(Sqrt[(a + b*x^2)^2]*(3*a^5 + 21*a^4*b*x^2 + 70*a^3*b^2*x^4 + 210*a^
2*b^3*x^6 - 105*a*b^4*x^8 - 7*b^5*x^10))/(x^7*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.38, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^8}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^8}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^8}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{a^5}{x^8}+\frac{5ba^4}{x^6}+\frac{10b^2a^3}{x^4}+\frac{10b^3a^2}{x^2}+5b^4a+b^5x^2\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

output
```
((-1/7*a^5/x^7 - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a
*b^4*x + (b^5*x^3)/3)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{\left(-7x^{10}b^{5}-105a\,x^{8}b^{4}+210a^{2}x^{6}b^{3}+70a^{3}x^{4}b^{2}+21x^{2}a^{4}b+3a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{21x^{7}\left(b\,x^{2}+a\right)^{5}}$	80
default	$-\frac{\left(-7x^{10}b^{5}-105a\,x^{8}b^{4}+210a^{2}x^{6}b^{3}+70a^{3}x^{4}b^{2}+21x^{2}a^{4}b+3a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{21x^{7}\left(b\,x^{2}+a\right)^{5}}$	80
orering	$-\frac{\left(-7x^{10}b^{5}-105a\,x^{8}b^{4}+210a^{2}x^{6}b^{3}+70a^{3}x^{4}b^{2}+21x^{2}a^{4}b+3a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{21x^{7}\left(b\,x^{2}+a\right)^{5}}$	89
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{4}\left(\frac{1}{3}b\,x^{3}+5ax\right)}{b\,x^{2}+a}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-10a^{2}x^{6}b^{3}-\frac{10}{3}a^{3}x^{4}b^{2}-x^{2}a^{4}b-\frac{1}{7}a^{5}\right)}{\left(b\,x^{2}+a\right)x^{7}}$	98

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21*(-7*b^5*x^10-105*a*b^4*x^8+210*a^2*b^3*x^6+70*a^3*b^2*x^4+21*a^4*b*x
^2+3*a^5)*((b*x^2+a)^2)^(5/2)/x^7/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^8}\,dx=\frac{7\,b^5x^{10}+105\,ab^4x^8-210\,a^2b^3x^6-70\,a^3b^2x^4-21\,a^4bx^2-3\,a^5}{21\,x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

output
```
1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x^4 - 21*a
^4*b*x^2 - 3*a^5)/x^7
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^8} dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{5}{2}}}{x^8} dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**8,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^8} dx = \frac{1}{3} b^5x^3 + 5 ab^4x - \frac{10 a^2b^3}{x} - \frac{10 a^3b^2}{3 x^3} - \frac{a^4b}{x^5} - \frac{a^5}{7 x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

output
```
1/3*b^5*x^3 + 5*a*b^4*x - 10*a^2*b^3/x - 10/3*a^3*b^2/x^3 - a^4*b/x^5 - 1/
7*a^5/x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.43

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^8} dx = \frac{1}{3} b^5x^3\mathrm{sgn}\left(bx^2 + a\right) + 5 ab^4x\mathrm{sgn}\left(bx^2 + a\right) - \frac{210 a^2b^3x^6\mathrm{sgn}(bx^2 + a) + 70 a^3b^2x^4\mathrm{sgn}(bx^2 + a) + 21 a^4bx^2\mathrm{sgn}(bx^2 + a) + 3 a^5\mathrm{sgn}(bx^2 + a)}{21 x^7}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x, algorithm="giac")
```

output
```
1/3*b^5*x^3*sgn(b*x^2 + a) + 5*a*b^4*x*sgn(b*x^2 + a) - 1/21*(210*a^2*b^3*
x^6*sgn(b*x^2 + a) + 70*a^3*b^2*x^4*sgn(b*x^2 + a) + 21*a^4*b*x^2*sgn(b*x^
2 + a) + 3*a^5*sgn(b*x^2 + a))/x^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^8} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{5/2}}{x^8} \, dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^8,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^8} \, dx = \frac{7b^5x^{10} + 105a\,b^4x^8 - 210a^2b^3x^6 - 70a^3b^2x^4 - 21a^4b\,x^2 - 3a^5}{21x^7}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x)
```

output
```
( - 3*a**5 - 21*a**4*b*x**2 - 70*a**3*b**2*x**4 - 210*a**2*b**3*x**6 + 105
*a*b**4*x**8 + 7*b**5*x**10)/(21*x**7)
```

3.563 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 246

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{10}}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)} - \frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)} - \frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)} - \frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)} + \frac{b^5x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

output
```
-1/9*a^5*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-5/7*a^4*b*((b*x^2+a)^2)^(1/2)/x
^7/(b*x^2+a)-2*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-10/3*a^2*b^3*((b*
x^2+a)^2)^(1/2)/x^3/(b*x^2+a)-5*a*b^4*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+b^5*
x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{10}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(7a^5+45a^4bx^2+126a^3b^2x^4+210a^2b^3x^6+315ab^4x^8-63b^5x^{10})}{63x^9(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

output
```
-1/63*(Sqrt[(a + b*x^2)^2]*(7*a^5 + 45*a^4*b*x^2 + 126*a^3*b^2*x^4 + 210*a
^2*b^3*x^6 + 315*a*b^4*x^8 - 63*b^5*x^10))/(x^9*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{10}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^{10}}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{10}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{a^5}{x^{10}}+\frac{5ba^4}{x^8}+\frac{10b^2a^3}{x^6}+\frac{10b^3a^2}{x^4}+\frac{5b^4a}{x^2}+b^5\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

output
```
((-1/9*a^5/x^9 - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3
) - (5*a*b^4)/x + b^5*x)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.33

method	result	size
gosper	$-\frac{\left(-63x^{10}b^{5}+315a\,x^{8}b^{4}+210a^{2}x^{6}b^{3}+126a^{3}x^{4}b^{2}+45x^{2}a^{4}b+7a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{63x^{9}\left(b\,x^{2}+a\right)^{5}}$	80
default	$-\frac{\left(-63x^{10}b^{5}+315a\,x^{8}b^{4}+210a^{2}x^{6}b^{3}+126a^{3}x^{4}b^{2}+45x^{2}a^{4}b+7a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{63x^{9}\left(b\,x^{2}+a\right)^{5}}$	80
orering	$-\frac{\left(-63x^{10}b^{5}+315a\,x^{8}b^{4}+210a^{2}x^{6}b^{3}+126a^{3}x^{4}b^{2}+45x^{2}a^{4}b+7a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{63x^{9}\left(b\,x^{2}+a\right)^{5}}$	89
risch	$\frac{b^{5}x\sqrt{\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-5a\,x^{8}b^{4}-\frac{10}{3}a^{2}x^{6}b^{3}-2a^{3}x^{4}b^{2}-\frac{5}{7}x^{2}a^{4}b-\frac{1}{9}a^{5}\right)}{\left(b\,x^{2}+a\right)x^{9}}$	97

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/63*(-63*b^5*x^10+315*a*b^4*x^8+210*a^2*b^3*x^6+126*a^3*b^2*x^4+45*a^4*b
*x^2+7*a^5)*((b*x^2+a)^2)^(5/2)/x^9/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{10}}\,dx=\frac{63\,b^5x^{10}-315\,ab^4x^8-210\,a^2b^3x^6-126\,a^3b^2x^4-45\,a^4bx^2-7\,a^5}{63\,x^9}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="fricas")
```

output
```
1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2*x^4 - 45
*a^4*b*x^2 - 7*a^5)/x^9
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{10}}\,dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**10,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**10, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{10}}\,dx = b^5x - \frac{5\,ab^4}{x} - \frac{10\,a^2b^3}{3\,x^3} - \frac{2\,a^3b^2}{x^5} - \frac{5\,a^4b}{7\,x^7} - \frac{a^5}{9\,x^9}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="maxima")
```

output
```
b^5*x - 5*a*b^4/x - 10/3*a^2*b^3/x^3 - 2*a^3*b^2/x^5 - 5/7*a^4*b/x^7 - 1/9
*a^5/x^9
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.43

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{10}}\,dx = b^5x\mathrm{sgn}\left(bx^2 + a\right) - \frac{315\,ab^4x^8\mathrm{sgn}(bx^2 + a) + 210\,a^2b^3x^6\mathrm{sgn}(bx^2 + a) + 126\,a^3b^2x^4\mathrm{sgn}(bx^2 + a) + 45\,a^4bx^2\mathrm{sgn}(bx^2 + a) + 7\,a}{63\,x^9}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x, algorithm="giac")
```

output
```
b^5*x*sgn(b*x^2 + a) - 1/63*(315*a*b^4*x^8*sgn(b*x^2 + a) + 210*a^2*b^3*x^
6*sgn(b*x^2 + a) + 126*a^3*b^2*x^4*sgn(b*x^2 + a) + 45*a^4*b*x^2*sgn(b*x^2
 + a) + 7*a^5*sgn(b*x^2 + a))/x^9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{10}} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{5/2}}{x^{10}} \, dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^10,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^10, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{10}} \, dx = \frac{63b^5x^{10} - 315a\,b^4x^8 - 210a^2b^3x^6 - 126a^3b^2x^4 - 45a^4b\,x^2 - 7a^5}{63x^9}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x)
```

output
```
( - 7*a**5 - 45*a**4*b*x**2 - 126*a**3*b**2*x**4 - 210*a**2*b**3*x**6 - 31
5*a*b**4*x**8 + 63*b**5*x**10)/(63*x**9)
```

3.564 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{12}}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)} - \frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)} - \frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

output

```
-1/11*a^5*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-5/9*a^4*b*((b*x^2+a)^2)^(1/2)
/x^9/(b*x^2+a)-10/7*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-2*a^2*b^3*((
b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)-5/3*a*b^4*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a
)-b^5*((b*x^2+a)^2)^(1/2)/x/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{12}}\,dx = -\frac{\sqrt{(a+bx^2)^2}\left(63a^5+385a^4bx^2+990a^3b^2x^4+1386a^2b^3x^6+1155ab^4x^8+693b^5x^{10}\right)}{693x^{11}\left(a+bx^2\right)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

output
```
-1/693*(Sqrt[(a + b*x^2)^2]*(63*a^5 + 385*a^4*b*x^2 + 990*a^3*b^2*x^4 + 13
86*a^2*b^3*x^6 + 1155*a*b^4*x^8 + 693*b^5*x^10))/(x^11*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{12}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5\left(bx^2+a\right)^5}{x^{12}}dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\left(bx^2+a\right)^5}{x^{12}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{a^5}{x^{12}}+\frac{5ba^4}{x^{10}}+\frac{10b^2a^3}{x^8}+\frac{10b^3a^2}{x^6}+\frac{5b^4a}{x^4}+\frac{b^5}{x^2}\right)dx}{a+bx^2}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\left(-\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

output
```
((-1/11*a^5/x^11 - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/
x^5 - (5*a*b^4)/(3*x^3) - b^5/x)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x
^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 5.86 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-x^{10}b^5-\frac{5}{3}a\,x^8b^4-2a^2x^6b^3-\frac{10}{7}a^3x^4b^2-\frac{5}{9}x^2a^4b-\frac{1}{11}a^5\right)}{(b\,x^2+a)x^{11}}$	79
gosper	$-\frac{(693x^{10}b^5+1155a\,x^8b^4+1386a^2x^6b^3+990a^3x^4b^2+385x^2a^4b+63a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{693x^{11}(b\,x^2+a)^5}$	80
default	$-\frac{(693x^{10}b^5+1155a\,x^8b^4+1386a^2x^6b^3+990a^3x^4b^2+385x^2a^4b+63a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{693x^{11}(b\,x^2+a)^5}$	80
orering	$-\frac{(693x^{10}b^5+1155a\,x^8b^4+1386a^2x^6b^3+990a^3x^4b^2+385x^2a^4b+63a^5)(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{2}}}{693x^{11}(b\,x^2+a)^5}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*(-x^10*b^5-5/3*a*x^8*b^4-2*a^2*x^6*b^3-10/7*
a^3*x^4*b^2-5/9*x^2*a^4*b-1/11*a^5)/x^11
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{12}}\,dx = -\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="fricas")
```

output
```
-1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
 + 385*a^4*b*x^2 + 63*a^5)/x^11
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{12}}\,dx=\int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^{12}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**12,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**12, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{12}}\,dx=-\frac{b^5}{x}-\frac{5\,ab^4}{3\,x^3}-\frac{2\,a^2b^3}{x^5}-\frac{10\,a^3b^2}{7\,x^7}-\frac{5\,a^4b}{9\,x^9}-\frac{a^5}{11\,x^{11}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="maxima")
```

output
```
-b^5/x - 5/3*a*b^4/x^3 - 2*a^2*b^3/x^5 - 10/7*a^3*b^2/x^7 - 5/9*a^4*b/x^9
- 1/11*a^5/x^11
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.43

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{12}}\,dx=$$
$$-\frac{693\,b^5x^{10}\mathrm{sgn}(bx^2+a)+1155\,ab^4x^8\mathrm{sgn}(bx^2+a)+1386\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+990\,a^3b^2x^4\mathrm{sgn}(bx^2+a)+}{693\,x^{11}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x, algorithm="giac")
```

output
```
-1/693*(693*b^5*x^10*sgn(b*x^2 + a) + 1155*a*b^4*x^8*sgn(b*x^2 + a) + 1386
*a^2*b^3*x^6*sgn(b*x^2 + a) + 990*a^3*b^2*x^4*sgn(b*x^2 + a) + 385*a^4*b*x
^2*sgn(b*x^2 + a) + 63*a^5*sgn(b*x^2 + a))/x^11
```

Mupad [B] (verification not implemented)

Time = 19.30 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.92

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{12}}\, dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x\,\left(b\,x^2+a\right)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^3\,\left(b\,x^2+a\right)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)} - \frac{2\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^5\,\left(b\,x^2+a\right)} - \frac{10\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^12,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(x*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(3*x^3*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (2*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(
1/2))/(x^5*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(
7*x^7*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{12}}\, dx = \frac{-693b^5x^{10} - 1155a\,b^4x^8 - 1386a^2b^3x^6 - 990a^3b^2x^4 - 385a^4b\,x^2 - 63a^5}{693x^{11}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x)
```

output
```
( - 63*a**5 - 385*a**4*b*x**2 - 990*a**3*b**2*x**4 - 1386*a**2*b**3*x**6 -
 1155*a*b**4*x**8 - 693*b**5*x**10)/(693*x**11)
```

3.565 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{14}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 253

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{14}}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)} - \frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)} - \frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)} - \frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

output
```
-1/13*a^5*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)-5/11*a^4*b*((b*x^2+a)^2)^(1/2
)/x^11/(b*x^2+a)-10/9*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-10/7*a^2*b
^3*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)-a*b^4*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+
a)-1/3*b^5*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{14}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(693a^5+4095a^4bx^2+10010a^3b^2x^4+12870a^2b^3x^6+9009ab^4x^8+3003b^5x^{10})}{9009x^{13}\,(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
```

output
```
-1/9009*(Sqrt[(a + b*x^2)^2]*(693*a^5 + 4095*a^4*b*x^2 + 10010*a^3*b^2*x^4
 + 12870*a^2*b^3*x^6 + 9009*a*b^4*x^8 + 3003*b^5*x^10))/(x^13*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{14}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^{14}}dx}{b^5\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{14}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{a^5}{x^{14}}+\frac{5ba^4}{x^{12}}+\frac{10b^2a^3}{x^{10}}+\frac{10b^3a^2}{x^8}+\frac{5b^4a}{x^6}+\frac{b^5}{x^4}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
```

output
```
((-1/13*a^5/x^13 - (5*a^4*b)/(11*x^11) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^
3)/(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 7.86 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^{2}+a)^{2}}\,\left(-\frac{1}{3}x^{10}b^{5}-a\,x^{8}b^{4}-\frac{10}{7}a^{2}x^{6}b^{3}-\frac{10}{9}a^{3}x^{4}b^{2}-\frac{5}{11}x^{2}a^{4}b-\frac{1}{13}a^{5}\right)}{(b\,x^{2}+a)\,x^{13}}$	79
gosper	$-\frac{\left(3003x^{10}b^{5}+9009a\,x^{8}b^{4}+12870a^{2}x^{6}b^{3}+10010a^{3}x^{4}b^{2}+4095x^{2}a^{4}b+693a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{9009x^{13}\left(b\,x^{2}+a\right)^{5}}$	80
default	$-\frac{\left(3003x^{10}b^{5}+9009a\,x^{8}b^{4}+12870a^{2}x^{6}b^{3}+10010a^{3}x^{4}b^{2}+4095x^{2}a^{4}b+693a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{9009x^{13}\left(b\,x^{2}+a\right)^{5}}$	80
orering	$-\frac{\left(3003x^{10}b^{5}+9009a\,x^{8}b^{4}+12870a^{2}x^{6}b^{3}+10010a^{3}x^{4}b^{2}+4095x^{2}a^{4}b+693a^{5}\right)\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{9009x^{13}\left(b\,x^{2}+a\right)^{5}}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*(-1/3*x^10*b^5-a*x^8*b^4-10/7*a^2*x^6*b^3-10
/9*a^3*x^4*b^2-5/11*x^2*a^4*b-1/13*a^5)/x^13
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{14}}\,dx = -\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="fricas")
```

output
```
-1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^
2*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{14}}\,dx=\int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^{14}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**14,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**14, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{14}}\,dx=-\frac{b^5}{3\,x^3}-\frac{ab^4}{x^5}-\frac{10\,a^2b^3}{7\,x^7}-\frac{10\,a^3b^2}{9\,x^9}-\frac{5\,a^4b}{11\,x^{11}}-\frac{a^5}{13\,x^{13}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="maxima")
```

output
```
-1/3*b^5/x^3 - a*b^4/x^5 - 10/7*a^2*b^3/x^7 - 10/9*a^3*b^2/x^9 - 5/11*a^4*
b/x^11 - 1/13*a^5/x^13
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{14}}\,dx=$$
$$-\frac{3003\,b^5x^{10}\mathrm{sgn}(bx^2+a)+9009\,ab^4x^8\mathrm{sgn}(bx^2+a)+12870\,a^2b^3x^6\mathrm{sgn}(bx^2+a)+10010\,a^3b^2x^4\mathrm{sgn}(bx^2+}{9009\,x^{13}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x, algorithm="giac")
```

output
```
-1/9009*(3003*b^5*x^10*sgn(b*x^2 + a) + 9009*a*b^4*x^8*sgn(b*x^2 + a) + 12
870*a^2*b^3*x^6*sgn(b*x^2 + a) + 10010*a^3*b^2*x^4*sgn(b*x^2 + a) + 4095*a
^4*b*x^2*sgn(b*x^2 + a) + 693*a^5*sgn(b*x^2 + a))/x^13
```

Mupad [B] (verification not implemented)

Time = 18.96 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{14}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^3\,\left(b\,x^2+a\right)} - \frac{a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{x^5\,\left(b\,x^2+a\right)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)} - \frac{10\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)} - \frac{10\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^14,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(3*x^3*(a + b*x^2)) - (a*b^4*(a^2 + b^2*x^
4 + 2*a*b*x^2)^(1/2))/(x^5*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^2
)^(1/2))/(7*x^7*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/
2))/(9*x^9*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{14}}\,dx = \frac{-3003b^5x^{10}-9009a\,b^4x^8-12870a^2b^3x^6-10010a^3b^2x^4-4095a^4b\,x^2-69}{9009x^{13}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x)
```

output
```
( - 693*a**5 - 4095*a**4*b*x**2 - 10010*a**3*b**2*x**4 - 12870*a**2*b**3*x
**6 - 9009*a*b**4*x**8 - 3003*b**5*x**10)/(9009*x**13)
```

3.566 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{16}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{16}}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)} - \frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)} - \frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)} - \frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}$$

output
```
-1/15*a^5*((b*x^2+a)^2)^(1/2)/x^15/(b*x^2+a)-5/13*a^4*b*((b*x^2+a)^2)^(1/2
)/x^13/(b*x^2+a)-10/11*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-10/9*a^2
*b^3*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)-5/7*a*b^4*((b*x^2+a)^2)^(1/2)/x^7/(
b*x^2+a)-1/5*b^5*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{16}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(3003a^5+17325a^4bx^2+40950a^3b^2x^4+50050a^2b^3x^6+32175ab^4x^8+9009b^5x^{10})}{45045x^{15}\,(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

output
```
-1/45045*(Sqrt[(a + b*x^2)^2]*(3003*a^5 + 17325*a^4*b*x^2 + 40950*a^3*b^2*
x^4 + 50050*a^2*b^3*x^6 + 32175*a*b^4*x^8 + 9009*b^5*x^10))/(x^15*(a + b*x
^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{16}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^{16}}dx}{b^5\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{16}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{a^5}{x^{16}}+\frac{5ba^4}{x^{14}}+\frac{10b^2a^3}{x^{12}}+\frac{10b^3a^2}{x^{10}}+\frac{5b^4a}{x^8}+\frac{b^5}{x^6}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

output
```
((-1/15*a^5/x^15 - (5*a^4*b)/(13*x^13) - (10*a^3*b^2)/(11*x^11) - (10*a^2*
b^3)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5))*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 10.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{(b x^2+a)^2}\,\left(-\frac{1}{15}a^5-\frac{5}{13}x^2a^4b-\frac{10}{11}a^3x^4b^2-\frac{10}{9}a^2x^6b^3-\frac{5}{7}a\,x^8b^4-\frac{1}{5}x^{10}b^5\right)}{(b x^2+a)x^{15}}$	79
gosper	$-\frac{(9009x^{10}b^5+32175a\,x^8b^4+50050a^2x^6b^3+40950a^3x^4b^2+17325x^2a^4b+3003a^5)\left((b x^2+a)^2\right)^{\frac{5}{2}}}{45045x^{15}(b x^2+a)^5}$	80
default	$-\frac{(9009x^{10}b^5+32175a\,x^8b^4+50050a^2x^6b^3+40950a^3x^4b^2+17325x^2a^4b+3003a^5)\left((b x^2+a)^2\right)^{\frac{5}{2}}}{45045x^{15}(b x^2+a)^5}$	80
orering	$-\frac{(9009x^{10}b^5+32175a\,x^8b^4+50050a^2x^6b^3+40950a^3x^4b^2+17325x^2a^4b+3003a^5)(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{2}}}{45045x^{15}(b x^2+a)^5}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*(-1/15*a^5-5/13*x^2*a^4*b-10/11*a^3*x^4*b^2-
10/9*a^2*x^6*b^3-5/7*a*x^8*b^4-1/5*x^10*b^5)/x^15
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{16}}\,dx = -\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="fricas")
```

output
```
-1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*
b^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{16}}\,dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{5}{2}}}{x^{16}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**16,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**16, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{16}}\,dx = -\frac{b^5}{5\,x^5} - \frac{5\,ab^4}{7\,x^7} - \frac{10\,a^2b^3}{9\,x^9} - \frac{10\,a^3b^2}{11\,x^{11}} - \frac{5\,a^4b}{13\,x^{13}} - \frac{a^5}{15\,x^{15}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="maxima")
```

output
```
-1/5*b^5/x^5 - 5/7*a*b^4/x^7 - 10/9*a^2*b^3/x^9 - 10/11*a^3*b^2/x^11 - 5/1
3*a^4*b/x^13 - 1/15*a^5/x^15
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{16}}\,dx = -\frac{9009\,b^5x^{10}\mathrm{sgn}(bx^2 + a) + 32175\,ab^4x^8\mathrm{sgn}(bx^2 + a) + 50050\,a^2b^3x^6\mathrm{sgn}(bx^2 + a) + 40950\,a^3b^2x^4\mathrm{sgn}(bx^2 +}{45045\,x^{15}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x, algorithm="giac")
```

output
```
-1/45045*(9009*b^5*x^10*sgn(b*x^2 + a) + 32175*a*b^4*x^8*sgn(b*x^2 + a) +
50050*a^2*b^3*x^6*sgn(b*x^2 + a) + 40950*a^3*b^2*x^4*sgn(b*x^2 + a) + 1732
5*a^4*b*x^2*sgn(b*x^2 + a) + 3003*a^5*sgn(b*x^2 + a))/x^15
```

Mupad [B] (verification not implemented)

Time = 20.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{16}}\,dx = -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{15\,x^{15}\,\left(b\,x^2+a\right)} - \frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{5\,x^5\,\left(b\,x^2+a\right)} - \frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)} - \frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)} - \frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)} - \frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^16,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(15*x^15*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(5*x^5*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2*
a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b
*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)
^(1/2))/(11*x^11*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{16}}\,dx = \frac{-9009b^5x^{10}-32175a\,b^4x^8-50050a^2b^3x^6-40950a^3b^2x^4-17325a^4b\,x^2-}{45045x^{15}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x)
```

output

```
( - 3003*a**5 - 17325*a**4*b*x**2 - 40950*a**3*b**2*x**4 - 50050*a**2*b**3
*x**6 - 32175*a*b**4*x**8 - 9009*b**5*x**10)/(45045*x**15)
```

3.567 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{18}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{18}}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)} - \frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)} - \frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)} - \frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)} - \frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}$$

output
```
-1/17*a^5*((b*x^2+a)^2)^(1/2)/x^17/(b*x^2+a)-1/3*a^4*b*((b*x^2+a)^2)^(1/2)
/x^15/(b*x^2+a)-10/13*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)-10/11*a^2
*b^3*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)-5/9*a*b^4*((b*x^2+a)^2)^(1/2)/x^9/
(b*x^2+a)-1/7*b^5*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{18}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(9009a^5+51051a^4bx^2+117810a^3b^2x^4+139230a^2b^3x^6+85085ab^4x^8+21879b^5x^{10})}{153153x^{17}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```

output
```
-1/153153*(Sqrt[(a + b*x^2)^2]*(9009*a^5 + 51051*a^4*b*x^2 + 117810*a^3*b^
2*x^4 + 139230*a^2*b^3*x^6 + 85085*a*b^4*x^8 + 21879*b^5*x^10))/(x^17*(a +
 b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{18}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^{18}}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{18}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{a^5}{x^{18}}+\frac{5ba^4}{x^{16}}+\frac{10b^2a^3}{x^{14}}+\frac{10b^3a^2}{x^{12}}+\frac{5b^4a}{x^{10}}+\frac{b^5}{x^8}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```

output
```
((-1/17*a^5/x^17 - (a^4*b)/(3*x^15) - (10*a^3*b^2)/(13*x^13) - (10*a^2*b^3
)/(11*x^11) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7))*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 13.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{17}a^5-\frac{1}{3}x^2a^4b-\frac{10}{13}a^3x^4b^2-\frac{10}{11}a^2x^6b^3-\frac{5}{9}a\,x^8b^4-\frac{1}{7}x^{10}b^5\right)}{(b\,x^2+a)x^{17}}$	79
gosper	$-\frac{(21879x^{10}b^5+85085a\,x^8b^4+139230a^2x^6b^3+117810a^3x^4b^2+51051x^2a^4b+9009a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{153153x^{17}(b\,x^2+a)^5}$	80
default	$-\frac{(21879x^{10}b^5+85085a\,x^8b^4+139230a^2x^6b^3+117810a^3x^4b^2+51051x^2a^4b+9009a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{153153x^{17}(b\,x^2+a)^5}$	80
orering	$-\frac{(21879x^{10}b^5+85085a\,x^8b^4+139230a^2x^6b^3+117810a^3x^4b^2+51051x^2a^4b+9009a^5)\,(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{2}}}{153153x^{17}(b\,x^2+a)^5}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*(-1/17*a^5-1/3*x^2*a^4*b-10/13*a^3*x^4*b^2-1
0/11*a^2*x^6*b^3-5/9*a*x^8*b^4-1/7*x^10*b^5)/x^17
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{18}}\,dx = -\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="fricas")
```

output
```
-1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*
a^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{18}} dx = \int \frac{\left((a + bx^2)^2\right)^{\frac{5}{2}}}{x^{18}} dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**18,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**18, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{18}} dx = -\frac{b^5}{7\,x^7} - \frac{5\,ab^4}{9\,x^9} - \frac{10\,a^2b^3}{11\,x^{11}} - \frac{10\,a^3b^2}{13\,x^{13}} - \frac{a^4b}{3\,x^{15}} - \frac{a^5}{17\,x^{17}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="maxima")
```

output
```
-1/7*b^5/x^7 - 5/9*a*b^4/x^9 - 10/11*a^2*b^3/x^11 - 10/13*a^3*b^2/x^13 - 1
/3*a^4*b/x^15 - 1/17*a^5/x^17
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{18}} dx = -\frac{21879\,b^5x^{10}\mathrm{sgn}(bx^2 + a) + 85085\,ab^4x^8\mathrm{sgn}(bx^2 + a) + 139230\,a^2b^3x^6\mathrm{sgn}(bx^2 + a) + 117810\,a^3b^2x^4\mathrm{sgn}(bx}{153153\,x^{17}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x, algorithm="giac")
```

output
```
-1/153153*(21879*b^5*x^10*sgn(b*x^2 + a) + 85085*a*b^4*x^8*sgn(b*x^2 + a)
+ 139230*a^2*b^3*x^6*sgn(b*x^2 + a) + 117810*a^3*b^2*x^4*sgn(b*x^2 + a) +
51051*a^4*b*x^2*sgn(b*x^2 + a) + 9009*a^5*sgn(b*x^2 + a))/x^17
```

Mupad [B] (verification not implemented)

Time = 18.58 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{18}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{17\,x^{17}\,\left(b\,x^2+a\right)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{7\,x^7\,\left(b\,x^2+a\right)} - \frac{5\,a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)} - \frac{a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^{15}\,\left(b\,x^2+a\right)} - \frac{10\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)} - \frac{10\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^18,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(17*x^17*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(7*x^7*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (a^4*b*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(3*x^15*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*b*x^
2)^(1/2))/(11*x^11*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^
(1/2))/(13*x^13*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{18}}\,dx = \frac{-21879b^5x^{10}-85085a\,b^4x^8-139230a^2b^3x^6-117810a^3b^2x^4-51051a^4b\,x}{153153x^{17}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x)
```

output
```
( - 9009*a**5 - 51051*a**4*b*x**2 - 117810*a**3*b**2*x**4 - 139230*a**2*b*
*3*x**6 - 85085*a*b**4*x**8 - 21879*b**5*x**10)/(153153*x**17)
```

3.568 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{20}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{20}}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)} - \frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}(a+bx^2)} - \frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)} - \frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)} - \frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}$$

output
```
-1/19*a^5*((b*x^2+a)^2)^(1/2)/x^19/(b*x^2+a)-5/17*a^4*b*((b*x^2+a)^2)^(1/2
)/x^17/(b*x^2+a)-2/3*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^15/(b*x^2+a)-10/13*a^2*
b^3*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)-5/11*a*b^4*((b*x^2+a)^2)^(1/2)/x^11
/(b*x^2+a)-1/9*b^5*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{20}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(21879a^5 + 122265a^4bx^2 + 277134a^3b^2x^4 + 319770a^2b^3x^6 + 188955ab^4x^8 + 46189b^5x^{10})}{415701x^{19}\,(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

output
```
-1/415701*(Sqrt[(a + b*x^2)^2]*(21879*a^5 + 122265*a^4*b*x^2 + 277134*a^3*
b^2*x^4 + 319770*a^2*b^3*x^6 + 188955*a*b^4*x^8 + 46189*b^5*x^10))/(x^19*(
a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{20}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^{20}}dx}{b^5\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{20}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{a^5}{x^{20}}+\frac{5ba^4}{x^{18}}+\frac{10b^2a^3}{x^{16}}+\frac{10b^3a^2}{x^{14}}+\frac{5b^4a}{x^{12}}+\frac{b^5}{x^{10}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

output
```
((-1/19*a^5/x^19 - (5*a^4*b)/(17*x^17) - (2*a^3*b^2)/(3*x^15) - (10*a^2*b^
3)/(13*x^13) - (5*a*b^4)/(11*x^11) - b^5/(9*x^9))*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 22.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{(b x^2+a)^2}\,(-\frac{1}{19}a^5-\frac{5}{17}x^2a^4b-\frac{2}{3}a^3x^4b^2-\frac{10}{13}a^2x^6b^3-\frac{5}{11}a\,x^8b^4-\frac{1}{9}x^{10}b^5)}{(b\,x^2+a)x^{19}}$	79
gosper	$-\frac{(46189x^{10}b^5+188955a\,x^8b^4+319770a^2x^6b^3+277134a^3x^4b^2+122265x^2a^4b+21879a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{415701x^{19}(b\,x^2+a)^5}$	80
default	$-\frac{(46189x^{10}b^5+188955a\,x^8b^4+319770a^2x^6b^3+277134a^3x^4b^2+122265x^2a^4b+21879a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{415701x^{19}(b\,x^2+a)^5}$	80
orering	$-\frac{(46189x^{10}b^5+188955a\,x^8b^4+319770a^2x^6b^3+277134a^3x^4b^2+122265x^2a^4b+21879a^5)(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{2}}}{415701x^{19}(b\,x^2+a)^5}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*(-1/19*a^5-5/17*x^2*a^4*b-2/3*a^3*x^4*b^2-10
/13*a^2*x^6*b^3-5/11*a*x^8*b^4-1/9*x^10*b^5)/x^19
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{20}}\,dx = -\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="fricas")
```

output
```
-1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134
*a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{20}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^{20}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**20,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**20, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{20}}\,dx = -\frac{b^5}{9\,x^9} - \frac{5\,ab^4}{11\,x^{11}} - \frac{10\,a^2b^3}{13\,x^{13}} - \frac{2\,a^3b^2}{3\,x^{15}} - \frac{5\,a^4b}{17\,x^{17}} - \frac{a^5}{19\,x^{19}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="maxima")
```

output
```
-1/9*b^5/x^9 - 5/11*a*b^4/x^11 - 10/13*a^2*b^3/x^13 - 2/3*a^3*b^2/x^15 - 5
/17*a^4*b/x^17 - 1/19*a^5/x^19
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{20}}\,dx =$$
$$-\frac{46189\,b^5x^{10}\mathrm{sgn}(bx^2+a) + 188955\,ab^4x^8\mathrm{sgn}(bx^2+a) + 319770\,a^2b^3x^6\mathrm{sgn}(bx^2+a) + 277134\,a^3b^2x^4\mathrm{sgn}(b}{415701\,x^{19}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x, algorithm="giac")
```

output
```
-1/415701*(46189*b^5*x^10*sgn(b*x^2 + a) + 188955*a*b^4*x^8*sgn(b*x^2 + a)
 + 319770*a^2*b^3*x^6*sgn(b*x^2 + a) + 277134*a^3*b^2*x^4*sgn(b*x^2 + a) +
 122265*a^4*b*x^2*sgn(b*x^2 + a) + 21879*a^5*sgn(b*x^2 + a))/x^19
```

Mupad [B] (verification not implemented)

Time = 17.69 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{20}}\,dx = -\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{19\,x^{19}\,\left(b\,x^2+a\right)} - \frac{b^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{9\,x^9\,\left(b\,x^2+a\right)} - \frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{11\,x^{11}\,\left(b\,x^2+a\right)} - \frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{17\,x^{17}\,\left(b\,x^2+a\right)} - \frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)} - \frac{2\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^{15}\,\left(b\,x^2+a\right)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^20,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(19*x^19*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(9*x^9*(a + b*x^2)) - (5*a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 +
2*a*b*x^2)^(1/2))/(17*x^17*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a
*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (2*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(3*x^15*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{20}}\,dx = \frac{-46189b^5x^{10}-188955a\,b^4x^8-319770a^2b^3x^6-277134a^3b^2x^4-122265a^4b}{415701x^{19}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x)
```

output
```
( - 21879*a**5 - 122265*a**4*b*x**2 - 277134*a**3*b**2*x**4 - 319770*a**2*
b**3*x**6 - 188955*a*b**4*x**8 - 46189*b**5*x**10)/(415701*x**19)
```

3.569 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{22}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{22}}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)} - \frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}(a+bx^2)} - \frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)} - \frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}$$

output
```
-1/21*a^5*((b*x^2+a)^2)^(1/2)/x^21/(b*x^2+a)-5/19*a^4*b*((b*x^2+a)^2)^(1/2
)/x^19/(b*x^2+a)-10/17*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^17/(b*x^2+a)-2/3*a^2*
b^3*((b*x^2+a)^2)^(1/2)/x^15/(b*x^2+a)-5/13*a*b^4*((b*x^2+a)^2)^(1/2)/x^13
/(b*x^2+a)-1/11*b^5*((b*x^2+a)^2)^(1/2)/x^11/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{22}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(46189a^5+255255a^4bx^2+570570a^3b^2x^4+646646a^2b^3x^6+373065ab^4x^8+88179b^5x^{10})}{969969x^{21}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

output
```
-1/969969*(Sqrt[(a + b*x^2)^2]*(46189*a^5 + 255255*a^4*b*x^2 + 570570*a^3*
b^2*x^4 + 646646*a^2*b^3*x^6 + 373065*a*b^4*x^8 + 88179*b^5*x^10))/(x^21*(
a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{22}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^{22}}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{22}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{a^5}{x^{22}}+\frac{5ba^4}{x^{20}}+\frac{10b^2a^3}{x^{18}}+\frac{10b^3a^2}{x^{16}}+\frac{5b^4a}{x^{14}}+\frac{b^5}{x^{12}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{a^5}{21x^{21}}-\frac{5a^4b}{19x^{19}}-\frac{10a^3b^2}{17x^{17}}-\frac{2a^2b^3}{3x^{15}}-\frac{5ab^4}{13x^{13}}-\frac{b^5}{11x^{11}}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

output
```
((-1/21*a^5/x^21 - (5*a^4*b)/(19*x^19) - (10*a^3*b^2)/(17*x^17) - (2*a^2*b
^3)/(3*x^15) - (5*a*b^4)/(13*x^13) - b^5/(11*x^11))*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 27.44 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{21}a^5-\frac{5}{19}x^2a^4b-\frac{10}{17}a^3x^4b^2-\frac{2}{3}a^2x^6b^3-\frac{5}{13}a\,x^8b^4-\frac{1}{11}x^{10}b^5\right)}{(b\,x^2+a)x^{21}}$	79
gosper	$-\frac{(88179x^{10}b^5+373065a\,x^8b^4+646646a^2x^6b^3+570570a^3x^4b^2+255255x^2a^4b+46189a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{969969x^{21}(b\,x^2+a)^5}$	80
default	$-\frac{(88179x^{10}b^5+373065a\,x^8b^4+646646a^2x^6b^3+570570a^3x^4b^2+255255x^2a^4b+46189a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{969969x^{21}(b\,x^2+a)^5}$	80
orering	$-\frac{(88179x^{10}b^5+373065a\,x^8b^4+646646a^2x^6b^3+570570a^3x^4b^2+255255x^2a^4b+46189a^5)\,(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{2}}}{969969x^{21}(b\,x^2+a)^5}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*(-1/21*a^5-5/19*x^2*a^4*b-10/17*a^3*x^4*b^2-
2/3*a^2*x^6*b^3-5/13*a*x^8*b^4-1/11*x^10*b^5)/x^21
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{22}}\,dx = -\frac{88179\,b^5x^{10}+373065\,ab^4x^8+646646\,a^2b^3x^6+570570\,a^3b^2x^4+255255\,a^4bx^2+46189\,a^5}{969969\,x^{21}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="fricas")
```

output
```
-1/969969*(88179*b^5*x^10 + 373065*a*b^4*x^8 + 646646*a^2*b^3*x^6 + 570570
*a^3*b^2*x^4 + 255255*a^4*b*x^2 + 46189*a^5)/x^21
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{22}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^{22}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**22,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**22, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{22}}\,dx = -\frac{b^5}{11\,x^{11}} - \frac{5\,ab^4}{13\,x^{13}} - \frac{2\,a^2b^3}{3\,x^{15}} - \frac{10\,a^3b^2}{17\,x^{17}} - \frac{5\,a^4b}{19\,x^{19}} - \frac{a^5}{21\,x^{21}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="maxima")
```

output
```
-1/11*b^5/x^11 - 5/13*a*b^4/x^13 - 2/3*a^2*b^3/x^15 - 10/17*a^3*b^2/x^17 -
 5/19*a^4*b/x^19 - 1/21*a^5/x^21
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{22}}\,dx = -\frac{88179\,b^5x^{10}\mathrm{sgn}(bx^2+a) + 373065\,ab^4x^8\mathrm{sgn}(bx^2+a) + 646646\,a^2b^3x^6\mathrm{sgn}(bx^2+a) + 570570\,a^3b^2x^4\mathrm{sgn}(b}{969969\,x^{21}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x, algorithm="giac")
```

output
```
-1/969969*(88179*b^5*x^10*sgn(b*x^2 + a) + 373065*a*b^4*x^8*sgn(b*x^2 + a)
 + 646646*a^2*b^3*x^6*sgn(b*x^2 + a) + 570570*a^3*b^2*x^4*sgn(b*x^2 + a) +
 255255*a^4*b*x^2*sgn(b*x^2 + a) + 46189*a^5*sgn(b*x^2 + a))/x^21
```

Mupad [B] (verification not implemented)

Time = 17.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{22}} \, dx = -\frac{a^5 \sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{21\,x^{21}\,(b\,x^2 + a)} - \frac{b^5 \sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{11\,x^{11}\,(b\,x^2 + a)} - \frac{5\,a\,b^4 \sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{13\,x^{13}\,(b\,x^2 + a)} - \frac{5\,a^4\,b \sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{19\,x^{19}\,(b\,x^2 + a)} - \frac{2\,a^2\,b^3 \sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{3\,x^{15}\,(b\,x^2 + a)} - \frac{10\,a^3\,b^2 \sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}}{17\,x^{17}\,(b\,x^2 + a)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^22,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(21*x^21*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(11*x^11*(a + b*x^2)) - (5*a*b^4*(a^2 + b^
2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4
+ 2*a*b*x^2)^(1/2))/(19*x^19*(a + b*x^2)) - (2*a^2*b^3*(a^2 + b^2*x^4 + 2*
a*b*x^2)^(1/2))/(3*x^15*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*
x^2)^(1/2))/(17*x^17*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{5/2}}{x^{22}} \, dx = \frac{-88179b^5x^{10} - 373065a\,b^4x^8 - 646646a^2b^3x^6 - 570570a^3b^2x^4 - 255255a^4b}{969969x^{21}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x)
```

output
```
( - 46189*a**5 - 255255*a**4*b*x**2 - 570570*a**3*b**2*x**4 - 646646*a**2*
b**3*x**6 - 373065*a*b**4*x**8 - 88179*b**5*x**10)/(969969*x**21)
```

3.570 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}}\,dx = -\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}(a+bx^2)} - \frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}(a+bx^2)} - \frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}(a+bx^2)} - \frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}(a+bx^2)} - \frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}(a+bx^2)}$$

output

```
-1/23*a^5*((b*x^2+a)^2)^(1/2)/x^23/(b*x^2+a)-5/21*a^4*b*((b*x^2+a)^2)^(1/2
)/x^21/(b*x^2+a)-10/19*a^3*b^2*((b*x^2+a)^2)^(1/2)/x^19/(b*x^2+a)-10/17*a^
2*b^3*((b*x^2+a)^2)^(1/2)/x^17/(b*x^2+a)-1/3*a*b^4*((b*x^2+a)^2)^(1/2)/x^1
5/(b*x^2+a)-1/13*b^5*((b*x^2+a)^2)^(1/2)/x^13/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}}\,dx = -\frac{\sqrt{(a+bx^2)^2}(88179a^5+482885a^4bx^2+1067430a^3b^2x^4+1193010a^2b^3x^6+676039ab^4x^8+156009b^5x^{10})}{2028117x^{23}(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

output
```
-1/2028117*(Sqrt[(a + b*x^2)^2]*(88179*a^5 + 482885*a^4*b*x^2 + 1067430*a^
3*b^2*x^4 + 1193010*a^2*b^3*x^6 + 676039*a*b^4*x^8 + 156009*b^5*x^10))/(x^
23*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{x^{24}}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{x^{24}}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{a^5}{x^{24}}+\frac{5ba^4}{x^{22}}+\frac{10b^2a^3}{x^{20}}+\frac{10b^3a^2}{x^{18}}+\frac{5b^4a}{x^{16}}+\frac{b^5}{x^{14}}\right)dx}{a+bx^2}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\left(-\frac{a^5}{23x^{23}}-\frac{5a^4b}{21x^{21}}-\frac{10a^3b^2}{19x^{19}}-\frac{10a^2b^3}{17x^{17}}-\frac{ab^4}{3x^{15}}-\frac{b^5}{13x^{13}}\right)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

output
```
((-1/23*a^5/x^23 - (5*a^4*b)/(21*x^21) - (10*a^3*b^2)/(19*x^19) - (10*a^2*
b^3)/(17*x^17) - (a*b^4)/(3*x^15) - b^5/(13*x^13))*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 33.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{1}{23}a^5-\frac{5}{21}x^2a^4b-\frac{10}{19}a^3x^4b^2-\frac{10}{17}a^2x^6b^3-\frac{1}{3}a\,x^8b^4-\frac{1}{13}x^{10}b^5\right)}{(b\,x^2+a)x^{23}}$	79
gosper	$-\frac{(156009x^{10}b^5+676039a\,x^8b^4+1193010a^2x^6b^3+1067430a^3x^4b^2+482885x^2a^4b+88179a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{2028117x^{23}(b\,x^2+a)^5}$	80
default	$-\frac{(156009x^{10}b^5+676039a\,x^8b^4+1193010a^2x^6b^3+1067430a^3x^4b^2+482885x^2a^4b+88179a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{2028117x^{23}(b\,x^2+a)^5}$	80
orering	$-\frac{(156009x^{10}b^5+676039a\,x^8b^4+1193010a^2x^6b^3+1067430a^3x^4b^2+482885x^2a^4b+88179a^5)(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{2}}}{2028117x^{23}(b\,x^2+a)^5}$	89

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*(-1/23*a^5-5/21*x^2*a^4*b-10/19*a^3*x^4*b^2-
10/17*a^2*x^6*b^3-1/3*a*x^8*b^4-1/13*x^10*b^5)/x^23
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}}\,dx = -\frac{156009\,b^5x^{10}+676039\,ab^4x^8+1193010\,a^2b^3x^6+1067430\,a^3b^2x^4+482885\,a^4bx^2+88179\,a^5}{2028117\,x^{23}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="fricas")
```

output
```
-1/2028117*(156009*b^5*x^10 + 676039*a*b^4*x^8 + 1193010*a^2*b^3*x^6 + 106
7430*a^3*b^2*x^4 + 482885*a^4*b*x^2 + 88179*a^5)/x^23
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{x^{24}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**24,x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/x**24, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}}\,dx = -\frac{b^5}{13\,x^{13}} - \frac{ab^4}{3\,x^{15}} - \frac{10\,a^2b^3}{17\,x^{17}} - \frac{10\,a^3b^2}{19\,x^{19}} - \frac{5\,a^4b}{21\,x^{21}} - \frac{a^5}{23\,x^{23}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="maxima")
```

output
```
-1/13*b^5/x^13 - 1/3*a*b^4/x^15 - 10/17*a^2*b^3/x^17 - 10/19*a^3*b^2/x^19
- 5/21*a^4*b/x^21 - 1/23*a^5/x^23
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.42

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}}\,dx = -\frac{156009\,b^5x^{10}\mathrm{sgn}(bx^2+a) + 676039\,ab^4x^8\mathrm{sgn}(bx^2+a) + 1193010\,a^2b^3x^6\mathrm{sgn}(bx^2+a) + 1067430\,a^3b^2x^4\mathrm{s}}{2028117\,x^{23}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x, algorithm="giac")
```

output
```
-1/2028117*(156009*b^5*x^10*sgn(b*x^2 + a) + 676039*a*b^4*x^8*sgn(b*x^2 +
a) + 1193010*a^2*b^3*x^6*sgn(b*x^2 + a) + 1067430*a^3*b^2*x^4*sgn(b*x^2 +
a) + 482885*a^4*b*x^2*sgn(b*x^2 + a) + 88179*a^5*sgn(b*x^2 + a))/x^23
```

Mupad [B] (verification not implemented)

Time = 18.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}}\,dx = -\frac{a^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{23\,x^{23}\,\left(b\,x^2+a\right)} - \frac{b^5\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13\,x^{13}\,\left(b\,x^2+a\right)} - \frac{a\,b^4\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{3\,x^{15}\,\left(b\,x^2+a\right)} - \frac{5\,a^4\,b\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{21\,x^{21}\,\left(b\,x^2+a\right)} - \frac{10\,a^2\,b^3\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{17\,x^{17}\,\left(b\,x^2+a\right)} - \frac{10\,a^3\,b^2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{19\,x^{19}\,\left(b\,x^2+a\right)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/x^24,x)
```

output
```
- (a^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(23*x^23*(a + b*x^2)) - (b^5*(a^
2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13*x^13*(a + b*x^2)) - (a*b^4*(a^2 + b^2*
x^4 + 2*a*b*x^2)^(1/2))/(3*x^15*(a + b*x^2)) - (5*a^4*b*(a^2 + b^2*x^4 + 2
*a*b*x^2)^(1/2))/(21*x^21*(a + b*x^2)) - (10*a^2*b^3*(a^2 + b^2*x^4 + 2*a*
b*x^2)^(1/2))/(17*x^17*(a + b*x^2)) - (10*a^3*b^2*(a^2 + b^2*x^4 + 2*a*b*x
^2)^(1/2))/(19*x^19*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{24}}\,dx = \frac{-156009b^5x^{10}-676039a\,b^4x^8-1193010a^2b^3x^6-1067430a^3b^2x^4-48288}{2028117x^{23}}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x)
```

output
```
( - 88179*a**5 - 482885*a**4*b*x**2 - 1067430*a**3*b**2*x**4 - 1193010*a**
2*b**3*x**6 - 676039*a*b**4*x**8 - 156009*b**5*x**10)/(2028117*x**23)
```

3.571 $\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 127

$$\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=-\frac{ax^2(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^4(a+bx^2)}{4b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2(a+bx^2)\log(a+bx^2)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-1/2*a*x^2*(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/4*x^4*(b*x^2+a)/b/((b*x^2+a
)^2)^(1/2)+1/2*a^2*(b*x^2+a)*ln(b*x^2+a)/b^3/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.31

$$\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\frac{bx^2(-2a+bx^2)\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(a+bx^2)^2}\right)\right)}{a^2+abx^2-\sqrt{a^2}\sqrt{(a+bx^2)^2}}-2a^2\log\left(\sqrt{a^2}-bx^2-\sqrt{(a+bx^2)^2}\right)+2a^2\log\left(b^3\left(\sqrt{a^2}+b\right.\right.}{4b^3}$$

input
```
Integrate[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
-1/4*((b*x^2*(-2*a + b*x^2)*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b*
x^2)^2])))/(a^2 + a*b*x^2 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2]) - 2*a^2*Log[Sqr
t[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] + 2*a^2*Log[b^3*(Sqrt[a^2] + b*x^2 -
 Sqrt[(a + b*x^2)^2])])/b^3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^2)\int \frac{x^5}{b(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{x^5}{bx^2+a}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{(a+bx^2)\int \frac{x^4}{bx^2+a}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx^2)\int \left(\frac{a^2}{b^2(bx^2+a)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^2)\left(\frac{a^2\log(a+bx^2)}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{2b}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*(-((a*x^2)/b^2) + x^4/(2*b) + (a^2*Log[a + b*x^2])/b^3))/(2*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.33

method	result	size
pseudoelliptic	$\frac{\left(b^{2} x^{4}-2 a b \,x^{2}+2 a^{2} \ln \left(b \,x^{2}+a \right)\right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{4 b^{3}}$	42
default	$\frac{\left(b \,x^{2}+a \right) \left(b^{2} x^{4}-2 a b \,x^{2}+2 a^{2} \ln \left(b \,x^{2}+a \right)\right)}{4 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, b^{3}}$	52
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(-b \,x^{2}+a \right)^{2}}{4 \left(b \,x^{2}+a \right) b^{3}}+\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, a^{2} \ln \left(b \,x^{2}+a \right)}{2 \left(b \,x^{2}+a \right) b^{3}}$	73

input
```
int(x^5/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(b^2*x^4-2*a*b*x^2+2*a^2*ln(b*x^2+a))*csgn(b*x^2+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{b^{2} x^{4} - 2 \, a b x^{2} + 2 \, a^{2} \log\left(b x^{2} + a\right)}{4 \, b^{3}}$$

input
```
integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(b^2*x^4 - 2*a*b*x^2 + 2*a^2*log(b*x^2 + a))/b^3
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{x^5}{\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate(x**5/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x**5/sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{a^2\log\left(bx^2+a\right)}{2\,b^3}+\frac{bx^4-2\,ax^2}{4\,b^2}$$

input
```
integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*a^2*log(b*x^2 + a)/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{a^2\log\left(|bx^2+a|\right)\operatorname{sgn}(bx^2+a)}{2\,b^3}+\frac{bx^4\operatorname{sgn}(bx^2+a)-2\,ax^2\operatorname{sgn}(bx^2+a)}{4\,b^2}$$

input
```
integrate(x^5/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*a^2*log(abs(b*x^2 + a))*sgn(b*x^2 + a)/b^3 + 1/4*(b*x^4*sgn(b*x^2 + a)
 - 2*a*x^2*sgn(b*x^2 + a))/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{a^2 + 2abx^2 + b^2x^4}}\,dx = \int \frac{x^5}{\sqrt{(b\,x^2 + a)^2}}\,dx$$

input
```
int(x^5/((a + b*x^2)^2)^(1/2),x)
```

output
```
int(x^5/((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{x^5}{\sqrt{a^2 + 2abx^2 + b^2x^4}}\,dx = \frac{2\log(b\,x^2 + a)\,a^2 - 2ab\,x^2 + b^2x^4}{4b^3}$$

input
```
int(x^5/((b*x^2+a)^2)^(1/2),x)
```

output
```
(2*log(a + b*x**2)*a**2 - 2*a*b*x**2 + b**2*x**4)/(4*b**3)
```

3.572 $\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{a(a+bx^2)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/2*((b*x^2+a)^2)^(1/2)/b^2-1/2*a*(b*x^2+a)*ln(b*x^2+a)/b^2/((b*x^2+a)^2)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.51

$$\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{\frac{bx^2\left(-\sqrt{a^2}(a+bx^2)+a\sqrt{(a+bx^2)^2}\right)}{a^2+abx^2-\sqrt{a^2}\sqrt{(a+bx^2)^2}}+2a\text{arctanh}\left(\frac{bx^2}{\sqrt{a^2}-\sqrt{(a+bx^2)^2}}\right)}{2b^2}$$

input
```
Integrate[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((b*x^2*(-(Sqrt[a^2]*(a + b*x^2)) + a*Sqrt[(a + b*x^2)^2]))/(a^2 + a*b*x^2
 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2]) + 2*a*ArcTanh[(b*x^2)/(Sqrt[a^2] - Sqrt[
(a + b*x^2)^2])])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1434, 1100, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{b^2x^4+2abx^2+a^2}}dx^2$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{2}\left(\frac{\sqrt{a^2+2abx^2+b^2x^4}}{b^2}-\frac{a\int \frac{1}{\sqrt{b^2x^4+2abx^2+a^2}}dx^2}{b}\right)$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{2}\left(\frac{\sqrt{a^2+2abx^2+b^2x^4}}{b^2}-\frac{a\left(a+bx^2\right)\int \frac{1}{b^2x^2+ab}dx^2}{\sqrt{a^2+2abx^2+b^2x^4}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{\sqrt{a^2+2abx^2+b^2x^4}}{b^2}-\frac{a\left(a+bx^2\right)\log\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}\right)$$

input
```
Int[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/b^2 - (a*(a + b*x^2)*Log[a + b*x^2])/(b^2
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.41

method	result	size
pseudoelliptic	$-\frac{\left(-b\,x^{2}+a\ln\left(b\,x^{2}+a\right)\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{2b^{2}}$	31
default	$-\frac{\left(b\,x^{2}+a\right)\left(-b\,x^{2}+a\ln\left(b\,x^{2}+a\right)\right)}{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b^{2}}$	41
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,x^{2}}{2\left(b\,x^{2}+a\right)b}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a\ln\left(b\,x^{2}+a\right)}{2\left(b\,x^{2}+a\right)b^{2}}$	64

input
```
int(x^3/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x^2+a*ln(b*x^2+a))*csgn(b*x^2+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.29

$$\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{bx^2-a\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{x^3}{\sqrt{\left(a+bx^2\right)^2}}\,dx$$

input
```
integrate(x**3/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.31

$$\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{x^2}{2\,b}-\frac{a\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.44

$$\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{2}\left(\frac{x^2}{b}-\frac{a\log\left(|bx^2+a|\right)}{b^2}\right)\operatorname{sgn}\left(bx^2+a\right)$$

input
```
integrate(x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(x^2/b - a*log(abs(b*x^2 + a))/b^2)*sgn(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 19.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,b^2} - \frac{a\,b\,\ln\left(a\,b+\sqrt{(b\,x^2+a)^2}\,\sqrt{b^2}+b^2\,x^2\right)}{2\,(b^2)^{3/2}}$$

input
```
int(x^3/((a + b*x^2)^2)^(1/2),x)
```

output
```
(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(2*b^2) - (a*b*log(a*b + ((a + b*x^2)^2)
^(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.29

$$\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{-\log(b\,x^2+a)\,a+b\,x^2}{2b^2}$$

input
```
int(x^3/((b*x^2+a)^2)^(1/2),x)
```

output

```
( - log(a + b*x**2)*a + b*x**2)/(2*b**2)
```

3.573 $\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 44

$$\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/2*(b*x^2+a)*ln(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\frac{\sqrt{a^2}}{b}-\frac{\sqrt{a^2+2abx^2+b^2x^4}}{b}}{x^2}\right)}{b}$$

input
```
Integrate[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
-(ArcTanh[(Sqrt[a^2]/b - Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/b)/x^2]/b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1432, 1079, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$
$$\Big\downarrow \text{1432}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{b^2x^4+2abx^2+a^2}}dx^2$$
$$\Big\downarrow \text{1079}$$
$$\frac{b(a+bx^2)\int \frac{1}{b^2x^2+ab}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$\Big\downarrow \text{16}$$
$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

method	result	size
pseudoelliptic	$\frac{\ln\left(b\,x^{2}+a\right)\operatorname{csgn}\left(b\,x^{2}+a\right)}{2b}$	22
default	$\frac{\left(b\,x^{2}+a\right)\ln\left(b\,x^{2}+a\right)}{2b\sqrt{\left(b\,x^{2}+a\right)^{2}}}$	32
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(b\,x^{2}+a\right)}{2\left(b\,x^{2}+a\right)b}$	34

input
```
int(x/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(b*x^2+a)/b*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

$$\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\log\left(bx^2+a\right)}{2\,b}$$

input
```
integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log(b*x^2 + a)/b
```

Sympy [F]

$$\int \frac{x}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \, dx = \int \frac{x}{\sqrt{(a + bx^2)^2}} \, dx$$

input
```
integrate(x/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x/sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

$$\int \frac{x}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \, dx = \frac{\log (bx^2 + a)}{2 \, b}$$

input
```
integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(b*x^2 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{x}{\sqrt{a^2 + 2abx^2 + b^2x^4}} \, dx = \frac{\log (|bx^2 + a|) \operatorname{sgn}(bx^2 + a)}{2 \, b}$$

input
```
integrate(x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*log(abs(b*x^2 + a))*sgn(b*x^2 + a)/b
```

Mupad [B] (verification not implemented)

Time = 18.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\ln\left(b^2\,x^2+a\,b\right)\,\mathrm{sign}(2\,b^2\,x^2+2\,a\,b)}{2\,\sqrt{b^2}}$$

input
```
int(x/((a + b*x^2)^2)^(1/2),x)
```

output
```
(log(a*b + b^2*x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

$$\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\log(b\,x^2+a)}{2b}$$

input
```
int(x/((b*x^2+a)^2)^(1/2),x)
```

output
```
log(a + b*x**2)/(2*b)
```

3.574 $\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{(a+bx^2)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(a+bx^2)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
(b*x^2+a)*ln(x)/a/((b*x^2+a)^2)^(1/2)-1/2*(b*x^2+a)*ln(b*x^2+a)/a/((b*x^2+
a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$$
$$=\frac{-2a\log(x^2)+\left(a-\sqrt{a^2}\right)\log\left(\sqrt{a^2}-bx^2-\sqrt{(a+bx^2)^2}\right)+a\log\left(\sqrt{a^2}+bx^2-\sqrt{(a+bx^2)^2}\right)+\sqrt{a}}{4a\sqrt{a^2}}$$

input
```
Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
(-2*a*Log[x^2] + (a - Sqrt[a^2])*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^
2]] + a*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]] + Sqrt[a^2]*Log[a*(Sq
rt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2])])/(4*a*Sqrt[a^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1384, 27, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^2)\int \frac{1}{bx(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{1}{x(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{(a+bx^2)\int \frac{1}{x^2(bx^2+a)}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{47}$$

$$\frac{(a+bx^2)\left(\frac{\int \frac{1}{x^2}dx^2}{a}-\frac{b\int \frac{1}{bx^2+a}dx^2}{a}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{14}$$

$$\frac{(a+bx^2)\left(\frac{\log(x^2)}{a}-\frac{b\int \frac{1}{bx^2+a}dx^2}{a}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{16}$$

$$\frac{(a+bx^2)\left(\frac{\log(x^2)}{a}-\frac{\log(a+bx^2)}{a}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
((a + b*x^2)*(Log[x^2]/a - Log[a + b*x^2]/a))/(2*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.36

method	result	size
pseudoelliptic	$-\frac{\left(-\ln \left(x^{2}\right)+\ln \left(b \,x^{2}+a \right)\right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{2 a}$	29
default	$-\frac{\left(b \,x^{2}+a \right) \left(\ln \left(b \,x^{2}+a \right)-2 \ln \left(x \right)\right)}{2 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, a}$	37
risch	$-\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \ln \left(b \,x^{2}+a \right)}{2 \left(b \,x^{2}+a \right) a}+\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \ln \left(x \right)}{\left(b \,x^{2}+a \right) a}$	61

input
```
int(1/x/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-ln(x^2)+ln(b*x^2+a))*csgn(b*x^2+a)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.22

$$\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx=-\frac{\log\left(bx^2+a\right)-2\,\log\left(x\right)}{2\,a}$$

input
```
integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(log(b*x^2 + a) - 2*log(x))/a
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{x\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate(1/x/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a + b*x**2)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.29

$$\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\log(bx^2+a)}{2\,a} + \frac{\log(x^2)}{2\,a}$$

input
```
integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*log(b*x^2 + a)/a + 1/2*log(x^2)/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.41

$$\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{2}\left(\frac{\log(x^2)}{a} - \frac{\log(|bx^2+a|)}{a}\right)\mathrm{sgn}(bx^2+a)$$

input
```
integrate(1/x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(log(x^2)/a - log(abs(b*x^2 + a))/a)*sgn(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 18.69 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\ln\left(\sqrt{(b\,x^2+a)^2}\,\sqrt{a^2}+a^2+a\,b\,x^2\right)+\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}}$$

input
```
int(1/(x*((a + b*x^2)^2)^(1/2)),x)
```

output
```
-(log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) + a^2 + a*b*x^2) + log(1/x^2))/(2*
(a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.25

$$\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{-\log(b\,x^2+a)+2\log(x)}{2a}$$

input
```
int(1/x/((b*x^2+a)^2)^(1/2),x)
```

output
```
( - log(a + b*x**2) + 2*log(x))/(2*a)
```

3.575 $\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 125

$$\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{-a-bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b(a+bx^2)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b(a+bx^2)\log(a+bx^2)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/2*(-b*x^2-a)/a/x^2/((b*x^2+a)^2)^(1/2)-b*(b*x^2+a)*ln(x)/a^2/((b*x^2+a)^
2)^(1/2)+1/2*b*(b*x^2+a)*ln(b*x^2+a)/a^2/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{a^2-\sqrt{a^2}\sqrt{(a+bx^2)^2}+2abx^2\log(x^2)+\left(-a+\sqrt{a^2}\right)bx^2\log\left(\sqrt{a^2}-bx^2-\sqrt{(a+bx^2)^2}\right)-abx^2\log}{4(a^2)^{3/2}x^2}$$

input
```
Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
(a^2 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2] + 2*a*b*x^2*Log[x^2] + (-a + Sqrt[a^2
])*b*x^2*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] - a*b*x^2*Log[Sqrt[a
^2] + b*x^2 - Sqrt[(a + b*x^2)^2]] - Sqrt[a^2]*b*x^2*Log[Sqrt[a^2] + b*x^2
 - Sqrt[(a + b*x^2)^2]])/(4*(a^2)^(3/2)*x^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^2)\int \frac{1}{bx^3(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{1}{x^3(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{(a+bx^2)\int \frac{1}{x^4(bx^2+a)}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx^2)\int \left(\frac{b^2}{a^2(bx^2+a)}-\frac{b}{a^2x^2}+\frac{1}{ax^4}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^2)\left(-\frac{b\log(x^2)}{a^2}+\frac{b\log(a+bx^2)}{a^2}-\frac{1}{ax^2}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
((a + b*x^2)*(-(1/(a*x^2)) - (b*Log[x^2])/a^2 + (b*Log[a + b*x^2])/a^2))/(
2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.34

method	result	size
pseudoelliptic	$-\frac{\left(b \ln \left(x^{2}\right) x^{2}-b \ln \left(b \,x^{2}+a \right) x^{2}+a \right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{2 a^{2} x^{2}}$	42
default	$\frac{\left(b \,x^{2}+a \right) \left(b \ln \left(b \,x^{2}+a \right) x^{2}-2 \ln \left(x \right) x^{2} b -a \right)}{2 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, a^{2} x^{2}}$	52
risch	$-\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}}{2 \left(b \,x^{2}+a \right) a \,x^{2}}-\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, b \ln \left(x \right)}{\left(b \,x^{2}+a \right) a^{2}}+\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, b \ln \left(-b \,x^{2}-a \right)}{2 \left(b \,x^{2}+a \right) a^{2}}$	95

input
```
int(1/x^3/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*ln(x^2)*x^2-b*ln(b*x^2+a)*x^2+a)*csgn(b*x^2+a)/a^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{bx^2\log\left(bx^2+a\right)-2\,bx^2\log\left(x\right)-a}{2\,a^2x^2}$$

input
```
integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{x^3\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate(1/x**3/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((a + b*x**2)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{b\log(bx^2+a)}{2\,a^2} - \frac{b\log(x^2)}{2\,a^2} - \frac{1}{2\,ax^2}$$

input
```
integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*log(b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{1}{2}\left(\frac{b\log(x^2)}{a^2} - \frac{b\log(|bx^2+a|)}{a^2} - \frac{bx^2-a}{a^2x^2}\right)\operatorname{sgn}(bx^2+a)$$

input
```
integrate(1/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(b*log(x^2)/a^2 - b*log(abs(b*x^2 + a))/a^2 - (b*x^2 - a)/(a^2*x^2))*
sgn(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 18.59 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^2}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}\right)}{2\,{\left(a^2\right)}^{3/2}} - \frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,a^2\,x^2}$$

input
```
int(1/(x^3*((a + b*x^2)^2)^(1/2)),x)
```

output
```
(a*b*atanh((a^2 + a*b*x^2)/((a^2)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))
))/(2*(a^2)^(3/2)) - (a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(2*a^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\log(b\,x^2+a)\,b\,x^2-2\log(x)\,b\,x^2-a}{2a^2x^2}$$

input
```
int(1/x^3/((b*x^2+a)^2)^(1/2),x)
```

output
```
(log(a + b*x**2)*b*x**2 - 2*log(x)*b*x**2 - a)/(2*a**2*x**2)
```

3.576 $\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 129

$$\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=-\frac{ax(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-a*x*(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/3*x^3*(b*x^2+a)/b/((b*x^2+a)^2)^(
1/2)+a^(3/2)*(b*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/b^(5/2)/((b*x^2+a)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{(a+bx^2)\left(\sqrt{b}x(-3a+bx^2)+3a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*(Sqrt[b]*x*(-3*a + b*x^2) + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]]))/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.56, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b(a+bx^2)\int \frac{x^4}{b(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{x^4}{bx^2+a}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{(a+bx^2)\int \left(\frac{a^2}{b^2(bx^2+a)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx^2)\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*(-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/b^(5/2)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{(b\,x^2+a)\left(-\sqrt{ab}\,b\,x^3+3\sqrt{ab}\,ax-3a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)}{3\sqrt{(b\,x^2+a)^2}\,b^2\sqrt{ab}}$	64
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(\frac{1}{3}b\,x^3-ax\right)}{(b\,x^2+a)b^2}+\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x+a\right)}{2(b\,x^2+a)b^3}-\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,a\ln\left(\sqrt{-ab}\,x+a\right)}{2(b\,x^2+a)b^3}$	124

input
```
int(x^4/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)*(-(a*b)^(1/2)*b*x^3+3*(a*b)^(1/2)*a*x-3*a^2*arctan(b/(a*b)^
(1/2)*x))/((b*x^2+a)^2)^(1/2)/b^2/(a*b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

$$\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \left[\frac{2\,bx^3+3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,ax}{6\,b^2}, \frac{bx^3+3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\,ax}{3\,b^2}\right]$$

input
```
integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(2*b*x^3 + 3*a*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
 a)) - 6*a*x)/b^2, 1/3*(b*x^3 + 3*a*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*
a*x)/b^2]
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{x^4}{\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate(x**4/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x**4/sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.29

$$\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{bx^3-3\,ax}{3\,b^2}$$

input
```
integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*x^3 - 3*a*x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\mathrm{sgn}(bx^2+a)}{\sqrt{ab}b^2} + \frac{b^2x^3\mathrm{sgn}(bx^2+a)-3\,abx\mathrm{sgn}(bx^2+a)}{3\,b^3}$$

input
```
integrate(x^4/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
a^2*arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/(sqrt(a*b)*b^2) + 1/3*(b^2*x^3*sg
n(b*x^2 + a) - 3*a*b*x*sgn(b*x^2 + a))/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{x^4}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

input
```
int(x^4/((a + b*x^2)^2)^(1/2),x)
```

output
```
int(x^4/((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.29

$$\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a-3abx+b^2x^3}{3b^3}$$

input
```
int(x^4/((b*x^2+a)^2)^(1/2),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a - 3*a*b*x + b**2*x**3)/
(3*b**3)
```

3.577 $\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{x(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
x*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)-a^(1/2)*(b*x^2+a)*arctan(b^(1/2)*x/a^(1/
2))/b^(3/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{(a+bx^2)\left(\sqrt{b}x-\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{b^{3/2}\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*(Sqrt[b]*x - Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(b^(3/2)*S
qrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1384, 27, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^2)\int \frac{x^2}{b(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{x^2}{bx^2+a}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{262}$$

$$\frac{(a+bx^2)\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx^2)\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)))/Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\left(b\,x^{2}+a\right)\left(x\sqrt{a b}-a \arctan\left(\frac{b x}{\sqrt{a b}}\right)\right)}{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, b \sqrt{a b}}$	48
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, x}{\left(b\,x^{2}+a\right) b}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, \sqrt{-a b}\, \ln\left(-\sqrt{-a b}\, x-a\right)}{2\left(b\,x^{2}+a\right) b^{2}}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\, \sqrt{-a b}\, \ln\left(\sqrt{-a b}\, x-a\right)}{2\left(b\,x^{2}+a\right) b^{2}}$	116

input
```
int(x^2/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)*(x*(a*b)^(1/2)-a*arctan(b/(a*b)^(1/2)*x))/((b*x^2+a)^2)^(1/2)/b/
(a*b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b},\ -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

input
```
integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
 -(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{x^2}{\sqrt{\left(a+bx^2\right)^2}}\,dx$$

input
```
integrate(x**2/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.29

$$\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{x}{b}$$

input
```
integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

$$\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)\operatorname{sgn}(bx^2+a)}{\sqrt{ab}b}+\frac{x\operatorname{sgn}(bx^2+a)}{b}$$

input
```
integrate(x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-a*arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/(sqrt(a*b)*b) + x*sgn(b*x^2 + a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{x^2}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

input
```
int(x^2/((a + b*x^2)^2)^(1/2),x)
```

output
```
int(x^2/((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.29

$$\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)+bx}{b^2}$$

input
```
int(x^2/((b*x^2+a)^2)^(1/2),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))) + b*x)/b**2
```

3.578 $\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
(b*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(1/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[(a + b*x^2
)^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1384, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$
$$\Big\downarrow \ 1384$$
$$\frac{b(a+bx^2)\int \frac{1}{b^2x^2+ab}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$\Big\downarrow \ 218$$
$$\frac{(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{(b\,x^2+a)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{(b\,x^2+a)^2}\,\sqrt{ab}}$	34
risch	$-\frac{\sqrt{(b\,x^2+a)^2}\,\ln\left(bx+\sqrt{-ab}\right)}{2(b\,x^2+a)\sqrt{-ab}}+\frac{\sqrt{(b\,x^2+a)^2}\,\ln\left(-bx+\sqrt{-ab}\right)}{2(b\,x^2+a)\sqrt{-ab}}$	81

input
```
int(1/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((b*x^2+a)^2)^(1/2)*(b*x^2+a)/(a*b)^(1/2)*arctan(b/(a*b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

input
```
integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt
(a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate(1/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(1/sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)\operatorname{sgn}(bx^2+a)}{\sqrt{ab}}$$

input
```
integrate(1/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
arctan(b*x/sqrt(a*b))*sgn(b*x^2 + a)/sqrt(a*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a^2 + 2abx^2 + b^2x^4}}\,dx = \int \frac{1}{\sqrt{(b\,x^2 + a)^2}}\,dx$$

input
```
int(1/((a + b*x^2)^2)^(1/2),x)
```

output
```
int(1/((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{a^2 + 2abx^2 + b^2x^4}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{ab}$$

input
```
int(1/((b*x^2+a)^2)^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))))/(a*b)
```

3.579 $\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 92

$$\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx=-\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-(b*x^2+a)/a/x/((b*x^2+a)^2)^(1/2)-b^(1/2)*(b*x^2+a)*arctan(b^(1/2)*x/a^(1
/2))/a^(3/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx=-\frac{(a+bx^2)\left(\sqrt{a}+\sqrt{b}x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{a^{3/2}x\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
-(((a + b*x^2)*(Sqrt[a] + Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(a^(3/2)
*x*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1384, 27, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b(a+bx^2)\int \frac{1}{bx^2(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{1}{x^2(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{(a+bx^2)\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(a+bx^2)\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
((a + b*x^2)*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))
/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{(b\,x^{2}+a)\left(b\arctan\left(\frac{bx}{\sqrt{ab}}\right)x+\sqrt{ab}\right)}{\sqrt{(b\,x^{2}+a)^{2}}\,a\sqrt{ab}\,x}$	50
risch	$-\frac{\sqrt{(b\,x^{2}+a)^{2}}}{(b\,x^{2}+a)ax}+\frac{\sqrt{(b\,x^{2}+a)^{2}}\,\sqrt{-ab}\,\ln(-bx+\sqrt{-ab})}{2(b\,x^{2}+a)a^{2}}-\frac{\sqrt{(b\,x^{2}+a)^{2}}\,\sqrt{-ab}\,\ln(-bx-\sqrt{-ab})}{2(b\,x^{2}+a)a^{2}}$	118

input
```
int(1/x^2/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)*(b*arctan(b/(a*b)^(1/2)*x)*x+(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a
/(a*b)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax}, -\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

input
```
integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a
*x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{x^2\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate(1/x**2/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((a + b*x**2)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{1}{ax}$$

input
```
integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\left(\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{1}{ax}\right)\operatorname{sgn}\left(bx^2+a\right)$$

input
```
integrate(1/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-(b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*x))*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{x^2\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

input
```
int(1/(x^2*((a + b*x^2)^2)^(1/2)),x)
```

output
```
int(1/(x^2*((a + b*x^2)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.33

$$\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)x-a}{a^2x}$$

input
```
int(1/x^2/((b*x^2+a)^2)^(1/2),x)
```

output
```
( - (sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*x + a))/(a**2*x)
```

3.580 $\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{-a-bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b(a+bx^2)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/2}(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/3*(-b*x^2-a)/a/x^3/((b*x^2+a)^2)^(1/2)+b*(b*x^2+a)/a^2/x/((b*x^2+a)^2)^(
1/2)+b^(3/2)*(b*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)/((b*x^2+a)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(a+bx^2)\left(\sqrt{a}(a-3bx^2)-3b^{3/2}x^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{3a^{5/2}x^3\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
-1/3*((a + b*x^2)*(Sqrt[a]*(a - 3*b*x^2) - 3*b^(3/2)*x^3*ArcTan[(Sqrt[b]*x
)/Sqrt[a]]))/(a^(5/2)*x^3*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^2)\int \frac{1}{bx^4(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{1}{x^4(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{264}$$

$$\frac{(a+bx^2)\left(-\frac{b\int \frac{1}{x^2(bx^2+a)}dx}{a}-\frac{1}{3ax^3}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{264}$$

$$\frac{(a+bx^2)\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx^2)\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
((a + b*x^2)*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/a^(3/2)))/a))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.51

method	result	size
default	$-\frac{\left(b\,x^{2}+a\right)\left(-3 b^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right) x^{3}-3 \sqrt{a b}\, b\,x^{2}+\sqrt{a b}\, a\right)}{3 \sqrt{\left(b\,x^{2}+a\right)^{2}}\, a^{2} \sqrt{a b}\, x^{3}}$	68
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{b\,x^{2}}{a^{2}}-\frac{1}{3 a}\right)}{\left(b\,x^{2}+a\right) x^{3}}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\sqrt{-a b}\, b \ln\left(-b x-\sqrt{-a b}\right)}{2 \left(b\,x^{2}+a\right) a^{3}}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\sqrt{-a b}\, b \ln\left(-b x+\sqrt{-a b}\right)}{2 \left(b\,x^{2}+a\right) a^{3}}$	130

input
```
int(1/x^4/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)*(-3*b^2*arctan(b/(a*b)^(1/2)*x)*x^3-3*(a*b)^(1/2)*b*x^2+(a*
b)^(1/2)*a)/((b*x^2+a)^2)^(1/2)/a^2/(a*b)^(1/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \left[\frac{3\,bx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,bx^2-2\,a}{6\,a^2x^3}, \frac{3\,bx^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^2-a}{3\,a^2x^3}\right]$$

input
```
integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
 6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*
b*x^2 - a)/(a^2*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{x^4\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate(1/x**4/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt((a + b*x**2)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^2-a}{3\,a^2x^3}$$

input
```
integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{3}\left(\frac{3\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^2-a}{a^2x^3}\right)\operatorname{sgn}\left(bx^2+a\right)$$

input
```
integrate(1/x^4/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(3*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*b*x^2 - a)/(a^2*x^3)
)*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{x^4\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

input
```
int(1/(x^4*((a + b*x^2)^2)^(1/2)),x)
```

output
```
int(1/(x^4*((a + b*x^2)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^3-a^2+3ab\,x^2}{3a^3x^3}$$

input
```
int(1/x^4/((b*x^2+a)^2)^(1/2),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*x**3 - a**2 + 3*a*b*x**
2)/(3*a**3*x**3)
```

3.581 $\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 158

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^3}{4b^4(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x^2(a+bx^2)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3a(a+bx^2)\log(a+bx^2)}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-3/2*a^2/b^4/((b*x^2+a)^2)^(1/2)+1/4*a^3/b^4/(b*x^2+a)/((b*x^2+a)^2)^(1/2)
+1/2*x^2*(b*x^2+a)/b^3/((b*x^2+a)^2)^(1/2)-3/2*a*(b*x^2+a)*ln(b*x^2+a)/b^4
/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.51

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{-5a^3-4a^2bx^2+4ab^2x^4+2b^3x^6-6a(a+bx^2)^2\log(a+bx^2)}{4b^4(a+bx^2)\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(-5*a^3 - 4*a^2*b*x^2 + 4*a*b^2*x^4 + 2*b^3*x^6 - 6*a*(a + b*x^2)^2*Log[a
+ b*x^2])/(4*b^4*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{3/2}}dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{b^3(a+bx^2)\int \frac{x^7}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$\Big\downarrow \text{27}$$
$$\frac{(a+bx^2)\int \frac{x^7}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$\Big\downarrow \text{243}$$
$$\frac{(a+bx^2)\int \frac{x^6}{(bx^2+a)^3}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$\Big\downarrow \text{49}$$
$$\frac{(a+bx^2)\int \left(-\frac{a^3}{b^3(bx^2+a)^3}+\frac{3a^2}{b^3(bx^2+a)^2}-\frac{3a}{b^3(bx^2+a)}+\frac{1}{b^3}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{(a+bx^2)\left(\frac{a^3}{2b^4(a+bx^2)^2}-\frac{3a^2}{b^4(a+bx^2)}-\frac{3a\log(a+bx^2)}{b^4}+\frac{x^2}{b^3}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(x^2/b^3 + a^3/(2*b^4*(a + b*x^2)^2) - (3*a^2)/(b^4*(a + b*x^
2)) - (3*a*Log[a + b*x^2])/b^4))/(2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.47

method	result	size
pseudoelliptic	$-\frac{3\,\mathrm{csgn}\left(b\,x^{2}+a\right)\left(a\left(b\,x^{2}+a\right)^{2}\ln\left(b\,x^{2}+a\right)-\frac{b^{3}x^{6}}{3}-\frac{2b^{2}x^{4}a}{3}+\frac{2a^{2}b\,x^{2}}{3}+\frac{5a^{3}}{6}\right)}{2\left(b\,x^{2}+a\right)^{2}b^{4}}$	74
default	$-\frac{\left(-2b^{3}x^{6}+6\ln\left(b\,x^{2}+a\right)x^{4}a\,b^{2}-4b^{2}x^{4}a+12\ln\left(b\,x^{2}+a\right)x^{2}a^{2}b+4a^{2}b\,x^{2}+6\ln\left(b\,x^{2}+a\right)a^{3}+5a^{3}\right)\left(b\,x^{2}+a\right)}{4b^{4}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$	103
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,x^{2}}{2\left(b\,x^{2}+a\right)b^{3}}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{3a^{2}x^{2}}{2}-\frac{5a^{3}}{4b}\right)}{\left(b\,x^{2}+a\right)^{3}b^{3}}-\frac{3\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a\ln\left(b\,x^{2}+a\right)}{2\left(b\,x^{2}+a\right)b^{4}}$	105

input
```
int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3/2*csgn(b*x^2+a)*(a*(b*x^2+a)^2*ln(b*x^2+a)-1/3*b^3*x^6-2/3*b^2*x^4*a+2/
3*a^2*b*x^2+5/6*a^3)/(b*x^2+a)^2/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.58

$$\int \frac{x^7}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{2\,b^3x^6+4\,ab^2x^4-4\,a^2bx^2-5\,a^3-6\left(ab^2x^4+2\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*b^3*x^6 + 4*a*b^2*x^4 - 4*a^2*b*x^2 - 5*a^3 - 6*(a*b^2*x^4 + 2*a^2*
b*x^2 + a^3)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [F]

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{x^7}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**7/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.42

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{6\,a^2bx^2+5\,a^3}{4\,(b^6x^4+2\,ab^5x^2+a^2b^4)} + \frac{x^2}{2\,b^3} - \frac{3\,a\log\left(bx^2+a\right)}{2\,b^4}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(6*a^2*b*x^2 + 5*a^3)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 1/2*x^2/b^3
 - 3/2*a*log(b*x^2 + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.58

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{x^2}{2\,b^3\mathrm{sgn}\left(bx^2+a\right)} - \frac{3\,a\log\left(|bx^2+a|\right)}{2\,b^4\mathrm{sgn}\left(bx^2+a\right)} + \frac{9\,ab^2x^4+12\,a^2bx^2+4\,a^3}{4\,(bx^2+a)^2b^4\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*x^2/(b^3*sgn(b*x^2 + a)) - 3/2*a*log(abs(b*x^2 + a))/(b^4*sgn(b*x^2 +
a)) + 1/4*(9*a*b^2*x^4 + 12*a^2*b*x^2 + 4*a^3)/((b*x^2 + a)^2*b^4*sgn(b*x^
2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\int \frac{x^7}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\frac{-6\log(b\,x^2+a)\,a^3-12\log(b\,x^2+a)\,a^2b\,x^2-6\log(b\,x^2+a)\,a\,b^2x^4-3a^3}{4b^4\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
( - 6*log(a + b*x**2)*a**3 - 12*log(a + b*x**2)*a**2*b*x**2 - 6*log(a + b*
x**2)*a*b**2*x**4 - 3*a**3 + 6*a*b**2*x**4 + 2*b**3*x**6)/(4*b**4*(a**2 +
2*a*b*x**2 + b**2*x**4))
```

3.582 $\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^2}{4b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)\log(a+bx^2)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
a/b^3/((b*x^2+a)^2)^(1/2)-1/4*a^2/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1/2*(b
*x^2+a)*ln(b*x^2+a)/b^3/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.70

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\frac{bx^2\left(a\sqrt{(a+bx^2)^2}(-2a^2-abx^2+b^2x^4)+\sqrt{a^2}(2a^3+3a^2bx^2+b^3x^6)\right)}{a^2(a+bx^2)\left(a^2+abx^2-\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)} + 2\log\left(\sqrt{a^2}-bx^2-\sqrt{\right.}{4b^3}$$

input
```
Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((b*x^2*(a*Sqrt[(a + b*x^2)^2]*(-2*a^2 - a*b*x^2 + b^2*x^4) + Sqrt[a^2]*(2
*a^3 + 3*a^2*b*x^2 + b^3*x^6)))/(a^2*(a + b*x^2)*(a^2 + a*b*x^2 - Sqrt[a^2
]*Sqrt[(a + b*x^2)^2])) + 2*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] -
 2*Log[b^3*(Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2])])/(4*b^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^2)\int \frac{x^5}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{x^5}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{(a+bx^2)\int \frac{x^4}{(bx^2+a)^3}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx^2)\int \left(\frac{a^2}{b^2(bx^2+a)^3}-\frac{2a}{b^2(bx^2+a)^2}+\frac{1}{b^2(bx^2+a)}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^2)\left(-\frac{a^2}{2b^3(a+bx^2)^2}+\frac{2a}{b^3(a+bx^2)}+\frac{\log(a+bx^2)}{b^3}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(-1/2*a^2/(b^3*(a + b*x^2)^2) + (2*a)/(b^3*(a + b*x^2)) + Log
[a + b*x^2]/b^3))/(2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.48

method	result	size
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(\left(b\,x^{2}+a\right)^{2}\ln\left(b\,x^{2}+a\right)+2ab\,x^{2}+\frac{3a^{2}}{2}\right)}{2\left(b\,x^{2}+a\right)^{2}b^{3}}$	54
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{a\,x^{2}}{b^{2}}+\frac{3a^{2}}{4b^{3}}\right)}{\left(b\,x^{2}+a\right)^{3}}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(b\,x^{2}+a\right)}{2\left(b\,x^{2}+a\right)b^{3}}$	73
default	$\frac{\left(2\ln\left(b\,x^{2}+a\right)b^{2}x^{4}+4\ln\left(b\,x^{2}+a\right)ab\,x^{2}+4ab\,x^{2}+2a^{2}\ln\left(b\,x^{2}+a\right)+3a^{2}\right)\left(b\,x^{2}+a\right)}{4b^{3}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$	81

input
```
int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*csgn(b*x^2+a)*((b*x^2+a)^2*ln(b*x^2+a)+2*a*b*x^2+3/2*a^2)/(b*x^2+a)^2/
b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.61

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{4\,abx^2+3\,a^2+2\,(b^2x^4+2\,abx^2+a^2)\log\left(bx^2+a\right)}{4\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(4*a*b*x^2 + 3*a^2 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^
5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F]

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{x^5}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**5/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{4\,abx^2+3\,a^2}{4\,(b^5x^4+2\,ab^4x^2+a^2b^3)} + \frac{\log\left(bx^2+a\right)}{2\,b^3}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*a*b*x^2 + 3*a^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + 1/2*log(b*x^2
+ a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\log\left(|bx^2+a|\right)}{2\,b^3\mathrm{sgn}\left(bx^2+a\right)} - \frac{3\,bx^4+2\,ax^2}{4\,(bx^2+a)^2b^2\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*log(abs(b*x^2 + a))/(b^3*sgn(b*x^2 + a)) - 1/4*(3*b*x^4 + 2*a*x^2)/((b
*x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{x^5}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{2\log(b\,x^2+a)\,a^2+4\log(b\,x^2+a)\,ab\,x^2+2\log(b\,x^2+a)\,b^2x^4+a^2-2b^2x^4}{4b^3\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(2*log(a + b*x**2)*a**2 + 4*log(a + b*x**2)*a*b*x**2 + 2*log(a + b*x**2)*b
**2*x**4 + a**2 - 2*b**2*x**4)/(4*b**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.583 $\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{x^4}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/4*x^4/a/(b*x^2+a)/((b*x^2+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 119 vs. 2(41) = 82.

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.90

$$\int \frac{x^3}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = -\frac{x^4\left(a^3+ab^2x^4-a\sqrt{a^2}\sqrt{\left(a+bx^2\right)^2}+\sqrt{a^2}bx^2\sqrt{\left(a+bx^2\right)^2}\right)}{4a^3\left(a+bx^2\right)\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{\left(a+bx^2\right)^2}\right)\right)}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
-1/4*(x^4*(a^3 + a*b^2*x^4 - a*Sqrt[a^2]*Sqrt[(a + b*x^2)^2] + Sqrt[a^2]*b
*x^2*Sqrt[(a + b*x^2)^2]))/(a^3*(a + b*x^2)*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2
] - Sqrt[(a + b*x^2)^2])))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1434, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{(b^2x^4+2abx^2+a^2)^{3/2}}dx^2$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{2}\left(-\frac{a\int \frac{1}{(b^2x^4+2abx^2+a^2)^{3/2}}dx^2}{b}-\frac{1}{b^2\sqrt{a^2+2abx^2+b^2x^4}}\right)$$

$$\Big\downarrow \text{1078}$$

$$\frac{1}{2}\left(\frac{a}{2b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1}{b^2\sqrt{a^2+2abx^2+b^2x^4}}\right)$$

input
```
Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(-(1/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + a/(2*b^2*(a + b*x^2)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]))/2
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$-\frac{\left(2 b \,x^{2}+a \right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{4 \left(b \,x^{2}+a \right)^{2} b^{2}}$	31
gosper	$-\frac{\left(b \,x^{2}+a \right) \left(2 b \,x^{2}+a \right)}{4 b^{2} \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}$	32
default	$-\frac{\left(b \,x^{2}+a \right) \left(2 b \,x^{2}+a \right)}{4 b^{2} \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}$	32
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(-\frac{x^{2}}{2 b}-\frac{a}{4 b^{2}}\right)}{\left(b \,x^{2}+a \right)^{3}}$	37
orering	$-\frac{\left(b \,x^{2}+a \right) \left(2 b \,x^{2}+a \right)}{4 b^{2} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{3}{2}}}$	41

input
```
int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(b*x^2+a)^2*(2*b*x^2+a)/b^2*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{2\,bx^2+a}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F]

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{x^3}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**3/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{2\,bx^2+a}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{2\,bx^2+a}{4\,(bx^2+a)^2 b^2 \mathrm{sgn}\,(bx^2+a)}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(2*b*x^2 + a)/((b*x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

Mupad [B] (verification not implemented)

Time = 18.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{(2\,b\,x^2+a)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,b^2\,(b\,x^2+a)^3}$$

input
```
int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
-((a + 2*b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4*b^2*(a + b*x^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{x^4}{4a\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
x**4/(4*a*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.584 $\int \frac{x}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{1}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-1/4/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{a+bx^2}{4b\left((a+bx^2)^2\right)^{3/2}}$$

input
```
Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
-1/4*(a + b*x^2)/(b*((a + b*x^2)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1432, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{(b^2x^4+2abx^2+a^2)^{3/2}}dx^2$$

$$\Big\downarrow \text{1078}$$

$$-\frac{1}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
-1/4*1/(b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)}{4\left(b\,x^{2}+a\right)^{2} b}$	23
gosper	$-\frac{b\,x^{2}+a}{4 b \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$	24
default	$-\frac{b\,x^{2}+a}{4 b \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$	24
risch	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}}{4\left(b\,x^{2}+a\right)^{3} b}$	26
orering	$-\frac{b\,x^{2}+a}{4 b \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{\frac{3}{2}}}$	33

input
```
int(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(b*x^2+a)^2/b*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=-\frac{1}{4\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Sympy [F]

$$\int \frac{x}{(a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = \int \frac{x}{\left((a + bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{x}{(a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = -\frac{1}{4\,(b^3x^4 + 2\,ab^2x^2 + a^2b)}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x}{(a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = -\frac{1}{4\,(bx^2 + a)^2 b\mathrm{sgn}\,(bx^2 + a)}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/4/((b*x^2 + a)^2*b*sgn(b*x^2 + a))
```

Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,b\,(b\,x^2+a)^3}$$

input
```
int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
-(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(4*b*(a + b*x^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{1}{4b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
( - 1)/(4*b*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.585 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a+bx^2\right)\log(x)}{a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/2/a^2/((b*x^2+a)^2)^(1/2)+1/4/a/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+(b*x^2+a)*
ln(x)/a^3/((b*x^2+a)^2)^(1/2)-1/2*(b*x^2+a)*ln(b*x^2+a)/a^3/((b*x^2+a)^2)^
(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 790 vs. 2(147) = 294.

Time = 1.21 (sec) , antiderivative size = 790, normalized size of antiderivative = 5.37

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{4a^4bx^2+3a^3b^2x^4-ab^4x^8-4\left(a^2\right)^{3/2}bx^2\sqrt{\left(a+bx^2\right)^2}+a\sqrt{a^2}b^2x^4\sqrt{(a+}}{}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
(4*a^4*b*x^2 + 3*a^3*b^2*x^4 - a*b^4*x^8 - 4*(a^2)^(3/2)*b*x^2*Sqrt[(a + b
*x^2)^2] + a*Sqrt[a^2]*b^2*x^4*Sqrt[(a + b*x^2)^2] - Sqrt[a^2]*b^3*x^6*Sqr
t[(a + b*x^2)^2] + 2*((a^2)^(3/2)*b^2*x^4 + a^4*(Sqrt[a^2] - Sqrt[(a + b*x
^2)^2]) + a^3*b*x^2*(2*Sqrt[a^2] - Sqrt[(a + b*x^2)^2]))*ArcTanh[(b*x^2)/(
Sqrt[a^2] - Sqrt[(a + b*x^2)^2])] - 2*(a^5 + 2*a^4*b*x^2 - (a^2)^(3/2)*b*x
^2*Sqrt[(a + b*x^2)^2] + a^3*(b^2*x^4 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2]))*Lo
g[x^2] + a^5*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] + 2*a^4*b*x^2*Lo
g[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] + a^3*b^2*x^4*Log[Sqrt[a^2] - b
*x^2 - Sqrt[(a + b*x^2)^2]] - a^3*Sqrt[a^2]*Sqrt[(a + b*x^2)^2]*Log[Sqrt[a
^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] - (a^2)^(3/2)*b*x^2*Sqrt[(a + b*x^2)^2]
*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] + a^5*Log[Sqrt[a^2] + b*x^2
- Sqrt[(a + b*x^2)^2]] + 2*a^4*b*x^2*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x
^2)^2]] + a^3*b^2*x^4*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]] - a^3*S
qrt[a^2]*Sqrt[(a + b*x^2)^2]*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]]
- (a^2)^(3/2)*b*x^2*Sqrt[(a + b*x^2)^2]*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a +
b*x^2)^2]])/(2*a^3*Sqrt[a^2]*(a^2 + a*b*x^2 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2
])^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.57, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3\left(a+bx^2\right)\int \frac{1}{b^3x\left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{1}{x(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{(a+bx^2)\int \frac{1}{x^2(bx^2+a)^3}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{(a+bx^2)\int \left(-\frac{b}{a^3(bx^2+a)}-\frac{b}{a^2(bx^2+a)^2}-\frac{b}{a(bx^2+a)^3}+\frac{1}{a^3x^2}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx^2)\left(-\frac{\log(a+bx^2)}{a^3}+\frac{\log(x^2)}{a^3}+\frac{1}{a^2(a+bx^2)}+\frac{1}{2a(a+bx^2)^2}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
((a + b*x^2)*(1/(2*a*(a + b*x^2)^2) + 1/(a^2*(a + b*x^2)) + Log[x^2]/a^3 -
 Log[a + b*x^2]/a^3))/(2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.46

method	result
pseudoelliptic	$\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(-\left(b\,x^{2}+a\right)^{2}\ln\left(b\,x^{2}+a\right)+\left(b\,x^{2}+a\right)^{2}\ln\left(x^{2}\right)+a b\,x^{2}+\frac{3 a^{2}}{2}\right)}{2\left(b\,x^{2}+a\right)^{2} a^{3}}$
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{b\,x^{2}}{2 a^{2}}+\frac{3}{4 a}\right)}{\left(b\,x^{2}+a\right)^{3}}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(x\right)}{\left(b\,x^{2}+a\right) a^{3}}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(b\,x^{2}+a\right)}{2\left(b\,x^{2}+a\right) a^{3}}$
default	$-\frac{\left(2\ln\left(b\,x^{2}+a\right) b^{2} x^{4}-4\ln\left(x\right) b^{2} x^{4}+4\ln\left(b\,x^{2}+a\right) a b\,x^{2}-8\ln\left(x\right) a b\,x^{2}-2 a b\,x^{2}+2 a^{2}\ln\left(b\,x^{2}+a\right)-4 a^{2}\ln\left(x\right)-3 a^{2}\right)\left(b\,x^{2}+a\right)}{4 a^{3}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$

input
```
int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*csgn(b*x^2+a)*(-(b*x^2+a)^2*ln(b*x^2+a)+(b*x^2+a)^2*ln(x^2)+a*b*x^2+3/
2*a^2)/(b*x^2+a)^2/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{2\,abx^2+3\,a^2-2\left(b^2x^4+2\,abx^2+a^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+2\,abx^2+a^2\right)}{4\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*a*b*x^2 + 3*a^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a) + 4*
(b^2*x^4 + 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)
```

Sympy [F]

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\int \frac{1}{x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(1/(x*((a + b*x**2)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{2\,bx^2+3\,a}{4\,\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^3}+\frac{\log\left(x\right)}{a^3}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 + a)
/a^3 + log(x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.61

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{\log\left(x^2\right)}{2\,a^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{3\,b^2x^4+8\,abx^2+6\,a^2}{4\,\left(bx^2+a\right)^2a^3\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*log(x^2)/(a^3*sgn(b*x^2 + a)) - 1/2*log(abs(b*x^2 + a))/(a^3*sgn(b*x^2
 + a)) + 1/4*(3*b^2*x^4 + 8*a*b*x^2 + 6*a^2)/((b*x^2 + a)^2*a^3*sgn(b*x^2
+ a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\int \frac{1}{x\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

input
```
int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

output
```
int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{-2\log\left(b\,x^2+a\right)a^2-4\log\left(b\,x^2+a\right)ab\,x^2-2\log\left(b\,x^2+a\right)b^2x^4+4\log(a}{4a^3\left(b^2x^4+2ab\,x^2+a^2\right)}$$

input
```
int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
( - 2*log(a + b*x**2)*a**2 - 4*log(a + b*x**2)*a*b*x**2 - 2*log(a + b*x**2
)*b**2*x**4 + 4*log(x)*a**2 + 8*log(x)*a*b*x**2 + 4*log(x)*b**2*x**4 + 2*a
**2 - b**2*x**4)/(4*a**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.586 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 189

$$\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b}{4a^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a+bx^2}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3b(a+bx^2)\log(x)}{a^4\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3b(a+bx^2)\log(a+bx^2)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-b/a^3/((b*x^2+a)^2)^(1/2)-1/4*b/a^2/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-1/2*(b*
x^2+a)/a^3/x^2/((b*x^2+a)^2)^(1/2)-3*b*(b*x^2+a)*ln(x)/a^4/((b*x^2+a)^2)^(
1/2)+3/2*b*(b*x^2+a)*ln(b*x^2+a)/a^4/((b*x^2+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 901 vs. $2(189) = 378$.

Time = 1.44 (sec) , antiderivative size = 901, normalized size of antiderivative = 4.77

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^{3/2}} dx =$$

$$-\frac{-2a^6 - 5a^5bx^2 + 2a^4b^2x^4 + 4a^3b^3x^6 - ab^5x^{10} + 2a^4\sqrt{a^2}\sqrt{(a+bx^2)^2} + 3a^3\sqrt{a^2}bx^2\sqrt{(a+bx^2)^2} - 5(a^2)^3}{}$$

input
```
Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
-1/2*(-2*a^6 - 5*a^5*b*x^2 + 2*a^4*b^2*x^4 + 4*a^3*b^3*x^6 - a*b^5*x^10 +
2*a^4*Sqrt[a^2]*Sqrt[(a + b*x^2)^2] + 3*a^3*Sqrt[a^2]*b*x^2*Sqrt[(a + b*x^
2)^2] - 5*(a^2)^(3/2)*b^2*x^4*Sqrt[(a + b*x^2)^2] + a*Sqrt[a^2]*b^3*x^6*Sq
rt[(a + b*x^2)^2] - Sqrt[a^2]*b^4*x^8*Sqrt[(a + b*x^2)^2] + 6*b*x^2*((a^2)
^(3/2)*b^2*x^4 + a^4*(Sqrt[a^2] - Sqrt[(a + b*x^2)^2]) + a^3*b*x^2*(2*Sqrt
[a^2] - Sqrt[(a + b*x^2)^2]))*ArcTanh[(b*x^2)/(Sqrt[a^2] - Sqrt[(a + b*x^2
)^2])] - 6*b*x^2*(a^5 + 2*a^4*b*x^2 - (a^2)^(3/2)*b*x^2*Sqrt[(a + b*x^2)^2
] + a^3*(b^2*x^4 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2]))*Log[x^2] + 3*a^5*b*x^2*
Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] + 6*a^4*b^2*x^4*Log[Sqrt[a^2]
 - b*x^2 - Sqrt[(a + b*x^2)^2]] + 3*a^3*b^3*x^6*Log[Sqrt[a^2] - b*x^2 - Sq
rt[(a + b*x^2)^2]] - 3*a^3*Sqrt[a^2]*b*x^2*Sqrt[(a + b*x^2)^2]*Log[Sqrt[a^
2] - b*x^2 - Sqrt[(a + b*x^2)^2]] - 3*(a^2)^(3/2)*b^2*x^4*Sqrt[(a + b*x^2)
^2]*Log[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] + 3*a^5*b*x^2*Log[Sqrt[a^
2] + b*x^2 - Sqrt[(a + b*x^2)^2]] + 6*a^4*b^2*x^4*Log[Sqrt[a^2] + b*x^2 -
Sqrt[(a + b*x^2)^2]] + 3*a^3*b^3*x^6*Log[Sqrt[a^2] + b*x^2 - Sqrt[(a + b*x
^2)^2]] - 3*a^3*Sqrt[a^2]*b*x^2*Sqrt[(a + b*x^2)^2]*Log[Sqrt[a^2] + b*x^2
- Sqrt[(a + b*x^2)^2]] - 3*(a^2)^(3/2)*b^2*x^4*Sqrt[(a + b*x^2)^2]*Log[Sqr
t[a^2] + b*x^2 - Sqrt[(a + b*x^2)^2]])/(a^4*Sqrt[a^2]*x^2*(a^2 + a*b*x^2 -
 Sqrt[a^2]*Sqrt[(a + b*x^2)^2])^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a^2+2abx^2+b^2x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3 (a+bx^2) \int \frac{1}{b^3x^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2) \int \frac{1}{x^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{(a+bx^2) \int \frac{1}{x^4(bx^2+a)^3}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx^2) \int \left(\frac{3b^2}{a^4(bx^2+a)}+\frac{2b^2}{a^3(bx^2+a)^2}+\frac{b^2}{a^2(bx^2+a)^3}-\frac{3b}{a^4x^2}+\frac{1}{a^3x^4}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^2)\left(-\frac{3b\log(x^2)}{a^4}+\frac{3b\log(a+bx^2)}{a^4}-\frac{2b}{a^3(a+bx^2)}-\frac{1}{a^3x^2}-\frac{b}{2a^2(a+bx^2)^2}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
((a + b*x^2)*(-(1/(a^3*x^2)) - b/(2*a^2*(a + b*x^2)^2) - (2*b)/(a^3*(a + b
*x^2)) - (3*b*Log[x^2])/a^4 + (3*b*Log[a + b*x^2])/a^4))/(2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.48

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(-3 b\,x^{2}\left(b\,x^{2}+a\right)^{2}\ln\left(b\,x^{2}+a\right)+3 b\,x^{2}\left(b\,x^{2}+a\right)^{2}\ln\left(x^{2}\right)+a\left(3 b^{2} x^{4}+\frac{9}{2} a b\,x^{2}+a^{2}\right)\right)}{2\left(b\,x^{2}+a\right)^{2} x^{2} a^{4}}$
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{3 b^{2} x^{4}}{2 a^{3}}-\frac{9 b\,x^{2}}{4 a^{2}}-\frac{1}{2 a}\right)}{\left(b\,x^{2}+a\right)^{3} x^{2}}-\frac{3 \sqrt{\left(b\,x^{2}+a\right)^{2}}\, b \ln\left(x\right)}{\left(b\,x^{2}+a\right) a^{4}}+\frac{3 \sqrt{\left(b\,x^{2}+a\right)^{2}}\, b \ln\left(-b\,x^{2}-a\right)}{2\left(b\,x^{2}+a\right) a^{4}}$
default	$\frac{(6 \ln\left(b\,x^{2}+a\right) x^{6} b^{3}-12 \ln\left(x\right) x^{6} b^{3}+12 \ln\left(b\,x^{2}+a\right) x^{4} a\,b^{2}-24 \ln\left(x\right) x^{4} a\,b^{2}-6 b^{2} x^{4} a+6 \ln\left(b\,x^{2}+a\right) x^{2} a^{2} b-12 \ln\left(x\right) x^{2} a^{2} b-9 a^{2}}{4 x^{2} a^{4}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$

input
```
int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*csgn(b*x^2+a)*(-3*b*x^2*(b*x^2+a)^2*ln(b*x^2+a)+3*b*x^2*(b*x^2+a)^2*l
n(x^2)+a*(3*b^2*x^4+9/2*a*b*x^2+a^2))/(b*x^2+a)^2/x^2/a^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = -\frac{6\,ab^2x^4+9\,a^2bx^2+2\,a^3-6\,(b^3x^6+2\,ab^2x^4+a^2bx^2)\log\left(bx^2+a\right)+12\,(b^3x^6+2\,ab^2x^4+a^2bx^2)\log\left(x\right)}{4\,(a^4b^2x^6+2\,a^5bx^4+a^6x^2)}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3 - 6*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b
*x^2)*log(b*x^2 + a) + 12*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4
*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \int \frac{1}{x^3 \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(1/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^{3/2}}\, dx = -\frac{6\, b^2x^4 + 9\, abx^2 + 2\, a^2}{4\, (a^3b^2x^6 + 2\, a^4bx^4 + a^5x^2)} + \frac{3\, b \log\left(bx^2 + a\right)}{2\, a^4} - \frac{3\, b \log\left(x\right)}{a^4}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)
 + 3/2*b*log(b*x^2 + a)/a^4 - 3*b*log(x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^{3/2}}\, dx = -\frac{3\, b \log\left(x^2\right)}{2\, a^4 \mathrm{sgn}\left(bx^2 + a\right)} + \frac{3\, b \log\left(|bx^2 + a|\right)}{2\, a^4 \mathrm{sgn}\left(bx^2 + a\right)} - \frac{9\, b^3x^4 + 22\, ab^2x^2 + 14\, a^2b}{4\, (bx^2 + a)^2 a^4 \mathrm{sgn}\left(bx^2 + a\right)} + \frac{3\, bx^2 - a}{2\, a^4x^2 \mathrm{sgn}\left(bx^2 + a\right)}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
-3/2*b*log(x^2)/(a^4*sgn(b*x^2 + a)) + 3/2*b*log(abs(b*x^2 + a))/(a^4*sgn(
b*x^2 + a)) - 1/4*(9*b^3*x^4 + 22*a*b^2*x^2 + 14*a^2*b)/((b*x^2 + a)^2*a^4
*sgn(b*x^2 + a)) + 1/2*(3*b*x^2 - a)/(a^4*x^2*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{1}{x^3\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

output
```
int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^3 (a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{6\log(b\,x^2+a)\,a^2b\,x^2+12\log(b\,x^2+a)\,a\,b^2x^4+6\log(b\,x^2+a)\,b^3x^6-12}{4a^4x^2\,(b^2x^4}$$

input
```
int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(6*log(a + b*x**2)*a**2*b*x**2 + 12*log(a + b*x**2)*a*b**2*x**4 + 6*log(a
+ b*x**2)*b**3*x**6 - 12*log(x)*a**2*b*x**2 - 24*log(x)*a*b**2*x**4 - 12*l
og(x)*b**3*x**6 - 2*a**3 - 6*a**2*b*x**2 + 3*b**3*x**6)/(4*a**4*x**2*(a**2
 + 2*a*b*x**2 + b**2*x**4))
```

3.587 $\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{9ax}{8b^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^2x}{4b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x(a+bx^2)}{b^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{15\sqrt{a}(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
9/8*a*x/b^3/((b*x^2+a)^2)^(1/2)-1/4*a^2*x/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2
)+x*(b*x^2+a)/b^3/((b*x^2+a)^2)^(1/2)-15/8*a^(1/2)*(b*x^2+a)*arctan(b^(1/2
)*x/a^(1/2))/b^(7/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.54

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\sqrt{b}x(15a^2+25abx^2+8b^2x^4)-15\sqrt{a}(a+bx^2)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}\left(a+bx^2\right)\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[b]*x*(15*a^2 + 25*a*b*x^2 + 8*b^2*x^4) - 15*Sqrt[a]*(a + b*x^2)^2*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(7/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1384, 27, 252, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3\left(a+bx^2\right)\int \frac{x^6}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{x^6}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{5\int \frac{x^4}{\left(bx^2+a\right)^2}dx}{4b}-\frac{x^5}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{3\int \frac{x^2}{bx^2+a}dx}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^5}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{3\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}\right)}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^5}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{3\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^5}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(-1/4*x^5/(b*(a + b*x^2)^2) + (5*(-1/2*x^3/(b*(a + b*x^2)) +
(3*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)))/(2*b)))/(4*b)))/
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

method	result
default	$\frac{\left(8\sqrt{ab}\, b^{2} x^{5}-15 \arctan\left(\frac{b x}{\sqrt{ab}}\right) a\, b^{2} x^{4}+25\sqrt{ab}\, a b\, x^{3}-30 \arctan\left(\frac{b x}{\sqrt{ab}}\right) a^{2} b\, x^{2}+15\sqrt{ab}\, a^{2} x-15 \arctan\left(\frac{b x}{\sqrt{ab}}\right) a^{3}\right)\left(b\, x^{2}+a\right)}{8\sqrt{ab}\, b^{3}\left(\left(b\, x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{\sqrt{\left(b\, x^{2}+a\right)^{2}}\, x}{\left(b\, x^{2}+a\right) b^{3}}+\frac{\sqrt{\left(b\, x^{2}+a\right)^{2}}\, \left(\frac{9}{8} a b\, x^{3}+\frac{7}{8} a^{2} x\right)}{\left(b\, x^{2}+a\right)^{3} b^{3}}+\frac{15\sqrt{\left(b\, x^{2}+a\right)^{2}}\, \sqrt{-ab}\, \ln\left(-\sqrt{-ab}\, x-a\right)}{16\left(b\, x^{2}+a\right) b^{4}}-\frac{15\sqrt{\left(b\, x^{2}+a\right)^{2}}\, \sqrt{-ab}\, \ln\left(\sqrt{-ab}\, x-\right.}{16\left(b\, x^{2}+a\right) b^{4}}$

input
```
int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(8*(a*b)^(1/2)*b^2*x^5-15*arctan(b/(a*b)^(1/2)*x)*a*b^2*x^4+25*(a*b)^(
1/2)*a*b*x^3-30*arctan(b/(a*b)^(1/2)*x)*a^2*b*x^2+15*(a*b)^(1/2)*a^2*x-15*
arctan(b/(a*b)^(1/2)*x)*a^3)*(b*x^2+a)/(a*b)^(1/2)/b^3/((b*x^2+a)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.23

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \left[\frac{16\,b^2x^5+50\,abx^3+30\,a^2x+15\,(b^2x^4+2\,abx^2+a^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-}}{bx^2+a}\right.}{16\,(b^5x^4+2\,ab^4x^2+a^2b^3)}\right.$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(16*b^2*x^5 + 50*a*b*x^3 + 30*a^2*x + 15*(b^2*x^4 + 2*a*b*x^2 + a^2)
*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*x^4 + 2*
a*b^4*x^2 + a^2*b^3), 1/8*(8*b^2*x^5 + 25*a*b*x^3 + 15*a^2*x - 15*(b^2*x^4
 + 2*a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^4 + 2*a*b^4*
x^2 + a^2*b^3)]
```

Sympy [F]

$$\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \int \frac{x^6}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**6/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.41

$$\int \frac{x^6}{(a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = \frac{9\,abx^3 + 7\,a^2x}{8\,(b^5x^4 + 2\,ab^4x^2 + a^2b^3)} - \frac{15\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} + \frac{x}{b^3}$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(9*a*b*x^3 + 7*a^2*x)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) - 15/8*a*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + x/b^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.51

$$\int \frac{x^6}{(a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = -\frac{15\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3\mathrm{sgn}\left(bx^2+a\right)} + \frac{x}{b^3\mathrm{sgn}\left(bx^2+a\right)} + \frac{9\,abx^3 + 7\,a^2x}{8\,(bx^2+a)^2b^3\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
-15/8*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3*sgn(b*x^2 + a)) + x/(b^3*sgn(
b*x^2 + a)) + 1/8*(9*a*b*x^3 + 7*a^2*x)/((b*x^2 + a)^2*b^3*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\int \frac{x^6}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.72

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2-30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2-15\sqrt{b}\,\sqrt{a}\,atan}{8b^4\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 - 30*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 - 15*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*b**2*x**4 + 15*a**2*b*x + 25*a*b**2*x**3 + 8*b**3*x**
5)/(8*b**4*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.588 $\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 128

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-3/8*x/b^2/((b*x^2+a)^2)^(1/2)-1/4*x^3/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+3/8
*(b*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(5/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.66

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{-\sqrt{a}\sqrt{b}x(3a+5bx^2)+3(a+bx^2)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{5/2}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(-(Sqrt[a]*Sqrt[b]*x*(3*a + 5*b*x^2)) + 3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(8*Sqrt[a]*b^(5/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 252, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^3(a+bx^2)\int \frac{x^4}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{x^4}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{(a+bx^2)\left(\frac{3\int \frac{x^2}{(bx^2+a)^2}dx}{4b}-\frac{x^3}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{(a+bx^2)\left(\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2b}-\frac{x}{2b(a+bx^2)}\right)}{4b}-\frac{x^3}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(a+bx^2)\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b(a+bx^2)}\right)}{4b}-\frac{x^3}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(-1/4*x^3/(b*(a + b*x^2)^2) + (3*(-1/2*x/(b*(a + b*x^2)) + Ar
cTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))))/(4*b)))/Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{\left(-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^{2}x^{4}+5\sqrt{ab}\,b\,x^{3}-6\arctan\left(\frac{bx}{\sqrt{ab}}\right)ab\,x^{2}+3\sqrt{ab}\,ax-3a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)\left(b\,x^{2}+a\right)}{8\sqrt{ab}\,b^{2}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$	97
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{5x^{3}}{8b}-\frac{3ax}{8b^{2}}\right)}{\left(b\,x^{2}+a\right)^{3}}-\frac{3\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(bx+\sqrt{-ab}\right)}{16\left(b\,x^{2}+a\right)\sqrt{-ab}\,b^{2}}+\frac{3\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(-bx+\sqrt{-ab}\right)}{16\left(b\,x^{2}+a\right)\sqrt{-ab}\,b^{2}}$	124

input
```
int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-3*arctan(b/(a*b)^(1/2)*x)*b^2*x^4+5*(a*b)^(1/2)*b*x^3-6*arctan(b/(a
*b)^(1/2)*x)*a*b*x^2+3*(a*b)^(1/2)*a*x-3*a^2*arctan(b/(a*b)^(1/2)*x))*(b*x
^2+a)/(a*b)^(1/2)/b^2/((b*x^2+a)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.47

$$\int \frac{x^4}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\left[-\frac{10\,ab^2x^3+6\,a^2bx+3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{16\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)},\right.$$
$$\left.-\frac{5\,ab^2x^3+3\,a^2bx-3\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{8\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)}\right]$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(10*a*b^2*x^3 + 6*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^5*x^4 + 2*a^2*b^4*x^
2 + a^3*b^3), -1/8*(5*a*b^2*x^3 + 3*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2
)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [F]

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{x^4}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**4/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.46

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{5\,bx^3+3\,ax}{8\,(b^4x^4+2\,ab^3x^2+a^2b^2)} + \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^2}$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/8*(5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) + 3/8*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^2\mathrm{sgn}\left(bx^2+a\right)} - \frac{5\,bx^3+3\,ax}{8\,(bx^2+a)^2b^2\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2*sgn(b*x^2 + a)) - 1/8*(5*b*x^3 +
3*a*x)/((b*x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{x^4}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

input
```
int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{8a\,b^3\left(b^2x^4+2ab\,x^2+a^2\right)}$$

input
```
int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 + 6*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*b**2*x**4 - 3*a**2*b*x - 5*a*b**2*x**3)/(8*a*b**3*(a**2 + 2
*a*b*x**2 + b**2*x**4))
```

3.589 $\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 129

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/8*x/a/b/((b*x^2+a)^2)^(1/2)-1/4*x/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1/8*(b
*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(3/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{b}x(-a+bx^2)+(a+bx^2)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}(a+bx^2)\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a]*Sqrt[b]*x*(-a + b*x^2) + (a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(8*a^(3/2)*b^(3/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 252, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^3(a+bx^2)\int \frac{x^2}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{x^2}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{(a+bx^2)\left(\frac{\int \frac{1}{\left(bx^2+a\right)^2}dx}{4b}-\frac{x}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{(a+bx^2)\left(\frac{\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}}{4b}-\frac{x}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(a+bx^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}}{4b}-\frac{x}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(-1/4*x/(b*(a + b*x^2)^2) + (x/(2*a*(a + b*x^2)) + ArcTan[(Sq
rt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]))/(4*b)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\left(-\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^{2}x^{4}-\sqrt{ab}\,b\,x^{3}-2\arctan\left(\frac{bx}{\sqrt{ab}}\right)ab\,x^{2}+\sqrt{ab}\,ax-a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)\left(b\,x^{2}+a\right)}{8\sqrt{ab}\,ba\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}$	99
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\left(\frac{x^{3}}{8a}-\frac{x}{8b}\right)}{\left(b\,x^{2}+a\right)^{3}}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(bx+\sqrt{-ab}\right)}{16\left(b\,x^{2}+a\right)\sqrt{-ab}\,ba}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(-bx+\sqrt{-ab}\right)}{16\left(b\,x^{2}+a\right)\sqrt{-ab}\,ba}$	129

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-arctan(b/(a*b)^(1/2)*x)*b^2*x^4-(a*b)^(1/2)*b*x^3-2*arctan(b/(a*b)^
(1/2)*x)*a*b*x^2+(a*b)^(1/2)*a*x-a^2*arctan(b/(a*b)^(1/2)*x))*(b*x^2+a)/(a
*b)^(1/2)/b/a/((b*x^2+a)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.47

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\left[\frac{2\,ab^2x^3-2\,a^2bx-\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{16\left(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2\right)},\frac{ab^2x^3}{}\right.$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(2*a*b^2*x^3 - 2*a^2*b*x - (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2
+ a^4*b^2), 1/8*(a*b^2*x^3 - a^2*b*x + (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a*
b)*arctan(sqrt(a*b)*x/a))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)]
```

Sympy [F]

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\int \frac{x^2}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(x**2/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.48

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{bx^3-ax}{8\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab}$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 1/8*arctan(b*x/sqr
t(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{(a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}ab\mathrm{sgn}\left(bx^2 + a\right)} + \frac{bx^3 - ax}{8\left(bx^2 + a\right)^2 ab\mathrm{sgn}\left(bx^2 + a\right)}$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b*sgn(b*x^2 + a)) + 1/8*(b*x^3 - a*
x)/((b*x^2 + a)^2*a*b*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = \int \frac{x^2}{{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)}^{3/2}}\,dx$$

input
```
int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{(a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2 + 2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2 + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{8a^2b^2\left(b^2x^4 + 2ab\,x^2 + a^2\right)}$$

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 + 2*sqrt(b)*sqrt(a)*at
an((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)
*sqrt(a)))*b**2*x**4 - a**2*b*x + a*b**2*x**3)/(8*a**2*b**2*(a**2 + 2*a*b*
x**2 + b**2*x**4))
```

3.590 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{3x}{8a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x}{4a\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
3/8*x/a^2/((b*x^2+a)^2)^(1/2)+1/4*x/a/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+3/8*(b
*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(1/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{b}x(5a+3bx^2)+3(a+bx^2)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,(a+bx^2)\,\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```

output
```
(Sqrt[a]*Sqrt[b]*x*(5*a + 3*b*x^2) + 3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(8*a^(5/2)*Sqrt[b]*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1384, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^2)\int \frac{1}{(b^2x^2+ab)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{215}$$

$$\frac{b^3(a+bx^2)\left(\frac{3\int \frac{1}{(b^2x^2+ab)^2}dx}{4ab}+\frac{x}{4ab^3(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{215}$$

$$\frac{b^3(a+bx^2)\left(\frac{3\left(\frac{\int \frac{1}{b^2x^2+ab}dx}{2ab}+\frac{x}{2ab^2(a+bx^2)}\right)}{4ab}+\frac{x}{4ab^3(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{b^3(a+bx^2)\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x}{2ab^2(a+bx^2)}\right)}{4ab}+\frac{x}{4ab^3(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```

output
```
(b^3*(a + b*x^2)*(x/(4*a*b^3*(a + b*x^2)^2) + (3*(x/(2*a*b^2*(a + b*x^2))
+ ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*b^(5/2))))/(4*a*b)))/Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\left(3 \arctan\left(\frac{b x}{\sqrt{a b}}\right) b^{2} x^{4}+3 \sqrt{a b}\, b \,x^{3}+6 \arctan\left(\frac{b x}{\sqrt{a b}}\right) a b \,x^{2}+5 \sqrt{a b}\, a x +3 a^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)\right) \left(b \,x^{2}+a \right)}{8 \sqrt{a b}\, a^{2} \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}$	97
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(\frac{3 b \,x^{3}}{8 a^{2}}+\frac{5 x}{8 a}\right)}{\left(b \,x^{2}+a \right)^{3}}-\frac{3 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, \ln\left(b x +\sqrt{-a b}\right)}{16 \left(b \,x^{2}+a \right) \sqrt{-a b}\, a^{2}}+\frac{3 \sqrt{\left(b \,x^{2}+a \right)^{2}}\, \ln\left(-b x +\sqrt{-a b}\right)}{16 \left(b \,x^{2}+a \right) \sqrt{-a b}\, a^{2}}$	124

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(3*arctan(b/(a*b)^(1/2)*x)*b^2*x^4+3*(a*b)^(1/2)*b*x^3+6*arctan(b/(a*b
)^(1/2)*x)*a*b*x^2+5*(a*b)^(1/2)*a*x+3*a^2*arctan(b/(a*b)^(1/2)*x))*(b*x^2
+a)/(a*b)^(1/2)/a^2/((b*x^2+a)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.49

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \left[\frac{6\,ab^2x^3+10\,a^2bx-3\,(b^2x^4+2\,abx^2+a^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{16\,(a^3b^3x^4+2\,a^4b^2x^2+a^5b)}, \frac{3\,ab\cdots}{\cdots}\right.$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(6*a*b^2*x^3 + 10*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3*x^4 + 2*a^4*b^2*x
^2 + a^5*b), 1/8*(3*a*b^2*x^3 + 5*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*
sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)]
```

Sympy [F]

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{3\,bx^3+5\,ax}{8\,(a^2b^2x^4+2\,a^3bx^2+a^4)} + \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3/8*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2\mathrm{sgn}\left(bx^2+a\right)} + \frac{3\,bx^3+5\,ax}{8\,(bx^2+a)^2a^2\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*sgn(b*x^2 + a)) + 1/8*(3*b*x^3 +
5*a*x)/((b*x^2 + a)^2*a^2*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{1}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{8a^3b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 + 6*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*b**2*x**4 + 5*a**2*b*x + 3*a*b**2*x**3)/(8*a**3*b*(a**2 + 2
*a*b*x**2 + b**2*x**4))
```

3.591 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=-\frac{7bx}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{bx}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{a^3x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15\sqrt{b}(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-7/8*b*x/a^3/((b*x^2+a)^2)^(1/2)-1/4*b*x/a^2/(b*x^2+a)/((b*x^2+a)^2)^(1/2)
-(b*x^2+a)/a^3/x/((b*x^2+a)^2)^(1/2)-15/8*b^(1/2)*(b*x^2+a)*arctan(b^(1/2)
*x/a^(1/2))/a^(7/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{3/2}} dx = \frac{-\sqrt{a}(8a^2 + 25abx^2 + 15b^2x^4) - 15\sqrt{b}x(a + bx^2)^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}x (a + bx^2) \sqrt{(a + bx^2)^2}}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
(-(Sqrt[a]*(8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)) - 15*Sqrt[b]*x*(a + b*x^2)^2
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*x*(a + b*x^2)*Sqrt[(a + b*x^2)^2]
)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1384, 27, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{3/2}} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3 (a + bx^2) \int \frac{1}{b^3x^2(bx^2+a)^3} dx}{\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a + bx^2) \int \frac{1}{x^2(bx^2+a)^3} dx}{\sqrt{a^2 + 2abx^2 + b^2x^4}}$$

$$\Big\downarrow \text{253}$$

$$\frac{(a+bx^2)\left(\frac{5\int \frac{1}{x^2(bx^2+a)^2}dx}{4a}+\frac{1}{4ax(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{3\int \frac{1}{x^2(bx^2+a)}dx}{2a}+\frac{1}{2ax(a+bx^2)}\right)}{4a}+\frac{1}{4ax(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 264

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{3\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a+bx^2)}\right)}{4a}+\frac{1}{4ax(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 218

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a+bx^2)}\right)}{4a}+\frac{1}{4ax(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
((a + b*x^2)*(1/(4*a*x*(a + b*x^2)^2) + (5*(1/(2*a*x*(a + b*x^2)) + (3*(-(
1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a)))
/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.01 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(-\frac{15b^2x^4}{8a^3}-\frac{25b\,x^2}{8a^2}-\frac{1}{a}\right)}{(b\,x^2+a)^3x}+\frac{15\sqrt{(b\,x^2+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^7\_Z^2+b\right)}\_R\ln\left(\left(3\_R^2a^7+2b\right)x+a^4\_R\right)\right)}{16(b\,x^2+a)}$	110
default	$-\frac{\left(15\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^3x^5+15\sqrt{ab}\,b^2x^4+30\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2x^3+25\sqrt{ab}\,ab\,x^2+15\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2bx+8\sqrt{ab}\,a^2\right)(b\,x^2+a)}{8x\sqrt{ab}\,a^3\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$	119

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)^3*(-15/8*b^2/a^3*x^4-25/8*b/a^2*x^2-1/a)/x+1
5/16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)*sum(_R*ln((3*_R^2*a^7+2*b)*x+a^4*_R),_R
=RootOf(_Z^2*a^7+b))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\left[-\frac{30\,b^2x^4+50\,abx^2-15\,(b^2x^5+2\,abx^3+a^2x)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-}{bx^2+a}\right.}{16\,(a^3b^2x^5+2\,a^4bx^3+a^5x)}\right.$$
$$\left.-\frac{15\,b^2x^4+25\,abx^2+15\,(b^2x^5+2\,abx^3+a^2x)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+8\,a^2}{8\,(a^3b^2x^5+2\,a^4bx^3+a^5x)}\right]$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(30*b^2*x^4 + 50*a*b*x^2 - 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(-b
/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 16*a^2)/(a^3*b^2*x^5
 + 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 15*(b^2*x^5 + 2*a
*b*x^3 + a^2*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 8*a^2)/(a^3*b^2*x^5 + 2*a^
4*b*x^3 + a^5*x)]
```

Sympy [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{3/2}} \, dx = \int \frac{1}{x^2 \left((a + bx^2)^2\right)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(1/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{3/2}} \, dx = -\frac{15 \, b^2 x^4 + 25 \, abx^2 + 8 \, a^2}{8 \, (a^3 b^2 x^5 + 2 \, a^4 b x^3 + a^5 x)} - \frac{15 \, b \arctan \left(\frac{bx}{\sqrt{ab}} \right)}{8 \, \sqrt{ab} a^3}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/8*(15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
 - 15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{3/2}} \, dx = -\frac{15 \, b \arctan \left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab} a^3 \mathrm{sgn}\left(bx^2 + a\right)} - \frac{7 \, b^2 x^3 + 9 \, abx}{8 \left(bx^2 + a\right)^2 a^3 \mathrm{sgn}\left(bx^2 + a\right)} - \frac{1}{a^3 x \mathrm{sgn}\left(bx^2 + a\right)}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
-15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*sgn(b*x^2 + a)) - 1/8*(7*b^2*
x^3 + 9*a*b*x)/((b*x^2 + a)^2*a^3*sgn(b*x^2 + a)) - 1/(a^3*x*sgn(b*x^2 + a
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{3/2}} \, dx = \int \frac{1}{x^2 \left(a^2 + 2 \, a \, b \, x^2 + b^2 \, x^4\right)^{3/2}} \, dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{3/2}} \, dx = \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^2 x - 30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) ab\,x^3 - 15\sqrt{b}\,\sqrt{a}\,a}{8a^4 x \left(b^2 x^4 + 2ab\,x^2 + a^2\right)}$$

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*x - 30*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**3 - 15*sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*b**2*x**5 - 8*a**3 - 25*a**2*b*x**2 - 15*a*b**2*x**
4)/(8*a**4*x*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.592 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 209

$$\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{11b^2x}{8a^4\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^2x}{4a^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a+bx^2}{3a^3x^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3b(a+bx^2)}{a^4x\sqrt{a^2+2abx^2+b^2x^4}} + \frac{35b^{3/2}(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
11/8*b^2*x/a^4/((b*x^2+a)^2)^(1/2)+1/4*b^2*x/a^3/(b*x^2+a)/((b*x^2+a)^2)^(
1/2)-1/3*(b*x^2+a)/a^3/x^3/((b*x^2+a)^2)^(1/2)+3*b*(b*x^2+a)/a^4/x/((b*x^2
+a)^2)^(1/2)+35/8*b^(3/2)*(b*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/a^(9/2)/((b*
x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^4 (a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\sqrt{a}(-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6)+105b^{3/2}x^3(a+bx^2)^2\,\mathrm{arcta}}{24a^{9/2}x^3(a+bx^2)\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[a]*(-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6) + 105*b^(3/
2)*x^3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(24*a^(9/2)*x^3*(a + b*x
^2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1384, 27, 253, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a^2+2abx^2+b^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^2)\int \frac{1}{b^3x^4(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{1}{x^4(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{253}$$

$$\frac{\left(a+bx^2\right)\left(\frac{7\int \frac{1}{x^4\left(bx^2+a\right)^2}dx}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{\left(a+bx^2\right)\left(\frac{7\left(\frac{5\int \frac{1}{x^4\left(bx^2+a\right)}dx}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 264

$$\frac{\left(a+bx^2\right)\left(\frac{7\left(\frac{5\left(-\frac{b\int \frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 264

$$\frac{\left(a+bx^2\right)\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 218

$$\frac{(a+bx^2)\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3(a+bx^2)}\right)}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
((a + b*x^2)*(1/(4*a*x^3*(a + b*x^2)^2) + (7*(1/(2*a*x^3*(a + b*x^2)) + (5
*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
a^(3/2)))/a))/(2*a)))/(4*a)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] && !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.48 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.61

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(\frac{35b^3x^6}{8a^4}+\frac{175b^2x^4}{24a^3}+\frac{7b\,x^2}{3a^2}-\frac{1}{3a}\right)}{(b\,x^2+a)^3x^3}+\frac{35\sqrt{(b\,x^2+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^9\_Z^2+b^3\right)}\_R\ln\left(\left(3\_R^2a^9+2b^3\right)x-a^5b\_R\right)\right.}{16(b\,x^2+a)}$
default	$-\frac{\left(-105\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^4x^7-105\sqrt{ab}\,b^3x^6-210\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^3x^5-175\sqrt{ab}\,a\,b^2x^4-105\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2b^2x^3-56\sqrt{ab}\,a^2b\,x^2+8\right.}{24x^3\sqrt{ab}\,a^4\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$

input
```
int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)^3*(35/8*b^3/a^4*x^6+175/24*b^2/a^3*x^4+7/3*b
/a^2*x^2-1/3/a)/x^3+35/16*((b*x^2+a)^2)^(1/2)/(b*x^2+a)*sum(_R*ln((3*_R^2*
a^9+2*b^3)*x-a^5*b*_R),_R=RootOf(_Z^2*a^9+b^3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4 (a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \left[\frac{210\,b^3x^6+350\,ab^2x^4+112\,a^2bx^2-16\,a^3+105\,(b^3x^7+2\,ab^2x^5+a^2bx}{48\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)}\right.$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(210*b^3*x^6 + 350*a*b^2*x^4 + 112*a^2*b*x^2 - 16*a^3 + 105*(b^3*x^7
 + 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3), 1/24*(105*b^3*x^6 + 1
75*a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3 + 105*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x
^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{1}{x^4 \left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(1/(x**4*((a + b*x**2)**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^{3/2}}\, dx = \frac{105\, b^3x^6 + 175\, ab^2x^4 + 56\, a^2bx^2 - 8\, a^3}{24\, (a^4b^2x^7 + 2\, a^5bx^5 + a^6x^3)} + \frac{35\, b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\, \sqrt{ab}a^4}$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/24*(105*b^3*x^6 + 175*a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3)/(a^4*b^2*x^7 + 2
*a^5*b*x^5 + a^6*x^3) + 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^{3/2}}\, dx = \frac{35\, b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\, \sqrt{ab}a^4 \mathrm{sgn}\,(bx^2 + a)} + \frac{11\, b^3x^3 + 13\, ab^2x}{8\, (bx^2 + a)^2 a^4 \mathrm{sgn}\,(bx^2 + a)} + \frac{9\, bx^2 - a}{3\, a^4x^3 \mathrm{sgn}\,(bx^2 + a)}$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*sgn(b*x^2 + a)) + 1/8*(11*b^
3*x^3 + 13*a*b^2*x)/((b*x^2 + a)^2*a^4*sgn(b*x^2 + a)) + 1/3*(9*b*x^2 - a)
/(a^4*x^3*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\int \frac{1}{x^4\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^3+210\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^5+105\sqrt{b}}{24a^5x^3\left(b^2x^4+2a\right.}$$

input
```
int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**3 + 210*sqrt(
b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*x**5 + 105*sqrt(b)*sqrt(a)
*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*x**7 - 8*a**4 + 56*a**3*b*x**2 + 175*a
**2*b**2*x**4 + 105*a*b**3*x**6)/(24*a**5*x**3*(a**2 + 2*a*b*x**2 + b**2*x
**4))
```

3.593 $\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 238

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{5a^2}{b^6\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^5}{8b^6(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5a^4}{6b^6(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5a^3}{2b^6(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x^2(a+bx^2)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5a(a+bx^2)\log(a+bx^2)}{2b^6\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-5*a^2/b^6/((b*x^2+a)^2)^(1/2)+1/8*a^5/b^6/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)
-5/6*a^4/b^6/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+5/2*a^3/b^6/(b*x^2+a)/((b*x^2
+a)^2)^(1/2)+1/2*x^2*(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)-5/2*a*(b*x^2+a)*ln(
b*x^2+a)/b^6/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.43

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{-77a^5-248a^4bx^2-252a^3b^2x^4-48a^2b^3x^6+48ab^4x^8+12b^5x^{10}-60a(a+}{24b^6(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(-77*a^5 - 248*a^4*b*x^2 - 252*a^3*b^2*x^4 - 48*a^2*b^3*x^6 + 48*a*b^4*x^8
 + 12*b^5*x^10 - 60*a*(a + b*x^2)^4*Log[a + b*x^2])/(24*b^6*(a + b*x^2)^3*
Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.53, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^2)\int \frac{x^{11}}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{x^{11}}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{(a+bx^2)\int \frac{x^{10}}{(bx^2+a)^5}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{49}$$

$$\frac{(a+bx^2)\int\left(-\frac{a^5}{b^5(bx^2+a)^5}+\frac{5a^4}{b^5(bx^2+a)^4}-\frac{10a^3}{b^5(bx^2+a)^3}+\frac{10a^2}{b^5(bx^2+a)^2}-\frac{5a}{b^5(bx^2+a)}+\frac{1}{b^5}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^2)\left(\frac{a^5}{4b^6(a+bx^2)^4}-\frac{5a^4}{3b^6(a+bx^2)^3}+\frac{5a^3}{b^6(a+bx^2)^2}-\frac{10a^2}{b^6(a+bx^2)}-\frac{5a\log(a+bx^2)}{b^6}+\frac{x^2}{b^5}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(x^2/b^5 + a^5/(4*b^6*(a + b*x^2)^4) - (5*a^4)/(3*b^6*(a + b*
x^2)^3) + (5*a^3)/(b^6*(a + b*x^2)^2) - (10*a^2)/(b^6*(a + b*x^2)) - (5*a*
Log[a + b*x^2])/b^6))/(2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
 - 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.40

method	result
pseudoelliptic	$-\frac{5\,\mathrm{csgn}\left(b\,x^{2}+a\right)\left(a\left(b\,x^{2}+a\right)^{4}\ln\left(b\,x^{2}+a\right)-\frac{x^{10}b^{5}}{5}-\frac{4a\,x^{8}b^{4}}{5}+\frac{4a^{2}x^{6}b^{3}}{5}+\frac{21a^{3}x^{4}b^{2}}{5}+\frac{62x^{2}a^{4}b}{15}+\frac{77a^{5}}{60}\right)}{2\left(b\,x^{2}+a\right)^{4}b^{6}}$
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,x^{2}}{2\left(b\,x^{2}+a\right)b^{5}}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\left(-5a^{2}b^{2}x^{6}-\frac{25a^{3}b\,x^{4}}{2}-\frac{65a^{4}x^{2}}{6}-\frac{77a^{5}}{24b}\right)}{\left(b\,x^{2}+a\right)^{5}b^{5}}-\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,a\ln\left(b\,x^{2}+a\right)}{2\left(b\,x^{2}+a\right)b^{6}}$
default	$-\frac{\left(-12x^{10}b^{5}+60\ln\left(b\,x^{2}+a\right)x^{8}a\,b^{4}-48a\,x^{8}b^{4}+240\ln\left(b\,x^{2}+a\right)x^{6}a^{2}b^{3}+48a^{2}x^{6}b^{3}+360\ln\left(b\,x^{2}+a\right)x^{4}a^{3}b^{2}+252a^{3}x^{4}b^{2}+24\right.}{24b^{6}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}$

input
```
int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-5/2*csgn(b*x^2+a)*(a*(b*x^2+a)^4*ln(b*x^2+a)-1/5*x^10*b^5-4/5*a*x^8*b^4+4
/5*a^2*x^6*b^3+21/5*a^3*x^4*b^2+62/15*x^2*a^4*b+77/60*a^5)/(b*x^2+a)^4/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.66

$$\int \frac{x^{11}}{\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{5/2}}\,dx=\frac{12\,b^{5}x^{10}+48\,ab^{4}x^{8}-48\,a^{2}b^{3}x^{6}-252\,a^{3}b^{2}x^{4}-248\,a^{4}bx^{2}-77\,a^{5}-60\,(ab^{4}}{24\,(b^{10}x^{8}+4\,ab^{9}x^{6}+6\,a^{2}b^{8}x^{4}+4\,a}$$

input
```
integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/24*(12*b^5*x^10 + 48*a*b^4*x^8 - 48*a^2*b^3*x^6 - 252*a^3*b^2*x^4 - 248*
a^4*b*x^2 - 77*a^5 - 60*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4
*b*x^2 + a^5)*log(b*x^2 + a))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*
a^3*b^7*x^2 + a^4*b^6)
```

Sympy [F]

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\int \frac{x^{11}}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**11/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.46

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = -\frac{120\,a^2b^3x^6+300\,a^3b^2x^4+260\,a^4bx^2+77\,a^5}{24\,\left(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6\right)}+\frac{x^2}{2\,b^5}-\frac{5\,a\log\left(bx^2+a\right)}{2\,b^6}$$

input
```
integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/24*(120*a^2*b^3*x^6 + 300*a^3*b^2*x^4 + 260*a^4*b*x^2 + 77*a^5)/(b^10*x
^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6) + 1/2*x^2/b^5
- 5/2*a*log(b*x^2 + a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.48

$$\int \frac{x^{11}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{x^2}{2\,b^5\mathrm{sgn}\left(bx^2+a\right)}-\frac{5\,a\log\left(\left|bx^2+a\right|\right)}{2\,b^6\mathrm{sgn}\left(bx^2+a\right)}+\frac{125\,ab^4x^8+380\,a^2b^3x^6+450\,a^3b^2x^4+240\,a^4bx^2+48\,a^5}{24\,\left(bx^2+a\right)^4b^6\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/2*x^2/(b^5*sgn(b*x^2 + a)) - 5/2*a*log(abs(b*x^2 + a))/(b^6*sgn(b*x^2 +
a)) + 1/24*(125*a*b^4*x^8 + 380*a^2*b^3*x^6 + 450*a^3*b^2*x^4 + 240*a^4*b*
x^2 + 48*a^5)/((b*x^2 + a)^4*b^6*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\int \frac{x^{11}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^11/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\frac{-60\log(b\,x^2+a)\,a^5-240\log(b\,x^2+a)\,a^4b\,x^2-360\log(b\,x^2+a)\,a^3b^2x^4-}{24b^6\,(b^4x}$$

input
```
int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
( - 60*log(a + b*x**2)*a**5 - 240*log(a + b*x**2)*a**4*b*x**2 - 360*log(a
+ b*x**2)*a**3*b**2*x**4 - 240*log(a + b*x**2)*a**2*b**3*x**6 - 60*log(a +
 b*x**2)*a*b**4*x**8 - 65*a**5 - 200*a**4*b*x**2 - 180*a**3*b**2*x**4 + 60
*a*b**4*x**8 + 12*b**5*x**10)/(24*b**6*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2
*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.594 $\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 196

$$\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{2a}{b^5\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^4}{8b^5\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2a^3}{3b^5\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3a^2}{2b^5\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
2*a/b^5/((b*x^2+a)^2)^(1/2)-1/8*a^4/b^5/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+2/
3*a^3/b^5/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-3/2*a^2/b^5/(b*x^2+a)/((b*x^2+a)
^2)^(1/2)+1/2*(b*x^2+a)*ln(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.33

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{\frac{bx^2\left(-a\sqrt{(a+bx^2)^2}(12a^6+30a^5bx^2+22a^4b^2x^4+3a^3b^3x^6-3a^2b^4x^8+3ab^5x^{10}-3b^6x^{12})+\sqrt{a^2}(12a^7+4}{a^4(a+bx^2)^3\left(a^2+abx^2-\sqrt{a^2}\sqrt{(a+bx^2)^2}\right)}}{}$$

input
```
Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((b*x^2*(-(a*Sqrt[(a + b*x^2)^2]*(12*a^6 + 30*a^5*b*x^2 + 22*a^4*b^2*x^4 +
 3*a^3*b^3*x^6 - 3*a^2*b^4*x^8 + 3*a*b^5*x^10 - 3*b^6*x^12)) + Sqrt[a^2]*(
12*a^7 + 42*a^6*b*x^2 + 52*a^5*b^2*x^4 + 25*a^4*b^3*x^6 + 3*b^7*x^14)))/(a
^4*(a + b*x^2)^3*(a^2 + a*b*x^2 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2])) + 12*Log
[Sqrt[a^2] - b*x^2 - Sqrt[(a + b*x^2)^2]] - 12*Log[b^5*(Sqrt[a^2] + b*x^2
- Sqrt[(a + b*x^2)^2])])/(24*b^5)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b^5(a+bx^2)\int \frac{x^9}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^2)\int \frac{x^9}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 243$$

$$\frac{(a+bx^2)\int \frac{x^8}{(bx^2+a)^5}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 49$$

$$\frac{(a+bx^2)\int \left(\frac{a^4}{b^4(bx^2+a)^5}-\frac{4a^3}{b^4(bx^2+a)^4}+\frac{6a^2}{b^4(bx^2+a)^3}-\frac{4a}{b^4(bx^2+a)^2}+\frac{1}{b^4(bx^2+a)}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 2009$$

$$\frac{(a+bx^2)\left(-\frac{a^4}{4b^5(a+bx^2)^4}+\frac{4a^3}{3b^5(a+bx^2)^3}-\frac{3a^2}{b^5(a+bx^2)^2}+\frac{4a}{b^5(a+bx^2)}+\frac{\log(a+bx^2)}{b^5}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/4*a^4/(b^5*(a + b*x^2)^4) + (4*a^3)/(3*b^5*(a + b*x^2)^3)
 - (3*a^2)/(b^5*(a + b*x^2)^2) + (4*a)/(b^5*(a + b*x^2)) + Log[a + b*x^2]/
b^5))/(2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.39

method	result
pseudoelliptic	$\frac{\operatorname{csgn}(b\,x^{2}+a)\left(\left(b\,x^{2}+a\right)^{4}\ln\left(b\,x^{2}+a\right)+4a\,b^{3}x^{6}+9a^{2}b^{2}x^{4}+\frac{22a^{3}b\,x^{2}}{3}+\frac{25a^{4}}{12}\right)}{2\left(b\,x^{2}+a\right)^{4}b^{5}}$
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\left(\frac{2a\,x^{6}}{b^{2}}+\frac{9a^{2}x^{4}}{2b^{3}}+\frac{11a^{3}x^{2}}{3b^{4}}+\frac{25a^{4}}{24b^{5}}\right)}{\left(b\,x^{2}+a\right)^{5}}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(b\,x^{2}+a\right)}{2\left(b\,x^{2}+a\right)b^{5}}$
default	$\frac{(12\ln(b\,x^{2}+a)b^{4}x^{8}+48\ln(b\,x^{2}+a)a\,b^{3}x^{6}+48a\,b^{3}x^{6}+72\ln(b\,x^{2}+a)a^{2}b^{2}x^{4}+108a^{2}b^{2}x^{4}+48\ln(b\,x^{2}+a)x^{2}a^{3}b+88a^{3}b\,x^{2}+}{24b^{5}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}$

input
```
int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*csgn(b*x^2+a)*((b*x^2+a)^4*ln(b*x^2+a)+4*a*b^3*x^6+9*a^2*b^2*x^4+22/3*
a^3*b*x^2+25/12*a^4)/(b*x^2+a)^4/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.69

$$\int \frac{x^9}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{48\,ab^3x^6+108\,a^2b^2x^4+88\,a^3bx^2+25\,a^4+12\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+\right.}{24\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}$$

input
```
integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/24*(48*a*b^3*x^6 + 108*a^2*b^2*x^4 + 88*a^3*b*x^2 + 25*a^4 + 12*(b^4*x^8
 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^9*x
^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

Sympy [F]

$$\int \frac{x^9}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \int \frac{x^9}{\left((a + bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**9/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.51

$$\int \frac{x^9}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \frac{48\,ab^3x^6 + 108\,a^2b^2x^4 + 88\,a^3bx^2 + 25\,a^4}{24\,(b^9x^8 + 4\,ab^8x^6 + 6\,a^2b^7x^4 + 4\,a^3b^6x^2 + a^4b^5)} + \frac{\log\left(bx^2 + a\right)}{2\,b^5}$$

input
```
integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/24*(48*a*b^3*x^6 + 108*a^2*b^2*x^4 + 88*a^3*b*x^2 + 25*a^4)/(b^9*x^8 + 4
*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5) + 1/2*log(b*x^2 + a)
/b^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.43

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{\log\left(|bx^2+a|\right)}{2\,b^5\mathrm{sgn}\left(bx^2+a\right)} - \frac{25\,b^3x^8+52\,ab^2x^6+42\,a^2bx^4+12\,a^3x^2}{24\,(bx^2+a)^4b^4\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/2*log(abs(b*x^2 + a))/(b^5*sgn(b*x^2 + a)) - 1/24*(25*b^3*x^8 + 52*a*b^2
*x^6 + 42*a^2*b*x^4 + 12*a^3*x^2)/((b*x^2 + a)^4*b^4*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{x^9}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^9/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.83

$$\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{12\log(b\,x^2+a)\,a^4+48\log(b\,x^2+a)\,a^3b\,x^2+72\log(b\,x^2+a)\,a^2b^2x^4+48\,\mathrm{l}}{24b^5\left(b^4x^8+4a\,b^3x^6\right.}$$

input
```
int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(12*log(a + b*x**2)*a**4 + 48*log(a + b*x**2)*a**3*b*x**2 + 72*log(a + b*x
**2)*a**2*b**2*x**4 + 48*log(a + b*x**2)*a*b**3*x**6 + 12*log(a + b*x**2)*
b**4*x**8 + 13*a**4 + 40*a**3*b*x**2 + 36*a**2*b**2*x**4 - 12*b**4*x**8)/(
24*b**5*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x*
*8))
```

3.595 $\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{x^8}{8a\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/8*x^8/a/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(41) = 82.

Time = 0.52 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.93

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{x^8\left(a^5+ab^4x^8-a^3\sqrt{a^2}\sqrt{(a+bx^2)^2}-a\sqrt{a^2}b^2x^4\sqrt{(a+bx^2)^2}+\sqrt{a^2}bx^2\sqrt{(a+bx^2)^2}(a^2+b^2x^4)\right)}{8a^5\,(a+bx^2)^3\left(\sqrt{a^2}bx^2+a\left(\sqrt{a^2}-\sqrt{(a+bx^2)^2}\right)\right)}$$

input
```
Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
-1/8*(x^8*(a^5 + a*b^4*x^8 - a^3*Sqrt[a^2]*Sqrt[(a + b*x^2)^2] - a*Sqrt[a^
2]*b^2*x^4*Sqrt[(a + b*x^2)^2] + Sqrt[a^2]*b*x^2*Sqrt[(a + b*x^2)^2]*(a^2
+ b^2*x^4)))/(a^5*(a + b*x^2)^3*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a
+ b*x^2)^2])))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1384, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{b^5(a+bx^2)\int \frac{x^7}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx^2)\int \frac{x^7}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 242$$

$$\frac{x^8}{8a(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
x^8/(8*a*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

method	result	size
pseudoelliptic	$-\frac{\left(2 b \,x^{2}+a \right) \left(2 b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{8 \left(b \,x^{2}+a \right)^{4} b^{4}}$	50
gosper	$-\frac{\left(b \,x^{2}+a \right) \left(4 b^{3} x^{6}+6 b^{2} x^{4} a +4 a^{2} b \,x^{2}+a^{3}\right)}{8 b^{4} \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}$	54
default	$-\frac{\left(b \,x^{2}+a \right) \left(4 b^{3} x^{6}+6 b^{2} x^{4} a +4 a^{2} b \,x^{2}+a^{3}\right)}{8 b^{4} \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}$	54
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(-\frac{x^{6}}{2 b}-\frac{3 a \,x^{4}}{4 b^{2}}-\frac{a^{2} x^{2}}{2 b^{3}}-\frac{a^{3}}{8 b^{4}}\right)}{\left(b \,x^{2}+a \right)^{5}}$	59
orering	$-\frac{\left(b \,x^{2}+a \right) \left(4 b^{3} x^{6}+6 b^{2} x^{4} a +4 a^{2} b \,x^{2}+a^{3}\right)}{8 b^{4} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{5}{2}}}$	63

input
```
int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(2*b*x^2+a)*(2*b^2*x^4+2*a*b*x^2+a^2)*csgn(b*x^2+a)/(b*x^2+a)^4/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(28) = 56$.

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

$$\int \frac{x^7}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\, dx = -\frac{4\, b^3x^6 + 6\, ab^2x^4 + 4\, a^2bx^2 + a^3}{8\, (b^8x^8 + 4\, ab^7x^6 + 6\, a^2b^6x^4 + 4\, a^3b^5x^2 + a^4b^4)}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/(b^8*x^8 + 4*a*b^7*x^6
+ 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [F]

$$\int \frac{x^7}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\, dx = \int \frac{x^7}{\left((a + bx^2)^2\right)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**7/((a + b*x**2)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/(b^8*x^8 + 4*a*b^7*x^6
+ 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,(bx^2+a)^4b^4\mathrm{sgn}\,(bx^2+a)}$$

input
```
integrate(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/((b*x^2 + a)^4*b^4*sgn(
b*x^2 + a))
```

Mupad [B] (verification not implemented)

Time = 19.85 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.51

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{a^3\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,b^4\,(b\,x^2+a)^5} - \frac{a^2\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,b^4\,(b\,x^2+a)^4} - \frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,b^4\,(b\,x^2+a)^2} + \frac{3\,a\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4\,b^4\,(b\,x^2+a)^3}$$

input
```
int(x^7/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
(a^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(8*b^4*(a + b*x^2)^5) - (a^2*(a^2
+ b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*b^4*(a + b*x^2)^4) - (a^2 + b^2*x^4 + 2*a
*b*x^2)^(1/2)/(2*b^4*(a + b*x^2)^2) + (3*a*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/
2))/(4*b^4*(a + b*x^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{x^8}{8a\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$$

input
```
int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
x**8/(8*a*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*
x**8))
```

3.596 $\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 74

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{x^6}{24a^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}+\frac{x^6}{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

output
```
1/24*x^6/a^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)+1/8*x^6/a/(b*x^2+a)/(b^2*x^4+2*
a*b*x^2+a^2)^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 206 vs. 2(74) = 148.

Time = 0.47 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.78

$$\int \frac{x^5}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{x^6\left(-4a^6-a^5bx^2+4a^4\sqrt{a^2}\sqrt{\left(a+bx^2\right)^2}-3a^3\sqrt{a^2}bx^2\sqrt{\left(a+bx^2\right)^2}+3\sqrt{a}\right.}{24a^6\left(a+bx^2\right)^3\left(\sqrt{a^2}bx^2+a\right.}$$

input
```
Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x^6*(-4*a^6 - a^5*b*x^2 + 4*a^4*Sqrt[a^2]*Sqrt[(a + b*x^2)^2] - 3*a^3*Sqr
t[a^2]*b*x^2*Sqrt[(a + b*x^2)^2] + 3*Sqrt[a^2]*b^2*x^4*Sqrt[(a + b*x^2)^2]
*(a^2 + b^2*x^4) + 3*a*b^3*x^6*(b^2*x^4 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2]))
/(24*a^6*(a + b*x^2)^3*(Sqrt[a^2]*b*x^2 + a*(Sqrt[a^2] - Sqrt[(a + b*x^2)^
2])))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1384, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^5(a+bx^2)\int \frac{x^5}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{x^5}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{(a+bx^2)\int \frac{x^4}{(bx^2+a)^5}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{(a+bx^2)\int \left(\frac{a^2}{b^2(bx^2+a)^5}-\frac{2a}{b^2(bx^2+a)^4}+\frac{1}{b^2(bx^2+a)^3}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx^2)\left(-\frac{a^2}{4b^3(a+bx^2)^4}+\frac{2a}{3b^3(a+bx^2)^3}-\frac{1}{2b^3(a+bx^2)^2}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/4*a^2/(b^3*(a + b*x^2)^4) + (2*a)/(3*b^3*(a + b*x^2)^3) -
 1/(2*b^3*(a + b*x^2)^2)))/(2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$-\frac{\left(6 b^{2} x^{4}+4 a b \,x^{2}+a^{2}\right) \operatorname{csgn}\left(b \,x^{2}+a \right)}{24 \left(b \,x^{2}+a \right)^{4} b^{3}}$	42
gosper	$-\frac{\left(b \,x^{2}+a \right) \left(6 b^{2} x^{4}+4 a b \,x^{2}+a^{2}\right)}{24 b^{3} \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}$	43
default	$-\frac{\left(b \,x^{2}+a \right) \left(6 b^{2} x^{4}+4 a b \,x^{2}+a^{2}\right)}{24 b^{3} \left(\left(b \,x^{2}+a \right)^{2}\right)^{\frac{5}{2}}}$	43
risch	$\frac{\sqrt{\left(b \,x^{2}+a \right)^{2}}\, \left(-\frac{x^{4}}{4 b}-\frac{a \,x^{2}}{6 b^{2}}-\frac{a^{2}}{24 b^{3}}\right)}{\left(b \,x^{2}+a \right)^{5}}$	48
orering	$-\frac{\left(b \,x^{2}+a \right) \left(6 b^{2} x^{4}+4 a b \,x^{2}+a^{2}\right)}{24 b^{3} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{\frac{5}{2}}}$	52

input
```
int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/(b*x^2+a)^4*(6*b^2*x^4+4*a*b*x^2+a^2)/b^3*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{6\,b^2x^4+4\,abx^2+a^2}{24\,(b^7x^8+4\,ab^6x^6+6\,a^2b^5x^4+4\,a^3b^4x^2+a^4b^3)}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/(b^7*x^8 + 4*a*b^6*x^6 + 6*a^2*b^5*x^4
 + 4*a^3*b^4*x^2 + a^4*b^3)
```

Sympy [F]

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\int \frac{x^5}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**5/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=-\frac{6\,b^2x^4+4\,abx^2+a^2}{24\,(b^7x^8+4\,ab^6x^6+6\,a^2b^5x^4+4\,a^3b^4x^2+a^4b^3)}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/(b^7*x^8 + 4*a*b^6*x^6 + 6*a^2*b^5*x^4
 + 4*a^3*b^4*x^2 + a^4*b^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=-\frac{6\,b^2x^4+4\,abx^2+a^2}{24\,(bx^2+a)^4b^3\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/((b*x^2 + a)^4*b^3*sgn(b*x^2 + a))
```

Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,(a^2+4\,a\,b\,x^2+6\,b^2\,x^4)}{24\,b^3\,(b\,x^2+a)^5}$$

input
```
int(x^5/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
-((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a^2 + 6*b^2*x^4 + 4*a*b*x^2))/(24*b^3
*(a + b*x^2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{-6b^2x^4-4ab\,x^2-a^2}{24b^3\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$$

input
```
int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
( - a**2 - 4*a*b*x**2 - 6*b**2*x**4)/(24*b**3*(a**4 + 4*a**3*b*x**2 + 6*a*
*2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.597 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{1}{6b^2(a^2+2abx^2+b^2x^4)^{3/2}} + \frac{a}{8b^2(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-1/6/b^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)+1/8*a/b^2/(b*x^2+a)^3/((b*x^2+a)^2)
^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(69) = 138.

Time = 0.48 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.36

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{x^4\left(3\sqrt{a^2}b^6x^{12}+3a^3b^3x^6\sqrt{(a+bx^2)^2}-3a^2b^4x^8\sqrt{(a+bx^2)^2}+3ab^5x^{10}\sqrt{(a+bx^2)^2}+a^4b^2x^4\left(\sqrt{a^2}-3\right.\right.}{24a^7(a+bx^2)^3\left(a^2+abx^2-\sqrt{a^2}\sqrt{\right.}$$

input
```
Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
-1/24*(x^4*(3*Sqrt[a^2]*b^6*x^12 + 3*a^3*b^3*x^6*Sqrt[(a + b*x^2)^2] - 3*a
^2*b^4*x^8*Sqrt[(a + b*x^2)^2] + 3*a*b^5*x^10*Sqrt[(a + b*x^2)^2] + a^4*b^
2*x^4*(Sqrt[a^2] - 3*Sqrt[(a + b*x^2)^2]) + 6*a^6*(Sqrt[a^2] - Sqrt[(a + b
*x^2)^2]) + 2*a^5*b*x^2*(2*Sqrt[a^2] + Sqrt[(a + b*x^2)^2])))/(a^7*(a + b*
x^2)^3*(a^2 + a*b*x^2 - Sqrt[a^2]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1434, 1100, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{(b^2x^4+2abx^2+a^2)^{5/2}}dx^2$$

$$\Big\downarrow \text{1100}$$

$$\frac{1}{2}\left(-\frac{a\int \frac{1}{(b^2x^4+2abx^2+a^2)^{5/2}}dx^2}{b}-\frac{1}{3b^2(a^2+2abx^2+b^2x^4)^{3/2}}\right)$$

$$\Big\downarrow \text{1078}$$

$$\frac{1}{2}\left(\frac{a}{4b^2(a+bx^2)(a^2+2abx^2+b^2x^4)^{3/2}}-\frac{1}{3b^2(a^2+2abx^2+b^2x^4)^{3/2}}\right)$$

input
```
Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(-1/3*1/(b^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + a/(4*b^2*(a + b*x^2)*(a^
2 + 2*a*b*x^2 + b^2*x^4)^(3/2)))/2
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.45

method	result	size
pseudoelliptic	$-\frac{(4b\,x^2+a)\operatorname{csgn}(b\,x^2+a)}{24(b\,x^2+a)^4 b^2}$	31
gosper	$-\frac{(b\,x^2+a)(4b\,x^2+a)}{24b^2\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}$	32
default	$-\frac{(b\,x^2+a)(4b\,x^2+a)}{24b^2\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}$	32
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(-\frac{x^2}{6b}-\frac{a}{24b^2}\right)}{(b\,x^2+a)^5}$	37
orering	$-\frac{(b\,x^2+a)(4b\,x^2+a)}{24b^2(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{2}}}$	41

input
```
int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/(b*x^2+a)^4*(4*b*x^2+a)/b^2*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{4\,bx^2+a}{24\,(b^6x^8+4\,ab^5x^6+6\,a^2b^4x^4+4\,a^3b^3x^2+a^4b^2)}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/24*(4*b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^4 + 4*a^3*b^3*x^2
 + a^4*b^2)
```

Sympy [F]

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{x^3}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**3/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{4\,bx^2+a}{24\,(b^6x^8+4\,ab^5x^6+6\,a^2b^4x^4+4\,a^3b^3x^2+a^4b^2)}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/24*(4*b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^4 + 4*a^3*b^3*x^2
 + a^4*b^2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{4\,bx^2+a}{24\,(bx^2+a)^4b^2\mathrm{sgn}\,(bx^2+a)}$$

input
```
integrate(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/24*(4*b*x^2 + a)/((b*x^2 + a)^4*b^2*sgn(b*x^2 + a))
```

Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{(4\,b\,x^2+a)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{24\,b^2\,(b\,x^2+a)^5}$$

input
```
int(x^3/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
-((a + 4*b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(24*b^2*(a + b*x^2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{(a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx = \frac{-4b\,x^2 - a}{24b^2\,(b^4x^8 + 4a\,b^3x^6 + 6a^2b^2x^4 + 4a^3b\,x^2 + a^4)}$$

input
```
int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
( - a - 4*b*x**2)/(24*b**2*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*
b**3*x**6 + b**4*x**8))
```

3.598 $\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=-\frac{1}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-1/8/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{x}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=-\frac{a+bx^2}{8b\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

input
```
Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
-1/8*(a + b*x^2)/(b*((a + b*x^2)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1432, 1078}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^{5/2}}dx$$
$$\Big\downarrow \; 1432$$
$$\frac{1}{2}\int \frac{1}{(b^2x^4+2abx^2+a^2)^{5/2}}dx^2$$
$$\Big\downarrow \; 1078$$
$$-\frac{1}{8b(a+bx^2)(a^2+2abx^2+b^2x^4)^{3/2}}$$

input
```
Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
-1/8*1/(b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Defintions of rubi rules used

rule 1078
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)}{8\left(b\,x^{2}+a\right)^{4} b}$	23
gosper	$-\frac{b\,x^{2}+a}{8 b \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}$	24
default	$-\frac{b\,x^{2}+a}{8 b \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}$	24
risch	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}}{8\left(b\,x^{2}+a\right)^{5} b}$	26
orering	$-\frac{b\,x^{2}+a}{8 b \left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{\frac{5}{2}}}$	33

input
```
int(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(b*x^2+a)^4/b*csgn(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{1}{8\,(b^5x^8+4\,ab^4x^6+6\,a^2b^3x^4+4\,a^3b^2x^2+a^4b)}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/8/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2 + a^4*b)
```

Sympy [F]

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{x}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{1}{8\,(b^5x^8+4\,ab^4x^6+6\,a^2b^3x^4+4\,a^3b^2x^2+a^4b)}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/8/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2 + a^4*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{1}{8\,(bx^2+a)^4 b\,\mathrm{sgn}\,(bx^2+a)}$$

input
```
integrate(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/8/((b*x^2 + a)^4*b*sgn(b*x^2 + a))
```

Mupad [B] (verification not implemented)

Time = 18.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{8\,b\,(b\,x^2+a)^5}$$

input
```
int(x/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
-(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)/(8*b*(a + b*x^2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{x}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{1}{8b\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$$

input
```
int(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
( - 1)/(8*b*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**
4*x**8))
```

3.599 $\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 223

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{1}{2a^4\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{6a^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1}{4a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(a+bx^2\right)\log(x)}{a^5\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^5\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/2/a^4/((b*x^2+a)^2)^(1/2)+1/8/a/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+1/6/a^2/
(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+1/4/a^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+(b*x
^2+a)*ln(x)/a^5/((b*x^2+a)^2)^(1/2)-1/2*(b*x^2+a)*ln(b*x^2+a)/a^5/((b*x^2+
a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.43

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{a(25a^3+52a^2bx^2+42ab^2x^4+12b^3x^6)+24(a+bx^2)^4\log(x)-12(a+b}{24a^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

input
```
Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
(a*(25*a^3 + 52*a^2*b*x^2 + 42*a*b^2*x^4 + 12*b^3*x^6) + 24*(a + b*x^2)^4*
Log[x] - 12*(a + b*x^2)^4*Log[a + b*x^2])/(24*a^5*(a + b*x^2)^3*Sqrt[(a +
b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.52, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5\left(a+bx^2\right)\int \frac{1}{b^5x(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{1}{x(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\left(a+bx^2\right)\int \frac{1}{x^2(bx^2+a)^5}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx^2)\int\left(-\frac{b}{a^5(bx^2+a)}-\frac{b}{a^4(bx^2+a)^2}-\frac{b}{a^3(bx^2+a)^3}-\frac{b}{a^2(bx^2+a)^4}-\frac{b}{a(bx^2+a)^5}+\frac{1}{a^5x^2}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^2)\left(-\frac{\log(a+bx^2)}{a^5}+\frac{\log(x^2)}{a^5}+\frac{1}{a^4(a+bx^2)}+\frac{1}{2a^3(a+bx^2)^2}+\frac{1}{3a^2(a+bx^2)^3}+\frac{1}{4a(a+bx^2)^4}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
((a + b*x^2)*(1/(4*a*(a + b*x^2)^4) + 1/(3*a^2*(a + b*x^2)^3) + 1/(2*a^3*(
a + b*x^2)^2) + 1/(a^4*(a + b*x^2)) + Log[x^2]/a^5 - Log[a + b*x^2]/a^5))/
(2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.40

method	result
pseudoelliptic	$\frac{\left(-(b\,x^2+a)^4 \ln(b\,x^2+a)+(b\,x^2+a)^4 \ln(x^2)+a\,b^3x^6+\frac{7a^2b^2x^4}{2}+\frac{13a^3b\,x^2}{3}+\frac{25a^4}{12}\right)\operatorname{csgn}(b\,x^2+a)}{2(b\,x^2+a)^4 a^5}$
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(\frac{b^3x^6}{2a^4}+\frac{7b^2x^4}{4a^3}+\frac{13b\,x^2}{6a^2}+\frac{25}{24a}\right)}{(b\,x^2+a)^5}+\frac{\sqrt{(b\,x^2+a)^2}\ln(x)}{(b\,x^2+a)a^5}-\frac{\sqrt{(b\,x^2+a)^2}\ln(b\,x^2+a)}{2(b\,x^2+a)a^5}$
default	$-\frac{(12\ln(b\,x^2+a)b^4x^8-24\ln(x)b^4x^8+48\ln(b\,x^2+a)a\,b^3x^6-96\ln(x)a\,b^3x^6-12a\,b^3x^6+72\ln(b\,x^2+a)a^2b^2x^4-144\ln(x)a^2}{24a^5\left((b\,x^2+a\right.}$

input
```
int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(b*x^2+a)^4*ln(b*x^2+a)+(b*x^2+a)^4*ln(x^2)+a*b^3*x^6+7/2*a^2*b^2*x^
4+13/3*a^3*b*x^2+25/12*a^4)*csgn(b*x^2+a)/(b*x^2+a)^4/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\,(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{12\,ab^3x^6+42\,a^2b^2x^4+52\,a^3bx^2+25\,a^4-12\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+}{24\,(a^5b^4x^8+4\,a^6b^3}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/24*(12*a*b^3*x^6 + 42*a^2*b^2*x^4 + 52*a^3*b*x^2 + 25*a^4 - 12*(b^4*x^8
+ 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(b*x^2 + a) + 24*(b^
4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(x))/(a^5*b^4*
x^8 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*b*x^2 + a^9)
```

Sympy [F]

$$\int \frac{1}{x\,(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{1}{x\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(1/(x*((a + b*x**2)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.45

$$\int \frac{1}{x\,(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{12\,b^3x^6+42\,ab^2x^4+52\,a^2bx^2+25\,a^3}{24\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)} - \frac{\log(bx^2+a)}{2\,a^5} + \frac{\log(x)}{a^5}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/24*(12*b^3*x^6 + 42*a*b^2*x^4 + 52*a^2*b*x^2 + 25*a^3)/(a^4*b^4*x^8 + 4*
a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8) - 1/2*log(b*x^2 + a)/a^5
+ log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.50

$$\int \frac{1}{x\,(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{\log(x^2)}{2\,a^5\mathrm{sgn}(bx^2+a)} - \frac{\log(|bx^2+a|)}{2\,a^5\mathrm{sgn}(bx^2+a)} + \frac{25\,b^4x^8+112\,ab^3x^6+192\,a^2b^2x^4+152\,a^3bx^2+50\,a^4}{24\,(bx^2+a)^4a^5\mathrm{sgn}(bx^2+a)}$$

input
```
integrate(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/2*log(x^2)/(a^5*sgn(b*x^2 + a)) - 1/2*log(abs(b*x^2 + a))/(a^5*sgn(b*x^2
 + a)) + 1/24*(25*b^4*x^8 + 112*a*b^3*x^6 + 192*a^2*b^2*x^4 + 152*a^3*b*x^
2 + 50*a^4)/((b*x^2 + a)^4*a^5*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\int \frac{1}{x\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

input
```
int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

output
```
int(1/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{-12\log\left(b\,x^2+a\right)a^4-48\log\left(b\,x^2+a\right)a^3b\,x^2-72\log\left(b\,x^2+a\right)a^2b^2x^4-}{}$$

input
```
int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
( - 12*log(a + b*x**2)*a**4 - 48*log(a + b*x**2)*a**3*b*x**2 - 72*log(a +
b*x**2)*a**2*b**2*x**4 - 48*log(a + b*x**2)*a*b**3*x**6 - 12*log(a + b*x**
2)*b**4*x**8 + 24*log(x)*a**4 + 96*log(x)*a**3*b*x**2 + 144*log(x)*a**2*b*
*2*x**4 + 96*log(x)*a*b**3*x**6 + 24*log(x)*b**4*x**8 + 22*a**4 + 40*a**3*
b*x**2 + 24*a**2*b**2*x**4 - 3*b**4*x**8)/(24*a**5*(a**4 + 4*a**3*b*x**2 +
 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.600 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 267

$$\begin{aligned}\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = & -\frac{2b}{a^5\sqrt{a^2+2abx^2+b^2x^4}} \\ & -\frac{b}{8a^2(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{3a^3(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} \\ & -\frac{3b}{4a^4(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^5x^2\sqrt{a^2+2abx^2+b^2x^4}} \\ & -\frac{5b(a+bx^2)\log(x)}{a^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5b(a+bx^2)\log(a+bx^2)}{2a^6\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
-2*b/a^5/((b*x^2+a)^2)^(1/2)-1/8*b/a^2/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)-1/3
*b/a^3/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-3/4*b/a^4/(b*x^2+a)/((b*x^2+a)^2)^(
1/2)-1/2*(b*x^2+a)/a^5/x^2/((b*x^2+a)^2)^(1/2)-5*b*(b*x^2+a)*ln(x)/a^6/((b
*x^2+a)^2)^(1/2)+5/2*b*(b*x^2+a)*ln(b*x^2+a)/a^6/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{-a(12a^4+125a^3bx^2+260a^2b^2x^4+210ab^3x^6+60b^4x^8)-120bx^2(a+b}{24a^6x^2\left(a+bx^2\right)^3\sqrt{(a+bx^2)}}$$

input
```
Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
(-(a*(12*a^4 + 125*a^3*b*x^2 + 260*a^2*b^2*x^4 + 210*a*b^3*x^6 + 60*b^4*x^
8)) - 120*b*x^2*(a + b*x^2)^4*Log[x] + 60*b*x^2*(a + b*x^2)^4*Log[a + b*x^
2])/(24*a^6*x^2*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.49, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1384, 27, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5\left(a+bx^2\right)\int \frac{1}{b^5x^3\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{1}{x^3\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\left(a+bx^2\right)\int \frac{1}{x^4\left(bx^2+a\right)^5}dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{54}$$

$$\frac{(a+bx^2)\int\left(\frac{5b^2}{a^6(bx^2+a)}+\frac{4b^2}{a^5(bx^2+a)^2}+\frac{3b^2}{a^4(bx^2+a)^3}+\frac{2b^2}{a^3(bx^2+a)^4}+\frac{b^2}{a^2(bx^2+a)^5}-\frac{5b}{a^6x^2}+\frac{1}{a^5x^4}\right)dx^2}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx^2)\left(-\frac{5b\log(x^2)}{a^6}+\frac{5b\log(a+bx^2)}{a^6}-\frac{4b}{a^5(a+bx^2)}-\frac{1}{a^5x^2}-\frac{3b}{2a^4(a+bx^2)^2}-\frac{2b}{3a^3(a+bx^2)^3}-\frac{b}{4a^2(a+bx^2)^4}\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
((a + b*x^2)*(-(1/(a^5*x^2)) - b/(4*a^2*(a + b*x^2)^4) - (2*b)/(3*a^3*(a +
 b*x^2)^3) - (3*b)/(2*a^4*(a + b*x^2)^2) - (4*b)/(a^5*(a + b*x^2)) - (5*b*
Log[x^2])/a^6 + (5*b*Log[a + b*x^2])/a^6))/(2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.42

method	result
pseudoelliptic	$-\frac{\operatorname{csgn}\left(b\,x^{2}+a\right)\left(-5 b\,x^{2}\left(b\,x^{2}+a\right)^{4}\ln\left(b\,x^{2}+a\right)+5 b\,x^{2}\left(b\,x^{2}+a\right)^{4}\ln\left(x^{2}\right)+a\left(5 b^{4} x^{8}+\frac{35}{2} a\,b^{3} x^{6}+\frac{65}{3} a^{2} b^{2} x^{4}+\frac{125}{12} a^{3} b\,x^{2}+a^{4}\right)\right)}{2\left(b\,x^{2}+a\right)^{4} x^{2} a^{6}}$
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{5 b^{4} x^{8}}{2 a^{5}}-\frac{35 b^{3} x^{6}}{4 a^{4}}-\frac{65 b^{2} x^{4}}{6 a^{3}}-\frac{125 b\,x^{2}}{24 a^{2}}-\frac{1}{2 a}\right)}{\left(b\,x^{2}+a\right)^{5} x^{2}}-\frac{5 \sqrt{\left(b\,x^{2}+a\right)^{2}}\,b\ln\left(x\right)}{\left(b\,x^{2}+a\right) a^{6}}+\frac{5 \sqrt{\left(b\,x^{2}+a\right)^{2}}\,b\ln\left(-b\,x^{2}-a\right)}{2\left(b\,x^{2}+a\right) a^{6}}$
default	$\frac{(60\ln\left(b\,x^{2}+a\right)x^{10}b^{5}-120\ln\left(x\right)x^{10}b^{5}+240\ln\left(b\,x^{2}+a\right)x^{8}a\,b^{4}-480\ln\left(x\right)x^{8}a\,b^{4}-60 a\,x^{8}b^{4}+360\ln\left(b\,x^{2}+a\right)x^{6}a^{2}b^{3}-720\ln(x}{2}$

input
```
int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*csgn(b*x^2+a)*(-5*b*x^2*(b*x^2+a)^4*ln(b*x^2+a)+5*b*x^2*(b*x^2+a)^4*l
n(x^2)+a*(5*b^4*x^8+35/2*a*b^3*x^6+65/3*a^2*b^2*x^4+125/12*a^3*b*x^2+a^4))
/(b*x^2+a)^4/x^2/a^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx =$$
$$-\frac{60\,ab^4x^8 + 210\,a^2b^3x^6 + 260\,a^3b^2x^4 + 125\,a^4bx^2 + 12\,a^5 - 60\,(b^5x^{10} + 4\,ab^4x^8 + 6\,a^2b^3x^6 + 4\,a^3b^2x^4 + a^4}{24\,(a^6b^4x^{10} + 4\,a^7b^3x^8 + 6\,a^8b^2x^6 + 4\,a^9}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/24*(60*a*b^4*x^8 + 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 + 125*a^4*b*x^2 +
12*a^5 - 60*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^4 + a^4*
b*x^2)*log(b*x^2 + a) + 120*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^
3*b^2*x^4 + a^4*b*x^2)*log(x))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x
^6 + 4*a^9*b*x^4 + a^10*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx = \int \frac{1}{x^3 \left((a + bx^2)^2\right)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(1/(x**3*((a + b*x**2)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx = -\frac{60 \, b^4x^8 + 210 \, ab^3x^6 + 260 \, a^2b^2x^4 + 125 \, a^3bx^2 + 12 \, a^4}{24 \, (a^5b^4x^{10} + 4 \, a^6b^3x^8 + 6 \, a^7b^2x^6 + 4 \, a^8bx^4 + a^9x^2)} + \frac{5 \, b \log (bx^2 + a)}{2 \, a^6} - \frac{5 \, b \log (x)}{a^6}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/24*(60*b^4*x^8 + 210*a*b^3*x^6 + 260*a^2*b^2*x^4 + 125*a^3*b*x^2 + 12*a
^4)/(a^5*b^4*x^10 + 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6 + 4*a^8*b*x^4 + a^9*x^2)
 + 5/2*b*log(b*x^2 + a)/a^6 - 5*b*log(x)/a^6
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx = -\frac{5 \, b \log (x^2)}{2 \, a^6 \text{sgn} (bx^2 + a)} + \frac{5 \, b \log (|bx^2 + a|)}{2 \, a^6 \text{sgn} (bx^2 + a)} + \frac{5 \, bx^2 - a}{2 \, a^6x^2 \text{sgn} (bx^2 + a)} - \frac{125 \, b^5x^8 + 548 \, ab^4x^6 + 912 \, a^2b^3x^4 + 688 \, a^3b^2x^2 + 202 \, a^4b}{24 \, (bx^2 + a)^4 a^6 \text{sgn} (bx^2 + a)}$$

input
```
integrate(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
-5/2*b*log(x^2)/(a^6*sgn(b*x^2 + a)) + 5/2*b*log(abs(b*x^2 + a))/(a^6*sgn(
b*x^2 + a)) + 1/2*(5*b*x^2 - a)/(a^6*x^2*sgn(b*x^2 + a)) - 1/24*(125*b^5*x
^8 + 548*a*b^4*x^6 + 912*a^2*b^3*x^4 + 688*a^3*b^2*x^2 + 202*a^4*b)/((b*x^
2 + a)^4*a^6*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \int \frac{1}{x^3\,(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{5/2}}\,dx$$

input
```
int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

output
```
int(1/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3 (a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \frac{60 \log(b\,x^2 + a)\,a^4 b\,x^2 + 240 \log(b\,x^2 + a)\,a^3 b^2 x^4 + 360 \log(b\,x^2 + a)\,a^2 b^3}{}$$

input
```
int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(60*log(a + b*x**2)*a**4*b*x**2 + 240*log(a + b*x**2)*a**3*b**2*x**4 + 360
*log(a + b*x**2)*a**2*b**3*x**6 + 240*log(a + b*x**2)*a*b**4*x**8 + 60*log
(a + b*x**2)*b**5*x**10 - 120*log(x)*a**4*b*x**2 - 480*log(x)*a**3*b**2*x*
*4 - 720*log(x)*a**2*b**3*x**6 - 480*log(x)*a*b**4*x**8 - 120*log(x)*b**5*
x**10 - 12*a**5 - 110*a**4*b*x**2 - 200*a**3*b**2*x**4 - 120*a**2*b**3*x**
6 + 15*b**5*x**10)/(24*a**6*x**2*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4
+ 4*a*b**3*x**6 + b**4*x**8))
```

3.601 $\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 248

$$\begin{aligned}\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx &= \frac{325ax}{128b^5\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{a^4x}{8b^5(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{11a^3x}{16b^5(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{105a^2x}{64b^5(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{x(a+bx^2)}{b^5\sqrt{a^2+2abx^2+b^2x^4}} - \frac{315\sqrt{a}(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
325/128*a*x/b^5/((b*x^2+a)^2)^(1/2)-1/8*a^4*x/b^5/(b*x^2+a)^3/((b*x^2+a)^2
)^(1/2)+11/16*a^3*x/b^5/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-105/64*a^2*x/b^5/(
b*x^2+a)/((b*x^2+a)^2)^(1/2)+x*(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)-315/128*a
^(1/2)*(b*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/b^(11/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.45

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\frac{\sqrt{b}x(315a^4+1155a^3bx^2+1533a^2b^2x^4+837ab^3x^6+128b^4x^8)-315\sqrt{a}(a}{128b^{11/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

input
```
Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[b]*x*(315*a^4 + 1155*a^3*b*x^2 + 1533*a^2*b^2*x^4 + 837*a*b^3*x^6 +
128*b^4*x^8) - 315*Sqrt[a]*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128
*b^(11/2)*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.68, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1384, 27, 252, 252, 252, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5\left(a+bx^2\right)\int \frac{x^{10}}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{x^{10}}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{9\int\frac{x^8}{(bx^2+a)^4}dx}{8b}-\frac{x^9}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{(a+bx^2)\left(\frac{9\left(\frac{7\int\frac{x^6}{(bx^2+a)^3}dx}{6b}-\frac{x^7}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^9}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{(a+bx^2)\left(\frac{9\left(\frac{7\left(\frac{5\int\frac{x^4}{(bx^2+a)^2}dx}{4b}-\frac{x^5}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^7}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^9}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{(a+bx^2)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{x^2}{bx^2+a}dx}{2b}-\frac{x^3}{2b(a+bx^2)}\right)}{4b}-\frac{x^5}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^7}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^9}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{262}$$

$$\frac{(a+bx^2)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{x}{b}-\frac{a\int\frac{1}{bx^2+a}dx}{b}\right)}{2b}-\frac{x^3}{2b(a+bx^2)}\right)}{4b}-\frac{x^5}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^7}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^9}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 218

$$\frac{(a+bx^2)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^5}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^9}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*x^9/(b*(a + b*x^2)^4) + (9*(-1/6*x^7/(b*(a + b*x^2)^3)
+ (7*(-1/4*x^5/(b*(a + b*x^2)^2) + (5*(-1/2*x^3/(b*(a + b*x^2)) + (3*(x/b
- (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)))/(2*b)))/(4*b)))/(6*b)))/
(8*b)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,x}{(b\,x^2+a)b^5}+\frac{\sqrt{(b\,x^2+a)^2}\,(\frac{325}{128}a\,b^3x^7+\frac{765}{128}a^2b^2x^5+\frac{643}{128}a^3b\,x^3+\frac{187}{128}a^4x)}{(b\,x^2+a)^5b^5}+\frac{315\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,\ln(-\sqrt{-ab}\,x-a)}{256(b\,x^2+a)b^6}-315$
default	$\frac{\left(128\sqrt{ab}\,b^4x^9-315\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^4x^8+837\sqrt{ab}\,a\,b^3x^7-1260\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2b^3x^6+1533\sqrt{ab}\,a^2b^2x^5-1890\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^3b^2x^4\right.}{128\sqrt{ab}\,b^5\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}$

input
```
int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)*x/b^5+((b*x^2+a)^2)^(1/2)/(b*x^2+a)^5*(325/1
28*a*b^3*x^7+765/128*a^2*b^2*x^5+643/128*a^3*b*x^3+187/128*a^4*x)/b^5+315/
256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/b^6*(-a*b)^(1/2)*ln(-(-a*b)^(1/2)*x-a)-3
15/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/b^6*(-a*b)^(1/2)*ln((-a*b)^(1/2)*x-a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.35

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \left[\frac{256\,b^4x^9+1674\,ab^3x^7+3066\,a^2b^2x^5+2310\,a^3bx^3+630\,a^4x+315\,(b^4x^8}{256\,(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4}\right.$$

input
```
integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/256*(256*b^4*x^9 + 1674*a*b^3*x^7 + 3066*a^2*b^2*x^5 + 2310*a^3*b*x^3 +
 630*a^4*x + 315*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^
4)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^9*x^8 +
4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), 1/128*(128*b^4*x^9
 + 837*a*b^3*x^7 + 1533*a^2*b^2*x^5 + 1155*a^3*b*x^3 + 315*a^4*x - 315*(b^
4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 +
 a^4*b^5)]
```

Sympy [F]

$$\int \frac{x^{10}}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \int \frac{x^{10}}{\left((a + bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**10/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.45

$$\int \frac{x^{10}}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \frac{325\,ab^3x^7 + 765\,a^2b^2x^5 + 643\,a^3bx^3 + 187\,a^4x}{128\,(b^9x^8 + 4\,ab^8x^6 + 6\,a^2b^7x^4 + 4\,a^3b^6x^2 + a^4b^5)} - \frac{315\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}b^5} + \frac{x}{b^5}$$

input
```
integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/128*(325*a*b^3*x^7 + 765*a^2*b^2*x^5 + 643*a^3*b*x^3 + 187*a^4*x)/(b^9*x
^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5) - 315/128*a*ar
ctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + x/b^5
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.43

$$\int \frac{x^{10}}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = -\frac{315\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}b^5\mathrm{sgn}\left(bx^2 + a\right)} + \frac{x}{b^5\mathrm{sgn}\left(bx^2 + a\right)} + \frac{325\,ab^3x^7 + 765\,a^2b^2x^5 + 643\,a^3bx^3 + 187\,a^4x}{128\,(bx^2 + a)^4b^5\mathrm{sgn}\left(bx^2 + a\right)}$$

input
```
integrate(x^10/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
-315/128*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5*sgn(b*x^2 + a)) + x/(b^5*s
gn(b*x^2 + a)) + 1/128*(325*a*b^3*x^7 + 765*a^2*b^2*x^5 + 643*a^3*b*x^3 +
187*a^4*x)/((b*x^2 + a)^4*b^5*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{x^{10}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^10/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.87

$$\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4 - 1260\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^2 - 1890\sqrt{b}\,\sqrt{}$$

input
```
int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
( - 315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4 - 1260*sqrt(b)*
sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*x**2 - 1890*sqrt(b)*sqrt(a)*a
tan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**4 - 1260*sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*a*b**3*x**6 - 315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*b**4*x**8 + 315*a**4*b*x + 1155*a**3*b**2*x**3 + 1533*a**2*b*
*3*x**5 + 837*a*b**4*x**7 + 128*b**5*x**9)/(128*b**6*(a**4 + 4*a**3*b*x**2
 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.602 $\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 210

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{35x}{128b^4\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^7}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7x^5}{48b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35x^3}{192b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{35(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\sqrt{a}b^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output

```
-35/128*x/b^4/((b*x^2+a)^2)^(1/2)-1/8*x^7/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2
)-7/48*x^5/b^2/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-35/192*x^3/b^3/(b*x^2+a)/((
b*x^2+a)^2)^(1/2)+35/128*(b*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(9/
2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.50

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{-\sqrt{a}\sqrt{b}x(105a^3+385a^2bx^2+511ab^2x^4+279b^3x^6)+105(a+bx^2)^4\arctan}{384\sqrt{a}b^{9/2}\,(a+bx^2)^3\,\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(-(Sqrt[a]*Sqrt[b]*x*(105*a^3 + 385*a^2*b*x^2 + 511*a*b^2*x^4 + 279*b^3*x^
6)) + 105*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*Sqrt[a]*b^(9/2)*
(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1384, 27, 252, 252, 252, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^2)\int \frac{x^8}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{x^8}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{7\int \frac{x^6}{(bx^2+a)^4}dx}{8b}-\frac{x^7}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{7\left(\frac{5\int \frac{x^4}{(bx^2+a)^3}dx}{6b}-\frac{x^5}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^7}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{x^2}{(bx^2+a)^2}dx}{4b}-\frac{x^3}{4b(a+bx^2)^2}\right)}{6b}-\frac{x^5}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^7}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\left(a+bx^2\right)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2b}-\frac{x}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^3}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^5}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^7}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 218$$

$$\frac{\left(a+bx^2\right)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^3}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^5}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^7}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*x^7/(b*(a + b*x^2)^4) + (7*(-1/6*x^5/(b*(a + b*x^2)^3)
+ (5*(-1/4*x^3/(b*(a + b*x^2)^2) + (3*(-1/2*x/(b*(a + b*x^2)) + ArcTan[(Sq
rt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))))/(4*b)))/(6*b)))/(8*b)))/Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-\frac{93 x^{7}}{128 b}-\frac{511 a\,x^{5}}{384 b^{2}}-\frac{385 a^{2} x^{3}}{384 b^{3}}-\frac{35 a^{3} x}{128 b^{4}}\right)}{\left(b\,x^{2}+a\right)^{5}}-\frac{35 \sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(b x+\sqrt{-a b}\right)}{256 \left(b\,x^{2}+a\right) \sqrt{-a b}\,b^{4}}+\frac{35 \sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(-b x+\sqrt{-a b}\right)}{256 \left(b\,x^{2}+a\right) \sqrt{-a b}\,b^{4}}$
default	$-\frac{\left(-105 \arctan\left(\frac{b x}{\sqrt{a b}}\right) b^{4} x^{8}+279 \sqrt{a b}\,b^{3} x^{7}-420 \arctan\left(\frac{b x}{\sqrt{a b}}\right) a\,b^{3} x^{6}+511 \sqrt{a b}\,a\,b^{2} x^{5}-630 \arctan\left(\frac{b x}{\sqrt{a b}}\right) a^{2} b^{2} x^{4}+385 \sqrt{a b}\,a^{2} b\,x^{3}-\right.}{384 \sqrt{a b}\,b^{4} \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}$

input
```
int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)^5*(-93/128/b*x^7-511/384*a/b^2*x^5-385/384*a
^2/b^3*x^3-35/128*a^3/b^4*x)-35/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(
1/2)/b^4*ln(b*x+(-a*b)^(1/2))+35/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^
(1/2)/b^4*ln(-b*x+(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.52

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \left[-\frac{558\,ab^4x^7+1022\,a^2b^3x^5+770\,a^3b^2x^3+210\,a^4bx+105\,(b^4x^8+4\,ab^3x^6}{768\,(ab^9x^8+4\,a^2b^8x^6+6\,a^3b^7x^4+}\right.$$

$$-\frac{279\,ab^4x^7+511\,a^2b^3x^5+385\,a^3b^2x^3+105\,a^4bx-105\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\sqrt{ab}\,\mathrm{arc}}{384\,(ab^9x^8+4\,a^2b^8x^6+6\,a^3b^7x^4+4\,a^4b^6x^2+a^5b^5)}$$

input
```
integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/768*(558*a*b^4*x^7 + 1022*a^2*b^3*x^5 + 770*a^3*b^2*x^3 + 210*a^4*b*x
+ 105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*
b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^9*x^8 + 4*a^2*b^8*x
^6 + 6*a^3*b^7*x^4 + 4*a^4*b^6*x^2 + a^5*b^5), -1/384*(279*a*b^4*x^7 + 511
*a^2*b^3*x^5 + 385*a^3*b^2*x^3 + 105*a^4*b*x - 105*(b^4*x^8 + 4*a*b^3*x^6
+ 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b
^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^4*b^6*x^2 + a^5*b^5)]
```

Sympy [F]

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{x^8}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**8/(b**2*x***4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**8/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{279\,b^3x^7+511\,ab^2x^5+385\,a^2bx^3+105\,a^3x}{384\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)} + \frac{35\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}b^4}$$

input
```
integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/384*(279*b^3*x^7 + 511*a*b^2*x^5 + 385*a^2*b*x^3 + 105*a^3*x)/(b^8*x^8
+ 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4) + 35/128*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{35\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}b^4\mathrm{sgn}\left(bx^2+a\right)} - \frac{279\,b^3x^7+511\,ab^2x^5+385\,a^2bx^3+105\,a^3x}{384\,(bx^2+a)^4b^4\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(x^8/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
35/128*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4*sgn(b*x^2 + a)) - 1/384*(279*b
^3*x^7 + 511*a*b^2*x^5 + 385*a^2*b*x^3 + 105*a^3*x)/((b*x^2 + a)^4*b^4*sgn
(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\int \frac{x^8}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^8/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4+420\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^2+630\sqrt{b}\,\sqrt{a}\,at}{}$$

input
```
int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4 + 420*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*x**2 + 630*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**4 + 420*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a*b**3*x**6 + 105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqr
t(a)))*b**4*x**8 - 105*a**4*b*x - 385*a**3*b**2*x**3 - 511*a**2*b**3*x**5
- 279*a*b**4*x**7)/(384*a*b**5*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 +
4*a*b**3*x**6 + b**4*x**8))
```

3.603 $\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 211

$$\begin{aligned}\int \frac{x^6}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx &= \frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{x^5}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5x^3}{48b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{5x}{64b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{3/2}b^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
5/128*x/a/b^3/((b*x^2+a)^2)^(1/2)-1/8*x^5/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2
)-5/48*x^3/b^2/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-5/64*x/b^3/(b*x^2+a)/((b*x^
2+a)^2)^(1/2)+5/128*(b*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(7/2)/((
b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.50

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{\sqrt{a}\sqrt{b}x(-15a^3-55a^2bx^2-73ab^2x^4+15b^3x^6)+15(a+bx^2)^4\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{384a^{3/2}b^{7/2}\,(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a]*Sqrt[b]*x*(-15*a^3 - 55*a^2*b*x^2 - 73*a*b^2*x^4 + 15*b^3*x^6) +
15*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(3/2)*b^(7/2)*(a + b*
x^2)^3*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1384, 27, 252, 252, 252, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^5(a+bx^2)\int \frac{x^6}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{x^6}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{(a+bx^2)\left(\frac{5\int \frac{x^4}{\left(bx^2+a\right)^4}dx}{8b}-\frac{x^5}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{\int \frac{x^2}{\left(bx^2+a\right)^3}dx}{2b}-\frac{x^3}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^5}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{\frac{\int \frac{1}{\left(bx^2+a\right)^2}dx}{4b}-\frac{x}{4b\left(a+bx^2\right)^2}}{2b}-\frac{x^3}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^5}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{215}$$

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{\frac{\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}}{4b}-\frac{x}{4b\left(a+bx^2\right)^2}}{2b}-\frac{x^3}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^5}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}}{4b}-\frac{x}{4b(a+bx^2)^2}}{2b}-\frac{x^3}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^5}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input

```
Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output

```
((a + b*x^2)*(-1/8*x^5/(b*(a + b*x^2)^4) + (5*(-1/6*x^3/(b*(a + b*x^2)^3)
+ (-1/4*x/(b*(a + b*x^2)^2) + (x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sq
rt[a]]/(2*a^(3/2)*Sqrt[b]))/(4*b))/(2*b)))/(8*b)))/Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{5x^{7}}{128a}-\frac{73x^{5}}{384b}-\frac{55a\,x^{3}}{384b^{2}}-\frac{5a^{2}x}{128b^{3}}\right)}{\left(b\,x^{2}+a\right)^{5}}-\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(bx+\sqrt{-ab}\right)}{256\left(b\,x^{2}+a\right)\sqrt{-ab}\,b^{3}a}+\frac{5\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(-bx+\sqrt{-ab}\right)}{256\left(b\,x^{2}+a\right)\sqrt{-ab}\,b^{3}a}$
default	$-\frac{\left(-15\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^{4}x^{8}-15\sqrt{ab}\,b^{3}x^{7}-60\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^{3}x^{6}+73\sqrt{ab}\,a\,b^{2}x^{5}-90\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^{2}b^{2}x^{4}+55\sqrt{ab}\,a^{2}b\,x^{3}-60\,\text{arcta}\right.}{384\sqrt{ab}\,a\,b^{3}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}$

input
```
int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)^5*(5/128/a*x^7-73/384/b*x^5-55/384*a/b^2*x^3
-5/128*a^2/b^3*x)-5/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)/b^3/a*l
n(b*x+(-a*b)^(1/2))+5/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)/b^3/a
*ln(-b*x+(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.54

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \left[\frac{30\,ab^4x^7-146\,a^2b^3x^5-110\,a^3b^2x^3-30\,a^4bx-15\,(b^4x^8+4\,ab^3x^6+6\,a^2}{768\,(a^2b^8x^8+4\,a^3b^7x^6+6\,a^4b^6x^4+4\,a^5}\right.$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(30*a*b^4*x^7 - 146*a^2*b^3*x^5 - 110*a^3*b^2*x^3 - 30*a^4*b*x - 15
*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^8*x^8 + 4*a^3*b^7*x^6
+ 6*a^4*b^6*x^4 + 4*a^5*b^5*x^2 + a^6*b^4), 1/384*(15*a*b^4*x^7 - 73*a^2*b
^3*x^5 - 55*a^3*b^2*x^3 - 15*a^4*b*x + 15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b
^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^8*x^8
+ 4*a^3*b^7*x^6 + 6*a^4*b^6*x^4 + 4*a^5*b^5*x^2 + a^6*b^4)]
```

Sympy [F]

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{x^6}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**6/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.52

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{15\,b^3x^7-73\,ab^2x^5-55\,a^2bx^3-15\,a^3x}{384\,(ab^7x^8+4\,a^2b^6x^6+6\,a^3b^5x^4+4\,a^4b^4x^2+a^5b^3)} + \frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}ab^3}$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/384*(15*b^3*x^7 - 73*a*b^2*x^5 - 55*a^2*b*x^3 - 15*a^3*x)/(a*b^7*x^8 + 4
*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3) + 5/128*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*a*b^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.44

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}ab^3\mathrm{sgn}\,(bx^2+a)} + \frac{15\,b^3x^7-73\,ab^2x^5-55\,a^2bx^3-15\,a^3x}{384\,(bx^2+a)^4ab^3\mathrm{sgn}\,(bx^2+a)}$$

input
```
integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
5/128*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^3*sgn(b*x^2 + a)) + 1/384*(15*b
^3*x^7 - 73*a*b^2*x^5 - 55*a^2*b*x^3 - 15*a^3*x)/((b*x^2 + a)^4*a*b^3*sgn(
b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{x^6}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^6/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4+60\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^2+90\sqrt{b}\,\sqrt{a}\,atan\left(\right.}{38}$$

input
```
int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4 + 60*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*x**2 + 90*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**4 + 60*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*a*b**3*x**6 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))
)*b**4*x**8 - 15*a**4*b*x - 55*a**3*b**2*x**3 - 73*a**2*b**3*x**5 + 15*a*b
**4*x**7)/(384*a**2*b**4*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b*
*3*x**6 + b**4*x**8))
```

3.604 $\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 212

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x^3}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{16b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x}{64ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{5/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
3/128*x/a^2/b^2/((b*x^2+a)^2)^(1/2)-1/8*x^3/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1
/2)-1/16*x/b^2/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+1/64*x/a/b^2/(b*x^2+a)/((b*
x^2+a)^2)^(1/2)+3/128*(b*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)/b^(5/2)/
((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{\sqrt{a}\sqrt{b}x(-3a^3-11a^2bx^2+11ab^2x^4+3b^3x^6)+3(a+bx^2)^4\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{5/2}b^{5/2}\,(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a]*Sqrt[b]*x*(-3*a^3 - 11*a^2*b*x^2 + 11*a*b^2*x^4 + 3*b^3*x^6) + 3*
(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2)*(a + b*x^2
)^3*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1384, 27, 252, 252, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^2)\int \frac{x^4}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{x^4}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{3\int\frac{x^2}{(bx^2+a)^4}dx}{8b}-\frac{x^3}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{(a+bx^2)\left(\frac{3\left(\frac{\int\frac{1}{(bx^2+a)^3}dx}{6b}-\frac{x}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^3}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 215

$$\frac{(a+bx^2)\left(\frac{3\left(\frac{\frac{3\int\frac{1}{(bx^2+a)^2}dx}{4a}+\frac{x}{4a(a+bx^2)^2}}{6b}-\frac{x}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^3}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 215

$$\frac{(a+bx^2)\left(\frac{3\left(\frac{\frac{3\left(\frac{\int\frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}\right)}{4a}+\frac{x}{4a(a+bx^2)^2}}{6b}-\frac{x}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^3}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 218

$$\frac{(a+bx^2)\left(\frac{3\left(\frac{3\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}\right)}{4a}+\frac{x}{4a(a+bx^2)^2}\right)}{6b}-\frac{x}{6b(a+bx^2)^3}\right)}{8b}-\frac{x^3}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*x^3/(b*(a + b*x^2)^4) + (3*(-1/6*x/(b*(a + b*x^2)^3) +
(x/(4*a*(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt
[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a))/(6*b)))/(8*b)))/Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.69

method	result
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{3b\,x^{7}}{128a^{2}}+\frac{11x^{5}}{128a}-\frac{11x^{3}}{128b}-\frac{3ax}{128b^{2}}\right)}{\left(b\,x^{2}+a\right)^{5}}-\frac{3\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(bx+\sqrt{-ab}\right)}{256\left(b\,x^{2}+a\right)\sqrt{-ab}\,b^{2}a^{2}}+\frac{3\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(-bx+\sqrt{-ab}\right)}{256\left(b\,x^{2}+a\right)\sqrt{-ab}\,b^{2}a^{2}}$
default	$-\frac{\left(-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^{4}x^{8}-3\sqrt{ab}\,b^{3}x^{7}-12\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^{3}x^{6}-11\sqrt{ab}\,a\,b^{2}x^{5}-18\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^{2}b^{2}x^{4}+11\sqrt{ab}\,a^{2}b\,x^{3}-12\arctan\left(\right.\right.}{128\sqrt{ab}\,b^{2}a^{2}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}$

input
```
int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)^5*(3/128*b/a^2*x^7+11/128/a*x^5-11/128/b*x^3
-3/128*a/b^2*x)-3/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)/b^2/a^2*l
n(b*x+(-a*b)^(1/2))+3/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)/b^2/a
^2*ln(-b*x+(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.53

$$\int \frac{x^4}{(a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx = \left[\frac{6\,ab^4x^7 + 22\,a^2b^3x^5 - 22\,a^3b^2x^3 - 6\,a^4bx - 3\,(b^4x^8 + 4\,ab^3x^6 + 6\,a^2b^2x^4 +}{256\,(a^3b^7x^8 + 4\,a^4b^6x^6 + 6\,a^5b^5x^4 + 4\,a^6b^4x}\right.$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/256*(6*a*b^4*x^7 + 22*a^2*b^3*x^5 - 22*a^3*b^2*x^3 - 6*a^4*b*x - 3*(b^4
*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*log((b*
x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^7*x^8 + 4*a^4*b^6*x^6 + 6*a
^5*b^5*x^4 + 4*a^6*b^4*x^2 + a^7*b^3), 1/128*(3*a*b^4*x^7 + 11*a^2*b^3*x^5
 - 11*a^3*b^2*x^3 - 3*a^4*b*x + 3*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 +
 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^7*x^8 + 4*a^4*
b^6*x^6 + 6*a^5*b^5*x^4 + 4*a^6*b^4*x^2 + a^7*b^3)]
```

Sympy [F]

$$\int \frac{x^4}{(a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx = \int \frac{x^4}{\left((a + bx^2)^2\right)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**4/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.52

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{3\,b^3x^7+11\,ab^2x^5-11\,a^2bx^3-3\,a^3x}{128\,(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2)} + \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^2b^2}$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/128*(3*b^3*x^7 + 11*a*b^2*x^5 - 11*a^2*b*x^3 - 3*a^3*x)/(a^2*b^6*x^8 + 4
*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2) + 3/128*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.44

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^2b^2\mathrm{sgn}\,(bx^2+a)} + \frac{3\,b^3x^7+11\,ab^2x^5-11\,a^2bx^3-3\,a^3x}{128\,(bx^2+a)^4a^2b^2\mathrm{sgn}\,(bx^2+a)}$$

input
```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
3/128*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2*sgn(b*x^2 + a)) + 1/128*(3*
b^3*x^7 + 11*a*b^2*x^5 - 11*a^2*b*x^3 - 3*a^3*x)/((b*x^2 + a)^4*a^2*b^2*sg
n(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{x^4}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^4/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4+12\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^2+18\sqrt{b}\,\sqrt{a}\,atan\left(\cdots\right.}{128a\cdots}$$

input
```
int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4 + 12*sqrt(b)*sqrt(a)
*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*x**2 + 18*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*a**2*b**2*x**4 + 12*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*a*b**3*x**6 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*
b**4*x**8 - 3*a**4*b*x - 11*a**3*b**2*x**3 + 11*a**2*b**3*x**5 + 3*a*b**4*
x**7)/(128*a**3*b**3*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x
**6 + b**4*x**8))
```

3.605 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 213

$$\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{x}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{x}{48ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5x}{192a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
5/128*x/a^3/b/((b*x^2+a)^2)^(1/2)-1/8*x/b/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+
1/48*x/a/b/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+5/192*x/a^2/b/(b*x^2+a)/((b*x^2
+a)^2)^(1/2)+5/128*(b*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(3/2)/((b
*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.49

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{\sqrt{a}\sqrt{b}x(-15a^3+73a^2bx^2+55ab^2x^4+15b^3x^6)+15(a+bx^2)^4\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{384a^{7/2}b^{3/2}\left(a+bx^2\right)^3\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a]*Sqrt[b]*x*(-15*a^3 + 73*a^2*b*x^2 + 55*a*b^2*x^4 + 15*b^3*x^6) +
15*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(7/2)*b^(3/2)*(a + b*
x^2)^3*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.70, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1384, 27, 252, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5\left(a+bx^2\right)\int \frac{x^2}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{x^2}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\left(a+bx^2\right)\left(\frac{\int \frac{1}{\left(bx^2+a\right)^4}dx}{8b}-\frac{x}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{215}$$

$$\frac{\left(a+bx^2\right)\left(\frac{\frac{5\int \frac{1}{\left(bx^2+a\right)^3}dx}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}}{8b}-\frac{x}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{215}$$

$$\frac{\left(a+bx^2\right)\left(\frac{\frac{5\left(\frac{3\int \frac{1}{\left(bx^2+a\right)^2}dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}}{8b}-\frac{x}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{215}$$

$$\frac{\left(a+bx^2\right)\left(\frac{\frac{5\left(\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}}{8b}-\frac{x}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(a+bx^2)\left(\frac{\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}}{8b}-\frac{x}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*x/(b*(a + b*x^2)^4) + (x/(6*a*(a + b*x^2)^3) + (5*(x/(4
*a*(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/
(2*a^(3/2)*Sqrt[b])))/(4*a)))/(6*a))/(8*b)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.70

method	result
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{5 b^{2} x^{7}}{128 a^{3}}+\frac{55 b\,x^{5}}{384 a^{2}}+\frac{73 x^{3}}{384 a}-\frac{5 x}{128 b}\right)}{\left(b\,x^{2}+a\right)^{5}}-\frac{5 \sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(b x+\sqrt{-a b}\right)}{256 \left(b\,x^{2}+a\right) \sqrt{-a b}\,b\,a^{3}}+\frac{5 \sqrt{\left(b\,x^{2}+a\right)^{2}}\,\ln\left(-b x+\sqrt{-a b}\right)}{256 \left(b\,x^{2}+a\right) \sqrt{-a b}\,b\,a^{3}}$
default	$-\frac{\left(-15 \arctan\left(\frac{b x}{\sqrt{a b}}\right) b^{4} x^{8}-15 \sqrt{a b}\,b^{3} x^{7}-60 \arctan\left(\frac{b x}{\sqrt{a b}}\right) a\,b^{3} x^{6}-55 \sqrt{a b}\,a\,b^{2} x^{5}-90 \arctan\left(\frac{b x}{\sqrt{a b}}\right) a^{2} b^{2} x^{4}-73 \sqrt{a b}\,a^{2} b\,x^{3}-60 \text{arcta}\right.}{384 \sqrt{a b}\,b\,a^{3} \left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}$

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)^5*(5/128*b^2/a^3*x^7+55/384*b/a^2*x^5+73/384
/a*x^3-5/128/b*x)-5/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)/b/a^3*l
n(b*x+(-a*b)^(1/2))+5/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1/2)/b/a^3
*ln(-b*x+(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \left[\frac{30\,ab^4x^7+110\,a^2b^3x^5+146\,a^3b^2x^3-30\,a^4bx-15\,(b^4x^8+4\,ab^3x^6+6\,a^2}{768\,(a^4b^6x^8+4\,a^5b^5x^6+6\,a^6b^4x^4+4\,a^7}\right.$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(30*a*b^4*x^7 + 110*a^2*b^3*x^5 + 146*a^3*b^2*x^3 - 30*a^4*b*x - 15
*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^6*x^8 + 4*a^5*b^5*x^6
+ 6*a^6*b^4*x^4 + 4*a^7*b^3*x^2 + a^8*b^2), 1/384*(15*a*b^4*x^7 + 55*a^2*b
^3*x^5 + 73*a^3*b^2*x^3 - 15*a^4*b*x + 15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b
^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^6*x^8
+ 4*a^5*b^5*x^6 + 6*a^6*b^4*x^4 + 4*a^7*b^3*x^2 + a^8*b^2)]
```

Sympy [F]

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{x^2}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(x**2/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.51

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{15\,b^3x^7+55\,ab^2x^5+73\,a^2bx^3-15\,a^3x}{384\,(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b)} + \frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^3b}$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/384*(15*b^3*x^7 + 55*a*b^2*x^5 + 73*a^2*b*x^3 - 15*a^3*x)/(a^3*b^5*x^8 +
 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b) + 5/128*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.44

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^3b\mathrm{sgn}\left(bx^2+a\right)} + \frac{15\,b^3x^7+55\,ab^2x^5+73\,a^2bx^3-15\,a^3x}{384\,(bx^2+a)^4a^3b\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
5/128*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b*sgn(b*x^2 + a)) + 1/384*(15*b
^3*x^7 + 55*a*b^2*x^5 + 73*a^2*b*x^3 - 15*a^3*x)/((b*x^2 + a)^4*a^3*b*sgn(
b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{x^2}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4+60\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^2+90\sqrt{b}\,\sqrt{a}\,atan\left(\right.}{38}$$

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4 + 60*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*x**2 + 90*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**4 + 60*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(
b)*sqrt(a)))*a*b**3*x**6 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))
)*b**4*x**8 - 15*a**4*b*x + 73*a**3*b**2*x**3 + 55*a**2*b**3*x**5 + 15*a*b
**4*x**7)/(384*a**4*b**2*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b*
*3*x**6 + b**4*x**8))
```

3.606 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 204

$$\begin{aligned}\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx &= \frac{35x}{128a^4\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{x}{8a(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{7x}{48a^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{35x}{192a^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{35(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
35/128*x/a^4/((b*x^2+a)^2)^(1/2)+1/8*x/a/(b*x^2+a)^3/((b*x^2+a)^2)^(1/2)+7
/48*x/a^2/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+35/192*x/a^3/(b*x^2+a)/((b*x^2+a
)^2)^(1/2)+35/128*(b*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/a^(9/2)/b^(1/2)/((b*
x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{\sqrt{a}\sqrt{b}x(279a^3+511a^2bx^2+385ab^2x^4+105b^3x^6)+105(a+bx^2)^4\arctan\left(\right.}{384a^{9/2}\sqrt{b}\,(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

output
```
(Sqrt[a]*Sqrt[b]*x*(279*a^3 + 511*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^3*x^6)
 + 105*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(9/2)*Sqrt[b]*(a
+ b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1384, 215, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^2)\int \frac{1}{(b^2x^2+ab)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{215}$$

$$\frac{b^5(a+bx^2)\left(\frac{7\int \frac{1}{(b^2x^2+ab)^4}dx}{8ab}+\frac{x}{8ab^5(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{215}$$

$$\frac{b^5\left(a+bx^2\right)\left(\frac{7\left(\frac{5\int\frac{1}{\left(b^2x^2+ab\right)^3}dx}{6ab}+\frac{x}{6ab^4\left(a+bx^2\right)^3}\right)}{8ab}+\frac{x}{8ab^5\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{215}$$

$$\frac{b^5\left(a+bx^2\right)\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{1}{\left(b^2x^2+ab\right)^2}dx}{4ab}+\frac{x}{4ab^3\left(a+bx^2\right)^2}\right)}{6ab}+\frac{x}{6ab^4\left(a+bx^2\right)^3}\right)}{8ab}+\frac{x}{8ab^5\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{215}$$

$$\frac{b^5\left(a+bx^2\right)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int\frac{1}{b^2x^2+ab}dx}{2ab}+\frac{x}{2ab^2\left(a+bx^2\right)}\right)}{4ab}+\frac{x}{4ab^3\left(a+bx^2\right)^2}\right)}{6ab}+\frac{x}{6ab^4\left(a+bx^2\right)^3}\right)}{8ab}+\frac{x}{8ab^5\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{218}$$

$$
\frac{b^5(a+bx^2)\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x}{2ab^2(a+bx^2)}\right)}{4ab}+\frac{x}{4ab^3(a+bx^2)^2}\right)}{6ab}+\frac{x}{6ab^4(a+bx^2)^3}\right)}{8ab}+\frac{x}{8ab^5(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}
$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

output
```
(b^5*(a + b*x^2)*(x/(8*a*b^5*(a + b*x^2)^4) + (7*(x/(6*a*b^4*(a + b*x^2)^3
) + (5*(x/(4*a*b^3*(a + b*x^2)^2) + (3*(x/(2*a*b^2*(a + b*x^2)) + ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*b^(5/2))))/(4*a*b)))/(6*a*b)))/(8*a*b)))/Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.72

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(\frac{35b^3x^7}{128a^4}+\frac{385b^2x^5}{384a^3}+\frac{511b\,x^3}{384a^2}+\frac{93x}{128a}\right)}{(b\,x^2+a)^5}-\frac{35\sqrt{(b\,x^2+a)^2}\,\ln(bx+\sqrt{-ab})}{256(b\,x^2+a)\sqrt{-ab}\,a^4}+\frac{35\sqrt{(b\,x^2+a)^2}\,\ln(-bx+\sqrt{-ab})}{256(b\,x^2+a)\sqrt{-ab}\,a^4}$
default	$\frac{\left(105\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^4x^8+105\sqrt{ab}\,b^3x^7+420\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^3x^6+385\sqrt{ab}\,a\,b^2x^5+630\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2b^2x^4+511\sqrt{ab}\,a^2b\,x^3+420\,a\right.}{384\sqrt{ab}\,a^4\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)^5*(35/128*b^3/a^4*x^7+385/384*b^2/a^3*x^5+51
1/384*b/a^2*x^3+93/128*x/a)-35/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(1
/2)/a^4*ln(b*x+(-a*b)^(1/2))+35/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(-a*b)^(
1/2)/a^4*ln(-b*x+(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.57

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \left[\frac{210\,ab^4x^7+770\,a^2b^3x^5+1022\,a^3b^2x^3+558\,a^4bx-105\,(b^4x^8+4\,ab^3x^6+}{768\,(a^5b^5x^8+4\,a^6b^4x^6+6\,a^7b^3x^4+4}\right.$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(210*a*b^4*x^7 + 770*a^2*b^3*x^5 + 1022*a^3*b^2*x^3 + 558*a^4*b*x -
 105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^5*x^8 + 4*a^6*b^4*
x^6 + 6*a^7*b^3*x^4 + 4*a^8*b^2*x^2 + a^9*b), 1/384*(105*a*b^4*x^7 + 385*a
^2*b^3*x^5 + 511*a^3*b^2*x^3 + 279*a^4*b*x + 105*(b^4*x^8 + 4*a*b^3*x^6 +
6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b
^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8*b^2*x^2 + a^9*b)]
```

Sympy [F]

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \frac{105\,b^3x^7 + 385\,ab^2x^5 + 511\,a^2bx^3 + 279\,a^3x}{384\,(a^4b^4x^8 + 4\,a^5b^3x^6 + 6\,a^6b^2x^4 + 4\,a^7bx^2 + a^8)} + \frac{35\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^4}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/384*(105*b^3*x^7 + 385*a*b^2*x^5 + 511*a^2*b*x^3 + 279*a^3*x)/(a^4*b^4*x
^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8) + 35/128*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\sqrt{ab}a^4\text{sgn}\,(bx^2+a)} + \frac{105\,b^3x^7+385\,ab^2x^5+511\,a^2bx^3+279\,a^3x}{384\,(bx^2+a)^4a^4\text{sgn}\,(bx^2+a)}$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
35/128*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*sgn(b*x^2 + a)) + 1/384*(105*b
^3*x^7 + 385*a*b^2*x^5 + 511*a^2*b*x^3 + 279*a^3*x)/((b*x^2 + a)^4*a^4*sgn
(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{1}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4+420\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^2+630\sqrt{b}\,\sqrt{a}\,at}{}$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output

```
(105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4 + 420*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*x**2 + 630*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**4 + 420*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a*b**3*x**6 + 105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqr
t(a)))*b**4*x**8 + 279*a**4*b*x + 511*a**3*b**2*x**3 + 385*a**2*b**3*x**5
+ 105*a*b**4*x**7)/(384*a**5*b*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 +
4*a*b**3*x**6 + b**4*x**8))
```

3.607 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 245

$$\begin{aligned}\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = &-\frac{187bx}{128a^5\sqrt{a^2+2abx^2+b^2x^4}} \\ &-\frac{bx}{8a^2(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5bx}{16a^3(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} \\ &-\frac{41bx}{64a^4(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} \\ &-\frac{a+bx^2}{a^5x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{315\sqrt{b}(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
-187/128*b*x/a^5/((b*x^2+a)^2)^(1/2)-1/8*b*x/a^2/(b*x^2+a)^3/((b*x^2+a)^2)
^(1/2)-5/16*b*x/a^3/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)-41/64*b*x/a^4/(b*x^2+a
)/((b*x^2+a)^2)^(1/2)-(b*x^2+a)/a^5/x/((b*x^2+a)^2)^(1/2)-315/128*b^(1/2)*
(b*x^2+a)*arctan(b^(1/2)*x/a^(1/2))/a^(11/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^2 (a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{-\sqrt{a}(128a^4+837a^3bx^2+1533a^2b^2x^4+1155ab^3x^6+315b^4x^8)-315\sqrt{b}}{128a^{11/2}x\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
(-(Sqrt[a]*(128*a^4 + 837*a^3*b*x^2 + 1533*a^2*b^2*x^4 + 1155*a*b^3*x^6 +
315*b^4*x^8)) - 315*Sqrt[b]*x*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
128*a^(11/2)*x*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.70, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1384, 27, 253, 253, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^2+2abx^2+b^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5\left(a+bx^2\right)\int \frac{1}{b^5x^2(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{1}{x^2(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{253}$$

$$\frac{(a+bx^2)\left(\frac{9\int\frac{1}{x^2(bx^2+a)^4}dx}{8a}+\frac{1}{8ax(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{9\left(\frac{7\int\frac{1}{x^2(bx^2+a)^3}dx}{6a}+\frac{1}{6ax(a+bx^2)^3}\right)}{8a}+\frac{1}{8ax(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{9\left(\frac{7\left(\frac{5\int\frac{1}{x^2(bx^2+a)^2}dx}{4a}+\frac{1}{4ax(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax(a+bx^2)^3}\right)}{8a}+\frac{1}{8ax(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{1}{x^2\left(bx^2+a\right)}dx}{2a}+\frac{1}{2ax\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 264$$

$$\frac{(a+bx^2)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{b\int\frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a+bx^2)}\right)}{4a}+\frac{1}{4ax(a+bx^2)^2}\right)}{6a}+\frac{1}{6ax(a+bx^2)^3}\right)}{8a}+\frac{1}{8ax(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 218$$

$$\frac{(a+bx^2)\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
((a + b*x^2)*(1/(8*a*x*(a + b*x^2)^4) + (9*(1/(6*a*x*(a + b*x^2)^3) + (7*(
1/(4*a*x*(a + b*x^2)^2) + (5*(1/(2*a*x*(a + b*x^2)) + (3*(-(1/(a*x)) - (Sq
rt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a)))/(6*a)))/(8*a)
))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.66

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(-\frac{315b^4x^8}{128a^5}-\frac{1155b^3x^6}{128a^4}-\frac{1533b^2x^4}{128a^3}-\frac{837b\,x^2}{128a^2}-\frac{1}{a}\right)}{(b\,x^2+a)^5x}+\frac{315\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,\ln(-bx+\sqrt{-ab})}{256(b\,x^2+a)a^6}-\frac{315\sqrt{(b\,x^2+a)^2}\,\sqrt{-a}}{256(b\,x^2}$
default	$-\frac{\left(315\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^5x^9+315\sqrt{ab}\,b^4x^8+1260\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^4x^7+1155\sqrt{ab}\,a\,b^3x^6+1890\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2b^3x^5+1533\sqrt{ab}\,a^2b^2x\right.}{128x\sqrt{ab}\,a^5\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}$

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)^5*(-315/128*b^4/a^5*x^8-1155/128*b^3/a^4*x^6
-1533/128*b^2/a^3*x^4-837/128*b/a^2*x^2-1/a)/x+315/256*((b*x^2+a)^2)^(1/2)
/(b*x^2+a)/a^6*(-a*b)^(1/2)*ln(-b*x+(-a*b)^(1/2))-315/256*((b*x^2+a)^2)^(1
/2)/(b*x^2+a)/a^6*(-a*b)^(1/2)*ln(-b*x-(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.36

$$\int\frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\left[-\frac{630\,b^4x^8+2310\,ab^3x^6+3066\,a^2b^2x^4+1674\,a^3bx^2+256\,a^4-315\,(b^4}{256\,(a^5b^4x^9+4\,a^6b^3x^7+}\right.$$

$$-\frac{315\,b^4x^8+1155\,ab^3x^6+1533\,a^2b^2x^4+837\,a^3bx^2+128\,a^4+315\,(b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a}{128\,(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x)}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/256*(630*b^4*x^8 + 2310*a*b^3*x^6 + 3066*a^2*b^2*x^4 + 1674*a^3*b*x^2
+ 256*a^4 - 315*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4
*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^4*x
^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), -1/128*(315*b^4
*x^8 + 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 + 837*a^3*b*x^2 + 128*a^4 + 315*(
b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*sqrt(b/a)*arc
tan(x*sqrt(b/a)))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x
^3 + a^9*x)]
```

Sympy [F]

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\int \frac{1}{x^2\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(1/(x**2*((a + b*x**2)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=-\frac{315\,b^4x^8+1155\,ab^3x^6+1533\,a^2b^2x^4+837\,a^3bx^2+128\,a^4}{128\,\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}-\frac{315\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^5}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/128*(315*b^4*x^8 + 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 + 837*a^3*b*x^2 +
128*a^4)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*
x) - 315/128*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^2 (a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{315\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^5\mathrm{sgn}\left(bx^2+a\right)} - \frac{1}{a^5x\mathrm{sgn}\left(bx^2+a\right)} - \frac{187\,b^4x^7+643\,ab^3x^5+765\,a^2b^2x^3+325\,a^3bx}{128\,(bx^2+a)^4a^5\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
-315/128*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5*sgn(b*x^2 + a)) - 1/(a^5*x
*sgn(b*x^2 + a)) - 1/128*(187*b^4*x^7 + 643*a*b^3*x^5 + 765*a^2*b^2*x^3 +
325*a^3*b*x)/((b*x^2 + a)^4*a^5*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{1}{x^2\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}} \,dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4x-1260\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^3-1890\sqrt{}}{}$$

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
( - 315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*x - 1260*sqrt(b
)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*x**3 - 1890*sqrt(b)*sqrt(a)
*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**5 - 1260*sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*a*b**3*x**7 - 315*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*b**4*x**9 - 128*a**5 - 837*a**4*b*x**2 - 1533*a**3*b**2*x**
4 - 1155*a**2*b**3*x**6 - 315*a*b**4*x**8)/(128*a**6*x*(a**4 + 4*a**3*b*x*
*2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.608 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 293

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{515b^2x}{128a^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^2x}{8a^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{23b^2x}{48a^4\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{259b^2x}{192a^5\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{3a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5b(a+bx^2)}{a^6x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155b^{3/2}(a+bx^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
515/128*b^2*x/a^6/((b*x^2+a)^2)^(1/2)+1/8*b^2*x/a^3/(b*x^2+a)^3/((b*x^2+a)
^2)^(1/2)+23/48*b^2*x/a^4/(b*x^2+a)^2/((b*x^2+a)^2)^(1/2)+259/192*b^2*x/a^
5/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-1/3*(b*x^2+a)/a^5/x^3/((b*x^2+a)^2)^(1/2)+
5*b*(b*x^2+a)/a^6/x/((b*x^2+a)^2)^(1/2)+1155/128*b^(3/2)*(b*x^2+a)*arctan(
b^(1/2)*x/a^(1/2))/a^(13/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\sqrt{a}(-128a^5+1408a^4bx^2+9207a^3b^2x^4+16863a^2b^3x^6+12705ab^4x^8+3}{384a^{13/2}x^3\left(a+bx^2\right)^3\sqrt{(a+}}$$

input
```
Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
(Sqrt[a]*(-128*a^5 + 1408*a^4*b*x^2 + 9207*a^3*b^2*x^4 + 16863*a^2*b^3*x^6
 + 12705*a*b^4*x^8 + 3465*b^5*x^10) + 3465*b^(3/2)*x^3*(a + b*x^2)^4*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(13/2)*x^3*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2
])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.65, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1384, 27, 253, 253, 253, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b^5\left(a+bx^2\right)\int \frac{1}{b^5x^4\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+bx^2\right)\int \frac{1}{x^4\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 253$$

$$\frac{(a+bx^2)\left(\frac{11\int\frac{1}{x^4(bx^2+a)^4}dx}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{11\left(\frac{3\int\frac{1}{x^4(bx^2+a)^3}dx}{2a}+\frac{1}{6ax^3(a+bx^2)^3}\right)}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{11\left(\frac{3\left(\frac{7\int\frac{1}{x^4(bx^2+a)^2}dx}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{2a}+\frac{1}{6ax^3(a+bx^2)^3}\right)}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{1}{x^4(bx^2+a)}dx}{2a}+\frac{1}{2ax^3(a+bx^2)}\right)}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{2a}+\frac{1}{6ax^3(a+bx^2)^3}\right)}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 264

$$\frac{(a+bx^2)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\int\frac{1}{x^2(bx^2+a)}\,dx}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3(a+bx^2)}\right)}{4a}+\frac{1}{4ax^3(a+bx^2)^2}\right)}{2a}+\frac{1}{6ax^3(a+bx^2)^3}\right)}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 264$$

$$(a+bx^2)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{b\int\frac{1}{bx^2+a}\,dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right)$$

↓ 218

$$(a+bx^2)\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3(a+bx^2)^4}\right.$$

input
```
Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
((a + b*x^2)*(1/(8*a*x^3*(a + b*x^2)^4) + (11*(1/(6*a*x^3*(a + b*x^2)^3) +
 (3*(1/(4*a*x^3*(a + b*x^2)^2) + (7*(1/(2*a*x^3*(a + b*x^2)) + (5*(-1/3*1/
(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))
/a))/(2*a)))/(4*a)))/(2*a)))/(8*a)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.60

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(\frac{1155b^5x^{10}}{128a^6}+\frac{4235b^4x^8}{128a^5}+\frac{5621b^3x^6}{128a^4}+\frac{3069b^2x^4}{128a^3}+\frac{11b\,x^2}{3a^2}-\frac{1}{3a}\right)}{(b\,x^2+a)^5x^3}+\frac{1155\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,b\ln(-bx-\sqrt{-ab})}{256(b\,x^2+a)a^7}-\frac{1155\sqrt{(}}{}$
default	$-\frac{\left(-3465\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^6x^{11}-3465\sqrt{ab}\,b^5x^{10}-13860\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^5x^9-12705\sqrt{ab}\,a\,b^4x^8-20790\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2b^4x^7-16863\right.}{384x^3\sqrt{ab}\,a^6(}$

input
```
int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^2)^(1/2)/(b*x^2+a)^5*(1155/128*b^5/a^6*x^10+4235/128*b^4/a^5*x^
8+5621/128*b^3/a^4*x^6+3069/128*b^2/a^3*x^4+11/3*b/a^2*x^2-1/3/a)/x^3+1155
/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/a^7*(-a*b)^(1/2)*b*ln(-b*x-(-a*b)^(1/2)
)-1155/256*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/a^7*(-a*b)^(1/2)*b*ln(-b*x+(-a*b)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\left[\frac{6930\,b^5x^{10}+25410\,ab^4x^8+33726\,a^2b^3x^6+18414\,a^3b^2x^4+2816\,a^4bx^2}{768\,(a^6b^4x^{11}+4}\right.$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(6930*b^5*x^10 + 25410*a*b^4*x^8 + 33726*a^2*b^3*x^6 + 18414*a^3*b^
2*x^4 + 2816*a^4*b*x^2 - 256*a^5 + 3465*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^
3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a
) - a)/(b*x^2 + a)))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9
*b*x^5 + a^10*x^3), 1/384*(3465*b^5*x^10 + 12705*a*b^4*x^8 + 16863*a^2*b^3
*x^6 + 9207*a^3*b^2*x^4 + 1408*a^4*b*x^2 - 128*a^5 + 3465*(b^5*x^11 + 4*a*
b^4*x^9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(b/a)*arctan(x*sq
rt(b/a)))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^
10*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \int \frac{1}{x^4 \left((a + bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(1/(x**4*((a + b*x**2)**2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \frac{3465\,b^5x^{10} + 12705\,ab^4x^8 + 16863\,a^2b^3x^6 + 9207\,a^3b^2x^4 + 1408\,a^4bx^2 -}{384\,(a^6b^4x^{11} + 4\,a^7b^3x^9 + 6\,a^8b^2x^7 + 4\,a^9bx^5 + a^{10}x^3)} + \frac{1155\,b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^6}$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/384*(3465*b^5*x^10 + 12705*a*b^4*x^8 + 16863*a^2*b^3*x^6 + 9207*a^3*b^2*
x^4 + 1408*a^4*b*x^2 - 128*a^5)/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*
x^7 + 4*a^9*b*x^5 + a^10*x^3) + 1155/128*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a^6)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^4 (a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{1155\, b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\,\sqrt{ab}a^6 \text{sgn}\left(bx^2+a\right)} + \frac{15\, bx^2-a}{3\, a^6x^3 \text{sgn}\left(bx^2+a\right)} + \frac{1545\, b^5x^7+5153\, ab^4x^5+5855\, a^2b^3x^3+2295\, a^3b^2x}{384\, (bx^2+a)^4 a^6 \text{sgn}\left(bx^2+a\right)}$$

input
```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1155/128*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6*sgn(b*x^2 + a)) + 1/3*(1
5*b*x^2 - a)/(a^6*x^3*sgn(b*x^2 + a)) + 1/384*(1545*b^5*x^7 + 5153*a*b^4*x
^5 + 5855*a^2*b^3*x^3 + 2295*a^3*b^2*x)/((b*x^2 + a)^4*a^6*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{1}{x^4\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

input
```
int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

output
```
int(1/(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^4 (a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4b\,x^3+13860\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b^2x^5+20}{}$$

input
```
int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*x**3 + 13860*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**5 + 20790*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*x**7 + 13860*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*x**9 + 3465*sqrt(b)*sqrt(a)*atan(
(b*x)/(sqrt(b)*sqrt(a)))*b**5*x**11 - 128*a**6 + 1408*a**5*b*x**2 + 9207*a
**4*b**2*x**4 + 16863*a**3*b**3*x**6 + 12705*a**2*b**4*x**8 + 3465*a*b**5*
x**10)/(384*a**7*x**3*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*
x**6 + b**4*x**8))
```

3.609 $\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 93

$$\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\,(a+bx^2)}+\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\,(a+bx^2)}$$

output
```
2/7*a*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/11*b*(d*x)^(11/2)*((b*
x^2+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.47

$$\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{2x(dx)^{5/2}\sqrt{(a+bx^2)^2}(11a+7bx^2)}{77\,(a+bx^2)}$$

input
```
Integrate[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(11*a + 7*b*x^2))/(77*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b(dx)^{5/2}\left(bx^2+a\right)dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{5/2}\left(bx^2+a\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{b(dx)^{9/2}}{d^2}+a(dx)^{5/2}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2a(dx)^{7/2}}{7d}+\frac{2b(dx)^{11/2}}{11d^3}\right)}{a+bx^2}$$

input
```
Int[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*a*(d*x)^(7/2))/(7*d) + (2*b*(d*x)^(11
/2))/(11*d^3)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{2x(7b\,x^2+11a)(dx)^{\frac{5}{2}}\sqrt{(b\,x^2+a)^2}}{77(b\,x^2+a)}$	39
orering	$\frac{2x(7b\,x^2+11a)(dx)^{\frac{5}{2}}\sqrt{(b\,x^2+a)^2}}{77(b\,x^2+a)}$	39
default	$\frac{2\sqrt{(b\,x^2+a)^2}\,(dx)^{\frac{7}{2}}(7b\,x^2+11a)}{77d(b\,x^2+a)}$	41
risch	$\frac{2d^3\sqrt{(b\,x^2+a)^2}\,x^4(7b\,x^2+11a)}{77(b\,x^2+a)\sqrt{dx}}$	44

input
```
int((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/77*x*(7*b*x^2+11*a)*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.28

$$\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{2}{77}\left(7\,bd^2x^5+11\,ad^2x^3\right)\sqrt{dx}$$

input
```
integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
2/77*(7*b*d^2*x^5 + 11*a*d^2*x^3)*sqrt(d*x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(5/2)*((b*x**2+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{2\left(7\,(dx)^{\frac{11}{2}}\,b+11\,(dx)^{\frac{7}{2}}\,ad^2\right)}{77\,d^3}$$

input
```
integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
2/77*(7*(d*x)^(11/2)*b + 11*(d*x)^(7/2)*a*d^2)/d^3
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

$$\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{2}{11}\sqrt{dx}bd^2x^5\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}ad^2x^3\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2/11*sqrt(d*x)*b*d^2*x^5*sgn(b*x^2 + a) + 2/7*sqrt(d*x)*a*d^2*x^3*sgn(b*x^
2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int (d\,x)^{5/2}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((d*x)^(5/2)*((a + b*x^2)^2)^(1/2),x)
```

output
```
int((d*x)^(5/2)*((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.24

$$\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,d^2x^3(7b\,x^2+11a)}{77}$$

input
```
int((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*d**2*x**3*(11*a + 7*b*x**2))/77
```

3.610 $\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 93

$$\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}+\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}$$

output
```
2/5*a*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/9*b*(d*x)^(9/2)*((b*x^
2+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.47

$$\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(9a+5bx^2\right)}{45\left(a+bx^2\right)}$$

input
```
Integrate[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(9*a + 5*b*x^2))/(45*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b(dx)^{3/2}\left(bx^2+a\right)dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{3/2}\left(bx^2+a\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{b(dx)^{7/2}}{d^2}+a(dx)^{3/2}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2a(dx)^{5/2}}{5d}+\frac{2b(dx)^{9/2}}{9d^3}\right)}{a+bx^2}$$

input
```
Int[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*a*(d*x)^(5/2))/(5*d) + (2*b*(d*x)^(9/
2))/(9*d^3)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{2x(5b\,x^2+9a)(dx)^{\frac{3}{2}}\sqrt{(b\,x^2+a)^2}}{45(b\,x^2+a)}$	39
orering	$\frac{2x(5b\,x^2+9a)(dx)^{\frac{3}{2}}\sqrt{(b\,x^2+a)^2}}{45(b\,x^2+a)}$	39
default	$\frac{2\sqrt{(b\,x^2+a)^2}\,(dx)^{\frac{5}{2}}(5b\,x^2+9a)}{45d(b\,x^2+a)}$	41
risch	$\frac{2d^2\sqrt{(b\,x^2+a)^2}\,x^3(5b\,x^2+9a)}{45(b\,x^2+a)\sqrt{dx}}$	44

input
```
int((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*x*(5*b*x^2+9*a)*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.24

$$\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{2}{45}\left(5\,bdx^4+9\,adx^2\right)\sqrt{dx}$$

input
```
integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(5*b*d*x^4 + 9*a*d*x^2)*sqrt(d*x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\text{Timed out}$$

input
```
integrate((d*x)**(3/2)*((b*x**2+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\frac{2\left(5\,(dx)^{\frac{9}{2}}\,b+9\,(dx)^{\frac{5}{2}}\,ad^2\right)}{45\,d^3}$$

input
```
integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
2/45*(5*(d*x)^(9/2)*b + 9*(d*x)^(5/2)*a*d^2)/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.45

$$\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{2}{45}\left(5\sqrt{dx}bx^4\mathrm{sgn}\left(bx^2+a\right)+9\sqrt{dx}ax^2\mathrm{sgn}\left(bx^2+a\right)\right)d$$

input
```
integrate((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2/45*(5*sqrt(d*x)*b*x^4*sgn(b*x^2 + a) + 9*sqrt(d*x)*a*x^2*sgn(b*x^2 + a))
*d
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int (d\,x)^{3/2}\,\sqrt{\left(b\,x^2+a\right)^2}\,dx$$

input
```
int((d*x)^(3/2)*((a + b*x^2)^2)^(1/2),x)
```

output
```
int((d*x)^(3/2)*((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.22

$$\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,d\,x^2(5b\,x^2+9a)}{45}$$

input
```
int((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*d*x**2*(9*a + 5*b*x**2))/45
```

3.611 $\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 93

$$\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\,(a+bx^2)} + \frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\,(a+bx^2)}$$

output
```
2/3*a*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/7*b*(d*x)^(7/2)*((b*x^
2+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.47

$$\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{2x\sqrt{dx}\sqrt{(a+bx^2)^2}(7a+3bx^2)}{21\,(a+bx^2)}$$

input
```
Integrate[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(2*x*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(7*a + 3*b*x^2))/(21*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b\sqrt{dx}\left(bx^2+a\right)dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\left(bx^2+a\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{b(dx)^{5/2}}{d^2}+a\sqrt{dx}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2a(dx)^{3/2}}{3d}+\frac{2b(dx)^{7/2}}{7d^3}\right)}{a+bx^2}$$

input
```
Int[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*a*(d*x)^(3/2))/(3*d) + (2*b*(d*x)^(7/
2))/(7*d^3)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{2x(3b\,x^{2}+7a)\sqrt{dx}\,\sqrt{(b\,x^{2}+a)^{2}}}{21(b\,x^{2}+a)}$	39
orering	$\frac{2x(3b\,x^{2}+7a)\sqrt{dx}\,\sqrt{(b\,x^{2}+a)^{2}}}{21(b\,x^{2}+a)}$	39
default	$\frac{2\sqrt{(b\,x^{2}+a)^{2}}\,(dx)^{\frac{3}{2}}\,(3b\,x^{2}+7a)}{21d(b\,x^{2}+a)}$	41
risch	$\frac{2d\sqrt{(b\,x^{2}+a)^{2}}\,(3b\,x^{2}+7a)x^{2}}{21(b\,x^{2}+a)\sqrt{dx}}$	42

input
```
int((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*x*(3*b*x^2+7*a)*(d*x)^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.19

$$\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{2}{21}\,(3\,bx^3+7\,ax)\sqrt{dx}$$

input
```
integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
2/21*(3*b*x^3 + 7*a*x)*sqrt(d*x)
```

Sympy [F]

$$\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int \sqrt{dx}\sqrt{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(sqrt(d*x)*sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{2\left(3\,(dx)^{\frac{7}{2}}\,b+7\,(dx)^{\frac{3}{2}}\,ad^2\right)}{21\,d^3}$$

input
```
integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
2/21*(3*(d*x)^(7/2)*b + 7*(d*x)^(3/2)*a*d^2)/d^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.40

$$\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{2}{7}\sqrt{dx}bx^3\mathrm{sgn}\left(bx^2+a\right)+\frac{2}{3}\sqrt{dx}ax\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
2/7*sqrt(d*x)*b*x^3*sgn(b*x^2 + a) + 2/3*sqrt(d*x)*a*x*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int \sqrt{d\,x}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((d*x)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

output
```
int((d*x)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.18

$$\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x(3b\,x^2+7a)}{21}$$

input
```
int((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x*(7*a + 3*b*x**2))/21
```

3.612 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 91

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx=\frac{2a\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}+\frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}$$

output
```
2*a*(d*x)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/5*b*(d*x)^(5/2)*((b*x^2+
a)^2)^(1/2)/d^3/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx=\frac{2x\sqrt{\left(a+bx^2\right)^2}\left(5a+bx^2\right)}{5\sqrt{dx}\left(a+bx^2\right)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]
```

output
```
(2*x*Sqrt[(a + b*x^2)^2]*(5*a + b*x^2))/(5*Sqrt[d*x]*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{\sqrt{dx}}dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{\sqrt{dx}}dx}{a+bx^2}$$

$$\Big\downarrow \ 244$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{b(dx)^{3/2}}{d^2}+\frac{a}{\sqrt{dx}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2a\sqrt{dx}}{d}+\frac{2b(dx)^{5/2}}{5d^3}\right)}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*a*Sqrt[d*x])/d + (2*b*(d*x)^(5/2))/(5
*d^3)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{2x\left(b\,x^{2}+5a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{5\left(b\,x^{2}+a\right)\sqrt{dx}}$	38
risch	$\frac{2x\left(b\,x^{2}+5a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{5\left(b\,x^{2}+a\right)\sqrt{dx}}$	38
orering	$\frac{2x\left(b\,x^{2}+5a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{5\left(b\,x^{2}+a\right)\sqrt{dx}}$	38
default	$\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\sqrt{dx}\,\left(b\,x^{2}+5a\right)}{5d\left(b\,x^{2}+a\right)}$	40

input
```
int(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x*(b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx=\frac{2\,(bx^2+5\,a)\sqrt{dx}}{5\,d}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(b*x^2 + 5*a)*sqrt(d*x)/d
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx=\int \frac{\sqrt{(a+bx^2)^2}}{\sqrt{dx}}\,dx$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(1/2),x)
```

output
```
Integral(sqrt((a + b*x**2)**2)/sqrt(d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx=\frac{2\left(5\,\sqrt{dx}a+\frac{(dx)^{\frac{5}{2}}b}{d^2}\right)}{5\,d}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(5*sqrt(d*x)*a + (d*x)^(5/2)*b/d^2)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{\sqrt{dx}}\,dx = \frac{2\left(\sqrt{dx}bx^2\mathrm{sgn}(bx^2+a) + 5\sqrt{dx}a\mathrm{sgn}(bx^2+a)\right)}{5\,d}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
2/5*(sqrt(d*x)*b*x^2*sgn(b*x^2 + a) + 5*sqrt(d*x)*a*sgn(b*x^2 + a))/d
```

Mupad [B] (verification not implemented)

Time = 18.58 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{\sqrt{dx}}\,dx = \frac{\left(\frac{2\,x^3}{5} + \frac{2\,a\,x}{b}\right)\sqrt{{\left(b\,x^2+a\right)}^2}}{x^2\,\sqrt{d\,x} + \frac{a\,\sqrt{d\,x}}{b}}$$

input
```
int(((a + b*x^2)^2)^(1/2)/(d*x)^(1/2),x)
```

output
```
(((2*x^3)/5 + (2*a*x)/b)*((a + b*x^2)^2)^(1/2))/(x^2*(d*x)^(1/2) + (a*(d*x
)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a^2 + 2abx^2 + b^2x^4}}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,(b\,x^2 + 5a)}{5d}$$

input
```
int(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*(5*a + b*x**2))/(5*d)
```

3.613 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 91

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx = -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)}+\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\,(a+bx^2)}$$

output
```
-2*a*((b*x^2+a)^2)^(1/2)/d/(d*x)^(1/2)/(b*x^2+a)+2/3*b*(d*x)^(3/2)*((b*x^2
+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx = -\frac{2x(3a-bx^2)\sqrt{(a+bx^2)^2}}{3(dx)^{3/2}\,(a+bx^2)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
```

output
```
(-2*x*(3*a - b*x^2)*Sqrt[(a + b*x^2)^2])/(3*(d*x)^(3/2)*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{(dx)^{3/2}}dx}{b\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{(dx)^{3/2}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a}{(dx)^{3/2}}+\frac{b\sqrt{dx}}{d^2}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2b(dx)^{3/2}}{3d^3}-\frac{2a}{d\sqrt{dx}}\right)}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((-2*a)/(d*Sqrt[d*x]) + (2*b*(d*x)^(3/2))
/(3*d^3)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.43

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+3a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{3\left(b\,x^{2}+a\right)\left(dx\right)^{\frac{3}{2}}}$	39
orering	$-\frac{2x\left(-b\,x^{2}+3a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{3\left(b\,x^{2}+a\right)\left(dx\right)^{\frac{3}{2}}}$	39
default	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-b\,x^{2}+3a\right)}{3d\left(b\,x^{2}+a\right)\sqrt{dx}}$	41
risch	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-b\,x^{2}+3a\right)}{3d\left(b\,x^{2}+a\right)\sqrt{dx}}$	41

input
```
int(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(-b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx=\frac{2\,(bx^2-3\,a)\sqrt{dx}}{3\,d^2x}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(b*x^2 - 3*a)*sqrt(d*x)/(d^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx=\int \frac{\sqrt{(a+bx^2)^2}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(3/2),x)
```

output
```
Integral(sqrt((a + b*x**2)**2)/(d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx=-\frac{2\left(\frac{3\,a}{\sqrt{dx}}-\frac{(dx)^{\frac{3}{2}}b}{d^2}\right)}{3\,d}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-2/3*(3*a/sqrt(d*x) - (d*x)^(3/2)*b/d^2)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{dx}bx\text{sgn}(bx^2+a)}{d}-\frac{3\,a\text{sgn}(bx^2+a)}{\sqrt{dx}}\right)}{3\,d}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
2/3*(sqrt(d*x)*b*x*sgn(b*x^2 + a)/d - 3*a*sgn(b*x^2 + a)/sqrt(d*x))/d
```

Mupad [B] (verification not implemented)

Time = 18.88 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx = \frac{\left(\frac{2\,x^2}{3\,d}-\frac{2\,a}{b\,d}\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{x^2\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

input
```
int(((a + b*x^2)^2)^(1/2)/(d*x)^(3/2),x)
```

output
```
(((2*x^2)/(3*d) - (2*a)/(b*d))*((a + b*x^2)^2)^(1/2))/(x^2*(d*x)^(1/2) + (
a*(d*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx = \frac{2\sqrt{d}\,(b\,x^2-3a)}{3\sqrt{x}\,d^2}$$

input
```
int(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x)
```

output
```
(2*sqrt(d)*( - 3*a + b*x**2))/(3*sqrt(x)*d**2)
```

3.614 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 91

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx = -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\,(a+bx^2)}+\frac{2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\,(a+bx^2)}$$

output
```
-2/3*a*((b*x^2+a)^2)^(1/2)/d/(d*x)^(3/2)/(b*x^2+a)+2*b*(d*x)^(1/2)*((b*x^2
+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx = -\frac{2x(a-3bx^2)\sqrt{(a+bx^2)^2}}{3(dx)^{5/2}\,(a+bx^2)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
```

output
```
(-2*x*(a - 3*b*x^2)*Sqrt[(a + b*x^2)^2])/(3*(d*x)^(5/2)*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{(dx)^{5/2}}dx}{b(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{(dx)^{5/2}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a}{(dx)^{5/2}}+\frac{b}{d^2\sqrt{dx}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2b\sqrt{dx}}{d^3}-\frac{2a}{3d(dx)^{3/2}}\right)}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((-2*a)/(3*d*(d*x)^(3/2)) + (2*b*Sqrt[d*x
])/d^3))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.41

method	result	size
gosper	$-\frac{2x\left(-3b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{3\left(b\,x^{2}+a\right)\left(dx\right)^{\frac{5}{2}}}$	37
orering	$-\frac{2x\left(-3b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{3\left(b\,x^{2}+a\right)\left(dx\right)^{\frac{5}{2}}}$	37
default	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-3b\,x^{2}+a\right)}{3d\left(b\,x^{2}+a\right)\left(dx\right)^{\frac{3}{2}}}$	39
risch	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-3b\,x^{2}+a\right)}{3d^{2}\left(b\,x^{2}+a\right)x\sqrt{dx}}$	42

input
```
int(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(-3*b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx=\frac{2\,(3\,bx^2-a)\sqrt{dx}}{3\,d^3x^2}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*b*x^2 - a)*sqrt(d*x)/(d^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx=-\frac{2\left(\frac{a}{(dx)^{\frac{3}{2}}}-\frac{3\sqrt{dx}b}{d^2}\right)}{3\,d}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(a/(d*x)^(3/2) - 3*sqrt(d*x)*b/d^2)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx = \frac{2\left(\frac{3\sqrt{dx}b\text{sgn}(bx^2+a)}{d^2} - \frac{a\text{sgn}(bx^2+a)}{\sqrt{dx}dx}\right)}{3\,d}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
2/3*(3*sqrt(d*x)*b*sgn(b*x^2 + a)/d^2 - a*sgn(b*x^2 + a)/(sqrt(d*x)*d*x))/
d
```

Mupad [B] (verification not implemented)

Time = 18.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx = \frac{\left(\frac{2\,x^2}{d^2} - \frac{2\,a}{3\,b\,d^2}\right)\sqrt{{\left(b\,x^2+a\right)}^2}}{x^3\,\sqrt{d\,x} + \frac{a\,x\,\sqrt{d\,x}}{b}}$$

input
```
int(((a + b*x^2)^2)^(1/2)/(d*x)^(5/2),x)
```

output
```
(((2*x^2)/d^2 - (2*a)/(3*b*d^2))*((a + b*x^2)^2)^(1/2))/(x^3*(d*x)^(1/2) +
 (a*x*(d*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx = \frac{2\sqrt{d}\,(3b\,x^2-a)}{3\sqrt{x}\,d^3x}$$

input
```
int(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x)
```

output
```
(2*sqrt(d)*( - a + 3*b*x**2))/(3*sqrt(x)*d**3*x)
```

3.615 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 91

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}} \, dx = -\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}(a+bx^2)} - \frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,(a+bx^2)}$$

output
```
-2/5*a*((b*x^2+a)^2)^(1/2)/d/(d*x)^(5/2)/(b*x^2+a)-2*b*((b*x^2+a)^2)^(1/2)
/d^3/(d*x)^(1/2)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}} \, dx = -\frac{2x\sqrt{(a+bx^2)^2}(a+5bx^2)}{5(dx)^{7/2}(a+bx^2)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
```

output
```
(-2*x*Sqrt[(a + b*x^2)^2]*(a + 5*b*x^2))/(5*(d*x)^(7/2)*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(bx^2+a)}{(dx)^{7/2}}dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{bx^2+a}{(dx)^{7/2}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a}{(dx)^{7/2}}+\frac{b}{d^2(dx)^{3/2}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{2a}{5d(dx)^{5/2}}-\frac{2b}{d^3\sqrt{dx}}\right)}{a+bx^2}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((-2*a)/(5*d*(d*x)^(5/2)) - (2*b)/(d^3*Sq
rt[d*x])))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.41

method	result	size
gosper	$-\frac{2x\left(5b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{5\left(b\,x^{2}+a\right)\left(dx\right)^{\frac{7}{2}}}$	37
orering	$-\frac{2x\left(5b\,x^{2}+a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{5\left(b\,x^{2}+a\right)\left(dx\right)^{\frac{7}{2}}}$	37
default	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(5b\,x^{2}+a\right)}{5d\left(b\,x^{2}+a\right)\left(dx\right)^{\frac{5}{2}}}$	39
risch	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(5b\,x^{2}+a\right)}{5d^{3}\left(b\,x^{2}+a\right)x^{2}\sqrt{dx}}$	42

input
```
int(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*x*(5*b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx = -\frac{2\,(5\,bx^2+a)\sqrt{dx}}{5\,d^4x^3}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(5*b*x^2 + a)*sqrt(d*x)/(d^4*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx = -\frac{2\,(5\,bd^2x^2+ad^2)}{5\,(dx)^{\frac{5}{2}}\,d^3}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
-2/5*(5*b*d^2*x^2 + a*d^2)/((d*x)^(5/2)*d^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx = -\frac{2\,(5\,bd^3x^2\mathrm{sgn}(bx^2+a)+ad^3\mathrm{sgn}(bx^2+a))}{5\,\sqrt{dx}d^6x^2}$$

input
```
integrate(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x, algorithm="giac")
```

output
```
-2/5*(5*b*d^3*x^2*sgn(b*x^2 + a) + a*d^3*sgn(b*x^2 + a))/(sqrt(d*x)*d^6*x^
2)
```

Mupad [B] (verification not implemented)

Time = 17.63 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx = -\frac{\left(\frac{2\,x^2}{d^3}+\frac{2\,a}{5\,b\,d^3}\right)\,\sqrt{{\left(b\,x^2+a\right)}^2}}{x^4\,\sqrt{d\,x}+\frac{a\,x^2\,\sqrt{d\,x}}{b}}$$

input
```
int(((a + b*x^2)^2)^(1/2)/(d*x)^(7/2),x)
```

output
```
-(((2*x^2)/d^3 + (2*a)/(5*b*d^3))*((a + b*x^2)^2)^(1/2))/(x^4*(d*x)^(1/2)
+ (a*x^2*(d*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx = \frac{2\sqrt{d}\,(-5b\,x^2-a)}{5\sqrt{x}\,d^4x^2}$$

input
```
int(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x)
```

output
```
(2*sqrt(d)*( - a - 5*b*x**2))/(5*sqrt(x)*d**4*x**2)
```

3.616 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 195

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{2a^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)} + \frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)} + \frac{2ab^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)} + \frac{2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}$$

output

```
2/7*a^3*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+6/11*a^2*b*(d*x)^(11/2
)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+2/5*a*b^2*(d*x)^(15/2)*((b*x^2+a)^2)^(
1/2)/d^5/(b*x^2+a)+2/19*b^3*(d*x)^(19/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.34

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{2x(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}\left(1045a^3+1995a^2bx^2+1463ab^2x^4+385b^3x^6\right)}{7315\left(a+bx^2\right)}$$

input
```
Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(1045*a^3 + 1995*a^2*b*x^2 + 1463*a*b
^2*x^4 + 385*b^3*x^6))/(7315*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^3(dx)^{5/2}\left(bx^2+a\right)^3dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{5/2}\left(bx^2+a\right)^3dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{b^3(dx)^{17/2}}{d^6}+\frac{3ab^2(dx)^{13/2}}{d^4}+\frac{3a^2b(dx)^{9/2}}{d^2}+a^3(dx)^{5/2}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2a^3(dx)^{7/2}}{7d}+\frac{6a^2b(dx)^{11/2}}{11d^3}+\frac{2ab^2(dx)^{15/2}}{5d^5}+\frac{2b^3(dx)^{19/2}}{19d^7}\right)}{a+bx^2}$$

input
```
Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*a^3*(d*x)^(7/2))/(7*d) + (6*a^2*b*(d*
x)^(11/2))/(11*d^3) + (2*a*b^2*(d*x)^(15/2))/(5*d^5) + (2*b^3*(d*x)^(19/2)
)/(19*d^7)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.31

method	result	size
gosper	$\frac{2x\left(385b^{3}x^{6}+1463b^{2}x^{4}a+1995a^{2}b\,x^{2}+1045a^{3}\right)\left(dx\right)^{\frac{5}{2}}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{7315\left(b\,x^{2}+a\right)^{3}}$	61
default	$\frac{2\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(dx\right)^{\frac{7}{2}}\left(385b^{3}x^{6}+1463b^{2}x^{4}a+1995a^{2}b\,x^{2}+1045a^{3}\right)}{7315d\left(b\,x^{2}+a\right)^{3}}$	63
risch	$\frac{2d^{3}\sqrt{\left(b\,x^{2}+a\right)^{2}}\,x^{4}\left(385b^{3}x^{6}+1463b^{2}x^{4}a+1995a^{2}b\,x^{2}+1045a^{3}\right)}{7315\left(b\,x^{2}+a\right)\sqrt{dx}}$	66
orering	$\frac{2x\left(385b^{3}x^{6}+1463b^{2}x^{4}a+1995a^{2}b\,x^{2}+1045a^{3}\right)\left(dx\right)^{\frac{5}{2}}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{7315\left(b\,x^{2}+a\right)^{3}}$	70

input
```
int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/7315*x*(385*b^3*x^6+1463*a*b^2*x^4+1995*a^2*b*x^2+1045*a^3)*(d*x)^(5/2)*
((b*x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.28

$$\int (dx)^{5/2} \left(a^2 + 2abx^2 + b^2x^4\right)^{3/2} dx = \frac{2}{7315} \left(385 b^3 d^2 x^9 + 1463 ab^2 d^2 x^7 + 1995 a^2 b d^2 x^5 + 1045 a^3 d^2 x^3\right) \sqrt{dx}$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
2/7315*(385*b^3*d^2*x^9 + 1463*a*b^2*d^2*x^7 + 1995*a^2*b*d^2*x^5 + 1045*a
^3*d^2*x^3)*sqrt(d*x)
```

Sympy [F]

$$\int (dx)^{5/2} \left(a^2 + 2abx^2 + b^2x^4\right)^{3/2} dx = \int (dx)^{\frac{5}{2}} \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d*x)**(5/2)*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.43

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{2}{285} \left(15 \, b^3 d^{\frac{5}{2}} x^3 + 19 \, a b^2 d^{\frac{5}{2}} x\right) x^{\frac{13}{2}} + \frac{4}{165} \left(11 \, a b^2 d^{\frac{5}{2}} x^3 + 15 \, a^2 b d^{\frac{5}{2}} x\right) x^{\frac{9}{2}} + \frac{2}{77} \left(7 \, a^2 b d^{\frac{5}{2}} x^3 + 11 \, a^3 d^{\frac{5}{2}} x\right) x^{\frac{5}{2}}$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
2/285*(15*b^3*d^(5/2)*x^3 + 19*a*b^2*d^(5/2)*x)*x^(13/2) + 4/165*(11*a*b^2
*d^(5/2)*x^3 + 15*a^2*b*d^(5/2)*x)*x^(9/2) + 2/77*(7*a^2*b*d^(5/2)*x^3 + 1
1*a^3*d^(5/2)*x)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.51

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{2}{19} \, \sqrt{dx} b^3 d^2 x^9 \mathrm{sgn}\left(bx^2+a\right) + \frac{2}{5} \, \sqrt{dx} a b^2 d^2 x^7 \mathrm{sgn}\left(bx^2+a\right) + \frac{6}{11} \, \sqrt{dx} a^2 b d^2 x^5 \mathrm{sgn}\left(bx^2+a\right) + \frac{2}{7} \, \sqrt{dx} a^3 d^2 x^3 \mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
2/19*sqrt(d*x)*b^3*d^2*x^9*sgn(b*x^2 + a) + 2/5*sqrt(d*x)*a*b^2*d^2*x^7*sg
n(b*x^2 + a) + 6/11*sqrt(d*x)*a^2*b*d^2*x^5*sgn(b*x^2 + a) + 2/7*sqrt(d*x)
*a^3*d^2*x^3*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \int (d\,x)^{5/2} \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2} dx$$

input
```
int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.23

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{2\sqrt{x}\,\sqrt{d}\,d^2x^3(385b^3x^6+1463a\,b^2x^4+1995a^2b\,x^2+1045a^3)}{7315}$$

input
```
int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*d**2*x**3*(1045*a**3 + 1995*a**2*b*x**2 + 1463*a*b**2*x
**4 + 385*b**3*x**6))/7315
```

3.617 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 195

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{2a^3(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)} + \frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\left(a+bx^2\right)} + \frac{6ab^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\left(a+bx^2\right)} + \frac{2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}$$

output
```
2/5*a^3*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2/3*a^2*b*(d*x)^(9/2)*
((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+6/13*a*b^2*(d*x)^(13/2)*((b*x^2+a)^2)^(1
/2)/d^5/(b*x^2+a)+2/17*b^3*(d*x)^(17/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.34

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{2x(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}\left(663a^3+1105a^2bx^2+765ab^2x^4+195b^3x^6\right)}{3315\left(a+bx^2\right)}$$

input
```
Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(663*a^3 + 1105*a^2*b*x^2 + 765*a*b^2
*x^4 + 195*b^3*x^6))/(3315*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int b^3 (dx)^{3/2} \left(bx^2+a\right)^3dx}{b^3 \left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int (dx)^{3/2} \left(bx^2+a\right)^3dx}{a+bx^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(\frac{b^3 (dx)^{15/2}}{d^6}+\frac{3ab^2 (dx)^{11/2}}{d^4}+\frac{3a^2b (dx)^{7/2}}{d^2}+a^3 (dx)^{3/2}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4} \left(\frac{2a^3 (dx)^{5/2}}{5d}+\frac{2a^2b (dx)^{9/2}}{3d^3}+\frac{6ab^2 (dx)^{13/2}}{13d^5}+\frac{2b^3 (dx)^{17/2}}{17d^7}\right)}{a+bx^2}$$

input
```
Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*a^3*(d*x)^(5/2))/(5*d) + (2*a^2*b*(d*
x)^(9/2))/(3*d^3) + (6*a*b^2*(d*x)^(13/2))/(13*d^5) + (2*b^3*(d*x)^(17/2))
/(17*d^7)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.31

method	result	size
gosper	$\frac{2x\left(195b^{3}x^{6}+765b^{2}x^{4}a+1105a^{2}b\,x^{2}+663a^{3}\right)\left(dx\right)^{\frac{3}{2}}\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{3315\left(b\,x^{2}+a\right)^{3}}$	61
default	$\frac{2\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(dx\right)^{\frac{5}{2}}\left(195b^{3}x^{6}+765b^{2}x^{4}a+1105a^{2}b\,x^{2}+663a^{3}\right)}{3315d\left(b\,x^{2}+a\right)^{3}}$	63
risch	$\frac{2d^{2}\sqrt{\left(b\,x^{2}+a\right)^{2}}\,x^{3}\left(195b^{3}x^{6}+765b^{2}x^{4}a+1105a^{2}b\,x^{2}+663a^{3}\right)}{3315\left(b\,x^{2}+a\right)\sqrt{dx}}$	66
orering	$\frac{2x\left(195b^{3}x^{6}+765b^{2}x^{4}a+1105a^{2}b\,x^{2}+663a^{3}\right)\left(dx\right)^{\frac{3}{2}}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{3315\left(b\,x^{2}+a\right)^{3}}$	70

input
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3315*x*(195*b^3*x^6+765*a*b^2*x^4+1105*a^2*b*x^2+663*a^3)*(d*x)^(3/2)*((
b*x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.24

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{2}{3315} \left(195 b^3 dx^8 + 765 ab^2 dx^6 + 1105 a^2 b dx^4 + 663 a^3 dx^2\right) \sqrt{dx}$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
2/3315*(195*b^3*d*x^8 + 765*a*b^2*d*x^6 + 1105*a^2*b*d*x^4 + 663*a^3*d*x^2
)*sqrt(d*x)
```

Sympy [F]

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \int (dx)^{\frac{3}{2}} \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d*x)**(3/2)*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.43

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{2}{221} \left(13\,b^3d^{\frac{3}{2}}x^3+17\,ab^2d^{\frac{3}{2}}x\right)x^{\frac{11}{2}} + \frac{4}{117} \left(9\,ab^2d^{\frac{3}{2}}x^3+13\,a^2bd^{\frac{3}{2}}x\right)x^{\frac{7}{2}} + \frac{2}{45} \left(5\,a^2bd^{\frac{3}{2}}x^3+9\,a^3d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
2/221*(13*b^3*d^(3/2)*x^3 + 17*a*b^2*d^(3/2)*x)*x^(11/2) + 4/117*(9*a*b^2*
d^(3/2)*x^3 + 13*a^2*b*d^(3/2)*x)*x^(7/2) + 2/45*(5*a^2*b*d^(3/2)*x^3 + 9*
a^3*d^(3/2)*x)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.46

$$\int (dx)^{3/2} \left(a^2+2abx^2 +b^2x^4\right)^{3/2} dx = \frac{2}{3315} \left(195\,\sqrt{dx}b^3x^8\text{sgn}\left(bx^2+a\right)+765\,\sqrt{dx}ab^2x^6\text{sgn}\left(bx^2+a\right)+1105\,\sqrt{dx}a^2bx^4\text{sgn}\left(bx^2+\right.\right.$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
2/3315*(195*sqrt(d*x)*b^3*x^8*sgn(b*x^2 + a) + 765*sqrt(d*x)*a*b^2*x^6*sgn
(b*x^2 + a) + 1105*sqrt(d*x)*a^2*b*x^4*sgn(b*x^2 + a) + 663*sqrt(d*x)*a^3*
x^2*sgn(b*x^2 + a))*d
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \int (d\,x)^{3/2} \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2} \,dx$$

input
```
int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.22

$$\int (dx)^{3/2} \left(a^2+2abx^2 +b^2x^4\right)^{3/2} \, dx = \frac{2\sqrt{x}\,\sqrt{d}\,d\,x^2 (195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3)}{3315}$$

input
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*d*x**2*(663*a**3 + 1105*a**2*b*x**2 + 765*a*b**2*x**4 +
 195*b**3*x**6))/3315
```

3.618 $\int \sqrt{dx}(a^2+2abx^2+b^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 195

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{2a^3(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{6ab^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\left(a+bx^2\right)}+\frac{2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^7\left(a+bx^2\right)}$$

output
```
2/3*a^3*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+6/7*a^2*b*(d*x)^(7/2)*
((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+6/11*a*b^2*(d*x)^(11/2)*((b*x^2+a)^2)^(1
/2)/d^5/(b*x^2+a)+2/15*b^3*(d*x)^(15/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.34

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{2x\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}\left(385a^3+495a^2bx^2+315ab^2x^4+77b^3x^6\right)}{1155\left(a+bx^2\right)}$$

input
```
Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(2*x*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(385*a^3 + 495*a^2*b*x^2 + 315*a*b^2*x^
4 + 77*b^3*x^6))/(1155*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.53, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^3\sqrt{dx}\left(bx^2+a\right)^3dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\left(bx^2+a\right)^3dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{b^3(dx)^{13/2}}{d^6}+\frac{3ab^2(dx)^{9/2}}{d^4}+\frac{3a^2b(dx)^{5/2}}{d^2}+a^3\sqrt{dx}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2a^3(dx)^{3/2}}{3d}+\frac{6a^2b(dx)^{7/2}}{7d^3}+\frac{6ab^2(dx)^{11/2}}{11d^5}+\frac{2b^3(dx)^{15/2}}{15d^7}\right)}{a+bx^2}$$

input
```
Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*a^3*(d*x)^(3/2))/(3*d) + (6*a^2*b*(d*
x)^(7/2))/(7*d^3) + (6*a*b^2*(d*x)^(11/2))/(11*d^5) + (2*b^3*(d*x)^(15/2))
/(15*d^7)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.31

method	result	size
gosper	$\frac{2x\left(77b^{3}x^{6}+315b^{2}x^{4}a+495a^{2}b\,x^{2}+385a^{3}\right)\sqrt{dx}\,\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{1155\left(b\,x^{2}+a\right)^{3}}$	61
default	$\frac{2\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(dx\right)^{\frac{3}{2}}\left(77b^{3}x^{6}+315b^{2}x^{4}a+495a^{2}b\,x^{2}+385a^{3}\right)}{1155d\left(b\,x^{2}+a\right)^{3}}$	63
risch	$\frac{2d\sqrt{\left(b\,x^{2}+a\right)^{2}}\,x^{2}\left(77b^{3}x^{6}+315b^{2}x^{4}a+495a^{2}b\,x^{2}+385a^{3}\right)}{1155\left(b\,x^{2}+a\right)\sqrt{dx}}$	64
orering	$\frac{2x\left(77b^{3}x^{6}+315b^{2}x^{4}a+495a^{2}b\,x^{2}+385a^{3}\right)\sqrt{dx}\,\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{1155\left(b\,x^{2}+a\right)^{3}}$	70

input
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*x*(77*b^3*x^6+315*a*b^2*x^4+495*a^2*b*x^2+385*a^3)*(d*x)^(1/2)*((b*
x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.21

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{2}{1155}\left(77\,b^3x^7+315\,ab^2x^5+495\,a^2bx^3+385\,a^3x\right)\sqrt{dx}$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(77*b^3*x^7 + 315*a*b^2*x^5 + 495*a^2*b*x^3 + 385*a^3*x)*sqrt(d*x)
```

Sympy [F]

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\int \sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x)**(1/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(sqrt(d*x)*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.43

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx = \frac{2}{165}\left(11\,b^3\sqrt{d}x^3+15\,ab^2\sqrt{d}x\right)x^{\frac{9}{2}} + \frac{4}{77}\left(7\,ab^2\sqrt{d}x^3+11\,a^2b\sqrt{d}x\right)x^{\frac{5}{2}} + \frac{2}{21}\left(3\,a^2b\sqrt{d}x^3+7\,a^3\sqrt{d}x\right)\sqrt{x}$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
2/165*(11*b^3*sqrt(d)*x^3 + 15*a*b^2*sqrt(d)*x)*x^(9/2) + 4/77*(7*a*b^2*sq
rt(d)*x^3 + 11*a^2*b*sqrt(d)*x)*x^(5/2) + 2/21*(3*a^2*b*sqrt(d)*x^3 + 7*a^
3*sqrt(d)*x)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.44

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx = \frac{2}{15}\sqrt{dx}b^3x^7\mathrm{sgn}\left(bx^2+a\right) + \frac{6}{11}\sqrt{dx}ab^2x^5\mathrm{sgn}\left(bx^2+a\right) + \frac{6}{7}\sqrt{dx}a^2bx^3\mathrm{sgn}\left(bx^2+a\right) + \frac{2}{3}\sqrt{dx}a^3x\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
2/15*sqrt(d*x)*b^3*x^7*sgn(b*x^2 + a) + 6/11*sqrt(d*x)*a*b^2*x^5*sgn(b*x^2
 + a) + 6/7*sqrt(d*x)*a^2*b*x^3*sgn(b*x^2 + a) + 2/3*sqrt(d*x)*a^3*x*sgn(b
*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\int \sqrt{d\,x}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

input
```
int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.20

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{2\sqrt{x}\,\sqrt{d}\,x(77b^3x^6+315a\,b^2x^4+495a^2b\,x^2+385a^3)}{1155}$$

input
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x*(385*a**3 + 495*a**2*b*x**2 + 315*a*b**2*x**4 + 77*b*
*3*x**6))/1155
```

3.619 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 193

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{\sqrt{dx}}\,dx = \frac{2a^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d(a+bx^2)} + \frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3(a+bx^2)} + \frac{2ab^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5(a+bx^2)} + \frac{2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7(a+bx^2)}$$

output
```
2*a^3*(d*x)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+6/5*a^2*b*(d*x)^(5/2)*((
b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+2/3*a*b^2*(d*x)^(9/2)*((b*x^2+a)^2)^(1/2)/
d^5/(b*x^2+a)+2/13*b^3*(d*x)^(13/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{\sqrt{dx}}\,dx = \frac{2x\sqrt{(a+bx^2)^2}(195a^3+117a^2bx^2+65ab^2x^4+15b^3x^6)}{195\sqrt{dx}\,(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```

output
```
(2*x*Sqrt[(a + b*x^2)^2]*(195*a^3 + 117*a^2*b*x^2 + 65*a*b^2*x^4 + 15*b^3*
x^6))/(195*Sqrt[d*x]*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3(bx^2+a)^3}{\sqrt{dx}}dx}{b^3(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{\sqrt{dx}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{b^3(dx)^{11/2}}{d^6}+\frac{3ab^2(dx)^{7/2}}{d^4}+\frac{3a^2b(dx)^{3/2}}{d^2}+\frac{a^3}{\sqrt{dx}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2a^3\sqrt{dx}}{d}+\frac{6a^2b(dx)^{5/2}}{5d^3}+\frac{2ab^2(dx)^{9/2}}{3d^5}+\frac{2b^3(dx)^{13/2}}{13d^7}\right)}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*a^3*Sqrt[d*x])/d + (6*a^2*b*(d*x)^(5/
2))/(5*d^3) + (2*a*b^2*(d*x)^(9/2))/(3*d^5) + (2*b^3*(d*x)^(13/2))/(13*d^7
)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.32

method	result	size
gosper	$\frac{2x(15b^3x^6+65b^2x^4a+117a^2b\,x^2+195a^3)\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}{195(b\,x^2+a)^3\sqrt{dx}}$	61
risch	$\frac{2\sqrt{(b\,x^2+a)^2}\,(15b^3x^6+65b^2x^4a+117a^2b\,x^2+195a^3)x}{195(b\,x^2+a)\sqrt{dx}}$	61
default	$\frac{2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}\sqrt{dx}\,(15b^3x^6+65b^2x^4a+117a^2b\,x^2+195a^3)}{195d(b\,x^2+a)^3}$	63
orering	$\frac{2(15b^3x^6+65b^2x^4a+117a^2b\,x^2+195a^3)x(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{2}}}{195(b\,x^2+a)^3\sqrt{dx}}$	70

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/195*x*(15*b^3*x^6+65*a*b^2*x^4+117*a^2*b*x^2+195*a^3)*((b*x^2+a)^2)^(3/2
)/(b*x^2+a)^3/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{\sqrt{dx}}\,dx=\frac{2\,(15\,b^3x^6+65\,ab^2x^4+117\,a^2bx^2+195\,a^3)\sqrt{dx}}{195\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/195*(15*b^3*x^6 + 65*a*b^2*x^4 + 117*a^2*b*x^2 + 195*a^3)*sqrt(d*x)/d
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{\sqrt{dx}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/sqrt(d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.45

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{\sqrt{dx}}\,dx = \frac{2\left(5\left(9\,b^3\sqrt{d}x^3+13\,ab^2\sqrt{d}x\right)x^{\frac{7}{2}}+26\left(5\,ab^2\sqrt{d}x^3+9\,a^2b\sqrt{d}x\right)x^{\frac{3}{2}}+\frac{117\,(a}{}\right.}{585\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2/585*(5*(9*b^3*sqrt(d)*x^3 + 13*a*b^2*sqrt(d)*x)*x^(7/2) + 26*(5*a*b^2*sq
rt(d)*x^3 + 9*a^2*b*sqrt(d)*x)*x^(3/2) + 117*(a^2*b*sqrt(d)*x^3 + 5*a^3*sq
rt(d)*x)/sqrt(x))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.46

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{\sqrt{dx}}\,dx = \frac{2\left(15\sqrt{dx}b^3x^6\mathrm{sgn}(bx^2+a)+65\sqrt{dx}ab^2x^4\mathrm{sgn}(bx^2+a)+117\sqrt{dx}a^2bx^2\mathrm{sg}\right.}{195\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
2/195*(15*sqrt(d*x)*b^3*x^6*sgn(b*x^2 + a) + 65*sqrt(d*x)*a*b^2*x^4*sgn(b*
x^2 + a) + 117*sqrt(d*x)*a^2*b*x^2*sgn(b*x^2 + a) + 195*sqrt(d*x)*a^3*sgn(
b*x^2 + a))/d
```

Mupad [B] (verification not implemented)

Time = 18.79 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.39

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{\sqrt{dx}}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\left(\frac{6\,a^2\,x^3}{5}+\frac{2\,b^2\,x^7}{13}+\frac{2\,a^3\,x}{b}+\frac{2\,a\,b\,x^5}{3}\right)}{x^2\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(1/2),x)
```

output
```
((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((6*a^2*x^3)/5 + (2*b^2*x^7)/13 + (2*a^
3*x)/b + (2*a*b*x^5)/3))/(x^2*(d*x)^(1/2) + (a*(d*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.21

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,(15b^3x^6+65a\,b^2x^4+117a^2b\,x^2+195a^3)}{195d}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*(195*a**3 + 117*a**2*b*x**2 + 65*a*b**2*x**4 + 15*b**3*
x**6))/(195*d)
```

3.620 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 191

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{3/2}}\,dx = -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)} + \frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\,(a+bx^2)} + \frac{6ab^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\,(a+bx^2)} + \frac{2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\,(a+bx^2)}$$

output
```
-2*a^3*((b*x^2+a)^2)^(1/2)/d/(d*x)^(1/2)/(b*x^2+a)+2*a^2*b*(d*x)^(3/2)*((b
*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+6/7*a*b^2*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d
^5/(b*x^2+a)+2/11*b^3*(d*x)^(11/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.35

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{3/2}}\,dx = -\frac{2x\left((a+bx^2)^2\right)^{3/2}(77a^3-77a^2bx^2-33ab^2x^4-7b^3x^6)}{77(dx)^{3/2}\,(a+bx^2)^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
```

output
```
(-2*x*((a + b*x^2)^2)^(3/2)*(77*a^3 - 77*a^2*b*x^2 - 33*a*b^2*x^4 - 7*b^3*
x^6))/(77*(d*x)^(3/2)*(a + b*x^2)^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3(bx^2+a)^3}{(dx)^{3/2}}dx}{b^3(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{(dx)^{3/2}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{b^3(dx)^{9/2}}{d^6}+\frac{3ab^2(dx)^{5/2}}{d^4}+\frac{3a^2b\sqrt{dx}}{d^2}+\frac{a^3}{(dx)^{3/2}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{2a^3}{d\sqrt{dx}}+\frac{2a^2b(dx)^{3/2}}{d^3}+\frac{6ab^2(dx)^{7/2}}{7d^5}+\frac{2b^3(dx)^{11/2}}{11d^7}\right)}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((-2*a^3)/(d*Sqrt[d*x]) + (2*a^2*b*(d*x)^
(3/2))/d^3 + (6*a*b^2*(d*x)^(7/2))/(7*d^5) + (2*b^3*(d*x)^(11/2))/(11*d^7)
))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{2x\left(-7b^{3}x^{6}-33b^{2}x^{4}a-77a^{2}b\,x^{2}+77a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{77\left(b\,x^{2}+a\right)^{3}\left(dx\right)^{\frac{3}{2}}}$	61
default	$-\frac{2\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(-7b^{3}x^{6}-33b^{2}x^{4}a-77a^{2}b\,x^{2}+77a^{3}\right)}{77d\left(b\,x^{2}+a\right)^{3}\sqrt{dx}}$	63
risch	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-7b^{3}x^{6}-33b^{2}x^{4}a-77a^{2}b\,x^{2}+77a^{3}\right)}{77d\left(b\,x^{2}+a\right)\sqrt{dx}}$	63
orering	$-\frac{2\left(-7b^{3}x^{6}-33b^{2}x^{4}a-77a^{2}b\,x^{2}+77a^{3}\right)x\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{77\left(b\,x^{2}+a\right)^{3}\left(dx\right)^{\frac{3}{2}}}$	70

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*x*(-7*b^3*x^6-33*a*b^2*x^4-77*a^2*b*x^2+77*a^3)*((b*x^2+a)^2)^(3/2)/
(b*x^2+a)^3/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{3/2}}\,dx=\frac{2\left(7\,b^3x^6+33\,ab^2x^4+77\,a^2bx^2-77\,a^3\right)\sqrt{dx}}{77\,d^2x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/77*(7*b^3*x^6 + 33*a*b^2*x^4 + 77*a^2*b*x^2 - 77*a^3)*sqrt(d*x)/(d^2*x)
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{3/2}}\,dx=\int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(3/2),x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{3/2}}\,dx=\frac{2\left(3\left(7\,b^3\sqrt{d}x^3+11\,ab^2\sqrt{d}x\right)x^{\frac{5}{2}}+22\left(3\,ab^2\sqrt{d}x^3+7\,a^2b\sqrt{d}x\right)\sqrt{x}+\frac{77\,(a}{}\right.}{231\,d^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
2/231*(3*(7*b^3*sqrt(d)*x^3 + 11*a*b^2*sqrt(d)*x)*x^(5/2) + 22*(3*a*b^2*sq
rt(d)*x^3 + 7*a^2*b*sqrt(d)*x)*sqrt(x) + 77*(a^2*b*sqrt(d)*x^3 - 3*a^3*sqr
t(d)*x)/x^(3/2))/d^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.53

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{3/2}}\,dx=-\frac{2\left(\frac{77\,a^3\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}}-\frac{7\,\sqrt{dx}b^3d^{65}x^5\mathrm{sgn}\left(bx^2+a\right)+33\,\sqrt{dx}ab^2d^{65}x^3\mathrm{sgn}\left(bx^2+a\right)+77\,\sqrt{dx}a^2bd^{65}x\mathrm{sgn}\left(bx^2+a\right)}{d^{66}}\right)}{77\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
-2/77*(77*a^3*sgn(b*x^2 + a)/sqrt(d*x) - (7*sqrt(d*x)*b^3*d^65*x^5*sgn(b*x
^2 + a) + 33*sqrt(d*x)*a*b^2*d^65*x^3*sgn(b*x^2 + a) + 77*sqrt(d*x)*a^2*b*
d^65*x*sgn(b*x^2 + a))/d^66)/d
```

Mupad [B] (verification not implemented)

Time = 18.87 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{3/2}}\,dx=\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{2\,a^2\,x^2}{d}-\frac{2\,a^3}{b\,d}+\frac{2\,b^2\,x^6}{11\,d}+\frac{6\,a\,b\,x^4}{7\,d}\right)}{x^2\,\sqrt{d\,x}+\frac{a\,\sqrt{d\,x}}{b}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(3/2),x)
```

output
```
((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((2*a^2*x^2)/d - (2*a^3)/(b*d) + (2*b^2
*x^6)/(11*d) + (6*a*b*x^4)/(7*d)))/(x^2*(d*x)^(1/2) + (a*(d*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{3/2}}\,dx=\frac{2\sqrt{d}\,(7b^3x^6+33a\,b^2x^4+77a^2b\,x^2-77a^3)}{77\sqrt{x}\,d^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x)
```

output
```
(2*sqrt(d)*( - 77*a**3 + 77*a**2*b*x**2 + 33*a*b**2*x**4 + 7*b**3*x**6))/(
77*sqrt(x)*d**2)
```

3.621 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 193

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{5/2}}\,dx = -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}(a+bx^2)} + \frac{6a^2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(a+bx^2)} + \frac{6ab^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5(a+bx^2)} + \frac{2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7(a+bx^2)}$$

output
```
-2/3*a^3*((b*x^2+a)^2)^(1/2)/d/(d*x)^(3/2)/(b*x^2+a)+6*a^2*b*(d*x)^(1/2)*(
(b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+6/5*a*b^2*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)
/d^5/(b*x^2+a)+2/9*b^3*(d*x)^(9/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.34

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{5/2}}\,dx = -\frac{2x\left((a+bx^2)^2\right)^{3/2}(15a^3-135a^2bx^2-27ab^2x^4-5b^3x^6)}{45(dx)^{5/2}(a+bx^2)^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```

output
```
(-2*x*((a + b*x^2)^2)^(3/2)*(15*a^3 - 135*a^2*b*x^2 - 27*a*b^2*x^4 - 5*b^3
*x^6))/(45*(d*x)^(5/2)*(a + b*x^2)^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3(bx^2+a)^3}{(dx)^{5/2}}dx}{b^3(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{(dx)^{5/2}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{b^3(dx)^{7/2}}{d^6}+\frac{3ab^2(dx)^{3/2}}{d^4}+\frac{3a^2b}{d^2\sqrt{dx}}+\frac{a^3}{(dx)^{5/2}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{2a^3}{3d(dx)^{3/2}}+\frac{6a^2b\sqrt{dx}}{d^3}+\frac{6ab^2(dx)^{5/2}}{5d^5}+\frac{2b^3(dx)^{9/2}}{9d^7}\right)}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((-2*a^3)/(3*d*(d*x)^(3/2)) + (6*a^2*b*Sq
rt[d*x])/d^3 + (6*a*b^2*(d*x)^(5/2))/(5*d^5) + (2*b^3*(d*x)^(9/2))/(9*d^7)
))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{2x\left(-5b^{3}x^{6}-27b^{2}x^{4}a-135a^{2}b\,x^{2}+15a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{45\left(b\,x^{2}+a\right)^{3}\left(dx\right)^{\frac{5}{2}}}$	61
default	$-\frac{2\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(-5b^{3}x^{6}-27b^{2}x^{4}a-135a^{2}b\,x^{2}+15a^{3}\right)}{45d\left(b\,x^{2}+a\right)^{3}\left(dx\right)^{\frac{3}{2}}}$	63
risch	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-5b^{3}x^{6}-27b^{2}x^{4}a-135a^{2}b\,x^{2}+15a^{3}\right)}{45d^{2}\left(b\,x^{2}+a\right)x\sqrt{dx}}$	66
orering	$-\frac{2\left(-5b^{3}x^{6}-27b^{2}x^{4}a-135a^{2}b\,x^{2}+15a^{3}\right)x\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{45\left(b\,x^{2}+a\right)^{3}\left(dx\right)^{\frac{5}{2}}}$	70

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*x*(-5*b^3*x^6-27*a*b^2*x^4-135*a^2*b*x^2+15*a^3)*((b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{3/2}}{(dx)^{5/2}}\,dx=\frac{2\left(5\,b^{3}x^{6}+27\,ab^{2}x^{4}+135\,a^{2}bx^{2}-15\,a^{3}\right)\sqrt{dx}}{45\,d^{3}x^{2}}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/45*(5*b^3*x^6 + 27*a*b^2*x^4 + 135*a^2*b*x^2 - 15*a^3)*sqrt(d*x)/(d^3*x^
2)
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{5/2}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(5/2),x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.45

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{5/2}}\,dx = \frac{2\left(\left(5\,b^3\sqrt{d}x^3+9\,ab^2\sqrt{d}x\right)x^{\frac{3}{2}}+\frac{18\left(ab^2\sqrt{d}x^3+5\,a^2b\sqrt{d}x\right)}{\sqrt{x}}+\frac{15\left(3\,a^2b\sqrt{d}x^3-a^3\sqrt{d}x\right)}{x^{\frac{5}{2}}}\right)}{45\,d^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
2/45*((5*b^3*sqrt(d)*x^3 + 9*a*b^2*sqrt(d)*x)*x^(3/2) + 18*(a*b^2*sqrt(d)*
x^3 + 5*a^2*b*sqrt(d)*x)/sqrt(x) + 15*(3*a^2*b*sqrt(d)*x^3 - a^3*sqrt(d)*x
)/x^(5/2))/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.55

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{5/2}}\,dx = -\frac{2\left(\frac{15\,a^3\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{dx}dx}-\frac{5\,\sqrt{dx}b^3d^{52}x^4\mathrm{sgn}\left(bx^2+a\right)+27\,\sqrt{dx}ab^2d^{52}x^2\mathrm{sgn}\left(bx^2+a\right)+135\,\sqrt{dx}a^2bd^{52}\mathrm{sgn}\left(bx^2+a\right)}{d^{54}}\right)}{45\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
-2/45*(15*a^3*sgn(b*x^2 + a)/(sqrt(d*x)*d*x) - (5*sqrt(d*x)*b^3*d^52*x^4*s
gn(b*x^2 + a) + 27*sqrt(d*x)*a*b^2*d^52*x^2*sgn(b*x^2 + a) + 135*sqrt(d*x)
*a^2*b*d^52*sgn(b*x^2 + a))/d^54)/d
```

Mupad [B] (verification not implemented)

Time = 18.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.46

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{5/2}}\,dx=\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{6\,a^2\,x^2}{d^2}-\frac{2\,a^3}{3\,b\,d^2}+\frac{2\,b^2\,x^6}{9\,d^2}+\frac{6\,a\,b\,x^4}{5\,d^2}\right)}{x^3\,\sqrt{d\,x}+\frac{a\,x\,\sqrt{d\,x}}{b}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(5/2),x)
```

output
```
((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((6*a^2*x^2)/d^2 - (2*a^3)/(3*b*d^2) +
(2*b^2*x^6)/(9*d^2) + (6*a*b*x^4)/(5*d^2)))/(x^3*(d*x)^(1/2) + (a*x*(d*x)^
(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.24

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{5/2}}\,dx=\frac{2\sqrt{d}\,(5b^3x^6+27a\,b^2x^4+135a^2b\,x^2-15a^3)}{45\sqrt{x}\,d^3x}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x)
```

output
```
(2*sqrt(d)*( - 15*a**3 + 135*a**2*b*x**2 + 27*a*b**2*x**4 + 5*b**3*x**6))/
(45*sqrt(x)*d**3*x)
```

3.622 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 191

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{7/2}}\,dx = -\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}(a+bx^2)} - \frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,(a+bx^2)} + \frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5(a+bx^2)} + \frac{2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7(a+bx^2)}$$

output
```
-2/5*a^3*((b*x^2+a)^2)^(1/2)/d/(d*x)^(5/2)/(b*x^2+a)-6*a^2*b*((b*x^2+a)^2)
^(1/2)/d^3/(d*x)^(1/2)/(b*x^2+a)+2*a*b^2*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d
^5/(b*x^2+a)+2/7*b^3*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.35

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{7/2}}\,dx = -\frac{2x\left((a+bx^2)^2\right)^{3/2}(7a^3+105a^2bx^2-35ab^2x^4-5b^3x^6)}{35(dx)^{7/2}(a+bx^2)^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

output
```
(-2*x*((a + b*x^2)^2)^(3/2)*(7*a^3 + 105*a^2*b*x^2 - 35*a*b^2*x^4 - 5*b^3*
x^6))/(35*(d*x)^(7/2)*(a + b*x^2)^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.52, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{7/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^3(bx^2+a)^3}{(dx)^{7/2}}dx}{b^3(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^3}{(dx)^{7/2}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(\frac{a^3}{(dx)^{7/2}}+\frac{3ba^2}{d^2(dx)^{3/2}}+\frac{3b^2\sqrt{dx}a}{d^4}+\frac{b^3(dx)^{5/2}}{d^6}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{2a^3}{5d(dx)^{5/2}}-\frac{6a^2b}{d^3\sqrt{dx}}+\frac{2ab^2(dx)^{3/2}}{d^5}+\frac{2b^3(dx)^{7/2}}{7d^7}\right)}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((-2*a^3)/(5*d*(d*x)^(5/2)) - (6*a^2*b)/(
d^3*Sqrt[d*x]) + (2*a*b^2*(d*x)^(3/2))/d^5 + (2*b^3*(d*x)^(7/2))/(7*d^7)))
/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.32

method	result	size
gosper	$-\frac{2x\left(-5b^{3}x^{6}-35b^{2}x^{4}a+105a^{2}b\,x^{2}+7a^{3}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{35\left(b\,x^{2}+a\right)^{3}\left(dx\right)^{\frac{7}{2}}}$	61
default	$-\frac{2\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(-5b^{3}x^{6}-35b^{2}x^{4}a+105a^{2}b\,x^{2}+7a^{3}\right)}{35d\left(b\,x^{2}+a\right)^{3}\left(dx\right)^{\frac{5}{2}}}$	63
risch	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-5b^{3}x^{6}-35b^{2}x^{4}a+105a^{2}b\,x^{2}+7a^{3}\right)}{35d^{3}\left(b\,x^{2}+a\right)x^{2}\sqrt{dx}}$	66
orering	$-\frac{2\left(-5b^{3}x^{6}-35b^{2}x^{4}a+105a^{2}b\,x^{2}+7a^{3}\right)x\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{3}{2}}}{35\left(b\,x^{2}+a\right)^{3}\left(dx\right)^{\frac{7}{2}}}$	70

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*x*(-5*b^3*x^6-35*a*b^2*x^4+105*a^2*b*x^2+7*a^3)*((b*x^2+a)^2)^(3/2)/
(b*x^2+a)^3/(d*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.24

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{7/2}}\,dx=\frac{2\left(5\,b^3x^6+35\,ab^2x^4-105\,a^2bx^2-7\,a^3\right)\sqrt{dx}}{35\,d^4x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^6 + 35*a*b^2*x^4 - 105*a^2*b*x^2 - 7*a^3)*sqrt(d*x)/(d^4*x^3
)
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{7/2}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(7/2),x)
```

output
```
Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.45

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{7/2}}\,dx = \frac{2\left(5\left(3\,b^3\sqrt{d}x^3+7\,ab^2\sqrt{d}x\right)\sqrt{x}+\frac{70\left(ab^2\sqrt{d}x^3-3\,a^2b\sqrt{d}x\right)}{x^{\frac{3}{2}}}-\frac{21\left(5\,a^2b\sqrt{d}x^3+a^3\sqrt{d}x\right)}{x^{\frac{7}{2}}}\right)}{105\,d^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
2/105*(5*(3*b^3*sqrt(d)*x^3 + 7*a*b^2*sqrt(d)*x)*sqrt(x) + 70*(a*b^2*sqrt(
d)*x^3 - 3*a^2*b*sqrt(d)*x)/x^(3/2) - 21*(5*a^2*b*sqrt(d)*x^3 + a^3*sqrt(d
)*x)/x^(7/2))/d^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.56

$$\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{(dx)^{7/2}}\,dx = -\frac{2\left(\frac{7\left(15\,a^2bd^3x^2\operatorname{sgn}(bx^2+a)+a^3d^3\operatorname{sgn}(bx^2+a)\right)}{\sqrt{dx}d^2x^2}-\frac{5\left(\sqrt{dx}b^3d^{21}x^3\operatorname{sgn}(bx^2+a)+7\sqrt{dx}ab^2d^{21}x\operatorname{sgn}(bx^2+a)\right)}{d^{21}}\right)}{35\,d^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x, algorithm="giac")
```

output
```
-2/35*(7*(15*a^2*b*d^3*x^2*sgn(b*x^2 + a) + a^3*d^3*sgn(b*x^2 + a))/(sqrt(
d*x)*d^2*x^2) - 5*(sqrt(d*x)*b^3*d^21*x^3*sgn(b*x^2 + a) + 7*sqrt(d*x)*a*b
^2*d^21*x*sgn(b*x^2 + a))/d^21)/d^4
```

Mupad [B] (verification not implemented)

Time = 18.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.48

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{(dx)^{7/2}}\,dx = -\frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{2\,a^3}{5\,b\,d^3}+\frac{6\,a^2\,x^2}{d^3}-\frac{2\,b^2\,x^6}{7\,d^3}-\frac{2\,a\,b\,x^4}{d^3}\right)}{x^4\,\sqrt{d\,x}+\frac{a\,x^2\,\sqrt{d\,x}}{b}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)/(d*x)^(7/2),x)
```

output
```
-((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((2*a^3)/(5*b*d^3) + (6*a^2*x^2)/d^3 -
 (2*b^2*x^6)/(7*d^3) - (2*a*b*x^4)/d^3))/(x^4*(d*x)^(1/2) + (a*x^2*(d*x)^(
1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.24

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^{3/2}}{(dx)^{7/2}}\,dx = \frac{2\sqrt{d}\,(5b^3x^6+35a\,b^2x^4-105a^2b\,x^2-7a^3)}{35\sqrt{x}\,d^4x^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x)
```

output
```
(2*sqrt(d)*( - 7*a**3 - 105*a**2*b*x**2 + 35*a*b**2*x**4 + 5*b**3*x**6))/(
35*sqrt(x)*d**4*x**2)
```

3.623 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 297

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx = \frac{2a^5(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)} + \frac{10a^4b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)} + \frac{4a^3b^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)} + \frac{20a^2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)} + \frac{10ab^4(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^9\left(a+bx^2\right)} + \frac{2b^5(dx)^{27/2}\sqrt{a^2+2abx^2+b^2x^4}}{27d^{11}\left(a+bx^2\right)}$$

output
```
2/7*a^5*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+10/11*a^4*b*(d*x)^(11/
2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+4/3*a^3*b^2*(d*x)^(15/2)*((b*x^2+a)^2
)^(1/2)/d^5/(b*x^2+a)+20/19*a^2*b^3*(d*x)^(19/2)*((b*x^2+a)^2)^(1/2)/d^7/(
b*x^2+a)+10/23*a*b^4*(d*x)^(23/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/27*b
^5*(d*x)^(27/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.30

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{2x(dx)^{5/2}\sqrt{(a+bx^2)^2}(129789a^5+412965a^4bx^2+605682a^3b^2x^4+478170a^2b^3x^6+19750\ldots}{908523\,(a+bx^2)}$$

input
```
Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(129789*a^5 + 412965*a^4*b*x^2 + 6056
82*a^3*b^2*x^4 + 478170*a^2*b^3*x^6 + 197505*a*b^4*x^8 + 33649*b^5*x^10))/
(908523*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5(dx)^{5/2}\left(bx^2+a\right)^5dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^{5/2}\left(bx^2+a\right)^5dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{b^5(dx)^{25/2}}{d^{10}}+\frac{5ab^4(dx)^{21/2}}{d^8}+\frac{10a^2b^3(dx)^{17/2}}{d^6}+\frac{10a^3b^2(dx)^{13/2}}{d^4}+\frac{5a^4b(dx)^{9/2}}{d^2}+a^5(dx)^{5/2}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2a^5(dx)^{7/2}}{7d}+\frac{10a^4b(dx)^{11/2}}{11d^3}+\frac{4a^3b^2(dx)^{15/2}}{3d^5}+\frac{20a^2b^3(dx)^{19/2}}{19d^7}+\frac{10ab^4(dx)^{23/2}}{23d^9}+\frac{2b^5(dx)^{27/2}}{27d^{11}}\right)}{a+bx^2}$$

input
```
Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*a^5*(d*x)^(7/2))/(7*d) + (10*a^4*b*(d
*x)^(11/2))/(11*d^3) + (4*a^3*b^2*(d*x)^(15/2))/(3*d^5) + (20*a^2*b^3*(d*x
)^(19/2))/(19*d^7) + (10*a*b^4*(d*x)^(23/2))/(23*d^9) + (2*b^5*(d*x)^(27/2
))/(27*d^11)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

method	result	size
gosper	$\frac{2x(33649x^{10}b^5+197505a\,x^8b^4+478170a^2x^6b^3+605682a^3x^4b^2+412965x^2a^4b+129789a^5)(dx)^{\frac{5}{2}}\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{908523(b\,x^2+a)^5}$	83
default	$\frac{2\left((b\,x^2+a)^2\right)^{\frac{5}{2}}(dx)^{\frac{7}{2}}(33649x^{10}b^5+197505a\,x^8b^4+478170a^2x^6b^3+605682a^3x^4b^2+412965x^2a^4b+129789a^5)}{908523d(b\,x^2+a)^5}$	85
risch	$\frac{2d^3\sqrt{(b\,x^2+a)^2}\,x^4(33649x^{10}b^5+197505a\,x^8b^4+478170a^2x^6b^3+605682a^3x^4b^2+412965x^2a^4b+129789a^5)}{908523(b\,x^2+a)\sqrt{dx}}$	88
orering	$\frac{2x(33649x^{10}b^5+197505a\,x^8b^4+478170a^2x^6b^3+605682a^3x^4b^2+412965x^2a^4b+129789a^5)(dx)^{\frac{5}{2}}(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{2}}}{908523(b\,x^2+a)^5}$	92

input
```
int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/908523*x*(33649*b^5*x^10+197505*a*b^4*x^8+478170*a^2*b^3*x^6+605682*a^3*
b^2*x^4+412965*a^4*b*x^2+129789*a^5)*(d*x)^(5/2)*((b*x^2+a)^2)^(5/2)/(b*x^
2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.28

$$\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{2}{908523}\left(33649\,b^5d^2x^{13}+197505\,ab^4d^2x^{11}+478170\,a^2b^3d^2x^9+605682\,a^3b^2d^2x^7+412965\,a\right.$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
2/908523*(33649*b^5*d^2*x^13 + 197505*a*b^4*d^2*x^11 + 478170*a^2*b^3*d^2*
x^9 + 605682*a^3*b^2*d^2*x^7 + 412965*a^4*b*d^2*x^5 + 129789*a^5*d^2*x^3)*
sqrt(d*x)
```

Sympy [F]

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \int (dx)^{\frac{5}{2}} \left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}} dx$$

input
```
integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral((d*x)**(5/2)*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.49

$$\begin{aligned}\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx &= \frac{2}{621} \left(23\, b^5 d^{\frac{5}{2}} x^3 + 27\, ab^4 d^{\frac{5}{2}} x\right) x^{\frac{21}{2}} \\ &+ \frac{8}{437} \left(19\, ab^4 d^{\frac{5}{2}} x^3 + 23\, a^2 b^3 d^{\frac{5}{2}} x\right) x^{\frac{17}{2}} + \frac{4}{95} \left(15\, a^2 b^3 d^{\frac{5}{2}} x^3 + 19\, a^3 b^2 d^{\frac{5}{2}} x\right) x^{\frac{13}{2}} \\ &+ \frac{8}{165} \left(11\, a^3 b^2 d^{\frac{5}{2}} x^3 + 15\, a^4 b d^{\frac{5}{2}} x\right) x^{\frac{9}{2}} + \frac{2}{77} \left(7\, a^4 b d^{\frac{5}{2}} x^3 + 11\, a^5 d^{\frac{5}{2}} x\right) x^{\frac{5}{2}}\end{aligned}$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
2/621*(23*b^5*d^(5/2)*x^3 + 27*a*b^4*d^(5/2)*x)*x^(21/2) + 8/437*(19*a*b^4
*d^(5/2)*x^3 + 23*a^2*b^3*d^(5/2)*x)*x^(17/2) + 4/95*(15*a^2*b^3*d^(5/2)*x
^3 + 19*a^3*b^2*d^(5/2)*x)*x^(13/2) + 8/165*(11*a^3*b^2*d^(5/2)*x^3 + 15*a
^4*b*d^(5/2)*x)*x^(9/2) + 2/77*(7*a^4*b*d^(5/2)*x^3 + 11*a^5*d^(5/2)*x)*x^
(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.52

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{2}{27}\sqrt{dx}b^5d^2x^{13}\mathrm{sgn}\left(bx^2+a\right) + \frac{10}{23}\sqrt{dx}ab^4d^2x^{11}\mathrm{sgn}\left(bx^2+a\right) + \frac{20}{19}\sqrt{dx}a^2b^3d^2x^9\mathrm{sgn}\left(bx^2+a\right) + \frac{4}{3}\sqrt{dx}a^3b^2d^2x^7\mathrm{sgn}\left(bx^2+a\right) + \frac{10}{11}\sqrt{dx}a^4bd^2x^5\mathrm{sgn}\left(bx^2+a\right) + \frac{2}{7}\sqrt{dx}a^5d^2x^3\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
2/27*sqrt(d*x)*b^5*d^2*x^13*sgn(b*x^2 + a) + 10/23*sqrt(d*x)*a*b^4*d^2*x^1
1*sgn(b*x^2 + a) + 20/19*sqrt(d*x)*a^2*b^3*d^2*x^9*sgn(b*x^2 + a) + 4/3*sq
rt(d*x)*a^3*b^2*d^2*x^7*sgn(b*x^2 + a) + 10/11*sqrt(d*x)*a^4*b*d^2*x^5*sgn
(b*x^2 + a) + 2/7*sqrt(d*x)*a^5*d^2*x^3*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \int {\left(d\,x\right)}^{5/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2} \,d x$$

input
```
int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.22

$$\int (dx)^{5/2} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \frac{2\sqrt{x}\,\sqrt{d}\,d^2x^3(33649b^5x^{10} + 197505a\,b^4x^8 + 478170a^2b^3x^6 + 605682a^3b^2x^4 + 412965a^4b\,x^2}{908523}$$

input
```
int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*d**2*x**3*(129789*a**5 + 412965*a**4*b*x**2 + 605682*a*
*3*b**2*x**4 + 478170*a**2*b**3*x**6 + 197505*a*b**4*x**8 + 33649*b**5*x**
10))/908523
```

3.624 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal result

Integrand size = 30, antiderivative size = 297

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{2a^5(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d(a+bx^2)} + \frac{10a^4b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3(a+bx^2)} + \frac{20a^3b^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5(a+bx^2)} + \frac{20a^2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7(a+bx^2)} + \frac{10ab^4(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^9(a+bx^2)} + \frac{2b^5(dx)^{25/2}\sqrt{a^2+2abx^2+b^2x^4}}{25d^{11}(a+bx^2)}$$

output
```
2/5*a^5*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+10/9*a^4*b*(d*x)^(9/2)
*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+20/13*a^3*b^2*(d*x)^(13/2)*((b*x^2+a)^2
)^(1/2)/d^5/(b*x^2+a)+20/17*a^2*b^3*(d*x)^(17/2)*((b*x^2+a)^2)^(1/2)/d^7/(
b*x^2+a)+10/21*a*b^4*(d*x)^(21/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/25*b
^5*(d*x)^(25/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.30

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \frac{2x(dx)^{3/2}\sqrt{(a+bx^2)^2}(69615a^5 + 193375a^4bx^2 + 267750a^3b^2x^4 + 204750a^2b^3x^6 + 82875ab}{348075\,(a+bx^2)}$$

input
```
Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(69615*a^5 + 193375*a^4*b*x^2 + 26775
0*a^3*b^2*x^4 + 204750*a^2*b^3*x^6 + 82875*a*b^4*x^8 + 13923*b^5*x^10))/(3
48075*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int b^5 (dx)^{3/2} \left(bx^2 + a\right)^5 dx}{b^5\left(a + bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2 + 2abx^2 + b^2x^4} \int (dx)^{3/2} \left(bx^2 + a\right)^5 dx}{a + bx^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{b^5(dx)^{23/2}}{d^{10}}+\frac{5ab^4(dx)^{19/2}}{d^8}+\frac{10a^2b^3(dx)^{15/2}}{d^6}+\frac{10a^3b^2(dx)^{11/2}}{d^4}+\frac{5a^4b(dx)^{7/2}}{d^2}+a^5(dx)^{3/2}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2a^5(dx)^{5/2}}{5d}+\frac{10a^4b(dx)^{9/2}}{9d^3}+\frac{20a^3b^2(dx)^{13/2}}{13d^5}+\frac{20a^2b^3(dx)^{17/2}}{17d^7}+\frac{10ab^4(dx)^{21/2}}{21d^9}+\frac{2b^5(dx)^{25/2}}{25d^{11}}\right)}{a+bx^2}$$

input
```
Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*a^5*(d*x)^(5/2))/(5*d) + (10*a^4*b*(d
*x)^(9/2))/(9*d^3) + (20*a^3*b^2*(d*x)^(13/2))/(13*d^5) + (20*a^2*b^3*(d*x
)^(17/2))/(17*d^7) + (10*a*b^4*(d*x)^(21/2))/(21*d^9) + (2*b^5*(d*x)^(25/2
))/(25*d^11)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

method	result	size
gosper	$\frac{2x(13923x^{10}b^5+82875a\,x^8b^4+204750a^2x^6b^3+267750a^3x^4b^2+193375x^2a^4b+69615a^5)(dx)^{\frac{3}{2}}\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{348075(b\,x^2+a)^5}$	83
default	$\frac{2\left((b\,x^2+a)^2\right)^{\frac{5}{2}}(dx)^{\frac{5}{2}}(13923x^{10}b^5+82875a\,x^8b^4+204750a^2x^6b^3+267750a^3x^4b^2+193375x^2a^4b+69615a^5)}{348075d(b\,x^2+a)^5}$	85
risch	$\frac{2d^2\sqrt{(b\,x^2+a)^2}\,x^3(13923x^{10}b^5+82875a\,x^8b^4+204750a^2x^6b^3+267750a^3x^4b^2+193375x^2a^4b+69615a^5)}{348075(b\,x^2+a)\sqrt{dx}}$	88
orering	$\frac{2x(13923x^{10}b^5+82875a\,x^8b^4+204750a^2x^6b^3+267750a^3x^4b^2+193375x^2a^4b+69615a^5)(dx)^{\frac{3}{2}}(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{2}}}{348075(b\,x^2+a)^5}$	92

input
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/348075*x*(13923*b^5*x^10+82875*a*b^4*x^8+204750*a^2*b^3*x^6+267750*a^3*b
^2*x^4+193375*a^4*b*x^2+69615*a^5)*(d*x)^(3/2)*((b*x^2+a)^2)^(5/2)/(b*x^2+
a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.24

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{2}{348075}\left(13923\,b^5dx^{12}+82875\,ab^4dx^{10}+204750\,a^2b^3dx^8+267750\,a^3b^2dx^6+193375\,a^4bdx\right.$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
2/348075*(13923*b^5*d*x^12 + 82875*a*b^4*d*x^10 + 204750*a^2*b^3*d*x^8 + 2
67750*a^3*b^2*d*x^6 + 193375*a^4*b*d*x^4 + 69615*a^5*d*x^2)*sqrt(d*x)
```

Sympy [F]

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \int (dx)^{\frac{3}{2}} \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}} dx$$

input
```
integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral((d*x)**(3/2)*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.49

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \frac{2}{525} \left(21\, b^5 d^{\frac{3}{2}} x^3 + 25\, ab^4 d^{\frac{3}{2}} x\right) x^{\frac{19}{2}} + \frac{8}{357} \left(17\, ab^4 d^{\frac{3}{2}} x^3 + 21\, a^2 b^3 d^{\frac{3}{2}} x\right) x^{\frac{15}{2}} + \frac{12}{221} \left(13\, a^2 b^3 d^{\frac{3}{2}} x^3 + 17\, a^3 b^2 d^{\frac{3}{2}} x\right) x^{\frac{11}{2}} + \frac{8}{117} \left(9\, a^3 b^2 d^{\frac{3}{2}} x^3 + 13\, a^4 b d^{\frac{3}{2}} x\right) x^{\frac{7}{2}} + \frac{2}{45} \left(5\, a^4 b d^{\frac{3}{2}} x^3 + 9\, a^5 d^{\frac{3}{2}} x\right) x^{\frac{3}{2}}$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
2/525*(21*b^5*d^(3/2)*x^3 + 25*a*b^4*d^(3/2)*x)*x^(19/2) + 8/357*(17*a*b^4
*d^(3/2)*x^3 + 21*a^2*b^3*d^(3/2)*x)*x^(15/2) + 12/221*(13*a^2*b^3*d^(3/2)
*x^3 + 17*a^3*b^2*d^(3/2)*x)*x^(11/2) + 8/117*(9*a^3*b^2*d^(3/2)*x^3 + 13*
a^4*b*d^(3/2)*x)*x^(7/2) + 2/45*(5*a^4*b*d^(3/2)*x^3 + 9*a^5*d^(3/2)*x)*x^
(3/2)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.46

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \frac{2}{348075} \left(13923 \sqrt{dx} b^5 x^{12} \text{sgn}\left(bx^2 + a\right) + 82875 \sqrt{dx} a b^4 x^{10} \text{sgn}\left(bx^2 + a\right) + 204750 \sqrt{dx} a^2 b^3$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
2/348075*(13923*sqrt(d*x)*b^5*x^12*sgn(b*x^2 + a) + 82875*sqrt(d*x)*a*b^4*
x^10*sgn(b*x^2 + a) + 204750*sqrt(d*x)*a^2*b^3*x^8*sgn(b*x^2 + a) + 267750
*sqrt(d*x)*a^3*b^2*x^6*sgn(b*x^2 + a) + 193375*sqrt(d*x)*a^4*b*x^4*sgn(b*x
^2 + a) + 69615*sqrt(d*x)*a^5*x^2*sgn(b*x^2 + a))*d
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \int (d\,x)^{3/2} \left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{5/2} dx$$

input
```
int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.22

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \frac{2\sqrt{x}\,\sqrt{d}\,d\,x^2(13923b^5x^{10} + 82875a\,b^4x^8 + 204750a^2b^3x^6 + 267750a^3b^2x^4 + 193375a^4b\,x^2 +}{348075}$$

input
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*d*x**2*(69615*a**5 + 193375*a**4*b*x**2 + 267750*a**3*b
**2*x**4 + 204750*a**2*b**3*x**6 + 82875*a*b**4*x**8 + 13923*b**5*x**10))/
348075
```

3.625 $\int \sqrt{dx}(a^2+2abx^2+b^2x^4)^{5/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 297

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{2a^5(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\,(a+bx^2)}+\frac{10a^4b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\,(a+bx^2)}+\frac{20a^3b^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\,(a+bx^2)}+\frac{4a^2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^7\,(a+bx^2)}+\frac{10ab^4(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^9\,(a+bx^2)}+\frac{2b^5(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^{11}\,(a+bx^2)}$$

output
```
2/3*a^5*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+10/7*a^4*b*(d*x)^(7/2)
*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+20/11*a^3*b^2*(d*x)^(11/2)*((b*x^2+a)^2
)^(1/2)/d^5/(b*x^2+a)+4/3*a^2*b^3*(d*x)^(15/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*
x^2+a)+10/19*a*b^4*(d*x)^(19/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/23*b^5
*(d*x)^(23/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.30

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{2x\sqrt{dx}\sqrt{(a+bx^2)^2}(33649a^5+72105a^4bx^2+91770a^3b^2x^4+67298a^2b^3x^6+26565ab^4x^8+\cdots}{100947\,(a+bx^2)}$$

input
```
Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(2*x*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(33649*a^5 + 72105*a^4*b*x^2 + 91770*a^
3*b^2*x^4 + 67298*a^2*b^3*x^6 + 26565*a*b^4*x^8 + 4389*b^5*x^10))/(100947*
(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5\sqrt{dx}\left(bx^2+a\right)^5dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \sqrt{dx}\left(bx^2+a\right)^5dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{b^5(dx)^{21/2}}{d^{10}}+\frac{5ab^4(dx)^{17/2}}{d^8}+\frac{10a^2b^3(dx)^{13/2}}{d^6}+\frac{10a^3b^2(dx)^{9/2}}{d^4}+\frac{5a^4b(dx)^{5/2}}{d^2}+a^5\sqrt{dx}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2a^5(dx)^{3/2}}{3d}+\frac{10a^4b(dx)^{7/2}}{7d^3}+\frac{20a^3b^2(dx)^{11/2}}{11d^5}+\frac{4a^2b^3(dx)^{15/2}}{3d^7}+\frac{10ab^4(dx)^{19/2}}{19d^9}+\frac{2b^5(dx)^{23/2}}{23d^{11}}\right)}{a+bx^2}$$

input
```
Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*a^5*(d*x)^(3/2))/(3*d) + (10*a^4*b*(d
*x)^(7/2))/(7*d^3) + (20*a^3*b^2*(d*x)^(11/2))/(11*d^5) + (4*a^2*b^3*(d*x)
^(15/2))/(3*d^7) + (10*a*b^4*(d*x)^(19/2))/(19*d^9) + (2*b^5*(d*x)^(23/2))
/(23*d^11)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

method	result	size
gosper	$\frac{2x\left(4389x^{10}b^{5}+26565a\,x^{8}b^{4}+67298a^{2}x^{6}b^{3}+91770a^{3}x^{4}b^{2}+72105x^{2}a^{4}b+33649a^{5}\right)\sqrt{dx}\,\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{100947\left(b\,x^{2}+a\right)^{5}}$	83
default	$\frac{2\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}\left(dx\right)^{\frac{3}{2}}\left(4389x^{10}b^{5}+26565a\,x^{8}b^{4}+67298a^{2}x^{6}b^{3}+91770a^{3}x^{4}b^{2}+72105x^{2}a^{4}b+33649a^{5}\right)}{100947d\left(b\,x^{2}+a\right)^{5}}$	85
risch	$\frac{2d\sqrt{\left(b\,x^{2}+a\right)^{2}}\,x^{2}\left(4389x^{10}b^{5}+26565a\,x^{8}b^{4}+67298a^{2}x^{6}b^{3}+91770a^{3}x^{4}b^{2}+72105x^{2}a^{4}b+33649a^{5}\right)}{100947\left(b\,x^{2}+a\right)\sqrt{dx}}$	86
orering	$\frac{2x\left(4389x^{10}b^{5}+26565a\,x^{8}b^{4}+67298a^{2}x^{6}b^{3}+91770a^{3}x^{4}b^{2}+72105x^{2}a^{4}b+33649a^{5}\right)\sqrt{dx}\,\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{100947\left(b\,x^{2}+a\right)^{5}}$	92

input
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/100947*x*(4389*b^5*x^10+26565*a*b^4*x^8+67298*a^2*b^3*x^6+91770*a^3*b^2*
x^4+72105*a^4*b*x^2+33649*a^5)*(d*x)^(1/2)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.21

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{2}{100947}\left(4389\,b^5x^{11}+26565\,ab^4x^9+67298\,a^2b^3x^7+91770\,a^3b^2x^5+72105\,a^4bx^3+33649\,a\right.$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
2/100947*(4389*b^5*x^11 + 26565*a*b^4*x^9 + 67298*a^2*b^3*x^7 + 91770*a^3*
b^2*x^5 + 72105*a^4*b*x^3 + 33649*a^5*x)*sqrt(d*x)
```

Sympy [F]

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int \sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((d*x)**(1/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(sqrt(d*x)*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.49

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{2}{437}\left(19\,b^5\sqrt{d}x^3+23\,ab^4\sqrt{d}x\right)x^{\frac{17}{2}}+\frac{8}{285}\left(15\,ab^4\sqrt{d}x^3+19\,a^2b^3\sqrt{d}x\right)x^{\frac{13}{2}}+\frac{4}{55}\left(11\,a^2b^3\sqrt{d}x^3+15\,a^3b^2\sqrt{d}x\right)x^{\frac{9}{2}}+\frac{8}{77}\left(7\,a^3b^2\sqrt{d}x^3+11\,a^4b\sqrt{d}x\right)x^{\frac{5}{2}}+\frac{2}{21}\left(3\,a^4b\sqrt{d}x^3+7\,a^5\sqrt{d}x\right)\sqrt{x}$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
2/437*(19*b^5*sqrt(d)*x^3 + 23*a*b^4*sqrt(d)*x)*x^(17/2) + 8/285*(15*a*b^4
*sqrt(d)*x^3 + 19*a^2*b^3*sqrt(d)*x)*x^(13/2) + 4/55*(11*a^2*b^3*sqrt(d)*x
^3 + 15*a^3*b^2*sqrt(d)*x)*x^(9/2) + 8/77*(7*a^3*b^2*sqrt(d)*x^3 + 11*a^4*
b*sqrt(d)*x)*x^(5/2) + 2/21*(3*a^4*b*sqrt(d)*x^3 + 7*a^5*sqrt(d)*x)*sqrt(x
)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.45

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{2}{23}\sqrt{dx}b^5x^{11}\text{sgn}\left(bx^2+a\right)+\frac{10}{19}\sqrt{dx}ab^4x^9\text{sgn}\left(bx^2+a\right)+\frac{4}{3}\sqrt{dx}a^2b^3x^7\text{sgn}\left(bx^2+a\right)+\frac{20}{11}\sqrt{dx}a^3b^2x^5\text{sgn}\left(bx^2+a\right)+\frac{10}{7}\sqrt{dx}a^4bx^3\text{sgn}\left(bx^2+a\right)+\frac{2}{3}\sqrt{dx}a^5x\text{sgn}\left(bx^2+a\right)$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
2/23*sqrt(d*x)*b^5*x^11*sgn(b*x^2 + a) + 10/19*sqrt(d*x)*a*b^4*x^9*sgn(b*x
^2 + a) + 4/3*sqrt(d*x)*a^2*b^3*x^7*sgn(b*x^2 + a) + 20/11*sqrt(d*x)*a^3*b
^2*x^5*sgn(b*x^2 + a) + 10/7*sqrt(d*x)*a^4*b*x^3*sgn(b*x^2 + a) + 2/3*sqrt
(d*x)*a^5*x*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\int \sqrt{d\,x}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

input
```
int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.21

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,x(4389b^5x^{10}+26565a\,b^4x^8+67298a^2b^3x^6+91770a^3b^2x^4+72105a^4b\,x^2+33649a}{100947}$$

input
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*x*(33649*a**5 + 72105*a**4*b*x**2 + 91770*a**3*b**2*x**
4 + 67298*a**2*b**3*x**6 + 26565*a*b**4*x**8 + 4389*b**5*x**10))/100947
```

3.626 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 293

$$\begin{aligned}\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\sqrt{dx}}\,dx &= \frac{2a^5\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)} \\ &+ \frac{2a^4b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)} + \frac{20a^3b^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^5\left(a+bx^2\right)} \\ &+ \frac{20a^2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)} \\ &+ \frac{10ab^4(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^9\left(a+bx^2\right)} + \frac{2b^5(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^{11}\left(a+bx^2\right)}\end{aligned}$$

output
```
2*a^5*(d*x)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+2*a^4*b*(d*x)^(5/2)*((b*
x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+20/9*a^3*b^2*(d*x)^(9/2)*((b*x^2+a)^2)^(1/2)
/d^5/(b*x^2+a)+20/13*a^2*b^3*(d*x)^(13/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a
)+10/17*a*b^4*(d*x)^(17/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/21*b^5*(d*x
)^(21/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.30

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{\sqrt{dx}}\,dx = \frac{2x\sqrt{(a+bx^2)^2}(13923a^5+13923a^4bx^2+15470a^3b^2x^4+10710a^2b^3x^6+40}{13923\sqrt{dx}\,(a+bx^2)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```

output
```
(2*x*Sqrt[(a + b*x^2)^2]*(13923*a^5 + 13923*a^4*b*x^2 + 15470*a^3*b^2*x^4
+ 10710*a^2*b^3*x^6 + 4095*a*b^4*x^8 + 663*b^5*x^10))/(13923*Sqrt[d*x]*(a
+ b*x^2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{\sqrt{dx}}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{\sqrt{dx}}dx}{a+bx^2}$$

$$\Big\downarrow \; 244$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{b^5(dx)^{19/2}}{d^{10}}+\frac{5ab^4(dx)^{15/2}}{d^8}+\frac{10a^2b^3(dx)^{11/2}}{d^6}+\frac{10a^3b^2(dx)^{7/2}}{d^4}+\frac{5a^4b(dx)^{3/2}}{d^2}+\frac{a^5}{\sqrt{dx}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{2a^5\sqrt{dx}}{d}+\frac{2a^4b(dx)^{5/2}}{d^3}+\frac{20a^3b^2(dx)^{9/2}}{9d^5}+\frac{20a^2b^3(dx)^{13/2}}{13d^7}+\frac{10ab^4(dx)^{17/2}}{17d^9}+\frac{2b^5(dx)^{21/2}}{21d^{11}}\right)}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((2*a^5*Sqrt[d*x])/d + (2*a^4*b*(d*x)^(5/
2))/d^3 + (20*a^3*b^2*(d*x)^(9/2))/(9*d^5) + (20*a^2*b^3*(d*x)^(13/2))/(13
*d^7) + (10*a*b^4*(d*x)^(17/2))/(17*d^9) + (2*b^5*(d*x)^(21/2))/(21*d^11))
)/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

method	result	size
gosper	$\frac{2x\left(663x^{10}b^{5}+4095a\,x^{8}b^{4}+10710a^{2}x^{6}b^{3}+15470a^{3}x^{4}b^{2}+13923x^{2}a^{4}b+13923a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{13923\left(b\,x^{2}+a\right)^{5}\sqrt{dx}}$	83
risch	$\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(663x^{10}b^{5}+4095a\,x^{8}b^{4}+10710a^{2}x^{6}b^{3}+15470a^{3}x^{4}b^{2}+13923x^{2}a^{4}b+13923a^{5}\right)x}{13923\left(b\,x^{2}+a\right)\sqrt{dx}}$	83
default	$\frac{2\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}\sqrt{dx}\,\left(663x^{10}b^{5}+4095a\,x^{8}b^{4}+10710a^{2}x^{6}b^{3}+15470a^{3}x^{4}b^{2}+13923x^{2}a^{4}b+13923a^{5}\right)}{13923d\left(b\,x^{2}+a\right)^{5}}$	85
orering	$\frac{2\left(663x^{10}b^{5}+4095a\,x^{8}b^{4}+10710a^{2}x^{6}b^{3}+15470a^{3}x^{4}b^{2}+13923x^{2}a^{4}b+13923a^{5}\right)x\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{13923\left(b\,x^{2}+a\right)^{5}\sqrt{dx}}$	92

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/13923*x*(663*b^5*x^10+4095*a*b^4*x^8+10710*a^2*b^3*x^6+15470*a^3*b^2*x^4
+13923*a^4*b*x^2+13923*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.22

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\sqrt{dx}}\,dx=\frac{2\left(663\,b^5x^{10}+4095\,ab^4x^8+10710\,a^2b^3x^6+15470\,a^3b^2x^4+13923\,a^4bx^2+\right.}{13923\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
2/13923*(663*b^5*x^10 + 4095*a*b^4*x^8 + 10710*a^2*b^3*x^6 + 15470*a^3*b^2
*x^4 + 13923*a^4*b*x^2 + 13923*a^5)*sqrt(d*x)/d
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{\sqrt{dx}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{\sqrt{dx}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/sqrt(d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.52

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{\sqrt{dx}}\,dx = \frac{2\left(195\left(17\,b^5\sqrt{d}x^3+21\,ab^4\sqrt{d}x\right)x^{\frac{15}{2}}+1260\left(13\,ab^4\sqrt{d}x^3+17\,a^2b^3\sqrt{d}x\right)\right.}{}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
2/69615*(195*(17*b^5*sqrt(d)*x^3 + 21*a*b^4*sqrt(d)*x)*x^(15/2) + 1260*(13
*a*b^4*sqrt(d)*x^3 + 17*a^2*b^3*sqrt(d)*x)*x^(11/2) + 3570*(9*a^2*b^3*sqrt
(d)*x^3 + 13*a^3*b^2*sqrt(d)*x)*x^(7/2) + 6188*(5*a^3*b^2*sqrt(d)*x^3 + 9*
a^4*b*sqrt(d)*x)*x^(3/2) + 13923*(a^4*b*sqrt(d)*x^3 + 5*a^5*sqrt(d)*x)/sqr
t(x))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.47

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{\sqrt{dx}}\,dx = \frac{2\left(663\sqrt{dx}b^5x^{10}\mathrm{sgn}(bx^2+a)+4095\sqrt{dx}ab^4x^8\mathrm{sgn}(bx^2+a)+10710\sqrt{dx}\right.}{}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
2/13923*(663*sqrt(d*x)*b^5*x^10*sgn(b*x^2 + a) + 4095*sqrt(d*x)*a*b^4*x^8*
sgn(b*x^2 + a) + 10710*sqrt(d*x)*a^2*b^3*x^6*sgn(b*x^2 + a) + 15470*sqrt(d
*x)*a^3*b^2*x^4*sgn(b*x^2 + a) + 13923*sqrt(d*x)*a^4*b*x^2*sgn(b*x^2 + a)
+ 13923*sqrt(d*x)*a^5*sgn(b*x^2 + a))/d
```

Mupad [B] (verification not implemented)

Time = 17.95 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.38

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{\sqrt{dx}}\,dx = \frac{2\,x\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,(5731\,a^4+8192\,a^3\,b\,x^2+7278\,a^2\,b^2\,x^4+3432\,a\,b^3}{13923\,\sqrt{d\,x}}$$
$$+\frac{16384\,a^5\,x\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{13923\,\sqrt{d\,x}\,\left(b\,x^2+a\right)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(1/2),x)
```

output
```
(2*x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(5731*a^4 + 663*b^4*x^8 + 8192*a^3*
b*x^2 + 3432*a*b^3*x^6 + 7278*a^2*b^2*x^4))/(13923*(d*x)^(1/2)) + (16384*a
^5*x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(13923*(d*x)^(1/2)*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{\sqrt{dx}}\,dx = \frac{2\sqrt{x}\,\sqrt{d}\,(663b^5x^{10}+4095a\,b^4x^8+10710a^2b^3x^6+15470a^3b^2x^4+13923a^4b}{13923d}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d)*(13923*a**5 + 13923*a**4*b*x**2 + 15470*a**3*b**2*x**4
+ 10710*a**2*b**3*x**6 + 4095*a*b**4*x**8 + 663*b**5*x**10))/(13923*d)
```

3.627 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 295

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}}\,dx = -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)} + \frac{10a^4b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\,(a+bx^2)} + \frac{20a^3b^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\,(a+bx^2)} + \frac{20a^2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\,(a+bx^2)} + \frac{2ab^4(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^9\,(a+bx^2)} + \frac{2b^5(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^{11}\,(a+bx^2)}$$

output
```
-2*a^5*((b*x^2+a)^2)^(1/2)/d/(d*x)^(1/2)/(b*x^2+a)+10/3*a^4*b*(d*x)^(3/2)*
((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+20/7*a^3*b^2*(d*x)^(7/2)*((b*x^2+a)^2)^(
1/2)/d^5/(b*x^2+a)+20/11*a^2*b^3*(d*x)^(11/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x
^2+a)+2/3*a*b^4*(d*x)^(15/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/19*b^5*(d
*x)^(19/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.30

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}}\,dx = -\frac{2x\left((a+bx^2)^2\right)^{5/2}(4389a^5-7315a^4bx^2-6270a^3b^2x^4-3990a^2b^3x^6-1463ab^4x^8-231b^5x^{10})}{4389(dx)^{3/2}(a+bx^2)^5}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```

output
```
(-2*x*((a + b*x^2)^2)^(5/2)*(4389*a^5 - 7315*a^4*b*x^2 - 6270*a^3*b^2*x^4
- 3990*a^2*b^3*x^6 - 1463*a*b^4*x^8 - 231*b^5*x^10))/(4389*(d*x)^(3/2)*(a
+ b*x^2)^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{(dx)^{3/2}}dx}{b^5(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{(dx)^{3/2}}dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{b^5(dx)^{17/2}}{d^{10}}+\frac{5ab^4(dx)^{13/2}}{d^8}+\frac{10a^2b^3(dx)^{9/2}}{d^6}+\frac{10a^3b^2(dx)^{5/2}}{d^4}+\frac{5a^4b\sqrt{dx}}{d^2}+\frac{a^5}{(dx)^{3/2}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{2a^5}{d\sqrt{dx}}+\frac{10a^4b(dx)^{3/2}}{3d^3}+\frac{20a^3b^2(dx)^{7/2}}{7d^5}+\frac{20a^2b^3(dx)^{11/2}}{11d^7}+\frac{2ab^4(dx)^{15/2}}{3d^9}+\frac{2b^5(dx)^{19/2}}{19d^{11}}\right)}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((-2*a^5)/(d*Sqrt[d*x]) + (10*a^4*b*(d*x)
^(3/2))/(3*d^3) + (20*a^3*b^2*(d*x)^(7/2))/(7*d^5) + (20*a^2*b^3*(d*x)^(11
/2))/(11*d^7) + (2*a*b^4*(d*x)^(15/2))/(3*d^9) + (2*b^5*(d*x)^(19/2))/(19*
d^11)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

method	result	size
gosper	$-\frac{2x\left(-231x^{10}b^{5}-1463a\,x^{8}b^{4}-3990a^{2}x^{6}b^{3}-6270a^{3}x^{4}b^{2}-7315x^{2}a^{4}b+4389a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{4389\left(b\,x^{2}+a\right)^{5}\left(dx\right)^{\frac{3}{2}}}$	83
default	$-\frac{2\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}\left(-231x^{10}b^{5}-1463a\,x^{8}b^{4}-3990a^{2}x^{6}b^{3}-6270a^{3}x^{4}b^{2}-7315x^{2}a^{4}b+4389a^{5}\right)}{4389d\left(b\,x^{2}+a\right)^{5}\sqrt{dx}}$	85
risch	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-231x^{10}b^{5}-1463a\,x^{8}b^{4}-3990a^{2}x^{6}b^{3}-6270a^{3}x^{4}b^{2}-7315x^{2}a^{4}b+4389a^{5}\right)}{4389d\left(b\,x^{2}+a\right)\sqrt{dx}}$	85
orering	$-\frac{2\left(-231x^{10}b^{5}-1463a\,x^{8}b^{4}-3990a^{2}x^{6}b^{3}-6270a^{3}x^{4}b^{2}-7315x^{2}a^{4}b+4389a^{5}\right)x\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{4389\left(b\,x^{2}+a\right)^{5}\left(dx\right)^{\frac{3}{2}}}$	92

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/4389*x*(-231*b^5*x^10-1463*a*b^4*x^8-3990*a^2*b^3*x^6-6270*a^3*b^2*x^4-
7315*a^4*b*x^2+4389*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{3/2}}\,dx = \frac{2\left(231\,b^5x^{10}+1463\,ab^4x^8+3990\,a^2b^3x^6+6270\,a^3b^2x^4+7315\,a^4bx^2-4389\right.}{4389\,d^2x}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
2/4389*(231*b^5*x^10 + 1463*a*b^4*x^8 + 3990*a^2*b^3*x^6 + 6270*a^3*b^2*x^
4 + 7315*a^4*b*x^2 - 4389*a^5)*sqrt(d*x)/(d^2*x)
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(3/2),x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.51

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}}\,dx = \frac{2\left(77\left(15\,b^5\sqrt{d}x^3+19\,ab^4\sqrt{d}x\right)x^{\frac{13}{2}}+532\left(11\,ab^4\sqrt{d}x^3+15\,a^2b^3\sqrt{d}x\right)x^{\frac{9}{2}}\right.}{}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
2/21945*(77*(15*b^5*sqrt(d)*x^3 + 19*a*b^4*sqrt(d)*x)*x^(13/2) + 532*(11*a
*b^4*sqrt(d)*x^3 + 15*a^2*b^3*sqrt(d)*x)*x^(9/2) + 1710*(7*a^2*b^3*sqrt(d)
*x^3 + 11*a^3*b^2*sqrt(d)*x)*x^(5/2) + 4180*(3*a^3*b^2*sqrt(d)*x^3 + 7*a^4
*b*sqrt(d)*x)*sqrt(x) + 7315*(a^4*b*sqrt(d)*x^3 - 3*a^5*sqrt(d)*x)/x^(3/2)
)/d^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.53

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}}\,dx =$$

$$-\frac{2\left(\frac{4389\,a^5\mathrm{sgn}(bx^2+a)}{\sqrt{dx}}-\frac{231\,\sqrt{dx}b^5d^{189}x^9\mathrm{sgn}(bx^2+a)+1463\,\sqrt{dx}ab^4d^{189}x^7\mathrm{sgn}(bx^2+a)+3990\,\sqrt{dx}a^2b^3d^{189}x^5\mathrm{sgn}(bx^2+a)+6270\,\sqrt{dx}a^3b^2}{d^{190}}\right.}{4389\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
-2/4389*(4389*a^5*sgn(b*x^2 + a)/sqrt(d*x) - (231*sqrt(d*x)*b^5*d^189*x^9*
sgn(b*x^2 + a) + 1463*sqrt(d*x)*a*b^4*d^189*x^7*sgn(b*x^2 + a) + 3990*sqrt
(d*x)*a^2*b^3*d^189*x^5*sgn(b*x^2 + a) + 6270*sqrt(d*x)*a^3*b^2*d^189*x^3*
sgn(b*x^2 + a) + 7315*sqrt(d*x)*a^4*b*d^189*x*sgn(b*x^2 + a))/d^190)/d
```

Mupad [B] (verification not implemented)

Time = 18.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.39

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}}\,dx = \frac{2\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,(3803\,a^4+3512\,a^3\,b\,x^2+2758\,a^2\,b^2\,x^4+1232\,a\,b^3\,x}{4389\,d\,\sqrt{d\,x}} - \frac{16384\,a^5\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{4389\,d\,\sqrt{d\,x}\,(b\,x^2+a)}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(3/2),x)
```

output
```
(2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(3803*a^4 + 231*b^4*x^8 + 3512*a^3*b*
x^2 + 1232*a*b^3*x^6 + 2758*a^2*b^2*x^4))/(4389*d*(d*x)^(1/2)) - (16384*a^
5*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(4389*d*(d*x)^(1/2)*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.22

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}}\,dx = \frac{2\sqrt{d}\,(231b^5x^{10}+1463a\,b^4x^8+3990a^2b^3x^6+6270a^3b^2x^4+7315a^4b\,x^2-43}{4389\sqrt{x}\,d^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x)
```

output
```
(2*sqrt(d)*( - 4389*a**5 + 7315*a**4*b*x**2 + 6270*a**3*b**2*x**4 + 3990*a
**2*b**3*x**6 + 1463*a*b**4*x**8 + 231*b**5*x**10))/(4389*sqrt(x)*d**2)
```

3.628 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 293

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{5/2}}\,dx = -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}(a+bx^2)} + \frac{10a^4b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(a+bx^2)} + \frac{4a^3b^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5(a+bx^2)} + \frac{20a^2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7(a+bx^2)} + \frac{10ab^4(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^9(a+bx^2)} + \frac{2b^5(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^{11}(a+bx^2)}$$

output

```
-2/3*a^5*((b*x^2+a)^2)^(1/2)/d/(d*x)^(3/2)/(b*x^2+a)+10*a^4*b*(d*x)^(1/2)*
((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)+4*a^3*b^2*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2
)/d^5/(b*x^2+a)+20/9*a^2*b^3*(d*x)^(9/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2+a)
+10/13*a*b^4*(d*x)^(13/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/17*b^5*(d*x)
^(17/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.30

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{5/2}}\,dx =$$
$$-\frac{2x\left((a+bx^2)^2\right)^{5/2}(663a^5-9945a^4bx^2-3978a^3b^2x^4-2210a^2b^3x^6-765ab^4x^8-117b^5x^{10})}{1989(dx)^{5/2}(a+bx^2)^5}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```

output
```
(-2*x*((a + b*x^2)^2)^(5/2)*(663*a^5 - 9945*a^4*b*x^2 - 3978*a^3*b^2*x^4 -
 2210*a^2*b^3*x^6 - 765*a*b^4*x^8 - 117*b^5*x^10))/(1989*(d*x)^(5/2)*(a +
b*x^2)^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{5/2}}\,dx$$
$$\Big\downarrow \; 1384$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{(dx)^{5/2}}dx}{b^5(a+bx^2)}$$
$$\Big\downarrow \; 27$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{(dx)^{5/2}}dx}{a+bx^2}$$
$$\Big\downarrow \; 244$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{b^5(dx)^{15/2}}{d^{10}}+\frac{5ab^4(dx)^{11/2}}{d^8}+\frac{10a^2b^3(dx)^{7/2}}{d^6}+\frac{10a^3b^2(dx)^{3/2}}{d^4}+\frac{5a^4b}{d^2\sqrt{dx}}+\frac{a^5}{(dx)^{5/2}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{2a^5}{3d(dx)^{3/2}}+\frac{10a^4b\sqrt{dx}}{d^3}+\frac{4a^3b^2(dx)^{5/2}}{d^5}+\frac{20a^2b^3(dx)^{9/2}}{9d^7}+\frac{10ab^4(dx)^{13/2}}{13d^9}+\frac{2b^5(dx)^{17/2}}{17d^{11}}\right)}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((-2*a^5)/(3*d*(d*x)^(3/2)) + (10*a^4*b*S
qrt[d*x])/d^3 + (4*a^3*b^2*(d*x)^(5/2))/d^5 + (20*a^2*b^3*(d*x)^(9/2))/(9*
d^7) + (10*a*b^4*(d*x)^(13/2))/(13*d^9) + (2*b^5*(d*x)^(17/2))/(17*d^11)))
/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

method	result	size
gosper	$-\frac{2x\left(-117x^{10}b^{5}-765a\,x^{8}b^{4}-2210a^{2}x^{6}b^{3}-3978a^{3}x^{4}b^{2}-9945x^{2}a^{4}b+663a^{5}\right)\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}}{1989\left(b\,x^{2}+a\right)^{5}\left(dx\right)^{\frac{5}{2}}}$	83
default	$-\frac{2\left(\left(b\,x^{2}+a\right)^{2}\right)^{\frac{5}{2}}\left(-117x^{10}b^{5}-765a\,x^{8}b^{4}-2210a^{2}x^{6}b^{3}-3978a^{3}x^{4}b^{2}-9945x^{2}a^{4}b+663a^{5}\right)}{1989d\left(b\,x^{2}+a\right)^{5}\left(dx\right)^{\frac{3}{2}}}$	85
risch	$-\frac{2\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-117x^{10}b^{5}-765a\,x^{8}b^{4}-2210a^{2}x^{6}b^{3}-3978a^{3}x^{4}b^{2}-9945x^{2}a^{4}b+663a^{5}\right)}{1989d^{2}\left(b\,x^{2}+a\right)x\sqrt{dx}}$	88
orering	$-\frac{2\left(-117x^{10}b^{5}-765a\,x^{8}b^{4}-2210a^{2}x^{6}b^{3}-3978a^{3}x^{4}b^{2}-9945x^{2}a^{4}b+663a^{5}\right)x\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{5}{2}}}{1989\left(b\,x^{2}+a\right)^{5}\left(dx\right)^{\frac{5}{2}}}$	92

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1989*x*(-117*b^5*x^10-765*a*b^4*x^8-2210*a^2*b^3*x^6-3978*a^3*b^2*x^4-9
945*a^4*b*x^2+663*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.23

$$\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{(dx)^{5/2}}\,dx = \frac{2\left(117\,b^5x^{10}+765\,ab^4x^8+2210\,a^2b^3x^6+3978\,a^3b^2x^4+9945\,a^4bx^2-663\,a\right.}{1989\,d^3x^2}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
2/1989*(117*b^5*x^10 + 765*a*b^4*x^8 + 2210*a^2*b^3*x^6 + 3978*a^3*b^2*x^4
 + 9945*a^4*b*x^2 - 663*a^5)*sqrt(d*x)/(d^3*x^2)
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{5/2}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(5/2),x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.52

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{5/2}}\,dx = \frac{2\left(45\left(13\,b^5\sqrt{d}x^3+17\,ab^4\sqrt{d}x\right)x^{\frac{11}{2}}+340\left(9\,ab^4\sqrt{d}x^3+13\,a^2b^3\sqrt{d}x\right)x^{\frac{7}{2}}\right.}{}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
2/9945*(45*(13*b^5*sqrt(d)*x^3 + 17*a*b^4*sqrt(d)*x)*x^(11/2) + 340*(9*a*b
^4*sqrt(d)*x^3 + 13*a^2*b^3*sqrt(d)*x)*x^(7/2) + 1326*(5*a^2*b^3*sqrt(d)*x
^3 + 9*a^3*b^2*sqrt(d)*x)*x^(3/2) + 7956*(a^3*b^2*sqrt(d)*x^3 + 5*a^4*b*sq
rt(d)*x)/sqrt(x) + 3315*(3*a^4*b*sqrt(d)*x^3 - a^5*sqrt(d)*x)/x^(5/2))/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.55

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{5/2}}\,dx =$$

$$-\frac{2\left(\frac{663\,a^5\mathrm{sgn}(bx^2+a)}{\sqrt{dx}dx}-\frac{117\,\sqrt{dx}b^5d^{168}x^8\mathrm{sgn}(bx^2+a)+765\,\sqrt{dx}ab^4d^{168}x^6\mathrm{sgn}(bx^2+a)+2210\,\sqrt{dx}a^2b^3d^{168}x^4\mathrm{sgn}(bx^2+a)+3978\,\sqrt{dx}a^3b^2d^{1}}{d^{170}}\right.}{1989\,d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
-2/1989*(663*a^5*sgn(b*x^2 + a)/(sqrt(d*x)*d*x) - (117*sqrt(d*x)*b^5*d^168
*x^8*sgn(b*x^2 + a) + 765*sqrt(d*x)*a*b^4*d^168*x^6*sgn(b*x^2 + a) + 2210*
sqrt(d*x)*a^2*b^3*d^168*x^4*sgn(b*x^2 + a) + 3978*sqrt(d*x)*a^3*b^2*d^168*
x^2*sgn(b*x^2 + a) + 9945*sqrt(d*x)*a^4*b*d^168*sgn(b*x^2 + a))/d^170)/d
```

Mupad [B] (verification not implemented)

Time = 19.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.40

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{5/2}}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{10\,a^4\,x^2}{d^2}-\frac{2\,a^5}{3\,b\,d^2}+\frac{2\,b^4\,x^{10}}{17\,d^2}+\frac{4\,a^3\,b\,x^4}{d^2}+\frac{10\,a\,b^3\,x^8}{13\,d^2}+\frac{20\,a^2\,b^2}{9\,d^2}\right.}{x^3\,\sqrt{d\,x}+\frac{a\,x\,\sqrt{d\,x}}{b}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(5/2),x)
```

output
```
((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((10*a^4*x^2)/d^2 - (2*a^5)/(3*b*d^2) +
 (2*b^4*x^10)/(17*d^2) + (4*a^3*b*x^4)/d^2 + (10*a*b^3*x^8)/(13*d^2) + (20
*a^2*b^2*x^6)/(9*d^2)))/(x^3*(d*x)^(1/2) + (a*x*(d*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{5/2}}\,dx = \frac{2\sqrt{d}\,(117b^5x^{10}+765a\,b^4x^8+2210a^2b^3x^6+3978a^3b^2x^4+9945a^4b\,x^2-663}{1989\sqrt{x}\,d^3x}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x)
```

output
```
(2*sqrt(d)*( - 663*a**5 + 9945*a**4*b*x**2 + 3978*a**3*b**2*x**4 + 2210*a*
*2*b**3*x**6 + 765*a*b**4*x**8 + 117*b**5*x**10))/(1989*sqrt(x)*d**3*x)
```

3.629 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 295

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}\,dx = -\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}(a+bx^2)} - \frac{10a^4b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}(a+bx^2)} + \frac{20a^3b^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5(a+bx^2)} + \frac{20a^2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7(a+bx^2)} + \frac{10ab^4(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^9(a+bx^2)} + \frac{2b^5(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^{11}(a+bx^2)}$$

output
```
-2/5*a^5*((b*x^2+a)^2)^(1/2)/d/(d*x)^(5/2)/(b*x^2+a)-10*a^4*b*((b*x^2+a)^2
)^(1/2)/d^3/(d*x)^(1/2)/(b*x^2+a)+20/3*a^3*b^2*(d*x)^(3/2)*((b*x^2+a)^2)^(
1/2)/d^5/(b*x^2+a)+20/7*a^2*b^3*(d*x)^(7/2)*((b*x^2+a)^2)^(1/2)/d^7/(b*x^2
+a)+10/11*a*b^4*(d*x)^(11/2)*((b*x^2+a)^2)^(1/2)/d^9/(b*x^2+a)+2/15*b^5*(d
*x)^(15/2)*((b*x^2+a)^2)^(1/2)/d^11/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.30

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}\,dx =$$
$$-\frac{2x\left((a+bx^2)^2\right)^{5/2}(231a^5+5775a^4bx^2-3850a^3b^2x^4-1650a^2b^3x^6-525ab^4x^8-77b^5x^{10})}{1155(dx)^{7/2}(a+bx^2)^5}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

output
```
(-2*x*((a + b*x^2)^2)^(5/2)*(231*a^5 + 5775*a^4*b*x^2 - 3850*a^3*b^2*x^4 -
 1650*a^2*b^3*x^6 - 525*a*b^4*x^8 - 77*b^5*x^10))/(1155*(d*x)^(7/2)*(a + b
*x^2)^5)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b^5(bx^2+a)^5}{(dx)^{7/2}}dx}{b^5(a+bx^2)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(bx^2+a)^5}{(dx)^{7/2}}dx}{a+bx^2}$$
$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(\frac{b^5(dx)^{13/2}}{d^{10}}+\frac{5ab^4(dx)^{9/2}}{d^8}+\frac{10a^2b^3(dx)^{5/2}}{d^6}+\frac{10a^3b^2\sqrt{dx}}{d^4}+\frac{5a^4b}{d^2(dx)^{3/2}}+\frac{a^5}{(dx)^{7/2}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(-\frac{2a^5}{5d(dx)^{5/2}}-\frac{10a^4b}{d^3\sqrt{dx}}+\frac{20a^3b^2(dx)^{3/2}}{3d^5}+\frac{20a^2b^3(dx)^{7/2}}{7d^7}+\frac{10ab^4(dx)^{11/2}}{11d^9}+\frac{2b^5(dx)^{15/2}}{15d^{11}}\right)}{a+bx^2}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((-2*a^5)/(5*d*(d*x)^(5/2)) - (10*a^4*b)/
(d^3*Sqrt[d*x]) + (20*a^3*b^2*(d*x)^(3/2))/(3*d^5) + (20*a^2*b^3*(d*x)^(7/
2))/(7*d^7) + (10*a*b^4*(d*x)^(11/2))/(11*d^9) + (2*b^5*(d*x)^(15/2))/(15*
d^11)))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

method	result	size
gosper	$-\frac{2x(-77x^{10}b^5-525a\,x^8b^4-1650a^2x^6b^3-3850a^3x^4b^2+5775x^2a^4b+231a^5)\left((b\,x^2+a)^2\right)^{\frac{5}{2}}}{1155(b\,x^2+a)^5(dx)^{\frac{7}{2}}}$	83
default	$-\frac{2\left((b\,x^2+a)^2\right)^{\frac{5}{2}}(-77x^{10}b^5-525a\,x^8b^4-1650a^2x^6b^3-3850a^3x^4b^2+5775x^2a^4b+231a^5)}{1155d(b\,x^2+a)^5(dx)^{\frac{5}{2}}}$	85
risch	$-\frac{2\sqrt{(b\,x^2+a)^2}\,(-77x^{10}b^5-525a\,x^8b^4-1650a^2x^6b^3-3850a^3x^4b^2+5775x^2a^4b+231a^5)}{1155d^3(b\,x^2+a)x^2\sqrt{dx}}$	88
orering	$-\frac{2(-77x^{10}b^5-525a\,x^8b^4-1650a^2x^6b^3-3850a^3x^4b^2+5775x^2a^4b+231a^5)x(b^2x^4+2ab\,x^2+a^2)^{\frac{5}{2}}}{1155(b\,x^2+a)^5(dx)^{\frac{7}{2}}}$	92

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1155*x*(-77*b^5*x^10-525*a*b^4*x^8-1650*a^2*b^3*x^6-3850*a^3*b^2*x^4+57
75*a^4*b*x^2+231*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}\,dx = \frac{2\left(77\,b^5x^{10}+525\,ab^4x^8+1650\,a^2b^3x^6+3850\,a^3b^2x^4-5775\,a^4bx^2-231\,a^5\right.}{1155\,d^4x^3}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
2/1155*(77*b^5*x^10 + 525*a*b^4*x^8 + 1650*a^2*b^3*x^6 + 3850*a^3*b^2*x^4
- 5775*a^4*b*x^2 - 231*a^5)*sqrt(d*x)/(d^4*x^3)
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}\,dx = \int \frac{\left((a+bx^2)^2\right)^{\frac{5}{2}}}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(7/2),x)
```

output
```
Integral(((a + b*x**2)**2)**(5/2)/(d*x)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.51

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}\,dx = \frac{2\left(7\left(11\,b^5\sqrt{d}x^3+15\,ab^4\sqrt{d}x\right)x^{\frac{9}{2}}+60\left(7\,ab^4\sqrt{d}x^3+11\,a^2b^3\sqrt{d}x\right)x^{\frac{5}{2}}+3\right.}{}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
2/1155*(7*(11*b^5*sqrt(d)*x^3 + 15*a*b^4*sqrt(d)*x)*x^(9/2) + 60*(7*a*b^4*
sqrt(d)*x^3 + 11*a^2*b^3*sqrt(d)*x)*x^(5/2) + 330*(3*a^2*b^3*sqrt(d)*x^3 +
 7*a^3*b^2*sqrt(d)*x)*sqrt(x) + 1540*(a^3*b^2*sqrt(d)*x^3 - 3*a^4*b*sqrt(d
)*x)/x^(3/2) - 231*(5*a^4*b*sqrt(d)*x^3 + a^5*sqrt(d)*x)/x^(7/2))/d^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.55

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}\,dx =$$

$$-\frac{2\left(\frac{231\,\left(25\,a^4bd^3x^2\mathrm{sgn}\left(bx^2+a\right)+a^5d^3\mathrm{sgn}\left(bx^2+a\right)\right)}{\sqrt{dx}d^2x^2}-\frac{77\,\sqrt{dx}b^5d^{105}x^7\mathrm{sgn}\left(bx^2+a\right)+525\,\sqrt{dx}ab^4d^{105}x^5\mathrm{sgn}\left(bx^2+a\right)+1650\,\sqrt{dx}a^2b^3d^{105}x^3\mathrm{sg}}{d^{105}}\right.}{1155\,d^4}$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x, algorithm="giac")
```

output
```
-2/1155*(231*(25*a^4*b*d^3*x^2*sgn(b*x^2 + a) + a^5*d^3*sgn(b*x^2 + a))/(s
qrt(d*x)*d^2*x^2) - (77*sqrt(d*x)*b^5*d^105*x^7*sgn(b*x^2 + a) + 525*sqrt(
d*x)*a*b^4*d^105*x^5*sgn(b*x^2 + a) + 1650*sqrt(d*x)*a^2*b^3*d^105*x^3*sgn
(b*x^2 + a) + 3850*sqrt(d*x)*a^3*b^2*d^105*x*sgn(b*x^2 + a))/d^105)/d^4
```

Mupad [B] (verification not implemented)

Time = 18.93 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.40

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}\,dx = \frac{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\,\left(\frac{2\,b^4\,x^{10}}{15\,d^3}-\frac{10\,a^4\,x^2}{d^3}-\frac{2\,a^5}{5\,b\,d^3}+\frac{20\,a^3\,b\,x^4}{3\,d^3}+\frac{10\,a\,b^3\,x^8}{11\,d^3}+\frac{20\,a^2\,b}{7\,d^3}\right.}{x^4\,\sqrt{d\,x}+\frac{a\,x^2\,\sqrt{d\,x}}{b}}$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)/(d*x)^(7/2),x)
```

output
```
((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*((2*b^4*x^10)/(15*d^3) - (10*a^4*x^2)/d
^3 - (2*a^5)/(5*b*d^3) + (20*a^3*b*x^4)/(3*d^3) + (10*a*b^3*x^8)/(11*d^3)
+ (20*a^2*b^2*x^6)/(7*d^3)))/(x^4*(d*x)^(1/2) + (a*x^2*(d*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.23

$$\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}\,dx = \frac{2\sqrt{d}\,(77b^5x^{10}+525a\,b^4x^8+1650a^2b^3x^6+3850a^3b^2x^4-5775a^4b\,x^2-231a}{1155\sqrt{x}\,d^4x^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x)
```

output
```
(2*sqrt(d)*( - 231*a**5 - 5775*a**4*b*x**2 + 3850*a**3*b**2*x**4 + 1650*a*
*2*b**3*x**6 + 525*a*b**4*x**8 + 77*b**5*x**10))/(1155*sqrt(x)*d**4*x**2)
```

3.630 $\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 348

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{2ad^3\sqrt{dx}(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2d(dx)^{5/2}(a+bx^2)}{5b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{5/4}d^{7/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^{5/4}d^{7/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^{5/4}d^{7/2}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}(\sqrt{a}+\sqrt{b}x)}\right)}{\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-2*a*d^3*(d*x)^(1/2)*(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+2/5*d*(d*x)^(5/2)*(
b*x^2+a)/b/((b*x^2+a)^2)^(1/2)-1/2*a^(5/4)*d^(7/2)*(b*x^2+a)*arctan(1-2^(1
/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(9/4)/((b*x^2+a)^2)^(1/
2)+1/2*a^(5/4)*d^(7/2)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1
/4)/d^(1/2))*2^(1/2)/b^(9/4)/((b*x^2+a)^2)^(1/2)+1/2*a^(5/4)*d^(7/2)*(b*x^
2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*
x))*2^(1/2)/b^(9/4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.47

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{d^3\sqrt{dx}(a+bx^2)\left(4\sqrt[4]{b}\sqrt{x}(-5a+bx^2)-5\sqrt{2}a^{5/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+5\cdots\right.}{10b^{9/4}\sqrt{x}\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(d^3*Sqrt[d*x]*(a + b*x^2)*(4*b^(1/4)*Sqrt[x]*(-5*a + b*x^2) - 5*Sqrt[2]*a
^(5/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5
*Sqrt[2]*a^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt
[b]*x)]))/(10*b^(9/4)*Sqrt[x]*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1384, 27, 262, 262, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b(a+bx^2)\int \frac{(dx)^{7/2}}{b(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{(dx)^{7/2}}{bx^2+a}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\left(a+bx^2\right)\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\int\frac{(dx)^{3/2}}{bx^2+a}dx}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\left(a+bx^2\right)\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{ad^2\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{b}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\left(a+bx^2\right)\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\int\frac{1}{bx^2+a}d\sqrt{dx}}{b}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{755}$$

$$\frac{\left(a+bx^2\right)\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{b}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1476

$$\frac{\left(a+bx^2\right)\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1082

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 217

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1479

$$\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx} \qquad \int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}$$

$$d\left(-\frac{\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right) \qquad d$$

$$2ad\left(\frac{\cdots}{2\sqrt{a}}+\cdots\right.$$

$$ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{\cdots}{b}\right.$$

$$\left(a+bx^2\right)\left(\frac{2d(dx)^{5/2}}{5b}-\frac{\cdots}{b}\right.$$

$$\sqrt{a^2+2abx^2+b^2x^4}$$

↓ 25

$$\frac{\left(a+bx^2\right)\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+d\left(\frac{\text{arct}\cdots}{\cdots}\right.\cdots}{b}\cdots}{b}\cdots}{b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 27

$$\frac{\left(a+bx^2\right)\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{}\right)}{b}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1103

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right)}{\cdots}\right)}{b}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*((2*d*(d*x)^(5/2))/(5*b) - (a*d^2*((2*d*Sqrt[d*x])/b - (2*a*d
*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Lo
g[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*S
qrt[a])))/b))/b))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.60

method	result
risch	$-\frac{2(-b x^2+5a) x d^4 \sqrt{(b x^2+a)^2}}{5b^2 \sqrt{dx}\,(b x^2+a)} + \frac{a d^3 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2} \left(\ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.\right.}{4b^2(b x^2+a)}$
default	$\frac{(b x^2+a) d \left(5a d^2 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \ln\left(\frac{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+10a d^2 \left(\frac{a d^2}{b}\right)^{\frac{1}{4}} \sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+10a d^2 \left(\frac{a d^2}{b}\right) \cdots\right.}{20\sqrt{(b x^2+a)^2}\, b^2}$

input
```
int((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-b*x^2+5*a)*x/b^2/(d*x)^(1/2)*d^4*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/4*
a/b^2*d^3*(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a
*d^2/b)^(1/4)*(d*x)^(1/2)-1))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.63

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{5\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}} b^2 \log\left(\sqrt{dx}ad^3+\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}} b^2\right)+5i\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}} b^2 \log\left(\sqrt{dx}ad \cdots\right.}{\cdots}$$

input
```
integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/10*(5*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3 + (-a^5*d^14/b^9)^(1
/4)*b^2) + 5*I*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3 + I*(-a^5*d^1
4/b^9)^(1/4)*b^2) - 5*I*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3 - I*
(-a^5*d^14/b^9)^(1/4)*b^2) - 5*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d
^3 - (-a^5*d^14/b^9)^(1/4)*b^2) + 4*(b*d^3*x^2 - 5*a*d^3)*sqrt(d*x))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.76

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\frac{5\left(\frac{\sqrt{2}d^6\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}d^6\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\sqrt{2}d^5\arctan\left(\frac{\sqrt{2}(\cdots}{\cdots}\right.}{\sqrt{\cdots}}\cdots\right.}{b^2}\cdots}{2\cdots}$$

input
```
integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/20*(5*(sqrt(2)*d^6*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(
1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^6*log(sqrt(b)*d*x -
sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4
)) + 2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*s
qrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a
)) + 2*sqrt(2)*d^5*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(
a)))*a^2/b^2 + 8*((d*x)^(5/2)*b*d^2 - 5*sqrt(d*x)*a*d^4)/b^2)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.81

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(\frac{10\sqrt{2}(ab^3d^2)^{\frac{1}{4}}ad^4\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}+\frac{10\sqrt{2}(ab^3d^2)^{\frac{1}{4}}ad^4\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right.\right.}{2\left(\frac{ad}{b}\right.}\right.}{b^3}\right.}{}$$

input
```
integrate((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/20*(10*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^4*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 + 10*sqrt(2)*(a*b^3*d^2)^(1
/4)*a*d^4*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/b^3 + 5*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^4*log(d*x + sqrt(2)*(a*
d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*
a*d^4*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 + 8
*(sqrt(d*x)*b^4*d^4*x^2 - 5*sqrt(d*x)*a*b^3*d^4)/b^5)*sgn(b*x^2 + a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{(d\,x)^{7/2}}{\sqrt{(b\,x^2+a)^2}}\,dx$$

input
```
int((d*x)^(7/2)/((a + b*x^2)^2)^(1/2),x)
```

output
```
int((d*x)^(7/2)/((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.47

$$\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{\sqrt{d}\,d^3\left(-10b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+10b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)\right.}{}$$

input
```
int((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x)
```

output
```
(sqrt(d)*d**3*( - 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 10*b**(3/4)*a**
(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/
4)*a**(1/4)*sqrt(2)))*a - 5*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1
/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a + 5*b**(3/4)*a**(1/4)*sqrt(2
)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a - 40*sqrt
(x)*a*b + 8*sqrt(x)*b**2*x**2))/(20*b**3)
```

3.631 $\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 303

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{2d(dx)^{3/2}(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^{3/4}d^{5/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{a^{3/4}d^{5/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{a^{3/4}d^{5/2}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
2/3*d*(d*x)^(3/2)*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)+1/2*a^(3/4)*d^(5/2)*(b*x
^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(7/4
)/((b*x^2+a)^2)^(1/2)-1/2*a^(3/4)*d^(5/2)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/
4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(7/4)/((b*x^2+a)^2)^(1/2)+1/2*a^
(3/4)*d^(5/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2
)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(7/4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.50

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(dx)^{5/2}\left(a+bx^2\right)\left(4b^{3/4}x^{3/2}+3\sqrt{2}a^{3/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+3\sqrt{2}a^{3/4}\text{arct}\right.}{6b^{7/4}x^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

input
```
Integrate[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((d*x)^(5/2)*(a + b*x^2)*(4*b^(3/4)*x^(3/2) + 3*Sqrt[2]*a^(3/4)*ArcTan[(Sq
rt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*a^(3/4)*
ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(6*b^(7
/4)*x^(5/2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {1384, 27, 262, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b\left(a+bx^2\right)\int \frac{(dx)^{5/2}}{b(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(a+bx^2\right)\int \frac{(dx)^{5/2}}{bx^2+a}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{3/2}}{3b}-\frac{ad^2\int\frac{\sqrt{dx}}{bx^2+a}dx}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 266

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad\int\frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 826

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1476

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1082

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 217

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1479

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}(\cdots}{\sqrt[4]{b}(\cdots}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}\cdots}$$

$\Big\downarrow$ 25

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\cdots\right.}{\sqrt[4]{b}\left(xd+\cdots\right.}\cdots}{2\cdots}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}\cdots}$$

$\Big\downarrow$ 27

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\cdots}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\sqrt[4]{b}\cdots}}\cdots}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}\cdots}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1103

$$\frac{(a+bx^2)\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots\right)}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input

```
Int[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*((2*d*(d*x)^(3/2))/(3*b) - (2*a*d^3*((-(ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) +
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) +
Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(
2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/b))/Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.66

method	result
risch	$\frac{2x^2 d^3 \sqrt{(b x^2+a)^2}}{3b\sqrt{dx}\,(b x^2+a)} - \frac{a\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}-1\right)\right) d^3\sqrt{(b x^2 -}}{4b^2\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}(b x^2+a)}$
default	$-\frac{(b x^2+a) d\left(3a\, d^2\sqrt{2}\,\ln\left(-\frac{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+6a\, d^2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)+6a\, d^2\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-(}{\left(\frac{a d^2}{b}\right.}\right.}{12\sqrt{(b x^2+a)^2}\, b^2\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}$

input
```
int((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*x^2/(d*x)^(1/2)*d^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-1/4*a/b^2/(a*d^2/b
)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^
(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x
)^(1/2)-1))*d^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.69

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{4\sqrt{dx}d^2x - 3\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}} b\log\left(\sqrt{dx}a^2d^7 + \left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{3}{4}} b^5\right) + 3i\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}} b\,\mathrm{l}}{}$$

input
```
integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(4*sqrt(d*x)*d^2*x - 3*(-a^3*d^10/b^7)^(1/4)*b*log(sqrt(d*x)*a^2*d^7 +
 (-a^3*d^10/b^7)^(3/4)*b^5) + 3*I*(-a^3*d^10/b^7)^(1/4)*b*log(sqrt(d*x)*a^
2*d^7 + I*(-a^3*d^10/b^7)^(3/4)*b^5) - 3*I*(-a^3*d^10/b^7)^(1/4)*b*log(sqr
t(d*x)*a^2*d^7 - I*(-a^3*d^10/b^7)^(3/4)*b^5) + 3*(-a^3*d^10/b^7)^(1/4)*b*
log(sqrt(d*x)*a^2*d^7 - (-a^3*d^10/b^7)^(3/4)*b^5))/b
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.80

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\frac{3ad^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log \cdots}{\cdots}}{b}}{12\,d}$$

input
```
integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/12*(3*a*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4
) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)
*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) -
 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sq
rt(b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*
(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/b -
8*(d*x)^(3/2)*d^2/b)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.84

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{12}\,d^2\left(\frac{8\,\sqrt{dx}x}{b} - \frac{6\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4d} - \frac{6\,\sqrt{2}(ab^3d^2\right.$$

$$+ a)$$

input
```
integrate((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/12*d^2*(8*sqrt(d*x)*x/b - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*d) - 6*sqrt
(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqr
t(d*x))/(a*d^2/b)^(1/4))/(b^4*d) + 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*d) - 3*sqrt(2)*(a*b
^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(b^4*d))*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{(d\,x)^{5/2}}{\sqrt{(b\,x^2+a)^2}}\,dx$$

input
```
int((d*x)^(5/2)/((a + b*x^2)^2)^(1/2),x)
```

output
```
int((d*x)^(5/2)/((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.50

$$\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\sqrt{d}\,d^2\left(6b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-6b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-$$

input
```
int((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x)
```

output
```
(sqrt(d)*d**2*(6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) - 6*b**(1/4)*a**(3/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2))) - 3*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1
/4)*sqrt(2) + sqrt(a) + sqrt(b)*x) + 3*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(
x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x) + 8*sqrt(x)*b*x))/(12*
b**2)
```

3.632 $\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 302

$$\begin{aligned}\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{2d\sqrt{dx}(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{\sqrt[4]{a}d^{3/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{\sqrt[4]{a}d^{3/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{\sqrt[4]{a}d^{3/2}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
2*d*(d*x)^(1/2)*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)+1/2*a^(1/4)*d^(3/2)*(b*x^2
+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(5/4)/
((b*x^2+a)^2)^(1/2)-1/2*a^(1/4)*d^(3/2)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)
*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(5/4)/((b*x^2+a)^2)^(1/2)-1/2*a^(1
/4)*d^(3/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/
(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(5/4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.50

$$\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{(dx)^{3/2}\left(a+bx^2\right)\left(4\sqrt[4]{b}\sqrt{x}+\sqrt{2}\sqrt[4]{a}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\sqrt{2}\sqrt[4]{a}\operatorname{arctanh}\left(\right.\right.}{2b^{5/4}x^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

input

```
Integrate[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output

```
((d*x)^(3/2)*(a + b*x^2)*(4*b^(1/4)*Sqrt[x] + Sqrt[2]*a^(1/4)*ArcTan[(Sqrt
[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - Sqrt[2]*a^(1/4)*ArcT
anh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(2*b^(5/4)*
x^(3/2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {1384, 27, 262, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b\left(a+bx^2\right)\int \frac{(dx)^{3/2}}{b(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{(dx)^{3/2}}{bx^2+a}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{262}$$

$$\frac{(a+bx^2)\left(\frac{2d\sqrt{dx}}{b}-\frac{ad^2\int\frac{1}{\sqrt{dx}(bx^2+a)}dx}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{(a+bx^2)\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\int\frac{1}{bx^2+a}d\sqrt{dx}}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{(a+bx^2)\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{\int\frac{d^2(\sqrt{a}d-\sqrt{b}dx)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2(\sqrt{b}xd+\sqrt{a}d)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{(a+bx^2)\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 1082

$$\frac{(a+bx^2)\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 217

$$\frac{(a+bx^2)\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1479

$$\frac{(a+bx^2)\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}\cdots\right)}{\cdots}\right)}{b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 25

$$\frac{\left(a+bx^2\right)\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{}\right.}{b}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 27

$$\frac{\left(a+bx^2\right)\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\sqrt[4]{\cdots}}\cdots\right)}{2\sqrt{a}}\cdots\right)}{b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}\cdots}$$

$\downarrow$ 1103

$$\frac{\left(a+bx^2\right)\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\cdots\right)}{2\sqrt{2}\cdots}\cdots\right)}{2\sqrt{a}}\cdots\right)}{b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}\cdots}$$

input
```
Int[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((a + b*x^2)*((2*d*Sqrt[d*x])/b - (2*a*d*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + L
og[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2
*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/b))/Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.65

method	result
risch	$\frac{2x\,d^{2}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{b\sqrt{dx}\,\left(b\,x^{2}+a\right)}-\frac{d\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)\right)\sqrt{}}{4b\left(b\,x^{2}+a\right)}$
default	$-\frac{\left(b\,x^{2}+a\right)d\left(\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+2\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\right.}{4\sqrt{\left(b\,x^{2}+a\right)^{2}}\,b}$

input
```
int((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*x/(d*x)^(1/2)*d^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-1/4/b*d*(a*d^2/b)^(1/4
)*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d
*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/
(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2
)-1))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.53

$$\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}} b\log\left(\sqrt{dx}d+\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}} b\right)+i\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}} b\log\left(\sqrt{dx}d+i\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}} b\right)-i\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}} b\log\left(\sqrt{dx}\right.}{2\,b}$$

input
```
integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((-a*d^6/b^5)^(1/4)*b*log(sqrt(d*x)*d + (-a*d^6/b^5)^(1/4)*b) + I*(-a
*d^6/b^5)^(1/4)*b*log(sqrt(d*x)*d + I*(-a*d^6/b^5)^(1/4)*b) - I*(-a*d^6/b^
5)^(1/4)*b*log(sqrt(d*x)*d - I*(-a*d^6/b^5)^(1/4)*b) - (-a*d^6/b^5)^(1/4)*
b*log(sqrt(d*x)*d - (-a*d^6/b^5)^(1/4)*b) - 4*sqrt(d*x)*d)/b
```

Sympy [F]

$$\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate((d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral((d*x)**(3/2)/sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.83

$$\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\frac{8\sqrt{dx}d^2}{b} - \frac{\left(\frac{\sqrt{2}d^4\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}d^4\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{2\sqrt{2}d^3}{}\right.}{b}}{4\,d}$$

input
```
integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(8*sqrt(d*x)*d^2/b - (sqrt(2)*d^4*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1
/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^4*l
og(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*
d^2)^(3/4)*b^(1/4)) + 2*sqrt(2)*d^3*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1
/4)*b^(1/4) + 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*
sqrt(b)*d)*sqrt(a)) + 2*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(
1/4)*b^(1/4) - 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)
*sqrt(b)*d)*sqrt(a)))*a/b)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.84

$$\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\left(\frac{2\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{2\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^2\log\left(dx\cdots\right.}{}\right.}{4d}$$

input
```
integrate((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(2*sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^2 + 2*sqrt(2)*(a*b^3*d^2)^(1/4)*
d^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^
(1/4))/b^2 + sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*log(d*x + sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/b^2 - sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*log(d*x
- sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^2 - 8*sqrt(d*x)*d^2
/b)*sgn(b*x^2 + a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{(d\,x)^{3/2}}{\sqrt{(b\,x^2+a)^2}}\,dx$$

input
```
int((d*x)^(3/2)/((a + b*x^2)^2)^(1/2),x)
```

output
```
int((d*x)^(3/2)/((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.49

$$\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\sqrt{d}\,d\left(2b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+b\right.}{}$$

input
```
int((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x)
```

output
```
(sqrt(d)*d*(2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) - 2*b**(3/4)*a**(1/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4
)*sqrt(2))) + b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*s
qrt(2) + sqrt(a) + sqrt(b)*x) - b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(
1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x) + 8*sqrt(x)*b))/(4*b**2)
```

3.633 $\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 260

$$\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\sqrt{d}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{d}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{d}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-1/2*d^(1/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))*2^(1/2)/a^(1/4)/b^(3/4)/((b*x^2+a)^2)^(1/2)+1/2*d^(1/2)*(b*x^2+a)*arcta
n(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(1/4)/b^(3/4)/(
(b*x^2+a)^2)^(1/2)-1/2*d^(1/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(
d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(1/4)/b^(3/4)/((b*x^2+a)
^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\sqrt{dx}(a+bx^2)\left(\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{x}\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
-((Sqrt[d*x]*(a + b*x^2)*(ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x])] + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqr
t[b]*x)]))/(Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[x]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1384, 27, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$$

$$\Big\downarrow \ 1384$$

$$\frac{b\left(a+bx^2\right)\int \frac{\sqrt{dx}}{b(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+bx^2\right)\int \frac{\sqrt{dx}}{bx^2+a}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 266$$

$$\frac{2(a+bx^2)\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{2d(a+bx^2)\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 826$$

$$\frac{2d(a+bx^2)\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 1476$$

$$\frac{2d(a+bx^2)\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 1082$$

$$\frac{2d(a+bx^2)\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 217$$

$$\frac{2d(a+bx^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 1479$$

$$\frac{2d(a+bx^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\ldots\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\ldots}{\ldots}\right)}\ldots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\ldots}}{2\sqrt{b}}\ldots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 25

$$\frac{2d(a+bx^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\ldots}{\sqrt[4]{b}}\right)}\ldots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\ldots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 27

$$\frac{2d(a+bx^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1103

$$\frac{2d(a+bx^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}\ldots\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\ldots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(2*d*(a + b*x^2)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) -
 (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[
d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d
]))/(2*Sqrt[b])))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{(b\,x^{2}+a) d \sqrt{2}\, \left(\ln \left(-\frac{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{d x}\, \sqrt{2}-d x -\sqrt{\frac{a\,d^{2}}{b}}}{d x +\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}} \sqrt{d x}\, \sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{d x}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{d x}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)\right)}{4 \sqrt{\left(b\,x^{2}+a\right)^{2}}\, b \left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}$	183

input
```
int((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/((b*x^2+a)^2)^(1/2)*(b*x^2+a)*d/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln(-((a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(
a*d^2/b)^(1/4)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{2}\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}+\sqrt{dx}d\right) - \frac{1}{2}i\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\log\left(i\,ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}+\sqrt{dx}d\right) + \frac{1}{2}i\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\log\left(-i\,ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}+\sqrt{dx}d\right) - \frac{1}{2}\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}+\sqrt{dx}d\right)$$

input
```
integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-d^2/(a*b^3))^(1/4)*log(a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)*d) - 1
/2*I*(-d^2/(a*b^3))^(1/4)*log(I*a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)*d)
+ 1/2*I*(-d^2/(a*b^3))^(1/4)*log(-I*a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)
*d) - 1/2*(-d^2/(a*b^3))^(1/4)*log(-a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)
*d)
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{\sqrt{dx}}{\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate((d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(sqrt(d*x)/sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{4}d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}\right.}{}\right.$$

input
```
integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqr
t(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d
*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b)) -
sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a
)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)^(
1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(\frac{2\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}+\frac{2\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}-\frac{\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\cdots\right.\right.}{ab^3}\cdots}{4\,d}$$

input
```
integrate((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4)
)/(a*b^3) - sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(a*b^3) + sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqr
t(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3))*sgn(b*x^2 + a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{\sqrt{d\,x}}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

input
```
int((d*x)^(1/2)/((a + b*x^2)^2)^(1/2),x)
```

output
```
int((d*x)^(1/2)/((a + b*x^2)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$$
$$=\frac{\sqrt{d}\,\sqrt{2}\left(-2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+\log\left(-\sqrt{x}\,b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+\sqrt{a}+\sqrt{b}\,x\right)-\log\left(\right.}{4b^{\frac{3}{4}}a^{\frac{1}{4}}}$$

input
```
int((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x)
```

output
```
(sqrt(d)*b**(1/4)*a**(3/4)*sqrt(2)*( - 2*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) + 2*atan((b**(1/4)*a**(1/
4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) + log( - sqrt
(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x) - log(sqrt(x)*b**(1/4
)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)))/(4*a*b)
```

3.634 $\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 259

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-1/2*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/
2)/a^(3/4)/b^(1/4)/d^(1/2)/((b*x^2+a)^2)^(1/2)+1/2*(b*x^2+a)*arctan(1+2^(1
/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(3/4)/b^(1/4)/d^(1/2)/(
(b*x^2+a)^2)^(1/2)+1/2*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/
2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(3/4)/b^(1/4)/d^(1/2)/((b*x^2+a)
^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\sqrt{x}(a+bx^2)\left(\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{dx}\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
-((Sqrt[x]*(a + b*x^2)*(ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x])] - ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[
b]*x)]))/(Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1384, 27, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^2)\int \frac{1}{b\sqrt{dx}(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{1}{\sqrt{dx}(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\left(a+bx^2\right)\int \frac{1}{bx^2+a}d\sqrt{dx}}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 755$$

$$\frac{2\left(a+bx^2\right)\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{2\left(a+bx^2\right)\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 1476$$

$$\frac{2\left(a+bx^2\right)\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 1082$$

$$\frac{2\left(a+bx^2\right)\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 217$$

$$\frac{2(a+bx^2)\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1479

$$\frac{2(a+bx^2)\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\right.}{2\sqrt{a}}\right.}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 25

$$\frac{2(a+bx^2)\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan}{\sqrt{}}\right.}{2\sqrt{a}}\right.}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 27

$$\frac{2(a+bx^2)\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1103

$$\frac{2(a+bx^2)\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\dots\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{d\sqrt{a^2+2abx^2+b^2x^4}}$$

input

```
Int[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output

```
(2*(a + b*x^2)*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])
 + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*
Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d])))/(2*Sqrt[a])))/(d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.70

method	result	si
default	$\frac{(b\,x^2+a)\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)\right)}{4\sqrt{(b\,x^2+a)^2}\,da}$	1

input
```
int(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/((b*x^2+a)^2)^(1/2)*(b*x^2+a)/d*(a*d^2/b)^(1/4)/a*2^(1/2)*(ln((d*x+(a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)
^(1/4))/(a*d^2/b)^(1/4))+2*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a
*d^2/b)^(1/4)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{2}\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\log\left(ad\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right) + \frac{1}{2}i\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\log\left(i\,ad\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right) - \frac{1}{2}i\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\log\left(-i\,ad\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right) - \frac{1}{2}\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\log\left(-ad\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)$$

input
```
integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-1/(a^3*b*d^2))^(1/4)*log(a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*x)) + 1
/2*I*(-1/(a^3*b*d^2))^(1/4)*log(I*a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*x))
- 1/2*I*(-1/(a^3*b*d^2))^(1/4)*log(-I*a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*
x)) - 1/2*(-1/(a^3*b*d^2))^(1/4)*log(-a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*
x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{\sqrt{dx}\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(d*x)*sqrt((a + b*x**2)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\frac{\sqrt{2}d^2\log\left(\sqrt{b}dx+\sqrt{2}(ad^2)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}d^2\log\left(\sqrt{b}dx-\sqrt{2}(ad^2)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{(ad^2)^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}}}{4\,d}$$

input
```
integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(sqrt(2)*d^2*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4)
 + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) - sqrt(2)*d^2*log(sqrt(b)*d*x - sqrt
(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(3/4)*b^(1/4)) +
 2*sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*
x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)) + 2
*sqrt(2)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x
)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)))/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{4}\left(\frac{2\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{abd} + \frac{2\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{abd} + \cdots + a\right)$$

input
```
integrate(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b*d) + 2*sqrt(2)*(a*b^3*d^2)^(1/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4)
)/(a*b*d) + sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(a*b*d) - sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x - sqr
t(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b*d))*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{\sqrt{d\,x}\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

input
```
int(1/((d*x)^(1/2)*((a + b*x^2)^2)^(1/2)),x)
```

output
```
int(1/((d*x)^(1/2)*((a + b*x^2)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\sqrt{d}\,\sqrt{2}\left(-2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-\log\left(-\sqrt{x}\,b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+\sqrt{a}+\sqrt{b}\,x\right)+\log\left(\right.}{4b^{\frac{1}{4}}a^{\frac{3}{4}}d}$$

input
```
int(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x)
```

output
```
(sqrt(d)*b**(3/4)*a**(1/4)*sqrt(2)*( - 2*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) + 2*atan((b**(1/4)*a**(1/
4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) - log( - sqrt
(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x) + log(sqrt(x)*b**(1/4
)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)))/(4*a*b*d)
```

3.635 $\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 303

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{2(a+bx^2)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{b}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt[4]{b}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt[4]{b}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
(-2*b*x^2-2*a)/a/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)+1/2*b^(1/4)*(b*x^2+a)*a
rctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(5/4)/d^(3/
2)/((b*x^2+a)^2)^(1/2)-1/2*b^(1/4)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*x
)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(5/4)/d^(3/2)/((b*x^2+a)^2)^(1/2)+1/2*b
^(1/4)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1
/2)+b^(1/2)*x))*2^(1/2)/a^(5/4)/d^(3/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.50

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{x(a+bx^2)\left(-4\sqrt[4]{a}+\sqrt{2}\sqrt[4]{b}\sqrt{x}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\sqrt{2}\sqrt[4]{b}\sqrt{x}\text{arc}\right.}{2a^{5/4}(dx)^{3/2}\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
(x*(a + b*x^2)*(-4*a^(1/4) + Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTan[(Sqrt[a] - Sqr
t[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTa
nh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(2*a^(5/4)*(
d*x)^(3/2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {1384, 27, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b(a+bx^2)\int \frac{1}{b(dx)^{3/2}(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{1}{(dx)^{3/2}(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{(a+bx^2)\left(-\frac{b\int \frac{\sqrt{dx}}{bx^2+a}dx}{ad^2}-\frac{2}{ad\sqrt{dx}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \textbf{266}$$

$$\frac{(a+bx^2)\left(-\frac{2b\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{ad^3}-\frac{2}{ad\sqrt{dx}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{(a+bx^2)\left(-\frac{2b\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \textbf{826}$$

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \textbf{1476}$$

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \textbf{1082}$$

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 217

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1479

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}}{2\sqrt{b}}\right)}{ad}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 25

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{d\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d\cdots}}{\sqrt[4]{b}}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 27

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1103

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right.}{ad}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
((a + b*x^2)*(-2/(a*d*Sqrt[d*x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[
a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(a*d)))/Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{2\sqrt{(b\,x^{2}+a)^{2}}}{a\sqrt{dx}\,d(b\,x^{2}+a)}-\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}-1\right)\right)\sqrt{(b\,x^{2}+a)}}{4a\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}d(b\,x^{2}+a)}$
default	$-\frac{(b\,x^{2}+a)\left(\sqrt{2}\,\sqrt{dx}\,\ln\left(-\frac{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\sqrt{2}\,\sqrt{dx}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)+2\sqrt{2}\,\sqrt{dx}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\right.}{\left(\frac{a\,d^{2}}{b}\right.}\right.}{4d\sqrt{(b\,x^{2}+a)^{2}}\,a\sqrt{dx}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}$

input
```
int(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a/(d*x)^(1/2)/d*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-1/4/a/(a*d^2/b)^(1/4)*2^(
1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^
2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1))
/d*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.66

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx =$$
$$-\frac{ad^2x\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\log\left(a^4d^5\left(-\frac{b}{a^5d^6}\right)^{\frac{3}{4}}+\sqrt{dx}b\right)-i\,ad^2x\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\log\left(i\,a^4d^5\left(-\frac{b}{a^5d^6}\right)^{\frac{3}{4}}+\sqrt{dx}b\right)+i\,ad^2x\left(-\right.}{2\,ad^2x}$$

input
```
integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(a*d^2*x*(-b/(a^5*d^6))^(1/4)*log(a^4*d^5*(-b/(a^5*d^6))^(3/4) + sqrt
(d*x)*b) - I*a*d^2*x*(-b/(a^5*d^6))^(1/4)*log(I*a^4*d^5*(-b/(a^5*d^6))^(3/
4) + sqrt(d*x)*b) + I*a*d^2*x*(-b/(a^5*d^6))^(1/4)*log(-I*a^4*d^5*(-b/(a^5
*d^6))^(3/4) + sqrt(d*x)*b) - a*d^2*x*(-b/(a^5*d^6))^(1/4)*log(-a^4*d^5*(-
b/(a^5*d^6))^(3/4) + sqrt(d*x)*b) + 4*sqrt(d*x))/(a*d^2*x)
```

Sympy [F]

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate(1/(d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral(1/((d*x)**(3/2)*sqrt((a + b*x**2)**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.77

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(ad^2)^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}dx+\sqrt{2}(ad^2)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{(ad^2)^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{\right.}{}}{}\cdots}{a}}{4\,d}$$

input
```
integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*s
qrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt
(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(b))
- sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt
(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*d^2)
^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/a + 8/(sqrt
(d*x)*a))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.87

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\left(\frac{8}{\sqrt{dx}a}+\frac{2\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2d^2}+\frac{2\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2d^2}-\frac{\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\log\left(\right.}{}\right.}{4d}$$

input
```
integrate(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(8/(sqrt(d*x)*a) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*d^2) + 2*sq
rt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*s
qrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*d^2) - sqrt(2)*(a*b^3*d^2)^(3/4)*log(d
*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^2*d^2) + sq
rt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/(a^2*b^2*d^2))*sgn(b*x^2 + a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{(d\,x)^{3/2}\,\sqrt{(b\,x^2+a)^2}}\,dx$$

input
```
int(1/((d*x)^(3/2)*((a + b*x^2)^2)^(1/2)),x)
```

output
```
int(1/((d*x)^(3/2)*((a + b*x^2)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.53

$$\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\sqrt{d}\left(2\sqrt{x}\,b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2\sqrt{x}\,b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{\cdots}}{b^{\frac{1}{4}}\cdots}\right.\cdots}{\cdots}$$

input
```
int(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x)
```

output
```
(sqrt(d)*(2*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) - 2*sqrt(x)*b**(1/4)
*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2))) - sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(
x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x) + sqrt(x)*b**(1/4)*a**
(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)
 - 8*a))/(4*sqrt(x)*a**2*d**2)
```

3.636 $\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 306

$$\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{2(a+bx^2)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{3/4}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b^{3/4}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b^{3/4}(a+bx^2)\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/3*(-2*b*x^2-2*a)/a/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+1/2*b^(3/4)*(b*x^2+
a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(7/4)/d
^(5/2)/((b*x^2+a)^2)^(1/2)-1/2*b^(3/4)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*
(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(7/4)/d^(5/2)/((b*x^2+a)^2)^(1/2)-1
/2*b^(3/4)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(
a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(7/4)/d^(5/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.50

$$\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{x(a+bx^2)\left(-4a^{3/4}+3\sqrt{2}b^{3/4}x^{3/2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-3\sqrt{2}b^{3/4}x}{6a^{7/4}(dx)^{5/2}\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
(x*(a + b*x^2)*(-4*a^(3/4) + 3*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTan[(Sqrt[a] - S
qrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 3*Sqrt[2]*b^(3/4)*x^(3/2)*A
rcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(6*a^(7/
4)*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {1384, 27, 264, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b(a+bx^2)\int \frac{1}{b(dx)^{5/2}(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{1}{(dx)^{5/2}(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{(a+bx^2)\left(-\frac{b\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{ad^2}-\frac{2}{3ad(dx)^{3/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 266

$$\frac{(a+bx^2)\left(-\frac{2b\int \frac{1}{bx^2+a}d\sqrt{dx}}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 755

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1476

$$\frac{\left(a+bx^2\right)\left(-\frac{2b\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\left(a+bx^2\right)\left(-\frac{2b\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 217$$

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1479

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{d\left(-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\right.}{2\sqrt{a}}\right.}{ad^3}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 25

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\text{arcta}\cdots}{\cdots}\right)}{2\sqrt{a}}\cdots\right)}{ad^3}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 27

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{d}\cdots}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots}{2\sqrt{a}}\cdots\right)}{ad^3}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1103

$$\frac{(a+bx^2)\left(-\frac{2b\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}\cdots\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\cdots\right)}{2\sqrt{a}}\cdots\right)}{ad^3}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
((a + b*x^2)*(-2/(3*a*d*(d*x)^(3/2)) - (2*b*((d*(-(ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])
+ Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]
/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(a*d^3)))/Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{2\sqrt{(b\,x^2+a)^2}}{3ax\sqrt{dx}\,d^2(b\,x^2+a)}-\frac{b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}-1\right)\right.}{4a^2d^3(b\,x^2+a)}$
default	$-\frac{(b\,x^2+a)\left(3b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,(dx)^{\frac{3}{2}}\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+6b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,(dx)^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)+6b\left(\frac{a\,d^2}{b}\right.\right.}{12d^3\sqrt{(b\,x^2+a)^2}\,a^2(dx)^{\frac{3}{2}}}$

input
```
int(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a/x/(d*x)^(1/2)/d^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-1/4/a^2*b/d^3*(a*d^
2/b)^(1/4)*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan
(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(
d*x)^(1/2)-1))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.74

$$\int\frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx=$$

$$-\frac{3\,ad^3x^2\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\log\left(a^2d^3\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}+\sqrt{dx}b\right)+3i\,ad^3x^2\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}\log\left(i\,a^2d^3\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac{1}{4}}+\sqrt{dx}b\right)}{}$$

input
```
integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(3*a*d^3*x^2*(-b^3/(a^7*d^10))^(1/4)*log(a^2*d^3*(-b^3/(a^7*d^10))^(1
/4) + sqrt(d*x)*b) + 3*I*a*d^3*x^2*(-b^3/(a^7*d^10))^(1/4)*log(I*a^2*d^3*(
-b^3/(a^7*d^10))^(1/4) + sqrt(d*x)*b) - 3*I*a*d^3*x^2*(-b^3/(a^7*d^10))^(1
/4)*log(-I*a^2*d^3*(-b^3/(a^7*d^10))^(1/4) + sqrt(d*x)*b) - 3*a*d^3*x^2*(-
b^3/(a^7*d^10))^(1/4)*log(-a^2*d^3*(-b^3/(a^7*d^10))^(1/4) + sqrt(d*x)*b)
+ 4*sqrt(d*x))/(a*d^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.79

$$\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\frac{3\left(\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}dx+\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}dx-\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}\sqrt{dx}b^{\frac{1}{4}}+\sqrt{a}d\right)}{\left(ad^2\right)^{\frac{3}{4}}}+\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{dx}\sqrt{b}\right)}{2\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{a}d}+\frac{2\sqrt{2}b\,\mathrm{arct}}{}}{a}}{12\,d}$$

input
```
integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/12*(3*(sqrt(2)*b^(3/4)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x
)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*d*x - s
qrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/(a*d^2)^(3/4) + 2*sqrt
(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) + 2*sqrt(d*x)*sqrt
(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d) + 2*sqrt
(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*sqrt(d*x)*sqr
t(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(a)*d))/a + 8/
((d*x)^(3/2)*a))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.84

$$\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{1}{12}\left(\frac{6\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^3} + \frac{6\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^3} + \cdots + a\right)$$

input
```
integrate(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/12*(6*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*d^3) + 6*sqrt(2)*(a*b^3*d^2)^(1/
4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^2*d^3) + 3*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^
(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*d^3) - 3*sqrt(2)*(a*b^3*d^2)^(1/4)*l
og(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*d^3) + 8/
(sqrt(d*x)*a*d^2*x))*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{1}{(d\,x)^{5/2}\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

input
```
int(1/((d*x)^(5/2)*((a + b*x^2)^2)^(1/2)),x)
```

output
```
int(1/((d*x)^(5/2)*((a + b*x^2)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.55

$$\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{\sqrt{d}\left(6\sqrt{x}\,b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x-6\sqrt{x}\,b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}}{b}\right.\right.}{}$$

input
```
int(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x)
```

output
```
(sqrt(d)*(6*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*x - 6*sqrt(x)*b**(3/
4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*x + 3*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log( -
 sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*x - 3*sqrt(x)*b*
*(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) +
sqrt(b)*x)*x - 8*a))/(12*sqrt(x)*a**2*d**3*x)
```

3.637 $\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 351

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{2(a+bx^2)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{2b(a+bx^2)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b^{5/4}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{b^{5/4}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{b^{5/4}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/5*(-2*b*x^2-2*a)/a/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)+2*b*(b*x^2+a)/a^2/d
^3/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)-1/2*b^(5/4)*(b*x^2+a)*arctan(1-2^(1/2)*
b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(9/4)/d^(7/2)/((b*x^2+a)^2)
^(1/2)+1/2*b^(5/4)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/
d^(1/2))*2^(1/2)/a^(9/4)/d^(7/2)/((b*x^2+a)^2)^(1/2)-1/2*b^(5/4)*(b*x^2+a)
*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*
2^(1/2)/a^(9/4)/d^(7/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.46

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = -\frac{x(a+bx^2)\left(4\sqrt[4]{a}(a-5bx^2)+5\sqrt{2}b^{5/4}x^{5/2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+5\sqrt{2}b^{5/4}x^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)\right.}{10a^{9/4}(dx)^{7/2}\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
-1/10*(x*(a + b*x^2)*(4*a^(1/4)*(a - 5*b*x^2) + 5*Sqrt[2]*b^(5/4)*x^(5/2)*
ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*Sqrt[2
]*b^(5/4)*x^(5/2)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqr
t[b]*x)]))/(a^(9/4)*(d*x)^(7/2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1384, 27, 264, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b(a+bx^2)\int \frac{1}{b(dx)^{7/2}(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^2)\int \frac{1}{(dx)^{7/2}(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 264$$

$$\frac{(a+bx^2)\left(-\frac{b\int \frac{1}{(dx)^{3/2}(bx^2+a)}dx}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 264$$

$$\frac{(a+bx^2)\left(-\frac{b\left(-\frac{b\int \frac{\sqrt{dx}}{bx^2+a}dx}{ad^2}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 266$$

$$\frac{(a+bx^2)\left(-\frac{b\left(-\frac{2b\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{ad^3}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{(a+bx^2)\left(-\frac{b\left(-\frac{2b\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 826$$

$$\frac{(a+bx^2)\left(-\frac{b\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 1476$$

$$\frac{(a+bx^2)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1082

$$\frac{\left(a+bx^2\right)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{217}$$

$$\frac{(a+bx^2)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1479

$$\frac{(a+bx^2)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\cdots\right.}}{2\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{b}}\cdots\right)}{ad}\cdots\right)}{ad^2}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 25

$$\frac{(a+bx^2)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}}{2\sqrt{b}}\right.}{ad}\right.}{ad^2}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 27

$$\frac{(a+bx^2)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\cdots}{2\sqrt{b}}\cdots\right)}{ad}\cdots\right)}{ad^2}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1103

$$\frac{(a+bx^2)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{ad}\cdots\right)}{ad^2}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

output
```
((a + b*x^2)*(-2/(5*a*d*(d*x)^(5/2)) - (b*(-2/(a*d*Sqrt[d*x]) - (2*b*((-(A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d +
 Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(a*d
)))/(a*d^2)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{2(-5bx^2+a)\sqrt{(bx^2+a)^2}}{5a^2\sqrt{dx}\,x^2d^3(bx^2+a)}+\frac{b\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{dx}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{dx}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}-1\right)\right)\cdots}{4a^2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}d^3(bx^2+a)}$
default	$-\frac{(bx^2+a)\left(-5b\sqrt{2}\,(dx)^{\frac{5}{2}}\ln\left(-\frac{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)-10b\sqrt{2}\,(dx)^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)-10b\sqrt{2}\,(dx)^{\frac{5}{2}}\arctan\cdots\right)}{20d^3\sqrt{(bx^2+a)^2}\,a^2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}(dx)^{\frac{5}{2}}}$

input
```
int(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-5*b*x^2+a)/a^2/(d*x)^(1/2)/x^2/d^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/
4*b/a^2/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d
^2/b)^(1/4)*(d*x)^(1/2)-1))/d^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.72

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{5\,a^2d^4x^3\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{1}{4}}\log\left(a^7d^{11}\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{3}{4}}+\sqrt{dx}b^4\right)-5i\,a^2d^4x^3\left(-\frac{b^5}{a^9d^{14}}\right.}{}$$

input
```
integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/10*(5*a^2*d^4*x^3*(-b^5/(a^9*d^14))^(1/4)*log(a^7*d^11*(-b^5/(a^9*d^14))
^(3/4) + sqrt(d*x)*b^4) - 5*I*a^2*d^4*x^3*(-b^5/(a^9*d^14))^(1/4)*log(I*a^
7*d^11*(-b^5/(a^9*d^14))^(3/4) + sqrt(d*x)*b^4) + 5*I*a^2*d^4*x^3*(-b^5/(a
^9*d^14))^(1/4)*log(-I*a^7*d^11*(-b^5/(a^9*d^14))^(3/4) + sqrt(d*x)*b^4) -
 5*a^2*d^4*x^3*(-b^5/(a^9*d^14))^(1/4)*log(-a^7*d^11*(-b^5/(a^9*d^14))^(3/
4) + sqrt(d*x)*b^4) + 4*(5*b*x^2 - a)*sqrt(d*x))/(a^2*d^4*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.74

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{5\,b^2\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(ad^2\right)^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{dx}\sqrt{b}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}d}}\right)}{\sqrt{\sqrt{a}\sqrt{b}d}\sqrt{b}}-\right.}{a^2d^2}$$

input
```
integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/20*(5*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) +
 2*sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sq
rt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2)^(1/4)*b^(1/4) - 2*
sqrt(d*x)*sqrt(b))/sqrt(sqrt(a)*sqrt(b)*d))/(sqrt(sqrt(a)*sqrt(b)*d)*sqrt(
b)) - sqrt(2)*log(sqrt(b)*d*x + sqrt(2)*(a*d^2)^(1/4)*sqrt(d*x)*b^(1/4) +
sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*d*x - sqrt(2)*(a*
d^2)^(1/4)*sqrt(d*x)*b^(1/4) + sqrt(a)*d)/((a*d^2)^(1/4)*b^(3/4)))/(a^2*d^
2) + 8*(5*b*d^2*x^2 - a*d^2)/((d*x)^(5/2)*a^2*d^2))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.81

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{20}\left(\frac{10\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3bd^5}+\frac{10\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ldots}{\ldots}\right.$$

$$+\,a)$$

input
```
integrate(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/20*(10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d^5) + 10*sqrt(2)*(a*b^3*d^2)^
(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/(a^3*b*d^5) - 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^
2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d^5) + 5*sqrt(2)*(a*b^3*d^2)^
(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*
d^5) + 8*(5*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^2*d^5*x^2))*sgn(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{(d\,x)^{7/2}\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

input
```
int(1/((d*x)^(7/2)*((a + b*x^2)^2)^(1/2)),x)
```

output
```
int(1/((d*x)^(7/2)*((a + b*x^2)^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.53

$$\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\sqrt{d}\left(-10\sqrt{x}\,b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2+10\sqrt{x}\,b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\right.\right.}{}$$

input
```
int(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x)
```

output
```
(sqrt(d)*( - 10*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**2 + 10*sqrt
(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*
sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**2 + 5*sqrt(x)*b**(1/4)*a**(3/4)
*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b
*x**2 - 5*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*
sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 - 8*a**2 + 40*a*b*x**2))/(20*sqrt(x)
*a**3*d**4*x**2)
```

3.638 $\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 445

$$
\begin{aligned}
\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx &= -\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{d(dx)^{13/2}}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117ad^7\sqrt{dx}(a+bx^2)}{16b^4\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{117d^5(dx)^{5/2}(a+bx^2)}{80b^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117a^{5/4}d^{15/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{117a^{5/4}d^{15/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{117a^{5/4}d^{15/2}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{32\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

output
```
-13/16*d^3*(d*x)^(9/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(13/2)/b/(b*x^2
+a)/((b*x^2+a)^2)^(1/2)-117/16*a*d^7*(d*x)^(1/2)*(b*x^2+a)/b^4/((b*x^2+a)^
2)^(1/2)+117/80*d^5*(d*x)^(5/2)*(b*x^2+a)/b^3/((b*x^2+a)^2)^(1/2)-117/64*a
^(5/4)*d^(15/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(
1/2))*2^(1/2)/b^(17/4)/((b*x^2+a)^2)^(1/2)+117/64*a^(5/4)*d^(15/2)*(b*x^2+
a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(17/4)/
((b*x^2+a)^2)^(1/2)+117/64*a^(5/4)*d^(15/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(1
/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(17/4)/((b*
x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.46

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{d^7\sqrt{dx}\Big(4\sqrt[4]{b}\sqrt{x}(-585a^3-1053a^2bx^2-416ab^2x^4+32b^3x^6)-585\sqrt{2}a^{5/4}(\cdots}{320b^{17/4}\sqrt{x}\,(a\cdots}$$

input
```
Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(d^7*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(-585*a^3 - 1053*a^2*b*x^2 - 416*a*b^2*x
^4 + 32*b^3*x^6) - 585*Sqrt[2]*a^(5/4)*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqr
t[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 585*Sqrt[2]*a^(5/4)*(a + b*x^
2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(3
20*b^(17/4)*Sqrt[x]*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.93, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1384, 27, 252, 252, 262, 262, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3\left(a+bx^2\right)\int \frac{(dx)^{15/2}}{b^3\left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{(dx)^{15/2}}{\left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\left(a+bx^2\right)\left(\frac{13d^2\int \frac{(dx)^{11/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\left(a+bx^2\right)\left(\frac{13d^2\left(\frac{9d^2\int \frac{(dx)^{7/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\left(a+bx^2\right)\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\int \frac{(dx)^{3/2}}{bx^2+a}dx}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{262}$$

$$\frac{(a+bx^2)\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{ad^2\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 266

$$\frac{(a+bx^2)\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\int\frac{1}{bx^2+a}d\sqrt{dx}}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 755

$$\frac{(a+bx^2)\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^2)\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 1476$$

$$13d^2 \Bigg(\frac{9d^2 \left(\frac{2d(dx)^{5/2}}{5b} - \frac{ad^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{2ad \left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}} + \frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}} + \frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d}{2\sqrt{b}}\right.}{2\sqrt{a}}\right.}{b}\right.}{b}\right.}{4b}$$

↓ 1082

$$13d^2\left(\frac{\left(9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{\left(ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)\cdots}{b}\cdots\right)\cdots}{4b}\cdots\right.$$

↓ 217

$$13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(\dots}{2b(\dots}\right.$$

↓ 1479

$$13d^2 \frac{\left(9d^2 \left(\frac{2d(dx)^{5/2}}{5b} - \frac{\left(ad^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{\left(2ad \frac{\left(d \left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{\cdots}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.\right.}{2\sqrt{a}}\right.}{b}\right.}{b}\right.}{4b}$$

↓ 25

$$13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right.}{b}\right.}{b}\right.}{4b}\right.$$

$\Big\downarrow$ 27

$$\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\ldots}{\ldots}\right)}{\sqrt{2}\sqrt[4]{\ldots}}\ldots\right)}{\ldots}$$

$$13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\ldots\right)}{b}\right)}{b}\right)\ldots}{4b}\ldots\right)$$

↓ 1103

$$13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{ad}\cdots\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right.}{\cdots}\right)}{b}\right.}{b}\right.}{4b}\right.$$

input
```
Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(-1/4*(d*(d*x)^(13/2))/(b*(a + b*x^2)^2) + (13*d^2*(-1/2*(d*(
d*x)^(9/2))/(b*(a + b*x^2)) + (9*d^2*((2*d*(d*x)^(5/2))/(5*b) - (a*d^2*((2
*d*Sqrt[d*x])/b - (2*a*d*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(
2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + S
qrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/b))/b))/(4*b)))/(8*b)))/Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{2\left(-b\,x^{2}+15a\right)x\,d^{8}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{5b^{4}\sqrt{dx}\,\left(b\,x^{2}+a\right)}+\frac{2a^{2}d^{9}\left(\frac{-\frac{25b\left(dx\right)^{\frac{5}{2}}}{32}-\frac{21\sqrt{dx}\,a\,d^{2}}{32}}{\left(b\,d^{2}x^{2}+a\,d^{2}\right)^{2}}+\frac{117\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\arctan\ldots}{256a\,d^{2}}\right.}{b^{4}\left(b\,x^{2}+a\right)}$
default	$-\frac{\left(-256\left(dx\right)^{\frac{5}{2}}b^{3}x^{4}-585\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)a\,b^{2}d^{2}x^{4}-1170\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)\ldots\right.}{\ldots}$

input
```
int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-b*x^2+15*a)*x/b^4/(d*x)^(1/2)*d^8*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+2*a
^2/b^4*d^9*((-25/32*b*(d*x)^(5/2)-21/32*(d*x)^(1/2)*a*d^2)/(b*d^2*x^2+a*d^
2)^2+117/256*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2
^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{585\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}(b^6x^4+2\,ab^5x^2+a^2b^4)\log\left(117\sqrt{dx}ad^7+117\left(-\frac{a^5d^{30}}{b^{17}}\right)^{\frac{1}{4}}b\right.}{}$$

input
```
integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

output
```
1/320*(585*(-a^5*d^30/b^17)^(1/4)*(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*log(11
7*sqrt(d*x)*a*d^7 + 117*(-a^5*d^30/b^17)^(1/4)*b^4) - 585*(-a^5*d^30/b^17)
^(1/4)*(-I*b^6*x^4 - 2*I*a*b^5*x^2 - I*a^2*b^4)*log(117*sqrt(d*x)*a*d^7 +
117*I*(-a^5*d^30/b^17)^(1/4)*b^4) - 585*(-a^5*d^30/b^17)^(1/4)*(I*b^6*x^4
+ 2*I*a*b^5*x^2 + I*a^2*b^4)*log(117*sqrt(d*x)*a*d^7 - 117*I*(-a^5*d^30/b^
17)^(1/4)*b^4) - 585*(-a^5*d^30/b^17)^(1/4)*(b^6*x^4 + 2*a*b^5*x^2 + a^2*b
^4)*log(117*sqrt(d*x)*a*d^7 - 117*(-a^5*d^30/b^17)^(1/4)*b^4) + 4*(32*b^3*
d^7*x^6 - 416*a*b^2*d^7*x^4 - 1053*a^2*b*d^7*x^2 - 585*a^3*d^7)*sqrt(d*x))
/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^{\frac{15}{2}}}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

output
```
-1/2*a^2*d^(15/2)*x^(5/2)/(a*b^4*x^2 + a^2*b^3 + (b^5*x^2 + a*b^4)*x^2) -
2*a*d^(15/2)*integrate(x^(3/2)/(b^4*x^2 + a*b^3), x) + d^(15/2)*integrate(
x^(7/2)/(b^3*x^2 + a*b^2), x) + 21/128*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sq
rt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/
4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b
)) + sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqr
t(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqr
t(b)*x + sqrt(a))/b^(1/4))*d^(15/2)/b^4 - 1/16*(17*a^2*b*d^(15/2)*x^(5/2)
+ 21*a^3*d^(15/2)*sqrt(x))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.87

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\frac{1170\sqrt{2}(ab^3d^2)^{\frac{1}{4}}ad^8\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5\mathrm{sgn}(bx^2+a)} + \frac{1170\sqrt{2}(ab^3d^2)^{\frac{1}{4}}ad^8\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{}\right.}{b^5\mathrm{sgn}(bx^2+a)}}{}$$

input
```
integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/640*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^8*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*sgn(b*x^2 + a)) + 1170*
sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^8*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*sgn(b*x^2 + a)) + 585*sqrt(2)*(a*b
^3*d^2)^(1/4)*a*d^8*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d
^2/b))/(b^5*sgn(b*x^2 + a)) - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^8*log(d*x
- sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^5*sgn(b*x^2 + a))
- 40*(25*sqrt(d*x)*a^2*b*d^12*x^2 + 21*sqrt(d*x)*a^3*d^12)/((b*d^2*x^2 + a
*d^2)^2*b^4*sgn(b*x^2 + a)) + 256*(sqrt(d*x)*b^12*d^8*x^2 - 15*sqrt(d*x)*a
*b^11*d^8)/(b^15*sgn(b*x^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\int \frac{(d\,x)^{15/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.13

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output

```
(sqrt(d)*d**7*( - 1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 - 2340*b**(3
/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/
(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 - 1170*b**(3/4)*a**(1/4)*sqrt(2)*
atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sq
rt(2)))*a*b**2*x**4 + 1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 2340*b
**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(
b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 + 1170*b**(3/4)*a**(1/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4
)*sqrt(2)))*a*b**2*x**4 - 585*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**
(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3 - 1170*b**(3/4)*a**(1/4
)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*
a**2*b*x**2 - 585*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/
4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 + 585*b**(3/4)*a**(1/4)*sqrt
(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3 + 11
70*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(
a) + sqrt(b)*x)*a**2*b*x**2 + 585*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b*
*(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 - 4680*sqrt(x)*
a**3*b - 8424*sqrt(x)*a**2*b**2*x**2 - 3328*sqrt(x)*a*b**3*x**4 + 256*s...
```

3.639 $\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 398

$$\begin{aligned}\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx =& -\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} \\ & -\frac{d(dx)^{11/2}}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^5(dx)^{3/2}(a+bx^2)}{48b^3\sqrt{a^2+2abx^2+b^2x^4}} \\ & +\frac{77a^{3/4}d^{13/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ & -\frac{77a^{3/4}d^{13/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ & +\frac{77a^{3/4}d^{13/2}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{32\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
-11/16*d^3*(d*x)^(7/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(11/2)/b/(b*x^2
+a)/((b*x^2+a)^2)^(1/2)+77/48*d^5*(d*x)^(3/2)*(b*x^2+a)/b^3/((b*x^2+a)^2)^
(1/2)+77/64*a^(3/4)*d^(13/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2
)/a^(1/4)/d^(1/2))*2^(1/2)/b^(15/4)/((b*x^2+a)^2)^(1/2)-77/64*a^(3/4)*d^(1
3/2)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/
2)/b^(15/4)/((b*x^2+a)^2)^(1/2)+77/64*a^(3/4)*d^(13/2)*(b*x^2+a)*arctanh(2
^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^
(15/4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.49

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{d^6\sqrt{dx}\Big(4b^{3/4}x^{3/2}(77a^2+121abx^2+32b^2x^4)+231\sqrt{2}a^{3/4}(a+bx^2)^2\arctan}{192b^{15/4}\sqrt{x}\,(a+bx^2)}$$

input
```
Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(d^6*Sqrt[d*x]*(4*b^(3/4)*x^(3/2)*(77*a^2 + 121*a*b*x^2 + 32*b^2*x^4) + 23
1*Sqrt[2]*a^(3/4)*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x])] + 231*Sqrt[2]*a^(3/4)*(a + b*x^2)^2*ArcTanh[(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(192*b^(15/4)*Sqrt[x]*(
a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1384, 27, 252, 252, 262, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}dx$$

$\Big\downarrow$ 1384

$$\frac{b^3\left(a+bx^2\right)\int \frac{(dx)^{13/2}}{b^3\left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{\left(a+bx^2\right)\int \frac{(dx)^{13/2}}{\left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{\left(a+bx^2\right)\left(\frac{11d^2\int \frac{(dx)^{9/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{\left(a+bx^2\right)\left(\frac{11d^2\left(\frac{7d^2\int \frac{(dx)^{5/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 262

$$\frac{\left(a+bx^2\right)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{ad^2\int \frac{\sqrt{dx}}{bx^2+a}dx}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 266

$$\frac{(a+bx^2)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad\int\frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 27

$$\frac{(a+bx^2)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 826

$$\frac{(a+bx^2)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1476

$$\frac{(a+bx^2)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)}{2b\left(a+\right.}}{8b}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1082

$$\frac{\left(a+bx^2\right)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b} -\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

↓ 217

$$\frac{\left(a+bx^2\right)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx}{4b(a+}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

↓ 1479

$$\left(a+bx^2\right)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\cdots \int \cdots}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8b}\cdots\right.$$

$\Big\downarrow$ 25

$$\left(a+bx^2\right)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\cdots}{\sqrt[4]{\cdots}}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8b}\cdots\right.$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\left(a+bx^2\right)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\cdots}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\cdots}\cdots}{2\cdots}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8b}\cdots\right.$$

↓ 1103

$$\frac{(a+bx^2)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\cdots\right.}{\cdots}}{2\sqrt{b}}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right)}{8b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(-1/4*(d*(d*x)^(11/2))/(b*(a + b*x^2)^2) + (11*d^2*(-1/2*(d*(
d*x)^(7/2))/(b*(a + b*x^2)) + (7*d^2*((2*d*(d*x)^(3/2))/(3*b) - (2*a*d^3*(
(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]
*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/
b))/(4*b)))/(8*b)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.61

method	result
risch	$\frac{2x^2d^7\sqrt{(bx^2+a)^2}}{3b^3\sqrt{dx}\,(bx^2+a)} - \frac{a\left(\frac{-\frac{19b(dx)^{\frac{7}{2}}}{16}-\frac{15ad^2(dx)^{\frac{3}{2}}}{16}}{(bd^2x^2+ad^2)^2} + \frac{77\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{dx}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(-\cdots\right)\right)}{128b\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3(bx^2+a)}$
default	$\frac{\left(256(dx)^{\frac{3}{2}}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}b^3d^2x^4-231\sqrt{2}\ln\left(-\frac{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}}{dx+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{ad^2}{b}}}\right)ab^2d^4x^4-462\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)ab^2d^4x^4-\cdots\right)}{\cdots}$

input
```
int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^2/b^3/(d*x)^(1/2)*d^7*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-a/b^3*(2*(-19/32
*b*(d*x)^(7/2)-15/32*a*d^2*(d*x)^(3/2))/(b*d^2*x^2+a*d^2)^2+77/128/b/(a*d^
2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan
(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(
d*x)^(1/2)-1)))*d^7*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.90

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{231\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}(b^5x^4+2\,ab^4x^2+a^2b^3)\log\left(456533\,\sqrt{dx}a^2d^{19}+456533\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{3}{4}}b^{11}\right)+231\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}}{}$$

input
```
integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

output
```
-1/192*(231*(-a^3*d^26/b^15)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(4
56533*sqrt(d*x)*a^2*d^19 + 456533*(-a^3*d^26/b^15)^(3/4)*b^11) + 231*(-a^3
*d^26/b^15)^(1/4)*(-I*b^5*x^4 - 2*I*a*b^4*x^2 - I*a^2*b^3)*log(456533*sqrt
(d*x)*a^2*d^19 + 456533*I*(-a^3*d^26/b^15)^(3/4)*b^11) + 231*(-a^3*d^26/b^
15)^(1/4)*(I*b^5*x^4 + 2*I*a*b^4*x^2 + I*a^2*b^3)*log(456533*sqrt(d*x)*a^2
*d^19 - 456533*I*(-a^3*d^26/b^15)^(3/4)*b^11) - 231*(-a^3*d^26/b^15)^(1/4)
*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(456533*sqrt(d*x)*a^2*d^19 - 456533*
(-a^3*d^26/b^15)^(3/4)*b^11) - 4*(32*b^2*d^6*x^5 + 121*a*b*d^6*x^3 + 77*a^
2*d^6*x)*sqrt(d*x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^{\frac{13}{2}}}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

output
```
-1/2*a^2*d^(13/2)*x^(3/2)/(a*b^4*x^2 + a^2*b^3 + (b^5*x^2 + a*b^4)*x^2) -
2*a*d^(13/2)*integrate(sqrt(x)/(b^4*x^2 + a*b^3), x) + d^(13/2)*integrate(
x^(5/2)/(b^3*x^2 + a*b^2), x) + 19/128*a*d^(13/2)*(2*sqrt(2)*arctan(1/2*sq
rt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*
a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*
sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt
(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^3 + 1/16*(19*a*b*d^(13/2)*
x^(7/2) + 23*a^2*d^(13/2)*x^(3/2))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.90

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{1}{384}\,d^6\left(\frac{256\,\sqrt{dx}x}{b^3\mathrm{sgn}\,(bx^2+a)} + \frac{24\left(19\,\sqrt{dx}abd^4x^3+15\,\sqrt{dx}a^2d^4x\right)}{(bd^2x^2+ad^2)^2b^3\mathrm{sgn}\,(bx^2+a)} - \frac{462\,\sqrt{2}(}{}\right.$$

input
```
integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/384*d^6*(256*sqrt(d*x)*x/(b^3*sgn(b*x^2 + a)) + 24*(19*sqrt(d*x)*a*b*d^4
*x^3 + 15*sqrt(d*x)*a^2*d^4*x)/((b*d^2*x^2 + a*d^2)^2*b^3*sgn(b*x^2 + a))
- 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*d*sgn(b*x^2 + a)) - 462*sqrt(2)*(a*
b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(b^6*d*sgn(b*x^2 + a)) + 231*sqrt(2)*(a*b^3*d^2)^(3/4)*l
og(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*d*sgn(b*x
^2 + a)) - 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)
*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*d*sgn(b*x^2 + a)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(d\,x)^{13/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.23

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(sqrt(d)*d**6*(462*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 924*b**(1/4)*a
**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*a*b*x**2 + 462*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b
**2*x**4 - 462*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 924*b**(1/4)*a**(3
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*a*b*x**2 - 462*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4
)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*
x**4 - 231*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt
(2) + sqrt(a) + sqrt(b)*x)*a**2 - 462*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqr
t(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 231*b**(1
/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) +
sqrt(b)*x)*b**2*x**4 + 231*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*
a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**2 + 462*b**(1/4)*a**(3/4)*sqrt(
2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 +
 231*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqr
t(a) + sqrt(b)*x)*b**2*x**4 + 616*sqrt(x)*a**2*b*x + 968*sqrt(x)*a*b**2*x*
*3 + 256*sqrt(x)*b**3*x**5))/(384*b**4*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.640 $\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 398

$$
\begin{aligned}
\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx &= -\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{d(dx)^{9/2}}{4b\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^5\sqrt{dx}(a+bx^2)}{16b^3\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{45\sqrt[4]{a}d^{11/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{45\sqrt[4]{a}d^{11/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{45\sqrt[4]{a}d^{11/2}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{32\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

output
```
-9/16*d^3*(d*x)^(5/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(9/2)/b/(b*x^2+a
)/((b*x^2+a)^2)^(1/2)+45/16*d^5*(d*x)^(1/2)*(b*x^2+a)/b^3/((b*x^2+a)^2)^(1
/2)+45/64*a^(1/4)*d^(11/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/
a^(1/4)/d^(1/2))*2^(1/2)/b^(13/4)/((b*x^2+a)^2)^(1/2)-45/64*a^(1/4)*d^(11/
2)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)
/b^(13/4)/((b*x^2+a)^2)^(1/2)-45/64*a^(1/4)*d^(11/2)*(b*x^2+a)*arctanh(2^(
1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(1
3/4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.49

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{d^5\sqrt{dx}\left(4\sqrt[4]{b}\sqrt{x}(45a^2+81abx^2+32b^2x^4)+45\sqrt{2}\sqrt[4]{a}(a+bx^2)^2\arctan\left(-\right.\right.}{64b^{13/4}\sqrt{x}\,(a+bx^2)\sqrt{(}}$$

input
```
Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(d^5*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(45*a^2 + 81*a*b*x^2 + 32*b^2*x^4) + 45*
Sqrt[2]*a^(1/4)*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x])] - 45*Sqrt[2]*a^(1/4)*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(64*b^(13/4)*Sqrt[x]*(a +
b*x^2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1384, 27, 252, 252, 262, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b^3(a+bx^2)\int \frac{(dx)^{11/2}}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^2)\int \frac{(dx)^{11/2}}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 252$$

$$\frac{(a+bx^2)\left(\frac{9d^2\int \frac{(dx)^{7/2}}{(bx^2+a)^2}dx}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 252$$

$$\frac{(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\int \frac{(dx)^{3/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{5/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 262$$

$$\frac{(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{ad^2\int \frac{1}{\sqrt{dx}(bx^2+a)}dx}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 266$$

$$\frac{(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\int \frac{1}{bx^2+a}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{755}$$

$$\frac{(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 1476$$

$$\left(a+bx^2\right)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx}{2b\left(a+\right.}}{8b}\right.$$

↓ 1082

$$(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}\right.$$

↓ 217

$$(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx}{4b(a+}\right.$$

↓ 1479

$$(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\cdots\right.}{\sqrt{2}\cdots}\right.}{\cdots}\right)}{b}\right.}{4b}\right.}{8b}\right.$$

↓ 25

$$(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.}{\sqrt{2}\sqrt[4]{a}\cdots}\right.}{\cdots}\right.}{b}\right.}{4b}\right.}{8b}\right.$$

$\Big\downarrow$ 27

$$\left(a+bx^2\right)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\mathrm{arc}\cdots}{\cdots}\right.}{2\sqrt{a}}\cdots}{b}\cdots\right)}{4b}\cdots\right)}{8b}\cdots\right.$$

↓ 1103

$$\left(a+bx^2\right)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{\cdots}\right.}{\cdots}\right.}{2\sqrt{a}}\cdots}{b}\right)}{4b}\cdots}{8b}\cdots\right.$$

input
```
Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(-1/4*(d*(d*x)^(9/2))/(b*(a + b*x^2)^2) + (9*d^2*(-1/2*(d*(d*
x)^(5/2))/(b*(a + b*x^2)) + (5*d^2*((2*d*Sqrt[d*x])/b - (2*a*d*((d*(-(ArcT
an[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d
+ Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/b)
)/(4*b)))/(8*b)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.61

method	result
risch	$\frac{2x\,d^{6}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{b^{3}\sqrt{dx}\,\left(b\,x^{2}+a\right)}-\frac{2a\,d^{7}\left(\frac{-\frac{17b\left(dx\right)^{\frac{5}{2}}}{32}-\frac{13\sqrt{dx}\,a\,d^{2}}{32}}{\left(b\,d^{2}x^{2}+a\,d^{2}\right)^{2}}+\frac{45\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}+1\right)+\cdots\right)}{256a\,d^{2}}\right)}{b^{3}\left(b\,x^{2}+a\right)}$
default	$-\frac{\left(45\sqrt{2}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)b^{2}d^{2}x^{4}+90\sqrt{2}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)b^{2}d^{2}x^{4}+90\sqrt{2}\left(\frac{a\,d^{2}}{b}\right)^{\cdots}\cdots\right)}{\cdots}$

input
```
int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^3*x/(d*x)^(1/2)*d^6*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-2*a/b^3*d^7*((-17/32
*b*(d*x)^(5/2)-13/32*(d*x)^(1/2)*a*d^2)/(b*d^2*x^2+a*d^2)^2+45/256*(a*d^2/
b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2
/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*ar
ctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/
4)*(d*x)^(1/2)-1)))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.83

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx =$$
$$-\frac{45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}(b^5x^4+2\,ab^4x^2+a^2b^3)\log\left(45\sqrt{dx}d^5+45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}b^3\right)+45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}(i\,b^5x^4+2i\,ab^4x^2}{}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

output
```
-1/64*(45*(-a*d^22/b^13)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(45*sq
rt(d*x)*d^5 + 45*(-a*d^22/b^13)^(1/4)*b^3) + 45*(-a*d^22/b^13)^(1/4)*(I*b^
5*x^4 + 2*I*a*b^4*x^2 + I*a^2*b^3)*log(45*sqrt(d*x)*d^5 + 45*I*(-a*d^22/b^
13)^(1/4)*b^3) + 45*(-a*d^22/b^13)^(1/4)*(-I*b^5*x^4 - 2*I*a*b^4*x^2 - I*a
^2*b^3)*log(45*sqrt(d*x)*d^5 - 45*I*(-a*d^22/b^13)^(1/4)*b^3) - 45*(-a*d^2
2/b^13)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(45*sqrt(d*x)*d^5 - 45*
(-a*d^22/b^13)^(1/4)*b^3) - 4*(32*b^2*d^5*x^4 + 81*a*b*d^5*x^2 + 45*a^2*d^
5)*sqrt(d*x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F]

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^{\frac{11}{2}}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d*x)**(11/2)/((a + b*x**2)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^{\frac{11}{2}}}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

output
```
1/2*a*d^(11/2)*x^(5/2)/(a*b^3*x^2 + a^2*b^2 + (b^4*x^2 + a*b^3)*x^2) + d^(
11/2)*integrate(x^(3/2)/(b^3*x^2 + a*b^2), x) - 13/128*(2*sqrt(2)*sqrt(a)*
arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*sqrt(2
)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqr
t(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
 sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))*d^(11/2)/b^3 + 1/16*(9*a*b*d^(1
1/2)*x^(5/2) + 13*a^2*d^(11/2)*sqrt(x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.90

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{\frac{90\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^6\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4\mathrm{sgn}(bx^2+a)}+\frac{90\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^6\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4\mathrm{sgn}(bx^2+a)}+\frac{45\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^6\log\left(\cdots\right.}{b^4\mathrm{sg}\cdots}}{1\cdots}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/128*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*d^6*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*sgn(b*x^2 + a)) + 90*sqrt(
2)*(a*b^3*d^2)^(1/4)*d^6*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*
sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*sgn(b*x^2 + a)) + 45*sqrt(2)*(a*b^3*d^2)^
(1/4)*d^6*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^
4*sgn(b*x^2 + a)) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*d^6*log(d*x - sqrt(2)*(a*
d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*sgn(b*x^2 + a)) - 256*sqrt(d*
x)*d^6/(b^3*sgn(b*x^2 + a)) - 8*(17*sqrt(d*x)*a*b*d^10*x^2 + 13*sqrt(d*x)*
a^2*d^10)/((b*d^2*x^2 + a*d^2)^2*b^3*sgn(b*x^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\int \frac{(d\,x)^{11/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.23

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(sqrt(d)*d**5*(90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 180*b**(3/4)*a*
*(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1
/4)*a**(1/4)*sqrt(2)))*a*b*x**2 + 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1
/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**
2*x**4 - 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 180*b**(3/4)*a**(1/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*a*b*x**2 - 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4
 + 45*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) +
 sqrt(a) + sqrt(b)*x)*a**2 + 90*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b
**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 45*b**(3/4)*a**
(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)
*x)*b**2*x**4 - 45*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)
*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**2 - 90*b**(3/4)*a**(1/4)*sqrt(2)*log(sq
rt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 45*b**(3
/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqr
t(b)*x)*b**2*x**4 + 360*sqrt(x)*a**2*b + 648*sqrt(x)*a*b**2*x**2 + 256*sqr
t(x)*b**3*x**4))/(128*b**4*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.641 $\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 352

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = -\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21d^{9/2}\left(a+bx^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{21d^{9/2}\left(a+bx^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21d^{9/2}\left(a+bx^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{32\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
-7/16*d^3*(d*x)^(3/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(7/2)/b/(b*x^2+a
)/((b*x^2+a)^2)^(1/2)-21/64*d^(9/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*
x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(1/4)/b^(11/4)/((b*x^2+a)^2)^(1/2)+21/
64*d^(9/2)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))
*2^(1/2)/a^(1/4)/b^(11/4)/((b*x^2+a)^2)^(1/2)-21/64*d^(9/2)*(b*x^2+a)*arct
anh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/
2)/a^(1/4)/b^(11/4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.52

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx =$$
$$-\frac{d^4\sqrt{dx}\left(4\sqrt[4]{a}b^{3/4}x^{3/2}(7a+11bx^2)+21\sqrt{2}(a+bx^2)^2\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+21\sqrt{2}(a+bx^2)^2\operatorname{arctanh}\left(\right.\right.}{64\sqrt[4]{a}b^{11/4}\sqrt{x}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
-1/64*(d^4*Sqrt[d*x]*(4*a^(1/4)*b^(3/4)*x^(3/2)*(7*a + 11*b*x^2) + 21*Sqrt
[2]*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x])] + 21*Sqrt[2]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
])/(Sqrt[a] + Sqrt[b]*x)]))/(a^(1/4)*b^(11/4)*Sqrt[x]*(a + b*x^2)*Sqrt[(a
+ b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1384, 27, 252, 252, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^2)\int \frac{(dx)^{9/2}}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{(dx)^{9/2}}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 252$$

$$\frac{(a+bx^2)\left(\frac{7d^2\int \frac{(dx)^{5/2}}{(bx^2+a)^2}dx}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 252$$

$$\frac{(a+bx^2)\left(\frac{7d^2\left(\frac{3d^2\int \frac{\sqrt{dx}}{bx^2+a}dx}{4b}-\frac{d(dx)^{3/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 266$$

$$\frac{(a+bx^2)\left(\frac{7d^2\left(\frac{3d\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2b}-\frac{d(dx)^{3/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^2)\left(\frac{7d^2\left(\frac{3d^3\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2b}-\frac{d(dx)^{3/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 826$$

$$\frac{(a+bx^2)\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1476

$$\frac{\left(a+bx^2\right)\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1082

$$\frac{\left(a+bx^2\right)\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 217

$$\frac{\left(a+bx^2\right)\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\left(a+bx^2\right)\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\cdots\right.}}{2\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{b}}\cdots}{2b}\cdots}{8b}\cdots}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 25

$$\frac{(a+bx^2)\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{\ldots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{\ldots}}{\ldots}\right.}\ldots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\ldots}}{2\sqrt{b}}\ldots\right.}{2b}\ldots\right.}{8b}\ldots\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 27

$$\frac{(a+bx^2)\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\cdots}{2\sqrt{b}}\cdots\right)}{2b}\cdots\right)}{8b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1103

$$\frac{\left(a+bx^2\right)\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}\cdots\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\cdots\right)}{2b}\cdots\right)}{8b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input

```
Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output

```
((a + b*x^2)*(-1/4*(d*(d*x)^(7/2))/(b*(a + b*x^2)^2) + (7*d^2*(-1/2*(d*(d*
x)^(3/2))/(b*(a + b*x^2)) + (3*d^3*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*
x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d
+ Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*b)))/(8*b)))/Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(225) = 450$.

Time = 0.14 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\left(-21\sqrt{2}\,\ln\left(-\frac{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)b^2d^4x^4-42\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)b^2d^4x^4-42\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a d^2}{b}\right.}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right.}{}$

input
```
int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/128*(-21*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)
^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^2*d^4
*x^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*b^2*d^4*x^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a
*d^2/b)^(1/4))*b^2*d^4*x^4+88*(a*d^2/b)^(1/4)*(d*x)^(7/2)*b^2-42*2^(1/2)*l
n(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b*d^4*x^2-84*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^4*x^2-84*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^
4*x^2+56*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a*b*d^2-21*2^(1/2)*ln(-((a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2
)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^4)*d*(b*x^2+a)/(a*d^2/b)^
(1/4)/b^3/((b*x^2+a)^2)^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.96

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{21\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\log\left(9261\,\sqrt{dx}d^{13}+9261\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{3}{4}}ab\right.}{}$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/64*(21*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^18/(a*b^11))^(1/4)*log(9261
*sqrt(d*x)*d^13 + 9261*(-d^18/(a*b^11))^(3/4)*a*b^8) - 21*(I*b^4*x^4 + 2*I
*a*b^3*x^2 + I*a^2*b^2)*(-d^18/(a*b^11))^(1/4)*log(9261*sqrt(d*x)*d^13 + 9
261*I*(-d^18/(a*b^11))^(3/4)*a*b^8) - 21*(-I*b^4*x^4 - 2*I*a*b^3*x^2 - I*a
^2*b^2)*(-d^18/(a*b^11))^(1/4)*log(9261*sqrt(d*x)*d^13 - 9261*I*(-d^18/(a*
b^11))^(3/4)*a*b^8) - 21*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^18/(a*b^11)
)^(1/4)*log(9261*sqrt(d*x)*d^13 - 9261*(-d^18/(a*b^11))^(3/4)*a*b^8) - 4*(
11*b*d^4*x^3 + 7*a*d^4*x)*sqrt(d*x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F]

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^{\frac{9}{2}}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d*x)**(9/2)/((a + b*x**2)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^{\frac{9}{2}}}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*a*d^(9/2)*x^(3/2)/(a*b^3*x^2 + a^2*b^2 + (b^4*x^2 + a*b^3)*x^2) + d^(9
/2)*integrate(sqrt(x)/(b^3*x^2 + a*b^2), x) - 11/128*d^(9/2)*(2*sqrt(2)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
 + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^2 - 1/16*(11*b*
d^(9/2)*x^(7/2) + 15*a*d^(9/2)*x^(3/2))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.98

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{1}{128}d^4\left(\frac{8\left(11\sqrt{dx}bd^4x^3+7\sqrt{dx}ad^4x\right)}{(bd^2x^2+ad^2)^2b^2\mathrm{sgn}\,(bx^2+a)} - \frac{42\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5d\mathrm{sgn}\,(bx^2+a)} - \frac{42\sqrt{2}(ab^3d\right.$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/128*d^4*(8*(11*sqrt(d*x)*b*d^4*x^3 + 7*sqrt(d*x)*a*d^4*x)/((b*d^2*x^2 +
 a*d^2)^2*b^2*sgn(b*x^2 + a)) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*d*sg
n(b*x^2 + a)) - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*d*sgn(b*x^2 + a)) +
 21*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
+ sqrt(a*d^2/b))/(a*b^5*d*sgn(b*x^2 + a)) - 21*sqrt(2)*(a*b^3*d^2)^(3/4)*l
og(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^5*d*sgn(b
*x^2 + a)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(d\,x)^{9/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.37

$$\int \frac{(dx)^{9/2}}{(a^2 + 2abx^2 + b^2x^4)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(sqrt(d)*d**4*( - 42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 84*b**(1/4)*
a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a*b*x**2 - 42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b
**2*x**4 + 42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 84*b**(1/4)*a**(3/4
)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a
**(1/4)*sqrt(2)))*a*b*x**2 + 42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**
4 + 21*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2)
+ sqrt(a) + sqrt(b)*x)*a**2 + 42*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*
b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 21*b**(1/4)*a*
*(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b
)*x)*b**2*x**4 - 21*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4
)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**2 - 42*b**(1/4)*a**(3/4)*sqrt(2)*log(s
qrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 21*b**(
1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sq
rt(b)*x)*b**2*x**4 - 56*sqrt(x)*a**2*b*x - 88*sqrt(x)*a*b**2*x**3))/(128*a
*b**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.642 $\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 352

$$\begin{aligned}\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx &= -\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{d(dx)^{5/2}}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^{7/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{5d^{7/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{5d^{7/2}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}(\sqrt{a}+\sqrt{b}x)}\right)}{32\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
-5/16*d^3*(d*x)^(1/2)/b^2/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(5/2)/b/(b*x^2+a
)/((b*x^2+a)^2)^(1/2)-5/64*d^(7/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x
)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(3/4)/b^(9/4)/((b*x^2+a)^2)^(1/2)+5/64*
d^(7/2)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^
(1/2)/a^(3/4)/b^(9/4)/((b*x^2+a)^2)^(1/2)+5/64*d^(7/2)*(b*x^2+a)*arctanh(2
^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^
(3/4)/b^(9/4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.52

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx =$$

$$-\frac{d^3\sqrt{dx}\left(4a^{3/4}\sqrt[4]{b}\sqrt{x}(5a+9bx^2)+5\sqrt{2}(a+bx^2)^2\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-5\sqrt{2}(a+bx^2)^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt{\cdots}}\right.\right.}{64a^{3/4}b^{9/4}\sqrt{x}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
-1/64*(d^3*Sqrt[d*x]*(4*a^(3/4)*b^(1/4)*Sqrt[x]*(5*a + 9*b*x^2) + 5*Sqrt[2
]*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])] - 5*Sqrt[2]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/
(Sqrt[a] + Sqrt[b]*x)]))/(a^(3/4)*b^(9/4)*Sqrt[x]*(a + b*x^2)*Sqrt[(a + b*
x^2)^2])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1384, 27, 252, 252, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^2)\int \frac{(dx)^{7/2}}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{(dx)^{7/2}}{\left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\left(a+bx^2\right)\left(\frac{5d^2\int \frac{(dx)^{3/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\left(a+bx^2\right)\left(\frac{5d^2\left(\frac{d^2\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\left(a+bx^2\right)\left(\frac{5d^2\left(\frac{d\int \frac{1}{bx^2+a}d\sqrt{dx}}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{755}$$

$$\frac{\left(a+bx^2\right)\left(\frac{5d^2\left(\frac{d\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\left(\frac{5d^2\left(\frac{d\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1476

$$\frac{\left(a+bx^2\right)\left(\frac{5d^2\left(\frac{d\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx}{4b(a+}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1082

$$\frac{\left(a+bx^2\right)\left(\frac{5d^2\left(\frac{d\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 217$$

$$\frac{\left(a+bx^2\right)\left(\frac{5d^2\left(\frac{d\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1479

$$\frac{\left(a+bx^2\right)\left(\frac{5d^2\left(\frac{d\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right)}{2\cdots}\right)}{2b}\cdots\right)}{8b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

↓ 25

$$\frac{\left(a+bx^2\right)\left(\frac{5d^2\left(\frac{d\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\text{arc}\cdots}{\cdots}\right)}{2\sqrt{a}}\cdots}{2b}\cdots\right)}{8b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}\cdots}$$

$$\Big\downarrow 27$$

$$\frac{\left(a+bx^2\right)\left(\frac{5d^2\left(\frac{d\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{a}\sqrt{\cdots}}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\cdots}}\cdots}{2\sqrt{a}}\cdots}{2b}\cdots\right)}{8b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 1103

$$\frac{(a+bx^2)\left(\frac{5d^2\left(\frac{d\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{d}\cdots\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\cdots\right)}{2\sqrt{a}}\cdots\right)}{2b}\cdots\right)}{8b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}\cdots}$$

input
```
Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(-1/4*(d*(d*x)^(5/2))/(b*(a + b*x^2)^2) + (5*d^2*(-1/2*(d*Sqr
t[d*x])/(b*(a + b*x^2)) + (d*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])
))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d
 + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*b)))/(8*b)))/Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(225) = 450$.

Time = 0.14 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.89

method	result
default	$\frac{\left(5\sqrt{2}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\ln\left(\frac{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}{dx-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)b^{2}d^{2}x^{4}+10\sqrt{2}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)b^{2}d^{2}x^{4}+10\sqrt{2}\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\mathrm{arc}\right.}{}$

input
```
int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/128*(5*2^(1/2)*(a*d^2/b)^(1/4)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))*b^2*d^2*x^4+10*2^(1/2)*(a*d^2/b)^(1/4)*arctan((2^(1/2)*(d*x)^(1/2)-(a
*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*d^2*x^4+10*2^(1/2)*(a*d^2/b)^(1/4)*arc
tan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*d^2*x^4+10*
2^(1/2)*(a*d^2/b)^(1/4)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2
/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b*
d^2*x^2+20*2^(1/2)*(a*d^2/b)^(1/4)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(
1/4))/(a*d^2/b)^(1/4))*a*b*d^2*x^2+20*2^(1/2)*(a*d^2/b)^(1/4)*arctan((2^(1
/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^2*x^2+5*2^(1/2)*(a
*d^2/b)^(1/4)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))
/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*d^2+10*2^(
1/2)*(a*d^2/b)^(1/4)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b
)^(1/4))*a^2*d^2+10*2^(1/2)*(a*d^2/b)^(1/4)*arctan((2^(1/2)*(d*x)^(1/2)+(a
*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^2-72*(d*x)^(5/2)*a*b-40*(d*x)^(1/2)*
a^2*d^2)*d*(b*x^2+a)/a/b^2/((b*x^2+a)^2)^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.95

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{5\,(b^4x^4+2\,ab^3x^2+a^2b^2)\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}\log\left(5\,\sqrt{dx}d^3+5\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}ab^2\right)-5\,(-$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/64*(5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b^9))^(1/4)*log(5*sq
rt(d*x)*d^3 + 5*(-d^14/(a^3*b^9))^(1/4)*a*b^2) - 5*(-I*b^4*x^4 - 2*I*a*b^3
*x^2 - I*a^2*b^2)*(-d^14/(a^3*b^9))^(1/4)*log(5*sqrt(d*x)*d^3 + 5*I*(-d^14
/(a^3*b^9))^(1/4)*a*b^2) - 5*(I*b^4*x^4 + 2*I*a*b^3*x^2 + I*a^2*b^2)*(-d^1
4/(a^3*b^9))^(1/4)*log(5*sqrt(d*x)*d^3 - 5*I*(-d^14/(a^3*b^9))^(1/4)*a*b^2
) - 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b^9))^(1/4)*log(5*sqrt
(d*x)*d^3 - 5*(-d^14/(a^3*b^9))^(1/4)*a*b^2) - 4*(9*b*d^3*x^2 + 5*a*d^3)*s
qrt(d*x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F]

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^{\frac{7}{2}}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d*x)**(7/2)/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.79

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{d^{\frac{7}{2}}x^{\frac{5}{2}}}{2\,(ab^2x^2+a^2b+(b^3x^2+ab^2)x^2)} + \frac{5\,d^3\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\cdots\right)}{128\,b^2} - \frac{bd^{\frac{7}{2}}x^{\frac{5}{2}}+5\,ad^{\frac{7}{2}}\sqrt{x}}{16\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*d^(7/2)*x^(5/2)/(a*b^2*x^2 + a^2*b + (b^3*x^2 + a*b^2)*x^2) + 5/128*d
^3*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sq
rt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*sqr
t(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b
^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/(a^(3/4)*b^(1/4)))/b^2 - 1/16*(b*d^(7/2)*x^(5/2) + 5*a*d^(7/2)
*sqrt(x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.98

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{10\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^4\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3\mathrm{sgn}(bx^2+a)} + \frac{10\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^4\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\cdots}\cdots\right)}{2\left(\frac{ad^2}{b}\right)^{\cdots}}\right)}{ab^3\mathrm{sgn}(bx^2+a)}\cdots$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/128*(10*sqrt(2)*(a*b^3*d^2)^(1/4)*d^4*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*sgn(b*x^2 + a)) + 10*sqrt
(2)*(a*b^3*d^2)^(1/4)*d^4*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2
*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*sgn(b*x^2 + a)) + 5*sqrt(2)*(a*b^3*d^2
)^(1/4)*d^4*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a*b^3*sgn(b*x^2 + a)) - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*d^4*log(d*x - sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3*sgn(b*x^2 + a)) - 8*(9*s
qrt(d*x)*b*d^8*x^2 + 5*sqrt(d*x)*a*d^8)/((b*d^2*x^2 + a*d^2)^2*b^2*sgn(b*x
^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(d\,x)^{7/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.37

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output

```
(sqrt(d)*d**3*( - 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 20*b**(3/4)*
a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a*b*x**2 - 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b
**2*x**4 + 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 20*b**(3/4)*a**(1/4
)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a
**(1/4)*sqrt(2)))*a*b*x**2 + 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**
4 - 5*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) +
 sqrt(a) + sqrt(b)*x)*a**2 - 10*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b
**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 5*b**(3/4)*a**(
1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*
x)*b**2*x**4 + 5*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*s
qrt(2) + sqrt(a) + sqrt(b)*x)*a**2 + 10*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt
(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 5*b**(3/4)
*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b
)*x)*b**2*x**4 - 40*sqrt(x)*a**2*b - 72*sqrt(x)*a*b**2*x**2))/(128*a*b**3*
(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.643 $\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 353

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{5/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3d^{5/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{5/2}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{32\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
3/16*d*(d*x)^(3/2)/a/b/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(3/2)/b/(b*x^2+a)/(
(b*x^2+a)^2)^(1/2)-3/64*d^(5/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(
1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(5/4)/b^(7/4)/((b*x^2+a)^2)^(1/2)+3/64*d^(
5/2)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/
2)/a^(5/4)/b^(7/4)/((b*x^2+a)^2)^(1/2)-3/64*d^(5/2)*(b*x^2+a)*arctanh(2^(1
/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(5/
4)/b^(7/4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.51

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{(dx)^{5/2}\left(4\sqrt[4]{a}b^{3/4}x^{3/2}(a-3bx^2)+3\sqrt{2}(a+bx^2)^2\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+3\sqrt{2}(a+bx^2)^2\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)\right)}{64a^{5/4}b^{7/4}x^{5/2}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
-1/64*((d*x)^(5/2)*(4*a^(1/4)*b^(3/4)*x^(3/2)*(a - 3*b*x^2) + 3*Sqrt[2]*(a
 + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
] + 3*Sqrt[2]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqr
t[a] + Sqrt[b]*x)]))/(a^(5/4)*b^(7/4)*x^(5/2)*(a + b*x^2)*Sqrt[(a + b*x^2)
^2])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {1384, 27, 252, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^3(a+bx^2)\int \frac{(dx)^{5/2}}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{(dx)^{5/2}}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \textbf{252}$$

$$\frac{(a+bx^2)\left(\frac{3d^2\int \frac{\sqrt{dx}}{(bx^2+a)^2}dx}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \textbf{253}$$

$$\frac{(a+bx^2)\left(\frac{3d^2\left(\frac{\int \frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad(a+bx^2)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \textbf{266}$$

$$\frac{(a+bx^2)\left(\frac{3d^2\left(\frac{\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad(a+bx^2)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{(a+bx^2)\left(\frac{3d^2\left(\frac{d\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad(a+bx^2)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \textbf{826}$$

$$\frac{(a+bx^2)\left(\frac{3d^2\left(\frac{d\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad(a+bx^2)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \textbf{1476}$$

$$\frac{\left(a+bx^2\right)\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\left(a+bx^2\right)\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 217

$$\frac{\left(a+bx^2\right)\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1479

$$\frac{(a+bx^2)\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}}{}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{}}}{2\sqrt{b}}\right.}{2a}\right.}{8b}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 25

$$\frac{(a+bx^2)\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\sqrt[4]{\cdots}}\right)}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\cdots}}}{2\sqrt{b}}\cdots\right)}{2a}\cdots\right)}{8b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 27

$$\frac{\left(a+bx^2\right)\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}\right)}{8b}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 1103$$

$$\frac{(a+bx^2)\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d\cdots\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\cdots\right)}{2a}\cdots\right)}{8b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(-1/4*(d*(d*x)^(3/2))/(b*(a + b*x^2)^2) + (3*d^2*((d*x)^(3/2)
/(2*a*d*(a + b*x^2)) + (d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*S
qrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]
*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d]))/(2*Sqrt[b])))/(2*a)))/(8*b)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(226) = 452$.

Time = 0.14 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.75

method	result
default	$\frac{\left(3\sqrt{2}\,\ln\left(-\frac{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a\,d^{2}}{b}}}{dx+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^{2}}{b}}}\right)b^{2}d^{4}x^{4}+6\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)b^{2}d^{4}x^{4}+6\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^{2}}{b}\right)^{\frac{1}{4}}}\right)b}{}$

input
```
int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/128*(3*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1
/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^2*d^4*x^
4+6*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
b^2*d^4*x^4+6*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*b^2*d^4*x^4+6*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d
*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))*a*b*d^4*x^2+12*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(
a*d^2/b)^(1/4))*a*b*d^4*x^2+12*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^4*x^2+24*(a*d^2/b)^(1/4)*(d*x)^(7/2)*b^2+
3*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d
*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*d^4+6*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^4+6*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d
^4-8*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a*b*d^2)/d*(b*x^2+a)/(a*d^2/b)^(1/4)/b^2/
a/((b*x^2+a)^2)^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.99

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{3\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}\log\left(27\,a^4b^5\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{3}{4}}+27\sqrt{dx}\,d^7\right)-}{}$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/64*(3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(27
*a^4*b^5*(-d^10/(a^5*b^7))^(3/4) + 27*sqrt(d*x)*d^7) - 3*(I*a*b^3*x^4 + 2*
I*a^2*b^2*x^2 + I*a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(27*I*a^4*b^5*(-d^10/(
a^5*b^7))^(3/4) + 27*sqrt(d*x)*d^7) - 3*(-I*a*b^3*x^4 - 2*I*a^2*b^2*x^2 -
I*a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(-27*I*a^4*b^5*(-d^10/(a^5*b^7))^(3/4)
 + 27*sqrt(d*x)*d^7) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b
^7))^(1/4)*log(-27*a^4*b^5*(-d^10/(a^5*b^7))^(3/4) + 27*sqrt(d*x)*d^7) + 4
*(3*b*d^2*x^3 - a*d^2*x)*sqrt(d*x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [F]

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d*x)**(5/2)/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.77

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = -\frac{d^{\frac{5}{2}}x^{\frac{3}{2}}}{2\,(ab^2x^2+a^2b+(b^3x^2+ab^2)x^2)} + \frac{3\,d^{\frac{5}{2}}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \sqrt{2}\log(- \cdots\right)}{128\,ab} + \frac{3\,bd^{\frac{5}{2}}x^{\frac{7}{2}}+7\,ad^{\frac{5}{2}}x^{\frac{3}{2}}}{16\,(ab^3x^4+2\,a^2b^2x^2+a^3b)}$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*d^(5/2)*x^(3/2)/(a*b^2*x^2 + a^2*b + (b^3*x^2 + a*b^2)*x^2) + 3/128*d
^(5/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(s
qrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*lo
g(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
))/(a*b) + 1/16*(3*b*d^(5/2)*x^(7/2) + 7*a*d^(5/2)*x^(3/2))/(a*b^3*x^4 + 2
*a^2*b^2*x^2 + a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.99

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{1}{128}\,d^2\left(\frac{8\left(3\sqrt{dx}bd^4x^3-\sqrt{dx}ad^4x\right)}{\left(bd^2x^2+ad^2\right)^2ab\text{sgn}\left(bx^2+a\right)}+\frac{6\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right.\right.}{2\left(\frac{a}{}\right.}\right.}{a^2b^4d\text{sgn}\left(bx^2+a\right)}\right.$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/128*d^2*(8*(3*sqrt(d*x)*b*d^4*x^3 - sqrt(d*x)*a*d^4*x)/((b*d^2*x^2 + a*d
^2)^2*a*b*sgn(b*x^2 + a)) + 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*d*sgn(b
*x^2 + a)) + 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*d*sgn(b*x^2 + a)) - 3
*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a^2*b^4*d*sgn(b*x^2 + a)) + 3*sqrt(2)*(a*b^3*d^2)^(3/4)*log
(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^4*d*sgn(b
*x^2 + a)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\int \frac{(d\,x)^{5/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.37

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(sqrt(d)*d**2*( - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 12*b**(1/4)*a
**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*a*b*x**2 - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1
/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**
2*x**4 + 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*s
qrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 12*b**(1/4)*a**(3/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*a*b*x**2 + 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1
/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 +
3*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqr
t(a) + sqrt(b)*x)*a**2 + 6*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/
4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 3*b**(1/4)*a**(3/4)*
sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*
*2*x**4 - 3*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2
) + sqrt(a) + sqrt(b)*x)*a**2 - 6*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b*
*(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 3*b**(1/4)*a**(3
/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b
**2*x**4 - 8*sqrt(x)*a**2*b*x + 24*sqrt(x)*a*b**2*x**3))/(128*a**2*b**2*(a
**2 + 2*a*b*x**2 + b**2*x**4))
```

3.644 $\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 353

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\sqrt{dx}}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^{3/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3d^{3/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3d^{3/2}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}(\sqrt{a}+\sqrt{b}x)}\right)}{32\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
1/16*d*(d*x)^(1/2)/a/b/((b*x^2+a)^2)^(1/2)-1/4*d*(d*x)^(1/2)/b/(b*x^2+a)/(
(b*x^2+a)^2)^(1/2)-3/64*d^(3/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(
1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(7/4)/b^(5/4)/((b*x^2+a)^2)^(1/2)+3/64*d^(
3/2)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/
2)/a^(7/4)/b^(5/4)/((b*x^2+a)^2)^(1/2)+3/64*d^(3/2)*(b*x^2+a)*arctanh(2^(1
/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(7/
4)/b^(5/4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.51

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx =$$
$$-\frac{(dx)^{3/2}\left(-4a^{3/4}\sqrt[4]{b}\sqrt{x}(-3a+bx^2)+3\sqrt{2}(a+bx^2)^2\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-3\sqrt{2}(a+bx^2)^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)\right)}{64a^{7/4}b^{5/4}x^{3/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

input
```
Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
-1/64*((d*x)^(3/2)*(-4*a^(3/4)*b^(1/4)*Sqrt[x]*(-3*a + b*x^2) + 3*Sqrt[2]*
(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
])] - 3*Sqrt[2]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(S
qrt[a] + Sqrt[b]*x)]))/(a^(7/4)*b^(5/4)*x^(3/2)*(a + b*x^2)*Sqrt[(a + b*x^
2)^2])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1384, 27, 252, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}dx$$
$$\Big\downarrow \text{1384}$$
$$\frac{b^3(a+bx^2)\int \frac{(dx)^{3/2}}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{(dx)^{3/2}}{\left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\left(a+bx^2\right)\left(\frac{d^2\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^2}dx}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{253}$$

$$\frac{\left(a+bx^2\right)\left(\frac{d^2\left(\frac{3\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\left(a+bx^2\right)\left(\frac{d^2\left(\frac{3\int \frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{755}$$

$$\frac{\left(a+bx^2\right)\left(\frac{d^2\left(\frac{3\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\left(\frac{d^2\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1476

$$\frac{\left(a+bx^2\right)\left(\frac{d^2\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{}}{4b(a+}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1082

$$\frac{(a+bx^2)\left(\frac{d^2\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 217

$$\frac{(a+bx^2)\left(\frac{d^2\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad(a+bx^2)}\right)}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1479

$$\frac{\left(a+bx^{2}\right)\left(d^{2}\left(\frac{3\left(\frac{d\left(-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\cdots\right.}{2\sqrt{\cdots}}\cdots\right)}{2ad}\cdots\right)}{8b}\cdots\right)}{\sqrt{a^{2}+2abx^{2}+b^{2}x^{4}}\cdots}$$

$\downarrow$ 25

$$\frac{(a+bx^2)\left(\frac{d^2\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\text{arcta}\cdots}{\cdots}\right)}{2\sqrt{a}}\cdots\right)}{2ad}\cdots\right)}{8b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{\left(a+bx^2\right)\left(\frac{d^2\left(\frac{3\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{d\cdots}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right)}{2\sqrt{a}}\cdots\right)}{2ad}\cdots\right)}{8b}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1103

$$\frac{(a+bx^2)\left(\frac{d^2\left(\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right.}{2\sqrt{a}}\right.}{2ad}\right.}{8b}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*(-1/4*(d*Sqrt[d*x])/(b*(a + b*x^2)^2) + (d^2*(Sqrt[d*x]/(2*a*
d*(a + b*x^2)) + (3*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqr
t[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b
]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*a*d)))/(8*b)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(226) = 452$.

Time = 0.14 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.89

method	result
default	$\frac{\left(3\sqrt{2}\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)b^2d^2x^4+6\sqrt{2}\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)b^2d^2x^4+6\sqrt{2}\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\text{arcta}}{}$

input
```
int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/128*(3*2^(1/2)*(a*d^2/b)^(1/4)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2)))*b^2*d^2*x^4+6*2^(1/2)*(a*d^2/b)^(1/4)*arctan((2^(1/2)*(d*x)^(1/2)-(a*
d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*d^2*x^4+6*2^(1/2)*(a*d^2/b)^(1/4)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*d^2*x^4+6*2^(
1/2)*(a*d^2/b)^(1/4)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b*d^2
*x^2+12*2^(1/2)*(a*d^2/b)^(1/4)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4
))/(a*d^2/b)^(1/4))*a*b*d^2*x^2+12*2^(1/2)*(a*d^2/b)^(1/4)*arctan((2^(1/2)
*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^2*x^2+3*2^(1/2)*(a*d^
2/b)^(1/4)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d
*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*d^2+6*2^(1/2)
*(a*d^2/b)^(1/4)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1
/4))*a^2*d^2+6*2^(1/2)*(a*d^2/b)^(1/4)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^2+8*(d*x)^(5/2)*a*b-24*(d*x)^(1/2)*a^2*d^
2)/d*(b*x^2+a)/b/a^2/((b*x^2+a)^2)^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.93

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{3\,(ab^3x^4+2\,a^2b^2x^2+a^3b)\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}\log\left(3\,a^2b\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}+3\,\sqrt{dx}d\right)-3\,(-}{}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/64*(3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(1/4)*log(3*a
^2*b*(-d^6/(a^7*b^5))^(1/4) + 3*sqrt(d*x)*d) - 3*(-I*a*b^3*x^4 - 2*I*a^2*b
^2*x^2 - I*a^3*b)*(-d^6/(a^7*b^5))^(1/4)*log(3*I*a^2*b*(-d^6/(a^7*b^5))^(1
/4) + 3*sqrt(d*x)*d) - 3*(I*a*b^3*x^4 + 2*I*a^2*b^2*x^2 + I*a^3*b)*(-d^6/(
a^7*b^5))^(1/4)*log(-3*I*a^2*b*(-d^6/(a^7*b^5))^(1/4) + 3*sqrt(d*x)*d) - 3
*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(1/4)*log(-3*a^2*b*(
-d^6/(a^7*b^5))^(1/4) + 3*sqrt(d*x)*d) + 4*(b*d*x^2 - 3*a*d)*sqrt(d*x))/(a
*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [F]

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d*x)**(3/2)/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.80

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{d^{\frac{3}{2}}x^{\frac{5}{2}}}{2\,(a^2bx^2+a^3+(ab^2x^2+a^2b)x^2)} - \frac{7\,bd^{\frac{3}{2}}x^{\frac{5}{2}}+3\,ad^{\frac{3}{2}}\sqrt{x}}{16\,(ab^3x^4+2\,a^2b^2x^2+a^3b)} + \frac{3\,d\left(\frac{2\,\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \cdots\right)}{128\,ab}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*d^(3/2)*x^(5/2)/(a^2*b*x^2 + a^3 + (a*b^2*x^2 + a^2*b)*x^2) - 1/16*(7*
b*d^(3/2)*x^(5/2) + 3*a*d^(3/2)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*
b) + 3/128*d*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*s
qrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + s
qrt(2)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/
(a^(3/4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
 sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.98

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\frac{6\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2\mathrm{sgn}(bx^2+a)} + \frac{6\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-\cdots\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2\mathrm{sgn}(bx^2+a)} \cdots}{\cdots}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/128*(6*sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/
b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*sgn(b*x^2 + a)) + 6*sqrt
(2)*(a*b^3*d^2)^(1/4)*d^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2
*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*sgn(b*x^2 + a)) + 3*sqrt(2)*(a*b^3*d
^2)^(1/4)*d^2*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a^2*b^2*sgn(b*x^2 + a)) - 3*sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*log(d*x - sqrt
(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^2*sgn(b*x^2 + a)) +
8*(sqrt(d*x)*b*d^6*x^2 - 3*sqrt(d*x)*a*d^6)/((b*d^2*x^2 + a*d^2)^2*a*b*sgn
(b*x^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(d\,x)^{3/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.36

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output

```
(sqrt(d)*d*( - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 12*b**(3/4)*a**(
1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4
)*a**(1/4)*sqrt(2)))*a*b*x**2 - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x
**4 + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt
(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 12*b**(3/4)*a**(1/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4
)*sqrt(2)))*a*b*x**2 + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 - 3*b
**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a
) + sqrt(b)*x)*a**2 - 6*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*
a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 3*b**(3/4)*a**(1/4)*sqr
t(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*
x**4 + 3*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) +
 sqrt(a) + sqrt(b)*x)*a**2 + 6*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1
/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 3*b**(3/4)*a**(1/4)
*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2
*x**4 - 24*sqrt(x)*a**2*b + 8*sqrt(x)*a*b**2*x**2))/(128*a**2*b**2*(a**2 +
 2*a*b*x**2 + b**2*x**4))
```

3.645 $\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 354

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(dx)^{3/2}}{4ad(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5\sqrt{d}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{d}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5\sqrt{d}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}(\sqrt{a}+\sqrt{b}x)}\right)}{32\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
5/16*(d*x)^(3/2)/a^2/d/((b*x^2+a)^2)^(1/2)+1/4*(d*x)^(3/2)/a/d/(b*x^2+a)/(
(b*x^2+a)^2)^(1/2)-5/64*d^(1/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(
1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(9/4)/b^(3/4)/((b*x^2+a)^2)^(1/2)+5/64*d^(
1/2)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/
2)/a^(9/4)/b^(3/4)/((b*x^2+a)^2)^(1/2)-5/64*d^(1/2)*(b*x^2+a)*arctanh(2^(1
/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(9/
4)/b^(3/4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\sqrt{dx}\left(4\sqrt[4]{a}b^{3/4}x^{3/2}(9a+5bx^2)-5\sqrt{2}(a+bx^2)^2\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-5\right.}{64a^{9/4}b^{3/4}\sqrt{x}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

input
```
Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[d*x]*(4*a^(1/4)*b^(3/4)*x^(3/2)*(9*a + 5*b*x^2) - 5*Sqrt[2]*(a + b*x
^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 5*
Sqrt[2]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
 Sqrt[b]*x)]))/(64*a^(9/4)*b^(3/4)*Sqrt[x]*(a + b*x^2)*Sqrt[(a + b*x^2)^2]
)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1384, 27, 253, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^2)\int \frac{\sqrt{dx}}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{\sqrt{dx}}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{253}$$

$$\frac{(a+bx^2)\left(\frac{5\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^2}dx}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{253}$$

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{\int \frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{d\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{826}$$

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{d\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\left(a+bx^2\right)\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 1082

$$\frac{\left(a+bx^2\right)\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{217}$$

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad(a+bx^2)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1479

$$\frac{\left(a+bx^2\right)\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\right.}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}}{2\sqrt{b}}\cdots\right)}{2a}\cdots\right)}{8a}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 25$$

$$\frac{\left(a+bx^2\right)\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\ldots}{\sqrt[4]{b}}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right.}{2a}\right.}{8a}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 27

$$\frac{(a+bx^2)\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}+\right.}{8a}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1103

$$\frac{\left(a+bx^2\right)\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\right)}{2a}}{8a}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((a + b*x^2)*((d*x)^(3/2)/(4*a*d*(a + b*x^2)^2) + (5*((d*x)^(3/2)/(2*a*d*(
a + b*x^2)) + (d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) -
 (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[
d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d
]))/(2*Sqrt[b])))/(2*a)))/(8*a)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(227) = 454$.

Time = 0.13 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.74

method	result
default	$\frac{\left(5\sqrt{2}\,\ln\left(-\frac{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)b^2d^2x^4+10\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)b^2d^2x^4+10\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right.\right.}{}$

input
```
int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/128*(5*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1
/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^2*d^2*x^
4+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))
*b^2*d^2*x^4+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*b^2*d^2*x^4+10*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2)))*a*b*d^2*x^2+20*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4)
)/(a*d^2/b)^(1/4))*a*b*d^2*x^2+20*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*d^2*x^2+40*(a*d^2/b)^(1/4)*(d*x)^(3/2)*b
^2*x^2+5*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1
/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*d^2+10
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2
*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*a^2*d^2+72*(d*x)^(3/2)*a*b*(a*d^2/b)^(1/4))/d*(b*x^2+a)/(a*d^2/b)^(1/4
)/b/a^2/((b*x^2+a)^2)^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{5\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}\log\left(125\,a^7b^2\left(-\frac{d^2}{a^9b^3}\right)^{\frac{3}{4}}+125\,\sqrt{dx}d\right)-\dots}{\dots}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
1/64*(5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)*log(125*a
^7*b^2*(-d^2/(a^9*b^3))^(3/4) + 125*sqrt(d*x)*d) - 5*(I*a^2*b^2*x^4 + 2*I*
a^3*b*x^2 + I*a^4)*(-d^2/(a^9*b^3))^(1/4)*log(125*I*a^7*b^2*(-d^2/(a^9*b^3
))^(3/4) + 125*sqrt(d*x)*d) - 5*(-I*a^2*b^2*x^4 - 2*I*a^3*b*x^2 - I*a^4)*(
-d^2/(a^9*b^3))^(1/4)*log(-125*I*a^7*b^2*(-d^2/(a^9*b^3))^(3/4) + 125*sqrt
(d*x)*d) - 5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)*log(
-125*a^7*b^2*(-d^2/(a^9*b^3))^(3/4) + 125*sqrt(d*x)*d) + 4*(5*b*x^3 + 9*a*
x)*sqrt(d*x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{dx}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(sqrt(d*x)/((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\sqrt{d}x^{\frac{3}{2}}}{2\,(a^2bx^2+a^3+(ab^2x^2+a^2b)x^2)} + \frac{5\,b\sqrt{d}x^{\frac{7}{2}}+a\sqrt{d}x^{\frac{3}{2}}}{16\,(a^2b^2x^4+2\,a^3bx^2+a^4)} + \frac{5\,\sqrt{d}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\right.}{}\right.}{128\,a^2}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(d)*x^(3/2)/(a^2*b*x^2 + a^3 + (a*b^2*x^2 + a^2*b)*x^2) + 1/16*(5*
b*sqrt(d)*x^(7/2) + a*sqrt(d)*x^(3/2))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) +
 5/128*sqrt(d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sq
rt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)
*b^(3/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{8\left(5\sqrt{dx}bd^5x^3+9\sqrt{dx}ad^5x\right)}{(bd^2x^2+ad^2)^2a^2\operatorname{sgn}(bx^2+a)} + \frac{10\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\operatorname{sgn}(bx^2+a)} + \frac{10\sqrt{2}(ab^3d^2}{}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/128*(8*(5*sqrt(d*x)*b*d^5*x^3 + 9*sqrt(d*x)*a*d^5*x)/((b*d^2*x^2 + a*d^2
)^2*a^2*sgn(b*x^2 + a)) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*x^
2 + a)) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*x^2 + a)) - 5*sqr
t(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a^3*b^3*sgn(b*x^2 + a)) + 5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x -
 sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^3*sgn(b*x^2 + a
)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{\sqrt{d\,x}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

input
```
int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output

```
(sqrt(d)*( - 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
- 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 20*b**(1/4)*a**(3
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*a*b*x**2 - 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x
**4 + 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqr
t(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 20*b**(1/4)*a**(3/4)*sqr
t(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/
4)*sqrt(2)))*a*b*x**2 + 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 + 5
*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt
(a) + sqrt(b)*x)*a**2 + 10*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/
4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 5*b**(1/4)*a**(3/4)*
sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*
*2*x**4 - 5*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2
) + sqrt(a) + sqrt(b)*x)*a**2 - 10*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b
**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 5*b**(1/4)*a**(
3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*
b**2*x**4 + 72*sqrt(x)*a**2*b*x + 40*sqrt(x)*a*b**2*x**3))/(128*a**3*b*(a*
*2 + 2*a*b*x**2 + b**2*x**4))
```

3.646 $\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 354

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{4ad\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} - \frac{21(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{21(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{21(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
7/16*(d*x)^(1/2)/a^2/d/((b*x^2+a)^2)^(1/2)+1/4*(d*x)^(1/2)/a/d/(b*x^2+a)/(
(b*x^2+a)^2)^(1/2)-21/64*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^
(1/4)/d^(1/2))*2^(1/2)/a^(11/4)/b^(1/4)/d^(1/2)/((b*x^2+a)^2)^(1/2)+21/64*
(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^
(11/4)/b^(1/4)/d^(1/2)/((b*x^2+a)^2)^(1/2)+21/64*(b*x^2+a)*arctanh(2^(1/2)
*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(11/4)
/b^(1/4)/d^(1/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\sqrt{x}\left(4a^{3/4}\sqrt[4]{b}\sqrt{x}(11a+7bx^2)-21\sqrt{2}(a+bx^2)^2\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right.\right.}{64a^{11/4}\sqrt[4]{b}\sqrt{dx}\left(a+bx^2\right)\sqrt{(a}}$$

input
```
Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
(Sqrt[x]*(4*a^(3/4)*b^(1/4)*Sqrt[x]*(11*a + 7*b*x^2) - 21*Sqrt[2]*(a + b*x
^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 21
*Sqrt[2]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a]
+ Sqrt[b]*x)]))/(64*a^(11/4)*b^(1/4)*Sqrt[d*x]*(a + b*x^2)*Sqrt[(a + b*x^2
)^2])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1384, 27, 253, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b^3\left(a+bx^2\right)\int \frac{1}{b^3\sqrt{dx}\left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+bx^2\right)\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 253$$

$$\frac{(a+bx^2)\left(\frac{7\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^2}dx}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 253$$

$$\frac{(a+bx^2)\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 266$$

$$\frac{(a+bx^2)\left(\frac{7\left(\frac{3\int \frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 755$$

$$\frac{(a+bx^2)\left(\frac{7\left(\frac{3\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^2)\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\left(a+bx^2\right)\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{a}\cdots}{4ad(a+\cdots}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\left(a+bx^2\right)\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 217

$$\frac{(a+bx^2)\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1479

$$\frac{(a+bx^2)\left(\frac{7\left(\frac{3\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\cdots\right.}{2\sqrt{a}}\cdots\right.}{2ad}\cdots\right.}{8a}\cdots\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\downarrow 25$$

$$\frac{\left(a+bx^2\right)\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan \cdots}{\cdots}\right.}{2\sqrt{a}}\cdots\right)}{2ad}\cdots\right)}{8a}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right)}{2\sqrt{a}}\cdots\right)}{2ad}\cdots\right)}{8a}\cdots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 1103$$

$$\frac{(a+bx^2)\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{}}\right.}{2\sqrt{a}}\right.}{2ad}\right.}{8a}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
((a + b*x^2)*(Sqrt[d*x]/(4*a*d*(a + b*x^2)^2) + (7*(Sqrt[d*x]/(2*a*d*(a +
b*x^2)) + (3*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) +
 (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sq
rt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d])))/(2*Sqrt[a])))/(2*a*d)))/(8*a)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(227) = 454$.

Time = 0.13 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.80

method	result
default	$\frac{\left(21\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)b^2x^4+42\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)b^2x^4+42\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\mathrm{arctan}}{}$

input
```
int(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/128*(21*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1
/2)))*b^2*x^4+42*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*x^4+42*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^
(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^2*x^4+42*(a*d^2/b)^(
1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/
(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b*x^2+84*(a*d
^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)
^(1/4))*a*b*x^2+84*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*
d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b*x^2+21*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2+42*(a*d^2/b)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2+42*(a*d^2/b)^
(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*a^2+56*(d*x)^(1/2)*a*b*x^2+88*(d*x)^(1/2)*a^2)/d*(b*x^2+a)/a^3/((b*x^2+a
)^2)^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{21\,(a^2b^2dx^4+2\,a^3bdx^2+a^4d)\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{1}{4}}\log\left(a^3d\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)}{}$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
1/64*(21*(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2))^(1/4)*l
og(a^3*d*(-1/(a^11*b*d^2))^(1/4) + sqrt(d*x)) - 21*(-I*a^2*b^2*d*x^4 - 2*I
*a^3*b*d*x^2 - I*a^4*d)*(-1/(a^11*b*d^2))^(1/4)*log(I*a^3*d*(-1/(a^11*b*d^
2))^(1/4) + sqrt(d*x)) - 21*(I*a^2*b^2*d*x^4 + 2*I*a^3*b*d*x^2 + I*a^4*d)*
(-1/(a^11*b*d^2))^(1/4)*log(-I*a^3*d*(-1/(a^11*b*d^2))^(1/4) + sqrt(d*x))
- 21*(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2))^(1/4)*log(-
a^3*d*(-1/(a^11*b*d^2))^(1/4) + sqrt(d*x)) + 4*(7*b*x^2 + 11*a)*sqrt(d*x))
/(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{1}{\sqrt{dx}\,\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(1/(sqrt(d*x)*((a + b*x**2)**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{1}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
-1/2*b*sqrt(d)*x^(5/2)/(a^3*b*d*x^2 + a^4*d + (a^2*b^2*d*x^2 + a^3*b*d)*x^
2) + 1/16*(15*b*x^(5/2) + 11*a*sqrt(x))/(a^2*b^2*sqrt(d)*x^4 + 2*a^3*b*sqr
t(d)*x^2 + a^4*sqrt(d)) - 11/128*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sq
rt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)
*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a
^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(s
qrt(a)*sqrt(b))) + sqrt(2)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*d) + int
egrate(1/((a^2*b*sqrt(d)*x^2 + a^3*sqrt(d))*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{7\sqrt{dx}bd^3x^2+11\sqrt{dx}ad^3}{16\,(bd^2x^2+ad^2)^2a^2\mathrm{sgn}\,(bx^2+a)} + \frac{21\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3bd\mathrm{sgn}\,(bx^2+a)} + \frac{21\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3bd\mathrm{sgn}\,(bx^2+a)} + \frac{21\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^3bd\mathrm{sgn}\,(bx^2+a)} - \frac{21\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^3bd\mathrm{sgn}\,(bx^2+a)}$$

input

```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output

```
1/16*(7*sqrt(d*x)*b*d^3*x^2 + 11*sqrt(d*x)*a*d^3)/((b*d^2*x^2 + a*d^2)^2*a
^2*sgn(b*x^2 + a)) + 21/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d*sgn(b*x^2
+ a)) + 21/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d*sgn(b*x^2 + a)) + 21/1
28*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/(a^3*b*d*sgn(b*x^2 + a)) - 21/128*sqrt(2)*(a*b^3*d^2)^(1/4
)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d*sg
n(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\int \frac{1}{\sqrt{d\,x}\,(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

output
```
int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(sqrt(d)*( - 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
- 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 84*b**(3/4)*a**(1
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*a*b*x**2 - 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x
**4 + 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqr
t(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 84*b**(3/4)*a**(1/4)*sqr
t(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/
4)*sqrt(2)))*a*b*x**2 + 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 - 2
1*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqr
t(a) + sqrt(b)*x)*a**2 - 42*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1
/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 21*b**(3/4)*a**(1/4
)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*
b**2*x**4 + 21*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqr
t(2) + sqrt(a) + sqrt(b)*x)*a**2 + 42*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x
)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 21*b**(3/4)*
a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)
*x)*b**2*x**4 + 88*sqrt(x)*a**2*b + 56*sqrt(x)*a*b**2*x**2))/(128*a**3*b*d
*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.647 $\int \frac{1}{(dx)^{3/2}(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 400

$$\begin{aligned}\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{1}{4ad\sqrt{dx}\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{45(a+bx^2)}{16a^3d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{45\sqrt[4]{b}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{45\sqrt[4]{b}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{45\sqrt[4]{b}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{32\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
9/16/a^2/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)+1/4/a/d/(d*x)^(1/2)/(b*x^2+a)/(
(b*x^2+a)^2)^(1/2)-45/16*(b*x^2+a)/a^3/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)+4
5/64*b^(1/4)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))*2^(1/2)/a^(13/4)/d^(3/2)/((b*x^2+a)^2)^(1/2)-45/64*b^(1/4)*(b*x^2+a)*ar
ctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(13/4)/d^(3/
2)/((b*x^2+a)^2)^(1/2)+45/64*b^(1/4)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(
1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(13/4)/d^(3/2)/((b
*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.48

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{x\left(-4\sqrt[4]{a}(32a^2+81abx^2+45b^2x^4)+45\sqrt{2}\sqrt[4]{b}\sqrt{x}(a+bx^2)^2\arctan\right.}{64a^{13/4}(dx)^{3/2}(a+}$$

input
```
Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
(x*(-4*a^(1/4)*(32*a^2 + 81*a*b*x^2 + 45*b^2*x^4) + 45*Sqrt[2]*b^(1/4)*Sqr
t[x]*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x])] + 45*Sqrt[2]*b^(1/4)*Sqrt[x]*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(64*a^(13/4)*(d*x)^(3/2)*(a +
 b*x^2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1384, 27, 253, 253, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3\left(a+bx^2\right)\int \frac{1}{b^3(dx)^{3/2}(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{1}{(dx)^{3/2}(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{253}$$

$$\frac{\left(a+bx^2\right)\left(\frac{9\int \frac{1}{(dx)^{3/2}\left(bx^2+a\right)^2}dx}{8a}+\frac{1}{4ad\sqrt{dx}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{253}$$

$$\frac{\left(a+bx^2\right)\left(\frac{9\left(\frac{5\int \frac{1}{(dx)^{3/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{264}$$

$$\frac{\left(a+bx^2\right)\left(\frac{9\left(\frac{5\left(-\frac{b\int \frac{\sqrt{dx}}{bx^2+a}dx}{ad^2}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\left(a+bx^2\right)\left(\frac{9\left(\frac{5\left(-\frac{2b\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{ad^3}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+bx^2\right)\left(\frac{9\left(\frac{5\left(-\frac{2b\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 826$$

$$\frac{\left(a+bx^2\right)\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 1476$$

$$\frac{\left(a+bx^2\right)\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}}{8a}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1082

$$\frac{(a+bx^2)\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{}{4a}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

↓ 217

$$\frac{(a+bx^2)\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}(a}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

↓ 1479

$$\left(a+bx^2\right)\left(\frac{9\left(\frac{5-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\cdots\right.}}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right)}{ad}\cdots}{4a}\cdots\right)}{8a}\cdots\right.$$

↓ 25

$$(a+bx^2)\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}}{2\sqrt{b}}\right.}{ad}\right.}{4a}\right.}{8a}\right.$$

$\downarrow$ 27

$$\left(a+bx^2\right)\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{d}\cdots}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\cdots}{2\sqrt{b}}\cdots\right)}{ad}\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right.$$

↓ 1103

$$\frac{\left(a+bx^2\right)\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{\ldots}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}\ldots\right)}{ad}\ldots\right)}{4a}\ldots\right)}{8a}\ldots\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
((a + b*x^2)*(1/(4*a*d*Sqrt[d*x]*(a + b*x^2)^2) + (9*(1/(2*a*d*Sqrt[d*x]*(
a + b*x^2)) + (5*(-2/(a*d*Sqrt[d*x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[S
qrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(a*d)))/(4*a)))/(8*a)))/Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.60

method	result
risch	$-\frac{2\sqrt{(b x^2+a)^2}}{a^3\sqrt{dx}\,d(b x^2+a)}-\frac{b\left(\frac{\frac{13b(dx)^{\frac{7}{2}}}{16}+\frac{17a d^2(dx)^{\frac{3}{2}}}{16}}{\left(b d^2x^2+a d^2\right)^2}+\frac{45\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{dx}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(-\ldots\right)\right)}{128b\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3d(b x^2+a)}$
default	$-\frac{\left(45\sqrt{dx}\sqrt{2}\ln\left(-\frac{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}-dx-\sqrt{\frac{a d^2}{b}}}{dx+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{a d^2}{b}}}\right)b^2x^4+90\sqrt{dx}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{dx}+\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a d^2}{b}\right)^{\frac{1}{4}}}\right)b^2x^4+90\sqrt{dx}\sqrt{2}\arctan\left(\frac{\sqrt{2}\ldots}{\ldots}\right)\ldots\right)}{\ldots}$

input
```
int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a^3/(d*x)^(1/2)/d*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-1/a^3*b*(2*(13/32*b*(d*
x)^(7/2)+17/32*a*d^2*(d*x)^(3/2))/(b*d^2*x^2+a*d^2)^2+45/128/b/(a*d^2/b)^(
1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))
/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/
2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(
1/2)-1)))/d*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.94

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = -\frac{45\left(a^3b^2d^2x^5+2\,a^4bd^2x^3+a^5d^2x\right)\left(-\frac{b}{a^{13}d^6}\right)^{\frac{1}{4}}\log\left(91125\,a^{10}d^5\left(-\frac{b}{a^{13}d^6}\right)^{\frac{3}{4}}+91125\,\sqrt{dx}b\right)+45\left(-i\,a^3b^2d^2x\right.}{}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
-1/64*(45*(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(-b/(a^13*d^6))^
(1/4)*log(91125*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(d*x)*b) + 45*(
-I*a^3*b^2*d^2*x^5 - 2*I*a^4*b*d^2*x^3 - I*a^5*d^2*x)*(-b/(a^13*d^6))^(1/4
)*log(91125*I*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(d*x)*b) + 45*(I*
a^3*b^2*d^2*x^5 + 2*I*a^4*b*d^2*x^3 + I*a^5*d^2*x)*(-b/(a^13*d^6))^(1/4)*l
og(-91125*I*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(d*x)*b) - 45*(a^3*
b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*(-b/(a^13*d^6))^(1/4)*log(-9112
5*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(d*x)*b) + 4*(45*b^2*x^4 + 81
*a*b*x^2 + 32*a^2)*sqrt(d*x))/(a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2
*x)
```

Sympy [F]

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(1/((d*x)**(3/2)*((a + b*x**2)**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{2}}(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
-1/2*b*x^(3/2)/(a^3*b*d^(3/2)*x^2 + a^4*d^(3/2) + (a^2*b^2*d^(3/2)*x^2 + a
^3*b*d^(3/2))*x^2) - 1/16*(13*b^2*x^(7/2) + 9*a*b*x^(3/2))/(a^3*b^2*d^(3/2
)*x^4 + 2*a^4*b*d^(3/2)*x^2 + a^5*d^(3/2)) - 13/128*b*(2*sqrt(2)*arctan(1/
2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt
(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt
(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^3*d^(3/2)) + integrat
e(1/((a^2*b*d^(3/2)*x^2 + a^3*d^(3/2))*x^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx =$$

$$-\frac{\frac{256}{\sqrt{dx}a^3\operatorname{sgn}(bx^2+a)} + \frac{8\left(13\sqrt{dx}b^2d^3x^3+17\sqrt{dx}abd^3x\right)}{(bd^2x^2+ad^2)^2a^3\operatorname{sgn}(bx^2+a)} + \frac{90\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^2d^2\operatorname{sgn}(bx^2+a)} + \frac{90\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\cdots\right)}{a^4b^2d^2\mathrm{s}\cdots}}{128\cdots}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/128*(256/(sqrt(d*x)*a^3*sgn(b*x^2 + a)) + 8*(13*sqrt(d*x)*b^2*d^3*x^3 +
 17*sqrt(d*x)*a*b*d^3*x)/((b*d^2*x^2 + a*d^2)^2*a^3*sgn(b*x^2 + a)) + 90*s
qrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*s
qrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*d^2*sgn(b*x^2 + a)) + 90*sqrt(2)*(a*b^
3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(
a*d^2/b)^(1/4))/(a^4*b^2*d^2*sgn(b*x^2 + a)) - 45*sqrt(2)*(a*b^3*d^2)^(3/4
)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^2*d^
2*sgn(b*x^2 + a)) + 45*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/
b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^2*d^2*sgn(b*x^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\int \frac{1}{{\left(d\,x\right)}^{3/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

input
```
int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

output
```
int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.28

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(sqrt(d)*(90*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 180*sqrt(x)*
b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*x**2 + 90*sqrt(x)*b**(1/4)*a**(3/4)*
sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**
(1/4)*sqrt(2)))*b**2*x**4 - 90*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a
**2 - 180*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*x**2 - 90*sqrt(x)*
b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 - 45*sqrt(x)*b**(1/4)*a**(3/4)
*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a
**2 - 90*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4
)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 45*sqrt(x)*b**(1/4)*a**(3/4)*s
qrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**
2*x**4 + 45*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4
)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**2 + 90*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(
2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 +
 45*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2
) + sqrt(a) + sqrt(b)*x)*b**2*x**4 - 256*a**3 - 648*a**2*b*x**2 - 360*a...
```

3.648 $\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 400

$$\begin{aligned}\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{77(a+bx^2)}{48a^3d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{77b^{3/4}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{77b^{3/4}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{77b^{3/4}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{32\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
11/16/a^2/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+1/4/a/d/(d*x)^(3/2)/(b*x^2+a)/
((b*x^2+a)^2)^(1/2)-77/48*(b*x^2+a)/a^3/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+
77/64*b^(3/4)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/
2))*2^(1/2)/a^(15/4)/d^(5/2)/((b*x^2+a)^2)^(1/2)-77/64*b^(3/4)*(b*x^2+a)*a
rctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(15/4)/d^(5
/2)/((b*x^2+a)^2)^(1/2)-77/64*b^(3/4)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^
(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(15/4)/d^(5/2)/((
b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.48

$$\int \frac{1}{(dx)^{5/2} (a^2 + 2abx^2 + b^2x^4)^{3/2}} \, dx = \frac{x\left(-4a^{3/4}(32a^2 + 121abx^2 + 77b^2x^4) + 231\sqrt{2}b^{3/4}x^{3/2}(a + bx^2)^2 \text{ar}\right.}{192a^{15/4}(dx)^{5/2}}$$

input
```
Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
(x*(-4*a^(3/4)*(32*a^2 + 121*a*b*x^2 + 77*b^2*x^4) + 231*Sqrt[2]*b^(3/4)*x
^(3/2)*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x])] - 231*Sqrt[2]*b^(3/4)*x^(3/2)*(a + b*x^2)^2*ArcTanh[(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(192*a^(15/4)*(d*x)^(5/2)*
(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1384, 27, 253, 253, 264, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3\left(a+bx^2\right)\int \frac{1}{b^3(dx)^{5/2}(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{1}{(dx)^{5/2}(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{253}$$

$$\frac{\left(a+bx^2\right)\left(\frac{11\int \frac{1}{(dx)^{5/2}\left(bx^2+a\right)^2}dx}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{253}$$

$$\frac{\left(a+bx^2\right)\left(\frac{11\left(\frac{7\int \frac{1}{(dx)^{5/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{264}$$

$$\frac{\left(a+bx^2\right)\left(\frac{11\left(\frac{7\left(-\frac{b\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{ad^2}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(a+bx^2)\left(\frac{11\left(\frac{7\left(-\frac{2b\int \frac{1}{bx^2+a}d\sqrt{dx}}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 755

$$\frac{(a+bx^2)\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 27

$$\frac{\left(a+bx^2\right)\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1476

$$\left(a+bx^2\right)\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{\cdots}{2ad(\cdots}}{8a}\right.$$

↓ 1082

$$(a+bx^2)\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a\cdots\right.}\right.}{8a}\right.$$

↓ 217

$$(a+bx^2)\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}\right.$$

↓ 1479

$$(a+bx^2)\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\cdots\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right.}{\cdots}\right)}{ad^3}\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right.$$

$$\Big\downarrow 25$$

$$(a+bx^2)\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right)}{\cdots}\right)}{ad^3}\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right)$$

$\downarrow$ 27

$$(a+bx^2)\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}\cdots\right)}{2\sqrt{a}}\cdots\right)}{ad^3}\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right.$$

$$\Bigg\downarrow$$ 1103

$$\left(a+bx^2\right)\left(\frac{11\left(\frac{7-\frac{2b\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\cdots}\right)}{2\sqrt{2}\sqrt[4]{a}\cdots}\right.}{2\sqrt{a}}\cdots\right)}{ad^3}\cdots}{4a}\cdots\right)}{8a}\cdots\right.$$

input
```
Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
((a + b*x^2)*(1/(4*a*d*(d*x)^(3/2)*(a + b*x^2)^2) + (11*(1/(2*a*d*(d*x)^(3
/2)*(a + b*x^2)) + (7*(-2/(3*a*d*(d*x)^(3/2)) - (2*b*((d*(-(ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*
d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*S
qrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(a*d^3)))/(4
*a)))/(8*a)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{2\sqrt{(b\,x^2+a)^2}}{3a^3x\sqrt{dx}\,d^2(b\,x^2+a)}-\frac{2b\left(\frac{\frac{15b(dx)^{\frac{5}{2}}}{32}+\frac{19\sqrt{dx}\,a\,d^2}{32}}{\left(b\,d^2x^2+a\,d^2\right)^2}+\frac{77\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)\right.}{256a\,d^2}}{a^3d(b\,x^2+a)}$
default	$-\frac{\left(231(dx)^{\frac{3}{2}}\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)b^3x^4+462(dx)^{\frac{3}{2}}\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)b^3x^4+462\right.}{}$

input
```
int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a^3/x/(d*x)^(1/2)/d^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-2/a^3*b/d*((15/32
*b*(d*x)^(5/2)+19/32*(d*x)^(1/2)*a*d^2)/(b*d^2*x^2+a*d^2)^2+77/256*(a*d^2/
b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2
/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*ar
ctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/
4)*(d*x)^(1/2)-1)))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^{3/2}}\,dx =$$

$$-\frac{231\,(a^3b^2d^3x^6+2\,a^4bd^3x^4+a^5d^3x^2)\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}\log\left(77\,a^4d^3\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}+77\,\sqrt{dx}b\right)+231\,(i\,a^3b^2d^3x^6 -}{}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
-1/192*(231*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*
d^10))^(1/4)*log(77*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) + 2
31*(I*a^3*b^2*d^3*x^6 + 2*I*a^4*b*d^3*x^4 + I*a^5*d^3*x^2)*(-b^3/(a^15*d^1
0))^(1/4)*log(77*I*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) + 23
1*(-I*a^3*b^2*d^3*x^6 - 2*I*a^4*b*d^3*x^4 - I*a^5*d^3*x^2)*(-b^3/(a^15*d^1
0))^(1/4)*log(-77*I*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) - 2
31*(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4 + a^5*d^3*x^2)*(-b^3/(a^15*d^10))^(1
/4)*log(-77*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) + 4*(77*b^2
*x^4 + 121*a*b*x^2 + 32*a^2)*sqrt(d*x))/(a^3*b^2*d^3*x^6 + 2*a^4*b*d^3*x^4
 + a^5*d^3*x^2)
```

Sympy [F]

$$\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{1}{(dx)^{\frac{5}{2}}\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(1/((d*x)**(5/2)*((a + b*x**2)**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{5/2}\,(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{1}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}\,(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
1/2*b^2*x^(5/2)/(a^4*b*d^(5/2)*x^2 + a^5*d^(5/2) + (a^3*b^2*d^(5/2)*x^2 +
a^4*b*d^(5/2))*x^2) - 2*b*integrate(1/((a^3*b*d^(5/2)*x^2 + a^4*d^(5/2))*s
qrt(x)), x) - 1/16*(23*b^2*x^(5/2) + 19*a*b*sqrt(x))/(a^3*b^2*d^(5/2)*x^4
+ 2*a^4*b*d^(5/2)*x^2 + a^5*d^(5/2)) + 19/128*(2*sqrt(2)*b*arctan(1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*
a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/(a^3*d^(5/2)) + integrate(1/((a^
2*b*d^(5/2)*x^2 + a^3*d^(5/2))*x^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.90

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = -\frac{15\sqrt{dx}b^2d^2x^2+19\sqrt{dx}abd^2}{16\left(bd^2x^2+ad^2\right)^2a^3d\mathrm{sgn}\left(bx^2+a\right)} - \frac{77\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4d^3\mathrm{sgn}\left(bx^2+a\right)} - \frac{77\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4d^3\mathrm{sgn}\left(bx^2+a\right)} - \frac{77\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^4d^3\mathrm{sgn}\left(bx^2+a\right)} + \frac{77\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^4d^3\mathrm{sgn}\left(bx^2+a\right)} - \frac{2}{3\sqrt{dx}a^3d^2x\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
-1/16*(15*sqrt(d*x)*b^2*d^2*x^2 + 19*sqrt(d*x)*a*b*d^2)/((b*d^2*x^2 + a*d^
2)^2*a^3*d*sgn(b*x^2 + a)) - 77/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*d^3*sg
n(b*x^2 + a)) - 77/64*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(
2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*d^3*sgn(b*x^2 + a)
) - 77/128*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqr
t(d*x) + sqrt(a*d^2/b))/(a^4*d^3*sgn(b*x^2 + a)) + 77/128*sqrt(2)*(a*b^3*d
^2)^(1/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
4*d^3*sgn(b*x^2 + a)) - 2/3/(sqrt(d*x)*a^3*d^2*x*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\int \frac{1}{(d\,x)^{5/2}\,(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

output
```
int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.30

$$\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output

```
(sqrt(d)*(462*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*x + 924*sqrt(
x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*s
qrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*x**3 + 462*sqrt(x)*b**(3/4)*a**(1
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*b**2*x**5 - 462*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*a**2*x - 924*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*x**3 - 462*
sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt
(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**5 + 231*sqrt(x)*b**(3/4)
*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqr
t(b)*x)*a**2*x + 462*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(
1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**3 + 231*sqrt(x)*b**(3/
4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + s
qrt(b)*x)*b**2*x**5 - 231*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b*
*(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**2*x - 462*sqrt(x)*b**(3/
4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt
(b)*x)*a*b*x**3 - 231*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/
4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**5 - 256*a**3 - 968*a...
```

3.649 $\int \frac{1}{(dx)^{7/2}(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 447

$$\begin{aligned}
\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117(a+bx^2)}{80a^3d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{117b(a+bx^2)}{16a^4d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{117b^{5/4}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{117b^{5/4}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{117b^{5/4}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{32\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

output
```
13/16/a^2/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)+1/4/a/d/(d*x)^(5/2)/(b*x^2+a)/
((b*x^2+a)^2)^(1/2)-117/80*(b*x^2+a)/a^3/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)
+117/16*b*(b*x^2+a)/a^4/d^3/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)-117/64*b^(5/4)
*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a
^(17/4)/d^(7/2)/((b*x^2+a)^2)^(1/2)+117/64*b^(5/4)*(b*x^2+a)*arctan(1+2^(1
/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(17/4)/d^(7/2)/((b*x^2+
a)^2)^(1/2)-117/64*b^(5/4)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)
^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(17/4)/d^(7/2)/((b*x^2+a)^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.46

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{x\left(4\sqrt[4]{a}(-32a^3+416a^2bx^2+1053ab^2x^4+585b^3x^6)-585\sqrt{2}b^{5/4}x^{5}\right.}{320a^{17/4}}$$

input
```
Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
(x*(4*a^(1/4)*(-32*a^3 + 416*a^2*b*x^2 + 1053*a*b^2*x^4 + 585*b^3*x^6) - 5
85*Sqrt[2]*b^(5/4)*x^(5/2)*(a + b*x^2)^2*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 585*Sqrt[2]*b^(5/4)*x^(5/2)*(a + b*x^2)^2
*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(320*a
^(17/4)*(d*x)^(7/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.93, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1384, 27, 253, 253, 264, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b^3\left(a+bx^2\right)\int \frac{1}{b^3(dx)^{7/2}(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(a+bx^2\right)\int \frac{1}{(dx)^{7/2}(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 253$$

$$\frac{\left(a+bx^2\right)\left(\frac{13\int \frac{1}{(dx)^{7/2}\left(bx^2+a\right)^2}dx}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 253$$

$$\frac{\left(a+bx^2\right)\left(\frac{13\left(\frac{9\int \frac{1}{(dx)^{7/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 264$$

$$\frac{\left(a+bx^2\right)\left(\frac{13\left(\frac{9\left(-\frac{b\int \frac{1}{(dx)^{3/2}\left(bx^2+a\right)}dx}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 264$$

$$\frac{(a+bx^2)\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{b\int\frac{\sqrt{dx}}{bx^2+a}dx}{ad^2}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 266$$

$$\frac{(a+bx^2)\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\int\frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{ad^3}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^2)\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2)^2}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{826}$$

$$\frac{(a+bx^2)\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}(a+bx^2}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1476

$$(a+bx^2)\left(\frac{13\left(\frac{9\left(\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}}{4a}}{8a}$$

↓ 1082

$$(a+bx^2)\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{}{2ad(}}{8a}$$

↓ 217

$$(a+bx^2)\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}(a\cdots}\right)}{8a}\right.$$

↓ 1479

$$\left(13 - \frac{\left(9 - \frac{\left(b - \frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} - \frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{\cdots}\right.}{\sqrt[4]{b}\left(xd+\cdots\right.}}{2\cdots}}{2\sqrt{b}}\cdots\right)}{ad}\right)}{ad^2}\right)}{4a}\cdots\right.$$

↓ 25

$$\left(13\left(9-\frac{\left(b-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\cdots\right.}}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right)}{ad}\cdots\right)}{ad^2}\cdots\right)}{4a}\cdots\right.$$

$$\Big\downarrow \text{27}$$

$$\left(13 \frac{\left(9 - \frac{\left(b - \frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} - \frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}} + \frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{\cdots}}{\sqrt[4]{b}}}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}} \cdots \right.}{ad} \cdots \right.}{ad^2} \cdots \right.}{4a} \cdots$$

↓ 1103

$$(a+bx^2)\left(\frac{13\left(\frac{9\left(\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right)}{ad}\cdots\right)}{ad^2}\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right.$$

input
```
Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

output
```
((a + b*x^2)*(1/(4*a*d*(d*x)^(5/2)*(a + b*x^2)^2) + (13*(1/(2*a*d*(d*x)^(5
/2)*(a + b*x^2)) + (9*(-2/(5*a*d*(d*x)^(5/2)) - (b*(-2/(a*d*Sqrt[d*x]) - (
2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sq
rt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b
])))/(a*d)))/(a*d^2)))/(4*a)))/(8*a)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
 - 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.57

method	result
risch	$-\frac{2(-15b\,x^2+a)\sqrt{(b\,x^2+a)^2}}{5a^4\sqrt{dx}\,x^2d^3(b\,x^2+a)}+\frac{b^2\left(\frac{\frac{21b(dx)^{\frac{7}{2}}}{16}+\frac{25a\,d^2(dx)^{\frac{3}{2}}}{16}}{\left(b\,d^2x^2+a\,d^2\right)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}+1\right)+ \cdots}{128b\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\cdots\right)}{a^4d^3(b\,x^2+a)}$
default	$-\frac{\left(-585\sqrt{2}\,\ln\left(-\frac{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}-dx-\sqrt{\frac{a\,d^2}{b}}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{a\,d^2}{b}}}\right)(dx)^{\frac{5}{2}}b^3x^4-1170\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}{\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}}\right)(dx)^{\frac{5}{2}}b^3x^4-1170\sqrt{2}\,\arctan\left(\cdots\right.\right.}{\cdots}$

input
```
int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-15*b*x^2+a)/a^4/(d*x)^(1/2)/x^2/d^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+b
^2/a^4*(2*(21/32*b*(d*x)^(7/2)+25/32*a*d^2*(d*x)^(3/2))/(b*d^2*x^2+a*d^2)^
2+117/128/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/
(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))/d^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.94

$$\int \frac{1}{(dx)^{7/2} (a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = \frac{585\,(a^4b^2d^4x^7 + 2\,a^5bd^4x^5 + a^6d^4x^3)\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}\log\left(1601613\,a^{13}d^{1}\right.}{}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas
")
```

output
```
1/320*(585*(a^4*b^2*d^4*x^7 + 2*a^5*b*d^4*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*d
^14))^(1/4)*log(1601613*a^13*d^11*(-b^5/(a^17*d^14))^(3/4) + 1601613*sqrt(
d*x)*b^4) - 585*(I*a^4*b^2*d^4*x^7 + 2*I*a^5*b*d^4*x^5 + I*a^6*d^4*x^3)*(-
b^5/(a^17*d^14))^(1/4)*log(1601613*I*a^13*d^11*(-b^5/(a^17*d^14))^(3/4) +
1601613*sqrt(d*x)*b^4) - 585*(-I*a^4*b^2*d^4*x^7 - 2*I*a^5*b*d^4*x^5 - I*a
^6*d^4*x^3)*(-b^5/(a^17*d^14))^(1/4)*log(-1601613*I*a^13*d^11*(-b^5/(a^17*
d^14))^(3/4) + 1601613*sqrt(d*x)*b^4) - 585*(a^4*b^2*d^4*x^7 + 2*a^5*b*d^4
*x^5 + a^6*d^4*x^3)*(-b^5/(a^17*d^14))^(1/4)*log(-1601613*a^13*d^11*(-b^5/
(a^17*d^14))^(3/4) + 1601613*sqrt(d*x)*b^4) + 4*(585*b^3*x^6 + 1053*a*b^2*
x^4 + 416*a^2*b*x^2 - 32*a^3)*sqrt(d*x))/(a^4*b^2*d^4*x^7 + 2*a^5*b*d^4*x^
5 + a^6*d^4*x^3)
```

Sympy [F]

$$\int \frac{1}{(dx)^{7/2} (a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = \int \frac{1}{(dx)^{\frac{7}{2}} \left((a + bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral(1/((d*x)**(7/2)*((a + b*x**2)**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{2}}(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima
")
```

output
```
1/2*b^2*x^(3/2)/(a^4*b*d^(7/2)*x^2 + a^5*d^(7/2) + (a^3*b^2*d^(7/2)*x^2 +
a^4*b*d^(7/2))*x^2) - 2*b*integrate(1/((a^3*b*d^(7/2)*x^2 + a^4*d^(7/2))*x
^(3/2)), x) + 1/16*(21*b^3*x^(7/2) + 17*a*b^2*x^(3/2))/(a^4*b^2*d^(7/2)*x^
4 + 2*a^5*b*d^(7/2)*x^2 + a^6*d^(7/2)) + 21/128*b^2*(2*sqrt(2)*arctan(1/2*
sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2
)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a
)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sq
rt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^4*d^(7/2)) + integrate(
1/((a^2*b*d^(7/2)*x^2 + a^3*d^(7/2))*x^(7/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.87

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{21\sqrt{dx}b^3d^3x^3+25\sqrt{dx}ab^2d^3x}{16\left(bd^2x^2+ad^2\right)^2a^4d^3\text{sgn}\left(bx^2+a\right)} + \frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5bd^5\text{sgn}\left(bx^2+a\right)} + \frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5bd^5\text{sgn}\left(bx^2+a\right)} - \frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^5bd^5\text{sgn}\left(bx^2+a\right)} + \frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^5bd^5\text{sgn}\left(bx^2+a\right)} + \frac{2\left(15\,bd^2x^2-ad^2\right)}{5\sqrt{dx}a^4d^5x^2\text{sgn}\left(bx^2+a\right)}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
1/16*(21*sqrt(d*x)*b^3*d^3*x^3 + 25*sqrt(d*x)*a*b^2*d^3*x)/((b*d^2*x^2 + a
*d^2)^2*a^4*d^3*sgn(b*x^2 + a)) + 117/64*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*
b*d^5*sgn(b*x^2 + a)) + 117/64*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(
2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d^5*sgn
(b*x^2 + a)) - 117/128*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/
b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b*d^5*sgn(b*x^2 + a)) + 117/128*s
qrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqr
t(a*d^2/b))/(a^5*b*d^5*sgn(b*x^2 + a)) + 2/5*(15*b*d^2*x^2 - a*d^2)/(sqrt(
d*x)*a^4*d^5*x^2*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\int \frac{1}{{\left(d\,x\right)}^{7/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

input
```
int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

output
```
int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.20

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output

```
(sqrt(d)*( - 1170*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 -
2340*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 - 1170*sqrt(x)*
b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 1170*sqrt(x)*b**(1/4)*a**(3/
4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*
a**(1/4)*sqrt(2)))*a**2*b*x**2 + 2340*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt
(2)))*a*b**2*x**4 + 1170*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x*
*6 + 585*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4
)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**2*b*x**2 + 1170*sqrt(x)*b**(1/4)*a**(3
/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x
)*a*b**2*x**4 + 585*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1
/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**3*x**6 - 585*sqrt(x)*b**(1/
4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt
(b)*x)*a**2*b*x**2 - 1170*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b*
*(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 - 585*sqrt(x)*b
**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a...
```

3.650 $\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 541

$$\begin{aligned}
\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx &= -\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{d(dx)^{21/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{17/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{119d^5(dx)^{13/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{13923ad^{11}\sqrt{dx}\left(a+bx^2\right)}{1024b^6\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13923d^9(dx)^{5/2}\left(a+bx^2\right)}{5120b^5\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{13923a^{5/4}d^{23/2}\left(a+bx^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{13923a^{5/4}d^{23/2}\left(a+bx^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{13923a^{5/4}d^{23/2}\left(a+bx^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

output
```
-1547/1024*d^7*(d*x)^(9/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(21/2)/b/(b
*x^2+a)^3/((b*x^2+a)^2)^(1/2)-7/32*d^3*(d*x)^(17/2)/b^2/(b*x^2+a)^2/((b*x^
2+a)^2)^(1/2)-119/256*d^5*(d*x)^(13/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-1
3923/1024*a*d^11*(d*x)^(1/2)*(b*x^2+a)/b^6/((b*x^2+a)^2)^(1/2)+13923/5120*
d^9*(d*x)^(5/2)*(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)-13923/4096*a^(5/4)*d^(23
/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2
)/b^(25/4)/((b*x^2+a)^2)^(1/2)+13923/4096*a^(5/4)*d^(23/2)*(b*x^2+a)*arcta
n(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(25/4)/((b*x^2+
a)^2)^(1/2)+13923/4096*a^(5/4)*d^(23/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*
b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(25/4)/((b*x^2+
a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.42

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{d^{11}\sqrt{dx}\left(4\sqrt[4]{b}\sqrt{x}(-69615a^5-264537a^4bx^2-369733a^3b^2x^4-220507a^2b^3\right.}{}$$

input
```
Integrate[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(d^11*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(-69615*a^5 - 264537*a^4*b*x^2 - 369733
*a^3*b^2*x^4 - 220507*a^2*b^3*x^6 - 43008*a*b^4*x^8 + 2048*b^5*x^10) + 696
15*Sqrt[2]*a^(5/4)*(a + b*x^2)^4*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])] + 69615*Sqrt[2]*a^(5/4)*(a + b*x^2)^4*ArcTanh[(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(20480*b^(25/4)*Sqr
t[x]*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.89, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1384, 27, 252, 252, 252, 252, 262, 262, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^2)\int \frac{(dx)^{23/2}}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{(dx)^{23/2}}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{21d^2\int \frac{(dx)^{19/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{21/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{21d^2\left(\frac{17d^2\int \frac{(dx)^{15/2}}{\left(bx^2+a\right)^3}dx}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{21/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\int \frac{(dx)^{11/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{21/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{(a+bx^2)\left(\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\int \frac{(dx)^{7/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{21/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{(a+bx^2)\left(\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\int\frac{(dx)^{3/2}}{bx^2+a}dx}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{21/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 262

$$(a+bx^2)\left(\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{ad^2\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(d}{8b(a}\right.$$

↓ 266

$$(a+bx^2)\left(\frac{21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\int\frac{1}{bx^2+a}d\sqrt{dx}}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{17/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^2}{8b(a+b}\right.$$

↓ 755

$$\left(21d^2\,\frac{\left(17d^2\,\frac{\left(13d^2\,\frac{\left(9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{b}\right)}{b}\right)\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(d\cdots}{4b\left(a\cdots\right.}\cdots\right)}{12b}\cdots\right.$$

$\Big\downarrow$ 27

$$21d^2\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4b}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)}}{12b}\right.$$

↓ 1476

$$\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx} \qquad \int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}}{}}$$

$$d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}}{}}}{2}\right.$$

$$2ad\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\cdots\right)}{2\sqrt{a}}\right.$$

$$ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\cdots\right)}{b}\right.$$

$$9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\cdots\right)}{b}\right.$$

$$13d^2\frac{\left(\cdots\right)}{4b}$$

↓ 1082

$$\Bigg(\Bigg(\Bigg(\Bigg(13d^2\Bigg(9d^2\Bigg(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\cdots\right)}{2\sqrt{a}}\cdots\right)}{b}\cdots\right)}{b}\cdots\Bigg)}{4b}\cdots$$

↓ 217

$$\left(\quad\left(\quad\left(\quad 13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right)}{2\sqrt{a}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{4b}\cdots\right.\right.\right.\right.$$

↓ 1479

$$d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{\ldots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{\ldots}}{\ldots}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\ldots}\right.$$

$$2ad\,\frac{\ldots}{2\sqrt{a}}$$

$$ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{\ldots}{b}\right.$$

$$9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{\ldots}{b}\right.$$

$$13d^2\,\frac{\ldots}{4b}$$

$$\Big\downarrow 25$$

$$13d^2\left(9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}}{\sqrt[4]{b}}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{2\sqrt{a}}\right.}{b}\right.}{b}\right.}{4b}\right.$$

$\Big\downarrow$ 27

$$13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\cdots\right)}{b}\right)}{b}\right)}{4b}\right)$$

↓ 1103

$$\left(\left(\left(13d^2\left(\frac{9d^2\left(\frac{2d(dx)^{5/2}}{5b}-\frac{ad^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}\right.}{2\sqrt{2}\cdots}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.}{4b}\cdots\right.\right.\right.\right.$$

input
```
Int[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*(d*(d*x)^(21/2))/(b*(a + b*x^2)^4) + (21*d^2*(-1/6*(d*(
d*x)^(17/2))/(b*(a + b*x^2)^3) + (17*d^2*(-1/4*(d*(d*x)^(13/2))/(b*(a + b*
x^2)^2) + (13*d^2*(-1/2*(d*(d*x)^(9/2))/(b*(a + b*x^2)) + (9*d^2*((2*d*(d*
x)^(5/2))/(5*b) - (a*d^2*((2*d*Sqrt[d*x])/b - (2*a*d*((d*(-(ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*
d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*S
qrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/b))/b))/(4*b
)))/(8*b)))/(12*b)))/(16*b)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{2(-b\,x^2+25a)\,x\,d^{12}\sqrt{(b\,x^2+a)^2}}{5b^6\sqrt{dx}\,(b\,x^2+a)}+\frac{2a^2d^{13}\left(\frac{-\frac{3683a^3d^6\sqrt{dx}}{2048}-\frac{12357b\,d^4a^2(dx)^{\frac{5}{2}}}{2048}-\frac{14145a\,d^2b^2(dx)^{\frac{9}{2}}}{2048}-\frac{5599b^3(dx)^{\frac{13}{2}}}{2048}}{\left(b\,d^2x^2+a\,d^2\right)^4}+\frac{13923\left(\frac{a\,d^2}{b}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-b*x^2+25*a)*x/b^6/(d*x)^(1/2)*d^12*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+2*
a^2/b^6*d^13*((-3683/2048*a^3*d^6*(d*x)^(1/2)-12357/2048*b*d^4*a^2*(d*x)^(
5/2)-14145/2048*a*d^2*b^2*(d*x)^(9/2)-5599/2048*b^3*(d*x)^(13/2))/(b*d^2*x
^2+a*d^2)^4+13923/16384*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)
+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))*((b*x^2+a)^2)^(1/2)/(b*
x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.93

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{69615\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{1}{4}}(b^{10}x^8+4\,ab^9x^6+6\,a^2b^8x^4+4\,a^3b^7x^2+a^4b^6)\log\Big(13923}{}$$

input
```
integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

output
```
1/20480*(69615*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*
x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*log(13923*sqrt(d*x)*a*d^11 + 13923*(-a^5*d^
46/b^25)^(1/4)*b^6) - 69615*(-a^5*d^46/b^25)^(1/4)*(-I*b^10*x^8 - 4*I*a*b^
9*x^6 - 6*I*a^2*b^8*x^4 - 4*I*a^3*b^7*x^2 - I*a^4*b^6)*log(13923*sqrt(d*x)
*a*d^11 + 13923*I*(-a^5*d^46/b^25)^(1/4)*b^6) - 69615*(-a^5*d^46/b^25)^(1/
4)*(I*b^10*x^8 + 4*I*a*b^9*x^6 + 6*I*a^2*b^8*x^4 + 4*I*a^3*b^7*x^2 + I*a^4
*b^6)*log(13923*sqrt(d*x)*a*d^11 - 13923*I*(-a^5*d^46/b^25)^(1/4)*b^6) - 6
9615*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^
3*b^7*x^2 + a^4*b^6)*log(13923*sqrt(d*x)*a*d^11 - 13923*(-a^5*d^46/b^25)^(
1/4)*b^6) + 4*(2048*b^5*d^11*x^10 - 43008*a*b^4*d^11*x^8 - 220507*a^2*b^3*
d^11*x^6 - 369733*a^3*b^2*d^11*x^4 - 264537*a^4*b*d^11*x^2 - 69615*a^5*d^1
1)*sqrt(d*x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^
4*b^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(dx)^{\frac{23}{2}}}{(b^2x^4+2\,abx^2+a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

output
```
-4*a*d^(23/2)*integrate(x^(3/2)/(b^6*x^2 + a*b^5), x) + d^(23/2)*integrate
(x^(7/2)/(b^5*x^2 + a*b^4), x) + 3683/8192*(2*sqrt(2)*a^(3/2)*arctan(1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b))
)/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a
^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sq
rt(b)) + sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
 sqrt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
 sqrt(b)*x + sqrt(a))/b^(1/4))*d^(23/2)/b^6 - 1/3072*(6925*a^2*b^3*d^(23/2
)*x^(13/2) + 23395*a^3*b^2*d^(23/2)*x^(9/2) + 27135*a^4*b*d^(23/2)*x^(5/2)
 + 11049*a^5*d^(23/2)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4
*a^3*b^7*x^2 + a^4*b^6) - 1/192*((617*a^2*b^4*d^(23/2)*x^5 + 1386*a^3*b^3*
d^(23/2)*x^3 + 801*a^4*b^2*d^(23/2)*x)*x^(11/2) + 2*(519*a^3*b^3*d^(23/2)*
x^5 + 1182*a^4*b^2*d^(23/2)*x^3 + 695*a^5*b*d^(23/2)*x)*x^(7/2) + (453*a^4
*b^2*d^(23/2)*x^5 + 1042*a^5*b*d^(23/2)*x^3 + 621*a^6*d^(23/2)*x)*x^(3/2))
/(a^3*b^8*x^6 + 3*a^4*b^7*x^4 + 3*a^5*b^6*x^2 + a^6*b^5 + (b^11*x^6 + 3*a*
b^10*x^4 + 3*a^2*b^9*x^2 + a^3*b^8)*x^6 + 3*(a*b^10*x^6 + 3*a^2*b^9*x^4 +
3*a^3*b^8*x^2 + a^4*b^7)*x^4 + 3*(a^2*b^9*x^6 + 3*a^3*b^8*x^4 + 3*a^4*b^7*
x^2 + a^5*b^6)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.78

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{139230\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad^{12}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7\mathrm{sgn}(bx^2+a)} + \frac{139230\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad^{12}\arctan\left(-\cdots\right)}{b^7\mathrm{sgn}(bx^2+\cdots}$$

input
```
integrate((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/40960*(139230*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^12*arctan(1/2*sqrt(2)*(sqrt(
2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*sgn(b*x^2 + a)) +
139230*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d^12*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^7*sgn(b*x^2 + a)) + 69615*sq
rt(2)*(a*b^3*d^2)^(1/4)*a*d^12*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
 + sqrt(a*d^2/b))/(b^7*sgn(b*x^2 + a)) - 69615*sqrt(2)*(a*b^3*d^2)^(1/4)*a
*d^12*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^7*sg
n(b*x^2 + a)) - 40*(5599*sqrt(d*x)*a^2*b^3*d^20*x^6 + 14145*sqrt(d*x)*a^3*
b^2*d^20*x^4 + 12357*sqrt(d*x)*a^4*b*d^20*x^2 + 3683*sqrt(d*x)*a^5*d^20)/(
(b*d^2*x^2 + a*d^2)^4*b^6*sgn(b*x^2 + a)) + 16384*(sqrt(d*x)*b^20*d^12*x^2
 - 25*sqrt(d*x)*a*b^19*d^12)/(b^25*sgn(b*x^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\int \frac{(d\,x)^{23/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int((d*x)^(23/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(23/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.56

$$\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output

```
(sqrt(d)*d**11*( - 139230*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 - 556920*
b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2 - 835380*b**(3/4)*a**(1/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*a**3*b**2*x**4 - 556920*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a
**2*b**3*x**6 - 139230*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 + 139
230*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*
sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5 + 556920*b**(3/4)*a**(1/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4
)*sqrt(2)))*a**4*b*x**2 + 835380*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*
*2*x**4 + 556920*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*x**6 + 139230
*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqr
t(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 69615*b**(3/4)*a**(1/4)*s
qrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**
5 - 278460*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*s...
```

3.651 $\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 494

$$\begin{aligned}\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx &= -\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} \\ &\quad -\frac{d(dx)^{19/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{19d^3(dx)^{15/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} \\ &\quad -\frac{95d^5(dx)^{11/2}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315d^9(dx)^{3/2}\left(a+bx^2\right)}{3072b^5\sqrt{a^2+2abx^2+b^2x^4}} \\ &\quad +\frac{7315a^{3/4}d^{21/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ &\quad -\frac{7315a^{3/4}d^{21/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ &\quad +\frac{7315a^{3/4}d^{21/2}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
-1045/1024*d^7*(d*x)^(7/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(19/2)/b/(b
*x^2+a)^3/((b*x^2+a)^2)^(1/2)-19/96*d^3*(d*x)^(15/2)/b^2/(b*x^2+a)^2/((b*x
^2+a)^2)^(1/2)-95/256*d^5*(d*x)^(11/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+7
315/3072*d^9*(d*x)^(3/2)*(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)+7315/4096*a^(3/
4)*d^(21/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2)
)*2^(1/2)/b^(23/4)/((b*x^2+a)^2)^(1/2)-7315/4096*a^(3/4)*d^(21/2)*(b*x^2+a
)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(23/4)/(
(b*x^2+a)^2)^(1/2)+7315/4096*a^(3/4)*d^(21/2)*(b*x^2+a)*arctanh(2^(1/2)*a^
(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(23/4)/((
b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.44

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{d^9(dx)^{3/2}\left(4b^{3/4}x^{3/2}(7315a^4+26125a^3bx^2+33345a^2b^2x^4+16967ab^3x^6+\right.}{1}$$

input
```
Integrate[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(d^9*(d*x)^(3/2)*(4*b^(3/4)*x^(3/2)*(7315*a^4 + 26125*a^3*b*x^2 + 33345*a^
2*b^2*x^4 + 16967*a*b^3*x^6 + 2048*b^4*x^8) - 21945*Sqrt[2]*a^(3/4)*(a + b
*x^2)^4*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] +
 21945*Sqrt[2]*a^(3/4)*(a + b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])/(Sqrt[a] + Sqrt[b]*x)]))/(12288*b^(23/4)*x^(3/2)*(a + b*x^2)^3*Sqrt[(
a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.567$, Rules used = {1384, 27, 252, 252, 252, 252, 262, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^5(a+bx^2)\int \frac{(dx)^{21/2}}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{(dx)^{21/2}}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{(a+bx^2)\left(\frac{19d^2\int \frac{(dx)^{17/2}}{(bx^2+a)^4}dx}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{(a+bx^2)\left(\frac{19d^2\left(\frac{5d^2\int \frac{(dx)^{13/2}}{(bx^2+a)^3}dx}{4b}-\frac{d(dx)^{15/2}}{6b(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{(a+bx^2)\left(\frac{19d^2\left(\frac{5d^2\left(\frac{11d^2\int \frac{(dx)^{9/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 252$$

$$\frac{(a+bx^2)\left(\frac{19d^2\left(\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\int \frac{(dx)^{5/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 262$$

$$\frac{(a+bx^2)\left(\frac{19d^2\left(\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{ad^2\int\frac{\sqrt{dx}}{bx^2+a}dx}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 266$$

$$\frac{(a+bx^2)\left(\frac{19d^2\left(\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad\int\frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{(a+bx^2)\left(\frac{19d^2\left(\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 826

$$\left(a+bx^2\right)\left(\frac{19d^2\left(\frac{5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{15/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}\right.$$

↓ 1476

$$19d^2\left(5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right.}{b}\right.}{4b}\right.}{8b}\right.}{4b}\right.$$

↓ 1082

$$\left(19d^2\left(5d^2\left(11d^2\left(7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)}{4b}-\frac{d(\cdots}{2b(\cdots}\right)}{8b}\cdots\right)}{4b}\cdots\right.\right.$$

↓ 217

$$\left(19d^2\frac{\left(5d^2\frac{\left(11d^2\frac{\left(7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{b}\right)\right)}{4b}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}\right)}{8b}\right)}{4b}\right.$$

↓ 1479

$$5d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{b}\right.}{4b}\right.}{8b}\right.$$

$$\Big\downarrow \text{25}$$

$$
5d^2 \frac{\left(11d^2 \frac{\left(7d^2 \left(\frac{2d(dx)^{3/2}}{3b} - \frac{2ad^3 \left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} - \frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)} d}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}
\right.}{b}
\right.}{4b}
\right.}{8b}
$$

$$\Big\downarrow \; 27$$

$$\Bigg(\;\Bigg(\;\Bigg(\; 5d^2 \frac{\Bigg(11d^2 \frac{\Bigg(7d^2 \Bigg(\frac{2d(dx)^{3/2}}{3b} - \frac{2ad^3 \left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} - \frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}} + \cdots \int \cdots}{2\sqrt{b}} \cdots}{b} \cdots}{4b} \cdots}{8b} \cdots$$

↓ 1103

$$\Bigg(19d^2\Bigg(\frac{5d^2\left(\frac{11d^2\left(7d^2\left(\frac{2d(dx)^{3/2}}{3b}-\frac{2ad^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots}{b}\cdots\right)}{4b}\cdots\right)}{8b}\cdots\right)}{4b}\cdots$$

input
```
Int[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*(d*(d*x)^(19/2))/(b*(a + b*x^2)^4) + (19*d^2*(-1/6*(d*(
d*x)^(15/2))/(b*(a + b*x^2)^3) + (5*d^2*(-1/4*(d*(d*x)^(11/2))/(b*(a + b*x
^2)^2) + (11*d^2*(-1/2*(d*(d*x)^(7/2))/(b*(a + b*x^2)) + (7*d^2*((2*d*(d*x
)^(3/2))/(3*b) - (2*a*d^3*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*S
qrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]
*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d]))/(2*Sqrt[b])))/b))/(4*b)))/(8*b)))/(4*b)))/(16*b)))/Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.56

method	result
risch	$\frac{2x^2 d^{11}\sqrt{(bx^2+a)^2}}{3b^5\sqrt{dx}\,(bx^2+a)} - \frac{a\left(\frac{-\frac{5267a^3d^6(dx)^{\frac{3}{2}}}{3072}-\frac{17933a^2d^4b(dx)^{\frac{7}{2}}}{3072}-\frac{7019a\,d^2b^2(dx)^{\frac{11}{2}}}{1024}-\frac{2925b^3(dx)^{\frac{15}{2}}}{1024}}{\left(b\,d^2x^2+a\,d^2\right)^4}+\frac{7315\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{}}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{}}\right.\right.}{}\right)}{b^5(bx^2+a)}$
default	Expression too large to display

input
```
int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^2/b^5/(d*x)^(1/2)*d^11*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-a/b^5*(2*(-5267
/6144*a^3*d^6*(d*x)^(3/2)-17933/6144*a^2*d^4*b*(d*x)^(7/2)-7019/2048*a*d^2
*b^2*(d*x)^(11/2)-2925/2048*b^3*(d*x)^(15/2))/(b*d^2*x^2+a*d^2)^4+7315/819
2/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))
+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b
)^(1/4)*(d*x)^(1/2)-1))*d^11*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.01

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{21945\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}}(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5)\log\left(391419980875\,\sqrt{dx}a^2d^{31}+3914199\right.}{}$$

input
```
integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

output
```
-1/12288*(21945*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*
x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*log(391419980875*sqrt(d*x)*a^2*d^31 + 39141
9980875*(-a^3*d^42/b^23)^(3/4)*b^17) + 21945*(-a^3*d^42/b^23)^(1/4)*(-I*b^
9*x^8 - 4*I*a*b^8*x^6 - 6*I*a^2*b^7*x^4 - 4*I*a^3*b^6*x^2 - I*a^4*b^5)*log
(391419980875*sqrt(d*x)*a^2*d^31 + 391419980875*I*(-a^3*d^42/b^23)^(3/4)*b
^17) + 21945*(-a^3*d^42/b^23)^(1/4)*(I*b^9*x^8 + 4*I*a*b^8*x^6 + 6*I*a^2*b
^7*x^4 + 4*I*a^3*b^6*x^2 + I*a^4*b^5)*log(391419980875*sqrt(d*x)*a^2*d^31
- 391419980875*I*(-a^3*d^42/b^23)^(3/4)*b^17) - 21945*(-a^3*d^42/b^23)^(1/
4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*log(3
91419980875*sqrt(d*x)*a^2*d^31 - 391419980875*(-a^3*d^42/b^23)^(3/4)*b^17)
 - 4*(2048*b^4*d^10*x^9 + 16967*a*b^3*d^10*x^7 + 33345*a^2*b^2*d^10*x^5 +
26125*a^3*b*d^10*x^3 + 7315*a^4*d^10*x)*sqrt(d*x))/(b^9*x^8 + 4*a*b^8*x^6
+ 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(dx)^{\frac{21}{2}}}{(b^2x^4+2\,abx^2+a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

output
```
-4*a*d^(21/2)*integrate(sqrt(x)/(b^6*x^2 + a*b^5), x) + d^(21/2)*integrate
(x^(5/2)/(b^5*x^2 + a*b^4), x) + 2925/8192*a*d^(21/2)*(2*sqrt(2)*arctan(1/
2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt
(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt
(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^5 + 1/3072*(8775*a*b^3
*d^(21/2)*x^(15/2) + 29649*a^2*b^2*d^(21/2)*x^(11/2) + 34285*a^3*b*d^(21/2
)*x^(7/2) + 13795*a^4*d^(21/2)*x^(3/2))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7
*x^4 + 4*a^3*b^6*x^2 + a^4*b^5) - 1/192*((537*a^2*b^4*d^(21/2)*x^5 + 1210*
a^3*b^3*d^(21/2)*x^3 + 705*a^4*b^2*d^(21/2)*x)*x^(9/2) + 2*(443*a^3*b^3*d^
(21/2)*x^5 + 1014*a^4*b^2*d^(21/2)*x^3 + 603*a^5*b*d^(21/2)*x)*x^(5/2) + (
381*a^4*b^2*d^(21/2)*x^5 + 882*a^5*b*d^(21/2)*x^3 + 533*a^6*d^(21/2)*x)*sq
rt(x))/(a^3*b^8*x^6 + 3*a^4*b^7*x^4 + 3*a^5*b^6*x^2 + a^6*b^5 + (b^11*x^6
+ 3*a*b^10*x^4 + 3*a^2*b^9*x^2 + a^3*b^8)*x^6 + 3*(a*b^10*x^6 + 3*a^2*b^9*
x^4 + 3*a^3*b^8*x^2 + a^4*b^7)*x^4 + 3*(a^2*b^9*x^6 + 3*a^3*b^8*x^4 + 3*a^
4*b^7*x^2 + a^5*b^6)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.80

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{1}{24576}d^{10}\left(\frac{16384\sqrt{dx}x}{b^5\mathrm{sgn}\,(bx^2+a)} - \frac{43890\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{\cdot}\right.}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{b^8d\mathrm{sgn}\,(bx^2+a)}\right.$$

input
```
integrate((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/24576*d^10*(16384*sqrt(d*x)*x/(b^5*sgn(b*x^2 + a)) - 43890*sqrt(2)*(a*b^
3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a
*d^2/b)^(1/4))/(b^8*d*sgn(b*x^2 + a)) - 43890*sqrt(2)*(a*b^3*d^2)^(3/4)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))
/(b^8*d*sgn(b*x^2 + a)) + 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^8*d*sgn(b*x^2 + a)) - 2194
5*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
sqrt(a*d^2/b))/(b^8*d*sgn(b*x^2 + a)) + 8*(8775*sqrt(d*x)*a*b^3*d^8*x^7 +
21057*sqrt(d*x)*a^2*b^2*d^8*x^5 + 17933*sqrt(d*x)*a^3*b*d^8*x^3 + 5267*sqr
t(d*x)*a^4*d^8*x)/((b*d^2*x^2 + a*d^2)^4*b^5*sgn(b*x^2 + a)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(d\,x)^{21/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int((d*x)^(21/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(21/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.68

$$\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(sqrt(d)*d**10*(43890*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 + 175560*b**(
1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))
/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 + 263340*b**(1/4)*a**(3/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*a**2*b**2*x**4 + 175560*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4
)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**
3*x**6 + 43890*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 - 43890*b**(1/4
)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*a**4 - 175560*b**(1/4)*a**(3/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
*a**3*b*x**2 - 263340*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 - 1
75560*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x
)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 - 43890*b**(1/4)*a**(3
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*b**4*x**8 - 21945*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqr
t(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**4 - 87780*b**(1/4
)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) +...
```

3.652 $\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 494

$$\begin{aligned}
\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx &= -\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{d(dx)^{17/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{17d^3(dx)^{13/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{221d^5(dx)^{9/2}}{768b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{3315d^9\sqrt{dx}(a+bx^2)}{1024b^5\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{3315\sqrt[4]{a}d^{19/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{3315\sqrt[4]{a}d^{19/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{3315\sqrt[4]{a}d^{19/2}(a+bx^2)\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

output
```
-663/1024*d^7*(d*x)^(5/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(17/2)/b/(b*
x^2+a)^3/((b*x^2+a)^2)^(1/2)-17/96*d^3*(d*x)^(13/2)/b^2/(b*x^2+a)^2/((b*x^
2+a)^2)^(1/2)-221/768*d^5*(d*x)^(9/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+33
15/1024*d^9*(d*x)^(1/2)*(b*x^2+a)/b^5/((b*x^2+a)^2)^(1/2)+3315/4096*a^(1/4
)*d^(19/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))
*2^(1/2)/b^(21/4)/((b*x^2+a)^2)^(1/2)-3315/4096*a^(1/4)*d^(19/2)*(b*x^2+a)
*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/b^(21/4)/((
b*x^2+a)^2)^(1/2)-3315/4096*a^(1/4)*d^(19/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(
1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(21/4)/((b
*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.44

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{d^9\sqrt{dx}\Big(4\sqrt[4]{b}\sqrt{x}(9945a^4+37791a^3bx^2+52819a^2b^2x^4+31501ab^3x^6+614}{1228}$$

input
```
Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(d^9*Sqrt[d*x]*(4*b^(1/4)*Sqrt[x]*(9945*a^4 + 37791*a^3*b*x^2 + 52819*a^2*
b^2*x^4 + 31501*a*b^3*x^6 + 6144*b^4*x^8) - 9945*Sqrt[2]*a^(1/4)*(a + b*x^
2)^4*ArcTan[(-Sqrt[a] + Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 99
45*Sqrt[2]*a^(1/4)*(a + b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
/(Sqrt[a] + Sqrt[b]*x)]))/(12288*b^(21/4)*Sqrt[x]*(a + b*x^2)^3*Sqrt[(a +
b*x^2)^2])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 459, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.567$, Rules used = {1384, 27, 252, 252, 252, 252, 262, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{19/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^5\left(a+bx^2\right)\int \frac{(dx)^{19/2}}{b^5\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(a+bx^2\right)\int \frac{(dx)^{19/2}}{\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\left(a+bx^2\right)\left(\frac{17d^2\int \frac{(dx)^{15/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\left(a+bx^2\right)\left(\frac{17d^2\left(\frac{13d^2\int \frac{(dx)^{11/2}}{\left(bx^2+a\right)^3}dx}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{(a+bx^2)\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\int\frac{(dx)^{7/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{17/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{(a+bx^2)\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\int\frac{(dx)^{3/2}}{bx^2+a}dx}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{17/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{(a+bx^2)\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{ad^2\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{17/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 266$$

$$\frac{(a+bx^2)\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\int\frac{1}{bx^2+a}d\sqrt{dx}}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{17/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 755

$$(a+bx^2)\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{\int\frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int\frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{13/}}{6b\left(a+bx^2\right.}}{16b}\right.$$

$$\Big\downarrow \, 27$$

$$(a+bx^2)\left(\frac{17d^2\left(\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{13/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}\right.$$

$\Big\downarrow$ 1476

$$\Bigg(\Bigg(13d^2\Bigg(9d^2\Bigg(5d^2\Bigg(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right.}{b}}{4b}}{8b}$$

↓ 1082

$$\Bigg(\quad\Bigg(\quad 13d^2 \Bigg(9d^2 \Bigg(\frac{5d^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{2ad \left(\frac{d \int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\, d\sqrt{dx}}{2\sqrt{a}} + \frac{d \left(\frac{\int \frac{1}{-dx-1}\, d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\int \frac{1}{-dx-1}\, d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} \right)}{b} \right)}{4b} - \frac{c}{2b} \cdots \Bigg) \Bigg/ 8b \cdots$$

↓ 217

$$\left(\left(\frac{13d^2\left(\frac{9d^2\left(\frac{5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}\right)}{8b}\right.\right.$$

↓ 1479

$$\left(\left(13d^2\frac{\left(9d^2\frac{\left(5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{2ad\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right.}{b}\right.}{4b}\right.}{8b}\right.\right.$$

$$\Big\downarrow 25$$

$$13d^2 \left(9d^2 \left(5d^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{2ad \left(\frac{d \left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} + \frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d(\cdots}{\cdots} \right)}{b} \right)}{4b} \right)}{8b}$$

$$\Big\downarrow \; 27$$

$$\left(\quad 13d^2 \frac{\left(9d^2 \frac{\left(5d^2 \left(\frac{2d\sqrt{dx}}{b} - \frac{2ad \frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}} + \frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}} \right)}{2\sqrt{a}} + \frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b} \cdots}{\sqrt[4]{a} \cdots} \right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b} \cdots} \cdots \right.}{\cdots}}{b} \cdots \right.}{4b} \cdots \right.}{8b} \cdots \right.$$

↓ 1103

$$2ad\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right.}{}\right.$$

$$5d^2\left(\frac{2d\sqrt{dx}}{b}-\frac{\cdots}{b}\right.$$

$$9d^2\ \frac{\cdots}{4b}$$

$$13d^2\ \frac{\cdots}{8b}$$

input
```
Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*(d*(d*x)^(17/2))/(b*(a + b*x^2)^4) + (17*d^2*(-1/6*(d*(
d*x)^(13/2))/(b*(a + b*x^2)^3) + (13*d^2*(-1/4*(d*(d*x)^(9/2))/(b*(a + b*x
^2)^2) + (9*d^2*(-1/2*(d*(d*x)^(5/2))/(b*(a + b*x^2)) + (5*d^2*((2*d*Sqrt[
d*x])/b - (2*a*d*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a
]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d
]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d])))/(2*Sqrt[a])))/b))/(4*b)))/(8*b)))/(12*b)))/(16*b)))/Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.56

method	result
risch	$\frac{2x\,d^{10}\sqrt{(b\,x^2+a)^2}}{b^5\sqrt{dx}\,(b\,x^2+a)} - \frac{2a\,d^{11}\left(\frac{-\frac{1267a^3d^6\sqrt{dx}}{2048}-\frac{4405b\,d^4a^2(dx)^{\frac{5}{2}}}{2048}-\frac{15955a\,d^2b^2(dx)^{\frac{9}{2}}}{6144}-\frac{6925b^3(dx)^{\frac{13}{2}}}{6144}}{\left(b\,d^2x^2+a\,d^2\right)^4}+\frac{3315\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{a\,d^2}{b}\right.}{dx-\left(\frac{a\,d^2}{b}\right.}\right.\right.}{}\right.}{b^5(b\,x^2+a)}$
default	Expression too large to display

input
```
int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^5*x/(d*x)^(1/2)*d^10*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-2*a/b^5*d^11*((-126
7/2048*a^3*d^6*(d*x)^(1/2)-4405/2048*b*d^4*a^2*(d*x)^(5/2)-15955/6144*a*d^
2*b^2*(d*x)^(9/2)-6925/6144*b^3*(d*x)^(13/2))/(b*d^2*x^2+a*d^2)^4+3315/163
84*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1
/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*
d^2/b)^(1/4)*(d*x)^(1/2)-1)))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.95

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx =$$

$$-\frac{9945\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{1}{4}}(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5)\log\left(3315\,\sqrt{dx}d^9+3315\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{1}{4}}b^5\right)+9}{}$$

input

```
integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

output

```
-1/12288*(9945*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4
 + 4*a^3*b^6*x^2 + a^4*b^5)*log(3315*sqrt(d*x)*d^9 + 3315*(-a*d^38/b^21)^(
1/4)*b^5) + 9945*(-a*d^38/b^21)^(1/4)*(I*b^9*x^8 + 4*I*a*b^8*x^6 + 6*I*a^2
*b^7*x^4 + 4*I*a^3*b^6*x^2 + I*a^4*b^5)*log(3315*sqrt(d*x)*d^9 + 3315*I*(-
a*d^38/b^21)^(1/4)*b^5) + 9945*(-a*d^38/b^21)^(1/4)*(-I*b^9*x^8 - 4*I*a*b^
8*x^6 - 6*I*a^2*b^7*x^4 - 4*I*a^3*b^6*x^2 - I*a^4*b^5)*log(3315*sqrt(d*x)*
d^9 - 3315*I*(-a*d^38/b^21)^(1/4)*b^5) - 9945*(-a*d^38/b^21)^(1/4)*(b^9*x^
8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*log(3315*sqrt(d
*x)*d^9 - 3315*(-a*d^38/b^21)^(1/4)*b^5) - 4*(6144*b^4*d^9*x^8 + 31501*a*b
^3*d^9*x^6 + 52819*a^2*b^2*d^9*x^4 + 37791*a^3*b*d^9*x^2 + 9945*a^4*d^9)*s
qrt(d*x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(dx)^{\frac{19}{2}}}{(b^2x^4+2\,abx^2+a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

output
```
d^(19/2)*integrate(x^(3/2)/(b^5*x^2 + a*b^4), x) - 1267/8192*(2*sqrt(2)*sq
rt(a)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*
sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)
))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqr
t(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))*d^(19/2)/b^5 + 1/3072*(18
53*a*b^3*d^(19/2)*x^(13/2) + 6515*a^2*b^2*d^(19/2)*x^(9/2) + 8079*a^3*b*d^
(19/2)*x^(5/2) + 3801*a^4*d^(19/2)*sqrt(x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2
*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5) + 1/192*((317*a*b^4*d^(19/2)*x^5 + 738
*a^2*b^3*d^(19/2)*x^3 + 453*a^3*b^2*d^(19/2)*x)*x^(11/2) + 2*(243*a^2*b^3*
d^(19/2)*x^5 + 582*a^3*b^2*d^(19/2)*x^3 + 371*a^4*b*d^(19/2)*x)*x^(7/2) +
(201*a^3*b^2*d^(19/2)*x^5 + 490*a^4*b*d^(19/2)*x^3 + 321*a^5*d^(19/2)*x)*x
^(3/2))/(a^3*b^7*x^6 + 3*a^4*b^6*x^4 + 3*a^5*b^5*x^2 + a^6*b^4 + (b^10*x^6
 + 3*a*b^9*x^4 + 3*a^2*b^8*x^2 + a^3*b^7)*x^6 + 3*(a*b^9*x^6 + 3*a^2*b^8*x
^4 + 3*a^3*b^7*x^2 + a^4*b^6)*x^4 + 3*(a^2*b^8*x^6 + 3*a^3*b^7*x^4 + 3*a^4
*b^6*x^2 + a^5*b^5)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.80

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{\frac{19890\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^{10}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6\operatorname{sgn}(bx^2+a)} + \frac{19890\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^{10}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6\operatorname{sgn}(bx^2+a)} + \frac{9945\sqrt{2}(ab^3d^2}{}}{}$$

input
```
integrate((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/24576*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*d^10*arctan(1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*sgn(b*x^2 + a)) + 19
890*sqrt(2)*(a*b^3*d^2)^(1/4)*d^10*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*sgn(b*x^2 + a)) + 9945*sqrt(2)*
(a*b^3*d^2)^(1/4)*d^10*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(b^6*sgn(b*x^2 + a)) - 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*d^10*log(d
*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*sgn(b*x^2 + a
)) - 49152*sqrt(d*x)*d^10/(b^5*sgn(b*x^2 + a)) - 8*(6925*sqrt(d*x)*a*b^3*d
^18*x^6 + 15955*sqrt(d*x)*a^2*b^2*d^18*x^4 + 13215*sqrt(d*x)*a^3*b*d^18*x^
2 + 3801*sqrt(d*x)*a^4*d^18)/((b*d^2*x^2 + a*d^2)^4*b^5*sgn(b*x^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\int \frac{(d\,x)^{19/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(19/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.68

$$\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output

```
(sqrt(d)*d**9*(19890*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 + 79560*b**(3/
4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 + 119340*b**(3/4)*a**(1/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*s
qrt(2)))*a**2*b**2*x**4 + 79560*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x
**6 + 19890*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 - 19890*b**(3/4)*a
**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*a**4 - 79560*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1
/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**
3*b*x**2 - 119340*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 - 79560
*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqr
t(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 - 19890*b**(3/4)*a**(1/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*b**4*x**8 + 9945*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b
**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**4 + 39780*b**(3/4)*a**(
1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(...
```

3.653 $\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 448

$$\begin{aligned}\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx &= -\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{d(dx)^{15/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3(dx)^{11/2}}{32b^2\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{55d^5(dx)^{7/2}}{256b^3\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{1155d^{17/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\sqrt[4]{a}b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{1155d^{17/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\sqrt[4]{a}b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{1155d^{17/2}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}\sqrt[4]{a}b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
-385/1024*d^7*(d*x)^(3/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(15/2)/b/(b*
x^2+a)^3/((b*x^2+a)^2)^(1/2)-5/32*d^3*(d*x)^(11/2)/b^2/(b*x^2+a)^2/((b*x^2
+a)^2)^(1/2)-55/256*d^5*(d*x)^(7/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-1155
/4096*d^(17/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1
/2))*2^(1/2)/a^(1/4)/b^(19/4)/((b*x^2+a)^2)^(1/2)+1155/4096*d^(17/2)*(b*x^
2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(1/4)
/b^(19/4)/((b*x^2+a)^2)^(1/2)-1155/4096*d^(17/2)*(b*x^2+a)*arctanh(2^(1/2)
*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(1/4)/
b^(19/4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.46

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{d^8\sqrt{dx}\Big(-4\sqrt[4]{a}b^{3/4}x^{3/2}(385a^3+1375a^2bx^2+1755ab^2x^4+893b^3x^6)+1155\cdots}{4096\sqrt[4]{a}b^{19/4}\sqrt{x}\cdots}$$

input
```
Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(d^8*Sqrt[d*x]*(-4*a^(1/4)*b^(3/4)*x^(3/2)*(385*a^3 + 1375*a^2*b*x^2 + 175
5*a*b^2*x^4 + 893*b^3*x^6) + 1155*Sqrt[2]*(a + b*x^2)^4*ArcTan[(-Sqrt[a] +
 Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 1155*Sqrt[2]*(a + b*x^2)^
4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(4096
*a^(1/4)*b^(19/4)*Sqrt[x]*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1384, 27, 252, 252, 252, 252, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$$

$$\Big\downarrow \; 1384$$

$$\frac{b^5(a+bx^2)\int \frac{(dx)^{17/2}}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^2)\int \frac{(dx)^{17/2}}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 252$$

$$\frac{(a+bx^2)\left(\frac{15d^2\int \frac{(dx)^{13/2}}{(bx^2+a)^4}dx}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 252$$

$$\frac{(a+bx^2)\left(\frac{15d^2\left(\frac{11d^2\int \frac{(dx)^{9/2}}{(bx^2+a)^3}dx}{12b}-\frac{d(dx)^{11/2}}{6b(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 252$$

$$\frac{(a+bx^2)\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\int \frac{(dx)^{5/2}}{(bx^2+a)^2}dx}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\int \frac{\sqrt{dx}}{bx^2+a}dx}{4b}-\frac{d(dx)^{3/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(a+bx^2)\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2b}-\frac{d(dx)^{3/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 826$$

$$\frac{(a+bx^2)\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{11/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 1476

$$(a+bx^2)\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}\right)}{8b}-}{12b}}{16b}$$

↓ 1082

$$(a+bx^2)\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{7/}}{4b(a+bx^2}}{12b}}{16b}$$

↓ 217

$$(a+bx^2)\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2b}-\frac{d(dx)^{3/2}}{2b(a+bx^2)}\right)}{8b}-\frac{d(dx)^{7/2}}{4b(a+bx^2)^2}\right)}{12b}-\frac{}{6}\right)}{16b}\right.$$

↓ 1479

$$15d^2\left(11d^2\left(7d^2\left(3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\cdots}{\sqrt[4]{b}\cdots}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right)}{2b}\cdots\right)}{8b}\cdots\right)}{12b}\cdots$$

$$\Big\downarrow \; 25$$

$$15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{\cdots}\right.}{\sqrt[4]{b}\left(xd+\cdots\right.}}{\cdots}}{2\sqrt{b}}\right.}{2b}\right.}{8b}\right.}{12b}\right.$$

$\downarrow$ 27

$$\left(15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{\cdots}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\sqrt[4]{\cdots}}}\cdots}{2\sqrt[4]{a}\sqrt{b}\cdots}}{2\sqrt{b}}\cdots\right.}{2b}\cdots\right.}{8b}\cdots\right.}{12b}\cdots\right.\right.$$

↓ 1103

$$
(a+bx^2)\left(\frac{15d^2\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}\right.}{2\sqrt{2}\cdots}}{2\sqrt{b}}\cdots}{2b}\cdots}{8b}\cdots}{12b}\cdots}{16b}\cdots
$$

input
```
Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*(d*(d*x)^(15/2))/(b*(a + b*x^2)^4) + (15*d^2*(-1/6*(d*(
d*x)^(11/2))/(b*(a + b*x^2)^3) + (11*d^2*(-1/4*(d*(d*x)^(7/2))/(b*(a + b*x
^2)^2) + (7*d^2*(-1/2*(d*(d*x)^(3/2))/(b*(a + b*x^2)) + (3*d^3*((-(ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[
b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d
]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*b)))/(8
*b)))/(12*b)))/(16*b)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. 2(291) = 582.

Time = 0.15 (sec) , antiderivative size = 1046, normalized size of antiderivative = 2.33

method	result	size
default	Expression too large to display	1046

input
```
int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8192*(-1155*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2
/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^4*
d^8*x^8-2310*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b
)^(1/4))*b^4*d^8*x^8-2310*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1
/4))/(a*d^2/b)^(1/4))*b^4*d^8*x^8+7144*(d*x)^(15/2)*(a*d^2/b)^(1/4)*b^4-46
20*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(
d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^8*x^6-92
40*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a
*b^3*d^8*x^6-9240*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*a*b^3*d^8*x^6+14040*(d*x)^(11/2)*(a*d^2/b)^(1/4)*a*b^3*d^2-6
930*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/
(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^8*x^4
-13860*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*a^2*b^2*d^8*x^4-13860*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/
4))/(a*d^2/b)^(1/4))*a^2*b^2*d^8*x^4+11000*(d*x)^(7/2)*(a*d^2/b)^(1/4)*a^2
*b^2*d^4-4620*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/
b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b
*d^8*x^2-9240*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*a^3*b*d^8*x^2-9240*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)
^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^8*x^2+3080*(d*x)^(3/2)*(a*d^2/b)^(1/4)...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.06

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{1155\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\log\left(15407988\right.}{}$$

input
```
integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

output
```
1/4096*(1155*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*
b^4)*(-d^34/(a*b^19))^(1/4)*log(1540798875*sqrt(d*x)*d^25 + 1540798875*(-d
^34/(a*b^19))^(3/4)*a*b^14) - 1155*(I*b^8*x^8 + 4*I*a*b^7*x^6 + 6*I*a^2*b^
6*x^4 + 4*I*a^3*b^5*x^2 + I*a^4*b^4)*(-d^34/(a*b^19))^(1/4)*log(1540798875
*sqrt(d*x)*d^25 + 1540798875*I*(-d^34/(a*b^19))^(3/4)*a*b^14) - 1155*(-I*b
^8*x^8 - 4*I*a*b^7*x^6 - 6*I*a^2*b^6*x^4 - 4*I*a^3*b^5*x^2 - I*a^4*b^4)*(-
d^34/(a*b^19))^(1/4)*log(1540798875*sqrt(d*x)*d^25 - 1540798875*I*(-d^34/(
a*b^19))^(3/4)*a*b^14) - 1155*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a
^3*b^5*x^2 + a^4*b^4)*(-d^34/(a*b^19))^(1/4)*log(1540798875*sqrt(d*x)*d^25
 - 1540798875*(-d^34/(a*b^19))^(3/4)*a*b^14) - 4*(893*b^3*d^8*x^7 + 1755*a
*b^2*d^8*x^5 + 1375*a^2*b*d^8*x^3 + 385*a^3*d^8*x)*sqrt(d*x))/(b^8*x^8 + 4
*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(dx)^{\frac{17}{2}}}{(b^2x^4+2\,abx^2+a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

output
```
d^(17/2)*integrate(sqrt(x)/(b^5*x^2 + a*b^4), x) - 893/8192*d^(17/2)*(2*sq
rt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)
*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^4 - 1/3
072*(2679*b^3*d^(17/2)*x^(15/2) + 9441*a*b^2*d^(17/2)*x^(11/2) + 11645*a^2
*b*d^(17/2)*x^(7/2) + 5267*a^3*d^(17/2)*x^(3/2))/(b^8*x^8 + 4*a*b^7*x^6 +
6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4) + 1/192*((261*a*b^4*d^(17/2)*x^5
+ 610*a^2*b^3*d^(17/2)*x^3 + 381*a^3*b^2*d^(17/2)*x)*x^(9/2) + 2*(191*a^2*
b^3*d^(17/2)*x^5 + 462*a^3*b^2*d^(17/2)*x^3 + 303*a^4*b*d^(17/2)*x)*x^(5/2
) + (153*a^3*b^2*d^(17/2)*x^5 + 378*a^4*b*d^(17/2)*x^3 + 257*a^5*d^(17/2)*
x)*sqrt(x))/(a^3*b^7*x^6 + 3*a^4*b^6*x^4 + 3*a^5*b^5*x^2 + a^6*b^4 + (b^10
*x^6 + 3*a*b^9*x^4 + 3*a^2*b^8*x^2 + a^3*b^7)*x^6 + 3*(a*b^9*x^6 + 3*a^2*b
^8*x^4 + 3*a^3*b^7*x^2 + a^4*b^6)*x^4 + 3*(a^2*b^8*x^6 + 3*a^3*b^7*x^4 + 3
*a^4*b^6*x^2 + a^5*b^5)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.85

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{1}{8192}\,d^8\left(\frac{2310\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^7d\,\mathrm{sgn}\left(bx^2+a\right)} + \frac{2310\,\sqrt{2}(ab^3d^2)}{\ }\right.$$

input
```
integrate((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/8192*d^8*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^7*d*sgn(b*x^2 + a)) + 23
10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
- 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^7*d*sgn(b*x^2 + a)) - 1155*sqrt(2)*(a
*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a*b^7*d*sgn(b*x^2 + a)) + 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqr
t(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^7*d*sgn(b*x^2 + a)) -
 8*(893*sqrt(d*x)*b^3*d^8*x^7 + 1755*sqrt(d*x)*a*b^2*d^8*x^5 + 1375*sqrt(d
*x)*a^2*b*d^8*x^3 + 385*sqrt(d*x)*a^3*d^8*x)/((b*d^2*x^2 + a*d^2)^4*b^4*sg
n(b*x^2 + a)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(d\,x)^{17/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(17/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.84

$$\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(sqrt(d)*d**8*( - 2310*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 - 9240*b**(1
/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/
(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 - 13860*b**(1/4)*a**(3/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*s
qrt(2)))*a**2*b**2*x**4 - 9240*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x*
*6 - 2310*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sq
rt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 + 2310*b**(1/4)*a**(
3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4
)*a**(1/4)*sqrt(2)))*a**4 + 9240*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*
x**2 + 13860*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 + 9240*b**(1
/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/
(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 + 2310*b**(1/4)*a**(3/4)*sqrt(2)*
atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sq
rt(2)))*b**4*x**8 + 1155*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)
*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**4 + 4620*b**(1/4)*a**(3/4)*sqr
t(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*...
```

3.654 $\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 448

$$
\begin{aligned}
\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx &= -\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}} \\
&\quad - \frac{d(dx)^{13/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{13d^3(dx)^{9/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} \\
&\quad - \frac{39d^5(dx)^{5/2}}{256b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} \\
&\quad - \frac{195d^{15/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&\quad + \frac{195d^{15/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&\quad + \frac{195d^{15/2}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

output
```
-195/1024*d^7*(d*x)^(1/2)/b^4/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(13/2)/b/(b*
x^2+a)^3/((b*x^2+a)^2)^(1/2)-13/96*d^3*(d*x)^(9/2)/b^2/(b*x^2+a)^2/((b*x^2
+a)^2)^(1/2)-39/256*d^5*(d*x)^(5/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-195/
4096*d^(15/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/
2))*2^(1/2)/a^(3/4)/b^(17/4)/((b*x^2+a)^2)^(1/2)+195/4096*d^(15/2)*(b*x^2+
a)*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(3/4)/b
^(17/4)/((b*x^2+a)^2)^(1/2)+195/4096*d^(15/2)*(b*x^2+a)*arctanh(2^(1/2)*a^
(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(3/4)/b^(
17/4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.45

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(dx)^{15/2}\,(a+bx^2)\left(-4\sqrt[4]{b}\sqrt{x}(585a^3+2223a^2bx^2+3107ab^2x^4+1853b^3x^6\right.}{12288b^{17/4}x^{15/2}}$$

input
```
Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((d*x)^(15/2)*(a + b*x^2)*(-4*b^(1/4)*Sqrt[x]*(585*a^3 + 2223*a^2*b*x^2 +
3107*a*b^2*x^4 + 1853*b^3*x^6) - (585*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[a
] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (585*Sqrt[2]*
(a + b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]
*x)])/a^(3/4)))/(12288*b^(17/4)*x^(15/2)*((a + b*x^2)^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1384, 27, 252, 252, 252, 252, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{15/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5\left(a+bx^2\right)\int \frac{(dx)^{15/2}}{b^5\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a+bx^2\right)\int \frac{(dx)^{15/2}}{\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\left(a+bx^2\right)\left(\frac{13d^2\int \frac{(dx)^{11/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\left(a+bx^2\right)\left(\frac{13d^2\left(\frac{3d^2\int \frac{(dx)^{7/2}}{\left(bx^2+a\right)^3}dx}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\left(a+bx^2\right)\left(\frac{13d^2\left(\frac{3d^2\left(\frac{5d^2\int \frac{(dx)^{3/2}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{13/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(a+bx^2)\left(\frac{13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d^2\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{13/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(a+bx^2)\left(\frac{13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\int \frac{1}{bx^2+a}d\sqrt{dx}}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{13/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{755}$$

$$\frac{(a+bx^2)\left(\frac{13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{13/\cdots}}{8b(a+bx^{\cdots}}\cdots\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{(a+bx^2)\left(\frac{13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{9/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{13/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1476

$$\left(13d^2 \frac{\left(3d^2 \frac{\left(5d^2 \frac{\left(d \left(\frac{d \int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2} d\sqrt{dx}}{2\sqrt{a}} + \frac{d \left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}} d\sqrt{dx}}{2\sqrt{b}} + \frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}} d\sqrt{dx}}{2\sqrt{b}} \right)}{2\sqrt{a}} \right) \right)}{2b} - \frac{d\sqrt{dx}}{2b\left(a+bx^2\right)} \right)}{8b} \cdots}{4b} \cdots \right.$$

↓ 1082

$$13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)}{4b\left(a+\ldots\right.}\ldots}{4b}\ldots\right.$$

$\Big\downarrow$ 217

$$\left(13d^2\frac{\left(3d^2\frac{\left(5d^2\frac{\left(d\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)\right)}{2b}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)^2}\right)}{4b}-$$

↓ 1479

$$\left(13d^2 \frac{\left(3d^2 \frac{\left(5d^2 \frac{\left(d \frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \right)}{2\sqrt{a}} + d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}} \cdots \right)}{\sqrt{2}\sqrt[4]{a}\cdots} \cdots \right) \cdots \right)}{2b} \cdots \right)}{8b} \cdots \right)}{4b} \cdots \right.$$

$$\Big\downarrow 25$$

$$13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{a}\sqrt{d}\cdots}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\right.}{\cdots}}{2b}\cdots}{8b}\cdots}{4b}\cdots\right.$$

$\downarrow$ 27

$$13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(\cdots\right.}{\cdots}\right.}{2\sqrt{a}}\right.}{2b}\right.}{8b}\right.}{4b}\right.$$

↓ 1103

$$13d^2\left(\frac{3d^2\left(\frac{5d^2\left(\frac{d\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{\cdots}\right.}{\cdots}\right.}{2\sqrt{a}}\cdots}{2b}\cdots}{8b}\cdots}{4b}\cdots\right.$$

input
```
Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*(d*(d*x)^(13/2))/(b*(a + b*x^2)^4) + (13*d^2*(-1/6*(d*(
d*x)^(9/2))/(b*(a + b*x^2)^3) + (3*d^2*(-1/4*(d*(d*x)^(5/2))/(b*(a + b*x^2
)^2) + (5*d^2*(-1/2*(d*Sqrt[d*x])/(b*(a + b*x^2)) + (d*((d*(-(ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b
]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]
*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*b)))/(8
*b)))/(4*b)))/(16*b)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1133 vs. $2(291) = 582$.

Time = 0.15 (sec) , antiderivative size = 1134, normalized size of antiderivative = 2.53

method	result	size
default	Expression too large to display	1134

input
```
int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24576*(585*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))*b^4*d^6*x^8+1170*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+1170*(a*d^2/b)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6
*x^8+2340*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1
/2)))*a*b^3*d^6*x^6+4680*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/
2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+4680*(a*d^2/b)^(1/4)*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*
d^6*x^6+3510*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))*a^2*b^2*d^6*x^4+7020*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x
)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4+7020*(a*d^2/b)^(
1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))
*a^2*b^2*d^6*x^4-14824*(d*x)^(13/2)*a*b^3+2340*(a*d^2/b)^(1/4)*2^(1/2)*ln(
(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^6*x^2+4680*(a*d^2/b)^(1
/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
a^3*b*d^6*x^2+4680*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.06

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{585\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)\left(-\frac{d^{30}}{a^3b^{17}}\right)^{\frac{1}{4}}\log\left(195\,\sqrt{dx}d\right.}{}$$

input
```
integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

output
```
1/12288*(585*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*
b^4)*(-d^30/(a^3*b^17))^(1/4)*log(195*sqrt(d*x)*d^7 + 195*(-d^30/(a^3*b^17
))^(1/4)*a*b^4) - 585*(-I*b^8*x^8 - 4*I*a*b^7*x^6 - 6*I*a^2*b^6*x^4 - 4*I*
a^3*b^5*x^2 - I*a^4*b^4)*(-d^30/(a^3*b^17))^(1/4)*log(195*sqrt(d*x)*d^7 +
195*I*(-d^30/(a^3*b^17))^(1/4)*a*b^4) - 585*(I*b^8*x^8 + 4*I*a*b^7*x^6 + 6
*I*a^2*b^6*x^4 + 4*I*a^3*b^5*x^2 + I*a^4*b^4)*(-d^30/(a^3*b^17))^(1/4)*log
(195*sqrt(d*x)*d^7 - 195*I*(-d^30/(a^3*b^17))^(1/4)*a*b^4) - 585*(b^8*x^8
+ 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^30/(a^3*b^17)
)^(1/4)*log(195*sqrt(d*x)*d^7 - 195*(-d^30/(a^3*b^17))^(1/4)*a*b^4) - 4*(1
853*b^3*d^7*x^6 + 3107*a*b^2*d^7*x^4 + 2223*a^2*b*d^7*x^2 + 585*a^3*d^7)*s
qrt(d*x))/(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. 2(291) = 582.

Time = 0.18 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.30

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{195\,d^7\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\right.}{8192\,b^4} - \frac{15\,b^3d^{\frac{15}{2}}x^{\frac{13}{2}}+65\,ab^2d^{\frac{15}{2}}x^{\frac{9}{2}}+117\,a^2bd^{\frac{15}{2}}x^{\frac{5}{2}}+195\,a^3d^{\frac{15}{2}}\sqrt{x}}{1024\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)} - \frac{\left(113\,b^4d^{\frac{15}{2}}x^5+282\,ab^3d^{\frac{15}{2}}x^3+201\,a^2b^2d^{\frac{15}{2}}x\right)x^{\frac{11}{2}}+2\left(63\,ab^3d^{\frac{15}{2}}x^5+174\,a^2b^2d^{\frac{15}{2}}x^3+143\,a^3bd^{\frac{15}{2}}\right.}{192\,(a^3b^6x^6+3\,a^4b^5x^4+3\,a^5b^4x^2+a^6b^3+(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)x^6+3\,(ab^8x^6+3\,a^2b^7x^4+}$$

input
```
integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

output
```
195/8192*d^7*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*s
qrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + s
qrt(2)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/
(a^(3/4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
 sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^4 - 1/1024*(15*b^3*d^(15/2)*x^(
13/2) + 65*a*b^2*d^(15/2)*x^(9/2) + 117*a^2*b*d^(15/2)*x^(5/2) + 195*a^3*d
^(15/2)*sqrt(x))/(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 +
a^4*b^4) - 1/192*((113*b^4*d^(15/2)*x^5 + 282*a*b^3*d^(15/2)*x^3 + 201*a^2
*b^2*d^(15/2)*x)*x^(11/2) + 2*(63*a*b^3*d^(15/2)*x^5 + 174*a^2*b^2*d^(15/2
)*x^3 + 143*a^3*b*d^(15/2)*x)*x^(7/2) + (45*a^2*b^2*d^(15/2)*x^5 + 130*a^3
*b*d^(15/2)*x^3 + 117*a^4*d^(15/2)*x)*x^(3/2))/(a^3*b^6*x^6 + 3*a^4*b^5*x^
4 + 3*a^5*b^4*x^2 + a^6*b^3 + (b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3
*b^6)*x^6 + 3*(a*b^8*x^6 + 3*a^2*b^7*x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^4 +
3*(a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4*b^5*x^2 + a^5*b^4)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.85

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{\frac{1170\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^8\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5\mathrm{sgn}(bx^2+a)} + \frac{1170\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^8\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(a\right.\right.}{2\left(\right.}\right.}{ab^5\mathrm{sgn}(bx^2+a)}}{}$$

input
```
integrate((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/24576*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*d^8*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sgn(b*x^2 + a)) + 117
0*sqrt(2)*(a*b^3*d^2)^(1/4)*d^8*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sgn(b*x^2 + a)) + 585*sqrt(2)*(a
*b^3*d^2)^(1/4)*d^8*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d
^2/b))/(a*b^5*sgn(b*x^2 + a)) - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*d^8*log(d*x
- sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^5*sgn(b*x^2 + a)
) - 8*(1853*sqrt(d*x)*b^3*d^16*x^6 + 3107*sqrt(d*x)*a*b^2*d^16*x^4 + 2223*
sqrt(d*x)*a^2*b*d^16*x^2 + 585*sqrt(d*x)*a^3*d^16)/((b*d^2*x^2 + a*d^2)^4*
b^4*sgn(b*x^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(d\,x)^{15/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(15/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.84

$$\int \frac{(dx)^{15/2}}{(a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx = \text{Too large to display}$$

input
```
int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(sqrt(d)*d**7*( - 1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 - 4680*b**(3
/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/
(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 - 7020*b**(3/4)*a**(1/4)*sqrt(2)*
atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sq
rt(2)))*a**2*b**2*x**4 - 4680*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**
(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**
6 - 1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 + 1170*b**(3/4)*a**(1
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*a**4 + 4680*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x
**2 + 7020*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*s
qrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 + 4680*b**(3/4
)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 + 1170*b**(3/4)*a**(1/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt
(2)))*b**4*x**8 - 585*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a*
*(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**4 - 2340*b**(3/4)*a**(1/4)*sqrt(2
)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3*...
```

3.655 $\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 451

$$\begin{aligned}\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx &= \frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{d(dx)^{11/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{11d^3(dx)^{7/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{77d^5(dx)^{3/2}}{768b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{77d^{13/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{77d^{13/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{77d^{13/2}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
77/1024*d^5*(d*x)^(3/2)/a/b^3/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(11/2)/b/(b*
x^2+a)^3/((b*x^2+a)^2)^(1/2)-11/96*d^3*(d*x)^(7/2)/b^2/(b*x^2+a)^2/((b*x^2
+a)^2)^(1/2)-77/768*d^5*(d*x)^(3/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-77/4
096*d^(13/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))*2^(1/2)/a^(5/4)/b^(15/4)/((b*x^2+a)^2)^(1/2)+77/4096*d^(13/2)*(b*x^2+a)
*arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(5/4)/b^(
15/4)/((b*x^2+a)^2)^(1/2)-77/4096*d^(13/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/
4)*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(5/4)/b^(15/
4)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.45

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(dx)^{13/2}\,(a+bx^2)\left(-4\sqrt[4]{a}b^{3/4}x^{3/2}(77a^3+275a^2bx^2+351ab^2x^4-231b^3x^6)\right.}{12288a^{5/4}b}$$

input
```
Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((d*x)^(13/2)*(a + b*x^2)*(-4*a^(1/4)*b^(3/4)*x^(3/2)*(77*a^3 + 275*a^2*b*
x^2 + 351*a*b^2*x^4 - 231*b^3*x^6) - 231*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqr
t[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 231*Sqrt[2]*(a + b*
x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/
(12288*a^(5/4)*b^(15/4)*x^(13/2)*((a + b*x^2)^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1384, 27, 252, 252, 252, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{13/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$$

$\Big\downarrow$ 1384

$$\frac{b^5\left(a+bx^2\right)\int \frac{(dx)^{13/2}}{b^5\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{\left(a+bx^2\right)\int \frac{(dx)^{13/2}}{\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{\left(a+bx^2\right)\left(\frac{11d^2\int \frac{(dx)^{9/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{\left(a+bx^2\right)\left(\frac{11d^2\left(\frac{7d^2\int \frac{(dx)^{5/2}}{\left(bx^2+a\right)^3}dx}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{\left(a+bx^2\right)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^2}dx}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{11/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{\int \frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 266$$

$$\frac{(a+bx^2)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^2)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 826

$$\frac{(a+bx^2)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{7/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1476

$$(a+bx^2)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{}{4b}\right.}{12b}\right.}{16b}\right.$$

↓ 1082

$$\left(a+bx^2\right)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\int \frac{1}{-dx-1}\,d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}\,d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\cdots\right)}{12b}\cdots\right)}{16b}\cdots\right.$$

↓ 217

$$\left(a+bx^2\right)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(\cdots}{6b(\cdots}\right)}{16b}\cdots$$

↓ 1479

$$\left(11d^2\frac{\left(7d^2\frac{\left(3d^2\frac{\left(d\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\cdots\right.}{\sqrt[4]{b}\left(xd\cdots\right.}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right.}{2a}\cdots\right.}{8b}\cdots\right.}{12b}\cdots\right.$$

↓ 25

$$\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}\cdots}{\sqrt{\cdots}}\right.}\cdots}{2\sqrt{\cdots}}}{2\sqrt{b}}\cdots\right)}{2a}$$

$$3d^2\,\frac{\left(\cdots\right)}{2a}\cdots$$

$$7d^2\,\frac{\left(\cdots\right)}{8b}\cdots$$

$$11d^2\,\frac{\left(\cdots\right)}{12b}\cdots$$

↓ 27

$$\Bigg(11d^2 \Bigg(7d^2 \Bigg(3d^2 \frac{\Bigg(d \Bigg(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} - \frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}} + \frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d\cdots}}{\sqrt[4]{b}}}\cdots}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}\cdots}}{2\sqrt{b}\cdots}\cdots}{2a\cdots}\cdots}{8b\cdots}\cdots}{12b\cdots}\cdots$$

↓ 1103

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}$$

$$\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots\right.}{2\sqrt{2}\sqrt[4]{\cdots}}$$

$$(a+bx^2)\left(\frac{11d^2\left(\frac{7d^2\left(\frac{3d^2\left(\frac{d\left(\frac{\cdots}{2\sqrt{b}}-\frac{\cdots}{2\sqrt{b}}\cdots\right.}{2a}\cdots\right.}{8b}\cdots\right.}{12b}\cdots\right.}{16b}\cdots\right.$$

input
```
Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*(d*(d*x)^(11/2))/(b*(a + b*x^2)^4) + (11*d^2*(-1/6*(d*(
d*x)^(7/2))/(b*(a + b*x^2)^3) + (7*d^2*(-1/4*(d*(d*x)^(3/2))/(b*(a + b*x^2
)^2) + (3*d^2*((d*x)^(3/2)/(2*a*d*(a + b*x^2)) + (d*((-(ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))
 + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])
 + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]
]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*a)))/(8*b)))/(12*b
)))/(16*b)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(294) = 588$.

Time = 0.15 (sec) , antiderivative size = 1051, normalized size of antiderivative = 2.33

method	result	size
default	Expression too large to display	1051

input
```
int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24576*(231*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b
)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^4*d^
8*x^8+462*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*b^4*d^8*x^8+462*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*b^4*d^8*x^8+1848*(d*x)^(15/2)*(a*d^2/b)^(1/4)*b^4+924*2^
(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^8*x^6+1848*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*
d^8*x^6+1848*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b
)^(1/4))*a*b^3*d^8*x^6-2808*(d*x)^(11/2)*(a*d^2/b)^(1/4)*a*b^3*d^2+1386*2^
(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^8*x^4+2772*
2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*
b^2*d^8*x^4+2772*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d
^2/b)^(1/4))*a^2*b^2*d^8*x^4-2200*(d*x)^(7/2)*(a*d^2/b)^(1/4)*a^2*b^2*d^4+
924*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/
(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^8*x^2+1
848*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
a^3*b*d^8*x^2+1848*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a
*d^2/b)^(1/4))*a^3*b*d^8*x^2-616*(d*x)^(3/2)*(a*d^2/b)^(1/4)*a^3*b*d^6+...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.10

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{231\,(ab^7x^8+4\,a^2b^6x^6+6\,a^3b^5x^4+4\,a^4b^4x^2+a^5b^3)\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{1}{4}}\log\left(456533\right.}{}$$

input
```
integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

output
```
1/12288*(231*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 +
a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 + 456533*(-d^2
6/(a^5*b^15))^(3/4)*a^4*b^11) - 231*(I*a*b^7*x^8 + 4*I*a^2*b^6*x^6 + 6*I*a
^3*b^5*x^4 + 4*I*a^4*b^4*x^2 + I*a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*log(456
533*sqrt(d*x)*d^19 + 456533*I*(-d^26/(a^5*b^15))^(3/4)*a^4*b^11) - 231*(-I
*a*b^7*x^8 - 4*I*a^2*b^6*x^6 - 6*I*a^3*b^5*x^4 - 4*I*a^4*b^4*x^2 - I*a^5*b
^3)*(-d^26/(a^5*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 - 456533*I*(-d^26/(
a^5*b^15))^(3/4)*a^4*b^11) - 231*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^
4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*log(456533*sqrt(d*x)
*d^19 - 456533*(-d^26/(a^5*b^15))^(3/4)*a^4*b^11) + 4*(231*b^3*d^6*x^7 - 3
51*a*b^2*d^6*x^5 - 275*a^2*b*d^6*x^3 - 77*a^3*d^6*x)*sqrt(d*x))/(a*b^7*x^8
 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.28

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{77\,d^{\frac{13}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{}}{}\right.}{8192\,ab^3} + \frac{77\,b^3d^{\frac{13}{2}}x^{\frac{15}{2}}+315\,ab^2d^{\frac{13}{2}}x^{\frac{11}{2}}+495\,a^2bd^{\frac{13}{2}}x^{\frac{7}{2}}+385\,a^3d^{\frac{13}{2}}x^{\frac{3}{2}}}{1024\,(ab^7x^8+4\,a^2b^6x^6+6\,a^3b^5x^4+4\,a^4b^4x^2+a^5b^3)} - \frac{\left(81\,b^4d^{\frac{13}{2}}x^5+202\,ab^3d^{\frac{13}{2}}x^3+153\,a^2b^2d^{\frac{13}{2}}x\right)x^{\frac{9}{2}}+2\left(35\,ab^3d^{\frac{13}{2}}x^5+102\,a^2b^2d^{\frac{13}{2}}x^3+99\,a^3bd^{\frac{13}{2}}\right.}{192\,(a^3b^6x^6+3\,a^4b^5x^4+3\,a^5b^4x^2+a^6b^3+(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)x^6+3\,(ab^8x^6+3\,a^2b^7x^4+}$$

input
```
integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

output
```
77/8192*d^(13/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) +
2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b))
+ 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(
x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) +
sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/
4)*b^(3/4)))/(a*b^3) + 1/1024*(77*b^3*d^(13/2)*x^(15/2) + 315*a*b^2*d^(13/
2)*x^(11/2) + 495*a^2*b*d^(13/2)*x^(7/2) + 385*a^3*d^(13/2)*x^(3/2))/(a*b^
7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3) - 1/192*(
(81*b^4*d^(13/2)*x^5 + 202*a*b^3*d^(13/2)*x^3 + 153*a^2*b^2*d^(13/2)*x)*x^
(9/2) + 2*(35*a*b^3*d^(13/2)*x^5 + 102*a^2*b^2*d^(13/2)*x^3 + 99*a^3*b*d^(
13/2)*x)*x^(5/2) + (21*a^2*b^2*d^(13/2)*x^5 + 66*a^3*b*d^(13/2)*x^3 + 77*a
^4*d^(13/2)*x)*sqrt(x))/(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6
*b^3 + (b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)*x^6 + 3*(a*b^8*x^
6 + 3*a^2*b^7*x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^4 + 3*(a^2*b^7*x^6 + 3*a^3*
b^6*x^4 + 3*a^4*b^5*x^2 + a^5*b^4)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.86

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{1}{24576}\,d^6\left(\frac{462\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^6d\,\mathrm{sgn}\left(bx^2+a\right)}+\frac{462\,\sqrt{2}(ab^3d^2\right.$$

input
```
integrate((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/24576*d^6*(462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^6*d*sgn(b*x^2 + a)) +
462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
 - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^6*d*sgn(b*x^2 + a)) - 231*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^2*b^6*d*sgn(b*x^2 + a)) + 231*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x -
sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^6*d*sgn(b*x^2 +
a)) + 8*(231*sqrt(d*x)*b^3*d^8*x^7 - 351*sqrt(d*x)*a*b^2*d^8*x^5 - 275*sqr
t(d*x)*a^2*b*d^8*x^3 - 77*sqrt(d*x)*a^3*d^8*x)/((b*d^2*x^2 + a*d^2)^4*a*b^
3*sgn(b*x^2 + a)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(d\,x)^{13/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(13/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.83

$$\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output

```
(sqrt(d)*d**6*( - 462*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 - 1848*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 - 2772*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*a**2*b**2*x**4 - 1848*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6
 - 462*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 + 462*b**(1/4)*a**(3/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*a**4 + 1848*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2
 + 2772*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt
(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 + 1848*b**(1/4)*a
**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 + 462*b**(1/4)*a**(3/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))
)*b**4*x**8 + 231*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/
4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**4 + 924*b**(1/4)*a**(3/4)*sqrt(2)*log
( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3*b*x**...
```

3.656 $\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 451

$$
\begin{aligned}
\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx &= \frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{d(dx)^{9/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3d^3(dx)^{5/2}}{32b^2\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{15d^5\sqrt{dx}}{256b^3\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{45d^{11/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{45d^{11/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{45d^{11/2}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

output

```
15/1024*d^5*(d*x)^(1/2)/a/b^3/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(9/2)/b/(b*x
^2+a)^3/((b*x^2+a)^2)^(1/2)-3/32*d^3*(d*x)^(5/2)/b^2/(b*x^2+a)^2/((b*x^2+a
)^2)^(1/2)-15/256*d^5*(d*x)^(1/2)/b^3/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-45/409
6*d^(11/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))
*2^(1/2)/a^(7/4)/b^(13/4)/((b*x^2+a)^2)^(1/2)+45/4096*d^(11/2)*(b*x^2+a)*a
rctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(7/4)/b^(13
/4)/((b*x^2+a)^2)^(1/2)+45/4096*d^(11/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)
*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(7/4)/b^(13/4)
/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.45

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{d(dx)^{9/2}\,(a+bx^2)\left(-4a^{3/4}\sqrt[4]{b}\sqrt{x}(45a^3+171a^2bx^2+239ab^2x^4-15b^3x^6)\right.}{4096a^{7/4}b^{13}}$$

input

```
Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output

```
(d*(d*x)^(9/2)*(a + b*x^2)*(-4*a^(3/4)*b^(1/4)*Sqrt[x]*(45*a^3 + 171*a^2*b
*x^2 + 239*a*b^2*x^4 - 15*b^3*x^6) - 45*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt
[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 45*Sqrt[2]*(a + b*x^
2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(4
096*a^(7/4)*b^(13/4)*x^(9/2)*((a + b*x^2)^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1384, 27, 252, 252, 252, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$$

$\big\downarrow$ 1384

$$\frac{b^5(a+bx^2)\int \frac{(dx)^{11/2}}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 27

$$\frac{(a+bx^2)\int \frac{(dx)^{11/2}}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 252

$$\frac{(a+bx^2)\left(\frac{9d^2\int \frac{(dx)^{7/2}}{(bx^2+a)^4}dx}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 252

$$\frac{(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\int \frac{(dx)^{3/2}}{(bx^2+a)^3}dx}{12b}-\frac{d(dx)^{5/2}}{6b(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 252

$$\frac{(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\int \frac{1}{\sqrt{dx}(bx^2+a)^2}dx}{8b}-\frac{d\sqrt{dx}}{4b(a+bx^2)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 266

$$\frac{(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\int\frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 755

$$\frac{\left(a+bx^2\right)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+bx^2)\left(\frac{9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\frac{d(dx)^{5/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 1476$$

$$\left(9d^2\left(\frac{\left(5d^2\,\frac{\left(d^2\,\frac{\left(3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}\,d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)\right.}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}\right.}{12b}\right.\right.$$

↓ 1082

$$9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{d}\cdots}{4b\left(a+b\cdots\right.}}{12b}\cdots\right.$$

↓ 217

$$9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8b}-\frac{d\sqrt{dx}}{4b\left(a+bx^2\right)^2}\right)}{12b}-\cdots\right.$$

↓ 1479

$$\left(9d^2\frac{\left(5d^2\frac{\left(d^2\frac{\left(3\frac{\left(d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}\right.}{\cdots}\right.}{2ad}\right.}{8b}\right.}{12b}\right.$$

$$\Big\downarrow 25$$

$$9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\cdots\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right.}{\cdots}\right)}{2ad}\cdots\right)}{8b}\cdots\right)}{12b}\cdots\right.$$

↓ 27

$$9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1\cdots\right.}{\sqrt{2}\sqrt[4]{\cdots}}\right.}{2\sqrt{a}}\right.}{2ad}\right.}{8b}\right.}{12b}\right.$$

↓ 1103

$$\left(9d^2\left(\frac{5d^2\left(\frac{d^2\left(\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\cdots\right.}{2\sqrt{\cdots}}\right.}{2\sqrt{a}}\cdots\right)}{2ad}\cdots\right)}{8b}\cdots\right)}{12b}\cdots\right.\right.$$

input
```
Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*(d*(d*x)^(9/2))/(b*(a + b*x^2)^4) + (9*d^2*(-1/6*(d*(d*
x)^(5/2))/(b*(a + b*x^2)^3) + (5*d^2*(-1/4*(d*Sqrt[d*x])/(b*(a + b*x^2)^2)
 + (d^2*(Sqrt[d*x]/(2*a*d*(a + b*x^2)) + (3*((d*(-(ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])
+ Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]
/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*a*d)))/(8*b)))/(12
*b)))/(16*b)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(294) = 588$.

Time = 0.15 (sec) , antiderivative size = 1136, normalized size of antiderivative = 2.52

method	result	size
default	Expression too large to display	1136

input
```
int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8192*(45*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*b^4*d^6*x^8+90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+90*(a*d^2/b)^(1/4)*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+1
80*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a
*b^3*d^6*x^6+360*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+360*(a*d^2/b)^(1/4)*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+1
20*(d*x)^(13/2)*a*b^3+270*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^6*x^4+540*(a*d^2/b)^(1/4)*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4+540*
(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*a^2*b^2*d^6*x^4-1912*(d*x)^(9/2)*a^2*b^2*d^2+180*(a*d^2/b)^(1/
4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d
*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^6*x^2+360
*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d
^2/b)^(1/4))*a^3*b*d^6*x^2+360*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.10

$$\int \frac{(dx)^{11/2}}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \frac{45\,(ab^7x^8 + 4\,a^2b^6x^6 + 6\,a^3b^5x^4 + 4\,a^4b^4x^2 + a^5b^3)\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{1}{4}}\log\left(45\,\sqrt{dx}a\right.}{}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas"
)
```

output
```
1/4096*(45*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^
5*b^3)*(-d^22/(a^7*b^13))^(1/4)*log(45*sqrt(d*x)*d^5 + 45*(-d^22/(a^7*b^13
))^(1/4)*a^2*b^3) - 45*(-I*a*b^7*x^8 - 4*I*a^2*b^6*x^6 - 6*I*a^3*b^5*x^4 -
 4*I*a^4*b^4*x^2 - I*a^5*b^3)*(-d^22/(a^7*b^13))^(1/4)*log(45*sqrt(d*x)*d^
5 + 45*I*(-d^22/(a^7*b^13))^(1/4)*a^2*b^3) - 45*(I*a*b^7*x^8 + 4*I*a^2*b^6
*x^6 + 6*I*a^3*b^5*x^4 + 4*I*a^4*b^4*x^2 + I*a^5*b^3)*(-d^22/(a^7*b^13))^(
1/4)*log(45*sqrt(d*x)*d^5 - 45*I*(-d^22/(a^7*b^13))^(1/4)*a^2*b^3) - 45*(a
*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^22
/(a^7*b^13))^(1/4)*log(45*sqrt(d*x)*d^5 - 45*(-d^22/(a^7*b^13))^(1/4)*a^2*
b^3) + 4*(15*b^3*d^5*x^6 - 239*a*b^2*d^5*x^4 - 171*a^2*b*d^5*x^2 - 45*a^3*
d^5)*sqrt(d*x))/(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2
 + a^5*b^3)
```

Sympy [F]

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\int \frac{(dx)^{\frac{11}{2}}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral((d*x)**(11/2)/((a + b*x**2)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(294) = 588.

Time = 0.17 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.32

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{45\,d^5\left(\frac{2\,\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} \right.}{8192\,ab^3}$$

$$-\frac{35\,b^3d^{\frac{11}{2}}x^{\frac{13}{2}}+173\,ab^2d^{\frac{11}{2}}x^{\frac{9}{2}}+657\,a^2bd^{\frac{11}{2}}x^{\frac{5}{2}}+135\,a^3d^{\frac{11}{2}}\sqrt{x}}{3072\,(ab^7x^8+4\,a^2b^6x^6+6\,a^3b^5x^4+4\,a^4b^4x^2+a^5b^3)}$$

$$+\frac{\left(5\,b^4d^{\frac{11}{2}}x^5+18\,ab^3d^{\frac{11}{2}}x^3+45\,a^2b^2d^{\frac{11}{2}}x\right)x^{\frac{11}{2}}-2\left(21\,ab^3d^{\frac{11}{2}}x^5+42\,a^2b^2d^{\frac{11}{2}}x^3-11\,a^3bd^{\frac{11}{2}}\right.}{192\,(a^4b^5x^6+3\,a^5b^4x^4+3\,a^6b^3x^2+a^7b^2+(ab^8x^6+3\,a^2b^7x^4+3\,a^3b^6x^2+a^4b^5)x^6+3\,(a^2b^7x^6+3\,a^3b^6x^4}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima"
)
```

output
```
45/8192*d^5*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sq
rt(2)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(
a^(3/4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^3) - 1/3072*(35*b^3*d^(11/2)*
x^(13/2) + 173*a*b^2*d^(11/2)*x^(9/2) + 657*a^2*b*d^(11/2)*x^(5/2) + 135*a
^3*d^(11/2)*sqrt(x))/(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^
4*x^2 + a^5*b^3) + 1/192*((5*b^4*d^(11/2)*x^5 + 18*a*b^3*d^(11/2)*x^3 + 45
*a^2*b^2*d^(11/2)*x)*x^(11/2) - 2*(21*a*b^3*d^(11/2)*x^5 + 42*a^2*b^2*d^(1
1/2)*x^3 - 11*a^3*b*d^(11/2)*x)*x^(7/2) - (15*a^2*b^2*d^(11/2)*x^5 + 38*a^
3*b*d^(11/2)*x^3 - 9*a^4*d^(11/2)*x)*x^(3/2))/(a^4*b^5*x^6 + 3*a^5*b^4*x^4
 + 3*a^6*b^3*x^2 + a^7*b^2 + (a*b^8*x^6 + 3*a^2*b^7*x^4 + 3*a^3*b^6*x^2 +
a^4*b^5)*x^6 + 3*(a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4*b^5*x^2 + a^5*b^4)*x
^4 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 + a^6*b^3)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.85

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{\frac{90\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^6\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4\mathrm{sgn}(bx^2+a)} + \frac{90\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^6\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\right.}{2\left(\frac{ad^2}{b}\right.}\right.}{a^2b^4\mathrm{sgn}(bx^2+a)}}{}$$

input
```
integrate((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/8192*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*d^6*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sgn(b*x^2 + a)) + 90*s
qrt(2)*(a*b^3*d^2)^(1/4)*d^6*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
- 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sgn(b*x^2 + a)) + 45*sqrt(2)*(a*b
^3*d^2)^(1/4)*d^6*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^2*b^4*sgn(b*x^2 + a)) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*d^6*log(d*x -
 sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^4*sgn(b*x^2 + a
)) + 8*(15*sqrt(d*x)*b^3*d^14*x^6 - 239*sqrt(d*x)*a*b^2*d^14*x^4 - 171*sqr
t(d*x)*a^2*b*d^14*x^2 - 45*sqrt(d*x)*a^3*d^14)/((b*d^2*x^2 + a*d^2)^4*a*b^
3*sgn(b*x^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(d\,x)^{11/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(11/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.83

$$\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(sqrt(d)*d**5*( - 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 - 360*b**(3/4)
*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 - 540*b**(3/4)*a**(1/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*a**2*b**2*x**4 - 360*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 - 9
0*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sq
rt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 + 90*b**(3/4)*a**(1/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*a**4 + 360*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 + 540*
b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 + 360*b**(3/4)*a**(1/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*a*b**3*x**6 + 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**
8 - 45*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2)
+ sqrt(a) + sqrt(b)*x)*a**4 - 180*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)
*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3*b*x**2 - 270*b**...
```

3.657 $\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 454

$$\begin{aligned}\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx &= \frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} \\ &- \frac{d(dx)^{7/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{3/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{7d^3(dx)^{3/2}}{256ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35d^{9/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}} \\ &+ \frac{35d^{9/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35d^{9/2}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}(\sqrt{a}+\sqrt{b}x)}\right)}{2048\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
35/1024*d^3*(d*x)^(3/2)/a^2/b^2/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(7/2)/b/(b
*x^2+a)^3/((b*x^2+a)^2)^(1/2)-7/96*d^3*(d*x)^(3/2)/b^2/(b*x^2+a)^2/((b*x^2
+a)^2)^(1/2)+7/256*d^3*(d*x)^(3/2)/a/b^2/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-35/
4096*d^(9/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))*2^(1/2)/a^(9/4)/b^(11/4)/((b*x^2+a)^2)^(1/2)+35/4096*d^(9/2)*(b*x^2+a)*
arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(9/4)/b^(1
1/4)/((b*x^2+a)^2)^(1/2)-35/4096*d^(9/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)
*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(9/4)/b^(11/4)
/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.44

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(dx)^{9/2}\,(a+bx^2)\left(4\sqrt[4]{a}b^{3/4}x^{3/2}(-35a^3-125a^2bx^2+399ab^2x^4+105b^3x^6)\right.}{12288a^{9/4}b}$$

input
```
Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((d*x)^(9/2)*(a + b*x^2)*(4*a^(1/4)*b^(3/4)*x^(3/2)*(-35*a^3 - 125*a^2*b*x
^2 + 399*a*b^2*x^4 + 105*b^3*x^6) - 105*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt
[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 105*Sqrt[2]*(a + b*x
^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(
12288*a^(9/4)*b^(11/4)*x^(9/2)*((a + b*x^2)^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.96, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1384, 27, 252, 252, 253, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{9/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}dx$$

$\Big\downarrow$ 1384

$$\frac{b^5\left(a+bx^2\right)\int \frac{(dx)^{9/2}}{b^5\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{\left(a+bx^2\right)\int \frac{(dx)^{9/2}}{\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{\left(a+bx^2\right)\left(\frac{7d^2\int \frac{(dx)^{5/2}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{\left(a+bx^2\right)\left(\frac{7d^2\left(\frac{d^2\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^3}dx}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{\left(a+bx^2\right)\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^2}dx}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{7/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{\left(a+bx^2\right)\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{\int \frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\left(a+bx^2\right)\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(a+bx^2\right)\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\int \frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{826}$$

$$\frac{(a+bx^2)\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad(a+bx^2)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4b}-\frac{d(dx)^{3/2}}{6b(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 1476

$$\left(a+bx^2\right)\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)\cdots}{4ad\left(a\cdots\right.}\cdots\right)}{4b}\cdots\right)}{16b}\cdots\right.$$

↓ 1082

$$(a+bx^2)\left|\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{-dx-1}\,d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}\,d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}\right.}{16b}\right.$$

$$\Big\downarrow \text{217}$$

$$(a+bx^2)\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4b}-\frac{d(dx)\cdots}{6b\left(a+b\cdots\right.}\right)}{16b}\right.$$

↓ 1479

$$7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt[4]{a}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\cdots\right.}}{2\sqrt{\cdots}}}{2\sqrt{b}}\cdots\right.}{2a}\cdots\right.}{8a}\cdots\right.}{4b}\cdots\right.\cdots\right.$$

↓ 25

$$7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\cdots\right.}}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right.}{2a}\cdots\right.}{8a}\cdots\right.}{4b}\cdots\right.$$

$\Big\downarrow$ 27

$$7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}}{2a}}{8a}}{4b}\right.$$

$$\downarrow 1103$$

$$(a+bx^2)\left(\frac{7d^2\left(\frac{d^2\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{d}\cdots\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right)}{2a}\cdots\right)}{8a}\cdots\right)}{4b}\cdots\right)}{16b}\cdots\right.$$

input
```
Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*(d*(d*x)^(7/2))/(b*(a + b*x^2)^4) + (7*d^2*(-1/6*(d*(d*
x)^(3/2))/(b*(a + b*x^2)^3) + (d^2*((d*x)^(3/2)/(4*a*d*(a + b*x^2)^2) + (5
*((d*x)^(3/2)/(2*a*d*(a + b*x^2)) + (d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 +
 (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a
]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*a)))/(8*a)))/(4*b)))/(16*b)))/S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(297) = 594$.

Time = 0.14 (sec) , antiderivative size = 1051, normalized size of antiderivative = 2.31

method	result	size
default	Expression too large to display	1051

input
```
int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24576*(105*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b
)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^4*d^
8*x^8+210*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*b^4*d^8*x^8+210*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*b^4*d^8*x^8+840*(d*x)^(15/2)*(a*d^2/b)^(1/4)*b^4+420*2^(
1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^8*x^6+840*2^(1
/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^
8*x^6+840*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*a*b^3*d^8*x^6+3192*(d*x)^(11/2)*(a*d^2/b)^(1/4)*a*b^3*d^2+630*2^(1/2
)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^8*x^4+1260*2^(1
/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*
d^8*x^4+1260*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b
)^(1/4))*a^2*b^2*d^8*x^4-1000*(d*x)^(7/2)*(a*d^2/b)^(1/4)*a^2*b^2*d^4+420*
2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x
+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^8*x^2+840*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b
*d^8*x^2+840*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b
)^(1/4))*a^3*b*d^8*x^2-280*(d*x)^(3/2)*(a*d^2/b)^(1/4)*a^3*b*d^6+105*2^...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{105\left(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{18}}{a^9b^{11}}\right)^{\frac{1}{4}}\log\left(42875\right.}{}$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12288*(105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2
+ a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*log(42875*a^7*b^8*(-d^18/(a^9*b^11))^(
3/4) + 42875*sqrt(d*x)*d^13) - 105*(I*a^2*b^6*x^8 + 4*I*a^3*b^5*x^6 + 6*I*
a^4*b^4*x^4 + 4*I*a^5*b^3*x^2 + I*a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*log(42
875*I*a^7*b^8*(-d^18/(a^9*b^11))^(3/4) + 42875*sqrt(d*x)*d^13) - 105*(-I*a
^2*b^6*x^8 - 4*I*a^3*b^5*x^6 - 6*I*a^4*b^4*x^4 - 4*I*a^5*b^3*x^2 - I*a^6*b
^2)*(-d^18/(a^9*b^11))^(1/4)*log(-42875*I*a^7*b^8*(-d^18/(a^9*b^11))^(3/4)
 + 42875*sqrt(d*x)*d^13) - 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^
4 + 4*a^5*b^3*x^2 + a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*log(-42875*a^7*b^8*(
-d^18/(a^9*b^11))^(3/4) + 42875*sqrt(d*x)*d^13) + 4*(105*b^3*d^4*x^7 + 399
*a*b^2*d^4*x^5 - 125*a^2*b*d^4*x^3 - 35*a^3*d^4*x)*sqrt(d*x))/(a^2*b^6*x^8
 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
```

Sympy [F]

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\int \frac{(dx)^{\frac{9}{2}}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral((d*x)**(9/2)/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.29

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{35\,d^{\frac{9}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\cdots}{\cdots}\right.}{8192\,a^2b^2\cdots}$$

$$+\frac{105\,b^3d^{\frac{9}{2}}x^{\frac{15}{2}}+447\,ab^2d^{\frac{9}{2}}x^{\frac{11}{2}}+803\,a^2bd^{\frac{9}{2}}x^{\frac{7}{2}}+77\,a^3d^{\frac{9}{2}}x^{\frac{3}{2}}}{3072\,(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2)}$$

$$-\frac{\left(3\,b^4d^{\frac{9}{2}}x^5+14\,ab^3d^{\frac{9}{2}}x^3-21\,a^2b^2d^{\frac{9}{2}}x\right)x^{\frac{9}{2}}+2\left(25\,ab^3d^{\frac{9}{2}}x^5+66\,a^2b^2d^{\frac{9}{2}}x^3+9\,a^3bd^{\frac{9}{2}}x\right)\cdots}{192\,(a^4b^5x^6+3\,a^5b^4x^4+3\,a^6b^3x^2+a^7b^2+(ab^8x^6+3\,a^2b^7x^4+3\,a^3b^6x^2+a^4b^5)x^6+3\,(a^2b^7x^6+3\,a^3b^6x\cdots}$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
35/8192*d^(9/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) +
 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + s
qrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4
)*b^(3/4)))/(a^2*b^2) + 1/3072*(105*b^3*d^(9/2)*x^(15/2) + 447*a*b^2*d^(9/
2)*x^(11/2) + 803*a^2*b*d^(9/2)*x^(7/2) + 77*a^3*d^(9/2)*x^(3/2))/(a^2*b^6
*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2) - 1/192*((
3*b^4*d^(9/2)*x^5 + 14*a*b^3*d^(9/2)*x^3 - 21*a^2*b^2*d^(9/2)*x)*x^(9/2) +
 2*(25*a*b^3*d^(9/2)*x^5 + 66*a^2*b^2*d^(9/2)*x^3 + 9*a^3*b*d^(9/2)*x)*x^(
5/2) + (15*a^2*b^2*d^(9/2)*x^5 + 54*a^3*b*d^(9/2)*x^3 + 7*a^4*d^(9/2)*x)*s
qrt(x))/(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2 + (a*b^8*x^
6 + 3*a^2*b^7*x^4 + 3*a^3*b^6*x^2 + a^4*b^5)*x^6 + 3*(a^2*b^7*x^6 + 3*a^3*
b^6*x^4 + 3*a^4*b^5*x^2 + a^5*b^4)*x^4 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 +
3*a^5*b^4*x^2 + a^6*b^3)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.85

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{1}{24576}d^4\left(\frac{210\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^5d\operatorname{sgn}(bx^2+a)}+\frac{210\sqrt{2}(ab^3d^2\right.$$

input
```
integrate((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/24576*d^4*(210*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^5*d*sgn(b*x^2 + a)) +
210*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
 - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^5*d*sgn(b*x^2 + a)) - 105*sqrt(2)*
(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^3*b^5*d*sgn(b*x^2 + a)) + 105*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x -
sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^5*d*sgn(b*x^2 +
a)) + 8*(105*sqrt(d*x)*b^3*d^8*x^7 + 399*sqrt(d*x)*a*b^2*d^8*x^5 - 125*sqr
t(d*x)*a^2*b*d^8*x^3 - 35*sqrt(d*x)*a^3*d^8*x)/((b*d^2*x^2 + a*d^2)^4*a^2*
b^2*sgn(b*x^2 + a)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(d\,x)^{9/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(9/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.82

$$\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(sqrt(d)*d**4*( - 210*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 - 840*b**(1/4
)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 - 1260*b**(1/4)*a**(3/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt
(2)))*a**2*b**2*x**4 - 840*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 -
 210*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)
*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 + 210*b**(1/4)*a**(3/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*a**4 + 840*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 +
1260*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)
*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 + 840*b**(1/4)*a**(3
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*a*b**3*x**6 + 210*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(
1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*
*4*x**8 + 105*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*s
qrt(2) + sqrt(a) + sqrt(b)*x)*a**4 + 420*b**(1/4)*a**(3/4)*sqrt(2)*log( -
sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3*b*x**2 + ...
```

3.658 $\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 454

$$\int \frac{(dx)^{7/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{5/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{5d^3\sqrt{dx}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5d^3\sqrt{dx}}{768ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35d^{7/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{35d^{7/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{35d^{7/2}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
35/3072*d^3*(d*x)^(1/2)/a^2/b^2/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(5/2)/b/(b
*x^2+a)^3/((b*x^2+a)^2)^(1/2)-5/96*d^3*(d*x)^(1/2)/b^2/(b*x^2+a)^2/((b*x^2
+a)^2)^(1/2)+5/768*d^3*(d*x)^(1/2)/a/b^2/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-35/
4096*d^(7/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2
))*2^(1/2)/a^(11/4)/b^(9/4)/((b*x^2+a)^2)^(1/2)+35/4096*d^(7/2)*(b*x^2+a)*
arctan(1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(11/4)/b^(
9/4)/((b*x^2+a)^2)^(1/2)+35/4096*d^(7/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)
*b^(1/4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(11/4)/b^(9/4)
/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.44

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(dx)^{7/2}\,(a+bx^2)\left(4a^{3/4}\sqrt[4]{b}\sqrt{x}(-105a^3-399a^2bx^2+125ab^2x^4+35b^3x^6)\right.}{12288a^{11/4}}$$

input
```
Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((d*x)^(7/2)*(a + b*x^2)*(4*a^(3/4)*b^(1/4)*Sqrt[x]*(-105*a^3 - 399*a^2*b*
x^2 + 125*a*b^2*x^4 + 35*b^3*x^6) - 105*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt
[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 105*Sqrt[2]*(a + b*x
^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(
12288*a^(11/4)*b^(9/4)*x^(7/2)*((a + b*x^2)^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1384, 27, 252, 252, 253, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$$

$\big\downarrow$ 1384

$$\frac{b^5(a+bx^2)\int \frac{(dx)^{7/2}}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 27

$$\frac{(a+bx^2)\int \frac{(dx)^{7/2}}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 252

$$\frac{(a+bx^2)\left(\frac{5d^2\int \frac{(dx)^{3/2}}{(bx^2+a)^4}dx}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 252

$$\frac{(a+bx^2)\left(\frac{5d^2\left(\frac{d^2\int \frac{1}{\sqrt{dx}(bx^2+a)^3}dx}{12b}-\frac{d\sqrt{dx}}{6b(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{5d^2\left(\frac{d^2\left(\frac{7\int \frac{1}{\sqrt{dx}(bx^2+a)^2}dx}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\big\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\int\frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 266

$$\frac{(a+bx^2)\left(\frac{5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\int\frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 755

$$\frac{\left(a+bx^2\right)\left(\frac{5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^{\cdot}}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\downarrow 27$$

$$\frac{(a+bx^2)\left(\frac{5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12b}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\downarrow 1476$$

$$5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\cdots\right)}{12b}\cdots\right.$$

↓ 1082

$$5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx\right.}\right.}{12b}\right.$$

$\Big\downarrow$ 217

$$\left(5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12b}-\frac{\ldots}{6\ldots}\right.\right.$$

↓ 1479

$$5d^2\left(d^2\left(7\left(3\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}{\sqrt[4]{a}\sqrt{d}\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\cdots\right.}{\cdots}\cdots\right.}{2ad}\cdots\right.}{8a}\cdots\right.}{12b}\cdots$$

↓ 25

$$\left(5d^2 \frac{\left(d^2 \frac{\left(7 \frac{\left(3 \frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}} + \frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1 \cdots\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} \cdots\right.}{\cdots} \cdots\right.}{2ad} \cdots\right.}{8a} \cdots\right.}{12b} \cdots\right.$$

↓ 27

$$5d^2\left(\frac{d^2\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\sqrt[4]{a}\cdots}\cdots\right)}{2\sqrt{a}}\cdots}{2ad}\cdots\right)}{8a}\cdots\right)}{12b}\cdots\right)\cdots$$

↓ 1103

$$5d^2\left(d^2\left(7\left(3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}\right.}{2\sqrt{2}\,\cdots}\right.}{2\sqrt{a}}\cdots\right.\right.\right.\right.$$

$$\frac{\cdots}{2ad}\qquad \frac{\cdots}{8a}\qquad \frac{\cdots}{12b}$$

input
```
Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*(d*(d*x)^(5/2))/(b*(a + b*x^2)^4) + (5*d^2*(-1/6*(d*Sqr
t[d*x])/(b*(a + b*x^2)^3) + (d^2*(Sqrt[d*x]/(4*a*d*(a + b*x^2)^2) + (7*(Sq
rt[d*x]/(2*a*d*(a + b*x^2)) + (3*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[
a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*a*d)))/(8*a)))/(12*b)))/(16*b
)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(297) = 594$.

Time = 0.14 (sec) , antiderivative size = 1136, normalized size of antiderivative = 2.50

method	result	size
default	Expression too large to display	1136

input
```
int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24576*(105*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))*b^4*d^6*x^8+210*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/
2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+210*(a*d^2/b)^(1/4)*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x
^8+420*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
))*a*b^3*d^6*x^6+840*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(
a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+840*(a*d^2/b)^(1/4)*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x
^6+280*(d*x)^(13/2)*a*b^3+630*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^6*x^4+1260*(a*d^2/b)^(1/4)*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4
+1260*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4+1000*(d*x)^(9/2)*a^2*b^2*d^2+420*(a*d^2/
b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^6*x
^2+840*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4)
)/(a*d^2/b)^(1/4))*a^3*b*d^6*x^2+840*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{105\left(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{1}{4}}\log\left(35\,a^3b\right.}{}$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12288*(105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2
+ a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*log(35*a^3*b^2*(-d^14/(a^11*b^9))^(1/4
) + 35*sqrt(d*x)*d^3) - 105*(-I*a^2*b^6*x^8 - 4*I*a^3*b^5*x^6 - 6*I*a^4*b^
4*x^4 - 4*I*a^5*b^3*x^2 - I*a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*log(35*I*a^3
*b^2*(-d^14/(a^11*b^9))^(1/4) + 35*sqrt(d*x)*d^3) - 105*(I*a^2*b^6*x^8 + 4
*I*a^3*b^5*x^6 + 6*I*a^4*b^4*x^4 + 4*I*a^5*b^3*x^2 + I*a^6*b^2)*(-d^14/(a^
11*b^9))^(1/4)*log(-35*I*a^3*b^2*(-d^14/(a^11*b^9))^(1/4) + 35*sqrt(d*x)*d
^3) - 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a
^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*log(-35*a^3*b^2*(-d^14/(a^11*b^9))^(1/4)
+ 35*sqrt(d*x)*d^3) + 4*(35*b^3*d^3*x^6 + 125*a*b^2*d^3*x^4 - 399*a^2*b*d^
3*x^2 - 105*a^3*d^3)*sqrt(d*x))/(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x
^4 + 4*a^5*b^3*x^2 + a^6*b^2)
```

Sympy [F]

$$\int \frac{(dx)^{7/2}}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \int \frac{(dx)^{\frac{7}{2}}}{\left((a + bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral((d*x)**(7/2)/((a + b*x**2)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(297) = 594.

Time = 0.16 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.31

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = -\frac{77\,b^3d^{\frac{7}{2}}x^{\frac{13}{2}}+803\,ab^2d^{\frac{7}{2}}x^{\frac{9}{2}}+447\,a^2bd^{\frac{7}{2}}x^{\frac{5}{2}}+105\,a^3d^{\frac{7}{2}}\sqrt{x}}{3072\,(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2)} + \frac{\left(7\,b^4d^{\frac{7}{2}}x^5+54\,ab^3d^{\frac{7}{2}}x^3+15\,a^2b^2d^{\frac{7}{2}}x\right)x^{\frac{11}{2}}+2\left(9\,ab^3d^{\frac{7}{2}}x^5+66\,a^2b^2d^{\frac{7}{2}}x^3+25\,a^3bd^{\frac{7}{2}}x\right)}{192\,(a^5b^4x^6+3\,a^6b^3x^4+3\,a^7b^2x^2+a^8b+(a^2b^7x^6+3\,a^3b^6x^4+3\,a^4b^5x^2+a^5b^4)x^6+3\,(a^3b^6x^6+3\,a^4b^5x^4}$$
$$+\frac{35\,d^3\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-}{8192\,a^2b^2}$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3072*(77*b^3*d^(7/2)*x^(13/2) + 803*a*b^2*d^(7/2)*x^(9/2) + 447*a^2*b*d
^(7/2)*x^(5/2) + 105*a^3*d^(7/2)*sqrt(x))/(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6
*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2) + 1/192*((7*b^4*d^(7/2)*x^5 + 54*a
*b^3*d^(7/2)*x^3 + 15*a^2*b^2*d^(7/2)*x)*x^(11/2) + 2*(9*a*b^3*d^(7/2)*x^5
 + 66*a^2*b^2*d^(7/2)*x^3 + 25*a^3*b*d^(7/2)*x)*x^(7/2) - (21*a^2*b^2*d^(7
/2)*x^5 - 14*a^3*b*d^(7/2)*x^3 - 3*a^4*d^(7/2)*x)*x^(3/2))/(a^5*b^4*x^6 +
3*a^6*b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b + (a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a
^4*b^5*x^2 + a^5*b^4)*x^6 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2
 + a^6*b^3)*x^4 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2
)*x^2) + 35/8192*d^3*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4
)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a
)*sqrt(b))) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b))) + sqrt(2)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*s
qrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.85

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{210\sqrt{2}(ab^3d^2)^{\frac14}d^4\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac14}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac14}}\right)}{a^3b^3\mathrm{sgn}(bx^2+a)} + \frac{210\sqrt{2}(ab^3d^2)^{\frac14}d^4\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right.\right.}{2\left(\frac{ad}{b}\right.}\right.}{a^3b^3\mathrm{sgn}(bx^2+a)}$$

input
```
integrate((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/24576*(210*sqrt(2)*(a*b^3*d^2)^(1/4)*d^4*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*x^2 + a)) + 21
0*sqrt(2)*(a*b^3*d^2)^(1/4)*d^4*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*sgn(b*x^2 + a)) + 105*sqrt(2)*
(a*b^3*d^2)^(1/4)*d^4*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^3*b^3*sgn(b*x^2 + a)) - 105*sqrt(2)*(a*b^3*d^2)^(1/4)*d^4*log(
d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^3*sgn(b*x^
2 + a)) + 8*(35*sqrt(d*x)*b^3*d^12*x^6 + 125*sqrt(d*x)*a*b^2*d^12*x^4 - 39
9*sqrt(d*x)*a^2*b*d^12*x^2 - 105*sqrt(d*x)*a^3*d^12)/((b*d^2*x^2 + a*d^2)^
4*a^2*b^2*sgn(b*x^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(d\,x)^{7/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(7/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.81

$$\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(sqrt(d)*d**3*( - 210*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 - 840*b**(3/4
)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b
**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 - 1260*b**(3/4)*a**(1/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt
(2)))*a**2*b**2*x**4 - 840*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 -
 210*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)
*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 + 210*b**(3/4)*a**(1/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*a**4 + 840*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 +
1260*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)
*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 + 840*b**(3/4)*a**(1
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)
*a**(1/4)*sqrt(2)))*a*b**3*x**6 + 210*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(
1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*
*4*x**8 - 105*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*s
qrt(2) + sqrt(a) + sqrt(b)*x)*a**4 - 420*b**(3/4)*a**(1/4)*sqrt(2)*log( -
sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3*b*x**2 - ...
```

3.659 $\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 451

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d(dx)^{3/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{d(dx)^{3/2}}{32ab(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{9d(dx)^{3/2}}{256a^2b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{45d^{5/2}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{45d^{5/2}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{45d^{5/2}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
45/1024*d*(d*x)^(3/2)/a^3/b/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(3/2)/b/(b*x^2
+a)^3/((b*x^2+a)^2)^(1/2)+1/32*d*(d*x)^(3/2)/a/b/(b*x^2+a)^2/((b*x^2+a)^2)
^(1/2)+9/256*d*(d*x)^(3/2)/a^2/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-45/4096*d^(
5/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/
2)/a^(13/4)/b^(7/4)/((b*x^2+a)^2)^(1/2)+45/4096*d^(5/2)*(b*x^2+a)*arctan(1
+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(13/4)/b^(7/4)/((b
*x^2+a)^2)^(1/2)-45/4096*d^(5/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)
*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(13/4)/b^(7/4)/((b*x^2
+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.45

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(dx)^{5/2}\,(a+bx^2)\left(4\sqrt[4]{a}b^{3/4}x^{3/2}(-15a^3+239a^2bx^2+171ab^2x^4+45b^3x^6)-\right.}{4096a^{13/4}b^{7}}$$

input
```
Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((d*x)^(5/2)*(a + b*x^2)*(4*a^(1/4)*b^(3/4)*x^(3/2)*(-15*a^3 + 239*a^2*b*x
^2 + 171*a*b^2*x^4 + 45*b^3*x^6) - 45*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[a
] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 45*Sqrt[2]*(a + b*x^2)
^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(409
6*a^(13/4)*b^(7/4)*x^(5/2)*((a + b*x^2)^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.96, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1384, 27, 252, 253, 253, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{5/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$$

$\Big\downarrow$ 1384

$$\frac{b^5\left(a+bx^2\right)\int \frac{(dx)^{5/2}}{b^5\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{\left(a+bx^2\right)\int \frac{(dx)^{5/2}}{\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{\left(a+bx^2\right)\left(\frac{3d^2\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^4}dx}{16b}-\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{\left(a+bx^2\right)\left(\frac{3d^2\left(\frac{3\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^3}dx}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{\left(a+bx^2\right)\left(\frac{3d^2\left(\frac{3\left(\frac{5\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^2}dx}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{\int\frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad(a+bx^2)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{3/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 266

$$\frac{(a+bx^2)\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{\int\frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad(a+bx^2)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad(a+bx^2)^3}\right)}{16b}-\frac{d(dx)^{3/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{\left(a+bx^2\right)\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{d\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{3/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 826

$$\frac{(a+bx^2)\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d(dx)^{3/2}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\downarrow 1476$$

$$\left(a+bx^2\right)\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3}\cdots}{4ad\left(a+\cdots\right.}\cdots\right.}{4a}\cdots\right.}{16b}\cdots\right.$$

↓ 1082

$$(a+bx^2)\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{-dx-1}\,d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}\,d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+}{16b}\right.$$

$$\Big\downarrow \text{217}$$

$$(a+bx^2)\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad(a+bx^2)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad(a+bx^2)^2}\right)}{4a}+\frac{(dx)^3}{6ad(a+b}\right)}{16b}$$

↓ 1479

$$3d^2\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}\cdots}{\sqrt{b}}\right.}\cdots}{2\sqrt{2}\cdots}}{2\sqrt{b}}\cdots\right)}{2a}\cdots\right)}{8a}\cdots\right)}{4a}\cdots\right.$$

$$\Big\downarrow 25$$

$$3d^2\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{\cdots}\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{\cdots}}{\cdots}\right.}}{2\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{b}}\cdots\right.}{2a}\cdots\right.}{8a}\cdots\right.}{4a}\cdots\right.\right.$$

↓ 27

$$3d^2 \left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}} - \frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}\,d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}} + \frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}\,d \cdots}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}} \cdots}{2\sqrt{b}} \cdots \right.}{2a} \cdots \right.}{8a} \cdots \right.}{4a} \cdots \right.$$

↓ 1103

$$\left(a+bx^2\right)\left(\frac{3d^2\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}\cdots\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}}{2\sqrt{b}}\cdots\right.}{2a}\cdots\right.}{8a}\cdots\right.}{4a}\cdots\right.}{16b}\cdots\right.$$

input
```
Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*(d*(d*x)^(3/2))/(b*(a + b*x^2)^4) + (3*d^2*((d*x)^(3/2)
/(6*a*d*(a + b*x^2)^3) + (3*((d*x)^(3/2)/(4*a*d*(a + b*x^2)^2) + (5*((d*x)
^(3/2)/(2*a*d*(a + b*x^2)) + (d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])
)/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + S
qrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(2*a)))/(8*a)))/(4*a)))/(16*b)))/Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(294) = 588$.

Time = 0.14 (sec) , antiderivative size = 1051, normalized size of antiderivative = 2.33

method	result	size
default	Expression too large to display	1051

input
```
int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8192*(45*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^
(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^4*d^8*
x^8+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*b^4*d^8*x^8+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*b^4*d^8*x^8+360*(d*x)^(15/2)*(a*d^2/b)^(1/4)*b^4+180*2^(1/2)
*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^8*x^6+360*2^(1/2)*
arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^8*x^
6+360*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*a*b^3*d^8*x^6+1368*(d*x)^(11/2)*(a*d^2/b)^(1/4)*a*b^3*d^2+270*2^(1/2)*ln
(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^8*x^4+540*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^8*x
^4+540*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*a^2*b^2*d^8*x^4+1912*(d*x)^(7/2)*(a*d^2/b)^(1/4)*a^2*b^2*d^4+180*2^(1/2
)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^8*x^2+360*2^(1/2)
*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^8*x
^2+360*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*a^3*b*d^8*x^2-120*(d*x)^(3/2)*(a*d^2/b)^(1/4)*a^3*b*d^6+45*2^(1/2)*l...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.10

$$\int \frac{(dx)^{5/2}}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \frac{45\,(a^3b^5x^8 + 4\,a^4b^4x^6 + 6\,a^5b^3x^4 + 4\,a^6b^2x^2 + a^7b)\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}\log\left(91125\,a^1\right.}{}$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4096*(45*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 +
a^7*b)*(-d^10/(a^13*b^7))^(1/4)*log(91125*a^10*b^5*(-d^10/(a^13*b^7))^(3/4
) + 91125*sqrt(d*x)*d^7) - 45*(I*a^3*b^5*x^8 + 4*I*a^4*b^4*x^6 + 6*I*a^5*b
^3*x^4 + 4*I*a^6*b^2*x^2 + I*a^7*b)*(-d^10/(a^13*b^7))^(1/4)*log(91125*I*a
^10*b^5*(-d^10/(a^13*b^7))^(3/4) + 91125*sqrt(d*x)*d^7) - 45*(-I*a^3*b^5*x
^8 - 4*I*a^4*b^4*x^6 - 6*I*a^5*b^3*x^4 - 4*I*a^6*b^2*x^2 - I*a^7*b)*(-d^10
/(a^13*b^7))^(1/4)*log(-91125*I*a^10*b^5*(-d^10/(a^13*b^7))^(3/4) + 91125*
sqrt(d*x)*d^7) - 45*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b
^2*x^2 + a^7*b)*(-d^10/(a^13*b^7))^(1/4)*log(-91125*a^10*b^5*(-d^10/(a^13*
b^7))^(3/4) + 91125*sqrt(d*x)*d^7) + 4*(45*b^3*d^2*x^7 + 171*a*b^2*d^2*x^5
 + 239*a^2*b*d^2*x^3 - 15*a^3*d^2*x)*sqrt(d*x))/(a^3*b^5*x^8 + 4*a^4*b^4*x
^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)
```

Sympy [F]

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral((d*x)**(5/2)/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.29

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{135\,b^3d^{\frac{5}{2}}x^{\frac{15}{2}}+657\,ab^2d^{\frac{5}{2}}x^{\frac{11}{2}}+173\,a^2bd^{\frac{5}{2}}x^{\frac{7}{2}}+35\,a^3d^{\frac{5}{2}}x^{\frac{3}{2}}}{3072\,(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b)}$$
$$-\frac{\left(9\,b^4d^{\frac{5}{2}}x^5-38\,ab^3d^{\frac{5}{2}}x^3-15\,a^2b^2d^{\frac{5}{2}}x\right)x^{\frac{9}{2}}+2\left(11\,ab^3d^{\frac{5}{2}}x^5-42\,a^2b^2d^{\frac{5}{2}}x^3-21\,a^3bd^{\frac{5}{2}}x\cdots\right.}{192\,(a^5b^4x^6+3\,a^6b^3x^4+3\,a^7b^2x^2+a^8b+(a^2b^7x^6+3\,a^3b^6x^4+3\,a^4b^5x^2+a^5b^4)x^6+3\,(a^3b^6x^6+3\,a^4b^5x\cdots}$$
$$+\frac{45\,d^{\frac{5}{2}}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\cdots}{\cdots}\right.}{8192\,a^3b}$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3072*(135*b^3*d^(5/2)*x^(15/2) + 657*a*b^2*d^(5/2)*x^(11/2) + 173*a^2*b*
d^(5/2)*x^(7/2) + 35*a^3*d^(5/2)*x^(3/2))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6
*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b) - 1/192*((9*b^4*d^(5/2)*x^5 - 38*a*b
^3*d^(5/2)*x^3 - 15*a^2*b^2*d^(5/2)*x)*x^(9/2) + 2*(11*a*b^3*d^(5/2)*x^5 -
 42*a^2*b^2*d^(5/2)*x^3 - 21*a^3*b*d^(5/2)*x)*x^(5/2) + (45*a^2*b^2*d^(5/2
)*x^5 + 18*a^3*b*d^(5/2)*x^3 + 5*a^4*d^(5/2)*x)*sqrt(x))/(a^5*b^4*x^6 + 3*
a^6*b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b + (a^2*b^7*x^6 + 3*a^3*b^6*x^4 + 3*a^4
*b^5*x^2 + a^5*b^4)*x^6 + 3*(a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5*b^4*x^2 +
 a^6*b^3)*x^4 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 + a^7*b^2)*
x^2) + 45/8192*d^(5/2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1
/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqr
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)
*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/
4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/
(a^(1/4)*b^(3/4)))/(a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.86

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{1}{8192}d^2\left(\frac{90\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^4d\,\mathrm{sgn}\left(bx^2+a\right)}+\frac{90\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\,a}{a}\right.$$

input
```
integrate((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/8192*d^2*(90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^4*d*sgn(b*x^2 + a)) + 90
*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) -
2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^4*d*sgn(b*x^2 + a)) - 45*sqrt(2)*(a*b
^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))
/(a^4*b^4*d*sgn(b*x^2 + a)) + 45*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(
2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^4*d*sgn(b*x^2 + a)) +
 8*(45*sqrt(d*x)*b^3*d^8*x^7 + 171*sqrt(d*x)*a*b^2*d^8*x^5 + 239*sqrt(d*x)
*a^2*b*d^8*x^3 - 15*sqrt(d*x)*a^3*d^8*x)/((b*d^2*x^2 + a*d^2)^4*a^3*b*sgn(
b*x^2 + a)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(d\,x)^{5/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(5/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.83

$$\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output

```
(sqrt(d)*d**2*( - 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 - 360*b**(1/4)
*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 - 540*b**(1/4)*a**(3/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*a**2*b**2*x**4 - 360*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 - 9
0*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sq
rt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 + 90*b**(1/4)*a**(3/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*a**4 + 360*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 + 540*
b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 + 360*b**(1/4)*a**(3/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*a*b**3*x**6 + 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**
8 + 45*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2)
+ sqrt(a) + sqrt(b)*x)*a**4 + 180*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)
*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3*b*x**2 + 270*b**...
```

3.660 $\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 451

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{11d\sqrt{dx}}{768a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{3/2}\left(a+bx^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}\left(a+bx^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}\left(a+bx^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
77/3072*d*(d*x)^(1/2)/a^3/b/((b*x^2+a)^2)^(1/2)-1/8*d*(d*x)^(1/2)/b/(b*x^2
+a)^3/((b*x^2+a)^2)^(1/2)+1/96*d*(d*x)^(1/2)/a/b/(b*x^2+a)^2/((b*x^2+a)^2)
^(1/2)+11/768*d*(d*x)^(1/2)/a^2/b/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-77/4096*d^
(3/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1
/2)/a^(15/4)/b^(5/4)/((b*x^2+a)^2)^(1/2)+77/4096*d^(3/2)*(b*x^2+a)*arctan(
1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(15/4)/b^(5/4)/((
b*x^2+a)^2)^(1/2)+77/4096*d^(3/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4
)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(15/4)/b^(5/4)/((b*x^
2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.45

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{(dx)^{3/2}\,(a+bx^2)\left(4a^{3/4}\sqrt[4]{b}\sqrt{x}(-231a^3+351a^2bx^2+275ab^2x^4+77b^3x^6)\right.}{12288a^{15/4}}$$

input
```
Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((d*x)^(3/2)*(a + b*x^2)*(4*a^(3/4)*b^(1/4)*Sqrt[x]*(-231*a^3 + 351*a^2*b*
x^2 + 275*a*b^2*x^4 + 77*b^3*x^6) - 231*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt
[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 231*Sqrt[2]*(a + b*x
^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(
12288*a^(15/4)*b^(5/4)*x^(3/2)*((a + b*x^2)^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1384, 27, 252, 253, 253, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$$

$\Big\downarrow$ 1384

$$\frac{b^5\left(a+bx^2\right)\int \frac{(dx)^{3/2}}{b^5\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{\left(a+bx^2\right)\int \frac{(dx)^{3/2}}{\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 252

$$\frac{\left(a+bx^2\right)\left(\frac{d^2\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^4}dx}{16b}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{\left(a+bx^2\right)\left(\frac{d^2\left(\frac{11\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^3}dx}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{\left(a+bx^2\right)\left(\frac{d^2\left(\frac{11\left(\frac{7\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^2}dx}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{\left(a+bx^2\right)\left(\frac{d^2\left(\frac{11\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d\sqrt{dx}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\left(a+bx^2\right)\left(\frac{d^2\left(\frac{11\left(\frac{7\left(\frac{3\int \frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d\sqrt{dx}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{\left(a+bx^2\right)\left(\frac{d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16b}-\frac{d\sqrt{dx}}{8b(a+bx^2)}$$

$$\sqrt{a^2+2abx^2+b^2x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\left(\frac{d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad(a+bx^2)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad(a+bx^2)^3}\right)}{16b}-\frac{d\sqrt{dx}}{8b(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{1476}$$

$$d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}\,d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\cdots}{4a\cdots}\cdots\right)}{12a}\cdots\right.\right.$$

↓ 1082

$$d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}\,d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}\,d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)}}{12a}$$

↓ 217

$$d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\quad}{6a\ldots}\right.\right.$$

↓ 1479

$$d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{d\cdots}}{\sqrt[4]{a}\sqrt{d}}\cdots\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\cdots}}\cdots\right.}{\cdots}\cdots\right)}{2ad}\cdots\right)}{8a}\cdots\right)}{12a}\cdots\right.$$

$$\Big\downarrow 25$$

$$d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right)}{2\cdots}\right)\cdots}{2ad}\cdots\right)\cdots}{8a}\cdots\right)\cdots}{12a}\cdots\right.$$

$$\Big\downarrow {\color{blue}27}$$

$$d^2\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\ldots\right.}{\sqrt{2}\sqrt[4]{a}\ldots}\right.}{2\sqrt{a}}\ldots\right.}{2ad}\ldots\right.}{8a}\ldots\right.}{12a}\ldots\right.$$

$\downarrow$ 1103

$$3\,\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots\right.}{2\sqrt{2}\sqrt[4]{\cdots}}\right.}{2\sqrt{a}}$$

$$7\,\frac{\left(\cdots\right.}{2ad}$$

$$11\,\frac{\left(\cdots\right.}{8a}$$

$$d^2\,\frac{\left(\cdots\right.}{12a}$$

input
```
Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*(-1/8*(d*Sqrt[d*x])/(b*(a + b*x^2)^4) + (d^2*(Sqrt[d*x]/(6*a*
d*(a + b*x^2)^3) + (11*(Sqrt[d*x]/(4*a*d*(a + b*x^2)^2) + (7*(Sqrt[d*x]/(2
*a*d*(a + b*x^2)) + (3*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*
Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqr
t[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(2*a*d)))/(8*a)))/(12*a)))/(16*b)))/Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(294) = 588$.

Time = 0.16 (sec) , antiderivative size = 1136, normalized size of antiderivative = 2.52

method	result	size
default	Expression too large to display	1136

input
```
int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24576*(231*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))*b^4*d^6*x^8+462*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/
2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+462*(a*d^2/b)^(1/4)*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x
^8+924*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
))*a*b^3*d^6*x^6+1848*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+1848*(a*d^2/b)^(1/4)*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6
*x^6+616*(d*x)^(13/2)*a*b^3+1386*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^6*x^4+2772*(a*d^2/b)^(1/4)*2^(1/2)*
arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*
x^4+2772*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/
4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4+2200*(d*x)^(9/2)*a^2*b^2*d^2+924*(a*d
^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^
6*x^2+1848*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(
1/4))/(a*d^2/b)^(1/4))*a^3*b*d^6*x^2+1848*(a*d^2/b)^(1/4)*2^(1/2)*arcta...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.05

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{231\,(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b)\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{1}{4}}\log\left(77\,a^4b\right(}{}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12288*(231*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2
+ a^7*b)*(-d^6/(a^15*b^5))^(1/4)*log(77*a^4*b*(-d^6/(a^15*b^5))^(1/4) + 77
*sqrt(d*x)*d) - 231*(-I*a^3*b^5*x^8 - 4*I*a^4*b^4*x^6 - 6*I*a^5*b^3*x^4 -
4*I*a^6*b^2*x^2 - I*a^7*b)*(-d^6/(a^15*b^5))^(1/4)*log(77*I*a^4*b*(-d^6/(a
^15*b^5))^(1/4) + 77*sqrt(d*x)*d) - 231*(I*a^3*b^5*x^8 + 4*I*a^4*b^4*x^6 +
 6*I*a^5*b^3*x^4 + 4*I*a^6*b^2*x^2 + I*a^7*b)*(-d^6/(a^15*b^5))^(1/4)*log(
-77*I*a^4*b*(-d^6/(a^15*b^5))^(1/4) + 77*sqrt(d*x)*d) - 231*(a^3*b^5*x^8 +
 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)*(-d^6/(a^15*b^5))^
(1/4)*log(-77*a^4*b*(-d^6/(a^15*b^5))^(1/4) + 77*sqrt(d*x)*d) + 4*(77*b^3*
d*x^6 + 275*a*b^2*d*x^4 + 351*a^2*b*d*x^2 - 231*a^3*d)*sqrt(d*x))/(a^3*b^5
*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)
```

Sympy [F]

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral((d*x)**(3/2)/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.30

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx =$$

$$-\frac{385\,b^3d^{\frac{3}{2}}x^{\frac{13}{2}}+495\,ab^2d^{\frac{3}{2}}x^{\frac{9}{2}}+315\,a^2bd^{\frac{3}{2}}x^{\frac{5}{2}}+77\,a^3d^{\frac{3}{2}}\sqrt{x}}{1024\,(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b)}$$

$$+\frac{\left(77\,b^4d^{\frac{3}{2}}x^5+66\,ab^3d^{\frac{3}{2}}x^3+21\,a^2b^2d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+2\left(99\,ab^3d^{\frac{3}{2}}x^5+102\,a^2b^2d^{\frac{3}{2}}x^3+35\,a^3bd^{\frac{3}{2}}x\right)x}{192\,(a^6b^3x^6+3\,a^7b^2x^4+3\,a^8bx^2+a^9+(a^3b^6x^6+3\,a^4b^5x^4+3\,a^5b^4x^2+a^6b^3)x^6+3\,(a^4b^5x^6+3\,a^5b^4x^4+}$$

$$+\frac{77\,d\left(\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\right.}{8192\,a^3b}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/1024*(385*b^3*d^(3/2)*x^(13/2) + 495*a*b^2*d^(3/2)*x^(9/2) + 315*a^2*b*
d^(3/2)*x^(5/2) + 77*a^3*d^(3/2)*sqrt(x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6
*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b) + 1/192*((77*b^4*d^(3/2)*x^5 + 66*a*
b^3*d^(3/2)*x^3 + 21*a^2*b^2*d^(3/2)*x)*x^(11/2) + 2*(99*a*b^3*d^(3/2)*x^5
 + 102*a^2*b^2*d^(3/2)*x^3 + 35*a^3*b*d^(3/2)*x)*x^(7/2) + (153*a^2*b^2*d^
(3/2)*x^5 + 202*a^3*b*d^(3/2)*x^3 + 81*a^4*d^(3/2)*x)*x^(3/2))/(a^6*b^3*x^
6 + 3*a^7*b^2*x^4 + 3*a^8*b*x^2 + a^9 + (a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a
^5*b^4*x^2 + a^6*b^3)*x^6 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2
 + a^7*b^2)*x^4 + 3*(a^5*b^4*x^6 + 3*a^6*b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b)*
x^2) + 77/8192*d*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^
(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(b))) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + sqrt(2)*sqrt(d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(
a))/(a^(3/4)*b^(1/4)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(
x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.85

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{462\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^2\mathrm{sgn}(bx^2+a)} + \frac{462\sqrt{2}(ab^3d^2)^{\frac{1}{4}}d^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right.\right.}{2\left(\frac{ad}{b}\right.}\right.}{a^4b^2\mathrm{sgn}(bx^2+a)}$$

input
```
integrate((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/24576*(462*sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*sgn(b*x^2 + a)) + 46
2*sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*sgn(b*x^2 + a)) + 231*sqrt(2)*
(a*b^3*d^2)^(1/4)*d^2*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^4*b^2*sgn(b*x^2 + a)) - 231*sqrt(2)*(a*b^3*d^2)^(1/4)*d^2*log(
d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^2*sgn(b*x^
2 + a)) + 8*(77*sqrt(d*x)*b^3*d^10*x^6 + 275*sqrt(d*x)*a*b^2*d^10*x^4 + 35
1*sqrt(d*x)*a^2*b*d^10*x^2 - 231*sqrt(d*x)*a^3*d^10)/((b*d^2*x^2 + a*d^2)^
4*a^3*b*sgn(b*x^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{(d\,x)^{3/2}}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(3/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 822, normalized size of antiderivative = 1.82

$$\int \frac{(dx)^{3/2}}{(a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx = \text{Too large to display}$$

input

```
int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output

```
(sqrt(d)*d*( - 462*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 - 1848*b**(3/4)*
a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 - 2772*b**(3/4)*a**(1/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*a**2*b**2*x**4 - 1848*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 -
462*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*
sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 + 462*b**(3/4)*a**(1/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1
/4)*sqrt(2)))*a**4 + 1848*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 +
2772*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)
*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 + 1848*b**(3/4)*a**(
1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4
)*a**(1/4)*sqrt(2)))*a*b**3*x**6 + 462*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b
**4*x**8 - 231*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*
sqrt(2) + sqrt(a) + sqrt(b)*x)*a**4 - 924*b**(3/4)*a**(1/4)*sqrt(2)*log( -
 sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3*b*x**2 -...
```

3.661 $\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 450

$$\begin{aligned}\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx &= \frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{(dx)^{3/2}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13(dx)^{3/2}}{96a^2d\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{39(dx)^{3/2}}{256a^3d\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{195\sqrt{d}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{195\sqrt{d}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{195\sqrt{d}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
195/1024*(d*x)^(3/2)/a^4/d/((b*x^2+a)^2)^(1/2)+1/8*(d*x)^(3/2)/a/d/(b*x^2+
a)^3/((b*x^2+a)^2)^(1/2)+13/96*(d*x)^(3/2)/a^2/d/(b*x^2+a)^2/((b*x^2+a)^2)
^(1/2)+39/256*(d*x)^(3/2)/a^3/d/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-195/4096*d^(
1/2)*(b*x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/
2)/a^(17/4)/b^(3/4)/((b*x^2+a)^2)^(1/2)+195/4096*d^(1/2)*(b*x^2+a)*arctan(
1+2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(17/4)/b^(3/4)/((
b*x^2+a)^2)^(1/2)-195/4096*d^(1/2)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/
4)*(d*x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(17/4)/b^(3/4)/((b*x
^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\sqrt{dx}(a+bx^2)\left(4\sqrt[4]{a}x^{3/2}(1853a^3+3107a^2bx^2+2223ab^2x^4+585b^3x^6)-\cdots\right.}{12288a^{17/4}\sqrt{x}\,((\cdots}$$

input
```
Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[d*x]*(a + b*x^2)*(4*a^(1/4)*x^(3/2)*(1853*a^3 + 3107*a^2*b*x^2 + 222
3*a*b^2*x^4 + 585*b^3*x^6) - (585*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[a] -
Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) - (585*Sqrt[2]*(a +
 b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]
)/b^(3/4)))/(12288*a^(17/4)*Sqrt[x]*((a + b*x^2)^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.96, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1384, 27, 253, 253, 253, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^5\left(a+bx^2\right)\int \frac{\sqrt{dx}}{b^5\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(a+bx^2\right)\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{\left(a+bx^2\right)\left(\frac{13\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^4}dx}{16a}+\frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{\left(a+bx^2\right)\left(\frac{13\left(\frac{3\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^3}dx}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{\left(a+bx^2\right)\left(\frac{13\left(\frac{3\left(\frac{5\int \frac{\sqrt{dx}}{\left(bx^2+a\right)^2}dx}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad\left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{\left(a+bx^2\right)\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{\int \frac{\sqrt{dx}}{bx^2+a}dx}{4a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 266

$$\frac{\left(a+bx^2\right)\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{2ad}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{(a+bx^2)\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 826

$$\frac{(a+bx^2)\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/2}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 1476$$

$$
\left(a+bx^2\right)\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/}}{4ad\left(a+bx\right.}\cdots\right)}{4a}\cdots\right)}{16a}\cdots\right.
$$

↓ 1082

$$(a+bx^2)\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\cdots\right)}{16a}\right.$$

$$\Big\downarrow \text{217}$$

$$\left(a+bx^2\right)\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{2a}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)^2}\right)}{4a}+\frac{(dx)^{3/}}{6ad\left(a+bx\right.}\right.}{16a}\right.$$

↓ 1479

$$13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}\cdots\right.}\cdots}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right.}{2a}\cdots\right.}{8a}\cdots\right.}{4a}\cdots\right.$$

↓ 25

$$13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\cdots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\cdots}{\cdots}\right.}\cdots}{2\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{b}}\cdots\right.}{2a}\cdots\right.}{8a}\cdots\right.}{4a}\cdots\right.\right.$$

$$\Big\downarrow \, 27$$

$$\Bigg(13\,\frac{\Bigg(3\,\frac{\Bigg(5\,\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{\ldots}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}}{2a}}{8a}}{4a}$$

↓ 1103

$$\left(a+bx^2\right)\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\cdots\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\cdots}}}{2\sqrt{b}}\right.}{2a}\cdots}{8a}\cdots}{4a}\cdots}{16a}\cdots\right.$$

input
```
Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((a + b*x^2)*((d*x)^(3/2)/(8*a*d*(a + b*x^2)^4) + (13*((d*x)^(3/2)/(6*a*d*
(a + b*x^2)^3) + (3*((d*x)^(3/2)/(4*a*d*(a + b*x^2)^2) + (5*((d*x)^(3/2)/(
2*a*d*(a + b*x^2)) + (d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4
)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqr
t[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d
]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d]))/(2*Sqrt[b])))/(2*a)))/(8*a)))/(4*a)))/(16*a)))/Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. 2(293) = 586.

Time = 0.14 (sec) , antiderivative size = 1051, normalized size of antiderivative = 2.34

method	result	size
default	Expression too large to display	1051

input
```
int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24576*(585*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b
)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*b^4*d^
8*x^8+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^
(1/4))*b^4*d^8*x^8+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4
))/(a*d^2/b)^(1/4))*b^4*d^8*x^8+4680*(d*x)^(15/2)*(a*d^2/b)^(1/4)*b^4+2340
*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*
x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a*b^3*d^8*x^6+4680
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b
^3*d^8*x^6+4680*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*a*b^3*d^8*x^6+17784*(d*x)^(11/2)*(a*d^2/b)^(1/4)*a*b^3*d^2+351
0*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d
*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^8*x^4+7
020*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
a^2*b^2*d^8*x^4+7020*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/
(a*d^2/b)^(1/4))*a^2*b^2*d^8*x^4+24856*(d*x)^(7/2)*(a*d^2/b)^(1/4)*a^2*b^2
*d^4+2340*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(
1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*b*d^8
*x^2+4680*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*a^3*b*d^8*x^2+4680*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/
4))/(a*d^2/b)^(1/4))*a^3*b*d^8*x^2+14824*(d*x)^(3/2)*(a*d^2/b)^(1/4)*a^...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{585\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}\log\left(7414875\,a\right.}{}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
1/12288*(585*(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 +
a^8)*(-d^2/(a^17*b^3))^(1/4)*log(7414875*a^13*b^2*(-d^2/(a^17*b^3))^(3/4)
+ 7414875*sqrt(d*x)*d) - 585*(I*a^4*b^4*x^8 + 4*I*a^5*b^3*x^6 + 6*I*a^6*b^
2*x^4 + 4*I*a^7*b*x^2 + I*a^8)*(-d^2/(a^17*b^3))^(1/4)*log(7414875*I*a^13*
b^2*(-d^2/(a^17*b^3))^(3/4) + 7414875*sqrt(d*x)*d) - 585*(-I*a^4*b^4*x^8 -
 4*I*a^5*b^3*x^6 - 6*I*a^6*b^2*x^4 - 4*I*a^7*b*x^2 - I*a^8)*(-d^2/(a^17*b^
3))^(1/4)*log(-7414875*I*a^13*b^2*(-d^2/(a^17*b^3))^(3/4) + 7414875*sqrt(d
*x)*d) - 585*(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 +
a^8)*(-d^2/(a^17*b^3))^(1/4)*log(-7414875*a^13*b^2*(-d^2/(a^17*b^3))^(3/4)
 + 7414875*sqrt(d*x)*d) + 4*(585*b^3*x^7 + 2223*a*b^2*x^5 + 3107*a^2*b*x^3
 + 1853*a^3*x)*sqrt(d*x))/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4
*a^7*b*x^2 + a^8)
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{\sqrt{dx}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(sqrt(d*x)/((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{195\,b^3\sqrt{d}x^{\frac{15}{2}}+117\,ab^2\sqrt{d}x^{\frac{11}{2}}+65\,a^2b\sqrt{d}x^{\frac{7}{2}}+15\,a^3\sqrt{d}x^{\frac{3}{2}}}{1024\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}$$

$$+\frac{\left(117\,b^4\sqrt{d}x^5+130\,ab^3\sqrt{d}x^3+45\,a^2b^2\sqrt{d}x\right)x^{\frac{9}{2}}+2\left(143\,ab^3\sqrt{d}x^5+174\,a^2b^2\sqrt{d}x^3+63\,a^3b\sqrt{d}x\right)}{192\,(a^6b^3x^6+3\,a^7b^2x^4+3\,a^8bx^2+a^9+(a^3b^6x^6+3\,a^4b^5x^4+3\,a^5b^4x^2+a^6b^3)x^6+3\,(a^4b^5x^6+3\,a^5b^4x^4+}$$

$$+\frac{195\,\sqrt{d}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\mathrm{l}}{}\right.}{8192\,a^4}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
1/1024*(195*b^3*sqrt(d)*x^(15/2) + 117*a*b^2*sqrt(d)*x^(11/2) + 65*a^2*b*s
qrt(d)*x^(7/2) + 15*a^3*sqrt(d)*x^(3/2))/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*
a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8) + 1/192*((117*b^4*sqrt(d)*x^5 + 130*a*b^3
*sqrt(d)*x^3 + 45*a^2*b^2*sqrt(d)*x)*x^(9/2) + 2*(143*a*b^3*sqrt(d)*x^5 +
174*a^2*b^2*sqrt(d)*x^3 + 63*a^3*b*sqrt(d)*x)*x^(5/2) + (201*a^2*b^2*sqrt(
d)*x^5 + 282*a^3*b*sqrt(d)*x^3 + 113*a^4*sqrt(d)*x)*sqrt(x))/(a^6*b^3*x^6
+ 3*a^7*b^2*x^4 + 3*a^8*b*x^2 + a^9 + (a^3*b^6*x^6 + 3*a^4*b^5*x^4 + 3*a^5
*b^4*x^2 + a^6*b^3)*x^6 + 3*(a^4*b^5*x^6 + 3*a^5*b^4*x^4 + 3*a^6*b^3*x^2 +
 a^7*b^2)*x^4 + 3*(a^5*b^4*x^6 + 3*a^6*b^3*x^4 + 3*a^7*b^2*x^2 + a^8*b)*x^
2) + 195/8192*sqrt(d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt
(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*
log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(
a^(1/4)*b^(3/4)))/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\frac{1170\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^3\mathrm{sgn}(bx^2+a)}+\frac{1170\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\right.}{2\left(\frac{ad^2}{b}\right.}\right.}{a^5b^3\mathrm{sgn}(bx^2+a)}}{}$$

input
```
integrate((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1/24576*(1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^3*sgn(b*x^2 + a)) + 1170*
sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2
*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^3*sgn(b*x^2 + a)) - 585*sqrt(2)*(a*b^3
*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a^5*b^3*sgn(b*x^2 + a)) + 585*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^3*sgn(b*x^2 + a)) + 8*(5
85*sqrt(d*x)*b^3*d^9*x^7 + 2223*sqrt(d*x)*a*b^2*d^9*x^5 + 3107*sqrt(d*x)*a
^2*b*d^9*x^3 + 1853*sqrt(d*x)*a^3*d^9*x)/((b*d^2*x^2 + a*d^2)^4*a^4*sgn(b*
x^2 + a)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \int \frac{\sqrt{d\,x}}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}}\,dx$$

input
```
int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^(1/2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 822, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{dx}}{(a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx = \text{Too large to display}$$

input
```
int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(sqrt(d)*( - 1170*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 - 4680*b**(1/4)*a
**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 - 7020*b**(1/4)*a**(3/4)*sqrt(2)*atan(
(b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)
))*a**2*b**2*x**4 - 4680*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 - 1
170*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*
sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 + 1170*b**(1/4)*a**(3/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(
1/4)*sqrt(2)))*a**4 + 4680*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 +
 7020*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x
)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 + 4680*b**(1/4)*a**
(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/
4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 + 1170*b**(1/4)*a**(3/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
*b**4*x**8 + 585*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4
)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**4 + 2340*b**(1/4)*a**(3/4)*sqrt(2)*log
( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3*b*x**...
```

3.662 $\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 450

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{dx}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{5\sqrt{dx}}{32a^2d\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{55\sqrt{dx}}{256a^3d\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{1155(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1155(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{1155(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
385/1024*(d*x)^(1/2)/a^4/d/((b*x^2+a)^2)^(1/2)+1/8*(d*x)^(1/2)/a/d/(b*x^2+
a)^3/((b*x^2+a)^2)^(1/2)+5/32*(d*x)^(1/2)/a^2/d/(b*x^2+a)^2/((b*x^2+a)^2)^
(1/2)+55/256*(d*x)^(1/2)/a^3/d/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-1155/4096*(b*
x^2+a)*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(19
/4)/b^(1/4)/d^(1/2)/((b*x^2+a)^2)^(1/2)+1155/4096*(b*x^2+a)*arctan(1+2^(1/
2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(19/4)/b^(1/4)/d^(1/2)/(
(b*x^2+a)^2)^(1/2)+1155/4096*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*
x)^(1/2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(19/4)/b^(1/4)/d^(1/2)/((b
*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{\sqrt{x}(a+bx^2)\left(4a^{3/4}\sqrt{x}(893a^3+1755a^2bx^2+1375ab^2x^4+385b^3x^6)\right.}{4096a^{19/4}\sqrt{d}}$$

input
```
Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
(Sqrt[x]*(a + b*x^2)*(4*a^(3/4)*Sqrt[x]*(893*a^3 + 1755*a^2*b*x^2 + 1375*a
*b^2*x^4 + 385*b^3*x^6) - (1155*Sqrt[2]*(a + b*x^2)^4*ArcTan[(Sqrt[a] - Sq
rt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (1155*Sqrt[2]*(a +
b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])
/b^(1/4)))/(4096*a^(19/4)*Sqrt[d*x]*((a + b*x^2)^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1384, 27, 253, 253, 253, 253, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{b^5(a+bx^2)\int \frac{1}{b^5\sqrt{dx}(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+bx^2)\int \frac{1}{\sqrt{dx}(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{(a+bx^2)\left(\frac{15\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^4}dx}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{(a+bx^2)\left(\frac{15\left(\frac{11\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^3}dx}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{(a+bx^2)\left(\frac{15\left(\frac{11\left(\frac{7\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)^2}dx}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{(a+bx^2)\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 266$$

$$\frac{(a+bx^2)\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\int \frac{1}{bx^2+a}d\sqrt{dx}}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 755$$

$$\frac{(a+bx^2)\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{\int \frac{d^2(\sqrt{a}d-\sqrt{b}dx)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2(\sqrt{b}xd+\sqrt{a}d)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{2ad}+\frac{\sqrt{dx}}{2ad(a+bx^2)}\right)}{8a}+\frac{\sqrt{dx}}{4ad(a+bx^2)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad(a+bx^2)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{(a+bx^2)\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}\right)}{16a}+\frac{\sqrt{dx}}{8ad(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 1476

$$15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\cdots}{4a\cdots}\right)}{12a}\cdots\right.\right.$$

↓ 1082

$$\left(15\,\frac{\left(11\,\frac{\left(7\,\frac{\left(3\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int\frac{1}{-dx-1}\,d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}\,d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right.}\right.}{12a}\right.$$

↓ 217

$$15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{2ad}+\frac{\sqrt{dx}}{2ad\left(a+bx^2\right)}\right)}{8a}+\frac{\sqrt{dx}}{4ad\left(a+bx^2\right)^2}\right)}{12a}+\frac{\cdots}{6a\cdots}\right.$$

↓ 1479

$$\left(\frac{15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{d}}{\sqrt[4]{a}\sqrt{d}}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{}}\right.}{}\right.}{2ad}\right.}{8a}\right.}{12a}\right.$$

$\downarrow$ 25

$$15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\cdots\right)}{2\cdots}\right)}{2ad}\cdots\right)}{8a}\cdots\right)}{12a}\cdots\right)$$

$\Big\downarrow$ 27

$$\left(15\frac{\left(11\frac{\left(7\frac{\left(3\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\ldots\right.}{\sqrt{2}\sqrt[4]{a}\ldots}\right.}{2\sqrt{a}}\ldots\right.}{2ad}\ldots\right.}{8a}\ldots\right.}{12a}\ldots\right.$$

↓ 1103

$$15\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots\right.}{2\sqrt{2}\sqrt[4]{\cdots}}\cdots\right.}{2\sqrt{a}}\cdots\right.}{2ad}\cdots\right.}{8a}\cdots\right.}{12a}\cdots\right.$$

input
```
Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
((a + b*x^2)*(Sqrt[d*x]/(8*a*d*(a + b*x^2)^4) + (15*(Sqrt[d*x]/(6*a*d*(a +
 b*x^2)^3) + (11*(Sqrt[d*x]/(4*a*d*(a + b*x^2)^2) + (7*(Sqrt[d*x]/(2*a*d*(
a + b*x^2)) + (3*((d*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*S
qrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a
]) + (d*(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d
]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d])))/(2*Sqrt[a])))/(2*a*d)))/(8*a)))/(12*a)))/(16*a)))/Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. $2(293) = 586$.

Time = 0.14 (sec) , antiderivative size = 1133, normalized size of antiderivative = 2.52

method	result	size
default	Expression too large to display	1133

input
```
int(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8192*(1155*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))*b^4*d^6*x^8+2310*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6*x^8+2310*(a*d^2/b)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*b^4*d^6
*x^8+4620*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1
/2)))*a*b^3*d^6*x^6+9240*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/
2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*d^6*x^6+9240*(a*d^2/b)^(1/4)*2^
(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a*b^3*
d^6*x^6+3080*(d*x)^(13/2)*a*b^3+6930*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*b^2*d^6*x^4+13860*(a*d^2/b)^(1/4)*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2
*d^6*x^4+13860*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))*a^2*b^2*d^6*x^4+11000*(d*x)^(9/2)*a^2*b^2*d^2+4
620*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*
a^3*b*d^6*x^2+9240*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)-(a*
d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^3*b*d^6*x^2+9240*(a*d^2/b)^(1/4)*2^(1/...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{1155\,(a^4b^4dx^8+4\,a^5b^3dx^6+6\,a^6b^2dx^4+4\,a^7bdx^2+a^8d)\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{1}{4}}\log\ldots}{\ldots}$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
1/4096*(1155*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4 + 4*a^7*b*
d*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*log(a^5*d*(-1/(a^19*b*d^2))^(1/4) +
 sqrt(d*x)) - 1155*(-I*a^4*b^4*d*x^8 - 4*I*a^5*b^3*d*x^6 - 6*I*a^6*b^2*d*x
^4 - 4*I*a^7*b*d*x^2 - I*a^8*d)*(-1/(a^19*b*d^2))^(1/4)*log(I*a^5*d*(-1/(a
^19*b*d^2))^(1/4) + sqrt(d*x)) - 1155*(I*a^4*b^4*d*x^8 + 4*I*a^5*b^3*d*x^6
 + 6*I*a^6*b^2*d*x^4 + 4*I*a^7*b*d*x^2 + I*a^8*d)*(-1/(a^19*b*d^2))^(1/4)*
log(-I*a^5*d*(-1/(a^19*b*d^2))^(1/4) + sqrt(d*x)) - 1155*(a^4*b^4*d*x^8 +
4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4 + 4*a^7*b*d*x^2 + a^8*d)*(-1/(a^19*b*d^2
))^(1/4)*log(-a^5*d*(-1/(a^19*b*d^2))^(1/4) + sqrt(d*x)) + 4*(385*b^3*x^6
+ 1375*a*b^2*x^4 + 1755*a^2*b*x^2 + 893*a^3)*sqrt(d*x))/(a^4*b^4*d*x^8 + 4
*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4 + 4*a^7*b*d*x^2 + a^8*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{1}{\sqrt{dx}\,\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(1/(sqrt(d*x)*((a + b*x**2)**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\int \frac{1}{(b^2x^4+2\,abx^2+a^2)^{\frac{5}{2}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
1/3072*(5267*b^3*x^(13/2) + 11645*a*b^2*x^(9/2) + 9441*a^2*b*x^(5/2) + 267
9*a^3*sqrt(x))/(a^4*b^4*sqrt(d)*x^8 + 4*a^5*b^3*sqrt(d)*x^6 + 6*a^6*b^2*sq
rt(d)*x^4 + 4*a^7*b*sqrt(d)*x^2 + a^8*sqrt(d)) - 1/192*((257*b^5*sqrt(d)*x
^5 + 378*a*b^4*sqrt(d)*x^3 + 153*a^2*b^3*sqrt(d)*x)*x^(11/2) + 2*(303*a*b^
4*sqrt(d)*x^5 + 462*a^2*b^3*sqrt(d)*x^3 + 191*a^3*b^2*sqrt(d)*x)*x^(7/2) +
 (381*a^2*b^3*sqrt(d)*x^5 + 610*a^3*b^2*sqrt(d)*x^3 + 261*a^4*b*sqrt(d)*x)
*x^(3/2))/(a^7*b^3*d*x^6 + 3*a^8*b^2*d*x^4 + 3*a^9*b*d*x^2 + a^10*d + (a^4
*b^6*d*x^6 + 3*a^5*b^5*d*x^4 + 3*a^6*b^4*d*x^2 + a^7*b^3*d)*x^6 + 3*(a^5*b
^5*d*x^6 + 3*a^6*b^4*d*x^4 + 3*a^7*b^3*d*x^2 + a^8*b^2*d)*x^4 + 3*(a^6*b^4
*d*x^6 + 3*a^7*b^3*d*x^4 + 3*a^8*b^2*d*x^2 + a^9*b*d)*x^2) - 893/8192*(2*s
qrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*s
qrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*sqrt(d)*lo
g(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4))
 - sqrt(2)*sqrt(d)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt
(a))/(a^(3/4)*b^(1/4)))/(a^4*d) + integrate(1/((a^4*b*sqrt(d)*x^2 + a^5*sq
rt(d))*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \frac{1155\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5bd\mathrm{sgn}\left(bx^2+a\right)} + \frac{1155\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5bd\mathrm{sgn}\left(bx^2+a\right)} + \frac{1155\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^5bd\mathrm{sgn}\left(bx^2+a\right)} - \frac{1155\,\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^5bd\mathrm{sgn}\left(bx^2+a\right)} + \frac{385\,\sqrt{dx}b^3d^7x^6+1375\,\sqrt{dx}ab^2d^7x^4+1755\,\sqrt{dx}a^2bd^7x^2+893\,\sqrt{dx}a^3d^7}{1024\,(bd^2x^2+ad^2)^4a^4\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
1155/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d*sgn(b*x^2 + a)) + 1155/4096
*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) -
2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d*sgn(b*x^2 + a)) + 1155/8192*sqrt(2)
*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^
2/b))/(a^5*b*d*sgn(b*x^2 + a)) - 1155/8192*sqrt(2)*(a*b^3*d^2)^(1/4)*log(d
*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b*d*sgn(b*x^2
 + a)) + 1/1024*(385*sqrt(d*x)*b^3*d^7*x^6 + 1375*sqrt(d*x)*a*b^2*d^7*x^4
+ 1755*sqrt(d*x)*a^2*b*d^7*x^2 + 893*sqrt(d*x)*a^3*d^7)/((b*d^2*x^2 + a*d^
2)^4*a^4*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{1}{\sqrt{d\,x}\,(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

output
```
int(1/((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.83

$$\int \frac{1}{\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output

```
(sqrt(d)*( - 2310*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 - 9240*b**(3/4)*a
**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 - 13860*b**(3/4)*a**(1/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*a**2*b**2*x**4 - 9240*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 -
2310*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)
*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*x**8 + 2310*b**(3/4)*a**(1/4)*
sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**
(1/4)*sqrt(2)))*a**4 + 9240*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1
/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2
+ 13860*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt
(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 + 9240*b**(3/4)*a
**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 + 2310*b**(3/4)*a**(1/4)*sqrt(2)*atan(
(b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)
))*b**4*x**8 - 1155*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(
1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**4 - 4620*b**(3/4)*a**(1/4)*sqrt(2)*
log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**3*b*...
```

3.663 $\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 496

$$\begin{aligned}
\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx &= \frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{1}{8ad\sqrt{dx}\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{17}{96a^2d\sqrt{dx}\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{221}{768a^3d\sqrt{dx}\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{3315(a+bx^2)}{1024a^5d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{3315\sqrt[4]{b}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{3315\sqrt[4]{b}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{3315\sqrt[4]{b}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

output
```
663/1024/a^4/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)+1/8/a/d/(d*x)^(1/2)/(b*x^2+
a)^3/((b*x^2+a)^2)^(1/2)+17/96/a^2/d/(d*x)^(1/2)/(b*x^2+a)^2/((b*x^2+a)^2)
^(1/2)+221/768/a^3/d/(d*x)^(1/2)/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-3315/1024*(
b*x^2+a)/a^5/d/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)+3315/4096*b^(1/4)*(b*x^2+a)
*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(21/4)/d^
(3/2)/((b*x^2+a)^2)^(1/2)-3315/4096*b^(1/4)*(b*x^2+a)*arctan(1+2^(1/2)*b^(
1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(21/4)/d^(3/2)/((b*x^2+a)^2)^(
1/2)+3315/4096*b^(1/4)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/
2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(21/4)/d^(3/2)/((b*x^2+a)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.43

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{x(a+bx^2)\left(-4\sqrt[4]{a}(6144a^4+31501a^3bx^2+52819a^2b^2x^4+37791a\right.}{}$$

input
```
Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
(x*(a + b*x^2)*(-4*a^(1/4)*(6144*a^4 + 31501*a^3*b*x^2 + 52819*a^2*b^2*x^4
 + 37791*a*b^3*x^6 + 9945*b^4*x^8) + 9945*Sqrt[2]*b^(1/4)*Sqrt[x]*(a + b*x
^2)^4*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 99
45*Sqrt[2]*b^(1/4)*Sqrt[x]*(a + b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(12288*a^(21/4)*(d*x)^(3/2)*((a + b*x^2)
^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.92, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.567$, Rules used = {1384, 27, 253, 253, 253, 253, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed

below.

$$\int \frac{1}{(dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2}} \, dx$$

$\Big\downarrow$ 1384

$$\frac{b^5 \left(a+bx^2\right) \int \frac{1}{b^5 (dx)^{3/2} \left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 27

$$\frac{\left(a+bx^2\right) \int \frac{1}{(dx)^{3/2} \left(bx^2+a\right)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{\left(a+bx^2\right) \left(\frac{17 \int \frac{1}{(dx)^{3/2} \left(bx^2+a\right)^4}dx}{16 a}+\frac{1}{8 a d \sqrt{dx} \left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{\left(a+bx^2\right) \left(\frac{17 \left(\frac{13 \int \frac{1}{(dx)^{3/2} \left(bx^2+a\right)^3}dx}{12 a}+\frac{1}{6 a d \sqrt{dx} \left(a+bx^2\right)^3}\right)}{16 a}+\frac{1}{8 a d \sqrt{dx} \left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{\left(a+bx^2\right) \left(\frac{17 \left(\frac{13 \left(\frac{9 \int \frac{1}{(dx)^{3/2} \left(bx^2+a\right)^2}dx}{8 a}+\frac{1}{4 a d \sqrt{dx} \left(a+bx^2\right)^2}\right)}{12 a}+\frac{1}{6 a d \sqrt{dx} \left(a+bx^2\right)^3}\right)}{16 a}+\frac{1}{8 a d \sqrt{dx} \left(a+bx^2\right)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\Big\downarrow$ 253

$$\frac{(a+bx^2)\left(\frac{17\left(\frac{13\left(\frac{9\left(\frac{5\int\frac{1}{(dx)^{3/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad\sqrt{dx}(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 264

$$\frac{(a+bx^2)\left(\frac{17\left(\frac{13\left(\frac{9\left(\frac{5\left(-\frac{b\int\frac{\sqrt{dx}}{bx^2+a}dx}{ad^2}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad\sqrt{dx}(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 266

$$\frac{\left(a+bx^2\right)\left(\frac{17\left(\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\int \frac{d^3x}{bx^2d^2+ad^2}d\sqrt{dx}}{ad^3}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad\sqrt{dx}(a+bx^2}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+bx^2\right)\left(\frac{17\left(\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad\sqrt{dx}(a+bx^2}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 826

$$(a+bx^2)\left(\frac{17\left(\frac{13\left(\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad\sqrt{dx}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad\sqrt{dx}\left(a+bx^2\right)}\cdots}{16a}\cdots\right.$$

↓ 1476

$$
\left(17\frac{\left(13\frac{\left(9\frac{\left(5\left(-\frac{2b\left(\frac{\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int\frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)\right)}{4a}+\frac{}{2ad\sqrt{dx}}\right)}{8a}\right)}{12a}\right)
$$

$\downarrow$ 1082

$$\left(\left(17\,\frac{\left(13\,\frac{\left(9\,\frac{\left(5\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-dx-1}\,d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}\,d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)} \cdots\right)}{8a} \cdots\right)}{12a} \cdots$$

$$\Big\downarrow$$ 217

$$\left(\left(17\frac{\left(13\frac{\left(9\frac{\left(5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}\,d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)\right)}{4a}+\frac{1}{2ad\sqrt{dx}\left(a+bx^2\right)}\right)}{8a}+\frac{\cdots}{4ad\sqrt{\cdots}}\right)}{12a}\cdots$$

↓ 1479

$$2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\cdots\right.}{\sqrt[4]{b}\left(xd\cdots\right.}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right.$$

$$5-\frac{\cdots}{ad}$$

$$9\frac{\cdots}{4a}$$

$$13\frac{\cdots}{8a}$$

↓ 25

$$\left(13\left(9\left(5-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt[4]{a}\ldots\right.}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}\ldots}{\sqrt{b}\ldots}\right.}}{2\sqrt{\ldots}}}{2\sqrt{b}}\right.}{ad}\right.}{4a}\right.}{8a}\right.$$

$\Big\downarrow$ 27

$$13\,\frac{9\,\frac{5-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{d\cdots}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d\cdots}}{\sqrt[4]{b}}}\cdots}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}}{2\sqrt{b}}\cdots\right)}{ad}\cdots}{4a}\cdots}{8a}\cdots$$

↓ 1103

$$\left(17\frac{\left(13\frac{\left(9\frac{\left(5-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{b}}\right.}{ad}\right.}{4a}\right.}{8a}\right.}{12a}\right.$$

input
```
Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
((a + b*x^2)*(1/(8*a*d*Sqrt[d*x]*(a + b*x^2)^4) + (17*(1/(6*a*d*Sqrt[d*x]*
(a + b*x^2)^3) + (13*(1/(4*a*d*Sqrt[d*x]*(a + b*x^2)^2) + (9*(1/(2*a*d*Sqr
t[d*x]*(a + b*x^2)) + (5*(-2/(a*d*Sqrt[d*x]) - (2*b*((-(ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))
 + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])
 + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]
]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b])))/(a*d)))/(4*a)))/(8*a)
))/(12*a)))/(16*a)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{2\sqrt{(b x^2+a)^2}}{a^5\sqrt{dx}\,d(b x^2+a)} - \frac{b\left(\frac{\frac{6925a^3d^6(dx)^{\frac{3}{2}}}{3072}+\frac{15955a^2d^4b(dx)^{\frac{7}{2}}}{3072}+\frac{4405a\,d^2b^2(dx)^{\frac{11}{2}}}{1024}+\frac{1267b^3(dx)^{\frac{15}{2}}}{1024}}{\left(b\,d^2x^2+a\,d^2\right)^4}+\frac{3315\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}\cdots}{dx+\left(\frac{a\,d^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}\cdots}\right)\cdots\right)}{\cdots}\right)}{a^5d(b x^2+a)}$
default	Expression too large to display

input
```
int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a^5/(d*x)^(1/2)/d*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-b/a^5*(2*(6925/6144*a^3
*d^6*(d*x)^(3/2)+15955/6144*a^2*d^4*b*(d*x)^(7/2)+4405/2048*a*d^2*b^2*(d*x
)^(11/2)+1267/2048*b^3*(d*x)^(15/2))/(b*d^2*x^2+a*d^2)^4+3315/8192/b/(a*d^
2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan
(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(
d*x)^(1/2)-1)))/d*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.08

$$\int \frac{1}{(dx)^{3/2} (a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx =$$
$$-\frac{9945 \left(a^5b^4d^2x^9 + 4\,a^6b^3d^2x^7 + 6\,a^7b^2d^2x^5 + 4\,a^8bd^2x^3 + a^9d^2x\right)\left(-\frac{b}{a^{21}d^6}\right)^{\frac{1}{4}} \log\left(36429280875\,a^{16}d^5\left(-\frac{b}{a^{21}d^6}\right.\right.}{}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
-1/12288*(9945*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 +
4*a^8*b*d^2*x^3 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*log(36429280875*a^16*d^
5*(-b/(a^21*d^6))^(3/4) + 36429280875*sqrt(d*x)*b) + 9945*(-I*a^5*b^4*d^2*
x^9 - 4*I*a^6*b^3*d^2*x^7 - 6*I*a^7*b^2*d^2*x^5 - 4*I*a^8*b*d^2*x^3 - I*a^
9*d^2*x)*(-b/(a^21*d^6))^(1/4)*log(36429280875*I*a^16*d^5*(-b/(a^21*d^6))^
(3/4) + 36429280875*sqrt(d*x)*b) + 9945*(I*a^5*b^4*d^2*x^9 + 4*I*a^6*b^3*d
^2*x^7 + 6*I*a^7*b^2*d^2*x^5 + 4*I*a^8*b*d^2*x^3 + I*a^9*d^2*x)*(-b/(a^21*
d^6))^(1/4)*log(-36429280875*I*a^16*d^5*(-b/(a^21*d^6))^(3/4) + 3642928087
5*sqrt(d*x)*b) - 9945*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2
*x^5 + 4*a^8*b*d^2*x^3 + a^9*d^2*x)*(-b/(a^21*d^6))^(1/4)*log(-36429280875
*a^16*d^5*(-b/(a^21*d^6))^(3/4) + 36429280875*sqrt(d*x)*b) + 4*(9945*b^4*x
^8 + 37791*a*b^3*x^6 + 52819*a^2*b^2*x^4 + 31501*a^3*b*x^2 + 6144*a^4)*sqr
t(d*x))/(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4*a^8*b
*d^2*x^3 + a^9*d^2*x)
```

Sympy [F]

$$\int \frac{1}{(dx)^{3/2} (a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx = \int \frac{1}{(dx)^{\frac{3}{2}} \left((a + bx^2)^2\right)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(1/((d*x)**(3/2)*((a + b*x**2)**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{5}{2}}(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
-1/3072*(3801*b^4*x^(15/2) + 8079*a*b^3*x^(11/2) + 6515*a^2*b^2*x^(7/2) +
1853*a^3*b*x^(3/2))/(a^5*b^4*d^(3/2)*x^8 + 4*a^6*b^3*d^(3/2)*x^6 + 6*a^7*b
^2*d^(3/2)*x^4 + 4*a^8*b*d^(3/2)*x^2 + a^9*d^(3/2)) - 1/192*((321*b^5*sqrt
(d)*x^5 + 490*a*b^4*sqrt(d)*x^3 + 201*a^2*b^3*sqrt(d)*x)*x^(9/2) + 2*(371*
a*b^4*sqrt(d)*x^5 + 582*a^2*b^3*sqrt(d)*x^3 + 243*a^3*b^2*sqrt(d)*x)*x^(5/
2) + (453*a^2*b^3*sqrt(d)*x^5 + 738*a^3*b^2*sqrt(d)*x^3 + 317*a^4*b*sqrt(d
)*x)*sqrt(x))/(a^7*b^3*d^2*x^6 + 3*a^8*b^2*d^2*x^4 + 3*a^9*b*d^2*x^2 + a^1
0*d^2 + (a^4*b^6*d^2*x^6 + 3*a^5*b^5*d^2*x^4 + 3*a^6*b^4*d^2*x^2 + a^7*b^3
*d^2)*x^6 + 3*(a^5*b^5*d^2*x^6 + 3*a^6*b^4*d^2*x^4 + 3*a^7*b^3*d^2*x^2 + a
^8*b^2*d^2)*x^4 + 3*(a^6*b^4*d^2*x^6 + 3*a^7*b^3*d^2*x^4 + 3*a^8*b^2*d^2*x
^2 + a^9*b*d^2)*x^2) - 1267/8192*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*
sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b
)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^5*d^(3/2)) + integrate(1/((a^4*b*d^(3/2)
*x^2 + a^5*d^(3/2))*x^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.82

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx =$$

$$-\frac{\frac{49152}{\sqrt{dx}a^5\mathrm{sgn}(bx^2+a)}+\frac{19890\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^2d^2\mathrm{sgn}(bx^2+a)}+\frac{19890\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^2d^2\mathrm{sgn}(bx^2+a)}-}{}$$

input
```
integrate(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
-1/24576*(49152/(sqrt(d*x)*a^5*sgn(b*x^2 + a)) + 19890*sqrt(2)*(a*b^3*d^2)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/(a^6*b^2*d^2*sgn(b*x^2 + a)) + 19890*sqrt(2)*(a*b^3*d^2)^(3/4)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))
/(a^6*b^2*d^2*sgn(b*x^2 + a)) - 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b^2*d^2*sgn(b*x^2 +
 a)) + 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(a^6*b^2*d^2*sgn(b*x^2 + a)) + 8*(3801*sqrt(d*x)*
b^4*d^7*x^7 + 13215*sqrt(d*x)*a*b^3*d^7*x^5 + 15955*sqrt(d*x)*a^2*b^2*d^7*
x^3 + 6925*sqrt(d*x)*a^3*b*d^7*x)/((b*d^2*x^2 + a*d^2)^4*a^5*sgn(b*x^2 + a
)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \int \frac{1}{{\left(d\,x\right)}^{3/2}\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{5/2}} \,d x$$

input
```
int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

output
```
int(1/((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 865, normalized size of antiderivative = 1.74

$$\int \frac{1}{(dx)^{3/2} (a^2 + 2abx^2 + b^2x^4)^{5/2}} dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(sqrt(d)*(19890*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 + 79560*sqr
t(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)
*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 + 119340*sqrt(x)*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 + 79560*sqrt(x)*b**(1/4)*a**(3/
4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*
a**(1/4)*sqrt(2)))*a*b**3*x**6 + 19890*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*b**4*x**8 - 19890*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4 -
79560*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**2 - 119340*sqrt(
x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*s
qrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**4 - 79560*sqrt(x)*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**6 - 19890*sqrt(x)*b**(1/4)*a**(3/4)*
sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**
(1/4)*sqrt(2)))*b**4*x**8 - 9945*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( -
sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**4 - 39780*s...
```

3.664 $\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 496

$$\begin{aligned}\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx &= \frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\\
&+\frac{1}{8ad(dx)^{3/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}\\
&+\frac{19}{96a^2d(dx)^{3/2}\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}\\
&+\frac{95}{256a^3d(dx)^{3/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}\\
&-\frac{7315(a+bx^2)}{3072a^5d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\\
&+\frac{7315b^{3/4}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\\
&-\frac{7315b^{3/4}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\\
&-\frac{7315b^{3/4}(a+bx^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

output
```
1045/1024/a^4/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+1/8/a/d/(d*x)^(3/2)/(b*x^2
+a)^3/((b*x^2+a)^2)^(1/2)+19/96/a^2/d/(d*x)^(3/2)/(b*x^2+a)^2/((b*x^2+a)^2
)^(1/2)+95/256/a^3/d/(d*x)^(3/2)/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-7315/3072*(
b*x^2+a)/a^5/d/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2)+7315/4096*b^(3/4)*(b*x^2+a)
*arctan(1-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(23/4)/d^
(5/2)/((b*x^2+a)^2)^(1/2)-7315/4096*b^(3/4)*(b*x^2+a)*arctan(1+2^(1/2)*b^(
1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(23/4)/d^(5/2)/((b*x^2+a)^2)^(
1/2)-7315/4096*b^(3/4)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/
2)/d^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(23/4)/d^(5/2)/((b*x^2+a)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.43

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{x(a+bx^2)\left(-4a^{3/4}(2048a^4+16967a^3bx^2+33345a^2b^2x^4+26125a\right.}{}$$

input
```
Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
(x*(a + b*x^2)*(-4*a^(3/4)*(2048*a^4 + 16967*a^3*b*x^2 + 33345*a^2*b^2*x^4
 + 26125*a*b^3*x^6 + 7315*b^4*x^8) + 21945*Sqrt[2]*b^(3/4)*x^(3/2)*(a + b*
x^2)^4*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 2
1945*Sqrt[2]*b^(3/4)*x^(3/2)*(a + b*x^2)^4*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(12288*a^(23/4)*(d*x)^(5/2)*((a + b*x^
2)^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.567$, Rules used = {1384, 27, 253, 253, 253, 253, 264, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed

below.

$$\int \frac{1}{(dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{b^5 \left(a+bx^2\right) \int \frac{1}{b^5 (dx)^{5/2} (bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+bx^2\right) \int \frac{1}{(dx)^{5/2} (bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 253$$

$$\frac{\left(a+bx^2\right) \left(\frac{19 \int \frac{1}{(dx)^{5/2} \left(bx^2+a\right)^4}dx}{16a}+\frac{1}{8ad(dx)^{3/2} (a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 253$$

$$\frac{\left(a+bx^2\right) \left(\frac{19 \left(\frac{5 \int \frac{1}{(dx)^{5/2} \left(bx^2+a\right)^3}dx}{4a}+\frac{1}{6ad(dx)^{3/2} \left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad(dx)^{3/2} (a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 253$$

$$\frac{\left(a+bx^2\right) \left(\frac{19 \left(\frac{5 \left(\frac{11 \int \frac{1}{(dx)^{5/2} \left(bx^2+a\right)^2}dx}{8a}+\frac{1}{4ad(dx)^{3/2} \left(a+bx^2\right)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2} \left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad(dx)^{3/2} (a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 253$$

$$\frac{(a+bx^2)\left(\frac{19\left(\frac{5\left(\frac{11\left(\frac{7\int\frac{1}{(dx)^{5/2}(bx^2+a)}dx}{4a}+\frac{1}{2ad(dx)^{3/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}(a+bx^2)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2}(a+bx^2)^3}\right)}{16a}+\frac{1}{8ad(dx)^{3/2}(a+bx^2)}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 264

$$\frac{(a+bx^2)\left(\frac{19\left(\frac{5\left(\frac{11\left(\frac{7\left(-\frac{b\int \frac{1}{\sqrt{dx}\left(bx^2+a\right)}dx}{ad^2}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{\cdots}{8ad\cdots}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 266

$$\frac{\left(a+bx^2\right)\left(\frac{19\left(\frac{5\left(\frac{11\left(\frac{7\left(-\frac{2b\int \frac{1}{bx^2+a}d\sqrt{dx}}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)^2}\right)}{4a}+\frac{1}{6ad(dx)^{3/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{}{8ad(dx}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 755

$$(a+bx^2)\left(\frac{19\left(\frac{5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{\int \frac{d^2\left(\sqrt{a}d-\sqrt{b}dx\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}+\frac{\int \frac{d^2\left(\sqrt{b}xd+\sqrt{a}d\right)}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}d}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)^2}\right.}{4a}\right.}{16a}\right.$$

$$\Big\downarrow \; 27$$

$$\left(a+bx^2\right)\left(\frac{19\left(\frac{5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{3/2}\left(a+bx^2\right)^2}\right)}{4a}+\frac{\cdots}{6a\cdots}\cdots\right)}{16a}\cdots\right.$$

↓ 1476

$$5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}}{8a}\right.$$

↓ 1082

$$5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)}{4a}+\frac{}{2ad(dx}\cdots}{8a}\cdots\right.$$

$$\Big\downarrow 217$$

$$\left(\left(\left(5\frac{\left(11\frac{\left(7\left(-\frac{2b\left(\frac{d\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}\right)}{ad^3}-\frac{2}{3ad(dx)^{3/2}}\right)\right)}{4a}+\frac{1}{2ad(dx)^{3/2}\left(a+b\right.}\right.}{8a}\right.\right.\right.$$

↓ 1479

$$5\left(11\left(7\left(-\frac{2b\left(\frac{d\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.}{\sqrt{2}\sqrt[4]{a}\cdots}\right.}{\cdots}\right.}{ad^3}\right.}{4a}\right.}{8a}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\left(\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}\right)}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{d}\cdots}{\sqrt[4]{a}\sqrt{d}\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}\cdots\right.}{\cdots}\right)}{\cdots}$$

$$7\left(-\frac{\cdots}{ad^3}\cdots\right)$$

$$11\left(\frac{\cdots}{4a}\cdots\right)$$

$$5\left(\frac{\cdots}{8a}\cdots\right)$$

↓ 27

$$2b\,\frac{d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt[4]{a}\sqrt{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(\cdots\right.}{\sqrt{2}\cdots}\right.}{2\sqrt{a}}$$

$$7-\frac{\cdots}{ad^3}$$

$$11\,\frac{\cdots}{4a}$$

$$5\,\frac{\cdots}{8a}$$

↓ 1103

$$5\left(\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}\right)}{2\sqrt{a}}+\frac{d\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{\cdots}\right.}{2\cdots}\right.}{2\sqrt{a}}\cdots\right)}{ad^3}\cdots\right)}{4a}\cdots\right)}{8a}\cdots\right)$$

input
```
Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
((a + b*x^2)*(1/(8*a*d*(d*x)^(3/2)*(a + b*x^2)^4) + (19*(1/(6*a*d*(d*x)^(3
/2)*(a + b*x^2)^3) + (5*(1/(4*a*d*(d*x)^(3/2)*(a + b*x^2)^2) + (11*(1/(2*a
*d*(d*x)^(3/2)*(a + b*x^2)) + (7*(-2/(3*a*d*(d*x)^(3/2)) - (2*b*((d*(-(Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a]) + (d*(-1/2*Log[Sqrt[a]*d
 + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])))/(2*Sqrt[a])))/(
a*d^3)))/(4*a)))/(8*a)))/(4*a)))/(16*a)))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.56

method	result
risch	$-\frac{2\sqrt{(bx^2+a)^2}}{3a^5x\sqrt{dx}\,d^2(bx^2+a)}-\frac{2b\left(\frac{\frac{2925a^3d^6\sqrt{dx}}{2048}+\frac{7019bd^4a^2(dx)^{\frac{5}{2}}}{2048}+\frac{17933ad^2b^2(dx)^{\frac{9}{2}}}{6144}+\frac{5267b^3(dx)^{\frac{13}{2}}}{6144}}{\left(bd^2x^2+ad^2\right)^4}+\frac{7315\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{ad^2}{b}\right)\cdots}{dx-\left(\frac{ad^2}{b}\right)\cdots}\right)\cdots\right)}{\cdots}\right)}{a^5d(bx^2+a)}$
default	Expression too large to display

input
```
int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a^5/x/(d*x)^(1/2)/d^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-2/a^5*b/d*((2925/
2048*a^3*d^6*(d*x)^(1/2)+7019/2048*b*d^4*a^2*(d*x)^(5/2)+17933/6144*a*d^2*
b^2*(d*x)^(9/2)+5267/6144*b^3*(d*x)^(13/2))/(b*d^2*x^2+a*d^2)^4+7315/16384
*(a*d^2/b)^(1/4)/a/d^2*2^(1/2)*(ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(a*d^
2/b)^(1/4)*(d*x)^(1/2)-1)))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.14

$$\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^{5/2}}\,dx =$$

$$-\frac{21945\left(a^5b^4d^3x^{10}+4\,a^6b^3d^3x^8+6\,a^7b^2d^3x^6+4\,a^8bd^3x^4+a^9d^3x^2\right)\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}\log\left(7315\,a^6d^3\left(-\frac{b^3}{a^{23}d^{10}}\right)^{\frac{1}{4}}\right.}{}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
-1/12288*(21945*(a^5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6
+ 4*a^8*b*d^3*x^4 + a^9*d^3*x^2)*(-b^3/(a^23*d^10))^(1/4)*log(7315*a^6*d^3
*(-b^3/(a^23*d^10))^(1/4) + 7315*sqrt(d*x)*b) + 21945*(I*a^5*b^4*d^3*x^10
+ 4*I*a^6*b^3*d^3*x^8 + 6*I*a^7*b^2*d^3*x^6 + 4*I*a^8*b*d^3*x^4 + I*a^9*d^
3*x^2)*(-b^3/(a^23*d^10))^(1/4)*log(7315*I*a^6*d^3*(-b^3/(a^23*d^10))^(1/4
) + 7315*sqrt(d*x)*b) + 21945*(-I*a^5*b^4*d^3*x^10 - 4*I*a^6*b^3*d^3*x^8 -
 6*I*a^7*b^2*d^3*x^6 - 4*I*a^8*b*d^3*x^4 - I*a^9*d^3*x^2)*(-b^3/(a^23*d^10
))^(1/4)*log(-7315*I*a^6*d^3*(-b^3/(a^23*d^10))^(1/4) + 7315*sqrt(d*x)*b)
- 21945*(a^5*b^4*d^3*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 + 4*a^8*
b*d^3*x^4 + a^9*d^3*x^2)*(-b^3/(a^23*d^10))^(1/4)*log(-7315*a^6*d^3*(-b^3/
(a^23*d^10))^(1/4) + 7315*sqrt(d*x)*b) + 4*(7315*b^4*x^8 + 26125*a*b^3*x^6
 + 33345*a^2*b^2*x^4 + 16967*a^3*b*x^2 + 2048*a^4)*sqrt(d*x))/(a^5*b^4*d^3
*x^10 + 4*a^6*b^3*d^3*x^8 + 6*a^7*b^2*d^3*x^6 + 4*a^8*b*d^3*x^4 + a^9*d^3*
x^2)
```

Sympy [F]

$$\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{1}{(dx)^{\frac{5}{2}}\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(1/((d*x)**(5/2)*((a + b*x**2)**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\int \frac{1}{(b^2x^4+2\,abx^2+a^2)^{\frac{5}{2}}\,(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
-4*b*integrate(1/((a^5*b*d^(5/2)*x^2 + a^6*d^(5/2))*sqrt(x)), x) - 1/3072*
(13795*b^4*x^(13/2) + 34285*a*b^3*x^(9/2) + 29649*a^2*b^2*x^(5/2) + 8775*a
^3*b*sqrt(x))/(a^5*b^4*d^(5/2)*x^8 + 4*a^6*b^3*d^(5/2)*x^6 + 6*a^7*b^2*d^(
5/2)*x^4 + 4*a^8*b*d^(5/2)*x^2 + a^9*d^(5/2)) + 1/192*((533*b^6*x^5 + 882*
a*b^5*x^3 + 381*a^2*b^4*x)*x^(11/2) + 2*(603*a*b^5*x^5 + 1014*a^2*b^4*x^3
+ 443*a^3*b^3*x)*x^(7/2) + (705*a^2*b^4*x^5 + 1210*a^3*b^3*x^3 + 537*a^4*b
^2*x)*x^(3/2))/(a^8*b^3*d^(5/2)*x^6 + 3*a^9*b^2*d^(5/2)*x^4 + 3*a^10*b*d^(
5/2)*x^2 + a^11*d^(5/2) + (a^5*b^6*d^(5/2)*x^6 + 3*a^6*b^5*d^(5/2)*x^4 + 3
*a^7*b^4*d^(5/2)*x^2 + a^8*b^3*d^(5/2))*x^6 + 3*(a^6*b^5*d^(5/2)*x^6 + 3*a
^7*b^4*d^(5/2)*x^4 + 3*a^8*b^3*d^(5/2)*x^2 + a^9*b^2*d^(5/2))*x^4 + 3*(a^7
*b^4*d^(5/2)*x^6 + 3*a^8*b^3*d^(5/2)*x^4 + 3*a^9*b^2*d^(5/2)*x^2 + a^10*b*
d^(5/2))*x^2) + 2925/8192*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)
*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)
*sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
sqrt(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))
/a^(3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/a^(3/4))/(a^5*d^(5/2)) + integrate(1/((a^4*b*d^(5/2)*x^2 + a^
5*d^(5/2))*x^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.80

$$\int \frac{1}{(dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx =$$

$$-\frac{7315\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^6d^3\mathrm{sgn}\left(bx^2+a\right)}$$
$$-\frac{7315\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^6d^3\mathrm{sgn}\left(bx^2+a\right)}$$
$$-\frac{7315\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^6d^3\mathrm{sgn}\left(bx^2+a\right)}$$
$$+\frac{7315\sqrt{2}(ab^3d^2)^{\frac{1}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^6d^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{2}{3\sqrt{dx}a^5d^2x\mathrm{sgn}\left(bx^2+a\right)}$$
$$-\frac{5267\sqrt{dx}b^4d^6x^6+17933\sqrt{dx}ab^3d^6x^4+21057\sqrt{dx}a^2b^2d^6x^2+8775\sqrt{dx}a^3bd^6}{3072\left(bd^2x^2+ad^2\right)^4a^5d\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
-7315/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*d^3*sgn(b*x^2 + a)) - 7315/409
6*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) -
 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*d^3*sgn(b*x^2 + a)) - 7315/8192*sqrt(2
)*(a*b^3*d^2)^(1/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d
^2/b))/(a^6*d^3*sgn(b*x^2 + a)) + 7315/8192*sqrt(2)*(a*b^3*d^2)^(1/4)*log(
d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*d^3*sgn(b*x^
2 + a)) - 2/3/(sqrt(d*x)*a^5*d^2*x*sgn(b*x^2 + a)) - 1/3072*(5267*sqrt(d*x
)*b^4*d^6*x^6 + 17933*sqrt(d*x)*a*b^3*d^6*x^4 + 21057*sqrt(d*x)*a^2*b^2*d^
6*x^2 + 8775*sqrt(d*x)*a^3*b*d^6)/((b*d^2*x^2 + a*d^2)^4*a^5*d*sgn(b*x^2 +
 a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{5/2} (a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx = \int \frac{1}{(d\,x)^{5/2} \left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{5/2}} \, dx$$

input
```
int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

output
```
int(1/((d*x)^(5/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 872, normalized size of antiderivative = 1.76

$$\int \frac{1}{(dx)^{5/2} (a^2 + 2abx^2 + b^2x^4)^{5/2}} \, dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(sqrt(d)*(43890*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*x + 175560*
sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt
(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**3 + 263340*sqrt(x)*b**
(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**5 + 175560*sqrt(x)*b**(3/4)*a*
*(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1
/4)*a**(1/4)*sqrt(2)))*a*b**3*x**7 + 43890*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*b**4*x**9 - 43890*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1
/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**
4*x - 175560*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*x**3 - 26334
0*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sq
rt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*x**5 - 175560*sqrt(x
)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sq
rt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*x**7 - 43890*sqrt(x)*b**(3/4)*a
**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*b**4*x**9 + 21945*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2
)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**4*...
```

3.665 $\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 543

$$
\begin{aligned}
\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx &= \frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{1}{8ad(dx)^{5/2}\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{7}{32a^2d(dx)^{5/2}\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{119}{256a^3d(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{13923(a+bx^2)}{5120a^5d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}} + \frac{13923b(a+bx^2)}{1024a^6d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{13923b^{5/4}(a+bx^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}} \\
&+ \frac{13923b^{5/4}(a+bx^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}} \\
&- \frac{13923b^{5/4}(a+bx^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}}{\sqrt{d}\left(\sqrt{a}+\sqrt{b}x\right)}\right)}{2048\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

output
```
1547/1024/a^4/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)+1/8/a/d/(d*x)^(5/2)/(b*x^2
+a)^3/((b*x^2+a)^2)^(1/2)+7/32/a^2/d/(d*x)^(5/2)/(b*x^2+a)^2/((b*x^2+a)^2)
^(1/2)+119/256/a^3/d/(d*x)^(5/2)/(b*x^2+a)/((b*x^2+a)^2)^(1/2)-13923/5120*
(b*x^2+a)/a^5/d/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2)+13923/1024*b*(b*x^2+a)/a^6
/d^3/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2)-13923/4096*b^(5/4)*(b*x^2+a)*arctan(1
-2^(1/2)*b^(1/4)*(d*x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(25/4)/d^(7/2)/((b
*x^2+a)^2)^(1/2)+13923/4096*b^(5/4)*(b*x^2+a)*arctan(1+2^(1/2)*b^(1/4)*(d*
x)^(1/2)/a^(1/4)/d^(1/2))*2^(1/2)/a^(25/4)/d^(7/2)/((b*x^2+a)^2)^(1/2)-139
23/4096*b^(5/4)*(b*x^2+a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x)^(1/2)/d^(1
/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(25/4)/d^(7/2)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.41

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{x(a+bx^2)\left(4\sqrt[4]{a}(-2048a^5+43008a^4bx^2+220507a^3b^2x^4+36973\right.}{}$$

input
```
Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
(x*(a + b*x^2)*(4*a^(1/4)*(-2048*a^5 + 43008*a^4*b*x^2 + 220507*a^3*b^2*x^
4 + 369733*a^2*b^3*x^6 + 264537*a*b^4*x^8 + 69615*b^5*x^10) - 69615*Sqrt[2
]*b^(5/4)*x^(5/2)*(a + b*x^2)^4*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x])] - 69615*Sqrt[2]*b^(5/4)*x^(5/2)*(a + b*x^2)^4*ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]))/(20480*a^(25/
4)*(d*x)^(7/2)*((a + b*x^2)^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.89, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1384, 27, 253, 253, 253, 253, 264, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed

below.

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}dx$$

$$\Big\downarrow \ 1384$$

$$\frac{b^5\left(a+bx^2\right)\int \frac{1}{b^5(dx)^{7/2}(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+bx^2\right)\int \frac{1}{(dx)^{7/2}(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 253$$

$$\frac{\left(a+bx^2\right)\left(\frac{21\int \frac{1}{(dx)^{7/2}\left(bx^2+a\right)^4}dx}{16a}+\frac{1}{8ad(dx)^{5/2}(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 253$$

$$\frac{\left(a+bx^2\right)\left(\frac{21\left(\frac{17\int \frac{1}{(dx)^{7/2}\left(bx^2+a\right)^3}dx}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad(dx)^{5/2}(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 253$$

$$\frac{\left(a+bx^2\right)\left(\frac{21\left(\frac{17\left(\frac{13\int \frac{1}{(dx)^{7/2}\left(bx^2+a\right)^2}dx}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad(dx)^{5/2}(a+bx^2)^4}\right)}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 253$$

$$\frac{(a+bx^2)\left(\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\int\frac{1}{(dx)^{7/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}\right)}{16a}+\frac{1}{8ad(dx)^{5/2}(a+bx^2}\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$\downarrow$ 264

$$\frac{\left(a+bx^2\right)\left(\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\int\frac{1}{(dx)^{3/2}\left(bx^2+a\right)}dx}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}\right)}{16a}+\right.}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\downarrow 264$$

$$\left(a+bx^2\right)\left(\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{b\int\frac{\sqrt{dx}}{bx^2+a}\,dx}{ad^2}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a+bx^2\right)^3}\right)}{16a}\right.$$

↓ 266

$$\left(a+bx^2\right)\left(\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\int\frac{d^3x}{bx^2d^2+ad^2}\,d\sqrt{dx}}{ad^3}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a\right.}\right.}{16a}\right.$$

$$\Big\downarrow \, 27$$

$$\left(a+bx^2\right)\left(\frac{21\left(\frac{17\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\int\frac{dx}{bx^2d^2+ad^2}d\sqrt{dx}}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)^2}\right)}{12a}+\frac{1}{6ad(dx)^{5/2}\left(a\right.}}{16a}\right.$$

↓ 826

$$\left(21\frac{\left(17\frac{\left(13\frac{\left(9\left(-\frac{b\left(-\frac{2b\left(\frac{\int\frac{\sqrt{b}xd+\sqrt{a}d}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}\right)-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ad(dx)^{5/2}\left(a\cdots\right.}\right.}{12a}\right.$$

↓ 1476

$$\left(\left(\left(\left(\left(\left(\frac{2b\left(\frac{\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}+\frac{\int \frac{1}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)b}{ad^2}-\frac{\cdots}{5a\cdots}\right)9}{4a}\cdots\right)13}{8a}\cdots\right)17 \cdots$$

↓ 1082

$$\left(\left(\left(\left(17\frac{\left(13\frac{\left(9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}}{8a}\right.\right.\right.\right.$$

↓ 217

$$\left(\left(\left(17\left(\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}d-\sqrt{b}dx}{bx^2d^2+ad^2}d\sqrt{dx}}{2\sqrt{b}}\right)}{ad}-\frac{2}{ad\sqrt{dx}}\right)}{ad^2}-\frac{2}{5ad(dx)^{5/2}}\right)}{4a}+\frac{2}{2ad(\cdots}\cdots\right)}{8a}\cdots\right.\right.\right.\right.$$

↓ 1479

$$13\left(\frac{9\left(\frac{b-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\int -\cdots}{\cdots}}{2\sqrt{b}}\cdots}{ad}}{ad^2}\cdots}{4a}\cdots$$

$\downarrow$ 25

$$\Bigg(\Bigg(\Bigg(\Bigg(13\left(9\left(b-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{b}\left(xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\cdots\right.}{\sqrt[4]{b}\left(x\cdots\right.}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right)}{ad}\cdots\right)}{ad^2}\cdots\right)}{4a}\cdots$$

$$\Big\downarrow \; 27$$

$$\left(\left(\left(\left(\frac{13\left(9-\frac{\left(b-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{-\frac{\int\frac{\sqrt{2}\sqrt[4]{a}\sqrt{d}-2\sqrt[4]{b}\sqrt{dx}}{xd+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{dx}\sqrt{d}}{\sqrt[4]{b}}}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}\sqrt{d}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{d}+\sqrt{2}\cdots}{xd+\frac{\sqrt{a}d}{\sqrt{b}}+\frac{\sqrt{2}\cdots}{\cdots}}\cdots}{2\sqrt[4]{a}\cdots}}{2\sqrt{b}}\cdots\right)}{ad}\cdots\right)}{ad^2}\cdots\right)}{4a}\cdots$$

↓ 1103

$$\left(\vphantom{\frac{1}{1}}\right.\left(\vphantom{\frac{1}{1}}\right.17\,\frac{\left(13\,\frac{\left(9\,-\frac{\left(b\,-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\cdots\right.}{2\sqrt{\cdots}}}{2\sqrt{b}}\cdots\right.}{ad}\cdots\right.}{ad^2}\cdots\right.}{4a}\cdots\right.}{8a}\cdots$$

input
```
Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

output
```
((a + b*x^2)*(1/(8*a*d*(d*x)^(5/2)*(a + b*x^2)^4) + (21*(1/(6*a*d*(d*x)^(5
/2)*(a + b*x^2)^3) + (17*(1/(4*a*d*(d*x)^(5/2)*(a + b*x^2)^2) + (13*(1/(2*
a*d*(d*x)^(5/2)*(a + b*x^2)) + (9*(-2/(5*a*d*(d*x)^(5/2)) - (b*(-2/(a*d*Sq
rt[d*x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
d*x])/(a^(1/4)*Sqrt[d])]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]))/(2*Sqrt[b]) -
(-1/2*Log[Sqrt[a]*d + Sqrt[b]*d*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d
*x]]/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]) + Log[Sqrt[a]*d + Sqrt[b]*d*x + Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d]
))/(2*Sqrt[b])))/(a*d)))/(a*d^2)))/(4*a)))/(8*a)))/(12*a)))/(16*a)))/Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.52

method	result
risch	$-\frac{2(-25b\,x^2+a)\sqrt{(b\,x^2+a)^2}}{5a^6\sqrt{dx}\,x^2d^3(b\,x^2+a)}+\frac{b^2\left(\frac{\frac{5599a^3d^6(dx)^{\frac{3}{2}}}{1024}+\frac{14145a^2d^4b(dx)^{\frac{7}{2}}}{1024}+\frac{12357a\,d^2b^2(dx)^{\frac{11}{2}}}{1024}+\frac{3683b^3(dx)^{\frac{15}{2}}}{1024}}{(b\,d^2x^2+a\,d^2)^4}+\frac{13923\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{a \cdots}{\cdots}\right.}{dx+\left(\frac{a \cdots}{\cdots}\right.}\cdots\right.\right.}{\cdots}\cdots\right)}{a^6d^3(b\,x^2+\cdots}$
default	Expression too large to display

input
```
int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-25*b*x^2+a)/a^6/(d*x)^(1/2)/x^2/d^3*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1
/a^6*b^2*(2*(5599/2048*a^3*d^6*(d*x)^(3/2)+14145/2048*a^2*d^4*b*(d*x)^(7/2
)+12357/2048*a*d^2*b^2*(d*x)^(11/2)+3683/2048*b^3*(d*x)^(15/2))/(b*d^2*x^2
+a*d^2)^4+13923/8192/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln((d*x-(a*d^2/b)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a*d^2/b)^(1/2)))+2*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+2*arct
an(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)))/d^3*((b*x^2+a)^2)^(1/2)/(b*x^2
+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.07

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{69615\left(a^6b^4d^4x^{11}+4\,a^7b^3d^4x^9+6\,a^8b^2d^4x^7+4\,a^9bd^4x^5+a^{10}d^4x^3\right)(}{}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas
")
```

output
```
1/20480*(69615*(a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 +
 4*a^9*b*d^4*x^5 + a^10*d^4*x^3)*(-b^5/(a^25*d^14))^(1/4)*log(269897256146
7*a^19*d^11*(-b^5/(a^25*d^14))^(3/4) + 2698972561467*sqrt(d*x)*b^4) - 6961
5*(I*a^6*b^4*d^4*x^11 + 4*I*a^7*b^3*d^4*x^9 + 6*I*a^8*b^2*d^4*x^7 + 4*I*a^
9*b*d^4*x^5 + I*a^10*d^4*x^3)*(-b^5/(a^25*d^14))^(1/4)*log(2698972561467*I
*a^19*d^11*(-b^5/(a^25*d^14))^(3/4) + 2698972561467*sqrt(d*x)*b^4) - 69615
*(-I*a^6*b^4*d^4*x^11 - 4*I*a^7*b^3*d^4*x^9 - 6*I*a^8*b^2*d^4*x^7 - 4*I*a^
9*b*d^4*x^5 - I*a^10*d^4*x^3)*(-b^5/(a^25*d^14))^(1/4)*log(-2698972561467*
I*a^19*d^11*(-b^5/(a^25*d^14))^(3/4) + 2698972561467*sqrt(d*x)*b^4) - 6961
5*(a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^8*b^2*d^4*x^7 + 4*a^9*b*d^4*
x^5 + a^10*d^4*x^3)*(-b^5/(a^25*d^14))^(1/4)*log(-2698972561467*a^19*d^11*
(-b^5/(a^25*d^14))^(3/4) + 2698972561467*sqrt(d*x)*b^4) + 4*(69615*b^5*x^1
0 + 264537*a*b^4*x^8 + 369733*a^2*b^3*x^6 + 220507*a^3*b^2*x^4 + 43008*a^4
*b*x^2 - 2048*a^5)*sqrt(d*x))/(a^6*b^4*d^4*x^11 + 4*a^7*b^3*d^4*x^9 + 6*a^
8*b^2*d^4*x^7 + 4*a^9*b*d^4*x^5 + a^10*d^4*x^3)
```

Sympy [F]

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \int \frac{1}{(dx)^{\frac{7}{2}}\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral(1/((d*x)**(7/2)*((a + b*x**2)**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{7/2}(a^2+2abx^2+b^2x^4)^{5/2}}\,dx = \int \frac{1}{(b^2x^4+2\,abx^2+a^2)^{\frac{5}{2}}(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima
")
```

output
```
-4*b*integrate(1/((a^5*b*d^(7/2)*x^2 + a^6*d^(7/2))*x^(3/2)), x) + 1/3072*
(11049*b^5*x^(15/2) + 27135*a*b^4*x^(11/2) + 23395*a^2*b^3*x^(7/2) + 6925*
a^3*b^2*x^(3/2))/(a^6*b^4*d^(7/2)*x^8 + 4*a^7*b^3*d^(7/2)*x^6 + 6*a^8*b^2*
d^(7/2)*x^4 + 4*a^9*b*d^(7/2)*x^2 + a^10*d^(7/2)) + 1/192*((621*b^6*x^5 +
1042*a*b^5*x^3 + 453*a^2*b^4*x)*x^(9/2) + 2*(695*a*b^5*x^5 + 1182*a^2*b^4*
x^3 + 519*a^3*b^3*x)*x^(5/2) + (801*a^2*b^4*x^5 + 1386*a^3*b^3*x^3 + 617*a
^4*b^2*x)*sqrt(x))/(a^8*b^3*d^(7/2)*x^6 + 3*a^9*b^2*d^(7/2)*x^4 + 3*a^10*b
*d^(7/2)*x^2 + a^11*d^(7/2) + (a^5*b^6*d^(7/2)*x^6 + 3*a^6*b^5*d^(7/2)*x^4
 + 3*a^7*b^4*d^(7/2)*x^2 + a^8*b^3*d^(7/2))*x^6 + 3*(a^6*b^5*d^(7/2)*x^6 +
 3*a^7*b^4*d^(7/2)*x^4 + 3*a^8*b^3*d^(7/2)*x^2 + a^9*b^2*d^(7/2))*x^4 + 3*
(a^7*b^4*d^(7/2)*x^6 + 3*a^8*b^3*d^(7/2)*x^4 + 3*a^9*b^2*d^(7/2)*x^2 + a^1
0*b*d^(7/2))*x^2) + 3683/8192*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a
^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*s
qrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
- 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)
) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^
(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(1/4)*b^(3/4)))/(a^6*d^(7/2)) + integrate(1/((a^4*b*d^(7/2)*
x^2 + a^5*d^(7/2))*x^(7/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.79

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \frac{13923\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^7bd^5\mathrm{sgn}\left(bx^2+a\right)}$$
$$+\frac{13923\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^7bd^5\mathrm{sgn}\left(bx^2+a\right)}$$
$$-\frac{13923\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\log\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^7bd^5\mathrm{sgn}\left(bx^2+a\right)}$$
$$+\frac{13923\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\log\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^7bd^5\mathrm{sgn}\left(bx^2+a\right)}$$
$$+\frac{3683\,\sqrt{dx}b^5d^7x^7+12357\,\sqrt{dx}ab^4d^7x^5+14145\,\sqrt{dx}a^2b^3d^7x^3+5599\,\sqrt{dx}a^3b^2d^7x}{1024\left(bd^2x^2+ad^2\right)^4a^6d^3\mathrm{sgn}\left(bx^2+a\right)}$$
$$+\frac{2\left(25\,bd^2x^2-ad^2\right)}{5\,\sqrt{dx}a^6d^5x^2\mathrm{sgn}\left(bx^2+a\right)}$$

input
```
integrate(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
13923/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b*d^5*sgn(b*x^2 + a)) + 13923/
4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4
) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b*d^5*sgn(b*x^2 + a)) - 13923/8192*
sqrt(2)*(a*b^3*d^2)^(3/4)*log(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sq
rt(a*d^2/b))/(a^7*b*d^5*sgn(b*x^2 + a)) + 13923/8192*sqrt(2)*(a*b^3*d^2)^(
3/4)*log(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*b*d
^5*sgn(b*x^2 + a)) + 1/1024*(3683*sqrt(d*x)*b^5*d^7*x^7 + 12357*sqrt(d*x)*
a*b^4*d^7*x^5 + 14145*sqrt(d*x)*a^2*b^3*d^7*x^3 + 5599*sqrt(d*x)*a^3*b^2*d
^7*x)/((b*d^2*x^2 + a*d^2)^4*a^6*d^3*sgn(b*x^2 + a)) + 2/5*(25*b*d^2*x^2 -
 a*d^2)/(sqrt(d*x)*a^6*d^5*x^2*sgn(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\int \frac{1}{(d\,x)^{7/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}}\,dx$$

input
```
int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)),x)
```

output
```
int(1/((d*x)^(7/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 891, normalized size of antiderivative = 1.64

$$\int \frac{1}{(dx)^{7/2}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output

```
(sqrt(d)*( - 139230*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1
/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*x**2
- 556920*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*x**4 - 835380
*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*x**6 - 556920*sqrt(x)
*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqr
t(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 - 139230*sqrt(x)*b**(1/4)*a
**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(
1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 139230*sqrt(x)*b**(1/4)*a**(3/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4
)*sqrt(2)))*a**4*b*x**2 + 556920*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
*a**3*b**2*x**4 + 835380*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*
*3*x**6 + 556920*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*x**8 + 1
39230*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**5*x**10 + 69615*sqrt(x)
*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + s...
```

3.666 $\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 298

$$\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \frac{3x(a+bx^2)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}} + \frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}\cdots}{1-\sqrt{3}\cdots}\right.\right.}{5b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}$$

output
```
3/5*x*(b*x^2+a)/b/(b^2*x^4+2*a*b*x^2+a^2)^(1/3)+3/5*3^(3/4)*(1/2*6^(1/2)-1
/2*2^(1/2))*a^2*(1+b*x^2/a)^(2/3)*(1-(1+b*x^2/a)^(1/3))*((1+(1+b*x^2/a)^(1
/3)+(1+b*x^2/a)^(2/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3))^2)^(1/2)*EllipticF((1
+3^(1/2)-(1+b*x^2/a)^(1/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3)),2*I-I*3^(1/2))/b
^2/x/(b^2*x^4+2*a*b*x^2+a^2)^(1/3)/(-(1-(1+b*x^2/a)^(1/3))/(1-3^(1/2)-(1+b
*x^2/a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.21

$$\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \frac{3x\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{5b\sqrt[3]{(a+bx^2)^2}}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3),x]
```

output
```
(3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2,
 -((b*x^2)/a)]))/(5*b*((a + b*x^2)^2)^(1/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1385, 262, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$$

$$\Big\downarrow \text{1385}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\int \frac{x^2}{\left(\frac{bx^2}{a}+1\right)^{2/3}}dx}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\left(\frac{3ax\sqrt[3]{\frac{bx^2}{a}+1}}{5b}-\frac{3a\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{2/3}}dx}{5b}\right)}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 234}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\left(\frac{3ax\sqrt[3]{\frac{bx^2}{a}+1}}{5b}-\frac{9a^2\sqrt{\frac{bx^2}{a}}\int \frac{1}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}}{10b^2x}\right)}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\left(\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right),-7+4\sqrt{\ldots}\right.}{5b^2x\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}\right.}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3),x]
```

output
```
((1 + (b*x^2)/a)^(2/3)*((3*a*x*(1 + (b*x^2)/a)^(1/3))/(5*b) + (3*3^(3/4)*S
qrt[2 - Sqrt[3]]*a^2*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)
^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*E
llipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 +
(b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*Sqrt[-((1 - (1 + (b*x^2)/a)^
(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])))/(a^2 + 2*a*b*x^2 + b^2
*x^4)^(1/3)
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{x^2}{(b^2x^4 + 2ab\,x^2 + a^2)^{\frac{1}{3}}}dx$$

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

output
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{x^2}{(b^2x^4+2\,abx^2+a^2)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="fricas")
```

output
```
integral(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{x^2}{\sqrt[3]{(a+bx^2)^2}}\,dx$$

input
```
integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

output
```
Integral(x**2/((a + b*x**2)**2)**(1/3), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{x^2}{(b^2x^4+2\,abx^2+a^2)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{x^2}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="giac")
```

output
```
integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{x^2}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{1/3}}\,dx$$

input
```
int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3),x)
```

output
```
int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{x^2}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{1}{3}}}dx$$

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

output
```
int(x**2/(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3),x)
```

3.667 $\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 256

$$\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = -\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{\cdots}}{1-\sqrt{3}-\sqrt[3]{\cdots}}\right.\right.}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}$$

output
```
-3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a*(1+b*x^2/a)^(2/3)*(1-(1+b*x^2/a)^(1/3
))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3))^
2)^(1/2)*EllipticF((1+3^(1/2)-(1+b*x^2/a)^(1/3))/(1-3^(1/2)-(1+b*x^2/a)^(1
/3)),2*I-I*3^(1/2))/b/x/(b^2*x^4+2*a*b*x^2+a^2)^(1/3)/(-(1-(1+b*x^2/a)^(1/
3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.19

$$\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx=\frac{x\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{(a+bx^2)^2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/3),x]
```

output
```
(x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(
(a + b*x^2)^2)^(1/3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1385, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \; 1385$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{2/3}}dx}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 234$$

$$\frac{3a\sqrt{\frac{bx^2}{a}}\left(\frac{bx^2}{a}+1\right)^{2/3}\int \frac{1}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}}{2bx\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \; 760$$

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}a\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}}\right.\right.}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/3),x]
```

output
```
-((3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^
(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[
3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2
)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*
(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 -
Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]))
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{1}{\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

output
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{1}{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$$

input
```
integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a^2 + 2abx^2 + b^2x^4}}\,dx = \int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a^2 + 2abx^2 + b^2x^4}}\,dx = \int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a^2 + 2abx^2 + b^2x^4}}\,dx = \int \frac{1}{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^{1/3}}\,dx$$

input
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3),x)
```

output
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

output
```
int(1/(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3),x)
```

3.668 $\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 289

$$\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = -\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}} + \frac{\sqrt{2-\sqrt{3}}\left(1+\frac{bx^2}{a}\right)^{2/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1+\cdots}}{1-\sqrt{3}-\sqrt[3]{1+\cdots}}\right.\right.}{\sqrt[4]{3}x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}$$

output
```
-(b*x^2+a)/a/x/(b^2*x^4+2*a*b*x^2+a^2)^(1/3)+1/3*(1/2*6^(1/2)-1/2*2^(1/2))
*(1+b*x^2/a)^(2/3)*(1-(1+b*x^2/a)^(1/3))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2/a)
^(2/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1+b*x
^2/a)^(1/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3)),2*I-I*3^(1/2))*3^(3/4)/x/(b^2*x
^4+2*a*b*x^2+a^2)^(1/3)/(-(1-(1+b*x^2/a)^(1/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/
3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[3]{(a+bx^2)^2}}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)),x]
```

output
```
-(((1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/2, 2/3, 1/2, -((b*x^2)/a)])/
(x*((a + b*x^2)^2)^(1/3)))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1385, 264, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$$

$$\Big\downarrow \text{ 1385}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\int \frac{1}{x^2\left(\frac{bx^2}{a}+1\right)^{2/3}}dx}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\left(-\frac{b\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{2/3}}dx}{3a}-\frac{\sqrt[3]{\frac{bx^2}{a}+1}}{x}\right)}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 234$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\left(-\frac{\sqrt{\frac{bx^2}{a}}\int \frac{1}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}}{2x}-\frac{\sqrt[3]{\frac{bx^2}{a}+1}}{x}\right)}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \ 760$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\left(\frac{\sqrt{2-\sqrt{3}}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}-\cdots\right)}{\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)),x]
```

output
```
((1 + (b*x^2)/a)^(2/3)*(-((1 + (b*x^2)/a)^(1/3)/x) + (Sqrt[2 - Sqrt[3]]*(1
 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a
)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sq
rt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7
+ 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] -
 (1 + (b*x^2)/a)^(1/3))^2)])))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{1}{x^2\,(b^2x^4+2ab\,x^2+a^2)^{\frac{1}{3}}}dx$$

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

output
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2)
, x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{x^2\sqrt[3]{(a+bx^2)^2}}\,dx$$

input
```
integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

output
```
Integral(1/(x**2*((a + b*x**2)**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx = \int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{1}{x^2\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{1/3}}\,dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{1}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{1}{3}}\,x^2}dx$$

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

output
```
int(1/((a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3)*x**2),x)
```

3.669 $\int \frac{x^2}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 618

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx = -\frac{3x(a+bx^2)}{2b\,(a^2+2abx^2+b^2x^4)^{2/3}} - \frac{9ax\left(1+\frac{bx^2}{a}\right)^{4/3}}{2b\,(a^2+2abx^2+b^2x^4)^{2/3}\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)}$$

$$+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^2\left(1+\frac{bx^2}{a}\right)^{4/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}{1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}\right.\right.}{4b^2x\,(a^2+2abx^2+b^2x^4)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}$$

$$-\frac{3\,3^{3/4}a^2\left(1+\frac{bx^2}{a}\right)^{4/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}{1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}\right.\right.}{\sqrt{2}b^2x\,(a^2+2abx^2+b^2x^4)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}$$

output
```
-3/2*x*(b*x^2+a)/b/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)-9/2*a*x*(1+b*x^2/a)^(4/3)
/b/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/(1-3^(1/2)-(1+b*x^2/a)^(1/3))+9/4*3^(1/4)
*(1/2*6^(1/2)+1/2*2^(1/2))*a^2*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*((1
+(1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3))^2)^(1/
2)*EllipticE((1+3^(1/2)-(1+b*x^2/a)^(1/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3)),2
*I-I*3^(1/2))/b^2/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/(-(1-(1+b*x^2/a)^(1/3))/
(1-3^(1/2)-(1+b*x^2/a)^(1/3))^2)^(1/2)-3/2*3^(3/4)*a^2*(1+b*x^2/a)^(4/3)*(
1-(1+b*x^2/a)^(1/3))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-3^(1/2)-(
1+b*x^2/a)^(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1+b*x^2/a)^(1/3))/(1-3^(1
/2)-(1+b*x^2/a)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/b^2/x/(b^2*x^4+2*a*b*x^2+a^2
)^(2/3)/(-(1-(1+b*x^2/a)^(1/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.10

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx=\frac{3x(a+bx^2)\left(-1+\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{2b\left((a+bx^2)^2\right)^{2/3}}$$

input
```
Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3),x]
```

output
```
(3*x*(a + b*x^2)*(-1 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3
/2, -((b*x^2)/a)]))/(2*b*((a + b*x^2)^2)^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 582, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1385, 252, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx$$

$$\Big\downarrow \ 1385$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\int \frac{x^2}{\left(\frac{bx^2}{a}+1\right)^{4/3}}dx}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$\Big\downarrow \ 252$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{3a\int \frac{1}{\sqrt[3]{\frac{bx^2}{a}+1}}dx}{2b}-\frac{3ax}{2b\sqrt[3]{\frac{bx^2}{a}+1}}\right)}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$\Big\downarrow \ 233$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{9a^2\sqrt{\frac{bx^2}{a}}\int \frac{\sqrt[3]{\frac{bx^2}{a}+1}}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}}{4b^2x}-\frac{3ax}{2b\sqrt[3]{\frac{bx^2}{a}+1}}\right)}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$\Big\downarrow \ 833$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{9a^2\sqrt{\frac{bx^2}{a}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}-\int \frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}\right)}{4b^2x}-\frac{3ax}{2b\sqrt[3]{\frac{bx^2}{a}+1}}\right)}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$\Big\downarrow \ 760$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{9a^2\sqrt{\frac{bx^2}{a}}\left(-\int \frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1\cdots}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\cdots}{\sqrt[4]{3}\sqrt{\frac{bx^2}{a}}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}\cdots}}{\left(-\sqrt[3]{\frac{bx^2}{a}+\cdots}\right)^{\cdots}}}\cdots}\right)}{4b^2x}\cdots\right)}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}\cdots}$$

$\Big\downarrow$ 2418

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{9a^2\sqrt{\frac{bx^2}{a}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right),\cdots\right)}{\sqrt[4]{3}\sqrt{\frac{bx^2}{a}}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}\cdots\right)}{\cdots}\cdots\right)}{\left(\cdots\right.}$$

input
```
Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3),x]
```

output
```
((1 + (b*x^2)/a)^(4/3)*((-3*a*x)/(2*b*(1 + (b*x^2)/a)^(1/3)) + (9*a^2*Sqrt
[(b*x^2)/a]*((-2*Sqrt[(b*x^2)/a])/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3)) +
(3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x
^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3)
)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] -
 (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[(b*x^2)/a]*Sqrt[-((1 - (1
 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) - (2*Sqrt[
2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x
^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3)
)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] -
 (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[(b*x^2)/a]*Sqrt[-
((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])))/
(4*b^2*x)))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^2}{(b^2x^4 + 2ab\,x^2 + a^2)^{\frac{2}{3}}}dx$$

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

output
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

Fricas [F]

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx = \int \frac{x^2}{(b^2x^4+2\,abx^2+a^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="fricas")
```

output
```
integral(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

Sympy [F]

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx = \int \frac{x^2}{\left((a+bx^2)^2\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

output
```
Integral(x**2/((a + b*x**2)**2)**(2/3), x)
```

Maxima [F]

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx = \int \frac{x^2}{(b^2x^4+2\,abx^2+a^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

Giac [F]

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx = \int \frac{x^2}{(b^2x^4+2\,abx^2+a^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="giac")
```

output
```
integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx = \int \frac{x^2}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{2/3}}\,dx$$

input
```
int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3),x)
```

output
```
int(x^2/(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx = \int \frac{x^2}{(b^2x^4+2ab\,x^2+a^2)^{\frac{2}{3}}}dx$$

input
```
int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

output
```
int(x**2/(a**2 + 2*a*b*x**2 + b**2*x**4)**(2/3),x)
```

3.670 $\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 609

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx = \frac{3x(a+bx^2)}{2a\,(a^2+2abx^2+b^2x^4)^{2/3}} + \frac{3x\left(1+\frac{bx^2}{a}\right)^{4/3}}{2\,(a^2+2abx^2+b^2x^4)^{2/3}\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)} - \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}a\left(1+\frac{bx^2}{a}\right)^{4/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}{1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}\right)\right.}{4bx\,(a^2+2abx^2+b^2x^4)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}} + \frac{3^{3/4}a\left(1+\frac{bx^2}{a}\right)^{4/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}{1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}\right),\right.}{\sqrt{2}bx\,(a^2+2abx^2+b^2x^4)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}$$

output
```
3/2*x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)+3/2*x*(1+b*x^2/a)^(4/3)/(b
^2*x^4+2*a*b*x^2+a^2)^(2/3)/(1-3^(1/2)-(1+b*x^2/a)^(1/3))-3/4*3^(1/4)*(1/2
*6^(1/2)+1/2*2^(1/2))*a*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*((1+(1+b*x
^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3))^2)^(1/2)*Elli
pticE((1+3^(1/2)-(1+b*x^2/a)^(1/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3)),2*I-I*3^
(1/2))/b/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/(-(1-(1+b*x^2/a)^(1/3))/(1-3^(1/2
)-(1+b*x^2/a)^(1/3))^2)^(1/2)+1/2*3^(3/4)*a*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/
a)^(1/3))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2/a)^(2/3))/(1-3^(1/2)-(1+b*x^2/a)^
(1/3))^2)^(1/2)*EllipticF((1+3^(1/2)-(1+b*x^2/a)^(1/3))/(1-3^(1/2)-(1+b*x^
2/a)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/b/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/(-(1-
(1+b*x^2/a)^(1/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.86 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.11

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx = -\frac{x(a+bx^2)\left(-3+\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{2a\left((a+bx^2)^2\right)^{2/3}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2/3),x]
```

output
```
-1/2*(x*(a + b*x^2)*(-3 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2
, 3/2, -((b*x^2)/a)]))/(a*((a + b*x^2)^2)^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1385, 215, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{2/3}}dx$$

$$\Big\downarrow \text{1385}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{4/3}}dx}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$\Big\downarrow \text{215}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{3x}{2\sqrt[3]{\frac{bx^2}{a}+1}}-\frac{1}{2}\int \frac{1}{\sqrt[3]{\frac{bx^2}{a}+1}}dx\right)}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$\Big\downarrow \text{233}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{3x}{2\sqrt[3]{\frac{bx^2}{a}+1}}-\frac{3a\sqrt{\frac{bx^2}{a}}\int \frac{\sqrt[3]{\frac{bx^2}{a}+1}}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}}{4bx}\right)}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$\Big\downarrow \text{833}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{3x}{2\sqrt[3]{\frac{bx^2}{a}+1}}-\frac{3a\sqrt{\frac{bx^2}{a}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}-\int\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}\right)}{4bx}\right)}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$\downarrow$ 760

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{3x}{2\sqrt[3]{\frac{bx^2}{a}+1}}-\frac{3a\sqrt{\frac{bx^2}{a}}\left(-\int\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\cdots}{\left(-\sqrt[3]{\frac{bx^2}{a}\cdots}\right)\cdots}}\cdots}{\sqrt[4]{3}\sqrt{\frac{bx^2}{a}}\cdots}\cdots\right)}{4bx}\cdots\right)}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$\downarrow$ 2418

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{3x}{2\sqrt[3]{\frac{bx^2}{a}+1}}-\frac{3a\sqrt{\frac{bx^2}{a}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{\cdots}}{-\sqrt[3]{\cdots}}\cdots\right.\right.}{\sqrt[4]{3}\sqrt{\frac{bx^2}{a}}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\cdots}\cdots\right)}{\cdots}\cdots\right)}{(a\cdots}$$

input

```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2/3),x]
```

output

```
((1 + (b*x^2)/a)^(4/3)*((3*x)/(2*(1 + (b*x^2)/a)^(1/3)) - (3*a*Sqrt[(b*x^2
)/a]*((-2*Sqrt[(b*x^2)/a])/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3)) + (3^(1/4
)*Sqrt[2 + Sqrt[3]]*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^
(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*El
lipticE[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (
b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[(b*x^2)/a]*Sqrt[-((1 - (1 + (b*x
^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) - (2*Sqrt[2 - Sqr
t[3]]*(1 + Sqrt[3])*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^
(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*El
lipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (
b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[(b*x^2)/a]*Sqrt[-((1 - (
1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])))/(4*b*x)
))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{\left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{\frac{2}{3}}}d x$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

output
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx=\int \frac{1}{\left(b^{2} x^{4}+2\,a b x^{2}+a^{2}\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

Sympy [F]

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{2/3}}\,dx = \int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{2/3}}\,dx = \int \frac{1}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{2/3}}\,dx = \int \frac{1}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx=\int \frac{1}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{2/3}}\,dx$$

input
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3),x)
```

output
```
int(1/(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx=\int \frac{1}{\left(b^2x^4+2ab\,x^2+a^2\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

output
```
int(1/(a**2 + 2*a*b*x**2 + b**2*x**4)**(2/3),x)
```

3.671 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^{2/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 649

$$\int \frac{1}{x^2 \left(a^2+2abx^2+b^2x^4\right)^{2/3}}\,dx = \frac{3(a+bx^2)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{5(a+bx^2)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}} - \frac{5bx\left(1+\frac{bx^2}{a}\right)^{4/3}}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)}$$

$$+\frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1+\frac{bx^2}{a}\right)^{4/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}{1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}\right)\right.}{4x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}$$

$$-\frac{5\left(1+\frac{bx^2}{a}\right)^{4/3}\left(1-\sqrt[3]{1+\frac{bx^2}{a}}\right)\sqrt{\frac{1+\sqrt[3]{1+\frac{bx^2}{a}}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}{1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}}\right),-7\right.}{\sqrt{2}\sqrt[4]{3}x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{1+\frac{bx^2}{a}}}{\left(1-\sqrt{3}-\sqrt[3]{1+\frac{bx^2}{a}}\right)^2}}}$$

output
```
3/2*(b*x^2+a)/a/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)-5/2*(b*x^2+a)^2/a^2/x/(b^2
*x^4+2*a*b*x^2+a^2)^(2/3)-5/2*b*x*(1+b*x^2/a)^(4/3)/a/(b^2*x^4+2*a*b*x^2+a
^2)^(2/3)/(1-3^(1/2)-(1+b*x^2/a)^(1/3))+5/4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/
2))*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*((1+(1+b*x^2/a)^(1/3)+(1+b*x^2
/a)^(2/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3))^2)^(1/2)*EllipticE((1+3^(1/2)-(1+
b*x^2/a)^(1/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3)),2*I-I*3^(1/2))/x/(b^2*x^4+2*
a*b*x^2+a^2)^(2/3)/(-(1-(1+b*x^2/a)^(1/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3))^2
)^(1/2)-5/6*(1+b*x^2/a)^(4/3)*(1-(1+b*x^2/a)^(1/3))*((1+(1+b*x^2/a)^(1/3)+
(1+b*x^2/a)^(2/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3))^2)^(1/2)*EllipticF((1+3^(
1/2)-(1+b*x^2/a)^(1/3))/(1-3^(1/2)-(1+b*x^2/a)^(1/3)),2*I-I*3^(1/2))*2^(1/
2)*3^(3/4)/x/(b^2*x^4+2*a*b*x^2+a^2)^(2/3)/(-(1-(1+b*x^2/a)^(1/3))/(1-3^(1
/2)-(1+b*x^2/a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.55 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{2/3}} \, dx = -\frac{(a + bx^2) \sqrt[3]{1 + \frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{4}{3}, \frac{1}{2}, -\frac{bx^2}{a}\right)}{ax \left((a + bx^2)^2\right)^{2/3}}$$

input
```
Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)),x]
```

output
```
-(((a + b*x^2)*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-1/2, 4/3, 1/2, -((
b*x^2)/a)])/(a*x*((a + b*x^2)^2)^(2/3)))
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 598, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1385, 253, 264, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a^2+2abx^2+b^2x^4)^{2/3}}dx$$

$$\Big\downarrow \text{1385}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\int \frac{1}{x^2\left(\frac{bx^2}{a}+1\right)^{4/3}}dx}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$\Big\downarrow \text{253}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{5}{2}\int \frac{1}{x^2\sqrt[3]{\frac{bx^2}{a}+1}}dx+\frac{3}{2x\sqrt[3]{\frac{bx^2}{a}+1}}\right)}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$\Big\downarrow \text{264}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{5}{2}\left(\frac{b\int \frac{1}{\sqrt[3]{\frac{bx^2}{a}+1}}dx}{3a}-\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}}{x}\right)+\frac{3}{2x\sqrt[3]{\frac{bx^2}{a}+1}}\right)}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$$\Big\downarrow \text{233}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{5}{2}\left(\frac{\sqrt{\frac{bx^2}{a}}\int \frac{\sqrt[3]{\frac{bx^2}{a}+1}}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}}{2x}-\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}}{x}\right)+\frac{3}{2x\sqrt[3]{\frac{bx^2}{a}+1}}\right)}{(a^2+2abx^2+b^2x^4)^{2/3}}$$

$\downarrow$ 833

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{5}{2}\left(\frac{\sqrt{\frac{bx^2}{a}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}-\int \frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}\right)}{2x}-\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}}{x}\right)+\frac{3}{2x\sqrt[3]{\frac{bx^2}{a}\cdots}}\cdots\right)}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$\downarrow$ 760

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}\left(\frac{5}{2}\left(\frac{\sqrt{\frac{bx^2}{a}}\left(-\int \frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{\sqrt{\frac{bx^2}{a}}}d\sqrt[3]{\frac{bx^2}{a}+1}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+\cdots}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\cdots}{\sqrt[4]{3}\sqrt{\frac{bx^2}{a}}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}\cdots}}{\left(-\sqrt[3]{\frac{bx^2}{a}+\cdots}\right)\cdots}}\cdots}\cdots\right)}{2x}\cdots\right)\cdots\right)}{\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$\downarrow$ 2418

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/3}}{\left(\left(\frac{5}{2}\frac{\sqrt{\frac{bx^2}{a}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\right.}{\sqrt[4]{3}\sqrt{\frac{bx^2}{a}}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}\right.}{}\right.\right.}$$

input
```
Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)),x]
```

output
```
((1 + (b*x^2)/a)^(4/3)*(3/(2*x*(1 + (b*x^2)/a)^(1/3)) + (5*(-((1 + (b*x^2)
/a)^(2/3)/x) + (Sqrt[(b*x^2)/a]*((-2*Sqrt[(b*x^2)/a])/(1 - Sqrt[3] - (1 +
(b*x^2)/a)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 + (b*x^2)/a)^(1/3))
*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (
1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(
1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[(b*x^
2)/a]*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1
/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 + (b*x^2)/a)^(1/3))
*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (
1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(
1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sq
rt[(b*x^2)/a]*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^
2)/a)^(1/3))^2)])))/(2*x)))/2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b^{2} x^{4}+2 a b\, x^{2}+a^{2}\right)^{\frac{2}{3}}}d x$$

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

output
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a^2+2abx^2+b^2x^4\right)^{2/3}}\, dx=\int \frac{1}{{\left(b^{2} x^{4}+2 \, a b x^{2}+a^{2}\right)}^{\frac{2}{3}} x^{2}}\,{d x}$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2)
, x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{2/3}}\,dx = \int \frac{1}{x^2 \left((a + bx^2)^2\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

output
```
Integral(1/(x**2*((a + b*x**2)**2)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{2/3}}\,dx = \int \frac{1}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{2}{3}} x^2}\,dx$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{2/3}}\,dx = \int \frac{1}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{2}{3}} x^2}\,dx$$

input
```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{2/3}} \, dx = \int \frac{1}{x^2 (a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^{2/3}} \, dx$$

input
```
int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3)),x)
```

output
```
int(1/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{2/3}} \, dx = \int \frac{1}{(b^2x^4 + 2ab\,x^2 + a^2)^{\frac{2}{3}}\,x^2} dx$$

input
```
int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

output
```
int(1/((a**2 + 2*a*b*x**2 + b**2*x**4)**(2/3)*x**2),x)
```

3.672 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 150

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{a^6(dx)^{1+m}}{d(1+m)}+\frac{6a^5b(dx)^{3+m}}{d^3(3+m)}+\frac{15a^4b^2(dx)^{5+m}}{d^5(5+m)}+\frac{20a^3b^3(dx)^{7+m}}{d^7(7+m)}+\frac{15a^2b^4(dx)^{9+m}}{d^9(9+m)}+\frac{6ab^5(dx)^{11+m}}{d^{11}(11+m)}+\frac{b^6(dx)^{13+m}}{d^{13}(13+m)}$$

output
```
a^6*(d*x)^(1+m)/d/(1+m)+6*a^5*b*(d*x)^(3+m)/d^3/(3+m)+15*a^4*b^2*(d*x)^(5+
m)/d^5/(5+m)+20*a^3*b^3*(d*x)^(7+m)/d^7/(7+m)+15*a^2*b^4*(d*x)^(9+m)/d^9/(
9+m)+6*a*b^5*(d*x)^(11+m)/d^11/(11+m)+b^6*(d*x)^(13+m)/d^13/(13+m)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.70

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = x(dx)^m \left(\frac{a^6}{1+m}+\frac{6a^5bx^2}{3+m}+\frac{15a^4b^2x^4}{5+m}+\frac{20a^3b^3x^6}{7+m}+\frac{15a^2b^4x^8}{9+m}+\frac{6ab^5x^{10}}{11+m}+\frac{b^6x^{12}}{13+m}\right)$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
x*(d*x)^m*(a^6/(1 + m) + (6*a^5*b*x^2)/(3 + m) + (15*a^4*b^2*x^4)/(5 + m)
+ (20*a^3*b^3*x^6)/(7 + m) + (15*a^2*b^4*x^8)/(9 + m) + (6*a*b^5*x^10)/(11
 + m) + (b^6*x^12)/(13 + m))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^2+b^2x^4\right)^3 (dx)^m \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int b^6 (dx)^m \left(bx^2+a\right)^6 dx}{b^6}$$

$$\Big\downarrow \text{ 27}$$

$$\int \left(a+bx^2\right)^6 (dx)^m dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^6 (dx)^m + \frac{6a^5 b (dx)^{m+2}}{d^2} + \frac{15a^4b^2(dx)^{m+4}}{d^4} + \frac{20a^3b^3(dx)^{m+6}}{d^6} + \frac{15a^2b^4(dx)^{m+8}}{d^8} + \frac{6ab^5(dx)^{m+10}}{d^{10}} + \frac{b^6(dx)^{r}}{d^{12}}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^6(dx)^{m+1}}{d(m+1)} + \frac{6a^5b(dx)^{m+3}}{d^3(m+3)} + \frac{15a^4b^2(dx)^{m+5}}{d^5(m+5)} + \frac{20a^3b^3(dx)^{m+7}}{d^7(m+7)} + \frac{15a^2b^4(dx)^{m+9}}{d^9(m+9)} + \frac{6ab^5(dx)^{m+11}}{d^{11}(m+11)} + \frac{b^6(dx)^{m+13}}{d^{13}(m+13)}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(a^6*(d*x)^(1 + m))/(d*(1 + m)) + (6*a^5*b*(d*x)^(3 + m))/(d^3*(3 + m)) +
(15*a^4*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (20*a^3*b^3*(d*x)^(7 + m))/(d^7
*(7 + m)) + (15*a^2*b^4*(d*x)^(9 + m))/(d^9*(9 + m)) + (6*a*b^5*(d*x)^(11
+ m))/(d^11*(11 + m)) + (b^6*(d*x)^(13 + m))/(d^13*(13 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(150) = 300$.

Time = 0.45 (sec) , antiderivative size = 602, normalized size of antiderivative = 4.01

method	result
gosper	$\frac{(dx)^m \left(b^6m^6x^{12}+36b^6m^5x^{12}+6a\,b^5m^6x^{10}+505b^6m^4x^{12}+228a\,b^5m^5x^{10}+3480b^6m^3x^{12}+15a^2b^4m^6x^8+3330a\,b^5m^4x^{10}+12\right.}{}$
risch	$\frac{(dx)^m \left(b^6m^6x^{12}+36b^6m^5x^{12}+6a\,b^5m^6x^{10}+505b^6m^4x^{12}+228a\,b^5m^5x^{10}+3480b^6m^3x^{12}+15a^2b^4m^6x^8+3330a\,b^5m^4x^{10}+12\right.}{}$
orering	$\frac{\left(b^6m^6x^{12}+36b^6m^5x^{12}+6a\,b^5m^6x^{10}+505b^6m^4x^{12}+228a\,b^5m^5x^{10}+3480b^6m^3x^{12}+15a^2b^4m^6x^8+3330a\,b^5m^4x^{10}+12139b^6\right.}{}$
parallelrisch	$\frac{6x^{11}(dx)^m a\,b^5m^6+228x^{11}(dx)^m a\,b^5m^5+3330x^{11}(dx)^m a\,b^5m^4+15x^9(dx)^m a^2b^4m^6+23640x^{11}(dx)^m a\,b^5m^3+600x^9(dx)^m}{}$

input
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(d*x)^m*(b^6*m^6*x^12+36*b^6*m^5*x^12+6*a*b^5*m^6*x^10+505*b^6*m^4*x^12+22
8*a*b^5*m^5*x^10+3480*b^6*m^3*x^12+15*a^2*b^4*m^6*x^8+3330*a*b^5*m^4*x^10+
12139*b^6*m^2*x^12+600*a^2*b^4*m^5*x^8+23640*a*b^5*m^3*x^10+19524*b^6*m*x^
12+20*a^3*b^3*m^6*x^6+9195*a^2*b^4*m^4*x^8+84234*a*b^5*m^2*x^10+10395*b^6*
x^12+840*a^3*b^3*m^5*x^6+67920*a^2*b^4*m^3*x^8+137412*a*b^5*m*x^10+15*a^4*
b^2*m^6*x^4+13580*a^3*b^3*m^4*x^6+249405*a^2*b^4*m^2*x^8+73710*a*b^5*x^10+
660*a^4*b^2*m^5*x^4+105840*a^3*b^3*m^3*x^6+415320*a^2*b^4*m*x^8+6*a^5*b*m^
6*x^2+11295*a^4*b^2*m^4*x^4+406700*a^3*b^3*m^2*x^6+225225*a^2*b^4*x^8+276*
a^5*b*m^5*x^2+94200*a^4*b^2*m^3*x^4+699720*a^3*b^3*m*x^6+a^6*m^6+5010*a^5*
b*m^4*x^2+389685*a^4*b^2*m^2*x^4+386100*a^3*b^3*x^6+48*a^6*m^5+45240*a^5*b
*m^3*x^2+711540*a^4*b^2*m*x^4+925*a^6*m^4+208554*a^5*b*m^2*x^2+405405*a^4*
b^2*x^4+9120*a^6*m^3+438324*a^5*b*m*x^2+48259*a^6*m^2+270270*a^5*b*x^2+129
072*a^6*m+135135*a^6)*x/(13+m)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(150) = 300$.

Time = 0.10 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.38

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right)^3 \, dx$$
$$= \frac{((b^6m^6 + 36\, b^6m^5 + 505\, b^6m^4 + 3480\, b^6m^3 + 12139\, b^6m^2 + 19524\, b^6m + 10395\, b^6)x^{13} + 6\, (ab^5m^6 + 38\, a}{}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
((b^6*m^6 + 36*b^6*m^5 + 505*b^6*m^4 + 3480*b^6*m^3 + 12139*b^6*m^2 + 1952
4*b^6*m + 10395*b^6)*x^13 + 6*(a*b^5*m^6 + 38*a*b^5*m^5 + 555*a*b^5*m^4 +
3940*a*b^5*m^3 + 14039*a*b^5*m^2 + 22902*a*b^5*m + 12285*a*b^5)*x^11 + 15*
(a^2*b^4*m^6 + 40*a^2*b^4*m^5 + 613*a^2*b^4*m^4 + 4528*a^2*b^4*m^3 + 16627
*a^2*b^4*m^2 + 27688*a^2*b^4*m + 15015*a^2*b^4)*x^9 + 20*(a^3*b^3*m^6 + 42
*a^3*b^3*m^5 + 679*a^3*b^3*m^4 + 5292*a^3*b^3*m^3 + 20335*a^3*b^3*m^2 + 34
986*a^3*b^3*m + 19305*a^3*b^3)*x^7 + 15*(a^4*b^2*m^6 + 44*a^4*b^2*m^5 + 75
3*a^4*b^2*m^4 + 6280*a^4*b^2*m^3 + 25979*a^4*b^2*m^2 + 47436*a^4*b^2*m + 2
7027*a^4*b^2)*x^5 + 6*(a^5*b*m^6 + 46*a^5*b*m^5 + 835*a^5*b*m^4 + 7540*a^5
*b*m^3 + 34759*a^5*b*m^2 + 73054*a^5*b*m + 45045*a^5*b)*x^3 + (a^6*m^6 + 4
8*a^6*m^5 + 925*a^6*m^4 + 9120*a^6*m^3 + 48259*a^6*m^2 + 129072*a^6*m + 13
5135*a^6)*x)*(d*x)^m/(m^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177
331*m^2 + 264207*m + 135135)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3104 vs. $2(138) = 276$.

Time = 0.82 (sec) , antiderivative size = 3104, normalized size of antiderivative = 20.69

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Piecewise(((-a**6/(12*x**12) - 3*a**5*b/(5*x**10) - 15*a**4*b**2/(8*x**8)
- 10*a**3*b**3/(3*x**6) - 15*a**2*b**4/(4*x**4) - 3*a*b**5/x**2 + b**6*log
(x))/d**13, Eq(m, -13)), ((-a**6/(10*x**10) - 3*a**5*b/(4*x**8) - 5*a**4*b
**2/(2*x**6) - 5*a**3*b**3/x**4 - 15*a**2*b**4/(2*x**2) + 6*a*b**5*log(x)
+ b**6*x**2/2)/d**11, Eq(m, -11)), ((-a**6/(8*x**8) - a**5*b/x**6 - 15*a**
4*b**2/(4*x**4) - 10*a**3*b**3/x**2 + 15*a**2*b**4*log(x) + 3*a*b**5*x**2
+ b**6*x**4/4)/d**9, Eq(m, -9)), ((-a**6/(6*x**6) - 3*a**5*b/(2*x**4) - 15
*a**4*b**2/(2*x**2) + 20*a**3*b**3*log(x) + 15*a**2*b**4*x**2/2 + 3*a*b**5
*x**4/2 + b**6*x**6/6)/d**7, Eq(m, -7)), ((-a**6/(4*x**4) - 3*a**5*b/x**2
+ 15*a**4*b**2*log(x) + 10*a**3*b**3*x**2 + 15*a**2*b**4*x**4/4 + a*b**5*x
**6 + b**6*x**8/8)/d**5, Eq(m, -5)), ((-a**6/(2*x**2) + 6*a**5*b*log(x) +
15*a**4*b**2*x**2/2 + 5*a**3*b**3*x**4 + 5*a**2*b**4*x**6/2 + 3*a*b**5*x**
8/4 + b**6*x**10/10)/d**3, Eq(m, -3)), ((a**6*log(x) + 3*a**5*b*x**2 + 15*
a**4*b**2*x**4/4 + 10*a**3*b**3*x**6/3 + 15*a**2*b**4*x**8/8 + 3*a*b**5*x*
*10/5 + b**6*x**12/12)/d, Eq(m, -1)), (a**6*m**6*x*(d*x)**m/(m**7 + 49*m**
6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 48*a**6*m**5*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*
m**3 + 177331*m**2 + 264207*m + 135135) + 925*a**6*m**4*x*(d*x)**m/(m**7 +
 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 1
35135) + 9120*a**6*m**3*x*(d*x)**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.96

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3 \, dx = \frac{b^6 d^m x^{13} x^m}{m+13} + \frac{6\, a b^5 d^m x^{11} x^m}{m+11} + \frac{15\, a^2 b^4 d^m x^9 x^m}{m+9} + \frac{20\, a^3 b^3 d^m x^7 x^m}{m+7} + \frac{15\, a^4 b^2 d^m x^5 x^m}{m+5} + \frac{6\, a^5 b d^m x^3 x^m}{m+3} + \frac{(dx)^{m+1} a^6}{d(m+1)}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
b^6*d^m*x^13*x^m/(m + 13) + 6*a*b^5*d^m*x^11*x^m/(m + 11) + 15*a^2*b^4*d^m
*x^9*x^m/(m + 9) + 20*a^3*b^3*d^m*x^7*x^m/(m + 7) + 15*a^4*b^2*d^m*x^5*x^m
/(m + 5) + 6*a^5*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^6/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(150) = 300$.

Time = 0.15 (sec) , antiderivative size = 847, normalized size of antiderivative = 5.65

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
((d*x)^m*b^6*m^6*x^13 + 36*(d*x)^m*b^6*m^5*x^13 + 6*(d*x)^m*a*b^5*m^6*x^11
 + 505*(d*x)^m*b^6*m^4*x^13 + 228*(d*x)^m*a*b^5*m^5*x^11 + 3480*(d*x)^m*b^
6*m^3*x^13 + 15*(d*x)^m*a^2*b^4*m^6*x^9 + 3330*(d*x)^m*a*b^5*m^4*x^11 + 12
139*(d*x)^m*b^6*m^2*x^13 + 600*(d*x)^m*a^2*b^4*m^5*x^9 + 23640*(d*x)^m*a*b
^5*m^3*x^11 + 19524*(d*x)^m*b^6*m*x^13 + 20*(d*x)^m*a^3*b^3*m^6*x^7 + 9195
*(d*x)^m*a^2*b^4*m^4*x^9 + 84234*(d*x)^m*a*b^5*m^2*x^11 + 10395*(d*x)^m*b^
6*x^13 + 840*(d*x)^m*a^3*b^3*m^5*x^7 + 67920*(d*x)^m*a^2*b^4*m^3*x^9 + 137
412*(d*x)^m*a*b^5*m*x^11 + 15*(d*x)^m*a^4*b^2*m^6*x^5 + 13580*(d*x)^m*a^3*
b^3*m^4*x^7 + 249405*(d*x)^m*a^2*b^4*m^2*x^9 + 73710*(d*x)^m*a*b^5*x^11 +
660*(d*x)^m*a^4*b^2*m^5*x^5 + 105840*(d*x)^m*a^3*b^3*m^3*x^7 + 415320*(d*x
)^m*a^2*b^4*m*x^9 + 6*(d*x)^m*a^5*b*m^6*x^3 + 11295*(d*x)^m*a^4*b^2*m^4*x^
5 + 406700*(d*x)^m*a^3*b^3*m^2*x^7 + 225225*(d*x)^m*a^2*b^4*x^9 + 276*(d*x
)^m*a^5*b*m^5*x^3 + 94200*(d*x)^m*a^4*b^2*m^3*x^5 + 699720*(d*x)^m*a^3*b^3
*m*x^7 + (d*x)^m*a^6*m^6*x + 5010*(d*x)^m*a^5*b*m^4*x^3 + 389685*(d*x)^m*a
^4*b^2*m^2*x^5 + 386100*(d*x)^m*a^3*b^3*x^7 + 48*(d*x)^m*a^6*m^5*x + 45240
*(d*x)^m*a^5*b*m^3*x^3 + 711540*(d*x)^m*a^4*b^2*m*x^5 + 925*(d*x)^m*a^6*m^
4*x + 208554*(d*x)^m*a^5*b*m^2*x^3 + 405405*(d*x)^m*a^4*b^2*x^5 + 9120*(d*
x)^m*a^6*m^3*x + 438324*(d*x)^m*a^5*b*m*x^3 + 48259*(d*x)^m*a^6*m^2*x + 27
0270*(d*x)^m*a^5*b*x^3 + 129072*(d*x)^m*a^6*m*x + 135135*(d*x)^m*a^6*x)/(m
^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m +...
```

Mupad [B] (verification not implemented)

Time = 18.45 (sec) , antiderivative size = 540, normalized size of antiderivative = 3.60

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$$
$$= \frac{a^6\,x\,(d\,x)^m\,(m^6+48\,m^5+925\,m^4+9120\,m^3+48259\,m^2+129072\,m+135135)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$
$$+ \frac{b^6\,x^{13}\,(d\,x)^m\,(m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$
$$+ \frac{6\,a\,b^5\,x^{11}\,(d\,x)^m\,(m^6+38\,m^5+555\,m^4+3940\,m^3+14039\,m^2+22902\,m+12285)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$
$$+ \frac{6\,a^5\,b\,x^3\,(d\,x)^m\,(m^6+46\,m^5+835\,m^4+7540\,m^3+34759\,m^2+73054\,m+45045)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$
$$+ \frac{15\,a^2\,b^4\,x^9\,(d\,x)^m\,(m^6+40\,m^5+613\,m^4+4528\,m^3+16627\,m^2+27688\,m+15015)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$
$$+ \frac{20\,a^3\,b^3\,x^7\,(d\,x)^m\,(m^6+42\,m^5+679\,m^4+5292\,m^3+20335\,m^2+34986\,m+19305)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$
$$+ \frac{15\,a^4\,b^2\,x^5\,(d\,x)^m\,(m^6+44\,m^5+753\,m^4+6280\,m^3+25979\,m^2+47436\,m+27027)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$

input
```
int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
(a^6*x*(d*x)^m*(129072*m + 48259*m^2 + 9120*m^3 + 925*m^4 + 48*m^5 + m^6 +
 135135))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^
6 + m^7 + 135135) + (b^6*x^13*(d*x)^m*(19524*m + 12139*m^2 + 3480*m^3 + 50
5*m^4 + 36*m^5 + m^6 + 10395))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*
m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (6*a*b^5*x^11*(d*x)^m*(22902*m +
14039*m^2 + 3940*m^3 + 555*m^4 + 38*m^5 + m^6 + 12285))/(264207*m + 177331
*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (6*a^5*b
*x^3*(d*x)^m*(73054*m + 34759*m^2 + 7540*m^3 + 835*m^4 + 46*m^5 + m^6 + 45
045))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 +
m^7 + 135135) + (15*a^2*b^4*x^9*(d*x)^m*(27688*m + 16627*m^2 + 4528*m^3 +
613*m^4 + 40*m^5 + m^6 + 15015))/(264207*m + 177331*m^2 + 57379*m^3 + 1004
5*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (20*a^3*b^3*x^7*(d*x)^m*(34986*
m + 20335*m^2 + 5292*m^3 + 679*m^4 + 42*m^5 + m^6 + 19305))/(264207*m + 17
7331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (15*
a^4*b^2*x^5*(d*x)^m*(47436*m + 25979*m^2 + 6280*m^3 + 753*m^4 + 44*m^5 + m
^6 + 27027))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49
*m^6 + m^7 + 135135)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 602, normalized size of antiderivative = 4.01

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3 dx$$
$$= \frac{x^m d^m x(b^6m^6x^{12}+36b^6m^5x^{12}+6a\,b^5m^6x^{10}+505b^6m^4x^{12}+228a\,b^5m^5x^{10}+3480b^6m^3x^{12}+15a^2b^4m^6x}{}$$

input
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
(x**m*d**m*x*(a**6*m**6 + 48*a**6*m**5 + 925*a**6*m**4 + 9120*a**6*m**3 +
48259*a**6*m**2 + 129072*a**6*m + 135135*a**6 + 6*a**5*b*m**6*x**2 + 276*a
**5*b*m**5*x**2 + 5010*a**5*b*m**4*x**2 + 45240*a**5*b*m**3*x**2 + 208554*
a**5*b*m**2*x**2 + 438324*a**5*b*m*x**2 + 270270*a**5*b*x**2 + 15*a**4*b**
2*m**6*x**4 + 660*a**4*b**2*m**5*x**4 + 11295*a**4*b**2*m**4*x**4 + 94200*
a**4*b**2*m**3*x**4 + 389685*a**4*b**2*m**2*x**4 + 711540*a**4*b**2*m*x**4
 + 405405*a**4*b**2*x**4 + 20*a**3*b**3*m**6*x**6 + 840*a**3*b**3*m**5*x**
6 + 13580*a**3*b**3*m**4*x**6 + 105840*a**3*b**3*m**3*x**6 + 406700*a**3*b
**3*m**2*x**6 + 699720*a**3*b**3*m*x**6 + 386100*a**3*b**3*x**6 + 15*a**2*
b**4*m**6*x**8 + 600*a**2*b**4*m**5*x**8 + 9195*a**2*b**4*m**4*x**8 + 6792
0*a**2*b**4*m**3*x**8 + 249405*a**2*b**4*m**2*x**8 + 415320*a**2*b**4*m*x*
*8 + 225225*a**2*b**4*x**8 + 6*a*b**5*m**6*x**10 + 228*a*b**5*m**5*x**10 +
 3330*a*b**5*m**4*x**10 + 23640*a*b**5*m**3*x**10 + 84234*a*b**5*m**2*x**1
0 + 137412*a*b**5*m*x**10 + 73710*a*b**5*x**10 + b**6*m**6*x**12 + 36*b**6
*m**5*x**12 + 505*b**6*m**4*x**12 + 3480*b**6*m**3*x**12 + 12139*b**6*m**2
*x**12 + 19524*b**6*m*x**12 + 10395*b**6*x**12))/(m**7 + 49*m**6 + 973*m**
5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
```

3.673 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2 dx$

Optimal result

Integrand size = 26, antiderivative size = 104

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2 dx = \frac{a^4(dx)^{1+m}}{d(1+m)}+\frac{4a^3b(dx)^{3+m}}{d^3(3+m)}+\frac{6a^2b^2(dx)^{5+m}}{d^5(5+m)}+\frac{4ab^3(dx)^{7+m}}{d^7(7+m)}+\frac{b^4(dx)^{9+m}}{d^9(9+m)}$$

output
```
a^4*(d*x)^(1+m)/d/(1+m)+4*a^3*b*(d*x)^(3+m)/d^3/(3+m)+6*a^2*b^2*(d*x)^(5+m
)/d^5/(5+m)+4*a*b^3*(d*x)^(7+m)/d^7/(7+m)+b^4*(d*x)^(9+m)/d^9/(9+m)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2 dx = x(dx)^m \left(\frac{a^4}{1+m}+\frac{4a^3bx^2}{3+m}+\frac{6a^2b^2x^4}{5+m}+\frac{4ab^3x^6}{7+m}+\frac{b^4x^8}{9+m}\right)$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
x*(d*x)^m*(a^4/(1 + m) + (4*a^3*b*x^2)/(3 + m) + (6*a^2*b^2*x^4)/(5 + m) +
 (4*a*b^3*x^6)/(7 + m) + (b^4*x^8)/(9 + m))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^2+b^2x^4\right)^2 (dx)^m \, dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int b^4 (dx)^m \left(bx^2+a\right)^4 dx}{b^4}$$

$$\Big\downarrow \text{ 27}$$

$$\int \left(a+bx^2\right)^4 (dx)^m dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^4 (dx)^m+\frac{4a^3b(dx)^{m+2}}{d^2}+\frac{6a^2b^2(dx)^{m+4}}{d^4}+\frac{4ab^3(dx)^{m+6}}{d^6}+\frac{b^4(dx)^{m+8}}{d^8}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^4(dx)^{m+1}}{d(m+1)}+\frac{4a^3b(dx)^{m+3}}{d^3(m+3)}+\frac{6a^2b^2(dx)^{m+5}}{d^5(m+5)}+\frac{4ab^3(dx)^{m+7}}{d^7(m+7)}+\frac{b^4(dx)^{m+9}}{d^9(m+9)}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(a^4*(d*x)^(1 + m))/(d*(1 + m)) + (4*a^3*b*(d*x)^(3 + m))/(d^3*(3 + m)) +
(6*a^2*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (4*a*b^3*(d*x)^(7 + m))/(d^7*(7
+ m)) + (b^4*(d*x)^(9 + m))/(d^9*(9 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(104) = 208.

Time = 0.18 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.81

method	result
gosper	$\frac{(dx)^m (b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176m\,x^8b^4+6a^2b^2m^4x^4+416a\,b^3m^2x^6+105b^4x^8+120a^2}{}$
risch	$\frac{(dx)^m (b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176m\,x^8b^4+6a^2b^2m^4x^4+416a\,b^3m^2x^6+105b^4x^8+120a^2}{}$
orering	$\frac{(b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176m\,x^8b^4+6a^2b^2m^4x^4+416a\,b^3m^2x^6+105b^4x^8+120a^2b^2m^3}{}$
parallelrisch	$\frac{x(dx)^m a^4m^4+24x(dx)^m a^4m^3+1260x^3(dx)^m a^3b+206x(dx)^m a^4m^2+744x(dx)^m a^4m+540x^7(dx)^m a\,b^3+x^9(dx)^m b^4m^4+16}{}$

input
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(d*x)^m*(b^4*m^4*x^8+16*b^4*m^3*x^8+4*a*b^3*m^4*x^6+86*b^4*m^2*x^8+72*a*b^
3*m^3*x^6+176*b^4*m*x^8+6*a^2*b^2*m^4*x^4+416*a*b^3*m^2*x^6+105*b^4*x^8+12
0*a^2*b^2*m^3*x^4+888*a*b^3*m*x^6+4*a^3*b*m^4*x^2+780*a^2*b^2*m^2*x^4+540*
a*b^3*x^6+88*a^3*b*m^3*x^2+1800*a^2*b^2*m*x^4+a^4*m^4+656*a^3*b*m^2*x^2+11
34*a^2*b^2*x^4+24*a^4*m^3+1832*a^3*b*m*x^2+206*a^4*m^2+1260*a^3*b*x^2+744*
a^4*m+945*a^4)*x/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(104) = 208$.

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.43

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right)^2 dx$$
$$= \frac{((b^4m^4 + 16\,b^4m^3 + 86\,b^4m^2 + 176\,b^4m + 105\,b^4)x^9 + 4\,(ab^3m^4 + 18\,ab^3m^3 + 104\,ab^3m^2 + 222\,ab^3m + 1}{}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
((b^4*m^4 + 16*b^4*m^3 + 86*b^4*m^2 + 176*b^4*m + 105*b^4)*x^9 + 4*(a*b^3*
m^4 + 18*a*b^3*m^3 + 104*a*b^3*m^2 + 222*a*b^3*m + 135*a*b^3)*x^7 + 6*(a^2
*b^2*m^4 + 20*a^2*b^2*m^3 + 130*a^2*b^2*m^2 + 300*a^2*b^2*m + 189*a^2*b^2)
*x^5 + 4*(a^3*b*m^4 + 22*a^3*b*m^3 + 164*a^3*b*m^2 + 458*a^3*b*m + 315*a^3
*b)*x^3 + (a^4*m^4 + 24*a^4*m^3 + 206*a^4*m^2 + 744*a^4*m + 945*a^4)*x)*(d
*x)^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1278 vs. $2(94) = 188$.

Time = 0.49 (sec) , antiderivative size = 1278, normalized size of antiderivative = 12.29

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right)^2 dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Piecewise(((-a**4/(8*x**8) - 2*a**3*b/(3*x**6) - 3*a**2*b**2/(2*x**4) - 2*
a*b**3/x**2 + b**4*log(x))/d**9, Eq(m, -9)), ((-a**4/(6*x**6) - a**3*b/x**
4 - 3*a**2*b**2/x**2 + 4*a*b**3*log(x) + b**4*x**2/2)/d**7, Eq(m, -7)), ((
-a**4/(4*x**4) - 2*a**3*b/x**2 + 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4
*x**4/4)/d**5, Eq(m, -5)), ((-a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**
2*x**2 + a*b**3*x**4 + b**4*x**6/6)/d**3, Eq(m, -3)), ((a**4*log(x) + 2*a*
*3*b*x**2 + 3*a**2*b**2*x**4/2 + 2*a*b**3*x**6/3 + b**4*x**8/8)/d, Eq(m, -
1)), (a**4*m**4*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
+ 945) + 24*a**4*m**3*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 206*a**4*m**2*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m
**2 + 1689*m + 945) + 744*a**4*m*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 9
50*m**2 + 1689*m + 945) + 945*a**4*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 +
 950*m**2 + 1689*m + 945) + 4*a**3*b*m**4*x**3*(d*x)**m/(m**5 + 25*m**4 +
230*m**3 + 950*m**2 + 1689*m + 945) + 88*a**3*b*m**3*x**3*(d*x)**m/(m**5 +
 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 656*a**3*b*m**2*x**3*(d*x
)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1832*a**3*b*m
*x**3*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 126
0*a**3*b*x**3*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
5) + 6*a**2*b**2*m**4*x**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
+ 1689*m + 945) + 120*a**2*b**2*m**3*x**5*(d*x)**m/(m**5 + 25*m**4 + 23...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2 \, dx = \frac{b^4 d^m x^9 x^m}{m+9} + \frac{4\,ab^3 d^m x^7 x^m}{m+7} + \frac{6\,a^2 b^2 d^m x^5 x^m}{m+5} + \frac{4\,a^3 b d^m x^3 x^m}{m+3} + \frac{(dx)^{m+1}\,a^4}{d(m+1)}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
b^4*d^m*x^9*x^m/(m + 9) + 4*a*b^3*d^m*x^7*x^m/(m + 7) + 6*a^2*b^2*d^m*x^5*
x^m/(m + 5) + 4*a^3*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^4/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(104) = 208$.

Time = 0.11 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.99

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right)^2 \, dx$$
$$= \frac{(dx)^m b^4m^4x^9 + 16\,(dx)^m b^4m^3x^9 + 4\,(dx)^m ab^3m^4x^7 + 86\,(dx)^m b^4m^2x^9 + 72\,(dx)^m ab^3m^3x^7 + 176\,(dx)}{}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
((d*x)^m*b^4*m^4*x^9 + 16*(d*x)^m*b^4*m^3*x^9 + 4*(d*x)^m*a*b^3*m^4*x^7 +
86*(d*x)^m*b^4*m^2*x^9 + 72*(d*x)^m*a*b^3*m^3*x^7 + 176*(d*x)^m*b^4*m*x^9
+ 6*(d*x)^m*a^2*b^2*m^4*x^5 + 416*(d*x)^m*a*b^3*m^2*x^7 + 105*(d*x)^m*b^4*
x^9 + 120*(d*x)^m*a^2*b^2*m^3*x^5 + 888*(d*x)^m*a*b^3*m*x^7 + 4*(d*x)^m*a^
3*b*m^4*x^3 + 780*(d*x)^m*a^2*b^2*m^2*x^5 + 540*(d*x)^m*a*b^3*x^7 + 88*(d*
x)^m*a^3*b*m^3*x^3 + 1800*(d*x)^m*a^2*b^2*m*x^5 + (d*x)^m*a^4*m^4*x + 656*
(d*x)^m*a^3*b*m^2*x^3 + 1134*(d*x)^m*a^2*b^2*x^5 + 24*(d*x)^m*a^4*m^3*x +
1832*(d*x)^m*a^3*b*m*x^3 + 206*(d*x)^m*a^4*m^2*x + 1260*(d*x)^m*a^3*b*x^3
+ 744*(d*x)^m*a^4*m*x + 945*(d*x)^m*a^4*x)/(m^5 + 25*m^4 + 230*m^3 + 950*m
^2 + 1689*m + 945)
```

Mupad [B] (verification not implemented)

Time = 18.42 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.53

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right)^2 \, dx$$
$$= (d\,x)^m \left(\frac{b^4\,x^9\,(m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105)}{m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945} + \frac{a^4\,x\,(m^4 + 24\,m^3 + 206\,m^2 + 744\,m + 945)}{m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945} + \frac{4\,a\,b^3\,x^7\,(m^4 + 18\,m^3 + 104\,m^2 + 222\,m + 135)}{m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945} + \frac{4\,a^3\,b\,x^3\,(m^4 + 22\,m^3 + 164\,m^2 + 458\,m + 315)}{m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945} + \frac{6\,a^2\,b^2\,x^5\,(m^4 + 20\,m^3 + 130\,m^2 + 300\,m + 189)}{m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945}\right)$$

input
```
int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
(d*x)^m*((b^4*x^9*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2
 + 230*m^3 + 25*m^4 + m^5 + 945) + (a^4*x*(744*m + 206*m^2 + 24*m^3 + m^4
+ 945))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (4*a*b^3*x^7*(
222*m + 104*m^2 + 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^
4 + m^5 + 945) + (4*a^3*b*x^3*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(168
9*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (6*a^2*b^2*x^5*(300*m + 13
0*m^2 + 20*m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 +
945))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.81

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right)^2 dx$$
$$= \frac{x^m d^m x(b^4m^4x^8 + 16b^4m^3x^8 + 4a\,b^3m^4x^6 + 86b^4m^2x^8 + 72a\,b^3m^3x^6 + 176b^4m\,x^8 + 6a^2b^2m^4x^4 + 416a\,b}{}$$

input
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
(x**m*d**m*x*(a**4*m**4 + 24*a**4*m**3 + 206*a**4*m**2 + 744*a**4*m + 945*
a**4 + 4*a**3*b*m**4*x**2 + 88*a**3*b*m**3*x**2 + 656*a**3*b*m**2*x**2 + 1
832*a**3*b*m*x**2 + 1260*a**3*b*x**2 + 6*a**2*b**2*m**4*x**4 + 120*a**2*b*
*2*m**3*x**4 + 780*a**2*b**2*m**2*x**4 + 1800*a**2*b**2*m*x**4 + 1134*a**2
*b**2*x**4 + 4*a*b**3*m**4*x**6 + 72*a*b**3*m**3*x**6 + 416*a*b**3*m**2*x*
*6 + 888*a*b**3*m*x**6 + 540*a*b**3*x**6 + b**4*m**4*x**8 + 16*b**4*m**3*x
**8 + 86*b**4*m**2*x**8 + 176*b**4*m*x**8 + 105*b**4*x**8))/(m**5 + 25*m**
4 + 230*m**3 + 950*m**2 + 1689*m + 945)
```

3.674 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right) \, dx=\frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{3+m}}{d^3(3+m)}+\frac{b^2(dx)^{5+m}}{d^5(5+m)}$$

output
```
a^2*(d*x)^(1+m)/d/(1+m)+2*a*b*(d*x)^(3+m)/d^3/(3+m)+b^2*(d*x)^(5+m)/d^5/(5
+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right) \, dx=x(dx)^m \left(\frac{a^2}{1+m}+\frac{2abx^2}{3+m}+\frac{b^2x^4}{5+m}\right)$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + (b^2*x^4)/(5 + m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1380, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^2+b^2x^4\right)(dx)^m\,dx$$

$$\Big\downarrow \ 1380$$

$$\frac{\int b^2(dx)^m\left(bx^2+a\right)^2dx}{b^2}$$

$$\Big\downarrow \ 27$$

$$\int \left(a+bx^2\right)^2(dx)^m dx$$

$$\Big\downarrow \ 244$$

$$\int \left(a^2(dx)^m+\frac{2ab(dx)^{m+2}}{d^2}+\frac{b^2(dx)^{m+4}}{d^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{b^2(dx)^{m+5}}{d^5(m+5)}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (b
^2*(d*x)^(5 + m))/(d^5*(5 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result
norman	$\frac{a^2 x\,\mathrm{e}^{m\ln(dx)}}{1+m}+\frac{b^2 x^5 \mathrm{e}^{m\ln(dx)}}{5+m}+\frac{2ab\,x^3 \mathrm{e}^{m\ln(dx)}}{3+m}$
gosper	$\frac{(dx)^m \left(b^2 m^2 x^4+4b^2 m\,x^4+2ab\,m^2 x^2+3b^2 x^4+12abm\,x^2+a^2 m^2+10ab\,x^2+8a^2 m+15a^2\right) x}{(5+m)(3+m)(1+m)}$
risch	$\frac{(dx)^m \left(b^2 m^2 x^4+4b^2 m\,x^4+2ab\,m^2 x^2+3b^2 x^4+12abm\,x^2+a^2 m^2+10ab\,x^2+8a^2 m+15a^2\right) x}{(5+m)(3+m)(1+m)}$
orering	$\frac{\left(b^2 m^2 x^4+4b^2 m\,x^4+2ab\,m^2 x^2+3b^2 x^4+12abm\,x^2+a^2 m^2+10ab\,x^2+8a^2 m+15a^2\right) x(dx)^m \left(b^2 x^4+2ab\,x^2+a^2\right)}{(5+m)(3+m)(1+m)\left(b\,x^2+a\right)^2}$
parallelrisch	$\frac{x^5 (dx)^m b^2 m^2+4x^5 (dx)^m b^2 m+3x^5 (dx)^m b^2+2x^3 (dx)^m ab\,m^2+12x^3 (dx)^m abm+10x^3 (dx)^m ab+x(dx)^m a^2 m^2+8x(dx)^m a^2 m}{(5+m)(3+m)(1+m)}$

input
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
a^2/(1+m)*x*exp(m*ln(d*x))+b^2/(5+m)*x^5*exp(m*ln(d*x))+2*a*b/(3+m)*x^3*ex
p(m*ln(d*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.50

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)\,dx = \frac{((b^2m^2+4\,b^2m+3\,b^2)x^5+2\,(abm^2+6\,abm+5\,ab)x^3+(a^2m^2+8\,a^2m+15\,a^2)x)(dx)^m}{m^3+9\,m^2+23\,m+15}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*x^3 + (a^
2*m^2 + 8*a^2*m + 15*a^2)*x)*(d*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(49) = 98$.

Time = 0.28 (sec) , antiderivative size = 330, normalized size of antiderivative = 5.69

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)\,dx = \begin{cases} \frac{-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)}{d^5} \\ \frac{-\frac{a^2}{2x^2}+2ab\log(x)+\frac{b^2x^2}{2}}{d^3} \\ \frac{a^2\log(x)+abx^2+\frac{b^2x^4}{4}}{d} \\ \frac{a^2m^2x(dx)^m}{m^3+9m^2+23m+15}+\frac{8a^2mx(dx)^m}{m^3+9m^2+23m+15}+\frac{15a^2x(dx)^m}{m^3+9m^2+23m+15}+\frac{2abm^2x^3(dx)^m}{m^3+9m^2+23m+15}+\frac{12abmx^3(dx)^m}{m^3+9m^2+23m+15}+\frac{10abx^3(dx)^m}{m^3+9m^2+23m+\cdots} \end{cases}$$

input
```
integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
Piecewise(((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x))/d**5, Eq(m, -5)), ((-
a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2)/d**3, Eq(m, -3)), ((a**2*log(x
) + a*b*x**2 + b**2*x**4/4)/d, Eq(m, -1)), (a**2*m**2*x*(d*x)**m/(m**3 + 9
*m**2 + 23*m + 15) + 8*a**2*m*x*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 15*
a**2*x*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 2*a*b*m**2*x**3*(d*x)**m/(m*
*3 + 9*m**2 + 23*m + 15) + 12*a*b*m*x**3*(d*x)**m/(m**3 + 9*m**2 + 23*m +
15) + 10*a*b*x**3*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + b**2*m**2*x**5*(d
*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*m*x**5*(d*x)**m/(m**3 + 9*m**2
 + 23*m + 15) + 3*b**2*x**5*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)\,dx=\frac{b^2d^mx^5x^m}{m+5}+\frac{2\,abd^mx^3x^m}{m+3}+\frac{(dx)^{m+1}\,a^2}{d(m+1)}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
b^2*d^m*x^5*x^m/(m + 5) + 2*a*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^2/(d
*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(58) = 116.

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.33

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)\,dx$$
$$=\frac{(dx)^m\,b^2m^2x^5+4\,(dx)^m\,b^2mx^5+2\,(dx)^m\,abm^2x^3+3\,(dx)^m\,b^2x^5+12\,(dx)^m\,abmx^3+(dx)^m\,a^2m^2x+10}{m^3+9\,m^2+23\,m+15}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
((d*x)^m*b^2*m^2*x^5 + 4*(d*x)^m*b^2*m*x^5 + 2*(d*x)^m*a*b*m^2*x^3 + 3*(d*
x)^m*b^2*x^5 + 12*(d*x)^m*a*b*m*x^3 + (d*x)^m*a^2*m^2*x + 10*(d*x)^m*a*b*x
^3 + 8*(d*x)^m*a^2*m*x + 15*(d*x)^m*a^2*x)/(m^3 + 9*m^2 + 23*m + 15)
```

Mupad [B] (verification not implemented)

Time = 18.89 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right) \, dx = (d\,x)^m \left(\frac{a^2\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15} + \frac{b^2\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15} + \frac{2\,a\,b\,x^3\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}\right)$$

input
```
int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
(d*x)^m*((a^2*x*(8*m + m^2 + 15))/(23*m + 9*m^2 + m^3 + 15) + (b^2*x^5*(4*
m + m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (2*a*b*x^3*(6*m + m^2 + 5))/(23*
m + 9*m^2 + m^3 + 15))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right) \, dx = \frac{x^m d^m x(b^2m^2x^4+4b^2m\,x^4+2ab\,m^2x^2+3b^2x^4+12abm\,x^2+a^2m^2+10ab\,x^2+8a^2m+15a^2)}{m^3+9m^2+23m+15}$$

input
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
(x**m*d**m*x*(a**2*m**2 + 8*a**2*m + 15*a**2 + 2*a*b*m**2*x**2 + 12*a*b*m*
x**2 + 10*a*b*x**2 + b**2*m**2*x**4 + 4*b**2*m*x**4 + 3*b**2*x**4))/(m**3
+ 9*m**2 + 23*m + 15)
```

3.675 $\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4} \, dx = \frac{(dx)^{1+m} \text{Hypergeometric2F1}\left(2, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{bx^2}{a}\right)}{a^2 d(1+m)}$$

output
```
(d*x)^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/d/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4} \, dx = \frac{x(dx)^m \text{Hypergeometric2F1}\left(2, \frac{1+m}{2}, 1+\frac{1+m}{2}, -\frac{bx^2}{a}\right)}{a^2(1+m)}$$

input
```
Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
(x*(d*x)^m*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(
a^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1380, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4}\,dx$$

$$\Big\downarrow \ 1380$$

$$b^2\int \frac{(dx)^m}{b^2\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{(dx)^m}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \ 278$$

$$\frac{(dx)^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a^2d(m+1)}$$

input
```
Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

output
```
((d*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(
a^2*d*(1 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(dx)^m}{b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4}\,dx=\int \frac{(dx)^m}{b^2x^4+2\,abx^2+a^2}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4}\,dx=\int \frac{(dx)^m}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2),x)
```

output
```
Integral((d*x)**m/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4}\,dx=\int \frac{(dx)^m}{b^2x^4+2\,abx^2+a^2}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4}\,dx=\int \frac{(dx)^m}{b^2x^4+2\,abx^2+a^2}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4}\,dx=\int \frac{(d\,x)^m}{a^2+2\,a\,b\,x^2+b^2\,x^4}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2),x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4}\,dx=d^m\left(\int \frac{x^m}{b^2x^4+2ab\,x^2+a^2}dx\right)$$

input
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x)
```

output
```
d**m*int(x**m/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.676 $\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^2} dx = \frac{(dx)^{1+m} \text{Hypergeometric2F1}\left(4, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{bx^2}{a}\right)}{a^4d(1+m)}$$

output
```
(d*x)^(1+m)*hypergeom([4, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^4/d/(1+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^2} dx = \frac{x(dx)^m \text{Hypergeometric2F1}\left(4, \frac{1+m}{2}, 1+\frac{1+m}{2}, -\frac{bx^2}{a}\right)}{a^4(1+m)}$$

input
```
Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
(x*(d*x)^m*Hypergeometric2F1[4, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(
a^4*(1 + m))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1380, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^2}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^4\int \frac{(dx)^m}{b^4(bx^2+a)^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(dx)^m}{(a+bx^2)^4}dx$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(dx)^{m+1}\text{Hypergeometric2F1}\left(4,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a^4d(m+1)}$$

input
```
Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

output
```
((d*x)^(1 + m)*Hypergeometric2F1[4, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(
a^4*d*(1 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(dx)^m}{(b^2x^4+2ab\,x^2+a^2)^2}dx$$

input
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^2}\,dx = \int \frac{(dx)^m}{(b^2x^4+2\,abx^2+a^2)^2}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^
4), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^2} \, dx = \int \frac{(dx)^m}{(a + bx^2)^4} \, dx$$

input
```
integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

output
```
Integral((d*x)**m/(a + b*x**2)**4, x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^2} \, dx = \int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^2} \, dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2, x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^2} \, dx = \int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^2} \, dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^2}\,dx=\int \frac{(d\,x)^m}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^2}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^2,x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^2}\,dx=d^m\left(\int \frac{x^m}{b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4}dx\right)$$

input
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

output
```
d**m*int(x**m/(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b
**4*x**8),x)
```

3.677 $\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^3} \, dx = \frac{(dx)^{1+m} \text{Hypergeometric2F1}\left(6,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a^6d(1+m)}$$

output
```
(d*x)^(1+m)*hypergeom([6, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^6/d/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^3} \, dx = \frac{x(dx)^m \text{Hypergeometric2F1}\left(6,\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{a^6(1+m)}$$

input
```
Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
(x*(d*x)^m*Hypergeometric2F1[6, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(
a^6*(1 + m))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1380, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^3}dx$$

$$\Big\downarrow \text{ 1380}$$

$$b^6\int \frac{(dx)^m}{b^6\,(bx^2+a)^6}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(dx)^m}{(a+bx^2)^6}dx$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(dx)^{m+1}\,\text{Hypergeometric2F1}\left(6,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a^6d(m+1)}$$

input
```
Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

output
```
((d*x)^(1 + m)*Hypergeometric2F1[6, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(
a^6*d*(1 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{(dx)^m}{(b^2x^4 + 2ab\, x^2 + a^2)^3} dx$$

input
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^3} \, dx = \int \frac{(dx)^m}{(b^2x^4 + 2\, abx^2 + a^2)^3} \, dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^6*x^12 + 6*a*b^5*x^10 + 15*a^2*b^4*x^8 + 20*a^3*b^3*x^
6 + 15*a^4*b^2*x^4 + 6*a^5*b*x^2 + a^6), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^3}\,dx = \int \frac{(dx)^m}{(a + bx^2)^6}\,dx$$

input
```
integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

output
```
Integral((d*x)**m/(a + b*x**2)**6, x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^3}\,dx = \int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^3}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3, x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^3}\,dx = \int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^3}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^3}\,dx = \int \frac{(d\,x)^m}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^3}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^3,x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^3}\,dx = d^m\left(\int \frac{x^m}{b^6x^{12}+6a\,b^5x^{10}+15a^2b^4x^8+20a^3b^3x^6+15a^4b^2x^4+6a^5b\,x^2+a^6}dx\right)$$

input
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

output
```
d**m*int(x**m/(a**6 + 6*a**5*b*x**2 + 15*a**4*b**2*x**4 + 20*a**3*b**3*x**
6 + 15*a**2*b**4*x**8 + 6*a*b**5*x**10 + b**6*x**12),x)
```

3.678 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal result

Integrand size = 28, antiderivative size = 313

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)(a+bx^2)} + \frac{5a^4b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)(a+bx^2)} + \frac{10a^3b^2(dx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^5(5+m)(a+bx^2)} + \frac{10a^2b^3(dx)^{7+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^7(7+m)(a+bx^2)} + \frac{5ab^4(dx)^{9+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^9(9+m)(a+bx^2)} + \frac{b^5(dx)^{11+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^{11}(11+m)(a+bx^2)}$$

output
```
a^5*(d*x)^(1+m)*((b*x^2+a)^2)^(1/2)/d/(1+m)/(b*x^2+a)+5*a^4*b*(d*x)^(3+m)*
((b*x^2+a)^2)^(1/2)/d^3/(3+m)/(b*x^2+a)+10*a^3*b^2*(d*x)^(5+m)*((b*x^2+a)^
2)^(1/2)/d^5/(5+m)/(b*x^2+a)+10*a^2*b^3*(d*x)^(7+m)*((b*x^2+a)^2)^(1/2)/d^
7/(7+m)/(b*x^2+a)+5*a*b^4*(d*x)^(9+m)*((b*x^2+a)^2)^(1/2)/d^9/(9+m)/(b*x^2
+a)+b^5*(d*x)^(11+m)*((b*x^2+a)^2)^(1/2)/d^11/(11+m)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.35

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx=\frac{x(dx)^m\left(\left(a+bx^2\right)^2\right)^{5/2}\left(\frac{a^5}{1+m}+\frac{5a^4bx^2}{3+m}+\frac{10a^3b^2x^4}{5+m}+\frac{10a^2b^3x^6}{7+m}+\frac{5ab^4x^8}{9+m}+\frac{b^5x^{10}}{11+m}\right)}{\left(a+bx^2\right)^5}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x*(d*x)^m*((a + b*x^2)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^2)/(3 + m) + (1
0*a^3*b^2*x^4)/(5 + m) + (10*a^2*b^3*x^6)/(7 + m) + (5*a*b^4*x^8)/(9 + m)
+ (b^5*x^10)/(11 + m)))/(a + b*x^2)^5
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.51, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^2+b^2x^4\right)^{5/2}(dx)^m\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^5(dx)^m\left(bx^2+a\right)^5dx}{b^5\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^m\left(bx^2+a\right)^5dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5(dx)^m+\frac{5a^4b(dx)^{m+2}}{d^2}+\frac{10a^3b^2(dx)^{m+4}}{d^4}+\frac{10a^2b^3(dx)^{m+6}}{d^6}+\frac{5ab^4(dx)^{m+8}}{d^8}+\frac{b^5(dx)^{m+10}}{d^{10}}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^5(dx)^{m+1}}{d(m+1)}+\frac{5a^4b(dx)^{m+3}}{d^3(m+3)}+\frac{10a^3b^2(dx)^{m+5}}{d^5(m+5)}+\frac{10a^2b^3(dx)^{m+7}}{d^7(m+7)}+\frac{5ab^4(dx)^{m+9}}{d^9(m+9)}+\frac{b^5(dx)^{m+11}}{d^{11}(m+11)}\right)}{a+bx^2}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^5*(d*x)^(1 + m))/(d*(1 + m)) + (5*a^4
*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (10*a^3*b^2*(d*x)^(5 + m))/(d^5*(5 + m))
 + (10*a^2*b^3*(d*x)^(7 + m))/(d^7*(7 + m)) + (5*a*b^4*(d*x)^(9 + m))/(d^9
*(9 + m)) + (b^5*(d*x)^(11 + m))/(d^11*(11 + m))))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.45

method	result
gosper	$\frac{x(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+1310a\,b^4m^3x^8+1689m\,x^{10}b^5+29}{}$
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+1310a\,b^4m^3x^8+1689m}{}$
orering	$\frac{(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+1310a\,b^4m^3x^8+1689m\,x^{10}b^5+290a}{}$

input
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b^5*m^5*x^10+25*b^5*m^4*x^10+5*a*b^4*m^5*x^8+230*b^5*m^3*x^10+135*a*b^4
*m^4*x^8+950*b^5*m^2*x^10+10*a^2*b^3*m^5*x^6+1310*a*b^4*m^3*x^8+1689*b^5*m
*x^10+290*a^2*b^3*m^4*x^6+5610*a*b^4*m^2*x^8+945*b^5*x^10+10*a^3*b^2*m^5*x
^4+3020*a^2*b^3*m^3*x^6+10205*a*b^4*m*x^8+310*a^3*b^2*m^4*x^4+13660*a^2*b^
3*m^2*x^6+5775*a*b^4*x^8+5*a^4*b*m^5*x^2+3500*a^3*b^2*m^3*x^4+25770*a^2*b^
3*m*x^6+165*a^4*b*m^4*x^2+17300*a^3*b^2*m^2*x^4+14850*a^2*b^3*x^6+a^5*m^5+
2030*a^4*b*m^3*x^2+34890*a^3*b^2*m*x^4+35*a^5*m^4+11310*a^4*b*m^2*x^2+2079
0*a^3*b^2*x^4+470*a^5*m^3+26765*a^4*b*m*x^2+3010*a^5*m^2+17325*a^4*b*x^2+9
129*a^5*m+10395*a^5)*(d*x)^m*((b*x^2+a)^2)^(5/2)/(11+m)/(9+m)/(7+m)/(5+m)/
(3+m)/(1+m)/(b*x^2+a)^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.18

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{((b^5m^5+25\,b^5m^4+230\,b^5m^3+950\,b^5m^2+1689\,b^5m+945\,b^5)x^{11}+5\,(ab^4m^5+27\,ab^4m^4}{}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
((b^5*m^5 + 25*b^5*m^4 + 230*b^5*m^3 + 950*b^5*m^2 + 1689*b^5*m + 945*b^5)
*x^11 + 5*(a*b^4*m^5 + 27*a*b^4*m^4 + 262*a*b^4*m^3 + 1122*a*b^4*m^2 + 204
1*a*b^4*m + 1155*a*b^4)*x^9 + 10*(a^2*b^3*m^5 + 29*a^2*b^3*m^4 + 302*a^2*b
^3*m^3 + 1366*a^2*b^3*m^2 + 2577*a^2*b^3*m + 1485*a^2*b^3)*x^7 + 10*(a^3*b
^2*m^5 + 31*a^3*b^2*m^4 + 350*a^3*b^2*m^3 + 1730*a^3*b^2*m^2 + 3489*a^3*b^
2*m + 2079*a^3*b^2)*x^5 + 5*(a^4*b*m^5 + 33*a^4*b*m^4 + 406*a^4*b*m^3 + 22
62*a^4*b*m^2 + 5353*a^4*b*m + 3465*a^4*b)*x^3 + (a^5*m^5 + 35*a^5*m^4 + 47
0*a^5*m^3 + 3010*a^5*m^2 + 9129*a^5*m + 10395*a^5)*x)*(d*x)^m/(m^6 + 36*m^
5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Sympy [F]

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} \, dx = \int (dx)^m \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}} \, dx$$

input
```
integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral((d*x)**m*((a + b*x**2)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.78

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \frac{((m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945)b^5d^mx^{11} + 5\,(m^5 + 27\,m^4 + 262\,m^3 + 11}{}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*d^m*x^11 + 5*(m^5 +
 27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*d^m*x^9 + 10*(m^5 + 29
*m^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*b^3*d^m*x^7 + 10*(m^5 + 31*
m^4 + 350*m^3 + 1730*m^2 + 3489*m + 2079)*a^3*b^2*d^m*x^5 + 5*(m^5 + 33*m^
4 + 406*m^3 + 2262*m^2 + 5353*m + 3465)*a^4*b*d^m*x^3 + (m^5 + 35*m^4 + 47
0*m^3 + 3010*m^2 + 9129*m + 10395)*a^5*d^m*x)*x^m/(m^6 + 36*m^5 + 505*m^4
+ 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(247) = 494$.

Time = 0.16 (sec) , antiderivative size = 900, normalized size of antiderivative = 2.88

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
((d*x)^m*b^5*m^5*x^11*sgn(b*x^2 + a) + 25*(d*x)^m*b^5*m^4*x^11*sgn(b*x^2 +
 a) + 5*(d*x)^m*a*b^4*m^5*x^9*sgn(b*x^2 + a) + 230*(d*x)^m*b^5*m^3*x^11*sg
n(b*x^2 + a) + 135*(d*x)^m*a*b^4*m^4*x^9*sgn(b*x^2 + a) + 950*(d*x)^m*b^5*
m^2*x^11*sgn(b*x^2 + a) + 10*(d*x)^m*a^2*b^3*m^5*x^7*sgn(b*x^2 + a) + 1310
*(d*x)^m*a*b^4*m^3*x^9*sgn(b*x^2 + a) + 1689*(d*x)^m*b^5*m*x^11*sgn(b*x^2
+ a) + 290*(d*x)^m*a^2*b^3*m^4*x^7*sgn(b*x^2 + a) + 5610*(d*x)^m*a*b^4*m^2
*x^9*sgn(b*x^2 + a) + 945*(d*x)^m*b^5*x^11*sgn(b*x^2 + a) + 10*(d*x)^m*a^3
*b^2*m^5*x^5*sgn(b*x^2 + a) + 3020*(d*x)^m*a^2*b^3*m^3*x^7*sgn(b*x^2 + a)
+ 10205*(d*x)^m*a*b^4*m*x^9*sgn(b*x^2 + a) + 310*(d*x)^m*a^3*b^2*m^4*x^5*s
gn(b*x^2 + a) + 13660*(d*x)^m*a^2*b^3*m^2*x^7*sgn(b*x^2 + a) + 5775*(d*x)^
m*a*b^4*x^9*sgn(b*x^2 + a) + 5*(d*x)^m*a^4*b*m^5*x^3*sgn(b*x^2 + a) + 3500
*(d*x)^m*a^3*b^2*m^3*x^5*sgn(b*x^2 + a) + 25770*(d*x)^m*a^2*b^3*m*x^7*sgn(
b*x^2 + a) + 165*(d*x)^m*a^4*b*m^4*x^3*sgn(b*x^2 + a) + 17300*(d*x)^m*a^3*
b^2*m^2*x^5*sgn(b*x^2 + a) + 14850*(d*x)^m*a^2*b^3*x^7*sgn(b*x^2 + a) + (d
*x)^m*a^5*m^5*x*sgn(b*x^2 + a) + 2030*(d*x)^m*a^4*b*m^3*x^3*sgn(b*x^2 + a)
 + 34890*(d*x)^m*a^3*b^2*m*x^5*sgn(b*x^2 + a) + 35*(d*x)^m*a^5*m^4*x*sgn(b
*x^2 + a) + 11310*(d*x)^m*a^4*b*m^2*x^3*sgn(b*x^2 + a) + 20790*(d*x)^m*a^3
*b^2*x^5*sgn(b*x^2 + a) + 470*(d*x)^m*a^5*m^3*x*sgn(b*x^2 + a) + 26765*(d*
x)^m*a^4*b*m*x^3*sgn(b*x^2 + a) + 3010*(d*x)^m*a^5*m^2*x*sgn(b*x^2 + a) +
17325*(d*x)^m*a^4*b*x^3*sgn(b*x^2 + a) + 9129*(d*x)^m*a^5*m*x*sgn(b*x^2...
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx = \int (d\,x)^m\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2} \,dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{x^m d^m x(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+}{}$$

input
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
(x**m*d**m*x*(a**5*m**5 + 35*a**5*m**4 + 470*a**5*m**3 + 3010*a**5*m**2 +
9129*a**5*m + 10395*a**5 + 5*a**4*b*m**5*x**2 + 165*a**4*b*m**4*x**2 + 203
0*a**4*b*m**3*x**2 + 11310*a**4*b*m**2*x**2 + 26765*a**4*b*m*x**2 + 17325*
a**4*b*x**2 + 10*a**3*b**2*m**5*x**4 + 310*a**3*b**2*m**4*x**4 + 3500*a**3
*b**2*m**3*x**4 + 17300*a**3*b**2*m**2*x**4 + 34890*a**3*b**2*m*x**4 + 207
90*a**3*b**2*x**4 + 10*a**2*b**3*m**5*x**6 + 290*a**2*b**3*m**4*x**6 + 302
0*a**2*b**3*m**3*x**6 + 13660*a**2*b**3*m**2*x**6 + 25770*a**2*b**3*m*x**6
 + 14850*a**2*b**3*x**6 + 5*a*b**4*m**5*x**8 + 135*a*b**4*m**4*x**8 + 1310
*a*b**4*m**3*x**8 + 5610*a*b**4*m**2*x**8 + 10205*a*b**4*m*x**8 + 5775*a*b
**4*x**8 + b**5*m**5*x**10 + 25*b**5*m**4*x**10 + 230*b**5*m**3*x**10 + 95
0*b**5*m**2*x**10 + 1689*b**5*m*x**10 + 945*b**5*x**10))/(m**6 + 36*m**5 +
 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
```

3.679 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 205

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx = \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\left(a+bx^2\right)} + \frac{3a^2b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\left(a+bx^2\right)} + \frac{3ab^2(dx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^5(5+m)\left(a+bx^2\right)} + \frac{b^3(dx)^{7+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^7(7+m)\left(a+bx^2\right)}$$

output

```
a^3*(d*x)^(1+m)*((b*x^2+a)^2)^(1/2)/d/(1+m)/(b*x^2+a)+3*a^2*b*(d*x)^(3+m)*
((b*x^2+a)^2)^(1/2)/d^3/(3+m)/(b*x^2+a)+3*a*b^2*(d*x)^(5+m)*((b*x^2+a)^2)^
(1/2)/d^5/(5+m)/(b*x^2+a)+b^3*(d*x)^(7+m)*((b*x^2+a)^2)^(1/2)/d^7/(7+m)/(b
*x^2+a)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{x(dx)^m\sqrt{(a+bx^2)^2}\left(a^3(105+71m+15m^2+m^3)+3a^2b(35+47m+13m^2+m^3)\,x^2+3a\cdots\right.}{(1+m)(3+m)(5+m)(7+m)\,(a\cdots}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(x*(d*x)^m*Sqrt[(a + b*x^2)^2]*(a^3*(105 + 71*m + 15*m^2 + m^3) + 3*a^2*b*
(35 + 47*m + 13*m^2 + m^3)*x^2 + 3*a*b^2*(21 + 31*m + 11*m^2 + m^3)*x^4 +
b^3*(15 + 23*m + 9*m^2 + m^3)*x^6))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*(a +
b*x^2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.55, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^2+b^2x^4\right)^{3/2} (dx)^m \, dx$$

$$\Big\downarrow \ 1384$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b^3(dx)^m \left(bx^2+a\right)^3 dx}{b^3\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^m \left(bx^2+a\right)^3 dx}{a+bx^2}$$

$$\Big\downarrow \ 244$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^3(dx)^m+\frac{3a^2b(dx)^{m+2}}{d^2}+\frac{3ab^2(dx)^{m+4}}{d^4}+\frac{b^3(dx)^{m+6}}{d^6}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2b(dx)^{m+3}}{d^3(m+3)}+\frac{3ab^2(dx)^{m+5}}{d^5(m+5)}+\frac{b^3(dx)^{m+7}}{d^7(m+7)}\right)}{a+bx^2}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a^3*(d*x)^(1 + m))/(d*(1 + m)) + (3*a^2
*b*(d*x)^(3 + m))/(d^3*(3 + m)) + (3*a*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) +
(b^3*(d*x)^(7 + m))/(d^7*(7 + m))))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.97

method	result
gosper	$\frac{x(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23m\,x^6b^3+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93m\,x^4b^2a+39a^2b\,m^2x^2+63b^2x^4a+a^3m^3+141m}{(7+m)(5+m)(3+m)(1+m)(b\,x^2+a)^3}$
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23m\,x^6b^3+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93m\,x^4b^2a+39a^2b\,m^2x^2+63b^2x^4a+}{(b\,x^2+a)(7+m)(5+m)(3+m)(1+m)}$
orering	$\frac{(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23m\,x^6b^3+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93m\,x^4b^2a+39a^2b\,m^2x^2+63b^2x^4a+a^3m^3+141m}{(7+m)(5+m)(3+m)(1+m)(b\,x^2+a)^3}$

input
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^2*x^4
+15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x^4+a
^3*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)*(d*x)^m*
((b*x^2+a)^2)^(3/2)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx = \frac{((b^3m^3+9\,b^3m^2+23\,b^3m+15\,b^3)x^7+3\,(ab^2m^3+11\,ab^2m^2+31\,ab^2m+21\,ab^2)x^5+3\,(a^2}{m^4+16\,m^3+86\,m}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
((b^3*m^3 + 9*b^3*m^2 + 23*b^3*m + 15*b^3)*x^7 + 3*(a*b^2*m^3 + 11*a*b^2*m
^2 + 31*a*b^2*m + 21*a*b^2)*x^5 + 3*(a^2*b*m^3 + 13*a^2*b*m^2 + 47*a^2*b*m
 + 35*a^2*b)*x^3 + (a^3*m^3 + 15*a^3*m^2 + 71*a^3*m + 105*a^3)*x)*(d*x)^m/
(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [F]

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \int (dx)^m \left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d*x)**m*((a + b*x**2)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.58

$$\int (dx)^m \left(a^2+2abx^2 +b^2x^4\right)^{3/2} dx = \frac{((m^3+9\,m^2+23\,m+15)b^3d^mx^7+3\,(m^3+11\,m^2+31\,m+21)ab^2d^mx^5+3\,(m^3+13\,m^2}{m^4+16\,m^3+86\,m^2+176\,m+}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
((m^3 + 9*m^2 + 23*m + 15)*b^3*d^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a*b^
2*d^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*d^m*x^3 + (m^3 + 15*m^2 + 7
1*m + 105)*a^3*d^m*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(161) = 322.

Time = 0.15 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.87

$$\int (dx)^m \left(a^2+2abx^2 +b^2x^4\right)^{3/2} dx = \frac{(dx)^m\,b^3m^3x^7\mathrm{sgn}(bx^2+a)+9\,(dx)^m\,b^3m^2x^7\mathrm{sgn}(bx^2+a)+3\,(dx)^m\,ab^2m^3x^5\mathrm{sgn}(bx^2+a)}{}$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
((d*x)^m*b^3*m^3*x^7*sgn(b*x^2 + a) + 9*(d*x)^m*b^3*m^2*x^7*sgn(b*x^2 + a)
 + 3*(d*x)^m*a*b^2*m^3*x^5*sgn(b*x^2 + a) + 23*(d*x)^m*b^3*m*x^7*sgn(b*x^2
 + a) + 33*(d*x)^m*a*b^2*m^2*x^5*sgn(b*x^2 + a) + 15*(d*x)^m*b^3*x^7*sgn(b
*x^2 + a) + 3*(d*x)^m*a^2*b*m^3*x^3*sgn(b*x^2 + a) + 93*(d*x)^m*a*b^2*m*x^
5*sgn(b*x^2 + a) + 39*(d*x)^m*a^2*b*m^2*x^3*sgn(b*x^2 + a) + 63*(d*x)^m*a*
b^2*x^5*sgn(b*x^2 + a) + (d*x)^m*a^3*m^3*x*sgn(b*x^2 + a) + 141*(d*x)^m*a^
2*b*m*x^3*sgn(b*x^2 + a) + 15*(d*x)^m*a^3*m^2*x*sgn(b*x^2 + a) + 105*(d*x)
^m*a^2*b*x^3*sgn(b*x^2 + a) + 71*(d*x)^m*a^3*m*x*sgn(b*x^2 + a) + 105*(d*x
)^m*a^3*x*sgn(b*x^2 + a))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\int (d\,x)^m\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}\,dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.87

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx=\frac{x^m d^m x(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23b^3m\,x^6+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+\ldots}{m^4+16m^3\ldots}$$

input
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
(x**m*d**m*x*(a**3*m**3 + 15*a**3*m**2 + 71*a**3*m + 105*a**3 + 3*a**2*b*m
**3*x**2 + 39*a**2*b*m**2*x**2 + 141*a**2*b*m*x**2 + 105*a**2*b*x**2 + 3*a
*b**2*m**3*x**4 + 33*a*b**2*m**2*x**4 + 93*a*b**2*m*x**4 + 63*a*b**2*x**4
+ b**3*m**3*x**6 + 9*b**3*m**2*x**6 + 23*b**3*m*x**6 + 15*b**3*x**6))/(m**
4 + 16*m**3 + 86*m**2 + 176*m + 105)
```

3.680 $\int (dx)^m \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 97

$$\int (dx)^m \sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{a(dx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{d(1+m)\,(a+bx^2)} + \frac{b(dx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(3+m)\,(a+bx^2)}$$

output
```
a*(d*x)^(1+m)*((b*x^2+a)^2)^(1/2)/d/(1+m)/(b*x^2+a)+b*(d*x)^(3+m)*((b*x^2+
a)^2)^(1/2)/d^3/(3+m)/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.55

$$\int (dx)^m \sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{x(dx)^m\sqrt{(a+bx^2)^2}(a(3+m)+b(1+m)x^2)}{(1+m)(3+m)\,(a+bx^2)}$$

input
```
Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(x*(d*x)^m*Sqrt[(a + b*x^2)^2]*(a*(3 + m) + b*(1 + m)*x^2))/((1 + m)*(3 +
m)*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1384, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx^2+b^2x^4}(dx)^m\,dx$$

$$\Big\downarrow \text{ 1384}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int b(dx)^m\left(bx^2+a\right)dx}{b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int (dx)^m\left(bx^2+a\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(a(dx)^m+\frac{b(dx)^{m+2}}{d^2}\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+3}}{d^3(m+3)}\right)}{a+bx^2}$$

input
```
Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*((a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)
^(3 + m))/(d^3*(3 + m))))/(a + b*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{x\left(bm\,x^{2}+b\,x^{2}+am+3a\right)\left(dx\right)^{m}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{\left(3+m\right)\left(1+m\right)\left(b\,x^{2}+a\right)}$	56
risch	$\frac{x\left(bm\,x^{2}+b\,x^{2}+am+3a\right)\left(dx\right)^{m}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{\left(3+m\right)\left(1+m\right)\left(b\,x^{2}+a\right)}$	56
orering	$\frac{x\left(bm\,x^{2}+b\,x^{2}+am+3a\right)\left(dx\right)^{m}\sqrt{\left(b\,x^{2}+a\right)^{2}}}{\left(3+m\right)\left(1+m\right)\left(b\,x^{2}+a\right)}$	56

input
```
int((d*x)^m*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(b*m*x^2+b*x^2+a*m+3*a)*(d*x)^m*((b*x^2+a)^2)^(1/2)/(3+m)/(1+m)/(b*x^2+a
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int (dx)^m \sqrt{a^2 + 2abx^2 + b^2x^4}\, dx = \frac{((bm + b)x^3 + (am + 3\,a)x)(dx)^m}{m^2 + 4\,m + 3}$$

input
```
integrate((d*x)^m*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
((b*m + b)*x^3 + (a*m + 3*a)*x)*(d*x)^m/(m^2 + 4*m + 3)
```

Sympy [F]

$$\int (dx)^m \sqrt{a^2 + 2abx^2 + b^2x^4}\, dx = \int (dx)^m \sqrt{(a + bx^2)^2}\, dx$$

input
```
integrate((d*x)**m*((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt((a + b*x**2)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.35

$$\int (dx)^m \sqrt{a^2 + 2abx^2 + b^2x^4}\, dx = \frac{bd^m x^3 x^m}{m + 3} + \frac{(dx)^{m+1}\, a}{d(m + 1)}$$

input
```
integrate((d*x)^m*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int (dx)^m \sqrt{a^2 + 2abx^2 + b^2x^4}\, dx = \frac{(dx)^m\, bmx^3 \mathrm{sgn}(bx^2 + a) + (dx)^m\, bx^3 \mathrm{sgn}(bx^2 + a) + (dx)^m\, amx \mathrm{sgn}(bx^2 + a) + 3\,(dx)^m\, ax \mathrm{sgn}(bx^2 + a)}{m^2 + 4\,m + 3}$$

input
```
integrate((d*x)^m*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
((d*x)^m*b*m*x^3*sgn(b*x^2 + a) + (d*x)^m*b*x^3*sgn(b*x^2 + a) + (d*x)^m*a
*m*x*sgn(b*x^2 + a) + 3*(d*x)^m*a*x*sgn(b*x^2 + a))/(m^2 + 4*m + 3)
```

Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.57

$$\int (dx)^m \sqrt{a^2 + 2abx^2 + b^2x^4}\, dx = \frac{x\,(d\,x)^m\,\sqrt{{\left(b\,x^2+a\right)}^2}\,\left(3\,a+a\,m+b\,x^2+b\,m\,x^2\right)}{\left(b\,x^2+a\right)\,\left(m^2+4\,m+3\right)}$$

input
```
int((d*x)^m*((a + b*x^2)^2)^(1/2),x)
```

output
```
(x*(d*x)^m*((a + b*x^2)^2)^(1/2)*(3*a + a*m + b*x^2 + b*m*x^2))/((a + b*x^
2)*(4*m + m^2 + 3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.37

$$\int (dx)^m \sqrt{a^2 + 2abx^2 + b^2x^4}\, dx = \frac{x^m d^m x(bm\,x^2 + b\,x^2 + am + 3a)}{m^2 + 4m + 3}$$

input
```
int((d*x)^m*((b*x^2+a)^2)^(1/2),x)
```

output
```
(x**m*d**m*x*(a*m + 3*a + b*m*x**2 + b*x**2))/(m**2 + 4*m + 3)
```

3.681 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\frac{(dx)^{1+m}\left(a+bx^2\right)\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
(d*x)^(1+m)*(b*x^2+a)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/d/(
1+m)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx =\frac{x(dx)^m\left(a+bx^2\right)\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{a(1+m)\sqrt{\left(a+bx^2\right)^2}}$$

input
```
Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
(x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b
*x^2)/a)])/(a*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1384, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b(a+bx^2)\int \frac{(dx)^m}{b(bx^2+a)}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{(dx)^m}{bx^2+a}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{278}$$

$$\frac{(a+bx^2)(dx)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

output
```
((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b
*x^2)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{(dx)^{m}}{\sqrt{\left(b\,x^{2}+a\right)^{2}}}dx$$

input
```
int((d*x)^m/((b*x^2+a)^2)^(1/2),x)
```

output
```
int((d*x)^m/((b*x^2+a)^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{(dx)^m}{\sqrt{(bx^2+a)^2}}\,dx$$

input
```
integrate((d*x)^m/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{(dx)^m}{\sqrt{(a+bx^2)^2}}\,dx$$

input
```
integrate((d*x)**m/((b*x**2+a)**2)**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt((a + b*x**2)**2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{(dx)^m}{\sqrt{(bx^2+a)^2}}\,dx$$

input
```
integrate((d*x)^m/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{(dx)^m}{\sqrt{(bx^2+a)^2}}\,dx$$

input
```
integrate((d*x)^m/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt((b*x^2 + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=\int \frac{(d\,x)^m}{\sqrt{(b\,x^2+a)^2}}\,dx$$

input
```
int((d*x)^m/((a + b*x^2)^2)^(1/2),x)
```

output
```
int((d*x)^m/((a + b*x^2)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx=d^m\left(\int \frac{x^m}{b\,x^2+a}dx\right)$$

input
```
int((d*x)^m/((b*x^2+a)^2)^(1/2),x)
```

output
```
d**m*int(x**m/(a + b*x**2),x)
```

3.682 $\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}} dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}} dx=\frac{(dx)^{1+m}\left(a+bx^2\right)\text{Hypergeometric2F1}\left(3,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
(d*x)^(1+m)*(b*x^2+a)*hypergeom([3, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/d
/(1+m)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}} dx=\frac{x(dx)^m\left(a+bx^2\right)\text{Hypergeometric2F1}\left(3,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a^3(1+m)\sqrt{\left(a+bx^2\right)^2}}$$

input
```
Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
(x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2
)/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1384, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^3(a+bx^2)\int \frac{(dx)^m}{b^3(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{(dx)^m}{(bx^2+a)^3}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{278}$$

$$\frac{(a+bx^2)(dx)^{m+1}\text{Hypergeometric2F1}\left(3,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

output
```
((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b
*x^2)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{(dx)^m}{(b^2x^4 + 2ab\,x^2 + a^2)^{\frac{3}{2}}}dx$$

input
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m/(b^4*x^8 + 4*a*b^3*x^6 +
6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^m}{\left((a + bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

output
```
Integral((d*x)**m/((a + b*x**2)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^{3/2}}\,dx = \int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \int \frac{(d\,x)^m}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{3/2}}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = d^m\left(\int \frac{x^m}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx\right)$$

input
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

output
```
d**m*int(x**m/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)
```

3.683 $\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{(dx)^{1+m}\left(a+bx^2\right)\text{Hypergeometric2F1}\left(5,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

output
```
(d*x)^(1+m)*(b*x^2+a)*hypergeom([5, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^5/d
/(1+m)/((b*x^2+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{(dx)^m}{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}\,dx=\frac{x(dx)^m\left(a+bx^2\right)\text{Hypergeometric2F1}\left(5,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a^5(1+m)\sqrt{\left(a+bx^2\right)^2}}$$

input
```
Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
(x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[5, (1 + m)/2, (3 + m)/2, -((b*x^2
)/a)])/(a^5*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1384, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^{5/2}}dx$$

$$\Big\downarrow \text{1384}$$

$$\frac{b^5(a+bx^2)\int \frac{(dx)^m}{b^5(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+bx^2)\int \frac{(dx)^m}{(bx^2+a)^5}dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$\Big\downarrow \text{278}$$

$$\frac{(a+bx^2)(dx)^{m+1}\text{Hypergeometric2F1}\left(5,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

input
```
Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

output
```
((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[5, (1 + m)/2, (3 + m)/2, -((b
*x^2)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1384
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*Frac
Part[p]))   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && NeQ[u, x^(n
- 1)] && NeQ[u, x^(2*n - 1)] &&  !(EqQ[p, 1/2] && EqQ[u, x^(-2*n - 1)])
```

Maple [F]

$$\int \frac{(dx)^m}{(b^2x^4 + 2ab\,x^2 + a^2)^{\frac{5}{2}}}dx$$

input
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\,dx = \int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m/(b^6*x^12 + 6*a*b^5*x^10
+ 15*a^2*b^4*x^8 + 20*a^3*b^3*x^6 + 15*a^4*b^2*x^4 + 6*a^5*b*x^2 + a^6), x
)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\, dx = \int \frac{(dx)^m}{\left((a + bx^2)^2\right)^{\frac{5}{2}}}\, dx$$

input
```
integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

output
```
Integral((d*x)**m/((a + b*x**2)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\, dx = \int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{5}{2}}}\, dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a^2 + 2abx^2 + b^2x^4)^{5/2}}\, dx = \int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{5}{2}}}\, dx$$

input
```
integrate((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=\int \frac{(d\,x)^m}{(a^2+2\,a\,b\,x^2+b^2\,x^4)^{5/2}}\,dx$$

input
```
int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
```

output
```
int((d*x)^m/(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx=d^m\left(\int \frac{x^m}{b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6+10a^3b^2x^4+5a^4b\,x^2+a^5}dx\right)$$

input
```
int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

output
```
d**m*int(x**m/(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3*x**
6 + 5*a*b**4*x**8 + b**5*x**10),x)
```

3.684 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 77

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{(dx)^{1+m}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p \text{Hypergeometric2F1}\left(\frac{1+m}{2},-2p,\frac{3+m}{2},-\frac{bx^2}{a}\right)}{d(1+m)}$$

output
```
(d*x)^(1+m)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-2*p, 1/2+1/2*m],[3/2+1/2
*m],-b*x^2/a)/d/(1+m)/((1+b*x^2/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{x(dx)^m\left(\left(a+bx^2\right)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-2p,1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{1+m}$$

input
```
Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(x*(d*x)^m*((a + b*x^2)^2)^p*Hypergeometric2F1[(1 + m)/2, -2*p, 1 + (1 + m
)/2, -((b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1385, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p dx$$

$$\Big\downarrow \text{ 1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p \int (dx)^m \left(\frac{bx^2}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(dx)^{m+1} \left(\frac{bx^2}{a}+1\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p \text{Hypergeometric2F1}\left(\frac{m+1}{2},-2p,\frac{m+3}{2},-\frac{bx^2}{a}\right)}{d(m+1)}$$

input
```
Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
((d*x)^(1 + m)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[(1 + m)/2,
-2*p, (3 + m)/2, -((b*x^2)/a)])/(d*(1 + m)*(1 + (b*x^2)/a)^(2*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int (dx)^m \left(b^2x^4 + 2ab\,x^2 + a^2\right)^p dx$$

input
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a^2 + 2abx^2 + b^2x^4\right)^p dx = \int \left(b^2x^4 + 2\,abx^2 + a^2\right)^p (dx)^m \,dx$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int (dx)^m \left(\left(a+bx^2\right)^2\right)^p\,dx$$

input
```
integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Integral((d*x)**m*((a + b*x**2)**2)**p, x)
```

Maxima [F]

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \left(b^2x^4+2\,abx^2+a^2\right)^p (dx)^m\,dx$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \left(b^2x^4+2\,abx^2+a^2\right)^p (dx)^m\,dx$$

input
```
integrate((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p\, dx = \int (d\,x)^m\, \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\, dx$$

input
```
int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
int((d*x)^m*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

Reduce [F]

$$\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p\, dx$$
$$= \frac{d^m \left(x^m (b^2x^4+2ab\,x^2+a^2)^p\, x + 4\left(\int \frac{x^m (b^2x^4+2ab\,x^2+a^2)^p}{bm\,x^2+4bp\,x^2+b\,x^2+am+4ap+a}dx\right) amp + 16\left(\int \frac{x^m (b^2x^4+2ab\,x^2+a^2)^p}{bm\,x^2+4bp\,x^2+b\,x^2+am+4ap+}\right.\right.}{m+4p+1}$$

input
```
int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
(d**m*(x**m*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*x + 4*int((x**m*(a**2 + 2*a
*b*x**2 + b**2*x**4)**p)/(a*m + 4*a*p + a + b*m*x**2 + 4*b*p*x**2 + b*x**2
),x)*a*m*p + 16*int((x**m*(a**2 + 2*a*b*x**2 + b**2*x**4)**p)/(a*m + 4*a*p
 + a + b*m*x**2 + 4*b*p*x**2 + b*x**2),x)*a*p**2 + 4*int((x**m*(a**2 + 2*a
*b*x**2 + b**2*x**4)**p)/(a*m + 4*a*p + a + b*m*x**2 + 4*b*p*x**2 + b*x**2
),x)*a*p))/(m + 4*p + 1)
```

3.685 $\int x^7(a^2+2abx^2+b^2x^4)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^p\,dx=-\frac{a^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(1+2p)}+\frac{3a^2(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(1+p)}-\frac{3a(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(3+2p)}+\frac{(a+bx^2)^4\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(2+p)}$$

output
```
-1/2*a^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^4/(1+2*p)+3/4*a^2*(b*x^2+a)
^2*(b^2*x^4+2*a*b*x^2+a^2)^p/b^4/(p+1)-3/2*a*(b*x^2+a)^3*(b^2*x^4+2*a*b*x^
2+a^2)^p/b^4/(3+2*p)+1/4*(b*x^2+a)^4*(b^2*x^4+2*a*b*x^2+a^2)^p/b^4/(2+p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.63

$$\int x^7 (a^2 + 2abx^2 + b^2x^4)^p \, dx$$
$$= \frac{(a+bx^2)\left((a+bx^2)^2\right)^p (-3a^3 + 3a^2b(1+2p)x^2 - 3ab^2(1+3p+2p^2)\,x^4 + b^3(3+11p+12p^2+4p^3)\,x^6}{4b^4(1+p)(2+p)(1+2p)(3+2p)}$$

input
```
Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
((a + b*x^2)*((a + b*x^2)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^2 - 3*a*b^2*(
1 + 3*p + 2*p^2)*x^4 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^6))/(4*b^4*(1 + p
)*(2 + p)*(1 + 2*p)*(3 + 2*p))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1385, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 (a^2 + 2abx^2 + b^2x^4)^p \, dx$$
$$\Big\downarrow \; 1385$$
$$\left(\frac{bx^2}{a}+1\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \int x^7 \left(\frac{bx^2}{a}+1\right)^{2p} dx$$
$$\Big\downarrow \; 243$$
$$\frac{1}{2}\left(\frac{bx^2}{a}+1\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \int x^6 \left(\frac{bx^2}{a}+1\right)^{2p} dx^2$$
$$\Big\downarrow \; 53$$

$$\frac{1}{2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\int\left(-\frac{a^3\left(\frac{bx^2}{a}+1\right)^{2p}}{b^3}+\frac{3a^3\left(\frac{bx^2}{a}+1\right)^{2p+1}}{b^3}-\frac{3a^3\left(\frac{bx^2}{a}+1\right)^{2p+2}}{b^3}+\frac{a^3\left(\frac{bx^2}{a}\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\left(\frac{3a^4\left(\frac{bx^2}{a}+1\right)^{2(p+1)}}{2b^4(p+1)}+\frac{a^4\left(\frac{bx^2}{a}+1\right)^{2(p+2)}}{2b^4(p+2)}-\frac{a^4\left(\frac{bx^2}{a}+1\right)^{2p+1}}{b^4(2p+1)}-\frac{3a^4\left(\frac{bx^2}{a}\right.}{b^4(2}\right.$$

input
```
Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
((a^2 + 2*a*b*x^2 + b^2*x^4)^p*((3*a^4*(1 + (b*x^2)/a)^(2*(1 + p)))/(2*b^4
*(1 + p)) + (a^4*(1 + (b*x^2)/a)^(2*(2 + p)))/(2*b^4*(2 + p)) - (a^4*(1 +
(b*x^2)/a)^(1 + 2*p))/(b^4*(1 + 2*p)) - (3*a^4*(1 + (b*x^2)/a)^(3 + 2*p))/
(b^4*(3 + 2*p))))/(2*(1 + (b*x^2)/a)^(2*p))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.86

method	result
gosper	$-\frac{(b^2x^4+2ab\,x^2+a^2)^p\,(-4b^3p^3x^6-12b^3p^2x^6-11b^3p\,x^6+6a\,b^2p^2x^4-3b^3x^6+9a\,b^2p\,x^4+3b^2x^4a-6a^2bp\,x^2-3a^2b\,x^2+3a^3)\,(b}{4b^4(4p^4+20p^3+35p^2+25p+6)}$
orering	$-\frac{(b^2x^4+2ab\,x^2+a^2)^p\,(-4b^3p^3x^6-12b^3p^2x^6-11b^3p\,x^6+6a\,b^2p^2x^4-3b^3x^6+9a\,b^2p\,x^4+3b^2x^4a-6a^2bp\,x^2-3a^2b\,x^2+3a^3)\,(b}{4b^4(4p^4+20p^3+35p^2+25p+6)}$
risch	$-\frac{(-4b^4p^3x^8-12b^4p^2x^8-4a\,b^3p^3x^6-11b^4p\,x^8-6a\,b^3p^2x^6-3b^4x^8-2ap\,x^6b^3+6a^2b^2p^2x^4+3a^2p\,x^4b^2-6a^3p\,x^2b+3a^4)\,\left((b\,x^2\right.}{4(3+2p)(2+p)(p+1)(1+2p)b^4}$
norman	$\frac{x^8e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4p+8}-\frac{3a^4e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4b^4(4p^4+20p^3+35p^2+25p+6)}+\frac{ap\,x^6e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2b(2p^2+7p+6)}-\frac{3a^2p\,x^4e^{p\ln\left(b^2x^4+2ab\,x^2+a\right.}}{4b^2(2p^3+9p^2+13p+6)}$
parallelrisch	$4x^8(b^2x^4+2ab\,x^2+a^2)^p a\,b^4p^3+12x^8(b^2x^4+2ab\,x^2+a^2)^p a\,b^4p^2+11x^8(b^2x^4+2ab\,x^2+a^2)^p a\,b^4p+4x^6(b^2x^4+2ab\,x^2+a^2)^p a^2$

input
```
int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-4*b^3*p^3*x^6-12*b^3*p^2*x^6-11*b^3*p*x^6
+6*a*b^2*p^2*x^4-3*b^3*x^6+9*a*b^2*p*x^4+3*a*b^2*x^4-6*a^2*b*p*x^2-3*a^2*b
*x^2+3*a^3)*(b*x^2+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.94

$$\int x^7\left(a^2+2abx^2+b^2x^4\right)^p\,dx$$
$$=\frac{((4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4)x^8+6\,a^3bpx^2+2\,(2\,ab^3p^3+3\,ab^3p^2+ab^3p)x^6-3\,(2\,a^2b^2p^2+a^2b^2p)x}{4\,(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4)}$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
1/4*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^8 + 6*a^3*b*p*x^2 + 2*(
2*a*b^3*p^3 + 3*a*b^3*p^2 + a*b^3*p)*x^6 - 3*(2*a^2*b^2*p^2 + a^2*b^2*p)*x
^4 - 3*a^4)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35*b^4
*p^2 + 25*b^4*p + 6*b^4)
```

Sympy [F]

$$\int x^7 \left(a^2 + 2abx^2 + b^2x^4\right)^p \, dx = \text{Too large to display}$$

input
```
integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Piecewise((x**8*(a**2)**p/8, Eq(b, 0)), (6*a**3*log(x - sqrt(-a/b))/(12*a*
*3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*log(
x + sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b*
*7*x**6) + 11*a**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12
*b**7*x**6) + 18*a**2*b*x**2*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b
**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(x + sqrt(-a
/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) +
27*a**2*b*x**2/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**
7*x**6) + 18*a*b**2*x**4*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*
x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(x + sqrt(-a/b))
/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a
*b**2*x**4/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x*
*6) + 6*b**3*x**6*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 +
36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(x + sqrt(-a/b))/(12*a**3*
b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -2)), (In
tegral(x**7/((a + b*x**2)**2)**(3/2), x), Eq(p, -3/2)), (6*a**3*log(x - sq
rt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**3*log(x + sqrt(-a/b))/(4*a*b**4
+ 4*b**5*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(x - s
qrt(-a/b))/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(x + sqrt(-a/b))/(4
*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b**4 + 4*b**5*x**2) + b**3*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.66

$$\int x^7 \left(a^2 + 2abx^2 + b^2x^4\right)^p \, dx = \frac{((4\,p^3 + 12\,p^2 + 11\,p + 3)b^4x^8 + 2\,(2\,p^3 + 3\,p^2 + p)ab^3x^6 - 3\,(2\,p^2 + p)a^2b^2x^4 + 6\,a^3bpx^2 - 3\,a^4)(bx^2 + a}{4\,(4\,p^4 + 20\,p^3 + 35\,p^2 + 25\,p + 6)b^4}$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
1/4*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^6
 - 3*(2*p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4)*(b*x^2 + a)^(2*p)/((
4*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(166) = 332.

Time = 0.16 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.16

$$\int x^7 \left(a^2 + 2abx^2 + b^2x^4\right)^p \, dx = \frac{4\,(b^2x^4 + 2\,abx^2 + a^2)^pb^4p^3x^8 + 12\,(b^2x^4 + 2\,abx^2 + a^2)^pb^4p^2x^8 + 4\,(b^2x^4 + 2\,abx^2 + a^2)^pab^3p^3x^6 + 11\,(b}{}$$

input
```
integrate(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
1/4*(4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p^3*x^8 + 12*(b^2*x^4 + 2*a*b*x^2
 + a^2)^p*b^4*p^2*x^8 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p^3*x^6 + 11
*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p*x^8 + 6*(b^2*x^4 + 2*a*b*x^2 + a^2)^p
*a*b^3*p^2*x^6 + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*x^8 + 2*(b^2*x^4 + 2*
a*b*x^2 + a^2)^p*a*b^3*p*x^6 - 6*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p^2
*x^4 - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p*x^4 + 6*(b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a^3*b*p*x^2 - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^4)/(4*b^4*p^4
 + 20*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p \left(\frac{x^8\,\left(p^3+3\,p^2+\frac{11\,p}{4}+\frac{3}{4}\right)}{4\,p^4+20\,p^3+35\,p^2+25\,p+6} - \frac{3\,a^4}{4\,b^4\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)} + \frac{3\,a^3\,p\,x^2}{2\,b^3\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)} + \frac{a\,p\,x^6\,\left(2\,p^2+3\,p+1\right)}{2\,b\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)} - \frac{3\,a^2\,p\,x^4\,\left(2\,p+1\right)}{4\,b^2\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}\right)$$

input
```
int(x^7*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
(a^2 + b^2*x^4 + 2*a*b*x^2)^p*((x^8*((11*p)/4 + 3*p^2 + p^3 + 3/4))/(25*p
+ 35*p^2 + 20*p^3 + 4*p^4 + 6) - (3*a^4)/(4*b^4*(25*p + 35*p^2 + 20*p^3 +
4*p^4 + 6)) + (3*a^3*p*x^2)/(2*b^3*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) +
 (a*p*x^6*(3*p + 2*p^2 + 1))/(2*b*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) -
(3*a^2*p*x^4*(2*p + 1))/(4*b^2*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.93

$$\int x^7 \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{\left(b^2x^4+2ab\,x^2+a^2\right)^p \left(4b^4p^3x^8+12b^4p^2x^8+4a\,b^3p^3x^6+11b^4p\,x^8+6a\,b^3p^2x^6+3b^4x^8+2a\,b^3p\,x^6-6a^2\right.}{4b^4\left(4p^4+20p^3+35p^2+25p+6\right)}$$

input
```
int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x**2 + b**2*x**4)**p*( - 3*a**4 + 6*a**3*b*p*x**2 - 6*a**2*
b**2*p**2*x**4 - 3*a**2*b**2*p*x**4 + 4*a*b**3*p**3*x**6 + 6*a*b**3*p**2*x
**6 + 2*a*b**3*p*x**6 + 4*b**4*p**3*x**8 + 12*b**4*p**2*x**8 + 11*b**4*p*x
**8 + 3*b**4*x**8))/(4*b**4*(4*p**4 + 20*p**3 + 35*p**2 + 25*p + 6))
```

3.686 $\int x^5(a^2+2abx^2+b^2x^4)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int x^5(a^2+2abx^2+b^2x^4)^p\,dx=\frac{a^2(a+bx^2)(a^2+2abx^2+b^2x^4)^p}{2b^3(1+2p)}-\frac{a(a+bx^2)^2(a^2+2abx^2+b^2x^4)^p}{2b^3(1+p)}+\frac{(a+bx^2)^3(a^2+2abx^2+b^2x^4)^p}{2b^3(3+2p)}$$

output
```
1/2*a^2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(1+2*p)-1/2*a*(b*x^2+a)^2*
(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(p+1)+1/2*(b*x^2+a)^3*(b^2*x^4+2*a*b*x^2+a^2
)^p/b^3/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.59

$$\int x^5(a^2+2abx^2+b^2x^4)^p\,dx=\frac{(a+bx^2)\left((a+bx^2)^2\right)^p(a^2-ab(1+2p)x^2+b^2(1+3p+2p^2)x^4)}{2b^3(1+p)(1+2p)(3+2p)}$$

input
```
Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
((a + b*x^2)*((a + b*x^2)^2)^p*(a^2 - a*b*(1 + 2*p)*x^2 + b^2*(1 + 3*p + 2
*p^2)*x^4))/(2*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1385, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a^2 + 2abx^2 + b^2x^4)^p \, dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \int x^5 \left(\frac{bx^2}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{bx^2}{a}+1\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \int x^4 \left(\frac{bx^2}{a}+1\right)^{2p} dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\left(\frac{bx^2}{a}+1\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \int \left(\frac{a^2\left(\frac{bx^2}{a}+1\right)^{2p}}{b^2} - \frac{2a^2\left(\frac{bx^2}{a}+1\right)^{2p+1}}{b^2} + \frac{a^2\left(\frac{bx^2}{a}+1\right)^{2p+2}}{b^2}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{bx^2}{a}+1\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \left(-\frac{a^3\left(\frac{bx^2}{a}+1\right)^{2(p+1)}}{b^3(p+1)} + \frac{a^3\left(\frac{bx^2}{a}+1\right)^{2p+1}}{b^3(2p+1)} + \frac{a^3\left(\frac{bx^2}{a}+1\right)^{2p+3}}{b^3(2p+3)}\right)$$

input
```
Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
((a^2 + 2*a*b*x^2 + b^2*x^4)^p*(-((a^3*(1 + (b*x^2)/a)^(2*(1 + p)))/(b^3*(
1 + p))) + (a^3*(1 + (b*x^2)/a)^(1 + 2*p))/(b^3*(1 + 2*p)) + (a^3*(1 + (b*
x^2)/a)^(3 + 2*p))/(b^3*(3 + 2*p))))/(2*(1 + (b*x^2)/a)^(2*p))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.74

method	result
gosper	$\frac{(b\,x^2+a)\,(2b^2p^2x^4+3b^2p\,x^4+b^2x^4-2abp\,x^2-ab\,x^2+a^2)\,(b^2x^4+2ab\,x^2+a^2)^p}{2b^3(4p^3+12p^2+11p+3)}$
orering	$\frac{(b\,x^2+a)\,(2b^2p^2x^4+3b^2p\,x^4+b^2x^4-2abp\,x^2-ab\,x^2+a^2)\,(b^2x^4+2ab\,x^2+a^2)^p}{2b^3(4p^3+12p^2+11p+3)}$
risch	$\frac{(2b^3p^2x^6+3b^3p\,x^6+2a\,b^2p^2x^4+b^3x^6+a\,b^2p\,x^4-2a^2bp\,x^2+a^3)\left(\left(b\,x^2+a\right)^2\right)^p}{2(p+1)(3+2p)(1+2p)b^3}$
norman	$\frac{x^6\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4p+6}+\frac{a^3\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2b^3(4p^3+12p^2+11p+3)}+\frac{ap\,x^4\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2b(2p^2+5p+3)}-\frac{p\,a^2x^2\mathrm{e}^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{b^2(4p^3+12p^2+11p+3)}$
parallelrisch	$\frac{2x^6(b^2x^4+2ab\,x^2+a^2)^pb^3p^2+3x^6(b^2x^4+2ab\,x^2+a^2)^pb^3p+x^6(b^2x^4+2ab\,x^2+a^2)^pb^3+2x^4(b^2x^4+2ab\,x^2+a^2)^pa\,b^2p^2+x^4(b^2}{2b^3(4p^3+12p^2+11p+3)}$

input
```
int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^2+a)*(2*b^2*p^2*x^4+3*b^2*p*x^4+b^2*x^4-2*a*b*p*x^2-a*b*x^2+a^2)*
(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.83

$$\int x^5\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \frac{((2\,b^3p^2+3\,b^3p+b^3)x^6-2\,a^2bpx^2+(2\,ab^2p^2+ab^2p)x^4+a^3)(b^2x^4+2\,abx^2+a^2)^p}{2\,(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3)}$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
1/2*((2*b^3*p^2 + 3*b^3*p + b^3)*x^6 - 2*a^2*b*p*x^2 + (2*a*b^2*p^2 + a*b^
2*p)*x^4 + a^3)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11
*b^3*p + 3*b^3)
```

Sympy [F]

$$\int x^5 (a^2 + 2abx^2 + b^2x^4)^p \, dx$$

$$= \begin{cases} \frac{x^6(a^2)^p}{6} \\ \int \frac{x^5}{\left((a+bx^2)^2\right)^{\frac{3}{2}}} \, dx \\ -\frac{2a^2 \log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2 \log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2a^2}{2ab^3+2b^4x^2} - \frac{2abx^2 \log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} - \frac{2abx^2 \log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^3+2b^4x^2} + \frac{b^2x^4}{2ab^3+2b^4x^2} \\ \int \frac{x^5}{\sqrt{(a+bx^2)^2}} \, dx \\ \frac{a^3(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3} - \frac{2a^2bpx^2(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3} + \frac{2ab^2p^2x^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3} + \frac{ab^2px^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+24b^3p^2+22b^3p+6b^3} + \frac{2b^3p^2x^6(}{8b^3p^3+2} \end{cases}$$

input
```
integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Piecewise((x**6*(a**2)**p/6, Eq(b, 0)), (Integral(x**5/((a + b*x**2)**2)**
(3/2), x), Eq(p, -3/2)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x
**2) - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b
**3 + 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**
2) - 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(
2*a*b**3 + 2*b**4*x**2), Eq(p, -1)), (Integral(x**5/sqrt((a + b*x**2)**2),
 x), Eq(p, -1/2)), (a**3*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 +
 24*b**3*p**2 + 22*b**3*p + 6*b**3) - 2*a**2*b*p*x**2*(a**2 + 2*a*b*x**2 +
 b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3) + 2*a*b**
2*p**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2
 + 22*b**3*p + 6*b**3) + a*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/
(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3) + 2*b**3*p**2*x**6*(a**2
 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*
b**3) + 3*b**3*p*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 24
*b**3*p**2 + 22*b**3*p + 6*b**3) + b**3*x**6*(a**2 + 2*a*b*x**2 + b**2*x**
4)**p/(8*b**3*p**3 + 24*b**3*p**2 + 22*b**3*p + 6*b**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.61

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{\left((2\,p^2+3\,p+1)b^3x^6+(2\,p^2+p)ab^2x^4-2\,a^2bpx^2+a^3\right)(bx^2+a)^{2\,p}}{2\,(4\,p^3+12\,p^2+11\,p+3)b^3}$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + a
^3)*(b*x^2 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.81

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{2\,(b^2x^4+2\,abx^2+a^2)^p b^3p^2x^6+3\,(b^2x^4+2\,abx^2+a^2)^p b^3px^6+2\,(b^2x^4+2\,abx^2+a^2)^p ab^2p^2x^4+(b^2x^4+}{2\,(4\,b^3p^3+12\,b}$$

input
```
integrate(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
1/2*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*p^2*x^6 + 3*(b^2*x^4 + 2*a*b*x^2
+ a^2)^p*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p^2*x^4 + (b^2*
x^4 + 2*a*b*x^2 + a^2)^p*b^3*x^6 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p*x
^4 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b*p*x^2 + (b^2*x^4 + 2*a*b*x^2 +
a^2)^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Mupad [B] (verification not implemented)

Time = 17.90 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p \left(\frac{x^6\,\left(p^2+\frac{3\,p}{2}+\frac{1}{2}\right)}{4\,p^3+12\,p^2+11\,p+3} + \frac{a^3}{2\,b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)} - \frac{a^2\,p\,x^2}{b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)} + \frac{a\,p\,x^4\,\left(2\,p+1\right)}{2\,b\,\left(4\,p^3+12\,p^2+11\,p+3\right)}\right)$$

input
```
int(x^5*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
(a^2 + b^2*x^4 + 2*a*b*x^2)^p*((x^6*((3*p)/2 + p^2 + 1/2))/(11*p + 12*p^2
+ 4*p^3 + 3) + a^3/(2*b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (a^2*p*x^2)/(b^2*
(11*p + 12*p^2 + 4*p^3 + 3)) + (a*p*x^4*(2*p + 1))/(2*b*(11*p + 12*p^2 + 4
*p^3 + 3)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.80

$$\int x^5 \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{\left(b^2x^4+2ab\,x^2+a^2\right)^p \left(2b^3p^2x^6+3b^3p\,x^6+2a\,b^2p^2x^4+b^3x^6+a\,b^2p\,x^4-2a^2bp\,x^2+a^3\right)}{2b^3\left(4p^3+12p^2+11p+3\right)}$$

input
```
int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x**2 + b**2*x**4)**p*(a**3 - 2*a**2*b*p*x**2 + 2*a*b**2*p**
2*x**4 + a*b**2*p*x**4 + 2*b**3*p**2*x**6 + 3*b**3*p*x**6 + b**3*x**6))/(2
*b**3*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.687 $\int x^3(a^2+2abx^2+b^2x^4)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^p\,dx=-\frac{a(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(1+2p)}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{1+p}}{4b^2(1+p)}$$

output
```
-1/2*a*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^2/(1+2*p)+1/4*(b^2*x^4+2*a*b*
x^2+a^2)^(p+1)/b^2/(p+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int x^3\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{(a+bx^2)\left((a+bx^2)^2\right)^p(-a+b(1+2p)x^2)}{4b^2(1+p)(1+2p)}$$

input
```
Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
((a + b*x^2)*((a + b*x^2)^2)^p*(-a + b*(1 + 2*p)*x^2))/(4*b^2*(1 + p)*(1 +
 2*p))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1434, 1100, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a^2+2 a b x^2+b^2 x^4\right)^p \, dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2} \int x^2 \left(b^2 x^4+2 a b x^2+a^2\right)^p dx^2$$

$$\Big\downarrow \text{ 1100}$$

$$\frac{1}{2} \left(\frac{\left(a^2+2 a b x^2+b^2 x^4\right)^{p+1}}{2 b^2 (p+1)}-\frac{a \int \left(b^2 x^4+2 a b x^2+a^2\right)^p dx^2}{b}\right)$$

$$\Big\downarrow \text{ 1079}$$

$$\frac{1}{2} \left(\frac{\left(a^2+2 a b x^2+b^2 x^4\right)^{p+1}}{2 b^2 (p+1)}-\frac{a \left(a b+b^2 x^2\right)^{-2 p} \left(a^2+2 a b x^2+b^2 x^4\right)^p \int \left(b^2 x^2+a b\right)^{2 p} dx^2}{b}\right)$$

$$\Big\downarrow \text{ 17}$$

$$\frac{1}{2} \left(\frac{\left(a^2+2 a b x^2+b^2 x^4\right)^{p+1}}{2 b^2 (p+1)}-\frac{a \left(a b+b^2 x^2\right) \left(a^2+2 a b x^2+b^2 x^4\right)^p}{b^3 (2 p+1)}\right)$$

input
```
Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(-((a*(a*b + b^2*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^3*(1 + 2*p))) + (a
^2 + 2*a*b*x^2 + b^2*x^4)^(1 + p)/(2*b^2*(1 + p)))/2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1100
```
Int[((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
 :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b*
e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[b^2 - 4*a*c, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\left(-2 b^{2} p \,x^{4}-b^{2} x^{4}-2 a b p \,x^{2}+a^{2}\right) \left(\left(b \,x^{2}+a \right)^{2}\right)^{p}}{4 b^{2} \left(p +1\right) \left(1+2 p \right)}$	58
gosper	$-\frac{\left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} \left(-2 p b \,x^{2}-b \,x^{2}+a \right) \left(b \,x^{2}+a \right)}{4 b^{2} \left(2 p^{2}+3 p +1\right)}$	60
orering	$-\frac{\left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} \left(-2 p b \,x^{2}-b \,x^{2}+a \right) \left(b \,x^{2}+a \right)}{4 b^{2} \left(2 p^{2}+3 p +1\right)}$	60
norman	$\frac{x^{4} {\mathrm e}^{p \ln \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}}{4 p +4}-\frac{a^{2} {\mathrm e}^{p \ln \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}}{4 b^{2} \left(2 p^{2}+3 p +1\right)}+\frac{p a \,x^{2} {\mathrm e}^{p \ln \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}}{2 b \left(2 p^{2}+3 p +1\right)}$	120
parallelrisch	$\frac{2 x^{4} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} a \,b^{2} p +x^{4} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} a \,b^{2}+2 x^{2} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} a^{2} b p -\left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} a^{3}}{4 \left(p +1\right) \left(1+2 p \right) b^{2} a}$	135

input
```
int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b^2*p*x^4-b^2*x^4-2*a*b*p*x^2+a^2)/b^2/(p+1)/(1+2*p)*((b*x^2+a)^2
)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int x^3 (a^2 + 2abx^2 + b^2x^4)^p \, dx = \frac{(2\,abpx^2 + (2\,b^2p + b^2)x^4 - a^2)(b^2x^4 + 2\,abx^2 + a^2)^p}{4\,(2\,b^2p^2 + 3\,b^2p + b^2)}$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
1/4*(2*a*b*p*x^2 + (2*b^2*p + b^2)*x^4 - a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^
p/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Sympy [F]

$$\int x^3 (a^2 + 2abx^2 + b^2x^4)^p \, dx$$
$$= \begin{cases} \frac{x^4 (a^2)^p}{4} & \text{for } b = 0 \\ \frac{a \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{a \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{a}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(x - \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} + \frac{bx^2 \log\left(x + \sqrt{-\frac{a}{b}}\right)}{2ab^2 + 2b^3x^2} & \text{for } p = -1 \\ \int \frac{x^3}{\sqrt{(a + bx^2)^2}}\, dx & \text{for } p = -\frac{1}{2} \\ -\frac{a^2 (a^2 + 2abx^2 + b^2x^4)^p}{8b^2p^2 + 12b^2p + 4b^2} + \frac{2abpx^2 (a^2 + 2abx^2 + b^2x^4)^p}{8b^2p^2 + 12b^2p + 4b^2} + \frac{2b^2px^4 (a^2 + 2abx^2 + b^2x^4)^p}{8b^2p^2 + 12b^2p + 4b^2} + \frac{b^2x^4 (a^2 + 2abx^2 + b^2x^4)^p}{8b^2p^2 + 12b^2p + 4b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Piecewise((x**4*(a**2)**p/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 +
 2*b**3*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b*
*2 + 2*b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) +
b*x**2*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -1)), (Integral
(x**3/sqrt((a + b*x**2)**2), x), Eq(p, -1/2)), (-a**2*(a**2 + 2*a*b*x**2 +
 b**2*x**4)**p/(8*b**2*p**2 + 12*b**2*p + 4*b**2) + 2*a*b*p*x**2*(a**2 + 2
*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 12*b**2*p + 4*b**2) + 2*b**2*p*x*
*4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 12*b**2*p + 4*b**2) +
 b**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 12*b**2*p + 4
*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{(b^2(2\,p+1)x^4+2\,abpx^2-a^2)(bx^2+a)^{2\,p}}{4\,(2\,p^2+3\,p+1)b^2}$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*
p + 1)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.71

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$$
$$= \frac{2\,(b^2x^4+2\,abx^2+a^2)^p b^2px^4+(b^2x^4+2\,abx^2+a^2)^p b^2x^4+2\,(b^2x^4+2\,abx^2+a^2)^p abpx^2-(b^2x^4+2\,abx}{4\,(2\,b^2p^2+3\,b^2p+b^2)}$$

input
```
integrate(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^2*p*x^4 + (b^2*x^4 + 2*a*b*x^2 + a^
2)^p*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b*p*x^2 - (b^2*x^4 + 2*a*
b*x^2 + a^2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Mupad [B] (verification not implemented)

Time = 18.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,\left(\frac{x^4\,\left(2\,p+1\right)}{4\,\left(2\,p^2+3\,p+1\right)}-\frac{a^2}{4\,b^2\,\left(2\,p^2+3\,p+1\right)}+\frac{a\,p\,x^2}{2\,b\,\left(2\,p^2+3\,p+1\right)}\right)$$

input
```
int(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
(a^2 + b^2*x^4 + 2*a*b*x^2)^p*((x^4*(2*p + 1))/(4*(3*p + 2*p^2 + 1)) - a^2
/(4*b^2*(3*p + 2*p^2 + 1)) + (a*p*x^2)/(2*b*(3*p + 2*p^2 + 1)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int x^3 \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{\left(b^2x^4+2ab\,x^2+a^2\right)^p\left(2b^2p\,x^4+b^2x^4+2abp\,x^2-a^2\right)}{4b^2\left(2p^2+3p+1\right)}$$

input
```
int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x**2 + b**2*x**4)**p*( - a**2 + 2*a*b*p*x**2 + 2*b**2*p*x**
4 + b**2*x**4))/(4*b**2*(2*p**2 + 3*p + 1))
```

3.688 $\int x(a^2+2abx^2+b^2x^4)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 41

$$\int x\left(a^2+2abx^2+b^2x^4\right)^p \, dx=\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p}{2b(1+2p)}$$

output
```
1/2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int x\left(a^2+2abx^2+b^2x^4\right)^p \, dx=\frac{(a+bx^2)\left((a+bx^2)^2\right)^p}{2b+4bp}$$

input
```
Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
((a + b*x^2)*((a + b*x^2)^2)^p)/(2*b + 4*b*p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1432, 1079, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a^2+2abx^2+b^2x^4)^p\,dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int (b^2x^4+2abx^2+a^2)^p\,dx^2$$

$$\Big\downarrow \text{1079}$$

$$\frac{1}{2}(ab+b^2x^2)^{-2p}(a^2+2abx^2+b^2x^4)^p\int (b^2x^2+ab)^{2p}\,dx^2$$

$$\Big\downarrow \text{17}$$

$$\frac{(ab+b^2x^2)(a^2+2abx^2+b^2x^4)^p}{2b^2(2p+1)}$$

input
```
Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
((a*b + b^2*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(1 + 2*p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 1079
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x + c
*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))   Int[(b/2 + c
*x)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{(b\,x^2+a)\left((b\,x^2+a)^2\right)^p}{2b(1+2p)}$	31
gosper	$\frac{(b\,x^2+a)\,(b^2x^4+2ab\,x^2+a^2)^p}{2b(1+2p)}$	40
orering	$\frac{(b\,x^2+a)\,(b^2x^4+2ab\,x^2+a^2)^p}{2b(1+2p)}$	40
parallelrisch	$\frac{x^2(b^2x^4+2ab\,x^2+a^2)^p b+(b^2x^4+2ab\,x^2+a^2)^p a}{2b(1+2p)}$	61
norman	$\frac{x^2 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4p+2}+\frac{a\,e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2b(1+2p)}$	71

input
```
int(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^2+a)/b/(1+2*p)*((b*x^2+a)^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int x(a^2+2abx^2+b^2x^4)^p\,dx=\frac{(bx^2+a)(b^2x^4+2\,abx^2+a^2)^p}{2\,(2\,bp+b)}$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
1/2*(b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b*p + b)
```

Sympy [F]

$$\int x\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \begin{cases} \frac{x^2}{2\sqrt{a^2}} & \text{for } b=0 \wedge p=-\frac{1}{2} \\ \frac{x^2(a^2)^p}{2} & \text{for } b=0 \\ \int \frac{x}{\sqrt{(a+bx^2)^2}}\,dx & \text{for } p=-\frac{1}{2} \\ \frac{a(a^2+2abx^2+b^2x^4)^p}{4bp+2b} + \frac{bx^2(a^2+2abx^2+b^2x^4)^p}{4bp+2b} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Piecewise((x**2/(2*sqrt(a**2)), Eq(b, 0) & Eq(p, -1/2)), (x**2*(a**2)**p/2
, Eq(b, 0)), (Integral(x/sqrt((a + b*x**2)**2), x), Eq(p, -1/2)), (a*(a**2
 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 2*b) + b*x**2*(a**2 + 2*a*b*x**2 +
b**2*x**4)**p/(4*b*p + 2*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int x\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \frac{(bx^2+a)(bx^2+a)^{2p}}{2b(2p+1)}$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)/(b*(2*p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int x(a^2 + 2abx^2 + b^2x^4)^p \, dx = \frac{(b^2x^4 + 2\,abx^2 + a^2)^p bx^2 + (b^2x^4 + 2\,abx^2 + a^2)^p a}{2\,(2\,bp + b)}$$

input
```
integrate(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
1/2*((b^2*x^4 + 2*a*b*x^2 + a^2)^p*b*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a
)/(2*b*p + b)
```

Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int x(a^2 + 2abx^2 + b^2x^4)^p \, dx = \left(\frac{x^2}{2\,(2\,p+1)} + \frac{a}{2\,b\,(2\,p+1)}\right)\,(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^p$$

input
```
int(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
(x^2/(2*(2*p + 1)) + a/(2*b*(2*p + 1)))*(a^2 + b^2*x^4 + 2*a*b*x^2)^p
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int x(a^2 + 2abx^2 + b^2x^4)^p \, dx = \frac{(b^2x^4 + 2ab\,x^2 + a^2)^p\,(b\,x^2 + a)}{2b\,(2p + 1)}$$

input
```
int(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x**2 + b**2*x**4)**p*(a + b*x**2))/(2*b*(2*p + 1))
```

3.689 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x}\,dx = -\frac{(a+bx^2)(a^2+2abx^2+b^2x^4)^p \text{Hypergeometric2F1}\left(1,1+2p,2(1+p),1+\frac{bx^2}{a}\right)}{2a(1+2p)}$$

output
```
-1/2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([1, 1+2*p],[2*p+2],1+b*
x^2/a)/a/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x}\,dx = -\frac{(a+bx^2)\left((a+bx^2)^2\right)^p \text{Hypergeometric2F1}\left(1,1+2p,2+2p,1+\frac{bx^2}{a}\right)}{2a(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x,x]
```

output
```
-1/2*((a + b*x^2)*((a + b*x^2)^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 + 2*p,
 1 + (b*x^2)/a])/(a*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1385, 243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x}dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\int \frac{\left(\frac{bx^2}{a}+1\right)^{2p}}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\int \frac{\left(\frac{bx^2}{a}+1\right)^{2p}}{x^2}dx^2$$

$$\Big\downarrow \text{75}$$

$$-\frac{\left(\frac{bx^2}{a}+1\right)(a^2+2abx^2+b^2x^4)^p\,\text{Hypergeometric2F1}\left(1,2p+1,2(p+1),\frac{bx^2}{a}+1\right)}{2(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x,x]
```

output
```
-1/2*((1 + (b*x^2)/a)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1, 1
 + 2*p, 2*(1 + p), 1 + (b*x^2)/a])/(1 + 2*p)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{(b^2x^4 + 2ab\,x^2 + a^2)^p}{x}dx$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/x,x)
```

output
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/x,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{x}\,dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{x}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x,x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{x} \, dx = \int \frac{\left((a + bx^2)^2\right)^p}{x} \, dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x,x)
```

output
```
Integral(((a + b*x**2)**2)**p/x, x)
```

Maxima [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{x} \, dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{x} \, dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x,x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{x} \, dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{x} \, dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x,x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x}\,dx=\int \frac{(a^2+2\,a\,b\,x^2+b^2\,x^4)^p}{x}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x, x)
```

Reduce [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x}\,dx=\frac{(b^2x^4+2ab\,x^2+a^2)^p+4\left(\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{b\,x^3+ax}dx\right)ap}{4p}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/x,x)
```

output
```
((a**2 + 2*a*b*x**2 + b**2*x**4)**p + 4*int((a**2 + 2*a*b*x**2 + b**2*x**4
)**p/(a*x + b*x**3),x)*a*p)/(4*p)
```

3.690 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3}\,dx = \frac{b(a+bx^2)(a^2+2abx^2+b^2x^4)^p \text{Hypergeometric2F1}\left(2,1+2p,2(1+p),1+\frac{bx^2}{a}\right)}{2a^2(1+2p)}$$

output
```
1/2*b*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([2, 1+2*p],[2*p+2],1+b
*x^2/a)/a^2/(1+2*p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3}\,dx = \frac{b(a+bx^2)\left((a+bx^2)^2\right)^p \text{Hypergeometric2F1}\left(2,1+2p,2+2p,1+\frac{bx^2}{a}\right)}{2a^2(1+2p)}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^3,x]
```

output
```
(b*(a + b*x^2)*((a + b*x^2)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, 1
+ (b*x^2)/a])/(2*a^2*(1 + 2*p))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1385, 243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3}dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\int \frac{\left(\frac{bx^2}{a}+1\right)^{2p}}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\int \frac{\left(\frac{bx^2}{a}+1\right)^{2p}}{x^4}dx^2$$

$$\Big\downarrow \text{75}$$

$$\frac{b\left(\frac{bx^2}{a}+1\right)(a^2+2abx^2+b^2x^4)^p\text{Hypergeometric2F1}\left(2,2p+1,2(p+1),\frac{bx^2}{a}+1\right)}{2a(2p+1)}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^3,x]
```

output
```
(b*(1 + (b*x^2)/a)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[2, 1 +
2*p, 2*(1 + p), 1 + (b*x^2)/a])/(2*a*(1 + 2*p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{(b^2x^4 + 2ab\,x^2 + a^2)^p}{x^3}dx$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x)
```

output
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{x^3}\,dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{x^3}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3}\,dx = \int \frac{\left((a+bx^2)^2\right)^p}{x^3}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**3,x)
```

output
```
Integral(((a + b*x**2)**2)**p/x**3, x)
```

Maxima [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3}\,dx = \int \frac{(b^2x^4+2\,abx^2+a^2)^p}{x^3}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

Giac [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3}\,dx = \int \frac{(b^2x^4+2\,abx^2+a^2)^p}{x^3}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3}\,dx=\int \frac{(a^2+2\,a\,b\,x^2+b^2\,x^4)^p}{x^3}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^3,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^3, x)
```

Reduce [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3}\,dx=\frac{-(b^2x^4+2ab\,x^2+a^2)^p+4\left(\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{b\,x^3+ax}dx\right)bp\,x^2}{2x^2}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x)
```

output
```
( - (a**2 + 2*a*b*x**2 + b**2*x**4)**p + 4*int((a**2 + 2*a*b*x**2 + b**2*x
**4)**p/(a*x + b*x**3),x)*b*p*x**2)/(2*x**2)
```

3.691 $\int x^4(a^2+2abx^2+b^2x^4)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{1}{5}x^5\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\text{Hypergeometric2F1}\left(\frac{5}{2},-2p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

output
```
1/5*x^5*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([5/2, -2*p],[7/2],-b*x^2/a)/((
1+b*x^2/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x^4\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{1}{5}x^5\left(\left(a+bx^2\right)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{5}{2},-2p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

input
```
Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(x^5*((a + b*x^2)^2)^p*Hypergeometric2F1[5/2, -2*p, 7/2, -((b*x^2)/a)])/(5
*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1385, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p \int x^4 \left(\frac{bx^2}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{5} x^5 \left(\frac{bx^2}{a}+1\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p \text{Hypergeometric2F1}\left(\frac{5}{2},-2p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/2, -2*p, 7/2, -((b*
x^2)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int x^4 (b^2x^4 + 2ab\,x^2 + a^2)^p \,dx$$

input
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Fricas [F]

$$\int x^4 (a^2 + 2abx^2 + b^2x^4)^p \, dx = \int (b^2x^4 + 2\,abx^2 + a^2)^p x^4 \,dx$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

Sympy [F]

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int x^4 \left(\left(a+bx^2\right)^2\right)^p\,dx$$

input
```
integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Integral(x**4*((a + b*x**2)**2)**p, x)
```

Maxima [F]

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \left(b^2x^4+2\,abx^2+a^2\right)^p x^4\,dx$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

Giac [F]

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \left(b^2x^4+2\,abx^2+a^2\right)^p x^4\,dx$$

input
```
integrate(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^p \,dx=\int x^4\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p \,dx$$

input
```
int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
int(x^4*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

Reduce [F]

$$\int x^4 \left(a^2+2abx^2+b^2x^4\right)^p \,dx$$
$$= \frac{-12(b^2x^4+2ab\,x^2+a^2)^p\,a^2px+16(b^2x^4+2ab\,x^2+a^2)^p\,ab\,p^2x^3+4(b^2x^4+2ab\,x^2+a^2)^p\,abp\,x^3+16(b}{}$$

input
```
int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
( - 12*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*a**2*p*x + 16*(a**2 + 2*a*b*x**2
 + b**2*x**4)**p*a*b*p**2*x**3 + 4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*a*b*
p*x**3 + 16*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*b**2*p**2*x**5 + 16*(a**2 +
 2*a*b*x**2 + b**2*x**4)**p*b**2*p*x**5 + 3*(a**2 + 2*a*b*x**2 + b**2*x**4
)**p*b**2*x**5 + 768*int((a**2 + 2*a*b*x**2 + b**2*x**4)**p/(64*a*p**3 + 1
44*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**2 + 144*b*p**2*x**2 + 92*b*p*x**2
 + 15*b*x**2),x)*a**3*p**4 + 1728*int((a**2 + 2*a*b*x**2 + b**2*x**4)**p/(
64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**2 + 144*b*p**2*x**2
+ 92*b*p*x**2 + 15*b*x**2),x)*a**3*p**3 + 1104*int((a**2 + 2*a*b*x**2 + b*
*2*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**2 + 144
*b*p**2*x**2 + 92*b*p*x**2 + 15*b*x**2),x)*a**3*p**2 + 180*int((a**2 + 2*a
*b*x**2 + b**2*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**
3*x**2 + 144*b*p**2*x**2 + 92*b*p*x**2 + 15*b*x**2),x)*a**3*p)/(b**2*(64*p
**3 + 144*p**2 + 92*p + 15))
```

3.692 $\int x^2(a^2+2abx^2+b^2x^4)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int x^2(a^2+2abx^2+b^2x^4)^p\,dx=\frac{1}{3}x^3\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,\text{Hypergeometric2F1}\left(\frac{3}{2},-2p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

output
```
1/3*x^3*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([3/2, -2*p],[5/2],-b*x^2/a)/((
1+b*x^2/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x^2(a^2+2abx^2+b^2x^4)^p\,dx=\frac{1}{3}x^3\left((a+bx^2)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{3}{2},-2p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

input
```
Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(x^3*((a + b*x^2)^2)^p*Hypergeometric2F1[3/2, -2*p, 5/2, -((b*x^2)/a)])/(3
*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p \int x^2 \left(\frac{bx^2}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{3}x^3 \left(\frac{bx^2}{a}+1\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p \text{Hypergeometric2F1}\left(\frac{3}{2},-2p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/2, -2*p, 5/2, -((b*
x^2)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int x^{2}\left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{p}dx$$

input
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Fricas [F]

$$\int x^2\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int\left(b^{2} x^{4}+2\,a b x^{2}+a^{2}\right)^{p} x^{2}\,dx$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

Sympy [F]

$$\int x^2(a^2+2abx^2+b^2x^4)^p\,dx=\int x^2\left((a+bx^2)^2\right)^p\,dx$$

input
```
integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Integral(x**2*((a + b*x**2)**2)**p, x)
```

Maxima [F]

$$\int x^2(a^2+2abx^2+b^2x^4)^p\,dx=\int (b^2x^4+2\,abx^2+a^2)^p x^2\,dx$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

Giac [F]

$$\int x^2(a^2+2abx^2+b^2x^4)^p\,dx=\int (b^2x^4+2\,abx^2+a^2)^p x^2\,dx$$

input
```
integrate(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a^2 + 2abx^2 + b^2x^4)^p \, dx = \int x^2 \left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)^p \,dx$$

input
```
int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
int(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

Reduce [F]

$$\int x^2 (a^2 + 2abx^2 + b^2x^4)^p \, dx$$
$$= \frac{4(b^2x^4 + 2ab\,x^2 + a^2)^p\,apx + 4(b^2x^4 + 2ab\,x^2 + a^2)^p\,bp\,x^3 + (b^2x^4 + 2ab\,x^2 + a^2)^p\,b\,x^3 - 64\Big(\int \frac{(}{16b\,p^2x^2+16}}{}$$

input
```
int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
(4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*a*p*x + 4*(a**2 + 2*a*b*x**2 + b**2*
x**4)**p*b*p*x**3 + (a**2 + 2*a*b*x**2 + b**2*x**4)**p*b*x**3 - 64*int((a*
*2 + 2*a*b*x**2 + b**2*x**4)**p/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**2
 + 16*b*p*x**2 + 3*b*x**2),x)*a**2*p**3 - 64*int((a**2 + 2*a*b*x**2 + b**2
*x**4)**p/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**2 + 16*b*p*x**2 + 3*b*x
**2),x)*a**2*p**2 - 12*int((a**2 + 2*a*b*x**2 + b**4)**p/(16*a*p**2 +
 16*a*p + 3*a + 16*b*p**2*x**2 + 16*b*p*x**2 + 3*b*x**2),x)*a**2*p)/(b*(16
*p**2 + 16*p + 3))
```

3.693 $\int (a^2 + 2abx^2 + b^2x^4)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int (a^2 + 2abx^2 + b^2x^4)^p \, dx = x\left(1 + \frac{bx^2}{a}\right)^{-2p} (a^2 + 2abx^2 + b^2x^4)^p \text{Hypergeometric2F1}\left(\frac{1}{2}, -2p, \frac{3}{2}, -\frac{bx^2}{a}\right)$$

output
```
x*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([1/2, -2*p],[3/2],-b*x^2/a)/((1+b*x^
2/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int (a^2 + 2abx^2 + b^2x^4)^p \, dx = x\left((a + bx^2)^2\right)^p \left(1 + \frac{bx^2}{a}\right)^{-2p} \text{Hypergeometric2F1}\left(\frac{1}{2}, -2p, \frac{3}{2}, -\frac{bx^2}{a}\right)$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(x*((a + b*x^2)^2)^p*Hypergeometric2F1[1/2, -2*p, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(2*p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1385, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^2+2abx^2+b^2x^4\right)^p\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\int \left(\frac{bx^2}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{237}$$

$$x\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2},-2p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/2, -2*p, 3/2, -((b*x^
2)/a)])/(1 + (b*x^2)/a)^(2*p)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int (b^2x^4 + 2ab\,x^2 + a^2)^p\,dx$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
int((b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Fricas [F]

$$\int (a^2 + 2abx^2 + b^2x^4)^p \, dx = \int (b^2x^4 + 2\,abx^2 + a^2)^p\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Sympy [F]

$$\int \left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \left(a^2+2abx^2+b^2x^4\right)^p\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**p, x)
```

Maxima [F]

$$\int \left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Giac [F]

$$\int \left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

Reduce [F]

$$\int \left(a^2+2abx^2+b^2x^4\right)^p\,dx$$
$$=\frac{\left(b^2x^4+2ab\,x^2+a^2\right)^p x+16\left(\int \frac{\left(b^2x^4+2ab\,x^2+a^2\right)^p}{4bp\,x^2+b\,x^2+4ap+a}dx\right)a\,p^2+4\left(\int \frac{\left(b^2x^4+2ab\,x^2+a^2\right)^p}{4bp\,x^2+b\,x^2+4ap+a}dx\right)ap}{4p+1}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
((a**2 + 2*a*b*x**2 + b**2*x**4)**p*x + 16*int((a**2 + 2*a*b*x**2 + b**2*x
**4)**p/(4*a*p + a + 4*b*p*x**2 + b*x**2),x)*a*p**2 + 4*int((a**2 + 2*a*b*
x**2 + b**2*x**4)**p/(4*a*p + a + 4*b*p*x**2 + b*x**2),x)*a*p)/(4*p + 1)
```

3.694 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^2}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-2p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

output
```
-(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-1/2, -2*p],[1/2],-b*x^2/a)/x/((1+b*
x^2/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^2}\,dx = -\frac{\left((a+bx^2)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-2p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^2,x]
```

output
```
-((((a + b*x^2)^2)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(2*p)))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1385, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^2} \, dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p} (a^2+2abx^2+b^2x^4)^p \int \frac{\left(\frac{bx^2}{a}+1\right)^{2p}}{x^2}dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p} (a^2+2abx^2+b^2x^4)^p \text{Hypergeometric2F1}\left(-\frac{1}{2},-2p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^2,x]
```

output
```
-(((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, -((b*x
^2)/a)])/(x*(1 + (b*x^2)/a)^(2*p)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{(b^2x^4 + 2ab\,x^2 + a^2)^p}{x^2}dx$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x)
```

output
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{x^2}\,dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{x^2}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^2}\,dx=\int \frac{\left((a+bx^2)^2\right)^p}{x^2}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**2,x)
```

output
```
Integral(((a + b*x**2)**2)**p/x**2, x)
```

Maxima [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^2}\,dx=\int \frac{(b^2x^4+2\,abx^2+a^2)^p}{x^2}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^2}\,dx=\int \frac{(b^2x^4+2\,abx^2+a^2)^p}{x^2}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{x^2} \, dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^p}{x^2} \, dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^2,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{x^2} \, dx = \frac{(b^2x^4 + 2ab\,x^2 + a^2)^p + 16\left(\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{4bp\,x^4-b\,x^4+4ap\,x^2-a\,x^2}dx\right) a\,p^2x - 4\left(\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{4bp\,x^4-b\,x^4+4ap\,x^2-a\,x^2}dx\right) apx}{x\,(4p-1)}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x)
```

output
```
((a**2 + 2*a*b*x**2 + b**2*x**4)**p + 16*int((a**2 + 2*a*b*x**2 + b**2*x**
4)**p/(4*a*p*x**2 - a*x**2 + 4*b*p*x**4 - b*x**4),x)*a*p**2*x - 4*int((a**
2 + 2*a*b*x**2 + b**2*x**4)**p/(4*a*p*x**2 - a*x**2 + 4*b*p*x**4 - b*x**4)
,x)*a*p*x)/(x*(4*p - 1))
```

3.695 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^4} dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p \text{Hypergeometric2F1}\left(-\frac{3}{2},-2p,-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3}$$

output
```
-1/3*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-3/2, -2*p],[-1/2],-b*x^2/a)/x^3
/((1+b*x^2/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^4} dx = -\frac{\left((a+bx^2)^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-2p} \text{Hypergeometric2F1}\left(-\frac{3}{2},-2p,-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^4,x]
```

output
```
-1/3*(((a + b*x^2)^2)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1385, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^4}\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\int \frac{\left(\frac{bx^2}{a}+1\right)^{2p}}{x^4}dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-2p,-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^4,x]
```

output
```
-1/3*((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, -(
(b*x^2)/a)])/(x^3*(1 + (b*x^2)/a)^(2*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{(b^2x^4 + 2ab\,x^2 + a^2)^p}{x^4}dx$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x)
```

output
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{x^4}\,dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{x^4}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{x^4} \, dx = \int \frac{\left((a + bx^2)^2\right)^p}{x^4} \, dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**4,x)
```

output
```
Integral(((a + b*x**2)**2)**p/x**4, x)
```

Maxima [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{x^4} \, dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{x^4} \, dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{x^4} \, dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{x^4} \, dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^4}\,dx=\int \frac{(a^2+2\,a\,b\,x^2+b^2\,x^4)^p}{x^4}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^4,x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/x^4, x)
```

Reduce [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^4}\,dx = \frac{(b^2x^4+2ab\,x^2+a^2)^p+16\left(\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{4bp\,x^6-3b\,x^6+4ap\,x^4-3a\,x^4}dx\right)a\,p^2x^3-12\left(\int \frac{(b^2x^4+2ab\,x^2+a^2)^p}{4bp\,x^6-3b\,x^6+4ap\,x^4-3a\,x^4}dx\right)ap\,x^3}{x^3\,(4p-3)}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x)
```

output
```
((a**2 + 2*a*b*x**2 + b**2*x**4)**p + 16*int((a**2 + 2*a*b*x**2 + b**2*x**
4)**p/(4*a*p*x**4 - 3*a*x**4 + 4*b*p*x**6 - 3*b*x**6),x)*a*p**2*x**3 - 12*
int((a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*a*p*x**4 - 3*a*x**4 + 4*b*p*x**6
 - 3*b*x**6),x)*a*p*x**3)/(x**3*(4*p - 3))
```

3.696 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 67

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{2(dx)^{5/2}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p \text{Hypergeometric2F1}\left(\frac{5}{4},-2p,\frac{9}{4},-\frac{bx^2}{a}\right)}{5d}$$

output
```
2/5*(d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([5/4, -2*p],[9/4],-b*x
^2/a)/d/((1+b*x^2/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{2}{5}x(dx)^{3/2}\left(\left(a+bx^2\right)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p} \text{Hypergeometric2F1}\left(\frac{5}{4},-2p,\frac{9}{4},-\frac{bx^2}{a}\right)$$

input
```
Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(2*x*(d*x)^(3/2)*((a + b*x^2)^2)^p*Hypergeometric2F1[5/4, -2*p, 9/4, -((b*
x^2)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1385, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^p \, dx$$

$$\Big\downarrow \text{ 1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p \int (dx)^{3/2} \left(\frac{bx^2}{a}+1\right)^{2p} dx$$

$$\Big\downarrow \text{ 278}$$

$$\frac{2(dx)^{5/2} \left(\frac{bx^2}{a}+1\right)^{-2p} \left(a^2+2abx^2+b^2x^4\right)^p \text{Hypergeometric2F1}\left(\frac{5}{4},-2p,\frac{9}{4},-\frac{bx^2}{a}\right)}{5d}$$

input
```
Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(2*(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/4, -2*p,
9/4, -((b*x^2)/a)])/(5*d*(1 + (b*x^2)/a)^(2*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int (dx)^{\frac{3}{2}} \left(b^2x^4 + 2ab\,x^2 + a^2\right)^p dx$$

input
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Fricas [F]

$$\int (dx)^{3/2} \left(a^2 + 2abx^2 + b^2x^4\right)^p \, dx = \int (dx)^{\frac{3}{2}} \left(b^2x^4 + 2\,abx^2 + a^2\right)^p dx$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*d*x, x)
```

Sympy [F]

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int (dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^p\,dx$$

input
```
integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Integral((d*x)**(3/2)*((a + b*x**2)**2)**p, x)
```

Maxima [F]

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int (dx)^{\frac{3}{2}}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Giac [F]

$$\int (dx)^{3/2}\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int (dx)^{\frac{3}{2}}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

input
```
integrate((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \int (d\,x)^{3/2}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,dx$$

input
```
int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
int((d*x)^(3/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

Reduce [F]

$$\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{2\sqrt{d}\,d\Big(8\sqrt{x}\,(b^2x^4+2ab\,x^2+a^2)^p\,ap+8\sqrt{x}\,(b^2x^4+2ab\,x^2+a^2)^p\,bp\,x^2+\sqrt{x}\,(b^2x^4+2ab\,x^2}{}$$

input
```
int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
(2*sqrt(d)*d*(8*sqrt(x)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*a*p + 8*sqrt(x)
*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*b*p*x**2 + sqrt(x)*(a**2 + 2*a*b*x**2
+ b**2*x**4)**p*b*x**2 - 256*int((sqrt(x)*(a**2 + 2*a*b*x**2 + b**2*x**4)*
*p)/(64*a*p**2*x + 48*a*p*x + 5*a*x + 64*b*p**2*x**3 + 48*b*p*x**3 + 5*b*x
**3),x)*a**2*p**3 - 192*int((sqrt(x)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p)/(
64*a*p**2*x + 48*a*p*x + 5*a*x + 64*b*p**2*x**3 + 48*b*p*x**3 + 5*b*x**3),
x)*a**2*p**2 - 20*int((sqrt(x)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p)/(64*a*p
**2*x + 48*a*p*x + 5*a*x + 64*b*p**2*x**3 + 48*b*p*x**3 + 5*b*x**3),x)*a**
2*p))/(b*(64*p**2 + 48*p + 5))
```

3.697 $\int \sqrt{dx}(a^2+2abx^2+b^2x^4)^p \, dx$

Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{2(dx)^{3/2}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p \text{Hypergeometric2F1}\left(\frac{3}{4},-2p,\frac{7}{4},-\frac{bx^2}{a}\right)}{3d}$$

output
```
2/3*(d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([3/4, -2*p],[7/4],-b*x
^2/a)/d/((1+b*x^2/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p \, dx = \frac{2}{3}x\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p} \text{Hypergeometric2F1}\left(\frac{3}{4},-2p,\frac{7}{4},-\frac{bx^2}{a}\right)$$

input
```
Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(2*x*Sqrt[d*x]*((a + b*x^2)^2)^p*Hypergeometric2F1[3/4, -2*p, 7/4, -((b*x^
2)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1385, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p \, dx$$

$$\Big\downarrow \text{ 1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\int \sqrt{dx}\left(\frac{bx^2}{a}+1\right)^{2p}dx$$

$$\Big\downarrow \text{ 278}$$

$$\frac{2(dx)^{3/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\text{Hypergeometric2F1}\left(\frac{3}{4},-2p,\frac{7}{4},-\frac{bx^2}{a}\right)}{3d}$$

input
```
Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

output
```
(2*(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/4, -2*p,
7/4, -((b*x^2)/a)])/(3*d*(1 + (b*x^2)/a)^(2*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \sqrt{dx}\,\left(b^{2} x^{4}+2 a b\,x^{2}+a^{2}\right)^{p} dx$$

input
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Fricas [F]

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \sqrt{dx}\left(b^{2} x^{4}+2\,a b x^{2}+a^{2}\right)^{p}\,dx$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Sympy [F]

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \sqrt{dx}\left(\left(a+bx^2\right)^2\right)^p\,dx$$

input
```
integrate((d*x)**(1/2)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

output
```
Integral(sqrt(d*x)*((a + b*x**2)**2)**p, x)
```

Maxima [F]

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Giac [F]

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

input
```
integrate((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\int \sqrt{d\,x}\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,dx$$

input
```
int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

output
```
int((d*x)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p, x)
```

Reduce [F]

$$\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^p\,dx = \frac{2\sqrt{d}\left(\sqrt{x}\,(b^2x^4+2ab\,x^2+a^2)^p\,x+32\left(\int\frac{\sqrt{x}\,(b^2x^4+2ab\,x^2+a^2)^p}{8bp\,x^2+3b\,x^2+8ap+3a}dx\right)a\,p^2+12\left(\int\frac{\sqrt{x}\,(b^2x^4+2ab\,x^2+a^2)^p}{8bp\,x^2+3b\,x^2+8ap+3a}dx\right)ap\right)}{8p+3}$$

input
```
int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

output
```
(2*sqrt(d)*(sqrt(x)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*x + 32*int((sqrt(x)
*(a**2 + 2*a*b*x**2 + b**2*x**4)**p)/(8*a*p + 3*a + 8*b*p*x**2 + 3*b*x**2)
,x)*a*p**2 + 12*int((sqrt(x)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p)/(8*a*p +
3*a + 8*b*p*x**2 + 3*b*x**2),x)*a*p))/(8*p + 3)
```

3.698 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p \text{Hypergeometric2F1}\left(\frac{1}{4},-2p,\frac{5}{4},-\frac{bx^2}{a}\right)}{d}$$

output
```
2*(d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([1/4, -2*p],[5/4],-b*x^2
/a)/d/((1+b*x^2/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{\sqrt{dx}}\,dx = \frac{2x\left((a+bx^2)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(\frac{1}{4},-2p,\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{dx}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/Sqrt[d*x],x]
```

output
```
(2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[1/4, -2*p, 5/4, -((b*x^2)/a)])/(S
qrt[d*x]*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1385, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\int \frac{\left(\frac{bx^2}{a}+1\right)^{2p}}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{278}$$

$$\frac{2\sqrt{dx}\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p \text{Hypergeometric2F1}\left(\frac{1}{4},-2p,\frac{5}{4},-\frac{bx^2}{a}\right)}{d}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/Sqrt[d*x],x]
```

output
```
(2*Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/4, -2*p, 5/
4, -((b*x^2)/a)])/(d*(1 + (b*x^2)/a)^(2*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{(b^2x^4 + 2ab\,x^2 + a^2)^p}{\sqrt{dx}}dx$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x)
```

output
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{\sqrt{dx}}\,dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{\sqrt{dx}}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x), x)
```

Sympy [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{\sqrt{dx}}\,dx=\int \frac{\left((a+bx^2)^2\right)^p}{\sqrt{dx}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(1/2),x)
```

output
```
Integral(((a + b*x**2)**2)**p/sqrt(d*x), x)
```

Maxima [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{\sqrt{dx}}\,dx=\int \frac{(b^2x^4+2\,abx^2+a^2)^p}{\sqrt{dx}}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x), x)
```

Giac [F]

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{\sqrt{dx}}\,dx=\int \frac{(b^2x^4+2\,abx^2+a^2)^p}{\sqrt{dx}}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{\sqrt{dx}}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^p}{\sqrt{d\,x}}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(1/2),x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{\sqrt{dx}}\,dx = \frac{2\sqrt{d}\left(\sqrt{x}\,(b^2x^4 + 2ab\,x^2 + a^2)^p + 32\left(\int \frac{\sqrt{x}\,(b^2x^4+2ab\,x^2+a^2)^p}{8bp\,x^3+b\,x^3+8apx+ax}dx\right)a\,p^2 + 4\left(\int \frac{\sqrt{x}\,(b^2x^4+2ab\,x^2+a^2)^p}{8bp\,x^3+b\,x^3+8apx+ax}dx\right)ap\right)}{d\,(8p+1)}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x)
```

output
```
(2*sqrt(d)*(sqrt(x)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p + 32*int((sqrt(x)*(
a**2 + 2*a*b*x**2 + b**2*x**4)**p)/(8*a*p*x + a*x + 8*b*p*x**3 + b*x**3),x
)*a*p**2 + 4*int((sqrt(x)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p)/(8*a*p*x + a
*x + 8*b*p*x**3 + b*x**3),x)*a*p))/(d*(8*p + 1))
```

3.699 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{3/2}}\,dx = -\frac{2\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,\text{Hypergeometric2F1}\left(-\frac{1}{4},-2p,\frac{3}{4},-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

output
```
-2*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-1/4, -2*p],[3/4],-b*x^2/a)/d/(d*x
)^(1/2)/((1+b*x^2/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{3/2}}\,dx = -\frac{2x\left((a+bx^2)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(-\frac{1}{4},-2p,\frac{3}{4},-\frac{bx^2}{a}\right)}{(dx)^{3/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(3/2),x]
```

output
```
(-2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[-1/4, -2*p, 3/4, -((b*x^2)/a)])/
((d*x)^(3/2)*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1385, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\int \frac{\left(\frac{bx^2}{a}+1\right)^{2p}}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p \text{Hypergeometric2F1}\left(-\frac{1}{4},-2p,\frac{3}{4},-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(3/2),x]
```

output
```
(-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/4, -2*p, 3/4, -((b*
x^2)/a)])/(d*Sqrt[d*x]*(1 + (b*x^2)/a)^(2*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{(b^2x^4 + 2ab\,x^2 + a^2)^p}{(dx)^{\frac{3}{2}}}dx$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x)
```

output
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{(dx)^{3/2}}\,dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d^2*x^2), x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{(dx)^{3/2}} \, dx = \int \frac{\left((a + bx^2)^2\right)^p}{(dx)^{\frac{3}{2}}} \, dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(3/2),x)
```

output
```
Integral(((a + b*x**2)**2)**p/(d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{(dx)^{3/2}} \, dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{(dx)^{\frac{3}{2}}} \, dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{(dx)^{3/2}} \, dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{(dx)^{\frac{3}{2}}} \, dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{(dx)^{3/2}}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^p}{(d\,x)^{3/2}}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(3/2),x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{(dx)^{3/2}}\,dx = \frac{2\sqrt{d}\left((b^2x^4 + 2ab\,x^2 + a^2)^p + 32\sqrt{x}\left(\int \frac{\sqrt{x}\,(b^2x^4+2ab\,x^2+a^2)^p}{8bp\,x^4-b\,x^4+8ap\,x^2-a\,x^2}dx\right)a\,p^2 - 4\sqrt{x}\,(\right.}{\sqrt{x}\,d^2\,(8p-1)}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x)
```

output
```
(2*sqrt(d)*((a**2 + 2*a*b*x**2 + b**2*x**4)**p + 32*sqrt(x)*int((sqrt(x)*(
a**2 + 2*a*b*x**2 + b**2*x**4)**p)/(8*a*p*x**2 - a*x**2 + 8*b*p*x**4 - b*x
**4),x)*a*p**2 - 4*sqrt(x)*int((sqrt(x)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p
)/(8*a*p*x**2 - a*x**2 + 8*b*p*x**4 - b*x**4),x)*a*p))/(sqrt(x)*d**2*(8*p
- 1))
```

3.700 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{5/2}}\,dx = -\frac{2\left(1+\frac{bx^2}{a}\right)^{-2p}(a^2+2abx^2+b^2x^4)^p \text{Hypergeometric2F1}\left(-\frac{3}{4},-2p,\frac{1}{4},-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

output
```
-2/3*(b^2*x^4+2*a*b*x^2+a^2)^p*hypergeom([-3/4, -2*p],[1/4],-b*x^2/a)/d/(d
*x)^(3/2)/((1+b*x^2/a)^(2*p))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{5/2}}\,dx = -\frac{2x\left((a+bx^2)^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(-\frac{3}{4},-2p,\frac{1}{4},-\frac{bx^2}{a}\right)}{3(dx)^{5/2}}$$

input
```
Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(5/2),x]
```

output
```
(-2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[-3/4, -2*p, 1/4, -((b*x^2)/a)])/
(3*(d*x)^(5/2)*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1385, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{1385}$$

$$\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\int \frac{\left(\frac{bx^2}{a}+1\right)^{2p}}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}(a^2+2abx^2+b^2x^4)^p\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-2p,\frac{1}{4},-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

input
```
Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(5/2),x]
```

output
```
(-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/4, -2*p, 1/4, -((b*
x^2)/a)])/(3*d*(d*x)^(3/2)*(1 + (b*x^2)/a)^(2*p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1385
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> S
imp[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*c*(x^n/b))^(2*
FracPart[p]))   Int[u*(1 + 2*c*(x^n/b))^(2*p), x], x] /; FreeQ[{a, b, c, n,
 p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[2*p] && NeQ[u,
 x^(n - 1)] && NeQ[u, x^(2*n - 1)]
```

Maple [F]

$$\int \frac{(b^2x^4 + 2ab\,x^2 + a^2)^p}{(dx)^{\frac{5}{2}}}dx$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x)
```

output
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{(dx)^{5/2}}\,dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d^3*x^3), x)
```

Sympy [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{(dx)^{5/2}}\,dx = \int \frac{\left((a + bx^2)^2\right)^p}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(5/2),x)
```

output
```
Integral(((a + b*x**2)**2)**p/(d*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{(dx)^{5/2}}\,dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{(dx)^{5/2}}\,dx = \int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{(dx)^{5/2}}\,dx = \int \frac{(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^p}{(d\,x)^{5/2}}\,dx$$

input
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(5/2),x)
```

output
```
int((a^2 + b^2*x^4 + 2*a*b*x^2)^p/(d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a^2 + 2abx^2 + b^2x^4)^p}{(dx)^{5/2}}\,dx = \frac{2\sqrt{d}\left((b^2x^4 + 2ab\,x^2 + a^2)^p + 32\sqrt{x}\left(\int \frac{\sqrt{x}\,(b^2x^4+2ab\,x^2+a^2)^p}{8bp\,x^5-3b\,x^5+8ap\,x^3-3a\,x^3}dx\right)a\,p^2x - 12}{\sqrt{x}\,d^3x\,(8p-3)}$$

input
```
int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x)
```

output
```
(2*sqrt(d)*((a**2 + 2*a*b*x**2 + b**2*x**4)**p + 32*sqrt(x)*int((sqrt(x)*(
a**2 + 2*a*b*x**2 + b**2*x**4)**p)/(8*a*p*x**3 - 3*a*x**3 + 8*b*p*x**5 - 3
*b*x**5),x)*a*p**2*x - 12*sqrt(x)*int((sqrt(x)*(a**2 + 2*a*b*x**2 + b**2*x
**4)**p)/(8*a*p*x**3 - 3*a*x**3 + 8*b*p*x**5 - 3*b*x**5),x)*a*p*x))/(sqrt(
x)*d**3*x*(8*p - 3))
```

3.701 $\int \frac{x^6}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 370

$$\begin{aligned}\int \frac{x^6}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx &= -\frac{(9abcd-8(bc+ad)^2)\,x(c+dx^2)}{15b^2d^3\sqrt{ac+(bc+ad)x^2+bdx^4}} \\ &\quad -\frac{4(bc+ad)x\sqrt{ac+(bc+ad)x^2+bdx^4}}{15b^2d^2}+\frac{x^3\sqrt{ac+(bc+ad)x^2+bdx^4}}{5bd} \\ &\quad +\frac{c(9abcd-8(bc+ad)^2)\,(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15\sqrt{a}b^{5/2}d^3\sqrt{ac+(bc+ad)x^2+bdx^4}} \\ &\quad +\frac{4\sqrt{a}c(bc+ad)\,(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{5/2}d^2\sqrt{ac+(bc+ad)x^2+bdx^4}}\end{aligned}$$

output
```
-1/15*(9*a*b*c*d-8*(a*d+b*c)^2)*x*(d*x^2+c)/b^2/d^3/(a*c+(a*d+b*c)*x^2+b*d
*x^4)^(1/2)-4/15*(a*d+b*c)*x*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)/b^2/d^2+1/5
*x^3*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)/b/d+1/15*c*(9*a*b*c*d-8*(a*d+b*c)^2
)*(b*x^2+a)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1
+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(5/2)/d^3/(a*c+(a*d+b*c)*x^2+
b*d*x^4)^(1/2)+4/15*a^(1/2)*c*(a*d+b*c)*(b*x^2+a)*(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/
2)/d^2/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.66

$$\int \frac{x^6}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(4bc+4ad-3bdx^2)-ic(8b^2c^2+7abcd+8a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\mathrm{arc}}{15a^2\left(\frac{b}{a}\right)^{5/2}d^3\sqrt{(a+}}$$

input
```
Integrate[x^6/Sqrt[(a + b*x^2)*(c + d*x^2)],x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*b*c + 4*a*d - 3*b*d*x^2)) - I*
c*(8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(8*b^2*c^2 + 3*a*
b*c*d + 4*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*(b/a)^(5/2)*d^3*Sqrt[(a + b*x^2)*
(c + d*x^2)])
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.66, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2048, 1442, 1602, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{(a+bx^2)(c+dx^2)}}dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{x^6}{\sqrt{x^2(ad+bc)+ac+bdx^4}}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x^3\sqrt{x^2(ad+bc)+ac+bdx^4}}{5bd}-\frac{\int \frac{x^2(4(bc+ad)x^2+3ac)}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{5bd}$$

$$\Big\downarrow \text{1602}$$

$$\frac{x^3\sqrt{x^2(ad+bc)+ac+bdx^4}}{5bd}-\frac{\frac{4x(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\int \frac{4ac(bc+ad)-\left(9abcd-8(bc+ad)^2\right)x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{3bd}}{5bd}$$

$$\Big\downarrow \text{1511}$$

$$\frac{x^3\sqrt{x^2(ad+bc)+ac+bdx^4}}{5bd}-\frac{\frac{4x(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt{a}\sqrt{c}\left(9abcd-8(ad+bc)^2\right)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{a}\sqrt{c}\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}\left(-8(ad+bc)^2-4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}(ad+bc)+9abcd\right)\int}{\sqrt{b}\sqrt{d}}}{3bd}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3\sqrt{x^2(ad+bc)+ac+bdx^4}}{5bd}-\frac{\frac{4x(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\left(9abcd-8(ad+bc)^2\right)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}\left(-8(ad+bc)^2-4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}(ad+bc)+9abcd\right)\int \frac{}{\sqrt{bdx^4+(bc}}}{\sqrt{b}\sqrt{d}}}{3bd}}{5bd}$$

$$\Big\downarrow \text{1416}$$

$$\frac{x^3\sqrt{x^2(ad+bc)+ac+bdx^4}}{5bd}-\frac{\frac{4x(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\left(9abcd-8(ad+bc)^2\right)\int\frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt[4]{a}\sqrt[4]{c}\left(-8(ad+bc)^2-4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}(ad+bc)+9abcd\right)\left(\sqrt{a}\sqrt{c}+\sqrt{}\right.}{2b^{3/}}}{3bd}}{5bd}$$

$\downarrow$ 1509

$$\frac{x^3\sqrt{x^2(ad+bc)+ac+bdx^4}}{5bd}-\frac{4x(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\left(9abcd-8(ad+bc)^2\right)\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right)|\frac{1}{4}\left(2-\frac{bc+}{\sqrt{a}\sqrt{b}\sqrt{}}\right.\right.}{\sqrt[4]{b}\sqrt[4]{d}\sqrt{x^2(ad+bc)+ac+bdx^4}}\right.}{\sqrt{b}\sqrt{d}}$$

input
```
Int[x^6/Sqrt[(a + b*x^2)*(c + d*x^2)],x]
```

output
```
(x^3*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(5*b*d) - ((4*(b*c + a*d)*x*Sq
rt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(3*b*d) - (((9*a*b*c*d - 8*(b*c + a*d
)^2)*(-((x*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt[
b]*Sqrt[d]*x^2)) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2
)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d
]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 -
(b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[a
*c + (b*c + a*d)*x^2 + b*d*x^4])))/(Sqrt[b]*Sqrt[d]) - (a^(1/4)*c^(1/4)*(9
*a*b*c*d - 4*Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]*(b*c + a*d) - 8*(b*c + a*d)^2
)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*
d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(b^(1
/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[
c]*Sqrt[d]))/4])/(2*b^(3/4)*d^(3/4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])
)/(3*b*d))/(5*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 5.26 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.93

method	result
default	$\frac{x^3\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{5bd}-\frac{(4ad+4bc)x\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{15b^2d^2}+\frac{(4ad+4bc)ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\ \mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1}\right.}{15b^2d^2\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}$
elliptic	$\frac{x^3\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{5bd}-\frac{(4ad+4bc)x\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{15b^2d^2}+\frac{(4ad+4bc)ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\ \mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1}\right.}{15b^2d^2\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}$
risch	$-\frac{x\left(-3bd\,x^2+4ad+4bc\right)\left(b\,x^2+a\right)\left(d\,x^2+c\right)}{15b^2d^2\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}}+\frac{-\frac{\left(8a^2d^2+7abcd+8b^2c^2\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\mathrm{Ellipticl}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}\,d}}{}$

input
```
int(x^6/((b*x^2+a)*(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/b/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/15/b^2/d^2*(4*a*d+4*b*c)
*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/15/b^2/d^2*(4*a*d+4*b*c)*a*c/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-3/5*a*c/b/d+1/
15/b^2/d^2*(4*a*d+4*b*c)*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(
1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.64

$$\int \frac{x^6}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = -\frac{(8\,b^2c^3+7\,abc^2d+8\,a^2cd^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(8\,b^2c^3+7\,abc^2d+4\,a^2d^3+4\,(2\,a^2+}{}$$

input
```
integrate(x^6/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((8*b^2*c^3 + 7*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elli
ptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^2*c^3 + 7*a*b*c^2*d + 4*a^2
*d^3 + 4*(2*a^2 + a*b)*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqr
t(-c/d)/x), a*d/(b*c)) - (3*b^2*d^3*x^4 + 8*b^2*c^2*d + 7*a*b*c*d^2 + 8*a^
2*d^3 - 4*(b^2*c*d^2 + a*b*d^3)*x^2)*sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c)
)/(b^3*d^4*x)
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{x^6}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$

input
```
integrate(x**6/((b*x**2+a)*(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**6/sqrt((a + b*x**2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{x^6}{\sqrt{(bx^2+a)(dx^2+c)}}\,dx$$

input
```
integrate(x^6/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt((b*x^2 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{x^6}{\sqrt{(bx^2+a)(dx^2+c)}}\,dx$$

input
```
integrate(x^6/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt((b*x^2 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{x^6}{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}}\,dx$$

input
```
int(x^6/((a + b*x^2)*(c + d*x^2))^(1/2),x)
```

output
```
int(x^6/((a + b*x^2)*(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$
$$= \frac{-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,adx - 4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bcx + 3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bd\,x^3 + 8\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x}}{bd\,x^4+ad\,x^2+b}\right.}{}$$

input
```
int(x^6/((b*x^2+a)*(d*x^2+c))^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*x - 4*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*b*c*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*x**3 + 8*int((sq
rt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**
4),x)*a**2*d**2 + 7*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*
d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c*d + 8*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c**2 + 4*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**
4),x)*a**2*c*d + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2
 + b*c*x**2 + b*d*x**4),x)*a*b*c**2)/(15*b**2*d**2)
```

3.702 $\int \frac{x^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 296

$$\begin{aligned}\int \frac{x^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx &= -\frac{2(bc+ad)x(c+dx^2)}{3bd^2\sqrt{ac+(bc+ad)x^2+bdx^4}}+\frac{x\sqrt{ac+(bc+ad)x^2+bdx^4}}{3bd}\\ &\quad+\frac{2c(bc+ad)(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{a}b^{3/2}d^2\sqrt{ac+(bc+ad)x^2+bdx^4}}\\ &\quad-\frac{\sqrt{a}c(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{3/2}d\sqrt{ac+(bc+ad)x^2+bdx^4}}\end{aligned}$$

output
```
-2/3*(a*d+b*c)*x*(d*x^2+c)/b/d^2/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)+1/3*x*(
a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)/b/d+2/3*c*(a*d+b*c)*(b*x^2+a)*(a*(d*x^2+c
)/c/(b*x^2+a))^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/
b/c)^(1/2))/a^(1/2)/b^(3/2)/d^2/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)-1/3*a^(1
/2)*c*(b*x^2+a)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*InverseJacobiAM(arctan(b^(
1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/d/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.54 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)+2ic(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic(2bc+ad)\sqrt{1+}}{3b\sqrt{\frac{b}{a}}d^2\sqrt{(a+bx^2)(c+dx^2)}}$$

input
```
Integrate[x^4/Sqrt[(a + b*x^2)*(c + d*x^2)],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + (2*I)*c*(b*c + a*d)*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*c*(2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*d^2*Sqrt[(a + b*x^2)*(c +
 d*x^2)])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.79, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2048, 1442, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$

$$\Big\downarrow \text{ 2048}$$

$$\int \frac{x^4}{\sqrt{x^2(ad+bc)+ac+bdx^4}}dx$$

$$\Big\downarrow \text{ 1442}$$

$$\frac{x\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\int\frac{2(bc+ad)x^2+ac}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{3bd}$$

$\downarrow$ 1511

$$\frac{x\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}+2ad+2bc\right)\int\frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{2\sqrt{a}\sqrt{c}(ad+bc)\int\frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{a}\sqrt{c}\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}}{3bd}$$

$\downarrow$ 27

$$\frac{x\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}+2ad+2bc\right)\int\frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{2(ad+bc)\int\frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}}{3bd}$$

$\downarrow$ 1416

$$\frac{x\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}+2ad+2bc\right)\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{2(ad+bc}{}}{3bd}$$

$\downarrow$ 1509

$$\frac{x\sqrt{x^2(ad+bc)+ac+bdx^4}}{3bd}-\frac{\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}+2ad+2bc\right)\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{2(ad+bc}{}}{3bd}$$

input
```
Int[x^4/Sqrt[(a + b*x^2)*(c + d*x^2)],x]
```

output
```
(x*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(3*b*d) - ((-2*(b*c + a*d)*(-((x
*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]
*x^2)) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*
c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*
EllipticE[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d
)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c
+ a*d)*x^2 + b*d*x^4])))/(Sqrt[b]*Sqrt[d]) + (a^(1/4)*c^(1/4)*(2*b*c + Sqr
t[a]*Sqrt[b]*Sqrt[c]*Sqrt[d] + 2*a*d)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x
^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt
[d]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2
- (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(3/4)*d^(3/4)*Sq
rt[a*c + (b*c + a*d)*x^2 + b*d*x^4]))/(3*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.90

method	result
default	$\frac{x\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{3bd}-\frac{ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3bd\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}+\frac{(2ad+2bc)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{Ellip}\cdots\right.}{3b\,\cdots}$
elliptic	$\frac{x\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{3bd}-\frac{ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3bd\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}+\frac{(2ad+2bc)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{Ellip}\cdots\right.}{3b\,\cdots}$
risch	$\frac{x\left(b\,x^2+a\right)\left(d\,x^2+c\right)}{3bd\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}}-\frac{\frac{ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}-\frac{(2ad+2bc)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+x^2\cdots}}}{3bd}$

input
```
int(x^4/((b*x^2+a)*(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*a*c/b/d/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellip
ticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/3/b/d^2*(2*a*d+2*b*c)*c/(-
b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.54

$$\int \frac{x^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \frac{2\,(bc^2+acd)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (2\,bc^2+2\,acd+ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}}{3\,b^2d^3x}$$

input
```
integrate(x^4/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*(b*c^2 + a*c*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)
/x), a*d/(b*c)) - (2*b*c^2 + 2*a*c*d + a*d^2)*sqrt(b*d)*x*sqrt(-c/d)*ellip
tic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + sqrt(b*d*x^4 + (b*c + a*d)*x^2 +
a*c)*(b*d^2*x^2 - 2*b*c*d - 2*a*d^2))/(b^2*d^3*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{x^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$

input
```
integrate(x**4/((b*x**2+a)*(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**4/sqrt((a + b*x**2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{x^4}{\sqrt{(bx^2+a)(dx^2+c)}}\,dx$$

input
```
integrate(x^4/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt((b*x^2 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{x^4}{\sqrt{(bx^2+a)(dx^2+c)}}\,dx$$

input
```
integrate(x^4/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt((b*x^2 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{x^4}{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}}\,dx$$

input
```
int(x^4/((a + b*x^2)*(c + d*x^2))^(1/2),x)
```

output
```
int(x^4/((a + b*x^2)*(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$
$$= \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)ad-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)bc-\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}}{bd\,x^4+ad\,x^2+bc}\right.}{3bd}$$

input
```
int(x^4/((b*x^2+a)*(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x - 2*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*d - 2*int((sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)
*b*c - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2
+ b*d*x**4),x)*a*c)/(3*b*d)
```

3.703 $\int \frac{x^2}{\sqrt{(a+bx^2)(c+dx^2)}} dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{x^2}{\sqrt{(a+bx^2)(c+dx^2)}} dx = \frac{x(c+dx^2)}{d\sqrt{ac+(bc+ad)x^2+bdx^4}} - \frac{c(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}d\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

output
```
x*(d*x^2+c)/d/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)-c*(b*x^2+a)*(a*(d*x^2+c)/c
/(b*x^2+a))^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c
)^(1/2))/a^(1/2)/b^(1/2)/d/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$
$$= -\frac{ic\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}d\sqrt{(a+bx^2)(c+dx^2)}}$$

input
```
Integrate[x^2/Sqrt[(a + b*x^2)*(c + d*x^2)],x]
```

output
```
((-I)*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(
Sqrt[b/a]*d*Sqrt[(a + b*x^2)*(c + d*x^2)])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 447 vs. 2(137) = 274.

Time = 0.88 (sec) , antiderivative size = 447, normalized size of antiderivative = 3.26, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2048, 1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{(a+bx^2)(c+dx^2)}}dx$$
$$\Big\downarrow \text{2048}$$
$$\int \frac{x^2}{\sqrt{x^2(ad+bc)+ac+bdx^4}}dx$$
$$\Big\downarrow \text{1459}$$
$$\frac{\sqrt{a}\sqrt{c}\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{a}\sqrt{c}\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}$$
$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a}\sqrt{c}\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}$$

$\Big\downarrow$ 1416

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}$$

$\Big\downarrow$ 1509

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right)|\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{\sqrt[4]{b}\sqrt[4]{d}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{x\sqrt{x^2(ad+bc)+ac+bdx^4}}{\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2}}{\sqrt{b}\sqrt{d}}$$

input
```
Int[x^2/Sqrt[(a + b*x^2)*(c + d*x^2)],x]
```

output
```
-((-((x*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt[b]*
Sqrt[d]*x^2)) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*S
qrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x
^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*
c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[a*c
+ (b*c + a*d)*x^2 + b*d*x^4]))/(Sqrt[b]*Sqrt[d])) + (a^(1/4)*c^(1/4)*(Sqrt
[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/
(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*d^(
1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt
[d]))/4])/(2*b^(3/4)*d^(3/4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}\,d}$	124
elliptic	$-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}\,d}$	124

input
```
int(x^2/((b*x^2+a)*(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-Ellipt
icE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx = -\frac{\sqrt{bd}cx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bd}cx\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bdx^4+(bc+ad)x^2+a}}{bd^2x}$$

input
```
integrate(x^2/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - s
qrt(b*d)*c*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt
(b*d*x^4 + (b*c + a*d)*x^2 + a*c)*d)/(b*d^2*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx = \int \frac{x^2}{\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx$$

input
```
integrate(x**2/((b*x**2+a)*(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**2/sqrt((a + b*x**2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx = \int \frac{x^2}{\sqrt{(bx^2+a)(dx^2+c)}}\,dx$$

input
```
integrate(x^2/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt((b*x^2 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx = \int \frac{x^2}{\sqrt{(bx^2+a)(dx^2+c)}}\,dx$$

input
```
integrate(x^2/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt((b*x^2 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{x^2}{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}}\,dx$$

input
```
int(x^2/((a + b*x^2)*(c + d*x^2))^(1/2),x)
```

output
```
int(x^2/((a + b*x^2)*(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx$$

input
```
int(x^2/((b*x^2+a)*(d*x^2+c))^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)
```

3.704 $\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \frac{(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

output
```
(b*x^2+a)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x
/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(1/2)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right),\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{(a+bx^2)(c+dx^2)}}$$

input
```
Integrate[1/Sqrt[(a + b*x^2)*(c + d*x^2)],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
 (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[(a + b*x^2)*(c + d*x^2)])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. $2(94) = 188$.

Time $= 0.41$ (sec) , antiderivative size $= 190$, normalized size of antiderivative $= 2.02$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used $= \{2048, 1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{1}{\sqrt{x^2(ad+bc)+ac+bdx^4}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x^2(ad+bc)+ac+bdx^4}}$$

input
```
Int[1/Sqrt[(a + b*x^2)*(c + d*x^2)],x]
```

output
```
((Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d
*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c
]*Sqrt[d]))/4])/(2*a^(1/4)*b^(1/4)*c^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*
x^2 + b*d*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}}$	87
elliptic	$\frac{\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+x^{2}da+bc\,x^{2}+ac}}$	87

input
```
int(1/((b*x^2+a)*(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx = -\frac{\sqrt{ac}\sqrt{-\frac{b}{a}}F\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right)}{bc}$$

input
```
integrate(1/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c))/(b*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$

input
```
integrate(1/((b*x**2+a)*(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/sqrt((a + b*x**2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(bx^2+a)(dx^2+c)}}\,dx$$

input
```
integrate(1/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^2 + a)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(bx^2+a)(dx^2+c)}}\,dx$$

input
```
integrate(1/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((b*x^2 + a)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(b\,x^2+a)\,(d\,x^2+c)}}\,dx$$

input
```
int(1/((a + b*x^2)*(c + d*x^2))^(1/2),x)
```

output
```
int(1/((a + b*x^2)*(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx$$

input
```
int(1/((b*x^2+a)*(d*x^2+c))^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x
**4),x)
```

3.705 $\int \frac{1}{x^2\sqrt{(a+bx^2)(c+dx^2)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{1}{x^2\sqrt{(a+bx^2)(c+dx^2)}}\,dx = -\frac{\sqrt{ac+(bc+ad)x^2+bdx^4}}{cx(a+bx^2)} - \frac{\sqrt{b}(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{a^{3/2}\sqrt{ac+(bc+ad)x^2+bdx^4}}$$

output
```
-(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)/c/x/(b*x^2+a)-b^(1/2)*(b*x^2+a)*(a*(d*x
^2+c)/c/(b*x^2+a))^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-
a*d/b/c)^(1/2))/a^(3/2)/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \frac{-\frac{(a+bx^2)(c+dx^2)}{cx} - ia\sqrt{\frac{b}{a}}\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) - \operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right.}{a\sqrt{(a+bx^2)(c+dx^2)}}$$

input
```
Integrate[1/(x^2*Sqrt[(a + b*x^2)*(c + d*x^2)]),x]
```

output
```
(-(((a + b*x^2)*(c + d*x^2))/(c*x)) - I*a*Sqrt[b/a]*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(a*Sqrt[(a + b*x^2)*(c + d*x^2)
])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 493 vs. 2(138) = 276.

Time = 0.99 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.57, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2048, 1443, 27, 1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{(a+bx^2)(c+dx^2)}}dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{1}{x^2\sqrt{x^2(ad+bc)+ac+bdx^4}}dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{\int \frac{bdx^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd\int \frac{x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}$$

$$\Big\downarrow \text{1459}$$

$$\frac{bd\left(\frac{\sqrt{a}\sqrt{c}\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{a}\sqrt{c}\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}\right)}{ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}$$

$$\Big\downarrow \; 27$$

$$\frac{bd\left(\frac{\sqrt{a}\sqrt{c}\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}\right)}{ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}$$

$$\Big\downarrow \; 1416$$

$$\frac{bd\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}\right)}{ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}$$

$$\Big\downarrow \; 1509$$

$$\frac{bd\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\cdots}}{\cdots}\right)}{ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}$$

input
```
Int[1/(x^2*Sqrt[(a + b*x^2)*(c + d*x^2)]),x]
```

output
```
-(Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4]/(a*c*x)) + (b*d*(-((-((x*Sqrt[a*c
+ (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)) + (
a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c +
 a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]
*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*x^2
 + b*d*x^4]))/(Sqrt[b]*Sqrt[d])) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqr
t[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c]
+ Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*
c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(3
/4)*d^(3/4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])))/(a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.16

method	result
default	$-\frac{\sqrt{b d \,x^{4}+x^{2} d a +b c \,x^{2}+a c}}{a c x}-\frac{b \sqrt{1+\frac{b \,x^{2}}{a}}\, \sqrt{1+\frac{d \,x^{2}}{c}}\, \left(\operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{a d +b c}{c b}}\right)-\operatorname{EllipticE}\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{a d +b c}{c b}}\right)\right)}{a \sqrt{-\frac{b}{a}}\, \sqrt{b d \,x^{4}+x^{2} d a +b c \,x^{2}+a c}}$
elliptic	$-\frac{\sqrt{b d \,x^{4}+x^{2} d a +b c \,x^{2}+a c}}{a c x}-\frac{b \sqrt{1+\frac{b \,x^{2}}{a}}\, \sqrt{1+\frac{d \,x^{2}}{c}}\, \left(\operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{a d +b c}{c b}}\right)-\operatorname{EllipticE}\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{a d +b c}{c b}}\right)\right)}{a \sqrt{-\frac{b}{a}}\, \sqrt{b d \,x^{4}+x^{2} d a +b c \,x^{2}+a c}}$
risch	$-\frac{\left(b \,x^{2}+a \right) \left(d \,x^{2}+c \right)}{a c x \sqrt{\left(b \,x^{2}+a \right) \left(d \,x^{2}+c \right)}}-\frac{b \sqrt{1+\frac{b \,x^{2}}{a}}\, \sqrt{1+\frac{d \,x^{2}}{c}}\, \left(\operatorname{EllipticF}\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{a d +b c}{c b}}\right)-\operatorname{EllipticE}\left(x \sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{a d +b c}{c b}}\right)\right)}{a \sqrt{-\frac{b}{a}}\, \sqrt{b d \,x^{4}+x^{2} d a +b c \,x^{2}+a c}}$

input
```
int(1/x^2/((b*x^2+a)*(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a/c*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x-b/a/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*
c)/c/b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$
$$= \frac{\sqrt{ac}bx\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - \sqrt{ac}bx\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - \sqrt{bdx^4+(bc+ad)x^2+c}}{a^2cx}$$

input
```
integrate(1/x^2/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(a*c)*b*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - sq
rt(a*c)*b*x*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - sqrt(
b*d*x^4 + (b*c + a*d)*x^2 + a*c)*a)/(a^2*c*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{x^2\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$

input
```
integrate(1/x**2/((b*x**2+a)*(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((a + b*x**2)*(c + d*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(bx^2+a)(dx^2+c)}x^2}\,dx$$

input
```
integrate(1/x^2/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*(d*x^2 + c))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(bx^2+a)(dx^2+c)}x^2}\,dx$$

input
```
integrate(1/x^2/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*(d*x^2 + c))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx = \int \frac{1}{x^2\,\sqrt{(b\,x^2+a)\,\,(d\,x^2+c)}}\,dx$$

input
```
int(1/(x^2*((a + b*x^2)*(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^2*((a + b*x^2)*(c + d*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^6+ad\,x^4+bc\,x^4+ac\,x^2}dx$$

input
```
int(1/x^2/((b*x^2+a)*(d*x^2+c))^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**2 + a*d*x**4 + b*c*x**4 +
b*d*x**6),x)
```

3.706 $\int \frac{1}{x^4\sqrt{(a+bx^2)(c+dx^2)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 301

$$\begin{aligned}\int \frac{1}{x^4\sqrt{(a+bx^2)(c+dx^2)}}\,dx &= -\frac{\sqrt{ac+(bc+ad)x^2+bdx^4}}{3acx^3}+\frac{2(bc+ad)\sqrt{ac+(bc+ad)x^2+bdx^4}}{3ac^2x\,(a+bx^2)} \\ &\quad+\frac{2\sqrt{b}(bc+ad)\,(a+bx^2)\,\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{5/2}c\sqrt{ac+(bc+ad)x^2+bdx^4}} \\ &\quad-\frac{\sqrt{b}d(a+bx^2)\,\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3a^{3/2}c\sqrt{ac+(bc+ad)x^2+bdx^4}}\end{aligned}$$

output
```
-1/3*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)/a/c/x^3+2/3*(a*d+b*c)*(a*c+(a*d+b*c
)*x^2+b*d*x^4)^(1/2)/a/c^2/x/(b*x^2+a)+2/3*b^(1/2)*(a*d+b*c)*(b*x^2+a)*(a*
(d*x^2+c)/c/(b*x^2+a))^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2)
,(1-a*d/b/c)^(1/2))/a^(5/2)/c/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)-1/3*b^(1/2
)*d*(b*x^2+a)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*InverseJacobiAM(arctan(b^(1/
2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(3/2)/c/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.72 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^4\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \frac{\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(-ac+2bcx^2+2adx^2)+2ibc(bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\right.}{3a^2\sqrt{\frac{b}{a}}c^2x^3\sqrt{(a+bx^2)(c+dx}}$$

input
```
Integrate[1/(x^4*Sqrt[(a + b*x^2)*(c + d*x^2)]),x]
```

output
```
(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-(a*c) + 2*b*c*x^2 + 2*a*d*x^2) + (2*I
)*b*c*(b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(2*b*c + a*d)*x^3*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])
/(3*a^2*Sqrt[b/a]*c^2*x^3*Sqrt[(a + b*x^2)*(c + d*x^2)])
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.95, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2048, 1443, 25, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{1}{x^4\sqrt{x^2(ad+bc)+ac+bdx^4}}dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{\int -\frac{bdx^2+2(bc+ad)}{x^2\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{3ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}$$

$\downarrow$ 25

$$-\frac{\int \frac{bdx^2+2(bc+ad)}{x^2\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{3ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}$$

$\downarrow$ 1604

$$-\frac{-\frac{\int -\frac{bd\left(2(bc+ad)x^2+ac\right)}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{ac}-\frac{2(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}}{3ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}$$

$\downarrow$ 25

$$-\frac{\frac{\int \frac{bd\left(2(bc+ad)x^2+ac\right)}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{ac}-\frac{2(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}}{3ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}$$

$\downarrow$ 27

$$-\frac{\frac{bd\int \frac{2(bc+ad)x^2+ac}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{ac}-\frac{2(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}}{3ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}$$

$\downarrow$ 1511

$$-\frac{\frac{bd\left(\frac{\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}+2ad+2bc\right)\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{2\sqrt{a}\sqrt{c}(ad+bc)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{a}\sqrt{c}\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}\right)}{ac}-\frac{2(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}}{3ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}$$

$\downarrow$ 27

$$-\frac{\frac{bd\left(\frac{\sqrt{a}\sqrt{c}\left(\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}+2ad+2bc\right)\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{2(ad+bc)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}\right)}{ac}-\frac{2(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}}{3ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}$$

$\downarrow$ 1416

$$-\frac{\frac{bd\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}+2ad+2bc\right)\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{2(ad+bc)\int \frac{\ldots}{\sqrt{b}\ldots}\ldots}{\ldots}\right)}{ac}}{3ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}$$

$\big\downarrow$ 1509

$$-\frac{\frac{bd\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}+2ad+2bc\right)\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right),\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{2(ad+bc)\left(\frac{\sqrt[4]{\ldots}}{\ldots}\ldots\right)}{\ldots}\right)}{ac}}{\ldots}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}$$

input

```
Int[1/(x^4*Sqrt[(a + b*x^2)*(c + d*x^2)]),x]
```

output

```
-1/3*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4]/(a*c*x^3) - ((-2*(b*c + a*d)*Sq
rt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(a*c*x) + (b*d*((-2*(b*c + a*d)*(-((x
*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]
*x^2)) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*
c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*
EllipticE[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d
)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(1/4)*d^(1/4)*Sqrt[a*c + (b*c
+ a*d)*x^2 + b*d*x^4])))/(Sqrt[b]*Sqrt[d]) + (a^(1/4)*c^(1/4)*(2*b*c + Sqr
t[a]*Sqrt[b]*Sqrt[c]*Sqrt[d] + 2*a*d)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x
^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt
[d]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2
- (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(3/4)*d^(3/4)*Sq
rt[a*c + (b*c + a*d)*x^2 + b*d*x^4])))/(a*c))/(3*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{(bx^2+a)(dx^2+c)(-2x^2da-2bcx^2+ac)}{3a^2c^2x^3\sqrt{(bx^2+a)(dx^2+c)}}-\frac{bd\left(\frac{ac\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}-\frac{(2ad+2bc)c\sqrt{1+\frac{bx^2}{a}}\sqrt{1+}}{}\right.}{3a^2c^2}$
default	$-\frac{\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{3ac\,x^3}+\frac{2(ad+bc)\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{3a^2c^2x}-\frac{bd\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3ac\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}+$
elliptic	$-\frac{\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{3ac\,x^3}+\frac{2(ad+bc)\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{3a^2c^2x}-\frac{bd\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3ac\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}+$

input
```
int(1/x^4/((b*x^2+a)*(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)*(d*x^2+c)*(-2*a*d*x^2-2*b*c*x^2+a*c)/a^2/c^2/x^3/((b*x^2+a)
*(d*x^2+c))^(1/2)-1/3*b*d/a^2/c^2*(a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d
*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))-(2*a*d+2*b*c)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^4\sqrt{(a+bx^2)(c+dx^2)}}\,dx = -\frac{2\,(b^2c+abd)\sqrt{ac}x^3\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}) - (2\,b^2c+(a^2+2\,ab)d)\sqrt{ac}x^3\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\right.}}{3\,a^3c^2x^3}$$

input
```
integrate(1/x^4/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(b^2*c + a*b*d)*sqrt(a*c)*x^3*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(
-b/a)), a*d/(b*c)) - (2*b^2*c + (a^2 + 2*a*b)*d)*sqrt(a*c)*x^3*sqrt(-b/a)*
elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + sqrt(b*d*x^4 + (b*c + a*d)*x
^2 + a*c)*(a^2*c - 2*(a*b*c + a^2*d)*x^2))/(a^3*c^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{x^4\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$

input
```
integrate(1/x**4/((b*x**2+a)*(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt((a + b*x**2)*(c + d*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(bx^2+a)(dx^2+c)}x^4}\,dx$$

input
```
integrate(1/x^4/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*(d*x^2 + c))*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(bx^2+a)(dx^2+c)}x^4}\,dx$$

input
```
integrate(1/x^4/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*(d*x^2 + c))*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx = \int \frac{1}{x^4\,\sqrt{(b\,x^2+a)\,(d\,x^2+c)}}\,dx$$

input
```
int(1/(x^4*((a + b*x^2)*(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^4*((a + b*x^2)*(c + d*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^8+ad\,x^6+bc\,x^6+ac\,x^4}dx$$

input
```
int(1/x^4/((b*x^2+a)*(d*x^2+c))^(1/2),x)
```

output
```
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**4 + a*d*x**6 + b*c*x**6 +
b*d*x**8),x)
```

3.707 $\int \frac{1}{x^6\sqrt{(a+bx^2)(c+dx^2)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 375

$$
\begin{aligned}
\int \frac{1}{x^6\sqrt{(a+bx^2)(c+dx^2)}}\,dx
&= -\frac{\sqrt{ac+(bc+ad)x^2+bdx^4}}{5acx^5}+\frac{4(bc+ad)\sqrt{ac+(bc+ad)x^2+bdx^4}}{15a^2c^2x^3} \\
&\quad+\frac{(9abcd-8(bc+ad)^2)\sqrt{ac+(bc+ad)x^2+bdx^4}}{15a^2c^3x(a+bx^2)} \\
&\quad+\frac{\sqrt{b}(9abcd-8(bc+ad)^2)(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{7/2}c^2\sqrt{ac+(bc+ad)x^2+bdx^4}} \\
&\quad+\frac{4\sqrt{b}d(bc+ad)(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15a^{5/2}c^2\sqrt{ac+(bc+ad)x^2+bdx^4}}
\end{aligned}
$$

output
```
-1/5*(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)/a/c/x^5+4/15*(a*d+b*c)*(a*c+(a*d+b*
c)*x^2+b*d*x^4)^(1/2)/a^2/c^2/x^3+1/15*(9*a*b*c*d-8*(a*d+b*c)^2)*(a*c+(a*d
+b*c)*x^2+b*d*x^4)^(1/2)/a^2/c^3/x/(b*x^2+a)+1/15*b^(1/2)*(9*a*b*c*d-8*(a*
d+b*c)^2)*(b*x^2+a)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*EllipticE(b^(1/2)*x/a^
(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(7/2)/c^2/(a*c+(a*d+b*c)*x^2+
b*d*x^4)^(1/2)+4/15*b^(1/2)*d*(a*d+b*c)*(b*x^2+a)*(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/a^(5/
2)/c^2/(a*c+(a*d+b*c)*x^2+b*d*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.31 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^6\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(8b^2c^2x^4+abcx^2(-4c+7dx^2)+a^2(3c^2-4cdx^2+8d^2x^4))-ibc(8b^2c^2+7abcd}{}$$

input
```
Integrate[1/(x^6*Sqrt[(a + b*x^2)*(c + d*x^2)]),x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(8*b^2*c^2*x^4 + a*b*c*x^2*(-4*c + 7*
d*x^2) + a^2*(3*c^2 - 4*c*d*x^2 + 8*d^2*x^4))) - I*b*c*(8*b^2*c^2 + 7*a*b*
c*d + 8*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(8*b^2*c^2 + 3*a*b*c*d + 4*a^2*d
^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)])/(15*a^3*Sqrt[b/a]*c^3*x^5*Sqrt[(a + b*x^2)*(c + d*x^
2)])
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.82, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2048, 1443, 25, 1604, 25, 1604, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{(a+bx^2)(c+dx^2)}}dx$$

$$\Big\downarrow \ 2048$$

$$\int \frac{1}{x^6\sqrt{x^2(ad+bc)+ac+bdx^4}}dx$$

$$\Big\downarrow \ 1443$$

$$\frac{\int -\frac{3bdx^2+4(bc+ad)}{x^4\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{5ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{5acx^5}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{3bdx^2+4(bc+ad)}{x^4\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{5ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{5acx^5}$$

$$\Big\downarrow \ 1604$$

$$-\frac{-\frac{\int -\frac{-8(bc+ad)^2-4bdx^2(bc+ad)+9abcd}{x^2\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{3ac}-\frac{4(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}}{5ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{5acx^5}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\int \frac{-8(bc+ad)^2-4bdx^2(bc+ad)+9abcd}{x^2\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{3ac}-\frac{4(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}}{5ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{5acx^5}$$

$$\Big\downarrow \ 1604$$

$$-\frac{\frac{-\frac{\int \frac{bd\left(4ac(bc+ad)-\left(9abcd-8(bc+ad)^2\right)x^2\right)}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{ac}-\frac{\left(9abcd-8(ad+bc)^2\right)\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}}{3ac}-\frac{4(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}}{5ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{5acx^5}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{bd\int \frac{4ac(bc+ad)-\left(9abcd-8(bc+ad)^2\right)x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{ac}-\frac{\left(9abcd-8(ad+bc)^2\right)\sqrt{x^2(ad+bc)+ac+bdx^4}}{acx}}{3ac}-\frac{4(ad+bc)\sqrt{x^2(ad+bc)+ac+bdx^4}}{3acx^3}}{5ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{5acx^5}$$

$\Big\downarrow$ 1511

$$-\frac{-\frac{-\frac{bd\left(\frac{\sqrt{a}\sqrt{c}\left(9abcd-8(ad+bc)^2\right)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{a}\sqrt{c}\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}\left(-8(ad+bc)^2-4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}(ad+bc)+9abcd\right)\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}\right)}{ac}-\frac{(9a}{}}{3ac}}{5ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{5acx^5}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{bd\left(\frac{\left(9abcd-8(ad+bc)^2\right)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{a}\sqrt{c}\left(-8(ad+bc)^2-4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}(ad+bc)+9abcd\right)\int \frac{1}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}\right)}{ac}-\frac{(9abcd-8(ad+}{}}{3ac}}{5ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{5acx^5}$$

$\Big\downarrow$ 1416

$$-\frac{-\frac{-\frac{bd\left(\frac{\left(9abcd-8(ad+bc)^2\right)\int \frac{\sqrt{a}\sqrt{c}-\sqrt{b}\sqrt{d}x^2}{\sqrt{bdx^4+(bc+ad)x^2+ac}}dx}{\sqrt{b}\sqrt{d}}-\frac{\sqrt[4]{a}\sqrt[4]{c}\left(-8(ad+bc)^2-4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}(ad+bc)+9abcd\right)\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}\text{E}}{2b^{3/4}d^{3/4}\sqrt{x^2(ad+bc)+ac+bdx^4}}}{ac}}{3ac}}{5ac}-\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{5acx^5}$$

$\Big\downarrow$ 1509

$$-\frac{-\frac{bd\left(\frac{\left(9abcd-8(ad+bc)^2\right)\left(\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)\sqrt{\frac{x^2(ad+bc)+ac+bdx^4}{\left(\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt[4]{d}x}{\sqrt[4]{a}\sqrt[4]{c}}\right)|\frac{1}{4}\left(2-\frac{bc+ad}{\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}}\right)\right)}{\sqrt[4]{b}\sqrt[4]{d}\sqrt{x^2(ad+bc)+ac+bdx^4}}-\frac{x\sqrt{x^2(ad+bc)+ac+bdx^4}}{\sqrt{a}\sqrt{c}+\sqrt{b}\sqrt{d}x^2}\cdots}{\sqrt{b}\sqrt{d}}\cdots}{\cdots}}{\cdots}$$

$$\frac{\sqrt{x^2(ad+bc)+ac+bdx^4}}{5acx^5}$$

input
```
Int[1/(x^6*Sqrt[(a + b*x^2)*(c + d*x^2)]),x]
```

output
```
-1/5*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4]/(a*c*x^5) - ((-4*(b*c + a*d)*Sq
rt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(3*a*c*x^3) + (-(((9*a*b*c*d - 8*(b*c
 + a*d)^2)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(a*c*x)) - (b*d*(((9*a*b
*c*d - 8*(b*c + a*d)^2)*(-((x*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])/(Sqrt
[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)) + (a^(1/4)*c^(1/4)*(Sqrt[a]*Sqrt[c] +
Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[
c] + Sqrt[b]*Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/
4)*c^(1/4))], (2 - (b*c + a*d)/(Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(b^(
1/4)*d^(1/4)*Sqrt[a*c + (b*c + a*d)*x^2 + b*d*x^4])))/(Sqrt[b]*Sqrt[d]) -
(a^(1/4)*c^(1/4)*(9*a*b*c*d - 4*Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]*(b*c + a*d
) - 8*(b*c + a*d)^2)*(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)*Sqrt[(a*c + (
b*c + a*d)*x^2 + b*d*x^4)/(Sqrt[a]*Sqrt[c] + Sqrt[b]*Sqrt[d]*x^2)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*d^(1/4)*x)/(a^(1/4)*c^(1/4))], (2 - (b*c + a*d)/(Sq
rt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]))/4])/(2*b^(3/4)*d^(3/4)*Sqrt[a*c + (b*c + a
*d)*x^2 + b*d*x^4])))/(a*c))/(3*a*c))/(5*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 5.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.03

method	result
default	$-\frac{\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{5ac\,x^5}+\frac{4(ad+bc)\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{15a^2c^2x^3}-\frac{(8a^2d^2+7abcd+8b^2c^2)\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{15a^3c^3x}+\frac{4(ad+b}{}$
elliptic	$-\frac{\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{5ac\,x^5}+\frac{4(ad+bc)\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{15a^2c^2x^3}-\frac{(8a^2d^2+7abcd+8b^2c^2)\sqrt{bd\,x^4+x^2da+bc\,x^2+ac}}{15a^3c^3x}+\frac{4(ad+b}{}$
risch	$-\frac{(b\,x^2+a)\,(d\,x^2+c)\,(8a^2d^2x^4+7abcd\,x^4+8b^2c^2x^4-4a^2cd\,x^2-4ab\,c^2x^2+3a^2c^2)}{15a^3c^3x^5\sqrt{(b\,x^2+a)(d\,x^2+c)}}+\frac{bd\left(-\frac{(8a^2d^2+7abcd+8b^2c^2)c\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}}{}\right.}{}$

input
```
int(1/x^6/((b*x^2+a)*(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/a/c*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^5+4/15*(a*d+b*c)/a^2/c^2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3-1/15*(8*a^2*d^2+7*a*b*c*d+8*b^2*c^2)
/a^3/c^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+4/15*(a*d+b*c)*b*d/a^2/c^2/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/15*b*(8*a^
2*d^2+7*a*b*c*d+8*b^2*c^2)/a^3/c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2
/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-
1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^6\sqrt{(a+bx^2)\,(c+dx^2)}}\,dx$$
$$=\frac{(8\,b^3c^2+7\,ab^2cd+8\,a^2bd^2)\sqrt{ac}x^5\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(8\,b^3c^2+(4\,a^2b+7\,ab^2)cd+4\,(a^3+2}{}$$

input
```
integrate(1/x^6/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
1/15*((8*b^3*c^2 + 7*a*b^2*c*d + 8*a^2*b*d^2)*sqrt(a*c)*x^5*sqrt(-b/a)*ell
iptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (8*b^3*c^2 + (4*a^2*b + 7*a*b^2
)*c*d + 4*(a^3 + 2*a^2*b)*d^2)*sqrt(a*c)*x^5*sqrt(-b/a)*elliptic_f(arcsin(
x*sqrt(-b/a)), a*d/(b*c)) - sqrt(b*d*x^4 + (b*c + a*d)*x^2 + a*c)*(3*a^3*c
^2 + (8*a*b^2*c^2 + 7*a^2*b*c*d + 8*a^3*d^2)*x^4 - 4*(a^2*b*c^2 + a^3*c*d)
*x^2))/(a^4*c^3*x^5)
```

Sympy [F]

$$\int \frac{1}{x^6\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{x^6\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$

input
```
integrate(1/x**6/((b*x**2+a)*(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x**6*sqrt((a + b*x**2)*(c + d*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(bx^2+a)(dx^2+c)}x^6}\,dx$$

input
```
integrate(1/x^6/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*(d*x^2 + c))*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{\sqrt{(bx^2+a)(dx^2+c)}x^6}\,dx$$

input
```
integrate(1/x^6/((b*x^2+a)*(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*(d*x^2 + c))*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{(a+bx^2)(c+dx^2)}}\,dx = \int \frac{1}{x^6\,\sqrt{(b\,x^2+a)\,(d\,x^2+c)}}\,dx$$

input
```
int(1/(x^6*((a + b*x^2)*(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^6*((a + b*x^2)*(c + d*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{(a+bx^2)(c+dx^2)}}\,dx$$
$$= \frac{-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ac+3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bd\,x^4-3\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)b^2d^2x^5-4\left(\int \frac{\sqrt{d\,x}}{bd\,x^8+a}\right.}{5a^2c^2x^5}$$

input
```
int(1/x^6/((b*x^2+a)*(d*x^2+c))^(1/2),x)
```

output
```
( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c + 3*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*b*d*x**4 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d
*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*d**2*x**5 - 4*int((sqrt(c + d*x**2)*s
qrt(a + b*x**2))/(a*c*x**4 + a*d*x**6 + b*c*x**6 + b*d*x**8),x)*a**2*c*d*x
**5 - 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**4 + a*d*x**6 + b*c
*x**6 + b*d*x**8),x)*a*b*c**2*x**5)/(5*a**2*c**2*x**5)
```

3.708 $\int \frac{x^6}{\sqrt{a+\frac{\left(cd^2+ae^2\right)x^2}{de}+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 428

$$
\begin{aligned}
\int \frac{x^6}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx
&= \frac{\left(7ac+\frac{8c^2d^2}{e^2}+\frac{8a^2e^2}{d^2}\right)x(ae+cdx^2)}{15c^3d\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}} \\
&\quad - \frac{4\left(\frac{d}{e}+\frac{ae}{cd}\right)x\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}{15c} + \frac{x^3\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}{5c} \\
&\quad - \frac{a(8c^2d^4+7acd^2e^2+8a^2e^4)\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{cd^2}{ae^2}\right)}{15c^3d^{7/2}e^{3/2}\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}} \\
&\quad + \frac{4a(cd^2+ae^2)\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),1-\frac{cd^2}{ae^2}\right)}{15c^2d^{3/2}e^{3/2}\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}
\end{aligned}
$$

output
```
1/15*(7*a*c+8*c^2*d^2/e^2+8*a^2*e^2/d^2)*x*(c*d*x^2+a*e)/c^3/d/(a+(c*d/e+a
*e/d)*x^2+c*x^4)^(1/2)-4/15*(d/e+a*e/c/d)*x*(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1
/2)/c+1/5*x^3*(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)/c-1/15*a*(8*a^2*e^4+7*a*c*
d^2*e^2+8*c^2*d^4)*(d*(c*d*x^2+a*e)/a/e/(e*x^2+d))^(1/2)*(e*x^2+d)*Ellipti
cE(e^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(1/2),(1-c*d^2/a/e^2)^(1/2))/c^3/d^(7/2)/
e^(3/2)/(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)+4/15*a*(a*e^2+c*d^2)*(d*(c*d*x^2
+a*e)/a/e/(e*x^2+d))^(1/2)*(e*x^2+d)*InverseJacobiAM(arctan(e^(1/2)*x/d^(1
/2)),(1-c*d^2/a/e^2)^(1/2))/c^2/d^(3/2)/e^(3/2)/(a+(c*d/e+a*e/d)*x^2+c*x^4
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.84 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx$$

$$= \frac{-\sqrt{\frac{cd}{ae}}ex(d+ex^2)\left(4a^2e^3+c^2d^2x^2(4d-3ex^2)+acde(4d+ex^2)\right)-id(8c^2d^4+7acd^2e^2+8a^2e^4)\sqrt{1+\cdots}}{15a^2\cdots}$$

input
```
Integrate[x^6/Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4],x]
```

output
```
(-(Sqrt[(c*d)/(a*e)]*e*x*(d + e*x^2)*(4*a^2*e^3 + c^2*d^2*x^2*(4*d - 3*e*x
^2) + a*c*d*e*(4*d + e*x^2))) - I*d*(8*c^2*d^4 + 7*a*c*d^2*e^2 + 8*a^2*e^4
)*Sqrt[1 + (c*d*x^2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[(
c*d)/(a*e)]*x], (a*e^2)/(c*d^2)] + I*d*(8*c^2*d^4 + 3*a*c*d^2*e^2 + 4*a^2*
e^4)*Sqrt[1 + (c*d*x^2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*EllipticF[I*ArcSinh[Sqr
t[(c*d)/(a*e)]*x], (a*e^2)/(c*d^2)])/(15*a^2*((c*d)/(a*e))^(5/2)*e^5*Sqrt[
((a*e + c*d*x^2)*(d + e*x^2))/(d*e)])
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1442, 1602, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{\frac{x^2(ae^2+cd^2)}{de}+a+cx^4}}dx$$

$$\Big\downarrow \text{ 1442}$$

$$\frac{x^3\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5c}-\frac{\int \frac{x^2\left(4\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+3a\right)}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{5c}$$

$$\Big\downarrow \text{ 1602}$$

$$\frac{x^3\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5c}-\frac{\frac{4x\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3c}-\frac{\int \frac{4a\left(\frac{cd}{e}+\frac{ae}{d}\right)-\left(9ac-8\left(\frac{cd}{e}+\frac{ae}{d}\right)^2\right)x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{3c}}{5c}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{x^3\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5c}-\frac{\frac{4x\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3c}-\frac{\frac{\sqrt{a}\left(9ac-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2-4\sqrt{a}\sqrt{c}\left(\frac{ae}{d}+\frac{cd}{e}\right)+9ac\right)\int \frac{\cdots}{\sqrt{cx^4+\left(\cdots\right.}}}{\sqrt{c}}}{3c}}{5c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^3\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5c}-\frac{\frac{4x\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3c}-\frac{\frac{\left(9ac-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2-4\sqrt{a}\sqrt{c}\left(\frac{ae}{d}+\frac{cd}{e}\right)+9ac\right)\int \frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\cdots\right.}}}{\sqrt{c}}}{3c}}{5c}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{x^3\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5c}-\frac{\frac{4x\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3c}-\frac{\frac{\left(9ac-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2-4\sqrt{a}\sqrt{c}\left(\frac{ae}{d}+\frac{cd}{e}\right)+9ac\right)\sqrt{\cdots}}{2c^{3/4}\sqrt{x^2\cdots}}}{3c}}{5c}$$

$\big\downarrow$ 1509

$$\frac{x^3\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5c}-\frac{\frac{4x\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3c}-\frac{\left(9ac-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{x\sqrt{x^2\cdots}}{\cdots}\right)}{\sqrt{c}}\cdots}{5c}$$

input
```
Int[x^6/Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4],x]
```

output
```
(x^3*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])/(5*c) - ((4*(((c*d)/e + (a*
e)/d)*x*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])/(3*c) - (((9*a*c - 8*((
c*d)/e + (a*e)/d)^2)*(-((x*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])/(Sqr
t[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e
 + (a*e)/d)*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - ((c*d)/e + (a*e)/d)/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4
)*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])))/Sqrt[c] - (a^(1/4)*(9*a*c -
 4*Sqrt[a]*Sqrt[c]*((c*d)/e + (a*e)/d) - 8*((c*d)/e + (a*e)/d)^2)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - ((c*d)/e + (a*e)
/d)/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c
*x^4]))/(3*c))/(5*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 6.20 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.03

method	result
default	$\frac{x^{3} \sqrt{a+\frac{x^{2} e a}{d}+\frac{x^{2} d c}{e}+c\,x^{4}}}{5 c}-\frac{\left(\frac{4 c d}{e}+\frac{4 a e}{d}\right) x \sqrt{a+\frac{x^{2} e a}{d}+\frac{x^{2} d c}{e}+c\,x^{4}}}{15 c^{2}}+\frac{\left(\frac{4 c d}{e}+\frac{4 a e}{d}\right) a \sqrt{1+\frac{x^{2} d c}{a e}}\, \sqrt{1+\frac{e\,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{c d}{a e}}, \sqrt{-\right.}{15 c^{2} \sqrt{-\frac{c d}{a e}}\, \sqrt{a+\frac{x^{2} e a}{d}+\frac{x^{2} d c}{e}+c\,x^{4}}}$
elliptic	$\frac{x^{3} \sqrt{a+\frac{x^{2} e a}{d}+\frac{x^{2} d c}{e}+c\,x^{4}}}{5 c}-\frac{\left(\frac{4 c d}{e}+\frac{4 a e}{d}\right) x \sqrt{a+\frac{x^{2} e a}{d}+\frac{x^{2} d c}{e}+c\,x^{4}}}{15 c^{2}}+\frac{\left(\frac{4 c d}{e}+\frac{4 a e}{d}\right) a \sqrt{1+\frac{x^{2} d c}{a e}}\, \sqrt{1+\frac{e\,x^{2}}{d}}\, \operatorname{EllipticF}\left(x \sqrt{-\frac{c d}{a e}}, \sqrt{-\right.}{15 c^{2} \sqrt{-\frac{c d}{a e}}\, \sqrt{a+\frac{x^{2} e a}{d}+\frac{x^{2} d c}{e}+c\,x^{4}}}$
risch	$-\frac{x \left(-3 c\,x^{2} d e+4 a\,e^{2}+4 c\,d^{2}\right) \left(e\,x^{2}+d\right) \left(c d\,x^{2}+a e\right)}{15 e^{2} c^{2} d^{2} \sqrt{\frac{\left(e\,x^{2}+d\right) \left(c d\,x^{2}+a e\right)}{d e}}}+\frac{\left(-\frac{2 \left(8 a^{2} e^{4}+7 a c\,d^{2} e^{2}+8 c^{2} d^{4}\right) a\,e^{2} d^{2} \sqrt{1+\frac{x^{2} d c}{a e}}\, \sqrt{1+\frac{e\,x^{2}}{d}}\, \left(\operatorname{EllipticF}\left(x \sqrt{-\frac{c d}{a e}}, \sqrt{-\right.\right.}{\sqrt{-\frac{c d}{a e}}\, \sqrt{c\,d^{2} e^{2} x^{4}+a\,e^{3} x^{2} d+c e\,x^{2} d^{3}+a\,d^{2} e^{2} \left(\right.}}\right.}{}$

input
```
int(x^6/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/c*x^3*(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)-1/15/c^2*(4*c*d/e+4/d*a*e)*x
*(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)+1/15/c^2*(4*c*d/e+4/d*a*e)*a/(-c*d/a/
e)^(1/2)*(1+1/a*x^2*d/e*c)^(1/2)*(1+e*x^2/d)^(1/2)/(a+x^2/d*e*a+x^2*d/e*c+
c*x^4)^(1/2)*EllipticF(x*(-c*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2)
)-2*(-3/5/c*a+1/15/c^2*(4*c*d/e+4/d*a*e)*(2*c*d/e+2/d*a*e))*a/(-c*d/a/e)^(
1/2)*(1+1/a*x^2*d/e*c)^(1/2)*(1+e*x^2/d)^(1/2)/(a+x^2/d*e*a+x^2*d/e*c+c*x^
4)^(1/2)/(c*d/e+1/d*a*e+(a*e^2-c*d^2)/d/e)*(EllipticF(x*(-c*d/a/e)^(1/2),(
-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2))-EllipticE(x*(-c*d/a/e)^(1/2),(-1+(c*d/e+1
/d*a*e)*e/d/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx =$$
$$-\frac{(8\,c^2d^5+7\,acd^3e^2+8\,a^2de^4)\sqrt{c}x\sqrt{-\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{x}\right)\mid\frac{ae^2}{cd^2})-(8\,c^2d^5+7\,acd^3e^2+4\,acd^2e^3+8\,a^2d}{}$$

input
```
integrate(x^6/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((8*c^2*d^5 + 7*a*c*d^3*e^2 + 8*a^2*d*e^4)*sqrt(c)*x*sqrt(-d/e)*elli
ptic_e(arcsin(sqrt(-d/e)/x), a*e^2/(c*d^2)) - (8*c^2*d^5 + 7*a*c*d^3*e^2 +
 4*a*c*d^2*e^3 + 8*a^2*d*e^4 + 4*a^2*e^5)*sqrt(c)*x*sqrt(-d/e)*elliptic_f(
arcsin(sqrt(-d/e)/x), a*e^2/(c*d^2)) - (3*c^2*d^2*e^3*x^4 + 8*c^2*d^4*e +
7*a*c*d^2*e^3 + 8*a^2*e^5 - 4*(c^2*d^3*e^2 + a*c*d*e^4)*x^2)*sqrt((c*d*e*x
^4 + a*d*e + (c*d^2 + a*e^2)*x^2)/(d*e)))/(c^3*d^2*e^3*x)
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{x^6}{\sqrt{a+\frac{aex^2}{d}+\frac{cdx^2}{e}+cx^4}}\,dx$$

input
```
integrate(x**6/(a+(a*e**2+c*d**2)*x**2/d/e+c*x**4)**(1/2),x)
```

output
```
Integral(x**6/sqrt(a + a*e*x**2/d + c*d*x**2/e + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{x^6}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}}\,dx$$

input
```
integrate(x^6/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{x^6}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}}\,dx$$

input
```
integrate(x^6/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{x^6}{\sqrt{a+c\,x^4+\frac{x^2\,\left(c\,d^2+a\,e^2\right)}{d\,e}}}\,dx$$

input
```
int(x^6/(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2),x)
```

output
```
int(x^6/(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx$$
$$=\frac{\sqrt{e}\,\sqrt{d}\left(-4\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}\,a\,e^2x-4\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}\,c\,d^2x+3\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}\,cde\,x\right.}{}$$

input
```
int(x^6/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x)
```

output
```
(sqrt(e)*sqrt(d)*( - 4*sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2)*a*e**2*x - 4*
sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2)*c*d**2*x + 3*sqrt(d + e*x**2)*sqrt(a
*e + c*d*x**2)*c*d*e*x**3 + 8*int((sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2)*x
**2)/(a*d*e + a*e**2*x**2 + c*d**2*x**2 + c*d*e*x**4),x)*a**2*e**4 + 7*int
((sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2)*x**2)/(a*d*e + a*e**2*x**2 + c*d**
2*x**2 + c*d*e*x**4),x)*a*c*d**2*e**2 + 8*int((sqrt(d + e*x**2)*sqrt(a*e +
 c*d*x**2)*x**2)/(a*d*e + a*e**2*x**2 + c*d**2*x**2 + c*d*e*x**4),x)*c**2*
d**4 + 4*int((sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2))/(a*d*e + a*e**2*x**2
+ c*d**2*x**2 + c*d*e*x**4),x)*a**2*d*e**3 + 4*int((sqrt(d + e*x**2)*sqrt(
a*e + c*d*x**2))/(a*d*e + a*e**2*x**2 + c*d**2*x**2 + c*d*e*x**4),x)*a*c*d
**3*e))/(15*c**2*d**2*e**2)
```

3.709 $\int \frac{x^4}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 331

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx &= -\frac{2\left(\frac{1}{e}+\frac{ae}{cd^2}\right)x(ae+cdx^2)}{3c\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}+\frac{x\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}{3c}\\ &\quad+\frac{2a(cd^2+ae^2)\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{cd^2}{ae^2}\right)}{3c^2d^{5/2}\sqrt{e}\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}\\ &\quad-\frac{a\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),1-\frac{cd^2}{ae^2}\right)}{3c\sqrt{d}\sqrt{e}\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}\end{aligned}$$

output
```
-2/3*(1/e+a*e/c/d^2)*x*(c*d*x^2+a*e)/c/(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)+1
/3*x*(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)/c+2/3*a*(a*e^2+c*d^2)*(d*(c*d*x^2+a
*e)/a/e/(e*x^2+d))^(1/2)*(e*x^2+d)*EllipticE(e^(1/2)*x/d^(1/2)/(1+e*x^2/d)
^(1/2),(1-c*d^2/a/e^2)^(1/2))/c^2/d^(5/2)/e^(1/2)/(a+(c*d/e+a*e/d)*x^2+c*x
^4)^(1/2)-1/3*a*(d*(c*d*x^2+a*e)/a/e/(e*x^2+d))^(1/2)*(e*x^2+d)*InverseJac
obiAM(arctan(e^(1/2)*x/d^(1/2)),(1-c*d^2/a/e^2)^(1/2))/c/d^(1/2)/e^(1/2)/(
a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx$$
$$= \frac{\sqrt{\frac{cd}{ae}}ex(ae+cdx^2)\left(d+ex^2\right)+2id(cd^2+ae^2)\sqrt{1+\frac{cdx^2}{ae}}\sqrt{1+\frac{ex^2}{d}}E\left(i\text{arcsinh}\left(\sqrt{\frac{cd}{ae}}x\right)|\frac{ae^2}{cd^2}\right)-id(2cd^2 -}{3a\left(\frac{cd}{ae}\right)^{3/2}e^3\sqrt{\frac{(ae+cdx^2)(d+ex^2)}{de}}}$$

input
```
Integrate[x^4/Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4],x]
```

output
```
(Sqrt[(c*d)/(a*e)]*e*x*(a*e + c*d*x^2)*(d + e*x^2) + (2*I)*d*(c*d^2 + a*e^
2)*Sqrt[1 + (c*d*x^2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[
(c*d)/(a*e)]*x], (a*e^2)/(c*d^2)] - I*d*(2*c*d^2 + a*e^2)*Sqrt[1 + (c*d*x^
2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[(c*d)/(a*e)]*x], (a
*e^2)/(c*d^2)])/(3*a*((c*d)/(a*e))^(3/2)*e^3*Sqrt[((a*e + c*d*x^2)*(d + e*
x^2))/(d*e)])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1442, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\frac{x^2(ae^2+cd^2)}{de}+a+cx^4}}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3c}-\frac{\int \frac{2\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{3c}$$

$$\Big\downarrow \text{1511}$$

$$\frac{x\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3c}-\frac{\sqrt{a}\left(\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)}{\sqrt{c}}+\sqrt{a}\right)\int \frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx-\frac{2\sqrt{a}\left(\frac{ae}{d}+\frac{cd}{e}\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3c}-\frac{\sqrt{a}\left(\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)}{\sqrt{c}}+\sqrt{a}\right)\int \frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx-\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}}{3c}$$

$$\Big\downarrow \text{1416}$$

$$\frac{x\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3c}-\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\left(\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)}{\sqrt{c}}+\sqrt{a}\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2 \cdots}}}{\sqrt{c}}}{3c}$$

$$\Big\downarrow \; 1509$$

$$\frac{x\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3c}-\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\left(\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)}{\sqrt{c}}+\sqrt{a}\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\cdots}}{\cdots}\right.}{\cdots}}{3c}$$

input
```
Int[x^4/Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4],x]
```

output
```
(x*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])/(3*c) - ((-2*((c*d)/e + (a*e
)/d)*(-((x*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x
^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e + (a*e)/d)*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - ((c*d)/e + (a*e)/d)/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + ((c*d
)/e + (a*e)/d)*x^2 + c*x^4])))/Sqrt[c] + (a^(1/4)*(Sqrt[a] + (2*((c*d)/e +
 (a*e)/d))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e + (a*e)/d)*
x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], (2 - ((c*d)/e + (a*e)/d)/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a +
 ((c*d)/e + (a*e)/d)*x^2 + c*x^4]))/(3*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.07

method	result
default	$\frac{x\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+cx^4}}{3c}-\frac{a\sqrt{1+\frac{x^2dc}{ae}}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\frac{ae}{d}\right)e}{dc}}\right)}{3c\sqrt{-\frac{cd}{ae}}\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+cx^4}}+\frac{2\left(\frac{2cd}{e}+\frac{2ae}{d}\right)a\sqrt{1+\frac{x^2dc}{ae}}\sqrt{1+\frac{ex^2}{d}}}{}$
elliptic	$\frac{x\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+cx^4}}{3c}-\frac{a\sqrt{1+\frac{x^2dc}{ae}}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\frac{ae}{d}\right)e}{dc}}\right)}{3c\sqrt{-\frac{cd}{ae}}\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+cx^4}}+\frac{2\left(\frac{2cd}{e}+\frac{2ae}{d}\right)a\sqrt{1+\frac{x^2dc}{ae}}\sqrt{1+\frac{ex^2}{d}}}{}$
risch	$\frac{x(ex^2+d)(cdx^2+ae)}{3cde\sqrt{\frac{(ex^2+d)(cdx^2+ae)}{de}}}-\frac{\left(-\frac{2(2ae^2+2cd^2)ae^2d^2\sqrt{1+\frac{x^2dc}{ae}}\sqrt{1+\frac{ex^2}{d}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{ade^3+cd^3e}{ed^3c}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{cd}{ae}}\right.\right.}{\sqrt{-\frac{cd}{ae}}\sqrt{cd^2e^2x^4+ae^3x^2d+cex^2d^3+ad^2e^2}\left(ade^3+cd^3e+de\left(ae^2-cd^2\right)\right)}\right.}{3ecd\sqrt{}}$

input
```
int(x^4/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/c*x*(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)-1/3/c*a/(-c*d/a/e)^(1/2)*(1+1/
a*x^2*d/e*c)^(1/2)*(1+e*x^2/d)^(1/2)/(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)*E
llipticF(x*(-c*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2))+2/3/c*(2*c*d
/e+2/d*a*e)*a/(-c*d/a/e)^(1/2)*(1+1/a*x^2*d/e*c)^(1/2)*(1+e*x^2/d)^(1/2)/(
a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)/(c*d/e+1/d*a*e+(a*e^2-c*d^2)/d/e)*(Elli
pticF(x*(-c*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2))-EllipticE(x*(-c
*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.56

$$\int\frac{x^4}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx$$

$$=\frac{2\,(cd^3+ade^2)\sqrt{c}x\sqrt{-\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{x}\right)\mid\frac{ae^2}{cd^2})-(2\,cd^3+2\,ade^2+ae^3)\sqrt{c}x\sqrt{-\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{x}\right)\mid\frac{ae}{cd}}{3\,c^2de^2x}$$

input
```
integrate(x^4/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*(c*d^3 + a*d*e^2)*sqrt(c)*x*sqrt(-d/e)*elliptic_e(arcsin(sqrt(-d/e)
/x), a*e^2/(c*d^2)) - (2*c*d^3 + 2*a*d*e^2 + a*e^3)*sqrt(c)*x*sqrt(-d/e)*e
lliptic_f(arcsin(sqrt(-d/e)/x), a*e^2/(c*d^2)) + (c*d*e^2*x^2 - 2*c*d^2*e
- 2*a*e^3)*sqrt((c*d*e*x^4 + a*d*e + (c*d^2 + a*e^2)*x^2)/(d*e)))/(c^2*d*e
^2*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{x^4}{\sqrt{a+\frac{aex^2}{d}+\frac{cdx^2}{e}+cx^4}}\,dx$$

input
```
integrate(x**4/(a+(a*e**2+c*d**2)*x**2/d/e+c*x**4)**(1/2),x)
```

output
```
Integral(x**4/sqrt(a + a*e*x**2/d + c*d*x**2/e + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{x^4}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}}\,dx$$

input
```
integrate(x^4/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\int \frac{x^4}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}}\,dx$$

input
```
integrate(x^4/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\int \frac{x^4}{\sqrt{a+c\,x^4+\frac{x^2\,(c\,d^2+a\,e^2)}{d\,e}}}\,dx$$

input
```
int(x^4/(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2),x)
```

output
```
int(x^4/(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \frac{\sqrt{e}\,\sqrt{d}\left(\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}\,x-2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}\,x^2}{cde\,x^4+a\,e^2x^2+c\,d^2x^2+ade}dx\right)a\,e^2-2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}\,x^2}{cde\,x^4+a\,e^2x^2+c\,d^2x^2+ade}dx\right)c\,\cdots\right.}{3cde}$$

input
```
int(x^4/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x)
```

output
```
(sqrt(e)*sqrt(d)*(sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2)*x - 2*int((sqrt(d
+ e*x**2)*sqrt(a*e + c*d*x**2)*x**2)/(a*d*e + a*e**2*x**2 + c*d**2*x**2 +
c*d*e*x**4),x)*a*e**2 - 2*int((sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2)*x**2)
/(a*d*e + a*e**2*x**2 + c*d**2*x**2 + c*d*e*x**4),x)*c*d**2 - int((sqrt(d
+ e*x**2)*sqrt(a*e + c*d*x**2))/(a*d*e + a*e**2*x**2 + c*d**2*x**2 + c*d*e
*x**4),x)*a*d*e))/(3*c*d*e)
```

3.710 $\int \frac{x^2}{\sqrt{a+\frac{\left(cd^2+ae^2\right)x^2}{de}+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 157

$$\int \frac{x^2}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \frac{x(ae+cdx^2)}{cd\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}} - \frac{a\sqrt{e}\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{cd^2}{ae^2}\right)}{cd^{3/2}\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}$$

output
```
x*(c*d*x^2+a*e)/c/d/(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)-a*e^(1/2)*(d*(c*d*x^
2+a*e)/a/e/(e*x^2+d))^(1/2)*(e*x^2+d)*EllipticE(e^(1/2)*x/d^(1/2)/(1+e*x^2
/d)^(1/2),(1-c*d^2/a/e^2)^(1/2))/c/d^(3/2)/(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = -\frac{id\sqrt{1+\frac{cdx^2}{ae}}\sqrt{1+\frac{ex^2}{d}}\left(E\left(i\text{arcsinh}\left(\sqrt{\frac{cd}{ae}}x\right)|\frac{ae^2}{cd^2}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{cd}{ae}}x\right),\frac{ae^2}{cd^2}\right)\right)}{\sqrt{\frac{cd}{ae}}e\sqrt{\frac{(ae+cdx^2)(d+ex^2)}{de}}}$$

input
```
Integrate[x^2/Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4],x]
```

output
```
((-I)*d*Sqrt[1 + (c*d*x^2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*(EllipticE[I*ArcSinh
[Sqrt[(c*d)/(a*e)]*x], (a*e^2)/(c*d^2)] - EllipticF[I*ArcSinh[Sqrt[(c*d)/(
a*e)]*x], (a*e^2)/(c*d^2)]))/(Sqrt[(c*d)/(a*e)]*e*Sqrt[((a*e + c*d*x^2)*(d
 + e*x^2))/(d*e)])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 354 vs. 2(157) = 314.

Time = 0.73 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.25, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\frac{x^2(ae^2+cd^2)}{de}+a+cx^4}}dx$$

$$\Big\downarrow \text{ 1459}$$

$$\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \; 1416$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \; 1509$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{x\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}}{\sqrt{c}}$$

input
```
Int[x^2/Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4],x]
```

output
```
-((-((x*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)
) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e + (a*e)/d)*x^2 + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - ((c*d)/e + (a*e)/d)/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + ((c*d)/e
 + (a*e)/d)*x^2 + c*x^4]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - ((c*d)/e + (a*e)/d)/(Sqrt[a]*Sqrt[c
]))/4])/(2*c^(3/4)*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{2a\sqrt{1+\frac{x^{2}dc}{ae}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\frac{ae}{d}\right)e}{dc}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\frac{ae}{d}\right)e}{dc}}\right)\right)}{\sqrt{-\frac{cd}{ae}}\,\sqrt{a+\frac{x^{2}ea}{d}+\frac{x^{2}dc}{e}+c\,x^{4}}\,\left(\frac{cd}{e}+\frac{ae}{d}+\frac{a\,e^{2}-c\,d^{2}}{de}\right)}$	188
elliptic	$-\frac{2a\sqrt{1+\frac{x^{2}dc}{ae}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\frac{ae}{d}\right)e}{dc}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\frac{ae}{d}\right)e}{dc}}\right)\right)}{\sqrt{-\frac{cd}{ae}}\,\sqrt{a+\frac{x^{2}ea}{d}+\frac{x^{2}dc}{e}+c\,x^{4}}\,\left(\frac{cd}{e}+\frac{ae}{d}+\frac{a\,e^{2}-c\,d^{2}}{de}\right)}$	188

input
```
int(x^2/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a/(-c*d/a/e)^(1/2)*(1+1/a*x^2*d/e*c)^(1/2)*(1+e*x^2/d)^(1/2)/(a+x^2/d*e
*a+x^2*d/e*c+c*x^4)^(1/2)/(c*d/e+1/d*a*e+(a*e^2-c*d^2)/d/e)*(EllipticF(x*(
-c*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2))-EllipticE(x*(-c*d/a/e)^(
1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = -\frac{\sqrt{c}dx\sqrt{-\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{x}\right) \mid \frac{ae^2}{cd^2}) - \sqrt{c}dx\sqrt{-\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{x}\right) \mid \frac{ae^2}{cd^2}) - e\sqrt{\frac{cdex^4+ade+(cd^2+ae^2)x^2}{de}}}{cex}$$

input
```
integrate(x^2/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(c)*d*x*sqrt(-d/e)*elliptic_e(arcsin(sqrt(-d/e)/x), a*e^2/(c*d^2)) -
 sqrt(c)*d*x*sqrt(-d/e)*elliptic_f(arcsin(sqrt(-d/e)/x), a*e^2/(c*d^2)) -
e*sqrt((c*d*e*x^4 + a*d*e + (c*d^2 + a*e^2)*x^2)/(d*e)))/(c*e*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{x^2}{\sqrt{a+\frac{aex^2}{d}+\frac{cdx^2}{e}+cx^4}}\,dx$$

input
```
integrate(x**2/(a+(a*e**2+c*d**2)*x**2/d/e+c*x**4)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + a*e*x**2/d + c*d*x**2/e + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\int \frac{x^2}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}}\,dx$$

input
```
integrate(x^2/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\int \frac{x^2}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}}\,dx$$

input
```
integrate(x^2/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\int \frac{x^2}{\sqrt{a+c\,x^4+\frac{x^2\,(c\,d^2+a\,e^2)}{d\,e}}}\,dx$$

input
```
int(x^2/(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2),x)
```

output
```
int(x^2/(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \sqrt{e}\,\sqrt{d}\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}\,x^2}{cde\,x^4+a\,e^2x^2+c\,d^2x^2+ade}dx\right)$$

input
```
int(x^2/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x)
```

output
```
sqrt(e)*sqrt(d)*int((sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2)*x**2)/(a*d*e +
a*e**2*x**2 + c*d**2*x**2 + c*d*e*x**4),x)
```

3.711 $\int \frac{1}{\sqrt{a+\frac{\left(cd^2+ae^2\right)x^2}{de}+cx^4}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 105

$$\int \frac{1}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\frac{\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),1-\frac{cd^2}{ae^2}\right)}{\sqrt{d}\sqrt{e}\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}$$

output
```
(d*(c*d*x^2+a*e)/a/e/(e*x^2+d))^(1/2)*(e*x^2+d)*InverseJacobiAM(arctan(e^(
1/2)*x/d^(1/2)),(1-c*d^2/a/e^2)^(1/2))/d^(1/2)/e^(1/2)/(a+(c*d/e+a*e/d)*x^
2+c*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\frac{\sqrt{1+\frac{cdx^2}{ae}}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{cd}{ae}}x\right),\frac{ae^2}{cd^2}\right)}{\sqrt{-\frac{cd}{ae}}\sqrt{\frac{(ae+cdx^2)(d+ex^2)}{de}}}$$

input
```
Integrate[1/Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4],x]
```

output
```
(Sqrt[1 + (c*d*x^2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[Sqrt[-((c*
d)/(a*e))]*x], (a*e^2)/(c*d^2)])/(Sqrt[-((c*d)/(a*e))]*Sqrt[((a*e + c*d*x^
2)*(d + e*x^2))/(d*e)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.47, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{x^2(ae^2+cd^2)}{de}+a+cx^4}}dx$$

$$\Big\downarrow \; 1416$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{cd^2+ae^2}{\sqrt{a}\sqrt{c}de}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}$$

input
```
Int[1/Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4],x]
```

output
```
((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - (c*d^2
+ a*e^2)/(Sqrt[a]*Sqrt[c]*d*e))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + ((c*d)/e +
 (a*e)/d)*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\sqrt{1+\frac{x^2 dc}{ae}}\,\sqrt{1+\frac{e\,x^2}{d}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\frac{ae}{d}\right)e}{dc}}\right)}{\sqrt{-\frac{cd}{ae}}\,\sqrt{a+\frac{x^2 ea}{d}+\frac{x^2 dc}{e}+c\,x^4}}$	109
elliptic	$\frac{\sqrt{1+\frac{x^2 dc}{ae}}\,\sqrt{1+\frac{e\,x^2}{d}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\frac{ae}{d}\right)e}{dc}}\right)}{\sqrt{-\frac{cd}{ae}}\,\sqrt{a+\frac{x^2 ea}{d}+\frac{x^2 dc}{e}+c\,x^4}}$	109

input
```
int(1/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-c*d/a/e)^(1/2)*(1+1/a*x^2*d/e*c)^(1/2)*(1+e*x^2/d)^(1/2)/(a+x^2/d*e*a+
x^2*d/e*c+c*x^4)^(1/2)*EllipticF(x*(-c*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/
d/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = -\frac{\sqrt{a}e\sqrt{-\frac{cd}{ae}}F\left(\arcsin\left(x\sqrt{-\frac{cd}{ae}}\right) \mid \frac{ae^2}{cd^2}\right)}{cd}$$

input
```
integrate(1/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a)*e*sqrt(-c*d/(a*e))*elliptic_f(arcsin(x*sqrt(-c*d/(a*e))), a*e^2/(
c*d^2))/(c*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\int \frac{1}{\sqrt{a+cx^4+\frac{x^2(ae^2+cd^2)}{de}}}\,dx$$

input
```
integrate(1/(a+(a*e**2+c*d**2)*x**2/d/e+c*x**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a + c*x**4 + x**2*(a*e**2 + c*d**2)/(d*e)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}}\,dx$$

input
```
integrate(1/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}}\,dx$$

input
```
integrate(1/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{1}{\sqrt{a+c\,x^4+\frac{x^2\,(c\,d^2+a\,e^2)}{d\,e}}}\,dx$$

input
```
int(1/(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2),x)
```

output
```
int(1/(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \sqrt{e}\,\sqrt{d}\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}}{cde\,x^4+a\,e^2x^2+c\,d^2x^2+ade}dx\right)$$

input
```
int(1/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x)
```

output
```
sqrt(e)*sqrt(d)*int((sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2))/(a*d*e + a*e**
2*x**2 + c*d**2*x**2 + c*d*e*x**4),x)
```

3.712 $\int \frac{1}{x^2\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 153

$$\int \frac{1}{x^2\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = -\frac{d\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}{ax\,(d+ex^2)} - \frac{\sqrt{e}\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{cd^2}{ae^2}\right)}{d^{3/2}\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}$$

output
```
-d*(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)/a/x/(e*x^2+d)-e^(1/2)*(d*(c*d*x^2+a*e
)/a/e/(e*x^2+d))^(1/2)*(e*x^2+d)*EllipticE(e^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(
1/2),(1-c*d^2/a/e^2)^(1/2))/d^(3/2)/(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \frac{-\frac{(ae+cdx^2)(d+ex^2)}{x}-iad\sqrt{\frac{cd}{ae}}e\sqrt{1+\frac{cdx^2}{ae}}\sqrt{1+\frac{ex^2}{d}}\left(E\left(i\text{arcsinh}\left(\sqrt{\frac{cd}{ae}}x\right)|\frac{ae^2}{cd^2}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{cd}{ae}}\right.\right.\right.}{ade\sqrt{\frac{(ae+cdx^2)(d+ex^2)}{de}}}$$

input
```
Integrate[1/(x^2*Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4]),x]
```

output
```
(-(((a*e + c*d*x^2)*(d + e*x^2))/x) - I*a*d*Sqrt[(c*d)/(a*e)]*e*Sqrt[1 + (
c*d*x^2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*(EllipticE[I*ArcSinh[Sqrt[(c*d)/(a*e)]
*x], (a*e^2)/(c*d^2)] - EllipticF[I*ArcSinh[Sqrt[(c*d)/(a*e)]*x], (a*e^2)/
(c*d^2)]))/(a*d*e*Sqrt[((a*e + c*d*x^2)*(d + e*x^2))/(d*e)])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 396 vs. 2(153) = 306.

Time = 0.85 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.59, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1443, 27, 1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{\frac{x^2(ae^2+cd^2)}{de}+a+cx^4}}\,dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{\int \frac{cx^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}$$

$$\Big\downarrow \; 1459$$

$$\frac{c\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{c\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}$$

$$\Big\downarrow \; 1416$$

$$\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}$$

$$\Big\downarrow \; 1509$$

$$\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+c}}\right.}{a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}$$

input
```
Int[1/(x^2*Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4]),x]
```

output
```
-(Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4]/(a*x)) + (c*(-((-((x*Sqrt[a +
((c*d)/e + (a*e)/d)*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - ((c*d)/e + (
a*e)/d)/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 +
 c*x^4]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e +
 (a*e)/d)*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - ((c*d)/e + (a*e)/d)/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4
)*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.46

method	result
default	$-\frac{\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+c\,x^4}}{ax}-\frac{2c\sqrt{1+\frac{x^2dc}{ae}}\,\sqrt{1+\frac{e\,x^2}{d}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\frac{ae}{d}\right)e}{dc}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\right.}{d}}\right.\right.}{\sqrt{-\frac{cd}{ae}}\,\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+c\,x^4}\left(\frac{cd}{e}+\frac{ae}{d}+\frac{a\,e^2-c\,d^2}{de}\right)}$
elliptic	$-\frac{\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+c\,x^4}}{ax}-\frac{2c\sqrt{1+\frac{x^2dc}{ae}}\,\sqrt{1+\frac{e\,x^2}{d}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\frac{ae}{d}\right)e}{dc}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{\left(\frac{cd}{e}+\right.}{d}}\right.\right.}{\sqrt{-\frac{cd}{ae}}\,\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+c\,x^4}\left(\frac{cd}{e}+\frac{ae}{d}+\frac{a\,e^2-c\,d^2}{de}\right)}$
risch	$-\frac{(e\,x^2+d)\,(cd\,x^2+ae)}{edax\sqrt{\frac{(e\,x^2+d)(cd\,x^2+ae)}{de}}}-\frac{2c\,e^2d^2\sqrt{1+\frac{x^2dc}{ae}}\,\sqrt{1+\frac{e\,x^2}{d}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-1+\frac{ad\,e^3+c\,d^3e}{e\,d^3c}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{cd}{ae}},\sqrt{-\right.\right.}{\sqrt{-\frac{cd}{ae}}\,\sqrt{c\,d^2e^2x^4+a\,e^3x^2d+ce\,x^2d^3+a\,d^2e^2}\,(ad\,e^3+c\,d^3e+de(a\,e^2-c\,d^2))}$

input
```
int(1/x^2/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)/x-2*c/(-c*d/a/e)^(1/2)*(1+1/a*x^2
*d/e*c)^(1/2)*(1+e*x^2/d)^(1/2)/(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)/(c*d/e
+1/d*a*e+(a*e^2-c*d^2)/d/e)*(EllipticF(x*(-c*d/a/e)^(1/2),(-1+(c*d/e+1/d*a
*e)*e/d/c)^(1/2))-EllipticE(x*(-c*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \frac{\sqrt{a}cdx\sqrt{-\frac{cd}{ae}}E(\arcsin\left(x\sqrt{-\frac{cd}{ae}}\right)\mid\frac{ae^2}{cd^2}) - \sqrt{a}cdx\sqrt{-\frac{cd}{ae}}F(\arcsin\left(x\sqrt{-\frac{cd}{ae}}\right)\mid\frac{ae^2}{cd^2}) - ae\sqrt{\frac{cdex^4+ade+(cd^2+a}{de}}}{a^2ex}$$

input
```
integrate(1/x^2/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="fricas
")
```

output
```
(sqrt(a)*c*d*x*sqrt(-c*d/(a*e))*elliptic_e(arcsin(x*sqrt(-c*d/(a*e))), a*e
^2/(c*d^2)) - sqrt(a)*c*d*x*sqrt(-c*d/(a*e))*elliptic_f(arcsin(x*sqrt(-c*d
/(a*e))), a*e^2/(c*d^2)) - a*e*sqrt((c*d*e*x^4 + a*d*e + (c*d^2 + a*e^2)*x
^2)/(d*e)))/(a^2*e*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{1}{x^2\sqrt{a+\frac{aex^2}{d}+\frac{cdx^2}{e}+cx^4}}\,dx$$

input
```
integrate(1/x**2/(a+(a*e**2+c*d**2)*x**2/d/e+c*x**4)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + a*e*x**2/d + c*d*x**2/e + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{1}{x^2\,\sqrt{a+c\,x^4+\frac{x^2\,\left(c\,d^2+a\,e^2\right)}{d\,e}}}\,dx$$

input
```
int(1/(x^2*(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2)),x)
```

output
```
int(1/(x^2*(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\sqrt{e}\,\sqrt{d}\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}}{cde\,x^6+a\,e^2x^4+c\,d^2x^4+ade\,x^2}dx\right)$$

input
```
int(1/x^2/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x)
```

output
```
sqrt(e)*sqrt(d)*int((sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2))/(a*d*e*x**2 +
a*e**2*x**4 + c*d**2*x**4 + c*d*e*x**6),x)
```

3.713 $\int \frac{1}{x^4\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 334

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx &= -\frac{\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}{3ax^3}+\frac{2(cd^2+ae^2)\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}{3a^2ex\,(d+ex^2)} \\ &+\frac{2(cd^2+ae^2)\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{cd^2}{ae^2}\right)}{3ad^{5/2}\sqrt{e}\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}} \\ &-\frac{c\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),1-\frac{cd^2}{ae^2}\right)}{3a\sqrt{d}\sqrt{e}\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}\end{aligned}$$

output
```
-1/3*(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)/a/x^3+2/3*(a*e^2+c*d^2)*(a+(c*d/e+a
*e/d)*x^2+c*x^4)^(1/2)/a^2/e/x/(e*x^2+d)+2/3*(a*e^2+c*d^2)*(d*(c*d*x^2+a*e
)/a/e/(e*x^2+d))^(1/2)*(e*x^2+d)*EllipticE(e^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(
1/2),(1-c*d^2/a/e^2)^(1/2))/a/d^(5/2)/e^(1/2)/(a+(c*d/e+a*e/d)*x^2+c*x^4)^
(1/2)-1/3*c*(d*(c*d*x^2+a*e)/a/e/(e*x^2+d))^(1/2)*(e*x^2+d)*InverseJacobiA
M(arctan(e^(1/2)*x/d^(1/2)),(1-c*d^2/a/e^2)^(1/2))/a/d^(1/2)/e^(1/2)/(a+(c
*d/e+a*e/d)*x^2+c*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \frac{(d+ex^2)\left(2c^2d^3x^4+a^2e^2(-d+2ex^2)+acdex^2(d+2ex^2)\right)+2iad\sqrt{\frac{cd}{ae}}e(cd^2+ae^2)\,x^3\sqrt{1+\frac{cdx^2}{ae}}\sqrt{1+}}{3a^2d^2e^2x^3}$$

input
```
Integrate[1/(x^4*Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4]),x]
```

output
```
((d + e*x^2)*(2*c^2*d^3*x^4 + a^2*e^2*(-d + 2*e*x^2) + a*c*d*e*x^2*(d + 2*
e*x^2)) + (2*I)*a*d*Sqrt[(c*d)/(a*e)]*e*(c*d^2 + a*e^2)*x^3*Sqrt[1 + (c*d*
x^2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[(c*d)/(a*e)]*x],
(a*e^2)/(c*d^2)] - I*a*d*Sqrt[(c*d)/(a*e)]*e*(2*c*d^2 + a*e^2)*x^3*Sqrt[1
+ (c*d*x^2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[(c*d)/(a*e
)]*x], (a*e^2)/(c*d^2)])/(3*a^2*d^2*e^2*x^3*Sqrt[((a*e + c*d*x^2)*(d + e*x
^2))/(d*e)])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.48, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1443, 25, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{\frac{x^2(ae^2+cd^2)}{de}+a+cx^4}}\,dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{\int -\frac{cx^2+2\left(\frac{cd}{e}+\frac{ae}{d}\right)}{x^2\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{3a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}$$

$\Big\downarrow$ 25

$$-\frac{\int \frac{cx^2+2\left(\frac{cd}{e}+\frac{ae}{d}\right)}{x^2\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{3a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}$$

$\Big\downarrow$ 1604

$$-\frac{-\frac{\int -\frac{c\left(2\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a\right)}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{a}-\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}}{3a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int \frac{c\left(2\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a\right)}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{a}-\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}}{3a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}$$

$\Big\downarrow$ 27

$$-\frac{\frac{c\int \frac{2\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{a}-\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}}{3a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}$$

$\Big\downarrow$ 1511

$$-\frac{\frac{c\left(\sqrt{a}\left(\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)}{\sqrt{c}}+\sqrt{a}\right)\int \frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx-\frac{2\sqrt{a}\left(\frac{ae}{d}+\frac{cd}{e}\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}}{3a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}$$

$\Big\downarrow$ 27

$$-\frac{\frac{c\left(\sqrt{a}\left(\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)}{\sqrt{c}}+\sqrt{a}\right)\int \frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx-\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}}{3a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}$$

$\downarrow$ 1416

$$-\frac{\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)}{\sqrt{c}}+\sqrt{a}\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}\cdots}{a}\cdots}{3a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}$$

$\downarrow$ 1509

$$-\frac{\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)}{\sqrt{c}}+\sqrt{a}\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{2\left(\frac{ae}{d}+\frac{cd}{e}\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{x^2\left(\cdots\right.}{\left(\cdots\right.}}\cdots}{\cdots}\cdots\right.}{\cdots}\cdots}{a}\cdots}{3a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}$$

input
```
Int[1/(x^4*Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4]),x]
```

output
```
-1/3*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4]/(a*x^3) - ((-2*((c*d)/e + (
a*e)/d)*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])/(a*x) + (c*((-2*((c*d)/
e + (a*e)/d)*(-((x*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])/(Sqrt[a] + S
qrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e + (a*e)
/d)*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - ((c*d)/e + (a*e)/d)/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a
 + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])))/Sqrt[c] + (a^(1/4)*(Sqrt[a] + (2*((
c*d)/e + (a*e)/d))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e + (
a*e)/d)*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - ((c*d)/e + (a*e)/d)/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*
Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])))/a)/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\sqrt{a+\frac{x^{2} e a}{d}+\frac{x^{2} d c}{e}+c\,x^{4}}}{3 a\,x^{3}}+\frac{2\left(a\,e^{2}+c\,d^{2}\right)\sqrt{a+\frac{x^{2} e a}{d}+\frac{x^{2} d c}{e}+c\,x^{4}}}{3 e d\,a^{2} x}-\frac{c\sqrt{1+\frac{x^{2} d c}{a e}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\ \mathrm{EllipticF}\left(x\sqrt{-\frac{c d}{a e}},\sqrt{-1+\frac{\left(\frac{c d}{e}+\frac{a e}{d}\right)}{d c}}\right.}{3 a\sqrt{-\frac{c d}{a e}}\,\sqrt{a+\frac{x^{2} e a}{d}+\frac{x^{2} d c}{e}+c\,x^{4}}}$
elliptic	$-\frac{\sqrt{a+\frac{x^{2} e a}{d}+\frac{x^{2} d c}{e}+c\,x^{4}}}{3 a\,x^{3}}+\frac{2\left(a\,e^{2}+c\,d^{2}\right)\sqrt{a+\frac{x^{2} e a}{d}+\frac{x^{2} d c}{e}+c\,x^{4}}}{3 e d\,a^{2} x}-\frac{c\sqrt{1+\frac{x^{2} d c}{a e}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\ \mathrm{EllipticF}\left(x\sqrt{-\frac{c d}{a e}},\sqrt{-1+\frac{\left(\frac{c d}{e}+\frac{a e}{d}\right)}{d c}}\right.}{3 a\sqrt{-\frac{c d}{a e}}\,\sqrt{a+\frac{x^{2} e a}{d}+\frac{x^{2} d c}{e}+c\,x^{4}}}$
risch	$-\frac{\left(e\,x^{2}+d\right)\left(c d\,x^{2}+a e\right)\left(-2 a\,e^{2} x^{2}-2 c\,d^{2} x^{2}+a d e\right)}{3 a^{2} d^{2} e^{2} x^{3}\sqrt{\frac{\left(e\,x^{2}+d\right)\left(c d\,x^{2}+a e\right)}{d e}}}-\frac{c\left(-\frac{2\left(2 a\,e^{2}+2 c\,d^{2}\right) a\,e^{2} d^{2}\sqrt{1+\frac{x^{2} d c}{a e}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{c d}{a e}},\sqrt{-1+\frac{a d\,e^{3}+}{e\,d^{3}}}\right.\right.}{\sqrt{-\frac{c d}{a e}}\,\sqrt{c\,d^{2} e^{2} x^{4}+a\,e^{3} x^{2} d+c e\,x^{2} d^{3}+a\,d^{2} e^{2}}\left(a d\,e^{3}\right.}\right.}{}$

input
```
int(1/x^4/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a*(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)/x^3+2/3*(a*e^2+c*d^2)/e/d/a^2*(
a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)/x-1/3*c/a/(-c*d/a/e)^(1/2)*(1+1/a*x^2*d
/e*c)^(1/2)*(1+e*x^2/d)^(1/2)/(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)*Elliptic
F(x*(-c*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2))+4/3*(a*e^2+c*d^2)/e
*c/d/a/(-c*d/a/e)^(1/2)*(1+1/a*x^2*d/e*c)^(1/2)*(1+e*x^2/d)^(1/2)/(a+x^2/d
*e*a+x^2*d/e*c+c*x^4)^(1/2)/(c*d/e+1/d*a*e+(a*e^2-c*d^2)/d/e)*(EllipticF(x
*(-c*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2))-EllipticE(x*(-c*d/a/e)
^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^4\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = -\frac{2\,(c^2d^3+acde^2)\sqrt{a}x^3\sqrt{-\frac{cd}{ae}}E(\arcsin\left(x\sqrt{-\frac{cd}{ae}}\right) \mid \frac{ae^2}{cd^2}) - (2\,c^2d^3+2\,acde^2+a^2e^3)\sqrt{a}x^3\sqrt{-\frac{cd}{ae}}F(\arcsi}{3\,a^3de^2x^3}$$

input
```
integrate(1/x^4/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="fricas
")
```

output
```
-1/3*(2*(c^2*d^3 + a*c*d*e^2)*sqrt(a)*x^3*sqrt(-c*d/(a*e))*elliptic_e(arcs
in(x*sqrt(-c*d/(a*e))), a*e^2/(c*d^2)) - (2*c^2*d^3 + 2*a*c*d*e^2 + a^2*e^
3)*sqrt(a)*x^3*sqrt(-c*d/(a*e))*elliptic_f(arcsin(x*sqrt(-c*d/(a*e))), a*e
^2/(c*d^2)) + (a^2*d*e^2 - 2*(a*c*d^2*e + a^2*e^3)*x^2)*sqrt((c*d*e*x^4 +
a*d*e + (c*d^2 + a*e^2)*x^2)/(d*e)))/(a^3*d*e^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{1}{x^4\sqrt{a+\frac{aex^2}{d}+\frac{cdx^2}{e}+cx^4}}\,dx$$

input
```
integrate(1/x**4/(a+(a*e**2+c*d**2)*x**2/d/e+c*x**4)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a + a*e*x**2/d + c*d*x**2/e + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}\,x^4}\,dx$$

input
```
integrate(1/x^4/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e))*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}\,x^4}\,dx$$

input
```
integrate(1/x^4/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e))*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\int \frac{1}{x^4\,\sqrt{a+c\,x^4+\frac{x^2\,(c\,d^2+a\,e^2)}{d\,e}}}\,dx$$

input
```
int(1/(x^4*(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2)),x)
```

output
```
int(1/(x^4*(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\sqrt{e}\,\sqrt{d}\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}}{cde\,x^8+a\,e^2x^6+c\,d^2x^6+ade\,x^4}dx\right)$$

input
```
int(1/x^4/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x)
```

output
```
sqrt(e)*sqrt(d)*int((sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2))/(a*d*e*x**4 +
a*e**2*x**6 + c*d**2*x**6 + c*d*e*x**8),x)
```

3.714 $\int \frac{1}{x^6\sqrt{a+\frac{\left(cd^2+ae^2\right)x^2}{de}+cx^4}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 426

$$\begin{aligned}\int \frac{1}{x^6\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx &= -\frac{\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}{5ax^5}+\frac{4\left(\frac{cd}{ae}+\frac{e}{d}\right)\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}{15ax^3}\\ &\quad+\frac{d\left(9ac-\frac{8(cd^2+ae^2)^2}{d^2e^2}\right)\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}{15a^3x\,(d+ex^2)}\\ &\quad+\frac{\sqrt{e}\left(9ac-\frac{8(cd^2+ae^2)^2}{d^2e^2}\right)\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{cd^2}{ae^2}\right)}{15a^2d^{3/2}\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}\\ &\quad+\frac{4c(cd^2+ae^2)\sqrt{\frac{d(ae+cdx^2)}{ae(d+ex^2)}}(d+ex^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),1-\frac{cd^2}{ae^2}\right)}{15a^2d^{3/2}e^{3/2}\sqrt{a+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+cx^4}}\end{aligned}$$

output
```
-1/5*(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)/a/x^5+4/15*(c*d/a/e+e/d)*(a+(c*d/e+
a*e/d)*x^2+c*x^4)^(1/2)/a/x^3+1/15*d*(9*a*c-8*(a*e^2+c*d^2)^2/d^2/e^2)*(a+
(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)/a^3/x/(e*x^2+d)+1/15*e^(1/2)*(9*a*c-8*(a*e^
2+c*d^2)^2/d^2/e^2)*(d*(c*d*x^2+a*e)/a/e/(e*x^2+d))^(1/2)*(e*x^2+d)*Ellipt
icE(e^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(1/2),(1-c*d^2/a/e^2)^(1/2))/a^2/d^(3/2)
/(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)+4/15*c*(a*e^2+c*d^2)*(d*(c*d*x^2+a*e)/a
/e/(e*x^2+d))^(1/2)*(e*x^2+d)*InverseJacobiAM(arctan(e^(1/2)*x/d^(1/2)),(1
-c*d^2/a/e^2)^(1/2))/a^2/d^(3/2)/e^(3/2)/(a+(c*d/e+a*e/d)*x^2+c*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.16 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^6\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx$$

$$= \frac{-\sqrt{\frac{cd}{ae}}(d+ex^2)\left(8c^3d^5x^6+ac^2d^3ex^4(4d+7ex^2)+a^3e^3(3d^2-4dex^2+8e^2x^4)+a^2cde^2x^2(-d^2+3dex^2\right.}{}$$

input
```
Integrate[1/(x^6*Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4]),x]
```

output
```
(-(Sqrt[(c*d)/(a*e)]*(d + e*x^2)*(8*c^3*d^5*x^6 + a*c^2*d^3*e*x^4*(4*d + 7
*e*x^2) + a^3*e^3*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4) + a^2*c*d*e^2*x^2*(-d^2
+ 3*d*e*x^2 + 8*e^2*x^4))) - I*c*d^2*(8*c^2*d^4 + 7*a*c*d^2*e^2 + 8*a^2*e^
4)*x^5*Sqrt[1 + (c*d*x^2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*EllipticE[I*ArcSinh[S
qrt[(c*d)/(a*e)]*x], (a*e^2)/(c*d^2)] + I*c*d^2*(8*c^2*d^4 + 3*a*c*d^2*e^2
 + 4*a^2*e^4)*x^5*Sqrt[1 + (c*d*x^2)/(a*e)]*Sqrt[1 + (e*x^2)/d]*EllipticF[
I*ArcSinh[Sqrt[(c*d)/(a*e)]*x], (a*e^2)/(c*d^2)])/(15*a^3*d^3*Sqrt[(c*d)/(
a*e)]*e^3*x^5*Sqrt[((a*e + c*d*x^2)*(d + e*x^2))/(d*e)])
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.39, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1443, 25, 1604, 25, 1604, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{\frac{x^2(ae^2+cd^2)}{de}+a+cx^4}}dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{\int -\frac{3cx^2+4\left(\frac{cd}{e}+\frac{ae}{d}\right)}{x^4\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{5a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5ax^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{3cx^2+4\left(\frac{cd}{e}+\frac{ae}{d}\right)}{x^4\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{5a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5ax^5}$$

$$\Big\downarrow \text{1604}$$

$$-\frac{-\frac{\int -\frac{-8\left(\frac{cd}{e}+\frac{ae}{d}\right)^2-4cx^2\left(\frac{cd}{e}+\frac{ae}{d}\right)+9ac}{x^2\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{3a}-\frac{4\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}}{5a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5ax^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{-8\left(\frac{cd}{e}+\frac{ae}{d}\right)^2-4cx^2\left(\frac{cd}{e}+\frac{ae}{d}\right)+9ac}{x^2\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{3a}-\frac{4\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}}{5a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5ax^5}$$

$$\Big\downarrow \text{1604}$$

$$-\frac{\frac{-\frac{\int \frac{c\left(4a\left(\frac{cd}{e}+\frac{ae}{d}\right)-\left(9ac-8\left(\frac{cd}{e}+\frac{ae}{d}\right)^2\right)x^2\right)}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{a}-\frac{\left(9ac-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}}{3a}-\frac{4\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}}{5a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5ax^5}$$

$\downarrow$ 27

$$-\frac{-\frac{-\frac{c\int \frac{4a\left(\frac{cd}{e}+\frac{ae}{d}\right)-\left(9ac-8\left(\frac{cd}{e}+\frac{ae}{d}\right)^2\right)x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{a}-\frac{\left(9ac-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{ax}}{3a}-\frac{4\left(\frac{ae}{d}+\frac{cd}{e}\right)\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{3ax^3}}{5a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5ax^5}$$

$\downarrow$ 1511

$$-\frac{-\frac{-\frac{c\left(\frac{\sqrt{a}\left(9ac-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2-4\sqrt{a}\sqrt{c}\left(\frac{ae}{d}+\frac{cd}{e}\right)+9ac\right)\int \frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\left(9ac-8\left(\frac{ae}{d}+\cdots\right.\right.}{\cdots}}{3a}\cdots}{5a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5ax^5}$$

$\downarrow$ 27

$$-\frac{-\frac{-\frac{c\left(\frac{\left(9ac-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2-4\sqrt{a}\sqrt{c}\left(\frac{ae}{d}+\frac{cd}{e}\right)+9ac\right)\int \frac{1}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\left(9ac-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2\right)\cdots}{\cdots}}{3a}\cdots}{5a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5ax^5}$$

$\downarrow$ 1416

$$-\frac{-\frac{-\frac{c\left(\frac{\left(9ac-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+\left(\frac{cd}{e}+\frac{ae}{d}\right)x^2+a}}dx}{\sqrt{c}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2-4\sqrt{a}\sqrt{c}\left(\frac{ae}{d}+\frac{cd}{e}\right)+9ac\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\,\text{arc}\cdots\right.}{2c^{3/4}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}\cdots}{a}\cdots}{3a}\cdots}{5a}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5ax^5}$$

$\downarrow$ 1509

$$-\frac{-\frac{c\left(\frac{\left(9ac-8\left(\frac{ae}{d}+\frac{cd}{e}\right)^2\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{\frac{cd}{e}+\frac{ae}{d}}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}-\frac{x\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{\cdots}\right.}{\cdots}\right)}{a}}{\cdots}-\frac{\sqrt{x^2\left(\frac{ae}{d}+\frac{cd}{e}\right)+a+cx^4}}{5ax^5}$$

input
```
Int[1/(x^6*Sqrt[a + ((c*d^2 + a*e^2)*x^2)/(d*e) + c*x^4]),x]
```

output
```
-1/5*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4]/(a*x^5) - ((-4*((c*d)/e + (
a*e)/d)*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])/(3*a*x^3) + (-(((9*a*c
- 8*((c*d)/e + (a*e)/d)^2)*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])/(a*x
)) - (c*(((9*a*c - 8*((c*d)/e + (a*e)/d)^2)*(-((x*Sqrt[a + ((c*d)/e + (a*e
)/d)*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - ((c*d)/e + (a*e)/d)/(Sqrt[a
]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + ((c*d)/e + (a*e)/d)*x^2 + c*x^4])))/Sqrt
[c] - (a^(1/4)*(9*a*c - 4*Sqrt[a]*Sqrt[c]*((c*d)/e + (a*e)/d) - 8*((c*d)/e
 + (a*e)/d)^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + ((c*d)/e + (a*e)/d)*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - ((c*d)/e + (a*e)/d)/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + ((c*
d)/e + (a*e)/d)*x^2 + c*x^4])))/a)/(3*a))/(5*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 6.07 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+c\,x^4}}{5a\,x^5}+\frac{4(a\,e^2+c\,d^2)\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+c\,x^4}}{15ed\,a^2x^3}-\frac{(8a^2e^4+7ac\,d^2e^2+8c^2d^4)\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+c\,x^4}}{15e^2d^2a^3x}+\frac{4(a\,e^2}{}$
elliptic	$-\frac{\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+c\,x^4}}{5a\,x^5}+\frac{4(a\,e^2+c\,d^2)\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+c\,x^4}}{15ed\,a^2x^3}-\frac{(8a^2e^4+7ac\,d^2e^2+8c^2d^4)\sqrt{a+\frac{x^2ea}{d}+\frac{x^2dc}{e}+c\,x^4}}{15e^2d^2a^3x}+\frac{4(a\,e^2}{}$
risch	$-\frac{(e\,x^2+d)(cd\,x^2+ae)(8a^2e^4x^4+7ac\,d^2e^2x^4+8c^2d^4x^4-4a^2d\,e^3x^2-4ac\,d^3e\,x^2+3a^2d^2e^2)}{15a^3d^3e^3x^5\sqrt{\frac{(e\,x^2+d)(cd\,x^2+ae)}{de}}}+\frac{c\left(-\frac{2(8a^2e^4+7ac\,d^2e^2+8c^2d^4)a\,e^2d}{}\right.}{}$

input
```
int(1/x^6/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/a*(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)/x^5+4/15*(a*e^2+c*d^2)/e/d/a^2*
(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)/x^3-1/15*(8*a^2*e^4+7*a*c*d^2*e^2+8*c^
2*d^4)/e^2/d^2/a^3*(a+x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)/x+4/15*(a*e^2+c*d^2
)/e*c/d/a^2/(-c*d/a/e)^(1/2)*(1+1/a*x^2*d/e*c)^(1/2)*(1+e*x^2/d)^(1/2)/(a+
x^2/d*e*a+x^2*d/e*c+c*x^4)^(1/2)*EllipticF(x*(-c*d/a/e)^(1/2),(-1+(c*d/e+1
/d*a*e)*e/d/c)^(1/2))-2/15*c*(8*a^2*e^4+7*a*c*d^2*e^2+8*c^2*d^4)/e^2/d^2/a
^2/(-c*d/a/e)^(1/2)*(1+1/a*x^2*d/e*c)^(1/2)*(1+e*x^2/d)^(1/2)/(a+x^2/d*e*a
+x^2*d/e*c+c*x^4)^(1/2)/(c*d/e+1/d*a*e+(a*e^2-c*d^2)/d/e)*(EllipticF(x*(-c
*d/a/e)^(1/2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2))-EllipticE(x*(-c*d/a/e)^(1/
2),(-1+(c*d/e+1/d*a*e)*e/d/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^6\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx$$

$$=\frac{(8\,c^3d^5+7\,ac^2d^3e^2+8\,a^2cde^4)\sqrt{a}x^5\sqrt{-\frac{cd}{ae}}E(\arcsin\left(x\sqrt{-\frac{cd}{ae}}\right)\mid\frac{ae^2}{cd^2})-(8\,c^3d^5+7\,ac^2d^3e^2+4\,a^2cd^2e^3+}{}$$

input
```
integrate(1/x^6/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="fricas
")
```

output
```
1/15*((8*c^3*d^5 + 7*a*c^2*d^3*e^2 + 8*a^2*c*d*e^4)*sqrt(a)*x^5*sqrt(-c*d/
(a*e))*elliptic_e(arcsin(x*sqrt(-c*d/(a*e))), a*e^2/(c*d^2)) - (8*c^3*d^5
+ 7*a*c^2*d^3*e^2 + 4*a^2*c*d^2*e^3 + 8*a^2*c*d*e^4 + 4*a^3*e^5)*sqrt(a)*x
^5*sqrt(-c*d/(a*e))*elliptic_f(arcsin(x*sqrt(-c*d/(a*e))), a*e^2/(c*d^2))
- (3*a^3*d^2*e^3 + (8*a*c^2*d^4*e + 7*a^2*c*d^2*e^3 + 8*a^3*e^5)*x^4 - 4*(
a^2*c*d^3*e^2 + a^3*d*e^4)*x^2)*sqrt((c*d*e*x^4 + a*d*e + (c*d^2 + a*e^2)*
x^2)/(d*e)))/(a^4*d^2*e^3*x^5)
```

Sympy [F]

$$\int \frac{1}{x^6\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{1}{x^6\sqrt{a+\frac{aex^2}{d}+\frac{cdx^2}{e}+cx^4}}\,dx$$

input
```
integrate(1/x**6/(a+(a*e**2+c*d**2)*x**2/d/e+c*x**4)**(1/2),x)
```

output
```
Integral(1/(x**6*sqrt(a + a*e*x**2/d + c*d*x**2/e + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}x^6}\,dx$$

input
```
integrate(1/x^6/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e))*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+a+\frac{(cd^2+ae^2)x^2}{de}}x^6}\,dx$$

input
```
integrate(1/x^6/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a + (c*d^2 + a*e^2)*x^2/(d*e))*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx=\int \frac{1}{x^6\sqrt{a+c\,x^4+\frac{x^2\,(c\,d^2+a\,e^2)}{d\,e}}}\,dx$$

input
```
int(1/(x^6*(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2)),x)
```

output
```
int(1/(x^6*(a + c*x^4 + (x^2*(a*e^2 + c*d^2))/(d*e))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{a+\frac{(cd^2+ae^2)x^2}{de}+cx^4}}\,dx$$
$$=\frac{\sqrt{e}\,\sqrt{d}\left(-\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}\,a+3\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}\,c\,x^4-3\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{cd\,x^2+ae}\,x^2}{cde\,x^4+a\,e^2x^2+c\,d^2x^2+ade}dx\right)c^2de\,x\right.}{5a^2de\,x^5}$$

input
```
int(1/x^6/(a+(a*e^2+c*d^2)*x^2/d/e+c*x^4)^(1/2),x)
```

output

```
(sqrt(e)*sqrt(d)*( - sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2)*a + 3*sqrt(d +
e*x**2)*sqrt(a*e + c*d*x**2)*c*x**4 - 3*int((sqrt(d + e*x**2)*sqrt(a*e + c
*d*x**2)*x**2)/(a*d*e + a*e**2*x**2 + c*d**2*x**2 + c*d*e*x**4),x)*c**2*d*
e*x**5 - 4*int((sqrt(d + e*x**2)*sqrt(a*e + c*d*x**2))/(a*d*e*x**4 + a*e**
2*x**6 + c*d**2*x**6 + c*d*e*x**8),x)*a**2*e**2*x**5 - 4*int((sqrt(d + e*x
**2)*sqrt(a*e + c*d*x**2))/(a*d*e*x**4 + a*e**2*x**6 + c*d**2*x**6 + c*d*e
*x**8),x)*a*c*d**2*x**5))/(5*a**2*d*e*x**5)
```

3.715 $\int \frac{x^6}{\sqrt{a+bx^2+\frac{\left(bde-ae^2\right)x^4}{d^2}}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 447

$$\begin{aligned}
&\int \frac{x^6}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx \\
&= \frac{d^2(8b^2d^2-9abde+9a^2e^2)\,x(ad+(bd-ae)x^2)}{15e^2(bd-ae)^3\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}} \\
&\quad - \frac{4bd^4x\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}{15e^2(bd-ae)^2} + \frac{d^2x^3\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}{5e(bd-ae)} \\
&\quad - \frac{ad^{5/2}(8b^2d^2-9abde+9a^2e^2)\,(d+ex^2)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|2-\frac{bd}{ae}\right)}{15e^{5/2}(bd-ae)^3\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}} \\
&\quad + \frac{4abd^{7/2}(d+ex^2)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),2-\frac{bd}{ae}\right)}{15e^{5/2}(bd-ae)^2\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}
\end{aligned}$$

output
```
1/15*d^2*(9*a^2*e^2-9*a*b*d*e+8*b^2*d^2)*x*(a*d+(-a*e+b*d)*x^2)/e^2/(-a*e+
b*d)^3/(a+b*x^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)-4/15*b*d^4*x*(a+b*x^2+e*(-a*e+
b*d)*x^4/d^2)^(1/2)/e^2/(-a*e+b*d)^2+1/5*d^2*x^3*(a+b*x^2+e*(-a*e+b*d)*x^4
/d^2)^(1/2)/e/(-a*e+b*d)-1/15*a*d^(5/2)*(9*a^2*e^2-9*a*b*d*e+8*b^2*d^2)*(e
*x^2+d)*((a*d+(-a*e+b*d)*x^2)/a/(e*x^2+d))^(1/2)*EllipticE(e^(1/2)*x/d^(1/
2)/(1+e*x^2/d)^(1/2),(2-b*d/a/e)^(1/2))/e^(5/2)/(-a*e+b*d)^3/(a+b*x^2+e*(-
a*e+b*d)*x^4/d^2)^(1/2)+4/15*a*b*d^(7/2)*(e*x^2+d)*((a*d+(-a*e+b*d)*x^2)/a
/(e*x^2+d))^(1/2)*InverseJacobiAM(arctan(e^(1/2)*x/d^(1/2)),(2-b*d/a/e)^(1
/2))/e^(5/2)/(-a*e+b*d)^2/(a+b*x^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.51 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.77

$$\int \frac{x^6}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx$$
$$= \frac{-\sqrt{\frac{e}{d}}(bd-ae)x(d+ex^2)\left(b^2d^2x^2(4d-3ex^2)+3a^2e^2x^2(d-ex^2)+abd(4d^2-7dex^2+6e^2x^4)\right)-iad^3(8}{}$$

input
```
Integrate[x^6/Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2],x]
```

output
```
(-(Sqrt[e/d]*(b*d - a*e)*x*(d + e*x^2)*(b^2*d^2*x^2*(4*d - 3*e*x^2) + 3*a^
2*e^2*x^2*(d - e*x^2) + a*b*d*(4*d^2 - 7*d*e*x^2 + 6*e^2*x^4))) - I*a*d^3*
(8*b^2*d^2 - 9*a*b*d*e + 9*a^2*e^2)*Sqrt[1 + (b*x^2)/a - (e*x^2)/d]*Sqrt[1
 + (e*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[e/d]*x], -1 + (b*d)/(a*e)] + I*a*d^
3*(4*b^2*d^2 - 5*a*b*d*e + 9*a^2*e^2)*Sqrt[1 + (b*x^2)/a - (e*x^2)/d]*Sqrt
[1 + (e*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[e/d]*x], -1 + (b*d)/(a*e)])/(15*d
^2*(e/d)^(5/2)*(b*d - a*e)^3*Sqrt[((d + e*x^2)*(b*d*x^2 + a*(d - e*x^2)))/
d^2])
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.63, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1442, 1602, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{\frac{x^4(bde-ae^2)}{d^2}+a+bx^2}}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{d^2x^3\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5e(bd-ae)}-\frac{d^2\int \frac{x^2(4bx^2+3a)}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{5e(bd-ae)}$$

$$\Big\downarrow \text{1602}$$

$$\frac{d^2x^3\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5e(bd-ae)}-\frac{d^2\left(\frac{4bd^2x\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3e(bd-ae)}-\frac{d^2\int \frac{\left(8b^2-\frac{9ae(bd-ae)}{d^2}\right)x^2+4ab}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{3e(bd-ae)}\right)}{5e(bd-ae)}$$

$$\Big\downarrow \text{1511}$$

$$\frac{d^2x^3\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5e(bd-ae)}-$$

$$\frac{d^2\left(\frac{4bd^2x\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3e(bd-ae)}-\frac{d^2\left(\sqrt{a}\left(\frac{8b^2d}{\sqrt{e}\sqrt{bd-ae}}-\frac{9a\sqrt{e}\sqrt{bd-ae}}{d}+4\sqrt{a}b\right)\int \frac{1}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx-\frac{\sqrt{a}d\left(8b^2-\frac{9ae(bd-ae)}{d^2}\right)\int \frac{\sqrt{a}d-\sqrt{\cdots}}{\sqrt{a}d\sqrt{\frac{e(bd\cdots}{}}}}{\sqrt{e}\sqrt{bd-ae}}\cdots\right.}{3e(bd-ae)}\cdots\right.}{5e(bd-ae)}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2x^3\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5e(bd-ae)}-\frac{d^2\left(\frac{4bd^2x\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3e(bd-ae)}-\frac{d^2\left(\sqrt{a}\left(\frac{8b^2d}{\sqrt{e}\sqrt{bd-ae}}-\frac{9a\sqrt{e}\sqrt{bd-ae}}{d}+4\sqrt{a}b\right)\int\frac{1}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx-\frac{\left(8b^2-\frac{9ae(bd-ae)}{d^2}\right)\int\frac{\sqrt{a}d-\sqrt{e}\sqrt{bd-}}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+}}}{\sqrt{e}\sqrt{bd-ae}}\right.}{3e(bd-ae)}\right.}{5e(bd-ae)}$$

$\downarrow$ 1416

$$\frac{d^2x^3\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5e(bd-ae)}-\frac{d^2\left(\frac{4bd^2x\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3e(bd-ae)}-\frac{d^2\left(\frac{\sqrt[4]{a}\left(\frac{8b^2d}{\sqrt{e}\sqrt{bd-ae}}-\frac{9a\sqrt{e}\sqrt{bd-ae}}{d}+4\sqrt{a}b\right)\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticF}\left(2\arctan\right.}{2\sqrt{d}\sqrt[4]{e}\sqrt[4]{bd-ae}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}\right.}{3e(bd-ae)}\right.}{5e(bd-ae)}$$

$\downarrow$ 1509

$$\frac{d^2x^3\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5e(bd-ae)}-\frac{d^2\left(\frac{4bd^2x\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3e(bd-ae)}-\frac{d^2\left(\frac{\sqrt[4]{a}\left(\frac{8b^2d}{\sqrt{e}\sqrt{bd-ae}}-\frac{9a\sqrt{e}\sqrt{bd-ae}}{d}+4\sqrt{a}b\right)\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticF}\left(2\arctan\right.}{2\sqrt{d}\sqrt[4]{e}\sqrt[4]{bd-ae}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}\right.}{}\right)}{}$$

input

```
Int[x^6/Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2],x]
```

output

```
(d^2*x^3*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])/(5*e*(b*d - a*e)) - (d
^2*((4*b*d^2*x*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])/(3*e*(b*d - a*e)
) - (d^2*(-(((8*b^2 - (9*a*e*(b*d - a*e))/d^2)*(-((d^2*x*Sqrt[a + b*x^2 +
(e*(b*d - a*e)*x^4)/d^2])/(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)) + (a^
(1/4)*Sqrt[d]*(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)*Sqrt[(d^2*(a + b*x
^2 + (e*(b*d - a*e)*x^4)/d^2))/(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)^2
]*EllipticE[2*ArcTan[(e^(1/4)*(b*d - a*e)^(1/4)*x)/(a^(1/4)*Sqrt[d])], (2
- (b*d)/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]))/4])/(e^(1/4)*(b*d - a*e)^(1/4)*
Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])))/(Sqrt[e]*Sqrt[b*d - a*e])) +
(a^(1/4)*(4*Sqrt[a]*b + (8*b^2*d)/(Sqrt[e]*Sqrt[b*d - a*e]) - (9*a*Sqrt[e]
*Sqrt[b*d - a*e])/d)*(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)*Sqrt[(d^2*(
a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2))/(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]
*x^2)^2]*EllipticF[2*ArcTan[(e^(1/4)*(b*d - a*e)^(1/4)*x)/(a^(1/4)*Sqrt[d]
)], (2 - (b*d)/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]))/4])/(2*Sqrt[d]*e^(1/4)*(
b*d - a*e)^(1/4)*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])))/(3*e*(b*d -
a*e))))/(5*e*(b*d - a*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.13

method	result
default	$\frac{x^3\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{-\frac{5a e^2}{d^2}+\frac{5eb}{d}}-\frac{4bx\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{15\left(-\frac{a e^2}{d^2}+\frac{eb}{d}\right)^2}+\frac{4ba\sqrt{1-\frac{(ae-bd)x^2}{ad}}\sqrt{1+\frac{e x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}},\sqrt{-1+\frac{}{d(}}\right.}{15\left(-\frac{a e^2}{d^2}+\frac{eb}{d}\right)^2\sqrt{\frac{ae-bd}{ad}}\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2}}$
elliptic	$\frac{x^3\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{-\frac{5a e^2}{d^2}+\frac{5eb}{d}}-\frac{4bx\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{15\left(-\frac{a e^2}{d^2}+\frac{eb}{d}\right)^2}+\frac{4ba\sqrt{1-\frac{(ae-bd)x^2}{ad}}\sqrt{1+\frac{e x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}},\sqrt{-1+\frac{}{d(}}\right.}{15\left(-\frac{a e^2}{d^2}+\frac{eb}{d}\right)^2\sqrt{\frac{ae-bd}{ad}}\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2}}$
risch	$-\frac{(-ae x^2+bd x^2+ad)(e x^2+d)x(3a e^2x^2-3bde x^2+4b d^2)\sqrt{(e x^2+d)(-ae x^2+bd x^2+ad)}}{15e^2\sqrt{-(e x^2+d)(ae x^2-bd x^2-ad)}(ae-bd)^2\sqrt{\frac{(e x^2+d)(-ae x^2+bd x^2+ad)}{d^2}}}+\frac{d^2\left(-\frac{2(9a^2e^2-9abde+8b^2d^2)a d^2\sqrt{}}{}\right.}{}$

input
```
int(x^6/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/(-a/d^2*e^2+e/d*b)*x^3*(-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)-4/15/(
-a/d^2*e^2+e/d*b)^2*b*x*(-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)+4/15/(-a/
d^2*e^2+e/d*b)^2*b*a/((a*e-b*d)/a/d)^(1/2)*(1-(a*e-b*d)/a/d*x^2)^(1/2)*(1+
e*x^2/d)^(1/2)/(-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)*EllipticF(x*((a*e-
b*d)/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e^2+e/d*b))^(1/2))-2*(-3/5/(-a/d^2*e^2+e
/d*b)*a+8/15/(-a/d^2*e^2+e/d*b)^2*b^2)*a/((a*e-b*d)/a/d)^(1/2)*(1-(a*e-b*d
)/a/d*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2
)/(b+(2*a*e-b*d)/d)*(EllipticF(x*((a*e-b*d)/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e
^2+e/d*b))^(1/2))-EllipticE(x*((a*e-b*d)/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e^2+
e/d*b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.01

$$\int \frac{x^6}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = -\frac{(8\,ab^2d^6-9\,a^2bd^5e+9\,a^3d^4e^2)\sqrt{bde-ae^2}\sqrt{-\frac{ad}{bd-ae}}xE\left(\arcsin\left(\frac{\sqrt{-\frac{ad}{bd-ae}}}{x}\right)\mid\frac{bd-ae}{ae}\right)-(4\,(2\,ab^2+b^3)d^6}{}$$

input
```
integrate(x^6/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/15*((8*a*b^2*d^6 - 9*a^2*b*d^5*e + 9*a^3*d^4*e^2)*sqrt(b*d*e - a*e^2)*s
qrt(-a*d/(b*d - a*e))*x*elliptic_e(arcsin(sqrt(-a*d/(b*d - a*e))/x), (b*d
- a*e)/(a*e)) - (4*(2*a*b^2 + b^3)*d^6 - (9*a^2*b + 8*a*b^2)*d^5*e + (9*a^
3 + 4*a^2*b)*d^4*e^2)*sqrt(b*d*e - a*e^2)*sqrt(-a*d/(b*d - a*e))*x*ellipti
c_f(arcsin(sqrt(-a*d/(b*d - a*e))/x), (b*d - a*e)/(a*e)) - (8*b^3*d^7 - 17
*a*b^2*d^6*e + 18*a^2*b*d^5*e^2 - 9*a^3*d^4*e^3 + 3*(b^3*d^5*e^2 - 3*a*b^2
*d^4*e^3 + 3*a^2*b*d^3*e^4 - a^3*d^2*e^5)*x^4 - 4*(b^3*d^6*e - 2*a*b^2*d^5
*e^2 + a^2*b*d^4*e^3)*x^2)*sqrt((b*d^2*x^2 + (b*d*e - a*e^2)*x^4 + a*d^2)/
d^2))/((b^4*d^4*e^3 - 4*a*b^3*d^3*e^4 + 6*a^2*b^2*d^2*e^5 - 4*a^3*b*d*e^6
+ a^4*e^7)*x)
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{x^6}{\sqrt{-\left(1+\frac{ex^2}{d}\right)\left(-a+\frac{aex^2}{d}-bx^2\right)}}\,dx$$

input
```
integrate(x**6/(a+b*x**2+(-a*e**2+b*d*e)*x**4/d**2)**(1/2),x)
```

output
```
Integral(x**6/sqrt(-(1 + e*x**2/d)*(-a + a*e*x**2/d - b*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{x^6}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}}\,dx$$

input
```
integrate(x^6/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^6/sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{x^6}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}}\,dx$$

input
```
integrate(x^6/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{x^6}{\sqrt{a+b\,x^2-\frac{x^4\,(a\,e^2-b\,d\,e)}{d^2}}}\,dx$$

input
```
int(x^6/(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2),x)
```

output
```
int(x^6/(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \text{Too large to display}$$

input
```
int(x^6/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x)
```

output
```
(d*( - 3*sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*a*e**2*x**3 - 4*
sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*b*d**2*x + 3*sqrt(d + e*x
**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*b*d*e*x**3 + 9*int((sqrt(d + e*x**2)*
sqrt(a*d - a*e*x**2 + b*d*x**2)*x**2)/(a**3*d**2*e**2 - a**3*e**4*x**4 - 2
*a**2*b*d**3*e + a**2*b*d**2*e**2*x**2 + 3*a**2*b*d*e**3*x**4 + a*b**2*d**
4 - 2*a*b**2*d**3*e*x**2 - 3*a*b**2*d**2*e**2*x**4 + b**3*d**4*x**2 + b**3
*d**3*e*x**4),x)*a**4*d**2*e**4 - 27*int((sqrt(d + e*x**2)*sqrt(a*d - a*e*
x**2 + b*d*x**2)*x**2)/(a**3*d**2*e**2 - a**3*e**4*x**4 - 2*a**2*b*d**3*e
+ a**2*b*d**2*e**2*x**2 + 3*a**2*b*d*e**3*x**4 + a*b**2*d**4 - 2*a*b**2*d*
*3*e*x**2 - 3*a*b**2*d**2*e**2*x**4 + b**3*d**4*x**2 + b**3*d**3*e*x**4),x
)*a**3*b*d**3*e**3 + 35*int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x*
*2)*x**2)/(a**3*d**2*e**2 - a**3*e**4*x**4 - 2*a**2*b*d**3*e + a**2*b*d**2
*e**2*x**2 + 3*a**2*b*d*e**3*x**4 + a*b**2*d**4 - 2*a*b**2*d**3*e*x**2 - 3
*a*b**2*d**2*e**2*x**4 + b**3*d**4*x**2 + b**3*d**3*e*x**4),x)*a**2*b**2*d
**4*e**2 - 25*int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*x**2)/
(a**3*d**2*e**2 - a**3*e**4*x**4 - 2*a**2*b*d**3*e + a**2*b*d**2*e**2*x**2
 + 3*a**2*b*d*e**3*x**4 + a*b**2*d**4 - 2*a*b**2*d**3*e*x**2 - 3*a*b**2*d*
*2*e**2*x**4 + b**3*d**4*x**2 + b**3*d**3*e*x**4),x)*a*b**3*d**5*e + 8*int
((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*x**2)/(a**3*d**2*e**2 -
 a**3*e**4*x**4 - 2*a**2*b*d**3*e + a**2*b*d**2*e**2*x**2 + 3*a**2*b*d*...
```

3.716 $\int \frac{x^4}{\sqrt{a+bx^2+\frac{\left(bde-ae^2\right)x^4}{d^2}}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 351

$$\begin{aligned}
&\int \frac{x^4}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx \\
&= -\frac{2bd^2x(ad+(bd-ae)x^2)}{3e(bd-ae)^2\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}+\frac{d^2x\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}{3e(bd-ae)} \\
&\quad+\frac{2abd^{5/2}(d+ex^2)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|2-\frac{bd}{ae}\right)}{3e^{3/2}(bd-ae)^2\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}} \\
&\quad-\frac{ad^{3/2}(d+ex^2)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),2-\frac{bd}{ae}\right)}{3e^{3/2}(bd-ae)\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}
\end{aligned}$$

output
```
-2/3*b*d^2*x*(a*d+(-a*e+b*d)*x^2)/e/(-a*e+b*d)^2/(a+b*x^2+e*(-a*e+b*d)*x^4
/d^2)^(1/2)+1/3*d^2*x*(a+b*x^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)/e/(-a*e+b*d)+2/
3*a*b*d^(5/2)*(e*x^2+d)*((a*d+(-a*e+b*d)*x^2)/a/(e*x^2+d))^(1/2)*EllipticE
(e^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(1/2),(2-b*d/a/e)^(1/2))/e^(3/2)/(-a*e+b*d)
^2/(a+b*x^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)-1/3*a*d^(3/2)*(e*x^2+d)*((a*d+(-a*
e+b*d)*x^2)/a/(e*x^2+d))^(1/2)*InverseJacobiAM(arctan(e^(1/2)*x/d^(1/2)),(
2-b*d/a/e)^(1/2))/e^(3/2)/(-a*e+b*d)/(a+b*x^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.51 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.73

$$\int \frac{x^4}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx$$
$$= \frac{\sqrt{\frac{e}{d}}(bd-ae)x(d+ex^2)\left(bdx^2+a(d-ex^2)\right)+2iabd^3\sqrt{1+\frac{bx^2}{a}-\frac{ex^2}{d}}\sqrt{1+\frac{ex^2}{d}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)|-1+\right.}{3e\sqrt{\frac{e}{d}}(bd-ae)^2\sqrt{\frac{(d+ex^2)(bdx^2-}{d^2}}}$$

input
```
Integrate[x^4/Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2],x]
```

output
```
(Sqrt[e/d]*(b*d - a*e)*x*(d + e*x^2)*(b*d*x^2 + a*(d - e*x^2)) + (2*I)*a*b
*d^3*Sqrt[1 + (b*x^2)/a - (e*x^2)/d]*Sqrt[1 + (e*x^2)/d]*EllipticE[I*ArcSi
nh[Sqrt[e/d]*x], -1 + (b*d)/(a*e)] - I*a*d^2*(b*d + a*e)*Sqrt[1 + (b*x^2)/
a - (e*x^2)/d]*Sqrt[1 + (e*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[e/d]*x], -1 +
(b*d)/(a*e)])/(3*e*Sqrt[e/d]*(b*d - a*e)^2*Sqrt[((d + e*x^2)*(b*d*x^2 + a*
(d - e*x^2)))/d^2])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.73, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1442, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\frac{x^4(bde-ae^2)}{d^2}+a+bx^2}}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{d^2x\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3e(bd-ae)}-\frac{d^2\int \frac{2bx^2+a}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{3e(bd-ae)}$$

$$\Big\downarrow \text{1511}$$

$$\frac{d^2x\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3e(bd-ae)}-\frac{d^2\left(\sqrt{a}\left(\frac{2bd}{\sqrt{e}\sqrt{bd-ae}}+\sqrt{a}\right)\int \frac{1}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx-\frac{2\sqrt{a}bd\int \frac{\sqrt{a}d-\sqrt{e}\sqrt{bd-ae}x^2}{\sqrt{a}d\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}\right)}{3e(bd-ae)}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2x\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3e(bd-ae)}-\frac{d^2\left(\sqrt{a}\left(\frac{2bd}{\sqrt{e}\sqrt{bd-ae}}+\sqrt{a}\right)\int \frac{1}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx-\frac{2b\int \frac{\sqrt{a}d-\sqrt{e}\sqrt{bd-ae}x^2}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}\right)}{3e(bd-ae)}$$

$$\Big\downarrow \text{1416}$$

$$\frac{d^2x\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3e(bd-ae)}-\frac{d^2\left(\frac{\sqrt[4]{a}\left(\frac{2bd}{\sqrt{e}\sqrt{bd-ae}}+\sqrt{a}\right)\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt[4]{bd-ae}x}{\sqrt[4]{a}\sqrt{d}}\right),\frac{1}{4}\left(2-\frac{bd}{\sqrt{a}\sqrt{e}\sqrt{bd-ae}}\right)\right)}{2\sqrt{d}\sqrt[4]{e}\sqrt[4]{bd-ae}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}\right.}{3e(bd-ae)}$$

$$\Big\downarrow \text{1509}$$

$$\frac{d^2x\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3e(bd-ae)}-\frac{d^2\left(\frac{\sqrt[4]{a}\left(\frac{2bd}{\sqrt{e}\sqrt{bd-ae}}+\sqrt{a}\right)\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt[4]{bd-ae}x}{\sqrt[4]{a}\sqrt{d}}\right),\frac{1}{4}\left(2-\frac{bd}{\sqrt{a}\sqrt{e}\sqrt{bd-ae}}\right)\right)}{2\sqrt{d}\sqrt[4]{e}\sqrt[4]{bd-ae}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}\right.}{3e(b}$$

input
```
Int[x^4/Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2],x]
```

output
```
(d^2*x*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])/(3*e*(b*d - a*e)) - (d^2
*((-2*b*(-((d^2*x*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])/(Sqrt[a]*d +
Sqrt[e]*Sqrt[b*d - a*e]*x^2)) + (a^(1/4)*Sqrt[d]*(Sqrt[a]*d + Sqrt[e]*Sqrt
[b*d - a*e]*x^2)*Sqrt[(d^2*(a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2))/(Sqrt[a]
*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)^2]*EllipticE[2*ArcTan[(e^(1/4)*(b*d - a*
e)^(1/4)*x)/(a^(1/4)*Sqrt[d])], (2 - (b*d)/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e
]))/4])/(e^(1/4)*(b*d - a*e)^(1/4)*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^
2])))/(Sqrt[e]*Sqrt[b*d - a*e]) + (a^(1/4)*(Sqrt[a] + (2*b*d)/(Sqrt[e]*Sqr
t[b*d - a*e]))*(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)*Sqrt[(d^2*(a + b*
x^2 + (e*(b*d - a*e)*x^4)/d^2))/(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)^
2]*EllipticF[2*ArcTan[(e^(1/4)*(b*d - a*e)^(1/4)*x)/(a^(1/4)*Sqrt[d])], (2
 - (b*d)/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]))/4])/(2*Sqrt[d]*e^(1/4)*(b*d -
a*e)^(1/4)*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])))/(3*e*(b*d - a*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 4.15 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.21

method	result
default	$\frac{x\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{-\frac{3 a e^2}{d^2}+\frac{3 e b}{d}} - \frac{a\sqrt{1-\frac{(ae-bd)x^2}{ad}}\sqrt{1+\frac{e x^2}{d}}\,\text{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}}, \sqrt{-1+\frac{be}{d\left(-\frac{a e^2}{d^2}+\frac{eb}{d}\right)}}\right)}{3\left(-\frac{a e^2}{d^2}+\frac{eb}{d}\right)\sqrt{\frac{ae-bd}{ad}}\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}} + \frac{4ba\sqrt{1-\frac{(ae-bd)x^2}{ad}}}{}$
elliptic	$\frac{x\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{-\frac{3 a e^2}{d^2}+\frac{3 e b}{d}} - \frac{a\sqrt{1-\frac{(ae-bd)x^2}{ad}}\sqrt{1+\frac{e x^2}{d}}\,\text{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}}, \sqrt{-1+\frac{be}{d\left(-\frac{a e^2}{d^2}+\frac{eb}{d}\right)}}\right)}{3\left(-\frac{a e^2}{d^2}+\frac{eb}{d}\right)\sqrt{\frac{ae-bd}{ad}}\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}} + \frac{4ba\sqrt{1-\frac{(ae-bd)x^2}{ad}}}{}$
risch	$-\frac{(-ae x^2+bd x^2+ad)(e x^2+d)x\sqrt{(e x^2+d)(-ae x^2+bd x^2+ad)}}{3e\sqrt{-(e x^2+d)(ae x^2-bd x^2-ad)}\,(ae-bd)\sqrt{\frac{(e x^2+d)(-ae x^2+bd x^2+ad)}{d^2}}} + \frac{d^2\left(\frac{a\sqrt{1-\frac{(ae-bd)x^2}{ad}}\sqrt{1+\frac{e x^2}{d}}\,\text{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}},\right.}{\sqrt{\frac{ae-bd}{ad}}\sqrt{-a e^2 x^4+bde x^4+b d^2 x^2}}\right.}{}$

input
```
int(x^4/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(-a/d^2*e^2+e/d*b)*x*(-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)-1/3/(-a/
d^2*e^2+e/d*b)*a/((a*e-b*d)/a/d)^(1/2)*(1-(a*e-b*d)/a/d*x^2)^(1/2)*(1+e*x^
2/d)^(1/2)/(-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)*EllipticF(x*((a*e-b*d)
/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e^2+e/d*b))^(1/2))+4/3/(-a/d^2*e^2+e/d*b)*b*
a/((a*e-b*d)/a/d)^(1/2)*(1-(a*e-b*d)/a/d*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(-x^
4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)/(b+(2*a*e-b*d)/d)*(EllipticF(x*((a*e-
b*d)/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e^2+e/d*b))^(1/2))-EllipticE(x*((a*e-b*d
)/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e^2+e/d*b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \frac{2\sqrt{bde-ae^2}ab\sqrt{-\frac{ad}{bd-ae}}d^4xE(\arcsin\left(\frac{\sqrt{-\frac{ad}{bd-ae}}}{x}\right) \mid \frac{bd-ae}{ae}) + (2\,abd^3e-a^2d^2e^2-(2\,ab+b^2)d^4)\sqrt{bde-a}}{3\,(b^3d^3e^2-3\,a}$$

input
```
integrate(x^4/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="fricas"
)
```

output
```
1/3*(2*sqrt(b*d*e - a*e^2)*a*b*sqrt(-a*d/(b*d - a*e))*d^4*x*elliptic_e(arc
sin(sqrt(-a*d/(b*d - a*e))/x), (b*d - a*e)/(a*e)) + (2*a*b*d^3*e - a^2*d^2
*e^2 - (2*a*b + b^2)*d^4)*sqrt(b*d*e - a*e^2)*sqrt(-a*d/(b*d - a*e))*x*ell
iptic_f(arcsin(sqrt(-a*d/(b*d - a*e))/x), (b*d - a*e)/(a*e)) - (2*b^2*d^5
- 2*a*b*d^4*e - (b^2*d^4*e - 2*a*b*d^3*e^2 + a^2*d^2*e^3)*x^2)*sqrt((b*d^2
*x^2 + (b*d*e - a*e^2)*x^4 + a*d^2)/d^2))/((b^3*d^3*e^2 - 3*a*b^2*d^2*e^3
+ 3*a^2*b*d*e^4 - a^3*e^5)*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{x^4}{\sqrt{-\left(1+\frac{ex^2}{d}\right)\left(-a+\frac{aex^2}{d}-bx^2\right)}}\,dx$$

input
```
integrate(x**4/(a+b*x**2+(-a*e**2+b*d*e)*x**4/d**2)**(1/2),x)
```

output
```
Integral(x**4/sqrt(-(1 + e*x**2/d)*(-a + a*e*x**2/d - b*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{x^4}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}}\,dx$$

input
```
integrate(x^4/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^4/sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{x^4}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}}\,dx$$

input
```
integrate(x^4/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{x^4}{\sqrt{a+b\,x^2-\frac{x^4\,(a\,e^2-b\,d\,e)}{d^2}}}\,dx$$

input
```
int(x^4/(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2),x)
```

output
```
int(x^4/(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx$$

$$=\frac{d\left(-\sqrt{e\,x^2+d}\,\sqrt{-ae\,x^2+bd\,x^2+ad}\,x+2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-ae\,x^2+bd\,x^2+ad}\,x^2}{-a^2e^3x^4+2abd\,e^2x^4-b^2d^2e\,x^4+ab\,d^2e\,x^2-b^2d^3x^2+a^2d^2e-ab\,d^3}dx\right)ab\,d^2 \cdots}{\cdots}$$

input
```
int(x^4/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x)
```

output
```
(d*( - sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*x + 2*int((sqrt(d
+ e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*x**2)/(a**2*d**2*e - a**2*e**3*x
**4 - a*b*d**3 + a*b*d**2*e*x**2 + 2*a*b*d*e**2*x**4 - b**2*d**3*x**2 - b*
*2*d**2*e*x**4),x)*a*b*d**2*e - 2*int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**
2 + b*d*x**2)*x**2)/(a**2*d**2*e - a**2*e**3*x**4 - a*b*d**3 + a*b*d**2*e*
x**2 + 2*a*b*d*e**2*x**4 - b**2*d**3*x**2 - b**2*d**2*e*x**4),x)*b**2*d**3
 + int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2))/(a**2*d**2*e - a
**2*e**3*x**4 - a*b*d**3 + a*b*d**2*e*x**2 + 2*a*b*d*e**2*x**4 - b**2*d**3
*x**2 - b**2*d**2*e*x**4),x)*a**2*d**2*e - int((sqrt(d + e*x**2)*sqrt(a*d
- a*e*x**2 + b*d*x**2))/(a**2*d**2*e - a**2*e**3*x**4 - a*b*d**3 + a*b*d**
2*e*x**2 + 2*a*b*d*e**2*x**4 - b**2*d**3*x**2 - b**2*d**2*e*x**4),x)*a*b*d
**3))/(3*e*(a*e - b*d))
```

3.717 $\int \frac{x^2}{\sqrt{a+bx^2+\frac{\left(bde-ae^2\right)x^4}{d^2}}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 172

$$\int \frac{x^2}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \frac{x(ad+(bd-ae)x^2)}{(bd-ae)\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}} - \frac{a\sqrt{d}(d+ex^2)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|2-\frac{bd}{ae}\right)}{\sqrt{e}(bd-ae)\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}$$

output

```
x*(a*d+(-a*e+b*d)*x^2)/(-a*e+b*d)/(a+b*x^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)-a*d
^(1/2)*(e*x^2+d)*((a*d+(-a*e+b*d)*x^2)/a/(e*x^2+d))^(1/2)*EllipticE(e^(1/2
)*x/d^(1/2)/(1+e*x^2/d)^(1/2),(2-b*d/a/e)^(1/2))/e^(1/2)/(-a*e+b*d)/(a+b*x
^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = -\frac{iad\sqrt{1+\frac{bx^2}{a}-\frac{ex^2}{d}}\sqrt{1+\frac{ex^2}{d}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)|-1+\frac{bd}{ae}\right)-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right),-1+\frac{bd}{ae}\right)\right)}{\sqrt{\frac{e}{d}}(bd-ae)\sqrt{\frac{(d+ex^2)(bdx^2+a(d-ex^2))}{d^2}}}$$

input
```
Integrate[x^2/Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2],x]
```

output
```
((-I)*a*d*Sqrt[1 + (b*x^2)/a - (e*x^2)/d]*Sqrt[1 + (e*x^2)/d]*(EllipticE[I
*ArcSinh[Sqrt[e/d]*x], -1 + (b*d)/(a*e)] - EllipticF[I*ArcSinh[Sqrt[e/d]*x
], -1 + (b*d)/(a*e)]))/(Sqrt[e/d]*(b*d - a*e)*Sqrt[((d + e*x^2)*(b*d*x^2 +
 a*(d - e*x^2)))/d^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 510 vs. 2(172) = 344.

Time = 0.95 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\frac{x^4(bde-ae^2)}{d^2}+a+bx^2}}dx$$

$$\Big\downarrow \text{ 1459}$$

$$\frac{\sqrt{a}d\int \frac{1}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}-\frac{\sqrt{a}d\int \frac{\sqrt{a}d-\sqrt{e}\sqrt{bd-ae}x^2}{\sqrt{a}d\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}$$

$\downarrow$ 27

$$\frac{\sqrt{a}d\int \frac{1}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}-\frac{\int \frac{\sqrt{a}d-\sqrt{e}\sqrt{bd-ae}x^2}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}$$

$\downarrow$ 1416

$$\frac{\sqrt[4]{a}\sqrt{d}\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt[4]{bd-ae}x}{\sqrt[4]{a}\sqrt{d}}\right),\frac{1}{4}\left(2-\frac{bd}{\sqrt{a}\sqrt{e}\sqrt{bd-ae}}\cdots\right.\right.}{2e^{3/4}(bd-ae)^{3/4}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}$$

$$\frac{\int \frac{\sqrt{a}d-\sqrt{e}\sqrt{bd-ae}x^2}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}$$

$\downarrow$ 1509

$$\frac{\sqrt[4]{a}\sqrt{d}\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt[4]{bd-ae}x}{\sqrt[4]{a}\sqrt{d}}\right),\frac{1}{4}\left(2-\frac{bd}{\sqrt{a}\sqrt{e}\sqrt{bd-ae}}\cdots\right.\right.}{2e^{3/4}(bd-ae)^{3/4}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{d}\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt[4]{bd-ae}x}{\sqrt[4]{a}\sqrt{d}}\right)|\frac{1}{4}\left(2-\frac{bd}{\sqrt{a}\sqrt{e}\sqrt{bd-ae}}\right)\right)}{\sqrt[4]{e}\sqrt[4]{bd-ae}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}-\frac{d^2x\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}\cdots}{\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d}}{\sqrt{e}\sqrt{bd-ae}}$$

input

```
Int[x^2/Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2],x]
```

output
```
-((-((d^2*x*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])/(Sqrt[a]*d + Sqrt[e
]*Sqrt[b*d - a*e]*x^2)) + (a^(1/4)*Sqrt[d]*(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d -
 a*e]*x^2)*Sqrt[(d^2*(a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2))/(Sqrt[a]*d + S
qrt[e]*Sqrt[b*d - a*e]*x^2)^2]*EllipticE[2*ArcTan[(e^(1/4)*(b*d - a*e)^(1/
4)*x)/(a^(1/4)*Sqrt[d])], (2 - (b*d)/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]))/4]
)/(e^(1/4)*(b*d - a*e)^(1/4)*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2]))/(
Sqrt[e]*Sqrt[b*d - a*e])) + (a^(1/4)*Sqrt[d]*(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d
 - a*e]*x^2)*Sqrt[(d^2*(a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2))/(Sqrt[a]*d +
 Sqrt[e]*Sqrt[b*d - a*e]*x^2)^2]*EllipticF[2*ArcTan[(e^(1/4)*(b*d - a*e)^(
1/4)*x)/(a^(1/4)*Sqrt[d])], (2 - (b*d)/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]))/
4])/(2*e^(3/4)*(b*d - a*e)^(3/4)*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2]
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.17

method	result
default	$-\frac{2a\sqrt{1-\frac{(ae-bd)x^2}{ad}}\,\sqrt{1+\frac{e\,x^2}{d}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}},\sqrt{-1+\frac{be}{d\left(-\frac{a\,e^2}{d^2}+\frac{eb}{d}\right)}}\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{ae-bd}{ad}},\sqrt{-1+\frac{be}{d\left(-\frac{a\,e^2}{d^2}+\frac{eb}{d}\right)}}\right)\right)}{\sqrt{\frac{ae-bd}{ad}}\,\sqrt{-\frac{x^4a\,e^2}{d^2}+\frac{e\,x^4b}{d}+b\,x^2+a}\,\left(b+\frac{2ae-bd}{d}\right)}$
elliptic	$-\frac{2a\sqrt{1-\frac{(ae-bd)x^2}{ad}}\,\sqrt{1+\frac{e\,x^2}{d}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}},\sqrt{-1+\frac{be}{d\left(-\frac{a\,e^2}{d^2}+\frac{eb}{d}\right)}}\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{ae-bd}{ad}},\sqrt{-1+\frac{be}{d\left(-\frac{a\,e^2}{d^2}+\frac{eb}{d}\right)}}\right)\right)}{\sqrt{\frac{ae-bd}{ad}}\,\sqrt{-\frac{x^4a\,e^2}{d^2}+\frac{e\,x^4b}{d}+b\,x^2+a}\,\left(b+\frac{2ae-bd}{d}\right)}$

input
```
int(x^2/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a/((a*e-b*d)/a/d)^(1/2)*(1-(a*e-b*d)/a/d*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(
-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)/(b+(2*a*e-b*d)/d)*(EllipticF(x*((a
*e-b*d)/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e^2+e/d*b))^(1/2))-EllipticE(x*((a*e-
b*d)/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e^2+e/d*b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx =$$

$$-\frac{\sqrt{bde-ae^2}a\sqrt{-\frac{ad}{bd-ae}}d^2xE(\arcsin\left(\frac{\sqrt{-\frac{ad}{bd-ae}}}{x}\right)\mid\frac{bd-ae}{ae})-\sqrt{bde-ae^2}a\sqrt{-\frac{ad}{bd-ae}}d^2xF(\arcsin\left(\frac{\sqrt{-\frac{ad}{bd-a}}}{x}\right.}{(b^2d^2e-2\,abde^2+a^2e^3)x}$$

input
```
integrate(x^2/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="fricas"
)
```

output
```
-(sqrt(b*d*e - a*e^2)*a*sqrt(-a*d/(b*d - a*e))*d^2*x*elliptic_e(arcsin(sqr
t(-a*d/(b*d - a*e))/x), (b*d - a*e)/(a*e)) - sqrt(b*d*e - a*e^2)*a*sqrt(-a
*d/(b*d - a*e))*d^2*x*elliptic_f(arcsin(sqrt(-a*d/(b*d - a*e))/x), (b*d -
a*e)/(a*e)) - (b*d^3 - a*d^2*e)*sqrt((b*d^2*x^2 + (b*d*e - a*e^2)*x^4 + a*
d^2)/d^2))/((b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3)*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{x^2}{\sqrt{-\left(1+\frac{ex^2}{d}\right)\left(-a+\frac{aex^2}{d}-bx^2\right)}}\,dx$$

input
```
integrate(x**2/(a+b*x**2+(-a*e**2+b*d*e)*x**4/d**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt(-(1 + e*x**2/d)*(-a + a*e*x**2/d - b*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{x^2}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}}\,dx$$

input
```
integrate(x^2/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^2/sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{x^2}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}}\,dx$$

input
```
integrate(x^2/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{x^2}{\sqrt{a+b\,x^2-\frac{x^4\,(a\,e^2-b\,d\,e)}{d^2}}}\,dx$$

input
```
int(x^2/(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2),x)
```

output
```
int(x^2/(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-ae\,x^2+bd\,x^2+ad}\,x^2}{-a\,e^2x^4+bde\,x^4+b\,d^2x^2+a\,d^2}dx\right)d$$

input
```
int(x^2/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*x**2)/(a*d**2 - a*e*
*2*x**4 + b*d**2*x**2 + b*d*e*x**4),x)*d
```

3.718 $\int \frac{1}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 104

$$\int \frac{1}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\frac{(d+ex^2)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),2-\frac{bd}{ae}\right)}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}$$

output

```
(e*x^2+d)*((a*d+(-a*e+b*d)*x^2)/a/(e*x^2+d))^(1/2)*InverseJacobiAM(arctan(
e^(1/2)*x/d^(1/2)),(2-b*d/a/e)^(1/2))/d^(1/2)/e^(1/2)/(a+b*x^2+e*(-a*e+b*d
)*x^4/d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx =\frac{\sqrt{1+\frac{bx^2}{a}-\frac{ex^2}{d}}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right),-1+\frac{bd}{ae}\right)}{\sqrt{-\frac{e}{d}}\sqrt{\frac{(d+ex^2)(bdx^2+a(d-ex^2))}{d^2}}}$$

input
```
Integrate[1/Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2],x]
```

output
```
(Sqrt[1 + (b*x^2)/a - (e*x^2)/d]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[Sqrt
[-(e/d)]*x], -1 + (b*d)/(a*e)])/(Sqrt[-(e/d)]*Sqrt[((d + e*x^2)*(b*d*x^2 +
 a*(d - e*x^2)))/d^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 214 vs. $2(104) = 208$.

Time = 0.45 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.06, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{x^4(bde-ae^2)}{d^2}+a+bx^2}}dx$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt[4]{bd-ae}x}{\sqrt[4]{a}\sqrt{d}}\right),\frac{1}{4}\left(2-\frac{bd}{\sqrt{a}\sqrt{e}\sqrt{bd-ae}}\right)\right)}{2\sqrt[4]{a}\sqrt{d}\sqrt[4]{e}\sqrt[4]{bd-ae}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}$$

input
```
Int[1/Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2],x]
```

output
```
((Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)*Sqrt[(d^2*(a + b*x^2 + (e*(b*d
- a*e)*x^4)/d^2))/(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)^2]*EllipticF[2
*ArcTan[(e^(1/4)*(b*d - a*e)^(1/4)*x)/(a^(1/4)*Sqrt[d])], (2 - (b*d)/(Sqrt
[a]*Sqrt[e]*Sqrt[b*d - a*e]))/4])/(2*a^(1/4)*Sqrt[d]*e^(1/4)*(b*d - a*e)^(
1/4)*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{\sqrt{1-\frac{(ae-bd)x^2}{ad}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}},\sqrt{-1+\frac{be}{d\left(-\frac{a\,e^2}{d^2}+\frac{eb}{d}\right)}}\right)}{\sqrt{\frac{ae-bd}{ad}}\,\sqrt{-\frac{x^4a\,e^2}{d^2}+\frac{e\,x^4b}{d}+b\,x^2+a}}$	132
elliptic	$\frac{\sqrt{1-\frac{(ae-bd)x^2}{ad}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}},\sqrt{-1+\frac{be}{d\left(-\frac{a\,e^2}{d^2}+\frac{eb}{d}\right)}}\right)}{\sqrt{\frac{ae-bd}{ad}}\,\sqrt{-\frac{x^4a\,e^2}{d^2}+\frac{e\,x^4b}{d}+b\,x^2+a}}$	132

input
```
int(1/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*e-b*d)/a/d)^(1/2)*(1-(a*e-b*d)/a/d*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(-x^
4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)*EllipticF(x*((a*e-b*d)/a/d)^(1/2),(-1
+b*e/d/(-a/d^2*e^2+e/d*b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = -\frac{\sqrt{ad^2}\sqrt{-\frac{bd-ae}{ad}}F\left(\arcsin\left(x\sqrt{-\frac{bd-ae}{ad}}\right) \mid \frac{ae}{bd-ae}\right)}{bd-ae}$$

input
```
integrate(1/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a*d^2)*sqrt(-(b*d - a*e)/(a*d))*elliptic_f(arcsin(x*sqrt(-(b*d - a*e
)/(a*d))), a*e/(b*d - a*e))/(b*d - a*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{1}{\sqrt{a+bx^2+\frac{x^4(-ae^2+bde)}{d^2}}}\,dx$$

input
```
integrate(1/(a+b*x**2+(-a*e**2+b*d*e)*x**4/d**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*x**2 + x**4*(-a*e**2 + b*d*e)/d**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{1}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}}\,dx$$

input
```
integrate(1/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{1}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}}\,dx$$

input
```
integrate(1/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{1}{\sqrt{a+b\,x^2-\frac{x^4\,\left(a\,e^2-b\,d\,e\right)}{d^2}}}\,dx$$

input
```
int(1/(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2),x)
```

output
```
int(1/(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-ae\,x^2+bd\,x^2+ad}}{-a\,e^2x^4+bde\,x^4+b\,d^2x^2+a\,d^2}dx\right)d$$

input
```
int(1/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2))/(a*d**2 - a*e**2*x*
*4 + b*d**2*x**2 + b*d*e*x**4),x)*d
```

3.719 $\int \frac{1}{x^2\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 151

$$\int \frac{1}{x^2\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = -\frac{d\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}{ax(d+ex^2)} - \frac{\sqrt{e}(d+ex^2)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|2-\frac{bd}{ae}\right)}{d^{3/2}\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}$$

output
```
-d*(a+b*x^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)/a/x/(e*x^2+d)-e^(1/2)*(e*x^2+d)*((
a*d+(-a*e+b*d)*x^2)/a/(e*x^2+d))^(1/2)*EllipticE(e^(1/2)*x/d^(1/2)/(1+e*x^
2/d)^(1/2),(2-b*d/a/e)^(1/2))/d^(3/2)/(a+b*x^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx$$
$$=\frac{-\sqrt{\frac{e}{d}}(d+ex^2)\left(ad+bdx^2-aex^2\right)-iadex\sqrt{1+\frac{bx^2}{a}-\frac{ex^2}{d}}\sqrt{1+\frac{ex^2}{d}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)|-1+\frac{bd}{ae}\right)+iad}{ad^2\sqrt{\frac{e}{d}}x\sqrt{\frac{(d+ex^2)(bdx^2+a(d-ex^2))}{d^2}}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2]),x]
```

output
```
(-(Sqrt[e/d]*(d + e*x^2)*(a*d + b*d*x^2 - a*e*x^2)) - I*a*d*e*x*Sqrt[1 + (
b*x^2)/a - (e*x^2)/d]*Sqrt[1 + (e*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[e/d]*x]
, -1 + (b*d)/(a*e)] + I*a*d*e*x*Sqrt[1 + (b*x^2)/a - (e*x^2)/d]*Sqrt[1 + (
e*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[e/d]*x], -1 + (b*d)/(a*e)])/(a*d^2*Sqrt
[e/d]*x*Sqrt[((d + e*x^2)*(b*d*x^2 + a*(d - e*x^2)))/d^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 562 vs. 2(151) = 302.

Time = 1.10 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.72, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1443, 27, 1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{\frac{x^4(bde-ae^2)}{d^2}+a+bx^2}}\,dx$$

$$\Big\downarrow \ 1443$$

$$\frac{\int \frac{e(bd-ae)x^2}{d^2\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{a}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}$$

$\downarrow$ 27

$$\frac{e(bd-ae)\int \frac{x^2}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}$$

$\downarrow$ 1459

$$\frac{e(bd-ae)\left(\frac{\sqrt{a}d\int \frac{1}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}-\frac{\sqrt{a}d\int \frac{\sqrt{a}d-\sqrt{e}\sqrt{bd-ae}x^2}{\sqrt{a}d\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}\right)}{ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}$$

$\downarrow$ 27

$$\frac{e(bd-ae)\left(\frac{\sqrt{a}d\int \frac{1}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}-\frac{\int \frac{\sqrt{a}d-\sqrt{e}\sqrt{bd-ae}x^2}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}\right)}{ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}$$

$\downarrow$ 1416

$$\frac{e(bd-ae)\left(\frac{\sqrt[4]{a}\sqrt{d}\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt[4]{bd-ae}x}{\sqrt[4]{a}\sqrt{d}}\right),\frac{1}{4}\left(2-\frac{bd}{\sqrt{a}\sqrt{e}\sqrt{bd-ae}}\right)\right)}{2e^{3/4}(bd-ae)^{3/4}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}-\cdots\right)}{ad^2}$$

$$\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}$$

$\downarrow$ 1509

$$\frac{e(bd-ae)\left(\frac{\sqrt[4]{a}\sqrt{d}(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt[4]{bd-ae}x}{\sqrt[4]{a}\sqrt{d}}\right),\frac{1}{4}\left(2-\frac{bd}{\sqrt{a}\sqrt{e}\sqrt{bd-ae}}\right)\right)}{2e^{3/4}(bd-ae)^{3/4}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}} - \cdots\right)}{ad^2}$$

$$\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2]),x]
```

output
```
-(Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2]/(a*x)) + (e*(b*d - a*e)*(-((-(
(d^2*x*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])/(Sqrt[a]*d + Sqrt[e]*Sqr
t[b*d - a*e]*x^2)) + (a^(1/4)*Sqrt[d]*(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]
*x^2)*Sqrt[(d^2*(a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2))/(Sqrt[a]*d + Sqrt[e
]*Sqrt[b*d - a*e]*x^2)^2]*EllipticE[2*ArcTan[(e^(1/4)*(b*d - a*e)^(1/4)*x)
/(a^(1/4)*Sqrt[d])], (2 - (b*d)/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]))/4])/(e^
(1/4)*(b*d - a*e)^(1/4)*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2]))/(Sqrt[
e]*Sqrt[b*d - a*e])) + (a^(1/4)*Sqrt[d]*(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*
e]*x^2)*Sqrt[(d^2*(a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2))/(Sqrt[a]*d + Sqrt
[e]*Sqrt[b*d - a*e]*x^2)^2]*EllipticF[2*ArcTan[(e^(1/4)*(b*d - a*e)^(1/4)*
x)/(a^(1/4)*Sqrt[d])], (2 - (b*d)/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]))/4])/(
2*e^(3/4)*(b*d - a*e)^(3/4)*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])))/(
a*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.67

method	result
default	$-\frac{\sqrt{-\frac{x^4 a\,e^2}{d^2}+\frac{e\,x^4 b}{d}+b\,x^2+a}}{a x}+\frac{2e(ae-bd)\sqrt{1-\frac{(ae-bd)x^2}{ad}}\,\sqrt{1+\frac{e\,x^2}{d}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}},\sqrt{-1+\frac{be}{d\left(-\frac{a\,e^2}{d^2}+\frac{eb}{d}\right)}}\right)-\operatorname{EllipticE}\left(\right.\right.}{d^2\sqrt{\frac{ae-bd}{ad}}\,\sqrt{-\frac{x^4 a\,e^2}{d^2}+\frac{e\,x^4 b}{d}+b\,x^2+a}\left(b+\frac{2ae-bd}{d}\right)}$
elliptic	$-\frac{\sqrt{-\frac{x^4 a\,e^2}{d^2}+\frac{e\,x^4 b}{d}+b\,x^2+a}}{a x}+\frac{2e(ae-bd)\sqrt{1-\frac{(ae-bd)x^2}{ad}}\,\sqrt{1+\frac{e\,x^2}{d}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}},\sqrt{-1+\frac{be}{d\left(-\frac{a\,e^2}{d^2}+\frac{eb}{d}\right)}}\right)-\operatorname{EllipticE}\left(\right.\right.}{d^2\sqrt{\frac{ae-bd}{ad}}\,\sqrt{-\frac{x^4 a\,e^2}{d^2}+\frac{e\,x^4 b}{d}+b\,x^2+a}\left(b+\frac{2ae-bd}{d}\right)}$
risch	$-\frac{(e\,x^2+d)\left(-ae\,x^2+bd\,x^2+ad\right)\sqrt{(e\,x^2+d)(-ae\,x^2+bd\,x^2+ad)}}{a\,d^2 x\sqrt{-(e\,x^2+d)(ae\,x^2-bd\,x^2-ad)}\,\sqrt{\frac{(e\,x^2+d)\left(-ae\,x^2+bd\,x^2+ad\right)}{d^2}}}+\frac{2e(ae-bd)\sqrt{1-\frac{(ae-bd)x^2}{ad}}\,\sqrt{1+\frac{e\,x^2}{d}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{ae}{}}\right.\right.}{\sqrt{\frac{ae-bd}{ad}}\,\sqrt{-a\,e^2x^4+ba}}$

input
```
int(1/x^2/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a*(-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)/x+2*e*(a*e-b*d)/d^2/((a*e-b*
d)/a/d)^(1/2)*(1-(a*e-b*d)/a/d*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(-x^4*a/d^2*e^
2+e/d*x^4*b+b*x^2+a)^(1/2)/(b+(2*a*e-b*d)/d)*(EllipticF(x*((a*e-b*d)/a/d)^
(1/2),(-1+b*e/d/(-a/d^2*e^2+e/d*b))^(1/2))-EllipticE(x*((a*e-b*d)/a/d)^(1/
2),(-1+b*e/d/(-a/d^2*e^2+e/d*b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^2\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \frac{\sqrt{ad^2}(bd-ae)x\sqrt{-\frac{bd-ae}{ad}}E\left(\arcsin\left(x\sqrt{-\frac{bd-ae}{ad}}\right) \mid \frac{ae}{bd-ae}\right) - \sqrt{ad^2}(bd-ae)x\sqrt{-\frac{bd-ae}{ad}}F\left(\arcsin\left(x\sqrt{-\frac{bd-}{a}}\right.\right.}{a^2d^2x}$$

input
```
integrate(1/x^2/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="frica
s")
```

output
```
(sqrt(a*d^2)*(b*d - a*e)*x*sqrt(-(b*d - a*e)/(a*d))*elliptic_e(arcsin(x*sq
rt(-(b*d - a*e)/(a*d))), a*e/(b*d - a*e)) - sqrt(a*d^2)*(b*d - a*e)*x*sqrt
(-(b*d - a*e)/(a*d))*elliptic_f(arcsin(x*sqrt(-(b*d - a*e)/(a*d))), a*e/(b
*d - a*e)) - a*d^2*sqrt((b*d^2*x^2 + (b*d*e - a*e^2)*x^4 + a*d^2)/d^2))/(a
^2*d^2*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{1}{x^2\sqrt{-\left(1+\frac{ex^2}{d}\right)\left(-a+\frac{aex^2}{d}-bx^2\right)}}\,dx$$

input
```
integrate(1/x**2/(a+b*x**2+(-a*e**2+b*d*e)*x**4/d**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(-(1 + e*x**2/d)*(-a + a*e*x**2/d - b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{1}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{1}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{1}{x^2\sqrt{a+b\,x^2-\frac{x^4\,(a\,e^2-b\,d\,e)}{d^2}}}\,dx$$

input
```
int(1/(x^2*(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-ae\,x^2+bd\,x^2+ad}}{-a\,e^2x^6+bde\,x^6+b\,d^2x^4+a\,d^2x^2}dx\right)d$$

input
```
int(1/x^2/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2))/(a*d**2*x**2 - a*e*
*2*x**6 + b*d**2*x**4 + b*d*e*x**6),x)*d
```

3.720 $\int \frac{1}{x^4\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 315

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx
&= -\frac{\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}{3ax^3}+\frac{2bd\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}{3a^2x\,(d+ex^2)} \\
&\quad+\frac{2b\sqrt{e}(d+ex^2)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}\,E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|2-\frac{bd}{ae}\right)}{3ad^{3/2}\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}} \\
&\quad-\frac{\sqrt{e}(bd-ae)\,(d+ex^2)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),2-\frac{bd}{ae}\right)}{3ad^{5/2}\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}
\end{aligned}
$$

output
```
-1/3*(a+b*x^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)/a/x^3+2/3*b*d*(a+b*x^2+e*(-a*e+b
*d)*x^4/d^2)^(1/2)/a^2/x/(e*x^2+d)+2/3*b*e^(1/2)*(e*x^2+d)*((a*d+(-a*e+b*d
)*x^2)/a/(e*x^2+d))^(1/2)*EllipticE(e^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(1/2),(2
-b*d/a/e)^(1/2))/a/d^(3/2)/(a+b*x^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)-1/3*e^(1/2
)*(-a*e+b*d)*(e*x^2+d)*((a*d+(-a*e+b*d)*x^2)/a/(e*x^2+d))^(1/2)*InverseJac
obiAM(arctan(e^(1/2)*x/d^(1/2)),(2-b*d/a/e)^(1/2))/a/d^(5/2)/(a+b*x^2+e*(-
a*e+b*d)*x^4/d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.37 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^4\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx$$
$$= \frac{\sqrt{\frac{e}{d}}(-a+2bx^2)\,(d+ex^2)\,(bdx^2+a(d-ex^2))+2iabdex^3\sqrt{1+\frac{bx^2}{a}-\frac{ex^2}{d}}\sqrt{1+\frac{ex^2}{d}}E\big(i\mathrm{arcsinh}\big(\sqrt{\frac{e}{d}}x\big)\,|-}{3a^2d^2\sqrt{\frac{e}{d}}x^3\sqrt{\frac{(d+ex^2)(bdx^2}{d^2}}}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2]),x]
```

output
```
(Sqrt[e/d]*(-a + 2*b*x^2)*(d + e*x^2)*(b*d*x^2 + a*(d - e*x^2)) + (2*I)*a*
b*d*e*x^3*Sqrt[1 + (b*x^2)/a - (e*x^2)/d]*Sqrt[1 + (e*x^2)/d]*EllipticE[I*
ArcSinh[Sqrt[e/d]*x], -1 + (b*d)/(a*e)] - I*a*e*(b*d + a*e)*x^3*Sqrt[1 + (
b*x^2)/a - (e*x^2)/d]*Sqrt[1 + (e*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[e/d]*x]
, -1 + (b*d)/(a*e)])/(3*a^2*d^2*Sqrt[e/d]*x^3*Sqrt[((d + e*x^2)*(b*d*x^2 +
 a*(d - e*x^2)))/d^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 639 vs. 2(315) = 630.

Time = 1.36 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {1443, 25, 27, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{\frac{x^4(bde-ae^2)}{d^2}+a+bx^2}}dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{\int -\frac{2bd^2+e(bd-ae)x^2}{d^2x^2\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{3a}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{2bd^2+e(bd-ae)x^2}{d^2x^2\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{3a}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{2bd^2+e(bd-ae)x^2}{x^2\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{3ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{1604}$$

$$-\frac{-\frac{\int -\frac{e\left(2b(bd-ae)x^2+a(bd-ae)\right)}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{a}-\frac{2bd^2\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}}{3ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{e(bd-ae)\left(2bx^2+a\right)}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{a}-\frac{2bd^2\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}}{3ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{e(bd-ae)\int \frac{2bx^2+a}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{a}-\frac{2bd^2\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}}{3ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{1511}$$

$$-\frac{\frac{e(bd-ae)\left(\sqrt{a}\left(\frac{2bd}{\sqrt{e}\sqrt{bd-ae}}+\sqrt{a}\right)\int \frac{1}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx-\frac{2\sqrt{a}bd\int \frac{\sqrt{a}d-\sqrt{e}\sqrt{bd-ae}x^2}{\sqrt{a}d\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}\right)}{a}-\frac{2bd^2\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}}{3ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{e(bd-ae)\left(\sqrt{a}\left(\frac{2bd}{\sqrt{e}\sqrt{bd-ae}}+\sqrt{a}\right)\int \frac{1}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx-\frac{2b\int \frac{\sqrt{a}d-\sqrt{e}\sqrt{bd-ae}x^2}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}\right)}{a}-\frac{2bd^2\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}}{3ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{\frac{e(bd-ae)\left(\frac{\sqrt[4]{a}\left(\frac{2bd}{\sqrt{e}\sqrt{bd-ae}}+\sqrt{a}\right)\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt[4]{bd-ae}x}{\sqrt[4]{a}\sqrt{d}}\right),\frac{1}{4}\left(2-\frac{bd}{\sqrt{a}\sqrt{e}\sqrt{bd-ae}}\right)\right)}{2\sqrt{d}\sqrt[4]{e}\sqrt[4]{bd-ae}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}\right.}{a}}{3ad^2}$$

$$-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{1509}$$

$$-\frac{\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}}{} \quad e(bd-ae)\left(\frac{\sqrt[4]{a}\left(\frac{2bd}{\sqrt{e}\sqrt{bd-ae}}+\sqrt{a}\right)\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt[4]{bd-ae}x}{\sqrt[4]{a}\sqrt{d}}\right),\frac{1}{4}\left(2-\frac{bd}{\sqrt{a}\sqrt{e}\sqrt{bd-ae}}\right)\right)}{2\sqrt{d}\sqrt[4]{e}\sqrt[4]{bd-ae}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}\right.$$

input
```
Int[1/(x^4*Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2]),x]
```

output
```
-1/3*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2]/(a*x^3) - ((-2*b*d^2*Sqrt[a
 + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])/(a*x) + (e*(b*d - a*e)*((-2*b*(-((d^2
*x*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])/(Sqrt[a]*d + Sqrt[e]*Sqrt[b*
d - a*e]*x^2)) + (a^(1/4)*Sqrt[d]*(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2
)*Sqrt[(d^2*(a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2))/(Sqrt[a]*d + Sqrt[e]*Sq
rt[b*d - a*e]*x^2)^2]*EllipticE[2*ArcTan[(e^(1/4)*(b*d - a*e)^(1/4)*x)/(a^
(1/4)*Sqrt[d])], (2 - (b*d)/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]))/4])/(e^(1/4
)*(b*d - a*e)^(1/4)*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])))/(Sqrt[e]*
Sqrt[b*d - a*e]) + (a^(1/4)*(Sqrt[a] + (2*b*d)/(Sqrt[e]*Sqrt[b*d - a*e]))*
(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)*Sqrt[(d^2*(a + b*x^2 + (e*(b*d -
 a*e)*x^4)/d^2))/(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)^2]*EllipticF[2*
ArcTan[(e^(1/4)*(b*d - a*e)^(1/4)*x)/(a^(1/4)*Sqrt[d])], (2 - (b*d)/(Sqrt[
a]*Sqrt[e]*Sqrt[b*d - a*e]))/4])/(2*Sqrt[d]*e^(1/4)*(b*d - a*e)^(1/4)*Sqrt
[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])))/a)/(3*a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 4.49 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.40

method	result
default	$-\frac{\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{3 a x^3}+\frac{2 b \sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{3 a^2 x}+\frac{e(ae-bd)\sqrt{1-\frac{(ae-bd)x^2}{ad}}\,\sqrt{1+\frac{e x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}},\sqrt{-1\right.}{3 a d^2 \sqrt{\frac{ae-bd}{ad}}\,\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}$
elliptic	$-\frac{\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{3 a x^3}+\frac{2 b \sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{3 a^2 x}+\frac{e(ae-bd)\sqrt{1-\frac{(ae-bd)x^2}{ad}}\,\sqrt{1+\frac{e x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}},\sqrt{-1\right.}{3 a d^2 \sqrt{\frac{ae-bd}{ad}}\,\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}$
risch	$-\frac{(e x^2+d)(-a e x^2+b d x^2+a d)(-2 b x^2+a)\sqrt{(e x^2+d)(-a e x^2+b d x^2+a d)}}{3 a^2 d^2 x^3 \sqrt{-(e x^2+d)(a e x^2-b d x^2-a d)}\,\sqrt{\frac{(e x^2+d)(-a e x^2+b d x^2+a d)}{d^2}}}+\frac{e\left(\frac{a^2 e\sqrt{1-\frac{(ae-bd)x^2}{ad}}\,\sqrt{1+\frac{e x^2}{d}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{ae-bd}{ad}}\right.}{\sqrt{\frac{ae-bd}{ad}}\,\sqrt{-a e^2 x^4+b d e x^4+b d^2 x^2}}\right.}{}$

input
```
int(1/x^4/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a*(-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)/x^3+2/3*b/a^2*(-x^4*a/d^2*
e^2+e/d*x^4*b+b*x^2+a)^(1/2)/x+1/3*e/a*(a*e-b*d)/d^2/((a*e-b*d)/a/d)^(1/2)
*(1-(a*e-b*d)/a/d*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(-x^4*a/d^2*e^2+e/d*x^4*b+b
*x^2+a)^(1/2)*EllipticF(x*((a*e-b*d)/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e^2+e/d*
b))^(1/2))-4/3*b*e*(a*e-b*d)/a/d^2/((a*e-b*d)/a/d)^(1/2)*(1-(a*e-b*d)/a/d*
x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)/(b+(
2*a*e-b*d)/d)*(EllipticF(x*((a*e-b*d)/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e^2+e/d
*b))^(1/2))-EllipticE(x*((a*e-b*d)/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e^2+e/d*b)
)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^4\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx =$$

$$-\frac{2\,(b^2d-abe)\sqrt{ad^2}x^3\sqrt{-\frac{bd-ae}{ad}}E(\arcsin\left(x\sqrt{-\frac{bd-ae}{ad}}\right)\mid\frac{ae}{bd-ae})-(2\,b^2d+(a^2-2\,ab)e)\sqrt{ad^2}x^3\sqrt{-\frac{bd-a}{ad}}}{3\,a^3d^2x^3}$$

input
```
integrate(1/x^4/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="frica
s")
```

output
```
-1/3*(2*(b^2*d - a*b*e)*sqrt(a*d^2)*x^3*sqrt(-(b*d - a*e)/(a*d))*elliptic_
e(arcsin(x*sqrt(-(b*d - a*e)/(a*d))), a*e/(b*d - a*e)) - (2*b^2*d + (a^2 -
 2*a*b)*e)*sqrt(a*d^2)*x^3*sqrt(-(b*d - a*e)/(a*d))*elliptic_f(arcsin(x*sq
rt(-(b*d - a*e)/(a*d))), a*e/(b*d - a*e)) - (2*a*b*d^2*x^2 - a^2*d^2)*sqrt
((b*d^2*x^2 + (b*d*e - a*e^2)*x^4 + a*d^2)/d^2))/(a^3*d^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{1}{x^4\sqrt{-\left(1+\frac{ex^2}{d}\right)\left(-a+\frac{aex^2}{d}-bx^2\right)}}\,dx$$

input
```
integrate(1/x**4/(a+b*x**2+(-a*e**2+b*d*e)*x**4/d**2)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(-(1 + e*x**2/d)*(-a + a*e*x**2/d - b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{1}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}\,x^4}\,dx$$

input
```
integrate(1/x^4/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{1}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+ax^4}}\,dx$$

input
```
integrate(1/x^4/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{1}{x^4\sqrt{a+b\,x^2-\frac{x^4\,(a\,e^2-b\,d\,e)}{d^2}}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{-ae\,x^2+bd\,x^2+ad}}{-a\,e^2x^8+bde\,x^8+b\,d^2x^6+a\,d^2x^4}dx\right)d$$

input
```
int(1/x^4/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2))/(a*d**2*x**4 - a*e*
*2*x**8 + b*d**2*x**6 + b*d*e*x**8),x)*d
```

3.721 $\int \frac{1}{x^6\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 396

$$\begin{aligned}
&\int \frac{1}{x^6\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx \\
&= -\frac{\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}{5ax^5}+\frac{4b\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}{15a^2x^3} \\
&\quad -\frac{d\left(8b^2-\frac{9ae(bd-ae)}{d^2}\right)\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}{15a^3x\left(d+ex^2\right)} \\
&\quad -\frac{\sqrt{e}(8b^2d^2-9abde+9a^2e^2)\left(d+ex^2\right)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|2-\frac{bd}{ae}\right)}{15a^2d^{7/2}\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}} \\
&\quad +\frac{4b\sqrt{e}(bd-ae)\left(d+ex^2\right)\sqrt{\frac{ad+(bd-ae)x^2}{a(d+ex^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right),2-\frac{bd}{ae}\right)}{15a^2d^{5/2}\sqrt{a+bx^2+\frac{e(bd-ae)x^4}{d^2}}}
\end{aligned}$$

output
```
-1/5*(a+b*x^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)/a/x^5+4/15*b*(a+b*x^2+e*(-a*e+b*
d)*x^4/d^2)^(1/2)/a^2/x^3-1/15*d*(8*b^2-9*a*e*(-a*e+b*d)/d^2)*(a+b*x^2+e*(
-a*e+b*d)*x^4/d^2)^(1/2)/a^3/x/(e*x^2+d)-1/15*e^(1/2)*(9*a^2*e^2-9*a*b*d*e
+8*b^2*d^2)*(e*x^2+d)*((a*d+(-a*e+b*d)*x^2)/a/(e*x^2+d))^(1/2)*EllipticE(e
^(1/2)*x/d^(1/2)/(1+e*x^2/d)^(1/2),(2-b*d/a/e)^(1/2))/a^2/d^(7/2)/(a+b*x^2
+e*(-a*e+b*d)*x^4/d^2)^(1/2)+4/15*b*e^(1/2)*(-a*e+b*d)*(e*x^2+d)*((a*d+(-a
*e+b*d)*x^2)/a/(e*x^2+d))^(1/2)*InverseJacobiAM(arctan(e^(1/2)*x/d^(1/2)),
(2-b*d/a/e)^(1/2))/a^2/d^(5/2)/(a+b*x^2+e*(-a*e+b*d)*x^4/d^2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.53 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^6\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx$$

$$= \frac{\sqrt{\frac{e}{d}}(d+ex^2)\left(-8b^3d^3x^6+ab^2d^2x^4(-4d+17ex^2)+a^2bdx^2(d^2+5dex^2-18e^2x^4)-3a^3(d^3-d^2ex^2+3e\right.}{}$$

input
```
Integrate[1/(x^6*Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2]),x]
```

output
```
(Sqrt[e/d]*(d + e*x^2)*(-8*b^3*d^3*x^6 + a*b^2*d^2*x^4*(-4*d + 17*e*x^2) +
 a^2*b*d*x^2*(d^2 + 5*d*e*x^2 - 18*e^2*x^4) - 3*a^3*(d^3 - d^2*e*x^2 + 3*d
*e^2*x^4 - 3*e^3*x^6)) - I*a*d*e*(8*b^2*d^2 - 9*a*b*d*e + 9*a^2*e^2)*x^5*S
qrt[1 + (b*x^2)/a - (e*x^2)/d]*Sqrt[1 + (e*x^2)/d]*EllipticE[I*ArcSinh[Sqr
t[e/d]*x], -1 + (b*d)/(a*e)] + I*a*d*e*(4*b^2*d^2 - 5*a*b*d*e + 9*a^2*e^2)
*x^5*Sqrt[1 + (b*x^2)/a - (e*x^2)/d]*Sqrt[1 + (e*x^2)/d]*EllipticF[I*ArcSi
nh[Sqrt[e/d]*x], -1 + (b*d)/(a*e)])/(15*a^3*d^4*Sqrt[e/d]*x^5*Sqrt[((d + e
*x^2)*(b*d*x^2 + a*(d - e*x^2)))/d^2])
```

Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.91, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1443, 25, 27, 1604, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{\frac{x^4(bde-ae^2)}{d^2}+a+bx^2}}dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{\int -\frac{4bd^2+3e(bd-ae)x^2}{d^2x^4\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{5a}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{4bd^2+3e(bd-ae)x^2}{d^2x^4\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{5a}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{4bd^2+3e(bd-ae)x^2}{x^4\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{5ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{1604}$$

$$-\frac{-\frac{\int \frac{8b^2d^2+4be(bd-ae)x^2-9ae(bd-ae)}{x^2\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{3a}-\frac{4bd^2\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}}{5ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{1604}$$

$$-\frac{-\frac{-\frac{\int -\frac{e\left(4ab(bd-ae)d^2+(bd-ae)\left(8b^2d^2-9ae(bd-ae)\right)x^2\right)}{d^2\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{a}-\frac{\left(8b^2d^2-9ae(bd-ae)\right)\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}}{3a}-\frac{4bd^2\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}}{5ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{\frac{\int \frac{e(bd-ae)\left(4abd^2+\left(8b^2d^2-9ae(bd-ae)\right)x^2\right)}{d^2\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{a}-\frac{\left(8b^2d^2-9ae(bd-ae)\right)\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}}{3a}-\frac{4bd^2\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}}{5ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5ax^5}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\frac{e(bd-ae)\int \frac{4abd^2+\left(8b^2d^2-9ae(bd-ae)\right)x^2}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{ad^2}-\frac{\left(8b^2d^2-9ae(bd-ae)\right)\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{ax}}{3a}-\frac{4bd^2\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{3ax^3}}{5ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5ax^5}$$

$\big\downarrow$ 1511

$$-\frac{-\frac{\frac{e(bd-ae)\left(\sqrt{a}d\left(\frac{8b^2d^2}{\sqrt{e}\sqrt{bd-ae}}-9a\sqrt{e}\sqrt{bd-ae}+4\sqrt{a}bd\right)\int\frac{1}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx-\frac{\sqrt{a}d\left(8b^2d^2-9ae(bd-ae)\right)\int\frac{\sqrt{a}d-\sqrt{e}\sqrt{bd-ae}x^2}{\sqrt{a}d\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}\right)}{ad^2}-\frac{\left(8b^2d^2\cdots\right.}{\cdots}}{3a}\cdots}{5ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5ax^5}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\frac{e(bd-ae)\left(\sqrt{a}d\left(\frac{8b^2d^2}{\sqrt{e}\sqrt{bd-ae}}-9a\sqrt{e}\sqrt{bd-ae}+4\sqrt{a}bd\right)\int\frac{1}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx-\frac{\left(8b^2d^2-9ae(bd-ae)\right)\int\frac{\sqrt{a}d-\sqrt{e}\sqrt{bd-ae}x^2}{\sqrt{\frac{e(bd-ae)x^4}{d^2}+bx^2+a}}dx}{\sqrt{e}\sqrt{bd-ae}}\right)}{ad^2}-\frac{\left(8b^2d^2-9ae(b\cdots\right.}{\cdots}}{3a}\cdots}{5ad^2}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5ax^5}$$

$\big\downarrow$ 1416

$$-\frac{-\frac{e(bd-ae)\left(\frac{\sqrt[4]{a}\sqrt{d}\left(\frac{8b^2d^2}{\sqrt{e}\sqrt{bd-ae}}-9a\sqrt{e}\sqrt{bd-ae}+4\sqrt{a}bd\right)\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt[4]{bd-ae}x}{\sqrt[4]{a}\sqrt{d}}\right),\ldots\right.}{2\sqrt[4]{e}\sqrt[4]{bd-ae}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}\ldots\right.}{ad^2}\ldots}{3a}\ldots}{5\ldots}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5ax^5}$$

$\Big\downarrow$ 1509

$$-\frac{-\frac{e(bd-ae)\left(\frac{\sqrt[4]{a}\sqrt{d}\left(\frac{8b^2d^2}{\sqrt{e}\sqrt{bd-ae}}-9a\sqrt{e}\sqrt{bd-ae}+4\sqrt{a}bd\right)\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)\sqrt{\frac{d^2\left(\frac{ex^4(bd-ae)}{d^2}+a+bx^2\right)}{\left(\sqrt{e}x^2\sqrt{bd-ae}+\sqrt{a}d\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt[4]{bd-ae}x}{\sqrt[4]{a}\sqrt{d}}\right),\ldots\right.}{2\sqrt[4]{e}\sqrt[4]{bd-ae}\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}\ldots\right.}{\ldots}}{\ldots}-\frac{\sqrt{\frac{ex^4(bd-ae)}{d^2}+a+bx^2}}{5ax^5}$$

input
```
Int[1/(x^6*Sqrt[a + b*x^2 + ((b*d*e - a*e^2)*x^4)/d^2]),x]
```

output
```
-1/5*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2]/(a*x^5) - ((-4*b*d^2*Sqrt[a
 + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])/(3*a*x^3) - (-(((8*b^2*d^2 - 9*a*e*(b
*d - a*e))*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])/(a*x)) + (e*(b*d - a
*e)*(-(((8*b^2*d^2 - 9*a*e*(b*d - a*e))*(-((d^2*x*Sqrt[a + b*x^2 + (e*(b*d
 - a*e)*x^4)/d^2])/(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)) + (a^(1/4)*S
qrt[d]*(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)*Sqrt[(d^2*(a + b*x^2 + (e
*(b*d - a*e)*x^4)/d^2))/(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)^2]*Ellip
ticE[2*ArcTan[(e^(1/4)*(b*d - a*e)^(1/4)*x)/(a^(1/4)*Sqrt[d])], (2 - (b*d)
/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]))/4])/(e^(1/4)*(b*d - a*e)^(1/4)*Sqrt[a
+ b*x^2 + (e*(b*d - a*e)*x^4)/d^2])))/(Sqrt[e]*Sqrt[b*d - a*e])) + (a^(1/4
)*Sqrt[d]*(4*Sqrt[a]*b*d + (8*b^2*d^2)/(Sqrt[e]*Sqrt[b*d - a*e]) - 9*a*Sqr
t[e]*Sqrt[b*d - a*e])*(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e]*x^2)*Sqrt[(d^2*
(a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2))/(Sqrt[a]*d + Sqrt[e]*Sqrt[b*d - a*e
]*x^2)^2]*EllipticF[2*ArcTan[(e^(1/4)*(b*d - a*e)^(1/4)*x)/(a^(1/4)*Sqrt[d
])], (2 - (b*d)/(Sqrt[a]*Sqrt[e]*Sqrt[b*d - a*e]))/4])/(2*e^(1/4)*(b*d - a
*e)^(1/4)*Sqrt[a + b*x^2 + (e*(b*d - a*e)*x^4)/d^2])))/(a*d^2))/(3*a))/(5*
a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 5.39 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{5a x^5}+\frac{4b\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{15a^2 x^3}-\frac{(9a^2e^2-9abde+8b^2d^2)\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{15a^3d^2x}-\frac{4be(ae-}{}$
elliptic	$-\frac{\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{5a x^5}+\frac{4b\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{15a^2 x^3}-\frac{(9a^2e^2-9abde+8b^2d^2)\sqrt{-\frac{x^4 a e^2}{d^2}+\frac{e x^4 b}{d}+b x^2+a}}{15a^3d^2x}-\frac{4be(ae-}{}$
risch	$-\frac{(e x^2+d)(-ae x^2+bd x^2+ad)(9a^2e^2x^4-9abde x^4+8b^2d^2x^4-4a d^2b x^2+3a^2d^2)\sqrt{(e x^2+d)(-ae x^2+bd x^2+ad)}}{15a^3d^4x^5\sqrt{-(e x^2+d)(ae x^2-bd x^2-ad)}\sqrt{\frac{(e x^2+d)(-ae x^2+bd x^2+ad)}{d^2}}}-\frac{e\left(-\frac{2(9a^3e^3}{}\right.}{}$

input
```
int(1/x^6/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/a*(-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)/x^5+4/15*b/a^2*(-x^4*a/d^2
*e^2+e/d*x^4*b+b*x^2+a)^(1/2)/x^3-1/15*(9*a^2*e^2-9*a*b*d*e+8*b^2*d^2)/a^3
/d^2*(-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)/x-4/15*b*e*(a*e-b*d)/a^2/d^2
/((a*e-b*d)/a/d)^(1/2)*(1-(a*e-b*d)/a/d*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(-x^4
*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)*EllipticF(x*((a*e-b*d)/a/d)^(1/2),(-1+
b*e/d/(-a/d^2*e^2+e/d*b))^(1/2))+2/15*e*(9*a^3*e^3-18*a^2*b*d*e^2+17*a*b^2
*d^2*e-8*b^3*d^3)/a^2/d^4/((a*e-b*d)/a/d)^(1/2)*(1-(a*e-b*d)/a/d*x^2)^(1/2
)*(1+e*x^2/d)^(1/2)/(-x^4*a/d^2*e^2+e/d*x^4*b+b*x^2+a)^(1/2)/(b+(2*a*e-b*d
)/d)*(EllipticF(x*((a*e-b*d)/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e^2+e/d*b))^(1/2
))-EllipticE(x*((a*e-b*d)/a/d)^(1/2),(-1+b*e/d/(-a/d^2*e^2+e/d*b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^6\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \frac{(8\,b^3d^3-17\,ab^2d^2e+18\,a^2bde^2-9\,a^3e^3)\sqrt{ad^2}x^5\sqrt{-\frac{bd-ae}{ad}}E\left(\arcsin\left(x\sqrt{-\frac{bd-ae}{ad}}\right)\mid\frac{ae}{bd-ae}\right)-(8\,b^3d^3+18}{}$$

input
```
integrate(1/x^6/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="frica
s")
```

output
```
1/15*((8*b^3*d^3 - 17*a*b^2*d^2*e + 18*a^2*b*d*e^2 - 9*a^3*e^3)*sqrt(a*d^2
)*x^5*sqrt(-(b*d - a*e)/(a*d))*elliptic_e(arcsin(x*sqrt(-(b*d - a*e)/(a*d)
)), a*e/(b*d - a*e)) - (8*b^3*d^3 + 18*a^2*b*d*e^2 - 9*a^3*e^3 + (4*a^2*b
- 17*a*b^2)*d^2*e)*sqrt(a*d^2)*x^5*sqrt(-(b*d - a*e)/(a*d))*elliptic_f(arc
sin(x*sqrt(-(b*d - a*e)/(a*d))), a*e/(b*d - a*e)) + (4*a^2*b*d^4*x^2 - 3*a
^3*d^4 - (8*a*b^2*d^4 - 9*a^2*b*d^3*e + 9*a^3*d^2*e^2)*x^4)*sqrt((b*d^2*x^
2 + (b*d*e - a*e^2)*x^4 + a*d^2)/d^2))/(a^4*d^4*x^5)
```

Sympy [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{1}{x^6\sqrt{-\left(1+\frac{ex^2}{d}\right)\left(-a+\frac{aex^2}{d}-bx^2\right)}}\,dx$$

input
```
integrate(1/x**6/(a+b*x**2+(-a*e**2+b*d*e)*x**4/d**2)**(1/2),x)
```

output
```
Integral(1/(x**6*sqrt(-(1 + e*x**2/d)*(-a + a*e*x**2/d - b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{1}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}x^6}\,dx$$

input
```
integrate(1/x^6/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/(sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx=\int \frac{1}{\sqrt{bx^2+\frac{(bde-ae^2)x^4}{d^2}+a}x^6}\,dx$$

input
```
integrate(1/x^6/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/(sqrt(b*x^2 + (b*d*e - a*e^2)*x^4/d^2 + a)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \int \frac{1}{x^6\,\sqrt{a+b\,x^2-\frac{x^4\,(a\,e^2-b\,d\,e)}{d^2}}}\,dx$$

input
```
int(1/(x^6*(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2)),x)
```

output
```
int(1/(x^6*(a + b*x^2 - (x^4*(a*e^2 - b*d*e))/d^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{a+bx^2+\frac{(bde-ae^2)x^4}{d^2}}}\,dx = \frac{-\sqrt{e\,x^2+d}\,\sqrt{-ae\,x^2+bd\,x^2+ad}\,a\,d^2-3\sqrt{e\,x^2+d}\,\sqrt{-ae\,x^2+bd\,x^2+ad}\,a\,e^2x^4+3\sqrt{e\,x^2+d}\,\sqrt{-ae\,x}}{}$$

input
```
int(1/x^6/(a+b*x^2+(-a*e^2+b*d*e)*x^4/d^2)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*a*d**2 - 3*sqrt(d + e
*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*a*e**2*x**4 + 3*sqrt(d + e*x**2)*sq
rt(a*d - a*e*x**2 + b*d*x**2)*b*d*e*x**4 - 3*int((sqrt(d + e*x**2)*sqrt(a*
d - a*e*x**2 + b*d*x**2)*x**2)/(a*d**2 - a*e**2*x**4 + b*d**2*x**2 + b*d*e
*x**4),x)*a**2*e**4*x**5 + 6*int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b
*d*x**2)*x**2)/(a*d**2 - a*e**2*x**4 + b*d**2*x**2 + b*d*e*x**4),x)*a*b*d*
e**3*x**5 - 3*int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2)*x**2)/
(a*d**2 - a*e**2*x**4 + b*d**2*x**2 + b*d*e*x**4),x)*b**2*d**2*e**2*x**5 -
 4*int((sqrt(d + e*x**2)*sqrt(a*d - a*e*x**2 + b*d*x**2))/(a*d**2*x**4 - a
*e**2*x**8 + b*d**2*x**6 + b*d*e*x**8),x)*a*b*d**4*x**5)/(5*a**2*d**3*x**5
)
```

3.722 $\int \frac{x^6}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 384

$$\begin{aligned}\int \frac{x^6}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx &= -\frac{4bx\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{15c^2}+\frac{x^3\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{5c}\\
&\quad+\frac{d\sqrt{cd-be}\,(9c^2d^2-9bcde+8b^2e^2)\sqrt{1+\frac{ex^2}{d}}\sqrt{1-\frac{cex^2}{cd-be}}\,E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)\middle|-1+\frac{be}{cd}\right)}{15c^{5/2}e^{7/2}\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}\\
&\quad-\frac{d\sqrt{cd-be}\,(9c^2d^2-5bcde+4b^2e^2)\sqrt{1+\frac{ex^2}{d}}\sqrt{1-\frac{cex^2}{cd-be}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),-1+\frac{be}{cd}\right)}{15c^{5/2}e^{7/2}\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}\end{aligned}$$

output
```
-4/15*b*x*(-d*(-b*e+c*d)/e^2+b*x^2+c*x^4)^(1/2)/c^2+1/5*x^3*(-d*(-b*e+c*d)
/e^2+b*x^2+c*x^4)^(1/2)/c+1/15*d*(-b*e+c*d)^(1/2)*(8*b^2*e^2-9*b*c*d*e+9*c
^2*d^2)*(1+e*x^2/d)^(1/2)*(1-c*e*x^2/(-b*e+c*d))^(1/2)*EllipticE(c^(1/2)*e
^(1/2)*x/(-b*e+c*d)^(1/2),(-1+b*e/c/d)^(1/2))/c^(5/2)/e^(7/2)/(-d*(-b*e+c*
d)/e^2+b*x^2+c*x^4)^(1/2)-1/15*d*(-b*e+c*d)^(1/2)*(4*b^2*e^2-5*b*c*d*e+9*c
^2*d^2)*(1+e*x^2/d)^(1/2)*(1-c*e*x^2/(-b*e+c*d))^(1/2)*EllipticF(c^(1/2)*e
^(1/2)*x/(-b*e+c*d)^(1/2),(-1+b*e/c/d)^(1/2))/c^(5/2)/e^(7/2)/(-d*(-b*e+c*
d)/e^2+b*x^2+c*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.82 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.84

$$\int \frac{x^6}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx$$

$$= \frac{ce\sqrt{\frac{e}{d}}x(-4b+3cx^2)(d+ex^2)(-cd+be+cex^2)+i(9c^3d^3-18bc^2d^2e+17b^2cde^2-8b^3e^3)\sqrt{\frac{-cd+be+cex}{-cd+be}}}{}$$

input
```
Integrate[x^6/Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4],x]
```

output
```
(c*e*Sqrt[e/d]*x*(-4*b + 3*c*x^2)*(d + e*x^2)*(-(c*d) + b*e + c*e*x^2) + I
*(9*c^3*d^3 - 18*b*c^2*d^2*e + 17*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[(-(c*d) +
b*e + c*e*x^2)/(-(c*d) + b*e)]*Sqrt[1 + (e*x^2)/d]*EllipticE[I*ArcSinh[Sqr
t[e/d]*x], (c*d)/(-(c*d) + b*e)] - I*(9*c^3*d^3 - 22*b*c^2*d^2*e + 21*b^2*
c*d*e^2 - 8*b^3*e^3)*Sqrt[(-(c*d) + b*e + c*e*x^2)/(-(c*d) + b*e)]*Sqrt[1
+ (e*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[e/d]*x], (c*d)/(-(c*d) + b*e)])/(15*
c^3*e^3*Sqrt[e/d]*Sqrt[((d + e*x^2)*(-(c*d) + b*e + c*e*x^2))/e^2])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1442, 25, 27, 1602, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{\frac{bde-cd^2}{e^2}+bx^2+cx^4}}dx$$

$$\Big\downarrow \textbf{1442}$$

$$\frac{x^3\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{5c}-\frac{\int -\frac{x^2(3d(cd-be)-4be^2x^2)}{e^2\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{5c}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{x^2(3d(cd-be)-4be^2x^2)}{e^2\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{5c}+\frac{x^3\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{x^2(3d(cd-be)-4be^2x^2)}{\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{5ce^2}+\frac{x^3\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \textbf{1602}$$

$$\frac{-\frac{\int \frac{4bd(cd-be)-\left(8b^2e^2+9cd(cd-be)\right)x^2}{\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{3c}-\frac{4be^2x\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3c}}{5ce^2}+\frac{x^3\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \textbf{1514}$$

$$\frac{-\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\int \frac{4bd(cd-be)-\left(8b^2e^2+9cd(cd-be)\right)x^2}{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}}dx}{3c\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}-\frac{4be^2x\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3c}}{5ce^2}+\frac{x^3\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \textbf{399}$$

$$-\frac{-\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\left(4b^2e^2-5bcde+9c^2d^2\right)\int\frac{1}{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}}dx}{e}-\frac{d\left(8b^2e^2+9cd(cd-be)\right)\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{cex^2}{cd-be}}}dx}{e}\right)}{3c\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}-\frac{4be^2x\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3c}}{5ce^2}+\frac{x^3\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \ 321$$

$$-\frac{-\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\sqrt{cd-be}\left(4b^2e^2-5bcde+9c^2d^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}e^{3/2}}-\frac{d\left(8b^2e^2+9cd(cd-be)\right)\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{cex^2}{cd-be}}}dx}{e}\right)}{3c\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}-\frac{4be^2x\sqrt{-\frac{d}{}}}{}}{5ce^2}+\frac{x^3\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{5c}$$

$$\Big\downarrow \ 327$$

$$-\frac{-\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\sqrt{cd-be}\left(4b^2e^2-5bcde+9c^2d^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}e^{3/2}}-\frac{d\sqrt{cd-be}\left(8b^2e^2+9cd(cd-be)\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)|\frac{be}{cd}-1\right)}{\sqrt{c}e^{3/2}}}{3c\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}}{5ce^2}+\frac{x^3\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{5c}$$

input
```
Int[x^6/Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4],x]
```

output
```
(x^3*Sqrt[-((d*(c*d - b*e))/e^2) + b*x^2 + c*x^4])/(5*c) + ((-4*b*e^2*x*Sq
rt[-((d*(c*d - b*e))/e^2) + b*x^2 + c*x^4])/(3*c) - (Sqrt[1 + (e*x^2)/d]*S
qrt[1 - (c*e*x^2)/(c*d - b*e)]*(-((d*Sqrt[c*d - b*e]*(8*b^2*e^2 + 9*c*d*(c
*d - b*e))*EllipticE[ArcSin[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]], -1 + (b*
e)/(c*d)])/(Sqrt[c]*e^(3/2))) + (d*Sqrt[c*d - b*e]*(9*c^2*d^2 - 5*b*c*d*e
+ 4*b^2*e^2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]], -1 + (
b*e)/(c*d)])/(Sqrt[c]*e^(3/2))))/(3*c*Sqrt[-((d*(c*d - b*e))/e^2) + b*x^2
+ c*x^4]))/(5*c*e^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 20.37 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.07

method	result
default	$\frac{x^{3}\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}{5c}-\frac{4bx\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}{15c^{2}}+\frac{4b\left(\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}\right)\sqrt{1+\frac{ce\,x^{2}}{be-cd}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\ \operatorname{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)}{15c^{2}\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}$
elliptic	$\frac{x^{3}\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}{5c}-\frac{4bx\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}{15c^{2}}+\frac{4b\left(\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}\right)\sqrt{1+\frac{ce\,x^{2}}{be-cd}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\ \operatorname{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)}{15c^{2}\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}$
risch	$-\frac{x\left(-3c\,x^{2}+4b\right)\left(e\,x^{2}+d\right)\left(ce\,x^{2}+be-cd\right)}{15c^{2}e^{2}\sqrt{\frac{\left(e\,x^{2}+d\right)\left(ce\,x^{2}+be-cd\right)}{e^{2}}}}+\frac{\left(-\frac{2\left(8b^{2}e^{2}-9bcde+9c^{2}d^{2}\right)\left(bde-c\,d^{2}\right)\sqrt{1+\frac{ce\,x^{2}}{be-cd}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\ldots}\right.\right.}{\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^{4}e^{2}+b\,x^{2}e^{2}+bde-c\,d^{2}}\,\left(b\,e^{2}+e(be-\ldots\right.}\right.}{\ldots}$

input
```
int(x^6/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/c*x^3*(c*x^4+b*x^2+b*d/e-c*d^2/e^2)^(1/2)-4/15*b/c^2*x*(c*x^4+b*x^2+b*
d/e-c*d^2/e^2)^(1/2)+4/15*b/c^2*(b*d/e-c*d^2/e^2)/(-c*e/(b*e-c*d))^(1/2)*(
1+c*e/(b*e-c*d)*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(c*x^4+b*x^2+b*d/e-c*d^2/e^2)
^(1/2)*EllipticF(x*(-c*e/(b*e-c*d))^(1/2),(-1+b*e/c/d)^(1/2))-2*(-1/5/c*(3
*b*d/e-3*c*d^2/e^2)+8/15/c^2*b^2)*(b*d/e-c*d^2/e^2)/(-c*e/(b*e-c*d))^(1/2)
*(1+c*e/(b*e-c*d)*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(c*x^4+b*x^2+b*d/e-c*d^2/e^
2)^(1/2)/(b+(b*e-2*c*d)/e)*(EllipticF(x*(-c*e/(b*e-c*d))^(1/2),(-1+b*e/c/d
)^(1/2))-EllipticE(x*(-c*e/(b*e-c*d))^(1/2),(-1+b*e/c/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.67

$$\int \frac{x^6}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = -\frac{(9\,c^2d^3-9\,bcd^2e+8\,b^2de^2)\sqrt{ce^2}x\sqrt{-\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{x}\right)\,|\,-\frac{cd-be}{cd})-(9\,c^2d^3-9\,bcd^2e+4\,b^2e^3+4\,(2$$

input
```
integrate(x^6/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((9*c^2*d^3 - 9*b*c*d^2*e + 8*b^2*d*e^2)*sqrt(c*e^2)*x*sqrt(-d/e)*el
liptic_e(arcsin(sqrt(-d/e)/x), -(c*d - b*e)/(c*d)) - (9*c^2*d^3 - 9*b*c*d^
2*e + 4*b^2*e^3 + 4*(2*b^2 - b*c)*d*e^2)*sqrt(c*e^2)*x*sqrt(-d/e)*elliptic
_f(arcsin(sqrt(-d/e)/x), -(c*d - b*e)/(c*d)) - (3*c^2*e^4*x^4 - 4*b*c*e^4*
x^2 + 9*c^2*d^2*e^2 - 9*b*c*d*e^3 + 8*b^2*e^4)*sqrt((c*e^2*x^4 + b*e^2*x^2
 - c*d^2 + b*d*e)/e^2))/(c^3*e^4*x)
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{x^6}{\sqrt{\left(\frac{d}{e}+x^2\right)\left(b-\frac{cd}{e}+cx^2\right)}}\,dx$$

input
```
integrate(x**6/((b*d*e-c*d**2)/e**2+b*x**2+c*x**4)**(1/2),x)
```

output
```
Integral(x**6/sqrt((d/e + x**2)*(b - c*d/e + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{x^6}{\sqrt{cx^4+bx^2-\frac{cd^2-bde}{e^2}}}\,dx$$

input
```
integrate(x^6/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{x^6}{\sqrt{cx^4+bx^2-\frac{cd^2-bde}{e^2}}}\,dx$$

input
```
integrate(x^6/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{x^6}{\sqrt{b\,x^2-\frac{c\,d^2-b\,d\,e}{e^2}+c\,x^4}}\,dx$$

input
```
int(x^6/(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(1/2),x)
```

output
```
int(x^6/(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \frac{-4\sqrt{e\,x^2+d}\,\sqrt{ce\,x^2+be-cd}\,bx+3\sqrt{e\,x^2+d}\,\sqrt{ce\,x^2+be-cd}\,c\,x^3+8\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{ce\,x^2+be-cd}\,x^2}{c\,e^2x^4+b\,e^2x^2+bde-c\,d^2}dx\right)b^2e}{}$$

input
```
int(x^6/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x)
```

output
```
( - 4*sqrt(d + e*x**2)*sqrt(b*e - c*d + c*e*x**2)*b*x + 3*sqrt(d + e*x**2)
*sqrt(b*e - c*d + c*e*x**2)*c*x**3 + 8*int((sqrt(d + e*x**2)*sqrt(b*e - c*
d + c*e*x**2)*x**2)/(b*d*e + b*e**2*x**2 - c*d**2 + c*e**2*x**4),x)*b**2*e
**2 - 9*int((sqrt(d + e*x**2)*sqrt(b*e - c*d + c*e*x**2)*x**2)/(b*d*e + b*
e**2*x**2 - c*d**2 + c*e**2*x**4),x)*b*c*d*e + 9*int((sqrt(d + e*x**2)*sqr
t(b*e - c*d + c*e*x**2)*x**2)/(b*d*e + b*e**2*x**2 - c*d**2 + c*e**2*x**4)
,x)*c**2*d**2 + 4*int((sqrt(d + e*x**2)*sqrt(b*e - c*d + c*e*x**2))/(b*d*e
 + b*e**2*x**2 - c*d**2 + c*e**2*x**4),x)*b**2*d*e - 4*int((sqrt(d + e*x**
2)*sqrt(b*e - c*d + c*e*x**2))/(b*d*e + b*e**2*x**2 - c*d**2 + c*e**2*x**4
),x)*b*c*d**2)/(15*c**2*e)
```

3.723 $\int \frac{x^4}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 306

$$\int \frac{x^4}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \frac{x\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3c} - \frac{2bd\sqrt{cd-be}\sqrt{1+\frac{ex^2}{d}}\sqrt{1-\frac{cex^2}{cd-be}}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)|-1+\frac{be}{cd}\right)}{3c^{3/2}e^{3/2}\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}} + \frac{d\sqrt{cd-be}(cd+be)\sqrt{1+\frac{ex^2}{d}}\sqrt{1-\frac{cex^2}{cd-be}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),-1+\frac{be}{cd}\right)}{3c^{3/2}e^{5/2}\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

output
```
1/3*x*(-d*(-b*e+c*d)/e^2+b*x^2+c*x^4)^(1/2)/c-2/3*b*d*(-b*e+c*d)^(1/2)*(1+
e*x^2/d)^(1/2)*(1-c*e*x^2/(-b*e+c*d))^(1/2)*EllipticE(c^(1/2)*e^(1/2)*x/(-
b*e+c*d)^(1/2),(-1+b*e/c/d)^(1/2))/c^(3/2)/e^(3/2)/(-d*(-b*e+c*d)/e^2+b*x^
2+c*x^4)^(1/2)+1/3*d*(-b*e+c*d)^(1/2)*(b*e+c*d)*(1+e*x^2/d)^(1/2)*(1-c*e*x
^2/(-b*e+c*d))^(1/2)*EllipticF(c^(1/2)*e^(1/2)*x/(-b*e+c*d)^(1/2),(-1+b*e/
c/d)^(1/2))/c^(3/2)/e^(5/2)/(-d*(-b*e+c*d)/e^2+b*x^2+c*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.26 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \frac{-c\sqrt{\frac{e}{d}}x(d+ex^2)\left(-be+c(d-ex^2)\right)+2ibe(-cd+be)\sqrt{\frac{-cd+be+cex^2}{-cd+be}}\sqrt{1+\frac{ex^2}{d}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)|\frac{cd}{-cd+be}\right)}{3c^2d^2\left(\frac{e}{d}\right)^{5/2}\sqrt{\frac{(d+ex^2)(-cd-}{e^2}}}$$

input
```
Integrate[x^4/Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4],x]
```

output
```
(-(c*Sqrt[e/d]*x*(d + e*x^2)*(-(b*e) + c*(d - e*x^2))) + (2*I)*b*e*(-(c*d)
 + b*e)*Sqrt[(-(c*d) + b*e + c*e*x^2)/(-(c*d) + b*e)]*Sqrt[1 + (e*x^2)/d]*
EllipticE[I*ArcSinh[Sqrt[e/d]*x], (c*d)/(-(c*d) + b*e)] - I*(c^2*d^2 - 3*b
*c*d*e + 2*b^2*e^2)*Sqrt[(-(c*d) + b*e + c*e*x^2)/(-(c*d) + b*e)]*Sqrt[1 +
 (e*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[e/d]*x], (c*d)/(-(c*d) + b*e)])/(3*c^
2*d^2*(e/d)^(5/2)*Sqrt[((d + e*x^2)*(-(c*d) + b*e + c*e*x^2))/e^2])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {1442, 25, 27, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\frac{bde-cd^2}{e^2}+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{ 1442}$$

$$\frac{x\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3c}-\frac{\int -\frac{d(cd-be)-2be^2x^2}{e^2\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{3c}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{d(cd-be)-2be^2x^2}{e^2\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{3c}+\frac{x\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{d(cd-be)-2be^2x^2}{\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{3ce^2}+\frac{x\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \ 1514$$

$$\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\int \frac{d(cd-be)-2be^2x^2}{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}}dx}{3ce^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}+\frac{x\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \ 399$$

$$\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(d(be+cd)\int \frac{1}{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}}dx-2bde\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{cex^2}{cd-be}}}dx\right)}{3ce^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}+\frac{x\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \ 321$$

$$\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\sqrt{cd-be}(be+cd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}\sqrt{e}}-2bde\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{cex^2}{cd-be}}}dx\right)}{3ce^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}+\frac{x\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3c}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\sqrt{cd-be}(be+cd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}\sqrt{e}}-\frac{2bd\sqrt{e}\sqrt{cd-be}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)|\frac{be}{cd}-1\right)}{\sqrt{c}}\right)}{3ce^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}+\frac{x\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3c}$$

input
```
Int[x^4/Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4],x]
```

output
```
(x*Sqrt[-((d*(c*d - b*e))/e^2) + b*x^2 + c*x^4])/(3*c) + (Sqrt[1 + (e*x^2)
/d]*Sqrt[1 - (c*e*x^2)/(c*d - b*e)]*((-2*b*d*Sqrt[e]*Sqrt[c*d - b*e]*Ellip
ticE[ArcSin[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]], -1 + (b*e)/(c*d)])/Sqrt[
c] + (d*Sqrt[c*d - b*e]*(c*d + b*e)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[e]*x)/S
qrt[c*d - b*e]], -1 + (b*e)/(c*d)])/(Sqrt[c]*Sqrt[e])))/(3*c*e^2*Sqrt[-((d
*(c*d - b*e))/e^2) + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 16.48 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.13

method	result
default	$\frac{x\sqrt{c\,x^4+b\,x^2+\frac{bd}{e}-\frac{c\,d^2}{e^2}}}{3c}-\frac{\left(\frac{bd}{e}-\frac{c\,d^2}{e^2}\right)\sqrt{1+\frac{ce\,x^2}{be-cd}}\,\sqrt{1+\frac{e\,x^2}{d}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)}{3c\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^4+b\,x^2+\frac{bd}{e}-\frac{c\,d^2}{e^2}}}+\frac{4b\left(\frac{bd}{e}-\frac{c\,d^2}{e^2}\right)\sqrt{1+\frac{ce\,x^2}{be-cd}}\,\sqrt{}$
elliptic	$\frac{x\sqrt{c\,x^4+b\,x^2+\frac{bd}{e}-\frac{c\,d^2}{e^2}}}{3c}-\frac{\left(\frac{bd}{e}-\frac{c\,d^2}{e^2}\right)\sqrt{1+\frac{ce\,x^2}{be-cd}}\,\sqrt{1+\frac{e\,x^2}{d}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)}{3c\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^4+b\,x^2+\frac{bd}{e}-\frac{c\,d^2}{e^2}}}+\frac{4b\left(\frac{bd}{e}-\frac{c\,d^2}{e^2}\right)\sqrt{1+\frac{ce\,x^2}{be-cd}}\,\sqrt{}$
risch	$\frac{x\left(e\,x^2+d\right)\left(ce\,x^2+be-cd\right)}{3c\,e^2\sqrt{\frac{\left(e\,x^2+d\right)\left(ce\,x^2+be-cd\right)}{e^2}}}-\left(\frac{bde\sqrt{1+\frac{ce\,x^2}{be-cd}}\,\sqrt{1+\frac{e\,x^2}{d}}\;\mathrm{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)}{\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^4e^2+b\,x^2e^2+bde-c\,d^2}}-\frac{c\,d^2\sqrt{1+\frac{ce\,x^2}{be-cd}}\,\sqrt{1+\frac{e\,x^2}{d}}\;\mathrm{EllipticF}\left(x\sqrt{}\right.}{\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^4e^2+b\,x^2e^2+}}\right.$

input
```
int(x^4/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/c*x*(c*x^4+b*x^2+b*d/e-c*d^2/e^2)^(1/2)-1/3/c*(b*d/e-c*d^2/e^2)/(-c*e/
(b*e-c*d))^(1/2)*(1+c*e/(b*e-c*d)*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(c*x^4+b*x^
2+b*d/e-c*d^2/e^2)^(1/2)*EllipticF(x*(-c*e/(b*e-c*d))^(1/2),(-1+b*e/c/d)^(
1/2))+4/3/c*b*(b*d/e-c*d^2/e^2)/(-c*e/(b*e-c*d))^(1/2)*(1+c*e/(b*e-c*d)*x^
2)^(1/2)*(1+e*x^2/d)^(1/2)/(c*x^4+b*x^2+b*d/e-c*d^2/e^2)^(1/2)/(b+(b*e-2*c
*d)/e)*(EllipticF(x*(-c*e/(b*e-c*d))^(1/2),(-1+b*e/c/d)^(1/2))-EllipticE(x
*(-c*e/(b*e-c*d))^(1/2),(-1+b*e/c/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.56

$$\int \frac{x^4}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \frac{2\sqrt{ce^2}bdx\sqrt{-\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{x}\right)\mid -\frac{cd-be}{cd}) - \sqrt{ce^2}((2b-c)d+be)x\sqrt{-\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{x}\right)\mid -\frac{cd-be}{cd}) + \cdots}{3c^2e^2x}$$

input
```
integrate(x^4/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(c*e^2)*b*d*x*sqrt(-d/e)*elliptic_e(arcsin(sqrt(-d/e)/x), -(c*d
 - b*e)/(c*d)) - sqrt(c*e^2)*((2*b - c)*d + b*e)*x*sqrt(-d/e)*elliptic_f(a
rcsin(sqrt(-d/e)/x), -(c*d - b*e)/(c*d)) + (c*e^2*x^2 - 2*b*e^2)*sqrt((c*e
^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)/e^2))/(c^2*e^2*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{\left(\frac{d}{e}+x^2\right)\left(b-\frac{cd}{e}+cx^2\right)}}\,dx$$

input
```
integrate(x**4/((b*d*e-c*d**2)/e**2+b*x**2+c*x**4)**(1/2),x)
```

output
```
Integral(x**4/sqrt((d/e + x**2)*(b - c*d/e + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{x^4}{\sqrt{cx^4+bx^2-\frac{cd^2-bde}{e^2}}}\,dx$$

input
```
integrate(x^4/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{x^4}{\sqrt{cx^4+bx^2-\frac{cd^2-bde}{e^2}}}\,dx$$

input
```
integrate(x^4/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{x^4}{\sqrt{b\,x^2-\frac{c\,d^2-b\,d\,e}{e^2}+c\,x^4}}\,dx$$

input
```
int(x^4/(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(1/2),x)
```

output
```
int(x^4/(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx$$
$$= \frac{\sqrt{e\,x^2+d}\,\sqrt{ce\,x^2+be-cd}\,x-2\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{ce\,x^2+be-cd}\,x^2}{c\,e^2x^4+b\,e^2x^2+bde-c\,d^2}dx\right)b\,e^2-\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{ce\,x^2+be-cd}}{c\,e^2x^4+b\,e^2x^2+bde-c\,d^2}dx\right)bde+\left(\int\right.}{3ce}$$

input
```
int(x^4/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*sqrt(b*e - c*d + c*e*x**2)*x - 2*int((sqrt(d + e*x**2)*s
qrt(b*e - c*d + c*e*x**2)*x**2)/(b*d*e + b*e**2*x**2 - c*d**2 + c*e**2*x**
4),x)*b*e**2 - int((sqrt(d + e*x**2)*sqrt(b*e - c*d + c*e*x**2))/(b*d*e +
b*e**2*x**2 - c*d**2 + c*e**2*x**4),x)*b*d*e + int((sqrt(d + e*x**2)*sqrt(
b*e - c*d + c*e*x**2))/(b*d*e + b*e**2*x**2 - c*d**2 + c*e**2*x**4),x)*c*d
**2)/(3*c*e)
```

3.724 $\int \frac{x^2}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 256

$$\begin{aligned}\int \frac{x^2}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx &= \frac{d\sqrt{cd-be}\sqrt{1+\frac{ex^2}{d}}\sqrt{1-\frac{cex^2}{cd-be}}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)|-1+\frac{be}{cd}\right)}{\sqrt{c}e^{3/2}\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}} \\ &\quad -\frac{d\sqrt{cd-be}\sqrt{1+\frac{ex^2}{d}}\sqrt{1-\frac{cex^2}{cd-be}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),-1+\frac{be}{cd}\right)}{\sqrt{c}e^{3/2}\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}\end{aligned}$$

output
```
d*(-b*e+c*d)^(1/2)*(1+e*x^2/d)^(1/2)*(1-c*e*x^2/(-b*e+c*d))^(1/2)*Elliptic
E(c^(1/2)*e^(1/2)*x/(-b*e+c*d)^(1/2),(-1+b*e/c/d)^(1/2))/c^(1/2)/e^(3/2)/(
-d*(-b*e+c*d)/e^2+b*x^2+c*x^4)^(1/2)-d*(-b*e+c*d)^(1/2)*(1+e*x^2/d)^(1/2)*
(1-c*e*x^2/(-b*e+c*d))^(1/2)*EllipticF(c^(1/2)*e^(1/2)*x/(-b*e+c*d)^(1/2),
(-1+b*e/c/d)^(1/2))/c^(1/2)/e^(3/2)/(-d*(-b*e+c*d)/e^2+b*x^2+c*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \frac{i(cd-be)\sqrt{1+\frac{ex^2}{d}}\sqrt{1+\frac{cex^2}{-cd+be}}\left(E\left(i\text{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)|\frac{cd}{-cd+be}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{e}{d}}x\right),\frac{cd}{-cd+be}\right)\right)}{ce\sqrt{\frac{e}{d}}\sqrt{\frac{(d+ex^2)(-cd+be+cex^2)}{e^2}}}$$

input
```
Integrate[x^2/Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4],x]
```

output
```
(I*(c*d - b*e)*Sqrt[1 + (e*x^2)/d]*Sqrt[1 + (c*e*x^2)/(-(c*d) + b*e)]*(Ell
ipticE[I*ArcSinh[Sqrt[e/d]*x], (c*d)/(-(c*d) + b*e)] - EllipticF[I*ArcSinh
[Sqrt[e/d]*x], (c*d)/(-(c*d) + b*e)]))/(c*e*Sqrt[e/d]*Sqrt[((d + e*x^2)*(-
(c*d) + b*e + c*e*x^2))/e^2])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1460, 389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\frac{bde-cd^2}{e^2}+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1460}$$

$$\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\int \frac{x^2}{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}}dx}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$$\downarrow \text{389}$$

$$\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{cex^2}{cd-be}}}dx}{e}-\frac{d\int \frac{1}{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}}dx}{e}\right)}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$$\downarrow \text{321}$$

$$\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{cex^2}{cd-be}}}dx}{e}-\frac{d\sqrt{cd-be}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}e^{3/2}}\right)}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$$\downarrow \text{327}$$

$$\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\sqrt{cd-be}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)|\frac{be}{cd}-1\right)}{\sqrt{c}e^{3/2}}-\frac{d\sqrt{cd-be}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}e^{3/2}}\right)}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

input
```
Int[x^2/Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4],x]
```

output
```
(Sqrt[1 + (e*x^2)/d]*Sqrt[1 - (c*e*x^2)/(c*d - b*e)]*((d*Sqrt[c*d - b*e]*E
llipticE[ArcSin[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]], -1 + (b*e)/(c*d)])/(
Sqrt[c]*e^(3/2)) - (d*Sqrt[c*d - b*e]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[e]*x)
/Sqrt[c*d - b*e]], -1 + (b*e)/(c*d)])/(Sqrt[c]*e^(3/2))))/Sqrt[-((d*(c*d -
 b*e))/e^2) + b*x^2 + c*x^4]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 1460
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(
b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[x^2/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sq
rt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{2\left(\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}\right)\sqrt{1+\frac{ce\,x^{2}}{be-cd}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)\right)}{\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}\,\left(b+\frac{be-2cd}{e}\right)}$	178
elliptic	$-\frac{2\left(\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}\right)\sqrt{1+\frac{ce\,x^{2}}{be-cd}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)\right)}{\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}\,\left(b+\frac{be-2cd}{e}\right)}$	178

input
```
int(x^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*d/e-c*d^2/e^2)/(-c*e/(b*e-c*d))^(1/2)*(1+c*e/(b*e-c*d)*x^2)^(1/2)*(1
+e*x^2/d)^(1/2)/(c*x^4+b*x^2+b*d/e-c*d^2/e^2)^(1/2)/(b+(b*e-2*c*d)/e)*(Ell
ipticF(x*(-c*e/(b*e-c*d))^(1/2),(-1+b*e/c/d)^(1/2))-EllipticE(x*(-c*e/(b*e
-c*d))^(1/2),(-1+b*e/c/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = -\frac{\sqrt{ce^2}dx\sqrt{-\frac{d}{e}}E(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{x}\right) | -\frac{cd-be}{cd}) - \sqrt{ce^2}dx\sqrt{-\frac{d}{e}}F(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{x}\right) | -\frac{cd-be}{cd}) - e^2\sqrt{\frac{ce^2x^4+be^2x^2}{e^2}}}{ce^2x}$$

input
```
integrate(x^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(c*e^2)*d*x*sqrt(-d/e)*elliptic_e(arcsin(sqrt(-d/e)/x), -(c*d - b*e)
/(c*d)) - sqrt(c*e^2)*d*x*sqrt(-d/e)*elliptic_f(arcsin(sqrt(-d/e)/x), -(c*
d - b*e)/(c*d)) - e^2*sqrt((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)/e^2))/(
c*e^2*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \int \frac{x^2}{\sqrt{\left(\frac{d}{e}+x^2\right)\left(b-\frac{cd}{e}+cx^2\right)}}\,dx$$

input
```
integrate(x**2/((b*d*e-c*d**2)/e**2+b*x**2+c*x**4)**(1/2),x)
```

output
```
Integral(x**2/sqrt((d/e + x**2)*(b - c*d/e + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{x^2}{\sqrt{cx^4+bx^2-\frac{cd^2-bde}{e^2}}}\,dx$$

input
```
integrate(x^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{x^2}{\sqrt{cx^4+bx^2-\frac{cd^2-bde}{e^2}}}\,dx$$

input
```
integrate(x^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{x^2}{\sqrt{b\,x^2-\frac{c\,d^2-b\,d\,e}{e^2}+c\,x^4}}\,dx$$

input
```
int(x^2/(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(1/2),x)
```

output
```
int(x^2/(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{ce\,x^2+be-cd}\,x^2}{c\,e^2x^4+b\,e^2x^2+bde-c\,d^2}dx\right)e$$

input
```
int(x^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(b*e - c*d + c*e*x**2)*x**2)/(b*d*e + b*e**2*x**
2 - c*d**2 + c*e**2*x**4),x)*e
```

3.725 $\int \frac{1}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 126

$$\int \frac{1}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \frac{\sqrt{cd-be}\sqrt{1+\frac{ex^2}{d}}\sqrt{1-\frac{cex^2}{cd-be}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),-1+\frac{be}{cd}\right)}{\sqrt{c}\sqrt{e}\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

output
```
(-b*e+c*d)^(1/2)*(1+e*x^2/d)^(1/2)*(1-c*e*x^2/(-b*e+c*d))^(1/2)*EllipticF(
c^(1/2)*e^(1/2)*x/(-b*e+c*d)^(1/2),(-1+b*e/c/d)^(1/2))/c^(1/2)/e^(1/2)/(-d
*(-b*e+c*d)/e^2+b*x^2+c*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \frac{\sqrt{\frac{-cd+be+cex^2}{-cd+be}}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right),\frac{cd}{-cd+be}\right)}{\sqrt{-\frac{e}{d}}\sqrt{\frac{(d+ex^2)(-cd+be+cex^2)}{e^2}}}$$

input
```
Integrate[1/Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4],x]
```

output
```
(Sqrt[(-(c*d) + b*e + c*e*x^2)/(-(c*d) + b*e)]*Sqrt[1 + (e*x^2)/d]*Ellipti
cF[ArcSin[Sqrt[-(e/d)]*x], (c*d)/(-(c*d) + b*e)])/(Sqrt[-(e/d)]*Sqrt[((d +
 e*x^2)*(-(c*d) + b*e + c*e*x^2))/e^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1417, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{bde-cd^2}{e^2}+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1417}$$

$$\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\int \frac{1}{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}}dx}{\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{\frac{ex^2}{d}+1}\sqrt{cd-be}\sqrt{1-\frac{cex^2}{cd-be}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}\sqrt{e}\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

input
```
Int[1/Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4],x]
```

output
```
(Sqrt[c*d - b*e]*Sqrt[1 + (e*x^2)/d]*Sqrt[1 - (c*e*x^2)/(c*d - b*e)]*Ellip
ticF[ArcSin[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]], -1 + (b*e)/(c*d)])/(Sqrt
[c]*Sqrt[e]*Sqrt[-((d*(c*d - b*e))/e^2) + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\sqrt{1+\frac{ce\,x^2}{be-cd}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)}{\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^4+b\,x^2+\frac{bd}{e}-\frac{c\,d^2}{e^2}}}$	110
elliptic	$\frac{\sqrt{1+\frac{ce\,x^2}{be-cd}}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)}{\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^4+b\,x^2+\frac{bd}{e}-\frac{c\,d^2}{e^2}}}$	110

input
```
int(1/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-c*e/(b*e-c*d))^(1/2)*(1+c*e/(b*e-c*d)*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(c*
x^4+b*x^2+b*d/e-c*d^2/e^2)^(1/2)*EllipticF(x*(-c*e/(b*e-c*d))^(1/2),(-1+b*
e/c/d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = -\frac{\sqrt{-cd^2+bde}\sqrt{\frac{ce}{cd-be}}F\left(\arcsin\left(\sqrt{\frac{ce}{cd-be}}x\right)\mid -\frac{cd-be}{cd}\right)}{cd}$$

input
```
integrate(1/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-c*d^2 + b*d*e)*sqrt(c*e/(c*d - b*e))*elliptic_f(arcsin(sqrt(c*e/(c*
d - b*e))*x), -(c*d - b*e)/(c*d))/(c*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{bx^2+cx^4+\frac{bde-cd^2}{e^2}}}\,dx$$

input
```
integrate(1/((b*d*e-c*d**2)/e**2+b*x**2+c*x**4)**(1/2),x)
```

output
```
Integral(1/sqrt(b*x**2 + c*x**4 + (b*d*e - c*d**2)/e**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2-\frac{cd^2-bde}{e^2}}}\,dx$$

input
```
integrate(1/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2-\frac{cd^2-bde}{e^2}}}\,dx$$

input
```
integrate(1/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{b\,x^2-\frac{c\,d^2-b\,d\,e}{e^2}+c\,x^4}}\,dx$$

input
```
int(1/(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(1/2),x)
```

output
```
int(1/(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{ce\,x^2+be-cd}}{c\,e^2x^4+b\,e^2x^2+bde-c\,d^2}dx\right)e$$

input
```
int(1/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(b*e - c*d + c*e*x**2))/(b*d*e + b*e**2*x**2 - c
*d**2 + c*e**2*x**4),x)*e
```

3.726 $\int \frac{1}{x^2\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 178

$$\int \frac{1}{x^2\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{(cd-be)x\left(d+ex^2\right)}-\frac{\sqrt{e}\left(d+ex^2\right)\sqrt{\frac{d(cd-be-cex^2)}{(cd-be)(d+ex^2)}}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|\frac{2cd-be}{cd-be}\right)}{d^{3/2}\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

output
```
e^2*(-d*(-b*e+c*d)/e^2+b*x^2+c*x^4)^(1/2)/(-b*e+c*d)/x/(e*x^2+d)-e^(1/2)*(
e*x^2+d)*(d*(-c*e*x^2-b*e+c*d)/(-b*e+c*d)/(e*x^2+d))^(1/2)*EllipticE(e^(1/
2)*x/d^(1/2)/(1+e*x^2/d)^(1/2),((-b*e+2*c*d)/(-b*e+c*d))^(1/2))/d^(3/2)/(-
d*(-b*e+c*d)/e^2+b*x^2+c*x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.44 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx$$

$$= \frac{\sqrt{\frac{e}{d}}\left(\sqrt{\frac{e}{d}}(d+ex^2)\left(cd-be-cex^2\right)+ie(cd-be)x\sqrt{\frac{-cd+be+cex^2}{-cd+be}}\sqrt{1+\frac{ex^2}{d}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)\mid\frac{cd}{-cd+be}\right)-\right.}{e(-cd+be)x\sqrt{\frac{(d+ex^2)(-cd+be+cex^2)}{e^2}}}$$

input
```
Integrate[1/(x^2*Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[e/d]*(Sqrt[e/d]*(d + e*x^2)*(c*d - b*e - c*e*x^2) + I*e*(c*d - b*e)*
x*Sqrt[(-(c*d) + b*e + c*e*x^2)/(-(c*d) + b*e)]*Sqrt[1 + (e*x^2)/d]*Ellipt
icE[I*ArcSinh[Sqrt[e/d]*x], (c*d)/(-(c*d) + b*e)] - I*e*(c*d - b*e)*x*Sqrt
[(-(c*d) + b*e + c*e*x^2)/(-(c*d) + b*e)]*Sqrt[1 + (e*x^2)/d]*EllipticF[I*
ArcSinh[Sqrt[e/d]*x], (c*d)/(-(c*d) + b*e)]))/(e*(-(c*d) + b*e)*x*Sqrt[((d
 + e*x^2)*(-(c*d) + b*e + c*e*x^2))/e^2])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.46, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1443, 27, 1460, 389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{\frac{bde-cd^2}{e^2}+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{dx(cd-be)}-\frac{e^2\int \frac{cx^2}{\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{d(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{dx(cd-be)}-\frac{ce^2\int \frac{x^2}{\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{d(cd-be)}$$

$$\Big\downarrow \text{1460}$$

$$\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{dx(cd-be)}-\frac{ce^2\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\int \frac{x^2}{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}}dx}{d(cd-be)\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$$\Big\downarrow \text{389}$$

$$\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{dx(cd-be)}-\frac{ce^2\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{cex^2}{cd-be}}}dx}{e}-\frac{d\int \frac{1}{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}}dx}{e}\right)}{d(cd-be)\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$$\Big\downarrow \text{321}$$

$$\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{dx(cd-be)}-\frac{ce^2\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{cex^2}{cd-be}}}dx}{e}-\frac{d\sqrt{cd-be}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}e^{3/2}}\right)}{d(cd-be)\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{dx(cd-be)}-\frac{ce^2\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\sqrt{cd-be}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)|\frac{be}{cd}-1\right)}{\sqrt{c}e^{3/2}}-\frac{d\sqrt{cd-be}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}e^{3/2}}\right)}{d(cd-be)\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}$$

input
```
Int[1/(x^2*Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4]),x]
```

output
```
(e^2*Sqrt[-((d*(c*d - b*e))/e^2) + b*x^2 + c*x^4])/(d*(c*d - b*e)*x) - (c*
e^2*Sqrt[1 + (e*x^2)/d]*Sqrt[1 - (c*e*x^2)/(c*d - b*e)]*((d*Sqrt[c*d - b*e
]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]], -1 + (b*e)/(c*d)]
)/(Sqrt[c]*e^(3/2)) - (d*Sqrt[c*d - b*e]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[e]
*x)/Sqrt[c*d - b*e]], -1 + (b*e)/(c*d)])/(Sqrt[c]*e^(3/2))))/(d*(c*d - b*e
)*Sqrt[-((d*(c*d - b*e))/e^2) + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1460
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(
b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[x^2/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sq
rt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.38

method	result
default	$-\frac{e^{2}\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}{d(be-cd)x}-\frac{2e^{2}c\left(\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}\right)\sqrt{1+\frac{ce\,x^{2}}{be-cd}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{ce}{be-cd}}\right.\right.}{d(be-cd)\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}\,\left(b+\frac{be-2cd}{e}\right)}$
elliptic	$-\frac{e^{2}\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}{d(be-cd)x}-\frac{2e^{2}c\left(\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}\right)\sqrt{1+\frac{ce\,x^{2}}{be-cd}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{ce}{be-cd}}\right.\right.}{d(be-cd)\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}\,\left(b+\frac{be-2cd}{e}\right)}$
risch	$-\frac{(e\,x^{2}+d)\left(ce\,x^{2}+be-cd\right)}{d(be-cd)x\sqrt{\frac{(e\,x^{2}+d)\left(ce\,x^{2}+be-cd\right)}{e^{2}}}}-\frac{2e^{2}c(bde-c\,d^{2})\sqrt{1+\frac{ce\,x^{2}}{be-cd}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)-\operatorname{EllipticE}\left(\right.\right.}{d(be-cd)\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^{4}e^{2}+b\,x^{2}e^{2}+bde-c\,d^{2}}\,(b\,e^{2}+e(be-2ca}$

input
```
int(1/x^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/(b*e-c*d)*e^2*(c*x^4+b*x^2+b*d/e-c*d^2/e^2)^(1/2)/x-2/d/(b*e-c*d)*e^2
*c*(b*d/e-c*d^2/e^2)/(-c*e/(b*e-c*d))^(1/2)*(1+c*e/(b*e-c*d)*x^2)^(1/2)*(1
+e*x^2/d)^(1/2)/(c*x^4+b*x^2+b*d/e-c*d^2/e^2)^(1/2)/(b+(b*e-2*c*d)/e)*(Ell
ipticF(x*(-c*e/(b*e-c*d))^(1/2),(-1+b*e/c/d)^(1/2))-EllipticE(x*(-c*e/(b*e
-c*d))^(1/2),(-1+b*e/c/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \frac{\sqrt{-cd^2+bde}c\sqrt{\frac{ce}{cd-be}}e^2xE(\arcsin\left(\sqrt{\frac{ce}{cd-be}}x\right)\mid -\frac{cd-be}{cd}) - \sqrt{-cd^2+bde}c\sqrt{\frac{ce}{cd-be}}e^2xF(\arcsin\left(\sqrt{\frac{ce}{cd-be}}x\right)}{(c^2d^3-2\,bcd^2e+b^2de^2)x}$$

input
```
integrate(1/x^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="fricas
")
```

output
```
(sqrt(-c*d^2 + b*d*e)*c*sqrt(c*e/(c*d - b*e))*e^2*x*elliptic_e(arcsin(sqrt
(c*e/(c*d - b*e))*x), -(c*d - b*e)/(c*d)) - sqrt(-c*d^2 + b*d*e)*c*sqrt(c*
e/(c*d - b*e))*e^2*x*elliptic_f(arcsin(sqrt(c*e/(c*d - b*e))*x), -(c*d - b
*e)/(c*d)) + (c*d*e^2 - b*e^3)*sqrt((c*e^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e
)/e^2))/((c^2*d^3 - 2*b*c*d^2*e + b^2*d*e^2)*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \int \frac{1}{x^2\sqrt{\left(\frac{d}{e}+x^2\right)\left(b-\frac{cd}{e}+cx^2\right)}}\,dx$$

input
```
integrate(1/x**2/((b*d*e-c*d**2)/e**2+b*x**2+c*x**4)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((d/e + x**2)*(b - c*d/e + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2-\frac{cd^2-bde}{e^2}}\,x^2}\,dx$$

input
```
integrate(1/x^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2-\frac{cd^2-bde}{e^2}}\,x^2}\,dx$$

input
```
integrate(1/x^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{1}{x^2\,\sqrt{b\,x^2-\frac{c\,d^2-b\,d\,e}{e^2}+c\,x^4}}\,dx$$

input
```
int(1/(x^2*(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^2*(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx = \left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{ce\,x^2+be-cd}}{c\,e^2x^6+b\,e^2x^4+bde\,x^2-c\,d^2x^2}dx\right)e$$

input
```
int(1/x^2/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(b*e - c*d + c*e*x**2))/(b*d*e*x**2 + b*e**2*x**
4 - c*d**2*x**2 + c*e**2*x**6),x)*e
```

3.727 $\int \frac{1}{x^4\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 378

$$\begin{aligned}\int \frac{1}{x^4\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx &= \frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3d(cd-be)x^3}+\frac{2be^4\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3d^2(cd-be)^2x}\\&\quad-\frac{2b\sqrt{c}e^{5/2}\sqrt{1+\frac{ex^2}{d}}\sqrt{1-\frac{cex^2}{cd-be}}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)|-1+\frac{be}{cd}\right)}{3d(cd-be)^{3/2}\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}\\&\quad+\frac{\sqrt{c}e^{3/2}(cd+be)\sqrt{1+\frac{ex^2}{d}}\sqrt{1-\frac{cex^2}{cd-be}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),-1+\frac{be}{cd}\right)}{3d(cd-be)^{3/2}\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}\end{aligned}$$

output
```
1/3*e^2*(-d*(-b*e+c*d)/e^2+b*x^2+c*x^4)^(1/2)/d/(-b*e+c*d)/x^3+2/3*b*e^4*(
-d*(-b*e+c*d)/e^2+b*x^2+c*x^4)^(1/2)/d^2/(-b*e+c*d)^2/x-2/3*b*c^(1/2)*e^(5
/2)*(1+e*x^2/d)^(1/2)*(1-c*e*x^2/(-b*e+c*d))^(1/2)*EllipticE(c^(1/2)*e^(1/
2)*x/(-b*e+c*d)^(1/2),(-1+b*e/c/d)^(1/2))/d/(-b*e+c*d)^(3/2)/(-d*(-b*e+c*d
)/e^2+b*x^2+c*x^4)^(1/2)+1/3*c^(1/2)*e^(3/2)*(b*e+c*d)*(1+e*x^2/d)^(1/2)*(
1-c*e*x^2/(-b*e+c*d))^(1/2)*EllipticF(c^(1/2)*e^(1/2)*x/(-b*e+c*d)^(1/2),(
-1+b*e/c/d)^(1/2))/d/(-b*e+c*d)^(3/2)/(-d*(-b*e+c*d)/e^2+b*x^2+c*x^4)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.02 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^4\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx$$
$$= \frac{-\sqrt{\frac{e}{d}}(d+ex^2)\left(b^2e^2(d-2ex^2)+c^2d^2(d-ex^2)+bce(-2d^2+3dex^2-2e^2x^4)\right)+2ibe^3(-cd+be)x^3\sqrt{=\cdots}}{3d^2\cdots}$$

input
```
Integrate[1/(x^4*Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4]),x]
```

output
```
(-(Sqrt[e/d]*(d + e*x^2)*(b^2*e^2*(d - 2*e*x^2) + c^2*d^2*(d - e*x^2) + b*
c*e*(-2*d^2 + 3*d*e*x^2 - 2*e^2*x^4))) + (2*I)*b*e^3*(-(c*d) + b*e)*x^3*Sq
rt[(-(c*d) + b*e + c*e*x^2)/(-(c*d) + b*e)]*Sqrt[1 + (e*x^2)/d]*EllipticE[
I*ArcSinh[Sqrt[e/d]*x], (c*d)/(-(c*d) + b*e)] - I*e^2*(c^2*d^2 - 3*b*c*d*e
 + 2*b^2*e^2)*x^3*Sqrt[(-(c*d) + b*e + c*e*x^2)/(-(c*d) + b*e)]*Sqrt[1 + (
e*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[e/d]*x], (c*d)/(-(c*d) + b*e)])/(3*d^2*
Sqrt[e/d]*(c*d - b*e)^2*x^3*Sqrt[((d + e*x^2)*(-(c*d) + b*e + c*e*x^2))/e^
2])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1443, 25, 1604, 27, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{\frac{bde-cd^2}{e^2}+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1443}$$

$$\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3dx^3(cd-be)}-\frac{e^2\int -\frac{cx^2+2b}{x^2\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{3d(cd-be)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{e^2\int \frac{cx^2+2b}{x^2\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{3d(cd-be)}+\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3dx^3(cd-be)}$$

$$\Big\downarrow \text{ 1604}$$

$$\frac{e^2\left(\frac{e^2\int \frac{c\left(d(cd-be)-2be^2x^2\right)}{e^2\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{d(cd-be)}+\frac{2be^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{dx(cd-be)}\right)}{3d(cd-be)}+\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3dx^3(cd-be)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^2\left(\frac{c\int \frac{d(cd-be)-2be^2x^2}{\sqrt{cx^4+bx^2-\frac{d(cd-be)}{e^2}}}dx}{d(cd-be)}+\frac{2be^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{dx(cd-be)}\right)}{3d(cd-be)}+\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3dx^3(cd-be)}$$

$$\Big\downarrow \text{ 1514}$$

$$\frac{e^2\left(\frac{c\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\int \frac{d(cd-be)-2be^2x^2}{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}}dx}{d(cd-be)\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}+\frac{2be^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{dx(cd-be)}\right)}{3d(cd-be)}+\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3dx^3(cd-be)}$$

$\Big\downarrow$ 399

$$\frac{e^2\left(\frac{c\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(d(be+cd)\int \frac{1}{\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}}dx-2bde\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{cex^2}{cd-be}}}dx\right)}{d(cd-be)\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}+\frac{2be^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{dx(cd-be)}\right)}{3d(cd-be)}+\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3dx^3(cd-be)}$$

$\Big\downarrow$ 321

$$\frac{e^2\left(\frac{c\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\sqrt{cd-be}(be+cd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}\sqrt{e}}-2bde\int \frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-\frac{cex^2}{cd-be}}}dx\right)}{d(cd-be)\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}+\frac{2be^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{dx(cd-be)}\right)}{3d(cd-be)}+\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3dx^3(cd-be)}$$

$\Big\downarrow$ 327

$$\frac{e^2\left(\frac{c\sqrt{\frac{ex^2}{d}+1}\sqrt{1-\frac{cex^2}{cd-be}}\left(\frac{d\sqrt{cd-be}(be+cd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right),\frac{be}{cd}-1\right)}{\sqrt{c}\sqrt{e}}-\frac{2bd\sqrt{e}\sqrt{cd-be}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{e}x}{\sqrt{cd-be}}\right)|\frac{be}{cd}-1\right)}{\sqrt{c}}\right)}{d(cd-be)\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}+\frac{2be^2\sqrt{-\frac{d(cd-be)}{e^2}\cdots}}{dx(cd-be\cdots}\right.}{3d(cd-be)}$$

$$\frac{e^2\sqrt{-\frac{d(cd-be)}{e^2}+bx^2+cx^4}}{3dx^3(cd-be)}$$

input
```
Int[1/(x^4*Sqrt[(-(c*d^2) + b*d*e)/e^2 + b*x^2 + c*x^4]),x]
```

output
```
(e^2*Sqrt[-((d*(c*d - b*e))/e^2) + b*x^2 + c*x^4])/(3*d*(c*d - b*e)*x^3) +
 (e^2*((2*b*e^2*Sqrt[-((d*(c*d - b*e))/e^2) + b*x^2 + c*x^4])/(d*(c*d - b*
e)*x) + (c*Sqrt[1 + (e*x^2)/d]*Sqrt[1 - (c*e*x^2)/(c*d - b*e)]*((-2*b*d*Sq
rt[e]*Sqrt[c*d - b*e]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]
], -1 + (b*e)/(c*d)])/Sqrt[c] + (d*Sqrt[c*d - b*e]*(c*d + b*e)*EllipticF[A
rcSin[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]], -1 + (b*e)/(c*d)])/(Sqrt[c]*Sq
rt[e])))/(d*(c*d - b*e)*Sqrt[-((d*(c*d - b*e))/e^2) + b*x^2 + c*x^4])))/(3
*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 16.56 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.12

method	result
default	$-\frac{e^{2}\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}{3d(be-cd)x^{3}}+\frac{2e^{4}b\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}{3d^{2}(be-cd)^{2}x}-\frac{e^{2}c\sqrt{1+\frac{ce\,x^{2}}{be-cd}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)}{3d(be-cd)\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}+\frac{4\cdots}{\cdots}$
elliptic	$-\frac{e^{2}\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}{3d(be-cd)x^{3}}+\frac{2e^{4}b\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}{3d^{2}(be-cd)^{2}x}-\frac{e^{2}c\sqrt{1+\frac{ce\,x^{2}}{be-cd}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)}{3d(be-cd)\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^{4}+b\,x^{2}+\frac{bd}{e}-\frac{c\,d^{2}}{e^{2}}}}+\frac{4\cdots}{\cdots}$
risch	$-\frac{(e\,x^{2}+d)(ce\,x^{2}+be-cd)(-2b\,x^{2}e^{2}+bde-c\,d^{2})}{3d^{2}(be-cd)^{2}x^{3}\sqrt{\frac{(e\,x^{2}+d)(ce\,x^{2}+be-cd)}{e^{2}}}}-\frac{e^{2}c\left(\frac{bde\sqrt{1+\frac{ce\,x^{2}}{be-cd}}\,\sqrt{1+\frac{e\,x^{2}}{d}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{ce}{be-cd}},\sqrt{-1+\frac{be}{cd}}\right)}{\sqrt{-\frac{ce}{be-cd}}\,\sqrt{c\,x^{4}e^{2}+b\,x^{2}e^{2}+bde-c\,d^{2}}}-\frac{c\,d^{2}\sqrt{1+\frac{ce\,x^{2}}{be-cd}}\cdots}{\sqrt{-\cdots}}\right.}{\cdots}$

input
```
int(1/x^4/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/d/(b*e-c*d)*e^2*(c*x^4+b*x^2+b*d/e-c*d^2/e^2)^(1/2)/x^3+2/3*e^4*b/d^2
/(b*e-c*d)^2*(c*x^4+b*x^2+b*d/e-c*d^2/e^2)^(1/2)/x-1/3/d/(b*e-c*d)*e^2*c/(
-c*e/(b*e-c*d))^(1/2)*(1+c*e/(b*e-c*d)*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(c*x^4
+b*x^2+b*d/e-c*d^2/e^2)^(1/2)*EllipticF(x*(-c*e/(b*e-c*d))^(1/2),(-1+b*e/c
/d)^(1/2))+4/3*e^4*b*c/d^2/(b*e-c*d)^2*(b*d/e-c*d^2/e^2)/(-c*e/(b*e-c*d))^
(1/2)*(1+c*e/(b*e-c*d)*x^2)^(1/2)*(1+e*x^2/d)^(1/2)/(c*x^4+b*x^2+b*d/e-c*d
^2/e^2)^(1/2)/(b+(b*e-2*c*d)/e)*(EllipticF(x*(-c*e/(b*e-c*d))^(1/2),(-1+b*
e/c/d)^(1/2))-EllipticE(x*(-c*e/(b*e-c*d))^(1/2),(-1+b*e/c/d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^4\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx$$

$$= \frac{2\sqrt{-cd^2+bde}bc\sqrt{\frac{ce}{cd-be}}e^4x^3E(\arcsin\left(\sqrt{\frac{ce}{cd-be}}x\right)\mid -\frac{cd-be}{cd}) - (c^2d^2e^2-2\,bcde^3+(b^2+2\,bc)e^4)\sqrt{-cd^2 \cdots}}{3\,(c^3d^5-3\,bc^2 \cdots}$$

input
```
integrate(1/x^4/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="fricas
")
```

output
```
1/3*(2*sqrt(-c*d^2 + b*d*e)*b*c*sqrt(c*e/(c*d - b*e))*e^4*x^3*elliptic_e(a
rcsin(sqrt(c*e/(c*d - b*e))*x), -(c*d - b*e)/(c*d)) - (c^2*d^2*e^2 - 2*b*c
*d*e^3 + (b^2 + 2*b*c)*e^4)*sqrt(-c*d^2 + b*d*e)*sqrt(c*e/(c*d - b*e))*x^3
*elliptic_f(arcsin(sqrt(c*e/(c*d - b*e))*x), -(c*d - b*e)/(c*d)) + (c^2*d^
3*e^2 - 2*b*c*d^2*e^3 + b^2*d*e^4 + 2*(b*c*d*e^4 - b^2*e^5)*x^2)*sqrt((c*e
^2*x^4 + b*e^2*x^2 - c*d^2 + b*d*e)/e^2))/((c^3*d^5 - 3*b*c^2*d^4*e + 3*b^
2*c*d^3*e^2 - b^3*d^2*e^3)*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{1}{x^4\sqrt{\left(\frac{d}{e}+x^2\right)\left(b-\frac{cd}{e}+cx^2\right)}}\,dx$$

input
```
integrate(1/x**4/((b*d*e-c*d**2)/e**2+b*x**2+c*x**4)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt((d/e + x**2)*(b - c*d/e + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2-\frac{cd^2-bde}{e^2}}x^4}\,dx$$

input
```
integrate(1/x^4/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2-\frac{cd^2-bde}{e^2}}x^4}\,dx$$

input
```
integrate(1/x^4/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 - (c*d^2 - b*d*e)/e^2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\int \frac{1}{x^4\,\sqrt{b\,x^2-\frac{c\,d^2-b\,d\,e}{e^2}+c\,x^4}}\,dx$$

input
```
int(1/(x^4*(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(b*x^2 - (c*d^2 - b*d*e)/e^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{\frac{-cd^2+bde}{e^2}+bx^2+cx^4}}\,dx=\left(\int \frac{\sqrt{e\,x^2+d}\,\sqrt{ce\,x^2+be-cd}}{c\,e^2x^8+b\,e^2x^6+bde\,x^4-c\,d^2x^4}dx\right)e$$

input
```
int(1/x^4/((b*d*e-c*d^2)/e^2+b*x^2+c*x^4)^(1/2),x)
```

output
```
int((sqrt(d + e*x**2)*sqrt(b*e - c*d + c*e*x**2))/(b*d*e*x**4 + b*e**2*x**
6 - c*d**2*x**4 + c*e**2*x**8),x)*e
```

3.728 $\int \frac{x^2}{\sqrt{-8+6x^2-x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{x^2}{\sqrt{-8+6x^2-x^4}}\,dx = \frac{2\sqrt{-2+x^2}E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{\sqrt{2-x^2}} - 2\sqrt{2}\,\text{EllipticF}\left(\arccos\left(\frac{x}{2}\right),2\right)$$

output
```
2*(x^2-2)^(1/2)*EllipticE(1/2*x*2^(1/2),1/2*2^(1/2))/(-x^2+2)^(1/2)-2*2^(1
/2)*InverseJacobiAM(arccos(1/2*x),2^(1/2))
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.34

$$\int \frac{x^2}{\sqrt{-8+6x^2-x^4}}\,dx = -\frac{2\sqrt{2-x^2}\sqrt{4-x^2}\left(E\left(\arcsin\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{2}\right)-\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),\frac{1}{2}\right)\right)}{\sqrt{-8+6x^2-x^4}}$$

input
```
Integrate[x^2/Sqrt[-8 + 6*x^2 - x^4],x]
```

output
```
(-2*Sqrt[2 - x^2]*Sqrt[4 - x^2]*(EllipticE[ArcSin[x/Sqrt[2]], 1/2] - Ellip
ticF[ArcSin[x/Sqrt[2]], 1/2]))/Sqrt[-8 + 6*x^2 - x^4]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1452, 27, 389, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{-x^4+6x^2-8}}dx$$

$$\Big\downarrow \text{1452}$$

$$2\int \frac{x^2}{2\sqrt{4-x^2}\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{x^2-2}}dx$$

$$\Big\downarrow \text{389}$$

$$2\int \frac{1}{\sqrt{4-x^2}\sqrt{x^2-2}}dx+\int \frac{\sqrt{x^2-2}}{\sqrt{4-x^2}}dx$$

$$\Big\downarrow \text{322}$$

$$\int \frac{\sqrt{x^2-2}}{\sqrt{4-x^2}}dx-\sqrt{2}\,\text{EllipticF}\left(\arccos\left(\frac{x}{2}\right),2\right)$$

$$\Big\downarrow \text{328}$$

$$-\sqrt{2}\,\text{EllipticF}\left(\arccos\left(\frac{x}{2}\right),2\right)-\sqrt{2}E\left(\arccos\left(\frac{x}{2}\right)\middle|2\right)$$

input
```
Int[x^2/Sqrt[-8 + 6*x^2 - x^4],x]
```

output
```
-(Sqrt[2]*EllipticE[ArcCos[x/2], 2]) - Sqrt[2]*EllipticF[ArcCos[x/2], 2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 1452
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt
[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
&& LtQ[c, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{-x^{2}+4}\,\sqrt{-2x^{2}+4}\,\left(\operatorname{EllipticF}\left(\frac{x}{2},\sqrt{2}\right)-\operatorname{EllipticE}\left(\frac{x}{2},\sqrt{2}\right)\right)}{\sqrt{-x^{4}+6x^{2}-8}}$	51
elliptic	$\frac{\sqrt{-x^{2}+4}\,\sqrt{-2x^{2}+4}\,\left(\operatorname{EllipticF}\left(\frac{x}{2},\sqrt{2}\right)-\operatorname{EllipticE}\left(\frac{x}{2},\sqrt{2}\right)\right)}{\sqrt{-x^{4}+6x^{2}-8}}$	51

input
```
int(x^2/(-x^4+6*x^2-8)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-x^2+4)^(1/2)*(-2*x^2+4)^(1/2)/(-x^4+6*x^2-8)^(1/2)*(EllipticF(1/2*x,2^(1
/2))-EllipticE(1/2*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\sqrt{-8+6x^2-x^4}}\,dx = \frac{-8i\,xE\left(\arcsin\left(\frac{2}{x}\right)\mid\frac{1}{2}\right)+8i\,xF\left(\arcsin\left(\frac{2}{x}\right)\mid\frac{1}{2}\right)-\sqrt{-x^4+6\,x^2-8}}{x}$$

input
```
integrate(x^2/(-x^4+6*x^2-8)^(1/2),x, algorithm="fricas")
```

output
```
(-8*I*x*elliptic_e(arcsin(2/x), 1/2) + 8*I*x*elliptic_f(arcsin(2/x), 1/2)
- sqrt(-x^4 + 6*x^2 - 8))/x
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{-8+6x^2-x^4}}\,dx = \int \frac{x^2}{\sqrt{-(x-2)\,(x+2)\,(x^2-2)}}\,dx$$

input
```
integrate(x**2/(-x**4+6*x**2-8)**(1/2),x)
```

output
```
Integral(x**2/sqrt(-(x - 2)*(x + 2)*(x**2 - 2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{-8+6x^2-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+6\,x^2-8}}\,dx$$

input
```
integrate(x^2/(-x^4+6*x^2-8)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(-x^4 + 6*x^2 - 8), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{-8+6x^2-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+6\,x^2-8}}\,dx$$

input
```
integrate(x^2/(-x^4+6*x^2-8)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(-x^4 + 6*x^2 - 8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{-8+6x^2-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+6\,x^2-8}}\,dx$$

input
```
int(x^2/(6*x^2 - x^4 - 8)^(1/2),x)
```

output
```
int(x^2/(6*x^2 - x^4 - 8)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{-8+6x^2-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4+6x^2-8}\,x^2}{x^4-6x^2+8}dx\right)$$

input
```
int(x^2/(-x^4+6*x^2-8)^(1/2),x)
```

output
```
- int((sqrt( - x**4 + 6*x**2 - 8)*x**2)/(x**4 - 6*x**2 + 8),x)
```

3.729 $\int x^2(a + bx^2 + cx^4)\ dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int x^2(a + bx^2 + cx^4)\ dx = \frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

output
```
1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2(a + bx^2 + cx^4)\ dx = \frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

input
```
Integrate[x^2*(a + b*x^2 + c*x^4),x]
```

output
```
(a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2+cx^4\right) \, dx$$

$$\Big\downarrow \; 1433$$

$$\int \left(ax^2+bx^4+cx^6\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

input
```
Int[x^2*(a + b*x^2 + c*x^4),x]
```

output
```
(a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
default	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
norman	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
risch	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
parallelrisch	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
orering	$\frac{x^{3}\left(15c\,x^{4}+21b\,x^{2}+35a\right)}{105}$	22

input
```
int(x^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2\left(a+bx^2+cx^4\right)\,dx=\frac{1}{7}\,cx^7+\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2 \left(a + bx^2 + cx^4\right) \, dx = \frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

input
```
integrate(x**2*(c*x**4+b*x**2+a),x)
```

output
```
a*x**3/3 + b*x**5/5 + c*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2 \left(a + bx^2 + cx^4\right) \, dx = \frac{1}{7} cx^7 + \frac{1}{5} bx^5 + \frac{1}{3} ax^3$$

input
```
integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2 \left(a + bx^2 + cx^4\right) \, dx = \frac{1}{7} cx^7 + \frac{1}{5} bx^5 + \frac{1}{3} ax^3$$

input
```
integrate(x^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2\left(a+bx^2+cx^4\right)\,dx=\frac{c\,x^7}{7}+\frac{b\,x^5}{5}+\frac{a\,x^3}{3}$$

input
```
int(x^2*(a + b*x^2 + c*x^4),x)
```

output
```
(a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x^2\left(a+bx^2+cx^4\right)\,dx=\frac{x^3(15c\,x^4+21b\,x^2+35a)}{105}$$

input
```
int(x^2*(c*x^4+b*x^2+a),x)
```

output
```
(x**3*(35*a + 21*b*x**2 + 15*c*x**4))/105
```

3.730 $\int x(a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int x(a+bx^2+cx^4)\,dx=\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

output
```
1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x(a+bx^2+cx^4)\,dx=\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

input
```
Integrate[x*(a + b*x^2 + c*x^4),x]
```

output
```
(a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1690, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2+cx^4)\,dx$$

$$\Big\downarrow \text{ 1690}$$

$$\frac{1}{2}\int (cx^4+bx^2+a)\,dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(ax^2+\frac{bx^4}{2}+\frac{cx^6}{3}\right)$$

input
```
Int[x*(a + b*x^2 + c*x^4),x]
```

output
```
(a*x^2 + (b*x^4)/2 + (c*x^6)/3)/2
```

Defintions of rubi rules used

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4+\frac{1}{6}c\,x^6$	20
default	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4+\frac{1}{6}c\,x^6$	20
norman	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4+\frac{1}{6}c\,x^6$	20
risch	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4+\frac{1}{6}c\,x^6$	20
parallelrisch	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4+\frac{1}{6}c\,x^6$	20
orering	$\frac{x^2(2c\,x^4+3b\,x^2+6a)}{12}$	22

input
```
int(x*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(a+bx^2+cx^4)\,dx=\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(a+bx^2+cx^4)\,dx=\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

input
```
integrate(x*(c*x**4+b*x**2+a),x)
```

output
```
a*x**2/2 + b*x**4/4 + c*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(a+bx^2+cx^4)\,dx=\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(a+bx^2+cx^4)\,dx=\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(a+bx^2+cx^4)\,dx=\frac{c\,x^6}{6}+\frac{b\,x^4}{4}+\frac{a\,x^2}{2}$$

input
```
int(x*(a + b*x^2 + c*x^4),x)
```

output
```
(a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int x(a+bx^2+cx^4)\,dx=\frac{x^2(2c\,x^4+3b\,x^2+6a)}{12}$$

input
```
int(x*(c*x^4+b*x^2+a),x)
```

output
```
(x**2*(6*a + 3*b*x**2 + 2*c*x**4))/12
```

3.731 $\int (a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int (a+bx^2+cx^4)\,dx = ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

output
```
a*x+1/3*b*x^3+1/5*c*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+bx^2+cx^4)\,dx = ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

input
```
Integrate[a + b*x^2 + c*x^4,x]
```

output
```
a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2+cx^4)\, dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

input
```
Int[a + b*x^2 + c*x^4,x]
```

output
```
a*x + (b*x^3)/3 + (c*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$ax + \frac{1}{3}b\,x^{3} + \frac{1}{5}c\,x^{5}$	17
default	$ax + \frac{1}{3}b\,x^{3} + \frac{1}{5}c\,x^{5}$	17
norman	$ax + \frac{1}{3}b\,x^{3} + \frac{1}{5}c\,x^{5}$	17
risch	$ax + \frac{1}{3}b\,x^{3} + \frac{1}{5}c\,x^{5}$	17
parallelrisch	$ax + \frac{1}{3}b\,x^{3} + \frac{1}{5}c\,x^{5}$	17
parts	$ax + \frac{1}{3}b\,x^{3} + \frac{1}{5}c\,x^{5}$	17
orering	$\frac{x\left(3c\,x^{4}+5b\,x^{2}+15a\right)}{15}$	20

input
```
int(c*x^4+b*x^2+a,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/3*b*x^3+1/5*c*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a + bx^{2} + cx^{4}\right)\,dx = \frac{1}{5}\,cx^{5} + \frac{1}{3}\,bx^{3} + ax$$

input
```
integrate(c*x^4+b*x^2+a,x, algorithm="fricas")
```

output
```
1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int (a+bx^2+cx^4)\ dx = ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

input
```
integrate(c*x**4+b*x**2+a,x)
```

output
```
a*x + b*x**3/3 + c*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (a+bx^2+cx^4)\ dx = \frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

input
```
integrate(c*x^4+b*x^2+a,x, algorithm="maxima")
```

output
```
1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (a+bx^2+cx^4)\ dx = \frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

input
```
integrate(c*x^4+b*x^2+a,x, algorithm="giac")
```

output
```
1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2+cx^4\right)\,dx=\frac{c\,x^5}{5}+\frac{b\,x^3}{3}+a\,x$$

input
```
int(a + b*x^2 + c*x^4,x)
```

output
```
a*x + (b*x^3)/3 + (c*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(a+bx^2+cx^4\right)\,dx=\frac{x(3c\,x^4+5b\,x^2+15a)}{15}$$

input
```
int(c*x^4+b*x^2+a,x)
```

output
```
(x*(15*a + 5*b*x**2 + 3*c*x**4))/15
```

3.732 $\int \frac{a+bx^2+cx^4}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{a+bx^2+cx^4}{x} \, dx = \frac{bx^2}{2} + \frac{cx^4}{4} + a \log(x)$$

output
```
1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x} \, dx = \frac{bx^2}{2} + \frac{cx^4}{4} + a \log(x)$$

input
```
Integrate[(a + b*x^2 + c*x^4)/x,x]
```

output
```
(b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x} \, dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a}{x}+bx+cx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

input
```
Int[(a + b*x^2 + c*x^4)/x,x]
```

output
```
(b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{b\,x^{2}}{2}+\frac{c\,x^{4}}{4}+a\ln\left(x\right)$	18
norman	$\frac{b\,x^{2}}{2}+\frac{c\,x^{4}}{4}+a\ln\left(x\right)$	18
parallelrisch	$\frac{b\,x^{2}}{2}+\frac{c\,x^{4}}{4}+a\ln\left(x\right)$	18
risch	$\frac{c\,x^{4}}{4}+\frac{b\,x^{2}}{2}+\frac{b^{2}}{4c}+a\ln\left(x\right)$	26

input
```
int((c*x^4+b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x}\,dx=\frac{1}{4}\,cx^{4}+\frac{1}{2}\,bx^{2}+a\log\left(x\right)$$

input
```
integrate((c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

output
```
1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x}\,dx=a\log\left(x\right)+\frac{bx^{2}}{2}+\frac{cx^{4}}{4}$$

input
```
integrate((c*x**4+b*x**2+a)/x,x)
```

output
```
a*log(x) + b*x**2/2 + c*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{x}\,dx = \frac{1}{4}\,cx^4 + \frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

input
```
integrate((c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

output
```
1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{x}\,dx = \frac{1}{4}\,cx^4 + \frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

input
```
integrate((c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

output
```
1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x}\,dx = \frac{b\,x^2}{2} + \frac{c\,x^4}{4} + a\,\ln\left(x\right)$$

input
```
int((a + b*x^2 + c*x^4)/x,x)
```

output
```
(b*x^2)/2 + (c*x^4)/4 + a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x}\,dx = \log(x)\,a + \frac{b\,x^2}{2} + \frac{c\,x^4}{4}$$

input
```
int((c*x^4+b*x^2+a)/x,x)
```

output
```
(4*log(x)*a + 2*b*x**2 + c*x**4)/4
```

3.733 $\int \frac{a+bx^2+cx^4}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{a+bx^2+cx^4}{x^2}\,dx = -\frac{a}{x}+bx+\frac{cx^3}{3}$$

output

```
-a/x+b*x+1/3*c*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^2}\,dx = -\frac{a}{x}+bx+\frac{cx^3}{3}$$

input

```
Integrate[(a + b*x^2 + c*x^4)/x^2,x]
```

output

```
-(a/x) + b*x + (c*x^3)/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^2}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a}{x^2}+b+cx^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

input
```
Int[(a + b*x^2 + c*x^4)/x^2,x]
```

output
```
-(a/x) + b*x + (c*x^3)/3
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a}{x}+bx+\frac{c\,x^{3}}{3}$	17
risch	$-\frac{a}{x}+bx+\frac{c\,x^{3}}{3}$	17
norman	$\frac{\frac{1}{3}c\,x^{4}+b\,x^{2}-a}{x}$	20
parallelrisch	$\frac{c\,x^{4}+3b\,x^{2}-3a}{3x}$	21
gosper	$-\frac{-c\,x^{4}-3b\,x^{2}+3a}{3x}$	22
orering	$-\frac{-c\,x^{4}-3b\,x^{2}+3a}{3x}$	22

input
```
int((c*x^4+b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*x+1/3*c*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+bx^2+cx^4}{x^2}\,dx=\frac{cx^4+3\,bx^2-3\,a}{3\,x}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

output
```
1/3*(c*x^4 + 3*b*x^2 - 3*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{a+bx^2+cx^4}{x^2}\,dx = -\frac{a}{x}+bx+\frac{cx^3}{3}$$

input
```
integrate((c*x**4+b*x**2+a)/x**2,x)
```

output
```
-a/x + b*x + c*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2+cx^4}{x^2}\,dx = \frac{1}{3}\,cx^3+bx-\frac{a}{x}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

output
```
1/3*c*x^3 + b*x - a/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2+cx^4}{x^2}\,dx = \frac{1}{3}\,cx^3+bx-\frac{a}{x}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

output
```
1/3*c*x^3 + b*x - a/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a + bx^2 + cx^4}{x^2} \, dx = b\,x - \frac{a}{x} + \frac{c\,x^3}{3}$$

input
```
int((a + b*x^2 + c*x^4)/x^2,x)
```

output
```
b*x - a/x + (c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + bx^2 + cx^4}{x^2} \, dx = \frac{c\,x^4 + 3b\,x^2 - 3a}{3x}$$

input
```
int((c*x^4+b*x^2+a)/x^2,x)
```

output
```
( - 3*a + 3*b*x**2 + c*x**4)/(3*x)
```

3.734 $\int \frac{a+bx^2+cx^4}{x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{a+bx^2+cx^4}{x^3} \, dx = -\frac{a}{2x^2} + \frac{cx^2}{2} + b \log(x)$$

output
```
-1/2*a/x^2+1/2*c*x^2+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^3} \, dx = -\frac{a}{2x^2} + \frac{cx^2}{2} + b \log(x)$$

input
```
Integrate[(a + b*x^2 + c*x^4)/x^3,x]
```

output
```
-1/2*a/x^2 + (c*x^2)/2 + b*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^3}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a}{x^3}+\frac{b}{x}+cx\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}$$

input
```
Int[(a + b*x^2 + c*x^4)/x^3,x]
```

output
```
-1/2*a/x^2 + (c*x^2)/2 + b*Log[x]
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a}{2x^{2}}+\frac{c\,x^{2}}{2}+b\ln\left(x\right)$	18
risch	$-\frac{a}{2x^{2}}+\frac{c\,x^{2}}{2}+b\ln\left(x\right)$	18
norman	$\frac{-\frac{a}{2}+\frac{c\,x^{4}}{2}}{x^{2}}+b\ln\left(x\right)$	20
parallelrisch	$\frac{c\,x^{4}+2\ln\left(x\right)x^{2}b-a}{2x^{2}}$	23

input
```
int((c*x^4+b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+1/2*c*x^2+b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{a+bx^2+cx^4}{x^3}\,dx=\frac{cx^4+2\,bx^2\log\left(x\right)-a}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

output
```
1/2*(c*x^4 + 2*b*x^2*log(x) - a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x^3}\,dx=-\frac{a}{2x^2}+b\log\left(x\right)+\frac{cx^2}{2}$$

input
```
integrate((c*x**4+b*x**2+a)/x**3,x)
```

output
```
-a/(2*x**2) + b*log(x) + c*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{x^3}\,dx=\frac{1}{2}\,cx^2+\frac{1}{2}\,b\log\left(x^2\right)-\frac{a}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

output
```
1/2*c*x^2 + 1/2*b*log(x^2) - 1/2*a/x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{a+bx^2+cx^4}{x^3}\,dx=\frac{1}{2}\,cx^2+\frac{1}{2}\,b\log\left(x^2\right)-\frac{bx^2+a}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

output
```
1/2*c*x^2 + 1/2*b*log(x^2) - 1/2*(b*x^2 + a)/x^2
```

Mupad [B] (verification not implemented)

Time = 17.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x^3}\,dx=\frac{c\,x^2}{2}-\frac{a}{2\,x^2}+b\,\ln\left(x\right)$$

input
```
int((a + b*x^2 + c*x^4)/x^3,x)
```

output
```
(c*x^2)/2 - a/(2*x^2) + b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{a+bx^2+cx^4}{x^3}\,dx=\frac{2\log(x)\,b\,x^2-a+c\,x^4}{2x^2}$$

input
```
int((c*x^4+b*x^2+a)/x^3,x)
```

output
```
(2*log(x)*b*x**2 - a + c*x**4)/(2*x**2)
```

3.735 $\int \frac{a+bx^2+cx^4}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{a+bx^2+cx^4}{x^4}\,dx=-\frac{a}{3x^3}-\frac{b}{x}+cx$$

output
```
-1/3*a/x^3-b/x+c*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^4}\,dx=-\frac{a}{3x^3}-\frac{b}{x}+cx$$

input
```
Integrate[(a + b*x^2 + c*x^4)/x^4,x]
```

output
```
-1/3*a/x^3 - b/x + c*x
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^4}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a}{x^4}+\frac{b}{x^2}+c\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{3x^3}-\frac{b}{x}+cx$$

input
```
Int[(a + b*x^2 + c*x^4)/x^4,x]
```

output
```
-1/3*a/x^3 - b/x + c*x
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a}{3x^3}-\frac{b}{x}+cx$	17
risch	$cx+\frac{-b\,x^2-\frac{a}{3}}{x^3}$	19
gosper	$-\frac{-3c\,x^4+3b\,x^2+a}{3x^3}$	20
norman	$\frac{c\,x^4-b\,x^2-\frac{1}{3}a}{x^3}$	20
orering	$-\frac{-3c\,x^4+3b\,x^2+a}{3x^3}$	20
parallelrisch	$\frac{3c\,x^4-3b\,x^2-a}{3x^3}$	22

input
```
int((c*x^4+b*x^2+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3-b/x+c*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2+cx^4}{x^4}\,dx=\frac{3\,cx^4-3\,bx^2-a}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*c*x^4 - 3*b*x^2 - a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+bx^2+cx^4}{x^4}\,dx=cx+\frac{-a-3bx^2}{3x^3}$$

input
```
integrate((c*x**4+b*x**2+a)/x**4,x)
```

output
```
c*x + (-a - 3*b*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+bx^2+cx^4}{x^4}\,dx=cx-\frac{3\,bx^2+a}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="maxima")
```

output
```
c*x - 1/3*(3*b*x^2 + a)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+bx^2+cx^4}{x^4}\,dx=cx-\frac{3\,bx^2+a}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4,x, algorithm="giac")
```

output
```
c*x - 1/3*(3*b*x^2 + a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^4}\,dx = c\,x - \frac{b\,x^2+\frac{a}{3}}{x^3}$$

input
```
int((a + b*x^2 + c*x^4)/x^4,x)
```

output
```
c*x - (a/3 + b*x^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2+cx^4}{x^4}\,dx = \frac{3c\,x^4-3b\,x^2-a}{3x^3}$$

input
```
int((c*x^4+b*x^2+a)/x^4,x)
```

output
```
( - a - 3*b*x**2 + 3*c*x**4)/(3*x**3)
```

3.736 $\int \frac{a+bx^2+cx^4}{x^5} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{a+bx^2+cx^4}{x^5} \, dx = -\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

output
```
-1/4*a/x^4-1/2*b/x^2+c*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^5} \, dx = -\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

input
```
Integrate[(a + b*x^2 + c*x^4)/x^5,x]
```

output
```
-1/4*a/x^4 - b/(2*x^2) + c*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^5}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a}{x^5}+\frac{b}{x^3}+\frac{c}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{4x^4}-\frac{b}{2x^2}+c\log(x)$$

input
```
Int[(a + b*x^2 + c*x^4)/x^5,x]
```

output
```
-1/4*a/x^4 - b/(2*x^2) + c*Log[x]
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a}{4x^{4}}-\frac{b}{2x^{2}}+c\ln\left(x\right)$	18
norman	$\frac{-\frac{b\,x^{2}}{2}-\frac{a}{4}}{x^{4}}+c\ln\left(x\right)$	20
risch	$\frac{-\frac{b\,x^{2}}{2}-\frac{a}{4}}{x^{4}}+c\ln\left(x\right)$	20
parallelrisch	$\frac{4c\ln\left(x\right)x^{4}-2b\,x^{2}-a}{4x^{4}}$	24

input
```
int((c*x^4+b*x^2+a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/x^4-1/2*b/x^2+c*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+bx^2+cx^4}{x^5}\,dx=\frac{4\,cx^4\log\left(x\right)-2\,bx^2-a}{4\,x^4}$$

input
```
integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="fricas")
```

output
```
1/4*(4*c*x^4*log(x) - 2*b*x^2 - a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+bx^2+cx^4}{x^5}\,dx=c\log\left(x\right)+\frac{-a-2bx^2}{4x^4}$$

input
```
integrate((c*x**4+b*x**2+a)/x**5,x)
```

output
```
c*log(x) + (-a - 2*b*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^5}\,dx = \frac{1}{2}\,c\log\left(x^2\right) - \frac{2\,bx^2+a}{4\,x^4}$$

input
```
integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="maxima")
```

output
```
1/2*c*log(x^2) - 1/4*(2*b*x^2 + a)/x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{a+bx^2+cx^4}{x^5}\,dx = \frac{1}{2}\,c\log\left(x^2\right) - \frac{3\,cx^4+2\,bx^2+a}{4\,x^4}$$

input
```
integrate((c*x^4+b*x^2+a)/x^5,x, algorithm="giac")
```

output
```
1/2*c*log(x^2) - 1/4*(3*c*x^4 + 2*b*x^2 + a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{x^5}\,dx = c\,\ln\left(x\right) - \frac{\frac{b\,x^2}{2}+\frac{a}{4}}{x^4}$$

input
```
int((a + b*x^2 + c*x^4)/x^5,x)
```

output
```
c*log(x) - (a/4 + (b*x^2)/2)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+bx^2+cx^4}{x^5}\,dx=\frac{4\log(x)\,c\,x^4-a-2b\,x^2}{4x^4}$$

input
```
int((c*x^4+b*x^2+a)/x^5,x)
```

output
```
(4*log(x)*c*x**4 - a - 2*b*x**2)/(4*x**4)
```

3.737 $\int \frac{a+bx^2+cx^4}{x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{a+bx^2+cx^4}{x^6} \, dx = -\frac{a}{5x^5} - \frac{b}{3x^3} - \frac{c}{x}$$

output
```
-1/5*a/x^5-1/3*b/x^3-c/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^6} \, dx = -\frac{a}{5x^5} - \frac{b}{3x^3} - \frac{c}{x}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/x^6,x]
```

output
```
-1/5*a/x^5 - b/(3*x^3) - c/x
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^6} \, dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a}{x^6}+\frac{b}{x^4}+\frac{c}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

input
```
Int[(a + b*x^2 + c*x^4)/x^6,x]
```

output
```
-1/5*a/x^5 - b/(3*x^3) - c/x
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$	20
norman	$\frac{-c\,x^4-\frac{1}{3}b\,x^2-\frac{1}{5}a}{x^5}$	21
risch	$\frac{-c\,x^4-\frac{1}{3}b\,x^2-\frac{1}{5}a}{x^5}$	21
gosper	$-\frac{15c\,x^4+5b\,x^2+3a}{15x^5}$	22
parallelrisch	$\frac{-15c\,x^4-5b\,x^2-3a}{15x^5}$	22
orering	$-\frac{15c\,x^4+5b\,x^2+3a}{15x^5}$	22

input
```
int((c*x^4+b*x^2+a)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5-1/3*b/x^3-c/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{x^6}\,dx=-\frac{15\,cx^4+5\,bx^2+3\,a}{15\,x^5}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="fricas")
```

output
```
-1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2+cx^4}{x^6}\,dx=\frac{-3a-5bx^2-15cx^4}{15x^5}$$

input
```
integrate((c*x**4+b*x**2+a)/x**6,x)
```

output
```
(-3*a - 5*b*x**2 - 15*c*x**4)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{x^6}\,dx=-\frac{15\,cx^4+5\,bx^2+3\,a}{15\,x^5}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="maxima")
```

output
```
-1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{x^6}\,dx=-\frac{15\,cx^4+5\,bx^2+3\,a}{15\,x^5}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6,x, algorithm="giac")
```

output
```
-1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^2+cx^4}{x^6}\,dx = -\frac{c\,x^4+\frac{b\,x^2}{3}+\frac{a}{5}}{x^5}$$

input
```
int((a + b*x^2 + c*x^4)/x^6,x)
```

output
```
-(a/5 + (b*x^2)/3 + c*x^4)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{x^6}\,dx = \frac{-15c\,x^4-5b\,x^2-3a}{15x^5}$$

input
```
int((c*x^4+b*x^2+a)/x^6,x)
```

output
```
( - 3*a - 5*b*x**2 - 15*c*x**4)/(15*x**5)
```

3.738 $\int \frac{a+bx^2+cx^4}{x^7} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{a+bx^2+cx^4}{x^7} \, dx = -\frac{a}{6x^6} - \frac{b}{4x^4} - \frac{c}{2x^2}$$

output
```
-1/6*a/x^6-1/4*b/x^4-1/2*c/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^7} \, dx = -\frac{a}{6x^6} - \frac{b}{4x^4} - \frac{c}{2x^2}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/x^7,x]
```

output
```
-1/6*a/x^6 - b/(4*x^4) - c/(2*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^7}\,dx$$

$$\Big\downarrow \; 1433$$

$$\int \left(\frac{a}{x^7}+\frac{b}{x^5}+\frac{c}{x^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

input
```
Int[(a + b*x^2 + c*x^4)/x^7,x]
```

output
```
-1/6*a/x^6 - b/(4*x^4) - c/(2*x^2)
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$	20
norman	$\frac{-\frac{1}{2}c\,x^4-\frac{1}{4}b\,x^2-\frac{1}{6}a}{x^6}$	21
risch	$\frac{-\frac{1}{2}c\,x^4-\frac{1}{4}b\,x^2-\frac{1}{6}a}{x^6}$	21
gosper	$-\frac{6c\,x^4+3b\,x^2+2a}{12x^6}$	22
parallelrisch	$\frac{-6c\,x^4-3b\,x^2-2a}{12x^6}$	22
orering	$-\frac{6c\,x^4+3b\,x^2+2a}{12x^6}$	22

input
```
int((c*x^4+b*x^2+a)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6-1/4*b/x^4-1/2*c/x^2
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{x^7}\,dx=-\frac{6\,cx^4+3\,bx^2+2\,a}{12\,x^6}$$

input
```
integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="fricas")
```

output
```
-1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2+cx^4}{x^7}\,dx=\frac{-2a-3bx^2-6cx^4}{12x^6}$$

input
```
integrate((c*x**4+b*x**2+a)/x**7,x)
```

output
```
(-2*a - 3*b*x**2 - 6*c*x**4)/(12*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{x^7}\,dx=-\frac{6\,cx^4+3\,bx^2+2\,a}{12\,x^6}$$

input
```
integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="maxima")
```

output
```
-1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{x^7}\,dx=-\frac{6\,cx^4+3\,bx^2+2\,a}{12\,x^6}$$

input
```
integrate((c*x^4+b*x^2+a)/x^7,x, algorithm="giac")
```

output
```
-1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{x^7}\,dx = -\frac{\frac{c\,x^4}{2}+\frac{b\,x^2}{4}+\frac{a}{6}}{x^6}$$

input
```
int((a + b*x^2 + c*x^4)/x^7,x)
```

output
```
-(a/6 + (b*x^2)/4 + (c*x^4)/2)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{x^7}\,dx = \frac{-6c\,x^4-3b\,x^2-2a}{12x^6}$$

input
```
int((c*x^4+b*x^2+a)/x^7,x)
```

output
```
( - 2*a - 3*b*x**2 - 6*c*x**4)/(12*x**6)
```

3.739 $\int \frac{a+bx^2+cx^4}{x^8} dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{a+bx^2+cx^4}{x^8} dx = -\frac{a}{7x^7} - \frac{b}{5x^5} - \frac{c}{3x^3}$$

output
```
-1/7*a/x^7-1/5*b/x^5-1/3*c/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^8} dx = -\frac{a}{7x^7} - \frac{b}{5x^5} - \frac{c}{3x^3}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/x^8,x]
```

output
```
-1/7*a/x^7 - b/(5*x^5) - c/(3*x^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^8}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a}{x^8}+\frac{b}{x^6}+\frac{c}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

input
```
Int[(a + b*x^2 + c*x^4)/x^8,x]
```

output
```
-1/7*a/x^7 - b/(5*x^5) - c/(3*x^3)
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{a}{7x^{7}}-\frac{b}{5x^{5}}-\frac{c}{3x^{3}}$	20
norman	$\frac{-\frac{1}{3}c\,x^{4}-\frac{1}{5}b\,x^{2}-\frac{1}{7}a}{x^{7}}$	21
risch	$\frac{-\frac{1}{3}c\,x^{4}-\frac{1}{5}b\,x^{2}-\frac{1}{7}a}{x^{7}}$	21
gosper	$-\frac{35c\,x^{4}+21b\,x^{2}+15a}{105x^{7}}$	22
parallelrisch	$\frac{-35c\,x^{4}-21b\,x^{2}-15a}{105x^{7}}$	22
orering	$-\frac{35c\,x^{4}+21b\,x^{2}+15a}{105x^{7}}$	22

input
```
int((c*x^4+b*x^2+a)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a/x^7-1/5*b/x^5-1/3*c/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{x^8}\,dx=-\frac{35\,cx^4+21\,bx^2+15\,a}{105\,x^7}$$

input
```
integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="fricas")
```

output
```
-1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2+cx^4}{x^8}\,dx=\frac{-15a-21bx^2-35cx^4}{105x^7}$$

input
```
integrate((c*x**4+b*x**2+a)/x**8,x)
```

output
```
(-15*a - 21*b*x**2 - 35*c*x**4)/(105*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{x^8}\,dx=-\frac{35\,cx^4+21\,bx^2+15\,a}{105\,x^7}$$

input
```
integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="maxima")
```

output
```
-1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{x^8}\,dx=-\frac{35\,cx^4+21\,bx^2+15\,a}{105\,x^7}$$

input
```
integrate((c*x^4+b*x^2+a)/x^8,x, algorithm="giac")
```

output
```
-1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{x^8}\,dx = -\frac{\frac{c\,x^4}{3}+\frac{b\,x^2}{5}+\frac{a}{7}}{x^7}$$

input
```
int((a + b*x^2 + c*x^4)/x^8,x)
```

output
```
-(a/7 + (b*x^2)/5 + (c*x^4)/3)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a+bx^2+cx^4}{x^8}\,dx = \frac{-35c\,x^4-21b\,x^2-15a}{105x^7}$$

input
```
int((c*x^4+b*x^2+a)/x^8,x)
```

output
```
( - 15*a - 21*b*x**2 - 35*c*x**4)/(105*x**7)
```

3.740 $\int x^2(a+bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int x^2\left(a+bx^2+cx^4\right)^2\,dx=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

output
```
1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2+cx^4\right)^2\,dx=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

input
```
Integrate[x^2*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2
*x^11)/11
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2+cx^4\right)^2 \, dx$$

$$\Big\downarrow \text{ 1433}$$

$$\int \left(a^2x^2+x^6\left(2ac+b^2\right)+2abx^4+2bcx^8+c^2x^{10}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

input
```
Int[x^2*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2
*x^11)/11
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2 x^3}{3}+\frac{2 a b \,x^5}{5}+\frac{\left(2 a c +b^2\right) x^7}{7}+\frac{2 b c \,x^9}{9}+\frac{c^2 x^{11}}{11}$	45
norman	$\frac{c^2 x^{11}}{11}+\frac{2 b c \,x^9}{9}+\left(\frac{2 a c}{7}+\frac{b^2}{7}\right) x^7+\frac{2 a b \,x^5}{5}+\frac{a^2 x^3}{3}$	46
gosper	$\frac{1}{11} c^2 x^{11}+\frac{2}{9} b c \,x^9+\frac{2}{7} x^7 a c +\frac{1}{7} b^2 x^7+\frac{2}{5} a b \,x^5+\frac{1}{3} a^2 x^3$	47
risch	$\frac{1}{11} c^2 x^{11}+\frac{2}{9} b c \,x^9+\frac{2}{7} x^7 a c +\frac{1}{7} b^2 x^7+\frac{2}{5} a b \,x^5+\frac{1}{3} a^2 x^3$	47
parallelrisch	$\frac{1}{11} c^2 x^{11}+\frac{2}{9} b c \,x^9+\frac{2}{7} x^7 a c +\frac{1}{7} b^2 x^7+\frac{2}{5} a b \,x^5+\frac{1}{3} a^2 x^3$	47
orering	$\frac{x^3 \left(315 c^2 x^8+770 b c \,x^6+990 a c \,x^4+495 b^2 x^4+1386 a b \,x^2+1155 a^2\right)}{3465}$	49

input
```
int(x^2*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2 \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{11} \, c^{2} x^{11} + \frac{2}{9} \, b c x^{9} + \frac{1}{7} \, {\left(b^{2} + 2 \, a c\right)} x^{7} + \frac{2}{5} \, a b x^{5} + \frac{1}{3} \, a^{2} x^{3}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*(b^2 + 2*a*c)*x^7 + 2/5*a*b*x^5 + 1/3*a^
2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int x^2 \left(a+bx^2+cx^4\right)^2 \, dx = \frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{2bcx^9}{9} + \frac{c^2x^{11}}{11} + x^7 \cdot \left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*(2*a*c/7
+ b**2/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2 \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{11}\,c^2x^{11} + \frac{2}{9}\,bcx^9 + \frac{1}{7}\,\left(b^2+2\,ac\right)x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*(b^2 + 2*a*c)*x^7 + 2/5*a*b*x^5 + 1/3*a^
2*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x^2 \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{11}\,c^2x^{11} + \frac{2}{9}\,bcx^9 + \frac{1}{7}\,b^2x^7 + \frac{2}{7}\,acx^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b*x^5 + 1/
3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x^2 (a + bx^2 + cx^4)^2 \, dx = x^7 \left(\frac{b^2}{7} + \frac{2\,a\,c}{7}\right) + \frac{a^2\,x^3}{3} + \frac{c^2\,x^{11}}{11} + \frac{2\,a\,b\,x^5}{5} + \frac{2\,b\,c\,x^9}{9}$$

input
```
int(x^2*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^7*((2*a*c)/7 + b^2/7) + (a^2*x^3)/3 + (c^2*x^11)/11 + (2*a*b*x^5)/5 + (2
*b*c*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int x^2 (a + bx^2 + cx^4)^2 \, dx = \frac{x^3(315c^2x^8 + 770bc\,x^6 + 990ac\,x^4 + 495b^2x^4 + 1386ab\,x^2 + 1155a^2)}{3465}$$

input
```
int(x^2*(c*x^4+b*x^2+a)^2,x)
```

output
```
(x**3*(1155*a**2 + 1386*a*b*x**2 + 990*a*c*x**4 + 495*b**2*x**4 + 770*b*c*
x**6 + 315*c**2*x**8))/3465
```

3.741 $\int x(a + bx^2 + cx^4)^2 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int x\left(a+bx^2+cx^4\right)^2 \, dx=\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

output
```
1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int x\left(a+bx^2+cx^4\right)^2 \, dx=\frac{1}{60}x^2\left(30a^2+30abx^2+10\left(b^2+2ac\right)x^4+15bcx^6+6c^2x^8\right)$$

input
```
Integrate[x*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6*c^2*x^8)
)/60
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1432, 1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \text{ 1432}$$

$$\frac{1}{2}\int \left(cx^4+bx^2+a\right)^2 dx^2$$

$$\Big\downarrow \text{ 1085}$$

$$\frac{1}{2}\int \left(c^2x^8+2bcx^6+b^2\left(\frac{2ac}{b^2}+1\right)x^4+2abx^2+a^2\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(a^2x^2+\frac{1}{3}x^6\left(2ac+b^2\right)+abx^4+\frac{1}{2}bcx^8+\frac{c^2x^{10}}{5}\right)$$

input
```
Int[x*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*x^2 + a*b*x^4 + ((b^2 + 2*a*c)*x^6)/3 + (b*c*x^8)/2 + (c^2*x^10)/5)/2
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^2}{2}+\frac{ab\,x^4}{2}+\frac{(2ac+b^2)x^6}{6}+\frac{bc\,x^8}{4}+\frac{c^2x^{10}}{10}$	45
norman	$\frac{c^2x^{10}}{10}+\frac{bc\,x^8}{4}+\left(\frac{ac}{3}+\frac{b^2}{6}\right)x^6+\frac{ab\,x^4}{2}+\frac{a^2x^2}{2}$	46
gosper	$\frac{1}{10}c^2x^{10}+\frac{1}{4}bc\,x^8+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	47
risch	$\frac{1}{10}c^2x^{10}+\frac{1}{4}bc\,x^8+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	47
parallelrisch	$\frac{1}{10}c^2x^{10}+\frac{1}{4}bc\,x^8+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	47
orering	$\frac{x^2(6c^2x^8+15bc\,x^6+20ac\,x^4+10b^2x^4+30ab\,x^2+30a^2)}{60}$	49

input
```
int(x*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,(b^2+2\,ac)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^
2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x(a+bx^2+cx^4)^2\,dx=\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}+x^6\left(\frac{ac}{3}+\frac{b^2}{6}\right)$$

input
```
integrate(x*(c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c/3 + b**2
/6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x(a+bx^2+cx^4)^2\,dx=\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,(b^2+2\,ac)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^
2*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x(a+bx^2+cx^4)^2\,dx=\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,b^2x^6+\frac{1}{3}\,acx^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b*x^4 + 1/
2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x(a+bx^2+cx^4)^2\,dx = x^6\left(\frac{b^2}{6}+\frac{a\,c}{3}\right)+\frac{a^2\,x^2}{2}+\frac{c^2\,x^{10}}{10}+\frac{a\,b\,x^4}{2}+\frac{b\,c\,x^8}{4}$$

input
```
int(x*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^6*((a*c)/3 + b^2/6) + (a^2*x^2)/2 + (c^2*x^10)/10 + (a*b*x^4)/2 + (b*c*x
^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int x(a+bx^2+cx^4)^2\,dx = \frac{x^2(6c^2x^8+15bc\,x^6+20ac\,x^4+10b^2x^4+30ab\,x^2+30a^2)}{60}$$

input
```
int(x*(c*x^4+b*x^2+a)^2,x)
```

output
```
(x**2*(30*a**2 + 30*a*b*x**2 + 20*a*c*x**4 + 10*b**2*x**4 + 15*b*c*x**6 +
6*c**2*x**8))/60
```

3.742 $\int (a + bx^2 + cx^4)^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int (a + bx^2 + cx^4)^2 \, dx = a^2x + \frac{2}{3}abx^3 + \frac{1}{5}(b^2 + 2ac) x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

output
```
a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int (a + bx^2 + cx^4)^2 \, dx = a^2x + \frac{2}{3}abx^3 + \frac{1}{5}(b^2 + 2ac) x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/
9
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \text{ 1403}$$

$$\int \left(a^2+b^2x^4\left(\frac{2ac}{b^2}+1\right)+2abx^2+2bcx^6+c^2x^8\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

input
```
Int[(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/
9
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
default	$a^2x+\frac{2ab\,x^3}{3}+\frac{(2ac+b^2)\,x^5}{5}+\frac{2bc\,x^7}{7}+\frac{c^2x^9}{9}$	42
norman	$\frac{c^2x^9}{9}+\frac{2bc\,x^7}{7}+\left(\frac{2ac}{5}+\frac{b^2}{5}\right)x^5+\frac{2ab\,x^3}{3}+a^2x$	43
gosper	$\frac{1}{9}c^2x^9+\frac{2}{7}bc\,x^7+\frac{2}{5}x^5ac+\frac{1}{5}x^5b^2+\frac{2}{3}ab\,x^3+a^2x$	44
risch	$\frac{1}{9}c^2x^9+\frac{2}{7}bc\,x^7+\frac{2}{5}x^5ac+\frac{1}{5}x^5b^2+\frac{2}{3}ab\,x^3+a^2x$	44
parallelrisch	$\frac{1}{9}c^2x^9+\frac{2}{7}bc\,x^7+\frac{2}{5}x^5ac+\frac{1}{5}x^5b^2+\frac{2}{3}ab\,x^3+a^2x$	44
orering	$\frac{x(35c^2x^8+90bc\,x^6+126ac\,x^4+63b^2x^4+210ab\,x^2+315a^2)}{315}$	47

input
```
int((c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,\left(b^2+2\,ac\right)x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \left(a+bx^2+cx^4\right)^2\,dx=a^2x+\frac{2abx^3}{3}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}+x^5\cdot\left(\frac{2ac}{5}+\frac{b^2}{5}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/
5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5+a^2x+\frac{2}{15}\,\left(3\,cx^5+5\,bx^3\right)a$$

input
```
integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 + 5*b*x^3)
*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5+\frac{2}{5}\,acx^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*
x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int (a + bx^2 + cx^4)^2 \, dx = a^2 x + x^5 \left(\frac{b^2}{5} + \frac{2 a c}{5}\right) + \frac{c^2 x^9}{9} + \frac{2 a b x^3}{3} + \frac{2 b c x^7}{7}$$

input
```
int((a + b*x^2 + c*x^4)^2,x)
```

output
```
a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7
)/7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int (a + bx^2 + cx^4)^2 \, dx = \frac{x(35c^2x^8 + 90bc\, x^6 + 126ac\, x^4 + 63b^2x^4 + 210ab\, x^2 + 315a^2)}{315}$$

input
```
int((c*x^4+b*x^2+a)^2,x)
```

output
```
(x*(315*a**2 + 210*a*b*x**2 + 126*a*c*x**4 + 63*b**2*x**4 + 90*b*c*x**6 +
35*c**2*x**8))/315
```

3.743 $\int \frac{(a+bx^2+cx^4)^2}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{(a+bx^2+cx^4)^2}{x} dx = abx^2 + \frac{1}{4}(b^2+2ac)\,x^4 + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8} + a^2\log(x)$$

output
```
a*b*x^2+1/4*(2*a*c+b^2)*x^4+1/3*b*c*x^6+1/8*c^2*x^8+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^2}{x} dx = abx^2 + \frac{1}{4}(b^2+2ac)\,x^4 + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8} + a^2\log(x)$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x,x]
```

output
```
a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^2}{x^2}dx^2$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{2}\int \left(c^2x^6+2bcx^4+(b^2+2ac)\,x^2+2ab+\frac{a^2}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^2\log(x^2)+\frac{1}{2}x^4(2ac+b^2)+2abx^2+\frac{2}{3}bcx^6+\frac{c^2x^8}{4}\right)$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x,x]
```

output
```
(2*a*b*x^2 + ((b^2 + 2*a*c)*x^4)/2 + (2*b*c*x^6)/3 + (c^2*x^8)/4 + a^2*Log
[x^2])/2
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result	size
norman	$\left(\frac{ac}{2}+\frac{b^{2}}{4}\right) x^{4}+ab\,x^{2}+\frac{c^{2} x^{8}}{8}+\frac{bc\,x^{6}}{3}+a^{2} \ln \left(x\right)$	43
default	$\frac{c^{2} x^{8}}{8}+\frac{bc\,x^{6}}{3}+\frac{ac\,x^{4}}{2}+\frac{b^{2} x^{4}}{4}+ab\,x^{2}+a^{2} \ln \left(x\right)$	44
parallelrisch	$\frac{c^{2} x^{8}}{8}+\frac{bc\,x^{6}}{3}+\frac{ac\,x^{4}}{2}+\frac{b^{2} x^{4}}{4}+ab\,x^{2}+a^{2} \ln \left(x\right)$	44
risch	$\frac{ac\,x^{4}}{2}+ab\,x^{2}+\frac{a\,b^{2}}{2c}+\frac{bc\,x^{6}}{3}+\frac{b^{2} x^{4}}{4}-\frac{b^{4}}{24c^{2}}+\frac{c^{2} x^{8}}{8}+a^{2} \ln \left(x\right)$	61

input
```
int((c*x^4+b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
(1/2*a*c+1/4*b^2)*x^4+a*b*x^2+1/8*c^2*x^8+1/3*b*c*x^6+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2+cx^4)^2}{x}\,dx = \frac{1}{8}\,c^2x^8 + \frac{1}{3}\,bcx^6 + \frac{1}{4}\,(b^2+2\,ac)x^4 + abx^2 + a^2\log\left(x\right)$$

input
```
integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="fricas")
```

output
```
1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*(b^2 + 2*a*c)*x^4 + a*b*x^2 + a^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2+cx^4)^2}{x}\,dx = a^2\log(x) + abx^2 + \frac{bcx^6}{3} + \frac{c^2x^8}{8} + x^4\left(\frac{ac}{2} + \frac{b^2}{4}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**2/x,x)
```

output
```
a**2*log(x) + a*b*x**2 + b*c*x**6/3 + c**2*x**8/8 + x**4*(a*c/2 + b**2/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2+cx^4)^2}{x}\,dx = \frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}(b^2+2ac)x^4 + abx^2 + \frac{1}{2}a^2\log(x^2)$$

input
```
integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="maxima")
```

output
```
1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*(b^2 + 2*a*c)*x^4 + a*b*x^2 + 1/2*a^2*log(
x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^2}{x}\,dx = \frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}b^2x^4 + \frac{1}{2}acx^4 + abx^2 + \frac{1}{2}a^2\log(x^2)$$

input
```
integrate((c*x^4+b*x^2+a)^2/x,x, algorithm="giac")
```

output
```
1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4 + 1/2*a*c*x^4 + a*b*x^2 + 1/2*a^2*
log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2+cx^4)^2}{x}\,dx = a^2 \ln(x) + x^4 \left(\frac{b^2}{4}+\frac{a\,c}{2}\right) + \frac{c^2\,x^8}{8} + a\,b\,x^2 + \frac{b\,c\,x^6}{3}$$

input
```
int((a + b*x^2 + c*x^4)^2/x,x)
```

output
```
a^2*log(x) + x^4*((a*c)/2 + b^2/4) + (c^2*x^8)/8 + a*b*x^2 + (b*c*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2+cx^4)^2}{x}\,dx = \log(x)\,a^2 + ab\,x^2 + \frac{ac\,x^4}{2} + \frac{b^2x^4}{4} + \frac{bc\,x^6}{3} + \frac{c^2x^8}{8}$$

input
```
int((c*x^4+b*x^2+a)^2/x,x)
```

output
```
(24*log(x)*a**2 + 24*a*b*x**2 + 12*a*c*x**4 + 6*b**2*x**4 + 8*b*c*x**6 + 3
*c**2*x**8)/24
```

3.744 $\int \frac{(a+bx^2+cx^4)^2}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{(a+bx^2+cx^4)^2}{x^2}\,dx = -\frac{a^2}{x}+2abx+\frac{1}{3}(b^2+2ac)\,x^3+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

output
```
-a^2/x+2*a*b*x+1/3*(2*a*c+b^2)*x^3+2/5*b*c*x^5+1/7*c^2*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^2}\,dx = -\frac{a^2}{x}+2abx+\frac{1}{3}(b^2+2ac)\,x^3+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^2,x]
```

output
```
-(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^2}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^2}{x^2}+x^2(2ac+b^2)+2ab+2bcx^4+c^2x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{x}+\frac{1}{3}x^3(2ac+b^2)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^2,x]
```

output
```
-(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{c^2x^7}{7}+\frac{2bc\,x^5}{5}+\frac{2ac\,x^3}{3}+\frac{b^2x^3}{3}+2abx-\frac{a^2}{x}$	45
risch	$\frac{c^2x^7}{7}+\frac{2bc\,x^5}{5}+\frac{2ac\,x^3}{3}+\frac{b^2x^3}{3}+2abx-\frac{a^2}{x}$	45
norman	$\frac{\frac{c^2x^8}{7}+\frac{2bc\,x^6}{5}+\left(\frac{2ac}{3}+\frac{b^2}{3}\right)x^4+2ab\,x^2-a^2}{x}$	47
gosper	$-\frac{-15c^2x^8-42bc\,x^6-70ac\,x^4-35b^2x^4-210ab\,x^2+105a^2}{105x}$	49
parallelrisch	$\frac{15c^2x^8+42bc\,x^6+70ac\,x^4+35b^2x^4+210ab\,x^2-105a^2}{105x}$	49
orering	$-\frac{-15c^2x^8-42bc\,x^6-70ac\,x^4-35b^2x^4-210ab\,x^2+105a^2}{105x}$	49

input
```
int((c*x^4+b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*c^2*x^7+2/5*b*c*x^5+2/3*a*c*x^3+1/3*b^2*x^3+2*a*b*x-a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2+cx^4)^2}{x^2}\,dx=\frac{15\,c^2x^8+42\,bcx^6+35\,(b^2+2\,ac)x^4+210\,abx^2-105\,a^2}{105\,x}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="fricas")
```

output
```
1/105*(15*c^2*x^8 + 42*b*c*x^6 + 35*(b^2 + 2*a*c)*x^4 + 210*a*b*x^2 - 105*
a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2+cx^4)^2}{x^2}\,dx = -\frac{a^2}{x} + 2abx + \frac{2bcx^5}{5} + \frac{c^2x^7}{7} + x^3\cdot\left(\frac{2ac}{3} + \frac{b^2}{3}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**2,x)
```

output
```
-a**2/x + 2*a*b*x + 2*b*c*x**5/5 + c**2*x**7/7 + x**3*(2*a*c/3 + b**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,(b^2+2\,ac)x^3 + 2\,abx - \frac{a^2}{x}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="maxima")
```

output
```
1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*(b^2 + 2*a*c)*x^3 + 2*a*b*x - a^2/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,b^2x^3 + \frac{2}{3}\,acx^3 + 2\,abx - \frac{a^2}{x}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^2,x, algorithm="giac")
```

output
```
1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3 + 2/3*a*c*x^3 + 2*a*b*x - a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^2}\,dx = x^3\left(\frac{b^2}{3}+\frac{2\,a\,c}{3}\right)-\frac{a^2}{x}+\frac{c^2\,x^7}{7}+2\,a\,b\,x+\frac{2\,b\,c\,x^5}{5}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^2,x)
```

output
```
x^3*((2*a*c)/3 + b^2/3) - a^2/x + (c^2*x^7)/7 + 2*a*b*x + (2*b*c*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^2}\,dx = \frac{15c^2x^8+42bc\,x^6+70ac\,x^4+35b^2x^4+210ab\,x^2-105a^2}{105x}$$

input
```
int((c*x^4+b*x^2+a)^2/x^2,x)
```

output
```
( - 105*a**2 + 210*a*b*x**2 + 70*a*c*x**4 + 35*b**2*x**4 + 42*b*c*x**6 + 1
5*c**2*x**8)/(105*x)
```

3.745 $\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx = -\frac{a^2}{2x^2}+\frac{1}{2}\left(b^2+2ac\right)x^2+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}+2ab\log(x)$$

output
```
-1/2*a^2/x^2+1/2*(2*a*c+b^2)*x^2+1/2*b*c*x^4+1/6*c^2*x^6+2*a*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{6}\left(-\frac{3a^2}{x^2}+3\left(b^2+2ac\right)x^2+3bcx^4+c^2x^6+12ab\log(x)\right)$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^3,x]
```

output
```
((-3*a^2)/x^2 + 3*(b^2 + 2*a*c)*x^2 + 3*b*c*x^4 + c^2*x^6 + 12*a*b*Log[x])
/6
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^3} dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^2}{x^4}dx^2$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{2}\int \left(c^2x^4+2bcx^2+b^2\left(\frac{2ac}{b^2}+1\right)+\frac{2ab}{x^2}+\frac{a^2}{x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{x^2}+x^2(2ac+b^2)+2ab\log(x^2)+bcx^4+\frac{c^2x^6}{3}\right)$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^3,x]
```

output
```
(-(a^2/x^2) + (b^2 + 2*a*c)*x^2 + b*c*x^4 + (c^2*x^6)/3 + 2*a*b*Log[x^2])/
2
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{c^2x^6}{6}+\frac{bc\,x^4}{2}+ac\,x^2+\frac{b^2x^2}{2}-\frac{a^2}{2x^2}+2ab\ln\left(x\right)$	45
risch	$\frac{c^2x^6}{6}+\frac{bc\,x^4}{2}+ac\,x^2+\frac{b^2x^2}{2}-\frac{a^2}{2x^2}+2ab\ln\left(x\right)$	45
norman	$\frac{\left(ac+\frac{b^2}{2}\right)x^4-\frac{a^2}{2}+\frac{c^2x^8}{6}+\frac{bc\,x^6}{2}}{x^2}+2ab\ln\left(x\right)$	46
parallelrisch	$\frac{c^2x^8+3bc\,x^6+6ac\,x^4+3b^2x^4+12ab\ln(x)x^2-3a^2}{6x^2}$	50

input
```
int((c*x^4+b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*x^6+1/2*b*c*x^4+a*c*x^2+1/2*b^2*x^2-1/2*a^2/x^2+2*a*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int\frac{(a+bx^2+cx^4)^2}{x^3}\,dx=\frac{c^2x^8+3\,bcx^6+3\,(b^2+2\,ac)x^4+12\,abx^2\log\left(x\right)-3\,a^2}{6\,x^2}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="fricas")
```

output
```
1/6*(c^2*x^8 + 3*b*c*x^6 + 3*(b^2 + 2*a*c)*x^4 + 12*a*b*x^2*log(x) - 3*a^2
)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx = -\frac{a^2}{2x^2} + 2ab\log(x) + \frac{bcx^4}{2} + \frac{c^2x^6}{6} + x^2\left(ac+\frac{b^2}{2}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**3,x)
```

output
```
-a**2/(2*x**2) + 2*a*b*log(x) + b*c*x**4/2 + c**2*x**6/6 + x**2*(a*c + b**
2/2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{6}\,c^2x^6 + \frac{1}{2}\,bcx^4 + \frac{1}{2}\,(b^2+2\,ac)x^2 + ab\log\left(x^2\right) - \frac{a^2}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="maxima")
```

output
```
1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*(b^2 + 2*a*c)*x^2 + a*b*log(x^2) - 1/2*a^2
/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{6}\,c^2x^6 + \frac{1}{2}\,bcx^4 + \frac{1}{2}\,b^2x^2 + acx^2 + ab\log\left(x^2\right) - \frac{2\,abx^2+a^2}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^3,x, algorithm="giac")
```

output
```
1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2 + a*c*x^2 + a*b*log(x^2) - 1/2*(2*
a*b*x^2 + a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx = x^2\left(\frac{b^2}{2}+a\,c\right)-\frac{a^2}{2\,x^2}+\frac{c^2\,x^6}{6}+2\,a\,b\,\ln(x)+\frac{b\,c\,x^4}{2}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^3,x)
```

output
```
x^2*(a*c + b^2/2) - a^2/(2*x^2) + (c^2*x^6)/6 + 2*a*b*log(x) + (b*c*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2+cx^4)^2}{x^3}\,dx = \frac{12\log(x)\,ab\,x^2-3a^2+6ac\,x^4+3b^2x^4+3bc\,x^6+c^2x^8}{6x^2}$$

input
```
int((c*x^4+b*x^2+a)^2/x^3,x)
```

output
```
(12*log(x)*a*b*x**2 - 3*a**2 + 6*a*c*x**4 + 3*b**2*x**4 + 3*b*c*x**6 + c**
2*x**8)/(6*x**2)
```

3.746 $\int \frac{(a+bx^2+cx^4)^2}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{(a+bx^2+cx^4)^2}{x^4}\,dx = -\frac{a^2}{3x^3}-\frac{2ab}{x}+\left(b^2+2ac\right)x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

output
```
-1/3*a^2/x^3-2*a*b/x+(2*a*c+b^2)*x+2/3*b*c*x^3+1/5*c^2*x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^4}\,dx = -\frac{a^2}{3x^3}-\frac{2ab}{x}+\left(b^2+2ac\right)x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^4,x]
```

output
```
-1/3*a^2/x^3 - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^4}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^2}{x^4}+b^2\left(\frac{2ac}{b^2}+1\right)+\frac{2ab}{x^2}+2bcx^2+c^2x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{3x^3}+x(2ac+b^2)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^4,x]
```

output
```
-1/3*a^2/x^3 - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{c^2x^5}{5}+\frac{2bc\,x^3}{3}+2acx+b^2x-\frac{a^2}{3x^3}-\frac{2ab}{x}$	42
risch	$\frac{c^2x^5}{5}+\frac{2bc\,x^3}{3}+2acx+b^2x+\frac{-2ab\,x^2-\frac{1}{3}a^2}{x^3}$	44
norman	$\frac{\frac{c^2x^8}{5}+\frac{2bc\,x^6}{3}+(2ac+b^2)x^4-2ab\,x^2-\frac{a^2}{3}}{x^3}$	45
gosper	$-\frac{-3c^2x^8-10bc\,x^6-30ac\,x^4-15b^2x^4+30ab\,x^2+5a^2}{15x^3}$	49
parallelrisch	$\frac{3c^2x^8+10bc\,x^6+30ac\,x^4+15b^2x^4-30ab\,x^2-5a^2}{15x^3}$	49
orering	$-\frac{-3c^2x^8-10bc\,x^6-30ac\,x^4-15b^2x^4+30ab\,x^2+5a^2}{15x^3}$	49

input
```
int((c*x^4+b*x^2+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*c^2*x^5+2/3*b*c*x^3+2*a*c*x+b^2*x-1/3*a^2/x^3-2*a*b/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^2}{x^4}\,dx=\frac{3\,c^2x^8+10\,bcx^6+15\,(b^2+2\,ac)x^4-30\,abx^2-5\,a^2}{15\,x^3}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="fricas")
```

output
```
1/15*(3*c^2*x^8 + 10*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 - 30*a*b*x^2 - 5*a^2)/
x^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^2}{x^4}\,dx = \frac{2bcx^3}{3} + \frac{c^2x^5}{5} + x(2ac+b^2) + \frac{-a^2-6abx^2}{3x^3}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**4,x)
```

output
```
2*b*c*x**3/3 + c**2*x**5/5 + x*(2*a*c + b**2) + (-a**2 - 6*a*b*x**2)/(3*x*
*3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + (b^2+2\,ac)x - \frac{6\,abx^2+a^2}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="maxima")
```

output
```
1/5*c^2*x^5 + 2/3*b*c*x^3 + (b^2 + 2*a*c)*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x + 2\,acx - \frac{6\,abx^2+a^2}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^4,x, algorithm="giac")
```

output
```
1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x + 2*a*c*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2+cx^4)^2}{x^4}\,dx = x\,(b^2+2\,a\,c)-\frac{\frac{a^2}{3}+2\,b\,a\,x^2}{x^3}+\frac{c^2\,x^5}{5}+\frac{2\,b\,c\,x^3}{3}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^4,x)
```

output
```
x*(2*a*c + b^2) - (a^2/3 + 2*a*b*x^2)/x^3 + (c^2*x^5)/5 + (2*b*c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2+cx^4)^2}{x^4}\,dx = \frac{3c^2x^8+10bc\,x^6+30ac\,x^4+15b^2x^4-30ab\,x^2-5a^2}{15x^3}$$

input
```
int((c*x^4+b*x^2+a)^2/x^4,x)
```

output
```
( - 5*a**2 - 30*a*b*x**2 + 30*a*c*x**4 + 15*b**2*x**4 + 10*b*c*x**6 + 3*c*
*2*x**8)/(15*x**3)
```

3.747 $\int \frac{(a+bx^2+cx^4)^2}{x^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 45

$$\int \frac{(a+bx^2+cx^4)^2}{x^5} dx = -\frac{a^2}{4x^4} - \frac{ab}{x^2} + bcx^2 + \frac{c^2x^4}{4} + (b^2+2ac)\log(x)$$

output
```
-1/4*a^2/x^4-a*b/x^2+b*c*x^2+1/4*c^2*x^4+(2*a*c+b^2)*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2+cx^4)^2}{x^5} dx = \frac{(-a+cx^4)(a+4bx^2+cx^4)}{4x^4} + (b^2+2ac)\log(x)$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^5,x]
```

output
```
((-a + c*x^4)*(a + 4*b*x^2 + c*x^4))/(4*x^4) + (b^2 + 2*a*c)*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^5} dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^2}{x^6}dx^2$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^6}+\frac{2ba}{x^4}+c^2x^2+2bc+\frac{b^2+2ac}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{2x^4}+\log\left(x^2\right)\left(2ac+b^2\right)-\frac{2ab}{x^2}+2bcx^2+\frac{c^2x^4}{2}\right)$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^5,x]
```

output
```
(-1/2*a^2/x^4 - (2*a*b)/x^2 + 2*b*c*x^2 + (c^2*x^4)/2 + (b^2 + 2*a*c)*Log[
x^2])/2
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+bc\,x^2+\frac{c^2x^4}{4}+\left(2ac+b^2\right)\ln\left(x\right)$	42
norman	$\frac{bc\,x^6-\frac{1}{4}a^2+\frac{1}{4}c^2x^8-ab\,x^2}{x^4}+\left(2ac+b^2\right)\ln\left(x\right)$	44
risch	$\frac{c^2x^4}{4}+bc\,x^2+b^2+\frac{-\frac{1}{4}a^2-ab\,x^2}{x^4}+2\ln\left(x\right)ac+b^2\ln\left(x\right)$	48
parallelrisch	$\frac{c^2x^8+4bc\,x^6+8\ln(x)x^4ac+4\ln(x)x^4b^2-4ab\,x^2-a^2}{4x^4}$	52

input
```
int((c*x^4+b*x^2+a)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/x^4-a*b/x^2+b*c*x^2+1/4*c^2*x^4+(2*a*c+b^2)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int\frac{(a+bx^2+cx^4)^2}{x^5}\,dx=\frac{c^2x^8+4\,bcx^6+4\,(b^2+2\,ac)x^4\log\left(x\right)-4\,abx^2-a^2}{4\,x^4}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="fricas")
```

output
```
1/4*(c^2*x^8 + 4*b*c*x^6 + 4*(b^2 + 2*a*c)*x^4*log(x) - 4*a*b*x^2 - a^2)/x
^4
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^2}{x^5}\,dx = bcx^2 + \frac{c^2x^4}{4} + (2ac+b^2)\log(x) + \frac{-a^2-4abx^2}{4x^4}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**5,x)
```

output
```
b*c*x**2 + c**2*x**4/4 + (2*a*c + b**2)*log(x) + (-a**2 - 4*a*b*x**2)/(4*x
**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{4}c^2x^4 + bcx^2 + \frac{1}{2}(b^2+2ac)\log(x^2) - \frac{4abx^2+a^2}{4x^4}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="maxima")
```

output
```
1/4*c^2*x^4 + b*c*x^2 + 1/2*(b^2 + 2*a*c)*log(x^2) - 1/4*(4*a*b*x^2 + a^2)
/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{4}c^2x^4 + bcx^2 + \frac{1}{2}(b^2+2ac)\log(x^2) - \frac{3b^2x^4+6acx^4+4abx^2+a^2}{4x^4}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^5,x, algorithm="giac")
```

output
```
1/4*c^2*x^4 + b*c*x^2 + 1/2*(b^2 + 2*a*c)*log(x^2) - 1/4*(3*b^2*x^4 + 6*a*
c*x^4 + 4*a*b*x^2 + a^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{(a + bx^2 + cx^4)^2}{x^5}\, dx = \ln(x)\ (b^2 + 2\,a\,c) - \frac{\frac{a^2}{4} + b\,a\,x^2}{x^4} + \frac{c^2\,x^4}{4} + b\,c\,x^2$$

input
```
int((a + b*x^2 + c*x^4)^2/x^5,x)
```

output
```
log(x)*(2*a*c + b^2) - (a^2/4 + a*b*x^2)/x^4 + (c^2*x^4)/4 + b*c*x^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{(a + bx^2 + cx^4)^2}{x^5}\, dx = \frac{8\log(x)\,ac\,x^4 + 4\log(x)\,b^2x^4 - a^2 - 4ab\,x^2 + 4bc\,x^6 + c^2x^8}{4x^4}$$

input
```
int((c*x^4+b*x^2+a)^2/x^5,x)
```

output
```
(8*log(x)*a*c*x**4 + 4*log(x)*b**2*x**4 - a**2 - 4*a*b*x**2 + 4*b*c*x**6 +
 c**2*x**8)/(4*x**4)
```

3.748 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^6} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^6} \, dx = -\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2+2ac}{x} + 2bcx + \frac{c^2x^3}{3}$$

output
```
-1/5*a^2/x^5-2/3*a*b/x^3-(2*a*c+b^2)/x+2*b*c*x+1/3*c^2*x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^6} \, dx = -\frac{a^2}{5x^5} - \frac{2ab}{3x^3} + \frac{-b^2-2ac}{x} + 2bcx + \frac{c^2x^3}{3}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^6,x]
```

output
```
-1/5*a^2/x^5 - (2*a*b)/(3*x^3) + (-b^2 - 2*a*c)/x + 2*b*c*x + (c^2*x^3)/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^6}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^2}{x^6}+\frac{2ac+b^2}{x^2}+\frac{2ab}{x^4}+2bc+c^2x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{5x^5}-\frac{2ac+b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^6,x]
```

output
```
-1/5*a^2/x^5 - (2*a*b)/(3*x^3) - (b^2 + 2*a*c)/x + 2*b*c*x + (c^2*x^3)/3
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^{2}}{5x^{5}}-\frac{2ab}{3x^{3}}-\frac{2ac+b^{2}}{x}+2bcx+\frac{c^{2}x^{3}}{3}$	43
risch	$\frac{c^{2}x^{3}}{3}+2bcx+\frac{\left(-2ac-b^{2}\right)x^{4}-\frac{2ab\,x^{2}}{3}-\frac{a^{2}}{5}}{x^{5}}$	46
norman	$\frac{\frac{c^{2}x^{8}}{3}+2bc\,x^{6}+\left(-2ac-b^{2}\right)x^{4}-\frac{2ab\,x^{2}}{3}-\frac{a^{2}}{5}}{x^{5}}$	47
gosper	$-\frac{-5c^{2}x^{8}-30bc\,x^{6}+30ac\,x^{4}+15b^{2}x^{4}+10ab\,x^{2}+3a^{2}}{15x^{5}}$	49
parallelrisch	$\frac{5c^{2}x^{8}+30bc\,x^{6}-30ac\,x^{4}-15b^{2}x^{4}-10ab\,x^{2}-3a^{2}}{15x^{5}}$	49
orering	$-\frac{-5c^{2}x^{8}-30bc\,x^{6}+30ac\,x^{4}+15b^{2}x^{4}+10ab\,x^{2}+3a^{2}}{15x^{5}}$	49

input
```
int((c*x^4+b*x^2+a)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^2/x^5-2/3*a*b/x^3-(2*a*c+b^2)/x+2*b*c*x+1/3*c^2*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2+cx^4)^2}{x^6}\,dx=\frac{5\,c^2x^8+30\,bcx^6-15\,(b^2+2\,ac)x^4-10\,abx^2-3\,a^2}{15\,x^5}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="fricas")
```

output
```
1/15*(5*c^2*x^8 + 30*b*c*x^6 - 15*(b^2 + 2*a*c)*x^4 - 10*a*b*x^2 - 3*a^2)/
x^5
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^6}\,dx = 2bcx + \frac{c^2x^3}{3} + \frac{-3a^2-10abx^2+x^4(-30ac-15b^2)}{15x^5}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**6,x)
```

output
```
2*b*c*x + c**2*x**3/3 + (-3*a**2 - 10*a*b*x**2 + x**4*(-30*a*c - 15*b**2))
/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2+cx^4)^2}{x^6}\,dx = \frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{15\,(b^2+2\,ac)x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="maxima")
```

output
```
1/3*c^2*x^3 + 2*b*c*x - 1/15*(15*(b^2 + 2*a*c)*x^4 + 10*a*b*x^2 + 3*a^2)/x
^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^2}{x^6}\,dx = \frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{15\,b^2x^4+30\,acx^4+10\,abx^2+3\,a^2}{15\,x^5}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^6,x, algorithm="giac")
```

output
```
1/3*c^2*x^3 + 2*b*c*x - 1/15*(15*b^2*x^4 + 30*a*c*x^4 + 10*a*b*x^2 + 3*a^2
)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2+cx^4)^2}{x^6}\,dx=\frac{c^2\,x^3}{3}-\frac{x^4\,(b^2+2\,a\,c)+\frac{a^2}{5}+\frac{2\,a\,b\,x^2}{3}}{x^5}+2\,b\,c\,x$$

input
```
int((a + b*x^2 + c*x^4)^2/x^6,x)
```

output
```
(c^2*x^3)/3 - (x^4*(2*a*c + b^2) + a^2/5 + (2*a*b*x^2)/3)/x^5 + 2*b*c*x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^6}\,dx=\frac{5c^2x^8+30bc\,x^6-30ac\,x^4-15b^2x^4-10ab\,x^2-3a^2}{15x^5}$$

input
```
int((c*x^4+b*x^2+a)^2/x^6,x)
```

output
```
( - 3*a**2 - 10*a*b*x**2 - 30*a*c*x**4 - 15*b**2*x**4 + 30*b*c*x**6 + 5*c*
*2*x**8)/(15*x**5)
```

3.749 $\int \frac{(a+bx^2+cx^4)^2}{x^7} dx$

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int \frac{(a+bx^2+cx^4)^2}{x^7} dx = -\frac{a^2}{6x^6} - \frac{ab}{2x^4} - \frac{b^2+2ac}{2x^2} + \frac{c^2x^2}{2} + 2bc\log(x)$$

output
```
-1/6*a^2/x^6-1/2*a*b/x^4-1/2*(2*a*c+b^2)/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^2}{x^7} dx = -\frac{a^2+3abx^2+3b^2x^4+6acx^4-3c^2x^8-12bcx^6\log(x)}{6x^6}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^7,x]
```

output
```
-1/6*(a^2 + 3*a*b*x^2 + 3*b^2*x^4 + 6*a*c*x^4 - 3*c^2*x^8 - 12*b*c*x^6*Log
[x])/x^6
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^7} dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^2}{x^8}dx^2$$

$$\Big\downarrow \text{ 1140}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^8}+\frac{2ba}{x^6}+c^2+\frac{2bc}{x^2}+\frac{b^2+2ac}{x^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{3x^6}-\frac{2ac+b^2}{x^2}-\frac{ab}{x^4}+2bc\log\left(x^2\right)+c^2x^2\right)$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^7,x]
```

output
```
(-1/3*a^2/x^6 - (a*b)/x^4 - (b^2 + 2*a*c)/x^2 + c^2*x^2 + 2*b*c*Log[x^2])/
2
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}-\frac{2ac+b^2}{2x^2}+\frac{c^2x^2}{2}+2bc\ln\left(x\right)$	44
norman	$\frac{\left(-ac-\frac{b^2}{2}\right)x^4-\frac{a^2}{6}+\frac{c^2x^8}{2}-\frac{ab\,x^2}{2}}{x^6}+2bc\ln\left(x\right)$	47
risch	$\frac{c^2x^2}{2}+\frac{\left(-ac-\frac{b^2}{2}\right)x^4-\frac{ab\,x^2}{2}-\frac{a^2}{6}}{x^6}+2bc\ln\left(x\right)$	47
parallelrisch	$\frac{3c^2x^8+12bc\ln\left(x\right)x^6-6ac\,x^4-3b^2x^4-3ab\,x^2-a^2}{6x^6}$	51

input
```
int((c*x^4+b*x^2+a)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2/x^6-1/2*a*b/x^4-1/2*(2*a*c+b^2)/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int\frac{\left(a+bx^2+cx^4\right)^2}{x^7}\,dx=\frac{3\,c^2x^8+12\,bcx^6\log\left(x\right)-3\,(b^2+2\,ac)x^4-3\,abx^2-a^2}{6\,x^6}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="fricas")
```

output
```
1/6*(3*c^2*x^8 + 12*b*c*x^6*log(x) - 3*(b^2 + 2*a*c)*x^4 - 3*a*b*x^2 - a^2
)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2+cx^4)^2}{x^7}\,dx = 2bc\log(x) + \frac{c^2x^2}{2} + \frac{-a^2-3abx^2+x^4(-6ac-3b^2)}{6x^6}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**7,x)
```

output
```
2*b*c*log(x) + c**2*x**2/2 + (-a**2 - 3*a*b*x**2 + x**4*(-6*a*c - 3*b**2))
/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2+cx^4)^2}{x^7}\,dx = \frac{1}{2}c^2x^2 + bc\log(x^2) - \frac{3(b^2+2ac)x^4+3abx^2+a^2}{6x^6}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="maxima")
```

output
```
1/2*c^2*x^2 + b*c*log(x^2) - 1/6*(3*(b^2 + 2*a*c)*x^4 + 3*a*b*x^2 + a^2)/x
^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2+cx^4)^2}{x^7}\,dx = \frac{1}{2}c^2x^2 + bc\log(x^2) - \frac{11bcx^6+3b^2x^4+6acx^4+3abx^2+a^2}{6x^6}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^7,x, algorithm="giac")
```

output
```
1/2*c^2*x^2 + b*c*log(x^2) - 1/6*(11*b*c*x^6 + 3*b^2*x^4 + 6*a*c*x^4 + 3*a
*b*x^2 + a^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 18.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2+cx^4)^2}{x^7}\,dx = \frac{c^2\,x^2}{2} - \frac{\frac{a^2}{6} + x^4\left(\frac{b^2}{2} + a\,c\right) + \frac{a\,b\,x^2}{2}}{x^6} + 2\,b\,c\,\ln(x)$$

input
```
int((a + b*x^2 + c*x^4)^2/x^7,x)
```

output
```
(c^2*x^2)/2 - (a^2/6 + x^4*(a*c + b^2/2) + (a*b*x^2)/2)/x^6 + 2*b*c*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^2}{x^7}\,dx = \frac{12\log(x)\,bc\,x^6 - a^2 - 3ab\,x^2 - 6ac\,x^4 - 3b^2x^4 + 3c^2x^8}{6x^6}$$

input
```
int((c*x^4+b*x^2+a)^2/x^7,x)
```

output
```
(12*log(x)*b*c*x**6 - a**2 - 3*a*b*x**2 - 6*a*c*x**4 - 3*b**2*x**4 + 3*c**
2*x**8)/(6*x**6)
```

3.750 $\int \frac{(a+bx^2+cx^4)^2}{x^8} dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{(a+bx^2+cx^4)^2}{x^8} dx = -\frac{a^2}{7x^7} - \frac{2ab}{5x^5} - \frac{b^2+2ac}{3x^3} - \frac{2bc}{x} + c^2x$$

output
```
-1/7*a^2/x^7-2/5*a*b/x^5-1/3*(2*a*c+b^2)/x^3-2*b*c/x+c^2*x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2+cx^4)^2}{x^8} dx = -\frac{a^2}{7x^7} - \frac{2ab}{5x^5} + \frac{-b^2-2ac}{3x^3} - \frac{2bc}{x} + c^2x$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^8,x]
```

output
```
-1/7*a^2/x^7 - (2*a*b)/(5*x^5) + (-b^2 - 2*a*c)/(3*x^3) - (2*b*c)/x + c^2*
x
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^8}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^2}{x^8}+\frac{2ac+b^2}{x^4}+\frac{2ab}{x^6}+\frac{2bc}{x^2}+c^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{7x^7}-\frac{2ac+b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^8,x]
```

output
```
-1/7*a^2/x^7 - (2*a*b)/(5*x^5) - (b^2 + 2*a*c)/(3*x^3) - (2*b*c)/x + c^2*x
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{2ac+b^2}{3x^3}-\frac{2bc}{x}+c^2x$	42
risch	$c^2x+\frac{-2bc\,x^6+\left(-\frac{2ac}{3}-\frac{b^2}{3}\right)x^4-\frac{2ab\,x^2}{5}-\frac{a^2}{7}}{x^7}$	45
norman	$\frac{c^2x^8-2bc\,x^6+\left(-\frac{2ac}{3}-\frac{b^2}{3}\right)x^4-\frac{2ab\,x^2}{5}-\frac{a^2}{7}}{x^7}$	46
gosper	$-\frac{-105c^2x^8+210bc\,x^6+70ac\,x^4+35b^2x^4+42ab\,x^2+15a^2}{105x^7}$	49
parallelrisch	$\frac{105c^2x^8-210bc\,x^6-70ac\,x^4-35b^2x^4-42ab\,x^2-15a^2}{105x^7}$	49
orering	$-\frac{-105c^2x^8+210bc\,x^6+70ac\,x^4+35b^2x^4+42ab\,x^2+15a^2}{105x^7}$	49

input
```
int((c*x^4+b*x^2+a)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^2/x^7-2/5*a*b/x^5-1/3*(2*a*c+b^2)/x^3-2/x*b*c+c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^2}{x^8}\,dx=\frac{105\,c^2x^8-210\,bcx^6-35\,(b^2+2\,ac)x^4-42\,abx^2-15\,a^2}{105\,x^7}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="fricas")
```

output
```
1/105*(105*c^2*x^8 - 210*b*c*x^6 - 35*(b^2 + 2*a*c)*x^4 - 42*a*b*x^2 - 15*
a^2)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^2}{x^8}\,dx = c^2x + \frac{-15a^2 - 42abx^2 - 210bcx^6 + x^4(-70ac - 35b^2)}{105x^7}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**8,x)
```

output
```
c**2*x + (-15*a**2 - 42*a*b*x**2 - 210*b*c*x**6 + x**4*(-70*a*c - 35*b**2)
)/(105*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2+cx^4)^2}{x^8}\,dx = c^2x - \frac{210\,bcx^6 + 35\,(b^2+2\,ac)x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="maxima")
```

output
```
c^2*x - 1/105*(210*b*c*x^6 + 35*(b^2 + 2*a*c)*x^4 + 42*a*b*x^2 + 15*a^2)/x
^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^2}{x^8}\,dx = c^2x - \frac{210\,bcx^6 + 35\,b^2x^4 + 70\,acx^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^8,x, algorithm="giac")
```

output
```
c^2*x - 1/105*(210*b*c*x^6 + 35*b^2*x^4 + 70*a*c*x^4 + 42*a*b*x^2 + 15*a^2
)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2+cx^4)^2}{x^8}\,dx = c^2\,x - \frac{\frac{a^2}{7}+x^4\left(\frac{b^2}{3}+\frac{2\,a\,c}{3}\right)+\frac{2\,a\,b\,x^2}{5}+2\,b\,c\,x^6}{x^7}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^8,x)
```

output
```
c^2*x - (a^2/7 + x^4*((2*a*c)/3 + b^2/3) + (2*a*b*x^2)/5 + 2*b*c*x^6)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2+cx^4)^2}{x^8}\,dx = \frac{105c^2x^8-210bc\,x^6-70ac\,x^4-35b^2x^4-42ab\,x^2-15a^2}{105x^7}$$

input
```
int((c*x^4+b*x^2+a)^2/x^8,x)
```

output
```
( - 15*a**2 - 42*a*b*x**2 - 70*a*c*x**4 - 35*b**2*x**4 - 210*b*c*x**6 + 10
5*c**2*x**8)/(105*x**7)
```

3.751 $\int \frac{(a+bx^2+cx^4)^2}{x^9} dx$

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{(a+bx^2+cx^4)^2}{x^9} dx = -\frac{a^2}{8x^8} - \frac{ab}{3x^6} - \frac{b^2+2ac}{4x^4} - \frac{bc}{x^2} + c^2\log(x)$$

output
```
-1/8*a^2/x^8-1/3*a*b/x^6-1/4*(2*a*c+b^2)/x^4-b*c/x^2+c^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2+cx^4)^2}{x^9} dx = -\frac{a^2}{8x^8} - \frac{ab}{3x^6} + \frac{-b^2-2ac}{4x^4} - \frac{bc}{x^2} + c^2\log(x)$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^9,x]
```

output
```
-1/8*a^2/x^8 - (a*b)/(3*x^6) + (-b^2 - 2*a*c)/(4*x^4) - (b*c)/x^2 + c^2*Lo
g[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^9} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^2}{x^{10}}dx^2$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^{10}}+\frac{2ba}{x^8}+\frac{c^2}{x^2}+\frac{2bc}{x^4}+\frac{b^2+2ac}{x^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{4x^8}-\frac{2ac+b^2}{2x^4}-\frac{2ab}{3x^6}-\frac{2bc}{x^2}+c^2\log\left(x^2\right)\right)$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^9,x]
```

output
```
(-1/4*a^2/x^8 - (2*a*b)/(3*x^6) - (b^2 + 2*a*c)/(2*x^4) - (2*b*c)/x^2 + c^
2*Log[x^2])/2
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{2ac+b^2}{4x^4}-\frac{bc}{x^2}+c^2\ln\left(x\right)$	43
norman	$\frac{\left(-\frac{ac}{2}-\frac{b^2}{4}\right)x^4-\frac{a^2}{8}-\frac{ab\,x^2}{3}-bc\,x^6}{x^8}+c^2\ln\left(x\right)$	46
risch	$\frac{\left(-\frac{ac}{2}-\frac{b^2}{4}\right)x^4-\frac{a^2}{8}-\frac{ab\,x^2}{3}-bc\,x^6}{x^8}+c^2\ln\left(x\right)$	46
parallelrisch	$\frac{24c^2\ln(x)x^8-24bc\,x^6-12ac\,x^4-6b^2x^4-8ab\,x^2-3a^2}{24x^8}$	51

input
```
int((c*x^4+b*x^2+a)^2/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^2/x^8-1/3*a*b/x^6-1/4*(2*a*c+b^2)/x^4-b*c/x^2+c^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int\frac{\left(a+bx^2+cx^4\right)^2}{x^9}\,dx=\frac{24\,c^2x^8\log\left(x\right)-24\,bcx^6-6\,(b^2+2\,ac)x^4-8\,abx^2-3\,a^2}{24\,x^8}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="fricas")
```

output
```
1/24*(24*c^2*x^8*log(x) - 24*b*c*x^6 - 6*(b^2 + 2*a*c)*x^4 - 8*a*b*x^2 - 3
*a^2)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^9}\,dx = c^2\log(x) + \frac{-3a^2-8abx^2-24bcx^6+x^4(-12ac-6b^2)}{24x^8}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**9,x)
```

output
```
c**2*log(x) + (-3*a**2 - 8*a*b*x**2 - 24*b*c*x**6 + x**4*(-12*a*c - 6*b**2
))/(24*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^2}{x^9}\,dx = \frac{1}{2}\,c^2\log\left(x^2\right) - \frac{24\,bcx^6+6\,(b^2+2\,ac)x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="maxima")
```

output
```
1/2*c^2*log(x^2) - 1/24*(24*b*c*x^6 + 6*(b^2 + 2*a*c)*x^4 + 8*a*b*x^2 + 3*
a^2)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2+cx^4)^2}{x^9}\,dx = \frac{1}{2}\,c^2\log\left(x^2\right) - \frac{25\,c^2x^8+24\,bcx^6+6\,b^2x^4+12\,acx^4+8\,abx^2+3\,a^2}{24\,x^8}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^9,x, algorithm="giac")
```

output
```
1/2*c^2*log(x^2) - 1/24*(25*c^2*x^8 + 24*b*c*x^6 + 6*b^2*x^4 + 12*a*c*x^4
+ 8*a*b*x^2 + 3*a^2)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2+cx^4)^2}{x^9}\,dx = c^2 \ln(x) - \frac{\frac{a^2}{8} + x^4\left(\frac{b^2}{4} + \frac{a\,c}{2}\right) + \frac{a\,b\,x^2}{3} + b\,c\,x^6}{x^8}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^9,x)
```

output
```
c^2*log(x) - (a^2/8 + x^4*((a*c)/2 + b^2/4) + (a*b*x^2)/3 + b*c*x^6)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2+cx^4)^2}{x^9}\,dx = \frac{24\log(x)\,c^2x^8 - 3a^2 - 8ab\,x^2 - 12ac\,x^4 - 6b^2x^4 - 24bc\,x^6}{24x^8}$$

input
```
int((c*x^4+b*x^2+a)^2/x^9,x)
```

output
```
(24*log(x)*c**2*x**8 - 3*a**2 - 8*a*b*x**2 - 12*a*c*x**4 - 6*b**2*x**4 - 2
4*b*c*x**6)/(24*x**8)
```

3.752 $\int \frac{(a+bx^2+cx^4)^2}{x^{10}} dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int \frac{(a+bx^2+cx^4)^2}{x^{10}} dx = -\frac{a^2}{9x^9} - \frac{2ab}{7x^7} - \frac{b^2+2ac}{5x^5} - \frac{2bc}{3x^3} - \frac{c^2}{x}$$

output
```
-1/9*a^2/x^9-2/7*a*b/x^7-1/5*(2*a*c+b^2)/x^5-2/3*b*c/x^3-c^2/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2+cx^4)^2}{x^{10}} dx = -\frac{35a^2+90abx^2+63b^2x^4+126acx^4+210bcx^6+315c^2x^8}{315x^9}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^10,x]
```

output
```
-1/315*(35*a^2 + 90*a*b*x^2 + 63*b^2*x^4 + 126*a*c*x^4 + 210*b*c*x^6 + 315
*c^2*x^8)/x^9
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^2}{x^{10}}+\frac{2ac+b^2}{x^6}+\frac{2ab}{x^8}+\frac{2bc}{x^4}+\frac{c^2}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{9x^9}-\frac{2ac+b^2}{5x^5}-\frac{2ab}{7x^7}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^10,x]
```

output
```
-1/9*a^2/x^9 - (2*a*b)/(7*x^7) - (b^2 + 2*a*c)/(5*x^5) - (2*b*c)/(3*x^3) -
 c^2/x
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{2ac+b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$	45
norman	$\frac{-c^2x^8-\frac{2bc\,x^6}{3}+\left(-\frac{2ac}{5}-\frac{b^2}{5}\right)x^4-\frac{2ab\,x^2}{7}-\frac{a^2}{9}}{x^9}$	47
risch	$\frac{-c^2x^8-\frac{2bc\,x^6}{3}+\left(-\frac{2ac}{5}-\frac{b^2}{5}\right)x^4-\frac{2ab\,x^2}{7}-\frac{a^2}{9}}{x^9}$	47
gosper	$-\frac{315c^2x^8+210bc\,x^6+126ac\,x^4+63b^2x^4+90ab\,x^2+35a^2}{315x^9}$	49
parallelrisch	$\frac{-315c^2x^8-210bc\,x^6-126ac\,x^4-63b^2x^4-90ab\,x^2-35a^2}{315x^9}$	49
orering	$-\frac{315c^2x^8+210bc\,x^6+126ac\,x^4+63b^2x^4+90ab\,x^2+35a^2}{315x^9}$	49

input
```
int((c*x^4+b*x^2+a)^2/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^2/x^9-2/7*a*b/x^7-1/5*(2*a*c+b^2)/x^5-2/3*b*c/x^3-c^2/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx=-\frac{315\,c^2x^8+210\,bcx^6+63\,(b^2+2\,ac)x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="fricas")
```

output
```
-1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*(b^2 + 2*a*c)*x^4 + 90*a*b*x^2 + 35
*a^2)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx = \frac{-35a^2-90abx^2-210bcx^6-315c^2x^8+x^4(-126ac-63b^2)}{315x^9}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**10,x)
```

output
```
(-35*a**2 - 90*a*b*x**2 - 210*b*c*x**6 - 315*c**2*x**8 + x**4*(-126*a*c -
63*b**2))/(315*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx = -\frac{315\,c^2x^8+210\,bcx^6+63\,(b^2+2\,ac)x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="maxima")
```

output
```
-1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*(b^2 + 2*a*c)*x^4 + 90*a*b*x^2 + 35
*a^2)/x^9
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx = -\frac{315\,c^2x^8+210\,bcx^6+63\,b^2x^4+126\,acx^4+90\,abx^2+35\,a^2}{315\,x^9}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^10,x, algorithm="giac")
```

output
```
-1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*b^2*x^4 + 126*a*c*x^4 + 90*a*b*x^2
+ 35*a^2)/x^9
```

Mupad [B] (verification not implemented)

Time = 17.92 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx = -\frac{\frac{a^2}{9}+x^4\left(\frac{b^2}{5}+\frac{2\,a\,c}{5}\right)+c^2\,x^8+\frac{2\,a\,b\,x^2}{7}+\frac{2\,b\,c\,x^6}{3}}{x^9}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^10,x)
```

output
```
-(a^2/9 + x^4*((2*a*c)/5 + b^2/5) + c^2*x^8 + (2*a*b*x^2)/7 + (2*b*c*x^6)/
3)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx = \frac{-315c^2x^8-210bc\,x^6-126ac\,x^4-63b^2x^4-90ab\,x^2-35a^2}{315x^9}$$

input
```
int((c*x^4+b*x^2+a)^2/x^10,x)
```

output
```
( - 35*a**2 - 90*a*b*x**2 - 126*a*c*x**4 - 63*b**2*x**4 - 210*b*c*x**6 - 3
15*c**2*x**8)/(315*x**9)
```

3.753 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{11}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{11}}\,dx=-\frac{a^2}{10x^{10}}-\frac{ab}{4x^8}-\frac{b^2+2ac}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

output
```
-1/10*a^2/x^10-1/4*a*b/x^8-1/6*(2*a*c+b^2)/x^6-1/2*b*c/x^4-1/2*c^2/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{11}}\,dx=-\frac{6a^2+5a(3bx^2+4cx^4)+10x^4(b^2+3bcx^2+3c^2x^4)}{60x^{10}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^11,x]
```

output
```
-1/60*(6*a^2 + 5*a*(3*b*x^2 + 4*c*x^4) + 10*x^4*(b^2 + 3*b*c*x^2 + 3*c^2*x
^4))/x^10
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^{11}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^2}{x^{12}}dx^2$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^{12}}+\frac{2ba}{x^{10}}+\frac{c^2}{x^4}+\frac{2bc}{x^6}+\frac{b^2+2ac}{x^8}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{5x^{10}}-\frac{2ac+b^2}{3x^6}-\frac{ab}{2x^8}-\frac{bc}{x^4}-\frac{c^2}{x^2}\right)$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^11,x]
```

output
```
(-1/5*a^2/x^10 - (a*b)/(2*x^8) - (b^2 + 2*a*c)/(3*x^6) - (b*c)/x^4 - c^2/x
^2)/2
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^2}{10x^{10}}-\frac{ab}{4x^8}-\frac{2ac+b^2}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$	45
norman	$\frac{-\frac{c^2x^8}{2}-\frac{bc\,x^6}{2}+\left(-\frac{ac}{3}-\frac{b^2}{6}\right)x^4-\frac{ab\,x^2}{4}-\frac{a^2}{10}}{x^{10}}$	47
risch	$\frac{-\frac{c^2x^8}{2}-\frac{bc\,x^6}{2}+\left(-\frac{ac}{3}-\frac{b^2}{6}\right)x^4-\frac{ab\,x^2}{4}-\frac{a^2}{10}}{x^{10}}$	47
gosper	$-\frac{30c^2x^8+30bc\,x^6+20ac\,x^4+10b^2x^4+15ab\,x^2+6a^2}{60x^{10}}$	49
parallelrisch	$\frac{-30c^2x^8-30bc\,x^6-20ac\,x^4-10b^2x^4-15ab\,x^2-6a^2}{60x^{10}}$	49
orering	$-\frac{30c^2x^8+30bc\,x^6+20ac\,x^4+10b^2x^4+15ab\,x^2+6a^2}{60x^{10}}$	49

input
```
int((c*x^4+b*x^2+a)^2/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^2/x^10-1/4*a*b/x^8-1/6*(2*a*c+b^2)/x^6-1/2*b*c/x^4-1/2*c^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2+cx^4)^2}{x^{11}}\,dx=-\frac{30\,c^2x^8+30\,bcx^6+10\,(b^2+2\,ac)x^4+15\,abx^2+6\,a^2}{60\,x^{10}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="fricas")
```

output
```
-1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*(b^2 + 2*a*c)*x^4 + 15*a*b*x^2 + 6*a^2
)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2+cx^4)^2}{x^{11}}\,dx = \frac{-6a^2-15abx^2-30bcx^6-30c^2x^8+x^4(-20ac-10b^2)}{60x^{10}}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**11,x)
```

output
```
(-6*a**2 - 15*a*b*x**2 - 30*b*c*x**6 - 30*c**2*x**8 + x**4*(-20*a*c - 10*b
**2))/(60*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2+cx^4)^2}{x^{11}}\,dx = -\frac{30\,c^2x^8+30\,bcx^6+10\,(b^2+2\,ac)x^4+15\,abx^2+6\,a^2}{60\,x^{10}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="maxima")
```

output
```
-1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*(b^2 + 2*a*c)*x^4 + 15*a*b*x^2 + 6*a^2
)/x^10
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2+cx^4)^2}{x^{11}}\,dx = -\frac{30\,c^2x^8+30\,bcx^6+10\,b^2x^4+20\,acx^4+15\,abx^2+6\,a^2}{60\,x^{10}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^11,x, algorithm="giac")
```

output
```
-1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*b^2*x^4 + 20*a*c*x^4 + 15*a*b*x^2 + 6*
a^2)/x^10
```

Mupad [B] (verification not implemented)

Time = 18.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2+cx^4)^2}{x^{11}}\,dx = -\frac{\frac{a^2}{10}+x^4\left(\frac{b^2}{6}+\frac{a\,c}{3}\right)+\frac{c^2\,x^8}{2}+\frac{a\,b\,x^2}{4}+\frac{b\,c\,x^6}{2}}{x^{10}}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^11,x)
```

output
```
-(a^2/10 + x^4*((a*c)/3 + b^2/6) + (c^2*x^8)/2 + (a*b*x^2)/4 + (b*c*x^6)/2
)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2+cx^4)^2}{x^{11}}\,dx = \frac{-30c^2x^8-30bc\,x^6-20ac\,x^4-10b^2x^4-15ab\,x^2-6a^2}{60x^{10}}$$

input
```
int((c*x^4+b*x^2+a)^2/x^11,x)
```

output
```
( - 6*a**2 - 15*a*b*x**2 - 20*a*c*x**4 - 10*b**2*x**4 - 30*b*c*x**6 - 30*c
**2*x**8)/(60*x**10)
```

3.754 $\int \frac{(a+bx^2+cx^4)^2}{x^{12}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{(a+bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{a^2}{11x^{11}}-\frac{2ab}{9x^9}-\frac{b^2+2ac}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

output
```
-1/11*a^2/x^11-2/9*a*b/x^9-1/7*(2*a*c+b^2)/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{a^2}{11x^{11}}-\frac{2ab}{9x^9}+\frac{-b^2-2ac}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^12,x]
```

output
```
-1/11*a^2/x^11 - (2*a*b)/(9*x^9) + (-b^2 - 2*a*c)/(7*x^7) - (2*b*c)/(5*x^5
) - c^2/(3*x^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^{12}}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^2}{x^{12}}+\frac{2ac+b^2}{x^8}+\frac{2ab}{x^{10}}+\frac{2bc}{x^6}+\frac{c^2}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{11x^{11}}-\frac{2ac+b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^12,x]
```

output
```
-1/11*a^2/x^11 - (2*a*b)/(9*x^9) - (b^2 + 2*a*c)/(7*x^7) - (2*b*c)/(5*x^5)
 - c^2/(3*x^3)
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^2}{11x^{11}}-\frac{2ab}{9x^9}-\frac{2ac+b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$	45
norman	$\frac{-\frac{c^2x^8}{3}-\frac{2bc\,x^6}{5}+\left(-\frac{2ac}{7}-\frac{b^2}{7}\right)x^4-\frac{2ab\,x^2}{9}-\frac{a^2}{11}}{x^{11}}$	47
risch	$\frac{-\frac{c^2x^8}{3}-\frac{2bc\,x^6}{5}+\left(-\frac{2ac}{7}-\frac{b^2}{7}\right)x^4-\frac{2ab\,x^2}{9}-\frac{a^2}{11}}{x^{11}}$	47
gosper	$-\frac{1155c^2x^8+1386bc\,x^6+990ac\,x^4+495b^2x^4+770ab\,x^2+315a^2}{3465x^{11}}$	49
parallelrisch	$\frac{-1155c^2x^8-1386bc\,x^6-990ac\,x^4-495b^2x^4-770ab\,x^2-315a^2}{3465x^{11}}$	49
orering	$-\frac{1155c^2x^8+1386bc\,x^6+990ac\,x^4+495b^2x^4+770ab\,x^2+315a^2}{3465x^{11}}$	49

input
```
int((c*x^4+b*x^2+a)^2/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*a^2/x^11-2/9*a*b/x^9-1/7*(2*a*c+b^2)/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{1155\,c^2x^8+1386\,bcx^6+495\,(b^2+2\,ac)x^4+770\,abx^2+315\,a^2}{3465\,x^{11}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="fricas")
```

output
```
-1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2
 + 315*a^2)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2+cx^4)^2}{x^{12}}\,dx = \frac{-315a^2-770abx^2-1386bcx^6-1155c^2x^8+x^4(-990ac-495b^2)}{3465x^{11}}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**12,x)
```

output
```
(-315*a**2 - 770*a*b*x**2 - 1386*b*c*x**6 - 1155*c**2*x**8 + x**4*(-990*a*
c - 495*b**2))/(3465*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{1155\,c^2x^8+1386\,bcx^6+495\,(b^2+2\,ac)x^4+770\,abx^2+315\,a^2}{3465\,x^{11}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="maxima")
```

output
```
-1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2
 + 315*a^2)/x^11
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{1155\,c^2x^8+1386\,bcx^6+495\,b^2x^4+990\,acx^4+770\,abx^2+315\,a^2}{3465\,x^{11}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^12,x, algorithm="giac")
```

output
```
-1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*b^2*x^4 + 990*a*c*x^4 + 770*a*b
*x^2 + 315*a^2)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{(a + bx^2 + cx^4)^2}{x^{12}} \, dx = -\frac{\frac{a^2}{11} + x^4 \left(\frac{b^2}{7} + \frac{2\,a\,c}{7}\right) + \frac{c^2\,x^8}{3} + \frac{2\,a\,b\,x^2}{9} + \frac{2\,b\,c\,x^6}{5}}{x^{11}}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^12,x)
```

output
```
-(a^2/11 + x^4*((2*a*c)/7 + b^2/7) + (c^2*x^8)/3 + (2*a*b*x^2)/9 + (2*b*c*
x^6)/5)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a + bx^2 + cx^4)^2}{x^{12}} \, dx = \frac{-1155c^2x^8 - 1386bc\,x^6 - 990ac\,x^4 - 495b^2x^4 - 770ab\,x^2 - 315a^2}{3465x^{11}}$$

input
```
int((c*x^4+b*x^2+a)^2/x^12,x)
```

output
```
( - 315*a**2 - 770*a*b*x**2 - 990*a*c*x**4 - 495*b**2*x**4 - 1386*b*c*x**6
 - 1155*c**2*x**8)/(3465*x**11)
```

3.755 $\int \frac{(a+bx^2+cx^4)^2}{x^{13}} dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{(a+bx^2+cx^4)^2}{x^{13}} dx = -\frac{a^2}{12x^{12}} - \frac{ab}{5x^{10}} - \frac{b^2+2ac}{8x^8} - \frac{bc}{3x^6} - \frac{c^2}{4x^4}$$

output
```
-1/12*a^2/x^12-1/5*a*b/x^10-1/8*(2*a*c+b^2)/x^8-1/3*b*c/x^6-1/4*c^2/x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2+cx^4)^2}{x^{13}} dx = -\frac{10a^2+24abx^2+15b^2x^4+30acx^4+40bcx^6+30c^2x^8}{120x^{12}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^13,x]
```

output
```
-1/120*(10*a^2 + 24*a*b*x^2 + 15*b^2*x^4 + 30*a*c*x^4 + 40*b*c*x^6 + 30*c^
2*x^8)/x^12
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^{13}}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^2}{x^{14}}dx^2$$

$$\Big\downarrow \text{ 1140}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^{14}}+\frac{2ba}{x^{12}}+\frac{c^2}{x^6}+\frac{2bc}{x^8}+\frac{b^2+2ac}{x^{10}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{6x^{12}}-\frac{2ac+b^2}{4x^8}-\frac{2ab}{5x^{10}}-\frac{2bc}{3x^6}-\frac{c^2}{2x^4}\right)$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^13,x]
```

output
```
(-1/6*a^2/x^12 - (2*a*b)/(5*x^10) - (b^2 + 2*a*c)/(4*x^8) - (2*b*c)/(3*x^6
) - c^2/(2*x^4))/2
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^{2}}{12 x^{12}}-\frac{a b}{5 x^{10}}-\frac{2 a c+b^{2}}{8 x^{8}}-\frac{b c}{3 x^{6}}-\frac{c^{2}}{4 x^{4}}$	45
norman	$\frac{-\frac{c^{2} x^{8}}{4}-\frac{b c \,x^{6}}{3}+\left(-\frac{a c}{4}-\frac{b^{2}}{8}\right) x^{4}-\frac{a b \,x^{2}}{5}-\frac{a^{2}}{12}}{x^{12}}$	47
risch	$\frac{-\frac{c^{2} x^{8}}{4}-\frac{b c \,x^{6}}{3}+\left(-\frac{a c}{4}-\frac{b^{2}}{8}\right) x^{4}-\frac{a b \,x^{2}}{5}-\frac{a^{2}}{12}}{x^{12}}$	47
gosper	$-\frac{30 c^{2} x^{8}+40 b c \,x^{6}+30 a c \,x^{4}+15 b^{2} x^{4}+24 a b \,x^{2}+10 a^{2}}{120 x^{12}}$	49
parallelrisch	$\frac{-30 c^{2} x^{8}-40 b c \,x^{6}-30 a c \,x^{4}-15 b^{2} x^{4}-24 a b \,x^{2}-10 a^{2}}{120 x^{12}}$	49
orering	$-\frac{30 c^{2} x^{8}+40 b c \,x^{6}+30 a c \,x^{4}+15 b^{2} x^{4}+24 a b \,x^{2}+10 a^{2}}{120 x^{12}}$	49

input
```
int((c*x^4+b*x^2+a)^2/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*a^2/x^12-1/5*a*b/x^10-1/8*(2*a*c+b^2)/x^8-1/3*b*c/x^6-1/4*c^2/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{13}}\,dx=-\frac{30\,c^2x^8+40\,bcx^6+15\,(b^2+2\,ac)x^4+24\,abx^2+10\,a^2}{120\,x^{12}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="fricas")
```

output
```
-1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 + 24*a*b*x^2 + 10*a
^2)/x^12
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2+cx^4)^2}{x^{13}}\,dx = \frac{-10a^2-24abx^2-40bcx^6-30c^2x^8+x^4(-30ac-15b^2)}{120x^{12}}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**13,x)
```

output
```
(-10*a**2 - 24*a*b*x**2 - 40*b*c*x**6 - 30*c**2*x**8 + x**4*(-30*a*c - 15*
b**2))/(120*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2+cx^4)^2}{x^{13}}\,dx = -\frac{30\,c^2x^8+40\,bcx^6+15\,(b^2+2\,ac)x^4+24\,abx^2+10\,a^2}{120\,x^{12}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="maxima")
```

output
```
-1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 + 24*a*b*x^2 + 10*a
^2)/x^12
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2+cx^4)^2}{x^{13}}\,dx = -\frac{30\,c^2x^8+40\,bcx^6+15\,b^2x^4+30\,acx^4+24\,abx^2+10\,a^2}{120\,x^{12}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^13,x, algorithm="giac")
```

output
```
-1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*b^2*x^4 + 30*a*c*x^4 + 24*a*b*x^2 + 1
0*a^2)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2+cx^4)^2}{x^{13}}\,dx = -\frac{\frac{a^2}{12}+x^4\left(\frac{b^2}{8}+\frac{a\,c}{4}\right)+\frac{c^2\,x^8}{4}+\frac{a\,b\,x^2}{5}+\frac{b\,c\,x^6}{3}}{x^{12}}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^13,x)
```

output
```
-(a^2/12 + x^4*((a*c)/4 + b^2/8) + (c^2*x^8)/4 + (a*b*x^2)/5 + (b*c*x^6)/3
)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2+cx^4)^2}{x^{13}}\,dx = \frac{-30c^2x^8-40bc\,x^6-30ac\,x^4-15b^2x^4-24ab\,x^2-10a^2}{120x^{12}}$$

input
```
int((c*x^4+b*x^2+a)^2/x^13,x)
```

output
```
( - 10*a**2 - 24*a*b*x**2 - 30*a*c*x**4 - 15*b**2*x**4 - 40*b*c*x**6 - 30*
c**2*x**8)/(120*x**12)
```

3.756 $\int x^2(a+bx^2+cx^4)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int x^2(a+bx^2+cx^4)^3\,dx = \frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}a(b^2+ac)\,x^7+\frac{1}{9}b(b^2+6ac)\,x^9+\frac{3}{11}c(b^2+ac)\,x^{11}+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

output
```
1/3*a^3*x^3+3/5*a^2*b*x^5+3/7*a*(a*c+b^2)*x^7+1/9*b*(6*a*c+b^2)*x^9+3/11*c
*(a*c+b^2)*x^11+3/13*b*c^2*x^13+1/15*c^3*x^15
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^2+cx^4)^3\,dx = \frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}a(b^2+ac)\,x^7+\frac{1}{9}b(b^2+6ac)\,x^9+\frac{3}{11}c(b^2+ac)\,x^{11}+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

input
```
Integrate[x^2*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2 + 6*a*c)
*x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13 + (c^3*x^15)/15
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2+cx^4\right)^3\,dx$$

$$\Big\downarrow \ 1433$$

$$\int \left(a^3x^2+3a^2bx^4+3cx^{10}\left(ac+b^2\right)+bx^8\left(6ac+b^2\right)+3ax^6\left(ac+b^2\right)+3bc^2x^{12}+c^3x^{14}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

input
```
Int[x^2*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2 + 6*a*c)
*x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13 + (c^3*x^15)/15
```


Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
norman	$\frac{a^3x^3}{3}+\frac{3a^2b\,x^5}{5}+\left(\frac{3}{7}c\,a^2+\frac{3}{7}b^2a\right)x^7+\left(\frac{2}{3}abc+\frac{1}{9}b^3\right)x^9+\left(\frac{3}{11}c^2a+\frac{3}{11}b^2c\right)x^{11}+\frac{3b\,c^2x^{13}}{13}+\frac{c^3x^{15}}{15}$
gosper	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}x^7c\,a^2+\frac{3}{7}a\,b^2x^7+\frac{2}{3}x^9abc+\frac{1}{9}b^3x^9+\frac{3}{11}x^{11}c^2a+\frac{3}{11}x^{11}b^2c+\frac{3}{13}b\,c^2x^{13}$
risch	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}x^7c\,a^2+\frac{3}{7}a\,b^2x^7+\frac{2}{3}x^9abc+\frac{1}{9}b^3x^9+\frac{3}{11}x^{11}c^2a+\frac{3}{11}x^{11}b^2c+\frac{3}{13}b\,c^2x^{13}$
parallelrisch	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}x^7c\,a^2+\frac{3}{7}a\,b^2x^7+\frac{2}{3}x^9abc+\frac{1}{9}b^3x^9+\frac{3}{11}x^{11}c^2a+\frac{3}{11}x^{11}b^2c+\frac{3}{13}b\,c^2x^{13}$
orering	$\frac{x^3\left(3003c^3x^{12}+10395b\,c^2x^{10}+12285a\,c^2x^8+12285b^2c\,x^8+30030abc\,x^6+5005b^3x^6+19305a^2c\,x^4+19305b^2x^4a+27027a^2b\,x^2\right.}{45045}$
default	$\frac{c^3x^{15}}{15}+\frac{3b\,c^2x^{13}}{13}+\frac{\left(c^2a+2b^2c+c(2ac+b^2)\right)x^{11}}{11}+\frac{\left(4abc+b(2ac+b^2)\right)x^9}{9}+\frac{\left(a(2ac+b^2)+2b^2a+c\,a^2\right)x^7}{7}+\frac{3a^2b\,x^5}{5}$

input
```
int(x^2*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^3*x^3+3/5*a^2*b*x^5+(3/7*c*a^2+3/7*b^2*a)*x^7+(2/3*a*b*c+1/9*b^3)*x^
9+(3/11*c^2*a+3/11*b^2*c)*x^11+3/13*b*c^2*x^13+1/15*c^3*x^15
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^2\left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{15}\,c^3x^{15}+\frac{3}{13}\,bc^2x^{13}+\frac{3}{11}\,\left(b^2c+ac^2\right)x^{11}+\frac{1}{9}\,\left(b^3+6\,abc\right)x^9+\frac{3}{5}\,a^2bx^5+\frac{3}{7}\,\left(ab^2+a^2c\right)x^7+\frac{1}{3}\,a^3x^3$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*(b^2*c + a*c^2)*x^11 + 1/9*(b^3 + 6
*a*b*c)*x^9 + 3/5*a^2*b*x^5 + 3/7*(a*b^2 + a^2*c)*x^7 + 1/3*a^3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int x^2(a+bx^2+cx^4)^3\,dx=\frac{a^3x^3}{3}+\frac{3a^2bx^5}{5}+\frac{3bc^2x^{13}}{13}+\frac{c^3x^{15}}{15}+x^{11}\cdot\left(\frac{3ac^2}{11}+\frac{3b^2c}{11}\right)+x^9\cdot\left(\frac{2abc}{3}+\frac{b^3}{9}\right)+x^7\cdot\left(\frac{3a^2c}{7}+\frac{3ab^2}{7}\right)$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)**3,x)
```

output
```
a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*b*c**2*x**13/13 + c**3*x**15/15 + x**11*
(3*a*c**2/11 + 3*b**2*c/11) + x**9*(2*a*b*c/3 + b**3/9) + x**7*(3*a**2*c/7
 + 3*a*b**2/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^2(a+bx^2+cx^4)^3\,dx=\frac{1}{15}\,c^3x^{15}+\frac{3}{13}\,bc^2x^{13}+\frac{3}{11}\,(b^2c+ac^2)x^{11}+\frac{1}{9}\,(b^3+6\,abc)x^9+\frac{3}{5}\,a^2bx^5+\frac{3}{7}\,(ab^2+a^2c)x^7+\frac{1}{3}\,a^3x^3$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*(b^2*c + a*c^2)*x^11 + 1/9*(b^3 + 6
*a*b*c)*x^9 + 3/5*a^2*b*x^5 + 3/7*(a*b^2 + a^2*c)*x^7 + 1/3*a^3*x^3
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int x^2 (a+bx^2+cx^4)^3 \, dx = \frac{1}{15}c^3x^{15} + \frac{3}{13}bc^2x^{13} + \frac{3}{11}b^2cx^{11} + \frac{3}{11}ac^2x^{11} + \frac{1}{9}b^3x^9 + \frac{2}{3}abcx^9 + \frac{3}{7}ab^2x^7 + \frac{3}{7}a^2cx^7 + \frac{3}{5}a^2bx^5 + \frac{1}{3}a^3x^3$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*b^2*c*x^11 + 3/11*a*c^2*x^11 + 1/9*
b^3*x^9 + 2/3*a*b*c*x^9 + 3/7*a*b^2*x^7 + 3/7*a^2*c*x^7 + 3/5*a^2*b*x^5 +
1/3*a^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int x^2 (a+bx^2+cx^4)^3 \, dx = x^9 \left(\frac{b^3}{9} + \frac{2\,a\,c\,b}{3}\right) + \frac{a^3\,x^3}{3} + \frac{c^3\,x^{15}}{15} + \frac{3\,a^2\,b\,x^5}{5} + \frac{3\,b\,c^2\,x^{13}}{13} + \frac{3\,a\,x^7\,(b^2+a\,c)}{7} + \frac{3\,c\,x^{11}\,(b^2+a\,c)}{11}$$

input
```
int(x^2*(a + b*x^2 + c*x^4)^3,x)
```

output
```
x^9*(b^3/9 + (2*a*b*c)/3) + (a^3*x^3)/3 + (c^3*x^15)/15 + (3*a^2*b*x^5)/5
+ (3*b*c^2*x^13)/13 + (3*a*x^7*(a*c + b^2))/7 + (3*c*x^11*(a*c + b^2))/11
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int x^2 \left(a + bx^2 + cx^4\right)^3 \, dx$$
$$= \frac{x^3(3003c^3x^{12} + 10395b\,c^2x^{10} + 12285a\,c^2x^8 + 12285b^2c\,x^8 + 30030abc\,x^6 + 5005b^3x^6 + 19305a^2c\,x^4 + 19}{45045}$$

input
```
int(x^2*(c*x^4+b*x^2+a)^3,x)
```

output
```
(x**3*(15015*a**3 + 27027*a**2*b*x**2 + 19305*a**2*c*x**4 + 19305*a*b**2*x
**4 + 30030*a*b*c*x**6 + 12285*a*c**2*x**8 + 5005*b**3*x**6 + 12285*b**2*c
*x**8 + 10395*b*c**2*x**10 + 3003*c**3*x**12))/45045
```

3.757 $\int x(a+bx^2+cx^4)^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int x\left(a+bx^2+cx^4\right)^3\,dx=\frac{a^3x^2}{2}+\frac{3}{4}a^2bx^4+\frac{1}{2}a\left(b^2+ac\right)x^6+\frac{1}{8}b\left(b^2+6ac\right)x^8+\frac{3}{10}c\left(b^2+ac\right)x^{10}+\frac{1}{4}bc^2x^{12}+\frac{c^3x^{14}}{14}$$

output
```
1/2*a^3*x^2+3/4*a^2*b*x^4+1/2*a*(a*c+b^2)*x^6+1/8*b*(6*a*c+b^2)*x^8+3/10*c
*(a*c+b^2)*x^10+1/4*b*c^2*x^12+1/14*c^3*x^14
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int x\left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{280}x^2\left(140a^3+210a^2bx^2+140a\left(b^2+ac\right)x^4+35b\left(b^2+6ac\right)x^6+84c\left(b^2+ac\right)x^8+70bc^2x^{10}+20c^3x^{12}\right)$$

input
```
Integrate[x*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(x^2*(140*a^3 + 210*a^2*b*x^2 + 140*a*(b^2 + a*c)*x^4 + 35*b*(b^2 + 6*a*c)
*x^6 + 84*c*(b^2 + a*c)*x^8 + 70*b*c^2*x^10 + 20*c^3*x^12))/280
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1432, 1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2+cx^4\right)^3\,dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \left(cx^4+bx^2+a\right)^3dx^2$$

$$\Big\downarrow \text{1085}$$

$$\frac{1}{2}\int \left(c^3x^{12}+3bc^2x^{10}+3b^2c\left(\frac{ac}{b^2}+1\right)x^8+b^3\left(\frac{6ac}{b^2}+1\right)x^6+3ab^2\left(\frac{ac}{b^2}+1\right)x^4+3a^2bx^2+a^3\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^3x^2+\frac{3}{2}a^2bx^4+\frac{3}{5}cx^{10}\left(ac+b^2\right)+\frac{1}{4}bx^8\left(6ac+b^2\right)+ax^6\left(ac+b^2\right)+\frac{1}{2}bc^2x^{12}+\frac{c^3x^{14}}{7}\right)$$

input
```
Int[x*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(a^3*x^2 + (3*a^2*b*x^4)/2 + a*(b^2 + a*c)*x^6 + (b*(b^2 + 6*a*c)*x^8)/4 +
 (3*c*(b^2 + a*c)*x^10)/5 + (b*c^2*x^12)/2 + (c^3*x^14)/7)/2
```

Defintions of rubi rules used

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
norman	$\frac{a^3x^2}{2}+\frac{3a^2b\,x^4}{4}+\left(\frac{1}{2}c\,a^2+\frac{1}{2}b^2a\right)x^6+\left(\frac{3}{4}abc+\frac{1}{8}b^3\right)x^8+\left(\frac{3}{10}c^2a+\frac{3}{10}b^2c\right)x^{10}+\frac{b\,c^2x^{12}}{4}+\frac{c^3x^{14}}{14}$
gosper	$\frac{1}{2}a^3x^2+\frac{3}{4}a^2b\,x^4+\frac{1}{2}x^6c\,a^2+\frac{1}{2}b^2x^6a+\frac{3}{4}x^8abc+\frac{1}{8}b^3x^8+\frac{3}{10}x^{10}c^2a+\frac{3}{10}x^{10}b^2c+\frac{1}{4}b\,c^2x^{12}+$
risch	$\frac{1}{2}a^3x^2+\frac{3}{4}a^2b\,x^4+\frac{1}{2}x^6c\,a^2+\frac{1}{2}b^2x^6a+\frac{3}{4}x^8abc+\frac{1}{8}b^3x^8+\frac{3}{10}x^{10}c^2a+\frac{3}{10}x^{10}b^2c+\frac{1}{4}b\,c^2x^{12}+$
parallelrisch	$\frac{1}{2}a^3x^2+\frac{3}{4}a^2b\,x^4+\frac{1}{2}x^6c\,a^2+\frac{1}{2}b^2x^6a+\frac{3}{4}x^8abc+\frac{1}{8}b^3x^8+\frac{3}{10}x^{10}c^2a+\frac{3}{10}x^{10}b^2c+\frac{1}{4}b\,c^2x^{12}+$
orering	$\frac{x^2\left(20c^3x^{12}+70b\,c^2x^{10}+84a\,c^2x^8+84b^2c\,x^8+210abc\,x^6+35b^3x^6+140a^2c\,x^4+140b^2x^4a+210a^2b\,x^2+140a^3\right)}{280}$
default	$\frac{c^3x^{14}}{14}+\frac{b\,c^2x^{12}}{4}+\frac{\left(c^2a+2b^2c+c\left(2ac+b^2\right)\right)x^{10}}{10}+\frac{\left(4abc+b\left(2ac+b^2\right)\right)x^8}{8}+\frac{\left(a\left(2ac+b^2\right)+2b^2a+c\,a^2\right)x^6}{6}+\frac{3a^2b\,x^4}{4}+$

input
```
int(x*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*x^2+3/4*a^2*b*x^4+(1/2*c*a^2+1/2*b^2*a)*x^6+(3/4*a*b*c+1/8*b^3)*x^
8+(3/10*c^2*a+3/10*b^2*c)*x^10+1/4*b*c^2*x^12+1/14*c^3*x^14
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x(a+bx^2+cx^4)^3\,dx=\frac{1}{14}c^3x^{14}+\frac{1}{4}bc^2x^{12}+\frac{3}{10}\left(b^2c+ac^2\right)x^{10}+\frac{1}{8}\left(b^3+6\,abc\right)x^8+\frac{3}{4}a^2bx^4+\frac{1}{2}\left(ab^2+a^2c\right)x^6+\frac{1}{2}a^3x^2$$

input
```
integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*(b^2*c + a*c^2)*x^10 + 1/8*(b^3 + 6*
a*b*c)*x^8 + 3/4*a^2*b*x^4 + 1/2*(a*b^2 + a^2*c)*x^6 + 1/2*a^3*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int x(a+bx^2+cx^4)^3\,dx=\frac{a^3x^2}{2}+\frac{3a^2bx^4}{4}+\frac{bc^2x^{12}}{4}+\frac{c^3x^{14}}{14}+x^{10}\cdot\left(\frac{3ac^2}{10}+\frac{3b^2c}{10}\right)+x^8\cdot\left(\frac{3abc}{4}+\frac{b^3}{8}\right)+x^6\left(\frac{a^2c}{2}+\frac{ab^2}{2}\right)$$

input
```
integrate(x*(c*x**4+b*x**2+a)**3,x)
```

output
```
a**3*x**2/2 + 3*a**2*b*x**4/4 + b*c**2*x**12/4 + c**3*x**14/14 + x**10*(3*
a*c**2/10 + 3*b**2*c/10) + x**8*(3*a*b*c/4 + b**3/8) + x**6*(a**2*c/2 + a*
b**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x(a+bx^2+cx^4)^3\,dx=\frac{1}{14}c^3x^{14}+\frac{1}{4}bc^2x^{12}+\frac{3}{10}(b^2c+ac^2)x^{10}+\frac{1}{8}(b^3+6abc)x^8+\frac{3}{4}a^2bx^4+\frac{1}{2}(ab^2+a^2c)x^6+\frac{1}{2}a^3x^2$$

input
```
integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*(b^2*c + a*c^2)*x^10 + 1/8*(b^3 + 6*
a*b*c)*x^8 + 3/4*a^2*b*x^4 + 1/2*(a*b^2 + a^2*c)*x^6 + 1/2*a^3*x^2
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int x(a+bx^2+cx^4)^3\,dx=\frac{1}{14}c^3x^{14}+\frac{1}{4}bc^2x^{12}+\frac{3}{10}b^2cx^{10}+\frac{3}{10}ac^2x^{10}+\frac{1}{8}b^3x^8+\frac{3}{4}abcx^8+\frac{1}{2}ab^2x^6+\frac{1}{2}a^2cx^6+\frac{3}{4}a^2bx^4+\frac{1}{2}a^3x^2$$

input
```
integrate(x*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*b^2*c*x^10 + 3/10*a*c^2*x^10 + 1/8*b
^3*x^8 + 3/4*a*b*c*x^8 + 1/2*a*b^2*x^6 + 1/2*a^2*c*x^6 + 3/4*a^2*b*x^4 + 1
/2*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int x(a+bx^2+cx^4)^3\,dx = x^8\left(\frac{b^3}{8}+\frac{3\,a\,c\,b}{4}\right)+\frac{a^3\,x^2}{2}+\frac{c^3\,x^{14}}{14}+\frac{3\,a^2\,b\,x^4}{4}+\frac{b\,c^2\,x^{12}}{4}+\frac{a\,x^6\,(b^2+a\,c)}{2}+\frac{3\,c\,x^{10}\,(b^2+a\,c)}{10}$$

input
```
int(x*(a + b*x^2 + c*x^4)^3,x)
```

output
```
x^8*(b^3/8 + (3*a*b*c)/4) + (a^3*x^2)/2 + (c^3*x^14)/14 + (3*a^2*b*x^4)/4
+ (b*c^2*x^12)/4 + (a*x^6*(a*c + b^2))/2 + (3*c*x^10*(a*c + b^2))/10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int x(a+bx^2+cx^4)^3\,dx = \frac{x^2(20c^3x^{12}+70b\,c^2x^{10}+84a\,c^2x^8+84b^2c\,x^8+210abc\,x^6+35b^3x^6+140a^2c\,x^4+140a\,b^2x^4+210a^2b\,x^2}{280}$$

input
```
int(x*(c*x^4+b*x^2+a)^3,x)
```

output
```
(x**2*(140*a**3 + 210*a**2*b*x**2 + 140*a**2*c*x**4 + 140*a*b**2*x**4 + 21
0*a*b*c*x**6 + 84*a*c**2*x**8 + 35*b**3*x**6 + 84*b**2*c*x**8 + 70*b*c**2*
x**10 + 20*c**3*x**12))/280
```

3.758 $\int \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 81

$$\int \left(a+bx^2+cx^4\right)^3 \, dx = a^3x+a^2bx^3+\frac{3}{5}a\left(b^2+ac\right)x^5+\frac{1}{7}b\left(b^2+6ac\right)x^7+\frac{1}{3}c\left(b^2+ac\right)x^9+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

output
```
a^3*x+a^2*b*x^3+3/5*a*(a*c+b^2)*x^5+1/7*b*(6*a*c+b^2)*x^7+1/3*c*(a*c+b^2)*
x^9+3/11*b*c^2*x^11+1/13*c^3*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2+cx^4\right)^3 \, dx = a^3x+a^2bx^3+\frac{3}{5}a\left(b^2+ac\right)x^5+\frac{1}{7}b\left(b^2+6ac\right)x^7+\frac{1}{3}c\left(b^2+ac\right)x^9+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^3,x]
```

output
```
a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c
*(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx^2 + cx^4)^3 \, dx$$

$$\Big\downarrow \text{ 1403}$$

$$\int \left(a^3 + 3a^2bx^2 + 3b^2cx^8\left(\frac{ac}{b^2} + 1\right) + 3ab^2x^4\left(\frac{ac}{b^2} + 1\right) + b^3x^6\left(\frac{6ac}{b^2} + 1\right) + 3bc^2x^{10} + c^3x^{12}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^3x + a^2bx^3 + \frac{1}{3}cx^9(ac + b^2) + \frac{1}{7}bx^7(6ac + b^2) + \frac{3}{5}ax^5(ac + b^2) + \frac{3}{11}bc^2x^{11} + \frac{c^3x^{13}}{13}$$

input
```
Int[(a + b*x^2 + c*x^4)^3,x]
```

output
```
a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^7)/7 + (c
*(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

method	result
norman	$\frac{c^{3} x^{13}}{13}+\frac{3 b \,c^{2} x^{11}}{11}+\left(\frac{1}{3} c^{2} a +\frac{1}{3} b^{2} c \right) x^{9}+\left(\frac{6}{7} a b c +\frac{1}{7} b^{3}\right) x^{7}+\left(\frac{3}{5} c \,a^{2}+\frac{3}{5} b^{2} a \right) x^{5}+a^{2} b \,x^{3}+a^{3} x$
gosper	$\frac{1}{13} c^{3} x^{13}+\frac{3}{11} b \,c^{2} x^{11}+\frac{1}{3} x^{9} c^{2} a +\frac{1}{3} x^{9} b^{2} c +\frac{6}{7} x^{7} a b c +\frac{1}{7} b^{3} x^{7}+\frac{3}{5} x^{5} c \,a^{2}+\frac{3}{5} b^{2} x^{5} a +a^{2} b \,x^{3}+a^{3}$
risch	$\frac{1}{13} c^{3} x^{13}+\frac{3}{11} b \,c^{2} x^{11}+\frac{1}{3} x^{9} c^{2} a +\frac{1}{3} x^{9} b^{2} c +\frac{6}{7} x^{7} a b c +\frac{1}{7} b^{3} x^{7}+\frac{3}{5} x^{5} c \,a^{2}+\frac{3}{5} b^{2} x^{5} a +a^{2} b \,x^{3}+a^{3}$
parallelrisch	$\frac{1}{13} c^{3} x^{13}+\frac{3}{11} b \,c^{2} x^{11}+\frac{1}{3} x^{9} c^{2} a +\frac{1}{3} x^{9} b^{2} c +\frac{6}{7} x^{7} a b c +\frac{1}{7} b^{3} x^{7}+\frac{3}{5} x^{5} c \,a^{2}+\frac{3}{5} b^{2} x^{5} a +a^{2} b \,x^{3}+a^{3}$
orering	$\frac{x \left(1155 c^{3} x^{12}+4095 b \,c^{2} x^{10}+5005 a \,c^{2} x^{8}+5005 b^{2} c \,x^{8}+12870 a b c \,x^{6}+2145 b^{3} x^{6}+9009 a^{2} c \,x^{4}+9009 b^{2} x^{4} a +15015 a^{2} b \,x^{2}+15015\right.}{15015}$
default	$\frac{c^{3} x^{13}}{13}+\frac{3 b \,c^{2} x^{11}}{11}+\frac{\left(c^{2} a +2 b^{2} c +c \left(2 a c +b^{2}\right)\right) x^{9}}{9}+\frac{\left(4 a b c +b \left(2 a c +b^{2}\right)\right) x^{7}}{7}+\frac{\left(a \left(2 a c +b^{2}\right)+2 b^{2} a +c \,a^{2}\right) x^{5}}{5}+a^{2} b \,x^{3}+$

input
```
int((c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*c^3*x^13+3/11*b*c^2*x^11+(1/3*c^2*a+1/3*b^2*c)*x^9+(6/7*a*b*c+1/7*b^3
)*x^7+(3/5*c*a^2+3/5*b^2*a)*x^5+a^2*b*x^3+a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{13}\,c^3x^{13}+\frac{3}{11}\,bc^2x^{11}+\frac{1}{3}\,\left(b^2c+ac^2\right)x^9+\frac{1}{7}\,\left(b^3+6\,abc\right)x^7+a^2bx^3+\frac{3}{5}\,\left(ab^2+a^2c\right)x^5+a^3x$$

input
```
integrate((c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*(b^2*c + a*c^2)*x^9 + 1/7*(b^3 + 6*a
*b*c)*x^7 + a^2*b*x^3 + 3/5*(a*b^2 + a^2*c)*x^5 + a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \left(a + bx^2 + cx^4\right)^3 \, dx = a^3x + a^2bx^3 + \frac{3bc^2x^{11}}{11} + \frac{c^3x^{13}}{13} + x^9\left(\frac{ac^2}{3} + \frac{b^2c}{3}\right) + x^7 \cdot \left(\frac{6abc}{7} + \frac{b^3}{7}\right) + x^5 \cdot \left(\frac{3a^2c}{5} + \frac{3ab^2}{5}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**3,x)
```

output
```
a**3*x + a**2*b*x**3 + 3*b*c**2*x**11/11 + c**3*x**13/13 + x**9*(a*c**2/3
+ b**2*c/3) + x**7*(6*a*b*c/7 + b**3/7) + x**5*(3*a**2*c/5 + 3*a*b**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \left(a + bx^2 + cx^4\right)^3 \, dx = \frac{1}{13}\,c^3x^{13} + \frac{3}{11}\,bc^2x^{11} + \frac{1}{3}\,b^2cx^9 + \frac{1}{7}\,b^3x^7 + a^3x + \frac{1}{5}\,\left(3\,cx^5 + 5\,bx^3\right)a^2 + \frac{1}{105}\,\left(35\,c^2x^9 + 90\,bcx^7 + 63\,b^2x^5\right)a$$

input
```
integrate((c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/7*b^3*x^7 + a^3*x + 1/
5*(3*c*x^5 + 5*b*x^3)*a^2 + 1/105*(35*c^2*x^9 + 90*b*c*x^7 + 63*b^2*x^5)*a
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int \left(a+bx^2+cx^4\right)^3 dx = \frac{1}{13}c^3x^{13}+\frac{3}{11}bc^2x^{11}+\frac{1}{3}b^2cx^9+\frac{1}{3}ac^2x^9+\frac{1}{7}b^3x^7 +\frac{6}{7}abcx^7+\frac{3}{5}ab^2x^5+\frac{3}{5}a^2cx^5+a^2bx^3+a^3x$$

input
```
integrate((c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/3*a*c^2*x^9 + 1/7*b^3*
x^7 + 6/7*a*b*c*x^7 + 3/5*a*b^2*x^5 + 3/5*a^2*c*x^5 + a^2*b*x^3 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2+cx^4\right)^3 dx = x^7\left(\frac{b^3}{7}+\frac{6\,a\,c\,b}{7}\right)+a^3\,x+\frac{c^3\,x^{13}}{13}+a^2\,b\,x^3 +\frac{3\,b\,c^2\,x^{11}}{11}+\frac{3\,a\,x^5\,\left(b^2+a\,c\right)}{5}+\frac{c\,x^9\,\left(b^2+a\,c\right)}{3}$$

input
```
int((a + b*x^2 + c*x^4)^3,x)
```

output
```
x^7*(b^3/7 + (6*a*b*c)/7) + a^3*x + (c^3*x^13)/13 + a^2*b*x^3 + (3*b*c^2*x
^11)/11 + (3*a*x^5*(a*c + b^2))/5 + (c*x^9*(a*c + b^2))/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \left(a+bx^2+cx^4\right)^3\,dx$$
$$=\frac{x(1155c^3x^{12}+4095b\,c^2x^{10}+5005a\,c^2x^8+5005b^2c\,x^8+12870abc\,x^6+2145b^3x^6+9009a^2c\,x^4+9009a\,b}{15015}$$

input
```
int((c*x^4+b*x^2+a)^3,x)
```

output
```
(x*(15015*a**3 + 15015*a**2*b*x**2 + 9009*a**2*c*x**4 + 9009*a*b**2*x**4 +
 12870*a*b*c*x**6 + 5005*a*c**2*x**8 + 2145*b**3*x**6 + 5005*b**2*c*x**8 +
 4095*b*c**2*x**10 + 1155*c**3*x**12))/15015
```

3.759 $\int \frac{(a+bx^2+cx^4)^3}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \frac{(a+bx^2+cx^4)^3}{x} dx = \frac{3}{2}a^2bx^2 + \frac{3}{4}a(b^2+ac)x^4 + \frac{1}{6}b(b^2+6ac)x^6 + \frac{3}{8}c(b^2+ac)x^8 + \frac{3}{10}bc^2x^{10} + \frac{c^3x^{12}}{12} + a^3\log(x)$$

output
```
3/2*a^2*b*x^2+3/4*a*(a*c+b^2)*x^4+1/6*b*(6*a*c+b^2)*x^6+3/8*c*(a*c+b^2)*x^
8+3/10*b*c^2*x^10+1/12*c^3*x^12+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^3}{x} dx = \frac{3}{2}a^2bx^2 + \frac{3}{4}a(b^2+ac)x^4 + \frac{1}{6}b(b^2+6ac)x^6 + \frac{3}{8}c(b^2+ac)x^8 + \frac{3}{10}bc^2x^{10} + \frac{c^3x^{12}}{12} + a^3\log(x)$$

input
```
Integrate[(a + b*x^2 + c*x^4)^3/x,x]
```

output
```
(3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)/6 + (3*c
*(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12 + a^3*Log[x]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{\left(cx^4+bx^2+a\right)^3}{x^2}dx^2$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{2}\int \left(c^3x^{10}+3bc^2x^8+3c\left(b^2+ac\right)x^6+b\left(b^2+6ac\right)x^4+3a\left(b^2+ac\right)x^2+3a^2b+\frac{a^3}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^3\log\left(x^2\right)+3a^2bx^2+\frac{3}{4}cx^8\left(ac+b^2\right)+\frac{1}{3}bx^6\left(6ac+b^2\right)+\frac{3}{2}ax^4\left(ac+b^2\right)+\frac{3}{5}bc^2x^{10}+\frac{c^3x^{12}}{6}\right)$$

input
```
Int[(a + b*x^2 + c*x^4)^3/x,x]
```

output
```
(3*a^2*b*x^2 + (3*a*(b^2 + a*c)*x^4)/2 + (b*(b^2 + 6*a*c)*x^6)/3 + (3*c*(b
^2 + a*c)*x^8)/4 + (3*b*c^2*x^10)/5 + (c^3*x^12)/6 + a^3*Log[x^2])/2
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

method	result
norman	$\left(\frac{3}{4}c\,a^{2}+\frac{3}{4}b^{2}a\right)x^{4}+\left(\frac{3}{8}c^{2}a+\frac{3}{8}b^{2}c\right)x^{8}+\left(abc+\frac{1}{6}b^{3}\right)x^{6}+\frac{c^{3}x^{12}}{12}+\frac{3a^{2}b\,x^{2}}{2}+\frac{3b\,c^{2}x^{10}}{10}+a^{3}\ln\left(x\right.$
default	$\frac{c^{3}x^{12}}{12}+\frac{3b\,c^{2}x^{10}}{10}+\frac{3a\,c^{2}x^{8}}{8}+\frac{3b^{2}c\,x^{8}}{8}+abc\,x^{6}+\frac{b^{3}x^{6}}{6}+\frac{3a^{2}c\,x^{4}}{4}+\frac{3b^{2}x^{4}a}{4}+\frac{3a^{2}b\,x^{2}}{2}+a^{3}\ln\left(x\right)$
parallelrisch	$\frac{c^{3}x^{12}}{12}+\frac{3b\,c^{2}x^{10}}{10}+\frac{3a\,c^{2}x^{8}}{8}+\frac{3b^{2}c\,x^{8}}{8}+abc\,x^{6}+\frac{b^{3}x^{6}}{6}+\frac{3a^{2}c\,x^{4}}{4}+\frac{3b^{2}x^{4}a}{4}+\frac{3a^{2}b\,x^{2}}{2}+a^{3}\ln\left(x\right)$
risch	$\frac{3a^{2}b\,x^{2}}{2}+abc\,x^{6}+\frac{3b^{2}x^{4}a}{4}+\frac{3b\,c^{2}x^{10}}{10}+\frac{c^{3}x^{12}}{12}+\frac{3b^{2}c\,x^{8}}{8}+\frac{3a\,c^{2}x^{8}}{8}-\frac{a\,b^{4}}{8c^{2}}+\frac{b^{3}x^{6}}{6}+\frac{b^{6}}{120c^{3}}+\frac{3a^{2}c\,x^{4}}{4}+$

input
```
int((c*x^4+b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
(3/4*c*a^2+3/4*b^2*a)*x^4+(3/8*c^2*a+3/8*b^2*c)*x^8+(a*b*c+1/6*b^3)*x^6+1/
12*c^3*x^12+3/2*a^2*b*x^2+3/10*b*c^2*x^10+a^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2+cx^4)^3}{x}\,dx = \frac{1}{12}c^3x^{12} + \frac{3}{10}bc^2x^{10} + \frac{3}{8}(b^2c+ac^2)x^8 + \frac{1}{6}(b^3+6\,abc)x^6 + \frac{3}{2}a^2bx^2 + \frac{3}{4}(ab^2+a^2c)x^4 + a^3\log(x)$$

input
```
integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="fricas")
```

output
```
1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*(b^2*c + a*c^2)*x^8 + 1/6*(b^3 + 6*a
*b*c)*x^6 + 3/2*a^2*b*x^2 + 3/4*(a*b^2 + a^2*c)*x^4 + a^3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2+cx^4)^3}{x}\,dx = a^3\log(x) + \frac{3a^2bx^2}{2} + \frac{3bc^2x^{10}}{10} + \frac{c^3x^{12}}{12} + x^8 \cdot \left(\frac{3ac^2}{8} + \frac{3b^2c}{8}\right) + x^6\left(abc + \frac{b^3}{6}\right) + x^4 \cdot \left(\frac{3a^2c}{4} + \frac{3ab^2}{4}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**3/x,x)
```

output
```
a**3*log(x) + 3*a**2*b*x**2/2 + 3*b*c**2*x**10/10 + c**3*x**12/12 + x**8*(
3*a*c**2/8 + 3*b**2*c/8) + x**6*(a*b*c + b**3/6) + x**4*(3*a**2*c/4 + 3*a*
b**2/4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2+cx^4)^3}{x}\,dx = \frac{1}{12}\,c^3x^{12} + \frac{3}{10}\,bc^2x^{10} + \frac{3}{8}\,(b^2c+ac^2)x^8 + \frac{1}{6}\,(b^3+6\,abc)x^6 + \frac{3}{2}\,a^2bx^2 + \frac{3}{4}\,(ab^2+a^2c)x^4 + \frac{1}{2}\,a^3\log\left(x^2\right)$$

input
```
integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="maxima")
```

output
```
1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*(b^2*c + a*c^2)*x^8 + 1/6*(b^3 + 6*a
*b*c)*x^6 + 3/2*a^2*b*x^2 + 3/4*(a*b^2 + a^2*c)*x^4 + 1/2*a^3*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2+cx^4)^3}{x}\,dx = \frac{1}{12}\,c^3x^{12} + \frac{3}{10}\,bc^2x^{10} + \frac{3}{8}\,b^2cx^8 + \frac{3}{8}\,ac^2x^8 + \frac{1}{6}\,b^3x^6 + abcx^6 + \frac{3}{4}\,ab^2x^4 + \frac{3}{4}\,a^2cx^4 + \frac{3}{2}\,a^2bx^2 + \frac{1}{2}\,a^3\log\left(x^2\right)$$

input
```
integrate((c*x^4+b*x^2+a)^3/x,x, algorithm="giac")
```

output
```
1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*b^2*c*x^8 + 3/8*a*c^2*x^8 + 1/6*b^3*
x^6 + a*b*c*x^6 + 3/4*a*b^2*x^4 + 3/4*a^2*c*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*
log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2+cx^4)^3}{x}\,dx = a^3\,\ln(x)+x^6\left(\frac{b^3}{6}+a\,c\,b\right)+\frac{c^3\,x^{12}}{12}+\frac{3\,a^2\,b\,x^2}{2}+\frac{3\,b\,c^2\,x^{10}}{10}+\frac{3\,a\,x^4\,(b^2+a\,c)}{4}+\frac{3\,c\,x^8\,(b^2+a\,c)}{8}$$

input
```
int((a + b*x^2 + c*x^4)^3/x,x)
```

output
```
a^3*log(x) + x^6*(b^3/6 + a*b*c) + (c^3*x^12)/12 + (3*a^2*b*x^2)/2 + (3*b*
c^2*x^10)/10 + (3*a*x^4*(a*c + b^2))/4 + (3*c*x^8*(a*c + b^2))/8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2+cx^4)^3}{x}\,dx = \log(x)\,a^3+\frac{3a^2b\,x^2}{2}+\frac{3a^2c\,x^4}{4}+\frac{3a\,b^2x^4}{4}+abc\,x^6+\frac{3a\,c^2x^8}{8}+\frac{b^3x^6}{6}+\frac{3b^2c\,x^8}{8}+\frac{3b\,c^2x^{10}}{10}+\frac{c^3x^{12}}{12}$$

input
```
int((c*x^4+b*x^2+a)^3/x,x)
```

output
```
(120*log(x)*a**3 + 180*a**2*b*x**2 + 90*a**2*c*x**4 + 90*a*b**2*x**4 + 120
*a*b*c*x**6 + 45*a*c**2*x**8 + 20*b**3*x**6 + 45*b**2*c*x**8 + 36*b*c**2*x
**10 + 10*c**3*x**12)/120
```

3.760 $\int \frac{(a+bx^2+cx^4)^3}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{(a+bx^2+cx^4)^3}{x^2} dx = -\frac{a^3}{x} + 3a^2bx + a(b^2+ac)x^3 + \frac{1}{5}b(b^2+6ac)x^5 + \frac{3}{7}c(b^2+ac)x^7 + \frac{1}{3}bc^2x^9 + \frac{c^3x^{11}}{11}$$

output
```
-a^3/x+3*a^2*b*x+a*(a*c+b^2)*x^3+1/5*b*(6*a*c+b^2)*x^5+3/7*c*(a*c+b^2)*x^7
+1/3*b*c^2*x^9+1/11*c^3*x^11
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^3}{x^2} dx = -\frac{a^3}{x} + 3a^2bx + a(b^2+ac)x^3 + \frac{1}{5}b(b^2+6ac)x^5 + \frac{3}{7}c(b^2+ac)x^7 + \frac{1}{3}bc^2x^9 + \frac{c^3x^{11}}{11}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^3/x^2,x]
```

output
```
-(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/5 + (3*c*
(b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^3}{x^2}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^3}{x^2}+3a^2b+3cx^6(ac+b^2)+bx^4(6ac+b^2)+3ax^2(ac+b^2)+3bc^2x^8+c^3x^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{x}+3a^2bx+\frac{3}{7}cx^7(ac+b^2)+\frac{1}{5}bx^5(6ac+b^2)+ax^3(ac+b^2)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

input
```
Int[(a + b*x^2 + c*x^4)^3/x^2,x]
```

output
```
-(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/5 + (3*c*
(b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

method	result	s
default	$\frac{c^3x^{11}}{11}+\frac{b\,c^2x^9}{3}+\frac{3a\,c^2x^7}{7}+\frac{3b^2c\,x^7}{7}+\frac{6abc\,x^5}{5}+\frac{b^3x^5}{5}+a^2c\,x^3+a\,b^2x^3+3a^2bx-\frac{a^3}{x}$	8
norman	$\frac{\frac{c^3x^{12}}{11}+\frac{b\,c^2x^{10}}{3}+\left(\frac{3}{7}c^2a+\frac{3}{7}b^2c\right)x^8+\left(\frac{6}{5}abc+\frac{1}{5}b^3\right)x^6+\left(c\,a^2+b^2a\right)x^4+3a^2b\,x^2-a^3}{x}$	8
risch	$\frac{c^3x^{11}}{11}+\frac{b\,c^2x^9}{3}+\frac{3a\,c^2x^7}{7}+\frac{3b^2c\,x^7}{7}+\frac{6abc\,x^5}{5}+\frac{b^3x^5}{5}+a^2c\,x^3+a\,b^2x^3+3a^2bx-\frac{a^3}{x}$	8
gosper	$-\frac{-105c^3x^{12}-385b\,c^2x^{10}-495a\,c^2x^8-495b^2c\,x^8-1386abc\,x^6-231b^3x^6-1155a^2c\,x^4-1155b^2x^4a-3465a^2b\,x^2+1155a^3}{1155x}$	9
parallelrisch	$\frac{105c^3x^{12}+385b\,c^2x^{10}+495a\,c^2x^8+495b^2c\,x^8+1386abc\,x^6+231b^3x^6+1155a^2c\,x^4+1155b^2x^4a+3465a^2b\,x^2-1155a^3}{1155x}$	9
orering	$-\frac{-105c^3x^{12}-385b\,c^2x^{10}-495a\,c^2x^8-495b^2c\,x^8-1386abc\,x^6-231b^3x^6-1155a^2c\,x^4-1155b^2x^4a-3465a^2b\,x^2+1155a^3}{1155x}$	9

input
```
int((c*x^4+b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*c^3*x^11+1/3*b*c^2*x^9+3/7*a*c^2*x^7+3/7*b^2*c*x^7+6/5*a*b*c*x^5+1/5*
b^3*x^5+a^2*c*x^3+a*b^2*x^3+3*a^2*b*x-a^3/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{(a + bx^2 + cx^4)^3}{x^2} \, dx = \frac{105 \, c^3 x^{12} + 385 \, bc^2 x^{10} + 495 \, (b^2 c + ac^2) x^8 + 231 \, (b^3 + 6 \, abc) x^6 + 3465 \, a^2 b x^2 + 1155 \, (ab^2 + a^2 c) x^4 - 115}{1155 \, x}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="fricas")
```

output
```
1/1155*(105*c^3*x^12 + 385*b*c^2*x^10 + 495*(b^2*c + a*c^2)*x^8 + 231*(b^3
 + 6*a*b*c)*x^6 + 3465*a^2*b*x^2 + 1155*(a*b^2 + a^2*c)*x^4 - 1155*a^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{(a + bx^2 + cx^4)^3}{x^2} \, dx = -\frac{a^3}{x} + 3a^2 bx + \frac{bc^2 x^9}{3} + \frac{c^3 x^{11}}{11} + x^7 \cdot \left(\frac{3ac^2}{7} + \frac{3b^2 c}{7}\right) + x^5 \cdot \left(\frac{6abc}{5} + \frac{b^3}{5}\right) + x^3 \left(a^2 c + ab^2\right)$$

input
```
integrate((c*x**4+b*x**2+a)**3/x**2,x)
```

output
```
-a**3/x + 3*a**2*b*x + b*c**2*x**9/3 + c**3*x**11/11 + x**7*(3*a*c**2/7 +
3*b**2*c/7) + x**5*(6*a*b*c/5 + b**3/5) + x**3*(a**2*c + a*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{11}\,c^3x^{11} + \frac{1}{3}\,bc^2x^9 + \frac{3}{7}\,(b^2c+ac^2)x^7 + \frac{1}{5}\,(b^3+6\,abc)x^5 + 3\,a^2bx + (ab^2+a^2c)x^3 - \frac{a^3}{x}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*(b^2*c + a*c^2)*x^7 + 1/5*(b^3 + 6*a*b
*c)*x^5 + 3*a^2*b*x + (a*b^2 + a^2*c)*x^3 - a^3/x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{11}\,c^3x^{11} + \frac{1}{3}\,bc^2x^9 + \frac{3}{7}\,b^2cx^7 + \frac{3}{7}\,ac^2x^7 + \frac{1}{5}\,b^3x^5 + \frac{6}{5}\,abcx^5 + ab^2x^3 + a^2cx^3 + 3\,a^2bx - \frac{a^3}{x}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^2,x, algorithm="giac")
```

output
```
1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 3/7*a*c^2*x^7 + 1/5*b^3*x^
5 + 6/5*a*b*c*x^5 + a*b^2*x^3 + a^2*c*x^3 + 3*a^2*b*x - a^3/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2+cx^4)^3}{x^2}\,dx = x^5\left(\frac{b^3}{5}+\frac{6\,a\,c\,b}{5}\right)-\frac{a^3}{x}+\frac{c^3\,x^{11}}{11}+\frac{b\,c^2\,x^9}{3} + a\,x^3\,(b^2+a\,c)+\frac{3\,c\,x^7\,(b^2+a\,c)}{7}+3\,a^2\,b\,x$$

input
```
int((a + b*x^2 + c*x^4)^3/x^2,x)
```

output
```
x^5*(b^3/5 + (6*a*b*c)/5) - a^3/x + (c^3*x^11)/11 + (b*c^2*x^9)/3 + a*x^3*
(a*c + b^2) + (3*c*x^7*(a*c + b^2))/7 + 3*a^2*b*x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2+cx^4)^3}{x^2}\,dx = \frac{105c^3x^{12}+385b\,c^2x^{10}+495a\,c^2x^8+495b^2c\,x^8+1386abc\,x^6+231b^3x^6+1155a^2c\,x^4+1155a\,b^2x^4+346}{1155x}$$

input
```
int((c*x^4+b*x^2+a)^3/x^2,x)
```

output
```
( - 1155*a**3 + 3465*a**2*b*x**2 + 1155*a**2*c*x**4 + 1155*a*b**2*x**4 + 1
386*a*b*c*x**6 + 495*a*c**2*x**8 + 231*b**3*x**6 + 495*b**2*c*x**8 + 385*b
*c**2*x**10 + 105*c**3*x**12)/(1155*x)
```

3.761 $\int \frac{(a+bx^2+cx^4)^3}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \frac{(a+bx^2+cx^4)^3}{x^3} dx = -\frac{a^3}{2x^2}+\frac{3}{2}a(b^2+ac)x^2+\frac{1}{4}b(b^2+6ac)x^4 + \frac{1}{2}c(b^2+ac)x^6+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}+3a^2b\log(x)$$

output
```
-1/2*a^3/x^2+3/2*a*(a*c+b^2)*x^2+1/4*b*(6*a*c+b^2)*x^4+1/2*c*(a*c+b^2)*x^6
+3/8*b*c^2*x^8+1/10*c^3*x^10+3*a^2*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2+cx^4)^3}{x^3} dx = \frac{1}{40}\left(-\frac{20a^3}{x^2}+60a(b^2+ac)x^2+10b(b^2+6ac)x^4 + 20c(b^2+ac)x^6+15bc^2x^8+4c^3x^{10}+120a^2b\log(x)\right)$$

input
```
Integrate[(a + b*x^2 + c*x^4)^3/x^3,x]
```

output
```
((-20*a^3)/x^2 + 60*a*(b^2 + a*c)*x^2 + 10*b*(b^2 + 6*a*c)*x^4 + 20*c*(b^2
 + a*c)*x^6 + 15*b*c^2*x^8 + 4*c^3*x^10 + 120*a^2*b*Log[x])/40
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1434, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^3}{x^3}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^3}{x^4}dx^2$$

$$\Big\downarrow \text{1140}$$

$$\frac{1}{2}\int \left(c^3x^8+3bc^2x^6+3c(b^2+ac)x^4+b(b^2+6ac)x^2+3a(b^2+ac)+\frac{3a^2b}{x^2}+\frac{a^3}{x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^3}{x^2}+3a^2b\log(x^2)+cx^6(ac+b^2)+\frac{1}{2}bx^4(6ac+b^2)+3ax^2(ac+b^2)+\frac{3}{4}bc^2x^8+\frac{c^3x^{10}}{5}\right)$$

input
```
Int[(a + b*x^2 + c*x^4)^3/x^3,x]
```

output
```
(-(a^3/x^2) + 3*a*(b^2 + a*c)*x^2 + (b*(b^2 + 6*a*c)*x^4)/2 + c*(b^2 + a*c
)*x^6 + (3*b*c^2*x^8)/4 + (c^3*x^10)/5 + 3*a^2*b*Log[x^2])/2
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

method	result	size
norman	$\frac{\left(\frac{3}{2} c\, a^{2}+\frac{3}{2} b^{2} a \right) x^{4}+\left(\frac{1}{2} c^{2} a +\frac{1}{2} b^{2} c \right) x^{8}+\left(\frac{3}{2} a b c +\frac{1}{4} b^{3}\right) x^{6}-\frac{a^{3}}{2}+\frac{c^{3} x^{12}}{10}+\frac{3 b\, c^{2} x^{10}}{8}}{x^{2}}+3 a^{2} b \ln \left(x \right)$	86
default	$\frac{c^{3} x^{10}}{10}+\frac{3 b\, c^{2} x^{8}}{8}+\frac{a\, c^{2} x^{6}}{2}+\frac{b^{2} c\, x^{6}}{2}+\frac{3 a b c\, x^{4}}{2}+\frac{b^{3} x^{4}}{4}+\frac{3 a^{2} c\, x^{2}}{2}+\frac{3 a\, b^{2} x^{2}}{2}-\frac{a^{3}}{2 x^{2}}+3 a^{2} b \ln \left(x \right)$	87
risch	$\frac{c^{3} x^{10}}{10}+\frac{3 b\, c^{2} x^{8}}{8}+\frac{a\, c^{2} x^{6}}{2}+\frac{b^{2} c\, x^{6}}{2}+\frac{3 a b c\, x^{4}}{2}+\frac{b^{3} x^{4}}{4}+\frac{3 a^{2} c\, x^{2}}{2}+\frac{3 a\, b^{2} x^{2}}{2}-\frac{a^{3}}{2 x^{2}}+3 a^{2} b \ln \left(x \right)$	87
parallelrisch	$\frac{4 c^{3} x^{12}+15 b\, c^{2} x^{10}+20 a\, c^{2} x^{8}+20 b^{2} c\, x^{8}+60 a b c\, x^{6}+10 b^{3} x^{6}+60 a^{2} c\, x^{4}+60 b^{2} x^{4} a +120 \ln \left(x \right) x^{2} a^{2} b -20 a^{3}}{40 x^{2}}$	92

input
```
int((c*x^4+b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
((3/2*c*a^2+3/2*b^2*a)*x^4+(1/2*c^2*a+1/2*b^2*c)*x^8+(3/2*a*b*c+1/4*b^3)*x
^6-1/2*a^3+1/10*c^3*x^12+3/8*b*c^2*x^10)/x^2+3*a^2*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2+cx^4)^3}{x^3}\,dx = \frac{4\,c^3x^{12}+15\,bc^2x^{10}+20\,(b^2c+ac^2)x^8+10\,(b^3+6\,abc)x^6+120\,a^2bx^2\log(x)+60\,(ab^2+a^2c)x^4-20\,a^3}{40\,x^2}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="fricas")
```

output
```
1/40*(4*c^3*x^12 + 15*b*c^2*x^10 + 20*(b^2*c + a*c^2)*x^8 + 10*(b^3 + 6*a*
b*c)*x^6 + 120*a^2*b*x^2*log(x) + 60*(a*b^2 + a^2*c)*x^4 - 20*a^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2+cx^4)^3}{x^3}\,dx = -\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{3bc^2x^8}{8}+\frac{c^3x^{10}}{10}+x^6\left(\frac{ac^2}{2}+\frac{b^2c}{2}\right) + x^4\cdot\left(\frac{3abc}{2}+\frac{b^3}{4}\right)+x^2\cdot\left(\frac{3a^2c}{2}+\frac{3ab^2}{2}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**3/x**3,x)
```

output
```
-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*b*c**2*x**8/8 + c**3*x**10/10 + x**6*
(a*c**2/2 + b**2*c/2) + x**4*(3*a*b*c/2 + b**3/4) + x**2*(3*a**2*c/2 + 3*a
*b**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{10}\,c^3x^{10} + \frac{3}{8}\,bc^2x^8 + \frac{1}{2}\,(b^2c+ac^2)x^6 + \frac{1}{4}\,(b^3+6\,abc)x^4 + \frac{3}{2}\,a^2b\log\left(x^2\right) + \frac{3}{2}\,(ab^2+a^2c)x^2 - \frac{a^3}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="maxima")
```

output
```
1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*(b^2*c + a*c^2)*x^6 + 1/4*(b^3 + 6*a*b
*c)*x^4 + 3/2*a^2*b*log(x^2) + 3/2*(a*b^2 + a^2*c)*x^2 - 1/2*a^3/x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{10}\,c^3x^{10} + \frac{3}{8}\,bc^2x^8 + \frac{1}{2}\,b^2cx^6 + \frac{1}{2}\,ac^2x^6 + \frac{1}{4}\,b^3x^4 + \frac{3}{2}\,abcx^4 + \frac{3}{2}\,ab^2x^2 + \frac{3}{2}\,a^2cx^2 + \frac{3}{2}\,a^2b\log\left(x^2\right) - \frac{3\,a^2bx^2+a^3}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^3,x, algorithm="giac")
```

output
```
1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/2*a*c^2*x^6 + 1/4*b^3*x^
4 + 3/2*a*b*c*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*c*x^2 + 3/2*a^2*b*log(x^2) - 1
/2*(3*a^2*b*x^2 + a^3)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2+cx^4)^3}{x^3}\,dx = x^4\left(\frac{b^3}{4}+\frac{3\,a\,c\,b}{2}\right)-\frac{a^3}{2\,x^2}+\frac{c^3\,x^{10}}{10}+\frac{3\,b\,c^2\,x^8}{8} + 3\,a^2\,b\,\ln(x)+\frac{3\,a\,x^2\,(b^2+a\,c)}{2}+\frac{c\,x^6\,(b^2+a\,c)}{2}$$

input
```
int((a + b*x^2 + c*x^4)^3/x^3,x)
```

output
```
x^4*(b^3/4 + (3*a*b*c)/2) - a^3/(2*x^2) + (c^3*x^10)/10 + (3*b*c^2*x^8)/8
+ 3*a^2*b*log(x) + (3*a*x^2*(a*c + b^2))/2 + (c*x^6*(a*c + b^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2+cx^4)^3}{x^3}\,dx = \frac{120\log(x)\,a^2b\,x^2-20a^3+60a^2c\,x^4+60a\,b^2x^4+60abc\,x^6+20a\,c^2x^8+10b^3x^6+20b^2c\,x^8+15b\,c^2x^{10}+}{40x^2}$$

input
```
int((c*x^4+b*x^2+a)^3/x^3,x)
```

output
```
(120*log(x)*a**2*b*x**2 - 20*a**3 + 60*a**2*c*x**4 + 60*a*b**2*x**4 + 60*a
*b*c*x**6 + 20*a*c**2*x**8 + 10*b**3*x**6 + 20*b**2*c*x**8 + 15*b*c**2*x**
10 + 4*c**3*x**12)/(40*x**2)
```

3.762 $\int \frac{(a+bx^2+cx^4)^3}{x^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \frac{(a+bx^2+cx^4)^3}{x^4} dx = -\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3a(b^2+ac)x + \frac{1}{3}b(b^2+6ac)x^3 + \frac{3}{5}c(b^2+ac)x^5 + \frac{3}{7}bc^2x^7 + \frac{c^3x^9}{9}$$

output
```
-1/3*a^3/x^3-3*a^2*b/x+3*a*(a*c+b^2)*x+1/3*b*(6*a*c+b^2)*x^3+3/5*c*(a*c+b^
2)*x^5+3/7*b*c^2*x^7+1/9*c^3*x^9
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^3}{x^4} dx = -\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3a(b^2+ac)x + \frac{1}{3}b(b^2+6ac)x^3 + \frac{3}{5}c(b^2+ac)x^5 + \frac{3}{7}bc^2x^7 + \frac{c^3x^9}{9}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^3/x^4,x]
```

output
```
-1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^3)/3 +
 (3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^3}{x^4} \, dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^2}+3cx^4(ac+b^2)+bx^2(6ac+b^2)+3a(ac+b^2)+3bc^2x^6+c^3x^8\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+\frac{3}{5}cx^5(ac+b^2)+\frac{1}{3}bx^3(6ac+b^2)+3ax(ac+b^2)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

input
```
Int[(a + b*x^2 + c*x^4)^3/x^4,x]
```

output

```
-1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)*x^3)/3 +
 (3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{c^3x^9}{9}+\frac{3b\,c^2x^7}{7}+\frac{3a\,c^2x^5}{5}+\frac{3b^2c\,x^5}{5}+2abc\,x^3+\frac{b^3x^3}{3}+3xc\,a^2+3x\,b^2a-\frac{a^3}{3x^3}-\frac{3a^2b}{x}$	84
norman	$\frac{\frac{c^3x^{12}}{9}+\frac{3b\,c^2x^{10}}{7}+\left(\frac{3}{5}c^2a+\frac{3}{5}b^2c\right)x^8+\left(2abc+\frac{1}{3}b^3\right)x^6+\left(3c\,a^2+3b^2a\right)x^4-3a^2b\,x^2-\frac{a^3}{3}}{x^3}$	86
risch	$\frac{c^3x^9}{9}+\frac{3b\,c^2x^7}{7}+\frac{3a\,c^2x^5}{5}+\frac{3b^2c\,x^5}{5}+2abc\,x^3+\frac{b^3x^3}{3}+3xc\,a^2+3x\,b^2a+\frac{-3a^2b\,x^2-\frac{1}{3}a^3}{x^3}$	86
gosper	$-\frac{-35c^3x^{12}-135b\,c^2x^{10}-189a\,c^2x^8-189b^2c\,x^8-630abc\,x^6-105b^3x^6-945a^2c\,x^4-945b^2x^4a+945a^2b\,x^2+105a^3}{315x^3}$	90
parallelrisch	$\frac{35c^3x^{12}+135b\,c^2x^{10}+189a\,c^2x^8+189b^2c\,x^8+630abc\,x^6+105b^3x^6+945a^2c\,x^4+945b^2x^4a-945a^2b\,x^2-105a^3}{315x^3}$	90
orering	$-\frac{-35c^3x^{12}-135b\,c^2x^{10}-189a\,c^2x^8-189b^2c\,x^8-630abc\,x^6-105b^3x^6-945a^2c\,x^4-945b^2x^4a+945a^2b\,x^2+105a^3}{315x^3}$	90

input
```
int((c*x^4+b*x^2+a)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/9*c^3*x^9+3/7*b*c^2*x^7+3/5*a*c^2*x^5+3/5*b^2*c*x^5+2*a*b*c*x^3+1/3*b^3*
x^3+3*x*c*a^2+3*x*b^2*a-1/3*a^3/x^3-3*a^2*b/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x^4}\,dx = \frac{35\,c^3x^{12}+135\,bc^2x^{10}+189\,(b^2c+ac^2)x^8+105\,(b^3+6\,abc)x^6-945\,a^2bx^2+945\,(ab^2+a^2c)x^4-105\,a^3}{315\,x^3}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="fricas")
```

output
```
1/315*(35*c^3*x^12 + 135*b*c^2*x^10 + 189*(b^2*c + a*c^2)*x^8 + 105*(b^3 +
 6*a*b*c)*x^6 - 945*a^2*b*x^2 + 945*(a*b^2 + a^2*c)*x^4 - 105*a^3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+bx^2+cx^4\right)^3}{x^4}\,dx = \frac{3bc^2x^7}{7}+\frac{c^3x^9}{9}+x^5\cdot\left(\frac{3ac^2}{5}+\frac{3b^2c}{5}\right)+x^3\cdot\left(2abc+\frac{b^3}{3}\right)+x\left(3a^2c+3ab^2\right)+\frac{-a^3-9a^2bx^2}{3x^3}$$

input
```
integrate((c*x**4+b*x**2+a)**3/x**4,x)
```

output
```
3*b*c**2*x**7/7 + c**3*x**9/9 + x**5*(3*a*c**2/5 + 3*b**2*c/5) + x**3*(2*a
*b*c + b**3/3) + x*(3*a**2*c + 3*a*b**2) + (-a**3 - 9*a**2*b*x**2)/(3*x**3
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2+cx^4)^3}{x^4}\,dx = \frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,(b^2c+ac^2)x^5+\frac{1}{3}\,(b^3+6\,abc)x^3+3\,(ab^2+a^2c)x-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="maxima")
```

output
```
1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*(b^2*c + a*c^2)*x^5 + 1/3*(b^3 + 6*a*b*c
)*x^3 + 3*(a*b^2 + a^2*c)*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2+cx^4)^3}{x^4}\,dx = \frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{3}{5}\,ac^2x^5+\frac{1}{3}\,b^3x^3+2\,abcx^3+3\,ab^2x+3\,a^2cx-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^4,x, algorithm="giac")
```

output
```
1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 3/5*a*c^2*x^5 + 1/3*b^3*x^3
+ 2*a*b*c*x^3 + 3*a*b^2*x + 3*a^2*c*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2+cx^4)^3}{x^4}\,dx = x^3\left(\frac{b^3}{3}+2\,a\,c\,b\right)-\frac{\frac{a^3}{3}+3\,b\,a^2\,x^2}{x^3}+\frac{c^3\,x^9}{9}+\frac{3\,b\,c^2\,x^7}{7}+3\,a\,x\,\left(b^2+a\,c\right)+\frac{3\,c\,x^5\,\left(b^2+a\,c\right)}{5}$$

input
```
int((a + b*x^2 + c*x^4)^3/x^4,x)
```

output
```
x^3*(b^3/3 + 2*a*b*c) - (a^3/3 + 3*a^2*b*x^2)/x^3 + (c^3*x^9)/9 + (3*b*c^2
*x^7)/7 + 3*a*x*(a*c + b^2) + (3*c*x^5*(a*c + b^2))/5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2+cx^4)^3}{x^4}\,dx = \frac{35c^3x^{12}+135b\,c^2x^{10}+189a\,c^2x^8+189b^2c\,x^8+630abc\,x^6+105b^3x^6+945a^2c\,x^4+945a\,b^2x^4-945a^2b\,x}{315x^3}$$

input
```
int((c*x^4+b*x^2+a)^3/x^4,x)
```

output
```
( - 105*a**3 - 945*a**2*b*x**2 + 945*a**2*c*x**4 + 945*a*b**2*x**4 + 630*a
*b*c*x**6 + 189*a*c**2*x**8 + 105*b**3*x**6 + 189*b**2*c*x**8 + 135*b*c**2
*x**10 + 35*c**3*x**12)/(315*x**3)
```

3.763 $\int \frac{x^7}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int \frac{x^7}{a+bx^2+cx^4}\,dx = -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

output
```
-1/2*b*x^2/c^2+1/4*x^4/c+1/2*b*(-3*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^
2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)+1/4*(-a*c+b^2)*ln(c*x^4+b*x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93

$$\int \frac{x^7}{a+bx^2+cx^4}\,dx = \frac{cx^2(-2b+cx^2)-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

input
```
Integrate[x^7/(a + b*x^2 + c*x^4),x]
```

output
```
(c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2
+ 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3
)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^6}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1143}$$

$$\frac{1}{2}\int \left(\frac{x^2}{c}+\frac{(b^2-ac)\,x^2+ab}{c^2\,(cx^4+bx^2+a)}-\frac{b}{c^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{2c^3}-\frac{bx^2}{c^2}+\frac{x^4}{2c}\right)$$

input
```
Int[x^7/(a + b*x^2 + c*x^4),x]
```

output
```
(-((b*x^2)/c^2) + x^4/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x^
4])/(2*c^3))/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

method	result
default	$-\frac{-\frac{1}{2}c\,x^{4}+b\,x^{2}}{2c^{2}}+\frac{\frac{\left(-ac+b^{2}\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(ab-\frac{\left(-ac+b^{2}\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2c^{2}}$
risch	$\frac{x^{4}}{4c}-\frac{b\,x^{2}}{2c^{2}}+\frac{b^{2}}{4c^{3}}-\frac{\ln\left(\left(\left(12a^{2}b\,c^{2}-7a\,b^{3}c+b^{5}+\sqrt{-b^{2}(4ac-b^{2})(3ac-b^{2})^{2}}\,b\right)x^{2}+2\sqrt{-b^{2}(4ac-b^{2})(3ac-b^{2})^{2}}\,a\right)a^{2}}{c(4ac-b^{2})}+\frac{5\ln\left(\left(\right.\right.}{}$

input
```
int(x^7/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2*(-1/2*c*x^4+b*x^2)+1/2/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^4+b*x^2+a)+2*
(a*b-1/2*(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.13

$$\int \frac{x^7}{a+bx^2+cx^4}\,dx = \left[\frac{(b^2c^2-4\,ac^3)x^4-2\,(b^3c-4\,abc^2)x^2-(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{4\,(b^2c^3-4\,ac^4)}\right.$$

input
```
integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 - (b^3 - 3*a*b*c
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 +
b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)
*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*x^4
 - 2*(b^3*c - 4*a*b*c^2)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(92) = 184$.

Time = 1.40 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.91

$$\int \frac{x^7}{a+bx^2+cx^4}\,dx = -\frac{bx^2}{2c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{4c^3}\right)\log\left(x^2 + \frac{2a^2c-ab^2+8ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac}{4}\right.}{3abc-b^3}\right. + \left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{4c^3}\right)\log\left(x^2 + \frac{2a^2c-ab^2+8ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{4c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right.}{3abc-b^3}\right. + \frac{x^4}{4c}$$

input
```
integrate(x**7/(c*x**4+b*x**2+a),x)
```

output
```
-b*x**2/(2*c**2) + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c -
 b**2)) - (a*c - b**2)/(4*c**3))*log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*
(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b*
*2)/(4*c**3)) - 2*b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3
*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*
a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)
)*log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c -
 b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(b*s
qrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(
4*c**3)))/(3*a*b*c - b**3)) + x**4/(4*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{a + bx^2 + cx^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{x^7}{a + bx^2 + cx^4} \, dx = \frac{cx^4 - 2\,bx^2}{4\,c^2} + \frac{(b^2 - ac)\log(cx^4 + bx^2 + a)}{4\,c^3} - \frac{(b^3 - 3\,abc)\arctan\left(\frac{2\,cx^2 + b}{\sqrt{-b^2 + 4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^3}$$

input
```
integrate(x^7/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/c^3 - 1
/2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4
*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 842, normalized size of antiderivative = 8.42

$$\int \frac{x^7}{a+bx^2+cx^4}\,dx = \frac{x^4}{4\,c} - \frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{2\,\left(16\,a\,c^4-4\,b^2\,c^3\right)} - \frac{b\,x^2}{2\,c^2}$$

$$+ \frac{b\,\mathrm{atan}\left(\frac{2\,c^4\,\left(4\,a\,c-b^2\right)\left(\frac{\frac{b\,\left(3\,a\,c-b^2\right)\,\left(\frac{8\,a^2\,c^4-8\,a\,b^2\,c^3}{c^4}-\frac{8\,a\,c^2\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{16\,a\,c^4-4\,b^2\,c^3}\right)}{8\,c^3\,\sqrt{4\,a\,c-b^2}}-\frac{a\,b\,\left(3\,a\,c-b^2\right)\,\left(8\,a^2\,c^2-10\,a\,b^2\,c+2\,b^4\right)}{c\,\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^4-4\,b^2\,c^3\right)}}{a}-x^2\left(\frac{b\left(6\cdots\right.}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
int(x^7/(a + b*x^2 + c*x^4),x)
```

output
```
x^4/(4*c) - (log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(
16*a*c^4 - 4*b^2*c^3)) - (b*x^2)/(2*c^2) + (b*atan((2*c^4*(4*a*c - b^2)*((
(b*(3*a*c - b^2)*((8*a^2*c^4 - 8*a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*
c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3)))/(8*c^3*(4*a*c - b^2)^(1/2)) -
(a*b*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(c*(4*a*c - b^2)^(1/2
)*(16*a*c^4 - 4*b^2*c^3)))/a - x^2*(((b*((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4
*b*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(3*a*c -
b^2))/(8*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2
- 10*a*b^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((
b^5 + 2*a^2*b*c^2 - 3*a*b^3*c)/c^4 + (((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b
*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4 + 8*
a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b^3*(3*a*c - b^2)^2)/
(2*c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((((8*a^2*c^4 - 8*
a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*
b^2*c^3))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (
a*b^4 + a^3*c^2 - 2*a^2*b^2*c)/c^4 + (a*b^2*(3*a*c - b^2)^2)/(c^4*(4*a*c -
 b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^6 + 9*a^2*b^2*c^2 - 6*a*b^4*c))*(3
*a*c - b^2))/(2*c^3*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 458, normalized size of antiderivative = 4.58

$$\int \frac{x^7}{a+bx^2+cx^4}\,dx = \frac{-6\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)abc+2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}}}{\sqrt{2}}\right.}{}$$

input
```
int(x^7/(c*x^4+b*x^2+a),x)
```

output
```
( - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c +
2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3 - 6*sqr
t(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*
sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c + 2*sqrt(2*
sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3 - 4*log( - sqrt(
2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a**2*c**2 + 5*log( - sq
rt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b**2*c - log( - sq
rt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**4 - 4*log(sqrt(2*
sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a**2*c**2 + 5*log(sqrt(2*
sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b**2*c - log(sqrt(2*sqr
t(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**4 - 8*a*b*c**2*x**2 + 4*a
*c**3*x**4 + 2*b**3*c*x**2 - b**2*c**2*x**4)/(4*c**3*(4*a*c - b**2))
```

3.764 $\int \frac{x^5}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{x^5}{a+bx^2+cx^4}\,dx=\frac{x^2}{2c}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log(a+bx^2+cx^4)}{4c^2}$$

output
```
1/2*x^2/c-1/2*(-2*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4
*a*c+b^2)^(1/2)-1/4*b*ln(c*x^4+b*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{a+bx^2+cx^4}\,dx=\frac{2cx^2+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log(a+bx^2+cx^4)}{4c^2}$$

input
```
Integrate[x^5/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^4}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1143}$$

$$\frac{1}{2}\int \left(\frac{1}{c}-\frac{bx^2+a}{c\,(cx^4+bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{2c^2}+\frac{x^2}{c}\right)$$

input
```
Int[x^5/(a + b*x^2 + c*x^4),x]
```

output
```
(x^2/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqr
t[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(2*c^2))/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

method	result
default	$\frac{x^2}{2c}+\frac{-\frac{b\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$
risch	$\frac{x^2}{2c}-\frac{\ln\left(\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)x^2+2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,a\right)ab}{c(4ac-b^2)}+\frac{\ln\left(\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{\right.\right.\right.}{}$

input
```
int(x^5/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c+1/2/c*(-1/2*b/c*ln(c*x^4+b*x^2+a)+2*(-a+1/2*b^2/c)/(4*a*c-b^2)^(
1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.14

$$\int \frac{x^5}{a+bx^2+cx^4}\,dx$$
$$=\left[\frac{2\,(b^2c-4\,ac^2)x^2-(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(b^3-4\,abc)\log\left(cx\right.}{4\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 +
 b*x^2 + a)) - (b^3 - 4*a*b*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)
, 1/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*
x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(71) = 142$.

Time = 1.03 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.90

$$\int \frac{x^5}{a+bx^2+cx^4}\,dx = \left(-\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\cdot}{4c^2\cdot(4ac}\right.}{2ac-b^2}\right. + \left(-\frac{b}{4c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\cdot}{4c^2\cdot(4ac}\right.}{2ac-b^2}\right. + \frac{x^2}{2c}$$

input
```
integrate(x**5/(c*x**4+b*x**2+a),x)
```

output
```
(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))
*log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b
**2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) - sqrt(-4*a*c + b**2
)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(4*c**2)
+ sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (
-a*b - 8*a*c**2*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*
(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{a+bx^2+cx^4}\,dx = \frac{x^2}{2\,c} - \frac{b\log\left(cx^4+bx^2+a\right)}{4\,c^2} + \frac{(b^2-2\,ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/2*x^2/c - 1/4*b*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2
*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 19.15 (sec) , antiderivative size = 655, normalized size of antiderivative = 8.09

$$\int \frac{x^5}{a+bx^2+cx^4}\,dx = \frac{x^2}{2\,c} + \frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,b^3-8\,a\,b\,c\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}$$

$$+ \frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\left(\frac{\frac{\left(8\,a\,b+\frac{8\,a\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)\,\left(2\,a\,c-b^2\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}+\frac{a\,\left(2\,b^3-8\,a\,b\,c\right)\,\left(2\,a\,c-b^2\right)}{\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}}{a}-x^2\left(\frac{\frac{\left(2\,a\,c-b^2\right)\,\left(\frac{4\,a\,c^3-6\,b^2\,c^2}{c^2}-\frac{4\,b\,c^2\,\left(2\,b^3-8\,\cdots\right)}{16\,a\,c^3-4\,b^2\cdots}\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}\cdots}{a}\cdots\right.\right.}{\cdots}\right)}{\cdots}$$

input

```
int(x^5/(a + b*x^2 + c*x^4),x)
```

output

```
x^2/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a*c^3 - 4*b^
2*c^2)) + (atan((2*c^2*(4*a*c - b^2)*((((8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c
))/(16*a*c^3 - 4*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a
*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c
^2)))/a - x^2*((((2*a*c - b^2)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^
3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(2
*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^
2)))/a + (b*(((2*b^3 - 8*a*b*c)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b
^3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b^3
- a*b*c)/c^2 + (b*(2*a*c - b^2)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b
^2)^(1/2))) + (b*((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c)*(8*a*b + (8*a*c^2*(2*b^
3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*(2*
a*c - b^2)^2)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^4 + 4*a
^2*c^2 - 4*a*b^2*c))*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.52

$$\int \frac{x^5}{a+bx^2+cx^4}\,dx$$
$$= \frac{4\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)ac-2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-}}{\sqrt{2\sqrt{c}}}\right)}{}$$

input
```
int(x^5/(c*x^4+b*x^2+a),x)
```

output
```
(4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c - 2*sqr
t(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*
sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2 + 4*sqrt(2*s
qrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c - 2*sqrt(2*sqrt(c)
*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b
) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2 - 4*log( - sqrt(2*sqrt(
c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*c + log( - sqrt(2*sqrt(c)*
sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**3 - 4*log(sqrt(2*sqrt(c)*sqrt(
a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*c + log(sqrt(2*sqrt(c)*sqrt(a) - b
)*x + sqrt(a) + sqrt(c)*x**2)*b**3 + 8*a*c**2*x**2 - 2*b**2*c*x**2)/(4*c**
2*(4*a*c - b**2))
```

3.765 $\int \frac{x^3}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{x^3}{a+bx^2+cx^4} dx = \frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{\log(a+bx^2+cx^4)}{4c}$$

output
```
1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)+1/4*ln(
c*x^4+b*x^2+a)/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{a+bx^2+cx^4} dx = \frac{-\frac{2b\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log(a+bx^2+cx^4)}{4c}$$

input
```
Integrate[x^3/(a + b*x^2 + c*x^4),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[
a + b*x^2 + c*x^4])/(4*c)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1434, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \ 1434$$

$$\frac{1}{2}\int \frac{x^2}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{2}\left(\frac{\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{c}+\frac{\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left(\frac{\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}+\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\left(\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{2c}\right)$$

input
```
Int[x^3/(a + b*x^2 + c*x^4),x]
```

output
```
((b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^2 + c*x^4]/(2*c))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
default	$\frac{\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{4c}-\frac{b\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{2c\sqrt{4ac-b^{2}}}$
risch	$\frac{\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}(4ac-b^{2})}\,b\right)x^{2}+2\sqrt{-b^{2}(4ac-b^{2})}\,a\right)a}{4ac-b^{2}}-\frac{\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}(4ac-b^{2})}\,b\right)x^{2}+2\sqrt{-b^{2}(4ac-b^{2})}\,a\right)b^{2}}{4c(4ac-b^{2})}+$

input
```
int(x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(c*x^4+b*x^2+a)/c-1/2/c*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.13

$$\int \frac{x^3}{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^4+bx^2+a\right)}{4\,\left(b^2c-4\,ac^2\right)}, \frac{2\,\sqrt{-b^2+4\,ac}b}{}\right.$$

input
```
integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*
x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4
 + b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c
*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^4 + b*
x^2 + a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.

Time = 0.47 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.54

$$\int \frac{x^3}{a+bx^2+cx^4}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)} + \frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)} + \frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)$$

input
```
integrate(x**3/(c*x**4+b*x**2+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c
*(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(-
b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c
 + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c
 + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{a+bx^2+cx^4}\,dx = -\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,acc}} + \frac{\log\left(cx^4+bx^2+a\right)}{4\,c}$$

input
```
integrate(x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1
/4*log(c*x^4 + b*x^2 + a)/c
```

Mupad [B] (verification not implemented)

Time = 18.35 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.87

$$\int \frac{x^3}{a+bx^2+cx^4}\,dx = \frac{4\,a\,c\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c} - \frac{b^2\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c} - \frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x^2}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^3/(a + b*x^2 + c*x^4),x)
```

output
```
(4*a*c*log(a + b*x^2 + c*x^4))/(16*a*c^2 - 4*b^2*c) - (b^2*log(a + b*x^2 +
 c*x^4))/(16*a*c^2 - 4*b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^2)/
(4*a*c - b^2)^(1/2)))/(2*c*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.75

$$\int \frac{x^3}{a+bx^2+cx^4}\,dx = \frac{2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)b+2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}{\sqrt{2\sqrt{c}\,\sqrt{}}}\right.}{}$$

input
```
int(x^3/(c*x^4+b*x^2+a),x)
```

output
```
(2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b + 2*sqrt(
2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b + 4*log( - sqrt(2
*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c - log( - sqrt(2*sqrt
(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**2 + 4*log(sqrt(2*sqrt(c)*s
qrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c - log(sqrt(2*sqrt(c)*sqrt(a) -
 b)*x + sqrt(a) + sqrt(c)*x**2)*b**2)/(4*c*(4*a*c - b**2))
```

3.766 $\int \frac{x}{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \frac{x}{a+bx^2+cx^4} \, dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
-arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{x}{a+bx^2+cx^4} \, dx = \frac{\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input
```
Integrate[x/(a + b*x^2 + c*x^4),x]
```

output
```
ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1432, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 1432}$$

$$\frac{1}{2}\int \frac{1}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{ 1083}$$

$$-\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[x/(a + b*x^2 + c*x^4),x]
```

output
```
-(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}$	36
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}-2a\right)}{2\sqrt{-4ac+b^{2}}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}+2a\right)}{2\sqrt{-4ac+b^{2}}}$	70

input
```
int(x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.58

$$\int \frac{x}{a+bx^2+cx^4}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\sqrt{b^2-4\,ac}}, -\frac{\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

input
```
integrate(x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(
-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(34) = 68$.

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.64

$$\int \frac{x}{a+bx^2+cx^4}\,dx = -\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2} + \frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}$$

input
```
integrate(x/(c*x**4+b*x**2+a),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2
*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2
+ (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c)
)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{x}{a+bx^2+cx^4}\,dx=\frac{\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

input
```
integrate(x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{x}{a+bx^2+cx^4}\,dx=\frac{\operatorname{atan}\left(\frac{2\,a\,c\,x^2+a\,b}{a\,\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

input
```
int(x/(a + b*x^2 + c*x^4),x)
```

output
```
atan((a*b + 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.64

$$\int \frac{x}{a+bx^2+cx^4}\,dx$$
$$= -\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\left(atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)+atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}+2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)\right)}{4ac-b^2}$$

input
```
int(x/(c*x^4+b*x^2+a),x)
```

output
```
( - sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*(atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b)) + atan((s
qrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))))/(
4*a*c - b**2)
```

3.767 $\int \frac{1}{x(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{1}{x(a+bx^2+cx^4)}\,dx = \frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log\left(a+bx^2+cx^4\right)}{4a}$$

output
```
1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+ln(x)/a
-1/4*ln(c*x^4+b*x^2+a)/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int \frac{1}{x(a+bx^2+cx^4)}\,dx = \frac{4\sqrt{b^2-4ac}\log(x) - \left(b+\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right) + \left(b-\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}\right.}{4a\sqrt{b^2-4ac}}$$

input
```
Integrate[1/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
(4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1434, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^2+cx^4)}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{1}{x^2(cx^4+bx^2+a)}dx^2$$

$$\Big\downarrow \text{ 1144}$$

$$\frac{1}{2}\left(\frac{\int -\frac{cx^2+b}{cx^4+bx^2+a}dx^2}{a}+\frac{\log(x^2)}{a}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\int \frac{cx^2+b}{cx^4+bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\frac{1}{2}b\int \frac{1}{cx^4+bx^2+a}dx^2+\frac{1}{2}\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\frac{1}{2}\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2-b\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{a}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\int\frac{2cx^2+b}{cx^4+bx^2+a}dx^2-\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a}-\frac{\frac{1}{2}\log\left(a+bx^2+cx^4\right)-\frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}\right)$$

input
```
Int[1/(x*(a + b*x^2 + c*x^4)),x]
```

output
```
(Log[x^2]/a - (-((b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4
*a*c]) + Log[a + b*x^2 + c*x^4]/2)/a)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\frac{\ln\left(c\,x^4+b\,x^2+a\right)}{2}+\frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a}$	65
risch	$\frac{\ln(x)}{a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4c\,a^2-b^2a\right)\_Z^2+\left(4ac-b^2\right)\_Z+c\right)}\_R\ln\left(\left(\left(10ac-3b^2\right)\_R+5c\right)x^2-ab\_R+2b\right)\right)}{2}$	77

input
```
int(1/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-1/2/a*(1/2*ln(c*x^4+b*x^2+a)+b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b
)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.23

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(b^2-4\,ac)\log\left(cx^4+bx^2+a\right)+4\,(b^2-4\,ac)\,\mathrm{lo}}{4\,(ab^2-4\,a^2c)}\right.$$

input
```
integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*
x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4
 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^
2 + 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^
2 - 4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2
*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(60) = 120.

Time = 2.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.67

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4a\,(4ac-b^2)} - \frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4a\,(4ac-b^2)} - \frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right) + \frac{\log(x)}{a}$$

input
```
integrate(1/x/(c*x**4+b*x**2+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**
2*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(-b
*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)
) + (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*
a**2*c*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(
b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c
)) + log(x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx=-\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a}-\frac{\log\left(cx^4+bx^2+a\right)}{4\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1
/4*log(c*x^4 + b*x^2 + a)/a + 1/2*log(x^2)/a
```

Mupad [B] (verification not implemented)

Time = 18.72 (sec) , antiderivative size = 1014, normalized size of antiderivative = 14.70

$$\int \frac{1}{x(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^2 + c*x^4)),x)
```

output
```
log(x)/a + (log(a + b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c
)) + (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*(((8*a*c - 2*b^2)^2*(10*b*c^3
 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/
(4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(
8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(16*a^2*(4*a*c - b^2)) + (b^2*(
12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*
a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b
^2*c)*((b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(
12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(
4*a*c - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b
*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c
)*(4*a*c - b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))
*(4*a*c - b^2)^(3/2))/(b^2*c^2) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)
*(((8*a*c - 2*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 -
 16*a^2*c)))/(4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(
8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(
8*a*c - 2*b^2))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(b^2*c^4*(25*a*
c - 6*b^2)) - (2*(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2
)/(16*a^2*(4*a*c - b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 -
 16*a^2*c)^2*(4*a*c - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.58

$$\int \frac{1}{x\,(a+bx^2+cx^4)}\,dx$$
$$= \frac{2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)b+2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}{\sqrt{2\sqrt{c}\,\sqrt{}}}\right.}{}$$

input
```
int(1/x/(c*x^4+b*x^2+a),x)
```

output
```
(2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b + 2*sqrt(
2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b - 4*log( - sqrt(2
*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c + log( - sqrt(2*sqrt
(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**2 - 4*log(sqrt(2*sqrt(c)*s
qrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c + log(sqrt(2*sqrt(c)*sqrt(a) -
 b)*x + sqrt(a) + sqrt(c)*x**2)*b**2 + 16*log(x)*a*c - 4*log(x)*b**2)/(4*a
*(4*a*c - b**2))
```

3.768 $\int \frac{1}{x^3(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{1}{x^3(a+bx^2+cx^4)}\,dx=-\frac{1}{2ax^2}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx^2+cx^4)}{4a^2}$$

output
```
-1/2/a/x^2-1/2*(-2*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-
4*a*c+b^2)^(1/2)-b*ln(x)/a^2+1/4*b*ln(c*x^4+b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^3(a+bx^2+cx^4)}\,dx=\frac{-\frac{2a}{x^2}-4b\log(x)+\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a^2}$$

input
```
Integrate[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^
2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^
2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1434, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)} dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int \frac{1}{x^4 (cx^4 + bx^2 + a)} dx^2$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{2} \left(\frac{\int -\frac{cx^2+b}{x^2(cx^4+bx^2+a)} dx^2}{a} - \frac{1}{ax^2} \right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(-\frac{\int \frac{cx^2+b}{x^2(cx^4+bx^2+a)} dx^2}{a} - \frac{1}{ax^2} \right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2} \left(-\frac{\int \left(\frac{b}{ax^2} + \frac{-b^2-cx^2b+ac}{a(cx^4+bx^2+a)} \right) dx^2}{a} - \frac{1}{ax^2} \right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log(a+bx^2+cx^4)}{2a}+\frac{b\log(x^2)}{a}}{a}-\frac{1}{ax^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

output
```
(-(1/(a*x^2)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(a*Sqrt[b^2 - 4*a*c]) + (b*Log[x^2])/a - (b*Log[a + b*x^2 + c*x^4])/(2*a))
/a)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
default	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}-\frac{-\frac{b\ln\left(c\,x^4+b\,x^2+a\right)}{2}+\frac{2\left(ac-\frac{b^2}{2}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a^2}$
risch	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c-a^2b^2\right)\_Z^2+\left(-4abc+b^3\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(10a^3c-3a^2b^2\right)\_R^2-4\_Rabc+2c^2\right)x^2-\right.\right.}{2}$

input
```
int(1/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2-b*ln(x)/a^2-1/2/a^2*(-1/2*b*ln(c*x^4+b*x^2+a)+2*(a*c-1/2*b^2)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.29

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx$$
$$=\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(b^3-4\,abc)x^2\log\left(cx^4+bx^2+a\right)}{4\,(a^2b^2-4\,a^3c)x^2}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x^2\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)x^2\log\left(cx^4+bx^2+a\right)+4\,(b^3-}{4\,(a^2b^2-4\,a^3c)x^2}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^
2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 -
 4*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*
b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2
+ 4*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^
3 - 4*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2
*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(82) = 164$.

Time = 80.87 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.88

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx = \left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\cdot\left(2ac-b^2\right)}{4a^2\cdot\left(4ac-b^2\right)}\right)\log\left(x^2 + \frac{-8a^3c\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2\cdot(4ac-b^2)}\right) + 2a^2b^2\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2\cdot(4ac-b^2)}\right.}{2ac^2-b^2c}\right. + \left(\frac{b}{4a^2} + \frac{\sqrt{-4ac+b^2}\cdot\left(2ac-b^2\right)}{4a^2\cdot\left(4ac-b^2\right)}\right)\log\left(x^2 + \frac{-8a^3c\left(\frac{b}{4a^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2\cdot(4ac-b^2)}\right) + 2a^2b^2\left(\frac{b}{4a^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2\cdot(4ac-b^2)}\right.}{2ac^2-b^2c}\right. - \frac{1}{2ax^2} - \frac{b\log\left(x\right)}{a^2}$$

input
```
integrate(1/x**3/(c*x**4+b*x**2+a),x)
```

output
```
(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*
log(x**2 + (-8*a**3*c*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
a**2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a
*c - b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c))
 + (b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)
))*log(x**2 + (-8*a**3*c*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*
c)) - 1/(2*a*x**2) - b*log(x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx=\frac{b\log\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*b*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*
arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b
*x^2 - a)/(a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 19.36 (sec) , antiderivative size = 2033, normalized size of antiderivative = 22.84

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)} dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^2 + c*x^4)),x)
```

output
```
(atan((16*a^6*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(c^5/a^3 + ((2*b^3 - 8*a
*b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^
3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c -
 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - (
(((2*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40
*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a
*c - b^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a
*c - b^2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^
2))/(4*a^2*(4*a*c - b^2)^(1/2)) - ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^
3*b^3*c^2)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2)))
)/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((((2*b^3 - 8*a*b*c)*(((2*a
*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b
*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b
^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b
^2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4
*a^2*b^2)) - ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^3)/(64*a^9*(4*
a*c - b^2)^(3/2)) + (((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*
a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*
a^3*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(
4*a^2*(4*a*c - b^2)^(1/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 409, normalized size of antiderivative = 4.60

$$\int \frac{1}{x^3 (a+bx^2+cx^4)}\,dx$$
$$= \frac{4\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)ac\,x^2-2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}}}{\sqrt{2}}\right.}{}$$

input
```
int(1/x^3/(c*x^4+b*x^2+a),x)
```

output
```
(4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*x**2 -
2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*x**2 +
4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*x**2 - 2
*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*x**2 + 4
*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*c*x**2
 - log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**3*x**
2 + 4*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*c*x*
*2 - log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**3*x**2
 - 16*log(x)*a*b*c*x**2 + 4*log(x)*b**3*x**2 - 8*a**2*c + 2*a*b**2)/(4*a**
2*x**2*(4*a*c - b**2))
```

3.769 $\int \frac{1}{x^5(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 114

$$\int \frac{1}{x^5(a+bx^2+cx^4)}\,dx = -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4a^3}$$

output
```
-1/4/a/x^4+1/2*b/a^2/x^2+1/2*b*(-3*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^
2)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)+(-a*c+b^2)*ln(x)/a^3-1/4*(-a*c+b^2)*ln(c*
x^4+b*x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.65

$$\int \frac{1}{x^5(a+bx^2+cx^4)}\,dx = \frac{-\frac{a^2}{x^4}+\frac{2ab}{x^2}+4(b^2-ac)\log(x)-\frac{\left(b^3-3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b^3-3abc-b^2\sqrt{b^2-4ac}+ac\sqrt{\vphantom{b}}\right.}{\sqrt{b}}}{4a^3}$$

input
```
Integrate[1/(x^5*(a + b*x^2 + c*x^4)),x]
```

output
```
(-(a^2/x^4) + (2*a*b)/x^2 + 4*(b^2 - a*c)*Log[x] - ((b^3 - 3*a*b*c + b^2*S
qrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*
x^2])/Sqrt[b^2 - 4*a*c] + ((b^3 - 3*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + a*c*Sq
rt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/
(4*a^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1434, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^2+cx^4)} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int \frac{1}{x^6 (cx^4+bx^2+a)} dx^2$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{2} \left(\frac{\int -\frac{cx^2+b}{x^4(cx^4+bx^2+a)} dx^2}{a} - \frac{1}{2ax^4} \right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(-\frac{\int \frac{cx^2+b}{x^4(cx^4+bx^2+a)} dx^2}{a} - \frac{1}{2ax^4} \right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2} \left(-\frac{\int \left(\frac{b}{ax^4} + \frac{c(b^2-ac)x^2+b(b^2-2ac)}{a^2(cx^4+bx^2+a)} + \frac{ac-b^2}{a^2x^2} \right) dx^2}{a} - \frac{1}{2ax^4} \right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{-\frac{b(b^2-3ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{\log(x^2)(b^2-ac)}{a^2}+\frac{(b^2-ac)\log(a+bx^2+cx^4)}{2a^2}-\frac{b}{ax^2}}{a}-\frac{1}{2ax^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^2 + c*x^4)),x]
```

output
```
(-1/2*1/(a*x^4) - (-(b/(a*x^2)) - (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/S
qrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - a*c)*Log[x^2])/a^2 +
((b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(2*a^2))/a)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

method	result
default	$-\frac{1}{4a\,x^{4}}+\frac{\left(-ac+b^{2}\right)\ln\left(x\right)}{a^{3}}+\frac{b}{2a^{2}x^{2}}+\frac{\frac{\left(c^{2}a-b^{2}c\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(2abc-b^{3}-\frac{\left(c^{2}a-b^{2}c\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{3}}$
risch	$\frac{\frac{b\,x^{2}}{2a^{2}}-\frac{1}{4a}}{x^{4}}-\frac{\ln\left(x\right)c}{a^{2}}+\frac{\ln\left(x\right)b^{2}}{a^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^{4}-a^{3}b^{2}\right)\_Z^{2}+\left(-4a^{2}c^{2}+5a\,b^{2}c-b^{4}\right)\_Z+c^{3}\right)}\_R\ln\left(\left(\left(10c\,a^{5}-3a^{4}b^{2}\right)\_R^{2}\right.\right.\right.}{2}$

input
```
int(1/x^5/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/x^4+(-a*c+b^2)*ln(x)/a^3+1/2*b/a^2/x^2+1/2/a^3*(1/2*(a*c^2-b^2*c)/c
*ln(c*x^4+b*x^2+a)+2*(2*a*b*c-b^3-1/2*(a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.28

$$\int\frac{1}{x^{5}\left(a+bx^{2}+cx^{4}\right)}\,dx$$
$$=\left[-\frac{(b^{3}-3\,abc)\sqrt{b^{2}-4\,ac}x^{4}\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac-(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)+(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2})x^{4}\log\left(cx^{4}\right.}{4\,(a^{3}b^{2}-4\,a^{4}c)x^{4}}\right.$$

input
```
integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^4 + 2*b*c*x^2 +
b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4
 - 5*a*b^2*c + 4*a^2*c^2)*x^4*log(c*x^4 + b*x^2 + a) - 4*(b^4 - 5*a*b^2*c
+ 4*a^2*c^2)*x^4*log(x) + a^2*b^2 - 4*a^3*c - 2*(a*b^3 - 4*a^2*b*c)*x^2)/(
(a^3*b^2 - 4*a^4*c)*x^4), 1/4*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^4*ar
ctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^4 - 5*a*b^2*c +
 4*a^2*c^2)*x^4*log(c*x^4 + b*x^2 + a) + 4*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x
^4*log(x) - a^2*b^2 + 4*a^3*c + 2*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2 - 4*a
^4*c)*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(107) = 214$.

Time = 175.64 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.71

$$\int \frac{1}{x^5 (a + bx^2 + cx^4)} \, dx = \left(-\frac{b\sqrt{-4ac + b^2} \cdot (3ac - b^2)}{4a^3 \cdot (4ac - b^2)} + \frac{ac - b^2}{4a^3} \right) \log \left(x^2 + \frac{8a^4c \left(-\frac{b\sqrt{-4ac+b^2} \cdot (3ac-b^2)}{4a^3 \cdot (4ac-b^2)} + \frac{ac-b^2}{4a^3} \right) - 2a^3b^2 \left(-\frac{b\sqrt{-4ac+b^2} \cdot (3ac-b^2)}{4a^3 \cdot (4ac-b^2)} + \frac{ac-b^2}{4a^3} \right) - 2a^2c^2}{3abc^2 - b^3c} \right. + \left(\frac{b\sqrt{-4ac + b^2} \cdot (3ac - b^2)}{4a^3 \cdot (4ac - b^2)} + \frac{ac - b^2}{4a^3} \right) \log \left(x^2 + \frac{8a^4c \left(\frac{b\sqrt{-4ac+b^2} \cdot (3ac-b^2)}{4a^3 \cdot (4ac-b^2)} + \frac{ac-b^2}{4a^3} \right) - 2a^3b^2 \left(\frac{b\sqrt{-4ac+b^2} \cdot (3ac-b^2)}{4a^3 \cdot (4ac-b^2)} + \frac{ac-b^2}{4a^3} \right) - 2a^2c^2 + 4}{3abc^2 - b^3c} \right. + \frac{-a + 2bx^2}{4a^2x^4} - \frac{(ac - b^2) \log(x)}{a^3}$$

input
```
integrate(1/x**5/(c*x**4+b*x**2+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*a**3*(4*a*c - b**2)) + (a*c - b*
*2)/(4*a**3))*log(x**2 + (8*a**4*c*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/
(4*a**3*(4*a*c - b**2)) + (a*c - b**2)/(4*a**3)) - 2*a**3*b**2*(-b*sqrt(-4
*a*c + b**2)*(3*a*c - b**2)/(4*a**3*(4*a*c - b**2)) + (a*c - b**2)/(4*a**3
)) - 2*a**2*c**2 + 4*a*b**2*c - b**4)/(3*a*b*c**2 - b**3*c)) + (b*sqrt(-4*
a*c + b**2)*(3*a*c - b**2)/(4*a**3*(4*a*c - b**2)) + (a*c - b**2)/(4*a**3)
)*log(x**2 + (8*a**4*c*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*a**3*(4*a*
c - b**2)) + (a*c - b**2)/(4*a**3)) - 2*a**3*b**2*(b*sqrt(-4*a*c + b**2)*(
3*a*c - b**2)/(4*a**3*(4*a*c - b**2)) + (a*c - b**2)/(4*a**3)) - 2*a**2*c*
*2 + 4*a*b**2*c - b**4)/(3*a*b*c**2 - b**3*c)) + (-a + 2*b*x**2)/(4*a**2*x
**4) - (a*c - b**2)*log(x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 (a + bx^2 + cx^4)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^5 (a + bx^2 + cx^4)} \, dx = -\frac{(b^2 - ac) \log (cx^4 + bx^2 + a)}{4 a^3} + \frac{(b^2 - ac) \log (x^2)}{2 a^3} - \frac{(b^3 - 3 abc) \arctan \left(\frac{2 cx^2 + b}{\sqrt{-b^2 + 4 ac}}\right)}{2 \sqrt{-b^2 + 4 ac} a^3} - \frac{3 b^2 x^4 - 3 acx^4 - 2 abx^2 + a^2}{4 a^3 x^4}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2 - a*c)*log(x^2)/a^3
 - 1/2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
 + 4*a*c)*a^3) - 1/4*(3*b^2*x^4 - 3*a*c*x^4 - 2*a*b*x^2 + a^2)/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 19.41 (sec) , antiderivative size = 2451, normalized size of antiderivative = 21.50

$$\int \frac{1}{x^5 (a + bx^2 + cx^4)} dx = \text{Too large to display}$$

input
```
int(1/(x^5*(a + b*x^2 + c*x^4)),x)
```

output
```
(log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4
*a^3*b^2)) - (1/(4*a) - (b*x^2)/(2*a^2))/x^4 - (log(x)*(a*c - b^2))/a^3 +
(b*atan((2*a^6*(4*a*c - b^2)*((((b*(3*a*c - b^2)*((4*a^4*b^4*c^2 - 8*a^5*b
^2*c^3)/a^6 - (2*a*b^2*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a^4*c - 4
*a^3*b^2)))/(4*a^3*(4*a*c - b^2)^(1/2)) - (b^3*c^2*(3*a*c - b^2)*(2*b^4 +
8*a^2*c^2 - 10*a*b^2*c))/(2*a^2*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)
))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)) + (b^5*c^2
*(3*a*c - b^2)^3)/(16*a^8*(4*a*c - b^2)^(3/2)) + (b*(3*a*c - b^2)*((4*a^2*
b^4*c^3 - 5*a^3*b^2*c^4)/a^6 + (((4*a^4*b^4*c^2 - 8*a^5*b^2*c^3)/a^6 - (2*
a*b^2*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a^4*c - 4*a^3*b^2))*(2*b^4
 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2))))/(4*a^3*(4*a*c - b
^2)^(1/2)))*(3*b^6 - 10*a^3*c^3 + 27*a^2*b^2*c^2 - 18*a*b^4*c))/(c^2*(b^6*
c^2 - 6*a*b^4*c^3 + 9*a^2*b^2*c^4)*(6*b^6 - 25*a^3*c^3 + 54*a^2*b^2*c^2 -
36*a*b^4*c)) - (16*a^9*x^2*((3*b*(b^4 + 3*a^2*c^2 - 4*a*b^2*c)*((((5*a^3*b
*c^5 - 6*a^2*b^3*c^4)/a^6 - (((10*a^5*b*c^4 + 2*a^4*b^3*c^3)/a^6 + ((40*a^
7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*a^6*(16*a^4
*c - 4*a^3*b^2)))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b
^2)))*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a^4*c - 4*a^3*b^2)) - (b^3*
c^5)/a^6 + (b*(3*a*c - b^2)*((b*((10*a^5*b*c^4 + 2*a^4*b^3*c^3)/a^6 + ((40
*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*a^6*(...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.54

$$\int \frac{1}{x^5 (a + bx^2 + cx^4)} \, dx$$
$$= \frac{-6\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)abc\,x^4 + 2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2}}{}\right.}{}$$

input
```
int(1/x^5/(c*x^4+b*x^2+a),x)
```

output
```
( - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*x*
*4 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*x*
*4 - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*x
**4 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*x
**4 + 4*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a**
2*c**2*x**4 - 5*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x
**2)*a*b**2*c*x**4 + log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt
(c)*x**2)*b**4*x**4 + 4*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt
(c)*x**2)*a**2*c**2*x**4 - 5*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) +
 sqrt(c)*x**2)*a*b**2*c*x**4 + log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a)
 + sqrt(c)*x**2)*b**4*x**4 - 16*log(x)*a**2*c**2*x**4 + 20*log(x)*a*b**2*c
*x**4 - 4*log(x)*b**4*x**4 - 4*a**3*c + a**2*b**2 + 8*a**2*b*c*x**2 - 2*a*
b**3*x**2)/(4*a**3*x**4*(4*a*c - b**2))
```

3.770 $\int \frac{x^6}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 203

$$\int \frac{x^6}{a+bx^2+cx^4}\,dx = -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-b*x/c^2+1/3*x^3/c+1/2*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(
2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)+1/2*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(
2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.23

$$\int \frac{x^6}{a+bx^2+cx^4}\,dx$$
$$= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(b^3-3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[x^6/(a + b*x^2 + c*x^4),x]
```

output
```
-((b*x)/c^2) + x^3/(3*c) + ((-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3
- 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1442, 27, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \; 1442$$
$$\frac{x^3}{3c}-\frac{\int \frac{3x^2(bx^2+a)}{cx^4+bx^2+a}dx}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3}{3c}-\frac{\int \frac{x^2(bx^2+a)}{cx^4+bx^2+a}dx}{c}$$

$$\Big\downarrow \text{1602}$$

$$\frac{x^3}{3c}-\frac{\frac{bx}{c}-\frac{\int \frac{(b^2-ac)x^2+ab}{cx^4+bx^2+a}dx}{c}}{c}$$

$$\Big\downarrow \text{1480}$$

$$\frac{x^3}{3c}-\frac{\frac{bx}{c}-\frac{\frac{1}{2}\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}}{c}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^3}{3c}-\frac{\frac{bx}{c}-\frac{\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{c}$$

input
```
Int[x^6/(a + b*x^2 + c*x^4),x]
```

output
```
x^3/(3*c) - ((b*x)/c - (((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]
*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.36

method	result
risch	$\frac{x^3}{3c}-\frac{bx}{c^2}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^4c+\_Z^2b+a\right)}\frac{\left(\left(-ac+b^2\right)\_R^2+ab\right)\ln\left(x-\_R\right)}{2\_R^3c+\_Rb}}{2c^2}$
default	$-\frac{-\frac{1}{3}c\,x^3+bx}{c^2}+\frac{\frac{\left(-ac\sqrt{-4ac+b^2}+b^2\sqrt{-4ac+b^2}-3abc+b^3\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-ac\sqrt{-4ac+b^2}+b^2\sqrt{-4ac+b^2}+3abc-b^3\right)}{2c\sqrt{-4ac+b^2}\,\sqrt{\left(-b\right.}}}{c}$

input
```
int(x^6/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/c-b*x/c^2+1/2/c^2*sum(((-a*c+b^2)*_R^2+a*b)/(2*_R^3*c+_R*b)*ln(x-_
R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. $2(167) = 334$.

Time = 0.13 (sec) , antiderivative size = 1564, normalized size of antiderivative = 7.70

$$\int \frac{x^6}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2
*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^
3*b^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3
*b*c^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 -
5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4
*a*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^
5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*
c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b
^2*c + a^4*c^2)*x - sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*
c^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*
c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 -
 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*
c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*...
```

Sympy [A] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.96

$$\int \frac{x^6}{a+bx^2+cx^4}\,dx = -\frac{bx}{c^2}$$
$$+\mathrm{RootSum}\left(t^4\cdot\left(256a^2c^7-128ab^2c^6+16b^4c^5\right)+t^2\left(-80a^3bc^3+100a^2b^3c^2-36ab^5c+4b^7\right)+a^5,\left(t\mapsto\right.\right.$$
$$+\frac{x^3}{3c}$$

input
```
integrate(x**6/(c*x**4+b*x**2+a),x)
```

output
```
-b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**5)
 + _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + a
**5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*b**2*c**6 - 8
*_t**3*b**4*c**5 + 14*_t*a**3*b*c**3 - 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5
*c - 2*_t*b**7)/(a**4*c**2 - 3*a**3*b**2*c + a**2*b**4))))) + x**3/(3*c)
```

Maxima [F]

$$\int \frac{x^6}{a+bx^2+cx^4}\,dx = \int \frac{x^6}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/3*(c*x^3 - 3*b*x)/c^2 - integrate(-((b^2 - a*c)*x^2 + a*b)/(c*x^4 + b*x^
2 + a), x)/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2457 vs. $2(167) = 334$.

Time = 0.67 (sec) , antiderivative size = 2457, normalized size of antiderivative = 12.10

$$\int \frac{x^6}{a + bx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^
2*c^5 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*
b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2...
```

Mupad [B] (verification not implemented)

Time = 19.05 (sec) , antiderivative size = 4127, normalized size of antiderivative = 20.33

$$\int \frac{x^6}{a+bx^2+cx^4} \, dx = \text{Too large to display}$$

input
```
int(x^6/(a + b*x^2 + c*x^4),x)
```

output
```
x^3/(3*c) - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 1
6*a*b*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b
^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-
(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c
^3 + 9*a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2
) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*
a*b^2*c^6)))^(1/2)*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c
^5 - 16*a*b*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25
*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c
^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2
+ a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2
*a^3*c^3 + 9*a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3
)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
 - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 761, normalized size of antiderivative = 3.75

$$\int \frac{x^6}{a+bx^2+cx^4} \, dx = \text{Too large to display}$$

input
```
int(x^6/(c*x^4+b*x^2+a),x)
```

output
```
(12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2 - 6*sqrt(a)*sqrt(2*sqrt
(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*s
qrt(c)*sqrt(a) + b))*b**2*c - 18*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a
*b*c + 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
- b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3 - 12*sqrt(a)*sqrt(2*
sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt
(2*sqrt(c)*sqrt(a) + b))*a*c**2 + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b)
)*b**2*c + 18*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqr
t(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c - 6*sqrt(c)*sq
rt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)
/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3 - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)
*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c**2 + 3
*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x
+ sqrt(a) + sqrt(c)*x**2)*b**2*c + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*l
og(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c**2 - 3*sqrt
(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a
) + sqrt(c)*x**2)*b**2*c - 9*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( -...
```

3.771 $\int \frac{x^4}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 179

$$\int \frac{x^4}{a+bx^2+cx^4}\,dx = \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
x/c-1/2*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*
(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^
2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.13

$$\int \frac{x^4}{a+bx^2+cx^4}\,dx = \frac{x}{c}-\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[x^4/(a + b*x^2 + c*x^4),x]
```

output
```
x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1442, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+bx^2+cx^4} \, dx$$

$$\Big\downarrow \text{ 1442}$$

$$\frac{x}{c}-\frac{\int \frac{bx^2+a}{cx^4+bx^2+a}dx}{c}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{x}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}$$

input
```
Int[x^4/(a + b*x^2 + c*x^4),x]
```

output
```
x/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.32

method	result
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}c+\_Z^{2}b+a\right)}\frac{\left(-\_R^{2}b-a\right)\ln\left(x-\_R\right)}{2\_R^{3}c+\_Rb}}{2c}$
default	$\frac{x}{c}+\frac{\left(-b\sqrt{-4ac+b^{2}}+2ac-b^{2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2c\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\left(-b\sqrt{-4ac+b^{2}}-2ac+b^{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right.}{2c\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}$

input
```
int(x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/2/c*sum((-_R^2*b-a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b
+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(143) = 286$.

Time = 0.08 (sec) , antiderivative size = 1059, normalized size of antiderivative = 5.92

$$\int \frac{x^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b
^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*
c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3
*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*
c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2
*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4
*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*
sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sq
rt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3
- 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*l
og(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^
3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*s...
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{a + bx^2 + cx^4} \, dx$$
$$= \text{RootSum}\left(t^4 \cdot \left(256a^2c^5 - 128ab^2c^4 + 16b^4c^3\right) + t^2 \cdot \left(48a^2bc^2 - 28ab^3c + 4b^5\right) + a^3, \left(t \mapsto t \log\left(x + \frac{32\right.\right.\right.$$
$$+ \frac{x}{c}$$

input
```
integrate(x**4/(c*x**4+b*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48
*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t*
*3*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b*
*4)/(a**2*c - a*b**2)))) + x/c
```

Maxima [F]

$$\int \frac{x^4}{a+bx^2+cx^4}\,dx=\int \frac{x^4}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2109 vs. $2(143) = 286$.

Time = 0.62 (sec) , antiderivative size = 2109, normalized size of antiderivative = 11.78

$$\int \frac{x^4}{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate(x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c
^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*
(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*s...
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 3026, normalized size of antiderivative = 16.91

$$\int \frac{x^4}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^4/(a + b*x^2 + c*x^4),x)
```

output
```
x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)
*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c
)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) -
 (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c +
 (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4
*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^
4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b
^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/(((
(16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 ...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 547, normalized size of antiderivative = 3.06

$$\int \frac{x^4}{a+bx^2+cx^4}\,dx$$

$$= \frac{2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)bc+4\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)ac-2\sqrt{c}\,\sqrt{2}}{}$$

input
```
int(x^4/(c*x^4+b*x^2+a),x)
```

output
```
(2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c + 4*sqrt(c)*sqrt(2*sqrt(c)*
sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2 - 2*
sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*
sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c - 4*sqrt(c)*sqrt(2*sqrt(c)*sqr
t(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + b))*a*c + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2 - sqrt(
a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt
(a) + sqrt(c)*x**2)*b*c + sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*s
qrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b*c + 2*sqrt(c)*sqrt(2*sqr
t(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)
*x**2)*a*c - sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqr
t(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**2 - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt
(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c +
 sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + s
qrt(a) + sqrt(c)*x**2)*b**2 + 16*a*c**2*x - 4*b**2*c*x)/(4*c**2*(4*a*c - b
**2))
```

3.772 $\int \frac{x^2}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\int \frac{x^2}{a+bx^2+cx^4} dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

output
```
-1/2*(b-(-4*a*c+b^2)^(1/2))^(1/2)*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)
^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*(b+(-4*a*c+b^2)^(1/2
))^(1/2)*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^
(1/2)/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{a+bx^2+cx^4} dx = \frac{\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

input
```
Integrate[x^2/(a + b*x^2 + c*x^4),x]
```

output
```
((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
 (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1450, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 1450$$

$$\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx$$

$$\Big\downarrow \; 218$$

$$\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

input
```
Int[x^2/(a + b*x^2 + c*x^4),x]
```

output
```
((1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((1 + b/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4c+\_Z^2b+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^3c+\_Rb}\right)}{2}$	41
default	$4c\left(\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$	149

input
```
int(x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R^2/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(115) = 230$.

Time = 0.09 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.73

$$\int \frac{x^2}{a+bx^2+cx^4}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)$$

input
```
integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
 - 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/
sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) -
 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*
c - 4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2
)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
 - 1/2*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^
2*c - 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^
2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x
) + 1/2*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b
^2*c - 4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*
c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) +
 x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.50

$$\int \frac{x^2}{a+bx^2+cx^4}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(256a^2c^3-128ab^2c^2+16b^4c\right)+t^2\left(-16abc+4b^3\right)+a,\left(t\mapsto t\log\left(64t^3ac^2-16t^3b^2c-2t\right.\right.\right.$$

input
```
integrate(x**2/(c*x**4+b*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(-16*a
*b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c -
2*_t*b + x)))
```

Maxima [F]

$$\int \frac{x^2}{a+bx^2+cx^4}\,dx = \int \frac{x^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(115) = 230.

Time = 0.69 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.35

$$\int \frac{x^2}{a+bx^2+cx^4}\,dx =$$

$$-\frac{\left(2\,b^2c^2-8\,ac^3-\sqrt{2}\sqrt{b^2-4\,ac}\sqrt{bc+\sqrt{b^2-4\,ac}c}b^2+4\,\sqrt{2}\sqrt{b^2-4\,ac}\sqrt{bc+\sqrt{b^2-4\,ac}c}ac+2\,\sqrt{2}\sqrt{}\right.}{2\,(b^4-8\,ab^2c-2\,b^3c+1}$$

$$-\frac{\left(2\,b^2c^2-8\,ac^3-\sqrt{2}\sqrt{b^2-4\,ac}\sqrt{bc-\sqrt{b^2-4\,ac}c}b^2+4\,\sqrt{2}\sqrt{b^2-4\,ac}\sqrt{bc-\sqrt{b^2-4\,ac}c}ac+2\,\sqrt{2}\sqrt{}\right.}{2\,(b^4-8\,ab^2c-2\,b^3c+1}$$

input
```
integrate(x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 -
 4*a*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((b^4 -
 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c))
 - 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*
c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2
 - 4*a*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((b^4
 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c
))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.77

$$\int \frac{x^2}{a+bx^2+cx^4}\,dx =$$

$$-2\,\mathrm{atanh}\left(\frac{\left(x\left(4\,a\,c^2-2\,b^2\,c\right)+\frac{x\left(8\,b^3\,c^2-32\,a\,b\,c^3\right)\left(b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\right)}{8\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\sqrt{-\frac{b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c}{8\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{a\,c}\right)\sqrt{}$$

$$-2\,\mathrm{atanh}\left(\frac{\left(x\left(4\,a\,c^2-2\,b^2\,c\right)-\frac{x\left(8\,b^3\,c^2-32\,a\,b\,c^3\right)\left(\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c\right)}{8\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\sqrt{\frac{\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c}{8\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{a\,c}\right)\sqrt{\frac{\sqrt{}}{8}}$$

input

```
int(x^2/(a + b*x^2 + c*x^4),x)
```

output

```
- 2*atanh(((x*(4*a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-(
4*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*
(-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*
b^2*c^2)))^(1/2))/(a*c))*(-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*b^2*c
) - (x*(8*b^3*c^2 - 32*a*b*c^3)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c)
)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(((-(4*a*c - b^2)^3)^(1/2) - b^3
 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(a*c))*(((-(4*a
*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.34

$$\int \frac{x^2}{a + bx^2 + cx^4} \, dx$$
$$= \frac{-4\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)c + 2\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)b + 4\sqrt{a}\,\sqrt{2}$$

input
```
int(x^2/(c*x^4+b*x^2+a),x)
```

output
```
( - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b
) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*c + 2*sqrt(c)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*b + 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*c - 2*sqrt
(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt
(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) -
 b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*c - 2*s
qrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqr
t(a) + sqrt(c)*x**2)*c + sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2
*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b - sqrt(c)*sqrt(2*sqrt(
c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2
)*b)/(4*c*(4*a*c - b**2))
```

3.773 $\int \frac{1}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{a+bx^2+cx^4}\,dx=\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
2^(1/2)*c^(1/2)*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))/(-4
*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2^(1/2)*c^(1/2)*arctan(2^(1/2
)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+bx^2+cx^4}\,dx=\frac{\sqrt{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/
Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1406, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1406}$$

$$\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

input
```
Int[(a + b*x^2 + c*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

method	result	size
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4 c+\_Z^2 b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^3 c+\_R b}\right)}{2}$	38
default	$4c\left(-\frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$	117

input
```
int(1/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(114) = 228$.

Time = 0.09 (sec) , antiderivative size = 613, normalized size of antiderivative = 4.09

$$\int \frac{1}{a+bx^2+cx^4}\,dx = -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx + \sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx - \sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx + \sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx - \sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

input
```
integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^
2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqr
t(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)
)/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^
2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c -
(a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/
sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a
*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sq
rt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-
(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*
sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4
*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2
*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*
b^2 - 4*a^2*c)))
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.58

$$\int \frac{1}{a+bx^2+cx^4}\,dx$$
$$= \operatorname{RootSum}\left(t^4\cdot\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3a\right.\right.\right.$$

input
```
integrate(1/(c*x**4+b*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a
*b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b
**3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Maxima [F]

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \int \frac{1}{cx^4+bx^2+a}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(114) = 228$.

Time = 0.32 (sec) , antiderivative size = 1026, normalized size of antiderivative = 6.84

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2
 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*
a^2*c^3)*abs(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2...
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 763, normalized size of antiderivative = 5.09

$$\int \frac{1}{a+bx^2+cx^4}\,dx =$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right.$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2}{4\,a\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-}\right.$$

input
```
int(1/(a + b*x^2 + c*x^4),x)
```

output
```
- atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 -
64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))
^(1/2) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(
b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8
*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*
b^2*c*x*8i)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^
3*c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a
*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6
- 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b
^2*c))^(1/2)*2i
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.34

$$\int \frac{1}{a+bx^2+cx^4}\,dx$$
$$= \frac{2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)b-4\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)a-2\sqrt{a}\,\sqrt{2\sqrt{}}}{}$$

input
```
int(1/(c*x^4+b*x^2+a),x)
```

output
```
(2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b - 4*sqrt(c)*sqrt(2*sqrt(c)*sq
rt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)
*sqrt(a) + b))*a - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b + 4*sqrt(c)
*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a - sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*l
og( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b + sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) +
sqrt(c)*x**2)*b - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt
(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a + 2*sqrt(c)*sqrt(2*sqrt(c)*
sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a
)/(4*a*(4*a*c - b**2))
```

3.774 $\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 174

$$\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx=-\frac{1}{ax}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/a/x-1/2*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*c^(1/2
)*(1-b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))
^(1/2))*2^(1/2)/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx=-\frac{\frac{2}{x}+\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a}$$

input
```
Integrate[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/2*(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2
- 4*a*c]]))/a
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1443, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2+cx^4)} \, dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{\int -\frac{cx^2+b}{cx^4+bx^2+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{cx^2+b}{cx^4+bx^2+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax}$$

input
```
Int[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

output
```
-(1/(a*x)) - ((Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

method	result
default	$-\frac{1}{ax}+\frac{4c\left(\frac{\left(b-\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-\sqrt{-4ac+b^2}-b\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^5c^2-8b^2c\,a^4+b^4a^3\right)\_Z^4+\left(12a^2b\,c^2-7a\,b^3c+b^5\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(40a^5c^2-22b^2c\,a^4+3b^4a^3\right)\_R^4+(2\right.\right.\right.}{2}$

input
```
int(1/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/a/x+4/a*c*(1/8*(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))-1/8*(-(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1116 vs. 2(137) = 274.

Time = 0.10 (sec) , antiderivative size = 1116, normalized size of antiderivative = 6.41

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b
^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 -
6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7
*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-
(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6
*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1
/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c
 + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c
)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2
- 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2
 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a
^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
- sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c
^2 - a*c^3)*x - sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6...
```

Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2 (a+bx^2+cx^4)}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(256a^5c^2-128a^4b^2c+16a^3b^4\right)+t^2\cdot\left(48a^2bc^2-28ab^3c+4b^5\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-6\cdots}{\cdots}\right.\right.\right.$$
$$-\frac{1}{ax}$$

input
```
integrate(1/x**2/(c*x**4+b*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(48
*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t
**3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 - 10*_t*a**2*b*c*
*2 + 10*_t*a*b**3*c - 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

Maxima [F]

$$\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1839 vs. 2(137) = 274.

Time = 0.56 (sec) , antiderivative size = 1839, normalized size of antiderivative = 10.57

$$\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
-1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*
b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 -
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*
b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 +
 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32
*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*
arctan(2*sqrt(1/2)*x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3...
```

Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 2997, normalized size of antiderivative = 17.22

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^2 + c*x^4)),x)
```

output
```
- atan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^
4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(3
2*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c
^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^
2 - 8*a^4*b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3
- 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*
b^2*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*
c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
)^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 560, normalized size of antiderivative = 3.22

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)} \, dx$$
$$= \frac{4\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)acx - 2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)b^2x + 2\sqrt{c}\ \cdots}{}$$

input
```
int(1/x^2/(c*x^4+b*x^2+a),x)
```

output
```
(4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*x - 2*sqrt(a)*sqrt(2*sqrt(c
)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqr
t(c)*sqrt(a) + b))*b**2*x + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*
x - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b
) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*x + 2*sqrt(a)*sqrt(2*sqr
t(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*
sqrt(c)*sqrt(a) + b))*b**2*x - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a
*b*x - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a)
 - b)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x + sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) -
 b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**2*x
+ 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x
+ sqrt(a) + sqrt(c)*x**2)*a*c*x - sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(
sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b**2*x + sqrt(c)*s
qrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a)
+ sqrt(c)*x**2)*a*b*x - sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*sqr
t(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*x - 16*a**2*c + 4*a*b**2
)/(4*a**2*x*(4*a*c - b**2))
```

3.775 $\int \frac{1}{x^4(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 196

$$\int \frac{1}{x^4\left(a+bx^2+cx^4\right)}\,dx=-\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
-1/3/a/x^3+b/a^2/x+1/2*c^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(
2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(b-(-4*a*c+b^2
)^(1/2))^(1/2)+1/2*c^(1/2)*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1
/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(b+(-4*a*c+b^2)^(1
/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^4 (a + bx^2 + cx^4)} \, dx = \frac{-\frac{2a}{x^3} + \frac{6b}{x} + \frac{3\sqrt{2}\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\sqrt{2}\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6a^2}$$

input
```
Integrate[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*a)/x^3 + (6*b)/x + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-b^2 + 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1443, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a + bx^2 + cx^4)} \, dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{\int -\frac{3(cx^2+b)}{x^2(cx^4+bx^2+a)}dx}{3a} - \frac{1}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{cx^2+b}{x^2(cx^4+bx^2+a)}dx}{a} - \frac{1}{3ax^3}$$

$$\Big\downarrow \text{1604}$$

$$-\frac{-\frac{\int \frac{b^2+cx^2b-ac}{cx^4+bx^2+a}dx}{a}-\frac{b}{ax}}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{-\frac{\frac{1}{2}c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{a}-\frac{b}{ax}}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{218}$$

$$-\frac{-\frac{\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{b}{ax}}{a}-\frac{1}{3ax^3}$$

input
```
Int[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/3*1/(a*x^3) - (-(b/(a*x)) - ((Sqrt[c]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]))/a)/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.91

method	result
default	$-\frac{1}{3a\,x^3}+\frac{b}{a^2x}+\frac{4c\left(\frac{\left(b\sqrt{-4ac+b^2}+2ac-b^2\right)\sqrt{2}\,\operatorname{arctan}\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(b\sqrt{-4ac+b^2}-2ac+b^2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right.}}\right.}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{a^2}$
risch	$\frac{\frac{b\,x^2}{a^2}-\frac{1}{3a}}{x^3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16c^2a^7-8a^6b^2c+a^5b^4\right)\_Z^4+\left(-20a^3b\,c^3+25a^2b^3c^2-9a\,b^5c+b^7\right)\_Z^2+c^5\right)}\_R\ln\left(\left(\left((40c^2a^7-22a^6b^2c+3a\right.\right.\right.\right.}{2}$

input
```
int(1/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3+b/a^2/x+4/a^2*c*(1/8*(b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1622 vs. $2(160) = 320$.

Time = 0.12 (sec) , antiderivative size = 1622, normalized size of antiderivative = 8.28

$$\int \frac{1}{x^4 (a + bx^2 + cx^4)} dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2
- 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3*a*b^2*c
^4 + a^2*c^5)*x + sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2
*c^3 + 4*a^4*c^4 - (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*
sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a
*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))
/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(
b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x - sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4 - (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^
2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10
*b^2 - 4*a^11*c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6
*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^1
0*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) + 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5
 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2
- 4*a^6*c))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x + sqrt(1/2)*(b^8 ...
```

Sympy [A] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (a + bx^2 + cx^4)} \, dx$$
$$= \text{RootSum}\left(t^4 \cdot (256a^7c^2 - 128a^6b^2c + 16a^5b^4) + t^2(-80a^3bc^3 + 100a^2b^3c^2 - 36ab^5c + 4b^7) + c^5, \left(t \mapsto t\right.\right.$$
$$+ \frac{-a + 3bx^2}{3a^2x^3}$$

input
```
integrate(1/x**4/(c*x**4+b*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**7*c**2 - 128*a**6*b**2*c + 16*a**5*b**4) + _t**2*(-8
0*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + c**5, Lambda(
_t, _t*log(x + (-96*_t**3*a**7*b*c**2 + 56*_t**3*a**6*b**3*c - 8*_t**3*a**
5*b**5 - 4*_t*a**4*c**4 + 32*_t*a**3*b**2*c**3 - 40*_t*a**2*b**4*c**2 + 16
*_t*a*b**6*c - 2*_t*b**8)/(a**2*c**5 - 3*a*b**2*c**4 + b**4*c**3)))) + (-a
 + 3*b*x**2)/(3*a**2*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + bx^2 + cx^4)} \, dx = \int \frac{1}{(cx^4 + bx^2 + a)x^4} \, dx$$

input
```
integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((b*c*x^2 + b^2 - a*c)/(c*x^4 + b*x^2 + a), x)/a^2 + 1/3*(3*b*x^2
 - a)/(a^2*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. $2(160) = 320$.

Time = 0.74 (sec) , antiderivative size = 1640, normalized size of antiderivative = 8.37

$$\int \frac{1}{x^4 (a + bx^2 + cx^4)} dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
*c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a^
2*b^2*c^3 - 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c
^4 + 32*a^3*c^4 + 24*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^4*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 +
2*(b^2 - 4*a*c)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*c
^2 + 8*(b^2 - 4*a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*arctan(2*sqrt(1/2)
*x/sqrt((a^2*b + sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*
c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(
c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt...
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 4160, normalized size of antiderivative = 21.22

$$\int \frac{1}{x^4 (a + bx^2 + cx^4)} dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^2 + c*x^4)),x)
```

output
```
- (1/(3*a) - (b*x^2)/a^2)/x^3 - atan(((((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^
7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a
*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*
a^6*b^2*c)))^(1/2)*(16*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*((b^4
*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*
(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))
/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20*a^9*
b^2*c^3) - x*(4*a^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*a*c -
 b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c -
b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^
4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i - (((b^4*(-(4*a*c - b^2)^3)^(1/2)
 - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
+ 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2
 - 8*a^6*b^2*c)))^(1/2)*(16*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*
((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2
*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 4*a^8*b^4*c^2 - 20
*a^9*b^2*c^3) + x*(4*a^8*c^5 + 2*a^6*b^4*c^3 - 8*a^7*b^2*c^4))*((b^4*(-(4*
a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a
*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8...
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 810, normalized size of antiderivative = 4.13

$$\int \frac{1}{x^4 (a + bx^2 + cx^4)} dx = \text{Too large to display}$$

input
```
int(1/x^4/(c*x^4+b*x^2+a),x)
```

output
```
( - 18*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*x**3 + 6*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*b**3*x**3 + 12*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
+ b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a**2*c*x**3 - 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*x*
*3 + 18*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*x**3 - 6*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*x**3 - 12*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
 + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + b))*a**2*c*x**3 + 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*x
**3 + 9*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a)
- b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*c*x**3 - 3*sqrt(a)*sqrt(2*sqrt(c)*sqr
t(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b
**3*x**3 - 9*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a
) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*c*x**3 + 3*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)...
```

3.776 $\int \frac{x^7}{a+fx^2+cx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{x^7}{a+fx^2+cx^4}\,dx = -\frac{fx^2}{2c^2}+\frac{x^4}{4c}+\frac{f(3ac-f^2)\arctan\left(\frac{f+2cx^2}{\sqrt{4ac-f^2}}\right)}{2c^3\sqrt{4ac-f^2}} - \frac{(ac-f^2)\log(a+fx^2+cx^4)}{4c^3}$$

output
```
-1/2*f*x^2/c^2+1/4*x^4/c+1/2*f*(3*a*c-f^2)*arctan((2*c*x^2+f)/(4*a*c-f^2)^
(1/2))/c^3/(4*a*c-f^2)^(1/2)-1/4*(a*c-f^2)*ln(c*x^4+f*x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.87

$$\int \frac{x^7}{a+fx^2+cx^4}\,dx = \frac{cx^2(-2f+cx^2)-\frac{2f(-3ac+f^2)\arctan\left(\frac{f+2cx^2}{\sqrt{4ac-f^2}}\right)}{\sqrt{4ac-f^2}}+(-ac+f^2)\log(a+fx^2+cx^4)}{4c^3}$$

input
```
Integrate[x^7/(a + f*x^2 + c*x^4),x]
```

output
```
(c*x^2*(-2*f + c*x^2) - (2*f*(-3*a*c + f^2)*ArcTan[(f + 2*c*x^2)/Sqrt[4*a*
c - f^2]])/Sqrt[4*a*c - f^2] + (-(a*c) + f^2)*Log[a + f*x^2 + c*x^4])/(4*c
^3)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{a+cx^4+fx^2}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^6}{cx^4+fx^2+a}dx^2$$

$$\Big\downarrow \text{1143}$$

$$\frac{1}{2}\int \left(\frac{x^2}{c}-\frac{f}{c^2}+\frac{af-(ac-f^2)x^2}{c^2(cx^4+fx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{f(3ac-f^2)\arctan\left(\frac{2cx^2+f}{\sqrt{4ac-f^2}}\right)}{c^3\sqrt{4ac-f^2}}-\frac{(ac-f^2)\log(a+cx^4+fx^2)}{2c^3}-\frac{fx^2}{c^2}+\frac{x^4}{2c}\right)$$

input
```
Int[x^7/(a + f*x^2 + c*x^4),x]
```

output
```
(-((f*x^2)/c^2) + x^4/(2*c) + (f*(3*a*c - f^2)*ArcTan[(f + 2*c*x^2)/Sqrt[4
*a*c - f^2]])/(c^3*Sqrt[4*a*c - f^2]) - ((a*c - f^2)*Log[a + f*x^2 + c*x^4
])/(2*c^3))/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

method	result
default	$\frac{\frac{1}{2}c\,x^4-f\,x^2}{2c^2}+\frac{\frac{\left(-ac+f^2\right)\ln\left(c\,x^4+f\,x^2+a\right)}{2c}+\frac{2\left(af-\frac{\left(-ac+f^2\right)f}{2c}\right)\arctan\left(\frac{2c\,x^2+f}{\sqrt{4ac-f^2}}\right)}{\sqrt{4ac-f^2}}}{2c^2}$
risch	$\frac{x^4}{4c}-\frac{f\,x^2}{2c^2}+\frac{f^2}{4c^3}-\frac{\ln\left(\left(\left(12a^2c^2f-7ac\,f^3+f^5+\sqrt{-f^2(4ac-f^2)(3ac-f^2)^2}\,f\right)x^2+2\sqrt{-f^2(4ac-f^2)(3ac-f^2)^2}\,a\right)a^2}{c(4ac-f^2)}+\frac{5\ln}{}$

input
```
int(x^7/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/c^2*(1/2*c*x^4-f*x^2)+1/2/c^2*(1/2*(-a*c+f^2)/c*ln(c*x^4+f*x^2+a)+2*(a
*f-1/2*(-a*c+f^2)*f/c)/(4*a*c-f^2)^(1/2)*arctan((2*c*x^2+f)/(4*a*c-f^2)^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.93

$$\int \frac{x^7}{a+fx^2+cx^4}\,dx = \left[\frac{(4\,ac^3-c^2f^2)x^4-2\,(4\,ac^2f-cf^3)x^2+(3\,acf-f^3)\sqrt{-4\,ac+f^2}\log\left(\frac{2\,c^2x^4+2\,cfx^2-2\,ac+f^2+(2\,cx^2+f)\sqrt{-4}}{cx^4+fx^2+a}\right.}{4\,(4\,ac^4-c^3f^2)}\right.$$

input
```
integrate(x^7/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((4*a*c^3 - c^2*f^2)*x^4 - 2*(4*a*c^2*f - c*f^3)*x^2 + (3*a*c*f - f^3
)*sqrt(-4*a*c + f^2)*log((2*c^2*x^4 + 2*c*f*x^2 - 2*a*c + f^2 + (2*c*x^2 +
 f)*sqrt(-4*a*c + f^2))/(c*x^4 + f*x^2 + a)) - (4*a^2*c^2 - 5*a*c*f^2 + f^
4)*log(c*x^4 + f*x^2 + a))/(4*a*c^4 - c^3*f^2), 1/4*((4*a*c^3 - c^2*f^2)*x
^4 - 2*(4*a*c^2*f - c*f^3)*x^2 - 2*(3*a*c*f - f^3)*sqrt(4*a*c - f^2)*arcta
n(-(2*c*x^2 + f)/sqrt(4*a*c - f^2)) - (4*a^2*c^2 - 5*a*c*f^2 + f^4)*log(c*
x^4 + f*x^2 + a))/(4*a*c^4 - c^3*f^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(92) = 184$.

Time = 1.42 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.65

$$\int \frac{x^7}{a+fx^2+cx^4}\,dx = \left(-\frac{f\sqrt{-4ac+f^2}\cdot(3ac-f^2)}{4c^3\cdot(4ac-f^2)} - \frac{ac-f^2}{4c^3}\right)\log\left(x^2 + \frac{2a^2c+8ac^3\left(-\frac{f\sqrt{-4ac+f^2}\cdot(3ac-f^2)}{4c^3\cdot(4ac-f^2)}-\frac{ac-f^2}{4c^3}\right)-af^2-2c^2f^2\left(-\frac{f\sqrt{-4ac+f^2}\cdot(3ac-f^2)}{4c^3\cdot(4ac-f^2)}\right.}{3acf-f^3}\right.$$
$$+\left(\frac{f\sqrt{-4ac+f^2}\cdot(3ac-f^2)}{4c^3\cdot(4ac-f^2)} - \frac{ac-f^2}{4c^3}\right)\log\left(x^2 + \frac{2a^2c+8ac^3\left(\frac{f\sqrt{-4ac+f^2}\cdot(3ac-f^2)}{4c^3\cdot(4ac-f^2)}-\frac{ac-f^2}{4c^3}\right)-af^2-2c^2f^2\left(\frac{f\sqrt{-4ac+f^2}\cdot(3ac-f^2)}{4c^3\cdot(4ac-f^2)}-\frac{ac}{4}\right.}{3acf-f^3}\right.$$
$$+\frac{x^4}{4c}-\frac{fx^2}{2c^2}$$

input
```
integrate(x**7/(c*x**4+f*x**2+a),x)
```

output
```
(-f*sqrt(-4*a*c + f**2)*(3*a*c - f**2)/(4*c**3*(4*a*c - f**2)) - (a*c - f*
*2)/(4*c**3))*log(x**2 + (2*a**2*c + 8*a*c**3*(-f*sqrt(-4*a*c + f**2)*(3*a
*c - f**2)/(4*c**3*(4*a*c - f**2)) - (a*c - f**2)/(4*c**3)) - a*f**2 - 2*c
**2*f**2*(-f*sqrt(-4*a*c + f**2)*(3*a*c - f**2)/(4*c**3*(4*a*c - f**2)) -
(a*c - f**2)/(4*c**3)))/(3*a*c*f - f**3)) + (f*sqrt(-4*a*c + f**2)*(3*a*c
- f**2)/(4*c**3*(4*a*c - f**2)) - (a*c - f**2)/(4*c**3))*log(x**2 + (2*a**
2*c + 8*a*c**3*(f*sqrt(-4*a*c + f**2)*(3*a*c - f**2)/(4*c**3*(4*a*c - f**2
)) - (a*c - f**2)/(4*c**3)) - a*f**2 - 2*c**2*f**2*(f*sqrt(-4*a*c + f**2)*
(3*a*c - f**2)/(4*c**3*(4*a*c - f**2)) - (a*c - f**2)/(4*c**3)))/(3*a*c*f
- f**3)) + x**4/(4*c) - f*x**2/(2*c**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{a+fx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(f^2-4*a*c>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \frac{x^7}{a+fx^2+cx^4}\,dx = \frac{cx^4-2\,fx^2}{4\,c^2} - \frac{(ac-f^2)\log\left(cx^4+fx^2+a\right)}{4\,c^3} + \frac{(3\,acf-f^3)\arctan\left(\frac{2\,cx^2+f}{\sqrt{4\,ac-f^2}}\right)}{2\,\sqrt{4\,ac-f^2}c^3}$$

input
```
integrate(x^7/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
1/4*(c*x^4 - 2*f*x^2)/c^2 - 1/4*(a*c - f^2)*log(c*x^4 + f*x^2 + a)/c^3 + 1
/2*(3*a*c*f - f^3)*arctan((2*c*x^2 + f)/sqrt(4*a*c - f^2))/(sqrt(4*a*c - f
^2)*c^3)
```

Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 842, normalized size of antiderivative = 7.87

$$\int \frac{x^7}{a+fx^2+cx^4}\,dx = \frac{x^4}{4\,c} - \frac{\ln\left(c\,x^4+f\,x^2+a\right)\,\left(8\,a^2\,c^2-10\,a\,c\,f^2+2\,f^4\right)}{2\,\left(16\,a\,c^4-4\,c^3\,f^2\right)} - \frac{f\,x^2}{2\,c^2}$$

$$+ \frac{f\,\mathrm{atan}\left(\frac{2\,c^4\,\left(4\,a\,c-f^2\right)\left(\frac{\frac{f\left(\frac{8\,a^2\,c^4-8\,a\,c^3\,f^2}{c^4}-\frac{8\,a\,c^2\,\left(8\,a^2\,c^2-10\,a\,c\,f^2+2\,f^4\right)}{16\,a\,c^4-4\,c^3\,f^2}\right)\left(3\,a\,c-f^2\right)}{8\,c^3\,\sqrt{4\,a\,c-f^2}}-\frac{a\,f\,\left(3\,a\,c-f^2\right)\left(8\,a^2\,c^2-10\,a\,c\,f^2+2\,f^4\right)}{c\,\sqrt{4\,a\,c-f^2}\,\left(16\,a\,c^4-4\,c^3\,f^2\right)}}{a}-x^2\left(\frac{f\ \cdots}{\cdots}\right.\right.}{\cdots}\right)}{\cdots}$$

input
```
int(x^7/(a + c*x^4 + f*x^2),x)
```

output
```
x^4/(4*c) - (log(a + c*x^4 + f*x^2)*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(2*(
16*a*c^4 - 4*c^3*f^2)) - (f*x^2)/(2*c^2) + (f*atan((2*c^4*(4*a*c - f^2)*((
(f*((8*a^2*c^4 - 8*a*c^3*f^2)/c^4 - (8*a*c^2*(2*f^4 + 8*a^2*c^2 - 10*a*c*f
^2))/(16*a*c^4 - 4*c^3*f^2))*(3*a*c - f^2))/(8*c^3*(4*a*c - f^2)^(1/2)) -
(a*f*(3*a*c - f^2)*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(c*(4*a*c - f^2)^(1/2
)*(16*a*c^4 - 4*c^3*f^2)))/a - x^2*(((f*(3*a*c - f^2)*((6*c^3*f^3 - 10*a*c
^4*f)/c^4 + (4*c^2*f*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(16*a*c^4 - 4*c^3*f
^2)))/(8*c^3*(4*a*c - f^2)^(1/2)) + (f^2*(3*a*c - f^2)*(2*f^4 + 8*a^2*c^2
- 10*a*c*f^2))/(2*c*(4*a*c - f^2)^(1/2)*(16*a*c^4 - 4*c^3*f^2)))/a + (f*((
f^5 + 2*a^2*c^2*f - 3*a*c*f^3)/c^4 + (((6*c^3*f^3 - 10*a*c^4*f)/c^4 + (4*c
^2*f*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(16*a*c^4 - 4*c^3*f^2))*(2*f^4 + 8*
a^2*c^2 - 10*a*c*f^2))/(2*(16*a*c^4 - 4*c^3*f^2)) - (f^3*(3*a*c - f^2)^2)/
(2*c^4*(4*a*c - f^2))))/(2*a*(4*a*c - f^2)^(1/2))) + (f*((((8*a^2*c^4 - 8*
a*c^3*f^2)/c^4 - (8*a*c^2*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(16*a*c^4 - 4*
c^3*f^2))*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(2*(16*a*c^4 - 4*c^3*f^2)) - (
a*f^4 + a^3*c^2 - 2*a^2*c*f^2)/c^4 + (a*f^2*(3*a*c - f^2)^2)/(c^4*(4*a*c -
 f^2))))/(2*a*(4*a*c - f^2)^(1/2))))/(f^6 + 9*a^2*c^2*f^2 - 6*a*c*f^4))*(3
*a*c - f^2))/(2*c^3*(4*a*c - f^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 458, normalized size of antiderivative = 4.28

$$\int \frac{x^7}{a+fx^2+cx^4}\,dx$$
$$= \frac{-6\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right)acf+2\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,atan\left(\frac{\sqrt{2}}{}\right.}{}$$

input
```
int(x^7/(c*x^4+f*x^2+a),x)
```

output
```
( - 6*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2
*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*f +
2*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sqr
t(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f**3 - 6*sqr
t(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sqrt(c)*
sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*f + 2*sqrt(2*
sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sqrt(c)*sqrt
(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f**3 - 4*log( - sqrt(
2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a**2*c**2 + 5*log( - sq
rt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c*f**2 - log( - sq
rt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f**4 - 4*log(sqrt(2*
sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a**2*c**2 + 5*log(sqrt(2*
sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c*f**2 - log(sqrt(2*sqr
t(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f**4 + 4*a*c**3*x**4 - 8*a*c
**2*f*x**2 - c**2*f**2*x**4 + 2*c*f**3*x**2)/(4*c**3*(4*a*c - f**2))
```

3.777 $\int \frac{x^5}{a+fx^2+cx^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{x^5}{a+fx^2+cx^4} dx = \frac{x^2}{2c} - \frac{(2ac-f^2)\arctan\left(\frac{f+2cx^2}{\sqrt{4ac-f^2}}\right)}{2c^2\sqrt{4ac-f^2}} - \frac{f\log(a+fx^2+cx^4)}{4c^2}$$

output
```
1/2*x^2/c-1/2*(2*a*c-f^2)*arctan((2*c*x^2+f)/(4*a*c-f^2)^(1/2))/c^2/(4*a*c
-f^2)^(1/2)-1/4*f*ln(c*x^4+f*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{a+fx^2+cx^4} dx = \frac{2cx^2 + \frac{2(-2ac+f^2)\arctan\left(\frac{f+2cx^2}{\sqrt{4ac-f^2}}\right)}{\sqrt{4ac-f^2}} - f\log(a+fx^2+cx^4)}{4c^2}$$

input
```
Integrate[x^5/(a + f*x^2 + c*x^4),x]
```

output
```
(2*c*x^2 + (2*(-2*a*c + f^2)*ArcTan[(f + 2*c*x^2)/Sqrt[4*a*c - f^2]])/Sqrt
[4*a*c - f^2] - f*Log[a + f*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+cx^4+fx^2} \, dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{x^4}{cx^4+fx^2+a}dx^2$$

$$\Big\downarrow \text{ 1143}$$

$$\frac{1}{2}\int \left(\frac{1}{c}-\frac{fx^2+a}{c\left(cx^4+fx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{\left(2ac-f^2\right)\arctan\left(\frac{2cx^2+f}{\sqrt{4ac-f^2}}\right)}{c^2\sqrt{4ac-f^2}}-\frac{f\log\left(a+cx^4+fx^2\right)}{2c^2}+\frac{x^2}{c}\right)$$

input
```
Int[x^5/(a + f*x^2 + c*x^4),x]
```

output
```
(x^2/c - ((2*a*c - f^2)*ArcTan[(f + 2*c*x^2)/Sqrt[4*a*c - f^2]])/(c^2*Sqrt
[4*a*c - f^2]) - (f*Log[a + f*x^2 + c*x^4])/(2*c^2))/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

method	result
default	$\frac{x^{2}}{2c}+\frac{-\frac{f\ln\left(c\,x^{4}+f\,x^{2}+a\right)}{2c}+\frac{2\left(-a+\frac{f^{2}}{2c}\right)\arctan\left(\frac{2c\,x^{2}+f}{\sqrt{4ac-f^{2}}}\right)}{\sqrt{4ac-f^{2}}}}{2c}$
risch	$\frac{x^{2}}{2c}-\frac{\ln\left(\left(-8a^{2}c^{2}+6ac\,f^{2}-f^{4}+\sqrt{-\left(4ac-f^{2}\right)\left(2ac-f^{2}\right)^{2}}\,f\right)x^{2}+2\sqrt{-\left(4ac-f^{2}\right)\left(2ac-f^{2}\right)^{2}}\,a\right)af}{c\left(4ac-f^{2}\right)}+\frac{\ln\left(\left(-8a^{2}c^{2}+6ac\,f^{2}-f^{4}-\right.\right.}{}$

input
```
int(x^5/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c+1/2/c*(-1/2*f/c*ln(c*x^4+f*x^2+a)+2*(-a+1/2*f^2/c)/(4*a*c-f^2)^(
1/2)*arctan((2*c*x^2+f)/(4*a*c-f^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.94

$$\int \frac{x^5}{a+fx^2+cx^4}\,dx = \left[\frac{2\,(4\,ac^2-cf^2)x^2+(2\,ac-f^2)\sqrt{-4\,ac+f^2}\log\left(\frac{2\,c^2x^4+2\,cfx^2-2\,ac+f^2-(2\,cx^2+f)\sqrt{-4\,ac+f^2}}{cx^4+fx^2+a}\right)-(4\,acf-f^3)}{4\,(4\,ac^3-c^2f^2)}\right.$$

input
```
integrate(x^5/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(2*(4*a*c^2 - c*f^2)*x^2 + (2*a*c - f^2)*sqrt(-4*a*c + f^2)*log((2*c^
2*x^4 + 2*c*f*x^2 - 2*a*c + f^2 - (2*c*x^2 + f)*sqrt(-4*a*c + f^2))/(c*x^4
 + f*x^2 + a)) - (4*a*c*f - f^3)*log(c*x^4 + f*x^2 + a))/(4*a*c^3 - c^2*f^
2), 1/4*(2*(4*a*c^2 - c*f^2)*x^2 + 2*sqrt(4*a*c - f^2)*(2*a*c - f^2)*arcta
n(-(2*c*x^2 + f)/sqrt(4*a*c - f^2)) - (4*a*c*f - f^3)*log(c*x^4 + f*x^2 +
a))/(4*a*c^3 - c^2*f^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(71) = 142.

Time = 1.04 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.63

$$\int \frac{x^5}{a+fx^2+cx^4}\,dx = \left(-\frac{f}{4c^2}\right.$$
$$\left.-\frac{\sqrt{-4ac+f^2}\cdot(2ac-f^2)}{4c^2\cdot(4ac-f^2)}\right)\log\left(x^2+\frac{-8ac^2\left(-\frac{f}{4c^2}-\frac{\sqrt{-4ac+f^2}\cdot(2ac-f^2)}{4c^2\cdot(4ac-f^2)}\right)-af+2cf^2\left(-\frac{f}{4c^2}-\frac{\sqrt{-4ac+}}{4c^2\cdot}\right.}{2ac-f^2}\right.$$
$$+\left(-\frac{f}{4c^2}\right.$$
$$\left.+\frac{\sqrt{-4ac+f^2}\cdot(2ac-f^2)}{4c^2\cdot(4ac-f^2)}\right)\log\left(x^2+\frac{-8ac^2\left(-\frac{f}{4c^2}+\frac{\sqrt{-4ac+f^2}\cdot(2ac-f^2)}{4c^2\cdot(4ac-f^2)}\right)-af+2cf^2\left(-\frac{f}{4c^2}+\frac{\sqrt{-4ac+}}{4c^2\cdot(}\right.}{2ac-f^2}\right.$$
$$+\frac{x^2}{2c}$$

input
```
integrate(x**5/(c*x**4+f*x**2+a),x)
```

output
```
(-f/(4*c**2) - sqrt(-4*a*c + f**2)*(2*a*c - f**2)/(4*c**2*(4*a*c - f**2)))
*log(x**2 + (-8*a*c**2*(-f/(4*c**2) - sqrt(-4*a*c + f**2)*(2*a*c - f**2)/(
4*c**2*(4*a*c - f**2))) - a*f + 2*c*f**2*(-f/(4*c**2) - sqrt(-4*a*c + f**2
)*(2*a*c - f**2)/(4*c**2*(4*a*c - f**2))))/(2*a*c - f**2)) + (-f/(4*c**2)
+ sqrt(-4*a*c + f**2)*(2*a*c - f**2)/(4*c**2*(4*a*c - f**2)))*log(x**2 + (
-8*a*c**2*(-f/(4*c**2) + sqrt(-4*a*c + f**2)*(2*a*c - f**2)/(4*c**2*(4*a*c
 - f**2))) - a*f + 2*c*f**2*(-f/(4*c**2) + sqrt(-4*a*c + f**2)*(2*a*c - f*
*2)/(4*c**2*(4*a*c - f**2))))/(2*a*c - f**2)) + x**2/(2*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{a + fx^2 + cx^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(f^2-4*a*c>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{a + fx^2 + cx^4} \, dx = \frac{x^2}{2\,c} - \frac{f \log\left(cx^4 + fx^2 + a\right)}{4\,c^2} - \frac{\left(2\,ac - f^2\right)\arctan\left(\frac{2\,cx^2 + f}{\sqrt{4\,ac - f^2}}\right)}{2\,\sqrt{4\,ac - f^2}c^2}$$

input
```
integrate(x^5/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
1/2*x^2/c - 1/4*f*log(c*x^4 + f*x^2 + a)/c^2 - 1/2*(2*a*c - f^2)*arctan((2
*c*x^2 + f)/sqrt(4*a*c - f^2))/(sqrt(4*a*c - f^2)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 655, normalized size of antiderivative = 7.53

$$\int \frac{x^5}{a+fx^2+cx^4}\,dx = \frac{x^2}{2\,c}+\frac{\ln\left(c\,x^4+f\,x^2+a\right)\,\left(2\,f^3-8\,a\,c\,f\right)}{2\,\left(16\,a\,c^3-4\,c^2\,f^2\right)}$$

$$+\frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-f^2\right)\left(\frac{\frac{\left(8\,a\,f+\frac{8\,a\,c^2\,\left(2\,f^3-8\,a\,c\,f\right)}{16\,a\,c^3-4\,c^2\,f^2}\right)\,\left(2\,a\,c-f^2\right)}{8\,c^2\,\sqrt{4\,a\,c-f^2}}+\frac{a\,\left(2\,f^3-8\,a\,c\,f\right)\,\left(2\,a\,c-f^2\right)}{\sqrt{4\,a\,c-f^2}\,\left(16\,a\,c^3-4\,c^2\,f^2\right)}}{a}-x^2\left(\frac{\left(\frac{4\,a\,c^3-6\,c^2\,f^2}{c^2}-\frac{4\,c^2\,f\,\left(2\,f^3-8\,a\,c\,f\right)}{16\,a\,c^3-4\,c^2\,f^2}\right)\cdots}{8\,c^2\,\sqrt{4\,a\,c-f^2}}\cdots\right.\right.}{\cdots}\right)}{\cdots}$$

input
```
int(x^5/(a + c*x^4 + f*x^2),x)
```

output
```
x^2/(2*c) + (log(a + c*x^4 + f*x^2)*(2*f^3 - 8*a*c*f))/(2*(16*a*c^3 - 4*c^
2*f^2)) + (atan((2*c^2*(4*a*c - f^2)*((((8*a*f + (8*a*c^2*(2*f^3 - 8*a*c*f
))/(16*a*c^3 - 4*c^2*f^2))*(2*a*c - f^2))/(8*c^2*(4*a*c - f^2)^(1/2)) + (a
*(2*f^3 - 8*a*c*f)*(2*a*c - f^2))/((4*a*c - f^2)^(1/2)*(16*a*c^3 - 4*c^2*f
^2)))/a - x^2*(((((4*a*c^3 - 6*c^2*f^2)/c^2 - (4*c^2*f*(2*f^3 - 8*a*c*f))/
(16*a*c^3 - 4*c^2*f^2))*(2*a*c - f^2))/(8*c^2*(4*a*c - f^2)^(1/2)) - (f*(2
*f^3 - 8*a*c*f)*(2*a*c - f^2))/(2*(4*a*c - f^2)^(1/2)*(16*a*c^3 - 4*c^2*f^
2)))/a + (f*((((4*a*c^3 - 6*c^2*f^2)/c^2 - (4*c^2*f*(2*f^3 - 8*a*c*f))/(16
*a*c^3 - 4*c^2*f^2))*(2*f^3 - 8*a*c*f))/(2*(16*a*c^3 - 4*c^2*f^2)) - (f^3
- a*c*f)/c^2 + (f*(2*a*c - f^2)^2)/(2*c^2*(4*a*c - f^2))))/(2*a*(4*a*c - f
^2)^(1/2))) + (f*((a*f^2)/c^2 + ((2*f^3 - 8*a*c*f)*(8*a*f + (8*a*c^2*(2*f^
3 - 8*a*c*f))/(16*a*c^3 - 4*c^2*f^2)))/(2*(16*a*c^3 - 4*c^2*f^2)) - (a*(2*
a*c - f^2)^2)/(c^2*(4*a*c - f^2))))/(2*a*(4*a*c - f^2)^(1/2))))/(f^4 + 4*a
^2*c^2 - 4*a*c*f^2))*(2*a*c - f^2))/(2*c^2*(4*a*c - f^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.21

$$\int \frac{x^5}{a+fx^2+cx^4}\,dx$$
$$= \frac{4\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right)ac-2\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{}}}{\sqrt{2\sqrt{}}}\right.}{}$$

input
```
int(x^5/(c*x^4+f*x^2+a),x)
```

output
```
(4*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sq
rt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c - 2*sqr
t(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sqrt(c)*
sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f**2 + 4*sqrt(2*s
qrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sqrt(c)*sqrt(
a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c - 2*sqrt(2*sqrt(c)
*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f
) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f**2 - 4*log( - sqrt(2*sqrt(
c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c*f + log( - sqrt(2*sqrt(c)*
sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f**3 - 4*log(sqrt(2*sqrt(c)*sqrt(
a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c*f + log(sqrt(2*sqrt(c)*sqrt(a) - f
)*x + sqrt(a) + sqrt(c)*x**2)*f**3 + 8*a*c**2*x**2 - 2*c*f**2*x**2)/(4*c**
2*(4*a*c - f**2))
```

3.778 $\int \frac{x^3}{a+fx^2+cx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int \frac{x^3}{a+fx^2+cx^4}\,dx = -\frac{f\arctan\left(\frac{f+2cx^2}{\sqrt{4ac-f^2}}\right)}{2c\sqrt{4ac-f^2}}+\frac{\log\left(a+fx^2+cx^4\right)}{4c}$$

output
```
-1/2*f*arctan((2*c*x^2+f)/(4*a*c-f^2)^(1/2))/c/(4*a*c-f^2)^(1/2)+1/4*ln(c*
x^4+f*x^2+a)/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{a+fx^2+cx^4}\,dx = \frac{-\frac{2f\arctan\left(\frac{f+2cx^2}{\sqrt{4ac-f^2}}\right)}{\sqrt{4ac-f^2}}+\log\left(a+fx^2+cx^4\right)}{4c}$$

input
```
Integrate[x^3/(a + f*x^2 + c*x^4),x]
```

output
```
((-2*f*ArcTan[(f + 2*c*x^2)/Sqrt[4*a*c - f^2]])/Sqrt[4*a*c - f^2] + Log[a
+ f*x^2 + c*x^4])/(4*c)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1434, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+cx^4+fx^2}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{cx^4+fx^2+a}dx^2$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{\int \frac{2cx^2+f}{cx^4+fx^2+a}dx^2}{2c}-\frac{f\int \frac{1}{cx^4+fx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{f\int \frac{1}{-x^4+f^2-4ac}d(2cx^2+f)}{c}+\frac{\int \frac{2cx^2+f}{cx^4+fx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\int \frac{2cx^2+f}{cx^4+fx^2+a}dx^2}{2c}-\frac{f\arctan\left(\frac{2cx^2+f}{\sqrt{4ac-f^2}}\right)}{c\sqrt{4ac-f^2}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\log\left(a+cx^4+fx^2\right)}{2c}-\frac{f\arctan\left(\frac{2cx^2+f}{\sqrt{4ac-f^2}}\right)}{c\sqrt{4ac-f^2}}\right)$$

input
```
Int[x^3/(a + f*x^2 + c*x^4),x]
```

output
```
(-((f*ArcTan[(f + 2*c*x^2)/Sqrt[4*a*c - f^2]])/(c*Sqrt[4*a*c - f^2])) + Lo
g[a + f*x^2 + c*x^4]/(2*c))/2
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

method	result
default	$-\frac{f \arctan\left(\frac{2c\,x^2+f}{\sqrt{4ac-f^2}}\right)}{2c\sqrt{4ac-f^2}}+\frac{\ln\left(c\,x^4+f\,x^2+a\right)}{4c}$
risch	$\frac{\ln\left(\left(-4acf+f^3+\sqrt{-f^2(4ac-f^2)}\,f\right)x^2+2\sqrt{-f^2(4ac-f^2)}\,a\right)a}{4ac-f^2}-\frac{\ln\left(\left(-4acf+f^3+\sqrt{-f^2(4ac-f^2)}\,f\right)x^2+2\sqrt{-f^2(4ac-f^2)}\,a\right)}{4c(4ac-f^2)}$

input
```
int(x^3/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*f*arctan((2*c*x^2+f)/(4*a*c-f^2)^(1/2))/c/(4*a*c-f^2)^(1/2)+1/4*ln(c*
x^4+f*x^2+a)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.90

$$\int \frac{x^3}{a+fx^2+cx^4}\,dx$$
$$=\left[-\frac{\sqrt{-4\,ac+f^2}f\log\left(\frac{2\,c^2x^4+2\,cfx^2-2\,ac+f^2+(2\,cx^2+f)\sqrt{-4\,ac+f^2}}{cx^4+fx^2+a}\right)-(4\,ac-f^2)\log\left(cx^4+fx^2+a\right)}{4\,(4\,ac^2-cf^2)},\ 2\sqrt{4\,ac}\right.$$

input
```
integrate(x^3/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-4*a*c + f^2)*f*log((2*c^2*x^4 + 2*c*f*x^2 - 2*a*c + f^2 + (2*
c*x^2 + f)*sqrt(-4*a*c + f^2))/(c*x^4 + f*x^2 + a)) - (4*a*c - f^2)*log(c*
x^4 + f*x^2 + a))/(4*a*c^2 - c*f^2), 1/4*(2*sqrt(4*a*c - f^2)*f*arctan(-(2
*c*x^2 + f)/sqrt(4*a*c - f^2)) + (4*a*c - f^2)*log(c*x^4 + f*x^2 + a))/(4*
a*c^2 - c*f^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.

Time = 0.47 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.33

$$\int \frac{x^3}{a+fx^2+cx^4}\,dx = \left(-\frac{f\sqrt{-4ac+f^2}}{4c\,(4ac-f^2)} + \frac{1}{4c}\right)\log\left(x^2 + \frac{-8ac\left(-\frac{f\sqrt{-4ac+f^2}}{4c(4ac-f^2)}+\frac{1}{4c}\right)+2a+2f^2\left(-\frac{f\sqrt{-4ac+f^2}}{4c(4ac-f^2)}+\frac{1}{4c}\right)}{f}\right) + \left(\frac{f\sqrt{-4ac+f^2}}{4c\,(4ac-f^2)} + \frac{1}{4c}\right)\log\left(x^2 + \frac{-8ac\left(\frac{f\sqrt{-4ac+f^2}}{4c(4ac-f^2)}+\frac{1}{4c}\right)+2a+2f^2\left(\frac{f\sqrt{-4ac+f^2}}{4c(4ac-f^2)}+\frac{1}{4c}\right)}{f}\right)$$

input
```
integrate(x**3/(c*x**4+f*x**2+a),x)
```

output
```
(-f*sqrt(-4*a*c + f**2)/(4*c*(4*a*c - f**2)) + 1/(4*c))*log(x**2 + (-8*a*c
*(-f*sqrt(-4*a*c + f**2)/(4*c*(4*a*c - f**2)) + 1/(4*c)) + 2*a + 2*f**2*(-
f*sqrt(-4*a*c + f**2)/(4*c*(4*a*c - f**2)) + 1/(4*c)))/f) + (f*sqrt(-4*a*c
 + f**2)/(4*c*(4*a*c - f**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(f*sqrt(-4*a*c
 + f**2)/(4*c*(4*a*c - f**2)) + 1/(4*c)) + 2*a + 2*f**2*(f*sqrt(-4*a*c + f
**2)/(4*c*(4*a*c - f**2)) + 1/(4*c)))/f)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a+fx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(f^2-4*a*c>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{a+fx^2+cx^4}\,dx = -\frac{f\arctan\left(\frac{2\,cx^2+f}{\sqrt{4\,ac-f^2}}\right)}{2\,\sqrt{4\,ac-f^2}c} + \frac{\log\left(cx^4+fx^2+a\right)}{4\,c}$$

input
```
integrate(x^3/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
-1/2*f*arctan((2*c*x^2 + f)/sqrt(4*a*c - f^2))/(sqrt(4*a*c - f^2)*c) + 1/4
*log(c*x^4 + f*x^2 + a)/c
```

Mupad [B] (verification not implemented)

Time = 17.63 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int \frac{x^3}{a+fx^2+cx^4}\,dx = \frac{4\,a\,c\,\ln\left(c\,x^4+f\,x^2+a\right)}{16\,a\,c^2-4\,c\,f^2} - \frac{f^2\,\ln\left(c\,x^4+f\,x^2+a\right)}{16\,a\,c^2-4\,c\,f^2} - \frac{f\,\mathrm{atan}\left(\frac{f}{\sqrt{4\,a\,c-f^2}}+\frac{2\,c\,x^2}{\sqrt{4\,a\,c-f^2}}\right)}{2\,c\,\sqrt{4\,a\,c-f^2}}$$

input
```
int(x^3/(a + c*x^4 + f*x^2),x)
```

output
```
(4*a*c*log(a + c*x^4 + f*x^2))/(16*a*c^2 - 4*c*f^2) - (f^2*log(a + c*x^4 +
 f*x^2))/(16*a*c^2 - 4*c*f^2) - (f*atan(f/(4*a*c - f^2)^(1/2) + (2*c*x^2)/
(4*a*c - f^2)^(1/2)))/(2*c*(4*a*c - f^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.52

$$\int \frac{x^3}{a + fx^2 + cx^4} \, dx$$
$$= \frac{2\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right)f + 2\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{c}}}{\sqrt{2\sqrt{}}}\right.}{}$$

input
```
int(x^3/(c*x^4+f*x^2+a),x)
```

output
```
(2*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sq
rt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f + 2*sqrt(
2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sqrt(c)*sq
rt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f + 4*log( - sqrt(2
*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c - log( - sqrt(2*sqrt
(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f**2 + 4*log(sqrt(2*sqrt(c)*s
qrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c - log(sqrt(2*sqrt(c)*sqrt(a) -
 f)*x + sqrt(a) + sqrt(c)*x**2)*f**2)/(4*c*(4*a*c - f**2))
```

3.779 $\int \frac{x}{a+fx^2+cx^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{x}{a+fx^2+cx^4} \, dx = \frac{\arctan\left(\frac{f+2cx^2}{\sqrt{4ac-f^2}}\right)}{\sqrt{4ac-f^2}}$$

output
```
arctan((2*c*x^2+f)/(4*a*c-f^2)^(1/2))/(4*a*c-f^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+fx^2+cx^4} \, dx = \frac{\arctan\left(\frac{f+2cx^2}{\sqrt{4ac-f^2}}\right)}{\sqrt{4ac-f^2}}$$

input
```
Integrate[x/(a + f*x^2 + c*x^4),x]
```

output
```
ArcTan[(f + 2*c*x^2)/Sqrt[4*a*c - f^2]]/Sqrt[4*a*c - f^2]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1432, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+cx^4+fx^2}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{cx^4+fx^2+a}dx^2$$

$$\Big\downarrow \text{1083}$$

$$-\int \frac{1}{-x^4+f^2-4ac}d(2cx^2+f)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2cx^2+f}{\sqrt{4ac-f^2}}\right)}{\sqrt{4ac-f^2}}$$

input
```
Int[x/(a + f*x^2 + c*x^4),x]
```

output
```
ArcTan[(f + 2*c*x^2)/Sqrt[4*a*c - f^2]]/Sqrt[4*a*c - f^2]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\arctan\left(\frac{2cx^2+f}{\sqrt{4ac-f^2}}\right)}{\sqrt{4ac-f^2}}$	36
risch	$-\frac{\ln\left(\left(\sqrt{-4ac+f^2}-f\right)x^2-2a\right)}{2\sqrt{-4ac+f^2}}+\frac{\ln\left(\left(\sqrt{-4ac+f^2}+f\right)x^2+2a\right)}{2\sqrt{-4ac+f^2}}$	70

input
```
int(x/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
arctan((2*c*x^2+f)/(4*a*c-f^2)^(1/2))/(4*a*c-f^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.10

$$\int \frac{x}{a+fx^2+cx^4}\,dx=\left[-\frac{\sqrt{-4\,ac+f^2}\log\left(\frac{2\,c^2x^4+2\,cfx^2-2\,ac+f^2-(2\,cx^2+f)\sqrt{-4\,ac+f^2}}{cx^4+fx^2+a}\right)}{2\,(4\,ac-f^2)},\; -\frac{\arctan\left(-\frac{2\,cx^2+f}{\sqrt{4\,ac-f^2}}\right)}{\sqrt{4\,ac-f^2}}\right]$$

input
```
integrate(x/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-4*a*c + f^2)*log((2*c^2*x^4 + 2*c*f*x^2 - 2*a*c + f^2 - (2*c*x
^2 + f)*sqrt(-4*a*c + f^2))/(c*x^4 + f*x^2 + a))/(4*a*c - f^2), -arctan(-(
2*c*x^2 + f)/sqrt(4*a*c - f^2))/sqrt(4*a*c - f^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(32) = 64.

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.36

$$\int \frac{x}{a+fx^2+cx^4}\,dx = -\frac{\sqrt{-\frac{1}{4ac-f^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-f^2}}+f^2\sqrt{-\frac{1}{4ac-f^2}}+f}{2c}\right)}{2} + \frac{\sqrt{-\frac{1}{4ac-f^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-f^2}}-f^2\sqrt{-\frac{1}{4ac-f^2}}+f}{2c}\right)}{2}$$

input
```
integrate(x/(c*x**4+f*x**2+a),x)
```

output
```
-sqrt(-1/(4*a*c - f**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - f**2)) + f**2
*sqrt(-1/(4*a*c - f**2)) + f)/(2*c))/2 + sqrt(-1/(4*a*c - f**2))*log(x**2
+ (4*a*c*sqrt(-1/(4*a*c - f**2)) - f**2*sqrt(-1/(4*a*c - f**2)) + f)/(2*c)
)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a+fx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(f^2-4*a*c>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{x}{a+fx^2+cx^4}\,dx=\frac{\arctan\left(\frac{2\,cx^2+f}{\sqrt{4\,ac-f^2}}\right)}{\sqrt{4\,ac-f^2}}$$

input
```
integrate(x/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
arctan((2*c*x^2 + f)/sqrt(4*a*c - f^2))/sqrt(4*a*c - f^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{x}{a+fx^2+cx^4}\,dx=\frac{\mathrm{atan}\left(\frac{2\,a\,c\,x^2+a\,f}{a\,\sqrt{4\,a\,c-f^2}}\right)}{\sqrt{4\,a\,c-f^2}}$$

input
```
int(x/(a + c*x^4 + f*x^2),x)
```

output
```
atan((a*f + 2*a*c*x^2)/(a*(4*a*c - f^2)^(1/2)))/(4*a*c - f^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \frac{x}{a+fx^2+cx^4}\,dx$$
$$= -\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,\left(atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right)+atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}+2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right)\right)}{4ac-f^2}$$

input
```
int(x/(c*x^4+f*x^2+a),x)
```

output
```
( - sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*(atan((sqrt(2*
sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f)) + atan((s
qrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))))/(
4*a*c - f**2)
```

3.780 $\int \frac{1}{x(a+fx^2+cx^4)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 73

$$\int \frac{1}{x(a+fx^2+cx^4)} \, dx = -\frac{f \arctan\left(\frac{f+2cx^2}{\sqrt{4ac-f^2}}\right)}{2a\sqrt{4ac-f^2}} + \frac{\log(x)}{a} - \frac{\log(a+fx^2+cx^4)}{4a}$$

output
```
-1/2*f*arctan((2*c*x^2+f)/(4*a*c-f^2)^(1/2))/a/(4*a*c-f^2)^(1/2)+ln(x)/a-1
/4*ln(c*x^4+f*x^2+a)/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.55

$$\int \frac{1}{x(a+fx^2+cx^4)} \, dx = \frac{4\sqrt{-4ac+f^2}\log(x) - \left(f+\sqrt{-4ac+f^2}\right)\log\left(f-\sqrt{-4ac+f^2}+2cx^2\right) + \left(f-\sqrt{-4ac+f^2}\right)\log\left(f+\right.}{4a\sqrt{-4ac+f^2}}$$

input
```
Integrate[1/(x*(a + f*x^2 + c*x^4)),x]
```

output
```
(4*Sqrt[-4*a*c + f^2]*Log[x] - (f + Sqrt[-4*a*c + f^2])*Log[f - Sqrt[-4*a*
c + f^2] + 2*c*x^2] + (f - Sqrt[-4*a*c + f^2])*Log[f + Sqrt[-4*a*c + f^2]
+ 2*c*x^2])/(4*a*Sqrt[-4*a*c + f^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1434, 1144, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+cx^4+fx^2)}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^2(cx^4+fx^2+a)}dx^2$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{2}\left(\frac{\int -\frac{cx^2+f}{cx^4+fx^2+a}dx^2}{a}+\frac{\log(x^2)}{a}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\int \frac{cx^2+f}{cx^4+fx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\frac{1}{2}f\int \frac{1}{cx^4+fx^2+a}dx^2+\frac{1}{2}\int \frac{2cx^2+f}{cx^4+fx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\frac{1}{2}\int \frac{2cx^2+f}{cx^4+fx^2+a}dx^2-f\int \frac{1}{-x^4+f^2-4ac}d(2cx^2+f)}{a}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\frac{1}{2}\int\frac{2cx^2+f}{cx^4+fx^2+a}dx^2+\frac{f\arctan\left(\frac{2cx^2+f}{\sqrt{4ac-f^2}}\right)}{\sqrt{4ac-f^2}}}{a}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\frac{f\arctan\left(\frac{2cx^2+f}{\sqrt{4ac-f^2}}\right)}{\sqrt{4ac-f^2}}+\frac{1}{2}\log\left(a+cx^4+fx^2\right)}{a}\right)$$

input
```
Int[1/(x*(a + f*x^2 + c*x^4)),x]
```

output
```
(Log[x^2]/a - ((f*ArcTan[(f + 2*c*x^2)/Sqrt[4*a*c - f^2]])/Sqrt[4*a*c - f^
2] + Log[a + f*x^2 + c*x^4]/2)/a)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\frac{\ln\left(c\,x^{4}+f\,x^{2}+a\right)}{2}+\frac{f\arctan\left(\frac{2c\,x^{2}+f}{\sqrt{4ac-f^{2}}}\right)}{\sqrt{4ac-f^{2}}}}{2a}+\frac{\ln\left(x\right)}{a}$	65
risch	$\frac{\ln\left(x\right)}{a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4c\,a^{2}-a\,f^{2}\right)\_Z^{2}+\left(4ac-f^{2}\right)\_Z+c\right)}\_R\ln\left(\left(\left(10ac-3f^{2}\right)\_R+5c\right)x^{2}-af\_R+2f\right)\right)}{2}$	77

input
```
int(1/x/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*(1/2*ln(c*x^4+f*x^2+a)+f/(4*a*c-f^2)^(1/2)*arctan((2*c*x^2+f)/(4*a*
c-f^2)^(1/2)))+ln(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.04

$$\int \frac{1}{x\,(a+fx^2+cx^4)}\,dx$$
$$= \left[-\frac{\sqrt{-4\,ac+f^2}f\log\left(\frac{2\,c^2x^4+2\,cfx^2-2\,ac+f^2+(2\,cx^2+f)\sqrt{-4\,ac+f^2}}{cx^4+fx^2+a}\right)+(4\,ac-f^2)\log\left(cx^4+fx^2+a\right)-4\,(4\,ac}{4\,(4\,a^2c-af^2)}\right.$$

input
```
integrate(1/x/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-4*a*c + f^2)*f*log((2*c^2*x^4 + 2*c*f*x^2 - 2*a*c + f^2 + (2*
c*x^2 + f)*sqrt(-4*a*c + f^2))/(c*x^4 + f*x^2 + a)) + (4*a*c - f^2)*log(c*
x^4 + f*x^2 + a) - 4*(4*a*c - f^2)*log(x))/(4*a^2*c - a*f^2), 1/4*(2*sqrt(
4*a*c - f^2)*f*arctan(-(2*c*x^2 + f)/sqrt(4*a*c - f^2)) - (4*a*c - f^2)*lo
g(c*x^4 + f*x^2 + a) + 4*(4*a*c - f^2)*log(x))/(4*a^2*c - a*f^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(60) = 120.

Time = 2.78 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.47

$$\int \frac{1}{x\,(a+fx^2+cx^4)}\,dx = \left(-\frac{f\sqrt{-4ac+f^2}}{4a\,(4ac-f^2)} - \frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(-\frac{f\sqrt{-4ac+f^2}}{4a(4ac-f^2)}-\frac{1}{4a}\right)-2ac+2af^2\left(-\frac{f\sqrt{-4ac+f^2}}{4a(4ac-f^2)}-\frac{1}{4a}\right)+f^2}{cf}\right)$$
$$+\left(\frac{f\sqrt{-4ac+f^2}}{4a\,(4ac-f^2)} - \frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(\frac{f\sqrt{-4ac+f^2}}{4a(4ac-f^2)}-\frac{1}{4a}\right)-2ac+2af^2\left(\frac{f\sqrt{-4ac+f^2}}{4a(4ac-f^2)}-\frac{1}{4a}\right)+f^2}{cf}\right)$$
$$+\frac{\log(x)}{a}$$

input
```
integrate(1/x/(c*x**4+f*x**2+a),x)
```

output
```
(-f*sqrt(-4*a*c + f**2)/(4*a*(4*a*c - f**2)) - 1/(4*a))*log(x**2 + (-8*a**
2*c*(-f*sqrt(-4*a*c + f**2)/(4*a*(4*a*c - f**2)) - 1/(4*a)) - 2*a*c + 2*a*
f**2*(-f*sqrt(-4*a*c + f**2)/(4*a*(4*a*c - f**2)) - 1/(4*a)) + f**2)/(c*f)
) + (f*sqrt(-4*a*c + f**2)/(4*a*(4*a*c - f**2)) - 1/(4*a))*log(x**2 + (-8*
a**2*c*(f*sqrt(-4*a*c + f**2)/(4*a*(4*a*c - f**2)) - 1/(4*a)) - 2*a*c + 2*
a*f**2*(f*sqrt(-4*a*c + f**2)/(4*a*(4*a*c - f**2)) - 1/(4*a)) + f**2)/(c*f
)) + log(x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+fx^2+cx^4\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(f^2-4*a*c>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(a+fx^2+cx^4\right)}\,dx=-\frac{f\arctan\left(\frac{2\,cx^2+f}{\sqrt{4\,ac-f^2}}\right)}{2\,\sqrt{4\,ac-f^2}a}-\frac{\log\left(cx^4+fx^2+a\right)}{4\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

input
```
integrate(1/x/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
-1/2*f*arctan((2*c*x^2 + f)/sqrt(4*a*c - f^2))/(sqrt(4*a*c - f^2)*a) - 1/4
*log(c*x^4 + f*x^2 + a)/a + 1/2*log(x^2)/a
```

Mupad [B] (verification not implemented)

Time = 18.00 (sec) , antiderivative size = 1012, normalized size of antiderivative = 13.86

$$\int \frac{1}{x\left(a+fx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + c*x^4 + f*x^2)),x)
```

output
```
log(x)/a - (log(a + c*x^4 + f*x^2)*(8*a*c - 2*f^2))/(2*(16*a^2*c - 4*a*f^2
)) - (f*atan((2*(3*f^3 - 8*a*c*f)*(4*a*c - f^2)^(3/2)*((f^2*(4*c^2*f^2 + (
2*a*c^2*f^2*(8*a*c - 2*f^2))/(16*a^2*c - 4*a*f^2)))/(16*a^2*(4*a*c - f^2))
 - ((8*a*c - 2*f^2)^2*(4*c^2*f^2 + (2*a*c^2*f^2*(8*a*c - 2*f^2))/(16*a^2*c
 - 4*a*f^2)))/(4*(16*a^2*c - 4*a*f^2)^2) + (c^2*f^4*(8*a*c - 2*f^2))/(4*a*
(16*a^2*c - 4*a*f^2)*(4*a*c - f^2))))/(c^4*f^2*(25*a*c - 6*f^2)) - (2*(4*a
*c - f^2)*(3*f^4 + 10*a^2*c^2 - 14*a*c*f^2)*((c^2*f^3*(8*a*c - 2*f^2)^2)/(
4*(16*a^2*c - 4*a*f^2)^2*(4*a*c - f^2)^(1/2)) - (c^2*f^5)/(16*a^2*(4*a*c -
 f^2)^(3/2)) + (f*(8*a*c - 2*f^2)*(4*c^2*f^2 + (2*a*c^2*f^2*(8*a*c - 2*f^2
))/(16*a^2*c - 4*a*f^2)))/(4*a*(16*a^2*c - 4*a*f^2)*(4*a*c - f^2)^(1/2))))
/(c^4*f^2*(25*a*c - 6*f^2)) + (16*a^3*x^2*(((3*f^3 - 8*a*c*f)*((f^2*(10*c^
3*f + ((12*c^2*f^3 - 40*a*c^3*f)*(8*a*c - 2*f^2))/(2*(16*a^2*c - 4*a*f^2))
))/(16*a^2*(4*a*c - f^2)) - ((8*a*c - 2*f^2)^2*(10*c^3*f + ((12*c^2*f^3 -
40*a*c^3*f)*(8*a*c - 2*f^2))/(2*(16*a^2*c - 4*a*f^2))))/(4*(16*a^2*c - 4*a
*f^2)^2) + (f^2*(12*c^2*f^3 - 40*a*c^3*f)*(8*a*c - 2*f^2))/(16*a^2*(16*a^2
*c - 4*a*f^2)*(4*a*c - f^2))))/(8*a^3*c^2*(25*a*c - 6*f^2)) - ((3*f^4 + 10
*a^2*c^2 - 14*a*c*f^2)*((f*(12*c^2*f^3 - 40*a*c^3*f)*(8*a*c - 2*f^2)^2)/(1
6*a*(16*a^2*c - 4*a*f^2)^2*(4*a*c - f^2)^(1/2)) - (f^3*(12*c^2*f^3 - 40*a*
c^3*f))/(64*a^3*(4*a*c - f^2)^(3/2)) + (f*(8*a*c - 2*f^2)*(10*c^3*f + ((12
*c^2*f^3 - 40*a*c^3*f)*(8*a*c - 2*f^2))/(2*(16*a^2*c - 4*a*f^2))))/(4*a...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.38

$$\int \frac{1}{x\left(a+fx^2+cx^4\right)}\,dx$$

$$= \frac{2\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right)f+2\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{c}}}{\sqrt{2\sqrt{}}}\right.}{}$$

input
```
int(1/x/(c*x^4+f*x^2+a),x)
```

output
```
(2*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sq
rt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f + 2*sqrt(
2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sqrt(c)*sq
rt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f - 4*log( - sqrt(2
*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c + log( - sqrt(2*sqrt
(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f**2 - 4*log(sqrt(2*sqrt(c)*s
qrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c + log(sqrt(2*sqrt(c)*sqrt(a) -
 f)*x + sqrt(a) + sqrt(c)*x**2)*f**2 + 16*log(x)*a*c - 4*log(x)*f**2)/(4*a
*(4*a*c - f**2))
```

3.781 $\int \frac{1}{x^3(a+fx^2+cx^4)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 95

$$\int \frac{1}{x^3\left(a+fx^2+cx^4\right)}\,dx=-\frac{1}{2ax^2}-\frac{\left(2ac-f^2\right)\arctan\left(\frac{f+2cx^2}{\sqrt{4ac-f^2}}\right)}{2a^2\sqrt{4ac-f^2}}-\frac{f\log(x)}{a^2}+\frac{f\log\left(a+fx^2+cx^4\right)}{4a^2}$$

output
```
-1/2/a/x^2-1/2*(2*a*c-f^2)*arctan((2*c*x^2+f)/(4*a*c-f^2)^(1/2))/a^2/(4*a*
c-f^2)^(1/2)-f*ln(x)/a^2+1/4*f*ln(c*x^4+f*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^3\left(a+fx^2+cx^4\right)}\,dx = \frac{-\frac{2a}{x^2}-4f\log(x)+\frac{\left(-2ac+f\left(f+\sqrt{-4ac+f^2}\right)\right)\log\left(f-\sqrt{-4ac+f^2}+2cx^2\right)}{\sqrt{-4ac+f^2}}+\frac{\left(2ac+f\left(-f+\sqrt{-4ac+f^2}\right)\right)\log\left(f+\sqrt{-4ac+f^2}+2cx\right.}{\sqrt{-4ac+f^2}}}{4a^2}$$

input
```
Integrate[1/(x^3*(a + f*x^2 + c*x^4)),x]
```

output
```
((-2*a)/x^2 - 4*f*Log[x] + ((-2*a*c + f*(f + Sqrt[-4*a*c + f^2]))*Log[f -
Sqrt[-4*a*c + f^2] + 2*c*x^2])/Sqrt[-4*a*c + f^2] + ((2*a*c + f*(-f + Sqrt
[-4*a*c + f^2]))*Log[f + Sqrt[-4*a*c + f^2] + 2*c*x^2])/Sqrt[-4*a*c + f^2]
)/(4*a^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1434, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+cx^4+fx^2)} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^4 (cx^4+fx^2+a)}dx^2$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{2}\left(\frac{\int -\frac{cx^2+f}{x^2(cx^4+fx^2+a)}dx^2}{a}-\frac{1}{ax^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{cx^2+f}{x^2(cx^4+fx^2+a)}dx^2}{a}-\frac{1}{ax^2}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(-\frac{\int \left(\frac{f}{ax^2}+\frac{-f^2-cx^2f+ac}{a(cx^4+fx^2+a)}\right)dx^2}{a}-\frac{1}{ax^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\frac{(2ac-f^2)\arctan\left(\frac{2cx^2+f}{\sqrt{4ac-f^2}}\right)}{a\sqrt{4ac-f^2}}-\frac{f\log(a+cx^4+fx^2)}{2a}+\frac{f\log(x^2)}{a}}{a}-\frac{1}{ax^2}\right)$$

input
```
Int[1/(x^3*(a + f*x^2 + c*x^4)),x]
```

output
```
(-(1/(a*x^2)) - (((2*a*c - f^2)*ArcTan[(f + 2*c*x^2)/Sqrt[4*a*c - f^2]])/(
a*Sqrt[4*a*c - f^2]) + (f*Log[x^2])/a - (f*Log[a + f*x^2 + c*x^4])/(2*a))/
a)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

method	result
default	$-\frac{-\frac{f\ln\left(c\,x^{4}+f\,x^{2}+a\right)}{2}+\frac{2\left(ac-\frac{f^{2}}{2}\right)\arctan\left(\frac{2c\,x^{2}+f}{\sqrt{4ac-f^{2}}}\right)}{\sqrt{4ac-f^{2}}}}{2a^{2}}-\frac{1}{2a\,x^{2}}-\frac{f\ln\left(x\right)}{a^{2}}$
risch	$-\frac{1}{2a\,x^{2}}-\frac{f\ln\left(x\right)}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-f^{2}a^{2}\right)\_Z^{2}+\left(-4acf+f^{3}\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(10a^{3}c-3f^{2}a^{2}\right)\_R^{2}-4\_Racf+2c^{2}\right)x^{2}\right.\right.}{2}$

input
```
int(1/x^3/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*(-1/2*f*ln(c*x^4+f*x^2+a)+2*(a*c-1/2*f^2)/(4*a*c-f^2)^(1/2)*arcta
n((2*c*x^2+f)/(4*a*c-f^2)^(1/2)))-1/2/a/x^2-f*ln(x)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.12

$$\int \frac{1}{x^3\left(a+fx^2+cx^4\right)}\,dx$$
$$=\left[\frac{(2\,ac-f^2)\sqrt{-4\,ac+f^2}x^2\log\left(\frac{2\,c^2x^4+2\,cfx^2-2\,ac+f^2-(2\,cx^2+f)\sqrt{-4\,ac+f^2}}{cx^4+fx^2+a}\right)+(4\,acf-f^3)x^2\log\left(cx^4+fx^2\right.}{4\,(4\,a^3c-a^2f^2)x^2}\right.$$

input
```
integrate(1/x^3/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((2*a*c - f^2)*sqrt(-4*a*c + f^2)*x^2*log((2*c^2*x^4 + 2*c*f*x^2 - 2*
a*c + f^2 - (2*c*x^2 + f)*sqrt(-4*a*c + f^2))/(c*x^4 + f*x^2 + a)) + (4*a*
c*f - f^3)*x^2*log(c*x^4 + f*x^2 + a) - 4*(4*a*c*f - f^3)*x^2*log(x) - 8*a
^2*c + 2*a*f^2)/((4*a^3*c - a^2*f^2)*x^2), 1/4*(2*sqrt(4*a*c - f^2)*(2*a*c
 - f^2)*x^2*arctan(-(2*c*x^2 + f)/sqrt(4*a*c - f^2)) + (4*a*c*f - f^3)*x^2
*log(c*x^4 + f*x^2 + a) - 4*(4*a*c*f - f^3)*x^2*log(x) - 8*a^2*c + 2*a*f^2
)/((4*a^3*c - a^2*f^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+f x^2+c x^4\right)}\, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(c*x**4+f*x**2+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(a+f x^2+c x^4\right)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(f^2-4*a*c>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^3 \left(a+f x^2+c x^4\right)}\, dx = \frac{f \log\left(c x^4+f x^2+a\right)}{4 a^2} - \frac{f \log\left(x^2\right)}{2 a^2} - \frac{\left(2 a c-f^2\right) \arctan\left(\frac{2 c x^2+f}{\sqrt{4 a c-f^2}}\right)}{2 \sqrt{4 a c-f^2} a^2} + \frac{f x^2-a}{2 a^2 x^2}$$

input
```
integrate(1/x^3/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
1/4*f*log(c*x^4 + f*x^2 + a)/a^2 - 1/2*f*log(x^2)/a^2 - 1/2*(2*a*c - f^2)*
arctan((2*c*x^2 + f)/sqrt(4*a*c - f^2))/(sqrt(4*a*c - f^2)*a^2) + 1/2*(f*x
^2 - a)/(a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 19.25 (sec) , antiderivative size = 2033, normalized size of antiderivative = 21.40

$$\int \frac{1}{x^3 (a + fx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + c*x^4 + f*x^2)),x)
```

output
```
(atan((16*a^6*x^2*(4*a*c - f^2)^(3/2)*(((3*f^4 + a^2*c^2 - 9*a*c*f^2)*(c^5
/a^3 + ((2*f^3 - 8*a*c*f)*((6*c^4*f)/a^2 + (((20*a^3*c^4 + 2*a^2*c^3*f^2)/
a^3 + ((2*f^3 - 8*a*c*f)*(40*a^4*c^3*f - 12*a^3*c^2*f^3))/(2*a^3*(16*a^3*c
 - 4*a^2*f^2)))*(2*f^3 - 8*a*c*f))/(2*(16*a^3*c - 4*a^2*f^2))))/(2*(16*a^3
*c - 4*a^2*f^2)) - (((((20*a^3*c^4 + 2*a^2*c^3*f^2)/a^3 + ((2*f^3 - 8*a*c*
f)*(40*a^4*c^3*f - 12*a^3*c^2*f^3))/(2*a^3*(16*a^3*c - 4*a^2*f^2)))*(2*a*c
 - f^2))/(4*a^2*(4*a*c - f^2)^(1/2)) + ((2*f^3 - 8*a*c*f)*(40*a^4*c^3*f -
12*a^3*c^2*f^3)*(2*a*c - f^2))/(8*a^5*(4*a*c - f^2)^(1/2)*(16*a^3*c - 4*a^
2*f^2)))*(2*a*c - f^2))/(4*a^2*(4*a*c - f^2)^(1/2)) - ((2*f^3 - 8*a*c*f)*(
40*a^4*c^3*f - 12*a^3*c^2*f^3)*(2*a*c - f^2)^2)/(32*a^7*(4*a*c - f^2)*(16*
a^3*c - 4*a^2*f^2))))/(8*a^3*c^2*(a^2*c^2 - 6*f^4 + 24*a*c*f^2)) + ((((2*f
^3 - 8*a*c*f)*((((20*a^3*c^4 + 2*a^2*c^3*f^2)/a^3 + ((2*f^3 - 8*a*c*f)*(40
*a^4*c^3*f - 12*a^3*c^2*f^3))/(2*a^3*(16*a^3*c - 4*a^2*f^2)))*(2*a*c - f^2
))/(4*a^2*(4*a*c - f^2)^(1/2)) + ((2*f^3 - 8*a*c*f)*(40*a^4*c^3*f - 12*a^3
*c^2*f^3)*(2*a*c - f^2))/(8*a^5*(4*a*c - f^2)^(1/2)*(16*a^3*c - 4*a^2*f^2)
)))/(2*(16*a^3*c - 4*a^2*f^2)) + (((6*c^4*f)/a^2 + (((20*a^3*c^4 + 2*a^2*c
^3*f^2)/a^3 + ((2*f^3 - 8*a*c*f)*(40*a^4*c^3*f - 12*a^3*c^2*f^3))/(2*a^3*(
16*a^3*c - 4*a^2*f^2)))*(2*f^3 - 8*a*c*f))/(2*(16*a^3*c - 4*a^2*f^2)))*(2*
a*c - f^2))/(4*a^2*(4*a*c - f^2)^(1/2)) - ((40*a^4*c^3*f - 12*a^3*c^2*f^3)
*(2*a*c - f^2)^3)/(64*a^9*(4*a*c - f^2)^(3/2)))*(3*f^5 + 13*a^2*c^2*f -...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 409, normalized size of antiderivative = 4.31

$$\int \frac{1}{x^3 (a + f x^2 + c x^4)} \, dx$$

$$= \frac{4\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right) ac\,x^2 - 2\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,atan\left(\frac{\sqrt{2}}{}\right.}{}$$

input
```
int(1/x^3/(c*x^4+f*x^2+a),x)
```

output
```
(4*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sq
rt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*x**2 -
2*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sqr
t(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f**2*x**2 +
4*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sqr
t(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*x**2 - 2
*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2*sqrt
(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f**2*x**2 + 4
*log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c*f*x**2
 - log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f**3*x**
2 + 4*log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c*f*x*
*2 - log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f**3*x**2
 - 16*log(x)*a*c*f*x**2 + 4*log(x)*f**3*x**2 - 8*a**2*c + 2*a*f**2)/(4*a**
2*x**2*(4*a*c - f**2))
```

3.782 $\int \frac{1}{x^5(a+fx^2+cx^4)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{1}{x^5(a+fx^2+cx^4)}\,dx = -\frac{1}{4ax^4}+\frac{f}{2a^2x^2}+\frac{f(3ac-f^2)\arctan\left(\frac{f+2cx^2}{\sqrt{4ac-f^2}}\right)}{2a^3\sqrt{4ac-f^2}} - \frac{(ac-f^2)\log(x)}{a^3}+\frac{(ac-f^2)\log(a+fx^2+cx^4)}{4a^3}$$

output
```
-1/4/a/x^4+1/2*f/a^2/x^2+1/2*f*(3*a*c-f^2)*arctan((2*c*x^2+f)/(4*a*c-f^2)^
(1/2))/a^3/(4*a*c-f^2)^(1/2)-(a*c-f^2)*ln(x)/a^3+1/4*(a*c-f^2)*ln(c*x^4+f*
x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^5(a+fx^2+cx^4)}\,dx = \frac{-\frac{a^2}{x^4}+\frac{2af}{x^2}+4(-ac+f^2)\log(x)+\frac{\left(-f^2\left(f+\sqrt{-4ac+f^2}\right)+ac\left(3f+\sqrt{-4ac+f^2}\right)\right)\log\left(f-\sqrt{-4ac+f^2}+2cx^2\right)}{\sqrt{-4ac+f^2}}+\frac{\left(f^2\left(f-\sqrt{-\right.\right.}{}}{4a^3}$$

input
```
Integrate[1/(x^5*(a + f*x^2 + c*x^4)),x]
```

output
```
(-(a^2/x^4) + (2*a*f)/x^2 + 4*(-(a*c) + f^2)*Log[x] + ((-(f^2*(f + Sqrt[-4
*a*c + f^2])) + a*c*(3*f + Sqrt[-4*a*c + f^2]))*Log[f - Sqrt[-4*a*c + f^2]
 + 2*c*x^2])/Sqrt[-4*a*c + f^2] + ((f^2*(f - Sqrt[-4*a*c + f^2]) + a*c*(-3
*f + Sqrt[-4*a*c + f^2]))*Log[f + Sqrt[-4*a*c + f^2] + 2*c*x^2])/Sqrt[-4*a
*c + f^2])/(4*a^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1434, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+cx^4+fx^2)}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^6 (cx^4+fx^2+a)}dx^2$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{2}\left(\frac{\int -\frac{cx^2+f}{x^4(cx^4+fx^2+a)}dx^2}{a}-\frac{1}{2ax^4}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{cx^2+f}{x^4(cx^4+fx^2+a)}dx^2}{a}-\frac{1}{2ax^4}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(-\frac{\int \left(\frac{f}{ax^4}+\frac{-c(ac-f^2)x^2-f(2ac-f^2)}{a^2(cx^4+fx^2+a)}+\frac{ac-f^2}{a^2x^2}\right)dx^2}{a}-\frac{1}{2ax^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{-\frac{f(3ac-f^2)\arctan\left(\frac{2cx^2+f}{\sqrt{4ac-f^2}}\right)}{a^2\sqrt{4ac-f^2}}+\frac{\log(x^2)(ac-f^2)}{a^2}-\frac{(ac-f^2)\log(a+cx^4+fx^2)}{2a^2}-\frac{f}{ax^2}}{a}-\frac{1}{2ax^4}\right)$$

input
```
Int[1/(x^5*(a + f*x^2 + c*x^4)),x]
```

output
```
(-1/2*1/(a*x^4) - (-(f/(a*x^2)) - (f*(3*a*c - f^2)*ArcTan[(f + 2*c*x^2)/Sq
rt[4*a*c - f^2]])/(a^2*Sqrt[4*a*c - f^2]) + ((a*c - f^2)*Log[x^2])/a^2 - (
(a*c - f^2)*Log[a + f*x^2 + c*x^4])/(2*a^2))/a)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

method	result
default	$\frac{\frac{(c^2a-cf^2)\ln(cx^4+fx^2+a)}{2c}+\frac{2\left(2acf-f^3-\frac{(c^2a-cf^2)f}{2c}\right)\arctan\left(\frac{2cx^2+f}{\sqrt{4ac-f^2}}\right)}{\sqrt{4ac-f^2}}}{2a^3}-\frac{1}{4a\,x^4}+\frac{(-ac+f^2)\ln(x)}{a^3}+\frac{f}{2a^2x^2}$
risch	$\frac{\frac{f\,x^2}{2a^2}-\frac{1}{4a}}{x^4}-\frac{\ln(x)c}{a^2}+\frac{\ln(x)f^2}{a^3}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4c\,a^4-f^2a^3\right)\_Z^2+\left(-4a^2c^2+5ac\,f^2-f^4\right)\_Z+c^3\right)}\_R\ln\left(\left(\left(10c\,a^5-3a^4f^2\right)\_R\right.\right.\right.}{}$

input
```
int(1/x^5/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^3*(1/2*(a*c^2-c*f^2)/c*ln(c*x^4+f*x^2+a)+2*(2*a*c*f-f^3-1/2*(a*c^2-c
*f^2)*f/c)/(4*a*c-f^2)^(1/2)*arctan((2*c*x^2+f)/(4*a*c-f^2)^(1/2)))-1/4/a/
x^4+1/a^3*(-a*c+f^2)*ln(x)+1/2*f/a^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.02

$$\int \frac{1}{x^5(a+fx^2+cx^4)}\,dx$$

$$= \left[\frac{(3\,acf-f^3)\sqrt{-4\,ac+f^2}x^4\log\left(\frac{2\,c^2x^4+2\,cfx^2-2\,ac+f^2+(2\,cx^2+f)\sqrt{-4\,ac+f^2}}{cx^4+fx^2+a}\right)+(4\,a^2c^2-5\,acf^2+f^4)x^4\log\left(\right.}{4\,(4\,a^4c-a^3f^2)}\right.$$

$$-\frac{2\,(3\,acf-f^3)\sqrt{4\,ac-f^2}x^4\arctan\left(-\frac{2\,cx^2+f}{\sqrt{4\,ac-f^2}}\right)-(4\,a^2c^2-5\,acf^2+f^4)x^4\log\left(cx^4+fx^2+a\right)+4\,(4}{4\,(4\,a^4c-a^3f^2)x^4}$$

input
```
integrate(1/x^5/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((3*a*c*f - f^3)*sqrt(-4*a*c + f^2)*x^4*log((2*c^2*x^4 + 2*c*f*x^2 -
2*a*c + f^2 + (2*c*x^2 + f)*sqrt(-4*a*c + f^2))/(c*x^4 + f*x^2 + a)) + (4*
a^2*c^2 - 5*a*c*f^2 + f^4)*x^4*log(c*x^4 + f*x^2 + a) - 4*(4*a^2*c^2 - 5*a
*c*f^2 + f^4)*x^4*log(x) - 4*a^3*c + a^2*f^2 + 2*(4*a^2*c*f - a*f^3)*x^2)/
((4*a^4*c - a^3*f^2)*x^4), -1/4*(2*(3*a*c*f - f^3)*sqrt(4*a*c - f^2)*x^4*a
rctan(-(2*c*x^2 + f)/sqrt(4*a*c - f^2)) - (4*a^2*c^2 - 5*a*c*f^2 + f^4)*x^
4*log(c*x^4 + f*x^2 + a) + 4*(4*a^2*c^2 - 5*a*c*f^2 + f^4)*x^4*log(x) + 4*
a^3*c - a^2*f^2 - 2*(4*a^2*c*f - a*f^3)*x^2)/((4*a^4*c - a^3*f^2)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a + fx^2 + cx^4)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**5/(c*x**4+f*x**2+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 (a + fx^2 + cx^4)} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(f^2-4*a*c>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^5 (a + fx^2 + cx^4)} \, dx = \frac{(ac - f^2) \log (cx^4 + fx^2 + a)}{4 a^3} - \frac{(ac - f^2) \log (x^2)}{2 a^3} + \frac{(3 acf - f^3) \arctan \left(\frac{2 cx^2 + f}{\sqrt{4 ac - f^2}}\right)}{2 \sqrt{4 ac - f^2} a^3} + \frac{3 acx^4 - 3 f^2 x^4 + 2 afx^2 - a^2}{4 a^3 x^4}$$

input
```
integrate(1/x^5/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
1/4*(a*c - f^2)*log(c*x^4 + f*x^2 + a)/a^3 - 1/2*(a*c - f^2)*log(x^2)/a^3
+ 1/2*(3*a*c*f - f^3)*arctan((2*c*x^2 + f)/sqrt(4*a*c - f^2))/(sqrt(4*a*c
- f^2)*a^3) + 1/4*(3*a*c*x^4 - 3*f^2*x^4 + 2*a*f*x^2 - a^2)/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 19.91 (sec) , antiderivative size = 2451, normalized size of antiderivative = 19.93

$$\int \frac{1}{x^5 (a + fx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^5*(a + c*x^4 + f*x^2)),x)
```

output
```
(log(a + c*x^4 + f*x^2)*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(2*(16*a^4*c - 4
*a^3*f^2)) - (1/(4*a) - (f*x^2)/(2*a^2))/x^4 - (log(x)*(a*c - f^2))/a^3 +
(f*atan((2*a^6*(4*a*c - f^2)*((((f*(3*a*c - f^2)*((4*a^4*c^2*f^4 - 8*a^5*c
^3*f^2)/a^6 - (2*a*c^2*f^2*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(16*a^4*c - 4
*a^3*f^2)))/(4*a^3*(4*a*c - f^2)^(1/2)) - (c^2*f^3*(3*a*c - f^2)*(2*f^4 +
8*a^2*c^2 - 10*a*c*f^2))/(2*a^2*(4*a*c - f^2)^(1/2)*(16*a^4*c - 4*a^3*f^2)
))*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(2*(16*a^4*c - 4*a^3*f^2)) + (f*(3*a*
c - f^2)*((4*a^2*c^3*f^4 - 5*a^3*c^4*f^2)/a^6 + (((4*a^4*c^2*f^4 - 8*a^5*c
^3*f^2)/a^6 - (2*a*c^2*f^2*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(16*a^4*c - 4
*a^3*f^2))*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(2*(16*a^4*c - 4*a^3*f^2))))/
(4*a^3*(4*a*c - f^2)^(1/2)) + (c^2*f^5*(3*a*c - f^2)^3)/(16*a^8*(4*a*c - f
^2)^(3/2)))*(3*f^6 - 10*a^3*c^3 + 27*a^2*c^2*f^2 - 18*a*c*f^4))/(c^2*(c^2*
f^6 - 6*a*c^3*f^4 + 9*a^2*c^4*f^2)*(6*f^6 - 25*a^3*c^3 + 54*a^2*c^2*f^2 -
36*a*c*f^4)) - (16*a^9*x^2*(4*a*c - f^2)^(3/2)*((((f^3*(40*a^7*c^3*f - 12*
a^6*c^2*f^3)*(3*a*c - f^2)^3)/(64*a^15*(4*a*c - f^2)^(3/2)) - (((f*(3*a*c
- f^2)*((10*a^5*c^4*f + 2*a^4*c^3*f^3)/a^6 + ((40*a^7*c^3*f - 12*a^6*c^2*f
^3)*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(2*a^6*(16*a^4*c - 4*a^3*f^2))))/(4*
a^3*(4*a*c - f^2)^(1/2)) + (f*(40*a^7*c^3*f - 12*a^6*c^2*f^3)*(3*a*c - f^2
)*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(8*a^9*(4*a*c - f^2)^(1/2)*(16*a^4*c -
 4*a^3*f^2)))*(2*f^4 + 8*a^2*c^2 - 10*a*c*f^2))/(2*(16*a^4*c - 4*a^3*f^...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.20

$$\int \frac{1}{x^5 (a+fx^2+cx^4)}\,dx$$

$$= \frac{-6\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right)acf\,x^4+2\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,\sqrt{2\sqrt{c}\,\sqrt{a}-f}\,atan\Big(}{}$$

input
```
int(1/x^5/(c*x^4+f*x^2+a),x)
```

output
```
( - 6*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(2
*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*f*x*
*4 + 2*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(
2*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f**3*x*
*4 - 6*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt(
2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*f*x
**4 + 2*sqrt(2*sqrt(c)*sqrt(a) + f)*sqrt(2*sqrt(c)*sqrt(a) - f)*atan((sqrt
(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f**3*x
**4 + 4*log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a**
2*c**2*x**4 - 5*log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x
**2)*a*c*f**2*x**4 + log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt
(c)*x**2)*f**4*x**4 + 4*log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt
(c)*x**2)*a**2*c**2*x**4 - 5*log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) +
 sqrt(c)*x**2)*a*c*f**2*x**4 + log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a)
 + sqrt(c)*x**2)*f**4*x**4 - 16*log(x)*a**2*c**2*x**4 + 20*log(x)*a*c*f**2
*x**4 - 4*log(x)*f**4*x**4 - 4*a**3*c + 8*a**2*c*f*x**2 + a**2*f**2 - 2*a*
f**3*x**2)/(4*a**3*x**4*(4*a*c - f**2))
```

3.783 $\int \frac{x^8}{a+fx^2+cx^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 355

$$\begin{aligned}\int \frac{x^8}{a+fx^2+cx^4}\,dx = & -\frac{(ac-f^2)\,x}{c^3}-\frac{fx^3}{3c^2}+\frac{x^5}{5c} \\ & -\frac{\left(2acf-f^3+\sqrt{a}\sqrt{c}(ac-f^2)\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2c^{7/2}\sqrt{2\sqrt{a}\sqrt{c}+f}} \\ & +\frac{\left(2acf-f^3+\sqrt{a}\sqrt{c}(ac-f^2)\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2c^{7/2}\sqrt{2\sqrt{a}\sqrt{c}+f}} \\ & +\frac{\left(a^{3/2}c^{3/2}-2acf-\sqrt{a}\sqrt{c}f^2+f^3\right)\operatorname{arctanh}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2c^{7/2}\sqrt{2\sqrt{a}\sqrt{c}-f}}\end{aligned}$$

output
```
-(a*c-f^2)*x/c^3-1/3*f*x^3/c^2+1/5*x^5/c-1/2*(2*a*c*f-f^3+a^(1/2)*c^(1/2)*
(a*c-f^2))*arctan(((2*a^(1/2)*c^(1/2)-f)^(1/2)-2*c^(1/2)*x)/(2*a^(1/2)*c^(
1/2)+f)^(1/2))/c^(7/2)/(2*a^(1/2)*c^(1/2)+f)^(1/2)+1/2*(2*a*c*f-f^3+a^(1/2
)*c^(1/2)*(a*c-f^2))*arctan(((2*a^(1/2)*c^(1/2)-f)^(1/2)+2*c^(1/2)*x)/(2*a
^(1/2)*c^(1/2)+f)^(1/2))/c^(7/2)/(2*a^(1/2)*c^(1/2)+f)^(1/2)+1/2*(a^(3/2)*
c^(3/2)-2*a*c*f-a^(1/2)*c^(1/2)*f^2+f^3)*arctanh((2*a^(1/2)*c^(1/2)-f)^(1/
2)*x/(a^(1/2)+c^(1/2)*x^2))/c^(7/2)/(2*a^(1/2)*c^(1/2)-f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{a+fx^2+cx^4}\,dx = \frac{(-ac+f^2)\,x}{c^3}-\frac{fx^3}{3c^2}+\frac{x^5}{5c} + \frac{\left(2a^2c^2+f^3\left(f-\sqrt{-4ac+f^2}\right)+2acf\left(-2f+\sqrt{-4ac+f^2}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f-\sqrt{-4ac+f^2}}}\right)}{\sqrt{2}c^{7/2}\sqrt{-4ac+f^2}\sqrt{f-\sqrt{-4ac+f^2}}} - \frac{\left(2a^2c^2+f^3\left(f+\sqrt{-4ac+f^2}\right)-2acf\left(2f+\sqrt{-4ac+f^2}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f+\sqrt{-4ac+f^2}}}\right)}{\sqrt{2}c^{7/2}\sqrt{-4ac+f^2}\sqrt{f+\sqrt{-4ac+f^2}}}$$

input
```
Integrate[x^8/(a + f*x^2 + c*x^4),x]
```

output
```
((-(a*c) + f^2)*x)/c^3 - (f*x^3)/(3*c^2) + x^5/(5*c) + ((2*a^2*c^2 + f^3*(
f - Sqrt[-4*a*c + f^2]) + 2*a*c*f*(-2*f + Sqrt[-4*a*c + f^2]))*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[f - Sqrt[-4*a*c + f^2]]])/(Sqrt[2]*c^(7/2)*Sqrt[-4*a*
c + f^2]*Sqrt[f - Sqrt[-4*a*c + f^2]]) - ((2*a^2*c^2 + f^3*(f + Sqrt[-4*a*
c + f^2]) - 2*a*c*f*(2*f + Sqrt[-4*a*c + f^2]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[f + Sqrt[-4*a*c + f^2]]])/(Sqrt[2]*c^(7/2)*Sqrt[-4*a*c + f^2]*Sqrt[f
 + Sqrt[-4*a*c + f^2]])
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.50, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {1442, 27, 1602, 27, 1602, 25, 1483, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{a+cx^4+fx^2}\,dx$$

$$\Big\downarrow \; 1442$$

$$\frac{x^5}{5c}-\frac{\int \frac{5x^4(fx^2+a)}{cx^4+fx^2+a}dx}{5c}$$

$\downarrow$ 27

$$\frac{x^5}{5c}-\frac{\int \frac{x^4(fx^2+a)}{cx^4+fx^2+a}dx}{c}$$

$\downarrow$ 1602

$$\frac{x^5}{5c}-\frac{\frac{fx^3}{3c}-\frac{\int \frac{3x^2\left(af-\left(ac-f^2\right)x^2\right)}{cx^4+fx^2+a}dx}{3c}}{c}$$

$\downarrow$ 27

$$\frac{x^5}{5c}-\frac{\frac{fx^3}{3c}-\frac{\int \frac{x^2\left(af-\left(ac-f^2\right)x^2\right)}{cx^4+fx^2+a}dx}{c}}{c}$$

$\downarrow$ 1602

$$\frac{x^5}{5c}-\frac{\frac{fx^3}{3c}-\frac{-\frac{\int -\frac{f\left(2ac-f^2\right)x^2+a\left(ac-f^2\right)}{cx^4+fx^2+a}dx}{c}-\frac{x\left(ac-f^2\right)}{c}}{c}}{c}$$

$\downarrow$ 25

$$\frac{x^5}{5c}-\frac{\frac{fx^3}{3c}-\frac{\frac{\int \frac{f\left(2ac-f^2\right)x^2+a\left(ac-f^2\right)}{cx^4+fx^2+a}dx}{c}-\frac{x\left(ac-f^2\right)}{c}}{c}}{c}$$

$\downarrow$ 1483

$$\frac{x^5}{5c}-\frac{\frac{fx^3}{3c}-\frac{\frac{\frac{\int \frac{\sqrt{a}\left(\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(ac-f^2\right)-\sqrt{c}\left(\sqrt{a}\left(ac-f^2\right)-\frac{f\left(2ac-f^2\right)}{\sqrt{c}}\right)x\right)}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\int \frac{\sqrt{a}\left(\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(ac-f^2\right)+\sqrt{c}\left(\sqrt{a}\left(ac-f^2\right)-\frac{f\left(2ac-f^2\right)}{\sqrt{c}}\right)x\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}}{c}-}{c}}{c}$$

$\downarrow$ 27

$$\frac{x^5}{5c}-\frac{\frac{fx^3}{3c}-\frac{\frac{\frac{\int \frac{\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(ac-f^2\right)-\left(f^3-\sqrt{a}\sqrt{c}f^2-2acf+a^{3/2}c^{3/2}\right)x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\int \frac{\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(ac-f^2\right)+\left(f^3-\sqrt{a}\sqrt{c}f^2-2acf+a^{3/2}c^{3/2}\right)x}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}}{c}-\frac{x\left(ac-f^2\right)}{c}}{c}}{c}$$

$\big\downarrow$ 1142

$$\frac{x^5}{5c}-\frac{\frac{fx^3}{3c}-\frac{\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(a^{3/2}c^{3/2}-\sqrt{a}\sqrt{c}f^2+2acf-f^3\right)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{1}{2}\left(a^{3/2}c^{3/2}-\sqrt{a}\sqrt{c}f^2-2acf+f^3\right)\int -\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}}{}}{}$$

$\big\downarrow$ 25

$$\frac{x^5}{5c}-\frac{\frac{fx^3}{3c}-\frac{\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(a^{3/2}c^{3/2}-\sqrt{a}\sqrt{c}f^2+2acf-f^3\right)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{1}{2}\left(a^{3/2}c^{3/2}-\sqrt{a}\sqrt{c}f^2-2acf+f^3\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+}{c}}{}$$

$\big\downarrow$ 27

$$\frac{x^5}{5c}-\frac{\frac{fx^3}{3c}-\frac{\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(a^{3/2}c^{3/2}-\sqrt{a}\sqrt{c}f^2+2acf-f^3\right)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\left(a^{3/2}c^{3/2}-\sqrt{a}\sqrt{c}f^2-2acf+f^3\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\sqrt{2\sqrt{a}}}{}}{c}}{c}$$

$\big\downarrow$ 1083

$$\frac{x^5}{5c}-\frac{\frac{fx^3}{3c}-\frac{\frac{\frac{\left(a^{3/2}c^{3/2}-\sqrt{a}\sqrt{c}f^2-2acf+f^3\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(a^{3/2}c^{3/2}-\sqrt{a}\sqrt{c}f^2+2acf-f^3\right)\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)^2-\frac{f+2\sqrt{a}\sqrt{c}}{c}}}{\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}}{}}{}$$

$\big\downarrow$ 217

$$\frac{fx^3}{3c}-\frac{\frac{x^5}{5c}-\frac{\frac{\frac{\left(a^{3/2}c^{3/2}-\sqrt{a}\sqrt{c}f^2-2acf+f^3\right)\int\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(a^{3/2}c^{3/2}-\sqrt{a}\sqrt{c}f^2+2acf-f^3\right)\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\cdots}{c}}{c}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{fx^3}{3c}-\frac{\frac{x^5}{5c}-\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(a^{3/2}c^{3/2}-\sqrt{a}\sqrt{c}f^2+2acf-f^3\right)\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}-\frac{1}{2}\left(a^{3/2}c^{3/2}-\sqrt{a}\sqrt{c}f^2-2acf+f^3\right)\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}-f}+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}\cdots}{\cdots}$$

input
```
Int[x^8/(a + f*x^2 + c*x^4),x]
```

output
```
x^5/(5*c) - ((f*x^3)/(3*c) - (-(((a*c - f^2)*x)/c) + (((Sqrt[2*Sqrt[a]*Sqr
t[c] - f]*(a^(3/2)*c^(3/2) + 2*a*c*f - Sqrt[a]*Sqrt[c]*f^2 - f^3)*ArcTan[(
Sqrt[c]*(-(Sqrt[2*Sqrt[a]*Sqrt[c] - f]/Sqrt[c]) + 2*x))/Sqrt[2*Sqrt[a]*Sqr
t[c] + f]])/Sqrt[2*Sqrt[a]*Sqrt[c] + f] - ((a^(3/2)*c^(3/2) - 2*a*c*f - Sq
rt[a]*Sqrt[c]*f^2 + f^3)*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] - f]*x + Sqr
t[c]*x^2])/2)/(2*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f]) + ((Sqrt[2*Sqrt[a]*S
qrt[c] - f]*(a^(3/2)*c^(3/2) + 2*a*c*f - Sqrt[a]*Sqrt[c]*f^2 - f^3)*ArcTan
[(Sqrt[c]*(Sqrt[2*Sqrt[a]*Sqrt[c] - f]/Sqrt[c] + 2*x))/Sqrt[2*Sqrt[a]*Sqrt
[c] + f]])/Sqrt[2*Sqrt[a]*Sqrt[c] + f] + ((a^(3/2)*c^(3/2) - 2*a*c*f - Sqr
t[a]*Sqrt[c]*f^2 + f^3)*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] - f]*x + Sqrt
[c]*x^2])/2)/(2*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f]))/c)/c)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.28

method	result
risch	$\frac{x^5}{5c}-\frac{f x^3}{3c^2}-\frac{ax}{c^2}+\frac{x f^2}{c^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 c+f\,\_Z^2+a\right)}\frac{\left(f\left(2ac-f^2\right)\_R^2+c\,a^2-a\,f^2\right)\ln\left(x-\_R\right)}{2\_R^3 c+\_R f}}{2c^3}$
default	$-\frac{-\frac{1}{5}c^2x^5+\frac{1}{3}f\,x^3c+acx-x\,f^2}{c^3}+\frac{\frac{\left(2acf\sqrt{-4ac+f^2}-f^3\sqrt{-4ac+f^2}-2a^2c^2+4ac\,f^2-f^4\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+f^2}+f\right)c}}\right)}{2c\sqrt{-4ac+f^2}\sqrt{\left(\sqrt{-4ac+f^2}+f\right)c}}-\frac{\left(2acf\sqrt{-\cdots\right.}{\cdots}}{c^2}$

input
```
int(x^8/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5/c-1/3*f*x^3/c^2-1/c^2*a*x+1/c^3*x*f^2+1/2/c^3*sum((f*(2*a*c-f^2)*_
R^2+c*a^2-a*f^2)/(2*_R^3*c+_R*f)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*f+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2133 vs. 2(270) = 540.

Time = 0.16 (sec) , antiderivative size = 2133, normalized size of antiderivative = 6.01

$$\int \frac{x^8}{a+fx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
1/30*(6*c^2*x^5 - 10*c*f*x^3 - 15*sqrt(1/2)*c^3*sqrt(-(7*a^3*c^3*f - 14*a^
2*c^2*f^3 + 7*a*c*f^5 - f^7 + (4*a*c^8 - c^7*f^2)*sqrt(-(a^6*c^6 - 12*a^5*
c^5*f^2 + 46*a^4*c^4*f^4 - 62*a^3*c^3*f^6 + 37*a^2*c^2*f^8 - 10*a*c*f^10 +
 f^12)/(4*a*c^15 - c^14*f^2)))/(4*a*c^8 - c^7*f^2))*log(-2*(a^6*c^3 - 6*a^
5*c^2*f^2 + 5*a^4*c*f^4 - a^3*f^6)*x + sqrt(1/2)*(4*a^5*c^5 - 29*a^4*c^4*f
^2 + 51*a^3*c^3*f^4 - 35*a^2*c^2*f^6 + 10*a*c*f^8 - f^10 + (12*a^2*c^9*f -
 7*a*c^8*f^3 + c^7*f^5)*sqrt(-(a^6*c^6 - 12*a^5*c^5*f^2 + 46*a^4*c^4*f^4 -
 62*a^3*c^3*f^6 + 37*a^2*c^2*f^8 - 10*a*c*f^10 + f^12)/(4*a*c^15 - c^14*f^
2)))*sqrt(-(7*a^3*c^3*f - 14*a^2*c^2*f^3 + 7*a*c*f^5 - f^7 + (4*a*c^8 - c^
7*f^2)*sqrt(-(a^6*c^6 - 12*a^5*c^5*f^2 + 46*a^4*c^4*f^4 - 62*a^3*c^3*f^6 +
 37*a^2*c^2*f^8 - 10*a*c*f^10 + f^12)/(4*a*c^15 - c^14*f^2)))/(4*a*c^8 - c
^7*f^2))) + 15*sqrt(1/2)*c^3*sqrt(-(7*a^3*c^3*f - 14*a^2*c^2*f^3 + 7*a*c*f
^5 - f^7 + (4*a*c^8 - c^7*f^2)*sqrt(-(a^6*c^6 - 12*a^5*c^5*f^2 + 46*a^4*c^
4*f^4 - 62*a^3*c^3*f^6 + 37*a^2*c^2*f^8 - 10*a*c*f^10 + f^12)/(4*a*c^15 -
c^14*f^2)))/(4*a*c^8 - c^7*f^2))*log(-2*(a^6*c^3 - 6*a^5*c^2*f^2 + 5*a^4*c
*f^4 - a^3*f^6)*x - sqrt(1/2)*(4*a^5*c^5 - 29*a^4*c^4*f^2 + 51*a^3*c^3*f^4
 - 35*a^2*c^2*f^6 + 10*a*c*f^8 - f^10 + (12*a^2*c^9*f - 7*a*c^8*f^3 + c^7*
f^5)*sqrt(-(a^6*c^6 - 12*a^5*c^5*f^2 + 46*a^4*c^4*f^4 - 62*a^3*c^3*f^6 + 3
7*a^2*c^2*f^8 - 10*a*c*f^10 + f^12)/(4*a*c^15 - c^14*f^2)))*sqrt(-(7*a^3*c
^3*f - 14*a^2*c^2*f^3 + 7*a*c*f^5 - f^7 + (4*a*c^8 - c^7*f^2)*sqrt(-(a^...
```

Sympy [A] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.74

$$\int \frac{x^8}{a+fx^2+cx^4}\,dx = x\left(-\frac{a}{c^2}+\frac{f^2}{c^3}\right)$$
$$+\text{RootSum}\left(t^4\cdot\left(256a^2c^9-128ac^8f^2+16c^7f^4\right)+t^2\cdot\left(112a^4c^4f-252a^3c^3f^3+168a^2c^2f^5-44acf^7+\right.\right.$$
$$+\frac{x^5}{5c}-\frac{fx^3}{3c^2}$$

input
```
integrate(x**8/(c*x**4+f*x**2+a),x)
```

output
```
x*(-a/c**2 + f**2/c**3) + RootSum(_t**4*(256*a**2*c**9 - 128*a*c**8*f**2 +
 16*c**7*f**4) + _t**2*(112*a**4*c**4*f - 252*a**3*c**3*f**3 + 168*a**2*c*
*2*f**5 - 44*a*c*f**7 + 4*f**9) + a**7, Lambda(_t, _t*log(x + (-96*_t**3*a
**2*c**9*f + 56*_t**3*a*c**8*f**3 - 8*_t**3*c**7*f**5 + 4*_t*a**5*c**5 - 5
0*_t*a**4*c**4*f**2 + 100*_t*a**3*c**3*f**4 - 70*_t*a**2*c**2*f**6 + 20*_t
*a*c*f**8 - 2*_t*f**10)/(a**6*c**3 - 6*a**5*c**2*f**2 + 5*a**4*c*f**4 - a*
*3*f**6)))) + x**5/(5*c) - f*x**3/(3*c**2)
```

Maxima [F]

$$\int \frac{x^8}{a+fx^2+cx^4}\,dx = \int \frac{x^8}{cx^4+fx^2+a}\,dx$$

input
```
integrate(x^8/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
1/15*(3*c^2*x^5 - 5*c*f*x^3 - 15*(a*c - f^2)*x)/c^3 - integrate(-(a^2*c -
a*f^2 + (2*a*c*f - f^3)*x^2)/(c*x^4 + f*x^2 + a), x)/c^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3064 vs. $2(270) = 540$.

Time = 0.65 (sec) , antiderivative size = 3064, normalized size of antiderivative = 8.63

$$\int \frac{x^8}{a+fx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
1/8*(16*a^3*c^7*f - 36*a^2*c^6*f^3 + 16*a*c^5*f^5 - 2*c^4*f^7 - 8*sqrt(2)*
sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^3*c^5*f + 2*sqrt(2)*
sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^6*f - 4*sqrt(2)*
sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^5*f^2 + 18*sqrt(
2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^4*f^3 - 4*sqr
t(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^5*f^3 + 8*sqr
t(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^4*f^4 - 8*sqr
t(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^3*f^5 + sqrt(
2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c^4*f^5 - 2*sqrt(2)
*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c^3*f^6 + sqrt(2)*sqr
t(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c^2*f^7 - 4*(4*a*c - f^2)
*a^2*c^6*f + 8*(4*a*c - f^2)*a*c^5*f^3 - 2*(4*a*c - f^2)*c^4*f^5 - (64*a^3
*c^5*f - 64*a^2*c^4*f^3 + 20*a*c^3*f^5 - 2*c^2*f^7 - 32*sqrt(2)*sqrt(-4*a*
c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^3*c^3*f + 8*sqrt(2)*sqrt(-4*a*
c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^4*f - 16*sqrt(2)*sqrt(-4*a
*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^3*f^2 + 32*sqrt(2)*sqrt(-
4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^2*f^3 - 6*sqrt(2)*sqrt
(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^3*f^3 + 12*sqrt(2)*sqr
t(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^2*f^4 - 10*sqrt(2)*sq
rt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c*f^5 + sqrt(2)*sqr...
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 5317, normalized size of antiderivative = 14.98

$$\int \frac{x^8}{a+fx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^8/(a + c*x^4 + f*x^2),x)
```

output
```
atan(((((16*a^3*c^6 + 4*a*c^4*f^4 - 20*a^2*c^5*f^2)/c^5 - (2*x*(4*c^7*f^3
- 16*a*c^8*f)*(-(f^9 + f^6*(-(4*a*c - f^2)^3)^(1/2) + 28*a^4*c^4*f + 42*a^
2*c^2*f^5 - 63*a^3*c^3*f^3 - a^3*c^3*(-(4*a*c - f^2)^3)^(1/2) - 11*a*c*f^7
 + 6*a^2*c^2*f^2*(-(4*a*c - f^2)^3)^(1/2) - 5*a*c*f^4*(-(4*a*c - f^2)^3)^(
1/2))/(8*(16*a^2*c^9 + c^7*f^4 - 8*a*c^8*f^2)))^(1/2))/c^5)*(-(f^9 + f^6*(
-(4*a*c - f^2)^3)^(1/2) + 28*a^4*c^4*f + 42*a^2*c^2*f^5 - 63*a^3*c^3*f^3 -
 a^3*c^3*(-(4*a*c - f^2)^3)^(1/2) - 11*a*c*f^7 + 6*a^2*c^2*f^2*(-(4*a*c -
f^2)^3)^(1/2) - 5*a*c*f^4*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^2*c^9 + c^7*f
^4 - 8*a*c^8*f^2)))^(1/2) - (2*x*(f^8 + 2*a^4*c^4 + 20*a^2*c^2*f^4 - 16*a^
3*c^3*f^2 - 8*a*c*f^6))/c^5)*(-(f^9 + f^6*(-(4*a*c - f^2)^3)^(1/2) + 28*a^
4*c^4*f + 42*a^2*c^2*f^5 - 63*a^3*c^3*f^3 - a^3*c^3*(-(4*a*c - f^2)^3)^(1/
2) - 11*a*c*f^7 + 6*a^2*c^2*f^2*(-(4*a*c - f^2)^3)^(1/2) - 5*a*c*f^4*(-(4*
a*c - f^2)^3)^(1/2))/(8*(16*a^2*c^9 + c^7*f^4 - 8*a*c^8*f^2)))^(1/2)*1i -
(((16*a^3*c^6 + 4*a*c^4*f^4 - 20*a^2*c^5*f^2)/c^5 + (2*x*(4*c^7*f^3 - 16*a
*c^8*f)*(-(f^9 + f^6*(-(4*a*c - f^2)^3)^(1/2) + 28*a^4*c^4*f + 42*a^2*c^2*
f^5 - 63*a^3*c^3*f^3 - a^3*c^3*(-(4*a*c - f^2)^3)^(1/2) - 11*a*c*f^7 + 6*a
^2*c^2*f^2*(-(4*a*c - f^2)^3)^(1/2) - 5*a*c*f^4*(-(4*a*c - f^2)^3)^(1/2))/
(8*(16*a^2*c^9 + c^7*f^4 - 8*a*c^8*f^2)))^(1/2))/c^5)*(-(f^9 + f^6*(-(4*a*
c - f^2)^3)^(1/2) + 28*a^4*c^4*f + 42*a^2*c^2*f^5 - 63*a^3*c^3*f^3 - a^3*c
^3*(-(4*a*c - f^2)^3)^(1/2) - 11*a*c*f^7 + 6*a^2*c^2*f^2*(-(4*a*c - f^2...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 992, normalized size of antiderivative = 2.79

$$\int \frac{x^8}{a + f x^2 + c x^4} \, dx = \text{Too large to display}$$

input
```
int(x^8/(c*x^4+f*x^2+a),x)
```

output
```
( - 90*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) -
f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c**2*f + 30*sqrt(a)*sqrt(
2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + f))*c*f**3 - 60*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)
*atan((sqrt(2*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 f))*a**2*c**2 + 120*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt
(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*f**2 - 30
*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) - 2
*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f**4 + 90*sqrt(a)*sqrt(2*sqrt(c)*
sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + f))*a*c**2*f - 30*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((s
qrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*c*f
**3 + 60*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a)
- f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a**2*c**2 - 120*sqrt(c)*s
qrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x
)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*f**2 + 30*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a
) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + f))*f**4 + 45*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - f)*log( - sqrt(2*sqr
t(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c**2*f - 15*sqrt(a)*sqrt(2
*sqrt(c)*sqrt(a) - f)*log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + ...
```

3.784 $\int \frac{x^6}{a+fx^2+cx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 304

$$\int \frac{x^6}{a+fx^2+cx^4}\,dx = -\frac{fx}{c^2}+\frac{x^3}{3c}+\frac{\left(ac-\sqrt{a}\sqrt{c}f-f^2\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2c^{5/2}\sqrt{2\sqrt{a}\sqrt{c}+f}} - \frac{\left(ac-\sqrt{a}\sqrt{c}f-f^2\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2c^{5/2}\sqrt{2\sqrt{a}\sqrt{c}+f}} + \frac{\left(ac+\sqrt{a}\sqrt{c}f-f^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\,x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2c^{5/2}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

output
```
-f*x/c^2+1/3*x^3/c+1/2*(a*c-a^(1/2)*c^(1/2)*f-f^2)*arctan(((2*a^(1/2)*c^(1
/2)-f)^(1/2)-2*c^(1/2)*x)/(2*a^(1/2)*c^(1/2)+f)^(1/2))/c^(5/2)/(2*a^(1/2)*
c^(1/2)+f)^(1/2)-1/2*(a*c-a^(1/2)*c^(1/2)*f-f^2)*arctan(((2*a^(1/2)*c^(1/2
)-f)^(1/2)+2*c^(1/2)*x)/(2*a^(1/2)*c^(1/2)+f)^(1/2))/c^(5/2)/(2*a^(1/2)*c^
(1/2)+f)^(1/2)+1/2*(a*c+a^(1/2)*c^(1/2)*f-f^2)*arctanh((2*a^(1/2)*c^(1/2)-
f)^(1/2)*x/(a^(1/2)+c^(1/2)*x^2))/c^(5/2)/(2*a^(1/2)*c^(1/2)-f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.82

$$\int \frac{x^6}{a+fx^2+cx^4}\,dx$$
$$= -\frac{fx}{c^2}+\frac{x^3}{3c} + \frac{\left(3acf-f^3-ac\sqrt{-4ac+f^2}+f^2\sqrt{-4ac+f^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f-\sqrt{-4ac+f^2}}}\right)}{\sqrt{2}c^{5/2}\sqrt{-4ac+f^2}\sqrt{f-\sqrt{-4ac+f^2}}} + \frac{\left(-3acf+f^3-ac\sqrt{-4ac+f^2}+f^2\sqrt{-4ac+f^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f+\sqrt{-4ac+f^2}}}\right)}{\sqrt{2}c^{5/2}\sqrt{-4ac+f^2}\sqrt{f+\sqrt{-4ac+f^2}}}$$

input
```
Integrate[x^6/(a + f*x^2 + c*x^4),x]
```

output
```
-((f*x)/c^2) + x^3/(3*c) + ((3*a*c*f - f^3 - a*c*Sqrt[-4*a*c + f^2] + f^2*
Sqrt[-4*a*c + f^2])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[f - Sqrt[-4*a*c + f^2]
]])/(Sqrt[2]*c^(5/2)*Sqrt[-4*a*c + f^2]*Sqrt[f - Sqrt[-4*a*c + f^2]]) + ((
-3*a*c*f + f^3 - a*c*Sqrt[-4*a*c + f^2] + f^2*Sqrt[-4*a*c + f^2])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[f + Sqrt[-4*a*c + f^2]]])/(Sqrt[2]*c^(5/2)*Sqrt[-4
*a*c + f^2]*Sqrt[f + Sqrt[-4*a*c + f^2]])
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.48, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {1442, 27, 1602, 1483, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{a+cx^4+fx^2}\,dx$$
$$\Big\downarrow \text{1442}$$

$$\frac{x^3}{3c}-\frac{\int \frac{3x^2\left(fx^2+a\right)}{cx^4+fx^2+a}dx}{3c}$$

$\downarrow$ 27

$$\frac{x^3}{3c}-\frac{\int \frac{x^2\left(fx^2+a\right)}{cx^4+fx^2+a}dx}{c}$$

$\downarrow$ 1602

$$\frac{x^3}{3c}-\frac{\frac{fx}{c}-\frac{\int \frac{af-\left(ac-f^2\right)x^2}{cx^4+fx^2+a}dx}{c}}{c}$$

$\downarrow$ 1483

$$\frac{x^3}{3c}-\frac{\frac{fx}{c}-\frac{\frac{\int \frac{\sqrt{a}\left(\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}f-\sqrt{c}\left(\sqrt{a}f+\frac{ac-f^2}{\sqrt{c}}\right)x\right)}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\int \frac{\sqrt{a}\left(\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}f+\sqrt{c}\left(\sqrt{a}f+\frac{ac-f^2}{\sqrt{c}}\right)x\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}}{c}}{c}$$

$\downarrow$ 27

$$\frac{x^3}{3c}-\frac{\frac{fx}{c}-\frac{\frac{\int \frac{\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}f-\left(-f^2+\sqrt{a}\sqrt{c}f+ac\right)x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\int \frac{\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}f+\left(-f^2+\sqrt{a}\sqrt{c}f+ac\right)x}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}}{c}}{c}$$

$\downarrow$ 1142

$$\frac{x^3}{3c}-\frac{\frac{fx}{c}-\frac{\frac{-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{1}{2}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int -\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \ldots}{\ldots}}{c}}{c}$$

$\downarrow$ 25

$$\frac{x^3}{3c}-\frac{\frac{fx}{c}-\frac{\frac{\frac{1}{2}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \ldots}{\ldots}}{c}}{c}$$

$\downarrow$ 27

$$\frac{x^3}{3c}-\frac{\frac{fx}{c}-\frac{\frac{\frac{\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{\cdots}}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-\cdots}}{\sqrt{c}}\cdots}\cdots}{2\sqrt{c}}\cdots}{c}}{c}$$

$$\Big\downarrow \; 1083$$

$$\frac{x^3}{3c}-\frac{\frac{fx}{c}-\frac{\frac{\frac{\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)^2-\frac{f+2\sqrt{a}\sqrt{c}}{c}}d\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\left(\sqrt{a}\cdots\right.}{\cdots}\cdots}{c}}{c}$$

$$\Big\downarrow \; 217$$

$$\frac{x^3}{3c}-\frac{\frac{fx}{c}-\frac{\frac{\frac{\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{2\sqrt{c}x+\cdots}{x^2+\frac{\sqrt{2\sqrt{\cdots}}}{\cdots}}\cdots}{2\sqrt{c}}\cdots}{c}}{c}$$

$$\Big\downarrow \; 1103$$

$$\frac{x^3}{3c}-\frac{\frac{fx}{c}-\frac{\frac{-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}-\frac{1}{2}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}-f}+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\log\left(\cdots\right.}{\cdots}\cdots}{c}}{c}$$

input
```
Int[x^6/(a + f*x^2 + c*x^4),x]
```

output
```
x^3/(3*c) - ((f*x)/c - ((-((Sqrt[2*Sqrt[a]*Sqrt[c] - f]*(a*c - Sqrt[a]*Sqr
t[c]*f - f^2)*ArcTan[(Sqrt[c]*(-(Sqrt[2*Sqrt[a]*Sqrt[c] - f]/Sqrt[c]) + 2*
x))/Sqrt[2*Sqrt[a]*Sqrt[c] + f]])/Sqrt[2*Sqrt[a]*Sqrt[c] + f]) - ((a*c + S
qrt[a]*Sqrt[c]*f - f^2)*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] - f]*x + Sqrt
[c]*x^2])/2)/(2*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f]) + (-((Sqrt[2*Sqrt[a]*
Sqrt[c] - f]*(a*c - Sqrt[a]*Sqrt[c]*f - f^2)*ArcTan[(Sqrt[c]*(Sqrt[2*Sqrt[
a]*Sqrt[c] - f]/Sqrt[c] + 2*x))/Sqrt[2*Sqrt[a]*Sqrt[c] + f]])/Sqrt[2*Sqrt[
a]*Sqrt[c] + f]) + ((a*c + Sqrt[a]*Sqrt[c]*f - f^2)*Log[Sqrt[a] + Sqrt[2*S
qrt[a]*Sqrt[c] - f]*x + Sqrt[c]*x^2])/2)/(2*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c]
 - f]))/c)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.24

method	result
risch	$\frac{x^3}{3c} - \frac{fx}{c^2} + \frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 c+f\_Z^2+a\right)} \frac{\left(\left(-ac+f^2\right)\_R^2+af\right)\ln\left(x-\_R\right)}{2\_R^3 c+\_R f}}{2c^2}$
default	$\frac{\frac{1}{3}c x^3-fx}{c^2} + \frac{\frac{\left(-ac\sqrt{-4ac+f^2}+f^2\sqrt{-4ac+f^2}-3acf+f^3\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+f^2}+f\right)c}}\right)}{2c\sqrt{-4ac+f^2}\sqrt{\left(\sqrt{-4ac+f^2}+f\right)c}} - \frac{\left(-ac\sqrt{-4ac+f^2}+f^2\sqrt{-4ac+f^2}+3acf-f^3 \ldots\right.}{2c\sqrt{-4ac+f^2}\sqrt{\left(\sqrt{\ldots\right.}}}{c}$

input
```
int(x^6/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/c-f*x/c^2+1/2/c^2*sum(((-a*c+f^2)*_R^2+a*f)/(2*_R^3*c+_R*f)*ln(x-_
R),_R=RootOf(_Z^4*c+_Z^2*f+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1604 vs. $2(225) = 450$.

Time = 0.12 (sec) , antiderivative size = 1604, normalized size of antiderivative = 5.28

$$\int \frac{x^6}{a + fx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
1/6*(2*c*x^3 + 3*sqrt(1/2)*c^2*sqrt((5*a^2*c^2*f - 5*a*c*f^3 + f^5 + (4*a*
c^6 - c^5*f^2)*sqrt(-(a^4*c^4 - 6*a^3*c^3*f^2 + 11*a^2*c^2*f^4 - 6*a*c*f^6
 + f^8)/(4*a*c^11 - c^10*f^2)))/(4*a*c^6 - c^5*f^2))*log(2*(a^4*c^2 - 3*a^
3*c*f^2 + a^2*f^4)*x + sqrt(1/2)*(4*a^3*c^3*f - 13*a^2*c^2*f^3 + 7*a*c*f^5
 - f^7 - (8*a^2*c^7 - 6*a*c^6*f^2 + c^5*f^4)*sqrt(-(a^4*c^4 - 6*a^3*c^3*f^
2 + 11*a^2*c^2*f^4 - 6*a*c*f^6 + f^8)/(4*a*c^11 - c^10*f^2)))*sqrt((5*a^2*
c^2*f - 5*a*c*f^3 + f^5 + (4*a*c^6 - c^5*f^2)*sqrt(-(a^4*c^4 - 6*a^3*c^3*f
^2 + 11*a^2*c^2*f^4 - 6*a*c*f^6 + f^8)/(4*a*c^11 - c^10*f^2)))/(4*a*c^6 -
c^5*f^2))) - 3*sqrt(1/2)*c^2*sqrt((5*a^2*c^2*f - 5*a*c*f^3 + f^5 + (4*a*c^
6 - c^5*f^2)*sqrt(-(a^4*c^4 - 6*a^3*c^3*f^2 + 11*a^2*c^2*f^4 - 6*a*c*f^6 +
 f^8)/(4*a*c^11 - c^10*f^2)))/(4*a*c^6 - c^5*f^2))*log(2*(a^4*c^2 - 3*a^3*
c*f^2 + a^2*f^4)*x - sqrt(1/2)*(4*a^3*c^3*f - 13*a^2*c^2*f^3 + 7*a*c*f^5 -
 f^7 - (8*a^2*c^7 - 6*a*c^6*f^2 + c^5*f^4)*sqrt(-(a^4*c^4 - 6*a^3*c^3*f^2
+ 11*a^2*c^2*f^4 - 6*a*c*f^6 + f^8)/(4*a*c^11 - c^10*f^2)))*sqrt((5*a^2*c^
2*f - 5*a*c*f^3 + f^5 + (4*a*c^6 - c^5*f^2)*sqrt(-(a^4*c^4 - 6*a^3*c^3*f^2
 + 11*a^2*c^2*f^4 - 6*a*c*f^6 + f^8)/(4*a*c^11 - c^10*f^2)))/(4*a*c^6 - c^
5*f^2))) + 3*sqrt(1/2)*c^2*sqrt((5*a^2*c^2*f - 5*a*c*f^3 + f^5 - (4*a*c^6
- c^5*f^2)*sqrt(-(a^4*c^4 - 6*a^3*c^3*f^2 + 11*a^2*c^2*f^4 - 6*a*c*f^6 + f
^8)/(4*a*c^11 - c^10*f^2)))/(4*a*c^6 - c^5*f^2))*log(2*(a^4*c^2 - 3*a^3*c*
f^2 + a^2*f^4)*x + sqrt(1/2)*(4*a^3*c^3*f - 13*a^2*c^2*f^3 + 7*a*c*f^5 ...
```

Sympy [A] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.64

$$\int \frac{x^6}{a+fx^2+cx^4}\,dx$$
$$=\mathrm{RootSum}\left(t^4\cdot\left(256a^2c^7-128ac^6f^2+16c^5f^4\right)+t^2\left(-80a^3c^3f+100a^2c^2f^3-36acf^5+4f^7\right)+a^5,\left(t\mapsto\right.\right.$$
$$+\frac{x^3}{3c}-\frac{fx}{c^2}$$

input
```
integrate(x**6/(c*x**4+f*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**2*c**7 - 128*a*c**6*f**2 + 16*c**5*f**4) + _t**2*(-8
0*a**3*c**3*f + 100*a**2*c**2*f**3 - 36*a*c*f**5 + 4*f**7) + a**5, Lambda(
_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*c**6*f**2 - 8*_t**3*c**5*
f**4 + 14*_t*a**3*c**3*f - 28*_t*a**2*c**2*f**3 + 14*_t*a*c*f**5 - 2*_t*f*
*7)/(a**4*c**2 - 3*a**3*c*f**2 + a**2*f**4)))) + x**3/(3*c) - f*x/c**2
```

Maxima [F]

$$\int \frac{x^6}{a+fx^2+cx^4}\,dx=\int \frac{x^6}{cx^4+fx^2+a}\,dx$$

input
```
integrate(x^6/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
1/3*(c*x^3 - 3*f*x)/c^2 + integrate(-((a*c - f^2)*x^2 - a*f)/(c*x^4 + f*x^
2 + a), x)/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2479 vs. $2(225) = 450$.

Time = 0.61 (sec) , antiderivative size = 2479, normalized size of antiderivative = 8.15

$$\int \frac{x^6}{a+fx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
1/8*(24*a^2*c^6*f^2 - 14*a*c^5*f^4 + 2*c^4*f^6 - 12*sqrt(2)*sqrt(-4*a*c +
f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^4*f^2 + 3*sqrt(2)*sqrt(-4*a*c
+ f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^5*f^2 - 6*sqrt(2)*sqrt(-4*a*c
+ f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^4*f^3 + 7*sqrt(2)*sqrt(-4*a*c
+ f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^3*f^4 - sqrt(2)*sqrt(-4*a*c +
f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c^4*f^4 + 2*sqrt(2)*sqrt(-4*a*c + f^
2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c^3*f^5 - sqrt(2)*sqrt(-4*a*c + f^2)*s
qrt(c*f + sqrt(-4*a*c + f^2)*c)*c^2*f^6 - 6*(4*a*c - f^2)*a*c^5*f^2 + 2*(4
*a*c - f^2)*c^4*f^4 + (32*a^3*c^5 - 48*a^2*c^4*f^2 + 18*a*c^3*f^4 - 2*c^2*
f^6 - 16*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^3*c
^3 + 4*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^4
 - 8*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^3*f
 + 24*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^2*
f^2 - 5*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^3*
f^2 + 10*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^2
*f^3 - 9*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c*f
^4 + sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c^2*f^4 -
 2*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c*f^5 + sqr
t(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*f^6 - 8*(4*a*c -
f^2)*a^2*c^4 + 10*(4*a*c - f^2)*a*c^3*f^2 - 2*(4*a*c - f^2)*c^2*f^4)*c^...
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 4127, normalized size of antiderivative = 13.58

$$\int \frac{x^6}{a+fx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^6/(a + c*x^4 + f*x^2),x)
```

output
```
x^3/(3*c) - atan((((((4*a*c^3*f^3 - 16*a^2*c^4*f)/c^3 - (2*x*(4*c^5*f^3 - 1
6*a*c^6*f)*(-(f^7 + f^4*(-(4*a*c - f^2)^3)^(1/2) - 20*a^3*c^3*f + 25*a^2*c
^2*f^3 + a^2*c^2*(-(4*a*c - f^2)^3)^(1/2) - 9*a*c*f^5 - 3*a*c*f^2*(-(4*a*c
 - f^2)^3)^(1/2))/(8*(16*a^2*c^7 + c^5*f^4 - 8*a*c^6*f^2)))^(1/2))/c^3)*(-
(f^7 + f^4*(-(4*a*c - f^2)^3)^(1/2) - 20*a^3*c^3*f + 25*a^2*c^2*f^3 + a^2*
c^2*(-(4*a*c - f^2)^3)^(1/2) - 9*a*c*f^5 - 3*a*c*f^2*(-(4*a*c - f^2)^3)^(1
/2))/(8*(16*a^2*c^7 + c^5*f^4 - 8*a*c^6*f^2)))^(1/2) - (2*x*(f^6 - 2*a^3*c
^3 + 9*a^2*c^2*f^2 - 6*a*c*f^4))/c^3)*(-(f^7 + f^4*(-(4*a*c - f^2)^3)^(1/2
) - 20*a^3*c^3*f + 25*a^2*c^2*f^3 + a^2*c^2*(-(4*a*c - f^2)^3)^(1/2) - 9*a
*c*f^5 - 3*a*c*f^2*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^2*c^7 + c^5*f^4 - 8*
a*c^6*f^2)))^(1/2)*1i - (((4*a*c^3*f^3 - 16*a^2*c^4*f)/c^3 + (2*x*(4*c^5*f
^3 - 16*a*c^6*f)*(-(f^7 + f^4*(-(4*a*c - f^2)^3)^(1/2) - 20*a^3*c^3*f + 25
*a^2*c^2*f^3 + a^2*c^2*(-(4*a*c - f^2)^3)^(1/2) - 9*a*c*f^5 - 3*a*c*f^2*(-
(4*a*c - f^2)^3)^(1/2))/(8*(16*a^2*c^7 + c^5*f^4 - 8*a*c^6*f^2)))^(1/2))/c
^3)*(-(f^7 + f^4*(-(4*a*c - f^2)^3)^(1/2) - 20*a^3*c^3*f + 25*a^2*c^2*f^3
+ a^2*c^2*(-(4*a*c - f^2)^3)^(1/2) - 9*a*c*f^5 - 3*a*c*f^2*(-(4*a*c - f^2)
^3)^(1/2))/(8*(16*a^2*c^7 + c^5*f^4 - 8*a*c^6*f^2)))^(1/2) + (2*x*(f^6 - 2
*a^3*c^3 + 9*a^2*c^2*f^2 - 6*a*c*f^4))/c^3)*(-(f^7 + f^4*(-(4*a*c - f^2)^3
)^(1/2) - 20*a^3*c^3*f + 25*a^2*c^2*f^3 + a^2*c^2*(-(4*a*c - f^2)^3)^(1/2)
 - 9*a*c*f^5 - 3*a*c*f^2*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^2*c^7 + c^5...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 761, normalized size of antiderivative = 2.50

$$\int \frac{x^6}{a+fx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^6/(c*x^4+f*x^2+a),x)
```

output
```
(12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c**2 - 6*sqrt(a)*sqrt(2*sqrt
(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*s
qrt(c)*sqrt(a) + f))*c*f**2 - 18*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a
*c*f + 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a)
- f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f**3 - 12*sqrt(a)*sqrt(2*
sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt
(2*sqrt(c)*sqrt(a) + f))*a*c**2 + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*at
an((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f)
)*c*f**2 + 18*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqr
t(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*f - 6*sqrt(c)*sq
rt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)
/sqrt(2*sqrt(c)*sqrt(a) + f))*f**3 - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - f)
*log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c**2 + 3
*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - f)*log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x
+ sqrt(a) + sqrt(c)*x**2)*c*f**2 + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - f)*l
og(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c**2 - 3*sqrt
(a)*sqrt(2*sqrt(c)*sqrt(a) - f)*log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a
) + sqrt(c)*x**2)*c*f**2 - 9*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - f)*log( -...
```

3.785 $\int \frac{x^4}{a+fx^2+cx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 268

$$\int \frac{x^4}{a+fx^2+cx^4}\,dx = \frac{x}{c}+\frac{\left(\sqrt{a}\sqrt{c}+f\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2c^{3/2}\sqrt{2\sqrt{a}\sqrt{c}+f}} - \frac{\left(\sqrt{a}\sqrt{c}+f\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2c^{3/2}\sqrt{2\sqrt{a}\sqrt{c}+f}} - \frac{\left(\sqrt{a}\sqrt{c}-f\right)\operatorname{arctanh}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2c^{3/2}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

output
```
x/c+1/2*(a^(1/2)*c^(1/2)+f)*arctan(((2*a^(1/2)*c^(1/2)-f)^(1/2)-2*c^(1/2)*
x)/(2*a^(1/2)*c^(1/2)+f)^(1/2))/c^(3/2)/(2*a^(1/2)*c^(1/2)+f)^(1/2)-1/2*(a
^(1/2)*c^(1/2)+f)*arctan(((2*a^(1/2)*c^(1/2)-f)^(1/2)+2*c^(1/2)*x)/(2*a^(1
/2)*c^(1/2)+f)^(1/2))/c^(3/2)/(2*a^(1/2)*c^(1/2)+f)^(1/2)-1/2*(a^(1/2)*c^(
1/2)-f)*arctanh((2*a^(1/2)*c^(1/2)-f)^(1/2)*x/(a^(1/2)+c^(1/2)*x^2))/c^(3/
2)/(2*a^(1/2)*c^(1/2)-f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{a+fx^2+cx^4}\,dx = \frac{x}{c} - \frac{\left(2ac-f^2+f\sqrt{-4ac+f^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f-\sqrt{-4ac+f^2}}}\right)}{\sqrt{2}c^{3/2}\sqrt{-4ac+f^2}\sqrt{f-\sqrt{-4ac+f^2}}} - \frac{\left(-2ac+f^2+f\sqrt{-4ac+f^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f+\sqrt{-4ac+f^2}}}\right)}{\sqrt{2}c^{3/2}\sqrt{-4ac+f^2}\sqrt{f+\sqrt{-4ac+f^2}}}$$

input
```
Integrate[x^4/(a + f*x^2 + c*x^4),x]
```

output
```
x/c - ((2*a*c - f^2 + f*Sqrt[-4*a*c + f^2])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[f - Sqrt[-4*a*c + f^2]]])/(Sqrt[2]*c^(3/2)*Sqrt[-4*a*c + f^2]*Sqrt[f - S
qrt[-4*a*c + f^2]]) - ((-2*a*c + f^2 + f*Sqrt[-4*a*c + f^2])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[f + Sqrt[-4*a*c + f^2]]])/(Sqrt[2]*c^(3/2)*Sqrt[-4*a*c
+ f^2]*Sqrt[f + Sqrt[-4*a*c + f^2]])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.50, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1442, 1483, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+cx^4+fx^2}\,dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x}{c} - \frac{\int \frac{fx^2+a}{cx^4+fx^2+a}dx}{c}$$

$$\Big\downarrow \text{1483}$$

$$\frac{x}{c}-\frac{\frac{\int \frac{\sqrt{a}\left(\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}-\sqrt{c}\left(\sqrt{a}-\frac{f}{\sqrt{c}}\right)x\right)}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\int \frac{\sqrt{a}\left(\sqrt{c}\left(\sqrt{a}-\frac{f}{\sqrt{c}}\right)x+\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{x}{c}-\frac{\frac{\int \frac{\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}-(\sqrt{a}\sqrt{c}-f)x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\int \frac{(\sqrt{a}\sqrt{c}-f)x+\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}}{c}$$

$$\Big\downarrow \; 1142$$

$$\frac{x}{c}-\frac{\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{1}{2}\left(\sqrt{a}\sqrt{c}-f\right)\int -\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\cdots}\cdots}{2\sqrt{c}}\cdots}{2\cdots}}{c}$$

$$\Big\downarrow \; 25$$

$$\frac{x}{c}-\frac{\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{1}{2}\left(\sqrt{a}\sqrt{c}-f\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\cdots}\cdots}{2\sqrt{c}}\cdots}{2\sqrt{c}\cdots}}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{x}{c}-\frac{\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{(\sqrt{a}\sqrt{c}-f)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\cdots}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}\cdots}}{c}$$

$$\Big\downarrow \; 1083$$

$$\frac{x}{c}-\frac{\frac{\frac{(\sqrt{a}\sqrt{c}-f)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)^2-\frac{f+2\sqrt{a}\sqrt{c}}{c}}d\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{(\sqrt{a}\sqrt{c}-f)\int \frac{2\sqrt{c}x+\sqrt{2}\cdots}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{}\cdots}}{\sqrt{c}}\cdots}\cdots}{2\sqrt{c}}\cdots}{\cdots}}{c}$$

$$\Big\downarrow \; 217$$

$$\frac{x}{c}-\frac{\frac{\frac{(\sqrt{a}\sqrt{c}-f)\int\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{(\sqrt{a}\sqrt{c}-f)\int\frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\sqrt{c}\cdots}}{\cdots}}{2\sqrt{c}\sqrt{2\sqrt{a}\cdots}}}{c}$$

$\Big\downarrow$ 1103

$$\frac{x}{c}-\frac{\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}-\frac{1}{2}(\sqrt{a}\sqrt{c}-f)\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}-f}+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\arctan\left(\frac{\sqrt{c}\left(\frac{\sqrt{2\cdots}}{\cdots}\right)}{\sqrt{2\cdots}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\cdots}{\cdots}}{c}$$

input

```
Int[x^4/(a + f*x^2 + c*x^4),x]
```

output

```
x/c - (((Sqrt[2*Sqrt[a]*Sqrt[c] - f]*(Sqrt[a]*Sqrt[c] + f)*ArcTan[(Sqrt[c]
*(-(Sqrt[2*Sqrt[a]*Sqrt[c] - f]/Sqrt[c]) + 2*x))/Sqrt[2*Sqrt[a]*Sqrt[c] +
f]])/Sqrt[2*Sqrt[a]*Sqrt[c] + f] - ((Sqrt[a]*Sqrt[c] - f)*Log[Sqrt[a] - Sq
rt[2*Sqrt[a]*Sqrt[c] - f]*x + Sqrt[c]*x^2])/2)/(2*Sqrt[c]*Sqrt[2*Sqrt[a]*S
qrt[c] - f]) + ((Sqrt[2*Sqrt[a]*Sqrt[c] - f]*(Sqrt[a]*Sqrt[c] + f)*ArcTan[
(Sqrt[c]*(Sqrt[2*Sqrt[a]*Sqrt[c] - f]/Sqrt[c] + 2*x))/Sqrt[2*Sqrt[a]*Sqrt[
c] + f]])/Sqrt[2*Sqrt[a]*Sqrt[c] + f] + ((Sqrt[a]*Sqrt[c] - f)*Log[Sqrt[a]
 + Sqrt[2*Sqrt[a]*Sqrt[c] - f]*x + Sqrt[c]*x^2])/2)/(2*Sqrt[c]*Sqrt[2*Sqrt
[a]*Sqrt[c] - f]))/c
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.21

method	result
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}c+f\_Z^{2}+a\right)}\frac{\left(-\_R^{2}f-a\right)\ln\left(x-\_R\right)}{2\_R^{3}c+\_Rf}}{2c}$
default	$\frac{x}{c}+\frac{\left(-f\sqrt{-4ac+f^{2}}+2ac-f^{2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+f^{2}}+f\right)c}}\right)}{2c\sqrt{-4ac+f^{2}}\sqrt{\left(\sqrt{-4ac+f^{2}}+f\right)c}}-\frac{\left(-f\sqrt{-4ac+f^{2}}-2ac+f^{2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+f^{2}}-\right.}}\right.}{2c\sqrt{-4ac+f^{2}}\sqrt{\left(\sqrt{-4ac+f^{2}}-f\right)c}}$

input
```
int(x^4/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/2/c*sum((-_R^2*f-a)/(2*_R^3*c+_R*f)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*f
+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1119 vs. 2(191) = 382.

Time = 0.11 (sec) , antiderivative size = 1119, normalized size of antiderivative = 4.18

$$\int \frac{x^4}{a+fx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*c*sqrt(-(3*a*c*f - f^3 + (4*a*c^4 - c^3*f^2)*sqrt(-(a^2*c^2
 - 2*a*c*f^2 + f^4)/(4*a*c^7 - c^6*f^2)))/(4*a*c^4 - c^3*f^2))*log(-2*(a^2
*c - a*f^2)*x + sqrt(1/2)*(4*a^2*c^2 - 5*a*c*f^2 + f^4 + (4*a*c^4*f - c^3*
f^3)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a*c^7 - c^6*f^2)))*sqrt(-(3*a*c*
f - f^3 + (4*a*c^4 - c^3*f^2)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a*c^7 -
 c^6*f^2)))/(4*a*c^4 - c^3*f^2))) - sqrt(1/2)*c*sqrt(-(3*a*c*f - f^3 + (4*
a*c^4 - c^3*f^2)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a*c^7 - c^6*f^2)))/(
4*a*c^4 - c^3*f^2))*log(-2*(a^2*c - a*f^2)*x - sqrt(1/2)*(4*a^2*c^2 - 5*a*
c*f^2 + f^4 + (4*a*c^4*f - c^3*f^3)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a
*c^7 - c^6*f^2)))*sqrt(-(3*a*c*f - f^3 + (4*a*c^4 - c^3*f^2)*sqrt(-(a^2*c^
2 - 2*a*c*f^2 + f^4)/(4*a*c^7 - c^6*f^2)))/(4*a*c^4 - c^3*f^2))) + sqrt(1/
2)*c*sqrt(-(3*a*c*f - f^3 - (4*a*c^4 - c^3*f^2)*sqrt(-(a^2*c^2 - 2*a*c*f^2
 + f^4)/(4*a*c^7 - c^6*f^2)))/(4*a*c^4 - c^3*f^2))*log(-2*(a^2*c - a*f^2)*
x + sqrt(1/2)*(4*a^2*c^2 - 5*a*c*f^2 + f^4 - (4*a*c^4*f - c^3*f^3)*sqrt(-(
a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a*c^7 - c^6*f^2)))*sqrt(-(3*a*c*f - f^3 - (4
*a*c^4 - c^3*f^2)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a*c^7 - c^6*f^2)))/
(4*a*c^4 - c^3*f^2))) - sqrt(1/2)*c*sqrt(-(3*a*c*f - f^3 - (4*a*c^4 - c^3*
f^2)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a*c^7 - c^6*f^2)))/(4*a*c^4 - c^
3*f^2))*log(-2*(a^2*c - a*f^2)*x - sqrt(1/2)*(4*a^2*c^2 - 5*a*c*f^2 + f^4
- (4*a*c^4*f - c^3*f^3)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a*c^7 - c^...
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.48

$$\int \frac{x^4}{a + fx^2 + cx^4} \, dx$$
$$= \text{RootSum}\left(t^4 \cdot \left(256a^2c^5 - 128ac^4f^2 + 16c^3f^4\right) + t^2 \cdot \left(48a^2c^2f - 28acf^3 + 4f^5\right) + a^3, \left(t \mapsto t \log\left(x + \right.\right.\right.$$
$$+ \frac{x}{c}$$

input
```
integrate(x**4/(c*x**4+f*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**2*c**5 - 128*a*c**4*f**2 + 16*c**3*f**4) + _t**2*(48
*a**2*c**2*f - 28*a*c*f**3 + 4*f**5) + a**3, Lambda(_t, _t*log(x + (32*_t*
*3*a*c**4*f - 8*_t**3*c**3*f**3 - 4*_t*a**2*c**2 + 8*_t*a*c*f**2 - 2*_t*f*
*4)/(a**2*c - a*f**2)))) + x/c
```

Maxima [F]

$$\int \frac{x^4}{a+fx^2+cx^4}\,dx = \int \frac{x^4}{cx^4+fx^2+a}\,dx$$

input
```
integrate(x^4/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
x/c - integrate((f*x^2 + a)/(c*x^4 + f*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2133 vs. 2(191) = 382.

Time = 0.59 (sec) , antiderivative size = 2133, normalized size of antiderivative = 7.96

$$\int \frac{x^4}{a+fx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
x/c - 1/8*(16*a^2*c^6*f - 12*a*c^5*f^3 + 2*c^4*f^5 - 8*sqrt(2)*sqrt(-4*a*c
 + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^4*f + 2*sqrt(2)*sqrt(-4*a*c
 + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^5*f - 4*sqrt(2)*sqrt(-4*a*c +
 f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^4*f^2 + 6*sqrt(2)*sqrt(-4*a*c +
 f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^3*f^3 - sqrt(2)*sqrt(-4*a*c + f
^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c^4*f^3 + 2*sqrt(2)*sqrt(-4*a*c + f^2
)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c^3*f^4 - sqrt(2)*sqrt(-4*a*c + f^2)*sq
rt(c*f + sqrt(-4*a*c + f^2)*c)*c^2*f^5 - 4*(4*a*c - f^2)*a*c^5*f + 2*(4*a*
c - f^2)*c^4*f^3 - (32*a^2*c^4*f - 16*a*c^3*f^3 + 2*c^2*f^5 - 16*sqrt(2)*s
qrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^2*f + 4*sqrt(2)*s
qrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^3*f - 8*sqrt(2)*sqr
t(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^2*f^2 + 8*sqrt(2)*sqr
t(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c*f^3 - sqrt(2)*sqrt(-4
*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c^2*f^3 + 2*sqrt(2)*sqrt(-4*a
*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c*f^4 - sqrt(2)*sqrt(-4*a*c + f
^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*f^5 - 8*(4*a*c - f^2)*a*c^3*f + 2*(4*
a*c - f^2)*c^2*f^3)*c^2 + 2*(16*sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a
^3*c^4 - 4*sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^5 - 32*a^3*c^5 +
 8*sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^4*f - 8*sqrt(2)*sqrt(c*f
 + sqrt(-4*a*c + f^2)*c)*a^2*c^3*f^2 + sqrt(2)*sqrt(c*f + sqrt(-4*a*c +...
```

Mupad [B] (verification not implemented)

Time = 18.53 (sec) , antiderivative size = 3026, normalized size of antiderivative = 11.29

$$\int \frac{x^4}{a + fx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
int(x^4/(a + c*x^4 + f*x^2),x)
```

output
```
x/c - atan(((((16*a^2*c^3 - 4*a*c^2*f^2)/c - (2*x*(4*c^3*f^3 - 16*a*c^4*f)
*(-(f^5 + f^2*(-(4*a*c - f^2)^3)^(1/2) + 12*a^2*c^2*f - 7*a*c*f^3 - a*c*(-
(4*a*c - f^2)^3)^(1/2))/(8*(16*a^2*c^5 + c^3*f^4 - 8*a*c^4*f^2)))^(1/2))/c
)*(-(f^5 + f^2*(-(4*a*c - f^2)^3)^(1/2) + 12*a^2*c^2*f - 7*a*c*f^3 - a*c*(
-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^2*c^5 + c^3*f^4 - 8*a*c^4*f^2)))^(1/2) -
 (2*x*(f^4 + 2*a^2*c^2 - 4*a*c*f^2))/c)*(-(f^5 + f^2*(-(4*a*c - f^2)^3)^(1
/2) + 12*a^2*c^2*f - 7*a*c*f^3 - a*c*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^2*
c^5 + c^3*f^4 - 8*a*c^4*f^2)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*c^2*f^2)/c +
 (2*x*(4*c^3*f^3 - 16*a*c^4*f)*(-(f^5 + f^2*(-(4*a*c - f^2)^3)^(1/2) + 12*
a^2*c^2*f - 7*a*c*f^3 - a*c*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^2*c^5 + c^3
*f^4 - 8*a*c^4*f^2)))^(1/2))/c)*(-(f^5 + f^2*(-(4*a*c - f^2)^3)^(1/2) + 12
*a^2*c^2*f - 7*a*c*f^3 - a*c*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^2*c^5 + c^
3*f^4 - 8*a*c^4*f^2)))^(1/2) + (2*x*(f^4 + 2*a^2*c^2 - 4*a*c*f^2))/c)*(-(f
^5 + f^2*(-(4*a*c - f^2)^3)^(1/2) + 12*a^2*c^2*f - 7*a*c*f^3 - a*c*(-(4*a*
c - f^2)^3)^(1/2))/(8*(16*a^2*c^5 + c^3*f^4 - 8*a*c^4*f^2)))^(1/2)*1i)/(((
(16*a^2*c^3 - 4*a*c^2*f^2)/c - (2*x*(4*c^3*f^3 - 16*a*c^4*f)*(-(f^5 + f^2*
(-(4*a*c - f^2)^3)^(1/2) + 12*a^2*c^2*f - 7*a*c*f^3 - a*c*(-(4*a*c - f^2)^
3)^(1/2))/(8*(16*a^2*c^5 + c^3*f^4 - 8*a*c^4*f^2)))^(1/2))/c)*(-(f^5 + f^2
*(-(4*a*c - f^2)^3)^(1/2) + 12*a^2*c^2*f - 7*a*c*f^3 - a*c*(-(4*a*c - f^2)
^3)^(1/2))/(8*(16*a^2*c^5 + c^3*f^4 - 8*a*c^4*f^2)))^(1/2) - (2*x*(f^4 ...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.04

$$\int \frac{x^4}{a + fx^2 + cx^4}\, dx$$
$$= \frac{2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+f}\; atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right) cf + 4\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+f}\; atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right) ac - 2\sqrt{c}\,\sqrt{}}{}$$

input
```
int(x^4/(c*x^4+f*x^2+a),x)
```

output

```
(2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*c*f + 4*sqrt(c)*sqrt(2*sqrt(c)*
sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + f))*a*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*
sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f**2 - 2*
sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*
sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*c*f - 4*sqrt(c)*sqrt(2*sqrt(c)*sqr
t(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + f))*a*c + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqr
t(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f**2 - sqrt(
a)*sqrt(2*sqrt(c)*sqrt(a) - f)*log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt
(a) + sqrt(c)*x**2)*c*f + sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - f)*log(sqrt(2*s
qrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*c*f + 2*sqrt(c)*sqrt(2*sqr
t(c)*sqrt(a) - f)*log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)
*x**2)*a*c - sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - f)*log( - sqrt(2*sqrt(c)*sqr
t(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f**2 - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt
(a) - f)*log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c +
 sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - f)*log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + s
qrt(a) + sqrt(c)*x**2)*f**2 + 16*a*c**2*x - 4*c*f**2*x)/(4*c**2*(4*a*c - f
**2))
```

3.786 $\int \frac{x^2}{a+fx^2+cx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 222

$$\int \frac{x^2}{a+fx^2+cx^4}\,dx = -\frac{\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+f}} + \frac{\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}+f}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

output
```
-1/2*arctan(((2*a^(1/2)*c^(1/2)-f)^(1/2)-2*c^(1/2)*x)/(2*a^(1/2)*c^(1/2)+f
)^(1/2))/c^(1/2)/(2*a^(1/2)*c^(1/2)+f)^(1/2)+1/2*arctan(((2*a^(1/2)*c^(1/2
)-f)^(1/2)+2*c^(1/2)*x)/(2*a^(1/2)*c^(1/2)+f)^(1/2))/c^(1/2)/(2*a^(1/2)*c^
(1/2)+f)^(1/2)-1/2*arctanh((2*a^(1/2)*c^(1/2)-f)^(1/2)*x/(a^(1/2)+c^(1/2)*
x^2))/c^(1/2)/(2*a^(1/2)*c^(1/2)-f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{a+fx^2+cx^4}\,dx = \frac{\left(-f+\sqrt{-4ac+f^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f-\sqrt{-4ac+f^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-4ac+f^2}\sqrt{f-\sqrt{-4ac+f^2}}} + \frac{\sqrt{f+\sqrt{-4ac+f^2}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f+\sqrt{-4ac+f^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-4ac+f^2}}$$

input
```
Integrate[x^2/(a + f*x^2 + c*x^4),x]
```

output
```
((-f + Sqrt[-4*a*c + f^2])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[f - Sqrt[-4*a*c
 + f^2]]])/(Sqrt[2]*Sqrt[c]*Sqrt[-4*a*c + f^2]*Sqrt[f - Sqrt[-4*a*c + f^2]
]) + (Sqrt[f + Sqrt[-4*a*c + f^2]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[f + Sqr
t[-4*a*c + f^2]]])/(Sqrt[2]*Sqrt[c]*Sqrt[-4*a*c + f^2])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.51, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1449, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+cx^4+fx^2}\,dx$$

$$\Big\downarrow \text{1449}$$

$$\frac{\int \frac{x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}} - \frac{\int \frac{x}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{1}{2}\int -\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}-\frac{\frac{1}{2}\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{1}{2}\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}-\frac{\frac{1}{2}\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}-\frac{\frac{\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)^2-\frac{f+2\sqrt{a}\sqrt{c}}{c}}d\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}-\frac{\frac{\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{-\left(2x+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)^2-\frac{f+2\sqrt{a}\sqrt{c}}{c}}d\left(2x+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}-\frac{\int\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}-\frac{\frac{\int\frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\arctan\left(\frac{\sqrt{c}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}+2x\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}+\frac{1}{2}\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}-f}+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}-\frac{\frac{1}{2}\log\left(x\sqrt{2\sqrt{a}\sqrt{c}-f}+\sqrt{a}+\sqrt{c}x^2\right)-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\arctan\left(\frac{\sqrt{c}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}+2x\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}}{2\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

input
```
Int[x^2/(a + f*x^2 + c*x^4),x]
```

output
```
((Sqrt[2*Sqrt[a]*Sqrt[c] - f]*ArcTan[(Sqrt[c]*(-(Sqrt[2*Sqrt[a]*Sqrt[c] -
f]/Sqrt[c]) + 2*x))/Sqrt[2*Sqrt[a]*Sqrt[c] + f]])/Sqrt[2*Sqrt[a]*Sqrt[c] +
 f] + Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] - f]*x + Sqrt[c]*x^2]/2)/(2*Sqr
t[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f]) - (-((Sqrt[2*Sqrt[a]*Sqrt[c] - f]*ArcTan
[(Sqrt[c]*(Sqrt[2*Sqrt[a]*Sqrt[c] - f]/Sqrt[c] + 2*x))/Sqrt[2*Sqrt[a]*Sqrt
[c] + f]])/Sqrt[2*Sqrt[a]*Sqrt[c] + f]) + Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqr
t[c] - f]*x + Sqrt[c]*x^2]/2)/(2*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1449
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Simp[1/(2*c*r)   Int[x^(m - 1)/(q + r*x + x^2), x],
 x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m,
3] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.18

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4c+f\,\_Z^2+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^3c+\_Rf}\right)}{2}$	41
default	$4c\left(\frac{\left(\sqrt{-4ac+f^2}+f\right)\sqrt{2}\,\mathrm{arctan}\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+f^2}+f\right)c}}\right)}{8c\sqrt{-4ac+f^2}\,\sqrt{\left(\sqrt{-4ac+f^2}+f\right)c}}-\frac{\left(\sqrt{-4ac+f^2}-f\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+f^2}-f\right)c}}\right)}{8c\sqrt{-4ac+f^2}\,\sqrt{\left(\sqrt{-4ac+f^2}-f\right)c}}\right)$	149

input
```
int(x^2/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R^2/(2*_R^3*c+_R*f)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*f+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(157) = 314$.

Time = 0.09 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.88

$$\int \frac{x^2}{a+fx^2+cx^4}\,dx$$
$$=\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{(4\,ac^2-cf^2)\sqrt{-\frac{1}{4\,ac^3-c^2f^2}}+f}{4\,ac^2-cf^2}}\log\left(\sqrt{\frac{1}{2}}(4\,ac^2-cf^2)\sqrt{\frac{(4\,ac^2-cf^2)\sqrt{-\frac{1}{4\,ac^3-c^2f^2}}+f}{4\,ac^2-cf^2}}\sqrt{-\frac{}{4\,a}}\right.$$
$$\left.+x\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{(4\,ac^2-cf^2)\sqrt{-\frac{1}{4\,ac^3-c^2f^2}}+f}{4\,ac^2-cf^2}}\log\left(-\sqrt{\frac{1}{2}}(4\,ac^2-cf^2)\sqrt{\frac{(4\,ac^2-cf^2)\sqrt{-\frac{1}{4\,ac^3-c^2f^2}}+f}{4\,ac^2-cf^2}}\sqrt{-}\right.$$
$$\left.+x\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{(4\,ac^2-cf^2)\sqrt{-\frac{1}{4\,ac^3-c^2f^2}}-f}{4\,ac^2-cf^2}}\log\left(\sqrt{\frac{1}{2}}(4\,ac^2-cf^2)\sqrt{-\frac{(4\,ac^2-cf^2)\sqrt{-\frac{1}{4\,ac^3-c^2f^2}}-f}{4\,ac^2-cf^2}}\right.$$
$$\left.+x\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{(4\,ac^2-cf^2)\sqrt{-\frac{1}{4\,ac^3-c^2f^2}}-f}{4\,ac^2-cf^2}}\log\left(-\sqrt{\frac{1}{2}}(4\,ac^2-cf^2)\sqrt{-\frac{(4\,ac^2-cf^2)\sqrt{-\frac{1}{4\,ac^3-c^2f^2}}-f}{4\,ac^2-cf^2}}\right.$$
$$\left.+x\right)$$

input
```
integrate(x^2/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt(((4*a*c^2 - c*f^2)*sqrt(-1/(4*a*c^3 - c^2*f^2)) + f)/(4
*a*c^2 - c*f^2))*log(sqrt(1/2)*(4*a*c^2 - c*f^2)*sqrt(((4*a*c^2 - c*f^2)*s
qrt(-1/(4*a*c^3 - c^2*f^2)) + f)/(4*a*c^2 - c*f^2))*sqrt(-1/(4*a*c^3 - c^2
*f^2)) + x) - 1/2*sqrt(1/2)*sqrt(((4*a*c^2 - c*f^2)*sqrt(-1/(4*a*c^3 - c^2
*f^2)) + f)/(4*a*c^2 - c*f^2))*log(-sqrt(1/2)*(4*a*c^2 - c*f^2)*sqrt(((4*a
*c^2 - c*f^2)*sqrt(-1/(4*a*c^3 - c^2*f^2)) + f)/(4*a*c^2 - c*f^2))*sqrt(-1
/(4*a*c^3 - c^2*f^2)) + x) - 1/2*sqrt(1/2)*sqrt(-((4*a*c^2 - c*f^2)*sqrt(-
1/(4*a*c^3 - c^2*f^2)) - f)/(4*a*c^2 - c*f^2))*log(sqrt(1/2)*(4*a*c^2 - c*
f^2)*sqrt(-((4*a*c^2 - c*f^2)*sqrt(-1/(4*a*c^3 - c^2*f^2)) - f)/(4*a*c^2 -
 c*f^2))*sqrt(-1/(4*a*c^3 - c^2*f^2)) + x) + 1/2*sqrt(1/2)*sqrt(-((4*a*c^2
 - c*f^2)*sqrt(-1/(4*a*c^3 - c^2*f^2)) - f)/(4*a*c^2 - c*f^2))*log(-sqrt(1
/2)*(4*a*c^2 - c*f^2)*sqrt(-((4*a*c^2 - c*f^2)*sqrt(-1/(4*a*c^3 - c^2*f^2)
) - f)/(4*a*c^2 - c*f^2))*sqrt(-1/(4*a*c^3 - c^2*f^2)) + x)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.34

$$\int \frac{x^2}{a+fx^2+cx^4}\,dx$$
$$= \operatorname{RootSum}\left(t^4\cdot\left(256a^2c^3-128ac^2f^2+16cf^4\right)+t^2\left(-16acf+4f^3\right)+a,\left(t\mapsto t\log\left(64t^3ac^2-16t^3cf^2-\right.\right.\right.$$

input
```
integrate(x**2/(c*x**4+f*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**2*c**3 - 128*a*c**2*f**2 + 16*c*f**4) + _t**2*(-16*a
*c*f + 4*f**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*c*f**2 -
2*_t*f + x)))
```

Maxima [F]

$$\int \frac{x^2}{a+fx^2+cx^4}\,dx = \int \frac{x^2}{cx^4+fx^2+a}\,dx$$

input
```
integrate(x^2/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^4 + f*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(157) = 314.

Time = 0.68 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.27

$$\int \frac{x^2}{a+fx^2+cx^4}\,dx$$

$$= \frac{\left(8\,ac^3-2\,c^2f^2-4\,\sqrt{2}\sqrt{-4\,ac+f^2}\sqrt{cf+\sqrt{-4\,ac+f^2}c}ac+\sqrt{2}\sqrt{-4\,ac+f^2}\sqrt{cf+\sqrt{-4\,ac+f^2}c}c^2\right.}{2\,(16\,a^2c^2-4\,ac}$$

$$-\frac{\left(8\,ac^3-2\,c^2f^2-4\,\sqrt{2}\sqrt{-4\,ac+f^2}\sqrt{cf-\sqrt{-4\,ac+f^2}c}ac+\sqrt{2}\sqrt{-4\,ac+f^2}\sqrt{cf-\sqrt{-4\,ac+f^2}c}c\right.}{2\,(16\,a^2c^2-4}$$

input
```
integrate(x^2/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
1/2*(8*a*c^3 - 2*c^2*f^2 - 4*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4
*a*c + f^2)*c)*a*c + sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f
^2)*c)*c^2 - 2*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)
*c*f + sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*f^2 - 2
*(4*a*c - f^2)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((f + sqrt(-4*a*c + f^2))/c))
/((16*a^2*c^2 - 4*a*c^3 + 8*a*c^2*f - 8*a*c*f^2 + c^2*f^2 - 2*c*f^3 + f^4)
*abs(c)) - 1/2*(8*a*c^3 - 2*c^2*f^2 - 4*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*
f - sqrt(-4*a*c + f^2)*c)*a*c + sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f - sqrt
(-4*a*c + f^2)*c)*c^2 - 2*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f - sqrt(-4*a*
c + f^2)*c)*c*f + sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f - sqrt(-4*a*c + f^2)
*c)*f^2 - 2*(4*a*c - f^2)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((f - sqrt(-4*a*c
+ f^2))/c))/((16*a^2*c^2 - 4*a*c^3 + 8*a*c^2*f - 8*a*c*f^2 + c^2*f^2 - 2*c
*f^3 + f^4)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.87

$$\int \frac{x^2}{a+fx^2+cx^4}\,dx = -2\,\mathrm{atanh}\left(\frac{\left(x\,(4\,a\,c^2-2\,c\,f^2)+\frac{x\,(8\,c^2\,f^3-32\,a\,c^3\,f)\,\left(f^3+\sqrt{-(4\,a\,c-f^2)^3}-4\,a\,c\,f\right)}{8\,(16\,a^2\,c^3-8\,a\,c^2\,f^2+c\,f^4)}\right)\sqrt{-\frac{f^3+\sqrt{-(4\,a\,c-f^2)^3}-4\,a\,c\,f}{8\,(16\,a^2\,c^3-8\,a\,c^2\,f^2+c\,f^4)}}}{a\,c}\right) -2\,\mathrm{atanh}\left(\frac{\left(x\,(4\,a\,c^2-2\,c\,f^2)-\frac{x\,(8\,c^2\,f^3-32\,a\,c^3\,f)\,\left(\sqrt{-(4\,a\,c-f^2)^3}-f^3+4\,a\,c\,f\right)}{8\,(16\,a^2\,c^3-8\,a\,c^2\,f^2+c\,f^4)}\right)\sqrt{\frac{\sqrt{-(4\,a\,c-f^2)^3}-f^3+4\,a\,c\,f}{8\,(16\,a^2\,c^3-8\,a\,c^2\,f^2+c\,f^4)}}}{a\,c}\right)\sqrt{}$$

input
```
int(x^2/(a + c*x^4 + f*x^2),x)
```

output
```
- 2*atanh(((x*(4*a*c^2 - 2*c*f^2) + (x*(8*c^2*f^3 - 32*a*c^3*f)*(f^3 + (-(
4*a*c - f^2)^3)^(1/2) - 4*a*c*f))/(8*(c*f^4 + 16*a^2*c^3 - 8*a*c^2*f^2)))*
(-(f^3 + (-(4*a*c - f^2)^3)^(1/2) - 4*a*c*f)/(8*(c*f^4 + 16*a^2*c^3 - 8*a*
c^2*f^2)))^(1/2))/(a*c))*(-(f^3 + (-(4*a*c - f^2)^3)^(1/2) - 4*a*c*f)/(8*(
c*f^4 + 16*a^2*c^3 - 8*a*c^2*f^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*c*f^2
) - (x*(8*c^2*f^3 - 32*a*c^3*f)*((-(4*a*c - f^2)^3)^(1/2) - f^3 + 4*a*c*f)
)/(8*(c*f^4 + 16*a^2*c^3 - 8*a*c^2*f^2)))*(((-(4*a*c - f^2)^3)^(1/2) - f^3
 + 4*a*c*f)/(8*(c*f^4 + 16*a^2*c^3 - 8*a*c^2*f^2)))^(1/2))/(a*c))*(((-(4*a
*c - f^2)^3)^(1/2) - f^3 + 4*a*c*f)/(8*(c*f^4 + 16*a^2*c^3 - 8*a*c^2*f^2))
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.58

$$\int \frac{x^2}{a+fx^2+cx^4}\,dx$$
$$= \frac{-4\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right)c+2\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right)f+4\sqrt{a}\,\sqrt{}}{}$$

input
```
int(x^2/(c*x^4+f*x^2+a),x)
```

output
```
( - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f
) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*c + 2*sqrt(c)*sqrt(2*sqrt(c)
*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + f))*f + 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*s
qrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*c - 2*sqrt
(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt
(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) -
 f)*log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*c - 2*s
qrt(a)*sqrt(2*sqrt(c)*sqrt(a) - f)*log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqr
t(a) + sqrt(c)*x**2)*c + sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - f)*log( - sqrt(2
*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f - sqrt(c)*sqrt(2*sqrt(
c)*sqrt(a) - f)*log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2
)*f)/(4*c*(4*a*c - f**2))
```

3.787 $\int \frac{1}{a+fx^2+cx^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 222

$$\int \frac{1}{a+fx^2+cx^4}\,dx = -\frac{\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+f}} + \frac{\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}+f}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

output
```
-1/2*arctan(((2*a^(1/2)*c^(1/2)-f)^(1/2)-2*c^(1/2)*x)/(2*a^(1/2)*c^(1/2)+f
)^(1/2))/a^(1/2)/(2*a^(1/2)*c^(1/2)+f)^(1/2)+1/2*arctan(((2*a^(1/2)*c^(1/2
)-f)^(1/2)+2*c^(1/2)*x)/(2*a^(1/2)*c^(1/2)+f)^(1/2))/a^(1/2)/(2*a^(1/2)*c^
(1/2)+f)^(1/2)+1/2*arctanh((2*a^(1/2)*c^(1/2)-f)^(1/2)*x/(a^(1/2)+c^(1/2)*
x^2))/a^(1/2)/(2*a^(1/2)*c^(1/2)-f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.58

$$\int \frac{1}{a+fx^2+cx^4}\,dx=\frac{\sqrt{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f-\sqrt{-4ac+f^2}}}\right)}{\sqrt{f-\sqrt{-4ac+f^2}}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f+\sqrt{-4ac+f^2}}}\right)}{\sqrt{f+\sqrt{-4ac+f^2}}}\right)}{\sqrt{-4ac+f^2}}$$

input
```
Integrate[(a + f*x^2 + c*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[f - Sqrt[-4*a*c + f^2]]]
/Sqrt[f - Sqrt[-4*a*c + f^2]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[f + Sqrt[-
4*a*c + f^2]]]/Sqrt[f + Sqrt[-4*a*c + f^2]]))/Sqrt[-4*a*c + f^2]
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.64, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {1407, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+cx^4+fx^2}\,dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\int \frac{\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\int \frac{\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx-\frac{1}{2}\sqrt{c}\int -\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\sqrt{c}\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\sqrt{c}\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\sqrt{c}\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx-\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)^2-\frac{f+2\sqrt{a}\sqrt{c}}{c}}d\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx-\sqrt{2\sqrt{a}\sqrt{c}-f}\int \frac{1}{-\left(2x+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)^2-\frac{f+2\sqrt{a}\sqrt{c}}{c}}d\left(2x+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\int \frac{2\sqrt{c}x+\sqrt{2\sqrt{a}\sqrt{c}-f}}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}\arctan\left(\frac{\sqrt{c}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}+2x\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{\frac{\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}-\frac{1}{2}\sqrt{c}\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}-f}+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}\arctan\left(\frac{\sqrt{c}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}+2x\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}+\frac{1}{2}\sqrt{c}\log\left(x\sqrt{2\sqrt{a}\sqrt{c}-f}+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

input
```
Int[(a + f*x^2 + c*x^4)^(-1),x]
```

output
```
((Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f]*ArcTan[(Sqrt[c]*(-(Sqrt[2*Sqrt[a]*Sq
rt[c] - f]/Sqrt[c]) + 2*x))/Sqrt[2*Sqrt[a]*Sqrt[c] + f]])/Sqrt[2*Sqrt[a]*S
qrt[c] + f] - (Sqrt[c]*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] - f]*x + Sqrt[
c]*x^2])/2)/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f]) + ((Sqrt[c]*Sq
rt[2*Sqrt[a]*Sqrt[c] - f]*ArcTan[(Sqrt[c]*(Sqrt[2*Sqrt[a]*Sqrt[c] - f]/Sqr
t[c] + 2*x))/Sqrt[2*Sqrt[a]*Sqrt[c] + f]])/Sqrt[2*Sqrt[a]*Sqrt[c] + f] + (
Sqrt[c]*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] - f]*x + Sqrt[c]*x^2])/2)/(2*
Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.17

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 c+f\,\_Z^2+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^3 c+\_R f}\right)}{2}$	38
default	$4c\left(-\frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+f^2}+f\right)c}}\right)}{4\sqrt{-4ac+f^2}\,\sqrt{\left(\sqrt{-4ac+f^2}+f\right)c}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+f^2}-f\right)c}}\right)}{4\sqrt{-4ac+f^2}\,\sqrt{\left(\sqrt{-4ac+f^2}-f\right)c}}\right)$	117

input
```
int(1/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3*c+_R*f)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*f+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 701 vs. $2(157) = 314$.

Time = 0.09 (sec) , antiderivative size = 701, normalized size of antiderivative = 3.16

$$\int \frac{1}{a+fx^2+cx^4}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{(4\,a^2c-af^2)\sqrt{-\frac{1}{4\,a^3c-a^2f^2}}+f}{4\,a^2c-af^2}}\log\left(2\,cx + \sqrt{\frac{1}{2}}\left(4\,ac-f^2+(4\,a^2cf-af^3)\sqrt{-\frac{1}{4\,a^3c-a^2f^2}}\right)\sqrt{\frac{(4\,a^2c-af^2)\sqrt{-\frac{1}{4\,a^3c-a^2f^2}}+f}{4\,a^2c-af^2}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{(4\,a^2c-af^2)\sqrt{-\frac{1}{4\,a^3c-a^2f^2}}+f}{4\,a^2c-af^2}}\log\left(2\,cx - \sqrt{\frac{1}{2}}\left(4\,ac-f^2+(4\,a^2cf-af^3)\sqrt{-\frac{1}{4\,a^3c-a^2f^2}}\right)\sqrt{\frac{(4\,a^2c-af^2)\sqrt{-\frac{1}{4\,a^3c-a^2f^2}}+f}{4\,a^2c-af^2}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{(4\,a^2c-af^2)\sqrt{-\frac{1}{4\,a^3c-a^2f^2}}-f}{4\,a^2c-af^2}}\log\left(2\,cx + \sqrt{\frac{1}{2}}\left(4\,ac-f^2-(4\,a^2cf-af^3)\sqrt{-\frac{1}{4\,a^3c-a^2f^2}}\right)\sqrt{-\frac{(4\,a^2c-af^2)\sqrt{-\frac{1}{4\,a^3c-a^2f^2}}-f}{4\,a^2c-af^2}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{(4\,a^2c-af^2)\sqrt{-\frac{1}{4\,a^3c-a^2f^2}}-f}{4\,a^2c-af^2}}\log\left(2\,cx - \sqrt{\frac{1}{2}}\left(4\,ac-f^2-(4\,a^2cf-af^3)\sqrt{-\frac{1}{4\,a^3c-a^2f^2}}\right)\sqrt{-\frac{(4\,a^2c-af^2)\sqrt{-\frac{1}{4\,a^3c-a^2f^2}}-f}{4\,a^2c-af^2}}\right)$$

input
```
integrate(1/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt(((4*a^2*c - a*f^2)*sqrt(-1/(4*a^3*c - a^2*f^2)) + f)/(4
*a^2*c - a*f^2))*log(2*c*x + sqrt(1/2)*(4*a*c - f^2 + (4*a^2*c*f - a*f^3)*
sqrt(-1/(4*a^3*c - a^2*f^2)))*sqrt(((4*a^2*c - a*f^2)*sqrt(-1/(4*a^3*c - a
^2*f^2)) + f)/(4*a^2*c - a*f^2))) - 1/2*sqrt(1/2)*sqrt(((4*a^2*c - a*f^2)*
sqrt(-1/(4*a^3*c - a^2*f^2)) + f)/(4*a^2*c - a*f^2))*log(2*c*x - sqrt(1/2)
*(4*a*c - f^2 + (4*a^2*c*f - a*f^3)*sqrt(-1/(4*a^3*c - a^2*f^2)))*sqrt(((4
*a^2*c - a*f^2)*sqrt(-1/(4*a^3*c - a^2*f^2)) + f)/(4*a^2*c - a*f^2))) + 1/
2*sqrt(1/2)*sqrt(-((4*a^2*c - a*f^2)*sqrt(-1/(4*a^3*c - a^2*f^2)) - f)/(4*
a^2*c - a*f^2))*log(2*c*x + sqrt(1/2)*(4*a*c - f^2 - (4*a^2*c*f - a*f^3)*s
qrt(-1/(4*a^3*c - a^2*f^2)))*sqrt(-((4*a^2*c - a*f^2)*sqrt(-1/(4*a^3*c - a
^2*f^2)) - f)/(4*a^2*c - a*f^2))) - 1/2*sqrt(1/2)*sqrt(-((4*a^2*c - a*f^2)
*sqrt(-1/(4*a^3*c - a^2*f^2)) - f)/(4*a^2*c - a*f^2))*log(2*c*x - sqrt(1/2
)*(4*a*c - f^2 - (4*a^2*c*f - a*f^3)*sqrt(-1/(4*a^3*c - a^2*f^2)))*sqrt(-(
(4*a^2*c - a*f^2)*sqrt(-1/(4*a^3*c - a^2*f^2)) - f)/(4*a^2*c - a*f^2)))
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.39

$$\int \frac{1}{a + fx^2 + cx^4} \, dx$$
$$= \text{RootSum}\left(t^4 \cdot \left(256a^3c^2 - 128a^2cf^2 + 16af^4\right) + t^2\left(-16acf + 4f^3\right) + c, \left(t \mapsto t \log\left(x + \frac{32t^3a^2cf - 8t\right.\right.\right.$$

input
```
integrate(1/(c*x**4+f*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**3*c**2 - 128*a**2*c*f**2 + 16*a*f**4) + _t**2*(-16*a
*c*f + 4*f**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*c*f - 8*_t**3*a*f
**3 + 4*_t*a*c - 2*_t*f**2)/c)))
```

Maxima [F]

$$\int \frac{1}{a + fx^2 + cx^4} \, dx = \int \frac{1}{cx^4 + fx^2 + a} \, dx$$

input
```
integrate(1/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^4 + f*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(157) = 314$.

Time = 0.35 (sec) , antiderivative size = 1038, normalized size of antiderivative = 4.68

$$\int \frac{1}{a + fx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
1/4*(16*sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^2 - 4*sqrt(2)*sqrt(
c*f + sqrt(-4*a*c + f^2)*c)*a*c^3 - 32*a^2*c^3 + 8*sqrt(2)*sqrt(c*f + sqrt
(-4*a*c + f^2)*c)*a*c^2*f - 8*a*c^3*f - 8*sqrt(2)*sqrt(c*f + sqrt(-4*a*c +
 f^2)*c)*a*c*f^2 + sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c^2*f^2 + 16*a
*c^2*f^2 - 2*sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c*f^3 + 2*c^2*f^3 +
sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*f^4 - 2*c*f^4 + 4*sqrt(2)*sqrt(-4
*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c*f - sqrt(2)*sqrt(-4*a*c +
 f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c^2*f + 2*sqrt(2)*sqrt(-4*a*c + f^2
)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c*f^2 - sqrt(2)*sqrt(-4*a*c + f^2)*sqrt
(c*f + sqrt(-4*a*c + f^2)*c)*f^3 + 8*(4*a*c - f^2)*a*c^2 + 2*(4*a*c - f^2)
*c^2*f - 2*(4*a*c - f^2)*c*f^2)*arctan(2*sqrt(1/2)*x/sqrt((f + sqrt(-4*a*c
 + f^2))/c))/((16*a^3*c^2 - 4*a^2*c^3 + 8*a^2*c^2*f - 8*a^2*c*f^2 + a*c^2*
f^2 - 2*a*c*f^3 + a*f^4)*abs(c)) + 1/4*(16*sqrt(2)*sqrt(c*f - sqrt(-4*a*c
+ f^2)*c)*a^2*c^2 - 4*sqrt(2)*sqrt(c*f - sqrt(-4*a*c + f^2)*c)*a*c^3 + 32*
a^2*c^3 + 8*sqrt(2)*sqrt(c*f - sqrt(-4*a*c + f^2)*c)*a*c^2*f + 8*a*c^3*f -
 8*sqrt(2)*sqrt(c*f - sqrt(-4*a*c + f^2)*c)*a*c*f^2 + sqrt(2)*sqrt(c*f - s
qrt(-4*a*c + f^2)*c)*c^2*f^2 - 16*a*c^2*f^2 - 2*sqrt(2)*sqrt(c*f - sqrt(-4
*a*c + f^2)*c)*c*f^3 - 2*c^2*f^3 + sqrt(2)*sqrt(c*f - sqrt(-4*a*c + f^2)*c
)*f^4 + 2*c*f^4 - 4*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f - sqrt(-4*a*c + f^
2)*c)*a*c*f + sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f - sqrt(-4*a*c + f^2)*...
```

Mupad [B] (verification not implemented)

Time = 18.00 (sec) , antiderivative size = 763, normalized size of antiderivative = 3.44

$$\int \frac{1}{a + fx^2 + cx^4} \, dx =$$

$$-\mathrm{atan}\left(\frac{f^4\,x\,1\mathrm{i} - f\,x\,\sqrt{-64\,a^3\,c^3 + 48\,a^2\,c^2\,f^2 - 12\,a\,c\,f^4 + f^6}\,1\mathrm{i} + a^2}{64\,a^3\,c^2\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,c^2\,f^2-12\,a\,c\,f^4+f^6}-f^3+4\,a\,c\,f}{128\,a^3\,c^2-64\,a^2\,c\,f^2+8\,a\,f^4}} + 4\,a\,f^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,c^2\,f^2-12\,a\,c\,f^4+f^6}-f^3+4\,a}{128\,a^3\,c^2-64\,a^2\,c\,f^2+8\,a\,f^4}}}\right.$$

$$-\mathrm{atan}\left(\frac{f^4\,x\,1\mathrm{i} + f\,x\,\sqrt{-64\,a^3\,c^3 + 48\,a^2\,c^2\,f^2 - 12\,a\,c\,f^4 + f^6}\,1\mathrm{i} +}{4\,a\,f^4\,\sqrt{-\frac{f^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,c^2\,f^2-12\,a\,c\,f^4+f^6}-4\,a\,c\,f}{128\,a^3\,c^2-64\,a^2\,c\,f^2+8\,a\,f^4}} + 64\,a^3\,c^2\,\sqrt{-\frac{f^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,c^2\,f^2-12\,a\,c\,f^4+f^6}}{128\,a^3\,c^2-64\,a^2\,c\,f^2+8\,a\,f^4}}}\right.$$

input
```
int(1/(a + c*x^4 + f*x^2),x)
```

output
```
- atan((f^4*x*1i - f*x*(f^6 - 64*a^3*c^3 + 48*a^2*c^2*f^2 - 12*a*c*f^4)^(1
/2)*1i + a^2*c^2*x*16i - a*c*f^2*x*8i)/(64*a^3*c^2*(((f^6 - 64*a^3*c^3 + 4
8*a^2*c^2*f^2 - 12*a*c*f^4)^(1/2) - f^3 + 4*a*c*f)/(8*a*f^4 + 128*a^3*c^2
- 64*a^2*c*f^2))^(1/2) + 4*a*f^4*(((f^6 - 64*a^3*c^3 + 48*a^2*c^2*f^2 - 12
*a*c*f^4)^(1/2) - f^3 + 4*a*c*f)/(8*a*f^4 + 128*a^3*c^2 - 64*a^2*c*f^2))^(
1/2) - 32*a^2*c*f^2*(((f^6 - 64*a^3*c^3 + 48*a^2*c^2*f^2 - 12*a*c*f^4)^(1/
2) - f^3 + 4*a*c*f)/(8*a*f^4 + 128*a^3*c^2 - 64*a^2*c*f^2))^(1/2)))*(((f^6
 - 64*a^3*c^3 + 48*a^2*c^2*f^2 - 12*a*c*f^4)^(1/2) - f^3 + 4*a*c*f)/(8*a*f
^4 + 128*a^3*c^2 - 64*a^2*c*f^2))^(1/2)*2i - atan((f^4*x*1i + f*x*(f^6 - 6
4*a^3*c^3 + 48*a^2*c^2*f^2 - 12*a*c*f^4)^(1/2)*1i + a^2*c^2*x*16i - a*c*f^
2*x*8i)/(4*a*f^4*(-(f^3 + (f^6 - 64*a^3*c^3 + 48*a^2*c^2*f^2 - 12*a*c*f^4)
^(1/2) - 4*a*c*f)/(8*a*f^4 + 128*a^3*c^2 - 64*a^2*c*f^2))^(1/2) + 64*a^3*c
^2*(-(f^3 + (f^6 - 64*a^3*c^3 + 48*a^2*c^2*f^2 - 12*a*c*f^4)^(1/2) - 4*a*c
*f)/(8*a*f^4 + 128*a^3*c^2 - 64*a^2*c*f^2))^(1/2) - 32*a^2*c*f^2*(-(f^3 +
(f^6 - 64*a^3*c^3 + 48*a^2*c^2*f^2 - 12*a*c*f^4)^(1/2) - 4*a*c*f)/(8*a*f^4
 + 128*a^3*c^2 - 64*a^2*c*f^2))^(1/2)))*(-(f^3 + (f^6 - 64*a^3*c^3 + 48*a^
2*c^2*f^2 - 12*a*c*f^4)^(1/2) - 4*a*c*f)/(8*a*f^4 + 128*a^3*c^2 - 64*a^2*c
*f^2))^(1/2)*2i
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.58

$$\int \frac{1}{a+fx^2+cx^4}\,dx$$
$$= \frac{2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right)f-4\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+f}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right)a-2\sqrt{a}\,\sqrt{2}}{}$$

input
```
int(1/(c*x^4+f*x^2+a),x)
```

output
```
(2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f - 4*sqrt(c)*sqrt(2*sqrt(c)*sq
rt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)
*sqrt(a) + f))*a - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt
(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*f + 4*sqrt(c)
*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a - sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - f)*l
og( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f + sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) - f)*log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) +
sqrt(c)*x**2)*f - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - f)*log( - sqrt(2*sqrt
(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a + 2*sqrt(c)*sqrt(2*sqrt(c)*
sqrt(a) - f)*log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a
)/(4*a*(4*a*c - f**2))
```

3.788 $\int \frac{1}{x^2(a+fx^2+cx^4)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 264

$$\int \frac{1}{x^2(a+fx^2+cx^4)}\,dx = -\frac{1}{ax}+\frac{\left(\sqrt{c}+\frac{f}{\sqrt{a}}\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2a\sqrt{2\sqrt{a}\sqrt{c}+f}} - \frac{\left(\sqrt{c}+\frac{f}{\sqrt{a}}\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2a\sqrt{2\sqrt{a}\sqrt{c}+f}} + \frac{\left(\sqrt{c}-\frac{f}{\sqrt{a}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2a\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

output
```
-1/a/x+1/2*(c^(1/2)+f/a^(1/2))*arctan(((2*a^(1/2)*c^(1/2)-f)^(1/2)-2*c^(1/
2)*x)/(2*a^(1/2)*c^(1/2)+f)^(1/2))/a/(2*a^(1/2)*c^(1/2)+f)^(1/2)-1/2*(c^(1
/2)+f/a^(1/2))*arctan(((2*a^(1/2)*c^(1/2)-f)^(1/2)+2*c^(1/2)*x)/(2*a^(1/2)
*c^(1/2)+f)^(1/2))/a/(2*a^(1/2)*c^(1/2)+f)^(1/2)+1/2*(c^(1/2)-f/a^(1/2))*a
rctanh((2*a^(1/2)*c^(1/2)-f)^(1/2)*x/(a^(1/2)+c^(1/2)*x^2))/a/(2*a^(1/2)*c
^(1/2)-f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^2(a+fx^2+cx^4)}\,dx$$
$$= -\frac{\frac{2}{x}+\frac{\sqrt{2}\sqrt{c}\left(f+\sqrt{-4ac+f^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f-\sqrt{-4ac+f^2}}}\right)}{\sqrt{-4ac+f^2}\sqrt{f-\sqrt{-4ac+f^2}}}+\frac{\sqrt{2}\sqrt{c}\left(-f+\sqrt{-4ac+f^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f+\sqrt{-4ac+f^2}}}\right)}{\sqrt{-4ac+f^2}\sqrt{f+\sqrt{-4ac+f^2}}}}{2a}$$

input
```
Integrate[1/(x^2*(a + f*x^2 + c*x^4)),x]
```

output
```
-1/2*(2/x + (Sqrt[2]*Sqrt[c]*(f + Sqrt[-4*a*c + f^2])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[f - Sqrt[-4*a*c + f^2]]])/(Sqrt[-4*a*c + f^2]*Sqrt[f - Sqrt[-4
*a*c + f^2]]) + (Sqrt[2]*Sqrt[c]*(-f + Sqrt[-4*a*c + f^2])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[f + Sqrt[-4*a*c + f^2]]])/(Sqrt[-4*a*c + f^2]*Sqrt[f + Sq
rt[-4*a*c + f^2]]))/a
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.65, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1443, 25, 1483, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+cx^4+fx^2)}\,dx$$
$$\Big\downarrow \text{1443}$$
$$\frac{\int -\frac{cx^2+f}{cx^4+fx^2+a}dx}{a}-\frac{1}{ax}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int \frac{cx^2+f}{cx^4+fx^2+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{1483}$$

$$-\frac{\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}f+\sqrt{c}(\sqrt{a}\sqrt{c}-f)x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}f-\sqrt{c}(\sqrt{a}\sqrt{c}-f)x}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}f+\sqrt{c}(\sqrt{a}\sqrt{c}-f)x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}f-\sqrt{c}(\sqrt{a}\sqrt{c}-f)x}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\sqrt{c}(\sqrt{a}\sqrt{c}-f)\int -\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{}{x^2}}{}}{a} \quad \frac{1}{ax}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx-\frac{1}{2}\sqrt{c}(\sqrt{a}\sqrt{c}-f)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{}{x^2+}}{}}{a} \quad \frac{1}{ax}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx-\frac{1}{2}(\sqrt{a}\sqrt{c}-f)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{1}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}}}{\sqrt{c}}}}{2\sqrt{}}}{a} \quad \frac{1}{ax}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\frac{-\frac{1}{2}(\sqrt{a}\sqrt{c}-f)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx-\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)^2-\frac{f+2\sqrt{a}\sqrt{c}}{c}}d\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{-\frac{1}{2}(\sqrt{a}\sqrt{c}-}{}}{a} \quad \frac{1}{ax}$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{\frac{\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}-\frac{1}{2}(\sqrt{a}\sqrt{c}-f)\int\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\arctan\left(\frac{\sqrt{c}\left(\frac{\sqrt{2\cdots}}{}\right)}{\sqrt{2\cdots}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}}{2\cdots}}{a} - \frac{1}{ax}$$

$$\Big\downarrow \; 1103$$

$$-\frac{\frac{\frac{\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}+\frac{1}{2}\sqrt{c}(\sqrt{a}\sqrt{c}-f)\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}-f}+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}(\sqrt{a}\sqrt{c}+f)\operatorname{arctan}\cdots}{\sqrt{2\sqrt{a}\sqrt{\cdots}}}}{}}{a} - \frac{1}{ax}$$

input
```
Int[1/(x^2*(a + f*x^2 + c*x^4)),x]
```

output
```
-(1/(a*x)) - (((Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f]*(Sqrt[a]*Sqrt[c] + f)*
ArcTan[(Sqrt[c]*(-(Sqrt[2*Sqrt[a]*Sqrt[c] - f]/Sqrt[c]) + 2*x))/Sqrt[2*Sqr
t[a]*Sqrt[c] + f]])/Sqrt[2*Sqrt[a]*Sqrt[c] + f] + (Sqrt[c]*(Sqrt[a]*Sqrt[c
] - f)*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] - f]*x + Sqrt[c]*x^2])/2)/(2*S
qrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f]) + ((Sqrt[c]*Sqrt[2*Sqrt[a]*Sqr
t[c] - f]*(Sqrt[a]*Sqrt[c] + f)*ArcTan[(Sqrt[c]*(Sqrt[2*Sqrt[a]*Sqrt[c] -
f]/Sqrt[c] + 2*x))/Sqrt[2*Sqrt[a]*Sqrt[c] + f]])/Sqrt[2*Sqrt[a]*Sqrt[c] +
f] - (Sqrt[c]*(Sqrt[a]*Sqrt[c] - f)*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] -
 f]*x + Sqrt[c]*x^2])/2)/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f]))/
a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.60

method	result
default	$\frac{4c\left(\frac{\left(-\sqrt{-4ac+f^2}+f\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+f^2}+f\right)c}}\right)}{8\sqrt{-4ac+f^2}\sqrt{\left(\sqrt{-4ac+f^2}+f\right)c}}-\frac{\left(-\sqrt{-4ac+f^2}-f\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+f^2}-f\right)c}}\right)}{8\sqrt{-4ac+f^2}\sqrt{\left(\sqrt{-4ac+f^2}-f\right)c}}\right)}{a}-\frac{1}{ax}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^5c^2-8cf^2a^4+f^4a^3\right)\_Z^4+\left(12a^2c^2f-7acf^3+f^5\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(40a^5c^2-22cf^2a^4+3f^4a^3\right)\_R^4+\right.\right.\right.}{2}$

input
```
int(1/x^2/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/a*c*(1/8*(-(-4*a*c+f^2)^(1/2)+f)/(-4*a*c+f^2)^(1/2)*2^(1/2)/(((-4*a*c+f^
2)^(1/2)+f)*c)^(1/2)*arctan(c*x*2^(1/2)/(((-4*a*c+f^2)^(1/2)+f)*c)^(1/2))-
1/8*(-(-4*a*c+f^2)^(1/2)-f)/(-4*a*c+f^2)^(1/2)*2^(1/2)/(((-4*a*c+f^2)^(1/2
)-f)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+f^2)^(1/2)-f)*c)^(1/2)))-1/a/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1172 vs. 2(193) = 386.

Time = 0.11 (sec) , antiderivative size = 1172, normalized size of antiderivative = 4.44

$$\int \frac{1}{x^2\left(a+fx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*a*x*sqrt(-(3*a*c*f - f^3 + (4*a^4*c - a^3*f^2)*sqrt(-(a^2*c
^2 - 2*a*c*f^2 + f^4)/(4*a^7*c - a^6*f^2)))/(4*a^4*c - a^3*f^2))*log(-2*(a
*c^3 - c^2*f^2)*x + sqrt(1/2)*(4*a^2*c^2*f - 5*a*c*f^3 + f^5 - (8*a^5*c^2
- 6*a^4*c*f^2 + a^3*f^4)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a^7*c - a^6*
f^2)))*sqrt(-(3*a*c*f - f^3 + (4*a^4*c - a^3*f^2)*sqrt(-(a^2*c^2 - 2*a*c*f
^2 + f^4)/(4*a^7*c - a^6*f^2)))/(4*a^4*c - a^3*f^2))) - sqrt(1/2)*a*x*sqrt
(-(3*a*c*f - f^3 + (4*a^4*c - a^3*f^2)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(
4*a^7*c - a^6*f^2)))/(4*a^4*c - a^3*f^2))*log(-2*(a*c^3 - c^2*f^2)*x - sqr
t(1/2)*(4*a^2*c^2*f - 5*a*c*f^3 + f^5 - (8*a^5*c^2 - 6*a^4*c*f^2 + a^3*f^4
)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a^7*c - a^6*f^2)))*sqrt(-(3*a*c*f -
 f^3 + (4*a^4*c - a^3*f^2)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a^7*c - a^
6*f^2)))/(4*a^4*c - a^3*f^2))) + sqrt(1/2)*a*x*sqrt(-(3*a*c*f - f^3 - (4*a
^4*c - a^3*f^2)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a^7*c - a^6*f^2)))/(4
*a^4*c - a^3*f^2))*log(-2*(a*c^3 - c^2*f^2)*x + sqrt(1/2)*(4*a^2*c^2*f - 5
*a*c*f^3 + f^5 + (8*a^5*c^2 - 6*a^4*c*f^2 + a^3*f^4)*sqrt(-(a^2*c^2 - 2*a*
c*f^2 + f^4)/(4*a^7*c - a^6*f^2)))*sqrt(-(3*a*c*f - f^3 - (4*a^4*c - a^3*f
^2)*sqrt(-(a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a^7*c - a^6*f^2)))/(4*a^4*c - a^3
*f^2))) - sqrt(1/2)*a*x*sqrt(-(3*a*c*f - f^3 - (4*a^4*c - a^3*f^2)*sqrt(-(
a^2*c^2 - 2*a*c*f^2 + f^4)/(4*a^7*c - a^6*f^2)))/(4*a^4*c - a^3*f^2))*log(
-2*(a*c^3 - c^2*f^2)*x - sqrt(1/2)*(4*a^2*c^2*f - 5*a*c*f^3 + f^5 + (8*...
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^2(a+fx^2+cx^4)}\,dx$$
$$=\operatorname{RootSum}\left(t^4\cdot\left(256a^5c^2-128a^4cf^2+16a^3f^4\right)+t^2\cdot\left(48a^2c^2f-28acf^3+4f^5\right)+c^3,\left(t\mapsto t\log\left(x+\right.\right.\right.$$
$$-\frac{1}{ax}$$

input
```
integrate(1/x**2/(c*x**4+f*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**5*c**2 - 128*a**4*c*f**2 + 16*a**3*f**4) + _t**2*(48
*a**2*c**2*f - 28*a*c*f**3 + 4*f**5) + c**3, Lambda(_t, _t*log(x + (-64*_t
**3*a**5*c**2 + 48*_t**3*a**4*c*f**2 - 8*_t**3*a**3*f**4 - 10*_t*a**2*c**2
*f + 10*_t*a*c*f**3 - 2*_t*f**5)/(a*c**3 - c**2*f**2)))) - 1/(a*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + fx^2 + cx^4)} \, dx = \int \frac{1}{(cx^4 + fx^2 + a)x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^2 + f)/(c*x^4 + f*x^2 + a), x)/a - 1/(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1857 vs. $2(193) = 386$.

Time = 0.66 (sec) , antiderivative size = 1857, normalized size of antiderivative = 7.03

$$\int \frac{1}{x^2 (a + fx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output
```
1/8*(8*a^3*c^3*f^2 - 2*a^2*c^2*f^4 - 4*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f
 + sqrt(-4*a*c + f^2)*c)*a^3*c*f^2 + sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f +
 sqrt(-4*a*c + f^2)*c)*a^2*c^2*f^2 - 2*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f
 + sqrt(-4*a*c + f^2)*c)*a^2*c*f^3 + sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f +
 sqrt(-4*a*c + f^2)*c)*a^2*f^4 - 2*(4*a*c - f^2)*a^2*c^2*f^2 - (32*a^2*c^4
 - 16*a*c^3*f^2 + 2*c^2*f^4 - 16*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqr
t(-4*a*c + f^2)*c)*a^2*c^2 + 4*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(
-4*a*c + f^2)*c)*a*c^3 - 8*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a
*c + f^2)*c)*a*c^2*f + 8*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c
 + f^2)*c)*a*c*f^2 - sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f
^2)*c)*c^2*f^2 + 2*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2
)*c)*c*f^3 - sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*f
^4 - 8*(4*a*c - f^2)*a*c^3 + 2*(4*a*c - f^2)*c^2*f^2)*a^2 - 2*(16*sqrt(2)*
sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^3*c^2*f - 4*sqrt(2)*sqrt(c*f + sqrt(-4*
a*c + f^2)*c)*a^2*c^3*f - 32*a^3*c^3*f + 8*sqrt(2)*sqrt(c*f + sqrt(-4*a*c
+ f^2)*c)*a^2*c^2*f^2 - 8*sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c*f
^3 + sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^2*f^3 + 16*a^2*c^2*f^3 -
 2*sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c*f^4 + sqrt(2)*sqrt(c*f + s
qrt(-4*a*c + f^2)*c)*a*f^5 - 2*a*c*f^5 + 8*(4*a*c - f^2)*a^2*c^2*f - 2*(4*
a*c - f^2)*a*c*f^3)*abs(a))*arctan(2*sqrt(1/2)*x/sqrt((a*f + sqrt(-4*a^...
```

Mupad [B] (verification not implemented)

Time = 18.79 (sec) , antiderivative size = 2997, normalized size of antiderivative = 11.35

$$\int \frac{1}{x^2 (a + fx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + c*x^4 + f*x^2)),x)
```

output
```
- atan((((4*a^4*c^2*f^3 - 16*a^5*c^3*f + x*(32*a^6*c^3*f - 8*a^5*c^2*f^3)*
(-(f^5 + f^2*(-(4*a*c - f^2)^3)^(1/2) + 12*a^2*c^2*f - 7*a*c*f^3 - a*c*(-(
4*a*c - f^2)^3)^(1/2))/(8*(16*a^5*c^2 + a^3*f^4 - 8*a^4*c*f^2)))^(1/2))*(-
(f^5 + f^2*(-(4*a*c - f^2)^3)^(1/2) + 12*a^2*c^2*f - 7*a*c*f^3 - a*c*(-(4*
a*c - f^2)^3)^(1/2))/(8*(16*a^5*c^2 + a^3*f^4 - 8*a^4*c*f^2)))^(1/2) + x*(
4*a^4*c^4 - 2*a^3*c^3*f^2))*(-(f^5 + f^2*(-(4*a*c - f^2)^3)^(1/2) + 12*a^2
*c^2*f - 7*a*c*f^3 - a*c*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^5*c^2 + a^3*f^
4 - 8*a^4*c*f^2)))^(1/2)*1i + ((16*a^5*c^3*f - 4*a^4*c^2*f^3 + x*(32*a^6*c
^3*f - 8*a^5*c^2*f^3)*(-(f^5 + f^2*(-(4*a*c - f^2)^3)^(1/2) + 12*a^2*c^2*f
 - 7*a*c*f^3 - a*c*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^5*c^2 + a^3*f^4 - 8*
a^4*c*f^2)))^(1/2))*(-(f^5 + f^2*(-(4*a*c - f^2)^3)^(1/2) + 12*a^2*c^2*f -
 7*a*c*f^3 - a*c*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^5*c^2 + a^3*f^4 - 8*a^
4*c*f^2)))^(1/2) + x*(4*a^4*c^4 - 2*a^3*c^3*f^2))*(-(f^5 + f^2*(-(4*a*c -
f^2)^3)^(1/2) + 12*a^2*c^2*f - 7*a*c*f^3 - a*c*(-(4*a*c - f^2)^3)^(1/2))/(
8*(16*a^5*c^2 + a^3*f^4 - 8*a^4*c*f^2)))^(1/2)*1i)/((((16*a^5*c^3*f - 4*a^4
*c^2*f^3 + x*(32*a^6*c^3*f - 8*a^5*c^2*f^3)*(-(f^5 + f^2*(-(4*a*c - f^2)^3
)^(1/2) + 12*a^2*c^2*f - 7*a*c*f^3 - a*c*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*
a^5*c^2 + a^3*f^4 - 8*a^4*c*f^2)))^(1/2))*(-(f^5 + f^2*(-(4*a*c - f^2)^3)^
(1/2) + 12*a^2*c^2*f - 7*a*c*f^3 - a*c*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^
5*c^2 + a^3*f^4 - 8*a^4*c*f^2)))^(1/2) + x*(4*a^4*c^4 - 2*a^3*c^3*f^2))...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.12

$$\int \frac{1}{x^2 (a + f x^2 + c x^4)} \, dx$$
$$= \frac{4\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+f}\; atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right) acx - 2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+f}\; atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-f}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+f}}\right) f^2 x + 2\sqrt{}}{}$$

input
```
int(1/x^2/(c*x^4+f*x^2+a),x)
```

output
```
(4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*x - 2*sqrt(a)*sqrt(2*sqrt(c
)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqr
t(c)*sqrt(a) + f))*f**2*x + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sq
rt(2*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*f*
x - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f
) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*x + 2*sqrt(a)*sqrt(2*sqr
t(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*
sqrt(c)*sqrt(a) + f))*f**2*x - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a
*f*x - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - f)*log( - sqrt(2*sqrt(c)*sqrt(a)
 - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x + sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) -
 f)*log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f**2*x
+ 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - f)*log(sqrt(2*sqrt(c)*sqrt(a) - f)*x
+ sqrt(a) + sqrt(c)*x**2)*a*c*x - sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - f)*log(
sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f**2*x + sqrt(c)*s
qrt(2*sqrt(c)*sqrt(a) - f)*log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a)
+ sqrt(c)*x**2)*a*f*x - sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - f)*log(sqrt(2*sqr
t(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*f*x - 16*a**2*c + 4*a*f**2
)/(4*a**2*x*(4*a*c - f**2))
```

3.789 $\int \frac{1}{x^4(a+fx^2+cx^4)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 305

$$\int \frac{1}{x^4(a+fx^2+cx^4)}\,dx = -\frac{1}{3ax^3}+\frac{f}{a^2x}-\frac{\left(\sqrt{c}f-\frac{ac-f^2}{\sqrt{a}}\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2a^2\sqrt{2\sqrt{a}\sqrt{c}+f}} + \frac{\left(\sqrt{c}f-\frac{ac-f^2}{\sqrt{a}}\right)\arctan\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}+2\sqrt{c}x}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{2a^2\sqrt{2\sqrt{a}\sqrt{c}+f}} - \frac{\left(ac+\sqrt{a}\sqrt{c}f-f^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\,x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2a^{5/2}\sqrt{2\sqrt{a}\sqrt{c}-f}}$$

output
```
-1/3/a/x^3+f/a^2/x-1/2*(c^(1/2)*f-(a*c-f^2)/a^(1/2))*arctan(((2*a^(1/2)*c^
(1/2)-f)^(1/2)-2*c^(1/2)*x)/(2*a^(1/2)*c^(1/2)+f)^(1/2))/a^2/(2*a^(1/2)*c^
(1/2)+f)^(1/2)+1/2*(c^(1/2)*f-(a*c-f^2)/a^(1/2))*arctan(((2*a^(1/2)*c^(1/2
)-f)^(1/2)+2*c^(1/2)*x)/(2*a^(1/2)*c^(1/2)+f)^(1/2))/a^2/(2*a^(1/2)*c^(1/2
)+f)^(1/2)-1/2*(a*c+a^(1/2)*c^(1/2)*f-f^2)*arctanh((2*a^(1/2)*c^(1/2)-f)^(
1/2)*x/(a^(1/2)+c^(1/2)*x^2))/a^(5/2)/(2*a^(1/2)*c^(1/2)-f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^4(a+fx^2+cx^4)}\,dx = \frac{-\frac{2a}{x^3}+\frac{6f}{x}+\frac{3\sqrt{2}\sqrt{c}\left(-2ac+f\left(f+\sqrt{-4ac+f^2}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f-\sqrt{-4ac+f^2}}}\right)}{\sqrt{-4ac+f^2}\sqrt{f-\sqrt{-4ac+f^2}}}+\frac{3\sqrt{2}\sqrt{c}\left(2ac+f\left(-f+\sqrt{-4ac+f^2}\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{f+\sqrt{-4ac}}}\right.}{\sqrt{-4ac+f^2}\sqrt{f+\sqrt{-4ac+f^2}}}}{6a^2}$$

input
```
Integrate[1/(x^4*(a + f*x^2 + c*x^4)),x]
```

output
```
((-2*a)/x^3 + (6*f)/x + (3*Sqrt[2]*Sqrt[c]*(-2*a*c + f*(f + Sqrt[-4*a*c +
f^2]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[f - Sqrt[-4*a*c + f^2]]])/(Sqrt[-4*
a*c + f^2]*Sqrt[f - Sqrt[-4*a*c + f^2]]) + (3*Sqrt[2]*Sqrt[c]*(2*a*c + f*(
-f + Sqrt[-4*a*c + f^2]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[f + Sqrt[-4*a*c
+ f^2]]])/(Sqrt[-4*a*c + f^2]*Sqrt[f + Sqrt[-4*a*c + f^2]]))/(6*a^2)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.58, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1443, 27, 1604, 25, 1483, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(a+cx^4+fx^2)}\,dx$$

$$\Big\downarrow \text{ 1443}$$

$$\frac{\int -\frac{3(cx^2+f)}{x^2(cx^4+fx^2+a)}dx}{3a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{cx^2+f}{x^2(cx^4+fx^2+a)}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \ 1604$$

$$-\frac{-\frac{\int -\frac{-f^2-cx^2f+ac}{cx^4+fx^2+a}dx}{a}-\frac{f}{ax}}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\int \frac{-f^2-cx^2f+ac}{cx^4+fx^2+a}dx}{a}-\frac{f}{ax}}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \ 1483$$

$$-\frac{\frac{\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(ac-f^2\right)-\sqrt{c}\left(-f^2+\sqrt{a}\sqrt{c}f+ac\right)x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(ac-f^2\right)+\sqrt{c}\left(-f^2+\sqrt{a}\sqrt{c}f+ac\right)x}{\sqrt{c}\left(x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{2\sqrt{a}\sqrt{c}-f}}}{a}-\frac{f}{ax}}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(ac-f^2\right)-\sqrt{c}\left(-f^2+\sqrt{a}\sqrt{c}f+ac\right)x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}\left(ac-f^2\right)+\sqrt{c}\left(-f^2+\sqrt{a}\sqrt{c}f+ac\right)x}{x^2+\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}}{a}-\frac{f}{ax}}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \ 1142$$

$$-\frac{\frac{\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx-\frac{1}{2}\sqrt{c}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int -\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac\cdots\right.}{\cdots}}{a}\cdots}{a}$$

$$\frac{1}{3ax^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\sqrt{c}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-\cdots\right.}{\cdots}}{a}\cdots}{a}$$

$$\frac{1}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{1}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{1}{2}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \cdots}{\cdots}}{a}}{a}$$

$$\frac{1}{3ax^3}$$

$\Big\downarrow$ 1083

$$-\frac{\frac{\frac{\frac{1}{2}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx-\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{1}{-\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)^2-\frac{f+2\sqrt{a}\sqrt{c}}{c}}d\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\left(\sqrt{a}\sqrt{c}\cdots\right.}{\cdots}}{a}}{a}$$

$$\frac{1}{3ax^3}$$

$\Big\downarrow$ 217

$$-\frac{\frac{\frac{\frac{1}{2}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{\sqrt{2\sqrt{a}\sqrt{c}-f}-2\sqrt{c}x}{x^2-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}x}{\sqrt{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx+\frac{\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{1}{2}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\int \frac{2\sqrt{c}x+\cdots}{x^2+\frac{\sqrt{2\cdots}}{\cdots}}}{\cdots}}{a}}{a}$$

$$\frac{1}{3ax^3}$$

$\Big\downarrow$ 1103

$$-\frac{\frac{\frac{\frac{\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f+ac-f^2\right)\arctan\left(\frac{\sqrt{c}\left(2x-\frac{\sqrt{2\sqrt{a}\sqrt{c}-f}}{\sqrt{c}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}\right)}{\sqrt{2\sqrt{a}\sqrt{c}+f}}-\frac{1}{2}\sqrt{c}\left(\sqrt{a}\sqrt{c}f+ac-f^2\right)\log\left(-x\sqrt{2\sqrt{a}\sqrt{c}-f}+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{a}\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}}+\frac{\frac{\sqrt{c}\sqrt{2\sqrt{a}\sqrt{c}-f}\left(-\sqrt{a}\sqrt{c}f\cdots\right.}{\cdots}}{\cdots}}{a}}{a}$$

$$\frac{1}{3ax^3}$$

input
```
Int[1/(x^4*(a + f*x^2 + c*x^4)),x]
```

output
```
-1/3*1/(a*x^3) - (-(f/(a*x)) + (((Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f]*(a*c
 - Sqrt[a]*Sqrt[c]*f - f^2)*ArcTan[(Sqrt[c]*(-(Sqrt[2*Sqrt[a]*Sqrt[c] - f]
/Sqrt[c]) + 2*x))/Sqrt[2*Sqrt[a]*Sqrt[c] + f]])/Sqrt[2*Sqrt[a]*Sqrt[c] + f
] - (Sqrt[c]*(a*c + Sqrt[a]*Sqrt[c]*f - f^2)*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*
Sqrt[c] - f]*x + Sqrt[c]*x^2])/2)/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c
] - f]) + ((Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f]*(a*c - Sqrt[a]*Sqrt[c]*f -
 f^2)*ArcTan[(Sqrt[c]*(Sqrt[2*Sqrt[a]*Sqrt[c] - f]/Sqrt[c] + 2*x))/Sqrt[2*
Sqrt[a]*Sqrt[c] + f]])/Sqrt[2*Sqrt[a]*Sqrt[c] + f] + (Sqrt[c]*(a*c + Sqrt[
a]*Sqrt[c]*f - f^2)*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] - f]*x + Sqrt[c]*
x^2])/2)/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - f]))/a)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.59

method	result
default	$\frac{4c\left(\frac{\left(f\sqrt{-4ac+f^2}+2ac-f^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+f^2}+f\right)c}}\right)}{8\sqrt{-4ac+f^2}\,\sqrt{\left(\sqrt{-4ac+f^2}+f\right)c}}-\frac{\left(f\sqrt{-4ac+f^2}-2ac+f^2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+f^2}-f\right)c}}\right)}{8\sqrt{-4ac+f^2}\,\sqrt{\left(\sqrt{-4ac+f^2}-f\right)c}}\right)}{a^2}-\frac{1}{3a\,x^3}$
risch	$\frac{\frac{f\,x^2}{a^2}-\frac{1}{3a}}{x^3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16c^2a^7-8a^6c\,f^2+a^5f^4\right)\_Z^4+\left(-20a^3c^3f+25a^2c^2f^3-9ac\,f^5+f^7\right)\_Z^2+c^5\right)}\_R\ln\left(\left(\left(40c^2a^7-22a^6c\,f^2-\right.\right.\right.\right.}{}$

input
```
int(1/x^4/(c*x^4+f*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/a^2*c*(1/8*(f*(-4*a*c+f^2)^(1/2)+2*a*c-f^2)/(-4*a*c+f^2)^(1/2)*2^(1/2)/(
((-4*a*c+f^2)^(1/2)+f)*c)^(1/2)*arctan(c*x*2^(1/2)/(((-4*a*c+f^2)^(1/2)+f)
*c)^(1/2))-1/8*(f*(-4*a*c+f^2)^(1/2)-2*a*c+f^2)/(-4*a*c+f^2)^(1/2)*2^(1/2)
/(((-4*a*c+f^2)^(1/2)-f)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+f^2)^(1/2)
-f)*c)^(1/2)))-1/3/a/x^3+f/a^2/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(230) = 460$.

Time = 0.11 (sec) , antiderivative size = 1654, normalized size of antiderivative = 5.42

$$\int \frac{1}{x^4(a + fx^2 + cx^4)} dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(c*x^4+f*x^2+a),x, algorithm="fricas")
```

output
```
-1/6*(3*sqrt(1/2)*a^2*x^3*sqrt((5*a^2*c^2*f - 5*a*c*f^3 + f^5 + (4*a^6*c -
 a^5*f^2)*sqrt(-(a^4*c^4 - 6*a^3*c^3*f^2 + 11*a^2*c^2*f^4 - 6*a*c*f^6 + f^
8)/(4*a^11*c - a^10*f^2)))/(4*a^6*c - a^5*f^2))*log(2*(a^2*c^5 - 3*a*c^4*f
^2 + c^3*f^4)*x + sqrt(1/2)*(4*a^4*c^4 - 17*a^3*c^3*f^2 + 20*a^2*c^2*f^4 -
 8*a*c*f^6 + f^8 + (12*a^7*c^2*f - 7*a^6*c*f^3 + a^5*f^5)*sqrt(-(a^4*c^4 -
 6*a^3*c^3*f^2 + 11*a^2*c^2*f^4 - 6*a*c*f^6 + f^8)/(4*a^11*c - a^10*f^2)))
*sqrt((5*a^2*c^2*f - 5*a*c*f^3 + f^5 + (4*a^6*c - a^5*f^2)*sqrt(-(a^4*c^4
- 6*a^3*c^3*f^2 + 11*a^2*c^2*f^4 - 6*a*c*f^6 + f^8)/(4*a^11*c - a^10*f^2))
)/(4*a^6*c - a^5*f^2))) - 3*sqrt(1/2)*a^2*x^3*sqrt((5*a^2*c^2*f - 5*a*c*f^
3 + f^5 + (4*a^6*c - a^5*f^2)*sqrt(-(a^4*c^4 - 6*a^3*c^3*f^2 + 11*a^2*c^2*
f^4 - 6*a*c*f^6 + f^8)/(4*a^11*c - a^10*f^2)))/(4*a^6*c - a^5*f^2))*log(2*
(a^2*c^5 - 3*a*c^4*f^2 + c^3*f^4)*x - sqrt(1/2)*(4*a^4*c^4 - 17*a^3*c^3*f^
2 + 20*a^2*c^2*f^4 - 8*a*c*f^6 + f^8 + (12*a^7*c^2*f - 7*a^6*c*f^3 + a^5*f
^5)*sqrt(-(a^4*c^4 - 6*a^3*c^3*f^2 + 11*a^2*c^2*f^4 - 6*a*c*f^6 + f^8)/(4*
a^11*c - a^10*f^2)))*sqrt((5*a^2*c^2*f - 5*a*c*f^3 + f^5 + (4*a^6*c - a^5*
f^2)*sqrt(-(a^4*c^4 - 6*a^3*c^3*f^2 + 11*a^2*c^2*f^4 - 6*a*c*f^6 + f^8)/(4
*a^11*c - a^10*f^2)))/(4*a^6*c - a^5*f^2))) + 3*sqrt(1/2)*a^2*x^3*sqrt((5*
a^2*c^2*f - 5*a*c*f^3 + f^5 - (4*a^6*c - a^5*f^2)*sqrt(-(a^4*c^4 - 6*a^3*c
^3*f^2 + 11*a^2*c^2*f^4 - 6*a*c*f^6 + f^8)/(4*a^11*c - a^10*f^2)))/(4*a^6*
c - a^5*f^2))*log(2*(a^2*c^5 - 3*a*c^4*f^2 + c^3*f^4)*x + sqrt(1/2)*(4*...
```

Sympy [A] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^4 (a + fx^2 + cx^4)} dx$$
$$= \text{RootSum}\left(t^4 \cdot (256a^7c^2 - 128a^6cf^2 + 16a^5f^4) + t^2(-80a^3c^3f + 100a^2c^2f^3 - 36acf^5 + 4f^7) + c^5, \left(t \mapsto\right.\right.$$
$$+ \frac{-a + 3fx^2}{3a^2x^3}$$

input
```
integrate(1/x**4/(c*x**4+f*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**7*c**2 - 128*a**6*c*f**2 + 16*a**5*f**4) + _t**2*(-8
0*a**3*c**3*f + 100*a**2*c**2*f**3 - 36*a*c*f**5 + 4*f**7) + c**5, Lambda(
_t, _t*log(x + (-96*_t**3*a**7*c**2*f + 56*_t**3*a**6*c*f**3 - 8*_t**3*a**
5*f**5 - 4*_t*a**4*c**4 + 32*_t*a**3*c**3*f**2 - 40*_t*a**2*c**2*f**4 + 16
*_t*a*c*f**6 - 2*_t*f**8)/(a**2*c**5 - 3*a*c**4*f**2 + c**3*f**4)))) + (-a
 + 3*f*x**2)/(3*a**2*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + fx^2 + cx^4)} dx = \int \frac{1}{(cx^4 + fx^2 + a)x^4} dx$$

input
```
integrate(1/x^4/(c*x^4+f*x^2+a),x, algorithm="maxima")
```

output
```
integrate((c*f*x^2 - a*c + f^2)/(c*x^4 + f*x^2 + a), x)/a^2 + 1/3*(3*f*x^2
 - a)/(a^2*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1664 vs. $2(230) = 460$.

Time = 0.82 (sec) , antiderivative size = 1664, normalized size of antiderivative = 5.46

$$\int \frac{1}{x^4 (a + fx^2 + cx^4)} \, dx = \text{Too large to display}$$

input

```
integrate(1/x^4/(c*x^4+f*x^2+a),x, algorithm="giac")
```

output

```
-1/4*(16*sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^3*c^3 - 4*sqrt(2)*sqrt
(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^4 - 32*a^3*c^4 + 8*sqrt(2)*sqrt(c*f + s
qrt(-4*a*c + f^2)*c)*a^2*c^3*f - 24*a^2*c^4*f - 24*sqrt(2)*sqrt(c*f + sqrt
(-4*a*c + f^2)*c)*a^2*c^2*f^2 + 5*sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)
*a*c^3*f^2 + 48*a^2*c^3*f^2 - 10*sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*
a*c^2*f^3 + 14*a*c^3*f^3 + 9*sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c*
f^4 - sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c^2*f^4 - 18*a*c^2*f^4 + 2*
sqrt(2)*sqrt(c*f + sqrt(-4*a*c + f^2)*c)*c*f^5 - 2*c^2*f^5 - sqrt(2)*sqrt(
c*f + sqrt(-4*a*c + f^2)*c)*f^6 + 2*c*f^6 + 12*sqrt(2)*sqrt(-4*a*c + f^2)*
sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a^2*c^2*f - 3*sqrt(2)*sqrt(-4*a*c + f^2)*
sqrt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^3*f + 6*sqrt(2)*sqrt(-4*a*c + f^2)*sq
rt(c*f + sqrt(-4*a*c + f^2)*c)*a*c^2*f^2 - 7*sqrt(2)*sqrt(-4*a*c + f^2)*sq
rt(c*f + sqrt(-4*a*c + f^2)*c)*a*c*f^3 + sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c
*f + sqrt(-4*a*c + f^2)*c)*c^2*f^3 - 2*sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f
 + sqrt(-4*a*c + f^2)*c)*c*f^4 + sqrt(2)*sqrt(-4*a*c + f^2)*sqrt(c*f + sqr
t(-4*a*c + f^2)*c)*f^5 + 8*(4*a*c - f^2)*a^2*c^3 + 6*(4*a*c - f^2)*a*c^3*f
 - 10*(4*a*c - f^2)*a*c^2*f^2 - 2*(4*a*c - f^2)*c^2*f^3 + 2*(4*a*c - f^2)*
c*f^4)*arctan(2*sqrt(1/2)*x/sqrt((a^2*f + sqrt(-4*a^5*c + a^4*f^2))/(a^2*c
)))/((16*a^5*c^2 - 4*a^4*c^3 + 8*a^4*c^2*f - 8*a^4*c*f^2 + a^3*c^2*f^2 - 2
*a^3*c*f^3 + a^3*f^4)*abs(c)) - 1/4*(16*sqrt(2)*sqrt(c*f - sqrt(-4*a*c ...
```

Mupad [B] (verification not implemented)

Time = 18.52 (sec) , antiderivative size = 4162, normalized size of antiderivative = 13.65

$$\int \frac{1}{x^4 (a + fx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + c*x^4 + f*x^2)),x)
```

output
```
- (1/(3*a) - (f*x^2)/a^2)/x^3 - atan(((x*(4*a^8*c^5 + 2*a^6*c^3*f^4 - 8*a^
7*c^4*f^2) + ((f^4*(-(4*a*c - f^2)^3)^(1/2) - f^7 + 20*a^3*c^3*f - 25*a^2*
c^2*f^3 + a^2*c^2*(-(4*a*c - f^2)^3)^(1/2) + 9*a*c*f^5 - 3*a*c*f^2*(-(4*a*
c - f^2)^3)^(1/2))/(8*(16*a^7*c^2 + a^5*f^4 - 8*a^6*c*f^2)))^(1/2)*(16*a^1
0*c^4 - x*(32*a^11*c^3*f - 8*a^10*c^2*f^3)*((f^4*(-(4*a*c - f^2)^3)^(1/2)
- f^7 + 20*a^3*c^3*f - 25*a^2*c^2*f^3 + a^2*c^2*(-(4*a*c - f^2)^3)^(1/2) +
 9*a*c*f^5 - 3*a*c*f^2*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^7*c^2 + a^5*f^4
- 8*a^6*c*f^2)))^(1/2) + 4*a^8*c^2*f^4 - 20*a^9*c^3*f^2))*((f^4*(-(4*a*c -
 f^2)^3)^(1/2) - f^7 + 20*a^3*c^3*f - 25*a^2*c^2*f^3 + a^2*c^2*(-(4*a*c -
f^2)^3)^(1/2) + 9*a*c*f^5 - 3*a*c*f^2*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^7
*c^2 + a^5*f^4 - 8*a^6*c*f^2)))^(1/2)*1i + (x*(4*a^8*c^5 + 2*a^6*c^3*f^4 -
 8*a^7*c^4*f^2) - ((f^4*(-(4*a*c - f^2)^3)^(1/2) - f^7 + 20*a^3*c^3*f - 25
*a^2*c^2*f^3 + a^2*c^2*(-(4*a*c - f^2)^3)^(1/2) + 9*a*c*f^5 - 3*a*c*f^2*(-
(4*a*c - f^2)^3)^(1/2))/(8*(16*a^7*c^2 + a^5*f^4 - 8*a^6*c*f^2)))^(1/2)*(1
6*a^10*c^4 + x*(32*a^11*c^3*f - 8*a^10*c^2*f^3)*((f^4*(-(4*a*c - f^2)^3)^(
1/2) - f^7 + 20*a^3*c^3*f - 25*a^2*c^2*f^3 + a^2*c^2*(-(4*a*c - f^2)^3)^(1
/2) + 9*a*c*f^5 - 3*a*c*f^2*(-(4*a*c - f^2)^3)^(1/2))/(8*(16*a^7*c^2 + a^5
*f^4 - 8*a^6*c*f^2)))^(1/2) + 4*a^8*c^2*f^4 - 20*a^9*c^3*f^2))*((f^4*(-(4*
a*c - f^2)^3)^(1/2) - f^7 + 20*a^3*c^3*f - 25*a^2*c^2*f^3 + a^2*c^2*(-(4*a
*c - f^2)^3)^(1/2) + 9*a*c*f^5 - 3*a*c*f^2*(-(4*a*c - f^2)^3)^(1/2))/(8...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 810, normalized size of antiderivative = 2.66

$$\int \frac{1}{x^4 (a + fx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
int(1/x^4/(c*x^4+f*x^2+a),x)
```

output
```
( - 18*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) -
f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*f*x**3 + 6*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + f))*f**3*x**3 + 12*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
+ f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + f))*a**2*c*x**3 - 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*
sqrt(c)*sqrt(a) - f) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*f**2*x*
*3 + 18*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*c*f*x**3 - 6*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + f))*f**3*x**3 - 12*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
 + f)*atan((sqrt(2*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + f))*a**2*c*x**3 + 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + f)*atan((sqrt(2
*sqrt(c)*sqrt(a) - f) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + f))*a*f**2*x
**3 + 9*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - f)*log( - sqrt(2*sqrt(c)*sqrt(a)
- f)*x + sqrt(a) + sqrt(c)*x**2)*a*c*f*x**3 - 3*sqrt(a)*sqrt(2*sqrt(c)*sqr
t(a) - f)*log( - sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)*f
**3*x**3 - 9*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - f)*log(sqrt(2*sqrt(c)*sqrt(a
) - f)*x + sqrt(a) + sqrt(c)*x**2)*a*c*f*x**3 + 3*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) - f)*log(sqrt(2*sqrt(c)*sqrt(a) - f)*x + sqrt(a) + sqrt(c)*x**2)...
```

3.790 $\int \frac{x^7}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 132

$$\int \frac{x^7}{(a+bx^2+cx^4)^2} dx = -\frac{bx^2}{2c(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}} + \frac{\log(a+bx^2+cx^4)}{4c^2}$$

output
```
-1/2*b*x^2/c/(-4*a*c+b^2)+1/2*x^4*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
+1/2*b*(-6*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^
2)^(3/2)+1/4*ln(c*x^4+b*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{x^7}{(a+bx^2+cx^4)^2} dx = \frac{\frac{2(-2a^2c+b^3x^2+ab(b-3cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + \log(a+bx^2+cx^4)}{4c^2}$$

input
```
Integrate[x^7/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b
^2 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1434, 1164, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \ 1434$$

$$\frac{1}{2}\int \frac{x^6}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \ 1164$$

$$\frac{1}{2}\left(\frac{x^4(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{x^2(bx^2+4a)}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \ 1200$$

$$\frac{1}{2}\left(\frac{x^4(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \left(\frac{b}{c}-\frac{(b^2-4ac)x^2+ab}{c(cx^4+bx^2+a)}\right)dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{x^4(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(b^2-4ac)\log(a+bx^2+cx^4)}{2c^2}+\frac{bx^2}{c}}{b^2-4ac}\right)$$

input
```
Int[x^7/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((x^4*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b*x^2)/c - (b
*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4
*a*c]) - ((b^2 - 4*a*c)*Log[a + b*x^2 + c*x^4])/(2*c^2))/(b^2 - 4*a*c))/2
```

Defintions of rubi rules used

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

method	result	size
default	$\frac{\frac{b(3ac-b^2)x^2}{c^2(4ac-b^2)}+\frac{a(2ac-b^2)}{(4ac-b^2)c^2}}{2c\,x^4+2b\,x^2+2a}+\frac{\frac{(4ac-b^2)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2\left(-ab-\frac{(4ac-b^2)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c(4ac-b^2)}$	179
risch	Expression too large to display	1017

input

```
int(x^7/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/2*(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x^2+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x
^4+b*x^2+a)+1/2/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^4+b*x^2+a)+2*(-a*b
-1/2*(4*a*c-b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(120) = 240.

Time = 0.09 (sec) , antiderivative size = 663, normalized size of antiderivative = 5.02

$$\int \frac{x^7}{(a+bx^2+cx^4)^2}\,dx$$
$$= \left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\,(b^5-7\,ab^3c+12\,a^2bc^2)x^2+((b^3c-6\,abc^2)x^4+ab^3-6\,a^2bc+(b^4-6\,a\cdots}{4\,(ab^4c^2-8\,a^2b\cdots}\right.$$

input

```
integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*
c^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c
)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (a*b^4 - 8*a^2*b^2*c + 1
6*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a
^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3
 + 16*a^2*b*c^4)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 -
 7*a*b^3*c + 12*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^
2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sq
rt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)
*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^
3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)
*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(117) = 234$.

Time = 11.60 (sec) , antiderivative size = 745, normalized size of antiderivative = 5.64

$$\int \frac{x^7}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x**7/(c*x**4+b*x**2+a)**2,x)
```

output
```
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3
*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a*
*2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c
**2*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48
*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(
-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (b*sq
rt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(b*sq
rt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b*
*2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(b*sqrt(-
(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**
2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*
b**2 + x**2*(3*a*b*c - b**3))/(8*a**2*c**3 - 2*a*b**2*c**2 + x**4*(8*a*c**
4 - 2*b**2*c**3) + x**2*(8*a*b*c**3 - 2*b**3*c**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.15

$$\int \frac{x^7}{(a+bx^2+cx^4)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} - \frac{b^2cx^4-4\,ac^2x^4-b^3x^2+2\,abcx^2-ab^2}{4\,(cx^4+bx^2+a)(b^2c^2-4\,ac^3)} + \frac{\log\left(cx^4+bx^2+a\right)}{4\,c^2}$$

input
```
integrate(x^7/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 -
4*a*c^3)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 - b^3*x^2 + 2*
a*b*c*x^2 - a*b^2)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/4*log(c*x
^4 + b*x^2 + a)/c^2
```

Mupad [B] (verification not implemented)

Time = 18.81 (sec) , antiderivative size = 1336, normalized size of antiderivative = 10.12

$$\int \frac{x^7}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^7/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((a*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)) + (b*x^2*(3*a*c - b^2))/(2*c^2*(4
*a*c - b^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a
^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*
b^4*c^3 - 192*a^2*b^2*c^4)) + (b*atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^
2*(4*a*c - b^2)^3)*(x^2*(((b*((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2)
 + ((8*b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*
b^4*c))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 1
92*a^2*b^2*c^4)))*(6*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(3/2)) + (b*(8*b^3*c
^4 - 32*a*b*c^5)*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*
a*b^4*c))/(16*c^2*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4
*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - (b*((b^3
- 5*a*b*c)/(4*a*c^3 - b^2*c^2) + (((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2
*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 -
24*a*b^4*c))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^
3 - 192*a^2*b^2*c^4)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)
)/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (b^2*((
b^3*c^4)/2 - 2*a*b*c^5)*(6*a*c - b^2)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3 - b
^2*c^2))))/(2*a*(4*a*c - b^2)^(3/2))) - ((b*(6*a*c - b^2)*(8*a + (8*a*c^2*
(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(256*a^3*c^5 - 4*b^6*
c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(8*c^2*(4*a*c - b^2)^(3/2)) + (...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1427, normalized size of antiderivative = 10.81

$$\int \frac{x^7}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^7/(c*x^4+b*x^2+a)^2,x)
```

output
```
(12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c -
 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3 + 12
*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c*x**2
 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**2
*x**4 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4
*x**2 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3
*c*x**4 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a
**2*b*c - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
b**3 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*
*2*c*x**2 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + ...
```

3.791 $\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx = \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2a\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*a*arctanh((2*c*x^2+b)/(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx = \frac{b^2x^2+a(b-2cx^2)}{2c(-b^2+4ac)(a+bx^2+cx^4)} + \frac{2a\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[x^5/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (2*
a*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1434, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^4}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{1153}$$

$$\frac{1}{2}\left(\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2a\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{4a\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}+\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{4a\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[x^5/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (4*a*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

method	result
default	$\frac{-\frac{\left(2ac-b^2\right)x^2}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{2c\,x^4+2b\,x^2+2a}+\frac{2a\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ac-b^2\right)x^2}{2c\left(4ac-b^2\right)}+\frac{ab}{2c\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{a\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^2+8c\,a^2-2b^2a\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{a\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)x^2-8c\,a^2+\right.}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$

input
```
int(x^5/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(2*a*c-b^2)/c/(4*a*c-b^2)*x^2+a*b/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2*a
/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(72) = 144$.

Time = 0.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 5.22

$$\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx$$

$$= \left[-\frac{ab^3 - 4\,a^2bc + (b^4 - 6\,ab^2c + 8\,a^2c^2)x^2 + 2\,(ac^2x^4 + abcx^2 + a^2c)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(}{cx^4+bx^2-}\right.}{2\,(ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^4 + (b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x} \right.$$

$$-\frac{ab^3 - 4\,a^2bc + (b^4 - 6\,ab^2c + 8\,a^2c^2)x^2 - 4\,(ac^2x^4 + abcx^2 + a^2c)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+}}{b^2-4\,ac}\right.}{2\,(ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^4 + (b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x^2)}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 + 2*(a*c^2*x^
4 + a*b*c*x^2 + a^2*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2
- 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4*c
- 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 +
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(a*b^3 - 4*a^2*b*c + (b^4
- 6*a*b^2*c + 8*a^2*c^2)*x^2 - 4*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(-b^2
 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4*
c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4
+ (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(70) = 140.

Time = 0.74 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.62

$$\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx =$$

$$-a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab}{2ac}\right)$$

$$+a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab}{2ac}\right)$$

$$+\frac{ab+x^2(-2ac+b^2)}{8a^2c^2-2ab^2c+x^4\cdot(8ac^3-2b^2c^2)+x^2\cdot(8abc^2-2b^3c)}$$

input
```
integrate(x**5/(c*x**4+b*x**2+a)**2,x)
```

output
```
-a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*b**4*sqrt(-1/(4*a*
c - b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (1
6*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b*
*2)**3) + a*b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) + (a*b + x**2*
(-2*a*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2)
 + x**2*(8*a*b*c**2 - 2*b**3*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx = -\frac{2\,a\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{b^2x^2-2\,acx^2+ab}{2\,(cx^4+bx^2+a)(b^2c-4\,ac^2)}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-2*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4
*a*c)) - 1/2*(b^2*x^2 - 2*a*c*x^2 + a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a
*c^2))
```

Mupad [B] (verification not implemented)

Time = 17.86 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.40

$$\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx = -\frac{\frac{x^2\,\left(2\,a\,c-b^2\right)}{2\,c\,\left(4\,a\,c-b^2\right)}-\frac{a\,b}{2\,c\,\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a} - \frac{2\,a\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{x^2\,\left(\frac{4\,a\,c^2}{{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{4\,a\,\left(b^3\,c^2-4\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{13/2}}\right)\,{\left(4\,a\,c-b^2\right)}^4}{8\,a^2\,c^2}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(x^5/(a + b*x^2 + c*x^4)^2,x)
```

output
```
- ((x^2*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b)/(2*c*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) - (2*a*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(
(4*a*c^2)/(4*a*c - b^2)^(7/2) + (4*a*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c)
)/(4*a*c - b^2)^(13/2))*(4*a*c - b^2)^4)/(8*a^2*c^2)))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 479, normalized size of antiderivative = 6.14

$$\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{-4\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)a^2b-4\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}}}{\sqrt{2}}\right.}{}$$

input
```
int(x^5/(c*x^4+b*x^2+a)^2,x)
```

output
```
( - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b -
 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*x**2
 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*x**
4 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b -
 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*x**2
 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*x**
4 + 8*a**3*c - 2*a**2*b**2 + 8*a**2*c**2*x**4 - 6*a*b**2*c*x**4 + b**4*x**
4)/(2*b*(16*a**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c**2*x**2 + 16*a**2*c**3
*x**4 + a*b**4 - 8*a*b**3*c*x**2 - 8*a*b**2*c**2*x**4 + b**5*x**2 + b**4*c
*x**4))
```

3.792 $\int \frac{x^3}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 75

$$\int \frac{x^3}{(a+bx^2+cx^4)^2} dx = \frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
1/2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*arctanh((2*c*x^2+b)/(-4*a*c
+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{(a+bx^2+cx^4)^2} dx = \frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{b\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[x^3/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(b + 2*c*x
^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1434, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{1159}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}+\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2a+bx^2}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2b\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\right)$$

input
```
Int[x^3/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*a + b*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*b*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

method	result
default	$\frac{-b\,x^{2}-2a}{2\left(4ac-b^{2}\right)\left(c\,x^{4}+b\,x^{2}+a\right)}-\frac{b\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{-\frac{b\,x^{2}}{2\left(4ac-b^{2}\right)}-\frac{a}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{b\ln\left(\left(-\left(-4ac+b^{2}\right)^{\frac{3}{2}}+4abc-b^{3}\right)x^{2}+8c\,a^{2}-2b^{2}a\right)}{2\left(-4ac+b^{2}\right)^{\frac{3}{2}}}-\frac{b\ln\left(\left(-\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}\right)x^{2}-8c\,a^{2}+2b\right.}{2\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$

input
```
int(x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(69) = 138$.

Time = 0.08 (sec) , antiderivative size = 360, normalized size of antiderivative = 4.80

$$\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx = \left[\frac{2\,ab^2-8\,a^2c+(b^3-4\,abc)x^2-(bcx^4+b^2x^2+ab)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)}\right.$$

input
```
integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - (b*c*x^4 + b^2*x^2 + a*b)*
sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)
*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c
^2)*x^2), 1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - 2*(b*c*x^4 + b^2*
x^2 + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^
2 - 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16
*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(63) = 126$.

Time = 0.64 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.59

$$\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx = \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)}{2} - \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)}{2} + \frac{-2a-bx^2}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

input
```
integrate(x**3/(c*x**4+b*x**2+a)**2,x)
```

output
```
b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/(4*a*c -
b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c -
b**2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16
*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2
)**3) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 + (-2*a - b*x**
2)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b
**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx=\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}+\frac{bx^2+2\,a}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

input
```
integrate(x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) + 1/2*(b*x^2 + 2*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 17.66 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.37

$$\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx=\frac{b\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{x^2\,{\left(4\,a\,c-b^2\right)}^4\,\left(\frac{b^2\,c^2}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{b^2\,\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)}{2\,a\,{\left(4\,a\,c-b^2\right)}^{13/2}}\right)}{2\,b^2\,c^2}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{\frac{a}{4\,a\,c-b^2}+\frac{b\,x^2}{2\,\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}$$

input
```
int(x^3/(a + b*x^2 + c*x^4)^2,x)
```

output
```
(b*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((b^2*c
^2)/(a*(4*a*c - b^2)^(7/2)) + (b^2*(2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c)
)/(2*a*(4*a*c - b^2)^(13/2))))/(2*b^2*c^2)))/(4*a*c - b^2)^(3/2) - (a/(4*a
*c - b^2) + (b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 457, normalized size of antiderivative = 6.09

$$\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx = \frac{2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)ab+2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-}{\sqrt{2\sqrt{c}}}\right.}{}$$

input
```
int(x^3/(c*x^4+b*x^2+a)^2,x)
```

output
```
(2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b + 2*sqr
t(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*
sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*x**2 + 2*sqr
t(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*
sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c*x**4 + 2*sqrt
(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b + 2*sqrt(2*sqr
t(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a)
 - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*x**2 + 2*sqrt(2*sqr
t(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a)
 - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c*x**4 - 4*a**2*c + a*
b**2 + 4*a*c**2*x**4 - b**2*c*x**4)/(2*(16*a**3*c**2 - 8*a**2*b**2*c + 16*
a**2*b*c**2*x**2 + 16*a**2*c**3*x**4 + a*b**4 - 8*a*b**3*c*x**2 - 8*a*b**2
*c**2*x**4 + b**5*x**2 + b**4*c*x**4))
```

3.793 $\int \frac{x}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 74

$$\int \frac{x}{(a+bx^2+cx^4)^2} dx = -\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
-1/2*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*c*arctanh((2*c*x^2+b)/(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{x}{(a+bx^2+cx^4)^2} dx = -\frac{\frac{b+2cx^2}{a+bx^2+cx^4} + \frac{4c\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)}$$

input
```
Integrate[x/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/Sqrt[-
b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1432, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{2}\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)$$

input
```
Int[x/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(-((b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (4*c*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

method	result
default	$\frac{2c\,x^{2}+b}{2\left(4ac-b^{2}\right)\left(c\,x^{4}+b\,x^{2}+a\right)}+\frac{2c\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{\frac{c\,x^{2}}{4ac-b^{2}}+\frac{b}{8ac-2b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{c\ln\left(\left(\left(-4ac+b^{2}\right)^{\frac{3}{2}}+4abc-b^{3}\right)x^{2}+8c\,a^{2}-2b^{2}a\right)}{\left(-4ac+b^{2}\right)^{\frac{3}{2}}}-\frac{c\ln\left(\left(\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}\right)x^{2}-8c\,a^{2}+2b^{2}a\right)}{\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$

input
```
int(x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(68) = 136$.

Time = 0.08 (sec) , antiderivative size = 361, normalized size of antiderivative = 4.88

$$\int \frac{x}{(a+bx^2+cx^4)^2}\,dx$$
$$= \left[-\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)x^2+2\,(c^2x^4+bcx^2+ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,c}}{cx^4+bx^2+a}\right.}{2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)}\right.$$
$$\left.-\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)x^2-4\,(c^2x^4+bcx^2+ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)}\right]$$

input
```
integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + 2*(c^2*x^4 + b*c*x^2 + a*
c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 +
 b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3
*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*x^2), -1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - 4*(c^2*x^4 +
b*c*x^2 + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)
/(b^2 - 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(66) = 132$.

Time = 0.62 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.61

$$\int \frac{x}{(a+bx^2+cx^4)^2}\,dx =$$
$$-c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc}{2c^2}\right)$$
$$+c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc}{2c^2}\right)$$
$$+\frac{b+2cx^2}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

input
```
integrate(x/(c*x**4+b*x**2+a)**2,x)
```

output
```
-c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*c**3*sqrt(-1/(4*a*c - b
**2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(-1/(4*a*
c - b**2)**3) + b*c)/(2*c**2)) + c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (
16*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b
**2)**3) + b**4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c)/(2*c**2)) + (b + 2*c*x
**2)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2
*b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^2+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{x}{(a+bx^2+cx^4)^2}\,dx = -\frac{2\,c\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,cx^2+b}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

input
```
integrate(x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-2*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4
*a*c)) - 1/2*(2*c*x^2 + b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.32

$$\int \frac{x}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{b}{2\,(4\,a\,c-b^2)}+\frac{c\,x^2}{4\,a\,c-b^2}}{c\,x^4+b\,x^2+a} - \frac{2\,c\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}}-\frac{x^2\,(4\,a\,c-b^2)^4\left(\frac{4\,c^4}{a\,(4\,a\,c-b^2)^{7/2}}+\frac{4\,c^2\,(b^3\,c^2-4\,a\,b\,c^3)\,(b^3-4\,a\,b\,c)}{a\,(4\,a\,c-b^2)^{13/2}}\right)}{8\,c^4}\right)}{(4\,a\,c-b^2)^{3/2}}$$

input
```
int(x/(a + b*x^2 + c*x^4)^2,x)
```

output
```
(b/(2*(4*a*c - b^2)) + (c*x^2)/(4*a*c - b^2))/(a + b*x^2 + c*x^4) - (2*c*a
tan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((4*c^4)/(a
*(4*a*c - b^2)^(7/2)) + (4*c^2*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(a*(
4*a*c - b^2)^(13/2))))/(8*c^4)))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 477, normalized size of antiderivative = 6.45

$$\int \frac{x}{(a+bx^2+cx^4)^2}\,dx = \frac{-4\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)abc-4\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}}}{\sqrt{2}}\right.}{}$$

input
```
int(x/(c*x^4+b*x^2+a)^2,x)
```

output
```
( - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c -
4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c*x**2
 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c**2*x**
4 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c -
4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c*x**2
 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*c**2*x**
4 - 8*a**2*c**2 + 6*a*b**2*c - 8*a*c**3*x**4 - b**4 + 2*b**2*c**2*x**4)/(2
*b*(16*a**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c**2*x**2 + 16*a**2*c**3*x**4
 + a*b**4 - 8*a*b**3*c*x**2 - 8*a*b**2*c**2*x**4 + b**5*x**2 + b**4*c*x**4
))
```

3.794 $\int \frac{1}{x(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 122

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx^2+cx^4\right)}{4a^2}$$

output
```
1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*b*(-6*a*c+b^2)*
arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1
/4*ln(c*x^4+b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.70

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx$$
$$=\frac{\frac{2a(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)}+4\log(x)-\frac{\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(b^3-6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}}{4a^2}$$

input
```
Integrate[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*Log
[x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*L
og[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c
 - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1434, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx$$
$$\Big\downarrow \text{1434}$$
$$\frac{1}{2}\int \frac{1}{x^2\left(cx^4+bx^2+a\right)^2}dx^2$$
$$\Big\downarrow \text{1165}$$
$$\frac{1}{2}\left(\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{b^2+cx^2b-4ac}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{b^2+cx^2b-4ac}{x^2(cx^4+bx^2+a)}dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{b^2-4ac}{ax^2}+\frac{-c(b^2-4ac)x^2-b(b^2-5ac)}{a(cx^4+bx^2+a)}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x^2)\left(b^2-4ac\right)}{a}-\frac{(b^2-4ac)\log(a+bx^2+cx^4)}{2a}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

input
```
Int[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*(b^2
- 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) +
 ((b^2 - 4*a*c)*Log[x^2])/a - ((b^2 - 4*a*c)*Log[a + b*x^2 + c*x^4])/(2*a)
)/(a*(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.52

method	result
default	$-\frac{\frac{\frac{abc\,x^2}{4ac-b^2}-\frac{a\left(2ac-b^2\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(4c^2a-b^2c\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(5abc-b^3-\frac{\left(4c^2a-b^2c\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^2}+\frac{\ln(x)}{a^2}$
risch	$\frac{-\frac{bc\,x^2}{2a\left(4ac-b^2\right)}+\frac{2ac-b^2}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(64a^5c^3-48a^4b^2c^2+12a^3b^4c-b^6a^2\right)\_Z^2+\left(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6\right)\_Z\right.}\right.}{}$

input
```
int(1/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*((a*b*c/(4*a*c-b^2)*x^2-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a
)+1/(4*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^4+b*x^2+a)+2*(5*a*b*c-b^3-1/
2*(4*a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2))))+ln(x)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(112) = 224$.

Time = 0.14 (sec) , antiderivative size = 813, normalized size of antiderivative = 6.66

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2
+ ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2
)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2
 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*a
*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + 2*((b^3
*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2
 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a*b^4
 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a
^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*
a*b^3*c + 16*a^2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 +
 (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 1
6*a^4*b*c^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{\left(b^3-6\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}} + \frac{b^2cx^4-4\,ac^2x^4+b^3x^2-2\,abcx^2+3\,ab^2-8\,a^2c}{4\left(cx^4+bx^2+a\right)\left(a^2b^2-4\,a^3c\right)} - \frac{\log\left(cx^4+bx^2+a\right)}{4\,a^2} + \frac{\log\left(x^2\right)}{2\,a^2}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 -
4*a^3*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 + b^3*x^2 - 2*
a*b*c*x^2 + 3*a*b^2 - 8*a^2*c)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c)) -
 1/4*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

Mupad [B] (verification not implemented)

Time = 22.18 (sec) , antiderivative size = 5048, normalized size of antiderivative = 41.38

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^2 + c*x^4)^2),x)
```

output
```
log(x)/a^2 + ((2*a*c - b^2)/(2*a*(4*a*c - b^2)) - (b*c*x^2)/(2*a*(4*a*c -
b^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3*c^3
+ 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c
 + 192*a^4*b^2*c^2)) + (b*atan((x^2*(((((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^
3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*
(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 26
88*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)
*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2
))/(4*a^2*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 9
6*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7
*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a
^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*
c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*
c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^
2*c^2)) + (b*((6*a*b^5*c^4 + 80*a^3*b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*
a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^
3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*
(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 -...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1533, normalized size of antiderivative = 12.57

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/x/(c*x^4+b*x^2+a)^2,x)
```

output
```
(12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c -
 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3 + 12
*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c*x**2
 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**2
*x**4 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4
*x**2 - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3
*c*x**4 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a
**2*b*c - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
b**3 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*
*2*c*x**2 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + ...
```

3.795 $\int \frac{1}{x^3(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 162

$$\int \frac{1}{x^3(a+bx^2+cx^4)^2}\,dx = -\frac{b^2-3ac}{a^2(b^2-4ac)\,x^2}+\frac{b^2-2ac+bcx^2}{2a(b^2-4ac)\,x^2(a+bx^2+cx^4)} - \frac{(b^4-6ab^2c+6a^2c^2)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2b\log(x)}{a^3}+\frac{b\log(a+bx^2+cx^4)}{2a^3}$$

output
```
-(-3*a*c+b^2)/a^2/(-4*a*c+b^2)/x^2+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/
x^2/(c*x^4+b*x^2+a)-(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x^2+b)/(-4*a*c+
b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+1/2*b*ln(c*x^4+b*x^2+a)/a
^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.53

$$\int \frac{1}{x^3 (a+bx^2+cx^4)^2} dx$$
$$= \frac{-\frac{a}{x^2} - \frac{a(b^3-3abc+b^2cx^2-2ac^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)} - 4b\log(x) + \frac{\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{\left(-b^4 \right.}{}}{2a^3}$$

input
```
Integrate[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

output
```
(-(a/x^2) - (a*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) - 4*b*Log[x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqr
t[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*
c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^
2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/(b^2 - 4*a*c)^(3/2))/(2*a^3)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1434, 1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2+cx^4)^2} dx$$
$$\Big\downarrow \ 1434$$
$$\frac{1}{2}\int \frac{1}{x^4 (cx^4+bx^2+a)^2} dx^2$$
$$\Big\downarrow \ 1165$$

$$\frac{1}{2}\left(\frac{-2ac+b^2+bcx^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{2\left(b^2+cx^2b-3ac\right)}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{2\int \frac{b^2+cx^2b-3ac}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{2}\left(\frac{2\int \left(\frac{b^2-3ac}{ax^4}+\frac{b^4-5acb^2+c\left(b^2-4ac\right)x^2b+3a^2c^2}{a^2\left(cx^4+bx^2+a\right)}+\frac{4abc-b^3}{a^2x^2}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2\left(-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log\left(x^2\right)\left(b^2-4ac\right)}{a^2}+\frac{b\left(b^2-4ac\right)\log\left(a+bx^2+cx^4\right)}{2a^2}-\frac{b^2-3ac}{ax^2}\right)}{a\left(b^2-4ac\right)}+\frac{-2ac}{ax^2\left(b^2-4a\right.}\right.$$

input
```
Int[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) + (2*(-
((b^2 - 3*a*c)/(a*x^2)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)*Log[x^
2])/a^2 + (b*(b^2 - 4*a*c)*Log[a + b*x^2 + c*x^4])/(2*a^2)))/(a*(b^2 - 4*a
*c)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.31

method	result
default	$-\frac{1}{2a^2x^2}-\frac{2b\ln(x)}{a^3}-\frac{\frac{\frac{ac(2ac-b^2)x^2}{4ac-b^2}+\frac{ab(3ac-b^2)}{4ac-b^2}}{cx^4+bx^2+a}+\frac{\frac{(-4abc^2+b^3c)\ln(cx^4+bx^2+a)}{c}+\frac{4\left(3a^2c^2-5ab^2c+b^4-\frac{(-4abc^2+b^3c)b}{2c}\right)\text{arcta}}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^3}$
risch	$\frac{-\frac{c(3ac-b^2)x^4}{a^2(4ac-b^2)}-\frac{b(7ac-2b^2)x^2}{2(4ac-b^2)a^2}-\frac{1}{2a}}{x^2(cx^4+bx^2+a)}-\frac{2b\ln(x)}{a^3}+\left(\sum_{\_R=\text{RootOf}((64a^6c^3-48b^2a^5c^2+12b^4a^4c-b^6a^3)\_Z^2+(-64a^3bc^3+48a^2b^3}\right.$

input
```
int(1/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/x^2-2*b*ln(x)/a^3-1/2/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x^2+a*b*(
3*a*c-b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3
*c)/c*ln(c*x^4+b*x^2+a)+2*(3*a^2*c^2-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*
b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. 2(154) = 308.

Time = 0.19 (sec) , antiderivative size = 1007, normalized size of antiderivative = 6.22

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 1
2*a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + ((b^4*c - 6
*a*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4
 - 6*a^2*b^2*c + 6*a^3*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*
x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
- ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^
2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 +
 a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*
a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3
*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5
*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/2*(a^2*b^4 - 8
*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (
2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a
^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c +
 6*a^3*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*
c)/(b^2 - 4*a*c)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a
*b^4*c + 16*a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*l
og(c*x^4 + b*x^2 + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6
 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*
x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)^2} dx = \frac{(b^4 - 6\,ab^2c + 6\,a^2c^2) \arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2 - 4\,a^4c)\sqrt{-b^2 + 4\,ac}} - \frac{2\,b^2cx^4 - 6\,ac^2x^4 + 2\,b^3x^2 - 7\,abcx^2 + ab^2 - 4\,a^2c}{2\,(cx^6 + bx^4 + ax^2)(a^2b^2 - 4\,a^3c)} + \frac{b\log(cx^4 + bx^2 + a)}{2\,a^3} - \frac{b\log(x^2)}{a^3}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a
^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^2*c*x^4 - 6*a*c^2*x^4 + 2
*b^3*x^2 - 7*a*b*c*x^2 + a*b^2 - 4*a^2*c)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^
2 - 4*a^3*c)) + 1/2*b*log(c*x^4 + b*x^2 + a)/a^3 - b*log(x^2)/a^3
```

Mupad [B] (verification not implemented)

Time = 22.49 (sec) , antiderivative size = 5491, normalized size of antiderivative = 33.90

$$\int \frac{1}{x^3 \left(a+bx^2+cx^4\right)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

output
```
(log(a + b*x^2 + c*x^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)
)/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (1/(2*a) -
(x^2*(2*b^3 - 7*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(3*a*c - b^2))/(a^2
*(4*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) - (2*b*log(x))/a^3 + (atan(((2*a^
9*b^6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4
*c*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 3*a
^3*c^3 + 36*a^2*b^2*c^2 - 21*a*b^4*c)*((4*(2*b^5*c^4 - 12*a*b^3*c^5 + 18*a
^2*b*c^6))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) + (((4*(9*a^5*c^6 - 4*a^2*
b^6*c^3 + 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*
b^2*c) - (((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*
c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c
^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(
(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2
*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*
c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2)) + (((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c
^3 - 46*a^6*b^3*c^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c
^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2
- 12*a*b^5*c))/((a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2043, normalized size of antiderivative = 12.61

$$\int \frac{1}{x^3 \left(a+bx^2+cx^4\right)^2} \, dx = \text{Too large to display}$$

input
```
int(1/x^3/(c*x^4+b*x^2+a)^2,x)
```

output
```
(12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**
2*x**2 - 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*2*b**3*c*x**2 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a**2*b**2*c**2*x**4 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*
sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a**2*b*c**3*x**6 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*s
qrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*a*b**5*x**2 - 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(
2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + b))*a*b**4*c*x**4 - 12*sqrt(2*sqrt(c)*sqrt(a) + b)*
sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c**2*x**6 + 2*sqrt(2*sqrt(c)*sqrt(a
) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**6*x**4 + 2*sqrt(2*sqrt(c)*sqrt(a
) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**5*c*x**6 + 12*sqrt(2*sqrt(c)*sqr
t(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)...
```

3.796 $\int \frac{x^8}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 303

$$\int \frac{x^8}{(a+bx^2+cx^4)^2} dx = \frac{x}{c^2} + \frac{x(a(b^2-2ac)+b(b^2-3ac)x^2)}{2c^2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3-13abc-\frac{3b^4-19ab^2c+20a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(3b^3-13abc+\frac{3b^4-19ab^2c+20a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
x/c^2+1/2*x*(a*(-2*a*c+b^2)+b*(-3*a*c+b^2)*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*
x^2+a)-1/4*(3*b^3-13*a*b*c-(20*a^2*c^2-19*a*b^2*c+3*b^4)/(-4*a*c+b^2)^(1/2
))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/
(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*(3*b^3-13*a*b*c+(20*a^2*c^2-
19*a*b^2*c+3*b^4)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+
b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.08

$$\int \frac{x^8}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{4\sqrt{c}x - \frac{2\sqrt{c}x(2a^2c-b^3x^2-ab(b-3cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\left(-3b^4+19ab^2c-20a^2c^2+3b^3\sqrt{b^2-4ac}-13abc\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}(}{}}{4c^{5/2}}$$

input
```
Integrate[x^8/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(4*Sqrt[c]*x - (2*Sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2)))/((b^2
 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4 + 19*a*b^2*c - 20*a^2*c^
2 + 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*Sqrt[
b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
))/(4*c^(5/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1440, 1602, 27, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^2+cx^4)^2}\,dx$$
$$\Big\downarrow \text{1440}$$
$$\frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{x^4(3bx^2+10a)}{cx^4+bx^2+a}dx}{2(b^2-4ac)}$$

$\big\downarrow$ 1602

$$\frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{bx^3}{c}-\frac{\int\frac{3x^2\left(\left(3b^2-10ac\right)x^2+3ab\right)}{cx^4+bx^2+a}dx}{3c}}{2(b^2-4ac)}$$

$\big\downarrow$ 27

$$\frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{bx^3}{c}-\frac{\int\frac{x^2\left(\left(3b^2-10ac\right)x^2+3ab\right)}{cx^4+bx^2+a}dx}{c}}{2(b^2-4ac)}$$

$\big\downarrow$ 1602

$$\frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{bx^3}{c}-\frac{\frac{x\left(3b^2-10ac\right)}{c}-\frac{\int\frac{b\left(3b^2-13ac\right)x^2+a\left(3b^2-10ac\right)}{cx^4+bx^2+a}dx}{c}}{c}}{2(b^2-4ac)}$$

$\big\downarrow$ 1480

$$\frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{bx^3}{c}-\frac{\frac{x\left(3b^2-10ac\right)}{c}-\frac{\frac{1}{2}\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}}{c}}{c}}{2(b^2-4ac)}$$

$\big\downarrow$ 218

$$\frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{bx^3}{c}-\frac{\frac{x\left(3b^2-10ac\right)}{c}-\frac{\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{c}}{2(b^2-4ac)}$$

input
```
Int[x^8/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b*x^3)/c - (
((3*b^2 - 10*a*c)*x)/c - (((3*b^3 - 13*a*b*c - (3*b^4 - 19*a*b^2*c + 20*a^
2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^3 - 13*a*b*
c + (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2
 - 4*a*c]]))/c)/c)/(2*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.57

method	result
risch	$\frac{x}{c^2}+\frac{\frac{b(3ac-b^2)x^3}{8ac-2b^2}+\frac{a(2ac-b^2)x}{8ac-2b^2}}{c^2(cx^4+bx^2+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4c+\_Z^2b+a)}\frac{\left(-\frac{b(13ac-3b^2)\_R^2}{4ac-b^2}-\frac{a(10ac-3b^2)}{4ac-b^2}\right)\ln(x-\_R)}{2\_R^3c+\_Rb}}{4c^2}$
default	$\frac{x}{c^2}-\frac{-\frac{b(3ac-b^2)x^3}{2(4ac-b^2)}-\frac{a(2ac-b^2)x}{2(4ac-b^2)}}{cx^4+bx^2+a}+\frac{2c\left(\frac{\left(13abc\sqrt{-4ac+b^2}-3b^3\sqrt{-4ac+b^2}-20a^2c^2+19ab^2c-3b^4\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\cdots\right.}{4ac-b^2 \cdots}\cdot\frac{1}{c^2}$

input
```
int(x^8/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2+(1/2*b*(3*a*c-b^2)/(4*a*c-b^2)*x^3+1/2*a*(2*a*c-b^2)/(4*a*c-b^2)*x)/
c^2/(c*x^4+b*x^2+a)+1/4/c^2*sum((-b*(13*a*c-3*b^2)/(4*a*c-b^2)*_R^2-a*(10*
a*c-3*b^2)/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2856 vs. $2(261) = 522$.

Time = 0.30 (sec) , antiderivative size = 2856, normalized size of antiderivative = 9.43

$$\int \frac{x^8}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + sqrt(1/2)*(a*b^2
*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sq
rt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12
*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 305
1*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
+ 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2
500*a^5*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 -
 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b
^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8
- 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^
10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^
2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 25
50*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
- 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)))
- sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 -
4*a*b*c^3)*x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c
^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 -
 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^8}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (
b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/2*integrate(-(3*a*
b^2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2
- 4*a*c^3) + x/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3335 vs. $2(261) = 522$.

Time $= 0.66$ (sec) , antiderivative size $= 3335$, normalized size of antiderivative $= 11.01$

$$\int \frac{x^8}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*x^3 - 3*a*b*c*x^3 + a*b^2*x - 2*a^2*c*x)/((c*x^4 + b*x^2 + a)*(b^
2*c^2 - 4*a*c^3)) + x/c^2 + 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c
^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c
)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9
- (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c...
```

Mupad [B] (verification not implemented)

Time = 19.48 (sec) , antiderivative size = 7599, normalized size of antiderivative = 25.08

$$\int \frac{x^8}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^8/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((b*x^3*(3*a*c - b^2))/(2*(4*a*c - b^2)) + (a*x*(2*a*c - b^2))/(2*(4*a*c -
 b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7 + 48*a*b^8*
c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c
^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 + 9*b^4*(-(4
*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*
c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9
)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^
6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3
840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 -
 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^
2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25
*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*
c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (
x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))
/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^
2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 302
40*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11...
```

Reduce [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 3084, normalized size of antiderivative = 10.18

$$\int \frac{x^8}{(a+bx^2+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(x^8/(c*x^4+b*x^2+a)^2,x)
```

output

```
(32*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**2 - 6*sqrt(a)*sqrt(2
*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqr
t(2*sqrt(c)*sqrt(a) + b))*a*b**3*c + 32*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a*b**2*c**2*x**2 + 32*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*
*3*x**4 - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*c*x**2 - 6*sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c
)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c**2*x**4 + 40*sqrt(c)*sqrt(2*sqrt(
c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sq
rt(c)*sqrt(a) + b))*a**3*c**2 - 38*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a**2*b**2*c + 40*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**2*x**2
 + 40*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b
) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**3*x**4 + 6*sqrt(c)*s
qrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x
)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4 - 38*sqrt(c)*sqrt(2*sqrt(c)*sqrt(...
```

3.797 $\int \frac{x^6}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 274

$$\int \frac{x^6}{(a+bx^2+cx^4)^2} dx = -\frac{x(ab+(b^2-2ac)x^2)}{2c(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(6a-\frac{b^2}{c}+\frac{b(b^2-8ac)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(6a-\frac{b^2}{c}-\frac{b(b^2-8ac)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
-1/2*x*(a*b+(-2*a*c+b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*(6*a-b^2/
c+b*(-8*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2)-1/4*(6*a-b^2/c-b*(-8*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(
1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.03

$$\int \frac{x^6}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{-\frac{2\sqrt{c}x(b^2x^2+a(b-2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^3+8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^3-8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-\right.}{(b^2-4ac)^{3/2}\sqrt{b+}}}{4c^{3/2}}$$

input
```
Integrate[x^6/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((-2*Sqrt[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*
x^4)) + (Sqrt[2]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 8*a*b*c + b^2
*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))/(4*c^(3/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1440, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{1440}$$

$$\frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{x^2(bx^2+6a)}{cx^4+bx^2+a}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \text{1602}$$

$$\frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{bx}{c}-\frac{\int\frac{(b^2-6ac)x^2+ab}{cx^4+bx^2+a}dx}{c}}{2(b^2-4ac)}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{bx}{c}-\frac{\frac{1}{2}\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}}{2(b^2-4ac)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{bx}{c}-\frac{\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{2(b^2-4ac)}$$

input

```
Int[x^6/(a + b*x^2 + c*x^4)^2,x]
```

output

```
(x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b*x)/c - (((
b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt
[b^2 - 4*a*c]]))/c)/(2*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.55

method	result
risch	$\frac{-\frac{(2ac-b^2)x^3}{2c(4ac-b^2)}+\frac{abx}{2c(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4c+\_Z^2b+a)}\frac{\left(\frac{(6ac-b^2)\_R^2}{4ac-b^2}-\frac{ab}{4ac-b^2}\right)\ln(x-\_R)}{2\_R^3c+\_Rb}}{4c}$
default	$\frac{-\frac{(2ac-b^2)x^3}{2c(4ac-b^2)}+\frac{abx}{2c(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\frac{\left(6ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}+8abc-b^3\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{(b+\sqrt{-4ac+b^2})c}}\right)}{4c\sqrt{-4ac+b^2}\sqrt{(b+\sqrt{-4ac+b^2})c}}-\frac{\left(6ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+\cdots}\right.}{4c\sqrt{-\cdots}}}{4ac-b^2}$

input
```
int(x^6/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(2*a*c-b^2)/c/(4*a*c-b^2)*x^3+1/2*a*b/c/(4*a*c-b^2)*x)/(c*x^4+b*x^2+
a)+1/4/c*sum(((6*a*c-b^2)/(4*a*c-b^2)*_R^2-a*b/(4*a*c-b^2))/(2*_R^3*c+_R*b
)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2257 vs. 2(232) = 464.

Time = 0.15 (sec) , antiderivative size = 2257, normalized size of antiderivative = 8.24

$$\int \frac{x^6}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(b^2 - 2*a*c)*x^3 + 2*a*b*x - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 +
 a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*lo
g((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5
*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^
4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15
*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^
3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a
^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a
^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^
3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 -
12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^
2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 -
 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c
+ 324*a^3*c^2)*x - 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*
a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 +
768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c...
```

Sympy [A] (verification not implemented)

Time = 20.55 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.38

$$\int \frac{x^6}{(a+bx^2+cx^4)^2}\,dx = \frac{abx+x^3(-2ac+b^2)}{8a^2c^2-2ab^2c+x^4\cdot(8ac^3-2b^2c^2)+x^2\cdot(8abc^2-2b^3c)}$$
$$+\mathrm{RootSum}\Big(t^4\cdot(1048576a^6c^9-1572864a^5b^2c^8+983040a^4b^4c^7-327680a^3b^6c^6+61440a^2b^8c^5-6144a$$

input
```
integrate(x**6/(c*x**4+b*x**2+a)**2,x)
```

output
```
(a*b*x + x**3*(-2*a*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3
- 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c)) + RootSum(_t**4*(1048576*a*
*6*c**9 - 1572864*a**5*b**2*c**8 + 983040*a**4*b**4*c**7 - 327680*a**3*b**
6*c**6 + 61440*a**2*b**8*c**5 - 6144*a*b**10*c**4 + 256*b**12*c**3) + _t**
2*(-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608
*a**2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**5*c**2 - 360*a**4*b**
2*c + 25*a**3*b**4, Lambda(_t, _t*log(x + (49152*_t**3*a**4*c**7 - 40960*_
t**3*a**3*b**2*c**6 + 12288*_t**3*a**2*b**4*c**5 - 1536*_t**3*a*b**6*c**4
+ 64*_t**3*b**8*c**3 - 1728*_t*a**3*b*c**3 + 656*_t*a**2*b**3*c**2 - 88*_t
*a*b**5*c + 4*_t*b**7)/(324*a**3*c**2 - 81*a**2*b**2*c + 5*a*b**4))))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2+cx^4)^2}\,dx=\int \frac{x^6}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^
2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-((b^2 - 6*a*c)*x^2 + a*b
)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2736 vs. $2(232) = 464$.

Time = 0.67 (sec) , antiderivative size = 2736, normalized size of antiderivative = 9.99

$$\int \frac{x^6}{(a+bx^2+cx^4)^2}\,dx=\text{Too large to display}$$

input
```
integrate(x^6/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*x^3 - 2*a*c*x^3 + a*b*x)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
 - 1/16*(2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*
b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b
^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3...
```

Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 6293, normalized size of antiderivative = 22.97

$$\int \frac{x^6}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(x^6/(a + b*x^2 + c*x^4)^2,x)
```

output
```
- ((x^3*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x)/(2*c*(4*a*c - b^2)))/
(a + b*x^2 + c*x^4) - atan(((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5
*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*
a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(
4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*
a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1
/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3
 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 -
 6144*a^5*b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*
b^4*c))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b
^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*
a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c
^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*
a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i - (((16*a*b^7*c^2 - 1024*a^4*b*
c^5 - 192*a^2*b^5*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4
*c^2 + 48*a^2*b^2*c^3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3...
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 2411, normalized size of antiderivative = 8.80

$$\int \frac{x^6}{(a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(x^6/(c*x^4+b*x^2+a)^2,x)
```

output
```
( - 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c**2 + 2*sqrt(a)*sqrt(
2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c - 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a*b*c**2*x**2 - 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**3*x
**4 + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c*x**2 + 2*sqrt(a)*sq
rt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)
/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c**2*x**4 + 16*sqrt(c)*sqrt(2*sqrt(c)*s
qrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*a**2*b*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**
3 + 16*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c*x**2 + 16*sqrt(c)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**2*x**4 - 2*sqrt(c)*sqrt(2*sqrt(c)*s
qrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*b**4*x**2 - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan(...
```

3.798 $\int \frac{x^4}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 237

$$\int \frac{x^4}{(a+bx^2+cx^4)^2} dx = \frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
1/2*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*(b-(4*a*c+b^2)/(-4*a*c+
b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)
/c^(1/2)/(-4*a*c+b^2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*(b^2+4*a*c+b*(-4*a*
c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/
2)/c^(1/2)/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.99

$$\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{2(2ax+bx^3)}{(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

input
```
Integrate[x^4/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-b^2
- 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]]))/4
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1440, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{1440}$$

$$\frac{x(2a+bx^2)}{2(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{2a-bx^2}{cx^4+bx^2+a}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{1}{2}\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2(b^2-4ac)}$$

$$\Big\downarrow \text{218}$$

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4ac+b^2}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{2(b^2-4ac)}$$

input
```
Int[x^4/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (-(((b - (b^2 +
4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - ((b + (b^2 + 4*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.52

method	result
risch	$\frac{-\frac{b x^3}{2(4ac-b^2)}-\frac{a x}{4ac-b^2}}{c x^4+b x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4 c+\_Z^2 b+a)}\frac{\left(-\frac{b \_R^2}{4ac-b^2}+\frac{2a}{4ac-b^2}\right)\ln(x-\_R)}{2\_R^3 c+\_R b}\right)}{4}$
default	$\frac{-\frac{b x^3}{2(4ac-b^2)}-\frac{a x}{4ac-b^2}}{c x^4+b x^2+a}+\frac{2c\left(\frac{\left(-b\sqrt{-4ac+b^2}-4ac-b^2\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-b\sqrt{-4ac+b^2}+4ac+b^2\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\ }{\sqrt{(-b+}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right.}}\right.}{4ac-b^2}$

input
```
int(x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b/(4*a*c-b^2)*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4*sum((-b/(4*a*
c-b^2)*_R^2+2*a/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^
2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1668 vs. 2(193) = 386.

Time = 0.11 (sec) , antiderivative size = 1668, normalized size of antiderivative = 7.04

$$\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*
c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^
5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a
*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2
+ 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*
b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^
3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^
2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 -
 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(
-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sq
rt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^
4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4
+ a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (b^6*c -
12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3...
```

Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.25

$$\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx = \frac{-2ax-bx^3}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$
$$+\mathrm{RootSum}\Big(t^4\cdot\big(1048576a^6c^7-1572864a^5b^2c^6+983040a^4b^4c^5-327680a^3b^6c^4+61440a^2b^8c^3-6144a$$

input
```
integrate(x**4/(c*x**4+b*x**2+a)**2,x)
```

output
```
(-2*a*x - b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2
*(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7 - 1572864*a**5*b**
2*c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b**6*c**4 + 61440*a**2*b**8*c
**3 - 6144*a*b**10*c**2 + 256*b**12*c) + _t**2*(-12288*a**4*b*c**4 + 8192*
a**3*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b
**2*c + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_
t**3*a**2*b**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a*
*2*c**2 - 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^4}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2) + 1/2*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a
*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2132 vs. $2(193) = 386$.

Time = 0.53 (sec) , antiderivative size = 2132, normalized size of antiderivative = 9.00

$$\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*x^3 + 2*a*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*b^7*c^2
- 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^
4 - (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2
*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2 + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^...
```

Mupad [B] (verification not implemented)

Time = 19.66 (sec) , antiderivative size = 4973, normalized size of antiderivative = 20.98

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(x^4/(a + b*x^2 + c*x^4)^2,x)
```

output
```
- atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^
4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c -
b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(3
2*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*
c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5
*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 51
2*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^
3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(
b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-
(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^
3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280
*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i - (((2048*a
^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a
^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b
^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*
a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^
4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b
*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c -
b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)...
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1795, normalized size of antiderivative = 7.57

$$\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^4/(c*x^4+b*x^2+a)^2,x)
```

output
```
(8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c + 8*sqrt(a)*sqrt(2*sqrt(c
)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqr
t(c)*sqrt(a) + b))*b**2*c*x**2 + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*b*c**2*x**4 - 8*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*
sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c - 2*sqrt(c
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c
)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2 - 8*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a
) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*a*b*c*x**2 - 8*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2*x
**4 - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*x**2 - 2*sqrt(c)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*b**2*c*x**4 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a)
 + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + b))*a*b*c - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c*x**2 -
8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)...
```

3.799 $\int \frac{x^2}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 221

$$\int \frac{x^2}{(a+bx^2+cx^4)^2} dx = -\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/2*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*c^(1/2)*(2*b-(-4*a*c+b
^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/
(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*c^(1/2)*(2*b+(-4*a*c+b
^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/
(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx = \frac{-bx-2cx^3}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[x^2/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(-2*b
+ Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(
2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1439, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \text{1439}$$

$$\frac{\int \frac{b-2cx^2}{cx^4+bx^2+a}dx}{2(b^2-4ac)} - \frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{-c\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-c\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2\left(b^2-4ac\right)}-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{\sqrt{2}\sqrt{c}\left(1-\frac{2b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{2b}{\sqrt{b^2-4ac}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2\left(b^2-4ac\right)}-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[x^2/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/2*(x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (-((Sqrt[2]*S
qrt[c]*(1 - (2*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(1 + (
2*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.55

method	result
risch	$\frac{\frac{c\,x^3}{4ac-b^2}+\frac{bx}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4c+\_Z^2b+a\right)}\frac{\left(\frac{2c\,\_R^2}{4ac-b^2}-\frac{b}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2\_R^3c+\_Rb}\right)}{4}$
default	$16c^2\left(\frac{\frac{\sqrt{-4ac+b^2}\,x}{8c\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}+\frac{\left(b+\frac{\sqrt{-4ac+b^2}}{2}\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{4c(4ac-b^2)\sqrt{-4ac+b^2}}+\frac{\frac{\sqrt{-4ac+b^2}\,x}{8c\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}-\frac{\left(-b+\frac{\sqrt{-4ac+b^2}}{2}\cdots\right.}{4\cdots}}{4c(4ac-b^2)\sqrt{-\cdots}}\right.$

input
```
int(x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(c/(4*a*c-b^2)*x^3+1/2*b/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4*sum((2*c/(4*a*
c-b^2)*_R^2-b/(4*a*c-b^2))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*
b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1680 vs. $2(180) = 360$.

Time = 0.11 (sec) , antiderivative size = 1680, normalized size of antiderivative = 7.60

$$\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*c*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2
*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c +
4*a*c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^
2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c +
 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - s
qrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*s
qrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3
)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*
a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*
sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4
*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 -
 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + sqrt(1/2)*((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*
b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6...
```

Sympy [A] (verification not implemented)

Time = 7.52 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.35

$$\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx = \frac{bx+2cx^3}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$
$$+\text{RootSum}\Big(t^4\cdot(1048576a^7c^6-1572864a^6b^2c^5+983040a^5b^4c^4-327680a^4b^6c^3+61440a^3b^8c^2-6144a$$

input
```
integrate(x**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
(b*x + 2*c*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*
(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**7*c**6 - 1572864*a**6*b**2
*c**5 + 983040*a**5*b**4*c**4 - 327680*a**4*b**6*c**3 + 61440*a**3*b**8*c*
*2 - 6144*a**2*b**10*c + 256*a*b**12) + _t**2*(-12288*a**4*b*c**4 + 8192*a
**3*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**2*c**3 + 24*a*b**2*
c**2 + 9*b**4*c, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4 - 8192*_t**3
*a**4*b**2*c**3 + 512*_t**3*a**2*b**6*c - 64*_t**3*a*b**8 - 128*_t*a**2*b*
c**2 - 16*_t*a*b**3*c - 4*_t*b**5)/(4*a*c**2 + 3*b**2*c))))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^2}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a
*b*c)*x^2) - 1/2*integrate((2*c*x^2 - b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*
a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1970 vs. $2(180) = 360$.

Time = 0.52 (sec) , antiderivative size = 1970, normalized size of antiderivative = 8.91

$$\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*c*x^3 + b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) + 1/8*(4*b^6*c^2
- 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^3 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*c)*a*b^2*c^3 - (2*b^2*c
^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*
(b^2 - 4*a*c)^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*
(b^2 - 4*a*c)*a*b*c^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/sqrt((b^3...
```

Mupad [B] (verification not implemented)

Time = 19.23 (sec) , antiderivative size = 4854, normalized size of antiderivative = 21.96

$$\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2/(a + b*x^2 + c*x^4)^2,x)
```

output
```
atan(((((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^
(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^1
2 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 38
40*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512
*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c
- b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/
(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^
6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2
*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 +
768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c
^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4
 - 6144*a^6*b^2*c^5)))^(1/2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*
c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) - (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 -
 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8
*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*
b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c
^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2
*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c ...
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1795, normalized size of antiderivative = 8.12

$$\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2/(c*x^4+b*x^2+a)^2,x)
```

output
```
( - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b
) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*c - 2*sqrt(a)*sqrt(2*sq
rt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2
*sqrt(c)*sqrt(a) + b))*a*b**2 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a*b*c*x**2 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c**2*x**4 - 2*sqr
t(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqr
t(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*x**2 - 2*sqrt(a)*sqrt(2*sqrt(c)*
sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*b**2*c*x**4 + 8*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a
**2*b + 8*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
 - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*x**2 + 8*sqrt(c)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c*x**4 + 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt
(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*s
qrt(a) + b))*a**2*c + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2 + 8
*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) ...
```

3.800 $\int \frac{1}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 252

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \frac{x(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(1/2)*(b^2-
12*a*c+b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*c^
(1/2)*(b^2-12*a*c-b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*
c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{2x(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

output
```
((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1405, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 1405$$

$$\frac{x(-2ac+b^2+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int -\frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{\frac{1}{2}c\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(a + b*x^2 + c*x^4)^(-2),x]
```

output
```
(x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((Sq
rt[c]*(b + (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
c]*(b - (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2
- 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

method	result
risch	$\frac{-\frac{bc\,x^3}{2a\left(4ac-b^2\right)}+\frac{\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4c+\_Z^2b+a\right)}\frac{\left(-\frac{bc\,\_R^2}{4ac-b^2}+\frac{6ac-b^2}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2\_R^3c+\_Rb}}{4a}$
default	$16c^2\left(-\frac{-\frac{\left(b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16a\,c^2\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}+\frac{\left(b\sqrt{-4ac+b^2}+12ac-b^2\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{16ac\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{4\sqrt{-4ac+b^2}\,\left(4ac-b^2\right)}-\frac{-\frac{\left(-b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16a\,c^2\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}}{}\right.$

input
```
int(1/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b/a/(4*a*c-b^2)*c*x^3+1/2*(2*a*c-b^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+
a)+1/4/a*sum((-b*c/(4*a*c-b^2)*_R^2+(6*a*c-b^2)/(4*a*c-b^2))/(2*_R^3*c+_R*
b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2309 vs. $2(206) = 412$.

Time = 0.16 (sec) , antiderivative size = 2309, normalized size of antiderivative = 9.16

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
+ (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b
^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c +
81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*
b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b
^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^
2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*
c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2
)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15
*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a
^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*
b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 -
12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^
2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 6
72*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 -
 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/...
```

Sympy [A] (verification not implemented)

Time = 103.35 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.56

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \frac{-bcx^3+x(2ac-b^2)}{8a^3c-2a^2b^2+x^4\cdot(8a^2c^2-2ab^2c)+x^2\cdot(8a^2bc-2ab^3)} + \text{RootSum}\left(t^4\cdot\left(1048576a^9c^6-1572864a^8b^2c^5+983040a^7b^4c^4-327680a^6b^6c^3+61440a^5b^8c^2-6144a\right.\right.$$

input
```
integrate(1/(c*x**4+b*x**2+a)**2,x)
```

output
```
(-b*c*x**3 + x*(2*a*c - b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2
 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + RootSum(_t**4*(1048576*a*
*9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**
6*c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**
2*(-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608
*a**2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c
**4 + 25*b**4*c**3, Lambda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4 - 28672
*_t**3*a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**4*b**7*c
 + 64*_t**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*b**2*c**3 + 740*_
t*a**2*b**4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(324*a**2*c**4 - 81*a*b**2*
c**3 + 5*b**4*c**2))))
```

Maxima [F]

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \int \frac{1}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a
^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c
*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. $2(206) = 412$.

Time = 0.36 (sec) , antiderivative size = 2682, normalized size of antiderivative = 10.64

$$\int \frac{1}{(a + bx^2 + cx^4)^2} dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) -
1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 -
 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*
b*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2
 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 - 2*(sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 6...
```

Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 6404, normalized size of antiderivative = 25.41

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3
 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 384
0*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a
*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24
*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144
*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 -
 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2
*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^
5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^
6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^
4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 +
 b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*
c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5
)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^...
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 2409, normalized size of antiderivative = 9.56

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(c*x^4+b*x^2+a)^2,x)
```

output
```
(16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c - 2*sqrt(a)*sqrt(2*sq
rt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2
*sqrt(c)*sqrt(a) + b))*a*b**3 + 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a*b**2*c*x**2 + 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**2*x**4
- 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*x**2 - 2*sqrt(a)*sqrt(2*s
qrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*b**3*c*x**4 - 24*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) +
b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a**3*c + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2 - 24*s
qrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c*x**2 - 24*sqrt(c)*sqrt(2*s
qrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*a**2*c**2*x**4 + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
+ b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a*b**3*x**2 + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt...
```

3.801 $\int \frac{1}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 308

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx &= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}\\&\quad-\frac{\sqrt{c}\left(3b^3-16abc+\left(3b^2-10ac\right)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\\&\quad+\frac{\sqrt{c}\left(3b^3-16abc-\left(3b^2-10ac\right)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output

```
-1/2*(-10*a*c+3*b^2)/a^2/(-4*a*c+b^2)/x+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+
b^2)/x/(c*x^4+b*x^2+a)-1/4*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c
+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2
)/a^2/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*c^(1/2)*(3*b^3-1
6*a*b*c-(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-
4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^2} \, dx$$
$$= \frac{-\frac{4}{x} - \frac{2x(b^3-3abc+b^2cx^2-2ac^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(3b^3-16abc\cdots\right.}{}}{4a^2}$$

input
```
Integrate[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
(-4/x - (2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 -
 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2
 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1441, 25, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^2} \, dx$$
$$\Big\downarrow \text{ 1441}$$
$$\frac{-2ac + b^2 + bcx^2}{2ax (b^2 - 4ac) (a + bx^2 + cx^4)} - \frac{\int -\frac{3b^2+3cx^2b-10ac}{x^2(cx^4+bx^2+a)}dx}{2a (b^2 - 4ac)}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3b^2+3cx^2b-10ac}{x^2(cx^4+bx^2+a)}dx}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 1604

$$\frac{-\frac{\int \frac{c\left(3b^2-10ac\right)x^2+b\left(3b^2-13ac\right)}{cx^4+bx^2+a}dx}{a}-\frac{3b^2-10ac}{ax}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 1480

$$\frac{-\frac{\frac{1}{2}c\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-\frac{c\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2\sqrt{b^2-4ac}}}{a}-\frac{3b^2-10ac}{ax}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$\Big\downarrow$ 218

$$\frac{-\frac{\frac{\sqrt{c}\left(-\frac{16abc}{\sqrt{b^2-4ac}}+\frac{3b^3}{\sqrt{b^2-4ac}}-10ac+3b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{3b^2-10ac}{ax}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
(b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) + (-((3*
b^2 - 10*a*c)/(a*x)) - ((Sqrt[c]*(3*b^2 - 10*a*c + (3*b^3)/Sqrt[b^2 - 4*a*
c] - (16*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^3
- 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b
^2 - 4*a*c]]))/a)/(2*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.95

method	result
default	$-\frac{1}{a^{2}x}-\frac{\frac{\frac{c\left(2ac-b^{2}\right)x^{3}}{8ac-2b^{2}}+\frac{b\left(3ac-b^{2}\right)x}{8ac-2b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{2c\left(\frac{\left(10ac\sqrt{-4ac+b^{2}}-3b^{2}\sqrt{-4ac+b^{2}}-16abc+3b^{3}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}-\frac{\left(10ac\sqrt{-4a\cdots}\right.}{\cdots}\right.}{4ac-b^{2}}}{a^{2}}$
risch	$\frac{-\frac{c\left(10ac-3b^{2}\right)x^{4}}{2a^{2}\left(4ac-b^{2}\right)}-\frac{b\left(11ac-3b^{2}\right)x^{2}}{2\left(4ac-b^{2}\right)a^{2}}-\frac{1}{a}}{x\left(c\,x^{4}+b\,x^{2}+a\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4096a^{11}c^{6}-6144a^{10}b^{2}c^{5}+3840a^{9}b^{4}c^{4}-1280a^{8}b^{6}c^{3}+240a^{7}b^{8}c^{2}-24a^{6}b^{10}c+a^{5}b\cdots\right.\right.}\cdots\right.}{\cdots}$

input
```
int(1/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/x-1/a^2*((1/2*c*(2*a*c-b^2)/(4*a*c-b^2)*x^3+1/2*b*(3*a*c-b^2)/(4*a*
c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(1/8*(10*a*c*(-4*a*c+b^2)^(1/2)-
3*b^2*(-4*a*c+b^2)^(1/2)-16*a*b*c+3*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))-1/8*(10*a*c*(-4*a*c+b^2)^(1/2)-3*b^2*(-4*a*c+b^2)^(1/2)+16*a*b*c-3*b
^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c
*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2912 vs. 2(260) = 520.

Time = 0.26 (sec) , antiderivative size = 2912, normalized size of antiderivative = 9.45

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*
c)*x^2 - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^
3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 -
420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqr
t((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^
4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 -
 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b
^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486
*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200
*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c
^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a
^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c
^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3
)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*
b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*
c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sq
rt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12
*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^2} \, dx = \int \frac{1}{(cx^4 + bx^2 + a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*x^
2)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4
*a^4*c)*x) + 1/2*integrate(-(3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*x^2)/
(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3087 vs. $2(260) = 520$.

Time = 0.62 (sec) , antiderivative size = 3087, normalized size of antiderivative = 10.02

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b^2*c*x^4 - 10*a*c^2*x^4 + 3*b^3*x^2 - 11*a*b*c*x^2 + 2*a*b^2 - 8*
a^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*(6*a^4*b^8*c^2 -
 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^
4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 +
 (6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*...
```

Mupad [B] (verification not implemented)

Time = 20.54 (sec) , antiderivative size = 7555, normalized size of antiderivative = 24.53

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^2} dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

output
```
- atan((((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 20
77*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5
 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7
*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2
)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b
^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7
+ x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^
2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25
*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*
c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(10
48576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9
*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7))
+ x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^
8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(
-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9
*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*
c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^...
```

Reduce [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 3104, normalized size of antiderivative = 10.08

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/x^2/(c*x^4+b*x^2+a)^2,x)
```

output
```
(40*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c**2*x - 38*sqrt(a)*sqrt(
2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c*x + 40*sqrt(a)*sqrt(2*sqrt(c)*sqrt(
a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq
rt(a) + b))*a**2*b*c**2*x**3 + 40*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**2*c**3*x**5 + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*x - 38*s
qrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c*x**3 - 38*sqrt(a)*sqrt(2*s
qrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*a*b**2*c**2*x**5 + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a
) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*b**5*x**3 + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*c*x*
*5 + 32*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c*x - 6*sqrt(c)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*x + 32*sqrt(c)*sqrt(2*sqrt(c)*sq...
```

3.802 $\int \frac{x^{11}}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 209

$$\int \frac{x^{11}}{(a+bx^2+cx^4)^3}\,dx = -\frac{b(b^2-7ac)\,x^2}{2c^2\,(b^2-4ac)^2}+\frac{x^8(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{x^4(a(b^2-16ac)+b(b^2-10ac)\,x^2)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{b(b^4-10ab^2c+30a^2c^2)\,\mathrm{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\,(b^2-4ac)^{5/2}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^3}$$

output
```
-1/2*b*(-7*a*c+b^2)*x^2/c^2/(-4*a*c+b^2)^2+1/4*x^8*(b*x^2+2*a)/(-4*a*c+b^2
)/(c*x^4+b*x^2+a)^2+1/4*x^4*(a*(-16*a*c+b^2)+b*(-10*a*c+b^2)*x^2)/c/(-4*a*
c+b^2)^2/(c*x^4+b*x^2+a)+1/2*b*(30*a^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x^
2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(5/2)+1/4*ln(c*x^4+b*x^2+a)/c^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}}{(a+bx^2+cx^4)^3}\,dx$$
$$=\frac{\frac{-b^6+11ab^4c-39a^2b^2c^2+32a^3c^3+4b^5cx^2-30ab^3c^2x^2+50a^2bc^3x^2}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{2a^3c^2+b^5x^2+ab^3(b-5cx^2)+a^2bc(-4b+5cx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{2bc(b^4-10ab^2c+30a^2}{(-b^2-}}{4c^4}$$

input
```
Integrate[x^11/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x^2 - 30*a*b^3
*c^2*x^2 + 50*a^2*b*c^3*x^2)/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (2*a^
3*c^2 + b^5*x^2 + a*b^3*(b - 5*c*x^2) + a^2*b*c*(-4*b + 5*c*x^2))/((b^2 -
4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*Arc
Tan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + b*
x^2 + c*x^4])/(4*c^4)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1434, 1164, 1233, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^2+cx^4)^3}\,dx$$
$$\Big\downarrow \text{1434}$$
$$\frac{1}{2}\int \frac{x^{10}}{(cx^4+bx^2+a)^3}dx^2$$
$$\Big\downarrow \text{1164}$$

$$\frac{1}{2}\left(\frac{x^8(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int\frac{x^6(bx^2+8a)}{(cx^4+bx^2+a)^2}dx^2}{2(b^2-4ac)}\right)$$

$\downarrow$ 1233

$$\frac{1}{2}\left(\frac{x^8(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{\int\frac{2x^2\left(b\left(b^2-7ac\right)x^2+a\left(b^2-16ac\right)\right)}{cx^4+bx^2+a}dx^2}{c(b^2-4ac)}-\frac{x^4(bx^2(b^2-10ac)+a(b^2-16ac))}{c(b^2-4ac)(a+bx^2+cx^4)}}{2(b^2-4ac)}\right)$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{x^8(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{2\int\frac{x^2\left(b\left(b^2-7ac\right)x^2+a\left(b^2-16ac\right)\right)}{cx^4+bx^2+a}dx^2}{c(b^2-4ac)}-\frac{x^4(bx^2(b^2-10ac)+a(b^2-16ac))}{c(b^2-4ac)(a+bx^2+cx^4)}}{2(b^2-4ac)}\right)$$

$\downarrow$ 1200

$$\frac{1}{2}\left(\frac{x^8(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{2\int\left(-b\left(7a-\frac{b^2}{c}\right)-\frac{\left(b^2-4ac\right)^2x^2+ab\left(b^2-7ac\right)}{c\left(cx^4+bx^2+a\right)}\right)dx^2}{c(b^2-4ac)}-\frac{x^4(bx^2(b^2-10ac)+a(b^2-16ac))}{c(b^2-4ac)(a+bx^2+cx^4)}}{2(b^2-4ac)}\right)$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{x^8(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{2\left(-\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{\left(b^2-4ac\right)^2\log\left(a+bx^2+cx^4\right)}{2c^2}-bx^2\left(7a-\frac{b^2}{c}\right)\right)}{c(b^2-4ac)}-}{2(b^2-4ac)}\right.$$

input
```
Int[x^11/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((x^8*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (-((x^4*(a*
(b^2 - 16*a*c) + b*(b^2 - 10*a*c)*x^2))/(c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4))) + (2*(-(b*(7*a - b^2/c)*x^2) - (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*Arc
Tanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4
*a*c)^2*Log[a + b*x^2 + c*x^4])/(2*c^2)))/(c*(b^2 - 4*a*c)))/(2*(b^2 - 4*a
*c)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.76

method	result
default	$\frac{\frac{b\left(25a^{2}c^{2}-15a\,b^{2}c+2b^{4}\right)x^{6}}{c^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(32a^{3}c^{3}+11a^{2}b^{2}c^{2}-19a\,b^{4}c+3b^{6}\right)x^{4}}{2c^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{ab\left(31a^{2}c^{2}-22a\,b^{2}c+3b^{4}\right)x^{2}}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{3}}+\frac{3a^{2}\left(8a^{2}c^{2}-7a\,b^{2}c+b^{4}\right)}{2c^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{\left(16a^{2}c^{2}-8\right.}{}$
risch	Expression too large to display

input
```
int(x^11/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+
1/2*(32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^4+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3
*x^2+3/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*
x^4+b*x^2+a)^2+1/2/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*a^2*c^2-8*a*b^2
*c+b^4)/c*ln(c*x^4+b*x^2+a)+2*(-7*a^2*b*c+b^3*a-1/2*(16*a^2*c^2-8*a*b^2*c+
b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(195) = 390$.

Time = 0.11 (sec) , antiderivative size = 1631, normalized size of antiderivative = 7.80

$$\int \frac{x^{11}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
 - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^6 + (3*b^8 - 31*a*b^6*
c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(3*a*b^7 - 34*a
^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x^2 + ((b^5*c^2 - 10*a*b^3*c^3
 + 30*a^2*b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c -
10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 6
0*a^3*b*c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(b^2
 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^
2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^
3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 -
10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7
 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 +
a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5
 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5
*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a
^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5
*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2), 1/4*(3*a^2*b^6 - 33*a^3*b^4*c
+ 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^
3 - 100*a^3*b*c^4)*x^6 + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**11/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.46

$$\int \frac{x^{11}}{(a+bx^2+cx^4)^3}\,dx = -\frac{(b^5-10\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^4c^3-8\,ab^2c^4+16\,a^2c^5)\sqrt{-b^2+4\,ac}} - \frac{3\,b^4c^2x^8-24\,ab^2c^3x^8+48\,a^2c^4x^8-2\,b^5cx^6+12\,ab^3c^2x^6-4\,a^2bc^3x^6-3\,b^6x^4+20\,ab^4cx^4-22\,a^2b^2c^2}{8\,(b^4c^3-8\,ab^2c^4+16\,a^2c^5)(cx^4+bx^2} + \frac{\log(cx^4+bx^2+a)}{4\,c^3}$$

input
```
integrate(x^11/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*
a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) - 1/8*(3*b
^4*c^2*x^8 - 24*a*b^2*c^3*x^8 + 48*a^2*c^4*x^8 - 2*b^5*c*x^6 + 12*a*b^3*c^
2*x^6 - 4*a^2*b*c^3*x^6 - 3*b^6*x^4 + 20*a*b^4*c*x^4 - 22*a^2*b^2*c^2*x^4
+ 32*a^3*c^3*x^4 - 6*a*b^5*x^2 + 40*a^2*b^3*c*x^2 - 28*a^3*b*c^2*x^2 - 3*a
^2*b^4 + 18*a^3*b^2*c)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^
2 + a)^2) + 1/4*log(c*x^4 + b*x^2 + a)/c^3
```

Mupad [B] (verification not implemented)

Time = 20.86 (sec) , antiderivative size = 2588, normalized size of antiderivative = 12.38

$$\int \frac{x^{11}}{(a + bx^2 + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int(x^11/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((x^4*(3*b^6 + 32*a^3*c^3 + 11*a^2*b^2*c^2 - 19*a*b^4*c))/(4*c^3*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (x^2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2))/(2*
c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*(a*b^4 + 8*a^3*c^2 - 7*a^2*b^2*
c))/(4*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^6*(2*b^4 + 25*a^2*c^2 -
15*a*b^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) +
a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (log((a/c^4 + ((c^3*(-(b^2*(b^4 +
 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^5))^(1/2) - 1)*((8*a)/c +
(2*(c^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^5))^(
1/2) - 1)*(2*a + b*x^2))/c + (2*b*x^2*(3*b^4 + 62*a^2*c^2 - 26*a*b^2*c))/(
c*(4*a*c - b^2)^2)))/(4*c^3) + (x^2*(b^5 + 23*a^2*b*c^2 - 9*a*b^3*c))/(c^4
*(4*a*c - b^2)^2))*(a/c^4 - ((c^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2
)/(c^6*(4*a*c - b^2)^5))^(1/2) + 1)*((8*a)/c - (2*(c^3*(-(b^2*(b^4 + 30*a^
2*c^2 - 10*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^5))^(1/2) + 1)*(2*a + b*x^2))/c
+ (2*b*x^2*(3*b^4 + 62*a^2*c^2 - 26*a*b^2*c))/(c*(4*a*c - b^2)^2)))/(4*c^3
) + (x^2*(b^5 + 23*a^2*b*c^2 - 9*a*b^3*c))/(c^4*(4*a*c - b^2)^2)))*(2*b^10
 - 2048*a^5*c^5 + 320*a^2*b^6*c^2 - 1280*a^3*b^4*c^3 + 2560*a^4*b^2*c^4 -
40*a*b^8*c))/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^
5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) - (b*atan(((x^2*(((b*((6*b^5*c^3
 - 52*a*b^3*c^4 + 124*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) + ((
8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(2*b^10 - 2048*a^5*c^5 + 320*...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3839, normalized size of antiderivative = 18.37

$$\int \frac{x^{11}}{(a + bx^2 + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int(x^11/(c*x^4+b*x^2+a)^3,x)
```

output
```
(60*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*c**
2 - 20*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*
*3*c + 120*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((s
qrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**
3*b**2*c**2*x**2 + 120*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
- b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a**3*b*c**3*x**4 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*s
qrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*a**2*b**5 - 40*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*
sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a**2*b**4*c*x**2 + 20*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*
sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt
(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c**2*x**4 + 120*sqrt(2*sqrt(c)*sqrt(a)
+ b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqr
t(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**3*x**6 + 60*sqrt(2*sqrt(
c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**4*x**8 + 4*sqrt(
2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)...
```

3.803 $\int \frac{x^9}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 121

$$\int \frac{x^9}{(a+bx^2+cx^4)^3}\,dx = \frac{x^6(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{3ax^2(2a+bx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{6a^2\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
1/4*x^6*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/2*a*x^2*(b*x^2+2*a)/(
-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-6*a^2*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2)
)/(-4*a*c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.60

$$\int \frac{x^9}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{4}\left(\frac{b^5-8ab^3c+22a^2bc^2-2b^4cx^2+16ab^2c^2x^2-20a^2c^3x^2}{c^3(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{b^4x^2+ab^2(b-4cx^2)+a^2c(-3b+2cx^2)}{c^3(-b^2+4ac)(a+bx^2+cx^4)^2} + \frac{24a^2\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

input
```
Integrate[x^9/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x^2 + 16*a*b^2*c^2*x^2 - 20*a^2
*c^3*x^2)/(c^3*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^4*x^2 + a*b^2*(b
- 4*c*x^2) + a^2*c*(-3*b + 2*c*x^2))/(c^3*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (24*a^2*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(
5/2))/4
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1434, 1153, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{x^8}{(cx^4+bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{ 1153}$$

$$\frac{1}{2}\left(\frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3a\int\frac{x^4}{\left(cx^4+bx^2+a\right)^2}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1153}$$

$$\frac{1}{2}\left(\frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3a\left(\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2a\int\frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3a\left(\frac{4a\int\frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}+\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3a\left(\frac{4a\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)}{b^2-4ac}\right)$$

input
```
Int[x^9/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((x^6*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*a*((x^2*
(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (4*a*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(113) = 226$.

Time = 0.17 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.21

method	result
default	$\frac{-\frac{\left(10a^2c^2-8a\,b^2c+b^4\right)x^6}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{b\left(2a^2c^2+8a\,b^2c-b^4\right)x^4}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(6a^2c^2-10a\,b^2c+b^4\right)x^2}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{a^2b\left(10ac-b^2\right)}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{2(c\,x^4+b\,x^2+a)^2}+\frac{6a^2\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(16a^2c^2-8a\,b^2c+b^4)\sqrt{4ac}}$
risch	$\frac{-\frac{\left(10a^2c^2-8a\,b^2c+b^4\right)x^6}{2c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{b\left(2a^2c^2+8a\,b^2c-b^4\right)x^4}{4c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(6a^2c^2-10a\,b^2c+b^4\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{a^2b\left(10ac-b^2\right)}{4c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{(c\,x^4+b\,x^2+a)^2}-\frac{3a^2\ln\left(\left(\left(-4ac+b^2\right)^{\frac{5}{2}}-1\right)\right)}{}$

input
```
int(x^9/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*b*
(2*a^2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-a*(6*a^2*c^2-
10*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2+1/2*a^2*b*(10*a*c-b^2)/
c^2/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+6*a^2/(16*a^2*c^2-8*a*b^
2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(113) = 226$.

Time = 0.10 (sec) , antiderivative size = 973, normalized size of antiderivative = 8.04

$$\int \frac{x^9}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/4*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 4
2*a^2*b^2*c^3 - 40*a^3*c^4)*x^6 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a
^3*b*c^3)*x^4 + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x^2
 - 12*(a^2*c^4*x^8 + 2*a^2*b*c^3*x^6 + 2*a^3*b*c^2*x^2 + a^4*c^2 + (a^2*b^
2*c^2 + 2*a^3*c^3)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2
 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a^2*b^6
*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*
c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^
2*b^3*c^5 - 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 +
 32*a^3*b^2*c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^
3*b^3*c^4 - 64*a^4*b*c^5)*x^2), -1/4*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^
2 + 2*(b^6*c - 12*a*b^4*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^6 + (b^7 - 12
*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3*b*c^3)*x^4 + 2*(a*b^6 - 14*a^2*b^4*c + 4
6*a^3*b^2*c^2 - 24*a^4*c^3)*x^2 + 24*(a^2*c^4*x^8 + 2*a^2*b*c^3*x^6 + 2*a^
3*b*c^2*x^2 + a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^4)*sqrt(-b^2 + 4*a*c)*
arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a^2*b^6*c^2 - 12
*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*
a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5
 - 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b
^2*c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(112) = 224$.

Time = 1.99 (sec) , antiderivative size = 554, normalized size of antiderivative = 4.58

$$\int \frac{x^9}{(a+bx^2+cx^4)^3}\,dx = -3a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-36a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}\right.}{6a^2c}$$
$$+3a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+36a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}-\right.}{6a^2c}$$
$$+\frac{10a^3bc-a^2b^3+x^6(-20a^2c^3+16ab^2c^2-2b^4c)+x^4\cdot(2a^2bc^2+8ab^3c}{64a^4c^4-32a^3b^2c^3+4a^2b^4c^2+x^8\cdot(64a^2c^6-32ab^2c^5+4b^4c^4)+x^6\cdot(128a^2bc^5-64ab^3c^4+8b^5c^3)+}$$

input
```
integrate(x**9/(c*x**4+b*x**2+a)**3,x)
```

output
```
-3*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**5*c**3*sqrt(-1/(4*a
*c - b**2)**5) + 144*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**3*b
**4*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5)
+ 3*a**2*b)/(6*a**2*c)) + 3*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (19
2*a**5*c**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**4*b**2*c**2*sqrt(-1/(4*a*c
 - b**2)**5) + 36*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a**2*b**6*sqr
t(-1/(4*a*c - b**2)**5) + 3*a**2*b)/(6*a**2*c)) + (10*a**3*b*c - a**2*b**3
 + x**6*(-20*a**2*c**3 + 16*a*b**2*c**2 - 2*b**4*c) + x**4*(2*a**2*b*c**2
+ 8*a*b**3*c - b**5) + x**2*(-12*a**3*c**2 + 20*a**2*b**2*c - 2*a*b**4))/(
64*a**4*c**4 - 32*a**3*b**2*c**3 + 4*a**2*b**4*c**2 + x**8*(64*a**2*c**6 -
 32*a*b**2*c**5 + 4*b**4*c**4) + x**6*(128*a**2*b*c**5 - 64*a*b**3*c**4 +
8*b**5*c**3) + x**4*(128*a**3*c**5 - 24*a*b**4*c**3 + 4*b**6*c**2) + x**2*
(128*a**3*b*c**4 - 64*a**2*b**3*c**3 + 8*a*b**5*c**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^9}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.75

$$\int \frac{x^9}{(a+bx^2+cx^4)^3}\,dx = \frac{6\,a^2\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{2\,b^4cx^6-16\,ab^2c^2x^6+20\,a^2c^3x^6+b^5x^4-8\,ab^3cx^4-2\,a^2bc^2x^4+2\,ab^4x^2-20\,a^2b^2cx^2+12\,a^3c^2x^2+a}{4\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)(cx^4+bx^2+a)^2}$$

input
```
integrate(x^9/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
6*a^2*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*
c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*b^4*c*x^6 - 16*a*b^2*c^2*x^6 + 20*a^2*c^
3*x^6 + b^5*x^4 - 8*a*b^3*c*x^4 - 2*a^2*b*c^2*x^4 + 2*a*b^4*x^2 - 20*a^2*b
^2*c*x^2 + 12*a^3*c^2*x^2 + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.67

$$\int \frac{x^9}{(a+bx^2+cx^4)^3}\,dx = \frac{6\,a^2\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{36\,a^3\,c^2}{\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{36\,a^3\,b\left(16\,a^2\,b\,c^4-8\,a\,b^3\,c^3+b^5\,c^2\right)}{\left(4\,a\,c-b^2\right)^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{72\,a^4\,b\,c^2}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\left(b^4\left(4\,a\,c-b^2\right)^5+16\,a^2\,c^2\left(4\right.\right.}{72\,a^4\,c^2}\right)}{\left(4\,a\,c-b^2\right)^{5/2}} - \frac{\frac{x^6\left(10\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{2\,c\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\left(b^3-10\,a\,b\,c\right)}{4\,c^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{x^4\left(2\,a^2\,b\,c^2+8\,a\,b^3\,c-b^5\right)}{4\,c^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x^2\left(6\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{2\,c^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

input
```
int(x^9/(a + b*x^2 + c*x^4)^3,x)
```

output
```
(6*a^2*atan(((x^2*((36*a^3*c^2)/((4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (36*a^3*b*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4))/((4*a*c - b
^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (72*a^4*b*c^2)/(4*a*c - b^2)
^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*
a*c - b^2)^5))/(72*a^4*c^2)))/(4*a*c - b^2)^(5/2) - ((x^6*(b^4 + 10*a^2*c^
2 - 8*a*b^2*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(b^3 - 10*a*b*
c))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^4*(2*a^2*b*c^2 - b^5 + 8*a
*b^3*c))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x^2*(b^4 + 6*a^2*c^2
- 10*a*b^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2)
+ a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1137, normalized size of antiderivative = 9.40

$$\int \frac{x^9}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^9/(c*x^4+b*x^2+a)^3,x)
```

output
```
( - 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*
c - 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*
*2*c*x**2 - 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a**3*b*c**2*x**4 - 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
- b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a**2*b**3*c*x**4 - 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*
sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a**2*b**2*c**2*x**6 - 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt
(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**3*x**8 - 24*sqrt(2*sqrt(c)*sqrt(a) +
 b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt
(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*c - 48*sqrt(2*sqrt(c)*sqrt(a) +
 b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt
(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c*x**2 - 48*sqrt(2*sqrt(c)*s
qrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
+ 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**2*x**4 - 24*sqrt(2*s
qrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sq...
```

3.804 $\int \frac{x^7}{(a+bx^2+cx^4)^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{x^7}{(a+bx^2+cx^4)^3} \, dx = -\frac{x^6(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3bx^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3ab\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
-1/4*x^6*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/4*b*x^2*(b*x^2+2*a)/
(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3*a*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int \frac{x^7}{(a+bx^2+cx^4)^3} \, dx = -\frac{8a^3c+b^4x^4+abx^2(2b^2+bcx^2+6c^2x^4)+a^2(b^2+10bcx^2+16c^2x^4)}{4c(b^2-4ac)^2(a+bx^2+cx^4)^2} - \frac{3ab\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}$$

input
```
Integrate[x^7/(a + b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(8*a^3*c + b^4*x^4 + a*b*x^2*(2*b^2 + b*c*x^2 + 6*c^2*x^4) + a^2*(b^2
 + 10*b*c*x^2 + 16*c^2*x^4))/(c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) - (
3*a*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1434, 1156, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^6}{(cx^4+bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{1156}$$

$$\frac{1}{2}\left(\frac{3b\int \frac{x^4}{(cx^4+bx^2+a)^2}dx^2}{2(b^2-4ac)}-\frac{x^6(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{1153}$$

$$\frac{1}{2}\left(\frac{3b\left(\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{2a\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}\right)}{2(b^2-4ac)}-\frac{x^6(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{3b\left(\frac{4a\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}+\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2(b^2-4ac)}-\frac{x^6(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow 219$$

$$\frac{1}{2}\left(\frac{3b\left(\frac{4a\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2\,(b^2-4ac)}-\frac{x^6\,(b+2cx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)^2}\right)$$

input
```
Int[x^7/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(-1/2*(x^6*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*b*((x
^2*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (4*a*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1156
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] + Simp[m*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c)))   Int[
(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(111) = 222$.

Time = 0.16 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.93

method	result
default	$\frac{-\frac{3abc\,x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{\left(16a^{2}c^{2}+a\,b^{2}c+b^{4}\right)x^{4}}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5ac+b^{2}\right)ab\,x^{2}}{c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a^{2}\left(8ac+b^{2}\right)}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}-\frac{3ab\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}$
risch	$\frac{-\frac{3abc\,x^{6}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(16a^{2}c^{2}+a\,b^{2}c+b^{4}\right)x^{4}}{4c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5ac+b^{2}\right)ab\,x^{2}}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a^{2}\left(8ac+b^{2}\right)}{4c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}-\frac{3ba\ln\left(\left(-\left(-4ac+b^{2}\right)^{\frac{5}{2}}-16a^{2}\right.\right.}{}$

input
```
int(x^7/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-1/2*(16*a^2*c^2+a*b^2*c+b^4)/
c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^2-1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^
2-3*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(111) = 222$.

Time = 0.10 (sec) , antiderivative size = 892, normalized size of antiderivative = 7.50

$$\int \frac{x^7}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/4*(6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^6 + a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^
2 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^4 + 2*(a*b^5 + a^2*b
^3*c - 20*a^3*b*c^2)*x^2 - 6*(a*b*c^3*x^8 + 2*a*b^2*c^2*x^6 + 2*a^2*b^2*c*
x^2 + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 +
b*x^2 + a)))/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (
b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 1
2*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 +
 24*a^2*b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*
b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2), -1/4*(6*(a*b^3*c^2 - 4*a^2*
b*c^3)*x^6 + a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + (b^6 - 3*a*b^4*c + 12*a^
2*b^2*c^2 - 64*a^3*c^3)*x^4 + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x^2 - 1
2*(a*b*c^3*x^8 + 2*a*b^2*c^2*x^6 + 2*a^2*b^2*c*x^2 + a^3*b*c + (a*b^3*c +
2*a^2*b*c^2)*x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a
*c)/(b^2 - 4*a*c)))/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*
c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*
c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^
6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c -
12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(112) = 224$.

Time = 1.49 (sec) , antiderivative size = 524, normalized size of antiderivative = 4.40

$$\int \frac{x^7}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3ab^7\sqrt{-\frac{1}{(4ac-b^2)^5}}+3ab^2}{6abc}\right.}{2}$$
$$-\frac{3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}}-3ab^7\sqrt{-\frac{1}{(4ac-b^2)^5}}+3a}{6abc}\right.}{2}$$
$$+\frac{-8a^3c-a^2b^2-6abc^2x^6+x^4(-16a^2c^2-ab^2c-b^4)+}{64a^4c^3-32a^3b^2c^2+4a^2b^4c+x^8\cdot(64a^2c^5-32ab^2c^4+4b^4c^3)+x^6\cdot(128a^2bc^4-64ab^3c^3+8b^5c^2)+x}$$

input
```
integrate(x**7/(c*x**4+b*x**2+a)**3,x)
```

output
```
3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**4*b*c**3*sqrt(-1/(4*a
*c - b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b
**5*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3
*a*b**2)/(6*a*b*c))/2 - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*a
**4*b*c**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c
- b**2)**5) + 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1
/(4*a*c - b**2)**5) + 3*a*b**2)/(6*a*b*c))/2 + (-8*a**3*c - a**2*b**2 - 6*
a*b*c**2*x**6 + x**4*(-16*a**2*c**2 - a*b**2*c - b**4) + x**2*(-10*a**2*b*
c - 2*a*b**3))/(64*a**4*c**3 - 32*a**3*b**2*c**2 + 4*a**2*b**4*c + x**8*(6
4*a**2*c**5 - 32*a*b**2*c**4 + 4*b**4*c**3) + x**6*(128*a**2*b*c**4 - 64*a
*b**3*c**3 + 8*b**5*c**2) + x**4*(128*a**3*c**4 - 24*a*b**4*c**2 + 4*b**6*
c) + x**2*(128*a**3*b*c**3 - 64*a**2*b**3*c**2 + 8*a*b**5*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.44

$$\int \frac{x^7}{(a+bx^2+cx^4)^3}\,dx$$
$$= -\frac{3\,ab\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,abc^2x^6+b^4x^4+ab^2cx^4+16\,a^2c^2x^4+2\,ab^3x^2+10\,a^2bcx^2+a^2b^2+8\,a^3c}{4\,(b^4c-8\,ab^2c^2+16\,a^2c^3)(cx^4+bx^2+a)^2}$$

input
```
integrate(x^7/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3*a*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2
*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*a*b*c^2*x^6 + b^4*x^4 + a*b^2*c*x^4 + 1
6*a^2*c^2*x^4 + 2*a*b^3*x^2 + 10*a^2*b*c*x^2 + a^2*b^2 + 8*a^3*c)/((b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 18.06 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.55

$$\int \frac{x^7}{(a+bx^2+cx^4)^3}\,dx$$
$$= -\frac{\frac{x^2\,(5\,c\,a^2\,b+a\,b^3)}{2\,c\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^4\,(16\,a^2\,c^2+a\,b^2\,c+b^4)}{4\,c\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{a\,(8\,c\,a^2+a\,b^2)}{4\,c\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,a\,b\,c\,x^6}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^4\,(b^2+2\,a\,c)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$
$$- \frac{3\,a\,b\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{9\,a\,b^2\,c^2}{{(4\,a\,c-b^2)}^{9/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{9\,a\,b^3\,(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2)}{2\,{(4\,a\,c-b^2)}^{15/2}\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}\right)+\frac{18\,a^2\,b^3\,c^2}{{(4\,a\,c-b^2)}^{15/2}}\right)\left(b^4\,{(4\,a\,c-b^2)}^5+16\,a\right.}{18\,a^2\,b^2\,c^2}\right.}{{(4\,a\,c-b^2)}^{5/2}}$$

input
```
int(x^7/(a + b*x^2 + c*x^4)^3,x)
```

output
```
- ((x^2*(a*b^3 + 5*a^2*b*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(
b^4 + 16*a^2*c^2 + a*b^2*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*(a*
b^2 + 8*a^2*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*b*c*x^6)/(2*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*
x^2 + 2*b*c*x^6) - (3*a*b*atan(((x^2*((9*a*b^2*c^2)/((4*a*c - b^2)^(9/2)*(
b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*a*b^3*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a
^2*b*c^4))/(2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (18*
a^2*b^3*c^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*
c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*a^2*b^2*c^2)))/(4*a*c - b^2)^
(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1091, normalized size of antiderivative = 9.17

$$\int \frac{x^7}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^7/(c*x^4+b*x^2+a)^3,x)
```

output

```
(12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c +
 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*
c*x**2 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*2*b*c**2*x**4 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*a*b**3*c*x**4 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a
) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*a*b**2*c**2*x**6 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c
)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqr
t(c)*sqrt(a) + b))*a*b*c**3*x**8 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*s
qrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*a**3*b*c + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*s
qrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c*x**2 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*
sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**2*x**4 + 12*sqrt(2*sqrt(c)*sqrt(
a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) +...
```

3.805 $\int \frac{x^5}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 130

$$\int \frac{x^5}{(a+bx^2+cx^4)^3}\,dx=\frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3ab+(b^2+2ac)x^2}{2(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{(b^2+2ac)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
1/4*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/2*(3*a*b+(2*a*c+b^2)*
x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-(2*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{x^5}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{4}\left(\frac{(b^2+2ac)(b+2cx^2)}{c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{b^2x^2+a(b-2cx^2)}{c(-b^2+4ac)(a+bx^2+cx^4)^2} + \frac{4(b^2+2ac)\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

input
```
Integrate[x^5/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(((b^2 + 2*a*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
b^2*x^2 + a*(b - 2*c*x^2))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(
b^2 + 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2
))/4
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1434, 1164, 27, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{x^4}{(cx^4+bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{ 1164}$$

$$\frac{1}{2}\left(\frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int \frac{2(a-bx^2)}{(cx^4+bx^2+a)^2}dx^2}{2(b^2-4ac)}\right)$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int \frac{a-bx^2}{(cx^4+bx^2+a)^2}dx^2}{b^2-4ac}\right)$$

$\downarrow$ 1159

$$\frac{1}{2}\left(\frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{-\frac{(2ac+b^2)\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{x^2(2ac+b^2)+3ab}{(b^2-4ac)(a+bx^2+cx^4)}}{b^2-4ac}\right)$$

$\downarrow$ 1083

$$\frac{1}{2}\left(\frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{2(2ac+b^2)\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{x^2(2ac+b^2)+3ab}{(b^2-4ac)(a+bx^2+cx^4)}}{b^2-4ac}\right)$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{2(2ac+b^2)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{x^2(2ac+b^2)+3ab}{(b^2-4ac)(a+bx^2+cx^4)}}{b^2-4ac}\right)$$

input

```
Int[x^5/(a + b*x^2 + c*x^4)^3,x]
```

output

```
((x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (-((3*a*b +
 (b^2 + 2*a*c)*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (2*(b^2 + 2*a*c
)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2))/(b^2 - 4*
a*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.66

method	result
default	$\frac{\frac{c\left(2ac+b^{2}\right)x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3b\left(2ac+b^{2}\right)x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(2ac-5b^{2}\right)x^{2}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3a^{2}b}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{\left(2ac+b^{2}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}$
risch	$\frac{\frac{c\left(2ac+b^{2}\right)x^{6}}{32a^{2}c^{2}-16a\,b^{2}c+2b^{4}}+\frac{3b\left(2ac+b^{2}\right)x^{4}}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(2ac-5b^{2}\right)x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{3a^{2}b}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}-\frac{\ln\left(\left(\left(-4ac+b^{2}\right)^{\frac{5}{2}}-16a^{2}b\,c^{2}+8a\,b^{3}c\right.\right.}{\left(-4ac\right.}$

input
```
int(x^5/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3/2*b*(2*a*c+b^2)/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^4-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+3*
a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+(2*a*c+b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(122) = 244.

Time = 0.09 (sec) , antiderivative size = 907, normalized size of antiderivative = 6.98

$$\int \frac{x^5}{\left(a+bx^2+cx^4\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/4*(2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^6 + 6*a^2*b^3 - 24*a^3*b*c + 3
*(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^4 + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c
^2)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^8 + 2*(b^3*c + 2*a*b*c^2)*x^6 + (b^4 +
4*a*b^2*c + 4*a^2*c^2)*x^4 + a^2*b^2 + 2*a^3*c + 2*(a*b^3 + 2*a^2*b*c)*x^2
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 +
b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6
+ (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 +
 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), 1/4*(2*(b^
4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^6 + 6*a^2*b^3 - 24*a^3*b*c + 3*(b^5 - 2*a
*b^3*c - 8*a^2*b*c^2)*x^4 + 2*(5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x^2 - 4
*((b^2*c^2 + 2*a*c^3)*x^8 + 2*(b^3*c + 2*a*b*c^2)*x^6 + (b^4 + 4*a*b^2*c +
 4*a^2*c^2)*x^4 + a^2*b^2 + 2*a^3*c + 2*(a*b^3 + 2*a^2*b*c)*x^2)*sqrt(-b^2
 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c
^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c
^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*
b*c^3)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(119) = 238$.

Time = 2.02 (sec) , antiderivative size = 580, normalized size of antiderivative = 4.46

$$\int \frac{x^5}{(a+bx^2+cx^4)^3}\,dx =$$

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)\log\left(x^2+\frac{-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)-12ab^4c\sqrt{-\frac{1}{(4ac-b}}}{4ac^2+2b^2c}\right.}{2}$$

$$+\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)\log\left(x^2+\frac{64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdot(2ac+b^2)+12ab^4c\sqrt{-\frac{1}{(4ac-b^2)}}}{4ac^2+2b^2c}\right.}{2}$$

$$+\frac{6a^2b+x^6\cdot(4ac^2+2b^2c)+x^4\cdot(6abc+3b^3)+x^2(-4a}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\cdot(64a^2c^4-32ab^2c^3+4b^4c^2)+x^6\cdot(128a^2bc^3-64ab^3c^2+8b^5c)+x^4\cdot}$$

input
```
integrate(x**5/(c*x**4+b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x**2 + (-64*a**3*c**3*sqrt(
-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c -
b**2)**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c +
 b**2) + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/
(4*a*c**2 + 2*b**2*c))/2 + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x
**2 + (64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b*
*2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4
*a*c - b**2)**5)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5
)*(2*a*c + b**2) + b**3)/(4*a*c**2 + 2*b**2*c))/2 + (6*a**2*b + x**6*(4*a*
c**2 + 2*b**2*c) + x**4*(6*a*b*c + 3*b**3) + x**2*(-4*a**2*c + 10*a*b**2))
/(64*a**4*c**2 - 32*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*
b**2*c**3 + 4*b**4*c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5
*c) + x**4*(128*a**3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2
- 64*a**2*b**3*c + 8*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.24

$$\int \frac{x^5}{(a+bx^2+cx^4)^3}\,dx = \frac{(b^2+2\,ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{2\,b^2cx^6+4\,ac^2x^6+3\,b^3x^4+6\,abcx^4+10\,ab^2x^2-4\,a^2cx^2+6\,a^2b}{4\,(cx^4+bx^2+a)^2(b^4-8\,ab^2c+16\,a^2c^2)}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
(b^2 + 2*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*b^2*c*x^6 + 4*a*c^2*x^6 + 3*b^3*
x^4 + 6*a*b*c*x^4 + 10*a*b^2*x^2 - 4*a^2*c*x^2 + 6*a^2*b)/((c*x^4 + b*x^2
+ a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

Mupad [B] (verification not implemented)

Time = 18.07 (sec) , antiderivative size = 460, normalized size of antiderivative = 3.54

$$\int \frac{x^5}{(a+bx^2+cx^4)^3}\,dx = \frac{\frac{3\,a^2\,b}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,(5\,a\,b^2-2\,a^2\,c)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,b\,x^4\,(b^2+2\,a\,c)}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{c\,x^6\,(b^2+2\,a\,c)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^4\,(b^2+2\,a\,c)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6} + \frac{\mathrm{atan}\left(\frac{\left(x^2\,\left(\frac{\left(b^2+2\,a\,c\right)\,\left(b^2\,c^2+2\,a\,c^3\right)}{a\,\left(4\,a\,c-b^2\right)^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b\,\left(b^2+2\,a\,c\right)^2\,\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\,a\,\left(4\,a\,c-b^2\right)^{15/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{2\,b\,c^2\,\left(b^2+2\,a\,c\right)^2}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\,\left(b^4\,\left(4\,a\,c-b^2\right)^5+16\,a \cdots\right.}{8\,a^2\,c^4+8\,a\,b^2\,c^3+2\,b^4\,c^2}\right.}{(4\,a\,c-b^2)^{5/2}}$$

input
```
int(x^5/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((3*a^2*b)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(5*a*b^2 - 2*a^2*c))/
(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*x^4*(2*a*c + b^2))/(4*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) + (c*x^6*(2*a*c + b^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + (ata
n(((x^2*(((2*a*c + b^2)*(2*a*c^3 + b^2*c^2))/(a*(4*a*c - b^2)^(9/2)*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)) + (b*(2*a*c + b^2)^2*(2*b^5*c^2 - 16*a*b^3*c^3 +
 32*a^2*b*c^4))/(2*a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))
 + (2*b*c^2*(2*a*c + b^2)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 +
16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(8*a^2*c^4 + 2*b^
4*c^2 + 8*a*b^2*c^3))*(2*a*c + b^2))/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1694, normalized size of antiderivative = 13.03

$$\int \frac{x^5}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^5/(c*x^4+b*x^2+a)^3,x)
```

output

```
( - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c
 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3
 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**
2*c*x**2 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**2*b*c**2*x**4 - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) -
b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a*b**4*x**2 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
 - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + b))*a*b**3*c*x**4 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sq
rt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)
*sqrt(a) + b))*a*b**2*c**2*x**6 - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqr
t(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*
sqrt(c)*sqrt(a) + b))*a*b*c**3*x**8 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2
*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqr
t(2*sqrt(c)*sqrt(a) + b))*b**5*x**4 - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2
*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/...
```

3.806 $\int \frac{x^3}{(a+bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int \frac{x^3}{(a+bx^2+cx^4)^3} dx = \frac{2a+bx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{3b(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3bc\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
1/4*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/4*b*(2*c*x^2+b)/(-4*a*c+b
^2)^2/(c*x^4+b*x^2+a)+3*b*c*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{(a+bx^2+cx^4)^3} dx = \frac{\frac{(b^2-4ac)(2a+bx^2)}{(a+bx^2+cx^4)^2} - \frac{3b(b+2cx^2)}{a+bx^2+cx^4} - \frac{12bc\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4(b^2-4ac)^2}$$

input
```
Integrate[x^3/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(((b^2 - 4*a*c)*(2*a + b*x^2))/(a + b*x^2 + c*x^4)^2 - (3*b*(b + 2*c*x^2))
/(a + b*x^2 + c*x^4) - (12*b*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/S
qrt[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1434, 1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{(cx^4+bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{1159}$$

$$\frac{1}{2}\left(\frac{3b\int \frac{1}{(cx^4+bx^2+a)^2}dx^2}{2(b^2-4ac)}+\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{2}\left(\frac{3b\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2(b^2-4ac)}+\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{3b\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2(b^2-4ac)}+\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{3b\left(\frac{4c\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{2\left(b^2-4ac\right)}+\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

input
```
Int[x^3/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*b*(-((b + 2*c*
x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (4*c*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13

method	result
default	$\frac{-b\,x^{2}-2a}{4(4ac-b^{2})(c\,x^{4}+b\,x^{2}+a)^{2}}-\frac{3b\left(\frac{2c\,x^{2}+b}{\left(4ac-b^{2}\right)\left(c\,x^{4}+b\,x^{2}+a\right)}+\frac{4c\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}\right)}{4(4ac-b^{2})}$
risch	$\frac{-\frac{3b\,c^{2}x^{6}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{9b^{2}c\,x^{4}}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5ac+b^{2}\right)b\,x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(8ac+b^{2}\right)}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}-\frac{3cb\ln\left(\left(-\left(-4ac+b^{2}\right)^{\frac{5}{2}}-16a^{2}b\,c^{\cdots}\right.\right.}{2(\cdots}$

input
```
int(x^3/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2-3/4*b/(4*a*c-b^2)*((2*c*x^2
+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(
4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(105) = 210.

Time = 0.09 (sec) , antiderivative size = 808, normalized size of antiderivative = 7.15

$$\int \frac{x^3}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*x^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*
(b^4*c - 4*a*b^2*c^2)*x^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x^2 - 6*(b*c^
3*x^8 + 2*b^2*c^2*x^6 + 2*a*b^2*c*x^2 + (b^3*c + 2*a*b*c^2)*x^4 + a^2*b*c)
*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 +
 (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 +
2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), -1/4*(6*(b^
3*c^2 - 4*a*b*c^3)*x^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a
*b^2*c^2)*x^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x^2 - 12*(b*c^3*x^8 + 2*b
^2*c^2*x^6 + 2*a*b^2*c*x^2 + (b^3*c + 2*a*b*c^2)*x^4 + a^2*b*c)*sqrt(-b^2
+ 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^
3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^
3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b
*c^3)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(105) = 210$.

Time = 1.21 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.35

$$\int \frac{x^3}{(a+bx^2+cx^4)^3}\,dx = \frac{3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^7c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^2c}{6bc^2}\right.}{2} - \frac{3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-3b^7c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^2}{6bc^2}\right.}{2} + \frac{-8a^2c-ab^2-9b^2cx^4-6bc^2x^6+x^2(-10abc-}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\cdot(64a^2c^4-32ab^2c^3+4b^4c^2)+x^6\cdot(128a^2bc^3-64ab^3c^2+8b^5c)+x^4\cdot(}$$

input
```
integrate(x**3/(c*x**4+b*x**2+a)**3,x)
```

output
```
3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**3*b*c**4*sqrt(-1/(4*a
*c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5
*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3
*b**2*c)/(6*b*c**2))/2 - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*
a**3*b*c**4*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c
 - b**2)**5) + 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-
1/(4*a*c - b**2)**5) + 3*b**2*c)/(6*b*c**2))/2 + (-8*a**2*c - a*b**2 - 9*b
**2*c*x**4 - 6*b*c**2*x**6 + x**2*(-10*a*b*c - 2*b**3))/(64*a**4*c**2 - 32
*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*
c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**
3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c +
8*a*b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.27

$$\int \frac{x^3}{(a+bx^2+cx^4)^3}\,dx = -\frac{3\,bc\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,bc^2x^6+9\,b^2cx^4+2\,b^3x^2+10\,abcx^2+ab^2+8\,a^2c}{4\,(cx^4+bx^2+a)^2(b^4-8\,ab^2c+16\,a^2c^2)}$$

input
```
integrate(x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3*b*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2
*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*b*c^2*x^6 + 9*b^2*c*x^4 + 2*b^3*x^2 + 1
0*a*b*c*x^2 + a*b^2 + 8*a^2*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 1
6*a^2*c^2))
```

Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.54

$$\int \frac{x^3}{(a+bx^2+cx^4)^3}\,dx = -\frac{\frac{8\,c\,a^2+a\,b^2}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,(b^3+5\,a\,c\,b)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{9\,b^2\,c\,x^4}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{3\,b\,c^2\,x^6}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^4\,(b^2+2\,a\,c)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$
$$-\frac{3\,b\,c\,\mathrm{atan}\left(\frac{\left(x^2\,\left(\frac{9\,b^2\,c^4}{a\,\left(4\,a\,c-b^2\right)^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b^3\,c^2\,\left(144\,a^2\,b\,c^4-72\,a\,b^3\,c^3+9\,b^5\,c^2\right)}{a\,\left(4\,a\,c-b^2\right)^{15/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{18\,b^3\,c^4}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\,\left(b^4\,\left(4\,a\,c-b^2\right)^5+16\,c\right.}{18\,b^2\,c^4}\right.}{\left(4\,a\,c-b^2\right)^{5/2}}$$

input
```
int(x^3/(a + b*x^2 + c*x^4)^3,x)
```

output
```
- ((a*b^2 + 8*a^2*c)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3 + 5*a*
b*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c*x^4)/(4*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) + (3*b*c^2*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4
*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (3*b*c*atan(((x^
2*((9*b^2*c^4)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b
^3*c^2*(9*b^5*c^2 - 72*a*b^3*c^3 + 144*a^2*b*c^4))/(a*(4*a*c - b^2)^(15/2)
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (18*b^3*c^4)/(4*a*c - b^2)^(15/2))*(b^
4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5
))/(18*b^2*c^4)))/(4*a*c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1055, normalized size of antiderivative = 9.34

$$\int \frac{x^3}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^3/(c*x^4+b*x^2+a)^3,x)
```

output
```
(12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c +
 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c*x
**2 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c
**2*x**4 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
b**3*c*x**4 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b
))*b**2*c**2*x**6 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
- b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*b*c**3*x**8 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(
a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq
rt(a) + b))*a**2*b*c + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(
a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq
rt(a) + b))*a*b**2*c*x**2 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*
sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a*b*c**2*x**4 + 12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqr
t(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt...
```

3.807 $\int \frac{x}{(a+bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int \frac{x}{(a+bx^2+cx^4)^3} dx = -\frac{b+2cx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3c(b+2cx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{6c^2\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

output
```
-1/4*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/2*c*(2*c*x^2+b)/(-4*a*c+
b^2)^2/(c*x^4+b*x^2+a)-6*c^2*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+bx^2+cx^4)^3} dx = \frac{-\frac{(b+2cx^2)(b^2-6bcx^2-2c(5a+3cx^4))}{(a+bx^2+cx^4)^2} + \frac{24c^2\arctan\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4(b^2-4ac)^2}$$

input
```
Integrate[x/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(-(((b + 2*c*x^2)*(b^2 - 6*b*c*x^2 - 2*c*(5*a + 3*c*x^4)))/(a + b*x^2 + c*
x^4)^2) + (24*c^2*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c])/(4*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1432, 1086, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2+cx^4)^3} \, dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{(cx^4+bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{2}\left(-\frac{3c\int \frac{1}{(cx^4+bx^2+a)^2}dx^2}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{1086}$$

$$\frac{1}{2}\left(-\frac{3c\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\frac{3c\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{3c\left(\frac{4c\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)}{b^2-4ac}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

input
```
Int[x/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(-1/2*(b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*c*(-((b + 2
*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (4*c*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(b^2 - 4*a*c))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

method	result
default	$\frac{2c\,x^{2}+b}{4(4ac-b^{2})(c\,x^{4}+b\,x^{2}+a)^{2}}+\frac{3c\left(\frac{2c\,x^{2}+b}{\left(4ac-b^{2}\right)\left(c\,x^{4}+b\,x^{2}+a\right)}+\frac{4c\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}\right)}{2(4ac-b^{2})}$
risch	$\frac{\frac{3c^{3}x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{9b\,c^{2}x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(5ac+b^{2}\right)c\,x^{2}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{b\left(10ac-b^{2}\right)}{64a^{2}c^{2}-32a\,b^{2}c+4b^{4}}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}-\frac{3c^{2}\ln\left(\left(\left(-4ac+b^{2}\right)^{\frac{5}{2}}-16a^{2}b\,c^{2}+8a\,b^{3}c-b\right.\right.}{\left(-4ac+b^{2}\right)^{\frac{5}{2}}}$

input
```
int(x/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3/2*c/(4*a*c-b^2)*((2*c*x^2+
b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(105) = 210.

Time = 0.09 (sec) , antiderivative size = 809, normalized size of antiderivative = 7.16

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/4*(12*(b^2*c^3 - 4*a*c^4)*x^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b
^3*c^2 - 4*a*b*c^3)*x^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x^2 + 12*(c^4
*x^8 + 2*b*c^3*x^6 + 2*a*b*c^2*x^2 + (b^2*c^2 + 2*a*c^3)*x^4 + a^2*c^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b
^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(
a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), 1/4*(12*(b^2*c
^3 - 4*a*c^4)*x^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*
c^3)*x^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*x^2 - 24*(c^4*x^8 + 2*b*c^3*
x^6 + 2*a*b*c^2*x^2 + (b^2*c^2 + 2*a*c^3)*x^4 + a^2*c^2)*sqrt(-b^2 + 4*a*c
)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*
a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128
*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x
^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(107) = 214$.

Time = 1.18 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.26

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx =$$

$$-3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{-192a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-36ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\cdots}{6c^3}\right.$$

$$+3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x^2+\frac{192a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+36ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-\cdots}{6c^3}\right.$$

$$+\frac{10abc-b^3+18bc^2x^4+12c^3x^6+x^2\cdot\left(20ac^2+\cdots\right.}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\cdot\left(64a^2c^4-32ab^2c^3+4b^4c^2\right)+x^6\cdot\left(128a^2bc^3-64ab^3c^2+8b^5c\right)+x^4\cdot\cdots}$$

input
```
integrate(x/(c*x**4+b*x**2+a)**3,x)
```

output
```
-3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**3*c**5*sqrt(-1/(4*a
*c - b**2)**5) + 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**4
*c**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5)
+ 3*b*c**2)/(6*c**3)) + 3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (192*
a**3*c**5*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*sqrt(-1/(4*a*c -
 b**2)**5) + 36*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*sqrt(
-1/(4*a*c - b**2)**5) + 3*b*c**2)/(6*c**3)) + (10*a*b*c - b**3 + 18*b*c**2
*x**4 + 12*c**3*x**6 + x**2*(20*a*c**2 + 4*b**2*c))/(64*a**4*c**2 - 32*a**
3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2
) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c*
*3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*
b**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^2+cx^4)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27

$$\int \frac{x}{(a+bx^2+cx^4)^3}\,dx = \frac{6\,c^2\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} + \frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2+20\,ac^2x^2-b^3+10\,abc}{4\,(cx^4+bx^2+a)^2(b^4-8\,ab^2c+16\,a^2c^2)}$$

input
```
integrate(x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
6*c^2*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*
c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(12*c^3*x^6 + 18*b*c^2*x^4 + 4*b^2*c*x^2 +
20*a*c^2*x^2 - b^3 + 10*a*b*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 1
6*a^2*c^2))
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.42

$$\int \frac{x}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{\frac{3\,c^3\,x^6}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{b^3-10\,a\,b\,c}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,(b^2\,c+5\,a\,c^2)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{9\,b\,c^2\,x^4}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^4\,(b^2+2\,a\,c)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$
$$+\frac{6\,c^2\,\mathrm{atan}\left(\frac{\left(x^2\,\left(\frac{36\,c^6}{a\,\left(4\,a\,c-b^2\right)^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{36\,b\,c^4\,\left(16\,a^2\,b\,c^4-8\,a\,b^3\,c^3+b^5\,c^2\right)}{a\,\left(4\,a\,c-b^2\right)^{15/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{72\,b\,c^6}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\,\left(b^4\,\left(4\,a\,c-b^2\right)^5+16\,a\right.}{72\,c^6}\right.}{\left(4\,a\,c-b^2\right)^{5/2}}$$

input
```
int(x/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((3*c^3*x^6)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3 - 10*a*b*c)/(4*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)) + (x^2*(5*a*c^2 + b^2*c))/(b^4 + 16*a^2*c^2 - 8*a*
b^2*c) + (9*b*c^2*x^4)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b
^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + (6*c^2*atan(((x^2*((36*c^6)
/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*(b^5*c
^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4))/(a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c))) + (72*b*c^6)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 +
 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(72*c^6)))/(4*a*
c - b^2)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1084, normalized size of antiderivative = 9.59

$$\int \frac{x}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x/(c*x^4+b*x^2+a)^3,x)
```

output
```
( - 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*
c**2 - 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*
*2*c**2*x**2 - 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*
atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
b))*a*b*c**3*x**4 - 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
- b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*b**3*c**2*x**4 - 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sq
rt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)
*sqrt(a) + b))*b**2*c**3*x**6 - 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt
(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*s
qrt(c)*sqrt(a) + b))*b*c**4*x**8 - 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*s
qrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*a**2*b*c**2 - 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(
2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**2*x**2 - 48*sqrt(2*sqrt(c)*sqrt(a) +
b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(
c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**3*x**4 - 24*sqrt(2*sqrt(c)*sqrt(
a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) +...
```

3.808 $\int \frac{1}{x(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 200

$$\int \frac{1}{x(a+bx^2+cx^4)^3}\,dx = \frac{b^2-2ac+bcx^2}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x^2}{4a^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{b(b^4-10ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3(b^2-4ac)^{5/2}} + \frac{\log(x)}{a^3} - \frac{\log(a+bx^2+cx^4)}{4a^3}$$

output
```
1/4*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*(2*b^4-15*a*b
^2*c+16*c^2*a^2+2*b*c*(-7*a*c+b^2)*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)
+1/2*b*(30*a^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))
/a^3/(-4*a*c+b^2)^(5/2)+ln(x)/a^3-1/4*ln(c*x^4+b*x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.71

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx$$
$$= \frac{a^2(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{a(2b^4-15ab^2c+16a^2c^2+2b^3cx^2-14abc^2x^2)}{(b^2-4ac)^2(a+bx^2+cx^4)} + 4\log(x) - \frac{\left(b^5-10ab^3c+30a^2bc^2+b^4\sqrt{b^2-4ac}-8ab^2c\sqrt{b^2-4\cdots}\right.}{(b^2-4ac\cdots}$$

input
```
Integrate[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

output
```
((a^2*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*
(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 - 14*a*b*c^2*x^2))/((b^2 -
4*a*c)^2*(a + b*x^2 + c*x^4)) + 4*Log[x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c
^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt
[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) +
 ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqr
t[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1434, 1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx$$
$$\Big\downarrow \; 1434$$
$$\frac{1}{2}\int \frac{1}{x^2\left(cx^4+bx^2+a\right)^3}dx^2$$
$$\Big\downarrow \; 1165$$

$$\frac{1}{2}\left(\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\int -\frac{3bcx^2+2\left(b^2-4ac\right)}{x^2\left(cx^4+bx^2+a\right)^2}dx^2}{2a\left(b^2-4ac\right)}\right)$$

$\big\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{3bcx^2+2\left(b^2-4ac\right)}{x^2\left(cx^4+bx^2+a\right)^2}dx^2}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$\big\downarrow$ 1235

$$\frac{1}{2}\left(\frac{\frac{16a^2c^2+2bcx^2\left(b^2-7ac\right)-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{2\left(\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)x^2\right)}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{\left(b^2-4ac\right)^2+bc\left(b^2-7ac\right)x^2}{x^2\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{16a^2c^2+2bcx^2\left(b^2-7ac\right)-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$\big\downarrow$ 1200

$$\frac{1}{2}\left(\frac{\frac{2\int \left(\frac{\left(4ac-b^2\right)^2}{ax^2}+\frac{-c\left(b^2-4ac\right)^2x^2-b\left(b^4-9acb^2+23a^2c^2\right)}{a\left(cx^4+bx^2+a\right)}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{16a^2c^2+2bcx^2\left(b^2-7ac\right)-15ab^2c+2b^4}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+\right.}\right.$$

$\big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{\frac{2\left(\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log\left(x^2\right)\left(b^2-4ac\right)^2}{a}-\frac{\left(b^2-4ac\right)^2\log\left(a+bx^2+cx^4\right)}{2a}\right)}{a\left(b^2-4ac\right)}+\frac{16a^2c^2+2bcx^2\left(b^2-7ac\right)-15ab^2c+2b}{a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}}{2a\left(b^2-4ac\right)}\right.$$

input
```
Int[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

output
```
((b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((2*b
^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x^2)/(a*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) + (2*((b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)^2*Log[
x^2])/a - ((b^2 - 4*a*c)^2*Log[a + b*x^2 + c*x^4])/(2*a)))/(a*(b^2 - 4*a*c
)))/(2*a*(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\frac{a b\,c^{2}\left(7 a c-b^{2}\right) x^{6}}{16 a^{2} c^{2}-8 a\,b^{2} c+b^{4}}-\frac{c a \left(16 a^{2} c^{2}-29 a\,b^{2} c+4 b^{4}\right) x^{4}}{2 \left(16 a^{2} c^{2}-8 a\,b^{2} c+b^{4}\right)}+\frac{a b \left(a^{2} c^{2}+6 a\,b^{2} c-b^{4}\right) x^{2}}{16 a^{2} c^{2}-8 a\,b^{2} c+b^{4}}-\frac{3 a^{2} \left(8 a^{2} c^{2}-7 a\,b^{2} c+b^{4}\right)}{2 \left(16 a^{2} c^{2}-8 a\,b^{2} c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{\left(16 a^{2} c^{3}-8 a\,b^{2} c^{2}+b^{4} c\right) \ln \left(c\,x^{4}+b \right.}{2 c}}{2 a^{3}}$
risch	$\frac{-\frac{b\,c^{2}\left(7 a c-b^{2}\right) x^{6}}{2 a^{2} \left(16 a^{2} c^{2}-8 a\,b^{2} c+b^{4}\right)}+\frac{c \left(16 a^{2} c^{2}-29 a\,b^{2} c+4 b^{4}\right) x^{4}}{4 \left(16 a^{2} c^{2}-8 a\,b^{2} c+b^{4}\right) a^{2}}-\frac{b \left(a^{2} c^{2}+6 a\,b^{2} c-b^{4}\right) x^{2}}{2 a^{2} \left(16 a^{2} c^{2}-8 a\,b^{2} c+b^{4}\right)}+\frac{6 a^{2} c^{2}-\frac{21}{4} a\,b^{2} c+\frac{3}{4} b^{4}}{a \left(16 a^{2} c^{2}-8 a\,b^{2} c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{\ln \left(x \right)}{a^{3}}+\frac{\left(\underline{\hphantom{R}}R=\operatorname{RootOf}\left(\right.\right.}{}$

input
```
int(1/x/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^3*((a*b*c^2*(7*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-1/2*c*a*(16*
a^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+a*b*(a^2*c^2+6*a*
b^2*c-b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-3/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4
)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^
4)*(1/2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)/c*ln(c*x^4+b*x^2+a)+2*(23*a^2*b*c^2
-9*a*b^3*c+b^5-1/2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))+ln(x)/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(188) = 376$.

Time = 0.37 (sec) , antiderivative size = 2017, normalized size of antiderivative = 10.08

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c
^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^6 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 13
2*a^3*b^2*c^3 - 64*a^4*c^4)*x^4 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2
 + 4*a^4*b*c^3)*x^2 + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8 + a^2*b
^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^
3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^4 + 2*(a*b^6
- 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2
*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 +
 a)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b
^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2
+ 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2
+ 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*
c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c
^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)*lo
g(x))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2
 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a
^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.62

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{\left(b^5-10\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^3b^4-8\,a^4b^2c+16\,a^5c^2\right)\sqrt{-b^2+4\,ac}} + \frac{3\,b^4c^2x^8-24\,ab^2c^3x^8+48\,a^2c^4x^8+6\,b^5cx^6-44\,ab^3c^2x^6+68\,a^2bc^3x^6+3\,b^6x^4-10\,ab^4cx^4-58\,a^2b^2c}{8\,\left(a^3b^4-8\,a^4b^2c+16\,a^5c^2\right)(cx} - \frac{\log\left(cx^4+bx^2+a\right)}{4\,a^3} + \frac{\log\left(x^2\right)}{2\,a^3}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*
a*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) + 1/8*(3*b
^4*c^2*x^8 - 24*a*b^2*c^3*x^8 + 48*a^2*c^4*x^8 + 6*b^5*c*x^6 - 44*a*b^3*c^
2*x^6 + 68*a^2*b*c^3*x^6 + 3*b^6*x^4 - 10*a*b^4*c*x^4 - 58*a^2*b^2*c^2*x^4
 + 128*a^3*c^3*x^4 + 10*a*b^5*x^2 - 72*a^2*b^3*c*x^2 + 92*a^3*b*c^2*x^2 +
9*a^2*b^4 - 66*a^3*b^2*c + 96*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*(c*x^4 + b*x^2 + a)^2) - 1/4*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*log(x^2)/
a^3
```

Mupad [B] (verification not implemented)

Time = 25.32 (sec) , antiderivative size = 9339, normalized size of antiderivative = 46.70

$$\int \frac{1}{x(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^2 + c*x^4)^3),x)
```

output
```
log(x)/a^3 + ((3*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(4*a*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (x^4*(4*b^4*c + 16*a^2*c^3 - 29*a*b^2*c^2))/(4*a^2*(b^4 + 16*
a^2*c^2 - 8*a*b^2*c)) - (b*x^2*(a^2*c^2 - b^4 + 6*a*b^2*c))/(2*a^2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^6*(7*a*c - b^2))/(2*a^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*
x^6) - (log((((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c -
 b^2)^5))^(1/2) + 1)*((b^2*c^3*(4*b^6 - 497*a^3*c^3 + 302*a^2*b^2*c^2 - 61
*a*b^4*c))/(a^4*(4*a*c - b^2)^4) - ((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b
^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) + 1)*((4*b^2*c^2*(b^4 + 23*a^2*c^2 -
 9*a*b^2*c))/(a^2*(4*a*c - b^2)^2) + (b*c^2*(a^3*(-(b^2*(b^4 + 30*a^2*c^2
- 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1/2) + 1)*(a*b + 3*b^2*x^2 - 10*a
*c*x^2))/a^3 + (2*b*c^3*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c -
b^2)^2)))/(4*a^3) + (b*c^4*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89
*a*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) - (b^2*c^4*(7*a*c - b^2)^2)/(a^
6*(4*a*c - b^2)^4) + (b^3*c^5*x^2*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6))*
(((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^2)^5))^(1
/2) - 1)*(((a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*a*c - b^
2)^5))^(1/2) - 1)*((4*b^2*c^2*(b^4 + 23*a^2*c^2 - 9*a*b^2*c))/(a^2*(4*a*c
- b^2)^2) - (b*c^2*(a^3*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*(4*
a*c - b^2)^5))^(1/2) - 1)*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (2*b*c^...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 4090, normalized size of antiderivative = 20.45

$$\int \frac{1}{x(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/x/(c*x^4+b*x^2+a)^3,x)
```

output
```
(60*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*c**
2 - 20*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*
*3*c + 120*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((s
qrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**
3*b**2*c**2*x**2 + 120*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
- b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a**3*b*c**3*x**4 + 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*s
qrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*a**2*b**5 - 40*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*
sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) + b))*a**2*b**4*c*x**2 + 20*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*
sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt
(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c**2*x**4 + 120*sqrt(2*sqrt(c)*sqrt(a)
+ b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqr
t(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**3*x**6 + 60*sqrt(2*sqrt(
c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**4*x**8 + 4*sqrt(
2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)...
```

3.809 $\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 255

$$\begin{aligned}\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx = & -\frac{3(b^2-5ac)(b^2-2ac)}{2a^3(b^2-4ac)^2x^2}+\frac{b^2-2ac+bcx^2}{4a(b^2-4ac)x^2(a+bx^2+cx^4)^2} \\ & +\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)x^2}{4a^2(b^2-4ac)^2x^2(a+bx^2+cx^4)} \\ & -\frac{3(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{5/2}} \\ & -\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx^2+cx^4)}{4a^4}\end{aligned}$$

output
```
-3/2*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/x^2+1/4*(b*c*x^2-2*a*c+b
^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2+a)^2+1/4*(3*b^4-20*a*b^2*c+20*c^2*a^2+
3*b*c*(-6*a*c+b^2)*x^2)/a^2/(-4*a*c+b^2)^2/x^2/(c*x^4+b*x^2+a)-3/2*(-20*a^
3*c^3+30*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2
))/a^4/(-4*a*c+b^2)^(5/2)-3*b*ln(x)/a^4+3/4*b*ln(c*x^4+b*x^2+a)/a^4
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{-\frac{2a}{x^2}+\frac{a^2(b^3-3abc+b^2cx^2-2ac^2x^2)}{(-b^2+4ac)(a+bx^2+cx^4)^2}-\frac{a(4b^5-29ab^3c+46a^2bc^2+4b^4cx^2-26ab^2c^2x^2+28a^2c^3x^2)}{(b^2-4ac)^2(a+bx^2+cx^4)}-12b\log(x)+\frac{3\left(b^6-10ab^4c+30a\right.}{}}{}$$

input
```
Integrate[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

output
```
((-2*a)/x^2 + (a^2*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((-b^2 + 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4
*c*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 +
 c*x^4)) - 12*b*Log[x] + (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^
3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqr
t[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2)
+ (3*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a
*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^4)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1434, 1165, 25, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{1}{x^4(cx^4+bx^2+a)^3}dx^2$$

$$\Big\downarrow \; 1165$$

$$\frac{1}{2}\left(\frac{-2ac+b^2+bcx^2}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\int -\frac{3b^2+4cx^2b-10ac}{x^4\left(cx^4+bx^2+a\right)^2}dx^2}{2a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int \frac{3b^2+4cx^2b-10ac}{x^4\left(cx^4+bx^2+a\right)^2}dx^2}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \; 1235$$

$$\frac{1}{2}\left(\frac{\frac{20a^2c^2+3bcx^2\left(b^2-6ac\right)-20ab^2c+3b^4}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{6\left(bc\left(b^2-6ac\right)x^2+\left(b^2-5ac\right)\left(b^2-2ac\right)\right)}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\frac{6\int \frac{bc\left(b^2-6ac\right)x^2+\left(b^2-5ac\right)\left(b^2-2ac\right)}{x^4\left(cx^4+bx^2+a\right)}dx^2}{a\left(b^2-4ac\right)}+\frac{20a^2c^2+3bcx^2\left(b^2-6ac\right)-20ab^2c+3b^4}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}}{2a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{2}\left(\frac{\frac{6\int \left(-\frac{b\left(4ac-b^2\right)^2}{a^2x^2}+\frac{b^6-9acb^4+23a^2c^2b^2+c\left(b^2-4ac\right)^2x^2b-10a^3c^3}{a^2\left(cx^4+bx^2+a\right)}+\frac{\left(b^2-5ac\right)\left(b^2-2ac\right)}{ax^4}\right)dx^2}{a\left(b^2-4ac\right)}+\frac{20a^2c^2+3bcx^2\left(b^2-6ac\right)-20ab^2c+3b^4}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}}{2a\left(b^2-4ac\right)}+\frac{}{2a}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{\frac{20a^2c^2+3bcx^2\left(b^2-6ac\right)-20ab^2c+3b^4}{ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{6\left(-\frac{b\log\left(x^2\right)\left(b^2-4ac\right)^2}{a^2}+\frac{b\left(b^2-4ac\right)^2\log\left(a+bx^2+cx^4\right)}{2a^2}-\frac{\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\text{arctanh}}{a^2\sqrt{b^2-4ac}}\right.}{a\left(b^2-4ac\right)}}{2a\left(b^2-4ac\right)}\right.$$

input
```
Int[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

output
```
((b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)^2) + (
(3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x^2)/(a*(b^2 - 4*a*
c)*x^2*(a + b*x^2 + c*x^4)) + (6*(-(((b^2 - 5*a*c)*(b^2 - 2*a*c))/(a*x^2))
 - ((b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x^2)
/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)^2*Log[x^2]
)/a^2 + (b*(b^2 - 4*a*c)^2*Log[a + b*x^2 + c*x^4])/(2*a^2)))/(a*(b^2 - 4*a
*c)))/(2*a*(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.62

method	result
default	$-\frac{\frac{\frac{c^2 a\left(14a^2c^2-13a\,b^2c+2b^4\right)x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{abc\left(74a^2c^2-55a\,b^2c+8b^4\right)x^4}{32a^2c^2-16a\,b^2c+2b^4}+\frac{a\left(18a^3c^3+7a^2b^2c^2-12a\,b^4c+2b^6\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{a^2b\left(58a^2c^2-36a\,b^2c+5b^4\right)}{32a^2c^2-16a\,b^2c+2b^4}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{3\left(-1\right.}{}}{2a^4}$
risch	$\frac{-\frac{3c^2\left(10a^2c^2-7a\,b^2c+b^4\right)x^8}{2a^3\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3bc\left(46a^2c^2-29a\,b^2c+4b^4\right)x^6}{4\left(16a^2c^2-8a\,b^2c+b^4\right)a^3}-\frac{\left(50a^3c^3+7a^2b^2c^2-18a\,b^4c+3b^6\right)x^4}{2a^3\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{b\left(122a^2c^2-68a\,b^2c+9b^4\right)x^2}{4a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{1}{2a}}{x^2\left(c\,x^4+b\,x^2+a\right)^2}-\frac{3}{}$

input
```
int(1/x^3/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^4*((c^2*a*(14*a^2*c^2-13*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^6+1/2*a*b*c*(74*a^2*c^2-55*a*b^2*c+8*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4
+a*(18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^2+1/2*a^2*b*(58*a^2*c^2-36*a*b^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c
*x^4+b*x^2+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(-16*a^2*b*c^3+8*a*b^3*c
^2-b^5*c)/c*ln(c*x^4+b*x^2+a)+2*(10*a^3*c^3-23*a^2*b^2*c^2+9*a*b^4*c-b^6-1
/2*(-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))))-1/2/a^3/x^2-3*b*ln(x)/a^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1144 vs. $2(241) = 482$.

Time = 0.57 (sec) , antiderivative size = 2312, normalized size of antiderivative = 9.07

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/4*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*
c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^8 + 3*(4*a*b^7*c - 4
5*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^6 + 2*(3*a*b^8 - 30*a^2
*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^4 + (9*a^2*b^7 -
 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x^2 + 3*((b^6*c^2 - 10*a
*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^10 + 2*(b^7*c - 10*a*b^5*c^2 + 3
0*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^8 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40
*a^3*b^2*c^3 - 40*a^4*c^4)*x^6 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2
- 20*a^4*b*c^3)*x^4 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^
3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*
x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^7*c^2 - 12*a*b^5*
c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a
^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32
*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c
^2 - 64*a^4*b^2*c^3)*x^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a
^5*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a) + 12*((b^7*c^2 - 12*a*b^5*c^3 + 48*a
^2*b^3*c^4 - 64*a^3*b*c^5)*x^10 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3
 - 64*a^3*b^2*c^4)*x^8 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c
^3 - 128*a^4*b*c^4)*x^6 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^
4*b^2*c^3)*x^4 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)^3} \, dx = \frac{3 \left(b^6 - 10 \, ab^4 c + 30 \, a^2 b^2 c^2 - 20 \, a^3 c^3\right) \arctan\left(\frac{2 \, cx^2 + b}{\sqrt{-b^2 + 4 \, ac}}\right)}{2 \left(a^4 b^4 - 8 \, a^5 b^2 c + 16 \, a^6 c^2\right) \sqrt{-b^2 + 4 \, ac}} - \frac{9 \, b^5 c^2 x^8 - 72 \, ab^3 c^3 x^8 + 144 \, a^2 bc^4 x^8 + 18 \, b^6 cx^6 - 136 \, ab^4 c^2 x^6 + 236 \, a^2 b^2 c^3 x^6 + 56 \, a^3 c^4 x^6 + 9 \, b^7 x^4 - 38}{8 \left(a^4 b^4 - 8\right.} + \frac{3 \, b \log\left(cx^4 + bx^2 + a\right)}{4 \, a^4} - \frac{3 \, b \log\left(x^2\right)}{2 \, a^4} + \frac{3 \, bx^2 - a}{2 \, a^4 x^2}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x^2 + b)/
sqrt(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*
c)) - 1/8*(9*b^5*c^2*x^8 - 72*a*b^3*c^3*x^8 + 144*a^2*b*c^4*x^8 + 18*b^6*c
*x^6 - 136*a*b^4*c^2*x^6 + 236*a^2*b^2*c^3*x^6 + 56*a^3*c^4*x^6 + 9*b^7*x^
4 - 38*a*b^5*c*x^4 - 110*a^2*b^3*c^2*x^4 + 436*a^3*b*c^3*x^4 + 26*a*b^6*x^
2 - 192*a^2*b^4*c*x^2 + 316*a^3*b^2*c^2*x^2 + 72*a^4*c^3*x^2 + 19*a^2*b^5
- 144*a^3*b^3*c + 260*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(c*
x^4 + b*x^2 + a)^2) + 3/4*b*log(c*x^4 + b*x^2 + a)/a^4 - 3/2*b*log(x^2)/a^
4 + 1/2*(3*b*x^2 - a)/(a^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 26.21 (sec) , antiderivative size = 10074, normalized size of antiderivative = 39.51

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)^3} dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^2 + c*x^4)^3),x)
```

output
```
(log(((27*c^5*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6)
- ((3*b - 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*
(4*a*c - b^2)^5))^(1/2))*((9*c^3*(4*b^10 - 100*a^5*c^5 + 342*a^2*b^6*c^2 -
 837*a^3*b^4*c^3 + 780*a^4*b^2*c^4 - 61*a*b^8*c))/(a^6*(4*a*c - b^2)^4) -
((3*b - 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4
*a*c - b^2)^5))^(1/2))*((6*c^3*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2
*a*b^4*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*(3*b - 3*a^4*(-(b^6 - 20*a^3*c^3
 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b
^2*x^2 - 10*a*c*x^2))/a^4 + (12*b*c^2*(b^6 - 10*a^3*c^3 + 23*a^2*b^2*c^2 -
 9*a*b^4*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4) + (9*b*c^4*x^2*(6*b^8 + 900*a
^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b
^2)^4)))/(4*a^4) + (27*b*c^4*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2)/(a^9*(4*a*c
 - b^2)^4))*((27*c^5*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b
^2)^6) - ((3*b + 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^
2/(a^8*(4*a*c - b^2)^5))^(1/2))*((9*c^3*(4*b^10 - 100*a^5*c^5 + 342*a^2*b^
6*c^2 - 837*a^3*b^4*c^3 + 780*a^4*b^2*c^4 - 61*a*b^8*c))/(a^6*(4*a*c - b^2
)^4) - ((3*b + 3*a^4*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/
(a^8*(4*a*c - b^2)^5))^(1/2))*((6*c^3*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*
c^2 - 2*a*b^4*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*(3*b + 3*a^4*(-(b^6 - 20*
a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 4941, normalized size of antiderivative = 19.38

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)^3} dx = \text{Too large to display}$$

input
```
int(1/x^3/(c*x^4+b*x^2+a)^3,x)
```

output
```
(120*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*b*c*
*3*x**2 - 180*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**4*b**3*c**2*x**2 + 240*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(
a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq
rt(a) + b))*a**4*b**2*c**3*x**4 + 240*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*s
qrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*a**4*b*c**4*x**6 + 60*sqrt(2*sqrt(c)*sqrt(a) + b)*
sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**5*c*x**2 - 360*sqrt(2*sqrt(c)*sqrt
(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2
*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**4*c**2*x**4 - 240*sqrt(2*
sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**3*c**3*x**6 +
 240*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2
*c**4*x**8 + 120*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b
))*a**3*b*c**5*x**10 - 6*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqr...
```

3.810 $\int \frac{x^{10}}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 382

$$\begin{aligned}\int \frac{x^{10}}{(a+bx^2+cx^4)^3}\,dx &= -\frac{bx}{4c^2(b^2-4ac)}+\frac{x^7(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}\\
&\quad-\frac{x(ab(b^2-16ac)+(b^4-17ab^2c+28a^2c^2)x^2)}{8c^2(b^2-4ac)^2(a+bx^2+cx^4)}\\
&\quad+\frac{3\left(b^4-9ab^2c+28a^2c^2-\frac{b(b^4-11ab^2c+44a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{5/2}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}\\
&\quad+\frac{3\left(b^4-9ab^2c+28a^2c^2+\frac{b(b^4-11ab^2c+44a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{5/2}(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-1/4*b*x/c^2/(-4*a*c+b^2)+1/4*x^7*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
^2-1/8*x*(a*b*(-16*a*c+b^2)+(28*a^2*c^2-17*a*b^2*c+b^4)*x^2)/c^2/(-4*a*c+b
^2)^2/(c*x^4+b*x^2+a)+3/16*(b^4-9*a*b^2*c+28*c^2*a^2-b*(44*a^2*c^2-11*a*b^
2*c+b^4)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*2^(1/2)/c^(5/2)/(-4*a*c+b^2)^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16
*(b^4-9*a*b^2*c+28*c^2*a^2+b*(44*a^2*c^2-11*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2
))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(5/2)/
(-4*a*c+b^2)^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.19

$$\int \frac{x^{10}}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{\frac{2x(2b^5-17ab^3c+48a^2bc^2-5b^4cx^2+37ab^2c^2x^2-44a^2c^3x^2)}{(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{4(b^4x^3+ab^2x(b-4cx^2)+a^2cx(-3b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3\sqrt{2}\sqrt{c}\left(-b^5+11ab^3c-44a^2bc^2\right.}{}}{}$$

input
```
Integrate[x^10/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((2*x*(2*b^5 - 17*a*b^3*c + 48*a^2*b*c^2 - 5*b^4*c*x^2 + 37*a*b^2*c^2*x^2
- 44*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*(b^4*x^3 + a
*b^2*x*(b - 4*c*x^2) + a^2*c*x*(-3*b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)^2) + (3*Sqrt[2]*Sqrt[c]*(-b^5 + 11*a*b^3*c - 44*a^2*b*c^2 + b^4
*Sqrt[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c] + 28*a^2*c^2*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*
a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^5 - 11*a*b
^3*c + 44*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 - 4*a*c]
+ 28*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^3)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1440, 1598, 27, 1602, 27, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx^2+cx^4)^3} \, dx$$

$$\Big\downarrow \text{1440}$$

$$\frac{x^7(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int \frac{x^6(bx^2+14a)}{(cx^4+bx^2+a)^2}dx}{4(b^2-4ac)}$$

$$\Big\downarrow \text{1598}$$

$$\frac{x^7(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{\int \frac{3x^4(20ab-(b^2-28ac)x^2)}{cx^4+bx^2+a}dx}{2(b^2-4ac)}-\frac{x^5(12ab-x^2(b^2-28ac))}{2(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^7(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{3\int \frac{x^4(20ab-(b^2-28ac)x^2)}{cx^4+bx^2+a}dx}{2(b^2-4ac)}-\frac{x^5(12ab-x^2(b^2-28ac))}{2(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)}$$

$$\Big\downarrow \text{1602}$$

$$\frac{x^7(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{3\left(-\frac{\int -\frac{3x^2(b(b^2-8ac)x^2+a(b^2-28ac))}{cx^4+bx^2+a}dx}{3c}-\frac{x^3(b^2-28ac)}{3c}\right)}{2(b^2-4ac)}-\frac{x^5(12ab-x^2(b^2-28ac))}{2(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\frac{3\left(\frac{\int\frac{x^2\left(b\left(b^2-8ac\right)x^2+a\left(b^2-28ac\right)\right)}{cx^4+bx^2+a}dx}{c}-\frac{x^3\left(b^2-28ac\right)}{3c}\right)}{2(b^2-4ac)}-\frac{x^5\left(12ab-x^2\left(b^2-28ac\right)\right)}{2(b^2-4ac)(a+bx^2+cx^4)}}{4\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1602$$

$$\frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\frac{3\left(\frac{bx\left(b^2-8ac\right)}{c}-\frac{\int\frac{\left(b^4-9acb^2+28a^2c^2\right)x^2+ab\left(b^2-8ac\right)}{c\left(cx^4+bx^2+a\right)}dx}{c}-\frac{x^3\left(b^2-28ac\right)}{3c}\right)}{2(b^2-4ac)}-\frac{x^5\left(12ab-x^2\left(b^2-28ac\right)\right)}{2(b^2-4ac)(a+bx^2+cx^4)}}{4\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1480$$

$$\frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{bx\left(b^2-8ac\right)}{c}-\frac{\frac{\frac{1}{2}\left(-\frac{b\left(44a^2c^2-11ab^2c+b^4\right)}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\int\frac{\cdots}{cx^2+\frac{1}{2}\left(b-\cdots\right.}}{c}}{c}\cdots\right)}{2(b^2-4ac)}\cdots}{4\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{bx\left(b^2-8ac\right)}{c}-\frac{\frac{\left(-\frac{b\left(44a^2c^2-11ab^2c+b^4\right)}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{\sqrt{b^2-\cdots}}}\right.}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{c}\cdots\right)}{2(b^2-4ac)}\cdots}{4\left(b^2-4ac\right)}$$

input
```
Int[x^10/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(x^7*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (-1/2*(x^5*(
12*a*b - (b^2 - 28*a*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*(-1
/3*((b^2 - 28*a*c)*x^3)/c + ((b*(b^2 - 8*a*c)*x)/c - (((b^4 - 9*a*b^2*c +
28*a^2*c^2 - (b*(b^4 - 11*a*b^2*c + 44*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + ((b^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^5 - 11*a*b^3*
c + 44*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)/c))/
(2*(b^2 - 4*a*c)))/(4*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.87

method	result
risch	$\frac{-\frac{\left(44a^2c^2-37a\,b^2c+5b^4\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)c}+\frac{b\left(4a^2c^2+20a\,b^2c-3b^4\right)x^5}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(28a^2c^2-49a\,b^2c+6b^4\right)x^3}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{3a^2b\left(8ac-b^2\right)x}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4\right.}\right.}{}$
default	$\frac{-\frac{\left(44a^2c^2-37a\,b^2c+5b^4\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)c}+\frac{b\left(4a^2c^2+20a\,b^2c-3b^4\right)x^5}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(28a^2c^2-49a\,b^2c+6b^4\right)x^3}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{3a^2b\left(8ac-b^2\right)x}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{\frac{3\left(28a^2c^2\sqrt{-4ac+b^2}-\right.}{}}{}$

input
```
int(x^10/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*(44*a^2*c^2-37*a*b^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^7+1/8*b
*(4*a^2*c^2+20*a*b^2*c-3*b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*a/c^2
*(28*a^2*c^2-49*a*b^2*c+6*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/8*a^2*b*(8
*a*c-b^2)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/16/c^2*sum
(((28*a^2*c^2-9*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2-a*b*(8*a*c-b^
2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_
Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4279 vs. $2(336) = 672$.

Time = 0.55 (sec) , antiderivative size = 4279, normalized size of antiderivative = 11.20

$$\int \frac{x^{10}}{(a + bx^2 + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a + bx^2 + cx^4)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**10/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{10}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{10}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*((5*b^4*c - 37*a*b^2*c^2 + 44*a^2*c^3)*x^7 + (3*b^5 - 20*a*b^3*c - 4*
a^2*b*c^2)*x^5 + (6*a*b^4 - 49*a^2*b^2*c + 28*a^3*c^2)*x^3 + 3*(a^2*b^3 -
8*a^3*b*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 +
(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 +
16*a^3*b*c^4)*x^2) + 3/8*integrate((a*b^3 - 8*a^2*b*c + (b^4 - 9*a*b^2*c +
 28*a^2*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*
c^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2430 vs. $2(336) = 672$.

Time = 2.06 (sec) , antiderivative size = 2430, normalized size of antiderivative = 6.36

$$\int \frac{x^{10}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^10/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^6*c - 2*b^7*c + 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 +
24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*a*b^5*c^2 - 2*b^6*c^2 - 128*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^
3 - 160*a^2*b^3*c^3 + 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^4 + 256*a^3*b*c^4 - 192*a^2*b^2*c^4 + 448*a^3*c^5 + sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 14*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 96*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 2*(b^2 - 4*a*c)*b^5*c - 24*(b^2 - 4*a*c
)*a*b^3*c^2 + 2*(b^2 - 4*a*c)*b^4*c^2 + 64*(b^2 - 4*a*c)*a^2*b*c^3 - 20...
```

Mupad [B] (verification not implemented)

Time = 22.98 (sec) , antiderivative size = 10912, normalized size of antiderivative = 28.57

$$\int \frac{x^{10}}{(a + bx^2 + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int(x^10/(a + b*x^2 + c*x^4)^3,x)
```

output
```
- ((x^3*(6*a*b^4 + 28*a^3*c^2 - 49*a^2*b^2*c))/(8*c^2*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)) + (x^7*(5*b^4 + 44*a^2*c^2 - 37*a*b^2*c))/(8*c*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)) - (b*x^5*(4*a^2*c^2 - 3*b^4 + 20*a*b^2*c))/(8*c^2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) - (3*a^2*b*x*(8*a*c - b^2))/(8*c^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*
x^6) - atan(((((3*(256*a*b^13*c^3 + 2097152*a^7*b*c^9 - 7168*a^2*b^11*c^4
+ 81920*a^3*b^9*c^5 - 491520*a^4*b^7*c^6 + 1638400*a^5*b^5*c^7 - 2883584*a
^6*b^3*c^8))/(512*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c
^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*((9*(b^
4*(-(4*a*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2
+ 8620*a^3*b^13*c^3 - 63440*a^4*b^11*c^4 + 316864*a^5*b^9*c^5 - 1069824*a^
6*b^7*c^6 + 2343936*a^7*b^5*c^7 - 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*
c - b^2)^15)^(1/2) + 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/
(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 76
80*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b
^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^
14)))^(1/2)*(256*b^11*c^5 - 5120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2
*b^7*c^7 - 163840*a^3*b^5*c^8 + 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^
8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))))*((9*(b^4*(-(4*a
*c - b^2)^15)^(1/2) - b^19 + 1720320*a^9*b*c^9 - 769*a^2*b^15*c^2 + 862...
```

Reduce [B] (verification not implemented)

Time = 4.85 (sec) , antiderivative size = 6796, normalized size of antiderivative = 17.79

$$\int \frac{x^{10}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^10/(c*x^4+b*x^2+a)^3,x)
```

output
```
( - 336*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*c**3 + 60*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**2 - 672*sqrt(a)*sqrt(2*sqrt(c)*s
qrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*a**3*b*c**3*x**2 - 672*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 b))*a**3*c**4*x**4 - 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**4*
c + 120*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
 b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c**2*x**2 - 216*
sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*
sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**3*x**4 - 672*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**4*x**6 - 336*sqrt(a)*sqrt(2*sqrt
(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*s
qrt(c)*sqrt(a) + b))*a**2*c**5*x**8 - 12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a*b**5*c*x**2 + 48*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**...
```

3.811 $\int \frac{x^8}{(a+bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 338

$$\int \frac{x^8}{(a+bx^2+cx^4)^3} dx = \frac{x^5(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x(a(b^2+20ac)+b(b^2+8ac)x^2)}{8c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\left(b^3-16abc-\frac{b^4-18ab^2c-40a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(16ab-\frac{b^3}{c}-\frac{b^4-18ab^2c-40a^2c^2}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
1/4*x^5*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x*(a*(20*a*c+b^2)+b
*(8*a*c+b^2)*x^2)/c/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*(b^3-16*a*b*c-(-40
*a^2*c^2-18*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(-4*a*c+b^2)^2/(b-(-4*a*c+b^2)^(
1/2))^(1/2)-1/16*(16*a*b-b^3/c-(-40*a^2*c^2-18*a*b^2*c+b^4)/c/(-4*a*c+b^2)
^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(
1/2)/(-4*a*c+b^2)^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.13

$$\int \frac{x^8}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{\frac{2x(-2b^4+11ab^2c-36a^2c^2+b^3cx^2-16abc^2x^2)}{(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{4(-2a^2cx+b^3x^3+abx(b-3cx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{\sqrt{2}\sqrt{c}\left(-b^4+18ab^2c+40a^2c^2+b^3\sqrt{b^2-4ac}-16abc\sqrt{b^2-}\right.}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4a}}}}{16c^2}$$

input
```
Integrate[x^8/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((2*x*(-2*b^4 + 11*a*b^2*c - 36*a^2*c^2 + b^3*c*x^2 - 16*a*b*c^2*x^2))/((b
^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (4*(-2*a^2*c*x + b^3*x^3 + a*b*x*(b -
 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-b^4
 + 18*a*b^2*c + 40*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 16*a*b*c*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*
a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b^4 - 18*a*b^2
*c - 40*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 16*a*b*c*Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1440, 1598, 1602, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{ 1440}$$

$$\frac{x^5(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{\int \frac{x^4(10a-bx^2)}{(cx^4+bx^2+a)^2}dx}{4(b^2-4ac)}$$

$$\Big\downarrow \; 1598$$

$$\frac{x^5(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{\int\frac{x^2((b^2+20ac)x^2+36ab)}{cx^4+bx^2+a}dx}{2(b^2-4ac)}-\frac{x^3(x^2(20ac+b^2)+12ab)}{2(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)}$$

$$\Big\downarrow \; 1602$$

$$\frac{x^5(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{\frac{x(20ac+b^2)}{c}-\frac{\int\frac{b(b^2-16ac)x^2+a(b^2+20ac)}{cx^4+bx^2+a}dx}{c}}{2(b^2-4ac)}-\frac{x^3(x^2(20ac+b^2)+12ab)}{2(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)}$$

$$\Big\downarrow \; 1480$$

$$\frac{x^5(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{\frac{x(20ac+b^2)}{c}-\frac{\frac{1}{2}\left(-\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{c}}{2(b^2-4ac)}-\frac{x^3(\cdots}{2(b\cdots}}{4(b^2-4ac)}$$

$$\Big\downarrow \; 218$$

$$\frac{x^5(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{\frac{x(20ac+b^2)}{c}-\frac{\frac{\left(-\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}}{2(b^2-4ac)}-\frac{x^3(x^2(20\cdots}{2(b^2-4a\cdots}}{4(b^2-4ac)}$$

input

```
Int[x^8/(a + b*x^2 + c*x^4)^3,x]
```

output

```
(x^5*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (-1/2*(x^3*(
12*a*b + (b^2 + 20*a*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (((b^2
 + 20*a*c)*x)/c - (((b^3 - 16*a*b*c - (b^4 - 18*a*b^2*c - 40*a^2*c^2)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3 - 16*a*b*c + (b^4 - 18
*a*b^2*c - 40*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/c)
/(2*(b^2 - 4*a*c)))/(4*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.86

method	result
risch	$\frac{-\frac{b(16ac-b^2)x^7}{8(16a^2c^2-8a\,b^2c+b^4)}-\frac{(36a^2c^2+5a\,b^2c+b^4)x^5}{8c(16a^2c^2-8a\,b^2c+b^4)}-\frac{ab(14ac+b^2)x^3}{4c(16a^2c^2-8a\,b^2c+b^4)}-\frac{a^2(20ac+b^2)x}{8c(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}+\sum_{\_R=\mathrm{RootOf}(\_Z^4c+\_Z^2b+a)}$
default	$\frac{-\frac{b(16ac-b^2)x^7}{8(16a^2c^2-8a\,b^2c+b^4)}-\frac{(36a^2c^2+5a\,b^2c+b^4)x^5}{8c(16a^2c^2-8a\,b^2c+b^4)}-\frac{ab(14ac+b^2)x^3}{4c(16a^2c^2-8a\,b^2c+b^4)}-\frac{a^2(20ac+b^2)x}{8c(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}+\frac{(-16abc\sqrt{-4ac+b^2}+b^3\sqrt{-4ac+b}}{8c\sqrt{}}$

input
```
int(x^8/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*b*(16*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-1/8*(36*a^2*c^2+5*a*b^
2*c+b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4/c*a*b*(14*a*c+b^2)/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^3-1/8*a^2*(20*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x)
/(c*x^4+b*x^2+a)^2+1/16/c*sum((-b*(16*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*
_R^2+a*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_
R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3725 vs. 2(294) = 588.

Time = 0.32 (sec) , antiderivative size = 3725, normalized size of antiderivative = 11.02

$$\int \frac{x^8}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^8}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((b^3*c - 16*a*b*c^2)*x^7 - (b^4 + 5*a*b^2*c + 36*a^2*c^2)*x^5 - 2*(a*
b^3 + 14*a^2*b*c)*x^3 - (a^2*b^2 + 20*a^3*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 +
16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*
a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2
*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2) - 1/8*integrate(-(a*b^2 + 2
0*a^2*c + (b^3 - 16*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4558 vs. $2(294) = 588$.

Time = 1.40 (sec) , antiderivative size = 4558, normalized size of antiderivative = 13.49

$$\int \frac{x^8}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/64*(2*b^13*c^4 - 68*a*b^11*c^5 + 688*a^2*b^9*c^6 - 2688*a^3*b^7*c^7 + 2
048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9 - 20480*a^6*b*c^10 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^13*c^2 + 34*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^11*c^3 + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^12*c^3 - 344*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 - 60*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 1344*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 448*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 30*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9*c^5 - 1024*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 896*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 224*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 - 5632*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 - 1536*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 448*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 10240*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 + 5120*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 + 768*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 -...
```

Mupad [B] (verification not implemented)

Time = 19.31 (sec) , antiderivative size = 9575, normalized size of antiderivative = 28.33

$$\int \frac{x^8}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^8/(a + b*x^2 + c*x^4)^3,x)
```

output
```
atan(((((5242880*a^7*c^8 - 256*a*b^12*c^2 + 61440*a^3*b^8*c^4 - 655360*a^4
*b^6*c^5 + 2949120*a^5*b^4*c^6 - 6291456*a^6*b^2*c^7)/(512*(b^12*c + 4096*
a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^
4*c^5 - 6144*a^5*b^2*c^6)) - (x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) -
1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9
*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a
*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20
*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^
12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 +
 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120
*a*b^9*c^4 - 262144*a^5*b*c^8 + 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 3
27680*a^4*b^3*c^7))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c
^3 - 256*a^3*b^2*c^4)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*
a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 4
3776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c
- 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^13 + b^20*c^3 - 4
0*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 -
 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120
*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(b^8 + 800*a^4*c^4 + 31
4*a^2*b^4*c^2 + 208*a^3*b^2*c^3 - 36*a*b^6*c))/(32*(b^8*c + 256*a^4*c^5...
```

Reduce [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 5719, normalized size of antiderivative = 16.92

$$\int \frac{x^8}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^8/(c*x^4+b*x^2+a)^3,x)
```

output

```
(104*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**2 - 2*sqrt(a)*sqrt(
2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sq
rt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c + 208*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a
) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*a**2*b**2*c**2*x**2 + 208*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*a
tan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b
))*a**2*b*c**3*x**4 - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*c*x
**2 + 100*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
 - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c**2*x**4 + 208*s
qrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**3*x**6 + 104*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**4*x**8 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(
a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq
rt(a) + b))*b**5*c*x**4 - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*c
**2*x**6 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c**3*x**8 - 8...
```

3.812 $\int \frac{x^6}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 298

$$\int \frac{x^6}{(a+bx^2+cx^4)^3}\,dx = \frac{x^3(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3x(4ab+(b^2+4ac)x^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3\left(b^2+4ac-\frac{b(b^2+12ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}} + \frac{3\left(b^2+4ac+\frac{b(b^2+12ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/4*x^3*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/8*x*(4*a*b+(4*a*c+b^2
)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/16*(b^2+4*a*c-b*(12*a*c+b^2)/(-4*a
*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1
/2)/c^(1/2)/(-4*a*c+b^2)^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16*(b^2+4*a*c+b*
(12*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^
(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.15

$$\int \frac{x^6}{(a+bx^2+cx^4)^3}\,dx$$
$$=\frac{\frac{4b^3x+8abcx+6b^2cx^3+24ac^2x^3}{(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{4(b^2x^3+ax(b-2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3\sqrt{2}\sqrt{c}\left(-b^3-12abc+b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+}{16c}$$

input
```
Integrate[x^6/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((4*b^3*x + 8*a*b*c*x + 6*b^2*c*x^3 + 24*a*c^2*x^3)/((b^2 - 4*a*c)^2*(a +
b*x^2 + c*x^4)) - (4*(b^2*x^3 + a*x*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)^2) + (3*Sqrt[2]*Sqrt[c]*(-b^3 - 12*a*b*c + b^2*Sqrt[b^2 - 4*a
*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[
2]*Sqrt[c]*(b^3 + 12*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1440, 27, 1598, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2+cx^4)^3}\,dx$$
$$\Big\downarrow \text{1440}$$
$$\frac{x^3(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int \frac{3x^2(2a-bx^2)}{(cx^4+bx^2+a)^2}dx}{4(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3\int \frac{x^2(2a-bx^2)}{(cx^4+bx^2+a)^2}dx}{4(b^2-4ac)}$$

$$\Big\downarrow \text{1598}$$

$$\frac{x^3(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3\left(\frac{\int \frac{4ab-(b^2+4ac)x^2}{cx^4+bx^2+a}dx}{2(b^2-4ac)}-\frac{x(x^2(4ac+b^2)+4ab)}{2(b^2-4ac)(a+bx^2+cx^4)}\right)}{4(b^2-4ac)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{x^3(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-$$
$$\frac{3\left(\frac{-\frac{1}{2}\left(\frac{b(12ac+b^2)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(-\frac{12abc+b^3}{\sqrt{b^2-4ac}}+4ac+b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{2(b^2-4ac)}-\frac{x(x^2(4ac+b^2)+4ab)}{2(b^2-4ac)(a+bx^2+cx^4)}\right)}{4(b^2-4ac)}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^3(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-$$
$$\frac{3\left(\frac{-\frac{\left(\frac{b(12ac+b^2)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\left(-\frac{12abc+b^3}{\sqrt{b^2-4ac}}+4ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}}{2(b^2-4ac)}-\frac{x(x^2(4ac+b^2)+4ab)}{2(b^2-4ac)(a+bx^2+cx^4)}\right)}{4(b^2-4ac)}$$

input
```
Int[x^6/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(x^3*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*(-1/2*(x*
(4*a*b + (b^2 + 4*a*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (-(((b^
2 + 4*a*c - (b^3 + 12*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]
])) - ((b^2 + 4*a*c + (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[
b^2 - 4*a*c]]))/(2*(b^2 - 4*a*c))))/(4*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.84

method	result
risch	$\frac{\frac{3c(4ac+b^2)x^7}{8(16a^2c^2-8a\,b^2c+b^4)}+\frac{b(16ac+5b^2)x^5}{128a^2c^2-64a\,b^2c+8b^4}-\frac{(4ac-19b^2)a\,x^3}{8(16a^2c^2-8a\,b^2c+b^4)}+\frac{3a^2bx}{2(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4c+\_Z^2b+a)}\left(\right.\right.}{}$
default	$\frac{\frac{3c(4ac+b^2)x^7}{8(16a^2c^2-8a\,b^2c+b^4)}+\frac{b(16ac+5b^2)x^5}{128a^2c^2-64a\,b^2c+8b^4}-\frac{(4ac-19b^2)a\,x^3}{8(16a^2c^2-8a\,b^2c+b^4)}+\frac{3a^2bx}{2(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}+\frac{3c\left(\frac{(4ac\sqrt{-4ac+b^2}+b^2\sqrt{-4ac+b^2}+1}{8c\sqrt{-4ac+b}}\right.}{}$

input
```
int(x^6/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/8*c*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*b*(16*a*c+5*b^2)/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(4*a*c-19*b^2)*a/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^3+3/2*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/16*sum((
(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2-4*a*b/(16*a^2*c^2-8*a*b^2*c+b^
4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3128 vs. 2(254) = 508.

Time = 0.15 (sec) , antiderivative size = 3128, normalized size of antiderivative = 10.50

$$\int \frac{x^6}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(292) = 584$.

Time = 10.74 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.10

$$\int \frac{x^6}{\left(a+bx^2+cx^4\right)^3}\,dx$$

$$= \frac{12a^2bx + x^7\cdot\left(12ac^2+3b^2c\right) + x^5\cdot\left(16abc+5b^3\right) + x^3(}{128a^4c^2 - 64a^3b^2c + 8a^2b^4 + x^8\cdot\left(128a^2c^4-64ab^2c^3+8b^4c^2\right) + x^6\cdot\left(256a^2bc^3-128ab^3c^2+16b^5c\right) + x^4}$$

$$+\mathrm{RootSum}\Big(t^4\cdot\left(68719476736a^{10}c^{11} - 171798691840a^9b^2c^{10} + 193273528320a^8b^4c^9 - 128849018880a^7\right.$$

input
```
integrate(x**6/(c*x**4+b*x**2+a)**3,x)
```

output
```
(12*a**2*b*x + x**7*(12*a*c**2 + 3*b**2*c) + x**5*(16*a*b*c + 5*b**3) + x*
*3*(-4*a**2*c + 19*a*b**2))/(128*a**4*c**2 - 64*a**3*b**2*c + 8*a**2*b**4
+ x**8*(128*a**2*c**4 - 64*a*b**2*c**3 + 8*b**4*c**2) + x**6*(256*a**2*b*c
**3 - 128*a*b**3*c**2 + 16*b**5*c) + x**4*(256*a**3*c**3 - 48*a*b**4*c + 8
*b**6) + x**2*(256*a**3*b*c**2 - 128*a**2*b**3*c + 16*a*b**5)) + RootSum(_
t**4*(68719476736*a**10*c**11 - 171798691840*a**9*b**2*c**10 + 19327352832
0*a**8*b**4*c**9 - 128849018880*a**7*b**6*c**8 + 56371445760*a**6*b**8*c**
7 - 16911433728*a**5*b**10*c**6 + 3523215360*a**4*b**12*c**5 - 503316480*a
**3*b**14*c**4 + 47185920*a**2*b**16*c**3 - 2621440*a*b**18*c**2 + 65536*b
**20*c) + _t**2*(-188743680*a**7*b*c**7 + 141557760*a**6*b**3*c**6 - 23592
96*a**5*b**5*c**5 - 26542080*a**4*b**7*c**4 + 9584640*a**3*b**9*c**3 - 129
0240*a**2*b**11*c**2 + 46080*a*b**13*c + 2304*b**15) + 20736*a**5*c**4 + 1
03680*a**4*b**2*c**3 + 142560*a**3*b**4*c**2 + 32400*a**2*b**6*c + 2025*a*
b**8, Lambda(_t, _t*log(x + (33554432*_t**3*a**6*c**7 - 16777216*_t**3*a**
5*b**2*c**6 - 10485760*_t**3*a**4*b**4*c**5 + 10485760*_t**3*a**3*b**6*c**
4 - 3276800*_t**3*a**2*b**8*c**3 + 458752*_t**3*a*b**10*c**2 - 24576*_t**3
*b**12*c - 64512*_t*a**3*b*c**3 - 43776*_t*a**2*b**3*c**2 - 21312*_t*a*b**
5*c - 144*_t*b**7)/(432*a**2*c**2 + 1080*a*b**2*c + 135*b**4))))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^6}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*(b^2*c + 4*a*c^2)*x^7 + (5*b^3 + 16*a*b*c)*x^5 + 12*a^2*b*x + (19*a
*b^2 - 4*a^2*c)*x^3)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (
b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)
*x^2) + 3/8*integrate(((b^2 + 4*a*c)*x^2 - 4*a*b)/(c*x^4 + b*x^2 + a), x)/
(b^4 - 8*a*b^2*c + 16*a^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1750 vs. $2(254) = 508$.

Time = 1.67 (sec) , antiderivative size = 1750, normalized size of antiderivative = 5.87

$$\int \frac{x^6}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3/16*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 16*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^4*c - 4*b^5*c + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 +
 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 32*a*b^3*c^2 + 6*b^4*c^2 - 8*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 64*a^2*b*c^3 - 16*a*b^2*c^3 - 32*a^
2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 16*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 + 4*(b^2 - 4*a*c)*b^3*c - 16*
(b^2 - 4*a*c)*a*b*c^2 - 6*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*c^3)*a
rctan(2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + sqrt((b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((b^8 - 16
*a*b^6*c - 2*b^7*c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*c^2 - 256*a^3*b^2
*c^3 - 96*a^2*b^3*c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a^3*b*c^4 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*abs(c)) - 3/16*(2*sqrt(2)*sqrt(b*c - sqrt(b^2 -...
```

Mupad [B] (verification not implemented)

Time = 18.06 (sec) , antiderivative size = 8521, normalized size of antiderivative = 28.59

$$\int \frac{x^6}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^6/(a + b*x^2 + c*x^4)^3,x)
```

output
```
atan(((((3*(1024*a*b^11*c^2 - 1048576*a^6*b*c^7 - 20480*a^2*b^9*c^3 + 1638
40*a^3*b^7*c^4 - 655360*a^4*b^5*c^5 + 1310720*a^5*b^3*c^6))/(512*(b^12 + 4
096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144
*a^5*b^2*c^5 - 24*a*b^10*c)) - (x*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) -
 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4
 - 1024*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 104
8576*a^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53
760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*
b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2)*(256*b^11*c^
2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b^5
*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*
a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)^15)^(1/2) - 81920
*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11520*a^4*b^7*c^4 - 102
4*a^5*b^5*c^5 + 61440*a^6*b^3*c^6 + 20*a*b^13*c))/(512*(b^20*c + 1048576*a
^10*c^11 - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^
4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^
8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10)))^(1/2) - (x*(9*b^6*c - 28
8*a^3*c^4 + 126*a*b^4*c^2 + 576*a^2*b^2*c^3))/(32*(b^8 + 256*a^4*c^4 + 96*
a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15 + (-(4*a*c - b^2)
^15)^(1/2) - 81920*a^7*b*c^7 - 560*a^2*b^11*c^2 + 4160*a^3*b^9*c^3 - 11...
```

Reduce [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 4644, normalized size of antiderivative = 15.58

$$\int \frac{x^6}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^6/(c*x^4+b*x^2+a)^3,x)
```

output
```
( - 48*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c**2 - 36*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c - 96*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a
) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*a**2*b*c**2*x**2 - 96*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a
**2*c**3*x**4 - 72*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c*x**2 -
 120*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**2*x**4 - 96*sqrt(a)
*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**3*x**6 - 48*sqrt(a)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*a*c**4*x**8 - 36*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*b**4*c*x**4 - 72*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c**2*x**6 -
 36*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**2*c**3*x**8 + 72*sqrt(c)...
```

3.813 $\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 289

$$\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx = \frac{x(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x(7b^2-4ac+12bcx^2)}{8(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{c}\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
1/4*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x*(12*b*c*x^2-4*a*c+7
*b^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/8*c^(1/2)*(3*b^2+4*a*c-2*b*(-4*a*c+
b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)
/(-4*a*c+b^2)^(5/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/8*c^(1/2)*(3*b^2+4*a*c+
2*b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1
/2))*2^(1/2)/(-4*a*c+b^2)^(5/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.99

$$\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{8}\left(\frac{2(2ax+bx^3)}{(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{-7b^2x+4acx-12bcx^3}{(b^2-4ac)^2\,(a+bx^2+cx^4)} + \frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

input
```
Integrate[x^4/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (-7*b^2*x + 4
*a*c*x - 12*b*c*x^3)/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sq
rt[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]))/8
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1440, 1492, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \; 1440$$

$$\frac{x(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int \frac{2a-5bx^2}{(cx^4+bx^2+a)^2}dx}{4(b^2-4ac)}$$

$\Big\downarrow$ 1492

$$\frac{x(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{x(-4ac+7b^2+12bcx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{3a\left(b^2-4cx^2b+4ac\right)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}}{4(b^2-4ac)}$$

$\Big\downarrow$ 27

$$\frac{x(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{x(-4ac+7b^2+12bcx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{3\int \frac{b^2-4cx^2b+4ac}{cx^4+bx^2+a}dx}{2(b^2-4ac)}}{4(b^2-4ac)}$$

$\Big\downarrow$ 1480

$$\frac{x(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{x(-4ac+7b^2+12bcx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{3\left(-c\left(2b-\frac{4ac+3b^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx-c\left(\frac{4ac+3b^2}{\sqrt{b^2-4ac}}+2b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx\right)}{2(b^2-4ac)}}{4(b^2-4ac)}$$

$\Big\downarrow$ 218

$$\frac{x(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{x(-4ac+7b^2+12bcx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{3\left(-\frac{\sqrt{2}\sqrt{c}\left(2b-\frac{4ac+3b^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\left(\frac{4ac+3b^2}{\sqrt{b^2-4ac}}+2b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2(b^2-4ac)}}{4(b^2-4ac)}$$

input
```
Int[x^4/(a + b*x^2 + c*x^4)^3,x]
```

output
```
(x*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - ((x*(7*b^2 - 4
*a*c + 12*b*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (3*(-((Sqrt[2]
*Sqrt[c]*(2*b - (3*b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*
Sqrt[c]*(2*b + (3*b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*(b^2 - 4
*a*c)))/(4*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.87

method	result
risch	$\frac{-\frac{3b\,c^{2}x^{7}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c\left(4ac-19b^{2}\right)x^{5}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}-\frac{b\left(16ac+5b^{2}\right)x^{3}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{3a\left(4ac+b^{2}\right)x}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}c+\_Z^{2}b+a\right)}\right.}{}$
default	$\frac{-\frac{3b\,c^{2}x^{7}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c\left(4ac-19b^{2}\right)x^{5}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}-\frac{b\left(16ac+5b^{2}\right)x^{3}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{3a\left(4ac+b^{2}\right)x}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{3c\left(\frac{\left(-2b\sqrt{-4ac+b^{2}}-4ac-3b^{2}\right)\sqrt{2}}{4\sqrt{-4ac+b^{2}}\,\sqrt{(b}}\right.}{}$

input
```
int(x^4/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/2*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*c*(4*a*c-19*b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^5-1/8*b*(16*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-3
/8*a*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/16*sum(
(-4*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+
b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3128 vs. 2(241) = 482.

Time = 0.17 (sec) , antiderivative size = 3128, normalized size of antiderivative = 10.82

$$\int \frac{x^4}{\left(a+bx^2+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(274) = 548$.

Time = 107.16 (sec) , antiderivative size = 648, normalized size of antiderivative = 2.24

$$\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{-12bc^2x^7 + x^5\cdot(4ac^2-19b^2c) + x^3(-16abc-5b^3) + x}{128a^4c^2 - 64a^3b^2c + 8a^2b^4 + x^8\cdot(128a^2c^4 - 64ab^2c^3 + 8b^4c^2) + x^6\cdot(256a^2bc^3 - 128ab^3c^2 + 16b^5c) + x^4}$$
$$+\mathrm{RootSum}\left(t^4\cdot\left(68719476736a^{11}c^{10} - 171798691840a^{10}b^2c^9 + 193273528320a^9b^4c^8 - 128849018880a^8\right.\right.$$

input
```
integrate(x**4/(c*x**4+b*x**2+a)**3,x)
```

output
```
(-12*b*c**2*x**7 + x**5*(4*a*c**2 - 19*b**2*c) + x**3*(-16*a*b*c - 5*b**3)
 + x*(-12*a**2*c - 3*a*b**2))/(128*a**4*c**2 - 64*a**3*b**2*c + 8*a**2*b**
4 + x**8*(128*a**2*c**4 - 64*a*b**2*c**3 + 8*b**4*c**2) + x**6*(256*a**2*b
*c**3 - 128*a*b**3*c**2 + 16*b**5*c) + x**4*(256*a**3*c**3 - 48*a*b**4*c +
 8*b**6) + x**2*(256*a**3*b*c**2 - 128*a**2*b**3*c + 16*a*b**5)) + RootSum
(_t**4*(68719476736*a**11*c**10 - 171798691840*a**10*b**2*c**9 + 193273528
320*a**9*b**4*c**8 - 128849018880*a**8*b**6*c**7 + 56371445760*a**7*b**8*c
**6 - 16911433728*a**6*b**10*c**5 + 3523215360*a**5*b**12*c**4 - 503316480
*a**4*b**14*c**3 + 47185920*a**3*b**16*c**2 - 2621440*a**2*b**18*c + 65536
*a*b**20) + _t**2*(-188743680*a**7*b*c**7 + 141557760*a**6*b**3*c**6 - 235
9296*a**5*b**5*c**5 - 26542080*a**4*b**7*c**4 + 9584640*a**3*b**9*c**3 - 1
290240*a**2*b**11*c**2 + 46080*a*b**13*c + 2304*b**15) + 20736*a**4*c**5 +
 103680*a**3*b**2*c**4 + 142560*a**2*b**4*c**3 + 32400*a*b**6*c**2 + 2025*
b**8*c, Lambda(_t, _t*log(x + (50331648*_t**3*a**7*b*c**6 - 58720256*_t**3
*a**6*b**3*c**5 + 26214400*_t**3*a**5*b**5*c**4 - 5242880*_t**3*a**4*b**7*
c**3 + 327680*_t**3*a**3*b**9*c**2 + 32768*_t**3*a**2*b**11*c - 4096*_t**3
*a*b**13 + 18432*_t*a**4*c**4 - 78336*_t*a**3*b**2*c**3 - 40320*_t*a**2*b*
*4*c**2 - 3168*_t*a*b**6*c - 144*_t*b**8)/(432*a**2*c**3 + 1080*a*b**2*c**
2 + 135*b**4*c))))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^4}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(12*b*c^2*x^7 + (19*b^2*c - 4*a*c^2)*x^5 + (5*b^3 + 16*a*b*c)*x^3 + 3
*(a*b^2 + 4*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
(b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2
)*x^2) - 3/8*integrate((4*b*c*x^2 - b^2 - 4*a*c)/(c*x^4 + b*x^2 + a), x)/(
b^4 - 8*a*b^2*c + 16*a^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1861 vs. $2(241) = 482$.

Time = 1.53 (sec) , antiderivative size = 1861, normalized size of antiderivative = 6.44

$$\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
5*c - 2*b^6*c - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*a*b^4*c^2 + 2*b^5*c^2 +
 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^2*b^2*c^3 + 16*a*b^3*c^3 - 16*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 128*a^3*c^4 - 96*a^2*b*c^4 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 48*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 24*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^4*c - 2*(b^2
 - 4*a*c)*b^3*c^2 - 32*(b^2 - 4*a*c)*a^2*c^3 - 24*(b^2 - 4*a*c)*a*b*c^3)*a
rctan(2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + sqrt((b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/((a*b^8 -
16*a^2*b^6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b^6*c^2 - 2
56*a^4*b^2*c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 + 128*a^...
```

Mupad [B] (verification not implemented)

Time = 22.00 (sec) , antiderivative size = 8397, normalized size of antiderivative = 29.06

$$\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^4/(a + b*x^2 + c*x^4)^3,x)
```

output
```
atan(((((3*(262144*a^6*c^8 - 64*b^12*c^2 + 1024*a*b^10*c^3 - 5120*a^2*b^8*
c^4 + 81920*a^4*b^4*c^6 - 262144*a^5*b^2*c^7))/(128*(b^12 + 4096*a^6*c^6 +
 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5
- 24*a*b^10*c)) - (x*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c
^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^
5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10
 - 40*a^2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c
^4 - 258048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949
120*a^9*b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2)*(128*b^11*c^2 - 2560*a*b^9
*c^3 - 131072*a^5*b*c^7 + 20480*a^2*b^7*c^4 - 81920*a^3*b^5*c^5 + 163840*a
^4*b^3*c^6))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 1
6*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560
*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 -
61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20 + 1048576*a^11*c^10 - 40*a^
2*b^18*c + 720*a^3*b^16*c^2 - 7680*a^4*b^14*c^3 + 53760*a^5*b^12*c^4 - 258
048*a^6*b^10*c^5 + 860160*a^7*b^8*c^6 - 1966080*a^8*b^6*c^7 + 2949120*a^9*
b^4*c^8 - 2621440*a^10*b^2*c^9)))^(1/2) - (x*(144*a^2*c^5 + 117*b^4*c^3 +
72*a*b^2*c^4))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 -
 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 5
60*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c...
```

Reduce [B] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 4644, normalized size of antiderivative = 16.07

$$\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^4/(c*x^4+b*x^2+a)^3,x)
```

output

```
(72*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c + 6*sqrt(a)*sqrt(2*sq
rt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2
*sqrt(c)*sqrt(a) + b))*a**2*b**3 + 144*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 b))*a**2*b**2*c*x**2 + 144*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt
(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b
*c**2*x**4 + 12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*x**2 + 84*s
qrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c*x**4 + 144*sqrt(a)*sqrt(2*
sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt
(2*sqrt(c)*sqrt(a) + b))*a*b**2*c**2*x**6 + 72*sqrt(a)*sqrt(2*sqrt(c)*sqrt
(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*s
qrt(a) + b))*a*b*c**3*x**8 + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((s
qrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**
5*x**4 + 12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*c*x**6 + 6*sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c
)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**3*c**2*x**8 - 48*sqrt(c)*sqrt(2*sq...
```

3.814 $\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 311

$$\begin{aligned}\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx &= -\frac{x(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}\\ &\quad+\frac{x(b(b^2+8ac)+c(b^2+20ac)x^2)}{8a(b^2-4ac)^2(a+bx^2+cx^4)}\\ &\quad+\frac{\sqrt{c}\left(b^2+20ac+\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}\\ &\quad+\frac{\sqrt{c}\left(b^2+20ac-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output

```
-1/4*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/8*x*(b*(8*a*c+b^2)+c*(
20*a*c+b^2)*x^2)/a/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*c^(1/2)*(b^2+20*a*c
+b*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b
^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/
16*c^(1/2)*(b^2+20*a*c-b*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*
c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^2/(b+(-4*a*
c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{1}{16}\left(-\frac{4x(b+2cx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x(b^3+8abc+b^2cx^2+20ac^2x^2)}{a(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac})\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}(-b^3+52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac})\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

input
```
Integrate[x^2/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((-4*x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(b^3 +
8*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2))/(a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4
)) + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^
3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]))/16
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1439, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{1439}$$

$$\frac{\int \frac{b-10cx^2}{(cx^4+bx^2+a)^2}dx}{4(b^2-4ac)} - \frac{x(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \text{1492}$$

$$\frac{\frac{x(cx^2(20ac+b^2)+b(8ac+b^2))}{2a(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int -\frac{c(b^2+20ac)x^2+b(b^2-16ac)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}}{4(b^2-4ac)} - \frac{x(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{c(b^2+20ac)x^2+b(b^2-16ac)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)} + \frac{x(cx^2(20ac+b^2)+b(8ac+b^2))}{2a(b^2-4ac)(a+bx^2+cx^4)}}{4(b^2-4ac)} - \frac{x(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{\frac{1}{2}c\left(-\frac{52abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(\frac{52abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a(b^2-4ac)} + \frac{x(cx^2(20ac}{2a(b^2-4}}{4(b^2-4ac)}$$

$$\frac{x(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\sqrt{c}\left(-\frac{52abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(\frac{52abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+20ac+b^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a(b^2-4ac)} + \frac{x(cx^2(20ac+b^2)+b}{2a(b^2-4ac)(a+b}}{4(b^2-4ac)}$$

$$\frac{x(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

input
```
Int[x^2/(a + b*x^2 + c*x^4)^3,x]
```

output
```
-1/4*(x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((x*(b*(b^2
 + 8*a*c) + c*(b^2 + 20*a*c)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 + ((Sqrt[c]*(b^2 + 20*a*c + b^3/Sqrt[b^2 - 4*a*c] - (52*a*b*c)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b^2 + 20*a*c - b^3/Sqrt[b^2 - 4*
a*c] + (52*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a
*c)))/(4*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.32 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.93

method	result
risch	$\frac{\frac{c^{2}\left(20ac+b^{2}\right)x^{7}}{8a\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{cb\left(14ac+b^{2}\right)x^{5}}{4a\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(36a^{2}c^{2}+5a\,b^{2}c+b^{4}\right)x^{3}}{8a\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{b\left(16ac-b^{2}\right)x}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}c+\_Z^{2}b+a\right)}\cdots$
default	$64c^{3}\left(-\frac{-\frac{\left(320\sqrt{-4ac+b^{2}}\,a^{3}c^{3}-144\sqrt{-4ac+b^{2}}\,a^{2}b^{2}c^{2}+12\sqrt{-4ac+b^{2}}\,a\,b^{4}c+\sqrt{-4ac+b^{2}}\,b^{6}+64a^{3}b\,c^{3}-48a^{2}b^{3}c^{2}+12a\,b^{5}c-b^{7}\right)x^{3}}{16a\,c^{2}}+\frac{\left(-96\sqrt{\cdots}\right.}{\cdots}}{\left(x^{2}+\frac{\sqrt{-4ac+b^{2}}}{2c}+\frac{b}{2c}\right)^{2}}\cdots\right.$

input
```
int(x^2/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*c^2*(20*a*c+b^2)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/4*c/a*b*(14*a*c+b
^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+1/8*(36*a^2*c^2+5*a*b^2*c+b^4)/a/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^3+1/8*b*(16*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/
(c*x^4+b*x^2+a)^2+1/16/a*sum((c*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R
^2-b*(16*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=
RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3777 vs. $2(267) = 534$.

Time = 0.30 (sec) , antiderivative size = 3777, normalized size of antiderivative = 12.14

$$\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^2}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((b^2*c^2 + 20*a*c^3)*x^7 + 2*(b^3*c + 14*a*b*c^2)*x^5 + (b^4 + 5*a*b^
2*c + 36*a^2*c^2)*x^3 - (a*b^3 - 16*a^2*b*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^
3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*
a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 +
 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) + 1/8*integrate((b^3 - 16*a
*b*c + (b^2*c + 20*a*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4270 vs. $2(267) = 534$.

Time = 1.42 (sec) , antiderivative size = 4270, normalized size of antiderivative = 13.73

$$\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/64*(2*a^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6
*c^5 + 27136*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2
*(b^2 - 4*a*c)*a^2*b^10*c^2 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 -
4*a*c)*a^4*b^6*c^4 + 5120*(b^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c...
```

Mupad [B] (verification not implemented)

Time = 22.54 (sec) , antiderivative size = 9731, normalized size of antiderivative = 31.29

$$\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^2/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((b*x*(16*a*c - b^2))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(b^4 + 36*
a^2*c^2 + 5*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^5*(14*a*
c^2 + b^2*c))/(4*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20*a*c^2 + b^
2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2
*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((256*a*b^13*c^2 + 4194304*a^7*b*c^
8 - 9216*a^2*b^11*c^3 + 122880*a^3*b^9*c^4 - 819200*a^4*b^7*c^5 + 2949120*
a^5*b^5*c^6 - 5505024*a^6*b^3*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*
b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*
b^2*c^5)) - (x*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8
 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*
b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*
(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^1
8*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a
^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4
*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*c^2
+ 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^6*b
^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*
a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8
 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*
b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a...
```

Reduce [B] (verification not implemented)

Time = 4.44 (sec) , antiderivative size = 5719, normalized size of antiderivative = 18.39

$$\int \frac{x^2}{(a + bx^2 + cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^2/(c*x^4+b*x^2+a)^3,x)
```

output
```
( - 80*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*c**2 - 36*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c - 160*sqrt(a)*sqrt(2*sqrt(c)*sqrt(
a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq
rt(a) + b))*a**3*b*c**2*x**2 - 160*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a**3*c**3*x**4 + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**4 - 72
*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2
*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c*x**2 - 152*sqrt(a)*sq
rt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)
/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**2*x**4 - 160*sqrt(a)*sqrt(2*sqr
t(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*
sqrt(c)*sqrt(a) + b))*a**2*b*c**3*x**6 - 80*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a)
 + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + b))*a**2*c**4*x**8 + 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**
5*x**2 - 32*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(
a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*c*x**4 - 72*...
```

3.815 $\int \frac{1}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 355

$$\begin{aligned}\int \frac{1}{(a+bx^2+cx^4)^3}\,dx &= \frac{x(b^2-2ac+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x((b^2-7ac)(3b^2-4ac)+3bc(b^2-8ac)x^2)}{8a^2(b^2-4ac)^2(a+bx^2+cx^4)}\\&\quad+\frac{3\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+b(b^2-8ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}\\&\quad+\frac{3\sqrt{c}\left(b^3-8abc-\frac{b^4-10ab^2c+56a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2(b^2-4ac)^2\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
1/4*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/8*x*((-7*a*c+
b^2)*(-4*a*c+3*b^2)+3*b*c*(-8*a*c+b^2)*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^
2+a)+3/16*c^(1/2)*(b^4-10*a*b^2*c+56*c^2*a^2+b*(-8*a*c+b^2)*(-4*a*c+b^2)^(
1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(
-4*a*c+b^2)^(5/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16*c^(1/2)*(b^3-8*a*b*c-(
56*a^2*c^2-10*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b
+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^2/(-4*a*c+b^2)^2/(b+(-4*a*c+b^2)^(1/
2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{\frac{4ax(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x(3b^4-25ab^2c+28a^2c^2+3b^3cx^2-24abc^2x^2)}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+b^3\sqrt{b^2-4ac}-8abc\sqrt{b^2-4ac}\right)\text{arcta}}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}}{16a^2}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-3),x]
```

output
```
((4*a*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (
2*x*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2))/((b^
2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c +
 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^
2 - b^3*Sqrt[b^2 - 4*a*c] + 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]))/(16*a^2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1405, 25, 1492, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx$$
$$\Big\downarrow \text{1405}$$
$$\frac{x(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int -\frac{3b^2+5cx^2b-14ac}{(cx^4+bx^2+a)^2}dx}{4a(b^2-4ac)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{3b^2+5cx^2b-14ac}{(cx^4+bx^2+a)^2}dx}{4a(b^2-4ac)}+\frac{x(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \ 1492$$

$$\frac{\frac{x(3bcx^2(b^2-8ac)+(b^2-7ac)(3b^2-4ac))}{2a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{3\left(b^4-9acb^2+c\left(b^2-8ac\right)x^2b+28a^2c^2\right)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}}{4a(b^2-4ac)}+\frac{x(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\int \frac{b^4-9acb^2+c\left(b^2-8ac\right)x^2b+28a^2c^2}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\frac{x(3bcx^2(b^2-8ac)+(b^2-7ac)(3b^2-4ac))}{2a(b^2-4ac)(a+bx^2+cx^4)}}{4a(b^2-4ac)}+\frac{x(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \ 1480$$

$$\frac{\frac{3\left(\frac{1}{2}c\left(\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}+b\left(b^2-8ac\right)\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(b\left(b^2-8ac\right)-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx\right)}{2a(b^2-4ac)}+\frac{x(3bcx^2}{}}{4a(b^2-4ac)}$$
$$\frac{x(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{3\left(\frac{\sqrt{c}\left(\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}+b\left(b^2-8ac\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b\left(b^2-8ac\right)-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2a(b^2-4ac)}+\frac{x(3bcx^2(b^2-8}{2a(b^2-}}{4a(b^2-4ac)}$$
$$\frac{x(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

input
```
Int[(a + b*x^2 + c*x^4)^(-3),x]
```

output
```
(x*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((
x*((b^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^2))/(2*a*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + (3*((Sqrt[c]*(b*(b^2 - 8*a*c) + (b^4 - 10*a*b
^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*(
b^2 - 8*a*c) - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]])))/(2*a*(b^2 - 4*a*c)))/(4*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.93

method	result
risch	$\frac{-\frac{3b\,c^{2}\left(8ac-b^{2}\right)x^{7}}{8a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c\left(28a^{2}c^{2}-49a\,b^{2}c+6b^{4}\right)x^{5}}{8a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{b\left(4a^{2}c^{2}+20a\,b^{2}c-3b^{4}\right)x^{3}}{8a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(44a^{2}c^{2}-37a\,b^{2}c+5b^{4}\right)x}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)a}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{3\left(\sum_{\_R=\text{RootOf}\left(\_Z^{4}c\right.}\right.}{}$
default	$64c^{3}\left(-\frac{\frac{3\left(128\sqrt{-4ac+b^{2}}\,a^{3}b\,c^{3}-80\sqrt{-4ac+b^{2}}\,a^{2}b^{3}c^{2}+16\sqrt{-4ac+b^{2}}\,a\,b^{5}c-\sqrt{-4ac+b^{2}}\,b^{7}+384a^{4}c^{4}-352a^{3}b^{2}c^{3}+120a^{2}b^{4}c^{2}-18a\,b^{6}c+b^{8}\right)}{64a^{2}c^{3}}}{\left(x^{2}+\frac{\sqrt{-4a}}{2c}\right.}\right.$

input
```
int(1/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/8*b*c^2*(8*a*c-b^2)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8/a^2*c*(28*a
^2*c^2-49*a*b^2*c+6*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*b*(4*a^2*c^2+2
0*a*b^2*c-3*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/8*(44*a^2*c^2-37*a*b
^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x)/(c*x^4+b*x^2+a)^2+3/16/a^2*sum
((-b*c*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+(28*a^2*c^2-9*a*b^2*c+b
^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+
_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4323 vs. $2(309) = 618$.

Time = 0.60 (sec) , antiderivative size = 4323, normalized size of antiderivative = 12.18

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx = \int \frac{1}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*(b^3*c^2 - 8*a*b*c^3)*x^7 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*x
^5 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^3 + (5*a*b^4 - 37*a^2*b^2*c + 44
*a^3*c^2)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8
*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6
 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 1
6*a^5*b*c^2)*x^2) - 3/8*integrate(-(b^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^3*c
- 8*a*b*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*
c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2707 vs. $2(309) = 618$.

Time = 0.97 (sec) , antiderivative size = 2707, normalized size of antiderivative = 7.63

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^7*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 +
 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
4*c^3 - 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4
 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^
4 + 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5
- 896*a^4*c^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^6*c - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c^2 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
*c^2 + 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
*c^3 + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
2*c^3 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*...
```

Mupad [B] (verification not implemented)

Time = 21.91 (sec) , antiderivative size = 10979, normalized size of antiderivative = 30.93

$$\int \frac{1}{(a + bx^2 + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((x*(5*b^4 + 44*a^2*c^2 - 37*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) + (x^5*(6*b^4*c + 28*a^2*c^3 - 49*a*b^2*c^2))/(8*a^2*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c)) - (x^3*(4*a^2*b*c^2 - 3*b^5 + 20*a*b^3*c))/(8*a^2*(b^4 + 16*a
^2*c^2 - 8*a*b^2*c)) + (3*c*x^7*(b^3*c - 8*a*b*c^2))/(8*a^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*
x^6) - atan(((((3*(7340032*a^9*c^9 - 256*a^2*b^14*c^2 + 7424*a^3*b^12*c^3
- 94208*a^4*b^10*c^4 + 675840*a^5*b^8*c^5 - 2949120*a^6*b^6*c^6 + 7798784*
a^7*b^4*c^7 - 11534336*a^8*b^2*c^8))/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a
^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a
^9*b^2*c^5)) - (x*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9
*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 31686
4*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^
3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-
(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18
*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^
10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^
4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*c^2
 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^8*
b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256
*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^...
```

Reduce [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 6794, normalized size of antiderivative = 19.14

$$\int \frac{1}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(c*x^4+b*x^2+a)^3,x)
```

output
```
(264*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b*c**2 - 66*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**3*c + 528*sqrt(a)*sqrt(2*sqrt(c)*sqrt(
a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq
rt(a) + b))*a**3*b**2*c**2*x**2 + 528*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*
atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
b))*a**3*b*c**3*x**4 + 6*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**5
 - 132*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) -
b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**4*c*x**2 + 132*sqrt
(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt
(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*c**2*x**4 + 528*sqrt(a)*sqrt
(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/s
qrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c**3*x**6 + 264*sqrt(a)*sqrt(2*sqrt(
c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sq
rt(c)*sqrt(a) + b))*a**2*b*c**4*x**8 + 12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a
) + b))*a*b**6*x**2 - 54*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**5...
```

3.816 $\int \frac{1}{x^2(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 425

$$\begin{aligned}
&\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^3}\,dx \\
&= -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2x}+\frac{b^2-2ac+bcx^2}{4a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)^2} \\
&\quad+\frac{5b^4-35ab^2c+36a^2c^2+bc(5b^2-32ac)x^2}{8a^2\left(b^2-4ac\right)^2x\left(a+bx^2+cx^4\right)} \\
&\quad-\frac{3\sqrt{c}\left((5b^2-12ac)\left(b^2-5ac\right)+\frac{b(5b^4-47ab^2c+124a^2c^2)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}} \\
&\quad-\frac{3\sqrt{c}\left((5b^2-12ac)\left(b^2-5ac\right)-\frac{5b^5-47ab^3c+124a^2bc^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^2\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

output
```
-3/8*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/x+1/4*(b*c*x^2-2*a*c+
b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)^2+1/8*(5*b^4-35*a*b^2*c+36*c^2*a^2+b
*c*(-32*a*c+5*b^2)*x^2)/a^2/(-4*a*c+b^2)^2/x/(c*x^4+b*x^2+a)-3/16*c^(1/2)*
((-12*a*c+5*b^2)*(-5*a*c+b^2)+b*(124*a^2*c^2-47*a*b^2*c+5*b^4)/(-4*a*c+b^2
)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^
3/(-4*a*c+b^2)^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*c^(1/2)*((-12*a*c+5*b^2
)*(-5*a*c+b^2)-(124*a^2*b*c^2-47*a*b^3*c+5*b^5)/(-4*a*c+b^2)^(1/2))*arctan
(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a^3/(-4*a*c+b^2)^
2/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2 \left(a+bx^2+cx^4\right)^3} \, dx =$$

$$-\frac{\frac{16}{x}+\frac{4ax\left(b^3-3abc+b^2cx^2-2ac^2x^2\right)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x\left(7b^5-52ab^3c+84a^2bc^2+7b^4cx^2-47ab^2c^2x^2+52a^2c^3x^2\right)}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2+\right.}{}}{}$$

input
```
Integrate[1/(x^2*(a + b*x^2 + c*x^4)^3),x]
```

output
```
-1/16*(16/x + (4*a*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*
a*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(7*b^5 - 52*a*b^3*c + 84*a^2*b*c^2 + 7*
b^4*c*x^2 - 47*a*b^2*c^2*x^2 + 52*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b
^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^5 +
47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2
 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
)/a^3
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1441, 25, 1600, 27, 1604, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2+cx^4)^3} \, dx$$

$$\Big\downarrow \text{ 1441}$$

$$\frac{-2ac+b^2+bcx^2}{4ax(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int -\frac{5b^2+7cx^2b-18ac}{x^2(cx^4+bx^2+a)^2}dx}{4a(b^2-4ac)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{5b^2+7cx^2b-18ac}{x^2(cx^4+bx^2+a)^2}dx}{4a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{4ax(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \text{ 1600}$$

$$\frac{\frac{36a^2c^2+bcx^2(5b^2-32ac)-35ab^2c+5b^4}{2ax(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{3\left(bc\left(5b^2-32ac\right)x^2+\left(5b^2-12ac\right)\left(b^2-5ac\right)\right)}{x^2\left(cx^4+bx^2+a\right)}dx}{2a(b^2-4ac)}}{4a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{4ax(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3\int \frac{bc\left(5b^2-32ac\right)x^2+\left(5b^2-12ac\right)\left(b^2-5ac\right)}{x^2\left(cx^4+bx^2+a\right)}dx}{2a(b^2-4ac)}+\frac{36a^2c^2+bcx^2(5b^2-32ac)-35ab^2c+5b^4}{2ax(b^2-4ac)(a+bx^2+cx^4)}}{4a(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{4ax(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$\Big\downarrow \text{ 1604}$$

$$\frac{\frac{3\left(-\frac{\int \frac{c(5b^2-12ac)(b^2-5ac)x^2+b(5b^4-42acb^2+92a^2c^2)}{cx^4+bx^2+a}dx}{a}-\frac{(5b^2-12ac)(b^2-5ac)}{ax}\right)}{2a(b^2-4ac)}+\frac{36a^2c^2+bcx^2(5b^2-32ac)-35ab^2c+5b^4}{2ax(b^2-4ac)(a+bx^2+cx^4)}}{4a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{4ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$\downarrow$ 1480

$$\frac{\frac{3\left(-\frac{\frac{1}{2}c\left(\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}+(5b^2-12ac)(b^2-5ac)\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left((5b^2-12ac)(b^2-5ac)-\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}\right)\int \frac{}{cx^2+\frac{1}{2}(}}{a}}{2a(b^2-4ac)}}{4a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx^2}{4ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$\downarrow$ 218

$$\frac{\frac{3\left(-\frac{\frac{\sqrt{c}\left(\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}+(5b^2-12ac)(b^2-5ac)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left((5b^2-12ac)(b^2-5ac)-\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{}{\sqrt{}}\right.}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}}{2a(b^2-4ac)}}{4a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx^2}{4ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

input
```
Int[1/(x^2*(a + b*x^2 + c*x^4)^3),x]
```

output
```
(b^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)^2) + ((5*
b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*x^2)/(2*a*(b^2 - 4*a*
c)*x*(a + b*x^2 + c*x^4)) + (3*(-(((5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(a*x))
- ((Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c + 124
*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*((5*b^2 - 12
*a*c)*(b^2 - 5*a*c) - (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/a))/(2*a*(b^2 - 4*a*c)))/(4*a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1600
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*(a + b*x^2 + c
*x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1))
- a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Int
egerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\frac{\frac{c^{2}\left(52a^{2}c^{2}-47a\,b^{2}c+7b^{4}\right)x^{7}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}+\frac{cb\left(136a^{2}c^{2}-99a\,b^{2}c+14b^{4}\right)x^{5}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}+\frac{\left(68a^{3}c^{3}+25a^{2}b^{2}c^{2}-43a\,b^{4}c+7b^{6}\right)x^{3}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}+\frac{3ab\left(36a^{2}c^{2}-22a\,b^{2}c+3b^{4}\right)x}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{3c\left(\cdots\right.}{\cdots}}{\cdots}$
risch	$\frac{-\frac{3c^{2}\left(60a^{2}c^{2}-37a\,b^{2}c+5b^{4}\right)x^{8}}{8a^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{cb\left(392a^{2}c^{2}-227a\,b^{2}c+30b^{4}\right)x^{6}}{8a^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(324a^{3}c^{3}+25a^{2}b^{2}c^{2}-91a\,b^{4}c+15b^{6}\right)x^{4}}{8a^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{b\left(364a^{2}c^{2}-194a\,b^{2}c+25b^{4}\right)x^{2}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)a^{2}}-\cdots}{x(c\,x^{4}+b\,x^{2}+a)^{2}}$

input
```
int(1/x^2/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*((1/8*c^2*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^7+1/8*c*b*(136*a^2*c^2-99*a*b^2*c+14*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5
+1/8*(68*a^3*c^3+25*a^2*b^2*c^2-43*a*b^4*c+7*b^6)/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3+3/8*a*b*(36*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)
/(c*x^4+b*x^2+a)^2+3/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*(1/8*(60*a^2*c^2*(-4*a
*c+b^2)^(1/2)-37*a*b^2*c*(-4*a*c+b^2)^(1/2)+5*b^4*(-4*a*c+b^2)^(1/2)-124*a
^2*b*c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/8*(60*
a^2*c^2*(-4*a*c+b^2)^(1/2)-37*a*b^2*c*(-4*a*c+b^2)^(1/2)+5*b^4*(-4*a*c+b^2
)^(1/2)+124*a^2*b*c^2-47*a*b^3*c+5*b^5)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))))-1/a^3/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4924 vs. $2(379) = 758$.

Time = 1.10 (sec) , antiderivative size = 4924, normalized size of antiderivative = 11.59

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^3} \, dx = \int \frac{1}{(cx^4 + bx^2 + a)^3 x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8 + (30*b^5*c - 227*a*b^
3*c^2 + 392*a^2*b*c^3)*x^6 + 8*a^2*b^4 - 64*a^3*b^2*c + 128*a^4*c^2 + (15*
b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*x^4 + (25*a*b^5 - 194*a^2
*b^3*c + 364*a^3*b*c^2)*x^2)/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x
^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b
^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a
^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x) - 3/8*integrate((5*b^5 - 42*a*b^3*c
+ 92*a^2*b*c^2 + (5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*x^2)/(c*x^4 + b*x^2
 + a), x)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5273 vs. $2(379) = 758$.

Time = 2.03 (sec) , antiderivative size = 5273, normalized size of antiderivative = 12.41

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3/64*(10*a^6*b^14*c^2 - 254*a^7*b^12*c^3 + 2712*a^8*b^10*c^4 - 15552*a^9*
b^8*c^5 + 50432*a^10*b^6*c^6 - 87552*a^11*b^4*c^7 + 63488*a^12*b^2*c^8 - 5
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^14 + 127*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^12*c + 10*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^13*c - 135
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^10*c^2 -
 214*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^11*c^
2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^12*c
^2 + 7776*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^
8*c^3 + 1856*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8
*b^9*c^3 + 107*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^7*b^10*c^3 - 25216*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^10*b^6*c^4 - 8128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^9*b^7*c^4 - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^8*b^8*c^4 + 43776*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^11*b^4*c^5 + 17920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^10*b^5*c^5 + 4064*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^9*b^6*c^5 - 31744*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^12*b^2*c^6 - 15872*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^11*b^3*c^6 - 8960*sqrt(2)*sqrt(b^2 - ...
```

Mupad [B] (verification not implemented)

Time = 23.51 (sec) , antiderivative size = 12130, normalized size of antiderivative = 28.54

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^2 + c*x^4)^3),x)
```

output
```
- atan(((x*(271790899200*a^20*c^14 - 230400*a^9*b^22*c^3 + 9861120*a^10*b^
20*c^4 - 191038464*a^11*b^18*c^5 + 2207803392*a^12*b^16*c^6 - 16878108672*
a^13*b^14*c^7 + 89374851072*a^14*b^12*c^8 - 333226967040*a^15*b^10*c^9 + 8
69815812096*a^16*b^8*c^10 - 1543847804928*a^17*b^6*c^11 + 1747313491968*a^
18*b^4*c^12 - 1101055131648*a^19*b^2*c^13) + (-(9*(25*b^21 - 25*b^6*(-(4*a
*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a
^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b
^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^
9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2
*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*
(a^7*b^20 + 1048576*a^17*c^10 - 40*a^8*b^18*c + 720*a^9*b^16*c^2 - 7680*a^
10*b^14*c^3 + 53760*a^11*b^12*c^4 - 258048*a^12*b^10*c^5 + 860160*a^13*b^8
*c^6 - 1966080*a^14*b^6*c^7 + 2949120*a^15*b^4*c^8 - 2621440*a^16*b^2*c^9)
))^(1/2)*(245760*a^12*b^23*c^2 - 1185410973696*a^23*b*c^13 - 10911744*a^13
*b^21*c^3 + 220397568*a^14*b^19*c^4 - 2673082368*a^15*b^17*c^5 + 216300257
28*a^16*b^15*c^6 - 122607894528*a^17*b^13*c^7 + 496773365760*a^18*b^11*c^8
 - 1438679826432*a^19*b^9*c^9 + 2918430277632*a^20*b^7*c^10 - 394922242867
2*a^21*b^5*c^11 + 3208340570112*a^22*b^3*c^12 + x*(-(9*(25*b^21 - 25*b^6*(
-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188
095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 199056...
```

Reduce [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 7940, normalized size of antiderivative = 18.68

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^3} dx = \text{Too large to display}$$

input
```
int(1/x^2/(c*x^4+b*x^2+a)^3,x)
```

output

```
(720*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**5*c**3*x - 996*sqrt(a)*sqr
t(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b**2*c**2*x + 1440*sqrt(a)*sqrt(2*sqrt(c
)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqr
t(c)*sqrt(a) + b))*a**4*b*c**3*x**3 + 1440*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a)
+ b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a**4*c**4*x**5 + 312*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3
*b**4*c*x - 1992*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*
sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**3*c**2*x*
*3 - 1272*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
 - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**3*x**5 + 14
40*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c**4*x**7 + 720*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*c**5*x**9 - 30*sqrt(a)*sqrt(2*sqrt(c)
*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*a**2*b**6*x + 624*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +...
```

3.817 $\int \frac{x^5}{a-bx^2+cx^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{x^5}{a-bx^2+cx^4} \, dx = \frac{x^2}{2c}+\frac{(b^2-2ac)\operatorname{arctanh}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log(a-bx^2+cx^4)}{4c^2}$$

output
```
1/2*x^2/c+1/2*(-2*a*c+b^2)*arctanh((-2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-
4*a*c+b^2)^(1/2)+1/4*b*ln(c*x^4-b*x^2+a)/c^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{a-bx^2+cx^4} \, dx = \frac{2cx^2+\frac{2(b^2-2ac)\arctan\left(\frac{-b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+b\log(a-bx^2+cx^4)}{4c^2}$$

input
```
Integrate[x^5/(a - b*x^2 + c*x^4),x]
```

output
```
(2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqr
t[-b^2 + 4*a*c] + b*Log[a - b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a-bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^4}{cx^4-bx^2+a}dx^2$$

$$\Big\downarrow \text{1143}$$

$$\frac{1}{2}\int \left(\frac{1}{c}-\frac{a-bx^2}{c\,(cx^4-bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{2c^2}+\frac{x^2}{c}\right)$$

input
```
Int[x^5/(a - b*x^2 + c*x^4),x]
```

output
```
(x^2/c + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqr
t[b^2 - 4*a*c]) + (b*Log[a - b*x^2 + c*x^4])/(2*c^2))/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

method	result
default	$\frac{x^2}{2c}+\frac{\frac{b\ln\left(c\,x^4-b\,x^2+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$
risch	$\frac{x^2}{2c}+\frac{\ln\left(\left(8a^2c^2-6a\,b^2c+b^4+\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,b\right)x^2-2\sqrt{-(4ac-b^2)(2ac-b^2)^2}\,a\right)ab}{c(4ac-b^2)}-\frac{\ln\left(\left(8a^2c^2-6a\,b^2c+b^4+\sqrt{-(4\right.\right.}{}$

input
```
int(x^5/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c+1/2/c*(1/2*b/c*ln(c*x^4-b*x^2+a)+2*(-a+1/2*b^2/c)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^2-b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.16

$$\int \frac{x^5}{a-bx^2+cx^4}\,dx$$
$$=\left[\frac{2\,(b^2c-4\,ac^2)x^2-(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac+(2\,cx^2-b)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+(b^3-4\,abc)\log\left(cx\right.}{4\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^4 - 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 -
 b*x^2 + a)) + (b^3 - 4*a*b*c)*log(c*x^4 - b*x^2 + a))/(b^2*c^2 - 4*a*c^3)
, 1/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*log(c*
x^4 - b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(71) = 142$.

Time = 1.04 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.79

$$\int \frac{x^5}{a - bx^2 + cx^4}\,dx = \left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2 + \frac{ab - 8ac^2\left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right) + 2b^2c\left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \left(\frac{b}{4c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2 + \frac{ab - 8ac^2\left(\frac{b}{4c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right) + 2b^2c\left(\frac{b}{4c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right) + \frac{x^2}{2c}$$

input
```
integrate(x**5/(c*x**4-b*x**2+a),x)
```

output
```
(b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*
log(x**2 + (a*b - 8*a*c**2*(b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2
)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(b/(4*c**2) - sqrt(-4*a*c + b**2)*(2
*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (b/(4*c**2) + sqr
t(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (a*b -
 8*a*c**2*(b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c
- b**2))) + 2*b**2*c*(b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c
**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{a - bx^2 + cx^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{a - bx^2 + cx^4} \, dx = \frac{x^2}{2\,c} + \frac{b \log\left(cx^4 - bx^2 + a\right)}{4\,c^2} + \frac{\left(b^2 - 2\,ac\right) \arctan\left(\frac{2\,cx^2 - b}{\sqrt{-b^2 + 4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^2}$$

input
```
integrate(x^5/(c*x^4-b*x^2+a),x, algorithm="giac")
```

output
```
1/2*x^2/c + 1/4*b*log(c*x^4 - b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2
*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 656, normalized size of antiderivative = 8.00

$$\int \frac{x^5}{a-bx^2+cx^4}\,dx = \frac{x^2}{2\,c} - \frac{\ln\left(c\,x^4-b\,x^2+a\right)\,\left(2\,b^3-8\,a\,b\,c\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}$$

$$-\frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\left(\frac{\frac{\left(8\,a\,b+\frac{8\,a\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)\,\left(2\,a\,c-b^2\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}+\frac{a\,\left(2\,b^3-8\,a\,b\,c\right)\,\left(2\,a\,c-b^2\right)}{\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}}{a}+x^2\left(\frac{\frac{\left(2\,a\,c-b^2\right)\,\left(\frac{4\,a\,c^3-6\,b^2\,c^2}{c^2}-\frac{4\,b\,c^2\,\left(2\,b^3-8\dots\right.}{16\,a\,c^3-4\,b^2\dots}\right.}{8\,c^2\,\sqrt{4\,a\,c-b^2}}\dots}{a}\right.\right.}{\dots}\right)}{\dots}$$

input
```
int(x^5/(a - b*x^2 + c*x^4),x)
```

output
```
x^2/(2*c) - (log(a - b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a*c^3 - 4*b^
2*c^2)) - (atan((2*c^2*(4*a*c - b^2)*((((8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c
))/(16*a*c^3 - 4*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a
*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c
^2)))/a + x^2*((((2*a*c - b^2)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^
3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(2
*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^
2)))/a + (b*(((2*b^3 - 8*a*b*c)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b
^3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b^3
- a*b*c)/c^2 + (b*(2*a*c - b^2)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b
^2)^(1/2))) + (b*((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c)*(8*a*b + (8*a*c^2*(2*b^
3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*(2*
a*c - b^2)^2)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^4 + 4*a
^2*c^2 - 4*a*b^2*c))*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 360, normalized size of antiderivative = 4.39

$$\int \frac{x^5}{a-bx^2+cx^4}\,dx$$
$$= \frac{4\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)ac-2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-}}{\sqrt{2\sqrt{c}}}\right.}{}$$

input
```
int(x^5/(c*x^4-b*x^2+a),x)
```

output
```
(4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*a*c - 2*sqr
t(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*
sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b**2 + 4*sqrt(2*s
qrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(
a) + b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*a*c - 2*sqrt(2*sqrt(c)
*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b
) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b**2 + 4*log( - sqrt(2*sqrt(
c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*c - log( - sqrt(2*sqrt(c)*
sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*b**3 + 4*log(sqrt(2*sqrt(c)*sqrt(
a) + b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*c - log(sqrt(2*sqrt(c)*sqrt(a) + b
)*x + sqrt(a) + sqrt(c)*x**2)*b**3 + 8*a*c**2*x**2 - 2*b**2*c*x**2)/(4*c**
2*(4*a*c - b**2))
```

3.818 $\int \frac{x^3}{a-bx^2+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 64

$$\int \frac{x^3}{a-bx^2+cx^4}\,dx=\frac{b\operatorname{arctanh}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}$$

output
```
1/2*b*arctanh((-2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)+1/4*ln
(c*x^4-b*x^2+a)/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{a-bx^2+cx^4}\,dx=\frac{\frac{2b\arctan\left(\frac{-b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a-bx^2+cx^4\right)}{4c}$$

input
```
Integrate[x^3/(a - b*x^2 + c*x^4),x]
```

output
```
((2*b*ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[
a - b*x^2 + c*x^4])/(4*c)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1434, 1142, 25, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a-bx^2+cx^4}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{x^2}{cx^4-bx^2+a}dx^2$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{cx^4-bx^2+a}dx^2}{2c}+\frac{\int -\frac{b-2cx^2}{cx^4-bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{cx^4-bx^2+a}dx^2}{2c}-\frac{\int \frac{b-2cx^2}{cx^4-bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(-\frac{b\int \frac{1}{-x^4+b^2-4ac}d(2cx^2-b)}{c}-\frac{\int \frac{b-2cx^2}{cx^4-bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(-\frac{\int \frac{b-2cx^2}{cx^4-bx^2+a}dx^2}{2c}-\frac{b\,\text{arctanh}\left(\frac{2cx^2-b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{\log\left(a-bx^2+cx^4\right)}{2c}-\frac{b\,\text{arctanh}\left(\frac{2cx^2-b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

input
```
Int[x^3/(a - b*x^2 + c*x^4),x]
```

output
```
(-((b*ArcTanh[(-b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c])) +
Log[a - b*x^2 + c*x^4]/(2*c))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

method	result
default	$\frac{\ln(c\,x^4-b\,x^2+a)}{4c}+\frac{b\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{2c\sqrt{4ac-b^2}}$
risch	$\frac{\ln\left(\left(4abc-b^3-\sqrt{-b^2(4ac-b^2)}\,b\right)x^2+2\sqrt{-b^2(4ac-b^2)}\,a\right)a}{4ac-b^2}-\frac{\ln\left(\left(4abc-b^3-\sqrt{-b^2(4ac-b^2)}\,b\right)x^2+2\sqrt{-b^2(4ac-b^2)}\,a\right)b^2}{4c(4ac-b^2)}+\frac{\ln}{}$

input
```
int(x^3/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(c*x^4-b*x^2+a)/c+1/2/c*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*
c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.22

$$\int \frac{x^3}{a-bx^2+cx^4}\,dx = \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-(2\,cx^2-b)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+(b^2-4\,ac)\log\left(cx^4-bx^2+a\right)}{4\,(b^2c-4\,ac^2)},\right.$$
$$\left.-\frac{2\,\sqrt{-b^2+4\,ac}b\arctan\left(-\frac{(2\,cx^2-b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^2-4\,ac)\log\left(cx^4-bx^2+a\right)}{4\,(b^2c-4\,ac^2)}\right]$$

input
```
integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*
x^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4
 - b*x^2 + a))/(b^2*c - 4*a*c^2), -1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*
c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^4 - b
*x^2 + a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.

Time = 0.48 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.48

$$\int \frac{x^3}{a - bx^2 + cx^4}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)} + \frac{1}{4c}\right)\log\left(x^2 + \frac{8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{1}{4c}\right) - 2a - 2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{1}{4c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)} + \frac{1}{4c}\right)\log\left(x^2 + \frac{8ac\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{1}{4c}\right) - 2a - 2b^2\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)} + \frac{1}{4c}\right)}{b}\right)$$

input
```
integrate(x**3/(c*x**4-b*x**2+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (8*a*c*
(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) - 2*a - 2*b**2*(-b
*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c
+ b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (8*a*c*(b*sqrt(-4*a*c +
 b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) - 2*a - 2*b**2*(b*sqrt(-4*a*c + b**
2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a - bx^2 + cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{a-bx^2+cx^4}\,dx=\frac{b\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,acc}}+\frac{\log\left(cx^4-bx^2+a\right)}{4\,c}$$

input
```
integrate(x^3/(c*x^4-b*x^2+a),x, algorithm="giac")
```

output
```
1/2*b*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
4*log(c*x^4 - b*x^2 + a)/c
```

Mupad [B] (verification not implemented)

Time = 18.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.88

$$\int \frac{x^3}{a-bx^2+cx^4}\,dx=\frac{4\,a\,c\,\ln\left(c\,x^4-b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b^2\,\ln\left(c\,x^4-b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}-\frac{2\,c\,x^2}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^3/(a - b*x^2 + c*x^4),x)
```

output
```
(4*a*c*log(a - b*x^2 + c*x^4))/(16*a*c^2 - 4*b^2*c) - (b^2*log(a - b*x^2 +
 c*x^4))/(16*a*c^2 - 4*b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) - (2*c*x^2)/
(4*a*c - b^2)^(1/2)))/(2*c*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.56

$$\int \frac{x^3}{a-bx^2+cx^4}\,dx$$
$$= \frac{-2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)b-2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}}{\sqrt{2\sqrt{c}}}\right.}{}$$

input
```
int(x^3/(c*x^4-b*x^2+a),x)
```

output
```
( - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b - 2*sq
rt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)
*sqrt(a) + b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b + 4*log( - sqr
t(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*a*c - log( - sqrt(2*s
qrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*b**2 + 4*log(sqrt(2*sqrt(c
)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*a*c - log(sqrt(2*sqrt(c)*sqrt(a
) + b)*x + sqrt(a) + sqrt(c)*x**2)*b**2)/(4*c*(4*a*c - b**2))
```

3.819 $\int \frac{x}{a-bx^2+cx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 35

$$\int \frac{x}{a-bx^2+cx^4}\,dx=\frac{\operatorname{arctanh}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
arctanh((-2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{x}{a-bx^2+cx^4}\,dx=\frac{\arctan\left(\frac{-b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input
```
Integrate[x/(a - b*x^2 + c*x^4),x]
```

output
```
ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1432, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a-bx^2+cx^4}dx$$
$$\Big\downarrow \text{1432}$$
$$\frac{1}{2}\int \frac{1}{cx^4-bx^2+a}dx^2$$
$$\Big\downarrow \text{1083}$$
$$-\int \frac{1}{-x^4+b^2-4ac}d(2cx^2-b)$$
$$\Big\downarrow \text{219}$$
$$-\frac{\text{arctanh}\left(\frac{2cx^2-b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[x/(a - b*x^2 + c*x^4),x]
```

output
```
-(ArcTanh[(-b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\arctan\left(\frac{2c\,x^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	38
risch	$-\frac{\ln\left(\left(b+\sqrt{-4ac+b^2}\right)x^2-2a\right)}{2\sqrt{-4ac+b^2}}+\frac{\ln\left(\left(-b+\sqrt{-4ac+b^2}\right)x^2+2a\right)}{2\sqrt{-4ac+b^2}}$	70

input
```
int(x/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.83

$$\int \frac{x}{a-bx^2+cx^4}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-(2\,cx^2-b)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}},\ -\frac{\sqrt{-b^2+4\,ac}\arctan\left(-\frac{(2\,cx^2-b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

input
```
integrate(x/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 - b)*sqrt(b^2 - 4
*a*c))/(c*x^4 - b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(
-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(32) = 64$.

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.74

$$\int \frac{x}{a-bx^2+cx^4}\,dx = -\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}-b}{2c}\right)}{2} + \frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}-b}{2c}\right)}{2}$$

input
```
integrate(x/(c*x**4-b*x**2+a),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2
*sqrt(-1/(4*a*c - b**2)) - b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2
+ (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) - b)/(2*c)
)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a-bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{x}{a-bx^2+cx^4}\,dx = \frac{\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

input
```
integrate(x/(c*x^4-b*x^2+a),x, algorithm="giac")
```

output
```
arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 17.61 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{x}{a-bx^2+cx^4}\,dx = -\frac{\mathrm{atan}\left(\frac{a\,b-2\,a\,c\,x^2}{a\,\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

input
```
int(x/(a - b*x^2 + c*x^4),x)
```

output
```
-atan((a*b - 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.71

$$\int \frac{x}{a - bx^2 + cx^4} \, dx$$
$$= -\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\left(atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)+atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}+2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)\right)}{4ac-b^2}$$

input
```
int(x/(c*x^4-b*x^2+a),x)
```

output
```
( - sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*(atan((sqrt(2*
sqrt(c)*sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b)) + atan((s
qrt(2*sqrt(c)*sqrt(a) + b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))))/(
4*a*c - b**2)
```

3.820 $\int \frac{1}{x(a-bx^2+cx^4)} dx$

Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{1}{x(a-bx^2+cx^4)} dx = \frac{b\operatorname{arctanh}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a-bx^2+cx^4)}{4a}$$

output
```
1/2*b*arctanh((-2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+ln(x)/
a-1/4*ln(c*x^4-b*x^2+a)/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.67

$$\int \frac{1}{x(a-bx^2+cx^4)} dx$$
$$= \frac{4\sqrt{b^2-4ac}\log(x) + \left(b-\sqrt{b^2-4ac}\right)\log\left(-b-\sqrt{b^2-4ac}+2cx^2\right) - \left(b+\sqrt{b^2-4ac}\right)\log\left(-b+\sqrt{b^2-\right.}{4a\sqrt{b^2-4ac}}$$

input
```
Integrate[1/(x*(a - b*x^2 + c*x^4)),x]
```

output
```
(4*Sqrt[b^2 - 4*a*c]*Log[x] + (b - Sqrt[b^2 - 4*a*c])*Log[-b - Sqrt[b^2 -
4*a*c] + 2*c*x^2] - (b + Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1434, 1144, 1142, 25, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a-bx^2+cx^4\right)}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(cx^4-bx^2+a\right)}dx^2$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{2}\left(\frac{\int \frac{b-cx^2}{cx^4-bx^2+a}dx^2}{a}+\frac{\log\left(x^2\right)}{a}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}b\int \frac{1}{cx^4-bx^2+a}dx^2-\frac{1}{2}\int -\frac{b-2cx^2}{cx^4-bx^2+a}dx^2}{a}+\frac{\log\left(x^2\right)}{a}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}b\int \frac{1}{cx^4-bx^2+a}dx^2+\frac{1}{2}\int \frac{b-2cx^2}{cx^4-bx^2+a}dx^2}{a}+\frac{\log\left(x^2\right)}{a}\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{b-2cx^2}{cx^4-bx^2+a}dx^2-b\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2-b\right)}{a}+\frac{\log\left(x^2\right)}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{b-2cx^2}{cx^4-bx^2+a}dx^2-\frac{b\text{arctanh}\left(\frac{2cx^2-b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}+\frac{\log\left(x^2\right)}{a}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{-\frac{b\text{arctanh}\left(\frac{2cx^2-b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{1}{2}\log\left(a-bx^2+cx^4\right)}{a}+\frac{\log\left(x^2\right)}{a}\right)$$

input
```
Int[1/(x*(a - b*x^2 + c*x^4)),x]
```

output
```
(Log[x^2]/a + (-((b*ArcTanh[(-b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 -
4*a*c]) - Log[a - b*x^2 + c*x^4]/2)/a)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{-\frac{\ln\left(c\,x^{4}-b\,x^{2}+a\right)}{2}+\frac{b\arctan\left(\frac{2c\,x^{2}-b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a}+\frac{\ln\left(x\right)}{a}$	68
risch	$\frac{\ln\left(x\right)}{a}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4c\,a^{2}-b^{2}a\right)\_Z^{2}+\left(4ac-b^{2}\right)\_Z+c\right)}\_R\ln\left(\left(\left(-10ac+3b^{2}\right)\_R-5c\right)x^{2}-ab\_R+2b\right)\right)}{2}$	77

input
```
int(1/x/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-1/2*ln(c*x^4-b*x^2+a)+b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*
c-b^2)^(1/2)))+ln(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.29

$$\int \frac{1}{x\,(a-bx^2+cx^4)}\,dx$$
$$= \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac-(2\,cx^2-b)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)-(b^2-4\,ac)\log\left(cx^4-bx^2+a\right)+4\,(b^2-4\,ac)\log\left(x\right)}{4\,(ab^2-4\,a^2c)}, -\frac{2\,\sqrt{-b^2+4\,ac}b\arctan\left(-\frac{(2\,cx^2-b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(b^2-4\,ac)\log\left(cx^4-bx^2+a\right)-4\,(b^2-4\,ac)\log\left(x\right)}{4\,(ab^2-4\,a^2c)}\right]$$

input
```
integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c - (2*c*
x^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4
 - b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), -1/4*(2*sqrt(-b
^2 + 4*a*c)*b*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b
^2 - 4*a*c)*log(c*x^4 - b*x^2 + a) - 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^
2*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 2.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.61

$$\int \frac{1}{x(a-bx^2+cx^4)}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a}\right)\log\left(x^2+\frac{8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ac-b^2}{bc}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a}\right)\log\left(x^2+\frac{8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ac-b^2}{bc}\right) + \frac{\log(x)}{a}$$

input
```
integrate(1/x/(c*x**4-b*x**2+a),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (8*a**2
*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*b**2*(-b*
sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*c - b**2)/(b*c))
 + (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (8*a*
*2*c*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*b**2*(b*
sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*c - b**2)/(b*c))
 + log(x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x(a-bx^2+cx^4)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{1}{x\left(a-bx^2+cx^4\right)}\,dx=\frac{b\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a}-\frac{\log\left(cx^4-bx^2+a\right)}{4\,a}+\frac{\log\left(x^2\right)}{2\,a}$$

input
```
integrate(1/x/(c*x^4-b*x^2+a),x, algorithm="giac")
```

output
```
1/2*b*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
4*log(c*x^4 - b*x^2 + a)/a + 1/2*log(x^2)/a
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 1015, normalized size of antiderivative = 14.50

$$\int \frac{1}{x\left(a-bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(x*(a - b*x^2 + c*x^4)),x)
```

output
```
log(x)/a + (log(a - b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c
)) - (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*(((8*a*c - 2*b^2)^2*(10*b*c^3
 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/
(4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(
8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(16*a^2*(4*a*c - b^2)) + (b^2*(
12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*
a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b
^2*c)*((b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(
12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(
4*a*c - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b
*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c
)*(4*a*c - b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))
*(4*a*c - b^2)^(3/2))/(b^2*c^2) - (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)
*(((8*a*c - 2*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 -
 16*a^2*c)))/(4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(
8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(
8*a*c - 2*b^2))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(b^2*c^4*(25*a*
c - 6*b^2)) + (2*(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2
)/(16*a^2*(4*a*c - b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 -
 16*a^2*c)^2*(4*a*c - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.41

$$\int \frac{1}{x\left(a-bx^2+cx^4\right)}\,dx$$
$$= \frac{-2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)b-2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}}}{\sqrt{2\sqrt{c}}}\right.}{}$$

input
```
int(1/x/(c*x^4-b*x^2+a),x)
```

output
```
( - 2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b - 2*sq
rt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)
*sqrt(a) + b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b - 4*log( - sqr
t(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*a*c + log( - sqrt(2*s
qrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*b**2 - 4*log(sqrt(2*sqrt(c
)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*a*c + log(sqrt(2*sqrt(c)*sqrt(a
) + b)*x + sqrt(a) + sqrt(c)*x**2)*b**2 + 16*log(x)*a*c - 4*log(x)*b**2)/(
4*a*(4*a*c - b**2))
```

3.821 $\int \frac{1}{x^3(a-bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{1}{x^3 (a-bx^2+cx^4)}\,dx = -\frac{1}{2ax^2}+\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log(x)}{a^2}-\frac{b\log(a-bx^2+cx^4)}{4a^2}$$

output
```
-1/2/a/x^2+1/2*(-2*a*c+b^2)*arctanh((-2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(
-4*a*c+b^2)^(1/2)+b*ln(x)/a^2-1/4*b*ln(c*x^4-b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^3 (a-bx^2+cx^4)}\,dx = \frac{-\frac{2a}{x^2}+4b\log(x)+\frac{\left(b^2-2ac-b\sqrt{b^2-4ac}\right)\log\left(-b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(-b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a^2}$$

input
```
Integrate[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

output
```
((-2*a)/x^2 + 4*b*Log[x] + ((b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*Log[-b - S
qrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((b^2 - 2*a*c + b*Sqrt[b^
2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a
^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1434, 1145, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a-bx^2+cx^4)} \, dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{1}{x^4 (cx^4-bx^2+a)}dx^2$$

$$\Big\downarrow \text{ 1145}$$

$$\frac{1}{2}\left(\frac{\int \frac{b-cx^2}{x^2(cx^4-bx^2+a)}dx^2}{a}-\frac{1}{ax^2}\right)$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{b}{ax^2}-\frac{-b^2+cx^2b+ac}{a(cx^4-bx^2+a)}\right)dx^2}{a}-\frac{1}{ax^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{\frac{(b^2-2ac)\text{arctanh}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log(a-bx^2+cx^4)}{2a}+\frac{b\log(x^2)}{a}}{a}-\frac{1}{ax^2}\right)$$

input
```
Int[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

output
```
(-(1/(a*x^2)) + (((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(a*Sqrt[b^2 - 4*a*c]) + (b*Log[x^2])/a - (b*Log[a - b*x^2 + c*x^4])/(2*a))
/a)/2
```

Defintions of rubi rules used

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

method	result
default	$-\frac{\frac{b\ln\left(c\,x^{4}-b\,x^{2}+a\right)}{2}+\frac{2\left(ac-\frac{b^{2}}{2}\right)\arctan\left(\frac{2c\,x^{2}-b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a^{2}}-\frac{1}{2a\,x^{2}}+\frac{b\ln\left(x\right)}{a^{2}}$
risch	$-\frac{1}{2a\,x^{2}}+\frac{b\ln\left(x\right)}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(4abc-b^{3}\right)\_Z+c^{2}\right)}\_R\ln\left(\left(\left(-10a^{3}c+3a^{2}b^{2}\right)\_R^{2}-4\_Rabc-2c^{2}\right)x^{2}-\right.\right.}{2}$

input
```
int(1/x^3/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*(1/2*b*ln(c*x^4-b*x^2+a)+2*(a*c-1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^2-b)/(4*a*c-b^2)^(1/2)))-1/2/a/x^2+b*ln(x)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.35

$$\int \frac{1}{x^3\left(a-bx^2+cx^4\right)}\,dx$$
$$= \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^4-2\,bcx^2+b^2-2\,ac+(2\,cx^2-b)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+(b^3-4\,abc)x^2\log\left(cx^4-bx^2+a\right)}{4\left(a^2b^2-4\,a^3c\right)x^2}\right.$$
$$-\frac{2\left(b^2-2\,ac\right)\sqrt{-b^2+4\,ac}x^2\arctan\left(-\frac{(2\,cx^2-b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(b^3-4\,abc)x^2\log\left(cx^4-bx^2+a\right)-4\,(b^3-}{4\left(a^2b^2-4\,a^3c\right)x^2}$$

input
```
integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 - 2*b*c*x^2 + b^
2 - 2*a*c + (2*c*x^2 - b)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a)) + (b^3 -
 4*a*b*c)*x^2*log(c*x^4 - b*x^2 + a) - 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*
b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2
+ 4*a*c)*x^2*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^
3 - 4*a*b*c)*x^2*log(c*x^4 - b*x^2 + a) - 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2
*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(82) = 164$.

Time = 80.91 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.93

$$\int \frac{1}{x^3 (a - bx^2 + cx^4)}\,dx = \left(-\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2\cdot(4ac-b^2)}\right)\log\left(x^2 + \frac{-8a^3c\left(-\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2\cdot(4ac-b^2)}\right) + 2a^2b^2\left(-\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2\cdot(4ac-b^2)}\right.}{2ac^2 - b^2c}\right. + \left(-\frac{b}{4a^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2\cdot(4ac-b^2)}\right)\log\left(x^2 + \frac{-8a^3c\left(-\frac{b}{4a^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2\cdot(4ac-b^2)}\right) + 2a^2b^2\left(-\frac{b}{4a^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4a^2\cdot(4ac-b^2)}\right.}{2ac^2 - b^2c}\right. - \frac{1}{2ax^2} + \frac{b\log(x)}{a^2}$$

input
```
integrate(1/x**3/(c*x**4-b*x**2+a),x)
```

output
```
(-b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))
*log(x**2 + (-8*a**3*c*(-b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(-b/(4*a**2) - sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) - 3*a*b*c + b**3)/(2*a*c**2 - b**2*
c)) + (-b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b
**2)))*log(x**2 + (-8*a**3*c*(-b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b
**2)/(4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(-b/(4*a**2) + sqrt(-4*a*c + b
**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) - 3*a*b*c + b**3)/(2*a*c**2 -
 b**2*c)) - 1/(2*a*x**2) + b*log(x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a - bx^2 + cx^4)} dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^3 (a - bx^2 + cx^4)} dx = -\frac{b \log(cx^4 - bx^2 + a)}{4 a^2} + \frac{b \log(x^2)}{2 a^2} + \frac{(b^2 - 2 ac) \arctan\left(\frac{2 cx^2 - b}{\sqrt{-b^2 + 4 ac}}\right)}{2 \sqrt{-b^2 + 4 ac} a^2} - \frac{bx^2 + a}{2 a^2 x^2}$$

input
```
integrate(1/x^3/(c*x^4-b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*b*log(c*x^4 - b*x^2 + a)/a^2 + 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)
*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*(
b*x^2 + a)/(a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 19.72 (sec) , antiderivative size = 2032, normalized size of antiderivative = 22.83

$$\int \frac{1}{x^3 (a - bx^2 + cx^4)} dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a - b*x^2 + c*x^4)),x)
```

output
```
(b*log(x))/a^2 - 1/(2*a*x^2) + (log(a - b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/
(2*(16*a^3*c - 4*a^2*b^2)) + (atan((16*a^6*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^
2*c)*(c^5/a^3 + ((2*b^3 - 8*a*b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((2
0*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3
*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2))))/(
2*(16*a^3*c - 4*a^2*b^2)) - ((((2*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)
/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*
c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4
*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3
*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(1/2)) - ((2*b^3 - 8
*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c -
b^2)*(16*a^3*c - 4*a^2*b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c))
+ ((((2*b^3 - 8*a*b*c)*(((2*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 +
 ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3
 - 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4
*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)
*(2*a*c - b^2)^3)/(64*a^9*(4*a*c - b^2)^(3/2)) + (((6*b*c^4)/a^2 + ((2*b^3
 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4
*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 399, normalized size of antiderivative = 4.48

$$\int \frac{1}{x^3 (a - bx^2 + cx^4)} dx$$

$$= \frac{4\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)ac\,x^2 - 2\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\cdots}}{\sqrt{2\cdots}}\right.}{}$$

input
```
int(1/x^3/(c*x^4-b*x^2+a),x)
```

output
```
(4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*a*c*x**2 -
2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqr
t(c)*sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b**2*x**2 +
4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqr
t(c)*sqrt(a) + b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*a*c*x**2 - 2
*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt
(c)*sqrt(a) + b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b**2*x**2 - 4
*log( - sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*c*x**2
 + log( - sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*b**3*x**
2 - 4*log(sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*c*x*
*2 + log(sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*b**3*x**2
 + 16*log(x)*a*b*c*x**2 - 4*log(x)*b**3*x**2 - 8*a**2*c + 2*a*b**2)/(4*a**
2*x**2*(4*a*c - b**2))
```

3.822 $\int \frac{x^4}{a-bx^2+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 179

$$\int \frac{x^4}{a-bx^2+cx^4}\,dx = \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
x/c-1/2*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c^(1/2)*x/(b-(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2
*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+
b^2)^(1/2))^(1/2))*2^(1/2)/c^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.16

$$\int \frac{x^4}{a-bx^2+cx^4}\,dx = \frac{x}{c}+\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}}+\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}}$$

input
```
Integrate[x^4/(a - b*x^2 + c*x^4),x]
```

output
```
x/c + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[-b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - Sq
rt[b^2 - 4*a*c]]) + ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[-b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1442, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a-bx^2+cx^4}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x}{c}-\frac{\int \frac{a-bx^2}{cx^4-bx^2+a}dx}{c}$$

$$\Big\downarrow \text{1480}$$

$$\frac{x}{c}-\frac{-\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(-b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(\sqrt{b^2-4ac}-b\right)}dx}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{x}{c}-\frac{\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}}{c}$$

input
```
Int[x^4/(a - b*x^2 + c*x^4),x]
```

output
```
x/c - (((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
c
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.32

method	result
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}-\_Z^{2}b+a\right)}\frac{\left(\_R^{2}b-a\right)\ln\left(x-\_R\right)}{2\_R^{3}c-\_Rb}}{2c}$
default	$\frac{x}{c}-\frac{\left(b\sqrt{-4ac+b^{2}}-2ac+b^{2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,c\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\left(b\sqrt{-4ac+b^{2}}+2ac-b^{2}\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{-4ac+b^{2}}\,c\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}$

input
```
int(x^4/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/2/c*sum((_R^2*b-a)/(2*_R^3*c-_R*b)*ln(x-_R),_R=RootOf(_Z^4*c-_Z^2*b+
a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. 2(143) = 286.

Time = 0.10 (sec) , antiderivative size = 1051, normalized size of antiderivative = 5.87

$$\int \frac{x^4}{a-bx^2+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^
2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c
^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a
*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a
*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c + (b^2*c^3
 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^
3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^
2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
4*a*c^7)))*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c
 + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt
((b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^
2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/
2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c - (b^2*c^3 - 4*a*
c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a
*c^4))) - sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(
a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a
*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^...
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{a - bx^2 + cx^4} \, dx$$
$$= \text{RootSum}\left(t^4 \cdot \left(256a^2c^5 - 128ab^2c^4 + 16b^4c^3\right) + t^2\left(-48a^2bc^2 + 28ab^3c - 4b^5\right) + a^3, \left(t \mapsto t\log\left(x + \frac{-3}{\ldots}\right.\right.\right.$$
$$+ \frac{x}{c}$$

input
```
integrate(x**4/(c*x**4-b*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(-4
8*a**2*b*c**2 + 28*a*b**3*c - 4*b**5) + a**3, Lambda(_t, _t*log(x + (-32*_
t**3*a*b*c**4 + 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*
b**4)/(a**2*c - a*b**2)))) + x/c
```

Maxima [F]

$$\int \frac{x^4}{a-bx^2+cx^4}\,dx = \int \frac{x^4}{cx^4-bx^2+a}\,dx$$

input
```
integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

output
```
x/c + integrate((b*x^2 - a)/(c*x^4 - b*x^2 + a), x)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2153 vs. 2(143) = 286.

Time = 0.57 (sec) , antiderivative size = 2153, normalized size of antiderivative = 12.03

$$\int \frac{x^4}{a-bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(c*x^4-b*x^2+a),x, algorithm="giac")
```

output
```
x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-
b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c
)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(-b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b
*c^3)*c^2 - 2*(sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt
(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 2*sqrt(2)*sqrt(-b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(-b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*c^4 - 8*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^...
```

Mupad [B] (verification not implemented)

Time = 18.44 (sec) , antiderivative size = 3000, normalized size of antiderivative = 16.76

$$\int \frac{x^4}{a - bx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
int(x^4/(a - b*x^2 + c*x^4),x)
```

output
```
x/c + atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)
*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)
*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (
2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5
 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2
*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*((b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^
2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*((b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*((b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.99

$$\int \frac{x^4}{a-bx^2+cx^4}\,dx$$
$$= \frac{-2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)bc+4\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)ac-2\sqrt{c}\,\sqrt{}}{}$$

input
```
int(x^4/(c*x^4-b*x^2+a),x)
```

output

```
( - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b
) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b*c + 4*sqrt(c)*sqrt(2*sqrt(
c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sq
rt(c)*sqrt(a) - b))*a*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt
(2*sqrt(c)*sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b**2 +
 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b) +
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b*c - 4*sqrt(c)*sqrt(2*sqrt(c)*
sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b) + 2*sqrt(c)*x)/sqrt(2*sqrt(
c)*sqrt(a) - b))*a*c + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*
sqrt(c)*sqrt(a) + b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b**2 + sq
rt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*log( - sqrt(2*sqrt(c)*sqrt(a) + b)*x + s
qrt(a) + sqrt(c)*x**2)*b*c - sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*log(sqrt(
2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*b*c + 2*sqrt(c)*sqrt(2*
sqrt(c)*sqrt(a) + b)*log( - sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt
(c)*x**2)*a*c - sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*log( - sqrt(2*sqrt(c)*
sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*b**2 - 2*sqrt(c)*sqrt(2*sqrt(c)*s
qrt(a) + b)*log(sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*a*
c + sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*log(sqrt(2*sqrt(c)*sqrt(a) + b)*x
+ sqrt(a) + sqrt(c)*x**2)*b**2 + 16*a*c**2*x - 4*b**2*c*x)/(4*c**2*(4*a*c
- b**2))
```

3.823 $\int \frac{x^2}{a-bx^2+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int \frac{x^2}{a-bx^2+cx^4}\,dx = \frac{\sqrt{b-\sqrt{b^2-4ac}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{b+\sqrt{b^2-4ac}}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

output
```
1/2*(b-(-4*a*c+b^2)^(1/2))^(1/2)*arctanh(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)
^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)-1/2*(b+(-4*a*c+b^2)^(1/2
))^(1/2)*arctanh(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c
^(1/2)/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{a-bx^2+cx^4}\,dx = \frac{-\sqrt{-b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)+\sqrt{-b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

input
```
Integrate[x^2/(a - b*x^2 + c*x^4),x]
```

output
```
(-(Sqrt[-b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b - Sqrt[
b^2 - 4*a*c]]]) + Sqrt[-b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1450, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a-bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1450}$$

$$\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(-b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(\sqrt{b^2-4ac}-b\right)}dx$$

$$\Big\downarrow \text{221}$$

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

input
```
Int[x^2/(a - b*x^2 + c*x^4),x]
```

output
```
-(((1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) - ((1 + b/Sqrt
[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^4-\_Z^2 b+a\right)}\frac{\_R^2 \ln\left(x-\_R\right)}{2\_R^3 c-\_R b}\right)}{2}$	43
default	$4c\left(-\frac{\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\text{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{2}\,\text{arctan}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$	149

input
```
int(x^2/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R^2/(2*_R^3*c-_R*b)*ln(x-_R),_R=RootOf(_Z^4*c-_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(115) = 230$.

Time = 0.09 (sec) , antiderivative size = 551, normalized size of antiderivative = 3.67

$$\int \frac{x^2}{a-bx^2+cx^4}\,dx = -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}(b^2c-4\,ac^2)\sqrt{\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)$$

input
```
integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
 - 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b + (b^2*c - 4*a*c^2)/s
qrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) +
1/2*sqrt(1/2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b + (b^2*c - 4*a*c^2)/s
qrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) +
1/2*sqrt(1/2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b - (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1
/2*sqrt(1/2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c -
 4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b - (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.50

$$\int \frac{x^2}{a-bx^2+cx^4}\,dx$$
$$=\mathrm{RootSum}\left(t^4\cdot\left(256a^2c^3-128ab^2c^2+16b^4c\right)+t^2\cdot\left(16abc-4b^3\right)+a,\left(t\mapsto t\log\left(64t^3ac^2-16t^3b^2c+2t\right.\right.\right.$$

input
```
integrate(x**2/(c*x**4-b*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(16*a*
b*c - 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c + 2
*_t*b + x)))
```

Maxima [F]

$$\int \frac{x^2}{a - bx^2 + cx^4}\,dx = \int \frac{x^2}{cx^4 - bx^2 + a}\,dx$$

input
```
integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^4 - b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(115) = 230.

Time = 0.55 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.42

$$\int \frac{x^2}{a - bx^2 + cx^4}\,dx =$$

$$-\frac{\left(2\,b^2c^2 - 8\,ac^3 - \sqrt{2}\sqrt{b^2 - 4\,ac}\sqrt{-bc - \sqrt{b^2 - 4\,ac}c}b^2 + 4\,\sqrt{2}\sqrt{b^2 - 4\,ac}\sqrt{-bc - \sqrt{b^2 - 4\,ac}c}ac - 2\,\cdots\right.}{2\,(b^4 - 8\,ab^2c + 2\,b^3\cdots}$$

$$-\frac{\left(2\,b^2c^2 - 8\,ac^3 - \sqrt{2}\sqrt{b^2 - 4\,ac}\sqrt{-bc + \sqrt{b^2 - 4\,ac}c}b^2 + 4\,\sqrt{2}\sqrt{b^2 - 4\,ac}\sqrt{-bc + \sqrt{b^2 - 4\,ac}c}ac - 2\,\cdots\right.}{2\,(b^4 - 8\,ab^2c + 2\,b^3\cdots}$$

input
```
integrate(x^2/(c*x^4-b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2
 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*
c)*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b
*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b
^2 - 4*a*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b + sqrt(b^2 - 4*a*c))/c))/((
b^4 - 8*a*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*ab
s(c)) - 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + s
qrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2
 - 4*a*c)*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*
c)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*c^2
 - 2*(b^2 - 4*a*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b - sqrt(b^2 - 4*a*c))
/c))/((b^4 - 8*a*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + b^2*c^2 - 4*a*
c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 17.88 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.77

$$\int \frac{x^2}{a-bx^2+cx^4}\,dx =$$

$$-2\,\mathrm{atanh}\left(\frac{\left(x\,\left(4\,a\,c^2-2\,b^2\,c\right)+\frac{x\,\left(8\,b^3\,c^2-32\,a\,b\,c^3\right)\,\left(b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{\frac{b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{a\,c}\right)\,\sqrt{\frac{b^3}{8}}$$

$$-2\,\mathrm{atanh}\left(\frac{\left(x\,\left(4\,a\,c^2-2\,b^2\,c\right)-\frac{x\,\left(8\,b^3\,c^2-32\,a\,b\,c^3\right)\,\left(\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{-\frac{\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{a\,c}\right)\,\sqrt{}$$

input
```
int(x^2/(a - b*x^2 + c*x^4),x)
```

output
```
- 2*atanh(((x*(4*a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-(
4*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*
((b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b
^2*c^2)))^(1/2))/(a*c))*((b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^
4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*b^2*c)
- (x*(8*b^3*c^2 - 32*a*b*c^3)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/
(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-((-(4*a*c - b^2)^3)^(1/2) - b^3
+ 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(a*c))*(-((-(4*a
*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2))
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.29

$$\int \frac{x^2}{a-bx^2+cx^4}\,dx = \frac{-4\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)c-2\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)b+4\sqrt{a}\,\sqrt{2}}{}$$

input
```
int(x^2/(c*x^4-b*x^2+a),x)
```

output
```
( - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b
) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*c - 2*sqrt(c)*sqrt(2*sqrt(c)
*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) - b))*b + 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) + b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*c + 2*sqrt
(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b) + 2*sqrt
(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
 b)*log( - sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*c - 2*s
qrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*log(sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqr
t(a) + sqrt(c)*x**2)*c - sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*log( - sqrt(2
*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*b + sqrt(c)*sqrt(2*sqrt(
c)*sqrt(a) + b)*log(sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2
)*b)/(4*c*(4*a*c - b**2))
```

3.824 $\int \frac{1}{a-bx^2+cx^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 150

$$\int \frac{1}{a-bx^2+cx^4} dx = \frac{\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
2^(1/2)*c^(1/2)*arctanh(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))/(-
4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2^(1/2)*c^(1/2)*arctanh(2^(1
/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int \frac{1}{a-bx^2+cx^4} dx = \frac{\sqrt{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{-b-\sqrt{b^2-4ac}}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[(a - b*x^2 + c*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]]
/Sqrt[-b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[
b^2 - 4*a*c]]]/Sqrt[-b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1406, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-bx^2+cx^4}\,dx$$

$$\Big\downarrow \ 1406$$

$$\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(-b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(\sqrt{b^2-4ac}-b\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \ 221$$

$$\frac{\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

input
```
Int[(a - b*x^2 + c*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan
h[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqr
t[b + Sqrt[b^2 - 4*a*c]])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4-\_Z^2b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^3c-\_Rb}\right)}{2}$	40
default	$4c\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$	117

input
```
int(1/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3*c-_R*b)*ln(x-_R),_R=RootOf(_Z^4*c-_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(114) = 228.

Time = 0.11 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.03

$$\int \frac{1}{a-bx^2+cx^4}\,dx = -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx+\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx-\sqrt{\frac{1}{2}}\left(b^2-4\,ac-\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{\frac{b+\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx+\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\log\left(2\,cx-\sqrt{\frac{1}{2}}\left(b^2-4\,ac+\frac{ab^3-4\,a^2bc}{\sqrt{a^2b^2-4\,a^3c}}\right)\sqrt{\frac{b-\frac{ab^2-4\,a^2c}{\sqrt{a^2b^2-4\,a^3c}}}{ab^2-4\,a^2c}}\right)$$

input
```
integrate(1/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt
(a^2*b^2 - 4*a^3*c))*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/
(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b
^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*
b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt
(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt((b - (a*b^2
- 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/
2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt((b - (
a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1
/2)*sqrt((b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c)
)*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 -
4*a^3*c))*sqrt((b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*
a^2*c)))
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.58

$$\int \frac{1}{a-bx^2+cx^4}\,dx$$
$$=\text{RootSum}\left(t^4\cdot\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\cdot\left(16abc-4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{-32t^3a^2bc+8t^3\right.\right.\right.$$

input
```
integrate(1/(c*x**4-b*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(16*a*
b*c - 4*b**3) + c, Lambda(_t, _t*log(x + (-32*_t**3*a**2*b*c + 8*_t**3*a*b
**3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Maxima [F]

$$\int \frac{1}{a - bx^2 + cx^4} \, dx = \int \frac{1}{cx^4 - bx^2 + a} \, dx$$

input
```
integrate(1/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^4 - b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(114) = 228$.

Time = 0.36 (sec) , antiderivative size = 1050, normalized size of antiderivative = 7.00

$$\int \frac{1}{a - bx^2 + cx^4} \, dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4-b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(-b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*
b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*
sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(-b*c - sqr
t(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(-b*c
 - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(-b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
- sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c
^2 - 2*(b^2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt(-(b + sqrt(b^2 - 4*a
*c))/c))/((a*b^4 - 8*a^2*b^2*c + 2*a*b^3*c + 16*a^3*c^2 - 8*a^2*b*c^2 + a*
b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*(sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*
c)*b^4 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sq
rt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(-b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c^2 + sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 -
2*b^3*c^2 - 4*sqrt(2)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3
+ 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*b
^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c...
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 763, normalized size of antiderivative = 5.09

$$\int \frac{1}{a-bx^2+cx^4}\,dx =$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,}{4\,a\,b^4\,\sqrt{\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-}\right.$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2}{4\,a\,b^4\,\sqrt{-\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right.$$

input
```
int(1/(a - b*x^2 + c*x^4),x)
```

output
```
- atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*((b^3 + (b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 6
4*a^2*b^2*c))^(1/2) + 64*a^3*c^2*((b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(
1/2) - 32*a^2*b^2*c*((b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*((b^3
+ (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b
^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 6
4*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*
c*x*8i)/(4*a*b^4*(-((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)
 - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c
^2*(-((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b
*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(-((b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4
 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-((b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b
^2*c))^(1/2)*2i
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.29

$$\int \frac{1}{a - bx^2 + cx^4}\,dx$$
$$= \frac{-2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)b - 4\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)a + 2\sqrt{a}\,\sqrt{2}}{}$$

input
```
int(1/(c*x^4-b*x^2+a),x)
```

output
```
( - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b
) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b - 4*sqrt(c)*sqrt(2*sqrt(c)
*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) - b))*a + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) + b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*b + 4*sqrt
(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b) + 2*sqrt
(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*a + sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b
)*log( - sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*b - sqrt(
a)*sqrt(2*sqrt(c)*sqrt(a) + b)*log(sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a)
 + sqrt(c)*x**2)*b - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*log( - sqrt(2*s
qrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*a + 2*sqrt(c)*sqrt(2*sqrt(
c)*sqrt(a) + b)*log(sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2
)*a)/(4*a*(4*a*c - b**2))
```

3.825 $\int \frac{1}{x^2(a-bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 172

$$\int \frac{1}{x^2\left(a-bx^2+cx^4\right)}\,dx=-\frac{1}{ax}+\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
-1/a/x+1/2*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c^(1/2)*x/(b-(
-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*c^(1/
2)*(1-b/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2
))^(1/2))*2^(1/2)/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^2\left(a-bx^2+cx^4\right)}\,dx=-\frac{\frac{2}{x}+\frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b+\sqrt{b^2-4ac}}}}{2a}$$

input
```
Integrate[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

output
```
-1/2*(2/x + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - Sqrt[b^
2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b + Sqrt[
b^2 - 4*a*c]]))/a
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1443, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a-bx^2+cx^4)} \, dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{\int \frac{b-cx^2}{cx^4-bx^2+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{1480}$$

$$\frac{-\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(-b-\sqrt{b^2-4ac}\right)}dx-\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(\sqrt{b^2-4ac}-b\right)}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{a}-\frac{1}{ax}$$

input
```
Int[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

output
```
-(1/(a*x)) + ((Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

method	result
default	$\frac{4c\left(-\frac{\left(b-\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-\sqrt{-4ac+b^2}-b\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a}-\frac{1}{ax}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^5c^2-8a^4b^2c+b^4a^3\right)\_Z^4+\left(-12a^2b\,c^2+7a\,b^3c-b^5\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(40a^5c^2-22a^4b^2c+3b^4a^3\right)\_R^4+\right.\right.\right.}{2}$

input
```
int(1/x^2/(c*x^4-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/a*c*(-1/8*(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
+1/8*(-(-4*a*c+b^2)^(1/2)-b)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/a
/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1108 vs. $2(137) = 274$.

Time = 0.12 (sec) , antiderivative size = 1108, normalized size of antiderivative = 6.44

$$\int \frac{1}{x^2 (a - bx^2 + cx^4)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2
*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*
a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c
)))*sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt((b^3
 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*
(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt(
(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 - 3*a*b*c + (a
^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(
a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^
4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a
^4*c))*log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c
^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(
1/2)*a*x*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c +
 a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c
^3)*x - sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2...
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2 (a - bx^2 + cx^4)} \, dx$$
$$= \text{RootSum}\left(t^4 \cdot (256a^5c^2 - 128a^4b^2c + 16a^3b^4) + t^2(-48a^2bc^2 + 28ab^3c - 4b^5) + c^3, \left(t \mapsto t \log\left(x + \frac{-\right.\right.\right.$$
$$- \frac{1}{ax}$$

input
```
integrate(1/x**2/(c*x**4-b*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(-4
8*a**2*b*c**2 + 28*a*b**3*c - 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_
t**3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 + 10*_t*a**2*b*c
**2 - 10*_t*a*b**3*c + 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

Maxima [F]

$$\int \frac{1}{x^2\,(a-bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4-bx^2+a)x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((c*x^2 - b)/(c*x^4 - b*x^2 + a), x)/a - 1/(a*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1877 vs. $2(137) = 274$.

Time = 0.69 (sec) , antiderivative size = 1877, normalized size of antiderivative = 10.91

$$\int \frac{1}{x^2\,(a-bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^4-b*x^2+a),x, algorithm="giac")
```

output
```
1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16
*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 -
 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)
*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^
2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2
 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(-b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^3*c + 2*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 1
6*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*sqrt(-b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^3*c^2 - 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(-b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^
2*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*x/sqrt(-(a*b + sqrt(a^2*b^2 - 4*a^3...
```

Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 2979, normalized size of antiderivative = 17.32

$$\int \frac{1}{x^2 (a - bx^2 + cx^4)} dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a - b*x^2 + c*x^4)),x)
```

output
```
- atan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32
*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2
 - 8*a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 -
 8*a^4*b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) - ((b^5 + b^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*
a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 + b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^
3) - ((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
*(16*a^5*b*c^3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*((b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b
^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*((b^5 +...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.19

$$\int \frac{1}{x^2 (a - bx^2 + cx^4)} \, dx$$
$$= \frac{4\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)acx - 2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}+b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}-b}}\right)b^2x - 2\sqrt{c}\cdots}{}$$

input
```
int(1/x^2/(c*x^4-b*x^2+a),x)
```

output
```
(4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*a*c*x - 2*sqrt(a)*sqrt(2*sqrt(c
)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sqr
t(c)*sqrt(a) - b))*b**2*x - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sq
rt(2*sqrt(c)*sqrt(a) + b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*a*b*
x - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b
) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*a*c*x + 2*sqrt(a)*sqrt(2*sqr
t(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) + b) + 2*sqrt(c)*x)/sqrt(2*
sqrt(c)*sqrt(a) - b))*b**2*x + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan(
(sqrt(2*sqrt(c)*sqrt(a) + b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) - b))*a
*b*x - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*log( - sqrt(2*sqrt(c)*sqrt(a)
 + b)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x + sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
 b)*log( - sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*b**2*x
+ 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*log(sqrt(2*sqrt(c)*sqrt(a) + b)*x
+ sqrt(a) + sqrt(c)*x**2)*a*c*x - sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*log(
sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*b**2*x - sqrt(c)*s
qrt(2*sqrt(c)*sqrt(a) + b)*log( - sqrt(2*sqrt(c)*sqrt(a) + b)*x + sqrt(a)
+ sqrt(c)*x**2)*a*b*x + sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*log(sqrt(2*sqr
t(c)*sqrt(a) + b)*x + sqrt(a) + sqrt(c)*x**2)*a*b*x - 16*a**2*c + 4*a*b**2
)/(4*a**2*x*(4*a*c - b**2))
```

3.826 $\int \frac{x^5}{a-b+2ax^2+ax^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \frac{x^5}{a-b+2ax^2+ax^4} dx = \frac{x^2}{2a} - \frac{(a+b)\text{arctanh}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}} - \frac{\log\left(a-b+2ax^2+ax^4\right)}{2a}$$

output
```
1/2*x^2/a-1/2*(a+b)*arctanh(a^(1/2)*(x^2+1)/b^(1/2))/a^(3/2)/b^(1/2)-1/2*l
n(a*x^4+2*a*x^2+a-b)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{a-b+2ax^2+ax^4} dx = -\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x^2 - \log\left(-b+a(1+x^2)^2\right)}{2a}$$

input
```
Integrate[x^5/(a - b + 2*a*x^2 + a*x^4),x]
```

output
```
-1/2*((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(a^(3/2)*Sqrt[b]) + (x
^2 - Log[-b + a*(1 + x^2)^2])/(2*a)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{ax^4+2ax^2+a-b}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^4}{ax^4+2ax^2+a-b}dx^2$$

$$\Big\downarrow \text{1143}$$

$$\frac{1}{2}\int \left(\frac{1}{a}-\frac{2ax^2+a-b}{a\left(ax^4+2ax^2+a-b\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{a}+\frac{x^2}{a}\right)$$

input
```
Int[x^5/(a - b + 2*a*x^2 + a*x^4),x]
```

output
```
(x^2/a - ((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(a^(3/2)*Sqrt[b])
- Log[a - b + 2*a*x^2 + a*x^4]/a)/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result
default	$\frac{x^2}{2a}+\frac{-\ln(a\,x^4+2a\,x^2+a-b)-\frac{(a+b)\,\mathrm{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{\sqrt{ab}}}{2a}$
risch	$\frac{x^2}{2a}-\frac{\ln\left(\left(a^2b+b^2a+\sqrt{ab(a+b)^2}\,a\right)x^2+\sqrt{ab(a+b)^2}\,a-\sqrt{ab(a+b)^2}\,b\right)}{2a}+\frac{\ln\left(\left(a^2b+b^2a+\sqrt{ab(a+b)^2}\,a\right)x^2+\sqrt{ab(a+b)^2}\,a-\sqrt{\right.}{4a^2b}$

input
```
int(x^5/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/a+1/2/a*(-ln(a*x^4+2*a*x^2+a-b)-(a+b)/(a*b)^(1/2)*arctanh(1/2*(2*a
*x^2+2*a)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.26

$$\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx = \left[\frac{2\,abx^2-2\,ab\log\left(ax^4+2\,ax^2+a-b\right)+\sqrt{ab}(a+b)\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{ab}(x^2+1)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,a^2b}, \frac{abx^2-ab\log\left(ax^4\right.}{}\right.$$

input
```
integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*x^2 - 2*a*b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(a*b)*(a + b)*l
og((a*x^4 + 2*a*x^2 - 2*sqrt(a*b)*(x^2 + 1) + a + b)/(a*x^4 + 2*a*x^2 + a
- b)))/(a^2*b), 1/2*(a*b*x^2 - a*b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(-a*
b)*(a + b)*arctan(sqrt(-a*b)/(a*x^2 + a)))/(a^2*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(60) = 120$.

Time = 0.63 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.00

$$\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx = \left(-\frac{1}{2a}-\frac{\sqrt{a^3b}(a+b)}{4a^3b}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{2a}-\frac{\sqrt{a^3b}(a+b)}{4a^3b}\right)+a-b}{a+b}\right) + \left(-\frac{1}{2a}+\frac{\sqrt{a^3b}(a+b)}{4a^3b}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{2a}+\frac{\sqrt{a^3b}(a+b)}{4a^3b}\right)+a-b}{a+b}\right)+\frac{x^2}{2a}$$

input
```
integrate(x**5/(a*x**4+2*a*x**2+a-b),x)
```

output
```
(-1/(2*a) - sqrt(a**3*b)*(a + b)/(4*a**3*b))*log(x**2 + (-4*a*b*(-1/(2*a)
- sqrt(a**3*b)*(a + b)/(4*a**3*b)) + a - b)/(a + b)) + (-1/(2*a) + sqrt(a*
*3*b)*(a + b)/(4*a**3*b))*log(x**2 + (-4*a*b*(-1/(2*a) + sqrt(a**3*b)*(a +
 b)/(4*a**3*b)) + a - b)/(a + b)) + x**2/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx = \frac{x^2}{2\,a}+\frac{(a+b)\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\,\sqrt{ab}a}-\frac{\log\left(ax^4+2\,ax^2+a-b\right)}{2\,a}$$

input
```
integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

output
```
1/2*x^2/a + 1/4*(a + b)*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)
))/(sqrt(a*b)*a) - 1/2*log(a*x^4 + 2*a*x^2 + a - b)/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx = \frac{x^2}{2\,a} + \frac{(a+b)\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}a} - \frac{\log\left(ax^4+2\,ax^2+a-b\right)}{2\,a}$$

input
```
integrate(x^5/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

output
```
1/2*x^2/a + 1/2*(a + b)*arctan((a*x^2 + a)/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/
2*log(a*x^4 + 2*a*x^2 + a - b)/a
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.41

$$\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx = \frac{x^2}{2\,a} - \ln\left(a\,\sqrt{a^3\,b} - b\,\sqrt{a^3\,b} - a^2\,b\,x^2 + a\,x^2\,\sqrt{a^3\,b}\right)\left(\frac{\frac{a^2}{2}+\frac{\sqrt{a^3\,b}}{4}}{a^3} + \frac{\sqrt{a^3\,b}}{4\,a^2\,b}\right) - \ln\left(a\,\sqrt{a^3\,b} - b\,\sqrt{a^3\,b} + a^2\,b\,x^2 + a\,x^2\,\sqrt{a^3\,b}\right)\left(\frac{\frac{a^2}{2}-\frac{\sqrt{a^3\,b}}{4}}{a^3} - \frac{\sqrt{a^3\,b}}{4\,a^2\,b}\right)$$

input
```
int(x^5/(a - b + 2*a*x^2 + a*x^4),x)
```

output
```
x^2/(2*a) - log(a*(a^3*b)^(1/2) - b*(a^3*b)^(1/2) - a^2*b*x^2 + a*x^2*(a^3
*b)^(1/2))*((a^2/2 + (a^3*b)^(1/2)/4)/a^3 + (a^3*b)^(1/2)/(4*a^2*b)) - log
(a*(a^3*b)^(1/2) - b*(a^3*b)^(1/2) + a^2*b*x^2 + a*x^2*(a^3*b)^(1/2))*((a^
2/2 - (a^3*b)^(1/2)/4)/a^3 - (a^3*b)^(1/2)/(4*a^2*b))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.00

$$\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{\sqrt{b}\,\sqrt{a}-a}+\sqrt{a}\,x\right)a+\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{\sqrt{b}\,\sqrt{a}-a}+\sqrt{a}\,x\right)b+\sqrt{b}\,\sqrt{a}\log\left(\sqrt{\sqrt{b}\,\sqrt{a}}\right.}{}$$

input
```
int(x^5/(a*x^4+2*a*x^2+a-b),x)
```

output
```
(sqrt(b)*sqrt(a)*log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*a + sqrt(b)
*sqrt(a)*log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*b + sqrt(b)*sqrt(a)
*log(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*a + sqrt(b)*sqrt(a)*log(sqrt(s
qrt(b)*sqrt(a) - a) + sqrt(a)*x)*b - sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) +
 a*x**2 + a)*a - sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) + a*x**2 + a)*b - 2*l
og( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*a*b - 2*log(sqrt(sqrt(b)*sqrt
(a) - a) + sqrt(a)*x)*a*b - 2*log(sqrt(b)*sqrt(a) + a*x**2 + a)*a*b + 2*a*
b*x**2)/(4*a**2*b)
```

3.827 $\int \frac{x^3}{a-b+2ax^2+ax^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 56

$$\int \frac{x^3}{a-b+2ax^2+ax^4} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}} + \frac{\log(a-b+2ax^2+ax^4)}{4a}$$

output
```
1/2*arctanh(a^(1/2)*(x^2+1)/b^(1/2))/a^(1/2)/b^(1/2)+1/4*ln(a*x^4+2*a*x^2+
a-b)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{a-b+2ax^2+ax^4} dx = \frac{\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{\sqrt{b}} + \log\left(-b+a(1+x^2)^2\right)}{4a}$$

input
```
Integrate[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

output
```
((2*Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Log[-b + a*(1
+ x^2)^2])/(4*a)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1434, 1142, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{ax^4+2ax^2+a-b}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{ax^4+2ax^2+a-b}dx^2$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{\int \frac{2a(x^2+1)}{ax^4+2ax^2+a-b}dx^2}{2a}-\int \frac{1}{ax^4+2ax^2+a-b}dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\int \frac{x^2+1}{ax^4+2ax^2+a-b}dx^2-\int \frac{1}{ax^4+2ax^2+a-b}dx^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(2\int \frac{1}{4ab-x^4}d(2ax^2+2a)+\int \frac{x^2+1}{ax^4+2ax^2+a-b}dx^2\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\int \frac{x^2+1}{ax^4+2ax^2+a-b}dx^2+\frac{\text{arctanh}\left(\frac{2ax^2+2a}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{2ax^2+2a}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}+\frac{\log\left(ax^4+2ax^2+a-b\right)}{2a}\right)$$

input
```
Int[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

output
```
(ArcTanh[(2*a + 2*a*x^2)/(2*Sqrt[a]*Sqrt[b])]/(Sqrt[a]*Sqrt[b]) + Log[a -
b + 2*a*x^2 + a*x^4]/(2*a))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result
default	$\frac{\ln(a\,x^4+2a\,x^2+a-b)}{4a}+\frac{\operatorname{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}$
risch	$\frac{\ln\left(\left(\sqrt{ab}\,a-ab\right)x^2+\sqrt{ab}\,a-\sqrt{ab}\,b\right)}{4a}+\frac{\ln\left(\left(\sqrt{ab}\,a-ab\right)x^2+\sqrt{ab}\,a-\sqrt{ab}\,b\right)\sqrt{ab}}{4ba}+\frac{\ln\left(\left(-\sqrt{ab}\,a-ab\right)x^2-\sqrt{ab}\,a+\sqrt{ab}\,b\right)}{4a}-\frac{\ln\left(\left(\right.\right.}{}$

input
```
int(x^3/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(a*x^4+2*a*x^2+a-b)/a+1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b
)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.39

$$\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx$$
$$=\left[\frac{b\log\left(ax^4+2\,ax^2+a-b\right)+\sqrt{ab}\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{ab}(x^2+1)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,ab},\frac{b\log\left(ax^4+2\,ax^2+a-b\right)-2\,\sqrt{-ab}}{4\,ab}\right.$$

input
```
integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

output
```
[1/4*(b*log(a*x^4 + 2*a*x^2 + a - b) + sqrt(a*b)*log((a*x^4 + 2*a*x^2 + 2*
sqrt(a*b)*(x^2 + 1) + a + b)/(a*x^4 + 2*a*x^2 + a - b)))/(a*b), 1/4*(b*log
(a*x^4 + 2*a*x^2 + a - b) - 2*sqrt(-a*b)*arctan(sqrt(-a*b)/(a*x^2 + a)))/(
a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(48) = 96$.

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.96

$$\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx = \left(\frac{1}{4a}-\frac{\sqrt{a^3b}}{4a^2b}\right)\log\left(x^2+\frac{4ab\left(\frac{1}{4a}-\frac{\sqrt{a^3b}}{4a^2b}\right)+a-b}{a}\right) + \left(\frac{1}{4a}+\frac{\sqrt{a^3b}}{4a^2b}\right)\log\left(x^2+\frac{4ab\left(\frac{1}{4a}+\frac{\sqrt{a^3b}}{4a^2b}\right)+a-b}{a}\right)$$

input
```
integrate(x**3/(a*x**4+2*a*x**2+a-b),x)
```

output
```
(1/(4*a) - sqrt(a**3*b)/(4*a**2*b))*log(x**2 + (4*a*b*(1/(4*a) - sqrt(a**3
*b)/(4*a**2*b)) + a - b)/a) + (1/(4*a) + sqrt(a**3*b)/(4*a**2*b))*log(x**2
 + (4*a*b*(1/(4*a) + sqrt(a**3*b)/(4*a**2*b)) + a - b)/a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx = -\frac{\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\sqrt{ab}} + \frac{\log\left(ax^4+2\,ax^2+a-b\right)}{4\,a}$$

input
```
integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

output
```
-1/4*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/sqrt(a*b) + 1/4*
log(a*x^4 + 2*a*x^2 + a - b)/a
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx = -\frac{\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}} + \frac{\log\left(ax^4+2\,ax^2+a-b\right)}{4\,a}$$

input
```
integrate(x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

output
```
-1/2*arctan((a*x^2 + a)/sqrt(-a*b))/sqrt(-a*b) + 1/4*log(a*x^4 + 2*a*x^2 +
 a - b)/a
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.73

$$\begin{aligned}\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx &= \frac{\ln\left(x^2\sqrt{a^3\,b}+a\,b-a^2-a^2\,x^2\right)}{4\,a} \\ &+ \frac{\ln\left(x^2\sqrt{a^3\,b}-a\,b+a^2+a^2\,x^2\right)}{4\,a} \\ &- \frac{\ln\left(x^2\sqrt{a^3\,b}-a\,b+a^2+a^2\,x^2\right)\sqrt{a^3\,b}}{4\,a^2\,b} \\ &+ \frac{\ln\left(x^2\sqrt{a^3\,b}+a\,b-a^2-a^2\,x^2\right)\sqrt{a^3\,b}}{4\,a^2\,b}\end{aligned}$$

input
```
int(x^3/(a - b + 2*a*x^2 + a*x^4),x)
```

output
```
log(x^2*(a^3*b)^(1/2) + a*b - a^2 - a^2*x^2)/(4*a) + log(x^2*(a^3*b)^(1/2)
 - a*b + a^2 + a^2*x^2)/(4*a) - (log(x^2*(a^3*b)^(1/2) - a*b + a^2 + a^2*x
^2)*(a^3*b)^(1/2))/(4*a^2*b) + (log(x^2*(a^3*b)^(1/2) + a*b - a^2 - a^2*x^
2)*(a^3*b)^(1/2))/(4*a^2*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.25

$$\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx$$
$$= \frac{-\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{\sqrt{b}\,\sqrt{a}-a}+\sqrt{a}\,x\right)-\sqrt{b}\,\sqrt{a}\log\left(\sqrt{\sqrt{b}\,\sqrt{a}-a}+\sqrt{a}\,x\right)+\sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}+a\,x^{2}\right.}{4ab}$$

input
```
int(x^3/(a*x^4+2*a*x^2+a-b),x)
```

output
```
( - sqrt(b)*sqrt(a)*log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x) - sqrt(b
)*sqrt(a)*log(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x) + sqrt(b)*sqrt(a)*log
(sqrt(b)*sqrt(a) + a*x**2 + a) + log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a
)*x)*b + log(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*b + log(sqrt(b)*sqrt(a
) + a*x**2 + a)*b)/(4*a*b)
```

3.828 $\int \frac{x}{a-b+2ax^2+ax^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 31

$$\int \frac{x}{a-b+2ax^2+ax^4}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

output
```
-1/2*arctanh(a^(1/2)*(x^2+1)/b^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x}{a-b+2ax^2+ax^4}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

input
```
Integrate[x/(a - b + 2*a*x^2 + a*x^4),x]
```

output
```
-1/2*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1432, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{ax^4+2ax^2+a-b}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{ax^4+2ax^2+a-b}dx^2$$

$$\Big\downarrow \text{1083}$$

$$-\int \frac{1}{4ab-x^4}d(2ax^2+2a)$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{2ax^2+2a}{2\sqrt{a}\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

input
```
Int[x/(a - b + 2*a*x^2 + a*x^4),x]
```

output
```
-1/2*ArcTanh[(2*a + 2*a*x^2)/(2*Sqrt[a]*Sqrt[b])]/(Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{2a\,x^{2}+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}$	26
risch	$\frac{\ln\left(\left(\sqrt{ab}+a\right)x^{2}+a-b\right)}{4\sqrt{ab}}-\frac{\ln\left(\left(\sqrt{ab}-a\right)x^{2}-a+b\right)}{4\sqrt{ab}}$	52

input
```
int(x/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.94

$$\int \frac{x}{a-b+2ax^2+ax^4}\,dx = \left[\frac{\sqrt{ab}\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{ab}(x^2+1)+a+b}{ax^4+2\,ax^2+a-b}\right)}{4\,ab}, \frac{\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{ax^2+a}\right)}{2\,ab}\right]$$

input
```
integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

output
```
[1/4*sqrt(a*b)*log((a*x^4 + 2*a*x^2 - 2*sqrt(a*b)*(x^2 + 1) + a + b)/(a*x^
4 + 2*a*x^2 + a - b))/(a*b), 1/2*sqrt(-a*b)*arctan(sqrt(-a*b)/(a*x^2 + a))
/(a*b)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{x}{a-b+2ax^2+ax^4}\,dx = \frac{\sqrt{\frac{1}{ab}}\log\left(-b\sqrt{\frac{1}{ab}}+x^2+1\right)}{4} - \frac{\sqrt{\frac{1}{ab}}\log\left(b\sqrt{\frac{1}{ab}}+x^2+1\right)}{4}$$

input
```
integrate(x/(a*x**4+2*a*x**2+a-b),x)
```

output
```
sqrt(1/(a*b))*log(-b*sqrt(1/(a*b)) + x**2 + 1)/4 - sqrt(1/(a*b))*log(b*sqr
t(1/(a*b)) + x**2 + 1)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{x}{a-b+2ax^2+ax^4}\,dx = \frac{\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\sqrt{ab}}$$

input
```
integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

output
```
1/4*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x}{a-b+2ax^2+ax^4}\,dx = \frac{\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}}$$

input
```
integrate(x/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

output
```
1/2*arctan((a*x^2 + a)/sqrt(-a*b))/sqrt(-a*b)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x}{a-b+2ax^2+ax^4}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a}\,\sqrt{b}\,x^2}{a\,x^2+a-b}\right)}{2\,\sqrt{a}\,\sqrt{b}}$$

input
```
int(x/(a - b + 2*a*x^2 + a*x^4),x)
```

output
```
atanh((a^(1/2)*b^(1/2)*x^2)/(a - b + a*x^2))/(2*a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.00

$$\int \frac{x}{a-b+2ax^2+ax^4}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\log\left(-\sqrt{\sqrt{b}\,\sqrt{a}-a}+\sqrt{a}\,x\right)+\log\left(\sqrt{\sqrt{b}\,\sqrt{a}-a}+\sqrt{a}\,x\right)-\log\left(\sqrt{b}\,\sqrt{a}+a\,x^2+a\right)\right)}{4ab}$$

input
```
int(x/(a*x^4+2*a*x^2+a-b),x)
```

output
```
(sqrt(b)*sqrt(a)*(log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x) + log(sqrt
(sqrt(b)*sqrt(a) - a) + sqrt(a)*x) - log(sqrt(b)*sqrt(a) + a*x**2 + a)))/(
4*a*b)
```

3.829 $\int \frac{1}{x(a-b+2ax^2+ax^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{1}{x\left(a-b+2ax^2+ax^4\right)}\,dx = \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2(a-b)\sqrt{b}}+\frac{\log(x)}{a-b}-\frac{\log\left(a-b+2ax^2+ax^4\right)}{4(a-b)}$$

output
```
1/2*a^(1/2)*arctanh(a^(1/2)*(x^2+1)/b^(1/2))/(a-b)/b^(1/2)+ln(x)/(a-b)-ln(
a*x^4+2*a*x^2+a-b)/(4*a-4*b)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\left(a-b+2ax^2+ax^4\right)}\,dx = \frac{-4\sqrt{b}\log(x)+\left(\sqrt{a}+\sqrt{b}\right)\log\left(-\sqrt{b}+\sqrt{a}(1+x^2)\right)+\left(-\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{b}+\sqrt{a}(1+x^2)\right)}{4\sqrt{b}(-a+b)}$$

input
```
Integrate[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

output
```
(-4*Sqrt[b]*Log[x] + (Sqrt[a] + Sqrt[b])*Log[-Sqrt[b] + Sqrt[a]*(1 + x^2)]
 + (-Sqrt[a] + Sqrt[b])*Log[Sqrt[b] + Sqrt[a]*(1 + x^2)])/(4*Sqrt[b]*(-a +
 b))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1434, 1144, 25, 27, 1142, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (ax^4+2ax^2+a-b)} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int \frac{1}{x^2 (ax^4+2ax^2+a-b)}dx^2$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{2} \left(\frac{\int -\frac{a(x^2+2)}{ax^4+2ax^2+a-b}dx^2}{a-b}+\frac{\log (x^2)}{a-b}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\frac{\log (x^2)}{a-b}-\frac{\int \frac{a(x^2+2)}{ax^4+2ax^2+a-b}dx^2}{a-b}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{\log (x^2)}{a-b}-\frac{a\int \frac{x^2+2}{ax^4+2ax^2+a-b}dx^2}{a-b}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a-b}-\frac{a\left(\int\frac{1}{ax^4+2ax^2+a-b}dx^2+\frac{\int\frac{2a(x^2+1)}{ax^4+2ax^2+a-b}dx^2}{2a}\right)}{a-b}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a-b}-\frac{a\left(\int\frac{1}{ax^4+2ax^2+a-b}dx^2+\int\frac{x^2+1}{ax^4+2ax^2+a-b}dx^2\right)}{a-b}\right)$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a-b}-\frac{a\left(\int\frac{x^2+1}{ax^4+2ax^2+a-b}dx^2-2\int\frac{1}{4ab-x^4}d(2ax^2+2a)\right)}{a-b}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a-b}-\frac{a\left(\int\frac{x^2+1}{ax^4+2ax^2+a-b}dx^2-\frac{\text{arctanh}\left(\frac{2ax^2+2a}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}\right)}{a-b}\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a-b}-\frac{a\left(\frac{\log(ax^4+2ax^2+a-b)}{2a}-\frac{\text{arctanh}\left(\frac{2ax^2+2a}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}\right)}{a-b}\right)$$

input
```
Int[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

output
```
(Log[x^2]/(a - b) - (a*(-(ArcTanh[(2*a + 2*a*x^2)/(2*Sqrt[a]*Sqrt[b])]/(Sq
rt[a]*Sqrt[b])) + Log[a - b + 2*a*x^2 + a*x^4]/(2*a)))/(a - b))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{\ln(x)}{a-b}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(-1+\left(ab-b^{2}\right)\_Z^{2}+2b\_Z\right)}\_R\ln\left(\left((-a-5b)\_R+5\right)x^{2}+(-a+b)\_R+4\right)\right)}{4}$	64
default	$\frac{\ln(x)}{a-b}-\frac{a\left(\frac{\ln\left(a\,x^{4}+2a\,x^{2}+a-b\right)}{2a}-\frac{\operatorname{arctanh}\left(\frac{2a\,x^{2}+2a}{2\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{2(a-b)}$	70

input
```
int(1/x/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/(a-b)+1/4*sum(_R*ln(((-a-5*b)*_R+5)*x^2+(-a+b)*_R+4),_R=RootOf(-1+(a
*b-b^2)*_Z^2+2*b*_Z))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\left(a-b+2ax^{2}+ax^{4}\right)}\,dx = \left[-\frac{\sqrt{\frac{a}{b}}\log\left(\frac{ax^{4}+2\,ax^{2}-2\,\left(bx^{2}+b\right)\sqrt{\frac{a}{b}}+a+b}{ax^{4}+2\,ax^{2}+a-b}\right)+\log\left(ax^{4}+2\,ax^{2}+a-b\right)-4\,\log\left(x\right)}{4\,(a-b)},\ -\frac{2\,\sqrt{-\frac{a}{b}}\arctan\left(\left(x^{2}+1\right)\sqrt{-\frac{a}{b}}\right)+\log\left(ax^{4}+2\,ax^{2}+a-b\right)-4\,\log\left(x\right)}{4\,(a-b)}\right]$$

input
```
integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(a/b) + a + b)/(
a*x^4 + 2*a*x^2 + a - b)) + log(a*x^4 + 2*a*x^2 + a - b) - 4*log(x))/(a -
b), -1/4*(2*sqrt(-a/b)*arctan((x^2 + 1)*sqrt(-a/b)) + log(a*x^4 + 2*a*x^2
+ a - b) - 4*log(x))/(a - b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(61) = 122$.

Time = 2.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.39

$$\int \frac{1}{x\,(a-b+2ax^2+ax^4)}\,dx = \left(-\frac{1}{4\,(a-b)} - \frac{\sqrt{ab}}{4b\,(a-b)}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)+a-4b^2\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)+b}{a}\right) + \left(-\frac{1}{4\,(a-b)} + \frac{\sqrt{ab}}{4b\,(a-b)}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{4(a-b)}+\frac{\sqrt{ab}}{4b(a-b)}\right)+a-4b^2\left(-\frac{1}{4(a-b)}+\frac{\sqrt{ab}}{4b(a-b)}\right)+b}{a}\right) + \frac{\log(x)}{a-b}$$

input
```
integrate(1/x/(a*x**4+2*a*x**2+a-b),x)
```

output
```
(-1/(4*(a - b)) - sqrt(a*b)/(4*b*(a - b)))*log(x**2 + (4*a*b*(-1/(4*(a - b
)) - sqrt(a*b)/(4*b*(a - b))) + a - 4*b**2*(-1/(4*(a - b)) - sqrt(a*b)/(4*
b*(a - b))) + b)/a) + (-1/(4*(a - b)) + sqrt(a*b)/(4*b*(a - b)))*log(x**2
+ (4*a*b*(-1/(4*(a - b)) + sqrt(a*b)/(4*b*(a - b))) + a - 4*b**2*(-1/(4*(a
 - b)) + sqrt(a*b)/(4*b*(a - b))) + b)/a) + log(x)/(a - b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a-b+2ax^2+ax^4)}\,dx = -\frac{a\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\sqrt{ab}(a-b)} - \frac{\log\left(ax^4+2\,ax^2+a-b\right)}{4\,(a-b)} + \frac{\log\left(x^2\right)}{2\,(a-b)}$$

input
```
integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

output
```
-1/4*a*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + sqrt(a*b)))/(sqrt(a*b)*(a
- b)) - 1/4*log(a*x^4 + 2*a*x^2 + a - b)/(a - b) + 1/2*log(x^2)/(a - b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(a-b+2ax^2+ax^4)}\,dx = -\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}(a-b)} - \frac{\log\left(ax^4+2\,ax^2+a-b\right)}{4\,(a-b)} + \frac{\log\left(x^2\right)}{2\,(a-b)}$$

input
```
integrate(1/x/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

output
```
-1/2*a*arctan((a*x^2 + a)/sqrt(-a*b))/(sqrt(-a*b)*(a - b)) - 1/4*log(a*x^4
 + 2*a*x^2 + a - b)/(a - b) + 1/2*log(x^2)/(a - b)
```

Mupad [B] (verification not implemented)

Time = 19.70 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.38

$$\int \frac{1}{x(a-b+2ax^2+ax^4)}\,dx = \frac{\ln(x)}{a-b} - \frac{\ln\left(16\,a^4+20\,a^4\,x^2+\frac{\left(b-\sqrt{a\,b}\right)\,\left(x^2\,\left(16\,a^5+80\,b\,a^4\right)-16\,a^4\,b+16\,a^5\right)}{4\,(a\,b-b^2)}\right)\,\left(b-\sqrt{a\,b}\right)}{4\,(a\,b-b^2)} - \frac{\ln\left(16\,a^4+20\,a^4\,x^2+\frac{\left(b+\sqrt{a\,b}\right)\,\left(x^2\,\left(16\,a^5+80\,b\,a^4\right)-16\,a^4\,b+16\,a^5\right)}{4\,(a\,b-b^2)}\right)\,\left(b+\sqrt{a\,b}\right)}{4\,(a\,b-b^2)}$$

input
```
int(1/(x*(a - b + 2*a*x^2 + a*x^4)),x)
```

output
```
log(x)/(a - b) - (log(16*a^4 + 20*a^4*x^2 + ((b - (a*b)^(1/2))*(x^2*(80*a^
4*b + 16*a^5) - 16*a^4*b + 16*a^5))/(4*(a*b - b^2)))*(b - (a*b)^(1/2)))/(4
*(a*b - b^2)) - (log(16*a^4 + 20*a^4*x^2 + ((b + (a*b)^(1/2))*(x^2*(80*a^4
*b + 16*a^5) - 16*a^4*b + 16*a^5))/(4*(a*b - b^2)))*(b + (a*b)^(1/2)))/(4*
(a*b - b^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.79

$$\int \frac{1}{x(a-b+2ax^2+ax^4)}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{\sqrt{b}\,\sqrt{a}-a}+\sqrt{a}\,x\right)-\sqrt{b}\,\sqrt{a}\log\left(\sqrt{\sqrt{b}\,\sqrt{a}-a}+\sqrt{a}\,x\right)+\sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}+a\,x^2 \cdots\right.}{4 \cdots}$$

input
```
int(1/x/(a*x^4+2*a*x^2+a-b),x)
```

output
```
( - sqrt(b)*sqrt(a)*log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x) - sqrt(b
)*sqrt(a)*log(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x) + sqrt(b)*sqrt(a)*log
(sqrt(b)*sqrt(a) + a*x**2 + a) - log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a
)*x)*b - log(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*b - log(sqrt(b)*sqrt(a
) + a*x**2 + a)*b + 4*log(x)*b)/(4*b*(a - b))
```

3.830 $\int \frac{1}{x^3(a-b+2ax^2+ax^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{1}{x^3\left(a-b+2ax^2+ax^4\right)}\,dx = -\frac{1}{2(a-b)x^2} - \frac{\sqrt{a}(a+b)\operatorname{arctanh}\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2(a-b)^2\sqrt{b}} - \frac{2a\log(x)}{(a-b)^2} + \frac{a\log\left(a-b+2ax^2+ax^4\right)}{2(a-b)^2}$$

output
```
-1/2/(a-b)/x^2-1/2*a^(1/2)*(a+b)*arctanh(a^(1/2)*(x^2+1)/b^(1/2))/(a-b)^2/
b^(1/2)-2*a*ln(x)/(a-b)^2+1/2*a*ln(a*x^4+2*a*x^2+a-b)/(a-b)^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^3\left(a-b+2ax^2+ax^4\right)}\,dx = \frac{-8a\sqrt{b}x^2\log(x)+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^2x^2\log\left(-\sqrt{b}+\sqrt{a}(1+x^2)\right)-\left(\sqrt{a}-\sqrt{b}\right)\left(2\left(\sqrt{a}\sqrt{b}+b\right)+\left(ax^2\right.\right.}{4(a-b)^2\sqrt{b}x^2}$$

input
```
Integrate[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

output
```
(-8*a*Sqrt[b]*x^2*Log[x] + Sqrt[a]*(Sqrt[a] + Sqrt[b])^2*x^2*Log[-Sqrt[b]
+ Sqrt[a]*(1 + x^2)] - (Sqrt[a] - Sqrt[b])*(2*(Sqrt[a]*Sqrt[b] + b) + (a*x
^2 - Sqrt[a]*Sqrt[b]*x^2)*Log[Sqrt[b] + Sqrt[a]*(1 + x^2)]))/(4*(a - b)^2*
Sqrt[b]*x^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1434, 1145, 25, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a x^4+2 a x^2+a-b\right)} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(a x^4+2 a x^2+a-b\right)}dx^2$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{2} \left(\frac{\int -\frac{a (x^2+2)}{x^2 (a x^4+2 a x^2+a-b)}dx^2}{a-b}-\frac{1}{x^2 (a-b)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(-\frac{\int \frac{a (x^2+2)}{x^2 (a x^4+2 a x^2+a-b)}dx^2}{a-b}-\frac{1}{x^2 (a-b)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{a \int \frac{x^2+2}{x^2 (a x^4+2 a x^2+a-b)}dx^2}{a-b}-\frac{1}{x^2 (a-b)}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2} \left(-\frac{a \int \left(\frac{-2 a x^2-3 a-b}{(a-b) (a x^4+2 a x^2+a-b)}+\frac{2}{(a-b) x^2}\right)dx^2}{a-b}-\frac{1}{x^2 (a-b)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a\left(\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}(a-b)}+\frac{2\log(x^2)}{a-b}-\frac{\log(ax^4+2ax^2+a-b)}{a-b}\right)}{a-b}-\frac{1}{x^2(a-b)}\right)$$

input
```
Int[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

output
```
(-(1/((a - b)*x^2)) - (a*(((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(
Sqrt[a]*(a - b)*Sqrt[b]) + (2*Log[x^2])/(a - b) - Log[a - b + 2*a*x^2 + a*
x^4]/(a - b)))/(a - b))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

method	result
default	$-\frac{1}{2(a-b)x^2}-\frac{2a\ln(x)}{(a-b)^2}+\frac{a\left(\ln(a\,x^4+2a\,x^2+a-b)-\frac{(a+b)\,\mathrm{arctanh}\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{2(a-b)^2}$
risch	$-\frac{1}{2(a-b)x^2}-\frac{2a\ln(x)}{a^2-2ab+b^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2b-2b^2a+b^3\right)\_Z^2-4ab\_Z-a\right)}\_R\ln\left(\left(\left(-a^3-3a^2b+9b^2a-5b^3\right)\_R^2+(-8a^2+\right.\right.\right.}{4}$

input
```
int(1/x^3/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a-b)/x^2-2*a*ln(x)/(a-b)^2+1/2/(a-b)^2*a*(ln(a*x^4+2*a*x^2+a-b)-(a+b
)/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.10

$$\int \frac{1}{x^3\,(a-b+2ax^2+ax^4)}\,dx$$
$$=\left[\frac{(a+b)x^2\sqrt{\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\,(bx^2+b)\sqrt{\frac{a}{b}}+a+b}{ax^4+2\,ax^2+a-b}\right)+2\,ax^2\log\left(ax^4+2\,ax^2+a-b\right)-8\,ax^2\log\left(x\right)-2\,a+}{4\,(a^2-2\,ab+b^2)x^2}\right.$$

input
```
integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

output
```
[1/4*((a + b)*x^2*sqrt(a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(a/b)
 + a + b)/(a*x^4 + 2*a*x^2 + a - b)) + 2*a*x^2*log(a*x^4 + 2*a*x^2 + a - b
) - 8*a*x^2*log(x) - 2*a + 2*b)/((a^2 - 2*a*b + b^2)*x^2), 1/2*((a + b)*x^
2*sqrt(-a/b)*arctan((x^2 + 1)*sqrt(-a/b)) + a*x^2*log(a*x^4 + 2*a*x^2 + a
- b) - 4*a*x^2*log(x) - a + b)/((a^2 - 2*a*b + b^2)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(83) = 166$.

Time = 13.10 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.84

$$\int \frac{1}{x^3 (a-b+2ax^2+ax^4)}\,dx = -\frac{2a\log(x)}{(a-b)^2} + \left(\frac{a}{2(a-b)^2} - \frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)\log\left(x^2 + \frac{-4a^2b\left(\frac{a}{2(a-b)^2} - \frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right) + a^2 + 8ab^2\left(\frac{a}{2(a-b)^2} - \frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right) +}{a^2+ab}\right. + \left(\frac{a}{2(a-b)^2} + \frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)\log\left(x^2 + \frac{-4a^2b\left(\frac{a}{2(a-b)^2} + \frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right) + a^2 + 8ab^2\left(\frac{a}{2(a-b)^2} + \frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right) +}{a^2+ab}\right. - \frac{1}{x^2\cdot(2a-2b)}$$

input
```
integrate(1/x**3/(a*x**4+2*a*x**2+a-b),x)
```

output
```
-2*a*log(x)/(a - b)**2 + (a/(2*(a - b)**2) - sqrt(a*b)*(a + b)/(4*b*(a**2
 - 2*a*b + b**2)))*log(x**2 + (-4*a**2*b*(a/(2*(a - b)**2) - sqrt(a*b)*(a +
 b)/(4*b*(a**2 - 2*a*b + b**2))) + a**2 + 8*a*b**2*(a/(2*(a - b)**2) - sqr
t(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))) + 3*a*b - 4*b**3*(a/(2*(a - b)
**2) - sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))))/(a**2 + a*b)) + (a/
(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2)))*log(x**2 +
 (-4*a**2*b*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**
2))) + a**2 + 8*a*b**2*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 -
2*a*b + b**2))) + 3*a*b - 4*b**3*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*
b*(a**2 - 2*a*b + b**2))))/(a**2 + a*b)) - 1/(x**2*(2*a - 2*b))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^3(a-b+2ax^2+ax^4)}\,dx = \frac{a\log(ax^4+2\,ax^2+a-b)}{2\,(a^2-2\,ab+b^2)} - \frac{a\log(x^2)}{a^2-2\,ab+b^2} + \frac{(a^2+ab)\log\left(\frac{ax^2+a-\sqrt{ab}}{ax^2+a+\sqrt{ab}}\right)}{4\,(a^2-2\,ab+b^2)\sqrt{ab}} - \frac{1}{2\,(a-b)x^2}$$

input
```
integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

output
```
1/2*a*log(a*x^4 + 2*a*x^2 + a - b)/(a^2 - 2*a*b + b^2) - a*log(x^2)/(a^2 -
 2*a*b + b^2) + 1/4*(a^2 + a*b)*log((a*x^2 + a - sqrt(a*b))/(a*x^2 + a + s
qrt(a*b)))/((a^2 - 2*a*b + b^2)*sqrt(a*b)) - 1/2/((a - b)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^3(a-b+2ax^2+ax^4)}\,dx = \frac{a\log(ax^4+2\,ax^2+a-b)}{2\,(a^2-2\,ab+b^2)} - \frac{a\log(x^2)}{a^2-2\,ab+b^2} + \frac{(a^2+ab)\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,(a^2-2\,ab+b^2)\sqrt{-ab}} + \frac{2\,ax^2-a+b}{2\,(a^2-2\,ab+b^2)x^2}$$

input
```
integrate(1/x^3/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

output
```
1/2*a*log(a*x^4 + 2*a*x^2 + a - b)/(a^2 - 2*a*b + b^2) - a*log(x^2)/(a^2 -
 2*a*b + b^2) + 1/2*(a^2 + a*b)*arctan((a*x^2 + a)/sqrt(-a*b))/((a^2 - 2*a
*b + b^2)*sqrt(-a*b)) + 1/2*(2*a*x^2 - a + b)/((a^2 - 2*a*b + b^2)*x^2)
```

Mupad [B] (verification not implemented)

Time = 19.51 (sec) , antiderivative size = 389, normalized size of antiderivative = 4.01

$$\int \frac{1}{x^3 (a-b+2ax^2+ax^4)} dx = \frac{\ln \left(100\,a\,(a\,b)^{7/2} - 198\,b\,(a\,b)^{7/2} - a^3\,(a\,b)^{5/2} + 100\,b^3\,(a\,b)^{5/2} - b^5\,(a\,b)^{3/2} + a^2\,b^6 - 100\,a^3\,b^5 + 198\,a^4\,b^4}{a^2\,b -} - \frac{2\,a\,\ln(x)}{a^2-2\,a\,b+b^2} - \frac{\ln \left(198\,b\,(a\,b)^{7/2} - 100\,a\,(a\,b)^{7/2} + a^3\,(a\,b)^{5/2} - 100\,b^3\,(a\,b)^{5/2} + b^5\,(a\,b)^{3/2} + a^2\,b^6 - 100\,a^3\,b^5 + 198\,a^4}{a^2\,b} - \frac{1}{2\,x^2\,(a-b)}$$

input
```
int(1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x)
```

output
```
(log(100*a*(a*b)^(7/2) - 198*b*(a*b)^(7/2) - a^3*(a*b)^(5/2) + 100*b^3*(a*
b)^(5/2) - b^5*(a*b)^(3/2) + a^2*b^6 - 100*a^3*b^5 + 198*a^4*b^4 - 100*a^5
*b^3 + a^6*b^2 + a^2*b^6*x^2 - 100*a^3*b^5*x^2 + 198*a^4*b^4*x^2 - 100*a^5
*b^3*x^2 + a^6*b^2*x^2)*((a*(a*b)^(1/2))/4 + b*(a/2 + (a*b)^(1/2)/4)))/(a^
2*b - 2*a*b^2 + b^3) - (2*a*log(x))/(a^2 - 2*a*b + b^2) - (log(198*b*(a*b)
^(7/2) - 100*a*(a*b)^(7/2) + a^3*(a*b)^(5/2) - 100*b^3*(a*b)^(5/2) + b^5*(
a*b)^(3/2) + a^2*b^6 - 100*a^3*b^5 + 198*a^4*b^4 - 100*a^5*b^3 + a^6*b^2 +
 a^2*b^6*x^2 - 100*a^3*b^5*x^2 + 198*a^4*b^4*x^2 - 100*a^5*b^3*x^2 + a^6*b
^2*x^2)*((a*(a*b)^(1/2))/4 - b*(a/2 - (a*b)^(1/2)/4)))/(a^2*b - 2*a*b^2 +
b^3) - 1/(2*x^2*(a - b))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.66

$$\int \frac{1}{x^3 (a-b+2ax^2+ax^4)} \, dx$$

$$= \frac{\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{\sqrt{b}\,\sqrt{a}-a}+\sqrt{a}\,x\right)a\,x^2+\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{\sqrt{b}\,\sqrt{a}-a}+\sqrt{a}\,x\right)b\,x^2+\sqrt{b}\,\sqrt{a}\log\left(\sqrt{\sqrt{}}\right.}{}$$

input

```
int(1/x^3/(a*x^4+2*a*x^2+a-b),x)
```

output

```
(sqrt(b)*sqrt(a)*log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*a*x**2 + sq
rt(b)*sqrt(a)*log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*b*x**2 + sqrt(
b)*sqrt(a)*log(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*a*x**2 + sqrt(b)*sqr
t(a)*log(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*b*x**2 - sqrt(b)*sqrt(a)*l
og(sqrt(b)*sqrt(a) + a*x**2 + a)*a*x**2 - sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt
(a) + a*x**2 + a)*b*x**2 + 2*log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)
*a*b*x**2 + 2*log(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*a*b*x**2 + 2*log(
sqrt(b)*sqrt(a) + a*x**2 + a)*a*b*x**2 - 8*log(x)*a*b*x**2 - 2*a*b + 2*b**
2)/(4*b*x**2*(a**2 - 2*a*b + b**2))
```

3.831 $\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx = \frac{x}{a}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}} - \frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}$$

output

```
x/a+1/2*(a^(1/2)-b^(1/2))^(3/2)*arctan(a^(1/4)*x/(a^(1/2)-b^(1/2))^(1/2))/
a^(5/4)/b^(1/2)-1/2*(a^(1/2)+b^(1/2))^(3/2)*arctan(a^(1/4)*x/(a^(1/2)+b^(1
/2))^(1/2))/a^(5/4)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx=\frac{x}{a}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\arctan\left(\frac{\sqrt{a}x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{a-\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\arctan\left(\frac{\sqrt{a}x}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
Integrate[x^4/(a - b + 2*a*x^2 + a*x^4),x]
```

output
```
x/a + ((Sqrt[a] - Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]]
)/(2*a*Sqrt[a - Sqrt[a]*Sqrt[b]]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])^2*ArcTan[
(Sqrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(2*a*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqr
t[b])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1442, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{ax^4+2ax^2+a-b}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x}{a}-\frac{\int \frac{2ax^2+a-b}{ax^4+2ax^2+a-b}dx}{a}$$

$$\Big\downarrow \text{1480}$$

$$\frac{x}{a}-\frac{\frac{1}{2}\sqrt{a}\left(2\sqrt{a}-\frac{a+b}{\sqrt{b}}\right)\int\frac{1}{ax^2+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}dx+\frac{1}{2}\sqrt{a}\left(\frac{a+b}{\sqrt{b}}+2\sqrt{a}\right)\int\frac{1}{ax^2+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}dx}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{a}-\frac{\frac{\left(2\sqrt{a}-\frac{a+b}{\sqrt{b}}\right)\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(\frac{a+b}{\sqrt{b}}+2\sqrt{a}\right)\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}}}{a}$$

input
```
Int[x^4/(a - b + 2*a*x^2 + a*x^4),x]
```

output
```
x/a - (((2*Sqrt[a] - (a + b)/Sqrt[b])*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sq
rt[b]]])/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]) + ((2*Sqrt[a] + (a + b)/Sqrt[
b])*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(1/4)*Sqrt[Sqrt[a] +
 Sqrt[b]]))/a
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))    Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))    Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))    Int[1/(b/2
 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

method	result	size
risch	$\frac{x}{a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4+2a\_Z^2+a-b\right)}\frac{\left(-2\_R^2a-a+b\right)\ln\left(x-\_R\right)}{\_R^3+\_R}}{4a^2}$	57
default	$\frac{x}{a}-\frac{\left(-2\sqrt{ab}+a+b\right)\operatorname{arctanh}\left(\frac{ax}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)a}}+\frac{\left(-2\sqrt{ab}-a-b\right)\arctan\left(\frac{ax}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)a}}$	101

input
```
int(x^4/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

output
```
x/a+1/4/a^2*sum((-2*_R^2*a-a+b)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4*a+2*_Z^2
*a+a-b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(74) = 148.

Time = 0.08 (sec) , antiderivative size = 603, normalized size of antiderivative = 5.29

$$\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx$$
$$= \frac{a\sqrt{-\frac{a^2b\sqrt{\frac{9\,a^2+6\,ab+b^2}{a^5b}}+a+3\,b}{a^2b}}\log\left(-(3\,a^2-2\,ab-b^2)x+\left(a^4b\sqrt{\frac{9\,a^2+6\,ab+b^2}{a^5b}}-3\,a^2b-ab^2\right)\sqrt{-\frac{a^2b\sqrt{\frac{9\,a^2+6\,ab+}{a^5b}}}{a^2b}}\right.}{}$$

input
```
integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

output
```
1/4*(a*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3*b)/(a^2*b)
)*log(-(3*a^2 - 2*a*b - b^2)*x + (a^4*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)
) - 3*a^2*b - a*b^2)*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a
+ 3*b)/(a^2*b))) - a*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a
+ 3*b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x - (a^4*b*sqrt((9*a^2 + 6*a*b
+ b^2)/(a^5*b)) - 3*a^2*b - a*b^2)*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)
/(a^5*b)) + a + 3*b)/(a^2*b))) - a*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/
(a^5*b)) - a - 3*b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x + (a^4*b*sqrt((9
*a^2 + 6*a*b + b^2)/(a^5*b)) + 3*a^2*b + a*b^2)*sqrt((a^2*b*sqrt((9*a^2 +
6*a*b + b^2)/(a^5*b)) - a - 3*b)/(a^2*b))) + a*sqrt((a^2*b*sqrt((9*a^2 + 6
*a*b + b^2)/(a^5*b)) - a - 3*b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x - (a
^4*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + 3*a^2*b + a*b^2)*sqrt((a^2*b*sq
rt((9*a^2 + 6*a*b + b^2)/(a^5*b)) - a - 3*b)/(a^2*b))) + 4*x)/a
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx$$
$$= \operatorname{RootSum}\left(256t^4a^5b^2 + t^2\cdot\left(32a^4b + 96a^3b^2\right) + a^3 - 3a^2b + 3ab^2 - b^3, \left(t \mapsto t\log\left(x + \frac{64t^3a^4b + 4ta^3 -}{3a^2 - 2}\right.\right.\right.$$
$$+ \frac{x}{a}$$

input
```
integrate(x**4/(a*x**4+2*a*x**2+a-b),x)
```

output
```
RootSum(256*_t**4*a**5*b**2 + _t**2*(32*a**4*b + 96*a**3*b**2) + a**3 - 3*
a**2*b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (64*_t**3*a**4*b + 4*_t*a*
*3 + 24*_t*a**2*b + 4*_t*a*b**2)/(3*a**2 - 2*a*b - b**2)))) + x/a
```

Maxima [F]

$$\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx = \int \frac{x^4}{ax^4+2\,ax^2+a-b}\,dx$$

input
```
integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

output
```
x/a - integrate((2*a*x^2 + a - b)/(a*x^4 + 2*a*x^2 + a - b), x)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(74) = 148$.

Time = 0.17 (sec) , antiderivative size = 511, normalized size of antiderivative = 4.48

$$\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx =$$

$$-\frac{\left(3\,\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}a^4-\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}a^3b-4\,\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}a^2b^2-2\left(3\,\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}ab-\right.\right.}{2}$$

$$+\frac{\left(3\,\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}a^4-\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}a^3b-4\,\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}a^2b^2-2\left(3\,\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}ab-\right.\right.}{2}$$

$$+\frac{x}{a}$$

input
```
integrate(x^4/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

output
```
-1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^4 - sqrt(a^2 + sqrt(a*b)*a)*sq
rt(a*b)*a^3*b - 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 2*(3*sqrt(a^
2 + sqrt(a*b)*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*b^2)*
a^2 + (3*sqrt(a^2 + sqrt(a*b)*a)*a^3*b - 7*sqrt(a^2 + sqrt(a*b)*a)*a^2*b^2
 + 4*sqrt(a^2 + sqrt(a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 + sqrt(a^4
- (a^2 - a*b)*a^2))/a^2))/(3*a^6*b - 7*a^5*b^2 + 4*a^4*b^3) + 1/2*(3*sqrt(
a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^4 - sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^3*b
 - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 2*(3*sqrt(a^2 - sqrt(a*b)
*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*b^2)*a^2 - (3*sqrt
(a^2 - sqrt(a*b)*a)*a^3*b - 7*sqrt(a^2 - sqrt(a*b)*a)*a^2*b^2 + 4*sqrt(a^2
 - sqrt(a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - sqrt(a^4 - (a^2 - a*b)
*a^2))/a^2))/(3*a^6*b - 7*a^5*b^2 + 4*a^4*b^3) + x/a
```

Mupad [B] (verification not implemented)

Time = 19.01 (sec) , antiderivative size = 1097, normalized size of antiderivative = 9.62

$$\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx = \text{Too large to display}$$

input
```
int(x^4/(a - b + 2*a*x^2 + a*x^4),x)
```

output
```
x/a - 2*atanh((24*x*(a^5*b^3)^(1/2)*(- 3/(16*a^2) - 1/(16*a*b) - (3*(a^5*b
^3)^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b^2 - (6
*(a^5*b^3)^(1/2))/a - 6*a^2*b + 2*b^3 + (2*b^2*(a^5*b^3)^(1/2))/a^3 + (4*b
*(a^5*b^3)^(1/2))/a^2) + (8*x*(a^5*b^3)^(1/2)*(- 3/(16*a^2) - 1/(16*a*b) -
 (3*(a^5*b^3)^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((4
*(a^5*b^3)^(1/2))/a - (6*(a^5*b^3)^(1/2))/b + 2*a*b^2 + 4*a^2*b - 6*a^3 +
(2*b*(a^5*b^3)^(1/2))/a^2) - (8*a*b^2*x*(- 3/(16*a^2) - 1/(16*a*b) - (3*(a
^5*b^3)^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b +
(4*(a^5*b^3)^(1/2))/a^2 - 6*a^2 + 2*b^2 - (6*(a^5*b^3)^(1/2))/(a*b) + (2*b
*(a^5*b^3)^(1/2))/a^3) - (24*a^2*b*x*(- 3/(16*a^2) - 1/(16*a*b) - (3*(a^5*
b^3)^(1/2))/(16*a^4*b^2) - (a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(4*a*b + (4*
(a^5*b^3)^(1/2))/a^2 - 6*a^2 + 2*b^2 - (6*(a^5*b^3)^(1/2))/(a*b) + (2*b*(a
^5*b^3)^(1/2))/a^3))*(-(3*a*(a^5*b^3)^(1/2) + b*(a^5*b^3)^(1/2) + a^4*b +
3*a^3*b^2)/(16*a^5*b^2))^(1/2) + 2*atanh((24*x*(a^5*b^3)^(1/2)*((3*(a^5*b^
3)^(1/2))/(16*a^4*b^2) - 1/(16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)/(16*a^5
*b))^(1/2))/((6*(a^5*b^3)^(1/2))/a + 4*a*b^2 - 6*a^2*b + 2*b^3 - (2*b^2*(a
^5*b^3)^(1/2))/a^3 - (4*b*(a^5*b^3)^(1/2))/a^2) - (8*x*(a^5*b^3)^(1/2)*((3
*(a^5*b^3)^(1/2))/(16*a^4*b^2) - 1/(16*a*b) - 3/(16*a^2) + (a^5*b^3)^(1/2)
/(16*a^5*b))^(1/2))/((4*(a^5*b^3)^(1/2))/a - (6*(a^5*b^3)^(1/2))/b - 2*a*b
^2 - 4*a^2*b + 6*a^3 + (2*b*(a^5*b^3)^(1/2))/a^2) + (8*a*b^2*x*((3*(a^5...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.77

$$\int \frac{x^4}{a-b+2ax^2+ax^4}\,dx$$
$$= \frac{-2\sqrt{a}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}\,atan\left(\frac{ax}{\sqrt{a}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}}\right)b-2\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}\,atan\left(\frac{ax}{\sqrt{a}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}}\right)a+\sqrt{a}\,\sqrt{\sqrt{b}\,\sqrt{a}-a}}{}$$

input
```
int(x^4/(a*x^4+2*a*x^2+a-b),x)
```

output

```
( - 2*sqrt(a)*sqrt(sqrt(b)*sqrt(a) + a)*atan((a*x)/(sqrt(a)*sqrt(sqrt(b)*s
qrt(a) + a)))*b - 2*sqrt(b)*sqrt(sqrt(b)*sqrt(a) + a)*atan((a*x)/(sqrt(a)*
sqrt(sqrt(b)*sqrt(a) + a)))*a + sqrt(a)*sqrt(sqrt(b)*sqrt(a) - a)*log( - s
qrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*b - sqrt(a)*sqrt(sqrt(b)*sqrt(a) - a
)*log(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*b - sqrt(b)*sqrt(sqrt(b)*sqrt
(a) - a)*log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*a + sqrt(b)*sqrt(sq
rt(b)*sqrt(a) - a)*log(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*a + 4*a*b*x)
/(4*a**2*b)
```

3.832 $\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx=-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\sqrt{\sqrt{a}+\sqrt{b}}\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

output
```
-1/2*(a^(1/2)-b^(1/2))^(1/2)*arctan(a^(1/4)*x/(a^(1/2)-b^(1/2))^(1/2))/a^(
3/4)/b^(1/2)+1/2*(a^(1/2)+b^(1/2))^(1/2)*arctan(a^(1/4)*x/(a^(1/2)+b^(1/2)
)^(1/2))/a^(3/4)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx=\frac{-\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{a}x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a-\sqrt{a}\sqrt{b}}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{a}x}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\sqrt{b}}$$

input
```
Integrate[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

output
```
(-(((Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]])/Sqr
t[a - Sqrt[a]*Sqrt[b]]) + ((Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a +
 Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1450, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{ax^4+2ax^2+a-b}dx$$

$$\Big\downarrow \text{1450}$$

$$\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\int \frac{1}{ax^2+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}dx+\frac{1}{2}\left(\frac{\sqrt{a}}{\sqrt{b}}+1\right)\int \frac{1}{ax^2+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}dx$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(\frac{\sqrt{a}}{\sqrt{b}}+1\right)\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}$$

input
```
Int[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

output
```
((1 - Sqrt[a]/Sqrt[b])*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^(
3/4)*Sqrt[Sqrt[a] - Sqrt[b]]) + ((1 + Sqrt[a]/Sqrt[b])*ArcTan[(a^(1/4)*x)/
Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^4+2a\_Z^2+a-b\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3+\_R}}{4a}$	43
default	$a\left(-\frac{\left(\sqrt{ab}-a\right)\operatorname{arctanh}\left(\frac{ax}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)}{2a\sqrt{ab}\sqrt{\left(\sqrt{ab}-a\right)a}}+\frac{\left(\sqrt{ab}+a\right)\arctan\left(\frac{ax}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)}{2a\sqrt{ab}\sqrt{\left(\sqrt{ab}+a\right)a}}\right)$	96

input
```
int(x^2/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*sum(_R^2/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4*a+2*_Z^2*a+a-b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(69) = 138$.

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.45

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx = \frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\log\left(a^2b\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right) - \frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\log\left(-a^2b\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right) - \frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\log\left(a^2b\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right) + \frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\log\left(-a^2b\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)$$

input
```
integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

output
```
1/4*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*log(a^2*b*sqrt(-(a*b*sqrt(1/(a^
3*b)) + 1)/(a*b))*sqrt(1/(a^3*b)) + x) - 1/4*sqrt(-(a*b*sqrt(1/(a^3*b)) +
1)/(a*b))*log(-a^2*b*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*sqrt(1/(a^3*b)
) + x) - 1/4*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*log(a^2*b*sqrt((a*b*sqr
t(1/(a^3*b)) - 1)/(a*b))*sqrt(1/(a^3*b)) + x) + 1/4*sqrt((a*b*sqrt(1/(a^3*
b)) - 1)/(a*b))*log(-a^2*b*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*sqrt(1/(a
^3*b)) + x)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.40

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3b^2+32t^2a^2b+a-b,\left(t\mapsto t\log\left(-64t^3a^2b-4ta+x\right)\right)\right)$$

input
```
integrate(x**2/(a*x**4+2*a*x**2+a-b),x)
```

output
```
RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b + a - b, Lambda(_t, _t*log(-
64*_t**3*a**2*b - 4*_t*a + x)))
```

Maxima [F]

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx=\int \frac{x^2}{ax^4+2\,ax^2+a-b}\,dx$$

input
```
integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

output
```
integrate(x^2/(a*x^4 + 2*a*x^2 + a - b), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(69) = 138$.

Time = 0.23 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.83

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx = \frac{\left(3\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}a-4\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{2a+\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^4b-4a^3b^2\right)} - \frac{\left(3\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}a-4\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{2a-\sqrt{-4(a-b)a+4a^2}}{a}}}\right)}{2\left(3a^4b-4a^3b^2\right)}$$

input
```
integrate(x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

output
```
1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a - 4*sqrt(a^2 + sqrt(a*b)*a)*sqr
t(a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a - b)*a + 4*a^
2))/a))/(3*a^4*b - 4*a^3*b^2) - 1/2*(3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a
 - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt
((2*a - sqrt(-4*(a - b)*a + 4*a^2))/a))/(3*a^4*b - 4*a^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.98

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx = -2\,\mathrm{atanh}\left(\frac{2\left(x\left(4a^3+4ba^2\right)-\frac{4ax\left(\sqrt{a^3b^3}+a^2b\right)}{b}\right)\sqrt{-\frac{\sqrt{a^3b^3}+a^2b}{16a^3b^2}}}{2ab-2a^2}\right)\sqrt{-\frac{\sqrt{a^3b^3}+a^2b}{16a^3b^2}} - 2\,\mathrm{atanh}\left(\frac{2\left(x\left(4a^3+4ba^2\right)+\frac{4ax\left(\sqrt{a^3b^3}-a^2b\right)}{b}\right)\sqrt{\frac{\sqrt{a^3b^3}-a^2b}{16a^3b^2}}}{2ab-2a^2}\right)\sqrt{\frac{\sqrt{a^3b^3}-a^2b}{16a^3b^2}}$$

input
```
int(x^2/(a - b + 2*a*x^2 + a*x^4),x)
```

output
```
- 2*atanh((2*(x*(4*a^2*b + 4*a^3) - (4*a*x*((a^3*b^3)^(1/2) + a^2*b))/b)*(
-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2))/(2*a*b - 2*a^2))*(-((a^3*b
^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2) - 2*atanh((2*(x*(4*a^2*b + 4*a^3) +
 (4*a*x*((a^3*b^3)^(1/2) - a^2*b))/b)*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b
^2))^(1/2))/(2*a*b - 2*a^2))*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2
)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx = \frac{\sqrt{b}\left(2\sqrt{\sqrt{b}\,\sqrt{a}+a}\,atan\left(\frac{ax}{\sqrt{a}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}}\right)+\sqrt{\sqrt{b}\,\sqrt{a}-a}\log\left(-\sqrt{\sqrt{b}\,\sqrt{a}-a}+\sqrt{a}\,x\right)-\sqrt{\sqrt{b}\,\sqrt{a}-a}\,\mathrm{l}\right.}{4ab}$$

input
```
int(x^2/(a*x^4+2*a*x^2+a-b),x)
```

output
```
(sqrt(b)*(2*sqrt(sqrt(b)*sqrt(a) + a)*atan((a*x)/(sqrt(a)*sqrt(sqrt(b)*sqr
t(a) + a))) + sqrt(sqrt(b)*sqrt(a) - a)*log( - sqrt(sqrt(b)*sqrt(a) - a) +
 sqrt(a)*x) - sqrt(sqrt(b)*sqrt(a) - a)*log(sqrt(sqrt(b)*sqrt(a) - a) + sq
rt(a)*x)))/(4*a*b)
```

3.833 $\int \frac{1}{a-b+2ax^2+ax^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{b}}$$

output
```
1/2*arctan(a^(1/4)*x/(a^(1/2)-b^(1/2))^(1/2))/a^(1/4)/(a^(1/2)-b^(1/2))^(1
/2)/b^(1/2)-1/2*arctan(a^(1/4)*x/(a^(1/2)+b^(1/2))^(1/2))/a^(1/4)/(a^(1/2)
+b^(1/2))^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx=\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a-\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
Integrate[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a - Sqrt[a]*Sqrt[b]]
*Sqrt[b]) - ArcTan[(Sqrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a + Sqrt
[a]*Sqrt[b]]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{1406, 218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^4+2ax^2+a-b}\,dx$$

$$\Big\downarrow \text{ 1406}$$

$$\frac{\sqrt{a}\int \frac{1}{ax^2+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}dx}{2\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{ax^2+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}dx}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}}$$

input
```
Int[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

output
```
ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt
[b]]*Sqrt[b]) - ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]]/(2*a^(1/4)*Sqr
t[Sqrt[a] + Sqrt[b]]*Sqrt[b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.37

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^4+2a\_Z^2+a-b\right)}\frac{\ln\left(x-\_R\right)}{\_R^3+\_R}}{4a}$	40
default	$a\left(-\frac{\operatorname{arctanh}\left(\frac{ax}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)a}}-\frac{\arctan\left(\frac{ax}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)}{2\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)a}}\right)$	74

input
```
int(1/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*sum(1/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4*a+2*_Z^2*a+a-b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(69) = 138$.

Time = 0.08 (sec) , antiderivative size = 553, normalized size of antiderivative = 5.07

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx$$
$$= -\frac{1}{4}\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}+1}{ab-b^2}}\log\left(\left(b-\frac{a^2b-ab^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}\right)\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}+1}{ab-b^2}}+x\right)$$
$$+\frac{1}{4}\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}+1}{ab-b^2}}\log\left(-\left(b-\frac{a^2b-ab^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}\right)\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}+1}{ab-b^2}}+x\right)$$
$$-\frac{1}{4}\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}-1}{ab-b^2}}\log\left(\left(b+\frac{a^2b-ab^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}\right)\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}-1}{ab-b^2}}+x\right)$$
$$+\frac{1}{4}\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}-1}{ab-b^2}}\log\left(-\left(b+\frac{a^2b-ab^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}\right)\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2\,a^2b^2+ab^3}}-1}{ab-b^2}}+x\right)$$

input
```
integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2))*
log((b - (a^2*b - a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3))*sqrt(-((a*b - b^
2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2)) + x) + 1/4*sqrt(-((a*
b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2))*log(-(b - (a^2*
b - a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3))*sqrt(-((a*b - b^2)/sqrt(a^3*b
- 2*a^2*b^2 + a*b^3) + 1)/(a*b - b^2)) + x) - 1/4*sqrt(((a*b - b^2)/sqrt(a
^3*b - 2*a^2*b^2 + a*b^3) - 1)/(a*b - b^2))*log((b + (a^2*b - a*b^2)/sqrt(
a^3*b - 2*a^2*b^2 + a*b^3))*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b
^3) - 1)/(a*b - b^2)) + x) + 1/4*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2
+ a*b^3) - 1)/(a*b - b^2))*log(-(b + (a^2*b - a*b^2)/sqrt(a^3*b - 2*a^2*b^
2 + a*b^3))*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) - 1)/(a*b -
b^2)) + x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.58

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx = \operatorname{RootSum}\left(t^4\cdot\left(256a^2b^2-256ab^3\right)+32t^2ab+1,\left(t\mapsto t\log\left(-64t^3a^2b+64t^3ab^2-4ta-4tb+x\right)\right)\right)$$

input
```
integrate(1/(a*x**4+2*a*x**2+a-b),x)
```

output
```
RootSum(_t**4*(256*a**2*b**2 - 256*a*b**3) + 32*_t**2*a*b + 1, Lambda(_t,
_t*log(-64*_t**3*a**2*b + 64*_t**3*a*b**2 - 4*_t*a - 4*_t*b + x)))
```

Maxima [F]

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx = \int \frac{1}{ax^4+2\,ax^2+a-b}\,dx$$

input
```
integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

output
```
integrate(1/(a*x^4 + 2*a*x^2 + a - b), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(69) = 138$.

Time = 0.17 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.74

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx$$
$$= \frac{\left(3\sqrt{a^2+\sqrt{ab}a}a^2b-4\sqrt{a^2+\sqrt{ab}a}ab^2+3\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}a^2-4\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}ab\right)|a|\arctan\left(\frac{\ }{\sqrt{\ }}\right.}{2\left(3a^5b-7a^4b^2+4a^3b^3\right)}$$
$$+\frac{\left(3\sqrt{a^2-\sqrt{ab}a}a^2b-4\sqrt{a^2-\sqrt{ab}a}ab^2+3\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}a^2-4\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}ab\right)|a|\arctan\left(\right.}{2\left(3a^5b-7a^4b^2+4a^3b^3\right)}$$

input
```
integrate(1/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

output
```
1/2*(3*sqrt(a^2 + sqrt(a*b)*a)*a^2*b - 4*sqrt(a^2 + sqrt(a*b)*a)*a*b^2 + 3
*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2 - 4*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*
b)*a*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a - b)*a + 4*a^2)
)/a))/(3*a^5*b - 7*a^4*b^2 + 4*a^3*b^3) + 1/2*(3*sqrt(a^2 - sqrt(a*b)*a)*a
^2*b - 4*sqrt(a^2 - sqrt(a*b)*a)*a*b^2 + 3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*
b)*a^2 - 4*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a*b)*abs(a)*arctan(2*sqrt(1/2
)*x/sqrt((2*a - sqrt(-4*(a - b)*a + 4*a^2))/a))/(3*a^5*b - 7*a^4*b^2 + 4*a
^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 20.37 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.95

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx = \frac{\ln\left(4\,a^3\,b\,\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}-4\,a^3\,x+\frac{4\,a^4\,b\,x}{a\,b+\sqrt{a\,b^3}}\right)\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}}{4}+\frac{\ln\left(4\,a^3\,x-4\,a^3\,b\,\sqrt{-\frac{1}{a\,b-\sqrt{a\,b^3}}}-\frac{4\,a^4\,b\,x}{a\,b-\sqrt{a\,b^3}}\right)\sqrt{-\frac{1}{a\,b-\sqrt{a\,b^3}}}}{4}-\ln\left(4\,a^3\,x+4\,a^3\,b\,\sqrt{-\frac{1}{a\,b+\sqrt{a\,b^3}}}-\frac{4\,a^4\,b\,x}{a\,b+\sqrt{a\,b^3}}\right)\sqrt{\frac{a\,b-\sqrt{a\,b^3}}{16\,\left(a\,b^3-a^2\,b^2\right)}}-\ln\left(4\,a^3\,x+16\,a^3\,b\,\sqrt{-\frac{1}{16\,a\,b-16\,\sqrt{a\,b^3}}}-\frac{4\,a^4\,b\,x}{a\,b-\sqrt{a\,b^3}}\right)\sqrt{\frac{a\,b+\sqrt{a\,b^3}}{16\,\left(a\,b^3-a^2\,b^2\right)}}$$

input

```
int(1/(a - b + 2*a*x^2 + a*x^4),x)
```

output

```
(log(4*a^3*b*(-1/(a*b + (a*b^3)^(1/2)))^(1/2) - 4*a^3*x + (4*a^4*b*x)/(a*b
 + (a*b^3)^(1/2)))*(-1/(a*b + (a*b^3)^(1/2)))^(1/2))/4 + (log(4*a^3*x - 4*
a^3*b*(-1/(a*b - (a*b^3)^(1/2)))^(1/2) - (4*a^4*b*x)/(a*b - (a*b^3)^(1/2))
)*(-1/(a*b - (a*b^3)^(1/2)))^(1/2))/4 - log(4*a^3*x + 4*a^3*b*(-1/(a*b + (
a*b^3)^(1/2)))^(1/2) - (4*a^4*b*x)/(a*b + (a*b^3)^(1/2)))*((a*b - (a*b^3)^
(1/2))/(16*(a*b^3 - a^2*b^2)))^(1/2) - log(4*a^3*x + 16*a^3*b*(-1/(16*a*b
- 16*(a*b^3)^(1/2)))^(1/2) - (4*a^4*b*x)/(a*b - (a*b^3)^(1/2)))*((a*b + (a
*b^3)^(1/2))/(16*(a*b^3 - a^2*b^2)))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.87

$$\int \frac{1}{a-b+2ax^2+ax^4}\,dx$$
$$= \frac{2\sqrt{a}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}\,atan\left(\frac{ax}{\sqrt{a}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}}\right)b-2\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}\,atan\left(\frac{ax}{\sqrt{a}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}}\right)a-\sqrt{a}\,\sqrt{\sqrt{b}\,\sqrt{a}-a}\,l}{}$$

input
```
int(1/(a*x^4+2*a*x^2+a-b),x)
```

output
```
(2*sqrt(a)*sqrt(sqrt(b)*sqrt(a) + a)*atan((a*x)/(sqrt(a)*sqrt(sqrt(b)*sqrt
(a) + a)))*b - 2*sqrt(b)*sqrt(sqrt(b)*sqrt(a) + a)*atan((a*x)/(sqrt(a)*sqr
t(sqrt(b)*sqrt(a) + a)))*a - sqrt(a)*sqrt(sqrt(b)*sqrt(a) - a)*log( - sqrt
(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*b + sqrt(a)*sqrt(sqrt(b)*sqrt(a) - a)*l
og(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*b - sqrt(b)*sqrt(sqrt(b)*sqrt(a)
 - a)*log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*a + sqrt(b)*sqrt(sqrt(
b)*sqrt(a) - a)*log(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*a)/(4*a*b*(a -
b))
```

3.834 $\int \frac{1}{x^2(a-b+2ax^2+ax^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{1}{x^2(a-b+2ax^2+ax^4)}\,dx = -\frac{1}{(a-b)x} - \frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}} + \frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}}$$

output

```
-1/(a-b)/x-1/2*a^(1/4)*arctan(a^(1/4)*x/(a^(1/2)-b^(1/2))^(1/2))/(a^(1/2)-
b^(1/2))^(3/2)/b^(1/2)+1/2*a^(1/4)*arctan(a^(1/4)*x/(a^(1/2)+b^(1/2))^(1/2
))/(a^(1/2)+b^(1/2))^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2(a-b+2ax^2+ax^4)}\,dx = \frac{\frac{2}{x}+\frac{\left(a+\sqrt{a}\sqrt{b}\right)\arctan\left(\frac{\sqrt{a}x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a-\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(a-\sqrt{a}\sqrt{b}\right)\arctan\left(\frac{\sqrt{a}x}{\sqrt{a+\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+\sqrt{a}\sqrt{b}}\sqrt{b}}}{2(-a+b)}$$

input
```
Integrate[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

output
```
(2/x + ((a + Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]
])/(Sqrt[a - Sqrt[a]*Sqrt[b]]*Sqrt[b]) - ((a - Sqrt[a]*Sqrt[b])*ArcTan[(Sq
rt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]))/
(2*(-a + b))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1443, 25, 27, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(ax^4+2ax^2+a-b)}\,dx$$

$$\Big\downarrow \text{ 1443}$$

$$\frac{\int -\frac{a(x^2+2)}{ax^4+2ax^2+a-b}dx}{a-b}-\frac{1}{x(a-b)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{a(x^2+2)}{ax^4+2ax^2+a-b}dx}{a-b}-\frac{1}{x(a-b)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x^2+2}{ax^4+2ax^2+a-b}dx}{a-b}-\frac{1}{x(a-b)}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{a\left(\frac{1}{2}\left(\frac{\sqrt{a}}{\sqrt{b}}+1\right)\int \frac{1}{ax^2+\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)}dx+\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\int \frac{1}{ax^2+\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)}dx\right)}{a-b}-\frac{1}{x(a-b)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a\left(\frac{\left(\frac{\sqrt{a}}{\sqrt{b}}+1\right)\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{a-b}-\frac{1}{x(a-b)}$$

input
```
Int[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

output
```
-(1/((a - b)*x)) - (a*(((1 + Sqrt[a]/Sqrt[b])*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt
[a] - Sqrt[b]]])/(2*a^(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]) + ((1 - Sqrt[a]/Sqrt[
b])*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/4)*Sqrt[Sqrt[a] +
 Sqrt[b]])))/(a - b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

method	result
default	$-\frac{1}{(a-b)x}-\frac{a^{2}\left(-\frac{\left(\sqrt{ab}+a\right)\operatorname{arctanh}\left(\frac{ax}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)}{2a\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)a}}+\frac{\left(\sqrt{ab}-a\right)\arctan\left(\frac{ax}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)}{2a\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)a}}\right)}{a-b}$
risch	$-\frac{1}{(a-b)x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^{3}b^{2}-3a^{2}b^{3}+3b^{4}a-b^{5}\right)\_Z^{4}+\left(2a^{2}b+6b^{2}a\right)\_Z^{2}+a\right)}\_R\ln\left(\left(\left(b\,a^{4}+2a^{3}b^{2}-12a^{2}b^{3}+14b^{4}a-5b^{5}\right)\_R^{4}\right.\right.\right.}{4}$

input
```
int(1/x^2/(a*x^4+2*a*x^2+a-b),x,method=_RETURNVERBOSE)
```

output
```
-1/(a-b)/x-1/(a-b)*a^2*(-1/2*((a*b)^(1/2)+a)/a/(a*b)^(1/2)/(((a*b)^(1/2)-a
)*a)^(1/2)*arctanh(a*x/(((a*b)^(1/2)-a)*a)^(1/2))+1/2*((a*b)^(1/2)-a)/a/(a
*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(a*x/(((a*b)^(1/2)+a)*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1612 vs. $2(81) = 162$.

Time = 0.09 (sec) , antiderivative size = 1612, normalized size of antiderivative = 13.32

$$\int \frac{1}{x^2 (a - b + 2ax^2 + ax^4)} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="fricas")
```

output
```
1/4*((a - b)*x*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*sq
rt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4
+ 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*log((
3*a^2 + a*b)*x + (6*a^2*b + 2*a*b^2 - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*
sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^
4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2
+ 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^
4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*
a*b^3 - b^4))) - (a - b)*x*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*b
^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 -
 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 -
 b^4))*log((3*a^2 + a*b)*x - (6*a^2*b + 2*a*b^2 - (a^4*b - 2*a^3*b^2 + 2*a
*b^4 - b^5)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3
 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b + (a^3*b
- 3*a^2*b^2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5
*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*
a^2*b^2 + 3*a*b^3 - b^4))) + (a - b)*x*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2
*b^2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 +
15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2
 + 3*a*b^3 - b^4))*log((3*a^2 + a*b)*x + (6*a^2*b + 2*a*b^2 + (a^4*b - ...
```

Sympy [A] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a-b+2ax^2+ax^4)} dx$$
$$= \text{RootSum}\left(t^4 \cdot (256a^3b^2 - 768a^2b^3 + 768ab^4 - 256b^5) + t^2 \cdot (32a^2b + 96ab^2) + a, \left(t \mapsto t \log\left(x + \frac{64t^3}{}\right.\right.\right.$$
$$- \frac{1}{x(a-b)}$$

input
```
integrate(1/x**2/(a*x**4+2*a*x**2+a-b),x)
```

output
```
RootSum(_t**4*(256*a**3*b**2 - 768*a**2*b**3 + 768*a*b**4 - 256*b**5) + _t
**2*(32*a**2*b + 96*a*b**2) + a, Lambda(_t, _t*log(x + (64*_t**3*a**4*b -
128*_t**3*a**3*b**2 + 128*_t**3*a*b**4 - 64*_t**3*b**5 + 4*_t*a**3 + 40*_t
*a**2*b + 20*_t*a*b**2)/(3*a**2 + a*b)))) - 1/(x*(a - b))
```

Maxima [F]

$$\int \frac{1}{x^2 (a-b+2ax^2+ax^4)} dx = \int \frac{1}{(ax^4+2\,ax^2+a-b)x^2} dx$$

input
```
integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="maxima")
```

output
```
-a*integrate((x^2 + 2)/(a*x^4 + 2*a*x^2 + a - b), x)/(a - b) - 1/((a - b)*
x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(81) = 162$.

Time = 0.21 (sec) , antiderivative size = 698, normalized size of antiderivative = 5.77

$$\int \frac{1}{x^2(a-b+2ax^2+ax^4)}\,dx$$

$$= \frac{\left(\left(3\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}ab-4\sqrt{a^2+\sqrt{ab}a}\sqrt{ab}b^2\right)(a-b)^2|a|-2\left(3\sqrt{a^2+\sqrt{ab}a}a^3b-7\sqrt{a^2+\sqrt{ab}a}a^2b\right.}{}$$

$$-\frac{\left(\left(3\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}ab-4\sqrt{a^2-\sqrt{ab}a}\sqrt{ab}b^2\right)(a-b)^2|a|+2\left(3\sqrt{a^2-\sqrt{ab}a}a^3b-7\sqrt{a^2-\sqrt{ab}a}\right.}{}$$

$$-\frac{1}{(a-b)x}$$

input
```
integrate(1/x^2/(a*x^4+2*a*x^2+a-b),x, algorithm="giac")
```

output
```
1/2*((3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 + sqrt(a*b)*a)*
sqrt(a*b)*b^2)*(a - b)^2*abs(a) - 2*(3*sqrt(a^2 + sqrt(a*b)*a)*a^3*b - 7*s
qrt(a^2 + sqrt(a*b)*a)*a^2*b^2 + 4*sqrt(a^2 + sqrt(a*b)*a)*a*b^3)*abs(a -
b)*abs(a) + (3*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^4 - 10*sqrt(a^2 + sqrt(
a*b)*a)*sqrt(a*b)*a^3*b + 11*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 4
*sqrt(a^2 + sqrt(a*b)*a)*sqrt(a*b)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - a*b
 + sqrt((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))/(a^2 - a*b)))/((
3*a^6*b - 13*a^5*b^2 + 21*a^4*b^3 - 15*a^3*b^4 + 4*a^2*b^5)*abs(a - b)) -
1/2*((3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a*b - 4*sqrt(a^2 - sqrt(a*b)*a)*
sqrt(a*b)*b^2)*(a - b)^2*abs(a) + 2*(3*sqrt(a^2 - sqrt(a*b)*a)*a^3*b - 7*s
qrt(a^2 - sqrt(a*b)*a)*a^2*b^2 + 4*sqrt(a^2 - sqrt(a*b)*a)*a*b^3)*abs(a -
b)*abs(a) + (3*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^4 - 10*sqrt(a^2 - sqrt(
a*b)*a)*sqrt(a*b)*a^3*b + 11*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a^2*b^2 - 4
*sqrt(a^2 - sqrt(a*b)*a)*sqrt(a*b)*a*b^3)*abs(a))*arctan(x/sqrt((a^2 - a*b
 - sqrt((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))/(a^2 - a*b)))/((
3*a^6*b - 13*a^5*b^2 + 21*a^4*b^3 - 15*a^3*b^4 + 4*a^2*b^5)*abs(a - b)) -
1/((a - b)*x)
```

Mupad [B] (verification not implemented)

Time = 19.59 (sec) , antiderivative size = 2774, normalized size of antiderivative = 22.93

$$\int \frac{1}{x^2 (a - b + 2ax^2 + ax^4)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x)
```

output
```
atan(((x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) + (-(3*a*b^2 + a^2*b +
3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*
b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*
b^2 - x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a
*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5
*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^2*b + 3*a
*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2
)))^(1/2)*1i + (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4 - 8*a^5*b^3) - (-(3*a*b^2 +
 a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b
^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 - 128*a^5*b^4 + 192*a^6*b^3 -
 128*a^7*b^2 + x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))
/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b - 64*a^4*b^6
+ 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - 320*a^8*b^2)))*(-(3*a*b^2 + a^
2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3
+ a^3*b^2)))^(1/2)*1i)/(6*a^6*b - 2*a^7 + (x*(8*a^7*b - 4*a^8 + 4*a^4*b^4
- 8*a^5*b^3) + (-(3*a*b^2 + a^2*b + 3*a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2))/(
16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 -
128*a^5*b^4 + 192*a^6*b^3 - 128*a^7*b^2 - x*(-(3*a*b^2 + a^2*b + 3*a*(a*b^
3)^(1/2) + b*(a*b^3)^(1/2))/(16*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))^(1
/2)*(64*a^9*b - 64*a^4*b^6 + 320*a^5*b^5 - 640*a^6*b^4 + 640*a^7*b^3 - ...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.67

$$\int \frac{1}{x^2 (a - b + 2ax^2 + ax^4)} \, dx$$
$$= \frac{-4\sqrt{a}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}\,atan\left(\frac{ax}{\sqrt{a}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}}\right)bx + 2\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}\,atan\left(\frac{ax}{\sqrt{a}\,\sqrt{\sqrt{b}\,\sqrt{a}+a}}\right)ax + 2\sqrt{b}\,\sqrt{\sqrt{b}\,\sqrt{a}}}{}$$

input
```
int(1/x^2/(a*x^4+2*a*x^2+a-b),x)
```

output
```
( - 4*sqrt(a)*sqrt(sqrt(b)*sqrt(a) + a)*atan((a*x)/(sqrt(a)*sqrt(sqrt(b)*s
qrt(a) + a)))*b*x + 2*sqrt(b)*sqrt(sqrt(b)*sqrt(a) + a)*atan((a*x)/(sqrt(a
)*sqrt(sqrt(b)*sqrt(a) + a)))*a*x + 2*sqrt(b)*sqrt(sqrt(b)*sqrt(a) + a)*at
an((a*x)/(sqrt(a)*sqrt(sqrt(b)*sqrt(a) + a)))*b*x + 2*sqrt(a)*sqrt(sqrt(b)
*sqrt(a) - a)*log( - sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*b*x - 2*sqrt(a
)*sqrt(sqrt(b)*sqrt(a) - a)*log(sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*b*x
 + sqrt(b)*sqrt(sqrt(b)*sqrt(a) - a)*log( - sqrt(sqrt(b)*sqrt(a) - a) + sq
rt(a)*x)*a*x + sqrt(b)*sqrt(sqrt(b)*sqrt(a) - a)*log( - sqrt(sqrt(b)*sqrt(
a) - a) + sqrt(a)*x)*b*x - sqrt(b)*sqrt(sqrt(b)*sqrt(a) - a)*log(sqrt(sqrt
(b)*sqrt(a) - a) + sqrt(a)*x)*a*x - sqrt(b)*sqrt(sqrt(b)*sqrt(a) - a)*log(
sqrt(sqrt(b)*sqrt(a) - a) + sqrt(a)*x)*b*x - 4*a*b + 4*b**2)/(4*b*x*(a**2
- 2*a*b + b**2))
```

3.835 $\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx=\frac{x^2}{2a}+\frac{(a-b)\arctan\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(a+b+2ax^2+ax^4\right)}{2a}$$

output
```
1/2*x^2/a+1/2*(a-b)*arctan(a^(1/2)*(x^2+1)/b^(1/2))/a^(3/2)/b^(1/2)-1/2*ln
(a*x^4+2*a*x^2+a+b)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx=\frac{\frac{(a-b)\arctan\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{\sqrt{b}}+\sqrt{a}\left(x^2-\log\left(b+a(1+x^2)^2\right)\right)}{2a^{3/2}}$$

input
```
Integrate[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

output
```
(((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Sqrt[a]*(x^2 - Lo
g[b + a*(1 + x^2)^2]))/(2*a^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{ax^4+2ax^2+a+b}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^4}{ax^4+2ax^2+a+b}dx^2$$

$$\Big\downarrow \text{1143}$$

$$\frac{1}{2}\int \left(\frac{1}{a}-\frac{2ax^2+a+b}{a\left(ax^4+2ax^2+a+b\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(a-b)\arctan\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{a}+\frac{x^2}{a}\right)$$

input
```
Int[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

output
```
(x^2/a + ((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(a^(3/2)*Sqrt[b]) -
 Log[a + b + 2*a*x^2 + a*x^4]/a)/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

method	result
default	$\frac{x^2}{2a}+\frac{-\ln(a\,x^4+2a\,x^2+a+b)+\frac{(a-b)\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{\sqrt{ab}}}{2a}$
risch	$\frac{x^2}{2a}-\frac{\ln\left(\left(a^2b-b^2a+\sqrt{-ab(a-b)^2}\,a\right)x^2+\sqrt{-ab(a-b)^2}\,a+\sqrt{-ab(a-b)^2}\,b\right)}{2a}+\frac{\ln\left(\left(a^2b-b^2a+\sqrt{-ab(a-b)^2}\,a\right)x^2+\sqrt{-ab(a-}\right.}{4a^2b}$

input
```
int(x^5/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/a+1/2/a*(-ln(a*x^4+2*a*x^2+a+b)+(a-b)/(a*b)^(1/2)*arctan(1/2*(2*a*
x^2+2*a)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.28

$$\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx$$
$$=\left[\frac{2\,abx^2-2\,ab\log\left(ax^4+2\,ax^2+a+b\right)+\sqrt{-ab}(a-b)\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{-ab}(x^2+1)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,a^2b},\frac{abx^2-ab\log\left(\right.}{}\right.$$

input
```
integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*x^2 - 2*a*b*log(a*x^4 + 2*a*x^2 + a + b) + sqrt(-a*b)*(a - b)*
log((a*x^4 + 2*a*x^2 + 2*sqrt(-a*b)*(x^2 + 1) + a - b)/(a*x^4 + 2*a*x^2 +
a + b)))/(a^2*b), 1/2*(a*b*x^2 - a*b*log(a*x^4 + 2*a*x^2 + a + b) - sqrt(a
*b)*(a - b)*arctan(sqrt(a*b)/(a*x^2 + a)))/(a^2*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(60) = 120$.

Time = 0.62 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.09

$$\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx = \left(-\frac{1}{2a}-\frac{\sqrt{-a^3b}(a-b)}{4a^3b}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{2a}-\frac{\sqrt{-a^3b}(a-b)}{4a^3b}\right)+a+b}{a-b}\right) + \left(-\frac{1}{2a}+\frac{\sqrt{-a^3b}(a-b)}{4a^3b}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{2a}+\frac{\sqrt{-a^3b}(a-b)}{4a^3b}\right)+a+b}{a-b}\right)+\frac{x^2}{2a}$$

input
```
integrate(x**5/(a*x**4+2*a*x**2+a+b),x)
```

output
```
(-1/(2*a) - sqrt(-a**3*b)*(a - b)/(4*a**3*b))*log(x**2 + (4*a*b*(-1/(2*a)
- sqrt(-a**3*b)*(a - b)/(4*a**3*b)) + a + b)/(a - b)) + (-1/(2*a) + sqrt(-
a**3*b)*(a - b)/(4*a**3*b))*log(x**2 + (4*a*b*(-1/(2*a) + sqrt(-a**3*b)*(a
 - b)/(4*a**3*b)) + a + b)/(a - b)) + x**2/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx = \frac{x^2}{2\,a}+\frac{(a-b)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a}-\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{2\,a}$$

input
```
integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

output
```
1/2*x^2/a + 1/2*(a - b)*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*
log(a*x^4 + 2*a*x^2 + a + b)/a
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx=\frac{x^2}{2\,a}+\frac{(a-b)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a}-\frac{\log\left(ax^4+2\,ax^2+a+b\right)}{2\,a}$$

input
```
integrate(x^5/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

output
```
1/2*x^2/a + 1/2*(a - b)*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*
log(a*x^4 + 2*a*x^2 + a + b)/a
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 302, normalized size of antiderivative = 4.38

$$\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx=\frac{x^2}{2\,a}-\frac{\ln\left(a\,x^4+2\,a\,x^2+a+b\right)}{2\,a}-\frac{\mathrm{atan}\left(\frac{a\,b\,\left(x^2\,\left(\frac{\frac{\sqrt{a}\,\left(2\,a-2\,b\right)}{\sqrt{b}}+\frac{\left(a-b\right)\,\left(4\,a\,b-12\,a^2\right)}{4\,a^{3/2}\,\sqrt{b}}}{a+b}+\frac{\sqrt{a}\,\left(6\,a-2\,b-\frac{\left(a-b\right)^2}{b}+\frac{2\,a\,b-6\,a^2}{a}\right)}{\sqrt{b}\,\left(a+b\right)}\right)-\frac{\frac{\left(a-b\right)\,\left(16\,a\,b-\frac{8\,a^3+8\,b\,a^2}{a}+16\,a^2\right)}{4\,a^{3/2}\,\sqrt{b}}-\frac{\left(16\,a^3+16\,b\,a^2\right)\cdots}{8\,a^{5/2}\,\sqrt{b}\cdots}}{a+b}\cdots}{a^2-2\,a\,b+b^2}\cdots\right)\cdots}{2\,a^{3/2}\,\sqrt{b}}$$

input
```
int(x^5/(a + b + 2*a*x^2 + a*x^4),x)
```

output
```
x^2/(2*a) - log(a + b + 2*a*x^2 + a*x^4)/(2*a) - (atan((a*b*(x^2*(((a^(1/2
)*(2*a - 2*b))/b^(1/2) + ((a - b)*(4*a*b - 12*a^2))/(4*a^(3/2)*b^(1/2)))/(
a + b) + (a^(1/2)*(6*a - 2*b - (a - b)^2/b + (2*a*b - 6*a^2)/a))/(b^(1/2)*
(a + b))) - (((a - b)*(16*a*b - (8*a^2*b + 8*a^3)/a + 16*a^2))/(4*a^(3/2)*
b^(1/2)) - ((16*a^2*b + 16*a^3)*(a - b))/(8*a^(5/2)*b^(1/2)))/(a + b) + (a
^(1/2)*(4*a + 4*b - (8*a*b - (8*a^2*b + 8*a^3)/(2*a) + 8*a^2)/a - ((a - b)
^2*(a^2*b + a^3))/(a^3*b)))/(b^(1/2)*(a + b))))/(a^2 - 2*a*b + b^2))*(a -
b))/(2*a^(3/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.84

$$\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx$$
$$= \frac{-\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{\sqrt{a}\,\sqrt{a+b}-a}\,atan\left(\frac{\sqrt{\sqrt{a}\,\sqrt{a+b}-a}\,\sqrt{2}-2\sqrt{a}\,x}{\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{2}}\right)a+\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{\sqrt{a}\,\sqrt{a+b}-}}{}$$

input
```
int(x^5/(a*x^4+2*a*x^2+a+b),x)
```

output
```
( - sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(sqrt(a)*sqrt(a + b) - a)*atan((sqrt
(sqrt(a)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b)
 + a)*sqrt(2)))*a + sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(sqrt(a)*sqrt(a + b)
 - a)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqr
t(a)*sqrt(a + b) + a)*sqrt(2)))*b - sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(sqr
t(a)*sqrt(a + b) - a)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) + 2*sqrt
(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))*a + sqrt(sqrt(a)*sqrt(a +
b) + a)*sqrt(sqrt(a)*sqrt(a + b) - a)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*
sqrt(2) + 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))*b - log( -
 sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*x + sqrt(a + b) + sqrt(a)*x**2)*a*b
 - log(sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*x + sqrt(a + b) + sqrt(a)*x**
2)*a*b + a*b*x**2)/(2*a**2*b)
```

3.836 $\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx=-\frac{\arctan\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\log\left(a+b+2ax^2+ax^4\right)}{4a}$$

output
```
-1/2*arctan(a^(1/2)*(x^2+1)/b^(1/2))/a^(1/2)/b^(1/2)+1/4*ln(a*x^4+2*a*x^2+
a+b)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx=\frac{-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{\sqrt{b}}+\log\left(b+a(1+x^2)^2\right)}{4a}$$

input
```
Integrate[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

output
```
((-2*Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Log[b + a*(1 +
 x^2)^2])/(4*a)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1434, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{ax^4+2ax^2+a+b}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{ax^4+2ax^2+a+b}dx^2$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{\int \frac{2a(x^2+1)}{ax^4+2ax^2+a+b}dx^2}{2a}-\int \frac{1}{ax^4+2ax^2+a+b}dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\int \frac{x^2+1}{ax^4+2ax^2+a+b}dx^2-\int \frac{1}{ax^4+2ax^2+a+b}dx^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(2\int \frac{1}{-x^4-4ab}d(2ax^2+2a)+\int \frac{x^2+1}{ax^4+2ax^2+a+b}dx^2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\int \frac{x^2+1}{ax^4+2ax^2+a+b}dx^2-\frac{\arctan\left(\frac{2ax^2+2a}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\log\left(ax^4+2ax^2+a+b\right)}{2a}-\frac{\arctan\left(\frac{2ax^2+2a}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}\right)$$

input
```
Int[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

output
```
(-(ArcTan[(2*a + 2*a*x^2)/(2*Sqrt[a]*Sqrt[b])]/(Sqrt[a]*Sqrt[b])) + Log[a
+ b + 2*a*x^2 + a*x^4]/(2*a))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

method	result
default	$\frac{\ln(a\,x^4+2a\,x^2+a+b)}{4a}-\frac{\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}$
risch	$\frac{\ln((a\sqrt{-ab}-ab)x^2+a\sqrt{-ab}+\sqrt{-ab}\,b)}{4a}+\frac{\ln((a\sqrt{-ab}-ab)x^2+a\sqrt{-ab}+\sqrt{-ab}\,b)\sqrt{-ab}}{4ba}+\frac{\ln((-a\sqrt{-ab}-ab)x^2-a\sqrt{-ab}-\sqrt{-al}}{4a}$

input
```
int(x^3/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(a*x^4+2*a*x^2+a+b)/a-1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.43

$$\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx$$
$$=\left[\frac{b\log\left(ax^4+2\,ax^2+a+b\right)-\sqrt{-ab}\log\left(\frac{ax^4+2\,ax^2+2\,\sqrt{-ab}(x^2+1)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,ab},\frac{b\log\left(ax^4+2\,ax^2+a+b\right)+2\,\sqrt{a}}{4\,ab}\right.$$

input
```
integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

output
```
[1/4*(b*log(a*x^4 + 2*a*x^2 + a + b) - sqrt(-a*b)*log((a*x^4 + 2*a*x^2 + 2
*sqrt(-a*b)*(x^2 + 1) + a - b)/(a*x^4 + 2*a*x^2 + a + b)))/(a*b), 1/4*(b*l
og(a*x^4 + 2*a*x^2 + a + b) + 2*sqrt(a*b)*arctan(sqrt(a*b)/(a*x^2 + a)))/(
a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(48) = 96$.

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.17

$$\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx = \left(\frac{1}{4a} - \frac{\sqrt{-a^3b}}{4a^2b}\right)\log\left(x^2 + \frac{-4ab\left(\frac{1}{4a} - \frac{\sqrt{-a^3b}}{4a^2b}\right) + a + b}{a}\right) + \left(\frac{1}{4a} + \frac{\sqrt{-a^3b}}{4a^2b}\right)\log\left(x^2 + \frac{-4ab\left(\frac{1}{4a} + \frac{\sqrt{-a^3b}}{4a^2b}\right) + a + b}{a}\right)$$

input
```
integrate(x**3/(a*x**4+2*a*x**2+a+b),x)
```

output
```
(1/(4*a) - sqrt(-a**3*b)/(4*a**2*b))*log(x**2 + (-4*a*b*(1/(4*a) - sqrt(-a
**3*b)/(4*a**2*b)) + a + b)/a) + (1/(4*a) + sqrt(-a**3*b)/(4*a**2*b))*log(
x**2 + (-4*a*b*(1/(4*a) + sqrt(-a**3*b)/(4*a**2*b)) + a + b)/a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx = -\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}} + \frac{\log\left(ax^4 + 2\,ax^2 + a + b\right)}{4\,a}$$

input
```
integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

output
```
-1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b) + 1/4*log(a*x^4 + 2*a*x^2 + a
 + b)/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx = -\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}} + \frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,a}$$

input
```
integrate(x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

output
```
-1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b) + 1/4*log(a*x^4 + 2*a*x^2 + a
 + b)/a
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.57

$$\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx = \frac{\ln\left(a\,x^4+2\,a\,x^2+a+b\right)}{4\,a} - \frac{\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}}{a+b}+\frac{a^{3/2}}{\sqrt{b}\,(a+b)}+\frac{\sqrt{a}\,\sqrt{b}\,x^2}{a+b}+\frac{a^{3/2}\,x^2}{\sqrt{b}\,(a+b)}\right)}{2\,\sqrt{a}\,\sqrt{b}}$$

input
```
int(x^3/(a + b + 2*a*x^2 + a*x^4),x)
```

output
```
log(a + b + 2*a*x^2 + a*x^4)/(4*a) - atan((a^(1/2)*b^(1/2))/(a + b) + a^(3
/2)/(b^(1/2)*(a + b)) + (a^(1/2)*b^(1/2)*x^2)/(a + b) + (a^(3/2)*x^2)/(b^(
1/2)*(a + b)))/(2*a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.63

$$\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx$$
$$= \frac{2\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{\sqrt{a}\,\sqrt{a+b}-a}\,atan\left(\frac{\sqrt{\sqrt{a}\,\sqrt{a+b}-a}\,\sqrt{2}-2\sqrt{a}\,x}{\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{2}}\right)+2\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{\sqrt{a}\,\sqrt{a+b}-a}$$

input
```
int(x^3/(a*x^4+2*a*x^2+a+b),x)
```

output
```
(2*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(sqrt(a)*sqrt(a + b) - a)*atan((sqrt(
sqrt(a)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b)
+ a)*sqrt(2))) + 2*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(sqrt(a)*sqrt(a + b)
- a)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) + 2*sqrt(a)*x)/(sqrt(sqrt
(a)*sqrt(a + b) + a)*sqrt(2))) + log( - sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt
(2)*x + sqrt(a + b) + sqrt(a)*x**2)*b + log(sqrt(sqrt(a)*sqrt(a + b) - a)*
sqrt(2)*x + sqrt(a + b) + sqrt(a)*x**2)*b)/(4*a*b)
```

3.837 $\int \frac{x}{a+b+2ax^2+ax^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{x}{a+b+2ax^2+ax^4} \, dx = \frac{\arctan\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

output
```
1/2*arctan(a^(1/2)*(x^2+1)/b^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b+2ax^2+ax^4} \, dx = \frac{\arctan\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

input
```
Integrate[x/(a + b + 2*a*x^2 + a*x^4),x]
```

output
```
ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1432, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{ax^4+2ax^2+a+b}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{ax^4+2ax^2+a+b}dx^2$$

$$\Big\downarrow \text{1083}$$

$$-\int \frac{1}{-x^4-4ab}d(2ax^2+2a)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2ax^2+2a}{2\sqrt{a}\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

input
```
Int[x/(a + b + 2*a*x^2 + a*x^4),x]
```

output
```
ArcTan[(2*a + 2*a*x^2)/(2*Sqrt[a]*Sqrt[b])]/(2*Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{2\sqrt{ab}}$	26
risch	$-\frac{\ln\left(\left(\sqrt{-ab}-a\right)x^2-a-b\right)}{4\sqrt{-ab}}+\frac{\ln\left(\left(\sqrt{-ab}+a\right)x^2+a+b\right)}{4\sqrt{-ab}}$	56

input
```
int(x/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.94

$$\int \frac{x}{a+b+2ax^2+ax^4}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{ax^4+2\,ax^2-2\,\sqrt{-ab}\left(x^2+1\right)+a-b}{ax^4+2\,ax^2+a+b}\right)}{4\,ab}, -\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{ax^2+a}\right)}{2\,ab}\right]$$

input
```
integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a*b)*log((a*x^4 + 2*a*x^2 - 2*sqrt(-a*b)*(x^2 + 1) + a - b)/(a
*x^4 + 2*a*x^2 + a + b))/(a*b), -1/2*sqrt(a*b)*arctan(sqrt(a*b)/(a*x^2 + a
))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \frac{x}{a+b+2ax^2+ax^4}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-b\sqrt{-\frac{1}{ab}}+x^2+1\right)}{4} + \frac{\sqrt{-\frac{1}{ab}}\log\left(b\sqrt{-\frac{1}{ab}}+x^2+1\right)}{4}$$

input
```
integrate(x/(a*x**4+2*a*x**2+a+b),x)
```

output
```
-sqrt(-1/(a*b))*log(-b*sqrt(-1/(a*b)) + x**2 + 1)/4 + sqrt(-1/(a*b))*log(b
*sqrt(-1/(a*b)) + x**2 + 1)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{x}{a+b+2ax^2+ax^4}\,dx = \frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}}$$

input
```
integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

output
```
1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{x}{a+b+2ax^2+ax^4}\,dx = \frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}}$$

input
```
integrate(x/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

output
```
1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 18.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x}{a+b+2ax^2+ax^4}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{a}+\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\sqrt{a}\,\sqrt{b}}$$

input
```
int(x/(a + b + 2*a*x^2 + a*x^4),x)
```

output
```
atan((a^(1/2) + a^(1/2)*x^2)/b^(1/2))/(2*a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{x}{a+b+2ax^2+ax^4}\,dx = -\frac{\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{\sqrt{a}\,\sqrt{a+b}-a}\left(atan\left(\frac{\sqrt{\sqrt{a}\,\sqrt{a+b}-a}\,\sqrt{2}-2\sqrt{a}\,x}{\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{2}}\right)+atan\left(\frac{\sqrt{\sqrt{a}\,\sqrt{a+b}-a}\,\sqrt{2}+2\sqrt{a}\,x}{\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{2}}\right)\right)}{2ab}$$

input
```
int(x/(a*x^4+2*a*x^2+a+b),x)
```

output
```
( - sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(sqrt(a)*sqrt(a + b) - a)*(atan((sqr
t(sqrt(a)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b
) + a)*sqrt(2))) + atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) + 2*sqrt(a)
*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))))/(2*a*b)
```

3.838 $\int \frac{1}{x(a+b+2ax^2+ax^4)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{1}{x(a+b+2ax^2+ax^4)}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)} + \frac{\log(x)}{a+b} - \frac{\log(a+b+2ax^2+ax^4)}{4(a+b)}$$

output
```
-1/2*a^(1/2)*arctan(a^(1/2)*(x^2+1)/b^(1/2))/b^(1/2)/(a+b)+ln(x)/(a+b)-ln(
a*x^4+2*a*x^2+a+b)/(4*a+4*b)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.52

$$\int \frac{1}{x(a+b+2ax^2+ax^4)}\,dx = \frac{4\sqrt{b}\log(x) + i\left(\sqrt{a}+i\sqrt{b}\right)\log\left(-i\sqrt{b}+\sqrt{a}(1+x^2)\right) + \left(-i\sqrt{a}-\sqrt{b}\right)\log\left(i\sqrt{b}+\sqrt{a}(1+x^2)\right)}{4\sqrt{b}(a+b)}$$

input
```
Integrate[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]
```

output
```
(4*Sqrt[b]*Log[x] + I*(Sqrt[a] + I*Sqrt[b])*Log[(-I)*Sqrt[b] + Sqrt[a]*(1
+ x^2)] + ((-I)*Sqrt[a] - Sqrt[b])*Log[I*Sqrt[b] + Sqrt[a]*(1 + x^2)])/(4*
Sqrt[b]*(a + b))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1434, 1144, 25, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a x^4+2 a x^2+a+b)} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int \frac{1}{x^2 (a x^4+2 a x^2+a+b)}dx^2$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{2} \left(\frac{\int -\frac{a (x^2+2)}{a x^4+2 a x^2+a+b}dx^2}{a+b}+\frac{\log (x^2)}{a+b}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\frac{\log (x^2)}{a+b}-\frac{\int \frac{a (x^2+2)}{a x^4+2 a x^2+a+b}dx^2}{a+b}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{\log (x^2)}{a+b}-\frac{a \int \frac{x^2+2}{a x^4+2 a x^2+a+b}dx^2}{a+b}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a+b}-\frac{a\left(\int\frac{1}{ax^4+2ax^2+a+b}dx^2+\frac{\int\frac{2a\left(x^2+1\right)}{ax^4+2ax^2+a+b}dx^2}{2a}\right)}{a+b}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a+b}-\frac{a\left(\int\frac{1}{ax^4+2ax^2+a+b}dx^2+\int\frac{x^2+1}{ax^4+2ax^2+a+b}dx^2\right)}{a+b}\right)$$

$\Big\downarrow$ 1083

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a+b}-\frac{a\left(\int\frac{x^2+1}{ax^4+2ax^2+a+b}dx^2-2\int\frac{1}{-x^4-4ab}d\left(2ax^2+2a\right)\right)}{a+b}\right)$$

$\Big\downarrow$ 217

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a+b}-\frac{a\left(\int\frac{x^2+1}{ax^4+2ax^2+a+b}dx^2+\frac{\arctan\left(\frac{2ax^2+2a}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}\right)}{a+b}\right)$$

$\Big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{\log\left(x^2\right)}{a+b}-\frac{a\left(\frac{\arctan\left(\frac{2ax^2+2a}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}+\frac{\log\left(ax^4+2ax^2+a+b\right)}{2a}\right)}{a+b}\right)$$

input
```
Int[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]
```

output
```
(Log[x^2]/(a + b) - (a*(ArcTan[(2*a + 2*a*x^2)/(2*Sqrt[a]*Sqrt[b])]/(Sqrt[
a]*Sqrt[b]) + Log[a + b + 2*a*x^2 + a*x^4]/(2*a)))/(a + b))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{\ln(x)}{a+b}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(ab+b^{2}\right)\_Z^{2}+2b\_Z\right)}\_R\ln\left(\left(\left(-a+5b\right)\_R+5\right)x^{2}+\left(-a-b\right)\_R+4\right)\right)}{4}$	62
default	$-\frac{a\left(\frac{\ln\left(a\,x^{4}+2a\,x^{2}+a+b\right)}{2a}+\frac{\arctan\left(\frac{2a\,x^{2}+2a}{2\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{2(a+b)}+\frac{\ln(x)}{a+b}$	63

input
```
int(1/x/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/(a+b)+1/4*sum(_R*ln(((-a+5*b)*_R+5)*x^2+(-a-b)*_R+4),_R=RootOf(1+(a*
b+b^2)*_Z^2+2*b*_Z))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.03

$$\int \frac{1}{x\left(a+b+2ax^2+ax^4\right)}\,dx = \left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\,(bx^2+b)\sqrt{-\frac{a}{b}}+a-b}{ax^4+2\,ax^2+a+b}\right)-\log\left(ax^4+2\,ax^2+a+b\right)+4\,\log\left(x\right)}{4\,(a+b)},\right.$$
$$\left.-\frac{2\,\sqrt{\frac{a}{b}}\arctan\left(\left(x^2+1\right)\sqrt{\frac{a}{b}}\right)+\log\left(ax^4+2\,ax^2+a+b\right)-4\,\log\left(x\right)}{4\,(a+b)}\right]$$

input
```
integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(-a/b) + a - b)/
(a*x^4 + 2*a*x^2 + a + b)) - log(a*x^4 + 2*a*x^2 + a + b) + 4*log(x))/(a +
 b), -1/4*(2*sqrt(a/b)*arctan((x^2 + 1)*sqrt(a/b)) + log(a*x^4 + 2*a*x^2 +
 a + b) - 4*log(x))/(a + b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(61) = 122$.

Time = 2.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.81

$$\int \frac{1}{x\,(a+b+2ax^2+ax^4)}\,dx = \left(-\frac{1}{4\,(a+b)} - \frac{\sqrt{-ab}}{4b\,(a+b)}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{4(a+b)}-\frac{\sqrt{-ab}}{4b(a+b)}\right)+a-4b^2\left(-\frac{1}{4(a+b)}-\frac{\sqrt{-ab}}{4b(a+b)}\right)-b}{a}\right) + \left(-\frac{1}{4\,(a+b)} + \frac{\sqrt{-ab}}{4b\,(a+b)}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{4(a+b)}+\frac{\sqrt{-ab}}{4b(a+b)}\right)+a-4b^2\left(-\frac{1}{4(a+b)}+\frac{\sqrt{-ab}}{4b(a+b)}\right)-b}{a}\right) + \frac{\log(x)}{a+b}$$

input
```
integrate(1/x/(a*x**4+2*a*x**2+a+b),x)
```

output
```
(-1/(4*(a + b)) - sqrt(-a*b)/(4*b*(a + b)))*log(x**2 + (-4*a*b*(-1/(4*(a +
 b)) - sqrt(-a*b)/(4*b*(a + b))) + a - 4*b**2*(-1/(4*(a + b)) - sqrt(-a*b)
/(4*b*(a + b))) - b)/a) + (-1/(4*(a + b)) + sqrt(-a*b)/(4*b*(a + b)))*log(
x**2 + (-4*a*b*(-1/(4*(a + b)) + sqrt(-a*b)/(4*b*(a + b))) + a - 4*b**2*(-
1/(4*(a + b)) + sqrt(-a*b)/(4*b*(a + b))) - b)/a) + log(x)/(a + b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(a+b+2ax^2+ax^4\right)}\,dx = -\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}(a+b)} - \frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,(a+b)} + \frac{\log\left(x^2\right)}{2\,(a+b)}$$

input
```
integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

output
```
-1/2*a*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/4*log(a*x^4 +
 2*a*x^2 + a + b)/(a + b) + 1/2*log(x^2)/(a + b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(a+b+2ax^2+ax^4\right)}\,dx = -\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}(a+b)} - \frac{\log\left(ax^4+2\,ax^2+a+b\right)}{4\,(a+b)} + \frac{\log\left(x^2\right)}{2\,(a+b)}$$

input
```
integrate(1/x/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

output
```
-1/2*a*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/4*log(a*x^4 +
 2*a*x^2 + a + b)/(a + b) + 1/2*log(x^2)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 18.59 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\,(a+b+2ax^2+ax^4)}\,dx = \frac{\ln(x)}{a+b} - \frac{4\,b\,\ln(a\,x^4+2\,a\,x^2+a+b)}{16\,b^2+16\,a\,b} - \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\,\sqrt{b}\,(a+b)}$$

input
```
int(1/(x*(a + b + 2*a*x^2 + a*x^4)),x)
```

output
```
log(x)/(a + b) - (4*b*log(a + b + 2*a*x^2 + a*x^4))/(16*a*b + 16*b^2) - (a
^(1/2)*atan(a^(1/2)/b^(1/2) + (a^(1/2)*x^2)/b^(1/2)))/(2*b^(1/2)*(a + b))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.97

$$\int \frac{1}{x\,(a+b+2ax^2+ax^4)}\,dx = \frac{2\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{\sqrt{a}\,\sqrt{a+b}-a}\,atan\left(\frac{\sqrt{\sqrt{a}\,\sqrt{a+b}-a}\,\sqrt{2}-2\sqrt{a}\,x}{\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{2}}\right)+2\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{\sqrt{a}\,\sqrt{a+b}-a}$$

input
```
int(1/x/(a*x^4+2*a*x^2+a+b),x)
```

output
```
(2*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(sqrt(a)*sqrt(a + b) - a)*atan((sqrt(
sqrt(a)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b)
+ a)*sqrt(2))) + 2*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(sqrt(a)*sqrt(a + b)
- a)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) + 2*sqrt(a)*x)/(sqrt(sqrt
(a)*sqrt(a + b) + a)*sqrt(2))) - log( - sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt
(2)*x + sqrt(a + b) + sqrt(a)*x**2)*b - log(sqrt(sqrt(a)*sqrt(a + b) - a)*
sqrt(2)*x + sqrt(a + b) + sqrt(a)*x**2)*b + 4*log(x)*b)/(4*b*(a + b))
```

3.839 $\int \frac{1}{x^3(a+b+2ax^2+ax^4)} dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{1}{x^3 (a+b+2ax^2+ax^4)} dx = -\frac{1}{2(a+b)x^2} + \frac{\sqrt{a}(a-b)\arctan\left(\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{a\log(a+b+2ax^2+ax^4)}{2(a+b)^2}$$

output
```
-1/2/(a+b)/x^2+1/2*a^(1/2)*(a-b)*arctan(a^(1/2)*(x^2+1)/b^(1/2))/b^(1/2)/(
a+b)^2-2*a*ln(x)/(a+b)^2+1/2*a*ln(a*x^4+2*a*x^2+a+b)/(a+b)^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.83

$$\int \frac{1}{x^3 (a+b+2ax^2+ax^4)} dx = -\frac{1}{2(a+b)x^2} - \frac{2a\log(x)}{(a+b)^2} + \frac{\left(-ia^2+2a^{3/2}\sqrt{b}+iab\right)\log\left(\sqrt{a}-i\sqrt{b}+\sqrt{a}x^2\right)}{4\sqrt{a}\sqrt{b}(a+b)^2} + \frac{\left(ia^2+2a^{3/2}\sqrt{b}-iab\right)\log\left(\sqrt{a}+i\sqrt{b}+\sqrt{a}x^2\right)}{4\sqrt{a}\sqrt{b}(a+b)^2}$$

input
```
Integrate[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
```

output
```
-1/2*1/((a + b)*x^2) - (2*a*Log[x])/(a + b)^2 + (((-I)*a^2 + 2*a^(3/2)*Sqr
t[b] + I*a*b)*Log[Sqrt[a] - I*Sqrt[b] + Sqrt[a]*x^2])/(4*Sqrt[a]*Sqrt[b]*(
a + b)^2) + ((I*a^2 + 2*a^(3/2)*Sqrt[b] - I*a*b)*Log[Sqrt[a] + I*Sqrt[b] +
 Sqrt[a]*x^2])/(4*Sqrt[a]*Sqrt[b]*(a + b)^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1434, 1145, 25, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a x^4+2 a x^2+a+b\right)} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(a x^4+2 a x^2+a+b\right)}dx^2$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{2} \left(\frac{\int -\frac{a \left(x^2+2\right)}{x^2 \left(a x^4+2 a x^2+a+b\right)}dx^2}{a+b}-\frac{1}{x^2 (a+b)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(-\frac{\int \frac{a \left(x^2+2\right)}{x^2 \left(a x^4+2 a x^2+a+b\right)}dx^2}{a+b}-\frac{1}{x^2 (a+b)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{a \int \frac{x^2+2}{x^2 \left(a x^4+2 a x^2+a+b\right)}dx^2}{a+b}-\frac{1}{x^2 (a+b)}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(-\frac{a\int\left(\frac{-2ax^2-3a+b}{(a+b)(ax^4+2ax^2+a+b)}+\frac{2}{(a+b)x^2}\right)dx^2}{a+b}-\frac{1}{x^2(a+b)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a\left(-\frac{(a-b)\arctan\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}(a+b)}+\frac{2\log(x^2)}{a+b}-\frac{\log(ax^4+2ax^2+a+b)}{a+b}\right)}{a+b}-\frac{1}{x^2(a+b)}\right)$$

input
```
Int[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
```

output
```
(-(1/((a + b)*x^2)) - (a*(-(((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/
(Sqrt[a]*Sqrt[b]*(a + b))) + (2*Log[x^2])/(a + b) - Log[a + b + 2*a*x^2 +
a*x^4]/(a + b)))/(a + b))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

method	result
default	$\frac{a\left(\ln\left(a\,x^4+2a\,x^2+a+b\right)+\frac{(a-b)\arctan\left(\frac{2a\,x^2+2a}{2\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{2(a+b)^2}-\frac{1}{2(a+b)x^2}-\frac{2a\ln(x)}{(a+b)^2}$
risch	$-\frac{1}{2(a+b)x^2}-\frac{2a\ln(x)}{a^2+2ab+b^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2b+2b^2a+b^3\right)\_Z^2-4ab\_Z+a\right)}\_R\ln\left(\left(\left(-a^3+3a^2b+9b^2a+5b^3\right)\_R^2+\left(-8a^2-\right.\right.\right.\right.}{4}$

input
```
int(1/x^3/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)^2*a*(ln(a*x^4+2*a*x^2+a+b)+(a-b)/(a*b)^(1/2)*arctan(1/2*(2*a*x^2
+2*a)/(a*b)^(1/2)))-1/2/(a+b)/x^2-2*a*ln(x)/(a+b)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.31

$$\int \frac{1}{x^3\left(a+b+2ax^2+ax^4\right)}\,dx$$
$$= \left[-\frac{(a-b)x^2\sqrt{-\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\,(bx^2+b)\sqrt{-\frac{a}{b}}+a-b}{ax^4+2\,ax^2+a+b}\right)-2\,ax^2\log\left(ax^4+2\,ax^2+a+b\right)+8\,ax^2\log\left(x\right)+2}{4\,(a^2+2\,ab+b^2)x^2}\right.$$

input
```
integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

output
```
[-1/4*((a - b)*x^2*sqrt(-a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(-a
/b) + a - b)/(a*x^4 + 2*a*x^2 + a + b)) - 2*a*x^2*log(a*x^4 + 2*a*x^2 + a
+ b) + 8*a*x^2*log(x) + 2*a + 2*b)/((a^2 + 2*a*b + b^2)*x^2), 1/2*((a - b)
*x^2*sqrt(a/b)*arctan((x^2 + 1)*sqrt(a/b)) + a*x^2*log(a*x^4 + 2*a*x^2 + a
 + b) - 4*a*x^2*log(x) - a - b)/((a^2 + 2*a*b + b^2)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(83) = 166$.

Time = 12.80 (sec) , antiderivative size = 386, normalized size of antiderivative = 4.34

$$\int \frac{1}{x^3\left(a+b+2ax^2+ax^4\right)}\,dx = -\frac{2a\log(x)}{(a+b)^2}+\left(\frac{a}{2\,(a+b)^2}\right.$$
$$\left.-\frac{\sqrt{-ab}(a-b)}{4b\,(a^2+2ab+b^2)}\right)\log\left(x^2+\frac{4a^2b\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)+a^2+8ab^2\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)-3a}{a^2-ab}\right.$$
$$+\left(\frac{a}{2\,(a+b)^2}\right.$$
$$\left.+\frac{\sqrt{-ab}(a-b)}{4b\,(a^2+2ab+b^2)}\right)\log\left(x^2+\frac{4a^2b\left(\frac{a}{2(a+b)^2}+\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)+a^2+8ab^2\left(\frac{a}{2(a+b)^2}+\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)-3a}{a^2-ab}\right.$$
$$-\frac{1}{x^2\cdot(2a+2b)}$$

input
```
integrate(1/x**3/(a*x**4+2*a*x**2+a+b),x)
```

output
```
-2*a*log(x)/(a + b)**2 + (a/(2*(a + b)**2) - sqrt(-a*b)*(a - b)/(4*b*(a**2
 + 2*a*b + b**2)))*log(x**2 + (4*a**2*b*(a/(2*(a + b)**2) - sqrt(-a*b)*(a
- b)/(4*b*(a**2 + 2*a*b + b**2))) + a**2 + 8*a*b**2*(a/(2*(a + b)**2) - sq
rt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))) - 3*a*b + 4*b**3*(a/(2*(a +
b)**2) - sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))))/(a**2 - a*b)) +
(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2)))*log(x*
*2 + (4*a**2*b*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b +
 b**2))) + a**2 + 8*a*b**2*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a*
*2 + 2*a*b + b**2))) - 3*a*b + 4*b**3*(a/(2*(a + b)**2) + sqrt(-a*b)*(a -
b)/(4*b*(a**2 + 2*a*b + b**2))))/(a**2 - a*b)) - 1/(x**2*(2*a + 2*b))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^3 (a+b+2ax^2+ax^4)}\,dx = \frac{a\log\left(ax^4+2\,ax^2+a+b\right)}{2\left(a^2+2\,ab+b^2\right)} - \frac{a\log\left(x^2\right)}{a^2+2\,ab+b^2} + \frac{\left(a^2-ab\right)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{ab}} - \frac{1}{2\,(a+b)x^2}$$

input
```
integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

output
```
1/2*a*log(a*x^4 + 2*a*x^2 + a + b)/(a^2 + 2*a*b + b^2) - a*log(x^2)/(a^2 +
 2*a*b + b^2) + 1/2*(a^2 - a*b)*arctan((a*x^2 + a)/sqrt(a*b))/((a^2 + 2*a*
b + b^2)*sqrt(a*b)) - 1/2/((a + b)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^3 (a+b+2ax^2+ax^4)} \, dx = \frac{a \log (ax^4+2\,ax^2+a+b)}{2\,(a^2+2\,ab+b^2)} - \frac{a \log (x^2)}{a^2+2\,ab+b^2} + \frac{(a^2-ab) \arctan \left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,(a^2+2\,ab+b^2)\sqrt{ab}} + \frac{2\,ax^2-a-b}{2\,(a^2+2\,ab+b^2)x^2}$$

input
```
integrate(1/x^3/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

output
```
1/2*a*log(a*x^4 + 2*a*x^2 + a + b)/(a^2 + 2*a*b + b^2) - a*log(x^2)/(a^2 +
 2*a*b + b^2) + 1/2*(a^2 - a*b)*arctan((a*x^2 + a)/sqrt(a*b))/((a^2 + 2*a*
b + b^2)*sqrt(a*b)) + 1/2*(2*a*x^2 - a - b)/((a^2 + 2*a*b + b^2)*x^2)
```

Mupad [B] (verification not implemented)

Time = 21.91 (sec) , antiderivative size = 3313, normalized size of antiderivative = 37.22

$$\int \frac{1}{x^3 (a+b+2ax^2+ax^4)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x)
```

output
```
(8*a*b*log(((2*a^5)/(a + b)^3 - (a/(2*(a + b)^2) - (-(a*(a - b)^2)/(b*(a +
 b)^4))^(1/2)/4)*((12*a^5*x^2)/(a + b)^2 - (a/(2*(a + b)^2) - (-(a*(a - b)
^2)/(b*(a + b)^4))^(1/2)/4)*((8*a^4*(3*a - b))/(a + b) + 16*a^4*(a/(2*(a +
 b)^2) - (-(a*(a - b)^2)/(b*(a + b)^4))^(1/2)/4)*(a + b + a*x^2 - 5*b*x^2)
 + (4*a^4*x^2*(7*a + 5*b))/(a + b)) + (a^4*(15*a - b))/(a + b)^2) + (a^5*x
^2)/(a + b)^3)*((2*a^5)/(a + b)^3 - (a/(2*(a + b)^2) + (-(a*(a - b)^2)/(b*
(a + b)^4))^(1/2)/4)*((12*a^5*x^2)/(a + b)^2 - (a/(2*(a + b)^2) + (-(a*(a
- b)^2)/(b*(a + b)^4))^(1/2)/4)*((8*a^4*(3*a - b))/(a + b) + 16*a^4*(a/(2*
(a + b)^2) + (-(a*(a - b)^2)/(b*(a + b)^4))^(1/2)/4)*(a + b + a*x^2 - 5*b*
x^2) + (4*a^4*x^2*(7*a + 5*b))/(a + b)) + (a^4*(15*a - b))/(a + b)^2) + (a
^5*x^2)/(a + b)^3)))/(32*a*b^2 + 16*a^2*b + 16*b^3) - (2*a*log(x))/(2*a*b
+ a^2 + b^2) - 1/(2*x^2*(a + b)) + (a^(1/2)*atan(((13*a^2 - 34*a*b + b^2)*
((8*a*b*((14*a^5*b + 15*a^6 - a^4*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (
8*a*b*((40*a^6*b + 24*a^7 - 8*a^4*b^3 + 8*a^5*b^2)/(3*a*b^2 + 3*a^2*b + a^
3 + b^3) + (8*a*b*(64*a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^
2))/((32*a*b^2 + 16*a^2*b + 16*b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))))/(32
*a*b^2 + 16*a^2*b + 16*b^3)))/(32*a*b^2 + 16*a^2*b + 16*b^3) - (2*a^5)/(3*
a*b^2 + 3*a^2*b + a^3 + b^3) + (a^(1/2)*((a^(1/2)*(a - b)*((40*a^6*b + 24*
a^7 - 8*a^4*b^3 + 8*a^5*b^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) + (8*a*b*(64*
a^7*b + 16*a^8 + 16*a^4*b^4 + 64*a^5*b^3 + 96*a^6*b^2))/((32*a*b^2 + 16...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 375, normalized size of antiderivative = 4.21

$$\int \frac{1}{x^3 (a+b+2ax^2+ax^4)} \, dx = \frac{-\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{\sqrt{a}\,\sqrt{a+b}-a}\,atan\left(\frac{\sqrt{\sqrt{a}\,\sqrt{a+b}-a}\,\sqrt{2}-2\sqrt{a}\,x}{\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{2}}\right)a\,x^2+\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{\sqrt{a}\,\sqrt{a+b}}$$

input
```
int(1/x^3/(a*x^4+2*a*x^2+a+b),x)
```

output
```
( - sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(sqrt(a)*sqrt(a + b) - a)*atan((sqrt
(sqrt(a)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b)
 + a)*sqrt(2)))*a*x**2 + sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(sqrt(a)*sqrt(a
 + b) - a)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqr
t(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))*b*x**2 - sqrt(sqrt(a)*sqrt(a + b) + a
)*sqrt(sqrt(a)*sqrt(a + b) - a)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2
) + 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))*a*x**2 + sqrt(sq
rt(a)*sqrt(a + b) + a)*sqrt(sqrt(a)*sqrt(a + b) - a)*atan((sqrt(sqrt(a)*sq
rt(a + b) - a)*sqrt(2) + 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(
2)))*b*x**2 + log( - sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*x + sqrt(a + b)
 + sqrt(a)*x**2)*a*b*x**2 + log(sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*x +
sqrt(a + b) + sqrt(a)*x**2)*a*b*x**2 - 4*log(x)*a*b*x**2 - a*b - b**2)/(2*
b*x**2*(a**2 + 2*a*b + b**2))
```

3.840 $\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 320

$$\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx = \frac{x}{a}+\frac{\left(a+b+2\sqrt{a}\sqrt{a+b}\right)\arctan\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}}-\frac{\left(a+b+2\sqrt{a}\sqrt{a+b}\right)\arctan\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}}+\frac{\left(2\sqrt{a}-\sqrt{a+b}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt{a+b}+\sqrt{a}x^2}\right)}{2\sqrt{2}a^{5/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

output
```
x/a+1/4*(a+b+2*a^(1/2)*(a+b)^(1/2))*arctan(((-a^(1/2)+(a+b)^(1/2))^(1/2)-2
^(1/2)*a^(1/4)*x)/(a^(1/2)+(a+b)^(1/2))^(1/2))*2^(1/2)/a^(5/4)/(a+b)^(1/2)
/(a^(1/2)+(a+b)^(1/2))^(1/2)-1/4*(a+b+2*a^(1/2)*(a+b)^(1/2))*arctan(((-a^(
1/2)+(a+b)^(1/2))^(1/2)+2^(1/2)*a^(1/4)*x)/(a^(1/2)+(a+b)^(1/2))^(1/2))*2^
(1/2)/a^(5/4)/(a+b)^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)+1/4*(2*a^(1/2)-(a+b)
^(1/2))*arctanh(2^(1/2)*a^(1/4)*(-a^(1/2)+(a+b)^(1/2))^(1/2)*x/((a+b)^(1/2
)+a^(1/2)*x^2))*2^(1/2)/a^(5/4)/(-a^(1/2)+(a+b)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx = \frac{x}{a} - \frac{i\left(\sqrt{a}-i\sqrt{b}\right)^2 \arctan\left(\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{a-i\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{i\left(\sqrt{a}+i\sqrt{b}\right)^2 \arctan\left(\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{a+i\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
Integrate[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

output
```
x/a - ((I/2)*(Sqrt[a] - I*Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]
*Sqrt[b]]])/(a*Sqrt[a - I*Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((I/2)*(Sqrt[a] + I*
Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b]]])/(a*Sqrt[a + I*
Sqrt[a]*Sqrt[b]]*Sqrt[b])
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.53, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1442, 1483, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{ax^4+2ax^2+a+b}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x}{a} - \frac{\int \frac{2ax^2+a+b}{ax^4+2ax^2+a+b}dx}{a}$$

$$\Big\downarrow \text{1483}$$

$$\frac{x}{a}-\frac{\frac{\int \frac{\sqrt{2}(a+b)\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt[4]{a}(a-2\sqrt{a+b}\sqrt{a}+b)x}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}(a+b)+\sqrt[4]{a}(a-2\sqrt{a+b}\sqrt{a}+b)x}{\sqrt[4]{a}\left(x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x}{a}-\frac{\frac{\int \frac{\sqrt{2}(a+b)\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt[4]{a}(a-2\sqrt{a+b}\sqrt{a}+b)x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}(a+b)+\sqrt[4]{a}(a-2\sqrt{a+b}\sqrt{a}+b)x}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}}{a}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{x}{a}-\frac{\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}(2\sqrt{a}\sqrt{a+b}+a+b)\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}-\frac{1}{2}\sqrt[4]{a}(-2\sqrt{a}\sqrt{a+b}+a+b)\int -\frac{\sqrt{2}\left(\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x\right)}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\sqrt{\sqrt{a+b}-\cdots}}{\cdots}}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x}{a}-\frac{\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}(2\sqrt{a}\sqrt{a+b}+a+b)\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}+\frac{1}{2}\sqrt[4]{a}(-2\sqrt{a}\sqrt{a+b}+a+b)\int \frac{\sqrt{2}\left(\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x\right)}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\sqrt{\sqrt{a+b}-\cdots}}{\cdots}}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x}{a}-\frac{\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}(2\sqrt{a}\sqrt{a+b}+a+b)\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}+\frac{(-2\sqrt{a}\sqrt{a+b}+a+b)\int \frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}(2\sqrt{a}\sqrt{a+b}+a+b\cdots}{\cdots}}{a}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{x}{a}-\frac{\frac{(-2\sqrt{a}\sqrt{a+b}+a+b)\int\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}(2\sqrt{a}\sqrt{a+b}+a+b)\int\frac{1}{-\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)^2-2\left(\frac{\sqrt{a+b}}{\sqrt{a}}+1\right)}d\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a}}}{\sqrt[4]{}}\right.}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$\Big\downarrow$ 217

$$\frac{x}{a}-\frac{\frac{\frac{(-2\sqrt{a}\sqrt{a+b}+a+b)\int\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}+\frac{\sqrt[4]{a}\sqrt{\sqrt{a+b}-\sqrt{a}}(2\sqrt{a}\sqrt{a+b}+a+b)\arctan\left(\frac{\sqrt[4]{a}\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{\sqrt{\sqrt{a+b}+\sqrt{a}}}}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\frac{(-2\sqrt{a}\sqrt{a+b}+a+b)}{}}{}$$

$$a$$

$\Big\downarrow$ 1103

$$\frac{x}{a}-\frac{\frac{\sqrt[4]{a}\sqrt{\sqrt{a+b}-\sqrt{a}}(2\sqrt{a}\sqrt{a+b}+a+b)\arctan\left(\frac{\sqrt[4]{a}\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{\sqrt{\sqrt{a+b}+\sqrt{a}}}-\frac{1}{2}\sqrt[4]{a}(-2\sqrt{a}\sqrt{a+b}+a+b)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

input
```
Int[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

output
```
x/a - (((a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*(a + b + 2*Sqrt[a]*Sqrt[a +
b])*ArcTan[(a^(1/4)*(-((Sqrt[2]*Sqrt[-Sqrt[a] + Sqrt[a + b]])/a^(1/4)) + 2
*x))/(Sqrt[2]*Sqrt[Sqrt[a] + Sqrt[a + b]])])/Sqrt[Sqrt[a] + Sqrt[a + b]] -
 (a^(1/4)*(a + b - 2*Sqrt[a]*Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4
)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/2)/(2*Sqrt[2]*Sqrt[a]*Sqr
t[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]]) + ((a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a
+ b]]*(a + b + 2*Sqrt[a]*Sqrt[a + b])*ArcTan[(a^(1/4)*((Sqrt[2]*Sqrt[-Sqrt
[a] + Sqrt[a + b]])/a^(1/4) + 2*x))/(Sqrt[2]*Sqrt[Sqrt[a] + Sqrt[a + b]])]
)/Sqrt[Sqrt[a] + Sqrt[a + b]] + (a^(1/4)*(a + b - 2*Sqrt[a]*Sqrt[a + b])*L
og[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*
x^2])/2)/(2*Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]]))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.18

method	result
risch	$\frac{x}{a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{4}+2a\_Z^{2}+a+b\right)}\frac{\left(-2\_R^{2}a-a-b\right)\ln\left(x-\_R\right)}{\_R^{3}+\_R}}{4a^{2}}$
default	$\frac{x}{a}+\frac{\frac{\left(-\sqrt{a+b}\,a^{\frac{3}{2}}\sqrt{2\sqrt{a^{2}+ab}-2a}-\sqrt{a+b}\,\sqrt{a}\,\sqrt{2\sqrt{a^{2}+ab}-2a}\,\sqrt{a^{2}+ab}+2\sqrt{2\sqrt{a^{2}+ab}-2a}\,\sqrt{a^{2}+ab}\,a+2\sqrt{2\sqrt{a^{2}+ab}-2a}\,a^{2}\right)\ln\left(\sqrt{a}\,x^{2}+x\sqrt{2\sqrt{a}}\right.}{2\sqrt{a}}}{}$

input
```
int(x^4/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

output
```
x/a+1/4/a^2*sum((-2*_R^2*a-a-b)/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4*a+2*_Z^2
*a+a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(222) = 444.

Time = 0.09 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.92

$$\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx = \frac{a\sqrt{\frac{a^2b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}+a-3b}{a^2b}}\log\left(-(3a^2+2ab-b^2)x+\left(a^4b\sqrt{-\frac{9a^2-6ab+b^2}{a^5b}}+3a^2b-ab^2\right)\sqrt{\frac{a^2b\sqrt{-\frac{9a^2-6ab}{a^5b}}}{a^2b}}\right.}{}$$

input
```
integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

output
```
1/4*(a*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a - 3*b)/(a^2*b)
)*log(-(3*a^2 + 2*a*b - b^2)*x + (a^4*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b
)) + 3*a^2*b - a*b^2)*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a
 - 3*b)/(a^2*b))) - a*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a
 - 3*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x - (a^4*b*sqrt(-(9*a^2 - 6*a*
b + b^2)/(a^5*b)) + 3*a^2*b - a*b^2)*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^
2)/(a^5*b)) + a - 3*b)/(a^2*b))) - a*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*a*b + b
^2)/(a^5*b)) - a + 3*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x + (a^4*b*sqr
t(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - 3*a^2*b + a*b^2)*sqrt(-(a^2*b*sqrt(-(9
*a^2 - 6*a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))) + a*sqrt(-(a^2*b*sqrt(-(
9*a^2 - 6*a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^
2)*x - (a^4*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - 3*a^2*b + a*b^2)*sqrt
(-(a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))) + 4*x)/
a
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.33

$$\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^5b^2+t^2\left(-32a^4b+96a^3b^2\right)+a^3+3a^2b+3ab^2+b^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^4b+4ta^3}{3a^2+}\right.\right.\right.$$
$$+\frac{x}{a}$$

input
```
integrate(x**4/(a*x**4+2*a*x**2+a+b),x)
```

output
```
RootSum(256*_t**4*a**5*b**2 + _t**2*(-32*a**4*b + 96*a**3*b**2) + a**3 + 3
*a**2*b + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b + 4*_t*
a**3 - 24*_t*a**2*b + 4*_t*a*b**2)/(3*a**2 + 2*a*b - b**2)))) + x/a
```

Maxima [F]

$$\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx = \int \frac{x^4}{ax^4+2\,ax^2+a+b}\,dx$$

input
```
integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

output
```
x/a - integrate((2*a*x^2 + a + b)/(a*x^4 + 2*a*x^2 + a + b), x)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(222) = 444.

Time = 0.19 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.67

$$\int \frac{x^4}{a+b+2ax^2+ax^4}\,dx$$

$$= \frac{\left(3\sqrt{a^2+\sqrt{-ab}a}\sqrt{-ab}a^4+\sqrt{a^2+\sqrt{-ab}a}\sqrt{-ab}a^3b-4\sqrt{a^2+\sqrt{-ab}a}\sqrt{-ab}a^2b^2+2\left(3\sqrt{a^2+\sqrt{-ab}\right.\right.}{}$$

$$-\frac{\left(3\sqrt{a^2-\sqrt{-ab}a}\sqrt{-ab}a^4+\sqrt{a^2-\sqrt{-ab}a}\sqrt{-ab}a^3b-4\sqrt{a^2-\sqrt{-ab}a}\sqrt{-ab}a^2b^2+2\left(3\sqrt{a^2-\sqrt{-}\right.\right.}{}$$

$$+\frac{x}{a}$$

input

```
integrate(x^4/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

output

```
1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^4 + sqrt(a^2 + sqrt(-a*b)*a)*
sqrt(-a*b)*a^3*b - 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b^2 + 2*(3*sq
rt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a*b + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a
*b)*b^2)*a^2 - (3*sqrt(a^2 + sqrt(-a*b)*a)*a^3*b + 7*sqrt(a^2 + sqrt(-a*b)
*a)*a^2*b^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2
 + sqrt(a^4 - (a^2 + a*b)*a^2))/a^2))/(3*a^6*b + 7*a^5*b^2 + 4*a^4*b^3) -
1/2*(3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^4 + sqrt(a^2 - sqrt(-a*b)*a)*
sqrt(-a*b)*a^3*b - 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b^2 + 2*(3*sq
rt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a
*b)*b^2)*a^2 + (3*sqrt(a^2 - sqrt(-a*b)*a)*a^3*b + 7*sqrt(a^2 - sqrt(-a*b)
*a)*a^2*b^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^3)*abs(a))*arctan(x/sqrt((a^2
 - sqrt(a^4 - (a^2 + a*b)*a^2))/a^2))/(3*a^6*b + 7*a^5*b^2 + 4*a^4*b^3) +
x/a
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1147, normalized size of antiderivative = 3.58

$$\int \frac{x^4}{a+b+2ax^2+ax^4} \, dx = \text{Too large to display}$$

input
```
int(x^4/(a + b + 2*a*x^2 + a*x^4),x)
```

output
```
x/a + 2*atanh((24*x*(-a^5*b^3)^(1/2)*(1/(16*a*b) - 3/(16*a^2) + (3*(-a^5*b
^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((6*(-a^5*b^
3)^(1/2))/a + 4*a*b^2 + 6*a^2*b - 2*b^3 - (2*b^2*(-a^5*b^3)^(1/2))/a^3 + (
4*b*(-a^5*b^3)^(1/2))/a^2) - (8*x*(-a^5*b^3)^(1/2)*(1/(16*a*b) - 3/(16*a^2
) + (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2)
)/((4*(-a^5*b^3)^(1/2))/a + (6*(-a^5*b^3)^(1/2))/b - 2*a*b^2 + 4*a^2*b + 6
*a^3 - (2*b*(-a^5*b^3)^(1/2))/a^2) - (8*a*b^2*x*(1/(16*a*b) - 3/(16*a^2) +
 (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(
4*a*b + (4*(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 + (6*(-a^5*b^3)^(1/2))/(a
*b) - (2*b*(-a^5*b^3)^(1/2))/a^3) + (24*a^2*b*x*(1/(16*a*b) - 3/(16*a^2) +
 (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) - (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/(
4*a*b + (4*(-a^5*b^3)^(1/2))/a^2 + 6*a^2 - 2*b^2 + (6*(-a^5*b^3)^(1/2))/(a
*b) - (2*b*(-a^5*b^3)^(1/2))/a^3))*((3*a*(-a^5*b^3)^(1/2) - b*(-a^5*b^3)^(
1/2) + a^4*b - 3*a^3*b^2)/(16*a^5*b^2))^(1/2) + 2*atanh((24*x*(-a^5*b^3)^(
1/2)*(1/(16*a*b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4*b^2) + (-a^5*
b^3)^(1/2)/(16*a^5*b))^(1/2))/((6*(-a^5*b^3)^(1/2))/a - 4*a*b^2 - 6*a^2*b
+ 2*b^3 - (2*b^2*(-a^5*b^3)^(1/2))/a^3 + (4*b*(-a^5*b^3)^(1/2))/a^2) - (8*
x*(-a^5*b^3)^(1/2)*(1/(16*a*b) - 3/(16*a^2) - (3*(-a^5*b^3)^(1/2))/(16*a^4
*b^2) + (-a^5*b^3)^(1/2)/(16*a^5*b))^(1/2))/((4*(-a^5*b^3)^(1/2))/a + (6*(
-a^5*b^3)^(1/2))/b + 2*a*b^2 - 4*a^2*b - 6*a^3 - (2*b*(-a^5*b^3)^(1/2))...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 640, normalized size of antiderivative = 2.00

$$\int \frac{x^4}{a+b+2ax^2+ax^4} \, dx = \text{Too large to display}$$

input
```
int(x^4/(a*x^4+2*a*x^2+a+b),x)
```

output
```
(2*sqrt(a + b)*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)*atan((sqrt(sqrt(a)*sq
rt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(
2)))*a - 2*sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)*atan((sqrt(sqrt(a
)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*s
qrt(2)))*a + 2*sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)*atan((sqrt(sq
rt(a)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) +
a)*sqrt(2)))*b - 2*sqrt(a + b)*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)*atan(
(sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) + 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a
 + b) + a)*sqrt(2)))*a + 2*sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)*a
tan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) + 2*sqrt(a)*x)/(sqrt(sqrt(a)*sq
rt(a + b) + a)*sqrt(2)))*a - 2*sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(
2)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) + 2*sqrt(a)*x)/(sqrt(sqrt(a
)*sqrt(a + b) + a)*sqrt(2)))*b - sqrt(a + b)*sqrt(sqrt(a)*sqrt(a + b) - a)
*sqrt(2)*log( - sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*x + sqrt(a + b) + sq
rt(a)*x**2)*a + sqrt(a + b)*sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*log(sqrt
(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*x + sqrt(a + b) + sqrt(a)*x**2)*a - sqrt
(a)*sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*log( - sqrt(sqrt(a)*sqrt(a + b)
- a)*sqrt(2)*x + sqrt(a + b) + sqrt(a)*x**2)*a + sqrt(a)*sqrt(sqrt(a)*sqrt
(a + b) - a)*sqrt(2)*log( - sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*x + sqrt
(a + b) + sqrt(a)*x**2)*b + sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(...
```

3.841 $\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 250

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx = -\frac{\arctan\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\arctan\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt{a+b}+\sqrt{a}x^2}\right)}{2\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

output
```
-1/4*arctan(((-a^(1/2)+(a+b)^(1/2))^(1/2)-2^(1/2)*a^(1/4)*x)/(a^(1/2)+(a+b
)^(1/2))^(1/2))*2^(1/2)/a^(3/4)/(a^(1/2)+(a+b)^(1/2))^(1/2)+1/4*arctan(((-
a^(1/2)+(a+b)^(1/2))^(1/2)+2^(1/2)*a^(1/4)*x)/(a^(1/2)+(a+b)^(1/2))^(1/2))
*2^(1/2)/a^(3/4)/(a^(1/2)+(a+b)^(1/2))^(1/2)-1/4*arctanh(2^(1/2)*a^(1/4)*(
-a^(1/2)+(a+b)^(1/2))^(1/2)*x/((a+b)^(1/2)+a^(1/2)*x^2))*2^(1/2)/a^(3/4)/(
-a^(1/2)+(a+b)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx = \frac{\frac{\left(i\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a-i\sqrt{a}\sqrt{b}}}+\frac{\left(-i\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+i\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\sqrt{b}}$$

input
```
Integrate[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

output
```
(((I*Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*Sqrt[b]]])/S
qrt[a - I*Sqrt[a]*Sqrt[b]] + (((-I)*Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/
Sqrt[a + I*Sqrt[a]*Sqrt[b]]])/Sqrt[a + I*Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*Sqrt
[b])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.51, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1449, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{ax^4+2ax^2+a+b}dx$$

$$\Big\downarrow \text{ 1449}$$

$$\frac{\int \frac{x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\int \frac{x}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt[4]{a}}+\frac{1}{2}\int -\frac{\sqrt{2}\left(\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x\right)}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x+\sqrt{\sqrt{a+b}-\sqrt{a}}\right)}{\sqrt[4]{a}\left(x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx-\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt[4]{a}}}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt[4]{a}}-\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x\right)}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x+\sqrt{\sqrt{a+b}-\sqrt{a}}\right)}{\sqrt[4]{a}\left(x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx-\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt[4]{a}}}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt[4]{a}}-\frac{\int \frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt[4]{a}}}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}x+\sqrt{\sqrt{a+b}-\sqrt{a}}}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt[4]{a}}}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$$\Big\downarrow \; 1083$$

$$\frac{-\frac{\int \frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{-\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)^2-2\left(\frac{\sqrt{a+b}}{\sqrt{a}}+1\right)}d\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}}}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}x+\sqrt{\sqrt{a+b}-\sqrt{a}}}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{-\left(2x+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)^2-2\left(\frac{\sqrt{a+b}}{\sqrt{a}}+1\right)}d\left(2x+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}}}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$\downarrow$ 217

$$\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\arctan\left(\frac{\sqrt[4]{a}\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{\sqrt{\sqrt{a+b}+\sqrt{a}}}-\frac{\int \frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt[4]{a}}}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}x+\sqrt{\sqrt{a+b}-\sqrt{a}}}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\arctan\left(\frac{\sqrt[4]{a}\left(\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}+2x\right)}{\sqrt{2}\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{\sqrt{\sqrt{a+b}+\sqrt{a}}}}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$\downarrow$ 1103

$$\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\arctan\left(\frac{\sqrt[4]{a}\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{1}{2}\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\frac{1}{2}\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)-\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\arctan\left(\frac{\sqrt[4]{a}\left(\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}+2x\right)}{\sqrt{2}\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{\sqrt{\sqrt{a+b}+\sqrt{a}}}}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

input
```
Int[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

output
```
((Sqrt[-Sqrt[a] + Sqrt[a + b]]*ArcTan[(a^(1/4)*(-((Sqrt[2]*Sqrt[-Sqrt[a] +
 Sqrt[a + b]])/a^(1/4)) + 2*x))/(Sqrt[2]*Sqrt[Sqrt[a] + Sqrt[a + b]])])/Sq
rt[Sqrt[a] + Sqrt[a + b]] + Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a
] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/2)/(2*Sqrt[2]*a^(3/4)*Sqrt[-Sqrt[a] + Sq
rt[a + b]]) - (-((Sqrt[-Sqrt[a] + Sqrt[a + b]]*ArcTan[(a^(1/4)*((Sqrt[2]*S
qrt[-Sqrt[a] + Sqrt[a + b]])/a^(1/4) + 2*x))/(Sqrt[2]*Sqrt[Sqrt[a] + Sqrt[
a + b]])])/Sqrt[Sqrt[a] + Sqrt[a + b]]) + Log[Sqrt[a + b] + Sqrt[2]*a^(1/4
)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/2)/(2*Sqrt[2]*a^(3/4)*Sqrt
[-Sqrt[a] + Sqrt[a + b]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1449
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Simp[1/(2*c*r)   Int[x^(m - 1)/(q + r*x + x^2), x],
 x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m,
3] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.16

method	result
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{4}+2a\_Z^{2}+a+b\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{\_R^{3}+\_R}}{4a}$
default	$-\frac{\sqrt{2\sqrt{a^{2}+ab}-2a}\left(\sqrt{a^{2}+ab}+a\right)\left(\frac{\ln\left(\sqrt{a}\,x^{2}+x\sqrt{2\sqrt{a(a+b)}-2a}+\sqrt{a+b}\right)}{2\sqrt{a}}-\frac{\sqrt{2\sqrt{a(a+b)}-2a}\,\arctan\left(\frac{2\sqrt{a}\,x+\sqrt{2\sqrt{a(a+b)}-2a}}{\sqrt{4\sqrt{a}\,\sqrt{a+b}-2\sqrt{a(a+b)}+2a}}\right)}{\sqrt{a}\,\sqrt{4\sqrt{a}\,\sqrt{a+b}-2\sqrt{a(a+b)}+2a}}\right)}{4ab}+$

input
```
int(x^2/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*sum(_R^2/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4*a+2*_Z^2*a+a+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx = \frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\log\left(a^2b\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right) - \frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\log\left(-a^2b\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right) - \frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\log\left(a^2b\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right) + \frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\log\left(-a^2b\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)$$

input
```
integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

output
```
1/4*sqrt((a*b*sqrt(-1/(a^3*b)) + 1)/(a*b))*log(a^2*b*sqrt((a*b*sqrt(-1/(a^
3*b)) + 1)/(a*b))*sqrt(-1/(a^3*b)) + x) - 1/4*sqrt((a*b*sqrt(-1/(a^3*b)) +
 1)/(a*b))*log(-a^2*b*sqrt((a*b*sqrt(-1/(a^3*b)) + 1)/(a*b))*sqrt(-1/(a^3*
b)) + x) - 1/4*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a*b))*log(a^2*b*sqrt(-(a*
b*sqrt(-1/(a^3*b)) - 1)/(a*b))*sqrt(-1/(a^3*b)) + x) + 1/4*sqrt(-(a*b*sqrt
(-1/(a^3*b)) - 1)/(a*b))*log(-a^2*b*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a*b)
)*sqrt(-1/(a^3*b)) + x)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.18

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx = \operatorname{RootSum}\left(256t^4a^3b^2-32t^2a^2b+a+b,\ \left(t\mapsto t\log\left(64t^3a^2b-4ta+x\right)\right)\right)$$

input
```
integrate(x**2/(a*x**4+2*a*x**2+a+b),x)
```

output
```
RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b + a + b, Lambda(_t, _t*log(6
4*_t**3*a**2*b - 4*_t*a + x)))
```

Maxima [F]

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx = \int \frac{x^2}{ax^4+2\,ax^2+a+b}\,dx$$

input
```
integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

output
```
integrate(x^2/(a*x^4 + 2*a*x^2 + a + b), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx = -\frac{\left(3\sqrt{a^2+\sqrt{-ab}a}\sqrt{-ab}a+4\sqrt{a^2+\sqrt{-ab}a}\sqrt{-ab}b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{2a+\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{2(3a^4b+4a^3b^2)} + \frac{\left(3\sqrt{a^2-\sqrt{-ab}a}\sqrt{-ab}a+4\sqrt{a^2-\sqrt{-ab}a}\sqrt{-ab}b\right)|a|\arctan\left(\frac{2\sqrt{\frac{1}{2}}x}{\sqrt{\frac{2a-\sqrt{-4(a+b)a+4a^2}}{a}}}\right)}{2(3a^4b+4a^3b^2)}$$

input
```
integrate(x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

output
```
-1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a + 4*sqrt(a^2 + sqrt(-a*b)*a)
*sqrt(-a*b)*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a + b)*a +
 4*a^2))/a))/(3*a^4*b + 4*a^3*b^2) + 1/2*(3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(
-a*b)*a + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*b)*abs(a)*arctan(2*sqrt(1/
2)*x/sqrt((2*a - sqrt(-4*(a + b)*a + 4*a^2))/a))/(3*a^4*b + 4*a^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx = -2\,\mathrm{atanh}\left(\frac{2\left(x\,(4\,a^2\,b-4\,a^3)+\frac{4\,a\,x\left(\sqrt{-a^3\,b^3}+a^2\,b\right)}{b}\right)\sqrt{\frac{\sqrt{-a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}}}{2\,a^2+2\,b\,a}\right)\sqrt{\frac{\sqrt{-a^3\,b^3}+a^2\,b}{16\,a^3\,b^2}} - 2\,\mathrm{atanh}\left(\frac{2\left(x\,(4\,a^2\,b-4\,a^3)-\frac{4\,a\,x\left(\sqrt{-a^3\,b^3}-a^2\,b\right)}{b}\right)\sqrt{-\frac{\sqrt{-a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}}{2\,a^2+2\,b\,a}\right)\sqrt{-\frac{\sqrt{-a^3\,b^3}-a^2\,b}{16\,a^3\,b^2}}$$

input
```
int(x^2/(a + b + 2*a*x^2 + a*x^4),x)
```

output
```
- 2*atanh((2*(x*(4*a^2*b - 4*a^3) + (4*a*x*((-a^3*b^3)^(1/2) + a^2*b))/b)*
(((-a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2))/(2*a*b + 2*a^2))*(((-a^3*
b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2) - 2*atanh((2*(x*(4*a^2*b - 4*a^3)
- (4*a*x*((-a^3*b^3)^(1/2) - a^2*b))/b)*(-((-a^3*b^3)^(1/2) - a^2*b)/(16*a
^3*b^2))^(1/2))/(2*a*b + 2*a^2))*(-((-a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2)
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.63

$$\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx = \frac{\sqrt{2}\left(-2\sqrt{a+b}\,\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\;atan\left(\frac{\sqrt{\sqrt{a}\,\sqrt{a+b}-a}\,\sqrt{2}-2\sqrt{a}\,x}{\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\,\sqrt{2}}\right)+2\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{a+b}+a}\;atan\left(\frac{\sqrt{\sqrt{a}\,\sqrt{a+b}-}}{\sqrt{\sqrt{a}\,\sqrt{a}}}\right.\right.}{}$$

input
```
int(x^2/(a*x^4+2*a*x^2+a+b),x)
```

output
```
(sqrt(2)*( - 2*sqrt(a + b)*sqrt(sqrt(a)*sqrt(a + b) + a)*atan((sqrt(sqrt(a
)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*s
qrt(2))) + 2*sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) + a)*atan((sqrt(sqrt(a)*sqrt
(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)
)) + 2*sqrt(a + b)*sqrt(sqrt(a)*sqrt(a + b) + a)*atan((sqrt(sqrt(a)*sqrt(a
 + b) - a)*sqrt(2) + 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))
 - 2*sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) + a)*atan((sqrt(sqrt(a)*sqrt(a + b)
- a)*sqrt(2) + 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2))) + sqr
t(a + b)*sqrt(sqrt(a)*sqrt(a + b) - a)*log( - sqrt(sqrt(a)*sqrt(a + b) - a
)*sqrt(2)*x + sqrt(a + b) + sqrt(a)*x**2) - sqrt(a + b)*sqrt(sqrt(a)*sqrt(
a + b) - a)*log(sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*x + sqrt(a + b) + sq
rt(a)*x**2) + sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) - a)*log( - sqrt(sqrt(a)*sq
rt(a + b) - a)*sqrt(2)*x + sqrt(a + b) + sqrt(a)*x**2) - sqrt(a)*sqrt(sqrt
(a)*sqrt(a + b) - a)*log(sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*x + sqrt(a
+ b) + sqrt(a)*x**2)))/(8*a*b)
```

3.842 $\int \frac{1}{a+b+2ax^2+ax^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 271

$$\int \frac{1}{a+b+2ax^2+ax^4} \, dx = -\frac{\arctan\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\arctan\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt{a+b}+\sqrt{a}x^2}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

output
```
-1/4*arctan(((-a^(1/2)+(a+b)^(1/2))^(1/2)-2^(1/2)*a^(1/4)*x)/(a^(1/2)+(a+b
)^(1/2))^(1/2))*2^(1/2)/a^(1/4)/(a+b)^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)+1/
4*arctan(((-a^(1/2)+(a+b)^(1/2))^(1/2)+2^(1/2)*a^(1/4)*x)/(a^(1/2)+(a+b)^(
1/2))^(1/2))*2^(1/2)/a^(1/4)/(a+b)^(1/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)+1/4*a
rctanh(2^(1/2)*a^(1/4)*(-a^(1/2)+(a+b)^(1/2))^(1/2)*x/((a+b)^(1/2)+a^(1/2)
*x^2))*2^(1/2)/a^(1/4)/(a+b)^(1/2)/(-a^(1/2)+(a+b)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.44

$$\int \frac{1}{a+b+2ax^2+ax^4}\,dx = -\frac{i\arctan\left(\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a-i\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{i\arctan\left(\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a+i\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
Integrate[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

output
```
((-1/2*I)*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*Sqrt[b]]])/(Sqrt[a - I*Sqr
t[a]*Sqrt[b]]*Sqrt[b]) + ((I/2)*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt
[b]]])/(Sqrt[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.52, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1407, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^4+2ax^2+a+b}\,dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt[4]{a}x}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\int \frac{\sqrt[4]{a}x+\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}\left(x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\int \frac{\sqrt[4]{a}x+\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}-\frac{1}{2}\sqrt[4]{a}\int -\frac{\sqrt{2}\left(\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x\right)}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}+\frac{1}{2}\sqrt[4]{a}\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x+\sqrt{\sqrt{a+b}-\sqrt{a}}\right)}{\sqrt[4]{a}\left(x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}+\frac{1}{2}\sqrt[4]{a}\int \frac{\sqrt{2}\left(\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x\right)}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}+\frac{1}{2}\sqrt[4]{a}\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x+\sqrt{\sqrt{a+b}-\sqrt{a}}\right)}{\sqrt[4]{a}\left(x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}+\frac{\int \frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{a}x+\sqrt{\sqrt{a+b}-\sqrt{a}}}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$$\Big\downarrow \ 1083$$

$$\frac{\frac{\int \frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{-\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)^2-2\left(\frac{\sqrt{a+b}}{\sqrt{a}}+1\right)}d\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}x+\sqrt{\sqrt{a+b}-\sqrt{a}}}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}\int \frac{1}{-\left(2x+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)^2-2\left(\frac{\sqrt{a+b}}{\sqrt{a}}+1\right)}d\left(2x+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{\int \frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}+\frac{\sqrt[4]{a}\sqrt{\sqrt{a+b}-\sqrt{a}}\arctan\left(\frac{\sqrt[4]{a}\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{\sqrt{\sqrt{a+b}+\sqrt{a}}}}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}x+\sqrt{\sqrt{a+b}-\sqrt{a}}}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}+\frac{\sqrt[4]{a}\sqrt{\sqrt{a+b}-\sqrt{a}}\arctan\left(\frac{\sqrt[4]{a}\left(\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}+2x\right)}{\sqrt{2}\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{\sqrt{\sqrt{a+b}+\sqrt{a}}}}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{\sqrt{a+b}-\sqrt{a}}\arctan\left(\frac{\sqrt[4]{a}\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{\sqrt{\sqrt{a+b}+\sqrt{a}}}-\frac{1}{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{\sqrt{a+b}-\sqrt{a}}\arctan\left(\frac{\sqrt[4]{a}\left(\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}+2x\right)}{\sqrt{2}\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{1}{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}$$

input
```
Int[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

output
```
((a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*ArcTan[(a^(1/4)*(-((Sqrt[2]*Sqrt[-S
qrt[a] + Sqrt[a + b]])/a^(1/4)) + 2*x))/(Sqrt[2]*Sqrt[Sqrt[a] + Sqrt[a + b
]])])/Sqrt[Sqrt[a] + Sqrt[a + b]] - (a^(1/4)*Log[Sqrt[a + b] - Sqrt[2]*a^(
1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/2)/(2*Sqrt[2]*Sqrt[a]*
Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]]) + ((a^(1/4)*Sqrt[-Sqrt[a] + Sqrt
[a + b]]*ArcTan[(a^(1/4)*((Sqrt[2]*Sqrt[-Sqrt[a] + Sqrt[a + b]])/a^(1/4) +
 2*x))/(Sqrt[2]*Sqrt[Sqrt[a] + Sqrt[a + b]])])/Sqrt[Sqrt[a] + Sqrt[a + b]]
 + (a^(1/4)*Log[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]
*x + Sqrt[a]*x^2])/2)/(2*Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[
a + b]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.14

method	result
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{4}+2a\_Z^{2}+a+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}+\_R}}{4a}$
default	$\frac{\frac{\left(\sqrt{2\sqrt{a^{2}+ab}-2a}\,\sqrt{a^{2}+ab}+\sqrt{2\sqrt{a^{2}+ab}-2a}\,a\right)\ln\left(\sqrt{a}\,x^{2}+x\sqrt{2\sqrt{a(a+b)}-2a}+\sqrt{a+b}\right)}{2\sqrt{a}}+\frac{2\left(2\sqrt{a}\,b-\frac{\left(\sqrt{2\sqrt{a^{2}+ab}-2a}\,\sqrt{a^{2}+ab}+\sqrt{2\sqrt{a^{2}+ab}-2a}\right.}{2\sqrt{a}}\right.}{\sqrt{4\sqrt{a}\,\sqrt{a+}}}}{4\sqrt{a+b}\,\sqrt{a}\,b}$

input
```
int(1/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*sum(1/(_R^3+_R)*ln(x-_R),_R=RootOf(_Z^4*a+2*_Z^2*a+a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(185) = 370$.

Time = 0.08 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.09

$$\int \frac{1}{a+b+2ax^2+ax^4}\,dx$$
$$= \frac{1}{4}\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2\,a^2b^2+ab^3}}+1}{ab+b^2}}\log\left(\left(\left(a^2b+ab^2\right)\sqrt{-\frac{1}{a^3b+2\,a^2b^2+ab^3}}+b\right)\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2}}}{ab+b^2}}\right.$$
$$\left.+x\right)$$
$$-\frac{1}{4}\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2\,a^2b^2+ab^3}}+1}{ab+b^2}}\log\left(-\left(\left(a^2b+ab^2\right)\sqrt{-\frac{1}{a^3b+2\,a^2b^2+ab^3}}+b\right)\sqrt{\frac{(ab+b^2)\sqrt{-}}{ab}}\right.$$
$$\left.+x\right)$$
$$-\frac{1}{4}\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2\,a^2b^2+ab^3}}-1}{ab+b^2}}\log\left(\left(\left(a^2b+ab^2\right)\sqrt{-\frac{1}{a^3b+2\,a^2b^2+ab^3}}-b\right)\sqrt{-\frac{(ab+b^2)\sqrt{-}}{a}}\right.$$
$$\left.+x\right)$$
$$+\frac{1}{4}\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2\,a^2b^2+ab^3}}-1}{ab+b^2}}\log\left(-\left(\left(a^2b+ab^2\right)\sqrt{-\frac{1}{a^3b+2\,a^2b^2+ab^3}}-b\right)\sqrt{-\frac{(ab+b^2)\sqrt{}}{}}\right.$$
$$\left.+x\right)$$

input
```
integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

output
```
1/4*sqrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2
))*log(((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + b)*sqrt(((a
*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)) + x) - 1/
4*sqrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2))
*log(-((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + b)*sqrt(((a*
b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)) + x) - 1/4
*sqrt(-((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2))
*log(((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - b)*sqrt(-((a*
b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2)) + x) + 1/4
*sqrt(-((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2))
*log(-((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - b)*sqrt(-((a
*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2)) + x)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.23

$$\int \frac{1}{a+b+2ax^2+ax^4}\,dx$$
$$=\mathrm{RootSum}\left(t^4\cdot\left(256a^2b^2+256ab^3\right)-32t^2ab+1,\left(t\mapsto t\log\left(64t^3a^2b+64t^3ab^2-4ta+4tb+x\right)\right)\right)$$

input
```
integrate(1/(a*x**4+2*a*x**2+a+b),x)
```

output
```
RootSum(_t**4*(256*a**2*b**2 + 256*a*b**3) - 32*_t**2*a*b + 1, Lambda(_t,
_t*log(64*_t**3*a**2*b + 64*_t**3*a*b**2 - 4*_t*a + 4*_t*b + x)))
```

Maxima [F]

$$\int \frac{1}{a+b+2ax^2+ax^4}\,dx=\int \frac{1}{ax^4+2\,ax^2+a+b}\,dx$$

input
```
integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

output
```
integrate(1/(a*x^4 + 2*a*x^2 + a + b), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.13

$$\int \frac{1}{a+b+2ax^2+ax^4}\,dx$$

$$= \frac{\left(3\sqrt{a^2+\sqrt{-ab}a}a^2b+4\sqrt{a^2+\sqrt{-ab}a}ab^2+3\sqrt{a^2+\sqrt{-ab}a}\sqrt{-ab}a^2+4\sqrt{a^2+\sqrt{-ab}a}\sqrt{-ab}ab\right)|a|}{2\left(3a^5b+7a^4b^2+4a^3b^3\right)}$$

$$+ \frac{\left(3\sqrt{a^2-\sqrt{-ab}a}a^2b+4\sqrt{a^2-\sqrt{-ab}a}ab^2+3\sqrt{a^2-\sqrt{-ab}a}\sqrt{-ab}a^2+4\sqrt{a^2-\sqrt{-ab}a}\sqrt{-ab}ab\right)|}{2\left(3a^5b+7a^4b^2+4a^3b^3\right)}$$

input
```
integrate(1/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

output
```
1/2*(3*sqrt(a^2 + sqrt(-a*b)*a)*a^2*b + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b^2 +
 3*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^2 + 4*sqrt(a^2 + sqrt(-a*b)*a)*sq
rt(-a*b)*a*b)*abs(a)*arctan(2*sqrt(1/2)*x/sqrt((2*a + sqrt(-4*(a + b)*a +
4*a^2))/a))/(3*a^5*b + 7*a^4*b^2 + 4*a^3*b^3) + 1/2*(3*sqrt(a^2 - sqrt(-a*
b)*a)*a^2*b + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^2 + 3*sqrt(a^2 - sqrt(-a*b)*a
)*sqrt(-a*b)*a^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b)*abs(a)*arcta
n(2*sqrt(1/2)*x/sqrt((2*a - sqrt(-4*(a + b)*a + 4*a^2))/a))/(3*a^5*b + 7*a
^4*b^2 + 4*a^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 18.63 (sec) , antiderivative size = 986, normalized size of antiderivative = 3.64

$$\int \frac{1}{a+b+2ax^2+ax^4}\,dx = 2\,\mathrm{atanh}\left(\frac{8\,a^3\,x\,\sqrt{\frac{a\,b}{16\,(a^2\,b^2+a\,b^3)}-\frac{\sqrt{-a\,b^3}}{16\,(a^2\,b^2+a\,b^3)}}}{\frac{2\,a^4\,b^2}{a^2\,b^2+a\,b^3}-\frac{2\,a^3\,b\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}} - \frac{8\,a^5\,b^2\,x\,\sqrt{\frac{a\,b}{16\,(a^2\,b^2+a\,b^3)}-\frac{\sqrt{-a\,b^3}}{16\,(a^2\,b^2+a\,b^3)}}}{\frac{2\,a^5\,b^5}{a^2\,b^2+a\,b^3}+\frac{2\,a^6\,b^4}{a^2\,b^2+a\,b^3}-\frac{2\,a^4\,b^4\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}-\frac{2\,a^5\,b^3\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}} + \frac{8\,a^4\,b\,x\,\sqrt{\frac{a\,b}{16\,(a^2\,b^2+a\,b^3)}-\frac{\sqrt{-a\,b^3}}{16\,(a^2\,b^2+a\,b^3)}}\,\sqrt{-a\,b^3}}{\frac{2\,a^5\,b^5}{a^2\,b^2+a\,b^3}+\frac{2\,a^6\,b^4}{a^2\,b^2+a\,b^3}-\frac{2\,a^4\,b^4\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}-\frac{2\,a^5\,b^3\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}}\right)\sqrt{\frac{a\,b-\sqrt{-a\,b^3}}{16\,(a^2\,b^2+a\,b^3)}} - 2\,\mathrm{atanh}\left(\frac{8\,a^5\,b^2\,x\,\sqrt{\frac{\sqrt{-a\,b^3}}{16\,(a^2\,b^2+a\,b^3)}+\frac{a\,b}{16\,(a^2\,b^2+a\,b^3)}}}{\frac{2\,a^5\,b^5}{a^2\,b^2+a\,b^3}+\frac{2\,a^6\,b^4}{a^2\,b^2+a\,b^3}+\frac{2\,a^4\,b^4\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}+\frac{2\,a^5\,b^3\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}} - \frac{8\,a^3\,x\,\sqrt{\frac{\sqrt{-a\,b^3}}{16\,(a^2\,b^2+a\,b^3)}+\frac{a\,b}{16\,(a^2\,b^2+a\,b^3)}}}{\frac{2\,a^4\,b^2}{a^2\,b^2+a\,b^3}+\frac{2\,a^3\,b\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}} + \frac{8\,a^4\,b\,x\,\sqrt{\frac{\sqrt{-a\,b^3}}{16\,(a^2\,b^2+a\,b^3)}+\frac{a\,b}{16\,(a^2\,b^2+a\,b^3)}}\,\sqrt{-a\,b^3}}{\frac{2\,a^5\,b^5}{a^2\,b^2+a\,b^3}+\frac{2\,a^6\,b^4}{a^2\,b^2+a\,b^3}+\frac{2\,a^4\,b^4\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}+\frac{2\,a^5\,b^3\,\sqrt{-a\,b^3}}{a^2\,b^2+a\,b^3}}\right)\sqrt{\frac{a\,b+\sqrt{-a\,b^3}}{16\,(a^2\,b^2+a\,b^3)}}$$

input
```
int(1/(a + b + 2*a*x^2 + a*x^4),x)
```

output
```
2*atanh((8*a^3*x*((a*b)/(16*(a*b^3 + a^2*b^2)) - (-a*b^3)^(1/2)/(16*(a*b^3
 + a^2*b^2)))^(1/2))/((2*a^4*b^2)/(a*b^3 + a^2*b^2) - (2*a^3*b*(-a*b^3)^(1
/2))/(a*b^3 + a^2*b^2)) - (8*a^5*b^2*x*((a*b)/(16*(a*b^3 + a^2*b^2)) - (-a
*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2)))^(1/2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2)
+ (2*a^6*b^4)/(a*b^3 + a^2*b^2) - (2*a^4*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*
b^2) - (2*a^5*b^3*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2)) + (8*a^4*b*x*((a*b)/(
16*(a*b^3 + a^2*b^2)) - (-a*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2)))^(1/2)*(-a*b
^3)^(1/2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2*a^6*b^4)/(a*b^3 + a^2*b^2)
- (2*a^4*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2) - (2*a^5*b^3*(-a*b^3)^(1/2)
)/(a*b^3 + a^2*b^2)))*((a*b - (-a*b^3)^(1/2))/(16*(a*b^3 + a^2*b^2)))^(1/2
) - 2*atanh((8*a^5*b^2*x*((-a*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2)) + (a*b)/(1
6*(a*b^3 + a^2*b^2)))^(1/2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2*a^6*b^4)/
(a*b^3 + a^2*b^2) + (2*a^4*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2) + (2*a^5*
b^3*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2)) - (8*a^3*x*((-a*b^3)^(1/2)/(16*(a*b
^3 + a^2*b^2)) + (a*b)/(16*(a*b^3 + a^2*b^2)))^(1/2))/((2*a^4*b^2)/(a*b^3
+ a^2*b^2) + (2*a^3*b*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2)) + (8*a^4*b*x*((-a
*b^3)^(1/2)/(16*(a*b^3 + a^2*b^2)) + (a*b)/(16*(a*b^3 + a^2*b^2)))^(1/2)*(
-a*b^3)^(1/2))/((2*a^5*b^5)/(a*b^3 + a^2*b^2) + (2*a^6*b^4)/(a*b^3 + a^2*b
^2) + (2*a^4*b^4*(-a*b^3)^(1/2))/(a*b^3 + a^2*b^2) + (2*a^5*b^3*(-a*b^3)^(
1/2))/(a*b^3 + a^2*b^2)))*((a*b + (-a*b^3)^(1/2))/(16*(a*b^3 + a^2*b^2)...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.28

$$\int \frac{1}{a + b + 2ax^2 + ax^4} \, dx = \text{Too large to display}$$

input
```
int(1/(a*x^4+2*a*x^2+a+b),x)
```

output
```
(sqrt(2)*(2*sqrt(a + b)*sqrt(sqrt(a)*sqrt(a + b) + a)*atan((sqrt(sqrt(a)*s
qrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt
(2)))*a - 2*sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) + a)*atan((sqrt(sqrt(a)*sqrt(
a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2))
)*a - 2*sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) + a)*atan((sqrt(sqrt(a)*sqrt(a +
b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))*b
- 2*sqrt(a + b)*sqrt(sqrt(a)*sqrt(a + b) + a)*atan((sqrt(sqrt(a)*sqrt(a +
b) - a)*sqrt(2) + 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))*a
+ 2*sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) + a)*atan((sqrt(sqrt(a)*sqrt(a + b) -
 a)*sqrt(2) + 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))*a + 2*
sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) + a)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*
sqrt(2) + 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))*b - sqrt(a
 + b)*sqrt(sqrt(a)*sqrt(a + b) - a)*log( - sqrt(sqrt(a)*sqrt(a + b) - a)*s
qrt(2)*x + sqrt(a + b) + sqrt(a)*x**2)*a + sqrt(a + b)*sqrt(sqrt(a)*sqrt(a
 + b) - a)*log(sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*x + sqrt(a + b) + sqr
t(a)*x**2)*a - sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) - a)*log( - sqrt(sqrt(a)*s
qrt(a + b) - a)*sqrt(2)*x + sqrt(a + b) + sqrt(a)*x**2)*a - sqrt(a)*sqrt(s
qrt(a)*sqrt(a + b) - a)*log( - sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*x + s
qrt(a + b) + sqrt(a)*x**2)*b + sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) - a)*log(s
qrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*x + sqrt(a + b) + sqrt(a)*x**2)*a ...
```

3.843 $\int \frac{1}{x^2(a+b+2ax^2+ax^4)} dx$

Optimal result

Integrand size = 20, antiderivative size = 328

$$\begin{aligned}\int \frac{1}{x^2(a+b+2ax^2+ax^4)} dx = & -\frac{1}{(a+b)x} \\ & + \frac{\sqrt[4]{a}(2\sqrt{a}+\sqrt{a+b}) \arctan\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a}+\sqrt{a+b}}} \\ & - \frac{\sqrt[4]{a}(2\sqrt{a}+\sqrt{a+b}) \arctan\left(\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a}+\sqrt{a+b}}} \\ & - \frac{\sqrt[4]{a}(2\sqrt{a}-\sqrt{a+b}) \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}x}{\sqrt{a+b}+\sqrt{a}x^2}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{-\sqrt{a}+\sqrt{a+b}}}\end{aligned}$$

output
```
-1/(a+b)/x+1/4*a^(1/4)*(2*a^(1/2)+(a+b)^(1/2))*arctan(((-a^(1/2)+(a+b)^(1/
2))^(1/2)-2^(1/2)*a^(1/4)*x)/(a^(1/2)+(a+b)^(1/2))^(1/2))*2^(1/2)/(a+b)^(3
/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)-1/4*a^(1/4)*(2*a^(1/2)+(a+b)^(1/2))*arctan
(((-a^(1/2)+(a+b)^(1/2))^(1/2)+2^(1/2)*a^(1/4)*x)/(a^(1/2)+(a+b)^(1/2))^(1
/2))*2^(1/2)/(a+b)^(3/2)/(a^(1/2)+(a+b)^(1/2))^(1/2)-1/4*a^(1/4)*(2*a^(1/2
)-(a+b)^(1/2))*arctanh(2^(1/2)*a^(1/4)*(-a^(1/2)+(a+b)^(1/2))^(1/2)*x/((a+
b)^(1/2)+a^(1/2)*x^2))*2^(1/2)/(a+b)^(3/2)/(-a^(1/2)+(a+b)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^2(a+b+2ax^2+ax^4)}\,dx = \frac{1}{(-a-b)x} + \frac{\left(ia-\sqrt{a}\sqrt{b}\right)\arctan\left(\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a-i\sqrt{a}\sqrt{b}}\sqrt{b}(a+b)} + \frac{\left(-ia-\sqrt{a}\sqrt{b}\right)\arctan\left(\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a+i\sqrt{a}\sqrt{b}}\sqrt{b}(a+b)}$$

input
```
Integrate[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```

output
```
1/((-a - b)*x) + ((I*a - Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sq
rt[a]*Sqrt[b]]])/(2*Sqrt[a - I*Sqrt[a]*Sqrt[b]]*Sqrt[b]*(a + b)) + (((-I)*
a - Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b]]])/(2*S
qrt[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b]*(a + b))
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {1443, 25, 27, 1483, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(ax^4+2ax^2+a+b\right)}dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{\int -\frac{a(x^2+2)}{ax^4+2ax^2+a+b}dx}{a+b}-\frac{1}{x(a+b)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{a(x^2+2)}{ax^4+2ax^2+a+b}dx}{a+b}-\frac{1}{x(a+b)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x^2+2}{ax^4+2ax^2+a+b}dx}{a+b}-\frac{1}{x(a+b)}$$

$$\Big\downarrow \text{1483}$$

$$-\frac{a\left(\frac{\int \frac{2\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt[4]{a}\left(2-\frac{\sqrt{a+b}}{\sqrt{a}}\right)x}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\int \frac{\sqrt[4]{a}\left(2-\frac{\sqrt{a+b}}{\sqrt{a}}\right)x+2\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}\left(x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}\right)}{a+b}-\frac{1}{x(a+b)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(\frac{\int \frac{2\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt[4]{a}\left(2-\frac{\sqrt{a+b}}{\sqrt{a}}\right)x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\int \frac{\sqrt[4]{a}\left(2-\frac{\sqrt{a+b}}{\sqrt{a}}\right)x+2\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}\right)}{a+b}-\frac{1}{x(a+b)}$$

$\Big\downarrow$ 1142

$$-\frac{a\left(\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}(\sqrt{a+b}+2\sqrt{a})\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt{a}}-\frac{1}{2}\sqrt[4]{a}\left(2-\frac{\sqrt{a+b}}{\sqrt{a}}\right)\int -\frac{\sqrt{2}\left(\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x\right)}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}(\sqrt{}}{}\right.}{a+b}$$

$$\frac{1}{x(a+b)}$$

$\Big\downarrow$ 25

$$-\frac{a\left(\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}(\sqrt{a+b}+2\sqrt{a})\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt{a}}+\frac{1}{2}\sqrt[4]{a}\left(2-\frac{\sqrt{a+b}}{\sqrt{a}}\right)\int \frac{\sqrt{2}\left(\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x\right)}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}(\sqrt{a}}{}\right.}{a+b}$$

$$\frac{1}{x(a+b)}$$

$\Big\downarrow$ 27

$$-\frac{a\left(\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}(\sqrt{a+b}+2\sqrt{a})\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}\sqrt{a}}+\frac{\left(2-\frac{\sqrt{a+b}}{\sqrt{a}}\right)\int \frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}(\sqrt{a+b}+2\sqrt{a})\int \frac{}{x^2+}}{\sqrt{2}\sqrt{}}}{}\right.}{a+b}$$

$$\frac{1}{x(a+b)}$$

$\Big\downarrow$ 1083

$$-\frac{a\left(\frac{\frac{\left(2-\frac{\sqrt{a+b}}{\sqrt{a}}\right)\int \frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}\left(\sqrt{a+b}+2\sqrt{a}\right)\int \frac{1}{-\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)^2-2\left(\frac{\sqrt{a+b}}{\sqrt{a}}+1\right)}d\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{\sqrt{a}}}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}\cdots\right)}{a\cdots}-\frac{1}{x(a+b)}$$

$\downarrow$ 217

$$-\frac{a\left(\frac{\frac{\left(2-\frac{\sqrt{a+b}}{\sqrt{a}}\right)\int \frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}x}{\sqrt[4]{a}}+\frac{\sqrt{a+b}}{\sqrt{a}}}dx}{\sqrt{2}}+\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\left(\sqrt{a+b}+2\sqrt{a}\right)\arctan\left(\frac{\sqrt[4]{a}\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{\sqrt[4]{a}\sqrt{\sqrt{a+b}+\sqrt{a}}}}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\frac{\left(2-\frac{\sqrt{a+b}}{\sqrt{a}}\right)\int \frac{\sqrt{2}\sqrt[4]{a}x\cdots}{x^2+\frac{\sqrt{2}\sqrt{\sqrt{a}\cdots}}{\sqrt[4]{\cdots}}}}{\sqrt{2}}\cdots\right)}{a+b}-\frac{1}{x(a+b)}$$

$\downarrow$ 1103

$$-\frac{a\left(\frac{\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}\left(\sqrt{a+b}+2\sqrt{a}\right)\arctan\left(\frac{\sqrt[4]{a}\left(2x-\frac{\sqrt{2}\sqrt{\sqrt{a+b}-\sqrt{a}}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{\sqrt[4]{a}\sqrt{\sqrt{a+b}+\sqrt{a}}}-\frac{1}{2}\sqrt[4]{a}\left(2-\frac{\sqrt{a+b}}{\sqrt{a}}\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\sqrt{\sqrt{a}\cdots}}{\cdots}\cdots\right)}{a+b}-\frac{1}{x(a+b)}$$

input
```
Int[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```

output
```
-(1/((a + b)*x)) - (a*(((Sqrt[-Sqrt[a] + Sqrt[a + b]]*(2*Sqrt[a] + Sqrt[a
+ b])*ArcTan[(a^(1/4)*(-((Sqrt[2]*Sqrt[-Sqrt[a] + Sqrt[a + b]])/a^(1/4)) +
 2*x))/(Sqrt[2]*Sqrt[Sqrt[a] + Sqrt[a + b]])])/(a^(1/4)*Sqrt[Sqrt[a] + Sqr
t[a + b]]) - (a^(1/4)*(2 - Sqrt[a + b]/Sqrt[a])*Log[Sqrt[a + b] - Sqrt[2]*
a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/2)/(2*Sqrt[2]*Sqrt[
a]*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]]) + ((Sqrt[-Sqrt[a] + Sqrt[a +
b]]*(2*Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*((Sqrt[2]*Sqrt[-Sqrt[a] + Sq
rt[a + b]])/a^(1/4) + 2*x))/(Sqrt[2]*Sqrt[Sqrt[a] + Sqrt[a + b]])])/(a^(1/
4)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + (a^(1/4)*(2 - Sqrt[a + b]/Sqrt[a])*Log[S
qrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]
)/2)/(2*Sqrt[2]*Sqrt[a]*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])))/(a + b
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.49

method	result
risch	$-\frac{1}{(a+b)x}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^{3}b^{2}+3a^{2}b^{3}+3b^{4}a+b^{5}\right)\_Z^{4}+\left(-2a^{2}b+6b^{2}a\right)\_Z^{2}+a\right)}\_R\ln\left(\left(\left(-b\,a^{4}+2a^{3}b^{2}+12a^{2}b^{3}+14b^{4}a+5b^{5}\right)\_\cdots\right)\right)\right)}{4}$
default	Expression too large to display

input
```
int(1/x^2/(a*x^4+2*a*x^2+a+b),x,method=_RETURNVERBOSE)
```

output
```
-1/(a+b)/x+1/4*sum(_R*ln(((-a^4*b+2*a^3*b^2+12*a^2*b^3+14*a*b^4+5*b^5)*_R^
4+(a^3-6*a^2*b+25*a*b^2)*_R^2+4*a)*x+(-3*a^3*b-5*a^2*b^2-a*b^3+b^4)*_R^3),
_R=RootOf((a^3*b^2+3*a^2*b^3+3*a*b^4+b^5)*_Z^4+(-2*a^2*b+6*a*b^2)*_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1582 vs. $2(228) = 456$.

Time = 0.10 (sec) , antiderivative size = 1582, normalized size of antiderivative = 4.82

$$\int \frac{1}{x^2 (a + b + 2ax^2 + ax^4)} dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="fricas")
```

output
```
1/4*((a + b)*x*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqr
t(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4
+ 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))*log(-
(3*a^2 - a*b)*x + (6*a^2*b - 2*a*b^2 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)
*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*
b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2
 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*
a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 +
3*a*b^3 + b^4))) - (a + b)*x*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a*
b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3
 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3
 + b^4))*log(-(3*a^2 - a*b)*x - (6*a^2*b - 2*a*b^2 + (a^4*b + 2*a^3*b^2 -
2*a*b^4 - b^5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4
*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b + (a^3
*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6
*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b
+ 3*a^2*b^2 + 3*a*b^3 + b^4))) + (a + b)*x*sqrt((a^2 - 3*a*b - (a^3*b + 3*
a^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^
2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2
*b^2 + 3*a*b^3 + b^4))*log(-(3*a^2 - a*b)*x + (6*a^2*b - 2*a*b^2 - (a^4...
```

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^2 (a+b+2ax^2+ax^4)} dx$$
$$= \text{RootSum}\left(t^4 \cdot \left(256a^3b^2 + 768a^2b^3 + 768ab^4 + 256b^5\right) + t^2\left(-32a^2b + 96ab^2\right) + a, \left(t \mapsto t \log\left(x + \frac{-64}{}\right.\right.\right.$$
$$- \frac{1}{x(a+b)}$$

input
```
integrate(1/x**2/(a*x**4+2*a*x**2+a+b),x)
```

output
```
RootSum(_t**4*(256*a**3*b**2 + 768*a**2*b**3 + 768*a*b**4 + 256*b**5) + _t
**2*(-32*a**2*b + 96*a*b**2) + a, Lambda(_t, _t*log(x + (-64*_t**3*a**4*b
- 128*_t**3*a**3*b**2 + 128*_t**3*a*b**4 + 64*_t**3*b**5 + 4*_t*a**3 - 40*
_t*a**2*b + 20*_t*a*b**2)/(3*a**2 - a*b)))) - 1/(x*(a + b))
```

Maxima [F]

$$\int \frac{1}{x^2 (a+b+2ax^2+ax^4)} dx = \int \frac{1}{(ax^4 + 2\,ax^2 + a + b)x^2} dx$$

input
```
integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="maxima")
```

output
```
-a*integrate((x^2 + 2)/(a*x^4 + 2*a*x^2 + a + b), x)/(a + b) - 1/((a + b)*
x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. $2(228) = 456$.

Time = 0.18 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.26

$$\int \frac{1}{x^2 (a + b + 2ax^2 + ax^4)} dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a*x^4+2*a*x^2+a+b),x, algorithm="giac")
```

output
```
1/2*((3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b + 4*sqrt(a^2 - sqrt(-a*b)*
a)*sqrt(-a*b)*b^2)*(a + b)^2*abs(a) - 2*(3*sqrt(a^2 - sqrt(-a*b)*a)*a^3*b
+ 7*sqrt(a^2 - sqrt(-a*b)*a)*a^2*b^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*a*b^3)*a
bs(a)*abs(-a - b) - (3*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^4 + 10*sqrt(a
^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a^3*b + 11*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*
b)*a^2*b^2 + 4*sqrt(a^2 - sqrt(-a*b)*a)*sqrt(-a*b)*a*b^3)*abs(a))*arctan(2
*sqrt(1/2)*x/sqrt((2*a^2 + 2*a*b + sqrt(-4*(a^2 + 2*a*b + b^2)*(a^2 + a*b)
 + 4*(a^2 + a*b)^2))/(a^2 + a*b)))/((3*a^6*b + 13*a^5*b^2 + 21*a^4*b^3 + 1
5*a^3*b^4 + 4*a^2*b^5)*abs(-a - b)) - 1/2*((3*sqrt(a^2 + sqrt(-a*b)*a)*sqr
t(-a*b)*a*b + 4*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*b^2)*(a + b)^2*abs(a)
+ 2*(3*sqrt(a^2 + sqrt(-a*b)*a)*a^3*b + 7*sqrt(a^2 + sqrt(-a*b)*a)*a^2*b^2
 + 4*sqrt(a^2 + sqrt(-a*b)*a)*a*b^3)*abs(a)*abs(-a - b) - (3*sqrt(a^2 + sq
rt(-a*b)*a)*sqrt(-a*b)*a^4 + 10*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^3*b
+ 11*sqrt(a^2 + sqrt(-a*b)*a)*sqrt(-a*b)*a^2*b^2 + 4*sqrt(a^2 + sqrt(-a*b)
*a)*sqrt(-a*b)*a*b^3)*abs(a))*arctan(2*sqrt(1/2)*x/sqrt((2*a^2 + 2*a*b - s
qrt(-4*(a^2 + 2*a*b + b^2)*(a^2 + a*b) + 4*(a^2 + a*b)^2))/(a^2 + a*b)))/(
(3*a^6*b + 13*a^5*b^2 + 21*a^4*b^3 + 15*a^3*b^4 + 4*a^2*b^5)*abs(-a - b))
- 1/((a + b)*x)
```

Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 2848, normalized size of antiderivative = 8.68

$$\int \frac{1}{x^2 (a + b + 2ax^2 + ax^4)} dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x)
```

output
```
- 1/(x*(a + b)) - atan((((-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b
^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 3
2*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 + x*(-(3*a*b^2 - a^2*b
 - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 +
 a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640
*a^7*b^3 + 320*a^8*b^2)) - x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 - 8*a^5*b^3))*(-
(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b
^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i - ((-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^
(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2
)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 + 192*a^6*b^3 + 128*a^7*b^2 - x*(-(
3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^
5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*b + 64*a^4*b^6 + 320*a^5*b^5 + 64
0*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)) + x*(8*a^7*b + 4*a^8 - 4*a^4*b^4 -
 8*a^5*b^3))*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(
16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*1i)/(6*a^6*b + 2*a^7 + ((
-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*(-a*b^3)^(1/2))/(16*(3*a*b^4 +
b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(32*a^8*b + 32*a^4*b^5 + 128*a^5*b^4 +
192*a^6*b^3 + 128*a^7*b^2 + x*(-(3*a*b^2 - a^2*b - 3*a*(-a*b^3)^(1/2) + b*
(-a*b^3)^(1/2))/(16*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))^(1/2)*(64*a^9*
b + 64*a^4*b^6 + 320*a^5*b^5 + 640*a^6*b^4 + 640*a^7*b^3 + 320*a^8*b^2)...
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 887, normalized size of antiderivative = 2.70

$$\int \frac{1}{x^2 (a + b + 2ax^2 + ax^4)} dx = \text{Too large to display}$$

input
```
int(1/x^2/(a*x^4+2*a*x^2+a+b),x)
```

output

```
( - 2*sqrt(a + b)*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)*atan((sqrt(sqrt(a)
*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sq
rt(2)))*a*x + 2*sqrt(a + b)*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)*atan((sq
rt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a +
b) + a)*sqrt(2)))*b*x + 2*sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)*at
an((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqr
t(a + b) + a)*sqrt(2)))*a*x + 2*sqrt(a)*sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt
(2)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) - 2*sqrt(a)*x)/(sqrt(sqrt(
a)*sqrt(a + b) + a)*sqrt(2)))*b*x + 2*sqrt(a + b)*sqrt(sqrt(a)*sqrt(a + b)
 + a)*sqrt(2)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) + 2*sqrt(a)*x)/(
sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))*a*x - 2*sqrt(a + b)*sqrt(sqrt(a)*s
qrt(a + b) + a)*sqrt(2)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2) + 2*sq
rt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))*b*x - 2*sqrt(a)*sqrt(sqr
t(a)*sqrt(a + b) + a)*sqrt(2)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)
+ 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))*a*x - 2*sqrt(a)*sq
rt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)*atan((sqrt(sqrt(a)*sqrt(a + b) - a)*sq
rt(2) + 2*sqrt(a)*x)/(sqrt(sqrt(a)*sqrt(a + b) + a)*sqrt(2)))*b*x + sqrt(a
 + b)*sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(2)*log( - sqrt(sqrt(a)*sqrt(a + b
) - a)*sqrt(2)*x + sqrt(a + b) + sqrt(a)*x**2)*a*x - sqrt(a + b)*sqrt(sqrt
(a)*sqrt(a + b) - a)*sqrt(2)*log( - sqrt(sqrt(a)*sqrt(a + b) - a)*sqrt(...
```

3.844 $\int \frac{x^9}{1+x^2+x^4} dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{x^9}{1+x^2+x^4} dx = -\frac{x^4}{4}+\frac{x^6}{6}-\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{4}\log\left(1+x^2+x^4\right)$$

output
```
-1/4*x^4+1/6*x^6-1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)+1/4*ln(x^4+x^2+
1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{x^9}{1+x^2+x^4} dx = \frac{1}{12}\left(-3x^4+2x^6-2\sqrt{3}\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)+3\log\left(1+x^2+x^4\right)\right)$$

input
```
Integrate[x^9/(1 + x^2 + x^4),x]
```

output
```
(-3*x^4 + 2*x^6 - 2*Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]] + 3*Log[1 + x^2 +
x^4])/12
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{x^4+x^2+1} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int \frac{x^8}{x^4+x^2+1}dx^2$$

$$\Big\downarrow \text{1143}$$

$$\frac{1}{2} \int \left(x^4+\frac{x^2}{x^4+x^2+1}-x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^6}{3}-\frac{x^4}{2}+\frac{1}{2}\log\left(x^4+x^2+1\right)\right)$$

input
```
Int[x^9/(1 + x^2 + x^4),x]
```

output
```
(-1/2*x^4 + x^6/3 - ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3] + Log[1 + x^2 + x^
4]/2)/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{x^4}{4}+\frac{x^6}{6}-\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(x^4+x^2+1)}{4}$	41
risch	$\frac{x^6}{6}-\frac{x^4}{4}+\frac{\ln(4x^4+4x^2+4)}{4}-\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	45

input
```
int(x^9/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^4+1/6*x^6-1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)+1/4*ln(x^4+x^2+
1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^9}{1+x^2+x^4}\,dx = \frac{1}{6}\,x^6 - \frac{1}{4}\,x^4 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) + \frac{1}{4}\,\log\left(x^4+x^2+1\right)$$

input
```
integrate(x^9/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/6*x^6 - 1/4*x^4 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(
x^4 + x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{x^9}{1+x^2+x^4}\,dx = \frac{x^6}{6} - \frac{x^4}{4} + \frac{\log\left(x^4+x^2+1\right)}{4} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**9/(x**4+x**2+1),x)
```

output
```
x**6/6 - x**4/4 + log(x**4 + x**2 + 1)/4 - sqrt(3)*atan(2*sqrt(3)*x**2/3 +
 sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^9}{1+x^2+x^4}\,dx = \frac{1}{6}\,x^6 - \frac{1}{4}\,x^4 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) + \frac{1}{4}\,\log\left(x^4+x^2+1\right)$$

input
```
integrate(x^9/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/6*x^6 - 1/4*x^4 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(
x^4 + x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^9}{1+x^2+x^4}\,dx = \frac{1}{6}\,x^6 - \frac{1}{4}\,x^4 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) + \frac{1}{4}\,\log\left(x^4+x^2+1\right)$$

input
```
integrate(x^9/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*x^6 - 1/4*x^4 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(
x^4 + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x^9}{1+x^2+x^4}\,dx = \frac{\ln\left(x^4+x^2+1\right)}{4} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{6} - \frac{x^4}{4} + \frac{x^6}{6}$$

input
```
int(x^9/(x^2 + x^4 + 1),x)
```

output
```
log(x^2 + x^4 + 1)/4 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/6 - x
^4/4 + x^6/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{x^9}{1+x^2+x^4}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} + \frac{\log(x^2-x+1)}{4} + \frac{\log(x^2+x+1)}{4} + \frac{x^6}{6} - \frac{x^4}{4}$$

input
```
int(x^9/(x^4+x^2+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))
+ 3*log(x**2 - x + 1) + 3*log(x**2 + x + 1) + 2*x**6 - 3*x**4)/12
```

3.845 $\int \frac{x^7}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{x^7}{1+x^2+x^4} \, dx = -\frac{x^2}{2}+\frac{x^4}{4}+\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-1/2*x^2+1/4*x^4+1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{1+x^2+x^4} \, dx = -\frac{x^2}{2}+\frac{x^4}{4}+\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[x^7/(1 + x^2 + x^4),x]
```

output
```
-1/2*x^2 + x^4/4 + ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{x^6}{x^4+x^2+1}dx^2$$

$$\Big\downarrow \text{ 1143}$$

$$\frac{1}{2}\int \left(x^2+\frac{1}{x^4+x^2+1}-1\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^4}{2}-x^2\right)$$

input
```
Int[x^7/(1 + x^2 + x^4),x]
```

output
```
(-x^2 + x^4/2 + (2*ArcTan[(1 + 2*x^2)/Sqrt[3]])/Sqrt[3])/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{x^2}{2}+\frac{x^4}{4}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	30
risch	$-\frac{x^2}{2}+\frac{x^4}{4}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	30

input
```
int(x^7/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2+1/4*x^4+1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^7}{1+x^2+x^4}\,dx = \frac{1}{4}\,x^4 - \frac{1}{2}\,x^2 + \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right)$$

input
```
integrate(x^7/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/4*x^4 - 1/2*x^2 + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{x^7}{1+x^2+x^4}\,dx = \frac{x^4}{4} - \frac{x^2}{2} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x**7/(x**4+x**2+1),x)
```

output
```
x**4/4 - x**2/2 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^7}{1+x^2+x^4}\,dx = \frac{1}{4}\,x^4 - \frac{1}{2}\,x^2 + \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right)$$

input
```
integrate(x^7/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/4*x^4 - 1/2*x^2 + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^7}{1+x^2+x^4}\,dx = \frac{1}{4}\,x^4 - \frac{1}{2}\,x^2 + \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right)$$

input
```
integrate(x^7/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/4*x^4 - 1/2*x^2 + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^7}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{3} - \frac{x^2}{2} + \frac{x^4}{4}$$

input
```
int(x^7/(x^2 + x^4 + 1),x)
```

output
```
(3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/3 - x^2/2 + x^4/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{x^7}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} + \frac{x^4}{4} - \frac{x^2}{2}$$

input
```
int(x^7/(x^4+x^2+1),x)
```

output
```
(4*sqrt(3)*atan((2*x - 1)/sqrt(3)) - 4*sqrt(3)*atan((2*x + 1)/sqrt(3)) + 3
*x**4 - 6*x**2)/12
```

3.846 $\int \frac{x^5}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 44

$$\int \frac{x^5}{1+x^2+x^4} \, dx = \frac{x^2}{2} - \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}\log\left(1+x^2+x^4\right)$$

output
```
1/2*x^2-1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)-1/4*ln(x^4+x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{1+x^2+x^4} \, dx = \frac{x^2}{2} - \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}\log\left(1+x^2+x^4\right)$$

input
```
Integrate[x^5/(1 + x^2 + x^4),x]
```

output
```
x^2/2 - ArcTan[(1 + 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - Log[1 + x^2 + x^4]/4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^4+x^2+1} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int \frac{x^4}{x^4+x^2+1} dx^2$$

$$\Big\downarrow \text{1143}$$

$$\frac{1}{2} \int \left(1-\frac{x^2+1}{x^4+x^2+1}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}+x^2-\frac{1}{2} \log \left(x^4+x^2+1\right)\right)$$

input
```
Int[x^5/(1 + x^2 + x^4),x]
```

output
```
(x^2 - ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3] - Log[1 + x^2 + x^4]/2)/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^2}{2}-\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln(x^4+x^2+1)}{4}$	36
risch	$\frac{x^2}{2}-\frac{\ln(4x^4+4x^2+4)}{4}-\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	40

input
```
int(x^5/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)-1/4*ln(x^4+x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{1+x^2+x^4}\,dx=\frac{1}{2}\,x^2-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right)-\frac{1}{4}\,\log\left(x^4+x^2+1\right)$$

input
```
integrate(x^5/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/4*log(x^4 + x^2
+ 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{1+x^2+x^4}\,dx = \frac{x^2}{2} - \frac{\log\left(x^4+x^2+1\right)}{4} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**5/(x**4+x**2+1),x)
```

output
```
x**2/2 - log(x**4 + x**2 + 1)/4 - sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/
3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{1+x^2+x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) - \frac{1}{4}\,\log\left(x^4+x^2+1\right)$$

input
```
integrate(x^5/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/4*log(x^4 + x^2
+ 1)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{1+x^2+x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right) - \frac{1}{4}\,\log\left(x^4+x^2+1\right)$$

input
```
integrate(x^5/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/4*log(x^4 + x^2
+ 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{1+x^2+x^4}\,dx = \frac{x^2}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{6} - \frac{\ln\left(x^4+x^2+1\right)}{4}$$

input
```
int(x^5/(x^2 + x^4 + 1),x)
```

output
```
x^2/2 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/6 - log(x^2 + x^4 +
1)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{x^5}{1+x^2+x^4}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} - \frac{\log(x^2-x+1)}{4} - \frac{\log(x^2+x+1)}{4} + \frac{x^2}{2}$$

input
```
int(x^5/(x^4+x^2+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))
- 3*log(x**2 - x + 1) - 3*log(x**2 + x + 1) + 6*x**2)/12
```

3.847 $\int \frac{x^3}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{x^3}{1+x^2+x^4} \, dx = -\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{4}\log\left(1+x^2+x^4\right)$$

output
```
-1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)+1/4*ln(x^4+x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+x^2+x^4} \, dx = -\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{4}\log\left(1+x^2+x^4\right)$$

input
```
Integrate[x^3/(1 + x^2 + x^4),x]
```

output
```
-1/2*ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3] + Log[1 + x^2 + x^4]/4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1434, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^4+x^2+1} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{x^4+x^2+1}dx^2$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2-\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\int \frac{1}{-x^4-3}d\left(2x^2+1\right)+\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2-\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\log\left(x^4+x^2+1\right)-\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

input
```
Int[x^3/(1 + x^2 + x^4),x]
```

output
```
(-(ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]) + Log[1 + x^2 + x^4]/2)/2
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(x^4+x^2+1)}{4}$	31
risch	$\frac{\ln(4x^4+4x^2+4)}{4}-\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	35

input
```
int(x^3/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)+1/4*ln(x^4+x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{1+x^2+x^4} \, dx = -\frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 x^2+1)\right) + \frac{1}{4} \log \left(x^4+x^2+1\right)$$

input
```
integrate(x^3/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1+x^2+x^4} \, dx = \frac{\log \left(x^4+x^2+1\right)}{4} - \frac{\sqrt{3} \operatorname{atan} \left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**3/(x**4+x**2+1),x)
```

output
```
log(x**4 + x**2 + 1)/4 - sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{1+x^2+x^4} \, dx = -\frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 x^2+1)\right) + \frac{1}{4} \log \left(x^4+x^2+1\right)$$

input
```
integrate(x^3/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{1+x^2+x^4}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

input
```
integrate(x^3/(x^4+x^2+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{1+x^2+x^4}\,dx = \frac{\ln\left(x^4+x^2+1\right)}{4}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
int(x^3/(x^2 + x^4 + 1),x)
```

output
```
log(x^2 + x^4 + 1)/4 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{1+x^2+x^4}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6}+\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6}+\frac{\log(x^2-x+1)}{4}+\frac{\log(x^2+x+1)}{4}$$

input
```
int(x^3/(x^4+x^2+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))
+ 3*log(x**2 - x + 1) + 3*log(x**2 + x + 1))/12
```

3.848 $\int \frac{x}{1+x^2+x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[x/(1 + x^2 + x^4),x]
```

output
```
ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1432, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx^2$$

$$\Big\downarrow \text{1083}$$

$$-\int \frac{1}{-x^4-3}d\left(2x^2+1\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[x/(1 + x^2 + x^4),x]
```

output
```
ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19
risch	$\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19

input
```
int(x/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x}{1+x^2+x^4} \, dx = \frac{1}{3} \, \sqrt{3} \arctan \left(\frac{1}{3} \, \sqrt{3} (2 \, x^2 + 1) \right)$$

input
```
integrate(x/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x/(x**4+x**2+1),x)
```

output
```
sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right)$$

input
```
integrate(x/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right)$$

input
```
integrate(x/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int(x/(x^2 + x^4 + 1),x)
```

output
```
(3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{x}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\left(atan\left(\frac{2x-1}{\sqrt{3}}\right)-atan\left(\frac{2x+1}{\sqrt{3}}\right)\right)}{3}$$

input
```
int(x/(x^4+x^2+1),x)
```

output
```
(sqrt(3)*(atan((2*x - 1)/sqrt(3)) - atan((2*x + 1)/sqrt(3))))/3
```

3.849 $\int \frac{1}{x(1+x^2+x^4)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{1}{x(1+x^2+x^4)}\,dx=-\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}+\log(x)-\frac{1}{4}\log\left(1+x^2+x^4\right)$$

output
```
-1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)+ln(x)-1/4*ln(x^4+x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int \frac{1}{x(1+x^2+x^4)}\,dx=\log(x)-\frac{1}{12}i\left(\left(-3i+\sqrt{3}\right)\log\left(-i+\sqrt{3}-2ix^2\right)-\left(3i+\sqrt{3}\right)\log\left(i+\sqrt{3}+2ix^2\right)\right)$$

input
```
Integrate[1/(x*(1 + x^2 + x^4)),x]
```

output
```
Log[x] - (I/12)*((-3*I + Sqrt[3])*Log[-I + Sqrt[3] - (2*I)*x^2] - (3*I + S
qrt[3])*Log[I + Sqrt[3] + (2*I)*x^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1434, 1144, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(x^4+x^2+1)}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^2(x^4+x^2+1)}dx^2$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{2}\left(\int -\frac{x^2+1}{x^4+x^2+1}dx^2+\log(x^2)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\log(x^2)-\int \frac{x^2+1}{x^4+x^2+1}dx^2\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^4+x^2+1}dx^2-\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2+\log(x^2)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\int \frac{1}{-x^4-3}d(2x^2+1)-\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2+\log(x^2)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{2x^2+1}{x^4+x^2+1}dx^2-\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x^2)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(-\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^2\right)-\frac{1}{2}\log\left(x^4+x^2+1\right)\right)$$

input
```
Int[1/(x*(1 + x^2 + x^4)),x]
```

output
```
(-(ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]) + Log[x^2] - Log[1 + x^2 + x^4]/2)
/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

method	result	size
risch	$\ln(x)-\frac{\ln(x^4+x^2+1)}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^2+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$	31
default	$-\frac{\ln(x^2+x+1)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln(x^2-x+1)}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\ln(x)$	56

input
```
int(1/x/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/4*ln(x^4+x^2+1)-1/6*3^(1/2)*arctan(2/3*(x^2+1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\,(1+x^2+x^4)}\,dx=-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2+1)\right)-\frac{1}{4}\,\log\left(x^4+x^2+1\right)+\log\left(x\right)$$

input
```
integrate(1/x/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/4*log(x^4 + x^2 + 1) + lo
g(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(1+x^2+x^4\right)}\,dx = \log\left(x\right) - \frac{\log\left(x^4+x^2+1\right)}{4} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(1/x/(x**4+x**2+1),x)
```

output
```
log(x) - log(x**4 + x**2 + 1)/4 - sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/
3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(1+x^2+x^4\right)}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) - \frac{1}{4}\log\left(x^4+x^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/4*log(x^4 + x^2 + 1) + 1/
2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(1+x^2+x^4\right)}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) - \frac{1}{4}\log\left(x^4+x^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x/(x^4+x^2+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/4*log(x^4 + x^2 + 1) + 1/
2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 18.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(1+x^2+x^4)}\,dx = \ln(x) - \frac{\ln(x^4+x^2+1)}{4} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
int(1/(x*(x^2 + x^4 + 1)),x)
```

output
```
log(x) - log(x^2 + x^4 + 1)/4 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/
3))/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \frac{1}{x(1+x^2+x^4)}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} - \frac{\log(x^2-x+1)}{4} - \frac{\log(x^2+x+1)}{4} + \log(x)$$

input
```
int(1/x/(x^4+x^2+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 2*sqrt(3)*atan((2*x + 1)/sqrt(3))
- 3*log(x**2 - x + 1) - 3*log(x**2 + x + 1) + 12*log(x))/12
```

3.850 $\int \frac{1}{x^3(1+x^2+x^4)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{x^3(1+x^2+x^4)}\,dx = -\frac{1}{2x^2} - \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \log(x) + \frac{1}{4}\log\left(1+x^2+x^4\right)$$

output
```
-1/2/x^2-1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)-ln(x)+1/4*ln(x^4+x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.62

$$\int \frac{1}{x^3(1+x^2+x^4)}\,dx = \frac{1}{12}\left(-\frac{6}{x^2} - 12\log(x) + \sqrt{3}\left(-i+\sqrt{3}\right)\log\left(-i+\sqrt{3}-2ix^2\right) + \sqrt{3}\left(i+\sqrt{3}\right)\log\left(i+\sqrt{3}+2ix^2\right)\right)$$

input
```
Integrate[1/(x^3*(1 + x^2 + x^4)),x]
```

output
```
(-6/x^2 - 12*Log[x] + Sqrt[3]*(-I + Sqrt[3])*Log[-I + Sqrt[3] - (2*I)*x^2]
 + Sqrt[3]*(I + Sqrt[3])*Log[I + Sqrt[3] + (2*I)*x^2])/12
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1434, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^4+x^2+1)} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int \frac{1}{x^4 (x^4+x^2+1)}dx^2$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{2} \left(\int -\frac{x^2+1}{x^2 (x^4+x^2+1)}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(-\int \frac{x^2+1}{x^2 (x^4+x^2+1)}dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2} \left(-\int \left(\frac{1}{x^2}-\frac{x^2}{x^4+x^2+1}\right)dx^2-\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x^2}-\log\left(x^2\right)+\frac{1}{2}\log\left(x^4+x^2+1\right)\right)$$

input
```
Int[1/(x^3*(1 + x^2 + x^4)),x]
```

output
```
(-x^(-2) - ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3] - Log[x^2] + Log[1 + x^2 +
x^4]/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{1}{2x^{2}}-\ln\left(x\right)+\frac{\ln\left(x^{4}+x^{2}+1\right)}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{2}+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$	38
default	$\frac{\ln\left(x^{2}+x+1\right)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln\left(x^{2}-x+1\right)}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{1}{2x^{2}}-\ln\left(x\right)$	63

input
```
int(1/x^3/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-ln(x)+1/4*ln(x^4+x^2+1)-1/6*3^(1/2)*arctan(2/3*(x^2+1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^3 (1+x^2+x^4)} \, dx = -\frac{2 \sqrt{3} x^2 \arctan \left(\frac{1}{3} \sqrt{3}(2 x^2+1)\right) - 3 x^2 \log \left(x^4+x^2+1\right) + 12 x^2 \log (x) + 6}{12 x^2}$$

input
```
integrate(1/x^3/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
-1/12*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 3*x^2*log(x^4 + x^2
 + 1) + 12*x^2*log(x) + 6)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (1+x^2+x^4)} \, dx = -\log (x) + \frac{\log (x^4+x^2+1)}{4} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{6} - \frac{1}{2x^2}$$

input
```
integrate(1/x**3/(x**4+x**2+1),x)
```

output
```
-log(x) + log(x**4 + x**2 + 1)/4 - sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)
/3)/6 - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3(1+x^2+x^4)}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) - \frac{1}{2x^2} + \frac{1}{4}\log\left(x^4+x^2+1\right) - \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x^3/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/2/x^2 + 1/4*log(x^4 + x^2
 + 1) - 1/2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3(1+x^2+x^4)}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right) + \frac{x^2-1}{2x^2} + \frac{1}{4}\log\left(x^4+x^2+1\right) - \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x^3/(x^4+x^2+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/2*(x^2 - 1)/x^2 + 1/4*log
(x^4 + x^2 + 1) - 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 (1+x^2+x^4)} \, dx = \frac{\ln(x^4+x^2+1)}{4} - \ln(x) - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3} + \frac{\sqrt{3}}{3}\right)}{6} - \frac{1}{2\,x^2}$$

input
```
int(1/(x^3*(x^2 + x^4 + 1)),x)
```

output
```
log(x^2 + x^4 + 1)/4 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/
3))/6 - 1/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^3 (1+x^2+x^4)} \, dx = \frac{-2\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^2 + 2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^2 + 3\log(x^2-x+1)\,x^2 + 3\log(x^2+x+1)\,x^2 - 12\log(x)\,x^2}{12x^2}$$

input
```
int(1/x^3/(x^4+x^2+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**2 + 2*sqrt(3)*atan((2*x + 1)/sqrt
(3))*x**2 + 3*log(x**2 - x + 1)*x**2 + 3*log(x**2 + x + 1)*x**2 - 12*log(x
)*x**2 - 6)/(12*x**2)
```

3.851 $\int \frac{1}{x^5(1+x^2+x^4)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{1}{x^5(1+x^2+x^4)}\,dx = -\frac{1}{4x^4}+\frac{1}{2x^2}+\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-1/4/x^4+1/2/x^2+1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.03

$$\int \frac{1}{x^5(1+x^2+x^4)}\,dx = -\frac{1}{4x^4}+\frac{1}{2x^2}+\frac{i\log\left(-i+\sqrt{3}-2ix^2\right)}{2\sqrt{3}}-\frac{i\log\left(i+\sqrt{3}+2ix^2\right)}{2\sqrt{3}}$$

input
```
Integrate[1/(x^5*(1 + x^2 + x^4)),x]
```

output
```
-1/4*1/x^4 + 1/(2*x^2) + ((I/2)*Log[-I + Sqrt[3] - (2*I)*x^2])/Sqrt[3] - (
(I/2)*Log[I + Sqrt[3] + (2*I)*x^2])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1434, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^4+x^2+1)} \, dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2} \int \frac{1}{x^6 (x^4+x^2+1)}dx^2$$

$$\Big\downarrow \text{ 1145}$$

$$\frac{1}{2} \left(\int -\frac{x^2+1}{x^4 (x^4+x^2+1)}dx^2-\frac{1}{2 x^4}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2} \left(-\int \frac{x^2+1}{x^4 (x^4+x^2+1)}dx^2-\frac{1}{2 x^4}\right)$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{2} \left(-\int \left(\frac{1}{x^4}+\frac{1}{-x^4-x^2-1}\right)dx^2-\frac{1}{2 x^4}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{2 \arctan \left(\frac{2 x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2 x^4}+\frac{1}{x^2}\right)$$

input
```
Int[1/(x^5*(1 + x^2 + x^4)),x]
```

output
```
(-1/2*1/x^4 + x^(-2) + (2*ArcTan[(1 + 2*x^2)/Sqrt[3]])/Sqrt[3])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{\frac{x^2}{2}-\frac{1}{4}}{x^4}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	31
default	$-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{1}{4x^4}+\frac{1}{2x^2}$	44

input
```
int(1/x^5/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(1/2*x^2-1/4)/x^4+1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5 (1 + x^2 + x^4)} \, dx = \frac{4 \sqrt{3} x^4 \arctan \left(\frac{1}{3} \sqrt{3}(2 x^2 + 1)\right) + 6 x^2 - 3}{12 x^4}$$

input
```
integrate(1/x^5/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/12*(4*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 6*x^2 - 3)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^5 (1 + x^2 + x^4)} \, dx = \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} + \frac{\sqrt{3}}{3}\right)}{3} + \frac{2x^2 - 1}{4x^4}$$

input
```
integrate(1/x**5/(x**4+x**2+1),x)
```

output
```
sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/3 + (2*x**2 - 1)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5 (1 + x^2 + x^4)} \, dx = \frac{1}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x^2 + 1)\right) + \frac{2 x^2 - 1}{4 x^4}$$

input
```
integrate(1/x^5/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*(2*x^2 - 1)/x^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5 (1 + x^2 + x^4)} \, dx = \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2 x^2 + 1)\right) + \frac{2 x^2 - 1}{4 x^4}$$

input
```
integrate(1/x^5/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*(2*x^2 - 1)/x^4
```

Mupad [B] (verification not implemented)

Time = 17.95 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^5 (1 + x^2 + x^4)} \, dx = \frac{\frac{x^2}{2} - \frac{1}{4}}{x^4} + \frac{\sqrt{3} \, \mathrm{atan}\left(\frac{2 \sqrt{3} x^2}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int(1/(x^5*(x^2 + x^4 + 1)),x)
```

output
```
(x^2/2 - 1/4)/x^4 + (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^5 (1 + x^2 + x^4)} \, dx = \frac{4\sqrt{3}\, atan\left(\frac{2x-1}{\sqrt{3}}\right) x^4 - 4\sqrt{3}\, atan\left(\frac{2x+1}{\sqrt{3}}\right) x^4 + 6x^2 - 3}{12x^4}$$

input
```
int(1/x^5/(x^4+x^2+1),x)
```

output
```
(4*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**4 - 4*sqrt(3)*atan((2*x + 1)/sqrt(3)
)*x**4 + 6*x**2 - 3)/(12*x**4)
```

3.852 $\int \frac{x^{22}}{1+x^2+x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 81

$$\int \frac{x^{22}}{1+x^2+x^4}\,dx = x-\frac{x^5}{5}+\frac{x^7}{7}-\frac{x^{11}}{11}+\frac{x^{13}}{13}-\frac{x^{17}}{17}+\frac{x^{19}}{19}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
x-1/5*x^5+1/7*x^7-1/11*x^11+1/13*x^13-1/17*x^17+1/19*x^19+1/3*arctan(1/3*(
1-2*x)*3^(1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\begin{aligned}\int \frac{x^{22}}{1+x^2+x^4}\,dx &= x-\frac{x^5}{5}+\frac{x^7}{7}-\frac{x^{11}}{11}+\frac{x^{13}}{13}-\frac{x^{17}}{17}+\frac{x^{19}}{19}\\ &\quad-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}}\\ &\quad-\frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}}\end{aligned}$$

input
```
Integrate[x^22/(1 + x^2 + x^4),x]
```

output
```
x - x^5/5 + x^7/7 - x^11/11 + x^13/13 - x^17/17 + x^19/19 - ((I + Sqrt[3])
*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)*Sqrt[3]] - ((-I + Sqrt[3])*A
rcTan[((I + Sqrt[3])*x)/2])/Sqrt[6 - (6*I)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {1442, 27, 1602, 27, 1442, 27, 1602, 27, 1442, 27, 1602, 27, 1442, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{22}}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x^{19}}{19}-\frac{1}{19}\int \frac{19x^{18}\left(x^2+1\right)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^{19}}{19}-\int \frac{x^{18}\left(x^2+1\right)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{17}\int \frac{17x^{16}}{x^4+x^2+1}dx+\frac{x^{19}}{19}-\frac{x^{17}}{17}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{16}}{x^4+x^2+1}dx+\frac{x^{19}}{19}-\frac{x^{17}}{17}$$

$$\Big\downarrow \text{1442}$$

$$-\frac{1}{13}\int \frac{13x^{12}\left(x^2+1\right)}{x^4+x^2+1}dx+\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^{12}\left(x^2+1\right)}{x^4+x^2+1}dx+\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{11}\int \frac{11x^{10}}{x^4+x^2+1}dx+\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{10}}{x^4+x^2+1}dx+\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}$$

$$\Big\downarrow \text{1442}$$

$$-\frac{1}{7}\int \frac{7x^6\left(x^2+1\right)}{x^4+x^2+1}dx+\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^6\left(x^2+1\right)}{x^4+x^2+1}dx+\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{5}\int \frac{5x^4}{x^4+x^2+1}dx+\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^4}{x^4+x^2+1}dx+\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}$$

$$\Big\downarrow \text{1442}$$

$$-\int \frac{x^2+1}{x^4+x^2+1}dx+\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}+x$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}+x$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{1}{-(2x-1)^2-3}d(2x-1)+\int \frac{1}{-(2x+1)^2-3}d(2x+1)+\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}+x$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}+x$$

input
```
Int[x^22/(1 + x^2 + x^4),x]
```

output
```
x - x^5/5 + x^7/7 - x^11/11 + x^13/13 - x^17/17 + x^19/19 - ArcTan[(-1 + 2
*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^{7}}{7}-\frac{x^{5}}{5}+x-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	65
risch	$\frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^{7}}{7}-\frac{x^{5}}{5}+x-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{x^{3}\sqrt{3}}{3}+\frac{2\sqrt{3}\,x}{3}\right)}{3}$	66

input
```
int(x^22/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/19*x^19-1/17*x^17+1/13*x^13-1/11*x^11+1/7*x^7-1/5*x^5+x-1/3*arctan(1/3*(
1+2*x)*3^(1/2))*3^(1/2)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \frac{x^{22}}{1+x^2+x^4}\,dx = \frac{1}{19}\,x^{19} - \frac{1}{17}\,x^{17} + \frac{1}{13}\,x^{13} - \frac{1}{11}\,x^{11} + \frac{1}{7}\,x^{7} - \frac{1}{5}\,x^{5} - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x^3+2\,x)\right) - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right) + x$$

input
```
integrate(x^22/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/19*x^19 - 1/17*x^17 + 1/13*x^13 - 1/11*x^11 + 1/7*x^7 - 1/5*x^5 - 1/3*sq
rt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x)
+ x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{x^{22}}{1+x^2+x^4}\,dx = \frac{x^{19}}{19} - \frac{x^{17}}{17} + \frac{x^{13}}{13} - \frac{x^{11}}{11} + \frac{x^7}{7} - \frac{x^5}{5} + x + \frac{\sqrt{3}\left(-2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right) - 2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3}\right)\right)}{6}$$

input
```
integrate(x**22/(x**4+x**2+1),x)
```

output
```
x**19/19 - x**17/17 + x**13/13 - x**11/11 + x**7/7 - x**5/5 + x + sqrt(3)*
(-2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{x^{22}}{1+x^2+x^4}\,dx = \frac{1}{19}\,x^{19} - \frac{1}{17}\,x^{17} + \frac{1}{13}\,x^{13} - \frac{1}{11}\,x^{11} + \frac{1}{7}\,x^7 - \frac{1}{5}\,x^5 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + x$$

input
```
integrate(x^22/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/19*x^19 - 1/17*x^17 + 1/13*x^13 - 1/11*x^11 + 1/7*x^7 - 1/5*x^5 - 1/3*sq
rt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x
- 1)) + x
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{x^{22}}{1+x^2+x^4}\,dx = \frac{1}{19}\,x^{19} - \frac{1}{17}\,x^{17} + \frac{1}{13}\,x^{13} - \frac{1}{11}\,x^{11} + \frac{1}{7}\,x^7 - \frac{1}{5}\,x^5 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + x$$

input
```
integrate(x^22/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/19*x^19 - 1/17*x^17 + 1/13*x^13 - 1/11*x^11 + 1/7*x^7 - 1/5*x^5 - 1/3*sq
rt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x
- 1)) + x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

$$\int \frac{x^{22}}{1+x^2+x^4}\,dx = x-\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\,\sqrt{3}\,x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{6} - \frac{x^5}{5}+\frac{x^7}{7}-\frac{x^{11}}{11}+\frac{x^{13}}{13}-\frac{x^{17}}{17}+\frac{x^{19}}{19}$$

input
```
int(x^22/(x^2 + x^4 + 1),x)
```

output
```
x - (3^(1/2)*(2*atan((2*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) + 2*atan((3^(1/2)*
x)/3)))/6 - x^5/5 + x^7/7 - x^11/11 + x^13/13 - x^17/17 + x^19/19
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \frac{x^{22}}{1+x^2+x^4}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3}-\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} + \frac{x^{19}}{19}-\frac{x^{17}}{17}+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}+x$$

input
```
int(x^22/(x^4+x^2+1),x)
```

output
```
( - 1616615*sqrt(3)*atan((2*x - 1)/sqrt(3)) - 1616615*sqrt(3)*atan((2*x +
1)/sqrt(3)) + 255255*x**19 - 285285*x**17 + 373065*x**13 - 440895*x**11 +
692835*x**7 - 969969*x**5 + 4849845*x)/4849845
```

3.853 $\int \frac{x^{16}}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{x^{16}}{1+x^2+x^4} \, dx = x-\frac{x^5}{5}+\frac{x^7}{7}-\frac{x^{11}}{11}+\frac{x^{13}}{13}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
x-1/5*x^5+1/7*x^7-1/11*x^11+1/13*x^13+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1
/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int \frac{x^{16}}{1+x^2+x^4} \, dx = x-\frac{x^5}{5}+\frac{x^7}{7}-\frac{x^{11}}{11}+\frac{x^{13}}{13}-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}}-\frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[x^16/(1 + x^2 + x^4),x]
```

output
```
x - x^5/5 + x^7/7 - x^11/11 + x^13/13 - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[
3])*x)/2])/Sqrt[6 + (6*I)*Sqrt[3]] - ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])
*x)/2])/Sqrt[6 - (6*I)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {1442, 27, 1602, 27, 1442, 27, 1602, 27, 1442, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{16}}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x^{13}}{13}-\frac{1}{13}\int \frac{13x^{12}\left(x^2+1\right)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^{13}}{13}-\int \frac{x^{12}\left(x^2+1\right)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{11}\int \frac{11x^{10}}{x^4+x^2+1}dx+\frac{x^{13}}{13}-\frac{x^{11}}{11}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{10}}{x^4+x^2+1}dx+\frac{x^{13}}{13}-\frac{x^{11}}{11}$$

$$\Big\downarrow \text{1442}$$

$$-\frac{1}{7}\int \frac{7x^6\left(x^2+1\right)}{x^4+x^2+1}dx+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^6\left(x^2+1\right)}{x^4+x^2+1}dx+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{5}\int \frac{5x^4}{x^4+x^2+1}dx+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^4}{x^4+x^2+1}dx+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}$$

$$\Big\downarrow \text{1442}$$

$$-\int \frac{x^2+1}{x^4+x^2+1}dx+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}+x$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}+x$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{1}{-(2x-1)^2-3}d(2x-1)+\int \frac{1}{-(2x+1)^2-3}d(2x+1)+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}+x$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}+x$$

input
```
Int[x^16/(1 + x^2 + x^4),x]
```

output
```
x - x^5/5 + x^7/7 - x^11/11 + x^13/13 - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]
 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^{7}}{7}-\frac{x^{5}}{5}+x-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	55
risch	$\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^{7}}{7}-\frac{x^{5}}{5}+x-\frac{\sqrt{3}\,\arctan\left(\frac{x^{3}\sqrt{3}}{3}+\frac{2\sqrt{3}\,x}{3}\right)}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{3}$	56

input
```
int(x^16/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/13*x^13-1/11*x^11+1/7*x^7-1/5*x^5+x-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1
/2)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \frac{x^{16}}{1+x^2+x^4}\,dx = \frac{1}{13}\,x^{13}-\frac{1}{11}\,x^{11}+\frac{1}{7}\,x^{7}-\frac{1}{5}\,x^{5}-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x^{3}+2\,x)\right) - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right)+x$$

input
```
integrate(x^16/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/13*x^13 - 1/11*x^11 + 1/7*x^7 - 1/5*x^5 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(x^3 + 2*x)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{x^{16}}{1+x^2+x^4}\,dx = \frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}-\frac{x^5}{5}+x+\frac{\sqrt{3}\left(-2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)-2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{6}$$

input
```
integrate(x**16/(x**4+x**2+1),x)
```

output
```
x**13/13 - x**11/11 + x**7/7 - x**5/5 + x + sqrt(3)*(-2*atan(sqrt(3)*x/3)
- 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{x^{16}}{1+x^2+x^4}\,dx = \frac{1}{13}\,x^{13}-\frac{1}{11}\,x^{11}+\frac{1}{7}\,x^7-\frac{1}{5}\,x^5-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+x$$

input
```
integrate(x^16/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/13*x^13 - 1/11*x^11 + 1/7*x^7 - 1/5*x^5 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{x^{16}}{1+x^2+x^4}\,dx = \frac{1}{13}\,x^{13} - \frac{1}{11}\,x^{11} + \frac{1}{7}\,x^7 - \frac{1}{5}\,x^5 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + x$$

input
```
integrate(x^16/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/13*x^13 - 1/11*x^11 + 1/7*x^7 - 1/5*x^5 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{x^{16}}{1+x^2+x^4}\,dx = x - \frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3} + \frac{2\,\sqrt{3}\,x}{3}\right) + 2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{6} - \frac{x^5}{5} + \frac{x^7}{7} - \frac{x^{11}}{11} + \frac{x^{13}}{13}$$

input
```
int(x^16/(x^2 + x^4 + 1),x)
```

output
```
x - (3^(1/2)*(2*atan((2*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) + 2*atan((3^(1/2)*
x)/3)))/6 - x^5/5 + x^7/7 - x^11/11 + x^13/13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \frac{x^{16}}{1+x^2+x^4} \, dx = -\frac{\sqrt{3}\, atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3} - \frac{\sqrt{3}\, atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} + \frac{x^{13}}{13} - \frac{x^{11}}{11} + \frac{x^7}{7} - \frac{x^5}{5} + x$$

input
```
int(x^16/(x^4+x^2+1),x)
```

output
```
( - 5005*sqrt(3)*atan((2*x - 1)/sqrt(3)) - 5005*sqrt(3)*atan((2*x + 1)/sqr
t(3)) + 1155*x**13 - 1365*x**11 + 2145*x**7 - 3003*x**5 + 15015*x)/15015
```

3.854 $\int \frac{x^{10}}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \frac{x^{10}}{1+x^2+x^4} \, dx = x-\frac{x^5}{5}+\frac{x^7}{7}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
x-1/5*x^5+1/7*x^7+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/3*arctan(1/3*(
1+2*x)*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.85

$$\int \frac{x^{10}}{1+x^2+x^4} \, dx = x-\frac{x^5}{5}+\frac{x^7}{7}-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}} - \frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[x^10/(1 + x^2 + x^4),x]
```

output
```
x - x^5/5 + x^7/7 - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 +
(6*I)*Sqrt[3]] - ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[6 - (6*
I)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1442, 27, 1602, 27, 1442, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{x^4+x^2+1}dx$$
$$\Big\downarrow \text{1442}$$
$$\frac{x^7}{7}-\frac{1}{7}\int \frac{7x^6\left(x^2+1\right)}{x^4+x^2+1}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{x^7}{7}-\int \frac{x^6\left(x^2+1\right)}{x^4+x^2+1}dx$$
$$\Big\downarrow \text{1602}$$
$$\frac{1}{5}\int \frac{5x^4}{x^4+x^2+1}dx+\frac{x^7}{7}-\frac{x^5}{5}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{x^4}{x^4+x^2+1}dx+\frac{x^7}{7}-\frac{x^5}{5}$$
$$\Big\downarrow \text{1442}$$
$$-\int \frac{x^2+1}{x^4+x^2+1}dx+\frac{x^7}{7}-\frac{x^5}{5}+x$$
$$\Big\downarrow \text{1475}$$
$$-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{x^7}{7}-\frac{x^5}{5}+x$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{1}{-(2x-1)^2-3}d(2x-1)+\int \frac{1}{-(2x+1)^2-3}d(2x+1)+\frac{x^7}{7}-\frac{x^5}{5}+x$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^7}{7}-\frac{x^5}{5}+x$$

input
```
Int[x^10/(1 + x^2 + x^4),x]
```

output
```
x - x^5/5 + x^7/7 - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/
Sqrt[3]]/Sqrt[3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{x^{7}}{7}-\frac{x^{5}}{5}+x-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	45
risch	$\frac{x^{7}}{7}-\frac{x^{5}}{5}+x-\frac{\sqrt{3}\,\arctan\left(\frac{x^{3}\sqrt{3}}{3}+\frac{2\sqrt{3}\,x}{3}\right)}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{3}$	46

input
```
int(x^10/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7-1/5*x^5+x-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/3*3^(1/2)*arct
an(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{x^{10}}{1+x^2+x^4}\,dx = \frac{1}{7}\,x^7 - \frac{1}{5}\,x^5 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x^3+2\,x)\right) - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right) + x$$

input
```
integrate(x^10/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/7*x^7 - 1/5*x^5 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) - 1/3*sqrt
(3)*arctan(1/3*sqrt(3)*x) + x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{x^{10}}{1+x^2+x^4}\,dx = \frac{x^7}{7} - \frac{x^5}{5} + x + \frac{\sqrt{3}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right) - 2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3}\right)\right)}{6}$$

input
```
integrate(x**10/(x**4+x**2+1),x)
```

output
```
x**7/7 - x**5/5 + x + sqrt(3)*(-2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/
3 + 2*sqrt(3)*x/3))/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{x^{10}}{1+x^2+x^4}\,dx = \frac{1}{7}\,x^7 - \frac{1}{5}\,x^5 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + x$$

input
```
integrate(x^10/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/7*x^7 - 1/5*x^5 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - 1)) + x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{x^{10}}{1+x^2+x^4}\,dx = \frac{1}{7}\,x^7 - \frac{1}{5}\,x^5 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + x$$

input
```
integrate(x^10/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/7*x^7 - 1/5*x^5 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - 1)) + x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^{10}}{1+x^2+x^4}\,dx = x - \frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\,\sqrt{3}\,x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{6} - \frac{x^5}{5} + \frac{x^7}{7}$$

input
```
int(x^10/(x^2 + x^4 + 1),x)
```

output
```
x - (3^(1/2)*(2*atan((2*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) + 2*atan((3^(1/2)*
x)/3)))/6 - x^5/5 + x^7/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{x^{10}}{1+x^2+x^4}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} + \frac{x^7}{7} - \frac{x^5}{5} + x$$

input
```
int(x^10/(x^4+x^2+1),x)
```

output
```
( - 35*sqrt(3)*atan((2*x - 1)/sqrt(3)) - 35*sqrt(3)*atan((2*x + 1)/sqrt(3)
) + 15*x**7 - 21*x**5 + 105*x)/105
```

3.855 $\int \frac{x^4}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{x^4}{1+x^2+x^4} \, dx = x+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
x+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*
3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.15

$$\int \frac{x^4}{1+x^2+x^4} \, dx = x-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}}-\frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[x^4/(1 + x^2 + x^4),x]
```

output
```
x - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)*Sqrt[3]] -
 ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[6 - (6*I)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1442, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1442}$$

$$x-\int \frac{x^2+1}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2+x+1}dx+x$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{1}{-(2x-1)^2-3}d(2x-1)+\int \frac{1}{-(2x+1)^2-3}d(2x+1)+x$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+x$$

input
```
Int[x^4/(1 + x^2 + x^4),x]
```

output
```
x - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

method	result	size
default	$x-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	35
risch	$x-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{x^{3}\sqrt{3}}{3}+\frac{2\sqrt{3}\,x}{3}\right)}{3}$	36

input
```
int(x^4/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
x-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{1+x^2+x^4}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x^3+2\,x)\right) - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right) + x$$

input
```
integrate(x^4/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) - 1/3*sqrt(3)*arctan(1/3*sqrt
(3)*x) + x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{x^4}{1+x^2+x^4}\,dx = x + \frac{\sqrt{3}\left(-2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right) - 2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3} + \frac{2\sqrt{3}x}{3}\right)\right)}{6}$$

input
```
integrate(x**4/(x**4+x**2+1),x)
```

output
```
x + sqrt(3)*(-2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3)
)/6
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{1+x^2+x^4}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x$$

input
```
integrate(x^4/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{1+x^2+x^4}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x$$

input
```
integrate(x^4/(x^4+x^2+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{1+x^2+x^4}\,dx = x-\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\sqrt{3}\,x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{6}$$

input
```
int(x^4/(x^2 + x^4 + 1),x)
```

output
```
x - (3^(1/2)*(2*atan((2*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) + 2*atan((3^(1/2)*
x)/3)))/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{1+x^2+x^4}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} + x$$

input
```
int(x^4/(x^4+x^2+1),x)
```

output
```
( - sqrt(3)*atan((2*x - 1)/sqrt(3)) - sqrt(3)*atan((2*x + 1)/sqrt(3)) + 3*
x)/3
```

3.856 $\int \frac{1}{x^2(1+x^2+x^4)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{1}{x^2(1+x^2+x^4)}\,dx = -\frac{1}{x}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-1/x+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2
))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^2(1+x^2+x^4)}\,dx = -\frac{1}{x}-\frac{(i+\sqrt{3})\arctan\left(\frac{1}{2}(-i+\sqrt{3})x\right)}{\sqrt{6+6i\sqrt{3}}} -\frac{(-i+\sqrt{3})\arctan\left(\frac{1}{2}(i+\sqrt{3})x\right)}{\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[1/(x^2*(1 + x^2 + x^4)),x]
```

output
```
-x^(-1) - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)*Sqrt
[3]] - ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[6 - (6*I)*Sqrt[3]
]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1443, 25, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^4+x^2+1)} \, dx$$

$$\Big\downarrow \text{1443}$$

$$\int -\frac{x^2+1}{x^4+x^2+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x^2+1}{x^4+x^2+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{2} \int \frac{1}{x^2-x+1}dx-\frac{1}{2} \int \frac{1}{x^2+x+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{1}{-(2x-1)^2-3}d(2x-1)+\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{x}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x}$$

input
```
Int[1/(x^2*(1 + x^2 + x^4)),x]
```

output
```
-x^(-1) - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/S
qrt[3]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{1}{x}$	39
risch	$-\frac{1}{x}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{x^3\sqrt{3}}{3}+\frac{2\sqrt{3}\,x}{3}\right)}{3}$	40

input
```
int(1/x^2/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^
(1/2))-1/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2\left(1+x^2+x^4\right)}\,dx = -\frac{\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}(x^3+2x)\right)+\sqrt{3}x\arctan\left(\frac{1}{3}\sqrt{3}x\right)+3}{3x}$$

input
```
integrate(1/x^2/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(3)*x*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + sqrt(3)*x*arctan(1/3*sqr
t(3)*x) + 3)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2\left(1+x^2+x^4\right)}\,dx = \frac{\sqrt{3}\left(-2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)-2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{6}-\frac{1}{x}$$

input
```
integrate(1/x**2/(x**4+x**2+1),x)
```

output
```
sqrt(3)*(-2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/6
- 1/x
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2 (1 + x^2 + x^4)}\, dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{x}$$

input
```
integrate(1/x^2/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/x
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2 (1 + x^2 + x^4)}\, dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{x}$$

input
```
integrate(1/x^2/(x^4+x^2+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/x
```

Mupad [B] (verification not implemented)

Time = 17.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 (1 + x^2 + x^4)} \, dx = -\frac{\sqrt{3} \left(2 \operatorname{atan}\left(\frac{\sqrt{3}\, x^3}{3} + \frac{2\sqrt{3}\, x}{3}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{3}\, x}{3}\right)\right)}{6} - \frac{1}{x}$$

input
```
int(1/(x^2*(x^2 + x^4 + 1)),x)
```

output
```
- (3^(1/2)*(2*atan((2*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) + 2*atan((3^(1/2)*x)
/3)))/6 - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 (1 + x^2 + x^4)} \, dx = \frac{-\sqrt{3}\, atan\left(\frac{2x-1}{\sqrt{3}}\right) x - \sqrt{3}\, atan\left(\frac{2x+1}{\sqrt{3}}\right) x - 3}{3x}$$

input
```
int(1/x^2/(x^4+x^2+1),x)
```

output
```
( - sqrt(3)*atan((2*x - 1)/sqrt(3))*x - sqrt(3)*atan((2*x + 1)/sqrt(3))*x
- 3)/(3*x)
```

3.857 $\int \frac{1}{x^8(1+x^2+x^4)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{1}{x^8 (1+x^2+x^4)}\,dx = -\frac{1}{7x^7}+\frac{1}{5x^5}-\frac{1}{x}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-1/7/x^7+1/5/x^5-1/x+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/3*arctan(1/
3*(1+2*x)*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.79

$$\int \frac{1}{x^8 (1+x^2+x^4)}\,dx = -\frac{1}{7x^7}+\frac{1}{5x^5}-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}}-\frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[1/(x^8*(1 + x^2 + x^4)),x]
```

output
```
-1/7*1/x^7 + 1/(5*x^5) - x^(-1) - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x)
/2])/Sqrt[6 + (6*I)*Sqrt[3]] - ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x)/2]
)/Sqrt[6 - (6*I)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1443, 27, 1604, 27, 1443, 25, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (x^4+x^2+1)} \, dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{7} \int -\frac{7 (x^2+1)}{x^6 (x^4+x^2+1)}dx-\frac{1}{7 x^7}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^2+1}{x^6 (x^4+x^2+1)}dx-\frac{1}{7 x^7}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{5} \int \frac{5}{x^2 (x^4+x^2+1)}dx-\frac{1}{7 x^7}+\frac{1}{5 x^5}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^2 (x^4+x^2+1)}dx-\frac{1}{7 x^7}+\frac{1}{5 x^5}$$

$$\Big\downarrow \text{1443}$$

$$\int -\frac{x^2+1}{x^4+x^2+1}dx-\frac{1}{7 x^7}+\frac{1}{5 x^5}-\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x^2+1}{x^4+x^2+1}dx-\frac{1}{7 x^7}+\frac{1}{5 x^5}-\frac{1}{x}$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{2}\int \frac{1}{x^2-x+1}dx - \frac{1}{2}\int \frac{1}{x^2+x+1}dx - \frac{1}{7x^7} + \frac{1}{5x^5} - \frac{1}{x}$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{1}{-(2x-1)^2-3}d(2x-1) + \int \frac{1}{-(2x+1)^2-3}d(2x+1) - \frac{1}{7x^7} + \frac{1}{5x^5} - \frac{1}{x}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{7x^7} + \frac{1}{5x^5} - \frac{1}{x}$$

input
```
Int[1/(x^8*(1 + x^2 + x^4)),x]
```

output
```
-1/7*1/x^7 + 1/(5*x^5) - x^(-1) - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Arc
Tan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{1}{7x^{7}}-\frac{1}{x}+\frac{1}{5x^{5}}$	49
risch	$\frac{-x^{6}+\frac{1}{5}x^{2}-\frac{1}{7}}{x^{7}}-\frac{\sqrt{3}\,\arctan\left(\frac{x^{3}\sqrt{3}}{3}+\frac{2\sqrt{3}\,x}{3}\right)}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{3}$	51

input
```
int(1/x^8/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^
(1/2))-1/7/x^7-1/x+1/5/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^8 (1+x^2+x^4)} \, dx = -\frac{35 \sqrt{3} x^7 \arctan\left(\frac{1}{3} \sqrt{3}(x^3+2x)\right) + 35 \sqrt{3} x^7 \arctan\left(\frac{1}{3} \sqrt{3} x\right) + 105 x^6 - 21 x^2 + 15}{105 x^7}$$

input
```
integrate(1/x^8/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
-1/105*(35*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 35*sqrt(3)*x^7*ar
ctan(1/3*sqrt(3)*x) + 105*x^6 - 21*x^2 + 15)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^8 (1+x^2+x^4)} \, dx = \frac{\sqrt{3} \left(-2 \operatorname{atan}\left(\frac{\sqrt{3} x}{3}\right) - 2 \operatorname{atan}\left(\frac{\sqrt{3} x^3}{3} + \frac{2 \sqrt{3} x}{3}\right)\right)}{6} + \frac{-35 x^6 + 7 x^2 - 5}{35 x^7}$$

input
```
integrate(1/x**8/(x**4+x**2+1),x)
```

output
```
sqrt(3)*(-2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/6
+ (-35*x**6 + 7*x**2 - 5)/(35*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^8(1+x^2+x^4)} \, dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{35x^6-7x^2+5}{35x^7}$$

input
```
integrate(1/x^8/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/35*(35*x^6 - 7*x^2 + 5)/x^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^8(1+x^2+x^4)} \, dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{35x^6-7x^2+5}{35x^7}$$

input
```
integrate(1/x^8/(x^4+x^2+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/35*(35*x^6 - 7*x^2 + 5)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^8 (1+x^2+x^4)} \, dx = -\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\,\sqrt{3}\,x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{6}-\frac{x^6-\frac{x^2}{5}+\frac{1}{7}}{x^7}$$

input
```
int(1/(x^8*(x^2 + x^4 + 1)),x)
```

output
```
- (3^(1/2)*(2*atan((2*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) + 2*atan((3^(1/2)*x)
/3)))/6 - (x^6 - x^2/5 + 1/7)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^8 (1+x^2+x^4)} \, dx = \frac{-35\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^7-35\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^7-105x^6+21x^2-15}{105x^7}$$

input
```
int(1/x^8/(x^4+x^2+1),x)
```

output
```
( - 35*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**7 - 35*sqrt(3)*atan((2*x + 1)/sq
rt(3))*x**7 - 105*x**6 + 21*x**2 - 15)/(105*x**7)
```

3.858 $\int \frac{1}{x^{14}(1+x^2+x^4)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{1}{x^{14}\left(1+x^2+x^4\right)}\,dx=-\frac{1}{13x^{13}}+\frac{1}{11x^{11}}-\frac{1}{7x^7}+\frac{1}{5x^5}-\frac{1}{x}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-1/13/x^13+1/11/x^11-1/7/x^7+1/5/x^5-1/x+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3
^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.63

$$\int \frac{1}{x^{14}\left(1+x^2+x^4\right)}\,dx=-\frac{1}{13x^{13}}+\frac{1}{11x^{11}}-\frac{1}{7x^7}+\frac{1}{5x^5}-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}}-\frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[1/(x^14*(1 + x^2 + x^4)),x]
```

output
```
-1/13*1/x^13 + 1/(11*x^11) - 1/(7*x^7) + 1/(5*x^5) - x^(-1) - ((I + Sqrt[3
])*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)*Sqrt[3]] - ((-I + Sqrt[3])
*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[6 - (6*I)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {1443, 27, 1604, 27, 1443, 27, 1604, 27, 1443, 25, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{14} \left(x^4+x^2+1\right)} \, dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{13} \int -\frac{13 \left(x^2+1\right)}{x^{12} \left(x^4+x^2+1\right)}dx-\frac{1}{13 x^{13}}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^2+1}{x^{12} \left(x^4+x^2+1\right)}dx-\frac{1}{13 x^{13}}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{11} \int \frac{11}{x^8 \left(x^4+x^2+1\right)}dx-\frac{1}{13 x^{13}}+\frac{1}{11 x^{11}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^8 \left(x^4+x^2+1\right)}dx-\frac{1}{13 x^{13}}+\frac{1}{11 x^{11}}$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{7} \int -\frac{7 \left(x^2+1\right)}{x^6 \left(x^4+x^2+1\right)}dx-\frac{1}{13 x^{13}}+\frac{1}{11 x^{11}}-\frac{1}{7 x^7}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^2+1}{x^6 \left(x^4+x^2+1\right)}dx-\frac{1}{13 x^{13}}+\frac{1}{11 x^{11}}-\frac{1}{7 x^7}$$

$$\Big\downarrow \text{ 1604}$$

$$\frac{1}{5} \int \frac{5}{x^2 \left(x^4+x^2+1\right)}dx-\frac{1}{13 x^{13}}+\frac{1}{11 x^{11}}-\frac{1}{7 x^7}+\frac{1}{5 x^5}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{x^2 \left(x^4+x^2+1\right)}dx-\frac{1}{13 x^{13}}+\frac{1}{11 x^{11}}-\frac{1}{7 x^7}+\frac{1}{5 x^5}$$

$$\Big\downarrow \text{ 1443}$$

$$\int -\frac{x^2+1}{x^4+x^2+1}dx-\frac{1}{13 x^{13}}+\frac{1}{11 x^{11}}-\frac{1}{7 x^7}+\frac{1}{5 x^5}-\frac{1}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{x^2+1}{x^4+x^2+1}dx-\frac{1}{13 x^{13}}+\frac{1}{11 x^{11}}-\frac{1}{7 x^7}+\frac{1}{5 x^5}-\frac{1}{x}$$

$$\Big\downarrow \text{ 1475}$$

$$-\frac{1}{2} \int \frac{1}{x^2-x+1}dx-\frac{1}{2} \int \frac{1}{x^2+x+1}dx-\frac{1}{13 x^{13}}+\frac{1}{11 x^{11}}-\frac{1}{7 x^7}+\frac{1}{5 x^5}-\frac{1}{x}$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{1}{-(2 x-1)^2-3}d(2 x-1)+\int \frac{1}{-(2 x+1)^2-3}d(2 x+1)-\frac{1}{13 x^{13}}+\frac{1}{11 x^{11}}-\frac{1}{7 x^7}+\frac{1}{5 x^5}-\frac{1}{x}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{\arctan \left(\frac{2 x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan \left(\frac{2 x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{13 x^{13}}+\frac{1}{11 x^{11}}-\frac{1}{7 x^7}+\frac{1}{5 x^5}-\frac{1}{x}$$

input
```
Int[1/(x^14*(1 + x^2 + x^4)),x]
```

output
```
-1/13*1/x^13 + 1/(11*x^11) - 1/(7*x^7) + 1/(5*x^5) - x^(-1) - ArcTan[(-1 +
 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{1}{13x^{13}}-\frac{1}{7x^{7}}-\frac{1}{x}+\frac{1}{11x^{11}}+\frac{1}{5x^{5}}$	59
risch	$\frac{-x^{12}+\frac{1}{5}x^{8}-\frac{1}{7}x^{6}+\frac{1}{11}x^{2}-\frac{1}{13}}{x^{13}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x}{3}\right)}{3}-\frac{\sqrt{3}\,\arctan\left(\frac{x^{3}\sqrt{3}}{3}+\frac{2\sqrt{3}\,x}{3}\right)}{3}$	61

input
```
int(1/x^14/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^
(1/2))-1/13/x^13-1/7/x^7-1/x+1/11/x^11+1/5/x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^{14}\left(1+x^{2}+x^{4}\right)}\,dx = -\frac{5005\,\sqrt{3}x^{13}\arctan\left(\frac{1}{3}\,\sqrt{3}(x^{3}+2\,x)\right)+5005\,\sqrt{3}x^{13}\arctan\left(\frac{1}{3}\,\sqrt{3}x\right)+15015\,x^{12}-3003\,x^{8}+2145\,x^{6}-}{15015\,x^{13}}$$

input
```
integrate(1/x^14/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
-1/15015*(5005*sqrt(3)*x^13*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 5005*sqrt(3)
*x^13*arctan(1/3*sqrt(3)*x) + 15015*x^12 - 3003*x^8 + 2145*x^6 - 1365*x^2
+ 1155)/x^13
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^{14}(1+x^2+x^4)}\,dx = \frac{\sqrt{3}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)-2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{6} + \frac{-5005x^{12}+1001x^8-715x^6+455x^2-385}{5005x^{13}}$$

input
```
integrate(1/x**14/(x**4+x**2+1),x)
```

output
```
sqrt(3)*(-2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/6
+ (-5005*x**12 + 1001*x**8 - 715*x**6 + 455*x**2 - 385)/(5005*x**13)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{14}(1+x^2+x^4)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{5005\,x^{12}-1001\,x^8+715\,x^6-455\,x^2+385}{5005\,x^{13}}$$

input
```
integrate(1/x^14/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/5005*(5005*x^12 - 1001*x^8 + 715*x^6 - 455*x^2 + 385)/x^1
3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{14}\left(1+x^2+x^4\right)}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{5005x^{12} - 1001x^8 + 715x^6 - 455x^2 + 385}{5005x^{13}}$$

input
```
integrate(1/x^14/(x^4+x^2+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/5005*(5005*x^12 - 1001*x^8 + 715*x^6 - 455*x^2 + 385)/x^1
3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{14}\left(1+x^2+x^4\right)}\,dx = -\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}\right)\right)}{6} - \frac{x^{12}-\frac{x^8}{5}+\frac{x^6}{7}-\frac{x^2}{11}+\frac{1}{13}}{x^{13}}$$

input
```
int(1/(x^14*(x^2 + x^4 + 1)),x)
```

output
```
- (3^(1/2)*(2*atan((2*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) + 2*atan((3^(1/2)*x)
/3)))/6 - (x^6/7 - x^2/11 - x^8/5 + x^12 + 1/13)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^{14}(1+x^2+x^4)}\,dx = \frac{-5005\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^{13}-5005\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^{13}-15015x^{12}+3003x^{8}-2145x^{6}+1365x^{2}-1155}{15015x^{13}}$$

input
```
int(1/x^14/(x^4+x^2+1),x)
```

output
```
( - 5005*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**13 - 5005*sqrt(3)*atan((2*x +
1)/sqrt(3))*x**13 - 15015*x**12 + 3003*x**8 - 2145*x**6 + 1365*x**2 - 1155
)/(15015*x**13)
```

3.859 $\int \frac{x^{14}}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \frac{x^{14}}{1+x^2+x^4} \, dx = -\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^9}{9}+\frac{x^{11}}{11}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\text{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
-1/3*x^3+1/5*x^5-1/9*x^9+1/11*x^11-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/2*arctanh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.28

$$\int \frac{x^{14}}{1+x^2+x^4} \, dx = -\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^9}{9}+\frac{x^{11}}{11}+\frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}}+\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[x^14/(1 + x^2 + x^4),x]
```

output
```
-1/3*x^3 + x^5/5 - x^9/9 + x^11/11 + ((-I + Sqrt[3])*ArcTan[((-I + Sqrt[3]
)*x)/2])/Sqrt[6 + (6*I)*Sqrt[3]] + ((I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x)
/2])/Sqrt[6 - (6*I)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {1442, 27, 1602, 27, 1442, 27, 1602, 27, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x^{11}}{11}-\frac{1}{11}\int \frac{11x^{10}\left(x^2+1\right)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^{11}}{11}-\int \frac{x^{10}\left(x^2+1\right)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{9}\int \frac{9x^8}{x^4+x^2+1}dx+\frac{x^{11}}{11}-\frac{x^9}{9}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^8}{x^4+x^2+1}dx+\frac{x^{11}}{11}-\frac{x^9}{9}$$

$$\Big\downarrow \text{1442}$$

$$-\frac{1}{5}\int \frac{5x^4\left(x^2+1\right)}{x^4+x^2+1}dx+\frac{x^{11}}{11}-\frac{x^9}{9}+\frac{x^5}{5}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^4 (x^2+1)}{x^4+x^2+1}dx+\frac{x^{11}}{11}-\frac{x^9}{9}+\frac{x^5}{5}$$

$$\Big\downarrow \ 1602$$

$$\frac{1}{3}\int \frac{3x^2}{x^4+x^2+1}dx+\frac{x^{11}}{11}-\frac{x^9}{9}+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \ 27$$

$$\int \frac{x^2}{x^4+x^2+1}dx+\frac{x^{11}}{11}-\frac{x^9}{9}+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \ 1447$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx+\frac{x^{11}}{11}-\frac{x^9}{9}+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \ 1475$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{x^{11}}{11}-\frac{x^9}{9}+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \ 1083$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(-\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)+\frac{x^{11}}{11}-\frac{x^9}{9}+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \ 217$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{x^{11}}{11}-\frac{x^9}{9}+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \ 1478$$

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx+\frac{1}{2}\int -\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{x^{11}}{11}-\frac{x^9}{9}+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{x^{11}}{11}-\frac{x^9}{9}+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{x^{11}}{11}-\frac{x^9}{9}+\frac{x^5}{5}-\frac{x^3}{3}+\frac{1}{2}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)$$

input
```
Int[x^14/(1 + x^2 + x^4),x]
```

output
```
-1/3*x^3 + x^5/5 - x^9/9 + x^11/11 + (ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] +
 ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3])/2 + (Log[1 - x + x^2]/2 - Log[1 + x +
x^2]/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x^{11}}{11}-\frac{x^{9}}{9}+\frac{x^{5}}{5}-\frac{x^{3}}{3}-\frac{\ln\left(x^{2}+x+1\right)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln\left(x^{2}-x+1\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	74
risch	$\frac{x^{11}}{11}-\frac{x^{9}}{9}+\frac{x^{5}}{5}-\frac{x^{3}}{3}+\frac{\ln\left(4x^{2}-4x+4\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln\left(4x^{2}+4x+4\right)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	80

input
```
int(x^14/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/11*x^11-1/9*x^9+1/5*x^5-1/3*x^3-1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3
^(1/2))*3^(1/2)+1/4*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{x^{14}}{1+x^2+x^4}\,dx = \frac{1}{11}\,x^{11}-\frac{1}{9}\,x^{9}+\frac{1}{5}\,x^{5}-\frac{1}{3}\,x^{3}+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{4}\,\log\left(x^{2}+x+1\right)+\frac{1}{4}\,\log\left(x^{2}-x+1\right)$$

input
```
integrate(x^14/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/11*x^11 - 1/9*x^9 + 1/5*x^5 - 1/3*x^3 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(
2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*log(x^2 + x +
1) + 1/4*log(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int \frac{x^{14}}{1+x^2+x^4}\,dx = \frac{x^{11}}{11} - \frac{x^9}{9} + \frac{x^5}{5} - \frac{x^3}{3} + \frac{\log\left(x^2-x+1\right)}{4} - \frac{\log\left(x^2+x+1\right)}{4} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**14/(x**4+x**2+1),x)
```

output
```
x**11/11 - x**9/9 + x**5/5 - x**3/3 + log(x**2 - x + 1)/4 - log(x**2 + x +
 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)
*x/3 + sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{x^{14}}{1+x^2+x^4}\,dx = \frac{1}{11}\,x^{11} - \frac{1}{9}\,x^9 + \frac{1}{5}\,x^5 - \frac{1}{3}\,x^3 + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{4}\,\log\left(x^2+x+1\right) + \frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^14/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/11*x^11 - 1/9*x^9 + 1/5*x^5 - 1/3*x^3 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(
2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*log(x^2 + x +
1) + 1/4*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{x^{14}}{1+x^2+x^4}\,dx = \frac{1}{11}\,x^{11} - \frac{1}{9}\,x^9 + \frac{1}{5}\,x^5 - \frac{1}{3}\,x^3 + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{4}\,\log\left(x^2+x+1\right) + \frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^14/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/11*x^11 - 1/9*x^9 + 1/5*x^5 - 1/3*x^3 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(
2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*log(x^2 + x +
1) + 1/4*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 18.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{x^{14}}{1+x^2+x^4}\,dx = \frac{x^5}{5} + \mathrm{atan}\left(\frac{\sqrt{3}\,x}{2} + \frac{x\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6} + \frac{1}{2}\mathrm{i}\right) - \frac{x^3}{3} - \mathrm{atan}\left(-\frac{\sqrt{3}\,x}{2} + \frac{x\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6} - \frac{1}{2}\mathrm{i}\right) - \frac{x^9}{9} + \frac{x^{11}}{11}$$

input
```
int(x^14/(x^2 + x^4 + 1),x)
```

output
```
atan((x*1i)/2 + (3^(1/2)*x)/2)*(3^(1/2)/6 + 1i/2) - atan((x*1i)/2 - (3^(1/
2)*x)/2)*(3^(1/2)/6 - 1i/2) - x^3/3 + x^5/5 - x^9/9 + x^11/11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{x^{14}}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} + \frac{\log(x^2-x+1)}{4} - \frac{\log(x^2+x+1)}{4} + \frac{x^{11}}{11} - \frac{x^9}{9} + \frac{x^5}{5} - \frac{x^3}{3}$$

input
```
int(x^14/(x^4+x^2+1),x)
```

output
```
(330*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 330*sqrt(3)*atan((2*x + 1)/sqrt(3))
 + 495*log(x**2 - x + 1) - 495*log(x**2 + x + 1) + 180*x**11 - 220*x**9 +
396*x**5 - 660*x**3)/1980
```

3.860 $\int \frac{x^{12}}{1+x^2+x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 81

$$\int \frac{x^{12}}{1+x^2+x^4}\,dx=-x+\frac{x^3}{3}-\frac{x^7}{7}+\frac{x^9}{9}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}\text{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
-x+1/3*x^3-1/7*x^7+1/9*x^9-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arc
tan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/2*arctanh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.22

$$\int \frac{x^{12}}{1+x^2+x^4}\,dx=-x+\frac{x^3}{3}-\frac{x^7}{7}+\frac{x^9}{9}+\frac{\text{arctanh}\left(\frac{1}{2}\left(x-i\sqrt{3}x\right)\right)}{\sqrt{\frac{3}{2}\left(1-i\sqrt{3}\right)}}+\frac{\text{arctanh}\left(\frac{1}{2}\left(x+i\sqrt{3}x\right)\right)}{\sqrt{\frac{3}{2}\left(1+i\sqrt{3}\right)}}$$

input
```
Integrate[x^12/(1 + x^2 + x^4),x]
```

output
```
-x + x^3/3 - x^7/7 + x^9/9 + ArcTanh[(x - I*Sqrt[3]*x)/2]/Sqrt[(3*(1 - I*S
qrt[3]))/2] + ArcTanh[(x + I*Sqrt[3]*x)/2]/Sqrt[(3*(1 + I*Sqrt[3]))/2]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {1442, 27, 1602, 27, 1442, 27, 1602, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x^9}{9}-\frac{1}{9}\int \frac{9x^8\left(x^2+1\right)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^9}{9}-\int \frac{x^8\left(x^2+1\right)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{7}\int \frac{7x^6}{x^4+x^2+1}dx+\frac{x^9}{9}-\frac{x^7}{7}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^6}{x^4+x^2+1}dx+\frac{x^9}{9}-\frac{x^7}{7}$$

$$\Big\downarrow \text{1442}$$

$$-\frac{1}{3}\int \frac{3x^2\left(x^2+1\right)}{x^4+x^2+1}dx+\frac{x^9}{9}-\frac{x^7}{7}+\frac{x^3}{3}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^2 (x^2+1)}{x^4+x^2+1}dx+\frac{x^9}{9}-\frac{x^7}{7}+\frac{x^3}{3}$$

$$\Big\downarrow \text{ 1602}$$

$$\int \frac{1}{x^4+x^2+1}dx+\frac{x^9}{9}-\frac{x^7}{7}+\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 1407}$$

$$\frac{1}{2}\int \frac{1-x}{x^2-x+1}dx+\frac{1}{2}\int \frac{x+1}{x^2+x+1}dx+\frac{x^9}{9}-\frac{x^7}{7}+\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{x^9}{9}-\frac{x^7}{7}+\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{x^9}{9}-\frac{x^7}{7}+\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)+\frac{x^9}{9}-\frac{x^7}{7}+\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{x^9}{9}-\frac{x^7}{7}+\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^2+x+1\right)\right)+\frac{x^9}{9}-\frac{x^7}{7}+\frac{x^3}{3}-x$$

input
```
Int[x^12/(1 + x^2 + x^4),x]
```

output
```
-x + x^3/3 - x^7/7 + x^9/9 + (ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 -
 x + x^2]/2)/2 + (ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x + x^2]/2)/
2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{x^9}{9}-\frac{x^7}{7}+\frac{x^3}{3}-x+\frac{\ln\left(x^2+x+1\right)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln\left(x^2-x+1\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	72
risch	$\frac{x^9}{9}-\frac{x^7}{7}+\frac{x^3}{3}-x-\frac{\ln\left(4x^2-4x+4\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(4x^2+4x+4\right)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	78

input
```
int(x^12/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/9*x^9-1/7*x^7+1/3*x^3-x+1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*
3^(1/2)-1/4*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{x^{12}}{1+x^2+x^4}\,dx = \frac{1}{9}\,x^9 - \frac{1}{7}\,x^7 + \frac{1}{3}\,x^3 + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - x + \frac{1}{4}\,\log\left(x^2+x+1\right) - \frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^12/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/9*x^9 - 1/7*x^7 + 1/3*x^3 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) +
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/4*log(x^2 + x + 1) - 1/4
*log(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \frac{x^{12}}{1+x^2+x^4}\,dx = \frac{x^9}{9} - \frac{x^7}{7} + \frac{x^3}{3} - x - \frac{\log\left(x^2-x+1\right)}{4} + \frac{\log\left(x^2+x+1\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**12/(x**4+x**2+1),x)
```

output
```
x**9/9 - x**7/7 + x**3/3 - x - log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 +
 sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 +
sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{x^{12}}{1+x^2+x^4}\,dx = \frac{1}{9}\,x^9 - \frac{1}{7}\,x^7 + \frac{1}{3}\,x^3 + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - x + \frac{1}{4}\,\log\left(x^2+x+1\right) - \frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^12/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/9*x^9 - 1/7*x^7 + 1/3*x^3 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) +
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/4*log(x^2 + x + 1) - 1/4
*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{x^{12}}{1+x^2+x^4}\,dx = \frac{1}{9}\,x^9 - \frac{1}{7}\,x^7 + \frac{1}{3}\,x^3 + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - x + \frac{1}{4}\,\log\left(x^2+x+1\right) - \frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^12/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/9*x^9 - 1/7*x^7 + 1/3*x^3 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) +
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - x + 1/4*log(x^2 + x + 1) - 1/4
*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int \frac{x^{12}}{1+x^2+x^4}\,dx = \frac{x^3}{3} + \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{2}\mathrm{i}\right) + \operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{2}\mathrm{i}\right) - x - \frac{x^7}{7} + \frac{x^9}{9}$$

input
```
int(x^12/(x^2 + x^4 + 1),x)
```

output
```
atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/6 + 1i/2) - x + atan((x*2i)/(3^(1/2
)*1i + 1))*(3^(1/2)/6 - 1i/2) + x^3/3 - x^7/7 + x^9/9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{x^{12}}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} - \frac{\log(x^2-x+1)}{4} + \frac{\log(x^2+x+1)}{4} + \frac{x^9}{9} - \frac{x^7}{7} + \frac{x^3}{3} - x$$

input
```
int(x^12/(x^4+x^2+1),x)
```

output
```
(42*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 42*sqrt(3)*atan((2*x + 1)/sqrt(3)) -
 63*log(x**2 - x + 1) + 63*log(x**2 + x + 1) + 28*x**9 - 36*x**7 + 84*x**3
 - 252*x)/252
```

3.861 $\int \frac{x^8}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{x^8}{1+x^2+x^4} \, dx = -\frac{x^3}{3}+\frac{x^5}{5}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\text{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
-1/3*x^3+1/5*x^5-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1
+2*x)*3^(1/2))*3^(1/2)-1/2*arctanh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.34

$$\int \frac{x^8}{1+x^2+x^4} \, dx = -\frac{x^3}{3}+\frac{x^5}{5}+\frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}} + \frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[x^8/(1 + x^2 + x^4),x]
```

output
```
-1/3*x^3 + x^5/5 + ((-I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 +
(6*I)*Sqrt[3]] + ((I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[6 - (6*I
)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {1442, 27, 1602, 27, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^4+x^2+1} \, dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x^5}{5}-\frac{1}{5}\int \frac{5x^4\left(x^2+1\right)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^5}{5}-\int \frac{x^4\left(x^2+1\right)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{3}\int \frac{3x^2}{x^4+x^2+1}dx+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2}{x^4+x^2+1}dx+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \text{1447}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(-\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx+\frac{1}{2}\int -\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{x^5}{5}-\frac{x^3}{3}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{x^5}{5}-\frac{x^3}{3}+\frac{1}{2}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)$$

input
```
Int[x^8/(1 + x^2 + x^4),x]
```

output
```
-1/3*x^3 + x^5/5 + (ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/
Sqrt[3]]/Sqrt[3])/2 + (Log[1 - x + x^2]/2 - Log[1 + x + x^2]/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{x^5}{5}-\frac{x^3}{3}-\frac{\ln(x^2+x+1)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(x^2-x+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	64
risch	$\frac{x^5}{5}-\frac{x^3}{3}+\frac{\ln(4x^2-4x+4)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(4x^2+4x+4)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	70

input
```
int(x^8/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5-1/3*x^3-1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/
4*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{1+x^2+x^4}\,dx = \frac{1}{5}\,x^5 - \frac{1}{3}\,x^3 + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{4}\,\log\left(x^2+x+1\right) + \frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^8/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/5*x^5 - 1/3*x^3 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x +
 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \frac{x^8}{1+x^2+x^4}\,dx = \frac{x^5}{5} - \frac{x^3}{3} + \frac{\log\left(x^2-x+1\right)}{4} - \frac{\log\left(x^2+x+1\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**8/(x**4+x**2+1),x)
```

output
```
x**5/5 - x**3/3 + log(x**2 - x + 1)/4 - log(x**2 + x + 1)/4 + sqrt(3)*atan
(2*sqrt(3)*x/3 - sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{1+x^2+x^4}\,dx = \frac{1}{5}\,x^5 - \frac{1}{3}\,x^3 + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{4}\,\log\left(x^2+x+1\right) + \frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^8/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/5*x^5 - 1/3*x^3 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x +
 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{1+x^2+x^4}\,dx = \frac{1}{5}\,x^5 - \frac{1}{3}\,x^3 + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{4}\,\log\left(x^2+x+1\right) + \frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^8/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/5*x^5 - 1/3*x^3 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x +
 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{x^8}{1+x^2+x^4}\,dx = \frac{x^5}{5} + \operatorname{atan}\left(\frac{\sqrt{3}\,x}{2} + \frac{x\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6} + \frac{1}{2}\mathrm{i}\right) - \frac{x^3}{3} - \operatorname{atan}\left(-\frac{\sqrt{3}\,x}{2} + \frac{x\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6} - \frac{1}{2}\mathrm{i}\right)$$

input
```
int(x^8/(x^2 + x^4 + 1),x)
```

output
```
atan((x*1i)/2 + (3^(1/2)*x)/2)*(3^(1/2)/6 + 1i/2) - atan((x*1i)/2 - (3^(1/
2)*x)/2)*(3^(1/2)/6 - 1i/2) - x^3/3 + x^5/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{x^8}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6} + \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} + \frac{\log(x^2-x+1)}{4} - \frac{\log(x^2+x+1)}{4} + \frac{x^5}{5} - \frac{x^3}{3}$$

input
```
int(x^8/(x^4+x^2+1),x)
```

output
```
(10*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 10*sqrt(3)*atan((2*x + 1)/sqrt(3)) +
 15*log(x**2 - x + 1) - 15*log(x**2 + x + 1) + 12*x**5 - 20*x**3)/60
```

3.862 $\int \frac{x^6}{1+x^2+x^4} dx$

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{x^6}{1+x^2+x^4} dx = -x+\frac{x^3}{3}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}\operatorname{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
-x+1/3*x^3-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*
3^(1/2))*3^(1/2)+1/2*arctanh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int \frac{x^6}{1+x^2+x^4} dx = \frac{1}{6}\left(2x(-3+x^2)+i\sqrt{6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right) - i\sqrt{6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)\right)$$

input
```
Integrate[x^6/(1 + x^2 + x^4),x]
```

output
```
(2*x*(-3 + x^2) + I*Sqrt[6 - (6*I)*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] -
 I*Sqrt[6 + (6*I)*Sqrt[3]]*ArcTan[((I + Sqrt[3])*x)/2])/6
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1442, 27, 1602, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^4+x^2+1} dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x^3}{3}-\frac{1}{3}\int \frac{3x^2(x^2+1)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3}{3}-\int \frac{x^2(x^2+1)}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1602}$$

$$\int \frac{1}{x^4+x^2+1}dx+\frac{x^3}{3}-x$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\int \frac{1-x}{x^2-x+1}dx+\frac{1}{2}\int \frac{x+1}{x^2+x+1}dx+\frac{x^3}{3}-x$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{x^3}{3}-x$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)+\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^2+x+1\right)\right)+\frac{x^3}{3}-x$$

input
```
Int[x^6/(1 + x^2 + x^4),x]
```

output
```
-x + x^3/3 + (ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x + x^2]/2)/2 +
 (ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x + x^2]/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x^{3}}{3}-x+\frac{\ln\left(x^{2}+x+1\right)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln\left(x^{2}-x+1\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	62
risch	$\frac{x^{3}}{3}-x+\frac{\ln\left(4x^{2}+4x+4\right)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln\left(4x^{2}-4x+4\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	68

input
```
int(x^6/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3-x+1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*ln(x
^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{1+x^2+x^4}\,dx = \frac{1}{3}\,x^{3}+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-x+\frac{1}{4}\,\log\left(x^{2}+x+1\right)-\frac{1}{4}\,\log\left(x^{2}-x+1\right)$$

input
```
integrate(x^6/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x - 1)) - x + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \frac{x^6}{1+x^2+x^4}\,dx = \frac{x^3}{3} - x - \frac{\log(x^2-x+1)}{4} + \frac{\log(x^2+x+1)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**6/(x**4+x**2+1),x)
```

output
```
x**3/3 - x - log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sq
rt(3)*x/3 - sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{1+x^2+x^4}\,dx = \frac{1}{3}\,x^3 + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - x + \frac{1}{4}\,\log\left(x^2+x+1\right) - \frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^6/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x - 1)) - x + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{1+x^2+x^4} \, dx = \frac{1}{3} x^3 + \frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2x+1)\right) + \frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2x-1)\right) - x + \frac{1}{4} \log \left(x^2+x+1\right) - \frac{1}{4} \log \left(x^2-x+1\right)$$

input
```
integrate(x^6/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/3*x^3 + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x - 1)) - x + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{x^6}{1+x^2+x^4} \, dx = \frac{x^3}{3} + \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right) \left(\frac{\sqrt{3}}{6}+\frac{1}{2}\mathrm{i}\right) + \mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right) \left(\frac{\sqrt{3}}{6}-\frac{1}{2}\mathrm{i}\right) - x$$

input
```
int(x^6/(x^2 + x^4 + 1),x)
```

output
```
atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/6 + 1i/2) - x + atan((x*2i)/(3^(1/2
)*1i + 1))*(3^(1/2)/6 - 1i/2) + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6}+\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} - \frac{\log(x^2-x+1)}{4}+\frac{\log(x^2+x+1)}{4}+\frac{x^3}{3}-x$$

input
```
int(x^6/(x^4+x^2+1),x)
```

output
```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - 3
*log(x**2 - x + 1) + 3*log(x**2 + x + 1) + 4*x**3 - 12*x)/12
```

3.863 $\int \frac{x^2}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{x^2}{1+x^2+x^4} \, dx = -\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\operatorname{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)-1/2*arctanh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{x^2}{1+x^2+x^4} \, dx = \frac{\sqrt{1-i\sqrt{3}}\left(-i+\sqrt{3}\right) \arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right) + \sqrt{1+i\sqrt{3}}\left(i+\sqrt{3}\right) \arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{2\sqrt{6}}$$

input
```
Integrate[x^2/(1 + x^2 + x^4),x]
```

output
```
(Sqrt[1 - I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x)/2] + Sqrt[1
+ I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x)/2])/(2*Sqrt[6])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx+\frac{1}{2}\int -\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)$$

input
```
Int[x^2/(1 + x^2 + x^4),x]
```

output
```
(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3])/2
 + (Log[1 - x + x^2]/2 - Log[1 + x + x^2]/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\ln\left(x^{2}+x+1\right)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln\left(x^{2}-x+1\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	54
risch	$-\frac{\ln\left(4x^{2}+4x+4\right)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln\left(4x^{2}-4x+4\right)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	60

input
```
int(x^2/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*ln(x^2-x+1)+1
/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{1+x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) - \frac{1}{4}\,\log\left(x^2+x+1\right)+\frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^2/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{1+x^2+x^4}\,dx = \frac{\log\left(x^2-x+1\right)}{4}-\frac{\log\left(x^2+x+1\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**2/(x**4+x**2+1),x)
```

output
```
log(x**2 - x + 1)/4 - log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - s
qrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{4}\log\left(x^2+x+1\right) + \frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(x^2/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{4}\log\left(x^2+x+1\right) + \frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(x^2/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{1+x^2+x^4}\,dx = -\mathrm{atan}\left(-\frac{\sqrt{3}\,x}{2}+\frac{x\,1\mathrm{i}}{2}\right)\,\left(\frac{\sqrt{3}}{6}-\frac{1}{2}\mathrm{i}\right) + \mathrm{atan}\left(\frac{\sqrt{3}\,x}{2}+\frac{x\,1\mathrm{i}}{2}\right)\,\left(\frac{\sqrt{3}}{6}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int(x^2/(x^2 + x^4 + 1),x)
```

output
```
atan((x*1i)/2 + (3^(1/2)*x)/2)*(3^(1/2)/6 + 1i/2) - atan((x*1i)/2 - (3^(1/
2)*x)/2)*(3^(1/2)/6 - 1i/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6}+\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} + \frac{\log(x^2-x+1)}{4}-\frac{\log(x^2+x+1)}{4}$$

input
```
int(x^2/(x^4+x^2+1),x)
```

output
```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) + 3
*log(x**2 - x + 1) - 3*log(x**2 + x + 1))/12
```

3.864 $\int \frac{1}{1+x^2+x^4} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 57

$$\int \frac{1}{1+x^2+x^4} \, dx = -\frac{\arctan \left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan \left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{2}\text{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)+1/2*arctanh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{1}{1+x^2+x^4} \, dx = \frac{i\left(\sqrt{1-i\sqrt{3}}\arctan \left(\frac{1}{2}\left(-i+\sqrt{3}\right) x\right) - \sqrt{1+i\sqrt{3}}\arctan \left(\frac{1}{2}\left(i+\sqrt{3}\right) x\right)\right)}{\sqrt{6}}$$

input
```
Integrate[(1 + x^2 + x^4)^(-1),x]
```

output
```
(I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I*Sqrt[3]]
*ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+x^2+1}dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{1}{2}\int \frac{1-x}{x^2-x+1}dx+\frac{1}{2}\int \frac{x+1}{x^2+x+1}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^2+x+1\right)\right)$$

input
```
Int[(1 + x^2 + x^4)^(-1),x]
```

output
```
(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x + x^2]/2)/2 + (ArcTan[(1 +
 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x + x^2]/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(x^2+x+1)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln(x^2-x+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	54
risch	$-\frac{\ln(4x^2-4x+4)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(4x^2+4x+4)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	60

input
```
int(1/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*ln(x^2-x+1)+1/
6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(1/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{1}{1+x^2+x^4}\,dx = -\frac{\log(x^2-x+1)}{4} + \frac{\log(x^2+x+1)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(1/(x**4+x**2+1),x)
```

output
```
-log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 -
sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{4}\log\left(x^2+x+1\right) - \frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(1/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{1}{4}\,\log\left(x^2+x+1\right) - \frac{1}{4}\,\log\left(x^2-x+1\right)$$

input
```
integrate(1/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 18.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+x^2+x^4}\,dx = \operatorname{atanh}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \operatorname{atanh}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(1/(x^2 + x^4 + 1),x)
```

output
```
atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atanh((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/6 + 1/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{6}+\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{6} - \frac{\log(x^2-x+1)}{4}+\frac{\log(x^2+x+1)}{4}$$

input
```
int(1/(x^4+x^2+1),x)
```

output
```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - 3
*log(x**2 - x + 1) + 3*log(x**2 + x + 1))/12
```

3.865 $\int \frac{1}{x^4(1+x^2+x^4)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{1}{x^4(1+x^2+x^4)}\,dx = -\frac{1}{3x^3}+\frac{1}{x}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\text{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
-1/3/x^3+1/x-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x
)*3^(1/2))*3^(1/2)-1/2*arctanh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^4(1+x^2+x^4)}\,dx = -\frac{1}{3x^3}+\frac{1}{x}+\frac{(-i+\sqrt{3})\arctan\left(\frac{1}{2}(-i+\sqrt{3})\,x\right)}{\sqrt{6+6i\sqrt{3}}}+\frac{(i+\sqrt{3})\arctan\left(\frac{1}{2}(i+\sqrt{3})\,x\right)}{\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[1/(x^4*(1 + x^2 + x^4)),x]
```

output
```
-1/3*1/x^3 + x^(-1) + ((-I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6
 + (6*I)*Sqrt[3]] + ((I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[6 - (
6*I)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {1443, 27, 1604, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^4+x^2+1)}dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{3}\int -\frac{3(x^2+1)}{x^2 (x^4+x^2+1)}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^2+1}{x^2 (x^4+x^2+1)}dx-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1604}$$

$$\int \frac{x^2}{x^4+x^2+1}dx-\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{1447}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx-\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx-\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(-\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx+\frac{1}{2}\int -\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{3x^3}+\frac{1}{2}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)+\frac{1}{x}$$

input
```
Int[1/(x^4*(1 + x^2 + x^4)),x]
```

output
```
-1/3*1/x^3 + x^(-1) + (ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*
x)/Sqrt[3]]/Sqrt[3])/2 + (Log[1 - x + x^2]/2 - Log[1 + x + x^2]/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{x^2-\frac{1}{3}}{x^3}+\frac{\ln(x^2-x+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}-\frac{\ln(x^2+x+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$	59
default	$-\frac{\ln(x^2+x+1)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(x^2-x+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{1}{3x^3}+\frac{1}{x}$	62

input
```
int(1/x^4/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(x^2-1/3)/x^3+1/4*ln(x^2-x+1)+1/6*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))-1/4*
ln(x^2+x+1)+1/6*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^4 (1+x^2+x^4)} dx = \frac{2\sqrt{3}x^3 \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + 2\sqrt{3}x^3 \arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - 3x^3 \log\left(x^2+x+1\right) + 3x^3 \log\left(x^2-\right.}{12x^3}$$

input
```
integrate(1/x^4/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/12*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x + 1)) + 2*sqrt(3)*x^3*arctan(1
/3*sqrt(3)*(2*x - 1)) - 3*x^3*log(x^2 + x + 1) + 3*x^3*log(x^2 - x + 1) +
12*x^2 - 4)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^4 (1+x^2+x^4)} dx = \frac{\log(x^2-x+1)}{4} - \frac{\log(x^2+x+1)}{4} + \frac{\sqrt{3}\,\text{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\,\text{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6} + \frac{3x^2-1}{3x^3}$$

input
```
integrate(1/x**4/(x**4+x**2+1),x)
```

output
```
log(x**2 - x + 1)/4 - log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - s
qrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6 + (3*x**2 - 1)/(3*
x**3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^4 (1+x^2+x^4)} \, dx = \frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x+1)\right) + \frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x-1)\right) + \frac{3 x^2-1}{3 x^3} - \frac{1}{4} \log \left(x^2+x+1\right) + \frac{1}{4} \log \left(x^2-x+1\right)$$

input
```
integrate(1/x^4/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/3*(3*x^2 - 1)/x^3 - 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^4 (1+x^2+x^4)} \, dx = \frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x+1)\right) + \frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2 x-1)\right) + \frac{3 x^2-1}{3 x^3} - \frac{1}{4} \log \left(x^2+x+1\right) + \frac{1}{4} \log \left(x^2-x+1\right)$$

input
```
integrate(1/x^4/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/3*(3*x^2 - 1)/x^3 - 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x
 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^4 (1+x^2+x^4)} \, dx = \frac{x^2-\frac{1}{3}}{x^3} + \operatorname{atan}\left(\frac{\sqrt{3}\, x}{2} + \frac{x\, 1\mathrm{i}}{2}\right) \left(\frac{\sqrt{3}}{6} + \frac{1}{2}\mathrm{i}\right) - \operatorname{atan}\left(-\frac{\sqrt{3}\, x}{2} + \frac{x\, 1\mathrm{i}}{2}\right) \left(\frac{\sqrt{3}}{6} - \frac{1}{2}\mathrm{i}\right)$$

input
```
int(1/(x^4*(x^2 + x^4 + 1)),x)
```

output
```
atan((x*1i)/2 + (3^(1/2)*x)/2)*(3^(1/2)/6 + 1i/2) - atan((x*1i)/2 - (3^(1/
2)*x)/2)*(3^(1/2)/6 - 1i/2) + (x^2 - 1/3)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^4 (1+x^2+x^4)} \, dx = \frac{2\sqrt{3}\, atan\left(\frac{2x-1}{\sqrt{3}}\right) x^3 + 2\sqrt{3}\, atan\left(\frac{2x+1}{\sqrt{3}}\right) x^3 + 3 \log(x^2-x+1)\, x^3 - 3 \log(x^2+x+1)\, x^3 + 12x^2 - 4}{12x^3}$$

input
```
int(1/x^4/(x^4+x^2+1),x)
```

output
```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**3 + 2*sqrt(3)*atan((2*x + 1)/sqrt(3)
)*x**3 + 3*log(x**2 - x + 1)*x**3 - 3*log(x**2 + x + 1)*x**3 + 12*x**2 - 4
)/(12*x**3)
```

3.866 $\int \frac{1}{x^6(1+x^2+x^4)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{1}{x^6\left(1+x^2+x^4\right)}\,dx=-\frac{1}{5x^5}+\frac{1}{3x^3}-\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}\text{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
-1/5/x^5+1/3/x^3-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1
+2*x)*3^(1/2))*3^(1/2)+1/2*arctanh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^6\left(1+x^2+x^4\right)}\,dx = \frac{-6+10x^2+5i\sqrt{6-6i\sqrt{3}}x^5\arctan\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)-5i\sqrt{6+6i\sqrt{3}}x^5\arctan\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{30x^5}$$

input
```
Integrate[1/(x^6*(1 + x^2 + x^4)),x]
```

output
```
(-6 + 10*x^2 + (5*I)*Sqrt[6 - (6*I)*Sqrt[3]]*x^5*ArcTan[((-I + Sqrt[3])*x)
/2] - (5*I)*Sqrt[6 + (6*I)*Sqrt[3]]*x^5*ArcTan[((I + Sqrt[3])*x)/2])/(30*x
^5)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {1443, 27, 1604, 27, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (x^4+x^2+1)} dx$$

$$\Big\downarrow \text{ 1443}$$

$$\frac{1}{5} \int -\frac{5(x^2+1)}{x^4 (x^4+x^2+1)} dx - \frac{1}{5x^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{x^2+1}{x^4 (x^4+x^2+1)} dx - \frac{1}{5x^5}$$

$$\Big\downarrow \text{ 1604}$$

$$\frac{1}{3} \int \frac{3}{x^4+x^2+1} dx - \frac{1}{5x^5} + \frac{1}{3x^3}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{x^4+x^2+1} dx - \frac{1}{5x^5} + \frac{1}{3x^3}$$

$$\Big\downarrow \text{ 1407}$$

$$\frac{1}{2} \int \frac{1-x}{x^2-x+1} dx + \frac{1}{2} \int \frac{x+1}{x^2+x+1} dx - \frac{1}{5x^5} + \frac{1}{3x^3}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{1}{5x^5}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{1}{5x^5}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{1}{5x^5}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{5x^5}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^2+x+1\right)\right)-\frac{1}{5x^5}+\frac{1}{3x^3}$$

input
```
Int[1/(x^6*(1 + x^2 + x^4)),x]
```

output
```
-1/5*1/x^5 + 1/(3*x^3) + (ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x +
 x^2]/2)/2 + (ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x + x^2]/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\ln(x^2+x+1)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln(x^2-x+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{1}{5x^5}+\frac{1}{3x^3}$	64
risch	$\frac{\frac{x^2}{3}-\frac{1}{5}}{x^5}+\frac{\ln(4x^2+4x+4)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln(x^2-x+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$	67

input
```
int(1/x^6/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*ln(x^2-x+1)+1/
6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/5/x^5+1/3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^6\,(1+x^2+x^4)}\,dx$$
$$=\frac{10\,\sqrt{3}x^5\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+10\,\sqrt{3}x^5\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+15\,x^5\log\left(x^2+x+1\right)-15\,x^5\log\left(\right.}{60\,x^5}$$

input
```
integrate(1/x^6/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/60*(10*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x + 1)) + 10*sqrt(3)*x^5*arctan
(1/3*sqrt(3)*(2*x - 1)) + 15*x^5*log(x^2 + x + 1) - 15*x^5*log(x^2 - x + 1
) + 20*x^2 - 12)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^6(1+x^2+x^4)}\,dx = -\frac{\log(x^2-x+1)}{4} + \frac{\log(x^2+x+1)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6} + \frac{5x^2-3}{15x^5}$$

input
```
integrate(1/x**6/(x**4+x**2+1),x)
```

output
```
-log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 -
sqrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6 + (5*x**2 - 3)/(1
5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^6(1+x^2+x^4)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{5x^2-3}{15x^5} + \frac{1}{4}\log\left(x^2+x+1\right) - \frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(1/x^6/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/15*(5*x^2 - 3)/x^5 + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 -
x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^6 (1+x^2+x^4)} \, dx = \frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2x+1) \right) + \frac{1}{6} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3}(2x-1) \right) + \frac{5x^2-3}{15x^5} + \frac{1}{4} \log \left(x^2+x+1 \right) - \frac{1}{4} \log \left(x^2-x+1 \right)$$

input
```
integrate(1/x^6/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/15*(5*x^2 - 3)/x^5 + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 -
x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^6 (1+x^2+x^4)} \, dx = \operatorname{atanh}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}} \right) \left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{6} \right) + \operatorname{atanh}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}} \right) \left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{6} \right) + \frac{\frac{x^2}{3} - \frac{1}{5}}{x^5}$$

input
```
int(1/(x^6*(x^2 + x^4 + 1)),x)
```

output
```
atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atanh((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/6 + 1/2) + (x^2/3 - 1/5)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^6 (1 + x^2 + x^4)} \, dx$$

$$= \frac{10\sqrt{3}\, atan\left(\frac{2x-1}{\sqrt{3}}\right) x^5 + 10\sqrt{3}\, atan\left(\frac{2x+1}{\sqrt{3}}\right) x^5 - 15 \log(x^2 - x + 1)\, x^5 + 15 \log(x^2 + x + 1)\, x^5 + 20x^2 - 1}{60x^5}$$

input
```
int(1/x^6/(x^4+x^2+1),x)
```

output
```
(10*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**5 + 10*sqrt(3)*atan((2*x + 1)/sqrt(
3))*x**5 - 15*log(x**2 - x + 1)*x**5 + 15*log(x**2 + x + 1)*x**5 + 20*x**2
 - 12)/(60*x**5)
```

3.867 $\int \frac{1}{x^{10}(1+x^2+x^4)} dx$

Optimal result

Integrand size = 14, antiderivative size = 81

$$\int \frac{1}{x^{10}(1+x^2+x^4)} dx = -\frac{1}{9x^9} + \frac{1}{7x^7} - \frac{1}{3x^3} + \frac{1}{x} - \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\text{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
-1/9/x^9+1/7/x^7-1/3/x^3+1/x-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*a
rctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/2*arctanh(x/(x^2+1))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^{10}(1+x^2+x^4)} dx = -\frac{1}{9x^9} + \frac{1}{7x^7} - \frac{1}{3x^3} + \frac{1}{x} + \frac{(-i+\sqrt{3})\arctan\left(\frac{1}{2}(-i+\sqrt{3})x\right)}{\sqrt{6+6i\sqrt{3}}} + \frac{(i+\sqrt{3})\arctan\left(\frac{1}{2}(i+\sqrt{3})x\right)}{\sqrt{6-6i\sqrt{3}}}$$

input
```
Integrate[1/(x^10*(1 + x^2 + x^4)),x]
```

output
```
-1/9*1/x^9 + 1/(7*x^7) - 1/(3*x^3) + x^(-1) + ((-I + Sqrt[3])*ArcTan[((-I
+ Sqrt[3])*x)/2])/Sqrt[6 + (6*I)*Sqrt[3]] + ((I + Sqrt[3])*ArcTan[((I + Sq
rt[3])*x)/2])/Sqrt[6 - (6*I)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {1443, 27, 1604, 27, 1443, 27, 1604, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}(x^4+x^2+1)}dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{9}\int -\frac{9(x^2+1)}{x^8(x^4+x^2+1)}dx-\frac{1}{9x^9}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^2+1}{x^8(x^4+x^2+1)}dx-\frac{1}{9x^9}$$

$$\Big\downarrow \text{1604}$$

$$\frac{1}{7}\int \frac{7}{x^4(x^4+x^2+1)}dx-\frac{1}{9x^9}+\frac{1}{7x^7}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{x^4(x^4+x^2+1)}dx-\frac{1}{9x^9}+\frac{1}{7x^7}$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{3}\int -\frac{3(x^2+1)}{x^2(x^4+x^2+1)}dx-\frac{1}{9x^9}+\frac{1}{7x^7}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^2+1}{x^2\left(x^4+x^2+1\right)}dx-\frac{1}{9x^9}+\frac{1}{7x^7}-\frac{1}{3x^3}$$

$\Big\downarrow$ 1604

$$\int \frac{x^2}{x^4+x^2+1}dx-\frac{1}{9x^9}+\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{x}$$

$\Big\downarrow$ 1447

$$-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx-\frac{1}{9x^9}+\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{x}$$

$\Big\downarrow$ 1475

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1}{x^2+x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx-\frac{1}{9x^9}+\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{x}$$

$\Big\downarrow$ 1083

$$-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(-\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{1}{9x^9}+\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{x}$$

$\Big\downarrow$ 217

$$-\frac{1}{2}\int \frac{1-x^2}{x^4+x^2+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{9x^9}+\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{x}$$

$\Big\downarrow$ 1478

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx+\frac{1}{2}\int -\frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{9x^9}+\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{x}$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{9x^9}+\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{x}$$

$\Big\downarrow$ 1103

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}\right)-\frac{1}{9x^9}+\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{2}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)+\frac{1}{x}$$

input
```
Int[1/(x^10*(1 + x^2 + x^4)),x]
```

output
```
-1/9*1/x^9 + 1/(7*x^7) - 1/(3*x^3) + x^(-1) + (ArcTan[(-1 + 2*x)/Sqrt[3]]/
Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3])/2 + (Log[1 - x + x^2]/2 - Log
[1 + x + x^2]/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\ln(x^2+x+1)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(x^2-x+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{1}{9x^9}-\frac{1}{3x^3}+\frac{1}{7x^7}+\frac{1}{x}$	72
risch	$\frac{x^8-\frac{1}{3}x^6+\frac{1}{7}x^2-\frac{1}{9}}{x^9}-\frac{\ln(4x^2+4x+4)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(x^2-x+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$	75

input
```
int(1/x^10/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/4*ln(x^2-x+1)+1
/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/9/x^9-1/3/x^3+1/7/x^7+1/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^{10}(1+x^2+x^4)}\,dx = \frac{42\sqrt{3}x^9\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+42\sqrt{3}x^9\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-63x^9\log\left(x^2+x+1\right)+63x^9\log\left(x\right.}{252x^9}$$

input
```
integrate(1/x^10/(x^4+x^2+1),x, algorithm="fricas")
```

output
```
1/252*(42*sqrt(3)*x^9*arctan(1/3*sqrt(3)*(2*x + 1)) + 42*sqrt(3)*x^9*arcta
n(1/3*sqrt(3)*(2*x - 1)) - 63*x^9*log(x^2 + x + 1) + 63*x^9*log(x^2 - x +
1) + 252*x^8 - 84*x^6 + 36*x^2 - 28)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^{10}\left(1+x^2+x^4\right)}\,dx = \frac{\log\left(x^2-x+1\right)}{4} - \frac{\log\left(x^2+x+1\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6} + \frac{63x^8-21x^6+9x^2-7}{63x^9}$$

input
```
integrate(1/x**10/(x**4+x**2+1),x)
```

output
```
log(x**2 - x + 1)/4 - log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - s
qrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6 + (63*x**8 - 21*x*
*6 + 9*x**2 - 7)/(63*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^{10}\left(1+x^2+x^4\right)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{63\,x^8-21\,x^6+9\,x^2-7}{63\,x^9} - \frac{1}{4}\log\left(x^2+x+1\right) + \frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(1/x^10/(x^4+x^2+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/63*(63*x^8 - 21*x^6 + 9*x^2 - 7)/x^9 - 1/4*log(x^2 + x + 1
) + 1/4*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^{10}(1+x^2+x^4)}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{63x^8-21x^6+9x^2-7}{63x^9} - \frac{1}{4}\log\left(x^2+x+1\right) + \frac{1}{4}\log\left(x^2-x+1\right)$$

input
```
integrate(1/x^10/(x^4+x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/63*(63*x^8 - 21*x^6 + 9*x^2 - 7)/x^9 - 1/4*log(x^2 + x + 1
) + 1/4*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{10}(1+x^2+x^4)}\,dx = \frac{x^8-\frac{x^6}{3}+\frac{x^2}{7}-\frac{1}{9}}{x^9} + \operatorname{atan}\left(\frac{\sqrt{3}\,x}{2}+\frac{x\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{2}\mathrm{i}\right) - \operatorname{atan}\left(-\frac{\sqrt{3}\,x}{2}+\frac{x\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int(1/(x^10*(x^2 + x^4 + 1)),x)
```

output
```
atan((x*1i)/2 + (3^(1/2)*x)/2)*(3^(1/2)/6 + 1i/2) - atan((x*1i)/2 - (3^(1/
2)*x)/2)*(3^(1/2)/6 - 1i/2) + (x^2/7 - x^6/3 + x^8 - 1/9)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{10}(1+x^2+x^4)}\,dx = \frac{42\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^9+42\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^9+63\log(x^2-x+1)\,x^9-63\log(x^2+x+1)\,x^9+252x^8-}{252x^9}$$

input
```
int(1/x^10/(x^4+x^2+1),x)
```

output
```
(42*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**9 + 42*sqrt(3)*atan((2*x + 1)/sqrt(
3))*x**9 + 63*log(x**2 - x + 1)*x**9 - 63*log(x**2 + x + 1)*x**9 + 252*x**
8 - 84*x**6 + 36*x**2 - 28)/(252*x**9)
```

3.868 $\int \frac{x^5}{1-x^2+x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{x^5}{1-x^2+x^4}\,dx = \frac{x^2}{2}+\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{4}\log\left(1-x^2+x^4\right)$$

output
```
1/2*x^2+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)+1/4*ln(x^4-x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{1-x^2+x^4}\,dx = \frac{x^2}{2}-\frac{\arctan\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{4}\log\left(1-x^2+x^4\right)$$

input
```
Integrate[x^5/(1 - x^2 + x^4),x]
```

output
```
x^2/2 - ArcTan[(-1 + 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 - x^2 + x^4]/4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1434, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{x^4}{x^4-x^2+1}dx^2$$

$$\Big\downarrow \text{ 1143}$$

$$\frac{1}{2}\int \left(1-\frac{1-x^2}{x^4-x^2+1}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}+x^2+\frac{1}{2}\log\left(x^4-x^2+1\right)\right)$$

input
```
Int[x^5/(1 - x^2 + x^4),x]
```

output
```
(x^2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] + Log[1 - x^2 + x^4]/2)/2
```

Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^2}{2}+\frac{\ln(x^4-x^2+1)}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{6}$	38
risch	$\frac{x^2}{2}+\frac{\ln(4x^4-4x^2+4)}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{6}$	40

input
```
int(x^5/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/4*ln(x^4-x^2+1)-1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{1-x^2+x^4}\,dx = \frac{1}{2}\,x^2-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right)+\frac{1}{4}\,\log\left(x^4-x^2+1\right)$$

input
```
integrate(x^5/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/4*log(x^4 - x^2
+ 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{1-x^2+x^4}\,dx = \frac{x^2}{2} + \frac{\log\left(x^4-x^2+1\right)}{4} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**5/(x**4-x**2+1),x)
```

output
```
x**2/2 + log(x**4 - x**2 + 1)/4 - sqrt(3)*atan(2*sqrt(3)*x**2/3 - sqrt(3)/
3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{1-x^2+x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) + \frac{1}{4}\,\log\left(x^4-x^2+1\right)$$

input
```
integrate(x^5/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/4*log(x^4 - x^2
+ 1)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{1-x^2+x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) + \frac{1}{4}\,\log\left(x^4-x^2+1\right)$$

input
```
integrate(x^5/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/4*log(x^4 - x^2
+ 1)
```

Mupad [B] (verification not implemented)

Time = 17.68 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{1-x^2+x^4}\,dx=\frac{\ln\left(x^4-x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^2}{3}\right)}{6}+\frac{x^2}{2}$$

input
```
int(x^5/(x^4 - x^2 + 1),x)
```

output
```
log(x^4 - x^2 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^2)/3))/6 + x
^2/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{1-x^2+x^4}\,dx=\frac{\sqrt{3}\,atan\left(\sqrt{3}-2x\right)}{6}+\frac{\sqrt{3}\,atan\left(\sqrt{3}+2x\right)}{6}+\frac{\log\left(-\sqrt{3}\,x+x^2+1\right)}{4}+\frac{\log\left(\sqrt{3}\,x+x^2+1\right)}{4}+\frac{x^2}{2}$$

input
```
int(x^5/(x^4-x^2+1),x)
```

output
```
(2*sqrt(3)*atan(sqrt(3) - 2*x) + 2*sqrt(3)*atan(sqrt(3) + 2*x) + 3*log( -
sqrt(3)*x + x**2 + 1) + 3*log(sqrt(3)*x + x**2 + 1) + 6*x**2)/12
```

3.869 $\int \frac{x^3}{1-x^2+x^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{x^3}{1-x^2+x^4} \, dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{4}\log\left(1-x^2+x^4\right)$$

output
```
-1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)+1/4*ln(x^4-x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{1-x^2+x^4} \, dx = \frac{\arctan\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{4}\log\left(1-x^2+x^4\right)$$

input
```
Integrate[x^3/(1 - x^2 + x^4),x]
```

output
```
ArcTan[(-1 + 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 - x^2 + x^4]/4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1434, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^4-x^2+1}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{x^4-x^2+1}dx^2$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx^2+\frac{1}{2}\int -\frac{1-2x^2}{x^4-x^2+1}dx^2\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx^2-\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\int \frac{1}{-x^4-3}d\left(2x^2-1\right)-\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^4-x^2+1\right)\right)$$

input
```
Int[x^3/(1 - x^2 + x^4),x]
```

output
```
(ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3] + Log[1 - x^2 + x^4]/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln(x^4-x^2+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{6}$	33
risch	$\frac{\ln(4x^4-4x^2+4)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{6}$	35

input
```
int(x^3/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^4-x^2+1)+1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1-x^2+x^4}\,dx=\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)+\frac{1}{4}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x^3/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/4*log(x^4 - x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{1-x^2+x^4}\,dx=\frac{\log\left(x^4-x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(x**3/(x**4-x**2+1),x)
```

output
```
log(x**4 - x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 - sqrt(3)/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1-x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)+\frac{1}{4}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x^3/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/4*log(x^4 - x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{1-x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)+\frac{1}{4}\log\left(x^4-x^2+1\right)$$

input
```
integrate(x^3/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/4*log(x^4 - x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{1-x^2+x^4}\,dx = \frac{\ln\left(x^4-x^2+1\right)}{4}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^2}{3}\right)}{6}$$

input
```
int(x^3/(x^4 - x^2 + 1),x)
```

output
```
log(x^4 - x^2 + 1)/4 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^2)/3))/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{x^3}{1-x^2+x^4}\,dx = -\frac{\sqrt{3}\,atan\left(\sqrt{3}-2x\right)}{6} - \frac{\sqrt{3}\,atan\left(\sqrt{3}+2x\right)}{6} + \frac{\log\left(-\sqrt{3}\,x+x^2+1\right)}{4} + \frac{\log\left(\sqrt{3}\,x+x^2+1\right)}{4}$$

input
```
int(x^3/(x^4-x^2+1),x)
```

output
```
( - 2*sqrt(3)*atan(sqrt(3) - 2*x) - 2*sqrt(3)*atan(sqrt(3) + 2*x) + 3*log(
 - sqrt(3)*x + x**2 + 1) + 3*log(sqrt(3)*x + x**2 + 1))/12
```

3.870 $\int \frac{x}{1-x^2+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{x}{1-x^2+x^4} \, dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-1/3*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{x}{1-x^2+x^4} \, dx = \frac{\arctan\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[x/(1 - x^2 + x^4),x]
```

output
```
ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1432, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx^2$$

$$\Big\downarrow \text{1083}$$

$$-\int \frac{1}{-x^4-3}d\left(2x^2-1\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[x/(1 - x^2 + x^4),x]
```

output
```
ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{3}$	19
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{3}$	19

input
```
int(x/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{x}{1-x^2+x^4}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right)$$

input
```
integrate(x/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{x}{1-x^2+x^4}\,dx = \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x/(x**4-x**2+1),x)
```

output
```
sqrt(3)*atan(2*sqrt(3)*x**2/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{x}{1-x^2+x^4}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)$$

input
```
integrate(x/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{x}{1-x^2+x^4}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)$$

input
```
integrate(x/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{x}{1-x^2+x^4}\,dx = -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^2}{3}\right)}{3}$$

input
```
int(x/(x^4 - x^2 + 1),x)
```

output
```
-(3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^2)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x}{1-x^2+x^4}\,dx = -\frac{\sqrt{3}\,\left(atan\left(\sqrt{3}-2x\right)+atan\left(\sqrt{3}+2x\right)\right)}{3}$$

input
```
int(x/(x^4-x^2+1),x)
```

output
```
( - sqrt(3)*(atan(sqrt(3) - 2*x) + atan(sqrt(3) + 2*x)))/3
```

3.871 $\int \frac{1}{x(1-x^2+x^4)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 41

$$\int \frac{1}{x(1-x^2+x^4)} \, dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \log(x) - \frac{1}{4}\log\left(1-x^2+x^4\right)$$

output
```
-1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)+ln(x)-1/4*ln(x^4-x^2+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1-x^2+x^4)} \, dx = -\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \log(x) - \frac{1}{4}\log\left(1-x^2+x^4\right)$$

input
```
Integrate[1/(x*(1 - x^2 + x^4)),x]
```

output
```
-1/2*ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[1 - x^2 + x^4]/4
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1434, 1144, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(x^4-x^2+1\right)}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(x^4-x^2+1\right)}dx^2$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{2}\left(\int \frac{1-x^2}{x^4-x^2+1}dx^2+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx^2-\frac{1}{2}\int -\frac{1-2x^2}{x^4-x^2+1}dx^2+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-x^2+1}dx^2+\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\int \frac{1}{-x^4-3}d\left(2x^2-1\right)+\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-2x^2}{x^4-x^2+1}dx^2+\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^2\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x^2-1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^2\right)-\frac{1}{2}\log\left(x^4-x^2+1\right)\right)$$

input
```
Int[1/(x*(1 - x^2 + x^4)),x]
```

output
```
(ArcTan[(-1 + 2*x^2)/Sqrt[3]]/Sqrt[3] + Log[x^2] - Log[1 - x^2 + x^4]/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln(x)-\frac{\ln(x^4-x^2+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^2-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$	33
default	$\ln(x)-\frac{\ln(x^4-x^2+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{6}$	35

input
```
int(1/x/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/4*ln(x^4-x^2+1)+1/6*3^(1/2)*arctan(2/3*(x^2-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(1-x^2+x^4)}\,dx=\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right)-\frac{1}{4}\log\left(x^4-x^2+1\right)+\log(x)$$

input
```
integrate(1/x/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/4*log(x^4 - x^2 + 1) + log
(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1-x^{2}+x^{4}\right)}\,dx = \log\left(x\right) - \frac{\log\left(x^{4}-x^{2}+1\right)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^{2}}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

input
```
integrate(1/x/(x**4-x**2+1),x)
```

output
```
log(x) - log(x**4 - x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 - sqrt(3)/
3)/6
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(1-x^{2}+x^{4}\right)}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^{2}-1)\right) - \frac{1}{4}\,\log\left(x^{4}-x^{2}+1\right) + \frac{1}{2}\,\log\left(x^{2}\right)$$

input
```
integrate(1/x/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/4*log(x^4 - x^2 + 1) + 1/2
*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(1-x^{2}+x^{4}\right)}\,dx = \frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^{2}-1)\right) - \frac{1}{4}\,\log\left(x^{4}-x^{2}+1\right) + \frac{1}{2}\,\log\left(x^{2}\right)$$

input
```
integrate(1/x/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/4*log(x^4 - x^2 + 1) + 1/2
*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 17.95 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\,(1-x^2+x^4)}\,dx = \ln(x) - \frac{\ln(x^4-x^2+1)}{4} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^2}{3}\right)}{6}$$

input
```
int(1/(x*(x^4 - x^2 + 1)),x)
```

output
```
log(x) - log(x^4 - x^2 + 1)/4 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^2)/
3))/6
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\,(1-x^2+x^4)}\,dx = -\frac{\sqrt{3}\,atan(\sqrt{3}-2x)}{6} - \frac{\sqrt{3}\,atan(\sqrt{3}+2x)}{6} - \frac{\log(-\sqrt{3}\,x+x^2+1)}{4} - \frac{\log(\sqrt{3}\,x+x^2+1)}{4} + \log(x)$$

input
```
int(1/x/(x^4-x^2+1),x)
```

output
```
( - 2*sqrt(3)*atan(sqrt(3) - 2*x) - 2*sqrt(3)*atan(sqrt(3) + 2*x) - 3*log(
 - sqrt(3)*x + x**2 + 1) - 3*log(sqrt(3)*x + x**2 + 1) + 12*log(x))/12
```

3.872 $\int \frac{1}{x^3(1-x^2+x^4)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{x^3(1-x^2+x^4)}\,dx = -\frac{1}{2x^2}+\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}+\log(x)-\frac{1}{4}\log\left(1-x^2+x^4\right)$$

output
```
-1/2/x^2+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)+ln(x)-1/4*ln(x^4-x^2+1
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^3(1-x^2+x^4)}\,dx = \frac{1}{12}\left(-\frac{6}{x^2}+12\log(x)-\sqrt{3}\left(i+\sqrt{3}\right)\log\left(i+\sqrt{3}-2ix^2\right) - \sqrt{3}\left(-i+\sqrt{3}\right)\log\left(-i+\sqrt{3}+2ix^2\right)\right)$$

input
```
Integrate[1/(x^3*(1 - x^2 + x^4)),x]
```

output
```
(-6/x^2 + 12*Log[x] - Sqrt[3]*(I + Sqrt[3])*Log[I + Sqrt[3] - (2*I)*x^2] -
 Sqrt[3]*(-I + Sqrt[3])*Log[-I + Sqrt[3] + (2*I)*x^2])/12
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1434, 1145, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^4-x^2+1)}dx$$
$$\Big\downarrow \text{1434}$$
$$\frac{1}{2}\int \frac{1}{x^4 (x^4-x^2+1)}dx^2$$
$$\Big\downarrow \text{1145}$$
$$\frac{1}{2}\left(\int \frac{1-x^2}{x^2 (x^4-x^2+1)}dx^2-\frac{1}{x^2}\right)$$
$$\Big\downarrow \text{1200}$$
$$\frac{1}{2}\left(\int \left(\frac{1}{x^2}-\frac{x^2}{x^4-x^2+1}\right)dx^2-\frac{1}{x^2}\right)$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{x^2}+\log\left(x^2\right)-\frac{1}{2}\log\left(x^4-x^2+1\right)\right)$$

input
```
Int[1/(x^3*(1 - x^2 + x^4)),x]
```

output
```
(-x^(-2) + ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] + Log[x^2] - Log[1 - x^2 +
x^4]/2)/2
```

Defintions of rubi rules used

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{1}{2x^2}+\ln\left(x\right)-\frac{\ln\left(x^4-x^2+1\right)}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^2-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$	38
default	$-\frac{1}{2x^2}+\ln\left(x\right)-\frac{\ln\left(x^4-x^2+1\right)}{4}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{6}$	40

input
```
int(1/x^3/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2+ln(x)-1/4*ln(x^4-x^2+1)-1/6*3^(1/2)*arctan(2/3*(x^2-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 (1 - x^2 + x^4)} \, dx = -\frac{2 \sqrt{3} x^2 \arctan\left(\frac{1}{3} \sqrt{3} (2 x^2 - 1)\right) + 3 x^2 \log\left(x^4 - x^2 + 1\right) - 12 x^2 \log\left(x\right) + 6}{12 x^2}$$

input
```
integrate(1/x^3/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
-1/12*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 3*x^2*log(x^4 - x^2
 + 1) - 12*x^2*log(x) + 6)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (1 - x^2 + x^4)} \, dx = \log(x) - \frac{\log(x^4 - x^2 + 1)}{4} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{6} - \frac{1}{2x^2}$$

input
```
integrate(1/x**3/(x**4-x**2+1),x)
```

output
```
log(x) - log(x**4 - x**2 + 1)/4 - sqrt(3)*atan(2*sqrt(3)*x**2/3 - sqrt(3)/
3)/6 - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3 (1 - x^2 + x^4)} \, dx = -\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x^2 - 1)\right) - \frac{1}{2 x^2} - \frac{1}{4} \log\left(x^4 - x^2 + 1\right) + \frac{1}{2} \log\left(x^2\right)$$

input
```
integrate(1/x^3/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/2/x^2 - 1/4*log(x^4 - x^2
 + 1) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(1-x^2+x^4)}\,dx = -\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) - \frac{x^2+1}{2x^2} - \frac{1}{4}\log\left(x^4-x^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x^3/(x^4-x^2+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/2*(x^2 + 1)/x^2 - 1/4*log
(x^4 - x^2 + 1) + 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3(1-x^2+x^4)}\,dx = \ln(x) - \frac{\ln(x^4-x^2+1)}{4} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^2}{3}\right)}{6} - \frac{1}{2x^2}$$

input
```
int(1/(x^3*(x^4 - x^2 + 1)),x)
```

output
```
log(x) - log(x^4 - x^2 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^2)/
3))/6 - 1/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^3 (1 - x^2 + x^4)} \, dx$$
$$= \frac{2\sqrt{3}\, atan(\sqrt{3} - 2x)\, x^2 + 2\sqrt{3}\, atan(\sqrt{3} + 2x)\, x^2 - 3\log(-\sqrt{3}\, x + x^2 + 1)\, x^2 - 3\log(\sqrt{3}\, x + x^2 + 1)\, x^2}{12x^2}$$

input
```
int(1/x^3/(x^4-x^2+1),x)
```

output
```
(2*sqrt(3)*atan(sqrt(3) - 2*x)*x**2 + 2*sqrt(3)*atan(sqrt(3) + 2*x)*x**2 -
 3*log( - sqrt(3)*x + x**2 + 1)*x**2 - 3*log(sqrt(3)*x + x**2 + 1)*x**2 +
12*log(x)*x**2 - 6)/(12*x**2)
```

3.873 $\int \frac{1}{x^5(1-x^2+x^4)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{1}{x^5(1-x^2+x^4)}\,dx = -\frac{1}{4x^4}-\frac{1}{2x^2}+\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-1/4/x^4-1/2/x^2+1/3*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.03

$$\int \frac{1}{x^5(1-x^2+x^4)}\,dx = -\frac{1}{4x^4}-\frac{1}{2x^2}-\frac{i\log\left(i+\sqrt{3}-2ix^2\right)}{2\sqrt{3}}+\frac{i\log\left(-i+\sqrt{3}+2ix^2\right)}{2\sqrt{3}}$$

input
```
Integrate[1/(x^5*(1 - x^2 + x^4)),x]
```

output
```
-1/4*1/x^4 - 1/(2*x^2) - ((I/2)*Log[I + Sqrt[3] - (2*I)*x^2])/Sqrt[3] + ((
I/2)*Log[-I + Sqrt[3] + (2*I)*x^2])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1434, 1145, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^4-x^2+1)} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^6 (x^4-x^2+1)}dx^2$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{2}\left(\int \frac{1-x^2}{x^4 (x^4-x^2+1)}dx^2-\frac{1}{2 x^4}\right)$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{2}\left(\int \left(\frac{1}{x^4}+\frac{1}{-x^4+x^2-1}\right)dx^2-\frac{1}{2 x^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2 \arctan \left(\frac{1-2 x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2 x^4}-\frac{1}{x^2}\right)$$

input
```
Int[1/(x^5*(1 - x^2 + x^4)),x]
```

output
```
(-1/2*1/x^4 - x^(-2) + (2*ArcTan[(1 - 2*x^2)/Sqrt[3]])/Sqrt[3])/2
```

Defintions of rubi rules used

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{1}{4x^4}-\frac{1}{2x^2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{3}$	30
risch	$\frac{-\frac{x^2}{2}-\frac{1}{4}}{x^4}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{3}$	31

input
```
int(1/x^5/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4-1/2/x^2-1/3*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5 (1-x^2+x^4)} \, dx = -\frac{4 \sqrt{3} x^4 \arctan \left(\frac{1}{3} \sqrt{3} (2 x^2-1)\right) + 6 x^2 + 3}{12 x^4}$$

input
```
integrate(1/x^5/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
-1/12*(4*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 6*x^2 + 3)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^5 (1-x^2+x^4)} \, dx = -\frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3} - \frac{\sqrt{3}}{3}\right)}{3} + \frac{-2x^2-1}{4x^4}$$

input
```
integrate(1/x**5/(x**4-x**2+1),x)
```

output
```
-sqrt(3)*atan(2*sqrt(3)*x**2/3 - sqrt(3)/3)/3 + (-2*x**2 - 1)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5 (1-x^2+x^4)} \, dx = -\frac{1}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2 x^2-1)\right) - \frac{2 x^2+1}{4 x^4}$$

input
```
integrate(1/x^5/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/4*(2*x^2 + 1)/x^4
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5\,(1-x^2+x^4)}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) - \frac{2\,x^2+1}{4\,x^4}$$

input
```
integrate(1/x^5/(x^4-x^2+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/4*(2*x^2 + 1)/x^4
```

Mupad [B] (verification not implemented)

Time = 18.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5\,(1-x^2+x^4)}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^2}{3}\right)}{3} - \frac{\frac{x^2}{2}+\frac{1}{4}}{x^4}$$

input
```
int(1/(x^5*(x^4 - x^2 + 1)),x)
```

output
```
(3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^2)/3))/3 - (x^2/2 + 1/4)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5\,(1-x^2+x^4)}\,dx = \frac{4\sqrt{3}\,atan\left(\sqrt{3}-2x\right)x^4+4\sqrt{3}\,atan\left(\sqrt{3}+2x\right)x^4-6x^2-3}{12x^4}$$

input
```
int(1/x^5/(x^4-x^2+1),x)
```

output
```
(4*sqrt(3)*atan(sqrt(3) - 2*x)*x**4 + 4*sqrt(3)*atan(sqrt(3) + 2*x)*x**4 -
 6*x**2 - 3)/(12*x**4)
```

3.874 $\int \frac{x^{22}}{1-x^2+x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{x^{22}}{1-x^2+x^4}\,dx=-x+\frac{x^5}{5}+\frac{x^7}{7}-\frac{x^{11}}{11}-\frac{x^{13}}{13}+\frac{x^{17}}{17}+\frac{x^{19}}{19}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{\sqrt{3}}$$

output
```
-x+1/5*x^5+1/7*x^7-1/11*x^11-1/13*x^13+1/17*x^17+1/19*x^19+1/3*arctanh(3^(
1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int \frac{x^{22}}{1-x^2+x^4}\,dx=-x+\frac{x^5}{5}+\frac{x^7}{7}-\frac{x^{11}}{11}-\frac{x^{13}}{13}+\frac{x^{17}}{17}+\frac{x^{19}}{19}-\frac{\log\left(-1+\sqrt{3}x-x^2\right)}{2\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

input
```
Integrate[x^22/(1 - x^2 + x^4),x]
```

output
```
-x + x^5/5 + x^7/7 - x^11/11 - x^13/13 + x^17/17 + x^19/19 - Log[-1 + Sqrt
[3]*x - x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {1442, 27, 1602, 27, 1442, 27, 1602, 27, 1442, 27, 1602, 27, 1442, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{22}}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x^{19}}{19}-\frac{1}{19}\int \frac{19x^{18}\left(1-x^2\right)}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^{19}}{19}-\int \frac{x^{18}\left(1-x^2\right)}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{17}\int -\frac{17x^{16}}{x^4-x^2+1}dx+\frac{x^{19}}{19}+\frac{x^{17}}{17}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^{16}}{x^4-x^2+1}dx+\frac{x^{19}}{19}+\frac{x^{17}}{17}$$

$$\Big\downarrow \text{1442}$$

$$\frac{1}{13}\int \frac{13x^{12}\left(1-x^2\right)}{x^4-x^2+1}dx+\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{12}\left(1-x^2\right)}{x^4-x^2+1}dx+\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}$$

$$\Big\downarrow \text{1602}$$

$$-\frac{1}{11}\int -\frac{11x^{10}}{x^4-x^2+1}dx+\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^{10}}{x^4-x^2+1}dx+\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}$$

$$\Big\downarrow \text{1442}$$

$$-\frac{1}{7}\int \frac{7x^6(1-x^2)}{x^4-x^2+1}dx+\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^6(1-x^2)}{x^4-x^2+1}dx+\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{5}\int -\frac{5x^4}{x^4-x^2+1}dx+\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}+\frac{x^5}{5}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^4}{x^4-x^2+1}dx+\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}+\frac{x^5}{5}$$

$$\Big\downarrow \text{1442}$$

$$\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}+\frac{x^5}{5}-x$$

$$\Big\downarrow \text{1478}$$

$$-\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}+\frac{x^5}{5}-x$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}+\frac{x^5}{5}-x$$

$$\Big\downarrow \text{1103}$$

$$\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}+\frac{x^5}{5}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-x$$

input
```
Int[x^22/(1 - x^2 + x^4),x]
```

output
```
-x + x^5/5 + x^7/7 - x^11/11 - x^13/13 + x^17/17 + x^19/19 - Log[1 - Sqrt[
3]*x + x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^{7}}{7}+\frac{x^{5}}{5}-x-\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{6}+\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{6}$	68
risch	$\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^{7}}{7}+\frac{x^{5}}{5}-x-\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{6}+\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{6}$	68

input
```
int(x^22/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/19*x^19+1/17*x^17-1/13*x^13-1/11*x^11+1/7*x^7+1/5*x^5-x-1/6*3^(1/2)*ln(x
^2-3^(1/2)*x+1)+1/6*3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{x^{22}}{1-x^2+x^4}\,dx = \frac{1}{19}\,x^{19}+\frac{1}{17}\,x^{17}-\frac{1}{13}\,x^{13}-\frac{1}{11}\,x^{11}+\frac{1}{7}\,x^{7}+\frac{1}{5}\,x^{5} + \frac{1}{6}\,\sqrt{3}\log\left(\frac{x^4+5\,x^2+2\,\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)-x$$

input
```
integrate(x^22/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/19*x^19 + 1/17*x^17 - 1/13*x^13 - 1/11*x^11 + 1/7*x^7 + 1/5*x^5 + 1/6*sq
rt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1)) - x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \frac{x^{22}}{1-x^2+x^4}\,dx = \frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}+\frac{x^5}{5}-x-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{6}$$

input
```
integrate(x**22/(x**4-x**2+1),x)
```

output
```
x**19/19 + x**17/17 - x**13/13 - x**11/11 + x**7/7 + x**5/5 - x - sqrt(3)*
log(x**2 - sqrt(3)*x + 1)/6 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/6
```

Maxima [F]

$$\int \frac{x^{22}}{1-x^2+x^4}\,dx = \int \frac{x^{22}}{x^4-x^2+1}\,dx$$

input
```
integrate(x^22/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/19*x^19 + 1/17*x^17 - 1/13*x^13 - 1/11*x^11 + 1/7*x^7 + 1/5*x^5 - x - in
tegrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{x^{22}}{1-x^2+x^4}\,dx = \frac{1}{19}\,x^{19}+\frac{1}{17}\,x^{17}-\frac{1}{13}\,x^{13}-\frac{1}{11}\,x^{11}+\frac{1}{7}\,x^{7}+\frac{1}{5}\,x^{5} +\frac{1}{6}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{6}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-x$$

input
```
integrate(x^22/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/19*x^19 + 1/17*x^17 - 1/13*x^13 - 1/11*x^11 + 1/7*x^7 + 1/5*x^5 + 1/6*sq
rt(3)*log(x^2 + sqrt(3)*x + 1) - 1/6*sqrt(3)*log(x^2 - sqrt(3)*x + 1) - x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{x^{22}}{1-x^2+x^4}\,dx = \frac{x^5}{5}-x+\frac{x^7}{7}-\frac{x^{11}}{11}-\frac{x^{13}}{13}+\frac{x^{17}}{17}+\frac{x^{19}}{19}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3\left(\frac{2\,x^2}{3}+\frac{2}{3}\right)}\right)\,1\mathrm{i}}{3}$$

input
```
int(x^22/(x^4 - x^2 + 1),x)
```

output
```
x^5/5 - (3^(1/2)*atan((3^(1/2)*x*2i)/(3*((2*x^2)/3 + 2/3)))*1i)/3 - x + x^
7/7 - x^11/11 - x^13/13 + x^17/17 + x^19/19
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{x^{22}}{1-x^2+x^4}\,dx = -\frac{\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)}{6}+\frac{\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{6} +\frac{x^{19}}{19}+\frac{x^{17}}{17}-\frac{x^{13}}{13}-\frac{x^{11}}{11}+\frac{x^7}{7}+\frac{x^5}{5}-x$$

input
```
int(x^22/(x^4-x^2+1),x)
```

output
```
( - 1616615*sqrt(3)*log( - sqrt(3)*x + x**2 + 1) + 1616615*sqrt(3)*log(sqr
t(3)*x + x**2 + 1) + 510510*x**19 + 570570*x**17 - 746130*x**13 - 881790*x
**11 + 1385670*x**7 + 1939938*x**5 - 9699690*x)/9699690
```

3.875 $\int \frac{x^{16}}{1-x^2+x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int \frac{x^{16}}{1-x^2+x^4} dx = x - \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^{11}}{11} + \frac{x^{13}}{13} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{\sqrt{3}}$$

output
```
x-1/5*x^5-1/7*x^7+1/11*x^11+1/13*x^13-1/3*arctanh(3^(1/2)*x/(x^2+1))*3^(1/
2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.46

$$\int \frac{x^{16}}{1-x^2+x^4} dx = x - \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^{11}}{11} + \frac{x^{13}}{13} + \frac{\log\left(-1+\sqrt{3}x-x^2\right)}{2\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

input
```
Integrate[x^16/(1 - x^2 + x^4),x]
```

output
```
x - x^5/5 - x^7/7 + x^11/11 + x^13/13 + Log[-1 + Sqrt[3]*x - x^2]/(2*Sqrt[
3]) - Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.44, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {1442, 27, 1602, 27, 1442, 27, 1602, 27, 1442, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{16}}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x^{13}}{13}-\frac{1}{13}\int \frac{13x^{12}\left(1-x^2\right)}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^{13}}{13}-\int \frac{x^{12}\left(1-x^2\right)}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{11}\int -\frac{11x^{10}}{x^4-x^2+1}dx+\frac{x^{13}}{13}+\frac{x^{11}}{11}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^{10}}{x^4-x^2+1}dx+\frac{x^{13}}{13}+\frac{x^{11}}{11}$$

$$\Big\downarrow \text{1442}$$

$$\frac{1}{7}\int \frac{7x^6\left(1-x^2\right)}{x^4-x^2+1}dx+\frac{x^{13}}{13}+\frac{x^{11}}{11}-\frac{x^7}{7}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^6\left(1-x^2\right)}{x^4-x^2+1}dx+\frac{x^{13}}{13}+\frac{x^{11}}{11}-\frac{x^7}{7}$$

$$\Big\downarrow \text{1602}$$

$$-\frac{1}{5}\int -\frac{5x^4}{x^4-x^2+1}dx+\frac{x^{13}}{13}+\frac{x^{11}}{11}-\frac{x^7}{7}-\frac{x^5}{5}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^4}{x^4-x^2+1}dx+\frac{x^{13}}{13}+\frac{x^{11}}{11}-\frac{x^7}{7}-\frac{x^5}{5}$$

$$\Big\downarrow \text{1442}$$

$$-\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{x^{13}}{13}+\frac{x^{11}}{11}-\frac{x^7}{7}-\frac{x^5}{5}+x$$

$$\Big\downarrow \text{1478}$$

$$\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{x^{13}}{13}+\frac{x^{11}}{11}-\frac{x^7}{7}-\frac{x^5}{5}+x$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{x^{13}}{13}+\frac{x^{11}}{11}-\frac{x^7}{7}-\frac{x^5}{5}+x$$

$$\Big\downarrow \text{1103}$$

$$\frac{x^{13}}{13}+\frac{x^{11}}{11}-\frac{x^7}{7}-\frac{x^5}{5}+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}+x$$

input
```
Int[x^16/(1 - x^2 + x^4),x]
```

output
```
x - x^5/5 - x^7/7 + x^11/11 + x^13/13 + Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3
]) - Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{x^{13}}{13}+\frac{x^{11}}{11}-\frac{x^{7}}{7}-\frac{x^{5}}{5}+x+\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{6}-\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{6}$	56
risch	$\frac{x^{13}}{13}+\frac{x^{11}}{11}-\frac{x^{7}}{7}-\frac{x^{5}}{5}+x+\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{6}-\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{6}$	56

input
```
int(x^16/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/13*x^13+1/11*x^11-1/7*x^7-1/5*x^5+x+1/6*3^(1/2)*ln(x^2-3^(1/2)*x+1)-1/6*
3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{x^{16}}{1-x^2+x^4}\,dx = \frac{1}{13}\,x^{13} + \frac{1}{11}\,x^{11} - \frac{1}{7}\,x^7 - \frac{1}{5}\,x^5 + \frac{1}{6}\,\sqrt{3}\log\left(\frac{x^4+5\,x^2-2\,\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right) + x$$

input
```
integrate(x^16/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/13*x^13 + 1/11*x^11 - 1/7*x^7 - 1/5*x^5 + 1/6*sqrt(3)*log((x^4 + 5*x^2 -
 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1)) + x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{x^{16}}{1-x^2+x^4}\,dx = \frac{x^{13}}{13} + \frac{x^{11}}{11} - \frac{x^7}{7} - \frac{x^5}{5} + x + \frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{6} - \frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{6}$$

input
```
integrate(x**16/(x**4-x**2+1),x)
```

output
```
x**13/13 + x**11/11 - x**7/7 - x**5/5 + x + sqrt(3)*log(x**2 - sqrt(3)*x +
 1)/6 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/6
```

Maxima [F]

$$\int \frac{x^{16}}{1-x^2+x^4}\,dx = \int \frac{x^{16}}{x^4-x^2+1}\,dx$$

input
```
integrate(x^16/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/13*x^13 + 1/11*x^11 - 1/7*x^7 - 1/5*x^5 + x + integrate((x^2 - 1)/(x^4 -
 x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{x^{16}}{1-x^2+x^4}\,dx = \frac{1}{13}\,x^{13} + \frac{1}{11}\,x^{11} - \frac{1}{7}\,x^7 - \frac{1}{5}\,x^5 - \frac{1}{6}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) + \frac{1}{6}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + x$$

input
```
integrate(x^16/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/13*x^13 + 1/11*x^11 - 1/7*x^7 - 1/5*x^5 - 1/6*sqrt(3)*log(x^2 + sqrt(3)*
x + 1) + 1/6*sqrt(3)*log(x^2 - sqrt(3)*x + 1) + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{x^{16}}{1-x^2+x^4}\,dx = x - \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^{11}}{11} + \frac{x^{13}}{13} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3\left(\frac{2\,x^2}{3}+\frac{2}{3}\right)}\right)\,1\mathrm{i}}{3}$$

input
```
int(x^16/(x^4 - x^2 + 1),x)
```

output
```
x + (3^(1/2)*atan((3^(1/2)*x*2i)/(3*((2*x^2)/3 + 2/3)))*1i)/3 - x^5/5 - x^
7/7 + x^11/11 + x^13/13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{x^{16}}{1-x^2+x^4}\,dx = \frac{\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)}{6} - \frac{\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{6} + \frac{x^{13}}{13} + \frac{x^{11}}{11} - \frac{x^7}{7} - \frac{x^5}{5} + x$$

input
```
int(x^16/(x^4-x^2+1),x)
```

output
```
(5005*sqrt(3)*log( - sqrt(3)*x + x**2 + 1) - 5005*sqrt(3)*log(sqrt(3)*x +
x**2 + 1) + 2310*x**13 + 2730*x**11 - 4290*x**7 - 6006*x**5 + 30030*x)/300
30
```

3.876 $\int \frac{x^{10}}{1-x^2+x^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{x^{10}}{1-x^2+x^4} \, dx = -x+\frac{x^5}{5}+\frac{x^7}{7}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{\sqrt{3}}$$

output
```
-x+1/5*x^5+1/7*x^7+1/3*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.64

$$\int \frac{x^{10}}{1-x^2+x^4} \, dx = -x+\frac{x^5}{5}+\frac{x^7}{7}-\frac{\log \left(-1+\sqrt{3}x-x^2\right)}{2\sqrt{3}}+\frac{\log \left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

input
```
Integrate[x^10/(1 - x^2 + x^4),x]
```

output
```
-x + x^5/5 + x^7/7 - Log[-1 + Sqrt[3]*x - x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[
3]*x + x^2]/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1442, 27, 1602, 27, 1442, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{x^4-x^2+1}dx$$

$$\Big\downarrow \textbf{1442}$$

$$\frac{x^7}{7}-\frac{1}{7}\int \frac{7x^6(1-x^2)}{x^4-x^2+1}dx$$

$$\Big\downarrow \textbf{27}$$

$$\frac{x^7}{7}-\int \frac{x^6(1-x^2)}{x^4-x^2+1}dx$$

$$\Big\downarrow \textbf{1602}$$

$$\frac{1}{5}\int -\frac{5x^4}{x^4-x^2+1}dx+\frac{x^7}{7}+\frac{x^5}{5}$$

$$\Big\downarrow \textbf{27}$$

$$-\int \frac{x^4}{x^4-x^2+1}dx+\frac{x^7}{7}+\frac{x^5}{5}$$

$$\Big\downarrow \textbf{1442}$$

$$\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{x^7}{7}+\frac{x^5}{5}-x$$

$$\Big\downarrow \textbf{1478}$$

$$-\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{x^7}{7}+\frac{x^5}{5}-x$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{x^7}{7}+\frac{x^5}{5}-x$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{x^7}{7}+\frac{x^5}{5}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-x$$

input
```
Int[x^10/(1 - x^2 + x^4),x]
```

output
```
-x + x^5/5 + x^7/7 - Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3
]*x + x^2]/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

method	result	size
default	$\frac{x^7}{7}+\frac{x^5}{5}-x-\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{6}+\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{6}$	48
risch	$\frac{x^7}{7}+\frac{x^5}{5}-x-\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{6}+\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{6}$	48

input
```
int(x^10/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7+1/5*x^5-x-1/6*3^(1/2)*ln(x^2-3^(1/2)*x+1)+1/6*3^(1/2)*ln(x^2+3^(1/
2)*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \frac{x^{10}}{1-x^2+x^4}\,dx = \frac{1}{7}\,x^7+\frac{1}{5}\,x^5+\frac{1}{6}\,\sqrt{3}\log\left(\frac{x^4+5\,x^2+2\,\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)-x$$

input
```
integrate(x^10/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/7*x^7 + 1/5*x^5 + 1/6*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1
)/(x^4 - x^2 + 1)) - x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \frac{x^{10}}{1-x^2+x^4}\,dx = \frac{x^7}{7}+\frac{x^5}{5}-x-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{6}$$

input
```
integrate(x**10/(x**4-x**2+1),x)
```

output
```
x**7/7 + x**5/5 - x - sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 + sqrt(3)*log(x*
*2 + sqrt(3)*x + 1)/6
```

Maxima [F]

$$\int \frac{x^{10}}{1-x^2+x^4}\,dx = \int \frac{x^{10}}{x^4-x^2+1}\,dx$$

input
```
integrate(x^10/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/7*x^7 + 1/5*x^5 - x - integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{x^{10}}{1-x^2+x^4}\,dx = \frac{1}{7}x^7+\frac{1}{5}x^5+\frac{1}{6}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{6}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-x$$

input
```
integrate(x^10/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/7*x^7 + 1/5*x^5 + 1/6*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/6*sqrt(3)*log
(x^2 - sqrt(3)*x + 1) - x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^{10}}{1-x^2+x^4}\,dx=\frac{x^5}{5}-x+\frac{x^7}{7}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3\left(\frac{2\,x^2}{3}+\frac{2}{3}\right)}\right)\,1\mathrm{i}}{3}$$

input
```
int(x^10/(x^4 - x^2 + 1),x)
```

output
```
x^5/5 - (3^(1/2)*atan((3^(1/2)*x*2i)/(3*((2*x^2)/3 + 2/3)))*1i)/3 - x + x^
7/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{x^{10}}{1-x^2+x^4}\,dx=-\frac{\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)}{6}+\frac{\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{6}+\frac{x^7}{7}+\frac{x^5}{5}-x$$

input
```
int(x^10/(x^4-x^2+1),x)
```

output
```
( - 35*sqrt(3)*log( - sqrt(3)*x + x**2 + 1) + 35*sqrt(3)*log(sqrt(3)*x + x
**2 + 1) + 30*x**7 + 42*x**5 - 210*x)/210
```

3.877 $\int \frac{x^4}{1-x^2+x^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 24

$$\int \frac{x^4}{1-x^2+x^4} \, dx = x - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{\sqrt{3}}$$

output
```
x-1/3*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^4}{1-x^2+x^4} \, dx = x + \frac{\log\left(-1+\sqrt{3}x-x^2\right)}{2\sqrt{3}} - \frac{\log\left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

input
```
Integrate[x^4/(1 - x^2 + x^4),x]
```

output
```
x + Log[-1 + Sqrt[3]*x - x^2]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(2*Sq
rt[3])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1442, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^4-x^2+1} \, dx$$

$$\Big\downarrow \; 1442$$

$$x-\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \; 1478$$

$$\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+x$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+x$$

$$\Big\downarrow \; 1103$$

$$\frac{\log \left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log \left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}+x$$

input
```
Int[x^4/(1 - x^2 + x^4),x]
```

output
```
x + Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(2*Sqr
t[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

method	result	size
default	$x+\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{6}-\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{6}$	36
risch	$x+\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{6}-\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{6}$	36

input
```
int(x^4/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
x+1/6*3^(1/2)*ln(x^2-3^(1/2)*x+1)-1/6*3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^4}{1-x^2+x^4}\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{x^4+5\,x^2-2\,\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)+x$$

input
```
integrate(x^4/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1)) +
 x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^4}{1-x^2+x^4}\,dx = x+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{6}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{6}$$

input
```
integrate(x**4/(x**4-x**2+1),x)
```

output
```
x + sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 - sqrt(3)*log(x**2 + sqrt(3)*x + 1
)/6
```

Maxima [F]

$$\int \frac{x^4}{1-x^2+x^4}\,dx = \int \frac{x^4}{x^4-x^2+1}\,dx$$

input
```
integrate(x^4/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
x + integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{x^4}{1-x^2+x^4}\,dx = -\frac{1}{6}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{6}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+x$$

input
```
integrate(x^4/(x^4-x^2+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/6*sqrt(3)*log(x^2 - sqrt(3)*x +
1) + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{1-x^2+x^4}\,dx = x-\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)}{3}$$

input
```
int(x^4/(x^4 - x^2 + 1),x)
```

output
```
x - (3^(1/2)*atanh((2*3^(1/2)*x)/(3*((2*x^2)/3 + 2/3))))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{x^4}{1-x^2+x^4}\,dx = \frac{\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)}{6} - \frac{\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{6} + x$$

input
```
int(x^4/(x^4-x^2+1),x)
```

output
```
(sqrt(3)*log( - sqrt(3)*x + x**2 + 1) - sqrt(3)*log(sqrt(3)*x + x**2 + 1)
+ 6*x)/6
```

3.878 $\int \frac{1}{x^2(1-x^2+x^4)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{1}{x^2(1-x^2+x^4)}\,dx = -\frac{1}{x}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{\sqrt{3}}$$

output
```
-1/x+1/3*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int \frac{1}{x^2(1-x^2+x^4)}\,dx = -\frac{1}{x}-\frac{\log\left(-1+\sqrt{3}x-x^2\right)}{2\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

input
```
Integrate[1/(x^2*(1 - x^2 + x^4)),x]
```

output
```
-x^(-1) - Log[-1 + Sqrt[3]*x - x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]
/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1443, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^4-x^2+1)} \, dx$$

$$\Big\downarrow \text{1443}$$

$$\int \frac{1-x^2}{x^4-x^2+1}dx-\frac{1}{x}$$

$$\Big\downarrow \text{1478}$$

$$-\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{1}{x}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{\log \left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{\log \left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{1}{x}$$

input
```
Int[1/(x^2*(1 - x^2 + x^4)),x]
```

output
```
-x^(-1) - Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/
(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

method	result	size
default	$-\frac{1}{x}-\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{6}+\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{6}$	40
risch	$-\frac{1}{x}-\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{6}+\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{6}$	40

input
```
int(1/x^2/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/x-1/6*3^(1/2)*ln(x^2-3^(1/2)*x+1)+1/6*3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^2 (1 - x^2 + x^4)} dx = \frac{\sqrt{3}x \log\left(\frac{x^4+5x^2+2\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right) - 6}{6x}$$

input
```
integrate(1/x^2/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/6*(sqrt(3)*x*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1)
) - 6)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^2 (1 - x^2 + x^4)} dx = -\frac{\sqrt{3}\log\left(x^2 - \sqrt{3}x + 1\right)}{6} + \frac{\sqrt{3}\log\left(x^2 + \sqrt{3}x + 1\right)}{6} - \frac{1}{x}$$

input
```
integrate(1/x**2/(x**4-x**2+1),x)
```

output
```
-sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/6
 - 1/x
```

Maxima [F]

$$\int \frac{1}{x^2 (1 - x^2 + x^4)} dx = \int \frac{1}{(x^4 - x^2 + 1)x^2} dx$$

input
```
integrate(1/x^2/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
-1/x - integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^2(1-x^2+x^4)}\,dx = \frac{1}{6}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{6}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) - \frac{1}{x}$$

input
```
integrate(1/x^2/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/6*sqrt(3)*log(x^2 - sqrt(3)*x + 1
) - 1/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(1-x^2+x^4)}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)}{3} - \frac{1}{x}$$

input
```
int(1/(x^2*(x^4 - x^2 + 1)),x)
```

output
```
(3^(1/2)*atanh((2*3^(1/2)*x)/(3*((2*x^2)/3 + 2/3))))/3 - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^2(1-x^2+x^4)}\,dx = \frac{-\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)x+\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)x-6}{6x}$$

input
```
int(1/x^2/(x^4-x^2+1),x)
```

output
```
( - sqrt(3)*log( - sqrt(3)*x + x**2 + 1)*x + sqrt(3)*log(sqrt(3)*x + x**2
+ 1)*x - 6)/(6*x)
```

3.879 $\int \frac{1}{x^8(1-x^2+x^4)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{1}{x^8(1-x^2+x^4)}\,dx = -\frac{1}{7x^7}-\frac{1}{5x^5}+\frac{1}{x}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{\sqrt{3}}$$

output
```
-1/7/x^7-1/5/x^5+1/x-1/3*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.60

$$\int \frac{1}{x^8(1-x^2+x^4)}\,dx = -\frac{1}{7x^7}-\frac{1}{5x^5}+\frac{1}{x}+\frac{\log\left(-1+\sqrt{3}x-x^2\right)}{2\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

input
```
Integrate[1/(x^8*(1 - x^2 + x^4)),x]
```

output
```
-1/7*1/x^7 - 1/(5*x^5) + x^(-1) + Log[-1 + Sqrt[3]*x - x^2]/(2*Sqrt[3]) -
Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.58, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1443, 27, 1604, 27, 1443, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \left(x^4-x^2+1\right)} \, dx$$

$$\Big\downarrow \text{ 1443}$$

$$\frac{1}{7} \int \frac{7 \left(1-x^2\right)}{x^6 \left(x^4-x^2+1\right)}dx-\frac{1}{7 x^7}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1-x^2}{x^6 \left(x^4-x^2+1\right)}dx-\frac{1}{7 x^7}$$

$$\Big\downarrow \text{ 1604}$$

$$-\frac{1}{5} \int \frac{5}{x^2 \left(x^4-x^2+1\right)}dx-\frac{1}{7 x^7}-\frac{1}{5 x^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{1}{x^2 \left(x^4-x^2+1\right)}dx-\frac{1}{7 x^7}-\frac{1}{5 x^5}$$

$$\Big\downarrow \text{ 1443}$$

$$-\int \frac{1-x^2}{x^4-x^2+1}dx-\frac{1}{7 x^7}-\frac{1}{5 x^5}+\frac{1}{x}$$

$$\Big\downarrow \text{ 1478}$$

$$\frac{\int -\frac{\sqrt{3}-2 x}{x^2-\sqrt{3} x+1}dx}{2 \sqrt{3}}+\frac{\int -\frac{2 x+\sqrt{3}}{x^2+\sqrt{3} x+1}dx}{2 \sqrt{3}}-\frac{1}{7 x^7}-\frac{1}{5 x^5}+\frac{1}{x}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\sqrt{3}-2 x}{x^2-\sqrt{3} x+1}dx}{2 \sqrt{3}}-\frac{\int \frac{2 x+\sqrt{3}}{x^2+\sqrt{3} x+1}dx}{2 \sqrt{3}}-\frac{1}{7 x^7}-\frac{1}{5 x^5}+\frac{1}{x}$$

$$\Big\downarrow \text{ 1103}$$

$$-\frac{1}{7x^7}-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{1}{x}$$

input
```
Int[1/(x^8*(1 - x^2 + x^4)),x]
```

output
```
-1/7*1/x^7 - 1/(5*x^5) + x^(-1) + Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) - L
og[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{1}{7x^7}-\frac{1}{5x^5}+\frac{1}{x}+\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{6}-\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{6}$	48
risch	$\frac{x^6-\frac{1}{5}x^2-\frac{1}{7}}{x^7}+\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{6}-\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{6}$	49

input
```
int(1/x^8/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/7/x^7-1/5/x^5+1/x+1/6*3^(1/2)*ln(x^2-3^(1/2)*x+1)-1/6*3^(1/2)*ln(x^2+3^
(1/2)*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^8\left(1-x^2+x^4\right)}\,dx=\frac{35\,\sqrt{3}x^7\log\left(\frac{x^4+5\,x^2-2\,\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)+210\,x^6-42\,x^2-30}{210\,x^7}$$

input
```
integrate(1/x^8/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/210*(35*sqrt(3)*x^7*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x
^2 + 1)) + 210*x^6 - 42*x^2 - 30)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^8 (1-x^2+x^4)} dx = \frac{\sqrt{3} \log \left(x^2 - \sqrt{3}x + 1\right)}{6} - \frac{\sqrt{3} \log \left(x^2 + \sqrt{3}x + 1\right)}{6} + \frac{35x^6 - 7x^2 - 5}{35x^7}$$

input
```
integrate(1/x**8/(x**4-x**2+1),x)
```

output
```
sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/6
+ (35*x**6 - 7*x**2 - 5)/(35*x**7)
```

Maxima [F]

$$\int \frac{1}{x^8 (1-x^2+x^4)} dx = \int \frac{1}{(x^4 - x^2 + 1)x^8} dx$$

input
```
integrate(1/x^8/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/35*(35*x^6 - 7*x^2 - 5)/x^7 + integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(32) = 64.

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^8 (1-x^2+x^4)} dx = -\frac{1}{12} \sqrt{3} \log \left(x^2 + \sqrt{3}x + 1\right) + \frac{1}{12} \sqrt{3} \log \left(x^2 - \sqrt{3}x + 1\right) + \frac{35 x^6 - 7 x^2 - 5}{35 x^7} + \frac{1}{2} \arctan \left(2 x + \sqrt{3}\right) + \frac{1}{2} \arctan \left(2 x - \sqrt{3}\right)$$

input
```
integrate(1/x^8/(x^4-x^2+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x
+ 1) + 1/35*(35*x^6 - 7*x^2 - 5)/x^7 + 1/2*arctan(2*x + sqrt(3)) + 1/2*arc
tan(2*x - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 17.73 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^8 (1-x^2+x^4)} \, dx = -\frac{-x^6+\frac{x^2}{5}+\frac{1}{7}}{x^7} - \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)}{3}$$

input
```
int(1/(x^8*(x^4 - x^2 + 1)),x)
```

output
```
- (x^2/5 - x^6 + 1/7)/x^7 - (3^(1/2)*atanh((2*3^(1/2)*x)/(3*((2*x^2)/3 + 2
/3))))/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^8 (1-x^2+x^4)} \, dx = \frac{35\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)x^7-35\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)x^7+210x^6-42x^2-30}{210x^7}$$

input
```
int(1/x^8/(x^4-x^2+1),x)
```

output
```
(35*sqrt(3)*log( - sqrt(3)*x + x**2 + 1)*x**7 - 35*sqrt(3)*log(sqrt(3)*x +
 x**2 + 1)*x**7 + 210*x**6 - 42*x**2 - 30)/(210*x**7)
```

3.880 $\int \frac{1}{x^{14}(1-x^2+x^4)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{1}{x^{14}\left(1-x^2+x^4\right)}\,dx=-\frac{1}{13x^{13}}-\frac{1}{11x^{11}}+\frac{1}{7x^7}+\frac{1}{5x^5}-\frac{1}{x}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{\sqrt{3}}$$

output
```
-1/13/x^13-1/11/x^11+1/7/x^7+1/5/x^5-1/x+1/3*arctanh(3^(1/2)*x/(x^2+1))*3^
(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^{14}\left(1-x^2+x^4\right)}\,dx=-\frac{1}{13x^{13}}-\frac{1}{11x^{11}}+\frac{1}{7x^7}+\frac{1}{5x^5}-\frac{1}{x}-\frac{\log\left(-1+\sqrt{3}x-x^2\right)}{2\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x+x^2\right)}{2\sqrt{3}}$$

input
```
Integrate[1/(x^14*(1 - x^2 + x^4)),x]
```

output
```
-1/13*1/x^13 - 1/(11*x^11) + 1/(7*x^7) + 1/(5*x^5) - x^(-1) - Log[-1 + Sqr
t[3]*x - x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {1443, 27, 1604, 27, 1443, 27, 1604, 27, 1443, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{14} (x^4-x^2+1)} dx$$
$$\Big\downarrow \text{1443}$$
$$\frac{1}{13} \int \frac{13(1-x^2)}{x^{12} (x^4-x^2+1)}dx - \frac{1}{13x^{13}}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1-x^2}{x^{12} (x^4-x^2+1)}dx - \frac{1}{13x^{13}}$$
$$\Big\downarrow \text{1604}$$
$$-\frac{1}{11} \int \frac{11}{x^8 (x^4-x^2+1)}dx - \frac{1}{13x^{13}} - \frac{1}{11x^{11}}$$
$$\Big\downarrow \text{27}$$
$$-\int \frac{1}{x^8 (x^4-x^2+1)}dx - \frac{1}{13x^{13}} - \frac{1}{11x^{11}}$$
$$\Big\downarrow \text{1443}$$
$$-\frac{1}{7} \int \frac{7(1-x^2)}{x^6 (x^4-x^2+1)}dx - \frac{1}{13x^{13}} - \frac{1}{11x^{11}} + \frac{1}{7x^7}$$
$$\Big\downarrow \text{27}$$
$$-\int \frac{1-x^2}{x^6 (x^4-x^2+1)}dx - \frac{1}{13x^{13}} - \frac{1}{11x^{11}} + \frac{1}{7x^7}$$
$$\Big\downarrow \text{1604}$$

$$\frac{1}{5}\int \frac{5}{x^2\left(x^4-x^2+1\right)}dx-\frac{1}{13x^{13}}-\frac{1}{11x^{11}}+\frac{1}{7x^7}+\frac{1}{5x^5}$$

$$\Big\downarrow \textbf{27}$$

$$\int \frac{1}{x^2\left(x^4-x^2+1\right)}dx-\frac{1}{13x^{13}}-\frac{1}{11x^{11}}+\frac{1}{7x^7}+\frac{1}{5x^5}$$

$$\Big\downarrow \textbf{1443}$$

$$\int \frac{1-x^2}{x^4-x^2+1}dx-\frac{1}{13x^{13}}-\frac{1}{11x^{11}}+\frac{1}{7x^7}+\frac{1}{5x^5}-\frac{1}{x}$$

$$\Big\downarrow \textbf{1478}$$

$$-\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{1}{13x^{13}}-\frac{1}{11x^{11}}+\frac{1}{7x^7}+\frac{1}{5x^5}-\frac{1}{x}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{1}{13x^{13}}-\frac{1}{11x^{11}}+\frac{1}{7x^7}+\frac{1}{5x^5}-\frac{1}{x}$$

$$\Big\downarrow \textbf{1103}$$

$$-\frac{1}{13x^{13}}-\frac{1}{11x^{11}}+\frac{1}{7x^7}+\frac{1}{5x^5}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{1}{x}$$

input
```
Int[1/(x^14*(1 - x^2 + x^4)),x]
```

output
```
-1/13*1/x^13 - 1/(11*x^11) + 1/(7*x^7) + 1/(5*x^5) - x^(-1) - Log[1 - Sqrt
[3]*x + x^2]/(2*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{1}{13x^{13}}-\frac{1}{11x^{11}}-\frac{1}{x}+\frac{1}{7x^{7}}+\frac{1}{5x^{5}}-\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{6}+\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{6}$	60
risch	$\frac{-x^{12}+\frac{1}{5}x^{8}+\frac{1}{7}x^{6}-\frac{1}{11}x^{2}-\frac{1}{13}}{x^{13}}-\frac{\sqrt{3}\,\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{6}+\frac{\sqrt{3}\,\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{6}$	61

input
```
int(1/x^14/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/13/x^13-1/11/x^11-1/x+1/7/x^7+1/5/x^5-1/6*3^(1/2)*ln(x^2-3^(1/2)*x+1)+1
/6*3^(1/2)*ln(x^2+3^(1/2)*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^{14}(1-x^2+x^4)}\,dx = \frac{5005\sqrt{3}x^{13}\log\left(\frac{x^4+5x^2+2\sqrt{3}(x^3+x)+1}{x^4-x^2+1}\right)-30030\,x^{12}+6006\,x^8+4290\,x^6-2730\,x^2-2310}{30030\,x^{13}}$$

input
```
integrate(1/x^14/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/30030*(5005*sqrt(3)*x^13*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^
4 - x^2 + 1)) - 30030*x^12 + 6006*x^8 + 4290*x^6 - 2730*x^2 - 2310)/x^13
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^{14}(1-x^2+x^4)}\,dx = -\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{6}+\frac{-5005x^{12}+1001x^8+715x^6-455x^2-385}{5005x^{13}}$$

input
```
integrate(1/x**14/(x**4-x**2+1),x)
```

output
```
-sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/6
 + (-5005*x**12 + 1001*x**8 + 715*x**6 - 455*x**2 - 385)/(5005*x**13)
```

Maxima [F]

$$\int \frac{1}{x^{14}(1-x^2+x^4)}\,dx=\int \frac{1}{(x^4-x^2+1)x^{14}}\,dx$$

input
```
integrate(1/x^14/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
-1/5005*(5005*x^12 - 1001*x^8 - 715*x^6 + 455*x^2 + 385)/x^13 - integrate(
(x^2 - 1)/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^{14}(1-x^2+x^4)}\,dx=\frac{1}{6}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{6}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{5005\,x^{12}-1001\,x^8-715\,x^6+455\,x^2+385}{5005\,x^{13}}$$

input
```
integrate(1/x^14/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/6*sqrt(3)*log(x^2 - sqrt(3)*x + 1
) - 1/5005*(5005*x^12 - 1001*x^8 - 715*x^6 + 455*x^2 + 385)/x^13
```

Mupad [B] (verification not implemented)

Time = 18.73 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{14}(1-x^2+x^4)}\,dx=\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2\,x^2}{3}+\frac{2}{3}\right)}\right)}{3}-\frac{x^{12}-\frac{x^8}{5}-\frac{x^6}{7}+\frac{x^2}{11}+\frac{1}{13}}{x^{13}}$$

input
```
int(1/(x^14*(x^4 - x^2 + 1)),x)
```

output
```
(3^(1/2)*atanh((2*3^(1/2)*x)/(3*((2*x^2)/3 + 2/3))))/3 - (x^2/11 - x^6/7 -
 x^8/5 + x^12 + 1/13)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^{14}(1-x^2+x^4)}\,dx$$
$$= \frac{-5005\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)x^{13}+5005\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)x^{13}-30030x^{12}+6006x^8+4290x^6-}{30030x^{13}}$$

input
```
int(1/x^14/(x^4-x^2+1),x)
```

output
```
( - 5005*sqrt(3)*log( - sqrt(3)*x + x**2 + 1)*x**13 + 5005*sqrt(3)*log(sqr
t(3)*x + x**2 + 1)*x**13 - 30030*x**12 + 6006*x**8 + 4290*x**6 - 2730*x**2
 - 2310)/(30030*x**13)
```

3.881 $\int \frac{x^{12}}{1-x^2+x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{x^{12}}{1-x^2+x^4}\,dx = -x-\frac{x^3}{3}+\frac{x^7}{7}+\frac{x^9}{9}-\frac{1}{2}\arctan\left(\sqrt{3}-2x\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2x\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{2\sqrt{3}}$$

output
```
-x-1/3*x^3+1/7*x^7+1/9*x^9+1/2*arctan(-3^(1/2)+2*x)+1/2*arctan(3^(1/2)+2*x
)+1/6*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.36

$$\int \frac{x^{12}}{1-x^2+x^4}\,dx = -x-\frac{x^3}{3}+\frac{x^7}{7}+\frac{x^9}{9}+\frac{i\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{\frac{3}{2}i\left(i+\sqrt{3}\right)}}-\frac{i\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-\frac{3}{2}i\left(-i+\sqrt{3}\right)}}$$

input
```
Integrate[x^12/(1 - x^2 + x^4),x]
```

output
```
-x - x^3/3 + x^7/7 + x^9/9 + (I*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[((3*I)
/2)*(I + Sqrt[3])] - (I*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[((-3*I)/2)*(-I
 + Sqrt[3])]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {1442, 27, 1602, 27, 1442, 27, 1602, 25, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x^9}{9}-\frac{1}{9}\int \frac{9x^8\left(1-x^2\right)}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^9}{9}-\int \frac{x^8\left(1-x^2\right)}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{7}\int -\frac{7x^6}{x^4-x^2+1}dx+\frac{x^9}{9}+\frac{x^7}{7}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^6}{x^4-x^2+1}dx+\frac{x^9}{9}+\frac{x^7}{7}$$

$$\Big\downarrow \text{1442}$$

$$\frac{1}{3}\int \frac{3x^2\left(1-x^2\right)}{x^4-x^2+1}dx+\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2\left(1-x^2\right)}{x^4-x^2+1}dx+\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}$$

$$\Big\downarrow \text{ 1602}$$

$$-\int -\frac{1}{x^4-x^2+1}dx+\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{1}{x^4-x^2+1}dx+\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 1407}$$

$$\frac{\int \frac{\sqrt{3}-x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)}{2\sqrt{3}}+\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+\sqrt{3}\arctan\left(2x+\sqrt{3}\right)}{2\sqrt{3}}+\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}-x$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}+$$
$$\frac{\sqrt{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}-x$$

input
```
Int[x^12/(1 - x^2 + x^4),x]
```

output
```
-x - x^3/3 + x^7/7 + x^9/9 + (-(Sqrt[3]*ArcTan[Sqrt[3] - 2*x]) - Log[1 - S
qrt[3]*x + x^2]/2)/(2*Sqrt[3]) + (Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + Log[1 +
Sqrt[3]*x + x^2]/2)/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}-x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^3-\_R}\right)}{2}$	54
default	$\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}-x-\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}+\frac{\arctan\left(2x-\sqrt{3}\right)}{2}+\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}+\frac{\arctan\left(2x+\sqrt{3}\right)}{2}$	75

input
```
int(x^12/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/9*x^9+1/7*x^7-1/3*x^3-x+1/2*sum(1/(2*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-_Z
^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{x^{12}}{1-x^2+x^4}\,dx = \frac{1}{9}\,x^9+\frac{1}{7}\,x^7-\frac{1}{3}\,x^3+\frac{1}{12}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{12}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-x + \frac{1}{2}\,\arctan\left(2\,x+\sqrt{3}\right)-\frac{1}{2}\,\arctan\left(-2\,x+\sqrt{3}\right)$$

input
```
integrate(x^12/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/9*x^9 + 1/7*x^7 - 1/3*x^3 + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12
*sqrt(3)*log(x^2 - sqrt(3)*x + 1) - x + 1/2*arctan(2*x + sqrt(3)) - 1/2*ar
ctan(-2*x + sqrt(3))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int \frac{x^{12}}{1-x^2+x^4}\,dx = \frac{x^9}{9}+\frac{x^7}{7}-\frac{x^3}{3}-x-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12} + \frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

input
```
integrate(x**12/(x**4-x**2+1),x)
```

output
```
x**9/9 + x**7/7 - x**3/3 - x - sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt
(3)*log(x**2 + sqrt(3)*x + 1)/12 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt
(3))/2
```

Maxima [F]

$$\int \frac{x^{12}}{1-x^2+x^4}\,dx = \int \frac{x^{12}}{x^4-x^2+1}\,dx$$

input
```
integrate(x^12/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/9*x^9 + 1/7*x^7 - 1/3*x^3 - x + integrate(1/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int \frac{x^{12}}{1-x^2+x^4}\,dx = \frac{1}{9}\,x^9 + \frac{1}{7}\,x^7 - \frac{1}{3}\,x^3 + \frac{1}{12}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{12}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) - x + \frac{1}{2}\,\arctan\left(2\,x+\sqrt{3}\right) + \frac{1}{2}\,\arctan\left(2\,x-\sqrt{3}\right)$$

input
```
integrate(x^12/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/9*x^9 + 1/7*x^7 - 1/3*x^3 + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12
*sqrt(3)*log(x^2 - sqrt(3)*x + 1) - x + 1/2*arctan(2*x + sqrt(3)) + 1/2*ar
ctan(2*x - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{x^{12}}{1-x^2+x^4}\,dx = \frac{x^7}{7} + \mathrm{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \mathrm{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \frac{x^3}{3} - x + \frac{x^9}{9}$$

input
```
int(x^12/(x^4 - x^2 + 1),x)
```

output
```
atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) - x + atan((2*x)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/6 + 1/2) - x^3/3 + x^7/7 + x^9/9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{x^{12}}{1-x^2+x^4}\,dx = -\frac{atan\left(\sqrt{3}-2x\right)}{2} + \frac{atan\left(\sqrt{3}+2x\right)}{2} - \frac{\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)}{12} + \frac{\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{12} + \frac{x^9}{9} + \frac{x^7}{7} - \frac{x^3}{3} - x$$

input
```
int(x^12/(x^4-x^2+1),x)
```

output
```
( - 126*atan(sqrt(3) - 2*x) + 126*atan(sqrt(3) + 2*x) - 21*sqrt(3)*log( -
sqrt(3)*x + x**2 + 1) + 21*sqrt(3)*log(sqrt(3)*x + x**2 + 1) + 28*x**9 + 3
6*x**7 - 84*x**3 - 252*x)/252
```

3.882 $\int \frac{x^8}{1-x^2+x^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{x^8}{1-x^2+x^4} \, dx = \frac{x^3}{3}+\frac{x^5}{5}+\frac{1}{2}\arctan\left(\sqrt{3}-2x\right) -\frac{1}{2}\arctan\left(\sqrt{3}+2x\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{2\sqrt{3}}$$

output
```
1/3*x^3+1/5*x^5-1/2*arctan(-3^(1/2)+2*x)-1/2*arctan(3^(1/2)+2*x)+1/6*arcta
nh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.51

$$\int \frac{x^8}{1-x^2+x^4} \, dx = \frac{x^3}{3}+\frac{x^5}{5}-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{-6+6i\sqrt{3}}} -\frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-6-6i\sqrt{3}}}$$

input
```
Integrate[x^8/(1 - x^2 + x^4),x]
```

output
```
x^3/3 + x^5/5 - ((I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[-6 + (6
*I)*Sqrt[3]] - ((-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[-6 - (6
*I)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1442, 27, 1602, 27, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x^5}{5}-\frac{1}{5}\int \frac{5x^4(1-x^2)}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^5}{5}-\int \frac{x^4(1-x^2)}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{3}\int -\frac{3x^2}{x^4-x^2+1}dx+\frac{x^5}{5}+\frac{x^3}{3}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^2}{x^4-x^2+1}dx+\frac{x^5}{5}+\frac{x^3}{3}$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4-x^2+1}dx+\frac{x^5}{5}+\frac{x^3}{3}$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int\frac{1}{x^2+\sqrt{3}x+1}dx\right)+\frac{1}{2}\int\frac{1-x^2}{x^4-x^2+1}dx+\frac{x^5}{5}+\frac{x^3}{3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\int\frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(\int\frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)+\int\frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)\right)+\frac{x^5}{5}+\frac{x^3}{3}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\int\frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)+\frac{x^5}{5}+\frac{x^3}{3}$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int-\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)+\frac{x^5}{5}+\frac{x^3}{3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)+\frac{x^5}{5}+\frac{x^3}{3}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)+\frac{x^5}{5}+\frac{x^3}{3}+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[x^8/(1 - x^2 + x^4),x]
```

output
```
x^3/3 + x^5/5 + (ArcTan[Sqrt[3] - 2*x] - ArcTan[Sqrt[3] + 2*x])/2 + (-1/2*
Log[1 - Sqrt[3]*x + x^2]/Sqrt[3] + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{x^5}{5}+\frac{x^3}{3}-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-\_Z^2+1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^3-\_R}\right)}{2}$	49
default	$\frac{x^5}{5}+\frac{x^3}{3}+\frac{\sqrt{3}\left(-\frac{\ln\left(x^2-\sqrt{3}\,x+1\right)}{2}-\sqrt{3}\,\arctan\left(2x-\sqrt{3}\right)\right)}{6}+\frac{\sqrt{3}\left(\frac{\ln\left(x^2+\sqrt{3}\,x+1\right)}{2}-\sqrt{3}\,\arctan\left(2x+\sqrt{3}\right)\right)}{6}$	79

input
```
int(x^8/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5+1/3*x^3-1/2*sum(_R^2/(2*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{1-x^2+x^4}\,dx = \frac{1}{5}\,x^5+\frac{1}{3}\,x^3+\frac{1}{12}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{12}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{1}{2}\,\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{2}\,\arctan\left(-2\,x+\sqrt{3}\right)$$

input
```
integrate(x^8/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/5*x^5 + 1/3*x^3 + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*l
og(x^2 - sqrt(3)*x + 1) - 1/2*arctan(2*x + sqrt(3)) + 1/2*arctan(-2*x + sq
rt(3))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{x^8}{1-x^2+x^4}\,dx = \frac{x^5}{5}+\frac{x^3}{3}-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}-\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}-\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

input
```
integrate(x**8/(x**4-x**2+1),x)
```

output
```
x**5/5 + x**3/3 - sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2
+ sqrt(3)*x + 1)/12 - atan(2*x - sqrt(3))/2 - atan(2*x + sqrt(3))/2
```

Maxima [F]

$$\int \frac{x^8}{1-x^2+x^4}\,dx = \int \frac{x^8}{x^4-x^2+1}\,dx$$

input
```
integrate(x^8/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/5*x^5 + 1/3*x^3 - integrate(x^2/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{x^8}{1-x^2+x^4}\,dx = \frac{1}{5}\,x^5 + \frac{1}{3}\,x^3 + \frac{1}{12}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{12}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) - \frac{1}{2}\,\arctan\left(2\,x+\sqrt{3}\right) - \frac{1}{2}\,\arctan\left(2\,x-\sqrt{3}\right)$$

input
```
integrate(x^8/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/5*x^5 + 1/3*x^3 + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*l
og(x^2 - sqrt(3)*x + 1) - 1/2*arctan(2*x + sqrt(3)) - 1/2*arctan(2*x - sqr
t(3))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{x^8}{1-x^2+x^4}\,dx = \mathrm{atan}\left(\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \mathrm{atan}\left(\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \frac{x^3}{3} + \frac{x^5}{5}$$

input
```
int(x^8/(x^4 - x^2 + 1),x)
```

output
```
atan(x/2 - (3^(1/2)*x*1i)/2)*((3^(1/2)*1i)/6 - 1/2) - atan(x/2 + (3^(1/2)*
x*1i)/2)*((3^(1/2)*1i)/6 + 1/2) + x^3/3 + x^5/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{x^8}{1-x^2+x^4}\,dx = \frac{atan\left(\sqrt{3}-2x\right)}{2} - \frac{atan\left(\sqrt{3}+2x\right)}{2} - \frac{\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)}{12} + \frac{\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{12} + \frac{x^5}{5} + \frac{x^3}{3}$$

input
```
int(x^8/(x^4-x^2+1),x)
```

output
```
(30*atan(sqrt(3) - 2*x) - 30*atan(sqrt(3) + 2*x) - 5*sqrt(3)*log( - sqrt(3
)*x + x**2 + 1) + 5*sqrt(3)*log(sqrt(3)*x + x**2 + 1) + 12*x**5 + 20*x**3)
/60
```

3.883 $\int \frac{x^6}{1-x^2+x^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int \frac{x^6}{1-x^2+x^4} \, dx = x+\frac{x^3}{3}+\frac{1}{2}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\arctan\left(\sqrt{3}+2x\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{2\sqrt{3}}$$

output
```
x+1/3*x^3-1/2*arctan(-3^(1/2)+2*x)-1/2*arctan(3^(1/2)+2*x)-1/6*arctanh(3^(
1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int \frac{x^6}{1-x^2+x^4} \, dx = \frac{1}{6}\left(2x(3+x^2)-i\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)+i\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)\right)$$

input
```
Integrate[x^6/(1 - x^2 + x^4),x]
```

output
```
(2*x*(3 + x^2) - I*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2]
+ I*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2])/6
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1442, 27, 1602, 25, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1442}$$

$$\frac{x^3}{3}-\frac{1}{3}\int \frac{3x^2(1-x^2)}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3}{3}-\int \frac{x^2(1-x^2)}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{1602}$$

$$\int -\frac{1}{x^4-x^2+1}dx+\frac{x^3}{3}+x$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{x^4-x^2+1}dx+\frac{x^3}{3}+x$$

$$\Big\downarrow \text{1407}$$

$$-\frac{\int \frac{\sqrt{3}-x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{x^3}{3}+x$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{x^3}{3}+x$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{x^3}{3}+x$$

$$\Big\downarrow \; 1083$$

$$-\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)}{2\sqrt{3}}+\frac{x^3}{3}+x$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+\sqrt{3}\arctan\left(2x+\sqrt{3}\right)}{2\sqrt{3}}+\frac{x^3}{3}+x$$

$$\Big\downarrow \; 1103$$

$$-\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\sqrt{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{x^3}{3}+x$$

input
```
Int[x^6/(1 - x^2 + x^4),x]
```

output
```
x + x^3/3 - (-(Sqrt[3]*ArcTan[Sqrt[3] - 2*x]) - Log[1 - Sqrt[3]*x + x^2]/2
)/(2*Sqrt[3]) - (Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + Log[1 + Sqrt[3]*x + x^2]/
2)/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x^3}{3}+x-\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-\_Z^2+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^3-\_R}\right)}{2}$	42
default	$\frac{x^3}{3}+x+\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\arctan\left(2x-\sqrt{3}\right)}{2}-\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}-\frac{\arctan\left(2x+\sqrt{3}\right)}{2}$	63

input
```
int(x^6/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+x-1/2*sum(1/(2*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int\frac{x^6}{1-x^2+x^4}\,dx=\frac{1}{3}\,x^3-\frac{1}{12}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+x-\frac{1}{2}\,\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{2}\,\arctan\left(-2\,x+\sqrt{3}\right)$$

input
```
integrate(x^6/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/3*x^3 - 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - s
qrt(3)*x + 1) + x - 1/2*arctan(2*x + sqrt(3)) + 1/2*arctan(-2*x + sqrt(3))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{x^6}{1-x^2+x^4} dx = \frac{x^3}{3} + x + \frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12} - \frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12} - \frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2} - \frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

input
```
integrate(x**6/(x**4-x**2+1),x)
```

output
```
x**3/3 + x + sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqr
t(3)*x + 1)/12 - atan(2*x - sqrt(3))/2 - atan(2*x + sqrt(3))/2
```

Maxima [F]

$$\int \frac{x^6}{1-x^2+x^4} dx = \int \frac{x^6}{x^4-x^2+1} dx$$

input
```
integrate(x^6/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/3*x^3 + x - integrate(1/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{x^6}{1-x^2+x^4}\,dx = \frac{1}{3}\,x^3 - \frac{1}{12}\,\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) + \frac{1}{12}\,\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + x - \frac{1}{2}\,\arctan\left(2\,x+\sqrt{3}\right) - \frac{1}{2}\,\arctan\left(2\,x-\sqrt{3}\right)$$

input
```
integrate(x^6/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/3*x^3 - 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - s
qrt(3)*x + 1) + x - 1/2*arctan(2*x + sqrt(3)) - 1/2*arctan(2*x - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{x^6}{1-x^2+x^4}\,dx = x - \operatorname{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \operatorname{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \frac{x^3}{3}$$

input
```
int(x^6/(x^4 - x^2 + 1),x)
```

output
```
x - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) - atan((2*x)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/6 + 1/2) + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{1-x^2+x^4}\,dx = \frac{atan\left(\sqrt{3}-2x\right)}{2} - \frac{atan\left(\sqrt{3}+2x\right)}{2} + \frac{\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)}{12} - \frac{\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{12} + \frac{x^3}{3} + x$$

input
```
int(x^6/(x^4-x^2+1),x)
```

output
```
(6*atan(sqrt(3) - 2*x) - 6*atan(sqrt(3) + 2*x) + sqrt(3)*log( - sqrt(3)*x
+ x**2 + 1) - sqrt(3)*log(sqrt(3)*x + x**2 + 1) + 4*x**3 + 12*x)/12
```

3.884 $\int \frac{x^2}{1-x^2+x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 53

$$\int \frac{x^2}{1-x^2+x^4}\,dx = -\frac{1}{2}\arctan\left(\sqrt{3}-2x\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2x\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{2\sqrt{3}}$$

output
```
1/2*arctan(-3^(1/2)+2*x)+1/2*arctan(3^(1/2)+2*x)-1/6*arctanh(3^(1/2)*x/(x^
2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.77

$$\int \frac{x^2}{1-x^2+x^4}\,dx = \frac{\sqrt{-1-i\sqrt{3}}\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)+\sqrt{-1+i\sqrt{3}}\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{2\sqrt{6}}$$

input
```
Integrate[x^2/(1 - x^2 + x^4),x]
```

output
```
(Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2] + Sqrt[-
1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])/(2*Sqrt[6])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 1447}$$

$$\frac{1}{2}\int \frac{x^2+1}{x^4-x^2+1}dx-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)-\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx$$

$$\Big\downarrow \text{ 1478}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)$$

input
```
Int[x^2/(1 - x^2 + x^4),x]
```

output
```
(-ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x])/2 + (Log[1 - Sqrt[3]*x + 
x^2]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}-\_Z^{2}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{3}-\_R}\right)}{2}$	38
default	$-\frac{\sqrt{3}\left(-\frac{\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{2}-\sqrt{3}\,\arctan\left(2x-\sqrt{3}\right)\right)}{6}-\frac{\sqrt{3}\left(\frac{\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{2}-\sqrt{3}\,\arctan\left(2x+\sqrt{3}\right)\right)}{6}$	69

input
```
int(x^2/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R^2/(2*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{1-x^2+x^4}\,dx = -\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \frac{1}{2}\arctan\left(2\,x+\sqrt{3}\right)-\frac{1}{2}\arctan\left(-2\,x+\sqrt{3}\right)$$

input
```
integrate(x^2/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
-1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x
+ 1) + 1/2*arctan(2*x + sqrt(3)) - 1/2*arctan(-2*x + sqrt(3))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{1-x^2+x^4}\,dx = \frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12} + \frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

input
```
integrate(x**2/(x**4-x**2+1),x)
```

output
```
sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

Maxima [F]

$$\int \frac{x^2}{1-x^2+x^4}\,dx = \int \frac{x^2}{x^4-x^2+1}\,dx$$

input
```
integrate(x^2/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{1-x^2+x^4}\,dx = -\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \frac{1}{2}\arctan\left(2\,x+\sqrt{3}\right)+\frac{1}{2}\arctan\left(2\,x-\sqrt{3}\right)$$

input
```
integrate(x^2/(x^4-x^2+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x
+ 1) + 1/2*arctan(2*x + sqrt(3)) + 1/2*arctan(2*x - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{1-x^2+x^4}\,dx = -\mathrm{atan}\left(\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \mathrm{atan}\left(\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(x^2/(x^4 - x^2 + 1),x)
```

output
```
atan(x/2 + (3^(1/2)*x*1i)/2)*((3^(1/2)*1i)/6 + 1/2) - atan(x/2 - (3^(1/2)*
x*1i)/2)*((3^(1/2)*1i)/6 - 1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{1-x^2+x^4}\,dx = -\frac{atan\left(\sqrt{3}-2x\right)}{2}+\frac{atan\left(\sqrt{3}+2x\right)}{2} + \frac{\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)}{12}-\frac{\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{12}$$

input
```
int(x^2/(x^4-x^2+1),x)
```

output
```
( - 6*atan(sqrt(3) - 2*x) + 6*atan(sqrt(3) + 2*x) + sqrt(3)*log( - sqrt(3)
*x + x**2 + 1) - sqrt(3)*log(sqrt(3)*x + x**2 + 1))/12
```

3.885 $\int \frac{1}{1-x^2+x^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{1}{1-x^2+x^4} dx = -\frac{1}{2}\arctan\left(\sqrt{3}-2x\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2x\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{2\sqrt{3}}$$

output
```
1/2*arctan(-3^(1/2)+2*x)+1/2*arctan(3^(1/2)+2*x)+1/6*arctanh(3^(1/2)*x/(x^
2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{1}{1-x^2+x^4} dx = \frac{i\left(\sqrt{-1-i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-\sqrt{-1+i\sqrt{3}}\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)\right)}{\sqrt{6}}$$

input
```
Integrate[(1 - x^2 + x^4)^(-1),x]
```

output
```
(I*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - Sqrt[-1 + I*Sqrt[
3]]*ArcTan[((1 + I*Sqrt[3])*x)/2]))/Sqrt[6]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-x^2+1} \, dx$$

$$\Big\downarrow \text{1407}$$

$$\frac{\int \frac{\sqrt{3}-x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)}{2\sqrt{3}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}+\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+\sqrt{3}\arctan\left(2x+\sqrt{3}\right)}{2\sqrt{3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}+\frac{\sqrt{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}$$

input
```
Int[(1 - x^2 + x^4)^(-1),x]
```

output
```
(-(Sqrt[3]*ArcTan[Sqrt[3] - 2*x]) - Log[1 - Sqrt[3]*x + x^2]/2)/(2*Sqrt[3]
) + (Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + Log[1 + Sqrt[3]*x + x^2]/2)/(2*Sqrt[3
])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-\_Z^2+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^3-\_R}\right)}{2}$	35
default	$-\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}+\frac{\arctan\left(2x-\sqrt{3}\right)}{2}+\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}+\frac{\arctan\left(2x+\sqrt{3}\right)}{2}$	57

input
```
int(1/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{1}{1-x^2+x^4}\,dx = \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{2}\arctan\left(2\,x+\sqrt{3}\right)-\frac{1}{2}\arctan\left(-2\,x+\sqrt{3}\right)$$

input
```
integrate(1/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + 1/2*arctan(2*x + sqrt(3)) - 1/2*arctan(-2*x + sqrt(3))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \frac{1}{1-x^2+x^4}\,dx = -\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12} + \frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

input
```
integrate(1/(x**4-x**2+1),x)
```

output
```
-sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/
12 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

Maxima [F]

$$\int \frac{1}{1-x^2+x^4}\,dx = \int \frac{1}{x^4-x^2+1}\,dx$$

input
```
integrate(1/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{1}{1-x^2+x^4}\,dx = \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \frac{1}{2}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\arctan\left(2x-\sqrt{3}\right)$$

input
```
integrate(1/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + 1/2*arctan(2*x + sqrt(3)) + 1/2*arctan(2*x - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{1}{1-x^2+x^4}\,dx = \operatorname{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \operatorname{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(1/(x^4 - x^2 + 1),x)
```

output
```
atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atan((2*x)/(3^(1/2)*
1i + 1))*((3^(1/2)*1i)/6 + 1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{1-x^2+x^4}\,dx = -\frac{atan\left(\sqrt{3}-2x\right)}{2}+\frac{atan\left(\sqrt{3}+2x\right)}{2} - \frac{\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(\sqrt{3}\,x+x^2+1\right)}{12}$$

input
```
int(1/(x^4-x^2+1),x)
```

output
```
( - 6*atan(sqrt(3) - 2*x) + 6*atan(sqrt(3) + 2*x) - sqrt(3)*log( - sqrt(3)
*x + x**2 + 1) + sqrt(3)*log(sqrt(3)*x + x**2 + 1))/12
```

3.886 $\int \frac{1}{x^4(1-x^2+x^4)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \frac{1}{x^4(1-x^2+x^4)}\,dx = -\frac{1}{3x^3}-\frac{1}{x}+\frac{1}{2}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\arctan\left(\sqrt{3}+2x\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{2\sqrt{3}}$$

output
```
-1/3/x^3-1/x-1/2*arctan(-3^(1/2)+2*x)-1/2*arctan(3^(1/2)+2*x)+1/6*arctanh(
3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^4(1-x^2+x^4)}\,dx = -\frac{1}{3x^3}-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{-6+6i\sqrt{3}}}-\frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-6-6i\sqrt{3}}}$$

input
```
Integrate[1/(x^4*(1 - x^2 + x^4)),x]
```

output
```
-1/3*1/x^3 - x^(-1) - ((I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[-
6 + (6*I)*Sqrt[3]] - ((-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[-
6 - (6*I)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.38, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1443, 27, 1604, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^4 - x^2 + 1)} \, dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{3} \int \frac{3(1 - x^2)}{x^2 (x^4 - x^2 + 1)} dx - \frac{1}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1 - x^2}{x^2 (x^4 - x^2 + 1)} dx - \frac{1}{3x^3}$$

$$\Big\downarrow \text{1604}$$

$$-\int \frac{x^2}{x^4 - x^2 + 1} dx - \frac{1}{3x^3} - \frac{1}{x}$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2} \int \frac{1 - x^2}{x^4 - x^2 + 1} dx - \frac{1}{2} \int \frac{x^2 + 1}{x^4 - x^2 + 1} dx - \frac{1}{3x^3} - \frac{1}{x}$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2} \left(-\frac{1}{2} \int \frac{1}{x^2 - \sqrt{3}x + 1} dx - \frac{1}{2} \int \frac{1}{x^2 + \sqrt{3}x + 1} dx \right) + \frac{1}{2} \int \frac{1 - x^2}{x^4 - x^2 + 1} dx - \frac{1}{3x^3} - \frac{1}{x}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)+\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)\right)-\frac{1}{3x^3}-\frac{1}{x}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)-\frac{1}{3x^3}-\frac{1}{x}$$

$$\Big\downarrow \; 1478$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)-\frac{1}{3x^3}-\frac{1}{x}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)-\frac{1}{3x^3}-\frac{1}{x}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{2}\left(\arctan\left(\sqrt{3}-2x\right)-\arctan\left(2x+\sqrt{3}\right)\right)-\frac{1}{3x^3}+\frac{1}{2}\left(\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}\right)-\frac{1}{x}$$

input
```
Int[1/(x^4*(1 - x^2 + x^4)),x]
```

output
```
-1/3*1/x^3 - x^(-1) + (ArcTan[Sqrt[3] - 2*x] - ArcTan[Sqrt[3] + 2*x])/2 +
(-1/2*Log[1 - Sqrt[3]*x + x^2]/Sqrt[3] + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[
3]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{-x^{2}-\frac{1}{3}}{x^{3}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{4}+3\_Z^{2}+1\right)}\_R\ln\left(-6\_R^{3}-\_R+x\right)\right)}{2}$	44
default	$-\frac{1}{3x^{3}}-\frac{1}{x}+\frac{\sqrt{3}\left(-\frac{\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{2}-\sqrt{3}\,\arctan\left(2x-\sqrt{3}\right)\right)}{6}+\frac{\sqrt{3}\left(\frac{\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{2}-\sqrt{3}\,\arctan\left(2x+\sqrt{3}\right)\right)}{6}$	79

input
```
int(1/x^4/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(-x^2-1/3)/x^3+1/2*sum(_R*ln(-6*_R^3-_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^4 (1-x^2+x^4)} dx = \frac{\sqrt{3}x^3 \log \left(x^2+\sqrt{3}x+1\right) - \sqrt{3}x^3 \log \left(x^2-\sqrt{3}x+1\right) - 6\,x^3 \arctan \left(2\,x+\sqrt{3}\right) + 6\,x^3 \arctan \left(-2\,x+\sqrt{3}\right)}{12\,x^3}$$

input
```
integrate(1/x^4/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/12*(sqrt(3)*x^3*log(x^2 + sqrt(3)*x + 1) - sqrt(3)*x^3*log(x^2 - sqrt(3)
*x + 1) - 6*x^3*arctan(2*x + sqrt(3)) + 6*x^3*arctan(-2*x + sqrt(3)) - 12*
x^2 - 4)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^4 (1-x^2+x^4)} dx = -\frac{\sqrt{3}\log \left(x^2-\sqrt{3}x+1\right)}{12} + \frac{\sqrt{3}\log \left(x^2+\sqrt{3}x+1\right)}{12} - \frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2} - \frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2} + \frac{-3x^2-1}{3x^3}$$

input
```
integrate(1/x**4/(x**4-x**2+1),x)
```

output
```
-sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/
12 - atan(2*x - sqrt(3))/2 - atan(2*x + sqrt(3))/2 + (-3*x**2 - 1)/(3*x**3
)
```

Maxima [F]

$$\int \frac{1}{x^4 (1-x^2+x^4)} \, dx = \int \frac{1}{(x^4-x^2+1)x^4} \, dx$$

input
```
integrate(1/x^4/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
-1/3*(3*x^2 + 1)/x^3 - integrate(x^2/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^4 (1-x^2+x^4)} \, dx = -\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) - \frac{3\,x^2+1}{3\,x^3}-\frac{1}{2}\arctan\left(2\,x+\sqrt{3}\right)-\frac{1}{2}\arctan\left(2\,x-\sqrt{3}\right)$$

input
```
integrate(1/x^4/(x^4-x^2+1),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 - sqrt(3)*x
+ 1) - 1/3*(3*x^2 + 1)/x^3 - 1/2*arctan(2*x + sqrt(3)) - 1/2*arctan(2*x -
sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^4 (1-x^2+x^4)} \, dx = \mathrm{atan}\left(\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \mathrm{atan}\left(\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\frac{x^2+\frac{1}{3}}{x^3}$$

input
```
int(1/(x^4*(x^4 - x^2 + 1)),x)
```

output
```
atan(x/2 - (3^(1/2)*x*1i)/2)*((3^(1/2)*1i)/6 - 1/2) - atan(x/2 + (3^(1/2)*
x*1i)/2)*((3^(1/2)*1i)/6 + 1/2) - (x^2 + 1/3)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4(1-x^2+x^4)}\,dx = \frac{6atan(\sqrt{3}-2x)\,x^3-6atan(\sqrt{3}+2x)\,x^3-\sqrt{3}\log(-\sqrt{3}\,x+x^2+1)\,x^3+\sqrt{3}\log(\sqrt{3}\,x+x^2+1)\,x^3-}{12x^3}$$

input
```
int(1/x^4/(x^4-x^2+1),x)
```

output
```
(6*atan(sqrt(3) - 2*x)*x**3 - 6*atan(sqrt(3) + 2*x)*x**3 - sqrt(3)*log( -
sqrt(3)*x + x**2 + 1)*x**3 + sqrt(3)*log(sqrt(3)*x + x**2 + 1)*x**3 - 12*x
**2 - 4)/(12*x**3)
```

3.887 $\int \frac{1}{x^6(1-x^2+x^4)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{1}{x^6(1-x^2+x^4)}\,dx = -\frac{1}{5x^5}-\frac{1}{3x^3}+\frac{1}{2}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\arctan\left(\sqrt{3}+2x\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{2\sqrt{3}}$$

output
```
-1/5/x^5-1/3/x^3-1/2*arctan(-3^(1/2)+2*x)-1/2*arctan(3^(1/2)+2*x)-1/6*arct
anh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^6(1-x^2+x^4)}\,dx = -\frac{6+10x^2+5i\sqrt{-6-6i\sqrt{3}}x^5\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-5i\sqrt{-6+6i\sqrt{3}}x^5\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{30x^5}$$

input
```
Integrate[1/(x^6*(1 - x^2 + x^4)),x]
```

output
```
-1/30*(6 + 10*x^2 + (5*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*x^5*ArcTan[((1 - I*Sqrt
[3])*x)/2] - (5*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*x^5*ArcTan[((1 + I*Sqrt[3])*x)
/2])/x^5
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.54, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1443, 27, 1604, 27, 1407, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (x^4-x^2+1)} \, dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{1}{5} \int \frac{5(1-x^2)}{x^4 (x^4-x^2+1)}dx-\frac{1}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1-x^2}{x^4 (x^4-x^2+1)}dx-\frac{1}{5x^5}$$

$$\Big\downarrow \text{1604}$$

$$-\frac{1}{3} \int \frac{3}{x^4-x^2+1}dx-\frac{1}{5x^5}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{1}{x^4-x^2+1}dx-\frac{1}{5x^5}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1407}$$

$$-\frac{\int \frac{\sqrt{3}-x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{3x^3}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx-\frac{1}{2}\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{1}{x^2+\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 1083$$

$$-\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx-\sqrt{3}\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 217$$

$$-\frac{\frac{1}{2}\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)}{2\sqrt{3}}-\frac{\frac{1}{2}\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx+\sqrt{3}\arctan\left(2x+\sqrt{3}\right)}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{3x^3}$$

$$\Big\downarrow \ 1103$$

$$-\frac{-\sqrt{3}\arctan\left(\sqrt{3}-2x\right)-\frac{1}{2}\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\sqrt{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{2}\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{1}{5x^5}-\frac{1}{3x^3}$$

input

```
Int[1/(x^6*(1 - x^2 + x^4)),x]
```

output

```
-1/5*1/x^5 - 1/(3*x^3) - (-(Sqrt[3]*ArcTan[Sqrt[3] - 2*x]) - Log[1 - Sqrt[
3]*x + x^2]/2)/(2*Sqrt[3]) - (Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + Log[1 + Sqrt
[3]*x + x^2]/2)/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{-\frac{x^2}{3}-\frac{1}{5}}{x^5}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(3\_R^3-\_R+x\right)\right)}{2}$	44
default	$-\frac{1}{5x^5}-\frac{1}{3x^3}+\frac{\sqrt{3}\,\ln\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\arctan\left(2x-\sqrt{3}\right)}{2}-\frac{\sqrt{3}\,\ln\left(x^2+\sqrt{3}\,x+1\right)}{12}-\frac{\arctan\left(2x+\sqrt{3}\right)}{2}$	67

input
```
int(1/x^6/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*x^2-1/5)/x^5+1/2*sum(_R*ln(3*_R^3-_R+x),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^6\left(1-x^2+x^4\right)}\,dx = -\frac{5\,\sqrt{3}x^5\log\left(x^2+\sqrt{3}x+1\right)-5\,\sqrt{3}x^5\log\left(x^2-\sqrt{3}x+1\right)+30\,x^5\arctan\left(2\,x+\sqrt{3}\right)-30\,x^5\arctan\left(-\right.}{60\,x^5}$$

input
```
integrate(1/x^6/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
-1/60*(5*sqrt(3)*x^5*log(x^2 + sqrt(3)*x + 1) - 5*sqrt(3)*x^5*log(x^2 - sq
rt(3)*x + 1) + 30*x^5*arctan(2*x + sqrt(3)) - 30*x^5*arctan(-2*x + sqrt(3)
) + 20*x^2 + 12)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^6 (1 - x^2 + x^4)} \, dx = \frac{\sqrt{3} \log \left(x^2 - \sqrt{3}x + 1\right)}{12} - \frac{\sqrt{3} \log \left(x^2 + \sqrt{3}x + 1\right)}{12} - \frac{\operatorname{atan}\left(2x - \sqrt{3}\right)}{2} - \frac{\operatorname{atan}\left(2x + \sqrt{3}\right)}{2} + \frac{-5x^2 - 3}{15x^5}$$

input
```
integrate(1/x**6/(x**4-x**2+1),x)
```

output
```
sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 - atan(2*x - sqrt(3))/2 - atan(2*x + sqrt(3))/2 + (-5*x**2 - 3)/(15*x**5
)
```

Maxima [F]

$$\int \frac{1}{x^6 (1 - x^2 + x^4)} \, dx = \int \frac{1}{(x^4 - x^2 + 1)x^6} \, dx$$

input
```
integrate(1/x^6/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
-1/15*(5*x^2 + 3)/x^5 - integrate(1/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^6 (1-x^2+x^4)} dx = -\frac{1}{6}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{6}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{5x^2+3}{15x^5}$$

input
```
integrate(1/x^6/(x^4-x^2+1),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/6*sqrt(3)*log(x^2 - sqrt(3)*x +
1) - 1/15*(5*x^2 + 3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^6 (1-x^2+x^4)} dx = -\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\frac{\frac{x^2}{3}+\frac{1}{5}}{x^5}$$

input
```
int(1/(x^6*(x^4 - x^2 + 1)),x)
```

output
```
- atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) - atan((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/6 + 1/2) - (x^2/3 + 1/5)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^6 (1-x^2+x^4)} dx = \frac{30atan(\sqrt{3}-2x)\,x^5-30atan(\sqrt{3}+2x)\,x^5+5\sqrt{3}\log(-\sqrt{3}\,x+x^2+1)\,x^5-5\sqrt{3}\log(\sqrt{3}\,x+x^2+1)\,x^5}{60x^5}$$

input
```
int(1/x^6/(x^4-x^2+1),x)
```

output
```
(30*atan(sqrt(3) - 2*x)*x**5 - 30*atan(sqrt(3) + 2*x)*x**5 + 5*sqrt(3)*log
( - sqrt(3)*x + x**2 + 1)*x**5 - 5*sqrt(3)*log(sqrt(3)*x + x**2 + 1)*x**5
- 20*x**2 - 12)/(60*x**5)
```

3.888 $\int \frac{1}{x^{10}(1-x^2+x^4)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{1}{x^{10}\left(1-x^2+x^4\right)}\,dx=-\frac{1}{9x^9}-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{x}-\frac{1}{2}\arctan\left(\sqrt{3}-2x\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2x\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{2\sqrt{3}}$$

output
```
-1/9/x^9-1/7/x^7+1/3/x^3+1/x+1/2*arctan(-3^(1/2)+2*x)+1/2*arctan(3^(1/2)+2
*x)-1/6*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^{10}\left(1-x^2+x^4\right)}\,dx=-\frac{1}{9x^9}-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{x}+\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{-6+6i\sqrt{3}}}+\frac{\left(-i+\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-6-6i\sqrt{3}}}$$

input
```
Integrate[1/(x^10*(1 - x^2 + x^4)),x]
```

output
```
-1/9*1/x^9 - 1/(7*x^7) + 1/(3*x^3) + x^(-1) + ((I + Sqrt[3])*ArcTan[((1 -
I*Sqrt[3])*x)/2])/Sqrt[-6 + (6*I)*Sqrt[3]] + ((-I + Sqrt[3])*ArcTan[((1 +
I*Sqrt[3])*x)/2])/Sqrt[-6 - (6*I)*Sqrt[3]]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.32, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {1443, 27, 1604, 27, 1443, 27, 1604, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10} \left(x^4-x^2+1\right)} \, dx$$
$$\Big\downarrow \text{1443}$$
$$\frac{1}{9} \int \frac{9 \left(1-x^2\right)}{x^8 \left(x^4-x^2+1\right)}dx-\frac{1}{9 x^9}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1-x^2}{x^8 \left(x^4-x^2+1\right)}dx-\frac{1}{9 x^9}$$
$$\Big\downarrow \text{1604}$$
$$-\frac{1}{7} \int \frac{7}{x^4 \left(x^4-x^2+1\right)}dx-\frac{1}{9 x^9}-\frac{1}{7 x^7}$$
$$\Big\downarrow \text{27}$$
$$-\int \frac{1}{x^4 \left(x^4-x^2+1\right)}dx-\frac{1}{9 x^9}-\frac{1}{7 x^7}$$
$$\Big\downarrow \text{1443}$$
$$-\frac{1}{3} \int \frac{3 \left(1-x^2\right)}{x^2 \left(x^4-x^2+1\right)}dx-\frac{1}{9 x^9}-\frac{1}{7 x^7}+\frac{1}{3 x^3}$$
$$\Big\downarrow \text{27}$$

$$-\int \frac{1-x^2}{x^2\left(x^4-x^2+1\right)}dx-\frac{1}{9x^9}-\frac{1}{7x^7}+\frac{1}{3x^3}$$

$$\Big\downarrow \text{1604}$$

$$\int \frac{x^2}{x^4-x^2+1}dx-\frac{1}{9x^9}-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{1447}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\int \frac{x^2+1}{x^4-x^2+1}dx-\frac{1}{9x^9}-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{3}x+1}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{3}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx-\frac{1}{9x^9}-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(-\int \frac{1}{-\left(2x-\sqrt{3}\right)^2-1}d\left(2x-\sqrt{3}\right)-\int \frac{1}{-\left(2x+\sqrt{3}\right)^2-1}d\left(2x+\sqrt{3}\right)\right)-\frac{1}{9x^9}-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{2}\int \frac{1-x^2}{x^4-x^2+1}dx+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)-\frac{1}{9x^9}-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}+\frac{\int -\frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)-\frac{1}{9x^9}-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{3}-2x}{x^2-\sqrt{3}x+1}dx}{2\sqrt{3}}-\frac{\int \frac{2x+\sqrt{3}}{x^2+\sqrt{3}x+1}dx}{2\sqrt{3}}\right)+\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)-\frac{1}{9x^9}-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{x}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\arctan\left(2x+\sqrt{3}\right)-\arctan\left(\sqrt{3}-2x\right)\right)-\frac{1}{9x^9}-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{3}x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{2\sqrt{3}}\right)+\frac{1}{x}$$

input
```
Int[1/(x^10*(1 - x^2 + x^4)),x]
```

output
```
-1/9*1/x^9 - 1/(7*x^7) + 1/(3*x^3) + x^(-1) + (-ArcTan[Sqrt[3] - 2*x] + Ar
cTan[Sqrt[3] + 2*x])/2 + (Log[1 - Sqrt[3]*x + x^2]/(2*Sqrt[3]) - Log[1 + S
qrt[3]*x + x^2]/(2*Sqrt[3]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.65

method	result
risch	$\frac{x^{8}+\frac{1}{3}x^{6}-\frac{1}{7}x^{2}-\frac{1}{9}}{x^{9}}+\frac{\left(\sum_{\_R =\mathrm{RootOf}\left(9\_Z^{4}+3\_Z^{2}+1\right)}\_R \ln\left(6\_R^{3}+\_R +x \right)\right)}{2}$
default	$-\frac{1}{9x^{9}}-\frac{1}{7x^{7}}+\frac{1}{3x^{3}}+\frac{1}{x}-\frac{\sqrt{3}\left(-\frac{\ln\left(x^{2}-\sqrt{3}\,x+1\right)}{2}-\sqrt{3}\,\arctan\left(2x-\sqrt{3}\right)\right)}{6}-\frac{\sqrt{3}\left(\frac{\ln\left(x^{2}+\sqrt{3}\,x+1\right)}{2}-\sqrt{3}\,\arctan\left(2x+\sqrt{3}\right)\right)}{6}$

input
```
int(1/x^10/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
(x^8+1/3*x^6-1/7*x^2-1/9)/x^9+1/2*sum(_R*ln(6*_R^3+_R+x),_R=RootOf(9*_Z^4+
3*_Z^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^{10}\left(1-x^{2}+x^{4}\right)}\,dx = -\frac{21\sqrt{3}x^{9}\log\left(x^{2}+\sqrt{3}x+1\right)-21\sqrt{3}x^{9}\log\left(x^{2}-\sqrt{3}x+1\right)-126\,x^{9}\arctan\left(2\,x+\sqrt{3}\right)+126\,x^{9}\,\mathrm{arcta}}{252\,x^{9}}$$

input
```
integrate(1/x^10/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
-1/252*(21*sqrt(3)*x^9*log(x^2 + sqrt(3)*x + 1) - 21*sqrt(3)*x^9*log(x^2 -
 sqrt(3)*x + 1) - 126*x^9*arctan(2*x + sqrt(3)) + 126*x^9*arctan(-2*x + sq
rt(3)) - 252*x^8 - 84*x^6 + 36*x^2 + 28)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^{10}\left(1-x^2+x^4\right)}\,dx = \frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12} - \frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12} + \frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2} + \frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2} + \frac{63x^8+21x^6-9x^2-7}{63x^9}$$

input
```
integrate(1/x**10/(x**4-x**2+1),x)
```

output
```
sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2 + (63*x**8 + 21*x**6 - 9
*x**2 - 7)/(63*x**9)
```

Maxima [F]

$$\int \frac{1}{x^{10}\left(1-x^2+x^4\right)}\,dx = \int \frac{1}{(x^4-x^2+1)x^{10}}\,dx$$

input
```
integrate(1/x^10/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/63*(63*x^8 + 21*x^6 - 9*x^2 - 7)/x^9 + integrate(x^2/(x^4 - x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^{10}\left(1-x^2+x^4\right)}\,dx = \frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \frac{63\,x^8+21\,x^6-9\,x^2-7}{63\,x^9} + \frac{1}{2}\arctan\left(2\,x+\sqrt{3}\right) + \frac{1}{2}\arctan\left(2\,x-\sqrt{3}\right)$$

input
```
integrate(1/x^10/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 - sqrt(3)*x +
 1) + 1/63*(63*x^8 + 21*x^6 - 9*x^2 - 7)/x^9 + 1/2*arctan(2*x + sqrt(3)) +
 1/2*arctan(2*x - sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^{10}\left(1-x^2+x^4\right)}\,dx = -\frac{-x^8-\frac{x^6}{3}+\frac{x^2}{7}+\frac{1}{9}}{x^9} - \operatorname{atan}\left(\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \operatorname{atan}\left(\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(1/(x^10*(x^4 - x^2 + 1)),x)
```

output
```
atan(x/2 + (3^(1/2)*x*1i)/2)*((3^(1/2)*1i)/6 + 1/2) - atan(x/2 - (3^(1/2)*
x*1i)/2)*((3^(1/2)*1i)/6 - 1/2) - (x^2/7 - x^6/3 - x^8 + 1/9)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{10}\left(1-x^2+x^4\right)}\,dx = \frac{-126atan\left(\sqrt{3}-2x\right)x^9+126atan\left(\sqrt{3}+2x\right)x^9+21\sqrt{3}\log\left(-\sqrt{3}\,x+x^2+1\right)x^9-21\sqrt{3}\log\left(\sqrt{3}\,x+x\right.}{252x^9}$$

input
```
int(1/x^10/(x^4-x^2+1),x)
```

output
```
( - 126*atan(sqrt(3) - 2*x)*x**9 + 126*atan(sqrt(3) + 2*x)*x**9 + 21*sqrt(
3)*log( - sqrt(3)*x + x**2 + 1)*x**9 - 21*sqrt(3)*log(sqrt(3)*x + x**2 + 1
)*x**9 + 252*x**8 + 84*x**6 - 36*x**2 - 28)/(252*x**9)
```

3.889 $\int \frac{x}{10+2x^2+x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{x}{10+2x^2+x^4}\,dx=\frac{1}{6}\arctan\left(\frac{1}{3}\left(1+x^2\right)\right)$$

output
```
1/6*arctan(1/3*x^2+1/3)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x}{10+2x^2+x^4}\,dx=\frac{1}{6}\arctan\left(\frac{1}{3}\left(1+x^2\right)\right)$$

input
```
Integrate[x/(10 + 2*x^2 + x^4),x]
```

output
```
ArcTan[(1 + x^2)/3]/6
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1432, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^4+2x^2+10}\,dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{x^4+2x^2+10}dx^2$$

$$\Big\downarrow \text{1083}$$

$$-\int \frac{1}{-x^4-36}d\left(2x^2+2\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{6}\arctan\left(\frac{1}{6}\left(2x^2+2\right)\right)$$

input
```
Int[x/(10 + 2*x^2 + x^4),x]
```

output
```
ArcTan[(2 + 2*x^2)/6]/6
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{1}{3}+\frac{x^2}{3}\right)}{6}$	11
risch	$\frac{\arctan\left(\frac{1}{3}+\frac{x^2}{3}\right)}{6}$	11
parallelrisch	$\frac{i\ln(x^2+3i+1)}{12}-\frac{i\ln(x^2-3i+1)}{12}$	24

input
```
int(x/(x^4+2*x^2+10),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctan(1/3+1/3*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{10+2x^2+x^4}\,dx=\frac{1}{6}\,\arctan\left(\frac{1}{3}\,x^2+\frac{1}{3}\right)$$

input
```
integrate(x/(x^4+2*x^2+10),x, algorithm="fricas")
```

output
```
1/6*arctan(1/3*x^2 + 1/3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{10+2x^2+x^4}\,dx = \frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

input
```
integrate(x/(x**4+2*x**2+10),x)
```

output
```
atan(x**2/3 + 1/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{10+2x^2+x^4}\,dx = \frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

input
```
integrate(x/(x^4+2*x^2+10),x, algorithm="maxima")
```

output
```
1/6*arctan(1/3*x^2 + 1/3)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{10+2x^2+x^4}\,dx = \frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

input
```
integrate(x/(x^4+2*x^2+10),x, algorithm="giac")
```

output
```
1/6*arctan(1/3*x^2 + 1/3)
```

Mupad [B] (verification not implemented)

Time = 19.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{x}{10+2x^2+x^4}\,dx = \frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

input
```
int(x/(2*x^2 + x^4 + 10),x)
```

output
```
atan(x^2/3 + 1/3)/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 4.50

$$\int \frac{x}{10+2x^2+x^4}\,dx = -\frac{\sqrt{\sqrt{10}+1}\,\sqrt{\sqrt{10}-1}\left(atan\left(\frac{\sqrt{\sqrt{10}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{10}+1}\,\sqrt{2}}\right)+atan\left(\frac{\sqrt{\sqrt{10}-1}\,\sqrt{2}+2x}{\sqrt{\sqrt{10}+1}\,\sqrt{2}}\right)\right)}{18}$$

input
```
int(x/(x^4+2*x^2+10),x)
```

output
```
( - sqrt(sqrt(10) + 1)*sqrt(sqrt(10) - 1)*(atan((sqrt(sqrt(10) - 1)*sqrt(2
) - 2*x)/(sqrt(sqrt(10) + 1)*sqrt(2))) + atan((sqrt(sqrt(10) - 1)*sqrt(2)
+ 2*x)/(sqrt(sqrt(10) + 1)*sqrt(2)))))/18
```

3.890 $\int \frac{x^2}{20+9x^2+x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{x^2}{20+9x^2+x^4}\,dx = -2\arctan\left(\frac{x}{2}\right)+\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)$$

output
```
-2*arctan(1/2*x)+5^(1/2)*arctan(1/5*x*5^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{20+9x^2+x^4}\,dx = -2\arctan\left(\frac{x}{2}\right)+\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)$$

input
```
Integrate[x^2/(20 + 9*x^2 + x^4),x]
```

output
```
-2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1450, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^4+9x^2+20}\,dx$$

$$\Big\downarrow \text{1450}$$

$$5\int \frac{1}{x^2+5}dx-4\int \frac{1}{x^2+4}dx$$

$$\Big\downarrow \text{216}$$

$$\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)-2\arctan\left(\frac{x}{2}\right)$$

input
```
Int[x^2/(20 + 9*x^2 + x^4),x]
```

output
```
-2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
default	$-2\arctan\left(\frac{x}{2}\right)+\sqrt{5}\,\arctan\left(\frac{x\sqrt{5}}{5}\right)$	19
risch	$-2\arctan\left(\frac{x}{2}\right)+\sqrt{5}\,\arctan\left(\frac{x\sqrt{5}}{5}\right)$	19

input
```
int(x^2/(x^4+9*x^2+20),x,method=_RETURNVERBOSE)
```

output
```
-2*arctan(1/2*x)+5^(1/2)*arctan(1/5*x*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{20+9x^2+x^4}\,dx=\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-2\arctan\left(\frac{1}{2}x\right)$$

input
```
integrate(x^2/(x^4+9*x^2+20),x, algorithm="fricas")
```

output
```
sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{20+9x^2+x^4}\,dx=-2\,\mathrm{atan}\left(\frac{x}{2}\right)+\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)$$

input
```
integrate(x**2/(x**4+9*x**2+20),x)
```

output
```
-2*atan(x/2) + sqrt(5)*atan(sqrt(5)*x/5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{20+9x^2+x^4}\,dx = \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - 2\arctan\left(\frac{1}{2}x\right)$$

input
```
integrate(x^2/(x^4+9*x^2+20),x, algorithm="maxima")
```

output
```
sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{20+9x^2+x^4}\,dx = \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - 2\arctan\left(\frac{1}{2}x\right)$$

input
```
integrate(x^2/(x^4+9*x^2+20),x, algorithm="giac")
```

output
```
sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

Mupad [B] (verification not implemented)

Time = 18.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{20+9x^2+x^4}\,dx = \sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right) - 2\,\mathrm{atan}\left(\frac{x}{2}\right)$$

input
```
int(x^2/(9*x^2 + x^4 + 20),x)
```

output
```
5^(1/2)*atan((5^(1/2)*x)/5) - 2*atan(x/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{20+9x^2+x^4}\,dx = \sqrt{5}\,atan\left(\frac{x}{\sqrt{5}}\right) - 2atan\left(\frac{x}{2}\right)$$

input
```
int(x^2/(x^4+9*x^2+20),x)
```

output
```
sqrt(5)*atan(x/sqrt(5)) - 2*atan(x/2)
```

3.891 $\int \frac{x^4}{4+5x^2+x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{x}{2}\right) + \frac{\arctan(x)}{3}$$

output
```
x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x + \frac{8}{3}\arctan\left(\frac{2}{x}\right) + \frac{\arctan(x)}{3}$$

input
```
Integrate[x^4/(4 + 5*x^2 + x^4),x]
```

output
```
x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1442, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^4+5x^2+4} \, dx$$

$$\Big\downarrow \text{ 1442}$$

$$x-\int \frac{5x^2+4}{x^4+5x^2+4}dx$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{3}\int \frac{1}{x^2+1}dx-\frac{16}{3}\int \frac{1}{x^2+4}dx+x$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{8}{3}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{3}+x$$

input
```
Int[x^4/(4 + 5*x^2 + x^4),x]
```

output
```
x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result	size
default	$x-\frac{8 \arctan \left(\frac{x}{2}\right)}{3}+\frac{\arctan \left(x \right)}{3}$	13
risch	$x-\frac{8 \arctan \left(\frac{x}{2}\right)}{3}+\frac{\arctan \left(x \right)}{3}$	13
parallelrisch	$x+\frac{i \ln \left(x +i\right)}{6}-\frac{i \ln \left(x -i\right)}{6}+\frac{4 i \ln \left(x -2 i\right)}{3}-\frac{4 i \ln \left(x +2 i\right)}{3}$	35

input
```
int(x^4/(x^4+5*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

input
```
integrate(x^4/(x^4+5*x^2+4),x, algorithm="fricas")
```

output
```
x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}(x)}{3}$$

input
```
integrate(x**4/(x**4+5*x**2+4),x)
```

output
```
x - 8*atan(x/2)/3 + atan(x)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

input
```
integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")
```

output
```
x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan\left(x\right)$$

input
```
integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")
```

output
```
x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 19.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}(x)}{3}$$

input
```
int(x^4/(5*x^2 + x^4 + 4),x)
```

output
```
x - (8*atan(x/2))/3 + atan(x)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = -\frac{8atan\left(\frac{x}{2}\right)}{3} + \frac{atan(x)}{3} + x$$

input
```
int(x^4/(x^4+5*x^2+4),x)
```

output
```
( - 8*atan(x/2) + atan(x) + 3*x)/3
```

3.892 $\int \frac{x^2}{2-2x^2+x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 147

$$\int \frac{x^2}{2-2x^2+x^4} dx = -\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x}{\sqrt{2}+x^2}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

output
```
-1/4*(2+2*2^(1/2))^(1/2)*arctan(((2+2*2^(1/2))^(1/2)-2*x)/(-2+2*2^(1/2))^(
1/2))+1/4*(2+2*2^(1/2))^(1/2)*arctan(((2+2*2^(1/2))^(1/2)+2*x)/(-2+2*2^(1/
2))^(1/2))-1/2*arctanh((2+2*2^(1/2))^(1/2)*x/(x^2+2^(1/2)))/(2+2*2^(1/2))^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.27

$$\int \frac{x^2}{2-2x^2+x^4}\,dx=-\frac{\arctan\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}}-\frac{\arctan\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

input
```
Integrate[x^2/(2 - 2*x^2 + x^4),x]
```

output
```
-(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)
^(3/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\int \frac{x^2+\sqrt{2}}{x^4-2x^2+2}dx-\frac{1}{2}\int \frac{\sqrt{2}-x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx\right)-\frac{1}{2}\int \frac{\sqrt{2}-x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\int \frac{1}{2\left(1-\sqrt{2}\right)-\left(2x-\sqrt{2\left(1+\sqrt{2}\right)}\right)^2}d\left(2x-\sqrt{2\left(1+\sqrt{2}\right)}\right)-\int \frac{1}{2\left(1-\sqrt{2}\right)-\left(2x+\sqrt{2\left(1+\sqrt{2}\right)}\right)}\right.$$

$$\frac{1}{2}\int \frac{\sqrt{2}-x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{2}\int \frac{\sqrt{2}-x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx}{2\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\int -\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)+$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\int \frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)$$

input
```
Int[x^2/(2 - 2*x^2 + x^4),x]
```

output
```
(ArcTan[(-Sqrt[2*(1 + Sqrt[2])] + 2*x)/Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(-1
+ Sqrt[2])] + ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x)/Sqrt[2*(-1 + Sqrt[2])]]
/Sqrt[2*(-1 + Sqrt[2])])/2 + (Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]
/(2*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/
(2*Sqrt[2*(1 + Sqrt[2])]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.24

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-2\_Z^2+2\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3-\_R}\right)}{4}$
default	$-\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(\frac{\ln\left(x^2+x\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{2}-\frac{\sqrt{2+2\sqrt{2}}\arctan\left(\frac{\sqrt{2+2\sqrt{2}}+2x}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}\right)}{4}-\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(-\frac{\ln\left(x^2-x\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{2}\right.}{4}$

input
```
int(x^2/(x^4-2*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^2/(_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-2*_Z^2+2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{2-2x^2+x^4}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\arctan\left(2\left(\left(\sqrt{2}+1\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}+x\right)\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\arctan\left(2\left(\left(\sqrt{2}+1\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}-x\right)\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\right) - \frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\log\left(x^2+2\left(\sqrt{2}x+x\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}+\sqrt{2}\right) + \frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\log\left(x^2-2\left(\sqrt{2}x+x\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}+\sqrt{2}\right)$$

input
```
integrate(x^2/(x^4-2*x^2+2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2*sqrt(2) + 1/2)*arctan(2*((sqrt(2) + 1)*sqrt(1/2*sqrt(2) - 1/2
) + x)*sqrt(1/2*sqrt(2) + 1/2)) - 1/2*sqrt(1/2*sqrt(2) + 1/2)*arctan(2*((s
qrt(2) + 1)*sqrt(1/2*sqrt(2) - 1/2) - x)*sqrt(1/2*sqrt(2) + 1/2)) - 1/4*sq
rt(1/2*sqrt(2) - 1/2)*log(x^2 + 2*(sqrt(2)*x + x)*sqrt(1/2*sqrt(2) - 1/2)
+ sqrt(2)) + 1/4*sqrt(1/2*sqrt(2) - 1/2)*log(x^2 - 2*(sqrt(2)*x + x)*sqrt(
1/2*sqrt(2) - 1/2) + sqrt(2))
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.16

$$\int \frac{x^2}{2-2x^2+x^4}\,dx = \operatorname{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\log\left(64t^3+4t+x\right)\right)\right)$$

input
```
integrate(x**2/(x**4-2*x**2+2),x)
```

output
```
RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))
```

Maxima [F]

$$\int \frac{x^2}{2-2x^2+x^4}\,dx = \int \frac{x^2}{x^4-2\,x^2+2}\,dx$$

input
```
integrate(x^2/(x^4-2*x^2+2),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^4 - 2*x^2 + 2), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{2-2x^2+x^4}\,dx = \frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2\,x+2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\,\sqrt{-\sqrt{2}+2}}\right) + \frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2\,x-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\,\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2+2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right) + \frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2-2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)$$

input
```
integrate(x^2/(x^4-2*x^2+2),x, algorithm="giac")
```

output
```
1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(sqrt(2) + 2
))/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x -
 2^(1/4)*sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*
log(x^2 + 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2)) + 1/8*sqrt(2*sqrt(2) - 2)
*log(x^2 - 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{2-2x^2+x^4}\,dx = \mathrm{atanh}\left(32\,x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right) + \mathrm{atanh}\left(32\,x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}} - 2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)$$

input
```
int(x^2/(x^4 - 2*x^2 + 2),x)
```

output
```
atanh(32*x*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 - 1/32)^(1/2))^3)*(2
*(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2)) + atanh(32*x*(
(- 2^(1/2)/32 - 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/
32 - 1/32)^(1/2) - 2*(2^(1/2)/32 - 1/32)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.57

$$\int \frac{x^2}{2-2x^2+x^4}\,dx = -\frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}-2x}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{4} - \frac{\sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}-2x}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{2} + \frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}+2x}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{4} + \frac{\sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}+2x}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{2} - \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{2}+1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)}{8} + \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\log\left(\sqrt{\sqrt{2}+1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)}{8} + \frac{\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)}{4} - \frac{\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)}{4}$$

input
```
int(x^2/(x^4-2*x^2+2),x)
```

output

```
( - 2*sqrt(sqrt(2) - 1)*sqrt(2)*atan((sqrt(sqrt(2) + 1)*sqrt(2) - 2*x)/(sq
rt(sqrt(2) - 1)*sqrt(2))) - 4*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(2) + 1)*sq
rt(2) - 2*x)/(sqrt(sqrt(2) - 1)*sqrt(2))) + 2*sqrt(sqrt(2) - 1)*sqrt(2)*at
an((sqrt(sqrt(2) + 1)*sqrt(2) + 2*x)/(sqrt(sqrt(2) - 1)*sqrt(2))) + 4*sqrt
(sqrt(2) - 1)*atan((sqrt(sqrt(2) + 1)*sqrt(2) + 2*x)/(sqrt(sqrt(2) - 1)*sq
rt(2))) - sqrt(sqrt(2) + 1)*sqrt(2)*log( - sqrt(sqrt(2) + 1)*sqrt(2)*x + s
qrt(2) + x**2) + sqrt(sqrt(2) + 1)*sqrt(2)*log(sqrt(sqrt(2) + 1)*sqrt(2)*x
 + sqrt(2) + x**2) + 2*sqrt(sqrt(2) + 1)*log( - sqrt(sqrt(2) + 1)*sqrt(2)*
x + sqrt(2) + x**2) - 2*sqrt(sqrt(2) + 1)*log(sqrt(sqrt(2) + 1)*sqrt(2)*x
+ sqrt(2) + x**2))/8
```

3.893 $\int \frac{x^2}{1+(-1+x^2)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 147

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = -\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}x}{\sqrt{2}+x^2}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

output
```
-1/4*(2+2*2^(1/2))^(1/2)*arctan(((2+2*2^(1/2))^(1/2)-2*x)/(-2+2*2^(1/2))^(
1/2))+1/4*(2+2*2^(1/2))^(1/2)*arctan(((2+2*2^(1/2))^(1/2)+2*x)/(-2+2*2^(1/
2))^(1/2))-1/2*arctanh((2+2*2^(1/2))^(1/2)*x/(x^2+2^(1/2)))/(2+2*2^(1/2))^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.27

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = -\frac{\arctan\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}} - \frac{\arctan\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

input
```
Integrate[x^2/(1 + (-1 + x^2)^2),x]
```

output
```
-(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)
^(3/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2086, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2-1)^2+1}\,dx$$

$$\Big\downarrow \text{2086}$$

$$\int \frac{x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{1447}$$

$$\frac{1}{2}\int \frac{x^2+\sqrt{2}}{x^4-2x^2+2}dx - \frac{1}{2}\int \frac{\sqrt{2}-x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx\right)-\frac{1}{2}\int \frac{\sqrt{2}-x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(-\int \frac{1}{2\left(1-\sqrt{2}\right)-\left(2x-\sqrt{2\left(1+\sqrt{2}\right)}\right)^2}d\left(2x-\sqrt{2\left(1+\sqrt{2}\right)}\right)-\int \frac{1}{2\left(1-\sqrt{2}\right)-\left(2x+\sqrt{2\left(1+\sqrt{2}\right)}\right)}\right.$$
$$\frac{1}{2}\int \frac{\sqrt{2}-x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)-\frac{1}{2}\int \frac{\sqrt{2}-x^2}{x^4-2x^2+2}dx$$

$$\Big\downarrow \text{1478}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx}{2\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\int -\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)+$$
$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\int \frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}}dx}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{2x-\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}\right)$$

input
```
Int[x^2/(1 + (-1 + x^2)^2),x]
```

output
```
(ArcTan[(-Sqrt[2*(1 + Sqrt[2])] + 2*x)/Sqrt[2*(-1 + Sqrt[2])]]/Sqrt[2*(-1
+ Sqrt[2])] + ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x)/Sqrt[2*(-1 + Sqrt[2])]]
/Sqrt[2*(-1 + Sqrt[2])])/2 + (Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]
/(2*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/
(2*Sqrt[2*(1 + Sqrt[2])]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 2086
```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.24

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-2\_Z^2+2\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3-\_R}\right)}{4}$
default	$-\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(\frac{\ln\left(x^2+x\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{2}-\frac{\sqrt{2+2\sqrt{2}}\arctan\left(\frac{\sqrt{2+2\sqrt{2}}+2x}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}\right)}{4}-\frac{\sqrt{2+2\sqrt{2}}\left(\sqrt{2}-1\right)\left(-\frac{\ln\left(x^2-x\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{2}\right.}{4}$

input
```
int(x^2/(1+(x^2-1)^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^2/(_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-2*_Z^2+2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\arctan\left(2\left(\left(\sqrt{2}+1\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}+x\right)\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\arctan\left(2\left(\left(\sqrt{2}+1\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}-x\right)\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\right) - \frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\log\left(x^2+2\left(\sqrt{2}x+x\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}+\sqrt{2}\right) + \frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\log\left(x^2-2\left(\sqrt{2}x+x\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}+\sqrt{2}\right)$$

input
```
integrate(x^2/(1+(x^2-1)^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2*sqrt(2) + 1/2)*arctan(2*((sqrt(2) + 1)*sqrt(1/2*sqrt(2) - 1/2
) + x)*sqrt(1/2*sqrt(2) + 1/2)) - 1/2*sqrt(1/2*sqrt(2) + 1/2)*arctan(2*((s
qrt(2) + 1)*sqrt(1/2*sqrt(2) - 1/2) - x)*sqrt(1/2*sqrt(2) + 1/2)) - 1/4*sq
rt(1/2*sqrt(2) - 1/2)*log(x^2 + 2*(sqrt(2)*x + x)*sqrt(1/2*sqrt(2) - 1/2)
+ sqrt(2)) + 1/4*sqrt(1/2*sqrt(2) - 1/2)*log(x^2 - 2*(sqrt(2)*x + x)*sqrt(
1/2*sqrt(2) - 1/2) + sqrt(2))
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.16

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = \operatorname{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\log\left(64t^3+4t+x\right)\right)\right)$$

input
```
integrate(x**2/(1+(x**2-1)**2),x)
```

output
```
RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))
```

Maxima [F]

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = \int \frac{x^2}{(x^2-1)^2+1}\,dx$$

input
```
integrate(x^2/(1+(x^2-1)^2),x, algorithm="maxima")
```

output
```
integrate(x^2/((x^2 - 1)^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{x^2}{1+(-1+x^2)^2}\,dx &= \frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x+2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)\\ &\quad+\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)\\ &\quad-\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2+2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)\\ &\quad+\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2-2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)\end{aligned}$$

input
```
integrate(x^2/(1+(x^2-1)^2),x, algorithm="giac")
```

output
```
1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(sqrt(2) + 2
))/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x -
 2^(1/4)*sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*
log(x^2 + 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2)) + 1/8*sqrt(2*sqrt(2) - 2)
*log(x^2 - 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = \operatorname{atanh}\left(32\,x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right) + \operatorname{atanh}\left(32\,x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)$$

input
```
int(x^2/((x^2 - 1)^2 + 1),x)
```

output
```
atanh(32*x*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 - 1/32)^(1/2))^3)*(2
*(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2)) + atanh(32*x*(
(- 2^(1/2)/32 - 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/
32 - 1/32)^(1/2) - 2*(2^(1/2)/32 - 1/32)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.57

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = -\frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}-2x}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{4} - \frac{\sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}-2x}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{2} + \frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}+2x}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{4} + \frac{\sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}+2x}{\sqrt{\sqrt{2}-1}\,\sqrt{2}}\right)}{2} - \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\log\left(-\sqrt{\sqrt{2}+1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)}{8} + \frac{\sqrt{\sqrt{2}+1}\,\sqrt{2}\log\left(\sqrt{\sqrt{2}+1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)}{8} + \frac{\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)}{4} - \frac{\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}\,\sqrt{2}\,x+\sqrt{2}+x^2\right)}{4}$$

input
```
int(x^2/(1+(x^2-1)^2),x)
```

output
```
( - 2*sqrt(sqrt(2) - 1)*sqrt(2)*atan((sqrt(sqrt(2) + 1)*sqrt(2) - 2*x)/(sq
rt(sqrt(2) - 1)*sqrt(2))) - 4*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(2) + 1)*sq
rt(2) - 2*x)/(sqrt(sqrt(2) - 1)*sqrt(2))) + 2*sqrt(sqrt(2) - 1)*sqrt(2)*at
an((sqrt(sqrt(2) + 1)*sqrt(2) + 2*x)/(sqrt(sqrt(2) - 1)*sqrt(2))) + 4*sqrt
(sqrt(2) - 1)*atan((sqrt(sqrt(2) + 1)*sqrt(2) + 2*x)/(sqrt(sqrt(2) - 1)*sq
rt(2))) - sqrt(sqrt(2) + 1)*sqrt(2)*log( - sqrt(sqrt(2) + 1)*sqrt(2)*x + s
qrt(2) + x**2) + sqrt(sqrt(2) + 1)*sqrt(2)*log(sqrt(sqrt(2) + 1)*sqrt(2)*x
 + sqrt(2) + x**2) + 2*sqrt(sqrt(2) + 1)*log( - sqrt(sqrt(2) + 1)*sqrt(2)*
x + sqrt(2) + x**2) - 2*sqrt(sqrt(2) + 1)*log(sqrt(sqrt(2) + 1)*sqrt(2)*x
+ sqrt(2) + x**2))/8
```

3.894 $\int \frac{2x^2}{1+2x^2-x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{2x^2}{1+2x^2-x^4}\,dx=-\frac{\arctan\left(\sqrt{1+\sqrt{2}}x\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{-1+\sqrt{2}}x\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}}$$

output
```
-arctan((1+2^(1/2))^(1/2)*x)/(2+2*2^(1/2))^(1/2)+arctanh((2^(1/2)-1)^(1/2)
*x)/(-2+2*2^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \frac{2x^2}{1+2x^2-x^4}\,dx = \frac{-\left(\left(-1+\sqrt{2}\right)\sqrt{1+\sqrt{2}}\arctan\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)\right)+\sqrt{-1+\sqrt{2}}\left(1+\sqrt{2}\right)\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}$$

input
```
Integrate[(2*x^2)/(1 + 2*x^2 - x^4),x]
```

output
```
(-((-1 + Sqrt[2])*Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]]) + Sqrt[-
1 + Sqrt[2]]*(1 + Sqrt[2])*ArcTanh[x/Sqrt[1 + Sqrt[2]]])/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {27, 1450, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2}{-x^4+2x^2+1}dx$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{x^2}{-x^4+2x^2+1}dx$$

$$\Big\downarrow \text{1450}$$

$$2\left(\frac{1}{4}\left(2-\sqrt{2}\right)\int \frac{1}{-x^2-\sqrt{2}+1}dx+\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{-x^2+\sqrt{2}+1}dx\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{-x^2+\sqrt{2}+1}dx-\frac{\left(2-\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{\left(2+\sqrt{2}\right)\text{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}-\frac{\left(2-\sqrt{2}\right)\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{4\sqrt{\sqrt{2}-1}}\right)$$

input
```
Int[(2*x^2)/(1 + 2*x^2 - x^4),x]
```

output
```
2*(-1/4*((2 - Sqrt[2])*ArcTan[x/Sqrt[-1 + Sqrt[2]]])/Sqrt[-1 + Sqrt[2]] +
((2 + Sqrt[2])*ArcTanh[x/Sqrt[1 + Sqrt[2]]])/(4*Sqrt[1 + Sqrt[2]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-2\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3-\_R}\right)}{2}$	36
default	$-\frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\,\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{2}+\frac{\sqrt{1+\sqrt{2}}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{2}$	46

input
```
int(2*x^2/(-x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*sum(_R^2/(_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-2*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.50

$$\int \frac{2x^2}{1+2x^2-x^4}\,dx = -\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\arctan\left(\left(\sqrt{2}x+2\,x\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(x+\sqrt{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(x-\sqrt{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\right)$$

input
```
integrate(2*x^2/(-x^4+2*x^2+1),x, algorithm="fricas")
```

output
```
-sqrt(1/2*sqrt(2) - 1/2)*arctan((sqrt(2)*x + 2*x)*sqrt(1/2*sqrt(2) - 1/2))
 + 1/2*sqrt(1/2*sqrt(2) + 1/2)*log(x + sqrt(2)*sqrt(1/2*sqrt(2) + 1/2)) -
1/2*sqrt(1/2*sqrt(2) + 1/2)*log(x - sqrt(2)*sqrt(1/2*sqrt(2) + 1/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(49) = 98$.

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.17

$$\int \frac{2x^2}{1+2x^2-x^4}\,dx = 2\sqrt{\frac{1}{32}+\frac{\sqrt{2}}{32}}\log\left(x-4\sqrt{\frac{1}{32}+\frac{\sqrt{2}}{32}}+128\left(\frac{1}{32}+\frac{\sqrt{2}}{32}\right)^{\frac{3}{2}}\right) - 2\sqrt{\frac{1}{32}+\frac{\sqrt{2}}{32}}\log\left(x-128\left(\frac{1}{32}+\frac{\sqrt{2}}{32}\right)^{\frac{3}{2}}+4\sqrt{\frac{1}{32}+\frac{\sqrt{2}}{32}}\right) - 4\sqrt{-\frac{1}{32}+\frac{\sqrt{2}}{32}}\,\mathrm{atan}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)$$

input
```
integrate(2*x**2/(-x**4+2*x**2+1),x)
```

output
```
2*sqrt(1/32 + sqrt(2)/32)*log(x - 4*sqrt(1/32 + sqrt(2)/32) + 128*(1/32 +
sqrt(2)/32)**(3/2)) - 2*sqrt(1/32 + sqrt(2)/32)*log(x - 128*(1/32 + sqrt(2
)/32)**(3/2) + 4*sqrt(1/32 + sqrt(2)/32)) - 4*sqrt(-1/32 + sqrt(2)/32)*ata
n(x/sqrt(-1 + sqrt(2)))
```

Maxima [F]

$$\int \frac{2x^2}{1+2x^2-x^4}\,dx = \int -\frac{2\,x^2}{x^4-2\,x^2-1}\,dx$$

input
```
integrate(2*x^2/(-x^4+2*x^2+1),x, algorithm="maxima")
```

output
```
-2*integrate(x^2/(x^4 - 2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \frac{2x^2}{1+2x^2-x^4}\,dx = -\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) + \frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(\left|x+\sqrt{\sqrt{2}+1}\right|\right) - \frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(\left|x-\sqrt{\sqrt{2}+1}\right|\right)$$

input
```
integrate(2*x^2/(-x^4+2*x^2+1),x, algorithm="giac")
```

output
```
-1/2*sqrt(2*sqrt(2) - 2)*arctan(x/sqrt(sqrt(2) - 1)) + 1/4*sqrt(2*sqrt(2)
+ 2)*log(abs(x + sqrt(sqrt(2) + 1))) - 1/4*sqrt(2*sqrt(2) + 2)*log(abs(x -
 sqrt(sqrt(2) + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.40

$$\int \frac{2x^2}{1+2x^2-x^4}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\left(\frac{3x}{2}+4x\left(\frac{\sqrt{2}}{8}-\frac{1}{8}\right)\right)\sqrt{1-\sqrt{2}}\right)\sqrt{1-\sqrt{2}}}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{-2\sqrt{2}-2}\left(\frac{3x}{2}-4x\left(\frac{\sqrt{2}}{8}+\frac{1}{8}\right)\right)\right)\sqrt{\sqrt{2}+1}\,1\mathrm{i}}{2}$$

input
```
int((2*x^2)/(2*x^2 - x^4 + 1),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*((3*x)/2 + 4*x*(2^(1/2)/8 - 1/8))*(1 - 2^(1/2))^(1/
2))*(1 - 2^(1/2))^(1/2))/2 - (2^(1/2)*atan((- 2*2^(1/2) - 2)^(1/2)*((3*x)/
2 - 4*x*(2^(1/2)/8 + 1/8)))*(2^(1/2) + 1)^(1/2)*1i)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{2x^2}{1+2x^2-x^4}\,dx = \frac{\sqrt{2}\left(-2\sqrt{\sqrt{2}-1}\,atan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)-\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}+x\right)+\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}+x\right)\right)}{4}$$

input
```
int(2*x^2/(-x^4+2*x^2+1),x)
```

output
```
(sqrt(2)*( - 2*sqrt(sqrt(2) - 1)*atan(x/sqrt(sqrt(2) - 1)) - sqrt(sqrt(2)
+ 1)*log( - sqrt(sqrt(2) + 1) + x) + sqrt(sqrt(2) + 1)*log(sqrt(sqrt(2) +
1) + x)))/4
```

3.895 $\int \frac{1}{1+\frac{1-x^4}{2x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int \frac{1}{1+\frac{1-x^4}{2x^2}}\,dx = -\frac{\arctan\left(\sqrt{1+\sqrt{2}}x\right)}{\sqrt{2\left(1+\sqrt{2}\right)}}+\frac{\operatorname{arctanh}\left(\sqrt{-1+\sqrt{2}}x\right)}{\sqrt{2\left(-1+\sqrt{2}\right)}}$$

output
```
-arctan((1+2^(1/2))^(1/2)*x)/(2+2*2^(1/2))^(1/2)+arctanh((2^(1/2)-1)^(1/2)
*x)/(-2+2*2^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \frac{1}{1+\frac{1-x^4}{2x^2}}\,dx = \frac{-\left(\left(-1+\sqrt{2}\right)\sqrt{1+\sqrt{2}}\arctan\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)\right)+\sqrt{-1+\sqrt{2}}\left(1+\sqrt{2}\right)\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{\sqrt{2}}$$

input
```
Integrate[(1 + (1 - x^4)/(2*x^2))^(-1),x]
```

output
```
(-((-1 + Sqrt[2])*Sqrt[1 + Sqrt[2]]*ArcTan[x/Sqrt[-1 + Sqrt[2]]]) + Sqrt[-
1 + Sqrt[2]]*(1 + Sqrt[2])*ArcTanh[x/Sqrt[1 + Sqrt[2]]])/Sqrt[2]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{1-x^4}{2x^2}+1} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{1}{\frac{1-x^4}{2x^2}+1}dx$$

input
```
Int[(1 + (1 - x^4)/(2*x^2))^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-2\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3-\_R}\right)}{2}$	36
default	$-\frac{\sqrt{\sqrt{2}-1}\,\sqrt{2}\,\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)}{2}+\frac{\sqrt{1+\sqrt{2}}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{x}{\sqrt{1+\sqrt{2}}}\right)}{2}$	46

input
```
int(1/(1+1/2*(-x^4+1)/x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*sum(_R^2/(_R^3-_R)*ln(x-_R),_R=RootOf(_Z^4-2*_Z^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.50

$$\int \frac{1}{1+\frac{1-x^4}{2x^2}}\,dx = -\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\arctan\left(\left(\sqrt{2}x+2\,x\right)\sqrt{\frac{1}{2}\sqrt{2}-\frac{1}{2}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(x+\sqrt{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\log\left(x-\sqrt{2}\sqrt{\frac{1}{2}\sqrt{2}+\frac{1}{2}}\right)$$

input
```
integrate(1/(1+1/2*(-x^4+1)/x^2),x, algorithm="fricas")
```

output
```
-sqrt(1/2*sqrt(2) - 1/2)*arctan((sqrt(2)*x + 2*x)*sqrt(1/2*sqrt(2) - 1/2))
 + 1/2*sqrt(1/2*sqrt(2) + 1/2)*log(x + sqrt(2)*sqrt(1/2*sqrt(2) + 1/2)) -
1/2*sqrt(1/2*sqrt(2) + 1/2)*log(x - sqrt(2)*sqrt(1/2*sqrt(2) + 1/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(49) = 98$.

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.17

$$\int \frac{1}{1+\frac{1-x^4}{2x^2}}\,dx = 2\sqrt{\frac{1}{32}+\frac{\sqrt{2}}{32}}\log\left(x-4\sqrt{\frac{1}{32}+\frac{\sqrt{2}}{32}}+128\left(\frac{1}{32}+\frac{\sqrt{2}}{32}\right)^{\frac{3}{2}}\right) - 2\sqrt{\frac{1}{32}+\frac{\sqrt{2}}{32}}\log\left(x-128\left(\frac{1}{32}+\frac{\sqrt{2}}{32}\right)^{\frac{3}{2}}+4\sqrt{\frac{1}{32}+\frac{\sqrt{2}}{32}}\right) - 4\sqrt{-\frac{1}{32}+\frac{\sqrt{2}}{32}}\operatorname{atan}\left(\frac{x}{\sqrt{-1+\sqrt{2}}}\right)$$

input
```
integrate(1/(1+1/2*(-x**4+1)/x**2),x)
```

output
```
2*sqrt(1/32 + sqrt(2)/32)*log(x - 4*sqrt(1/32 + sqrt(2)/32) + 128*(1/32 +
sqrt(2)/32)**(3/2)) - 2*sqrt(1/32 + sqrt(2)/32)*log(x - 128*(1/32 + sqrt(2
)/32)**(3/2) + 4*sqrt(1/32 + sqrt(2)/32)) - 4*sqrt(-1/32 + sqrt(2)/32)*ata
n(x/sqrt(-1 + sqrt(2)))
```

Maxima [F]

$$\int \frac{1}{1+\frac{1-x^4}{2x^2}}\,dx = \int -\frac{2}{\frac{x^4-1}{x^2}-2}\,dx$$

input
```
integrate(1/(1+1/2*(-x^4+1)/x^2),x, algorithm="maxima")
```

output
```
-2*integrate(1/((x^4 - 1)/x^2 - 2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \frac{1}{1+\frac{1-x^4}{2x^2}}\,dx = -\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right) + \frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(\left|x+\sqrt{\sqrt{2}+1}\right|\right) - \frac{1}{4}\sqrt{2\sqrt{2}+2}\log\left(\left|x-\sqrt{\sqrt{2}+1}\right|\right)$$

input
```
integrate(1/(1+1/2*(-x^4+1)/x^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2*sqrt(2) - 2)*arctan(x/sqrt(sqrt(2) - 1)) + 1/4*sqrt(2*sqrt(2)
+ 2)*log(abs(x + sqrt(sqrt(2) + 1))) - 1/4*sqrt(2*sqrt(2) + 2)*log(abs(x -
 sqrt(sqrt(2) + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.40

$$\int \frac{1}{1+\frac{1-x^4}{2x^2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\left(\frac{3x}{2}+4x\left(\frac{\sqrt{2}}{8}-\frac{1}{8}\right)\right)\sqrt{1-\sqrt{2}}\right)\sqrt{1-\sqrt{2}}}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{-2\sqrt{2}-2}\left(\frac{3x}{2}-4x\left(\frac{\sqrt{2}}{8}+\frac{1}{8}\right)\right)\right)\sqrt{\sqrt{2}+1}\,1\mathrm{i}}{2}$$

input
```
int(-1/((x^4/2 - 1/2)/x^2 - 1),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*((3*x)/2 + 4*x*(2^(1/2)/8 - 1/8))*(1 - 2^(1/2))^(1/
2))*(1 - 2^(1/2))^(1/2))/2 - (2^(1/2)*atan((- 2*2^(1/2) - 2)^(1/2)*((3*x)/
2 - 4*x*(2^(1/2)/8 + 1/8)))*(2^(1/2) + 1)^(1/2)*1i)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{1}{1+\frac{1-x^4}{2x^2}}\,dx = \frac{\sqrt{2}\left(-2\sqrt{\sqrt{2}-1}\,atan\left(\frac{x}{\sqrt{\sqrt{2}-1}}\right)-\sqrt{\sqrt{2}+1}\log\left(-\sqrt{\sqrt{2}+1}+x\right)+\sqrt{\sqrt{2}+1}\log\left(\sqrt{\sqrt{2}+1}+x\right)\right)}{4}$$

input
```
int(1/(1+1/2*(-x^4+1)/x^2),x)
```

output
```
(sqrt(2)*( - 2*sqrt(sqrt(2) - 1)*atan(x/sqrt(sqrt(2) - 1)) - sqrt(sqrt(2)
+ 1)*log( - sqrt(sqrt(2) + 1) + x) + sqrt(sqrt(2) + 1)*log(sqrt(sqrt(2) +
1) + x)))/4
```

3.896 $\int \frac{x^2}{a+(b+d)x^2+cx^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 196

$$\int \frac{x^2}{a+(b+d)x^2+cx^4} dx = -\frac{\sqrt{b+d-\sqrt{b^2-4ac+2bd+d^2}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+d-\sqrt{b^2-4ac+2bd+d^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac+2bd+d^2}} + \frac{\sqrt{b+d+\sqrt{b^2-4ac+2bd+d^2}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+d+\sqrt{b^2-4ac+2bd+d^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac+2bd+d^2}}$$

output

```
-1/2*(b+d-(-4*a*c+b^2+2*b*d+d^2)^(1/2))^(1/2)*arctan(2^(1/2)*c^(1/2)*x/(b+
d-(-4*a*c+b^2+2*b*d+d^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2+2*b*d+d
^2)^(1/2)+1/2*(b+d+(-4*a*c+b^2+2*b*d+d^2)^(1/2))^(1/2)*arctan(2^(1/2)*c^(1
/2)*x/(b+d+(-4*a*c+b^2+2*b*d+d^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^
2+2*b*d+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.13

$$\int \frac{x^2}{a+(b+d)x^2+cx^4}\,dx = \frac{\left(-b-d+\sqrt{b^2-4ac+2bd+d^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+d-\sqrt{b^2-4ac+2bd+d^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac+2bd+d^2}\sqrt{b+d-\sqrt{b^2-4ac+2bd+d^2}}} + \frac{\sqrt{b+d+\sqrt{b^2-4ac+2bd+d^2}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+d+\sqrt{b^2-4ac+2bd+d^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac+2bd+d^2}}$$

input
```
Integrate[x^2/(a + (b + d)*x^2 + c*x^4),x]
```

output
```
((-b - d + Sqrt[b^2 - 4*a*c + 2*b*d + d^2])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + d - Sqrt[b^2 - 4*a*c + 2*b*d + d^2]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4
*a*c + 2*b*d + d^2]*Sqrt[b + d - Sqrt[b^2 - 4*a*c + 2*b*d + d^2]]) + (Sqrt
[b + d + Sqrt[b^2 - 4*a*c + 2*b*d + d^2]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + d + Sqrt[b^2 - 4*a*c + 2*b*d + d^2]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a
*c + 2*b*d + d^2])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1450, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+x^2(b+d)+cx^4}\,dx$$

$$\Big\downarrow \text{1450}$$

$$\frac{1}{2}\left(1-\frac{b+d}{\sqrt{-4ac+b^2+2bd+d^2}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+d-\sqrt{b^2+2db+d^2-4ac}\right)}dx+$$
$$\frac{1}{2}\left(\frac{b+d}{\sqrt{-4ac+b^2+2bd+d^2}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+d+\sqrt{b^2+2db+d^2-4ac}\right)}dx$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(1-\frac{b+d}{\sqrt{-4ac+b^2+2bd+d^2}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{-4ac+b^2+2bd+d^2}+b+d}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-\sqrt{-4ac+b^2+2bd+d^2}+b+d}}+\frac{\left(\frac{b+d}{\sqrt{-4ac+b^2+2bd+d^2}}+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{-4ac+b^2+2bd+d^2}+b+d}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{-4ac+b^2+2bd+d^2}+b+d}}$$

input
```
Int[x^2/(a + (b + d)*x^2 + c*x^4),x]
```

output
```
((1 - (b + d)/Sqrt[b^2 - 4*a*c + 2*b*d + d^2])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + d - Sqrt[b^2 - 4*a*c + 2*b*d + d^2]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b +
d - Sqrt[b^2 - 4*a*c + 2*b*d + d^2]]) + ((1 + (b + d)/Sqrt[b^2 - 4*a*c + 2
*b*d + d^2])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + d + Sqrt[b^2 - 4*a*c + 2*
b*d + d^2]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + d + Sqrt[b^2 - 4*a*c + 2*b*d + d^2
]])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+(b+d)\_Z^{2}+a\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{3}c+\_Rb+\_Rd}\right)}{2}$
default	$4c\left(-\frac{\left(\sqrt{-4ac+b^{2}+2bd+d^{2}}-b-d\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^{2}+2bd+d^{2}}-b-d\right)c}}\right)}{8c\sqrt{-4ac+b^{2}+2bd+d^{2}}\,\sqrt{\left(\sqrt{-4ac+b^{2}+2bd+d^{2}}-b-d\right)c}}+\frac{\left(b+d+\sqrt{-4ac+b^{2}+2bd+d^{2}}\right)\sqrt{2}\,\arctan\left(\right.}{8\sqrt{-4ac+b^{2}+2bd+d^{2}}\,c\sqrt{\left(b+d+\right.}}\right.$

input
```
int(x^2/(a+(b+d)*x^2+c*x^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R^2/(2*_R^3*c+_R*b+_R*d)*ln(x-_R),_R=RootOf(c*_Z^4+(b+d)*_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(161) = 322$.

Time = 0.08 (sec) , antiderivative size = 935, normalized size of antiderivative = 4.77

$$\int \frac{x^2}{a+(b+d)x^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+(b+d)*x^2+c*x^4),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt(-(b + d + (b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2)/sqrt(b^2*
c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2))/(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2))
*log(sqrt(1/2)*(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2)*sqrt(-(b + d + (b^2*c -
 4*a*c^2 + 2*b*c*d + c*d^2)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2))
/(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2))/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d +
 c^2*d^2) + x) - 1/2*sqrt(1/2)*sqrt(-(b + d + (b^2*c - 4*a*c^2 + 2*b*c*d +
 c*d^2)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2))/(b^2*c - 4*a*c^2 +
2*b*c*d + c*d^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2)*sqrt(
-(b + d + (b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b
*c^2*d + c^2*d^2))/(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2))/sqrt(b^2*c^2 - 4*a
*c^3 + 2*b*c^2*d + c^2*d^2) + x) - 1/2*sqrt(1/2)*sqrt(-(b + d - (b^2*c - 4
*a*c^2 + 2*b*c*d + c*d^2)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2))/(
b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2 + 2*b*c
*d + c*d^2)*sqrt(-(b + d - (b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2)/sqrt(b^2*c^
2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2))/(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2))/s
qrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2) + x) + 1/2*sqrt(1/2)*sqrt(-(b
 + d - (b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^
2*d + c^2*d^2))/(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2))*log(-sqrt(1/2)*(b^2*c
 - 4*a*c^2 + 2*b*c*d + c*d^2)*sqrt(-(b + d - (b^2*c - 4*a*c^2 + 2*b*c*d +
c*d^2)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2))/(b^2*c - 4*a*c^2 ...
```

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{a+(b+d)x^2+cx^4}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(256a^2c^3-128ab^2c^2-256abc^2d-128ac^2d^2+16b^4c+64b^3cd+96b^2cd^2+64bcd^3+16cd\right.\right.$$

input
```
integrate(x**2/(a+(b+d)*x**2+c*x**4),x)
```

output
```
RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 - 256*a*b*c**2*d - 128*a*c*
*2*d**2 + 16*b**4*c + 64*b**3*c*d + 96*b**2*c*d**2 + 64*b*c*d**3 + 16*c*d*
*4) + _t**2*(-16*a*b*c - 16*a*c*d + 4*b**3 + 12*b**2*d + 12*b*d**2 + 4*d**
3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c - 32*_t**3*b*c
*d - 16*_t**3*c*d**2 - 2*_t*b - 2*_t*d + x)))
```

Maxima [F]

$$\int \frac{x^2}{a+(b+d)x^2+cx^4}\,dx=\int \frac{x^2}{cx^4+(b+d)x^2+a}\,dx$$

input
```
integrate(x^2/(a+(b+d)*x^2+c*x^4),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^4 + (b + d)*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(161) = 322$.

Time = 2.42 (sec) , antiderivative size = 1160, normalized size of antiderivative = 5.92

$$\int \frac{x^2}{a+(b+d)x^2+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate(x^2/(a+(b+d)*x^2+c*x^4),x, algorithm="giac")
```

output
```
1/2*(2*b^2*c^2 - 8*a*c^3 + 4*b*c^2*d + 2*c^2*d^2 - sqrt(2)*sqrt(b^2 - 4*a*
c + 2*b*d + d^2)*sqrt(b*c + c*d + sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*b^2 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqrt(b*c + c*d + sqrt(b^2 - 4*a
*c + 2*b*d + d^2)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqrt(
b*c + c*d + sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*
a*c + 2*b*d + d^2)*sqrt(b*c + c*d + sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*c^2
 - 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqrt(b*c + c*d + sqrt(b^2 - 4
*a*c + 2*b*d + d^2)*c)*b*d + 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqr
t(b*c + c*d + sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*c*d - sqrt(2)*sqrt(b^2 -
4*a*c + 2*b*d + d^2)*sqrt(b*c + c*d + sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*d
^2 - 2*(b^2 - 4*a*c + 2*b*d + d^2)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + d +
 sqrt((b + d)^2 - 4*a*c))/c))/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8
*a*b*c^2 + b^2*c^2 - 4*a*c^3 + 4*b^3*d - 16*a*b*c*d - 6*b^2*c*d + 8*a*c^2*
d + 2*b*c^2*d + 6*b^2*d^2 - 8*a*c*d^2 - 6*b*c*d^2 + c^2*d^2 + 4*b*d^3 - 2*
c*d^3 + d^4)*abs(c)) + 1/2*(2*b^2*c^2 - 8*a*c^3 + 4*b*c^2*d + 2*c^2*d^2 -
sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqrt(b*c + c*d - sqrt(b^2 - 4*a*c
+ 2*b*d + d^2)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqrt(b*c
 + c*d - sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c + 2*b*d + d^2)*sqrt(b*c + c*d - sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*b*c
- sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqrt(b*c + c*d - sqrt(b^2 - 4...
```

Mupad [B] (verification not implemented)

Time = 18.92 (sec) , antiderivative size = 921, normalized size of antiderivative = 4.70

$$\int \frac{x^2}{a+(b+d)x^2+cx^4}\,dx$$

$$= 2\,\mathrm{atanh}\left(\frac{\left(x\,(2\,b^2\,c+4\,b\,c\,d-4\,a\,c^2+2\,c\,d^2)-\frac{x\,(8\,b^3\,c^2+24\,b^2\,c^2\,d-32\,a\,b\,c^3+24\,b\,c^2\,d^2-32\,a\,c^3\,d+8\,c^2\,d^3)\left(3\,b\,d^2+3\,b^2\,d+\cdots\right.}{8\,(16\,a^2\,c^3-8\,a\,b^2\,c^2-16\,a\,b\,c^2\,d-8\,a\,c^2\,d^2+b^4\,c+4\,b^3\,c\cdots}\right.}{\cdots}\right.$$

$$+ 2\,\mathrm{atanh}\left(\frac{\left(x\,(2\,b^2\,c+4\,b\,c\,d-4\,a\,c^2+2\,c\,d^2)-\frac{x\,(8\,b^3\,c^2+24\,b^2\,c^2\,d-32\,a\,b\,c^3+24\,b\,c^2\,d^2-32\,a\,c^3\,d+8\,c^2\,d^3)\left(3\,b\,d^2+3\,b^2\cdots\right.}{8\,(16\,a^2\,c^3-8\,a\,b^2\,c^2-16\,a\,b\,c^2\,d-8\,a\,c^2\,d^2+b^4\,c+4\,b\cdots}\right.}{\cdots}\right.$$

input
```
int(x^2/(a + x^2*(b + d) + c*x^4),x)
```

output
```
2*atanh(((x*(2*b^2*c - 4*a*c^2 + 2*c*d^2 + 4*b*c*d) - (x*(8*b^3*c^2 + 8*c^
2*d^3 + 24*b*c^2*d^2 + 24*b^2*c^2*d - 32*a*b*c^3 - 32*a*c^3*d)*(3*b*d^2 +
3*b^2*d + b^3 + d^3 + ((2*b*d - 4*a*c + b^2 + d^2)^3)^(1/2) - 4*a*b*c - 4*
a*c*d))/(8*(b^4*c + c*d^4 + 16*a^2*c^3 - 8*a*b^2*c^2 - 8*a*c^2*d^2 + 6*b^2
*c*d^2 + 4*b*c*d^3 + 4*b^3*c*d - 16*a*b*c^2*d)))*(-(3*b*d^2 + 3*b^2*d + b^
3 + d^3 + ((2*b*d - 4*a*c + b^2 + d^2)^3)^(1/2) - 4*a*b*c - 4*a*c*d)/(8*(b
^4*c + c*d^4 + 16*a^2*c^3 - 8*a*b^2*c^2 - 8*a*c^2*d^2 + 6*b^2*c*d^2 + 4*b*
c*d^3 + 4*b^3*c*d - 16*a*b*c^2*d)))^(1/2))/(a*c))*(-(3*b*d^2 + 3*b^2*d + b
^3 + d^3 + ((2*b*d - 4*a*c + b^2 + d^2)^3)^(1/2) - 4*a*b*c - 4*a*c*d)/(8*(
b^4*c + c*d^4 + 16*a^2*c^3 - 8*a*b^2*c^2 - 8*a*c^2*d^2 + 6*b^2*c*d^2 + 4*b
*c*d^3 + 4*b^3*c*d - 16*a*b*c^2*d)))^(1/2) + 2*atanh(((x*(2*b^2*c - 4*a*c^
2 + 2*c*d^2 + 4*b*c*d) - (x*(8*b^3*c^2 + 8*c^2*d^3 + 24*b*c^2*d^2 + 24*b^2
*c^2*d - 32*a*b*c^3 - 32*a*c^3*d)*(3*b*d^2 + 3*b^2*d + b^3 + d^3 - ((2*b*d
 - 4*a*c + b^2 + d^2)^3)^(1/2) - 4*a*b*c - 4*a*c*d))/(8*(b^4*c + c*d^4 + 1
6*a^2*c^3 - 8*a*b^2*c^2 - 8*a*c^2*d^2 + 6*b^2*c*d^2 + 4*b*c*d^3 + 4*b^3*c*
d - 16*a*b*c^2*d)))*(-(3*b*d^2 + 3*b^2*d + b^3 + d^3 - ((2*b*d - 4*a*c + b
^2 + d^2)^3)^(1/2) - 4*a*b*c - 4*a*c*d)/(8*(b^4*c + c*d^4 + 16*a^2*c^3 - 8
*a*b^2*c^2 - 8*a*c^2*d^2 + 6*b^2*c*d^2 + 4*b*c*d^3 + 4*b^3*c*d - 16*a*b*c^
2*d)))^(1/2))/(a*c))*(-(3*b*d^2 + 3*b^2*d + b^3 + d^3 - ((2*b*d - 4*a*c +
b^2 + d^2)^3)^(1/2) - 4*a*b*c - 4*a*c*d)/(8*(b^4*c + c*d^4 + 16*a^2*c^3...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.02

$$\int \frac{x^2}{a+(b+d)x^2+cx^4}\,dx$$
$$= \frac{-4\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b+d}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b-d}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b+d}}\right)c+2\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b+d}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b-d}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b+d}}\right)}{}$$

input
```
int(x^2/(a+(b+d)*x^2+c*x^4),x)
```

output

```
( - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b + d)*atan((sqrt(2*sqrt(c)*sqrt(a)
 - b - d) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b + d))*c + 2*sqrt(c)*sq
rt(2*sqrt(c)*sqrt(a) + b + d)*atan((sqrt(2*sqrt(c)*sqrt(a) - b - d) - 2*sq
rt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b + d))*b + 2*sqrt(c)*sqrt(2*sqrt(c)*sqr
t(a) + b + d)*atan((sqrt(2*sqrt(c)*sqrt(a) - b - d) - 2*sqrt(c)*x)/sqrt(2*
sqrt(c)*sqrt(a) + b + d))*d + 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b + d)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b - d) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b + d))*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b + d)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b - d) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b + d))*b - 2*
sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b + d)*atan((sqrt(2*sqrt(c)*sqrt(a) - b -
 d) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b + d))*d + 2*sqrt(a)*sqrt(2*s
qrt(c)*sqrt(a) - b - d)*log( - sqrt(2*sqrt(c)*sqrt(a) - b - d)*x + sqrt(a)
 + sqrt(c)*x**2)*c - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b - d)*log(sqrt(2*
sqrt(c)*sqrt(a) - b - d)*x + sqrt(a) + sqrt(c)*x**2)*c + sqrt(c)*sqrt(2*sq
rt(c)*sqrt(a) - b - d)*log( - sqrt(2*sqrt(c)*sqrt(a) - b - d)*x + sqrt(a)
+ sqrt(c)*x**2)*b + sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b - d)*log( - sqrt(2*
sqrt(c)*sqrt(a) - b - d)*x + sqrt(a) + sqrt(c)*x**2)*d - sqrt(c)*sqrt(2*sq
rt(c)*sqrt(a) - b - d)*log(sqrt(2*sqrt(c)*sqrt(a) - b - d)*x + sqrt(a) + s
qrt(c)*x**2)*b - sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b - d)*log(sqrt(2*sqrt(c
)*sqrt(a) - b - d)*x + sqrt(a) + sqrt(c)*x**2)*d)/(4*c*(4*a*c - b**2 - ...
```

3.897 $\int \frac{1}{d+\frac{a+bx^2+cx^4}{x^2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 196

$$\int \frac{1}{d+\frac{a+bx^2+cx^4}{x^2}} dx = -\frac{\sqrt{b+d-\sqrt{b^2-4ac+2bd+d^2}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+d-\sqrt{b^2-4ac+2bd+d^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac+2bd+d^2}} + \frac{\sqrt{b+d+\sqrt{b^2-4ac+2bd+d^2}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+d+\sqrt{b^2-4ac+2bd+d^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac+2bd+d^2}}$$

output
```
-1/2*(b+d-(-4*a*c+b^2+2*b*d+d^2)^(1/2))^(1/2)*arctan(2^(1/2)*c^(1/2)*x/(b+
d-(-4*a*c+b^2+2*b*d+d^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^2+2*b*d+d
^2)^(1/2)+1/2*(b+d+(-4*a*c+b^2+2*b*d+d^2)^(1/2))^(1/2)*arctan(2^(1/2)*c^(1
/2)*x/(b+d+(-4*a*c+b^2+2*b*d+d^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(-4*a*c+b^
2+2*b*d+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.13

$$\int \frac{1}{d+\frac{a+bx^2+cx^4}{x^2}}\,dx = \frac{\left(-b-d+\sqrt{b^2-4ac+2bd+d^2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+d-\sqrt{b^2-4ac+2bd+d^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac+2bd+d^2}\sqrt{b+d-\sqrt{b^2-4ac+2bd+d^2}}} + \frac{\sqrt{b+d+\sqrt{b^2-4ac+2bd+d^2}}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+d+\sqrt{b^2-4ac+2bd+d^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac+2bd+d^2}}$$

input
```
Integrate[(d + (a + b*x^2 + c*x^4)/x^2)^(-1),x]
```

output
```
((-b - d + Sqrt[b^2 - 4*a*c + 2*b*d + d^2])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + d - Sqrt[b^2 - 4*a*c + 2*b*d + d^2]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4
*a*c + 2*b*d + d^2]*Sqrt[b + d - Sqrt[b^2 - 4*a*c + 2*b*d + d^2]]) + (Sqrt
[b + d + Sqrt[b^2 - 4*a*c + 2*b*d + d^2]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + d + Sqrt[b^2 - 4*a*c + 2*b*d + d^2]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a
*c + 2*b*d + d^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{a+bx^2+cx^4}{x^2}+d}\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{1}{\frac{a+bx^2+cx^4}{x^2}+d}dx$$

input
```
Int[(d + (a + b*x^2 + c*x^4)/x^2)^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^4+(b+d)\_Z^2+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^3c+\_Rb+\_Rd}\right)}{2}$
default	$4c\left(-\frac{\left(\sqrt{-4ac+b^2+2bd+d^2}-b-d\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^2+2bd+d^2}-b-d\right)c}}\right)}{8c\sqrt{-4ac+b^2+2bd+d^2}\sqrt{\left(\sqrt{-4ac+b^2+2bd+d^2}-b-d\right)c}}+\frac{\left(b+d+\sqrt{-4ac+b^2+2bd+d^2}\right)\sqrt{2}\arctan\left(\right.}{8\sqrt{-4ac+b^2+2bd+d^2}\,c\sqrt{\left(b+d+\right.}}\right.$

input
```
int(1/(d+(c*x^4+b*x^2+a)/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R^2/(2*_R^3*c+_R*b+_R*d)*ln(x-_R),_R=RootOf(c*_Z^4+(b+d)*_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. 2(161) = 322.

Time = 0.08 (sec) , antiderivative size = 935, normalized size of antiderivative = 4.77

$$\int \frac{1}{d+\frac{a+bx^2+cx^4}{x^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d+(c*x^4+b*x^2+a)/x^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt(-(b + d + (b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2)/sqrt(b^2*
c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2))/(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2))
*log(sqrt(1/2)*(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2)*sqrt(-(b + d + (b^2*c -
 4*a*c^2 + 2*b*c*d + c*d^2)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2))
/(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2))/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d +
 c^2*d^2) + x) - 1/2*sqrt(1/2)*sqrt(-(b + d + (b^2*c - 4*a*c^2 + 2*b*c*d +
 c*d^2)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2))/(b^2*c - 4*a*c^2 +
2*b*c*d + c*d^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2)*sqrt(
-(b + d + (b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b
*c^2*d + c^2*d^2))/(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2))/sqrt(b^2*c^2 - 4*a
*c^3 + 2*b*c^2*d + c^2*d^2) + x) - 1/2*sqrt(1/2)*sqrt(-(b + d - (b^2*c - 4
*a*c^2 + 2*b*c*d + c*d^2)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2))/(
b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2 + 2*b*c
*d + c*d^2)*sqrt(-(b + d - (b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2)/sqrt(b^2*c^
2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2))/(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2))/s
qrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2) + x) + 1/2*sqrt(1/2)*sqrt(-(b
 + d - (b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^
2*d + c^2*d^2))/(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2))*log(-sqrt(1/2)*(b^2*c
 - 4*a*c^2 + 2*b*c*d + c*d^2)*sqrt(-(b + d - (b^2*c - 4*a*c^2 + 2*b*c*d +
c*d^2)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2))/(b^2*c - 4*a*c^2 ...
```

Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.92

$$\int \frac{1}{d+\frac{a+bx^2+cx^4}{x^2}}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(256a^2c^3-128ab^2c^2-256abc^2d-128ac^2d^2+16b^4c+64b^3cd+96b^2cd^2+64bcd^3+16cd\right.\right.$$

input
```
integrate(1/(d+(c*x**4+b*x**2+a)/x**2),x)
```

output
```
RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 - 256*a*b*c**2*d - 128*a*c*
*2*d**2 + 16*b**4*c + 64*b**3*c*d + 96*b**2*c*d**2 + 64*b*c*d**3 + 16*c*d*
*4) + _t**2*(-16*a*b*c - 16*a*c*d + 4*b**3 + 12*b**2*d + 12*b*d**2 + 4*d**
3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c - 32*_t**3*b*c
*d - 16*_t**3*c*d**2 - 2*_t*b - 2*_t*d + x)))
```

Maxima [F]

$$\int \frac{1}{d+\frac{a+bx^2+cx^4}{x^2}}\,dx=\int \frac{1}{d+\frac{cx^4+bx^2+a}{x^2}}\,dx$$

input
```
integrate(1/(d+(c*x^4+b*x^2+a)/x^2),x, algorithm="maxima")
```

output
```
integrate(1/(d + (c*x^4 + b*x^2 + a)/x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(161) = 322$.

Time = 1.26 (sec) , antiderivative size = 1160, normalized size of antiderivative = 5.92

$$\int \frac{1}{d+\frac{a+bx^2+cx^4}{x^2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/(d+(c*x^4+b*x^2+a)/x^2),x, algorithm="giac")
```

output
```
1/2*(2*b^2*c^2 - 8*a*c^3 + 4*b*c^2*d + 2*c^2*d^2 - sqrt(2)*sqrt(b^2 - 4*a*
c + 2*b*d + d^2)*sqrt(b*c + c*d + sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*b^2 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqrt(b*c + c*d + sqrt(b^2 - 4*a
*c + 2*b*d + d^2)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqrt(
b*c + c*d + sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*
a*c + 2*b*d + d^2)*sqrt(b*c + c*d + sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*c^2
 - 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqrt(b*c + c*d + sqrt(b^2 - 4
*a*c + 2*b*d + d^2)*c)*b*d + 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqr
t(b*c + c*d + sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*c*d - sqrt(2)*sqrt(b^2 -
4*a*c + 2*b*d + d^2)*sqrt(b*c + c*d + sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*d
^2 - 2*(b^2 - 4*a*c + 2*b*d + d^2)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + d +
 sqrt((b + d)^2 - 4*a*c))/c))/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8
*a*b*c^2 + b^2*c^2 - 4*a*c^3 + 4*b^3*d - 16*a*b*c*d - 6*b^2*c*d + 8*a*c^2*
d + 2*b*c^2*d + 6*b^2*d^2 - 8*a*c*d^2 - 6*b*c*d^2 + c^2*d^2 + 4*b*d^3 - 2*
c*d^3 + d^4)*abs(c)) - 1/2*(2*b^2*c^2 - 8*a*c^3 + 4*b*c^2*d + 2*c^2*d^2 -
sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqrt(b*c + c*d - sqrt(b^2 - 4*a*c
+ 2*b*d + d^2)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqrt(b*c
 + c*d - sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c + 2*b*d + d^2)*sqrt(b*c + c*d - sqrt(b^2 - 4*a*c + 2*b*d + d^2)*c)*b*c
- sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2)*sqrt(b*c + c*d - sqrt(b^2 - 4...
```

Mupad [B] (verification not implemented)

Time = 17.35 (sec) , antiderivative size = 921, normalized size of antiderivative = 4.70

$$\int \frac{1}{d+\frac{a+bx^2+cx^4}{x^2}}\,dx$$

$$= 2\,\mathrm{atanh}\left(\frac{\left(x\,(2\,b^2\,c+4\,b\,c\,d-4\,a\,c^2+2\,c\,d^2)-\frac{x\,(8\,b^3\,c^2+24\,b^2\,c^2\,d-32\,a\,b\,c^3+24\,b\,c^2\,d^2-32\,a\,c^3\,d+8\,c^2\,d^3)\left(3\,b\,d^2+3\,b^2\,d+\cdots\right.}{8\,(16\,a^2\,c^3-8\,a\,b^2\,c^2-16\,a\,b\,c^2\,d-8\,a\,c^2\,d^2+b^4\,c+4\,b^3\,c\cdots}\right.}{\cdots}\right.$$

$$+2\,\mathrm{atanh}\left(\frac{\left(x\,(2\,b^2\,c+4\,b\,c\,d-4\,a\,c^2+2\,c\,d^2)-\frac{x\,(8\,b^3\,c^2+24\,b^2\,c^2\,d-32\,a\,b\,c^3+24\,b\,c^2\,d^2-32\,a\,c^3\,d+8\,c^2\,d^3)\left(3\,b\,d^2+3\,b^2\cdots\right.}{8\,(16\,a^2\,c^3-8\,a\,b^2\,c^2-16\,a\,b\,c^2\,d-8\,a\,c^2\,d^2+b^4\,c+4\,b\cdots}\right.}{\cdots}\right.$$

input
```
int(1/(d + (a + b*x^2 + c*x^4)/x^2),x)
```

output
```
2*atanh(((x*(2*b^2*c - 4*a*c^2 + 2*c*d^2 + 4*b*c*d) - (x*(8*b^3*c^2 + 8*c^
2*d^3 + 24*b*c^2*d^2 + 24*b^2*c^2*d - 32*a*b*c^3 - 32*a*c^3*d)*(3*b*d^2 +
3*b^2*d + b^3 + d^3 + ((2*b*d - 4*a*c + b^2 + d^2)^3)^(1/2) - 4*a*b*c - 4*
a*c*d))/(8*(b^4*c + c*d^4 + 16*a^2*c^3 - 8*a*b^2*c^2 - 8*a*c^2*d^2 + 6*b^2
*c*d^2 + 4*b*c*d^3 + 4*b^3*c*d - 16*a*b*c^2*d)))*(-(3*b*d^2 + 3*b^2*d + b^
3 + d^3 + ((2*b*d - 4*a*c + b^2 + d^2)^3)^(1/2) - 4*a*b*c - 4*a*c*d)/(8*(b
^4*c + c*d^4 + 16*a^2*c^3 - 8*a*b^2*c^2 - 8*a*c^2*d^2 + 6*b^2*c*d^2 + 4*b*
c*d^3 + 4*b^3*c*d - 16*a*b*c^2*d)))^(1/2))/(a*c))*(-(3*b*d^2 + 3*b^2*d + b
^3 + d^3 + ((2*b*d - 4*a*c + b^2 + d^2)^3)^(1/2) - 4*a*b*c - 4*a*c*d)/(8*(
b^4*c + c*d^4 + 16*a^2*c^3 - 8*a*b^2*c^2 - 8*a*c^2*d^2 + 6*b^2*c*d^2 + 4*b
*c*d^3 + 4*b^3*c*d - 16*a*b*c^2*d)))^(1/2) + 2*atanh(((x*(2*b^2*c - 4*a*c^
2 + 2*c*d^2 + 4*b*c*d) - (x*(8*b^3*c^2 + 8*c^2*d^3 + 24*b*c^2*d^2 + 24*b^2
*c^2*d - 32*a*b*c^3 - 32*a*c^3*d)*(3*b*d^2 + 3*b^2*d + b^3 + d^3 - ((2*b*d
 - 4*a*c + b^2 + d^2)^3)^(1/2) - 4*a*b*c - 4*a*c*d))/(8*(b^4*c + c*d^4 + 1
6*a^2*c^3 - 8*a*b^2*c^2 - 8*a*c^2*d^2 + 6*b^2*c*d^2 + 4*b*c*d^3 + 4*b^3*c*
d - 16*a*b*c^2*d)))*(-(3*b*d^2 + 3*b^2*d + b^3 + d^3 - ((2*b*d - 4*a*c + b
^2 + d^2)^3)^(1/2) - 4*a*b*c - 4*a*c*d)/(8*(b^4*c + c*d^4 + 16*a^2*c^3 - 8
*a*b^2*c^2 - 8*a*c^2*d^2 + 6*b^2*c*d^2 + 4*b*c*d^3 + 4*b^3*c*d - 16*a*b*c^
2*d)))^(1/2))/(a*c))*(-(3*b*d^2 + 3*b^2*d + b^3 + d^3 - ((2*b*d - 4*a*c +
b^2 + d^2)^3)^(1/2) - 4*a*b*c - 4*a*c*d)/(8*(b^4*c + c*d^4 + 16*a^2*c^3...
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.02

$$\int \frac{1}{d+\frac{a+bx^2+cx^4}{x^2}}\,dx = \frac{-4\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b+d}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b-d}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b+d}}\right)c+2\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b+d}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b-d}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b+d}}\right)}{}$$

input
```
int(1/(d+(c*x^4+b*x^2+a)/x^2),x)
```

output
```
( - 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b + d)*atan((sqrt(2*sqrt(c)*sqrt(a)
 - b - d) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b + d))*c + 2*sqrt(c)*sq
rt(2*sqrt(c)*sqrt(a) + b + d)*atan((sqrt(2*sqrt(c)*sqrt(a) - b - d) - 2*sq
rt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b + d))*b + 2*sqrt(c)*sqrt(2*sqrt(c)*sqr
t(a) + b + d)*atan((sqrt(2*sqrt(c)*sqrt(a) - b - d) - 2*sqrt(c)*x)/sqrt(2*
sqrt(c)*sqrt(a) + b + d))*d + 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b + d)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b - d) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b + d))*c - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b + d)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b - d) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b + d))*b - 2*
sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b + d)*atan((sqrt(2*sqrt(c)*sqrt(a) - b -
 d) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b + d))*d + 2*sqrt(a)*sqrt(2*s
qrt(c)*sqrt(a) - b - d)*log( - sqrt(2*sqrt(c)*sqrt(a) - b - d)*x + sqrt(a)
 + sqrt(c)*x**2)*c - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b - d)*log(sqrt(2*
sqrt(c)*sqrt(a) - b - d)*x + sqrt(a) + sqrt(c)*x**2)*c + sqrt(c)*sqrt(2*sq
rt(c)*sqrt(a) - b - d)*log( - sqrt(2*sqrt(c)*sqrt(a) - b - d)*x + sqrt(a)
+ sqrt(c)*x**2)*b + sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b - d)*log( - sqrt(2*
sqrt(c)*sqrt(a) - b - d)*x + sqrt(a) + sqrt(c)*x**2)*d - sqrt(c)*sqrt(2*sq
rt(c)*sqrt(a) - b - d)*log(sqrt(2*sqrt(c)*sqrt(a) - b - d)*x + sqrt(a) + s
qrt(c)*x**2)*b - sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b - d)*log(sqrt(2*sqrt(c
)*sqrt(a) - b - d)*x + sqrt(a) + sqrt(c)*x**2)*d)/(4*c*(4*a*c - b**2 - ...
```

3.898 $\int \frac{x^2}{a+(b+d)x^2+(c+e)x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 216

$$\int \frac{x^2}{a+(b+d)x^2+(c+e)x^4}\,dx = -\frac{\sqrt{b+d-\sqrt{b^2+2bd+d^2-4a(c+e)}}\arctan\left(\frac{\sqrt{2}\sqrt{c+e}x}{\sqrt{b+d-\sqrt{b^2+2bd+d^2-4a(c+e)}}}\right)}{\sqrt{2}\sqrt{c+e}\sqrt{b^2+2bd+d^2-4a(c+e)}} + \frac{\sqrt{b+d+\sqrt{b^2+2bd+d^2-4a(c+e)}}\arctan\left(\frac{\sqrt{2}\sqrt{c+e}x}{\sqrt{b+d+\sqrt{b^2+2bd+d^2-4a(c+e)}}}\right)}{\sqrt{2}\sqrt{c+e}\sqrt{b^2+2bd+d^2-4a(c+e)}}$$

output
```
-1/2*(b+d-(b^2+2*b*d+d^2-4*a*(c+e))^(1/2))^(1/2)*arctan(2^(1/2)*(c+e)^(1/2
)*x/(b+d-(b^2+2*b*d+d^2-4*a*(c+e))^(1/2))^(1/2))*2^(1/2)/(c+e)^(1/2)/(b^2+
2*b*d+d^2-4*a*(c+e))^(1/2)+1/2*(b+d+(b^2+2*b*d+d^2-4*a*(c+e))^(1/2))^(1/2)
*arctan(2^(1/2)*(c+e)^(1/2)*x/(b+d+(b^2+2*b*d+d^2-4*a*(c+e))^(1/2))^(1/2))
*2^(1/2)/(c+e)^(1/2)/(b^2+2*b*d+d^2-4*a*(c+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{a+(b+d)x^2+(c+e)x^4}\,dx$$
$$= \frac{\left(-b-d+\sqrt{b^2+2bd+d^2-4a(c+e)}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c+e}x}{\sqrt{b+d-\sqrt{b^2+2bd+d^2-4a(c+e)}}}\right)+\sqrt{b+d-\sqrt{b^2+2bd+d}}}{\sqrt{2}\sqrt{c+e}\sqrt{b^2+2bd+d^2-4a(c+e)}\sqrt{b+d}}$$

input
```
Integrate[x^2/(a + (b + d)*x^2 + (c + e)*x^4),x]
```

output
```
((-b - d + Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)])*ArcTan[(Sqrt[2]*Sqrt[c +
 e]*x)/Sqrt[b + d - Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)]]] + Sqrt[b + d -
 Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)]]*Sqrt[b + d + Sqrt[b^2 + 2*b*d + d^
2 - 4*a*(c + e)]]*ArcTan[(Sqrt[2]*Sqrt[c + e]*x)/Sqrt[b + d + Sqrt[b^2 + 2
*b*d + d^2 - 4*a*(c + e)]]])/(Sqrt[2]*Sqrt[c + e]*Sqrt[b^2 + 2*b*d + d^2 -
 4*a*(c + e)]*Sqrt[b + d - Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)]])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1450, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+x^2(b+d)+x^4(c+e)}\,dx$$
$$\Big\downarrow \; 1450$$

$$\frac{1}{2}\left(1-\frac{b+d}{\sqrt{-4a(c+e)+b^2+2bd+d^2}}\right)\int\frac{1}{(c+e)x^2+\frac{1}{2}\left(b+d-\sqrt{b^2+2db+d^2-4a(c+e)}\right)}dx+$$
$$\frac{1}{2}\left(\frac{b+d}{\sqrt{-4a(c+e)+b^2+2bd+d^2}}+1\right)\int\frac{1}{(c+e)x^2+\frac{1}{2}\left(b+d+\sqrt{b^2+2db+d^2-4a(c+e)}\right)}dx$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(1-\frac{b+d}{\sqrt{-4a(c+e)+b^2+2bd+d^2}}\right)\arctan\left(\frac{\sqrt{2}x\sqrt{c+e}}{\sqrt{-\sqrt{-4a(c+e)+b^2+2bd+d^2}+b+d}}\right)}{\sqrt{2}\sqrt{c+e}\sqrt{-\sqrt{-4a(c+e)+b^2+2bd+d^2}+b+d}}+$$
$$\frac{\left(\frac{b+d}{\sqrt{-4a(c+e)+b^2+2bd+d^2}}+1\right)\arctan\left(\frac{\sqrt{2}x\sqrt{c+e}}{\sqrt{\sqrt{-4a(c+e)+b^2+2bd+d^2}+b+d}}\right)}{\sqrt{2}\sqrt{c+e}\sqrt{\sqrt{-4a(c+e)+b^2+2bd+d^2}+b+d}}$$

input
```
Int[x^2/(a + (b + d)*x^2 + (c + e)*x^4),x]
```

output
```
((1 - (b + d)/Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)])*ArcTan[(Sqrt[2]*Sqrt[
c + e]*x)/Sqrt[b + d - Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)]]])/(Sqrt[2]*S
qrt[c + e]*Sqrt[b + d - Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)]]) + ((1 + (b
 + d)/Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)])*ArcTan[(Sqrt[2]*Sqrt[c + e]*x
)/Sqrt[b + d + Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)]]])/(Sqrt[2]*Sqrt[c +
e]*Sqrt[b + d + Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)]])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.25

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((c+e)\_Z^4+(b+d)\_Z^2+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^3c+2\_R^3e+\_Rb+\_Rd}\right)}{2}$
default	$(4c+4e)\left(\frac{\left(\sqrt{-4ac-4ae+b^2+2bd+d^2}-b-d\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{(-2c-2e)x\sqrt{2}}{2\sqrt{\left(\sqrt{-4ac-4ae+b^2+2bd+d^2}-b-d\right)(c+e)}}\right)}{8\sqrt{-4ac-4ae+b^2+2bd+d^2}\,(c+e)\sqrt{\left(\sqrt{-4ac-4ae+b^2+2bd+d^2}-b-d\right)(c+e)}}+\frac{\left(\sqrt{-4ac-4ae+b^2+\right.}{8\sqrt{-4ac-4ae}}\right.$

input
```
int(x^2/(a+(b+d)*x^2+(c+e)*x^4),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R^2/(2*_R^3*c+2*_R^3*e+_R*b+_R*d)*ln(x-_R),_R=RootOf((c+e)*_Z^4+(
b+d)*_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1995 vs. 2(181) = 362.

Time = 0.09 (sec) , antiderivative size = 1995, normalized size of antiderivative = 9.24

$$\int \frac{x^2}{a+(b+d)x^2+(c+e)x^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+(b+d)*x^2+(c+e)*x^4),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt(-(b + d + (b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2 - 4*a*e^2
+ (b^2 - 8*a*c + 2*b*d + d^2)*e)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*
d^2 - 4*a*e^3 + (b^2 - 12*a*c + 2*b*d + d^2)*e^2 + 2*(b^2*c - 6*a*c^2 + 2*
b*c*d + c*d^2)*e))/(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2 - 4*a*e^2 + (b^2 - 8
*a*c + 2*b*d + d^2)*e))*log(sqrt(1/2)*(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2 -
 4*a*e^2 + (b^2 - 8*a*c + 2*b*d + d^2)*e)*sqrt(-(b + d + (b^2*c - 4*a*c^2
+ 2*b*c*d + c*d^2 - 4*a*e^2 + (b^2 - 8*a*c + 2*b*d + d^2)*e)/sqrt(b^2*c^2
- 4*a*c^3 + 2*b*c^2*d + c^2*d^2 - 4*a*e^3 + (b^2 - 12*a*c + 2*b*d + d^2)*e
^2 + 2*(b^2*c - 6*a*c^2 + 2*b*c*d + c*d^2)*e))/(b^2*c - 4*a*c^2 + 2*b*c*d
+ c*d^2 - 4*a*e^2 + (b^2 - 8*a*c + 2*b*d + d^2)*e))/sqrt(b^2*c^2 - 4*a*c^3
 + 2*b*c^2*d + c^2*d^2 - 4*a*e^3 + (b^2 - 12*a*c + 2*b*d + d^2)*e^2 + 2*(b
^2*c - 6*a*c^2 + 2*b*c*d + c*d^2)*e) + x) - 1/2*sqrt(1/2)*sqrt(-(b + d + (
b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2 - 4*a*e^2 + (b^2 - 8*a*c + 2*b*d + d^2)*
e)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2 - 4*a*e^3 + (b^2 - 12*a*c
+ 2*b*d + d^2)*e^2 + 2*(b^2*c - 6*a*c^2 + 2*b*c*d + c*d^2)*e))/(b^2*c - 4*
a*c^2 + 2*b*c*d + c*d^2 - 4*a*e^2 + (b^2 - 8*a*c + 2*b*d + d^2)*e))*log(-s
qrt(1/2)*(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2 - 4*a*e^2 + (b^2 - 8*a*c + 2*b
*d + d^2)*e)*sqrt(-(b + d + (b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2 - 4*a*e^2 +
 (b^2 - 8*a*c + 2*b*d + d^2)*e)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d
^2 - 4*a*e^3 + (b^2 - 12*a*c + 2*b*d + d^2)*e^2 + 2*(b^2*c - 6*a*c^2 + ...
```

Sympy [A] (verification not implemented)

Time = 7.45 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.74

$$\int \frac{x^2}{a+(b+d)x^2+(c+e)x^4}\,dx$$
$$= \operatorname{RootSum}\left(t^4\cdot\left(256a^2c^3+768a^2c^2e+768a^2ce^2+256a^2e^3-128ab^2c^2-256ab^2ce-128ab^2e^2-256abc^2\right.\right.$$

input
```
integrate(x**2/(a+(b+d)*x**2+(c+e)*x**4),x)
```

output
```
RootSum(_t**4*(256*a**2*c**3 + 768*a**2*c**2*e + 768*a**2*c*e**2 + 256*a**
2*e**3 - 128*a*b**2*c**2 - 256*a*b**2*c*e - 128*a*b**2*e**2 - 256*a*b*c**2
*d - 512*a*b*c*d*e - 256*a*b*d*e**2 - 128*a*c**2*d**2 - 256*a*c*d**2*e - 1
28*a*d**2*e**2 + 16*b**4*c + 16*b**4*e + 64*b**3*c*d + 64*b**3*d*e + 96*b*
*2*c*d**2 + 96*b**2*d**2*e + 64*b*c*d**3 + 64*b*d**3*e + 16*c*d**4 + 16*d*
*4*e) + _t**2*(-16*a*b*c - 16*a*b*e - 16*a*c*d - 16*a*d*e + 4*b**3 + 12*b*
*2*d + 12*b*d**2 + 4*d**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 + 128*_t
**3*a*c*e + 64*_t**3*a*e**2 - 16*_t**3*b**2*c - 16*_t**3*b**2*e - 32*_t**3
*b*c*d - 32*_t**3*b*d*e - 16*_t**3*c*d**2 - 16*_t**3*d**2*e - 2*_t*b - 2*_
t*d + x)))
```

Maxima [F]

$$\int \frac{x^2}{a+(b+d)x^2+(c+e)x^4}\,dx = \int \frac{x^2}{(c+e)x^4+(b+d)x^2+a}\,dx$$

input
```
integrate(x^2/(a+(b+d)*x^2+(c+e)*x^4),x, algorithm="maxima")
```

output
```
integrate(x^2/((c + e)*x^4 + (b + d)*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3423 vs. $2(181) = 362$.

Time = 38.88 (sec) , antiderivative size = 3423, normalized size of antiderivative = 15.85

$$\int \frac{x^2}{a+(b+d)x^2+(c+e)x^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+(b+d)*x^2+(c+e)*x^4),x, algorithm="giac")
```

output
```
-1/4*(2*b^2*c^2 - 8*a*c^3 + 4*b*c^2*d + 2*c^2*d^2 + 4*b^2*c*e - 24*a*c^2*e
 + 8*b*c*d*e + 4*c*d^2*e + 2*b^2*e^2 - 24*a*c*e^2 + 4*b*d*e^2 + 2*d^2*e^2
- 8*a*e^3 - sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*sqrt(b*c + c*d
 + b*e + d*e + sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*(c + e))*b^2 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*sqrt(b*c + c*d + b*e + d*e +
 sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*(c + e))*a*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c + 2*b*d + d^2 - 4*a*e)*sqrt(b*c + c*d + b*e + d*e + sqrt(b^2 - 4*
a*c + 2*b*d + d^2 - 4*a*e)*(c + e))*b*c - sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d
 + d^2 - 4*a*e)*sqrt(b*c + c*d + b*e + d*e + sqrt(b^2 - 4*a*c + 2*b*d + d^
2 - 4*a*e)*(c + e))*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e
)*sqrt(b*c + c*d + b*e + d*e + sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*(c
+ e))*b*d + 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*sqrt(b*c + c
*d + b*e + d*e + sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*(c + e))*c*d - sq
rt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*sqrt(b*c + c*d + b*e + d*e +
 sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*(c + e))*d^2 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c + 2*b*d + d^2 - 4*a*e)*sqrt(b*c + c*d + b*e + d*e + sqrt(b^2 - 4*
a*c + 2*b*d + d^2 - 4*a*e)*(c + e))*a*e + 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b
*d + d^2 - 4*a*e)*sqrt(b*c + c*d + b*e + d*e + sqrt(b^2 - 4*a*c + 2*b*d +
d^2 - 4*a*e)*(c + e))*b*e - 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a
*e)*sqrt(b*c + c*d + b*e + d*e + sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e...
```

Mupad [B] (verification not implemented)

Time = 19.88 (sec) , antiderivative size = 1956, normalized size of antiderivative = 9.06

$$\int \frac{x^2}{a+(b+d)x^2+(c+e)x^4}\,dx = \text{Too large to display}$$

input
```
int(x^2/(a + x^2*(b + d) + x^4*(c + e)),x)
```

output
```
2*atanh((2*(x*(2*b^2*c - 4*a*c^2 - 4*a*e^2 + 2*c*d^2 + 2*b^2*e + 2*d^2*e -
 8*a*c*e + 4*b*c*d + 4*b*d*e) - (x*(((2*b*d - 4*a*e - 4*a*c + b^2 + d^2)^3
)^(1/2) + 3*b*d^2 + 3*b^2*d + b^3 + d^3 - 4*a*b*c - 4*a*b*e - 4*a*c*d - 4*
a*d*e)*(8*b^3*c^2 + 8*b^3*e^2 + 8*c^2*d^3 + 8*d^3*e^2 + 24*b*c^2*d^2 + 24*
b^2*c^2*d + 24*b*d^2*e^2 + 24*b^2*d*e^2 - 32*a*b*c^3 - 32*a*b*e^3 - 32*a*c
^3*d - 32*a*d*e^3 + 16*b^3*c*e + 16*c*d^3*e - 96*a*b*c*e^2 - 96*a*b*c^2*e
- 96*a*c*d*e^2 - 96*a*c^2*d*e + 48*b*c*d^2*e + 48*b^2*c*d*e))/(8*(b^4*c +
c*d^4 + b^4*e + d^4*e + 16*a^2*c^3 + 16*a^2*e^3 - 8*a*b^2*c^2 - 8*a*b^2*e^
2 - 8*a*c^2*d^2 + 48*a^2*c*e^2 + 48*a^2*c^2*e + 6*b^2*c*d^2 - 8*a*d^2*e^2
+ 6*b^2*d^2*e + 4*b*c*d^3 + 4*b^3*c*d + 4*b*d^3*e + 4*b^3*d*e - 16*a*b*c^2
*d - 16*a*b^2*c*e - 16*a*b*d*e^2 - 16*a*c*d^2*e - 32*a*b*c*d*e)))*(-(((2*b
*d - 4*a*e - 4*a*c + b^2 + d^2)^3)^(1/2) + 3*b*d^2 + 3*b^2*d + b^3 + d^3 -
 4*a*b*c - 4*a*b*e - 4*a*c*d - 4*a*d*e)/(8*(b^4*c + c*d^4 + b^4*e + d^4*e
+ 16*a^2*c^3 + 16*a^2*e^3 - 8*a*b^2*c^2 - 8*a*b^2*e^2 - 8*a*c^2*d^2 + 48*a
^2*c*e^2 + 48*a^2*c^2*e + 6*b^2*c*d^2 - 8*a*d^2*e^2 + 6*b^2*d^2*e + 4*b*c*
d^3 + 4*b^3*c*d + 4*b*d^3*e + 4*b^3*d*e - 16*a*b*c^2*d - 16*a*b^2*c*e - 16
*a*b*d*e^2 - 16*a*c*d^2*e - 32*a*b*c*d*e)))^(1/2))/(2*a*c + 2*a*e))*(-(((2
*b*d - 4*a*e - 4*a*c + b^2 + d^2)^3)^(1/2) + 3*b*d^2 + 3*b^2*d + b^3 + d^3
 - 4*a*b*c - 4*a*b*e - 4*a*c*d - 4*a*d*e)/(8*(b^4*c + c*d^4 + b^4*e + d^4*
e + 16*a^2*c^3 + 16*a^2*e^3 - 8*a*b^2*c^2 - 8*a*b^2*e^2 - 8*a*c^2*d^2 +...
```

Reduce [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 938, normalized size of antiderivative = 4.34

$$\int \frac{x^2}{a + (b + d)x^2 + (c + e)x^4} \, dx = \text{Too large to display}$$

input
```
int(x^2/(a+(b+d)*x^2+(c+e)*x^4),x)
```

output
```
(2*sqrt(c + e)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(2*sqrt(a)*sq
rt(c + e) - b - d) - 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c + e) + b + d))
*b + 2*sqrt(c + e)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(2*sqrt(a
)*sqrt(c + e) - b - d) - 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c + e) + b +
 d))*d - 4*sqrt(a)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(2*sqrt(a
)*sqrt(c + e) - b - d) - 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c + e) + b +
 d))*c - 4*sqrt(a)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(2*sqrt(a
)*sqrt(c + e) - b - d) - 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c + e) + b +
 d))*e - 2*sqrt(c + e)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(2*sq
rt(a)*sqrt(c + e) - b - d) + 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c + e) +
 b + d))*b - 2*sqrt(c + e)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(
2*sqrt(a)*sqrt(c + e) - b - d) + 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c +
e) + b + d))*d + 4*sqrt(a)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(
2*sqrt(a)*sqrt(c + e) - b - d) + 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c +
e) + b + d))*c + 4*sqrt(a)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(
2*sqrt(a)*sqrt(c + e) - b - d) + 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c +
e) + b + d))*e + sqrt(c + e)*sqrt(2*sqrt(a)*sqrt(c + e) - b - d)*log( - sq
rt(2*sqrt(a)*sqrt(c + e) - b - d)*x + sqrt(c + e)*x**2 + sqrt(a))*b + sqrt
(c + e)*sqrt(2*sqrt(a)*sqrt(c + e) - b - d)*log( - sqrt(2*sqrt(a)*sqrt(c +
 e) - b - d)*x + sqrt(c + e)*x**2 + sqrt(a))*d - sqrt(c + e)*sqrt(2*sqr...
```

3.899 $\int \frac{1}{d+ex^2+\frac{a+bx^2+cx^4}{x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 216

$$\int \frac{1}{d+ex^2+\frac{a+bx^2+cx^4}{x^2}}\,dx = -\frac{\sqrt{b+d-\sqrt{b^2+2bd+d^2-4a(c+e)}}\arctan\left(\frac{\sqrt{2}\sqrt{c+e}x}{\sqrt{b+d-\sqrt{b^2+2bd+d^2-4a(c+e)}}}\right)}{\sqrt{2}\sqrt{c+e}\sqrt{b^2+2bd+d^2-4a(c+e)}} + \frac{\sqrt{b+d+\sqrt{b^2+2bd+d^2-4a(c+e)}}\arctan\left(\frac{\sqrt{2}\sqrt{c+e}x}{\sqrt{b+d+\sqrt{b^2+2bd+d^2-4a(c+e)}}}\right)}{\sqrt{2}\sqrt{c+e}\sqrt{b^2+2bd+d^2-4a(c+e)}}$$

output
```
-1/2*(b+d-(b^2+2*b*d+d^2-4*a*(c+e))^(1/2))^(1/2)*arctan(2^(1/2)*(c+e)^(1/2
)*x/(b+d-(b^2+2*b*d+d^2-4*a*(c+e))^(1/2))^(1/2))*2^(1/2)/(c+e)^(1/2)/(b^2+
2*b*d+d^2-4*a*(c+e))^(1/2)+1/2*(b+d+(b^2+2*b*d+d^2-4*a*(c+e))^(1/2))^(1/2)
*arctan(2^(1/2)*(c+e)^(1/2)*x/(b+d+(b^2+2*b*d+d^2-4*a*(c+e))^(1/2))^(1/2))
*2^(1/2)/(c+e)^(1/2)/(b^2+2*b*d+d^2-4*a*(c+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\int \frac{1}{d+ex^2+\frac{a+bx^2+cx^4}{x^2}}\,dx$$
$$=\frac{\left(-b-d+\sqrt{b^2+2bd+d^2-4a(c+e)}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c+ex}}{\sqrt{b+d-\sqrt{b^2+2bd+d^2-4a(c+e)}}}\right)+\sqrt{b+d-\sqrt{b^2+2bd+d}}}{\sqrt{2}\sqrt{c+e}\sqrt{b^2+2bd+d^2-4a(c+e)}\sqrt{b+d}}$$

input
```
Integrate[(d + e*x^2 + (a + b*x^2 + c*x^4)/x^2)^(-1),x]
```

output
```
((-b - d + Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)])*ArcTan[(Sqrt[2]*Sqrt[c +
 e]*x)/Sqrt[b + d - Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)]]] + Sqrt[b + d -
 Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)]]*Sqrt[b + d + Sqrt[b^2 + 2*b*d + d^
2 - 4*a*(c + e)]]*ArcTan[(Sqrt[2]*Sqrt[c + e]*x)/Sqrt[b + d + Sqrt[b^2 + 2
*b*d + d^2 - 4*a*(c + e)]]])/(Sqrt[2]*Sqrt[c + e]*Sqrt[b^2 + 2*b*d + d^2 -
 4*a*(c + e)]*Sqrt[b + d - Sqrt[b^2 + 2*b*d + d^2 - 4*a*(c + e)]])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{a+bx^2+cx^4}{x^2}+d+ex^2}\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{1}{\frac{a+bx^2+cx^4}{x^2}+d+ex^2}dx$$

input
```
Int[(d + e*x^2 + (a + b*x^2 + c*x^4)/x^2)^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.25

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left((c+e)\_Z^4+(b+d)\_Z^2+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^3 c+2\_R^3 e+\_R b+\_R d}\right)}{2}$
default	$(4c+4e)\left(\frac{\left(\sqrt{-4ac-4ae+b^2+2bd+d^2}-b-d\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{(-2c-2e)x\sqrt{2}}{2\sqrt{\left(\sqrt{-4ac-4ae+b^2+2bd+d^2}-b-d\right)(c+e)}}\right)}{8\sqrt{-4ac-4ae+b^2+2bd+d^2}\,(c+e)\sqrt{\left(\sqrt{-4ac-4ae+b^2+2bd+d^2}-b-d\right)(c+e)}}+\frac{\left(\sqrt{-4ac-4ae+b^2+\cdots\right.}{8\sqrt{-4ac-4a\cdots}}\right.$

input
```
int(1/(d+e*x^2+(c*x^4+b*x^2+a)/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R^2/(2*_R^3*c+2*_R^3*e+_R*b+_R*d)*ln(x-_R),_R=RootOf((c+e)*_Z^4+(
b+d)*_Z^2+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1995 vs. 2(181) = 362.

Time = 0.09 (sec) , antiderivative size = 1995, normalized size of antiderivative = 9.24

$$\int \frac{1}{d+ex^2+\frac{a+bx^2+cx^4}{x^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d+e*x^2+(c*x^4+b*x^2+a)/x^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*sqrt(-(b + d + (b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2 - 4*a*e^2
+ (b^2 - 8*a*c + 2*b*d + d^2)*e)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*
d^2 - 4*a*e^3 + (b^2 - 12*a*c + 2*b*d + d^2)*e^2 + 2*(b^2*c - 6*a*c^2 + 2*
b*c*d + c*d^2)*e))/(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2 - 4*a*e^2 + (b^2 - 8
*a*c + 2*b*d + d^2)*e))*log(sqrt(1/2)*(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2 -
 4*a*e^2 + (b^2 - 8*a*c + 2*b*d + d^2)*e)*sqrt(-(b + d + (b^2*c - 4*a*c^2
+ 2*b*c*d + c*d^2 - 4*a*e^2 + (b^2 - 8*a*c + 2*b*d + d^2)*e)/sqrt(b^2*c^2
- 4*a*c^3 + 2*b*c^2*d + c^2*d^2 - 4*a*e^3 + (b^2 - 12*a*c + 2*b*d + d^2)*e
^2 + 2*(b^2*c - 6*a*c^2 + 2*b*c*d + c*d^2)*e))/(b^2*c - 4*a*c^2 + 2*b*c*d
+ c*d^2 - 4*a*e^2 + (b^2 - 8*a*c + 2*b*d + d^2)*e))/sqrt(b^2*c^2 - 4*a*c^3
 + 2*b*c^2*d + c^2*d^2 - 4*a*e^3 + (b^2 - 12*a*c + 2*b*d + d^2)*e^2 + 2*(b
^2*c - 6*a*c^2 + 2*b*c*d + c*d^2)*e) + x) - 1/2*sqrt(1/2)*sqrt(-(b + d + (
b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2 - 4*a*e^2 + (b^2 - 8*a*c + 2*b*d + d^2)*
e)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d^2 - 4*a*e^3 + (b^2 - 12*a*c
+ 2*b*d + d^2)*e^2 + 2*(b^2*c - 6*a*c^2 + 2*b*c*d + c*d^2)*e))/(b^2*c - 4*
a*c^2 + 2*b*c*d + c*d^2 - 4*a*e^2 + (b^2 - 8*a*c + 2*b*d + d^2)*e))*log(-s
qrt(1/2)*(b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2 - 4*a*e^2 + (b^2 - 8*a*c + 2*b
*d + d^2)*e)*sqrt(-(b + d + (b^2*c - 4*a*c^2 + 2*b*c*d + c*d^2 - 4*a*e^2 +
 (b^2 - 8*a*c + 2*b*d + d^2)*e)/sqrt(b^2*c^2 - 4*a*c^3 + 2*b*c^2*d + c^2*d
^2 - 4*a*e^3 + (b^2 - 12*a*c + 2*b*d + d^2)*e^2 + 2*(b^2*c - 6*a*c^2 + ...
```

Sympy [A] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.74

$$\int \frac{1}{d+ex^2+\frac{a+bx^2+cx^4}{x^2}}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(256a^2c^3+768a^2c^2e+768a^2ce^2+256a^2e^3-128ab^2c^2-256ab^2ce-128ab^2e^2-256abc^2\right.\right.$$

input
```
integrate(1/(d+e*x**2+(c*x**4+b*x**2+a)/x**2),x)
```

output
```
RootSum(_t**4*(256*a**2*c**3 + 768*a**2*c**2*e + 768*a**2*c*e**2 + 256*a**
2*e**3 - 128*a*b**2*c**2 - 256*a*b**2*c*e - 128*a*b**2*e**2 - 256*a*b*c**2
*d - 512*a*b*c*d*e - 256*a*b*d*e**2 - 128*a*c**2*d**2 - 256*a*c*d**2*e - 1
28*a*d**2*e**2 + 16*b**4*c + 16*b**4*e + 64*b**3*c*d + 64*b**3*d*e + 96*b*
*2*c*d**2 + 96*b**2*d**2*e + 64*b*c*d**3 + 64*b*d**3*e + 16*c*d**4 + 16*d*
*4*e) + _t**2*(-16*a*b*c - 16*a*b*e - 16*a*c*d - 16*a*d*e + 4*b**3 + 12*b*
*2*d + 12*b*d**2 + 4*d**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 + 128*_t
**3*a*c*e + 64*_t**3*a*e**2 - 16*_t**3*b**2*c - 16*_t**3*b**2*e - 32*_t**3
*b*c*d - 32*_t**3*b*d*e - 16*_t**3*c*d**2 - 16*_t**3*d**2*e - 2*_t*b - 2*_
t*d + x)))
```

Maxima [F]

$$\int \frac{1}{d+ex^2+\frac{a+bx^2+cx^4}{x^2}}\,dx=\int \frac{1}{ex^2+d+\frac{cx^4+bx^2+a}{x^2}}\,dx$$

input
```
integrate(1/(d+e*x^2+(c*x^4+b*x^2+a)/x^2),x, algorithm="maxima")
```

output
```
integrate(1/(e*x^2 + d + (c*x^4 + b*x^2 + a)/x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3423 vs. $2(181) = 362$.

Time = 20.09 (sec) , antiderivative size = 3423, normalized size of antiderivative = 15.85

$$\int \frac{1}{d+ex^2+\frac{a+bx^2+cx^4}{x^2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/(d+e*x^2+(c*x^4+b*x^2+a)/x^2),x, algorithm="giac")
```

output
```
-1/4*(2*b^2*c^2 - 8*a*c^3 + 4*b*c^2*d + 2*c^2*d^2 + 4*b^2*c*e - 24*a*c^2*e
 + 8*b*c*d*e + 4*c*d^2*e + 2*b^2*e^2 - 24*a*c*e^2 + 4*b*d*e^2 + 2*d^2*e^2
- 8*a*e^3 - sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*sqrt(b*c + c*d
 + b*e + d*e + sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*(c + e))*b^2 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*sqrt(b*c + c*d + b*e + d*e +
 sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*(c + e))*a*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c + 2*b*d + d^2 - 4*a*e)*sqrt(b*c + c*d + b*e + d*e + sqrt(b^2 - 4*
a*c + 2*b*d + d^2 - 4*a*e)*(c + e))*b*c - sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d
 + d^2 - 4*a*e)*sqrt(b*c + c*d + b*e + d*e + sqrt(b^2 - 4*a*c + 2*b*d + d^
2 - 4*a*e)*(c + e))*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e
)*sqrt(b*c + c*d + b*e + d*e + sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*(c
+ e))*b*d + 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*sqrt(b*c + c
*d + b*e + d*e + sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*(c + e))*c*d - sq
rt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*sqrt(b*c + c*d + b*e + d*e +
 sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e)*(c + e))*d^2 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c + 2*b*d + d^2 - 4*a*e)*sqrt(b*c + c*d + b*e + d*e + sqrt(b^2 - 4*
a*c + 2*b*d + d^2 - 4*a*e)*(c + e))*a*e + 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b
*d + d^2 - 4*a*e)*sqrt(b*c + c*d + b*e + d*e + sqrt(b^2 - 4*a*c + 2*b*d +
d^2 - 4*a*e)*(c + e))*b*e - 2*sqrt(2)*sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a
*e)*sqrt(b*c + c*d + b*e + d*e + sqrt(b^2 - 4*a*c + 2*b*d + d^2 - 4*a*e...
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 1956, normalized size of antiderivative = 9.06

$$\int \frac{1}{d + ex^2 + \frac{a+bx^2+cx^4}{x^2}}\,dx = \text{Too large to display}$$

input
```
int(1/(d + (a + b*x^2 + c*x^4)/x^2 + e*x^2),x)
```

output
```
2*atanh((2*(x*(2*b^2*c - 4*a*c^2 - 4*a*e^2 + 2*c*d^2 + 2*b^2*e + 2*d^2*e -
 8*a*c*e + 4*b*c*d + 4*b*d*e) - (x*(((2*b*d - 4*a*e - 4*a*c + b^2 + d^2)^3
)^(1/2) + 3*b*d^2 + 3*b^2*d + b^3 + d^3 - 4*a*b*c - 4*a*b*e - 4*a*c*d - 4*
a*d*e)*(8*b^3*c^2 + 8*b^3*e^2 + 8*c^2*d^3 + 8*d^3*e^2 + 24*b*c^2*d^2 + 24*
b^2*c^2*d + 24*b*d^2*e^2 + 24*b^2*d*e^2 - 32*a*b*c^3 - 32*a*b*e^3 - 32*a*c
^3*d - 32*a*d*e^3 + 16*b^3*c*e + 16*c*d^3*e - 96*a*b*c*e^2 - 96*a*b*c^2*e
- 96*a*c*d*e^2 - 96*a*c^2*d*e + 48*b*c*d^2*e + 48*b^2*c*d*e))/(8*(b^4*c +
c*d^4 + b^4*e + d^4*e + 16*a^2*c^3 + 16*a^2*e^3 - 8*a*b^2*c^2 - 8*a*b^2*e^
2 - 8*a*c^2*d^2 + 48*a^2*c*e^2 + 48*a^2*c^2*e + 6*b^2*c*d^2 - 8*a*d^2*e^2
+ 6*b^2*d^2*e + 4*b*c*d^3 + 4*b^3*c*d + 4*b*d^3*e + 4*b^3*d*e - 16*a*b*c^2
*d - 16*a*b^2*c*e - 16*a*b*d*e^2 - 16*a*c*d^2*e - 32*a*b*c*d*e)))*(-(((2*b
*d - 4*a*e - 4*a*c + b^2 + d^2)^3)^(1/2) + 3*b*d^2 + 3*b^2*d + b^3 + d^3 -
 4*a*b*c - 4*a*b*e - 4*a*c*d - 4*a*d*e)/(8*(b^4*c + c*d^4 + b^4*e + d^4*e
+ 16*a^2*c^3 + 16*a^2*e^3 - 8*a*b^2*c^2 - 8*a*b^2*e^2 - 8*a*c^2*d^2 + 48*a
^2*c*e^2 + 48*a^2*c^2*e + 6*b^2*c*d^2 - 8*a*d^2*e^2 + 6*b^2*d^2*e + 4*b*c*
d^3 + 4*b^3*c*d + 4*b*d^3*e + 4*b^3*d*e - 16*a*b*c^2*d - 16*a*b^2*c*e - 16
*a*b*d*e^2 - 16*a*c*d^2*e - 32*a*b*c*d*e)))^(1/2))/(2*a*c + 2*a*e))*(-(((2
*b*d - 4*a*e - 4*a*c + b^2 + d^2)^3)^(1/2) + 3*b*d^2 + 3*b^2*d + b^3 + d^3
 - 4*a*b*c - 4*a*b*e - 4*a*c*d - 4*a*d*e)/(8*(b^4*c + c*d^4 + b^4*e + d^4*
e + 16*a^2*c^3 + 16*a^2*e^3 - 8*a*b^2*c^2 - 8*a*b^2*e^2 - 8*a*c^2*d^2 +...
```

Reduce [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 938, normalized size of antiderivative = 4.34

$$\int \frac{1}{d+ex^2+\frac{a+bx^2+cx^4}{x^2}}\,dx = \text{Too large to display}$$

input
```
int(1/(d+e*x^2+(c*x^4+b*x^2+a)/x^2),x)
```

output
```
(2*sqrt(c + e)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(2*sqrt(a)*sq
rt(c + e) - b - d) - 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c + e) + b + d))
*b + 2*sqrt(c + e)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(2*sqrt(a
)*sqrt(c + e) - b - d) - 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c + e) + b +
 d))*d - 4*sqrt(a)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(2*sqrt(a
)*sqrt(c + e) - b - d) - 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c + e) + b +
 d))*c - 4*sqrt(a)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(2*sqrt(a
)*sqrt(c + e) - b - d) - 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c + e) + b +
 d))*e - 2*sqrt(c + e)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(2*sq
rt(a)*sqrt(c + e) - b - d) + 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c + e) +
 b + d))*b - 2*sqrt(c + e)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(
2*sqrt(a)*sqrt(c + e) - b - d) + 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c +
e) + b + d))*d + 4*sqrt(a)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(
2*sqrt(a)*sqrt(c + e) - b - d) + 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c +
e) + b + d))*c + 4*sqrt(a)*sqrt(2*sqrt(a)*sqrt(c + e) + b + d)*atan((sqrt(
2*sqrt(a)*sqrt(c + e) - b - d) + 2*sqrt(c + e)*x)/sqrt(2*sqrt(a)*sqrt(c +
e) + b + d))*e + sqrt(c + e)*sqrt(2*sqrt(a)*sqrt(c + e) - b - d)*log( - sq
rt(2*sqrt(a)*sqrt(c + e) - b - d)*x + sqrt(c + e)*x**2 + sqrt(a))*b + sqrt
(c + e)*sqrt(2*sqrt(a)*sqrt(c + e) - b - d)*log( - sqrt(2*sqrt(a)*sqrt(c +
 e) - b - d)*x + sqrt(c + e)*x**2 + sqrt(a))*d - sqrt(c + e)*sqrt(2*sqr...
```

3.900 $\int x^{5/2}(a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

output
```
2/7*a*x^(7/2)+2/11*b*x^(11/2)+2/15*c*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2x^{7/2}(165a+105bx^2+77cx^4)}{1155}$$

input
```
Integrate[x^(5/2)*(a + b*x^2 + c*x^4),x]
```

output
```
(2*x^(7/2)*(165*a + 105*b*x^2 + 77*c*x^4))/1155
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(ax^{5/2}+bx^{9/2}+cx^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}+\frac{2}{15}cx^{15/2}$$

input
```
Int[x^(5/2)*(a + b*x^2 + c*x^4),x]
```

output
```
(2*a*x^(7/2))/7 + (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{11}{2}}}{11}+\frac{2c\,x^{\frac{15}{2}}}{15}$	20
default	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{11}{2}}}{11}+\frac{2c\,x^{\frac{15}{2}}}{15}$	20
gosper	$\frac{2x^{\frac{7}{2}}\left(77c\,x^{4}+105b\,x^{2}+165a\right)}{1155}$	22
trager	$\frac{2x^{\frac{7}{2}}\left(77c\,x^{4}+105b\,x^{2}+165a\right)}{1155}$	22
risch	$\frac{2x^{\frac{7}{2}}\left(77c\,x^{4}+105b\,x^{2}+165a\right)}{1155}$	22
orering	$\frac{2x^{\frac{7}{2}}\left(77c\,x^{4}+105b\,x^{2}+165a\right)}{1155}$	22

input
```
int(x^(5/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/7*a*x^(7/2)+2/11*b*x^(11/2)+2/15*c*x^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2}{1155}\left(77\,cx^7+105\,bx^5+165\,ax^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
2/1155*(77*c*x^7 + 105*b*x^5 + 165*a*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{11}{2}}}{11}+\frac{2cx^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(5/2)*(c*x**4+b*x**2+a),x)
```

output
```
2*a*x**(7/2)/7 + 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
2/15*c*x^(15/2) + 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
2/15*c*x^(15/2) + 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 17.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2\,x^{7/2}\,\left(77\,c\,x^4+105\,b\,x^2+165\,a\right)}{1155}$$

input
```
int(x^(5/2)*(a + b*x^2 + c*x^4),x)
```

output
```
(2*x^(7/2)*(165*a + 105*b*x^2 + 77*c*x^4))/1155
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^{5/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2\sqrt{x}\,x^3(77c\,x^4+105b\,x^2+165a)}{1155}$$

input
```
int(x^(5/2)*(c*x^4+b*x^2+a),x)
```

output
```
(2*sqrt(x)*x**3*(165*a + 105*b*x**2 + 77*c*x**4))/1155
```

3.901 $\int x^{3/2}(a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int x^{3/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

output
```
2/5*a*x^(5/2)+2/9*b*x^(9/2)+2/13*c*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2}{585}x^{5/2}\left(117a+65bx^2+45cx^4\right)$$

input
```
Integrate[x^(3/2)*(a + b*x^2 + c*x^4),x]
```

output
```
(2*x^(5/2)*(117*a + 65*b*x^2 + 45*c*x^4))/585
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^2+cx^4\right)\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(ax^{3/2}+bx^{7/2}+cx^{11/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}+\frac{2}{13}cx^{13/2}$$

input
```
Int[x^(3/2)*(a + b*x^2 + c*x^4),x]
```

output
```
(2*a*x^(5/2))/5 + (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{9}{2}}}{9}+\frac{2c\,x^{\frac{13}{2}}}{13}$	20
default	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{9}{2}}}{9}+\frac{2c\,x^{\frac{13}{2}}}{13}$	20
gosper	$\frac{2x^{\frac{5}{2}}\left(45c\,x^{4}+65b\,x^{2}+117a\right)}{585}$	22
trager	$\frac{2x^{\frac{5}{2}}\left(45c\,x^{4}+65b\,x^{2}+117a\right)}{585}$	22
risch	$\frac{2x^{\frac{5}{2}}\left(45c\,x^{4}+65b\,x^{2}+117a\right)}{585}$	22
orering	$\frac{2x^{\frac{5}{2}}\left(45c\,x^{4}+65b\,x^{2}+117a\right)}{585}$	22

input
```
int(x^(3/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*a*x^(5/2)+2/9*b*x^(9/2)+2/13*c*x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int x^{3/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2}{585}\left(45\,cx^6+65\,bx^4+117\,ax^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
2/585*(45*c*x^6 + 65*b*x^4 + 117*a*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int x^{3/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2ax^{\frac{5}{2}}}{5}+\frac{2bx^{\frac{9}{2}}}{9}+\frac{2cx^{\frac{13}{2}}}{13}$$

input
```
integrate(x**(3/2)*(c*x**4+b*x**2+a),x)
```

output
```
2*a*x**(5/2)/5 + 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int x^{3/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
2/13*c*x^(13/2) + 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int x^{3/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
2/13*c*x^(13/2) + 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int x^{3/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2\,x^{5/2}\,\left(45\,c\,x^4+65\,b\,x^2+117\,a\right)}{585}$$

input
```
int(x^(3/2)*(a + b*x^2 + c*x^4),x)
```

output
```
(2*x^(5/2)*(117*a + 65*b*x^2 + 45*c*x^4))/585
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^{3/2}\left(a+bx^2+cx^4\right)\,dx=\frac{2\sqrt{x}\,x^2(45c\,x^4+65b\,x^2+117a)}{585}$$

input
```
int(x^(3/2)*(c*x^4+b*x^2+a),x)
```

output
```
(2*sqrt(x)*x**2*(117*a + 65*b*x**2 + 45*c*x**4))/585
```

3.902 $\int \sqrt{x}(a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)\,dx=\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

output
```
2/3*a*x^(3/2)+2/7*b*x^(7/2)+2/11*c*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)\,dx=\frac{2}{231}x^{3/2}\left(77a+33bx^2+21cx^4\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^2 + c*x^4),x]
```

output
```
(2*x^(3/2)*(77*a + 33*b*x^2 + 21*c*x^4))/231
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^2+cx^4\right) dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(a\sqrt{x}+bx^{5/2}+cx^{9/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

input
```
Int[Sqrt[x]*(a + b*x^2 + c*x^4),x]
```

output
```
(2*a*x^(3/2))/3 + (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2b\,x^{\frac{7}{2}}}{7}+\frac{2c\,x^{\frac{11}{2}}}{11}$	20
default	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2b\,x^{\frac{7}{2}}}{7}+\frac{2c\,x^{\frac{11}{2}}}{11}$	20
gosper	$\frac{2x^{\frac{3}{2}}\left(21c\,x^{4}+33b\,x^{2}+77a\right)}{231}$	22
trager	$\frac{2x^{\frac{3}{2}}\left(21c\,x^{4}+33b\,x^{2}+77a\right)}{231}$	22
risch	$\frac{2x^{\frac{3}{2}}\left(21c\,x^{4}+33b\,x^{2}+77a\right)}{231}$	22
orering	$\frac{2x^{\frac{3}{2}}\left(21c\,x^{4}+33b\,x^{2}+77a\right)}{231}$	22

input
```
int(x^(1/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*a*x^(3/2)+2/7*b*x^(7/2)+2/11*c*x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)\,dx=\frac{2}{231}\,\left(21\,cx^5+33\,bx^3+77\,ax\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
2/231*(21*c*x^5 + 33*b*x^3 + 77*a*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \sqrt{x}(a+bx^2+cx^4) \, dx = \frac{2ax^{\frac{3}{2}}}{3} + \frac{2bx^{\frac{7}{2}}}{7} + \frac{2cx^{\frac{11}{2}}}{11}$$

input
```
integrate(x**(1/2)*(c*x**4+b*x**2+a),x)
```

output
```
2*a*x**(3/2)/3 + 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \sqrt{x}(a+bx^2+cx^4) \, dx = \frac{2}{11}\,cx^{\frac{11}{2}} + \frac{2}{7}\,bx^{\frac{7}{2}} + \frac{2}{3}\,ax^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
2/11*c*x^(11/2) + 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \sqrt{x}(a+bx^2+cx^4) \, dx = \frac{2}{11}\,cx^{\frac{11}{2}} + \frac{2}{7}\,bx^{\frac{7}{2}} + \frac{2}{3}\,ax^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
2/11*c*x^(11/2) + 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \sqrt{x}(a + bx^2 + cx^4) \; dx = \frac{2\,x^{3/2}\,(21\,c\,x^4 + 33\,b\,x^2 + 77\,a)}{231}$$

input
```
int(x^(1/2)*(a + b*x^2 + c*x^4),x)
```

output
```
(2*x^(3/2)*(77*a + 33*b*x^2 + 21*c*x^4))/231
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \sqrt{x}(a + bx^2 + cx^4) \; dx = \frac{2\sqrt{x}\,x(21c\,x^4 + 33b\,x^2 + 77a)}{231}$$

input
```
int(x^(1/2)*(c*x^4+b*x^2+a),x)
```

output
```
(2*sqrt(x)*x*(77*a + 33*b*x**2 + 21*c*x**4))/231
```

3.903 $\int \frac{a+bx^2+cx^4}{\sqrt{x}} dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{a+bx^2+cx^4}{\sqrt{x}} dx = 2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

output
```
2*a*x^(1/2)+2/5*b*x^(5/2)+2/9*c*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2+cx^4}{\sqrt{x}} dx = \frac{2}{45}\sqrt{x}\left(45a+9bx^2+5cx^4\right)$$

input
```
Integrate[(a + b*x^2 + c*x^4)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(45*a + 9*b*x^2 + 5*c*x^4))/45
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx$$
$$\Big\downarrow \text{1433}$$
$$\int \left(\frac{a}{\sqrt{x}}+bx^{3/2}+cx^{7/2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

input
```
Int[(a + b*x^2 + c*x^4)/Sqrt[x],x]
```

output
```
2*a*Sqrt[x] + (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$2a\sqrt{x}+\frac{2b\,x^{\frac{5}{2}}}{5}+\frac{2c\,x^{\frac{9}{2}}}{9}$	20
default	$2a\sqrt{x}+\frac{2b\,x^{\frac{5}{2}}}{5}+\frac{2c\,x^{\frac{9}{2}}}{9}$	20
trager	$\left(\frac{2}{9}c\,x^{4}+\frac{2}{5}b\,x^{2}+2a\right)\sqrt{x}$	21
gosper	$\frac{2\sqrt{x}\,\left(5c\,x^{4}+9b\,x^{2}+45a\right)}{45}$	22
risch	$\frac{2\sqrt{x}\,\left(5c\,x^{4}+9b\,x^{2}+45a\right)}{45}$	22
orering	$\frac{2\sqrt{x}\,\left(5c\,x^{4}+9b\,x^{2}+45a\right)}{45}$	22

input
```
int((c*x^4+b*x^2+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*x^(1/2)+2/5*b*x^(5/2)+2/9*c*x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx=\frac{2}{45}\left(5\,cx^4+9\,bx^2+45\,a\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

output
```
2/45*(5*c*x^4 + 9*b*x^2 + 45*a)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx = 2a\sqrt{x} + \frac{2bx^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{9}{2}}}{9}$$

input
```
integrate((c*x**4+b*x**2+a)/x**(1/2),x)
```

output
```
2*a*sqrt(x) + 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx = \frac{2}{9}\,cx^{\frac{9}{2}} + \frac{2}{5}\,bx^{\frac{5}{2}} + 2\,a\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

output
```
2/9*c*x^(9/2) + 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx = \frac{2}{9}\,cx^{\frac{9}{2}} + \frac{2}{5}\,bx^{\frac{5}{2}} + 2\,a\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2+a)/x^(1/2),x, algorithm="giac")
```

output
```
2/9*c*x^(9/2) + 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(5\,c\,x^4+9\,b\,x^2+45\,a)}{45}$$

input
```
int((a + b*x^2 + c*x^4)/x^(1/2),x)
```

output
```
(2*x^(1/2)*(45*a + 9*b*x^2 + 5*c*x^4))/45
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(5c\,x^4+9b\,x^2+45a)}{45}$$

input
```
int((c*x^4+b*x^2+a)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(45*a + 9*b*x**2 + 5*c*x**4))/45
```

3.904 $\int \frac{a+bx^2+cx^4}{x^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{a+bx^2+cx^4}{x^{3/2}}\,dx=-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

output
```
-2*a/x^(1/2)+2/3*b*x^(3/2)+2/7*c*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2+cx^4}{x^{3/2}}\,dx=-\frac{2(21a-7bx^2-3cx^4)}{21\sqrt{x}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/x^(3/2),x]
```

output
```
(-2*(21*a - 7*b*x^2 - 3*c*x^4))/(21*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a}{x^{3/2}}+b\sqrt{x}+cx^{5/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

input
```
Int[(a + b*x^2 + c*x^4)/x^(3/2),x]
```

output
```
(-2*a)/Sqrt[x] + (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{2a}{\sqrt{x}}+\frac{2b\,x^{\frac{3}{2}}}{3}+\frac{2c\,x^{\frac{7}{2}}}{7}$	20
default	$-\frac{2a}{\sqrt{x}}+\frac{2b\,x^{\frac{3}{2}}}{3}+\frac{2c\,x^{\frac{7}{2}}}{7}$	20
gosper	$-\frac{2\left(-3c\,x^{4}-7b\,x^{2}+21a\right)}{21\sqrt{x}}$	22
trager	$-\frac{2\left(-3c\,x^{4}-7b\,x^{2}+21a\right)}{21\sqrt{x}}$	22
risch	$-\frac{2\left(-3c\,x^{4}-7b\,x^{2}+21a\right)}{21\sqrt{x}}$	22
orering	$-\frac{2\left(-3c\,x^{4}-7b\,x^{2}+21a\right)}{21\sqrt{x}}$	22

input
```
int((c*x^4+b*x^2+a)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a/x^(1/2)+2/3*b*x^(3/2)+2/7*c*x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{a+bx^2+cx^4}{x^{3/2}}\,dx=\frac{2\left(3\,cx^4+7\,bx^2-21\,a\right)}{21\,\sqrt{x}}$$

input
```
integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="fricas")
```

output
```
2/21*(3*c*x^4 + 7*b*x^2 - 21*a)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{a + bx^2 + cx^4}{x^{3/2}}\,dx = -\frac{2a}{\sqrt{x}} + \frac{2bx^{\frac{3}{2}}}{3} + \frac{2cx^{\frac{7}{2}}}{7}$$

input
```
integrate((c*x**4+b*x**2+a)/x**(3/2),x)
```

output
```
-2*a/sqrt(x) + 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{a + bx^2 + cx^4}{x^{3/2}}\,dx = \frac{2}{7}\,cx^{\frac{7}{2}} + \frac{2}{3}\,bx^{\frac{3}{2}} - \frac{2\,a}{\sqrt{x}}$$

input
```
integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="maxima")
```

output
```
2/7*c*x^(7/2) + 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{a + bx^2 + cx^4}{x^{3/2}}\,dx = \frac{2}{7}\,cx^{\frac{7}{2}} + \frac{2}{3}\,bx^{\frac{3}{2}} - \frac{2\,a}{\sqrt{x}}$$

input
```
integrate((c*x^4+b*x^2+a)/x^(3/2),x, algorithm="giac")
```

output
```
2/7*c*x^(7/2) + 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 17.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{a+bx^2+cx^4}{x^{3/2}}\,dx=\frac{6\,c\,x^4+14\,b\,x^2-42\,a}{21\,\sqrt{x}}$$

input
```
int((a + b*x^2 + c*x^4)/x^(3/2),x)
```

output
```
(14*b*x^2 - 42*a + 6*c*x^4)/(21*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^2+cx^4}{x^{3/2}}\,dx=\frac{\frac{2}{7}c\,x^4+\frac{2}{3}b\,x^2-2a}{\sqrt{x}}$$

input
```
int((c*x^4+b*x^2+a)/x^(3/2),x)
```

output
```
(2*( - 21*a + 7*b*x**2 + 3*c*x**4))/(21*sqrt(x))
```

3.905 $\int \frac{a+bx^2+cx^4}{x^{5/2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx = -\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

output
```
-2/3*a/x^(3/2)+2*b*x^(1/2)+2/5*c*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx = -\frac{2(5a-15bx^2-3cx^4)}{15x^{3/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/x^(5/2),x]
```

output
```
(-2*(5*a - 15*b*x^2 - 3*c*x^4))/(15*x^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a}{x^{5/2}}+\frac{b}{\sqrt{x}}+cx^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

input
```
Int[(a + b*x^2 + c*x^4)/x^(5/2),x]
```

output
```
(-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}+\frac{2c\,x^{\frac{5}{2}}}{5}$	20
default	$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}+\frac{2c\,x^{\frac{5}{2}}}{5}$	20
gosper	$-\frac{2\left(-3c\,x^{4}-15b\,x^{2}+5a\right)}{15x^{\frac{3}{2}}}$	22
trager	$-\frac{2\left(-3c\,x^{4}-15b\,x^{2}+5a\right)}{15x^{\frac{3}{2}}}$	22
risch	$-\frac{2\left(-3c\,x^{4}-15b\,x^{2}+5a\right)}{15x^{\frac{3}{2}}}$	22
orering	$-\frac{2\left(-3c\,x^{4}-15b\,x^{2}+5a\right)}{15x^{\frac{3}{2}}}$	22

input
```
int((c*x^4+b*x^2+a)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a/x^(3/2)+2*b*x^(1/2)+2/5*c*x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx=\frac{2\left(3\,cx^4+15\,bx^2-5\,a\right)}{15\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*c*x^4 + 15*b*x^2 - 5*a)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx = -\frac{2a}{3x^{\frac{3}{2}}} + 2b\sqrt{x} + \frac{2cx^{\frac{5}{2}}}{5}$$

input
```
integrate((c*x**4+b*x**2+a)/x**(5/2),x)
```

output
```
-2*a/(3*x**(3/2)) + 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx = \frac{2}{5}\,cx^{\frac{5}{2}} + 2\,b\sqrt{x} - \frac{2\,a}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="maxima")
```

output
```
2/5*c*x^(5/2) + 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx = \frac{2}{5}\,cx^{\frac{5}{2}} + 2\,b\sqrt{x} - \frac{2\,a}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)/x^(5/2),x, algorithm="giac")
```

output
```
2/5*c*x^(5/2) + 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx=\frac{6\,c\,x^4+30\,b\,x^2-10\,a}{15\,x^{3/2}}$$

input
```
int((a + b*x^2 + c*x^4)/x^(5/2),x)
```

output
```
(30*b*x^2 - 10*a + 6*c*x^4)/(15*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2+cx^4}{x^{5/2}}\,dx=\frac{\frac{2}{5}c\,x^4+2b\,x^2-\frac{2}{3}a}{\sqrt{x}\,x}$$

input
```
int((c*x^4+b*x^2+a)/x^(5/2),x)
```

output
```
(2*( - 5*a + 15*b*x**2 + 3*c*x**4))/(15*sqrt(x)*x)
```

3.906 $\int \frac{a+bx^2+cx^4}{x^{7/2}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{a+bx^2+cx^4}{x^{7/2}} \, dx = -\frac{2a}{5x^{5/2}} - \frac{2b}{\sqrt{x}} + \frac{2}{3}cx^{3/2}$$

output
```
-2/5*a/x^(5/2)-2*b/x^(1/2)+2/3*c*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2+cx^4}{x^{7/2}} \, dx = \frac{2(-3a-15bx^2+5cx^4)}{15x^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/x^(7/2),x]
```

output
```
(2*(-3*a - 15*b*x^2 + 5*c*x^4))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a}{x^{7/2}}+\frac{b}{x^{3/2}}+c\sqrt{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}$$

input
```
Int[(a + b*x^2 + c*x^4)/x^(7/2),x]
```

output
```
(-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}+\frac{2c\,x^{\frac{3}{2}}}{3}$	20
default	$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}+\frac{2c\,x^{\frac{3}{2}}}{3}$	20
gosper	$-\frac{2(-5c\,x^{4}+15b\,x^{2}+3a)}{15x^{\frac{5}{2}}}$	22
trager	$-\frac{2(-5c\,x^{4}+15b\,x^{2}+3a)}{15x^{\frac{5}{2}}}$	22
risch	$-\frac{2(-5c\,x^{4}+15b\,x^{2}+3a)}{15x^{\frac{5}{2}}}$	22
orering	$-\frac{2(-5c\,x^{4}+15b\,x^{2}+3a)}{15x^{\frac{5}{2}}}$	22

input
```
int((c*x^4+b*x^2+a)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a/x^(5/2)-2*b/x^(1/2)+2/3*c*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{a+bx^2+cx^4}{x^{7/2}}\,dx=\frac{2\,(5\,cx^4-15\,bx^2-3\,a)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(5*c*x^4 - 15*b*x^2 - 3*a)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^2+cx^4}{x^{7/2}}\,dx = -\frac{2a}{5x^{\frac{5}{2}}} - \frac{2b}{\sqrt{x}} + \frac{2cx^{\frac{3}{2}}}{3}$$

input
```
integrate((c*x**4+b*x**2+a)/x**(7/2),x)
```

output
```
-2*a/(5*x**(5/2)) - 2*b/sqrt(x) + 2*c*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{a+bx^2+cx^4}{x^{7/2}}\,dx = \frac{2}{3}\,cx^{\frac{3}{2}} - \frac{2\,(5\,bx^2+a)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="maxima")
```

output
```
2/3*c*x^(3/2) - 2/5*(5*b*x^2 + a)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{a+bx^2+cx^4}{x^{7/2}}\,dx = \frac{2}{3}\,cx^{\frac{3}{2}} - \frac{2\,(5\,bx^2+a)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)/x^(7/2),x, algorithm="giac")
```

output
```
2/3*c*x^(3/2) - 2/5*(5*b*x^2 + a)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 17.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{a+bx^2+cx^4}{x^{7/2}}\,dx=-\frac{-10\,c\,x^4+30\,b\,x^2+6\,a}{15\,x^{5/2}}$$

input
```
int((a + b*x^2 + c*x^4)/x^(7/2),x)
```

output
```
-(6*a + 30*b*x^2 - 10*c*x^4)/(15*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2+cx^4}{x^{7/2}}\,dx=\frac{\frac{2}{3}c\,x^4-2b\,x^2-\frac{2}{5}a}{\sqrt{x}\,x^2}$$

input
```
int((c*x^4+b*x^2+a)/x^(7/2),x)
```

output
```
(2*( - 3*a - 15*b*x**2 + 5*c*x**4))/(15*sqrt(x)*x**2)
```

3.907 $\int x^{5/2}(a+bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}\left(b^2+2ac\right)x^{15/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

output
```
2/7*a^2*x^(7/2)+4/11*a*b*x^(11/2)+2/15*(2*a*c+b^2)*x^(15/2)+4/19*b*c*x^(19
/2)+2/23*c^2*x^(23/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2\left(72105a^2x^{7/2}+91770abx^{11/2}+33649b^2x^{15/2}+67298acx^{15/2}+53130bcx^{19/2}+21945c^2x^{23/2}\right.}{504735}$$

input
```
Integrate[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(2*(72105*a^2*x^(7/2) + 91770*a*b*x^(11/2) + 33649*b^2*x^(15/2) + 67298*a*
c*x^(15/2) + 53130*b*c*x^(19/2) + 21945*c^2*x^(23/2)))/504735
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx$$

$$\Big\downarrow \ 1433$$

$$\int \left(a^2x^{5/2}+x^{13/2}\left(2ac+b^2\right)+2abx^{9/2}+2bcx^{17/2}+c^2x^{21/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{7}a^2x^{7/2}+\frac{2}{15}x^{15/2}\left(2ac+b^2\right)+\frac{4}{11}abx^{11/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

input
```
Int[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*(b^2 + 2*a*c)*x^(15/2))/15 +
(4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2a^{2}x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{11}{2}}}{11}+\frac{2(2ac+b^{2})x^{\frac{15}{2}}}{15}+\frac{4bc\,x^{\frac{19}{2}}}{19}+\frac{2c^{2}x^{\frac{23}{2}}}{23}$	45
default	$\frac{2a^{2}x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{11}{2}}}{11}+\frac{2(2ac+b^{2})x^{\frac{15}{2}}}{15}+\frac{4bc\,x^{\frac{19}{2}}}{19}+\frac{2c^{2}x^{\frac{23}{2}}}{23}$	45
gosper	$\frac{2x^{\frac{7}{2}}\left(21945c^{2}x^{8}+53130bc\,x^{6}+67298ac\,x^{4}+33649b^{2}x^{4}+91770ab\,x^{2}+72105a^{2}\right)}{504735}$	49
trager	$\frac{2x^{\frac{7}{2}}\left(21945c^{2}x^{8}+53130bc\,x^{6}+67298ac\,x^{4}+33649b^{2}x^{4}+91770ab\,x^{2}+72105a^{2}\right)}{504735}$	49
risch	$\frac{2x^{\frac{7}{2}}\left(21945c^{2}x^{8}+53130bc\,x^{6}+67298ac\,x^{4}+33649b^{2}x^{4}+91770ab\,x^{2}+72105a^{2}\right)}{504735}$	49
orering	$\frac{2x^{\frac{7}{2}}\left(21945c^{2}x^{8}+53130bc\,x^{6}+67298ac\,x^{4}+33649b^{2}x^{4}+91770ab\,x^{2}+72105a^{2}\right)}{504735}$	49

input
```
int(x^(5/2)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/7*a^2*x^(7/2)+4/11*a*b*x^(11/2)+2/15*(2*a*c+b^2)*x^(15/2)+4/19*b*c*x^(19
/2)+2/23*c^2*x^(23/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2}{504735}\left(21945\,c^2x^{11}+53130\,bcx^9+33649\left(b^2+2\,ac\right)x^7+91770\,abx^5+72105\,a^2x^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/504735*(21945*c^2*x^11 + 53130*b*c*x^9 + 33649*(b^2 + 2*a*c)*x^7 + 91770
*a*b*x^5 + 72105*a^2*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{11}{2}}}{11}+\frac{4acx^{\frac{15}{2}}}{15}+\frac{2b^2x^{\frac{15}{2}}}{15}+\frac{4bcx^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}$$

input
```
integrate(x**(5/2)*(c*x**4+b*x**2+a)**2,x)
```

output
```
2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 4*a*c*x**(15/2)/15 + 2*b**2*x**(1
5/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,\left(b^2+2\,ac\right)x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*(b^2 + 2*a*c)*x^(15/2) + 4/11
*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^2x^{\frac{15}{2}}+\frac{4}{15}\,acx^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2) + 4/15*a*c*x^(15
/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 17.98 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx=x^{15/2}\left(\frac{2\,b^2}{15}+\frac{4\,a\,c}{15}\right)+\frac{2\,a^2\,x^{7/2}}{7}+\frac{2\,c^2\,x^{23/2}}{23}+\frac{4\,a\,b\,x^{11/2}}{11}+\frac{4\,b\,c\,x^{19/2}}{19}$$

input
```
int(x^(5/2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^(15/2)*((4*a*c)/15 + (2*b^2)/15) + (2*a^2*x^(7/2))/7 + (2*c^2*x^(23/2))/
23 + (4*a*b*x^(11/2))/11 + (4*b*c*x^(19/2))/19
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2\sqrt{x}\,x^3(21945c^2x^8+53130bc\,x^6+67298ac\,x^4+33649b^2x^4+91770ab\,x^2+72105a^2)}{504735}$$

input
```
int(x^(5/2)*(c*x^4+b*x^2+a)^2,x)
```

output
```
(2*sqrt(x)*x**3*(72105*a**2 + 91770*a*b*x**2 + 67298*a*c*x**4 + 33649*b**2
*x**4 + 53130*b*c*x**6 + 21945*c**2*x**8))/504735
```

3.908 $\int x^{3/2}(a+bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}\left(b^2+2ac\right)x^{13/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

output
```
2/5*a^2*x^(5/2)+4/9*a*b*x^(9/2)+2/13*(2*a*c+b^2)*x^(13/2)+4/17*b*c*x^(17/2
)+2/21*c^2*x^(21/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2x^{5/2}(13923a^2+1190a(13bx^2+9cx^4)+15x^4(357b^2+546bcx^2+221c^2x^4))}{69615}$$

input
```
Integrate[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(2*x^(5/2)*(13923*a^2 + 1190*a*(13*b*x^2 + 9*c*x^4) + 15*x^4*(357*b^2 + 54
6*b*c*x^2 + 221*c^2*x^4)))/69615
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^2\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(a^2x^{3/2}+x^{11/2}\left(2ac+b^2\right)+2abx^{7/2}+2bcx^{15/2}+c^2x^{19/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^2x^{5/2}+\frac{2}{13}x^{13/2}\left(2ac+b^2\right)+\frac{4}{9}abx^{9/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

input
```
Int[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*(b^2 + 2*a*c)*x^(13/2))/13 + (4
*b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{2(2ac+b^2)\,x^{\frac{13}{2}}}{13}+\frac{4bc\,x^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$	45
default	$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{2(2ac+b^2)\,x^{\frac{13}{2}}}{13}+\frac{4bc\,x^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$	45
gosper	$\frac{2x^{\frac{5}{2}}(3315c^2x^8+8190bc\,x^6+10710ac\,x^4+5355b^2x^4+15470ab\,x^2+13923a^2)}{69615}$	49
trager	$\frac{2x^{\frac{5}{2}}(3315c^2x^8+8190bc\,x^6+10710ac\,x^4+5355b^2x^4+15470ab\,x^2+13923a^2)}{69615}$	49
risch	$\frac{2x^{\frac{5}{2}}(3315c^2x^8+8190bc\,x^6+10710ac\,x^4+5355b^2x^4+15470ab\,x^2+13923a^2)}{69615}$	49
orering	$\frac{2x^{\frac{5}{2}}(3315c^2x^8+8190bc\,x^6+10710ac\,x^4+5355b^2x^4+15470ab\,x^2+13923a^2)}{69615}$	49

input
```
int(x^(3/2)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*a^2*x^(5/2)+4/9*a*b*x^(9/2)+2/13*(2*a*c+b^2)*x^(13/2)+4/17*b*c*x^(17/2
)+2/21*c^2*x^(21/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2}{69615}\left(3315\,c^2x^{10}+8190\,bcx^8+5355\,\left(b^2+2\,ac\right)x^6+15470\,abx^4+13923\,a^2x^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/69615*(3315*c^2*x^10 + 8190*b*c*x^8 + 5355*(b^2 + 2*a*c)*x^6 + 15470*a*b
*x^4 + 13923*a^2*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{4acx^{\frac{13}{2}}}{13}+\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bcx^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(3/2)*(c*x**4+b*x**2+a)**2,x)
```

output
```
2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 4*a*c*x**(13/2)/13 + 2*b**2*x**(13/
2)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,\left(b^2+2\,ac\right)x^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*(b^2 + 2*a*c)*x^(13/2) + 4/9*
a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{13}\,acx^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2) + 4/13*a*c*x^(13
/2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^2\,dx=x^{13/2}\left(\frac{2\,b^2}{13}+\frac{4\,a\,c}{13}\right)+\frac{2\,a^2\,x^{5/2}}{5}+\frac{2\,c^2\,x^{21/2}}{21}+\frac{4\,a\,b\,x^{9/2}}{9}+\frac{4\,b\,c\,x^{17/2}}{17}$$

input
```
int(x^(3/2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^(13/2)*((4*a*c)/13 + (2*b^2)/13) + (2*a^2*x^(5/2))/5 + (2*c^2*x^(21/2))/
21 + (4*a*b*x^(9/2))/9 + (4*b*c*x^(17/2))/17
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2\sqrt{x}\,x^2(3315c^2x^8+8190bc\,x^6+10710ac\,x^4+5355b^2x^4+15470ab\,x^2+13923a^2)}{69615}$$

input
```
int(x^(3/2)*(c*x^4+b*x^2+a)^2,x)
```

output
```
(2*sqrt(x)*x**2*(13923*a**2 + 15470*a*b*x**2 + 10710*a*c*x**4 + 5355*b**2*
x**4 + 8190*b*c*x**6 + 3315*c**2*x**8))/69615
```

3.909 $\int \sqrt{x}\left(a+bx^2+cx^4\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}\left(b^2+2ac\right)x^{11/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

output
```
2/3*a^2*x^(3/2)+4/7*a*b*x^(7/2)+2/11*(2*a*c+b^2)*x^(11/2)+4/15*b*c*x^(15/2
)+2/19*c^2*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^2\,dx$$
$$=\frac{2x^{3/2}(7315a^2+570a(11bx^2+7cx^4)+7(285b^2x^4+418bcx^6+165c^2x^8))}{21945}$$

input
```
Integrate[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(2*x^(3/2)*(7315*a^2 + 570*a*(11*b*x^2 + 7*c*x^4) + 7*(285*b^2*x^4 + 418*b
*c*x^6 + 165*c^2*x^8)))/21945
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(a^2\sqrt{x}+x^{9/2}\left(2ac+b^2\right)+2abx^{5/2}+2bcx^{13/2}+c^2x^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^2x^{3/2}+\frac{2}{11}x^{11/2}\left(2ac+b^2\right)+\frac{4}{7}abx^{7/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

input
```
Int[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*(b^2 + 2*a*c)*x^(11/2))/11 + (4
*b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2(2ac+b^2)\,x^{\frac{11}{2}}}{11}+\frac{4bc\,x^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}$	45
default	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2(2ac+b^2)\,x^{\frac{11}{2}}}{11}+\frac{4bc\,x^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}$	45
gosper	$\frac{2x^{\frac{3}{2}}(1155c^2x^8+2926bc\,x^6+3990ac\,x^4+1995b^2x^4+6270ab\,x^2+7315a^2)}{21945}$	49
trager	$\frac{2x^{\frac{3}{2}}(1155c^2x^8+2926bc\,x^6+3990ac\,x^4+1995b^2x^4+6270ab\,x^2+7315a^2)}{21945}$	49
risch	$\frac{2x^{\frac{3}{2}}(1155c^2x^8+2926bc\,x^6+3990ac\,x^4+1995b^2x^4+6270ab\,x^2+7315a^2)}{21945}$	49
orering	$\frac{2x^{\frac{3}{2}}(1155c^2x^8+2926bc\,x^6+3990ac\,x^4+1995b^2x^4+6270ab\,x^2+7315a^2)}{21945}$	49

input
```
int(x^(1/2)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*a^2*x^(3/2)+4/7*a*b*x^(7/2)+2/11*(2*a*c+b^2)*x^(11/2)+4/15*b*c*x^(15/2
)+2/19*c^2*x^(19/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^2\,dx$$
$$=\frac{2}{21945}\left(1155\,c^2x^9+2926\,bcx^7+1995\,\left(b^2+2\,ac\right)x^5+6270\,abx^3+7315\,a^2x\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/21945*(1155*c^2*x^9 + 2926*b*c*x^7 + 1995*(b^2 + 2*a*c)*x^5 + 6270*a*b*x
^3 + 7315*a^2*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}+\frac{2x^{\frac{11}{2}}\cdot(2ac+b^2)}{11}$$

input
```
integrate(x**(1/2)*(c*x**4+b*x**2+a)**2,x)
```

output
```
2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 4*b*c*x**(15/2)/15 + 2*c**2*x**(19/
2)/19 + 2*x**(11/2)*(2*a*c + b**2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,\left(b^2+2\,ac\right)x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*(b^2 + 2*a*c)*x^(11/2) + 4/7*
a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{11}\,acx^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2) + 4/11*a*c*x^(11
/2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^2\,dx=x^{11/2}\left(\frac{2\,b^2}{11}+\frac{4\,a\,c}{11}\right)+\frac{2\,a^2\,x^{3/2}}{3}+\frac{2\,c^2\,x^{19/2}}{19}+\frac{4\,a\,b\,x^{7/2}}{7}+\frac{4\,b\,c\,x^{15/2}}{15}$$

input
```
int(x^(1/2)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^(11/2)*((4*a*c)/11 + (2*b^2)/11) + (2*a^2*x^(3/2))/3 + (2*c^2*x^(19/2))/
19 + (4*a*b*x^(7/2))/7 + (4*b*c*x^(15/2))/15
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^2\,dx=\frac{2\sqrt{x}\,x(1155c^2x^8+2926bc\,x^6+3990ac\,x^4+1995b^2x^4+6270ab\,x^2+7315a^2)}{21945}$$

input
```
int(x^(1/2)*(c*x^4+b*x^2+a)^2,x)
```

output
```
(2*sqrt(x)*x*(7315*a**2 + 6270*a*b*x**2 + 3990*a*c*x**4 + 1995*b**2*x**4 +
 2926*b*c*x**6 + 1155*c**2*x**8))/21945
```

3.910 $\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}} \, dx = 2a^2\sqrt{x} + \frac{4}{5}abx^{5/2} + \frac{2}{9}(b^2+2ac)\,x^{9/2} + \frac{4}{13}bcx^{13/2} + \frac{2}{17}c^2x^{17/2}$$

output
```
2*a^2*x^(1/2)+4/5*a*b*x^(5/2)+2/9*(2*a*c+b^2)*x^(9/2)+4/13*b*c*x^(13/2)+2/
17*c^2*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}} \, dx = \frac{2\sqrt{x}(9945a^2+442a(9bx^2+5cx^4)+5x^4(221b^2+306bcx^2+117c^2x^4))}{9945}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(9945*a^2 + 442*a*(9*b*x^2 + 5*c*x^4) + 5*x^4*(221*b^2 + 306*b*
c*x^2 + 117*c^2*x^4)))/9945
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{\sqrt{x}}dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^2}{\sqrt{x}}+x^{7/2}\left(2ac+b^2\right)+2abx^{3/2}+2bcx^{11/2}+c^2x^{15/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a^2\sqrt{x}+\frac{2}{9}x^{9/2}\left(2ac+b^2\right)+\frac{4}{5}abx^{5/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```

output
```
2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*(b^2 + 2*a*c)*x^(9/2))/9 + (4*b*c*x
^(13/2))/13 + (2*c^2*x^(17/2))/17
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$2a^2\sqrt{x}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2(2ac+b^2)x^{\frac{9}{2}}}{9}+\frac{4bc\,x^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{17}{2}}}{17}$	45
default	$2a^2\sqrt{x}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2(2ac+b^2)x^{\frac{9}{2}}}{9}+\frac{4bc\,x^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{17}{2}}}{17}$	45
trager	$\left(\frac{2}{17}c^2x^8+\frac{4}{13}bc\,x^6+\frac{4}{9}ac\,x^4+\frac{2}{9}b^2x^4+\frac{4}{5}ab\,x^2+2a^2\right)\sqrt{x}$	48
gosper	$\frac{2\sqrt{x}\,(585c^2x^8+1530bc\,x^6+2210ac\,x^4+1105b^2x^4+3978ab\,x^2+9945a^2)}{9945}$	49
risch	$\frac{2\sqrt{x}\,(585c^2x^8+1530bc\,x^6+2210ac\,x^4+1105b^2x^4+3978ab\,x^2+9945a^2)}{9945}$	49
orering	$\frac{2\sqrt{x}\,(585c^2x^8+1530bc\,x^6+2210ac\,x^4+1105b^2x^4+3978ab\,x^2+9945a^2)}{9945}$	49

input
```
int((c*x^4+b*x^2+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2*x^(1/2)+4/5*a*b*x^(5/2)+2/9*(2*a*c+b^2)*x^(9/2)+4/13*b*c*x^(13/2)+2/
17*c^2*x^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{9945}\left(585\,c^2x^8+1530\,bcx^6+1105\,(b^2+2\,ac)x^4+3978\,abx^2+9945\,a^2\right)\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/9945*(585*c^2*x^8 + 1530*b*c*x^6 + 1105*(b^2 + 2*a*c)*x^4 + 3978*a*b*x^2
 + 9945*a^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}}\,dx = 2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{4acx^{\frac{9}{2}}}{9} + \frac{2b^2x^{\frac{9}{2}}}{9} + \frac{4bcx^{\frac{13}{2}}}{13} + \frac{2c^2x^{\frac{17}{2}}}{17}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**(1/2),x)
```

output
```
2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 4*a*c*x**(9/2)/9 + 2*b**2*x**(9/2)/9 +
 4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{17}c^2x^{\frac{17}{2}} + \frac{4}{13}bcx^{\frac{13}{2}} + \frac{2}{9}b^2x^{\frac{9}{2}} + 2a^2\sqrt{x} + \frac{4}{45}\left(5cx^{\frac{9}{2}} + 9bx^{\frac{5}{2}}\right)a$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

output
```
2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2) + 2*a^2*sqrt(x) +
4/45*(5*c*x^(9/2) + 9*b*x^(5/2))*a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{17}c^2x^{\frac{17}{2}} + \frac{4}{13}bcx^{\frac{13}{2}} + \frac{2}{9}b^2x^{\frac{9}{2}} + \frac{4}{9}acx^{\frac{9}{2}} + \frac{4}{5}abx^{\frac{5}{2}} + 2a^2\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

output
```
2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2) + 4/9*a*c*x^(9/2)
+ 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}}\,dx = x^{9/2}\left(\frac{2\,b^2}{9}+\frac{4\,a\,c}{9}\right)+2\,a^2\,\sqrt{x}+\frac{2\,c^2\,x^{17/2}}{17}+\frac{4\,a\,b\,x^{5/2}}{5}+\frac{4\,b\,c\,x^{13/2}}{13}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^(1/2),x)
```

output
```
x^(9/2)*((4*a*c)/9 + (2*b^2)/9) + 2*a^2*x^(1/2) + (2*c^2*x^(17/2))/17 + (4
*a*b*x^(5/2))/5 + (4*b*c*x^(13/2))/13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(585c^2x^8+1530bc\,x^6+2210ac\,x^4+1105b^2x^4+3978ab\,x^2+9945a^2)}{9945}$$

input
```
int((c*x^4+b*x^2+a)^2/x^(1/2),x)
```

output
```
(2*sqrt(x)*(9945*a**2 + 3978*a*b*x**2 + 2210*a*c*x**4 + 1105*b**2*x**4 + 1
530*b*c*x**6 + 585*c**2*x**8))/9945
```

3.911 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{3/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{3/2}} \, dx = -\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}\left(b^2+2ac\right)x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

output
```
-2*a^2/x^(1/2)+4/3*a*b*x^(3/2)+2/7*(2*a*c+b^2)*x^(7/2)+4/11*b*c*x^(11/2)+2
/15*c^2*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{3/2}} \, dx = -\frac{2\left(1155a^2-770abx^2-165b^2x^4-330acx^4-210bcx^6-77c^2x^8\right)}{1155\sqrt{x}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

output
```
(-2*(1155*a^2 - 770*a*b*x^2 - 165*b^2*x^4 - 330*a*c*x^4 - 210*b*c*x^6 - 77
*c^2*x^8))/(1155*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^2}{x^{3/2}}+x^{5/2}(2ac+b^2)+2ab\sqrt{x}+2bcx^{9/2}+c^2x^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2}{\sqrt{x}}+\frac{2}{7}x^{7/2}(2ac+b^2)+\frac{4}{3}abx^{3/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

output
```
(-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*(b^2 + 2*a*c)*x^(7/2))/7 + (4*b*
c*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2c^2x^{\frac{15}{2}}}{15}+\frac{4bc\,x^{\frac{11}{2}}}{11}+\frac{4ac\,x^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{3}{2}}}{3}-\frac{2a^2}{\sqrt{x}}$	47
default	$\frac{2c^2x^{\frac{15}{2}}}{15}+\frac{4bc\,x^{\frac{11}{2}}}{11}+\frac{4ac\,x^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{3}{2}}}{3}-\frac{2a^2}{\sqrt{x}}$	47
gosper	$-\frac{2(-77c^2x^8-210bc\,x^6-330ac\,x^4-165b^2x^4-770ab\,x^2+1155a^2)}{1155\sqrt{x}}$	49
trager	$-\frac{2(-77c^2x^8-210bc\,x^6-330ac\,x^4-165b^2x^4-770ab\,x^2+1155a^2)}{1155\sqrt{x}}$	49
risch	$-\frac{2(-77c^2x^8-210bc\,x^6-330ac\,x^4-165b^2x^4-770ab\,x^2+1155a^2)}{1155\sqrt{x}}$	49
orering	$-\frac{2(-77c^2x^8-210bc\,x^6-330ac\,x^4-165b^2x^4-770ab\,x^2+1155a^2)}{1155\sqrt{x}}$	49

input
```
int((c*x^4+b*x^2+a)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*c^2*x^(15/2)+4/11*b*c*x^(11/2)+4/7*a*c*x^(7/2)+2/7*b^2*x^(7/2)+4/3*a*
b*x^(3/2)-2*a^2/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}}\,dx=\frac{2\,(77\,c^2x^8+210\,bcx^6+165\,(b^2+2\,ac)x^4+770\,abx^2-1155\,a^2)}{1155\,\sqrt{x}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(77*c^2*x^8 + 210*b*c*x^6 + 165*(b^2 + 2*a*c)*x^4 + 770*a*b*x^2 - 1
155*a^2)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}}\,dx = -\frac{2a^2}{\sqrt{x}} + \frac{4abx^{\frac{3}{2}}}{3} + \frac{4acx^{\frac{7}{2}}}{7} + \frac{2b^2x^{\frac{7}{2}}}{7} + \frac{4bcx^{\frac{11}{2}}}{11} + \frac{2c^2x^{\frac{15}{2}}}{15}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**(3/2),x)
```

output
```
-2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 4*a*c*x**(7/2)/7 + 2*b**2*x**(7/2)/7
+ 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2}{15}\,c^2x^{\frac{15}{2}} + \frac{4}{11}\,bcx^{\frac{11}{2}} + \frac{2}{7}\,(b^2+2\,ac)x^{\frac{7}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} - \frac{2\,a^2}{\sqrt{x}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="maxima")
```

output
```
2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*(b^2 + 2*a*c)*x^(7/2) + 4/3*a*
b*x^(3/2) - 2*a^2/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2}{15}\,c^2x^{\frac{15}{2}} + \frac{4}{11}\,bcx^{\frac{11}{2}} + \frac{2}{7}\,b^2x^{\frac{7}{2}} + \frac{4}{7}\,acx^{\frac{7}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} - \frac{2\,a^2}{\sqrt{x}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^(3/2),x, algorithm="giac")
```

output
```
2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2) + 4/7*a*c*x^(7/2)
+ 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}}\,dx = x^{7/2}\left(\frac{2\,b^2}{7}+\frac{4\,a\,c}{7}\right)-\frac{2\,a^2}{\sqrt{x}}+\frac{2\,c^2\,x^{15/2}}{15}+\frac{4\,a\,b\,x^{3/2}}{3}+\frac{4\,b\,c\,x^{11/2}}{11}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^(3/2),x)
```

output
```
x^(7/2)*((4*a*c)/7 + (2*b^2)/7) - (2*a^2)/x^(1/2) + (2*c^2*x^(15/2))/15 +
(4*a*b*x^(3/2))/3 + (4*b*c*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{\frac{2}{15}c^2x^8+\frac{4}{11}bc\,x^6+\frac{4}{7}ac\,x^4+\frac{2}{7}b^2x^4+\frac{4}{3}ab\,x^2-2a^2}{\sqrt{x}}$$

input
```
int((c*x^4+b*x^2+a)^2/x^(3/2),x)
```

output
```
(2*( - 1155*a**2 + 770*a*b*x**2 + 330*a*c*x**4 + 165*b**2*x**4 + 210*b*c*x
**6 + 77*c**2*x**8))/(1155*sqrt(x))
```

3.912 $\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{5/2}}\,dx=-\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}\left(b^2+2ac\right)x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

output
```
-2/3*a^2/x^(3/2)+4*a*b*x^(1/2)+2/5*(2*a*c+b^2)*x^(5/2)+4/9*b*c*x^(9/2)+2/1
3*c^2*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+bx^2+cx^4\right)^2}{x^{5/2}}\,dx=-\frac{2\left(195a^2-1170abx^2-117b^2x^4-234acx^4-130bcx^6-45c^2x^8\right)}{585x^{3/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

output
```
(-2*(195*a^2 - 1170*a*b*x^2 - 117*b^2*x^4 - 234*a*c*x^4 - 130*b*c*x^6 - 45
*c^2*x^8))/(585*x^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^2}{x^{5/2}}+x^{3/2}(2ac+b^2)+\frac{2ab}{\sqrt{x}}+2bcx^{7/2}+c^2x^{11/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2}{3x^{3/2}}+\frac{2}{5}x^{5/2}(2ac+b^2)+4ab\sqrt{x}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

output
```
(-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*(b^2 + 2*a*c)*x^(5/2))/5 + (4*b*
c*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2c^2x^{\frac{13}{2}}}{13}+\frac{4bc\,x^{\frac{9}{2}}}{9}+\frac{4ac\,x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{5}{2}}}{5}+4ab\sqrt{x}-\frac{2a^2}{3x^{\frac{3}{2}}}$	47
default	$\frac{2c^2x^{\frac{13}{2}}}{13}+\frac{4bc\,x^{\frac{9}{2}}}{9}+\frac{4ac\,x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{5}{2}}}{5}+4ab\sqrt{x}-\frac{2a^2}{3x^{\frac{3}{2}}}$	47
gosper	$-\frac{2\left(-45c^2x^8-130bc\,x^6-234ac\,x^4-117b^2x^4-1170ab\,x^2+195a^2\right)}{585x^{\frac{3}{2}}}$	49
trager	$-\frac{2\left(-45c^2x^8-130bc\,x^6-234ac\,x^4-117b^2x^4-1170ab\,x^2+195a^2\right)}{585x^{\frac{3}{2}}}$	49
risch	$-\frac{2\left(-45c^2x^8-130bc\,x^6-234ac\,x^4-117b^2x^4-1170ab\,x^2+195a^2\right)}{585x^{\frac{3}{2}}}$	49
orering	$-\frac{2\left(-45c^2x^8-130bc\,x^6-234ac\,x^4-117b^2x^4-1170ab\,x^2+195a^2\right)}{585x^{\frac{3}{2}}}$	49

input
```
int((c*x^4+b*x^2+a)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/13*c^2*x^(13/2)+4/9*b*c*x^(9/2)+4/5*a*c*x^(5/2)+2/5*b^2*x^(5/2)+4*a*b*x^
(1/2)-2/3*a^2/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2+cx^4)^2}{x^{5/2}}\,dx=\frac{2\,(45\,c^2x^8+130\,bcx^6+117\,(b^2+2\,ac)x^4+1170\,abx^2-195\,a^2)}{585\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="fricas")
```

output
```
2/585*(45*c^2*x^8 + 130*b*c*x^6 + 117*(b^2 + 2*a*c)*x^4 + 1170*a*b*x^2 - 1
95*a^2)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2+cx^4)^2}{x^{5/2}}\,dx = -\frac{2a^2}{3x^{\frac{3}{2}}} + 4ab\sqrt{x} + \frac{4acx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{5}{2}}}{5} + \frac{4bcx^{\frac{9}{2}}}{9} + \frac{2c^2x^{\frac{13}{2}}}{13}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**(5/2),x)
```

output
```
-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 4*a*c*x**(5/2)/5 + 2*b**2*x**(5/2)/
5 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{13}\,c^2x^{\frac{13}{2}} + \frac{4}{9}\,bcx^{\frac{9}{2}} + \frac{2}{5}\,(b^2+2\,ac)x^{\frac{5}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="maxima")
```

output
```
2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*(b^2 + 2*a*c)*x^(5/2) + 4*a*b*sq
rt(x) - 2/3*a^2/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{13}\,c^2x^{\frac{13}{2}} + \frac{4}{9}\,bcx^{\frac{9}{2}} + \frac{2}{5}\,b^2x^{\frac{5}{2}} + \frac{4}{5}\,acx^{\frac{5}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^(5/2),x, algorithm="giac")
```

output
```
2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2) + 4/5*a*c*x^(5/2) +
4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2+cx^4)^2}{x^{5/2}}\,dx = x^{5/2}\left(\frac{2\,b^2}{5}+\frac{4\,a\,c}{5}\right)-\frac{2\,a^2}{3\,x^{3/2}}+\frac{2\,c^2\,x^{13/2}}{13}+4\,a\,b\,\sqrt{x}+\frac{4\,b\,c\,x^{9/2}}{9}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^(5/2),x)
```

output
```
x^(5/2)*((4*a*c)/5 + (2*b^2)/5) - (2*a^2)/(3*x^(3/2)) + (2*c^2*x^(13/2))/1
3 + 4*a*b*x^(1/2) + (4*b*c*x^(9/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{\frac{2}{13}c^2x^8+\frac{4}{9}bc\,x^6+\frac{4}{5}ac\,x^4+\frac{2}{5}b^2x^4+4ab\,x^2-\frac{2}{3}a^2}{\sqrt{x}\,x}$$

input
```
int((c*x^4+b*x^2+a)^2/x^(5/2),x)
```

output
```
(2*( - 195*a**2 + 1170*a*b*x**2 + 234*a*c*x**4 + 117*b**2*x**4 + 130*b*c*x
**6 + 45*c**2*x**8))/(585*sqrt(x)*x)
```

3.913 $\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx = -\frac{2a^2}{5x^{5/2}} - \frac{4ab}{\sqrt{x}} + \frac{2}{3}\left(b^2+2ac\right)x^{3/2} + \frac{4}{7}bcx^{7/2} + \frac{2}{11}c^2x^{11/2}$$

output
```
-2/5*a^2/x^(5/2)-4*a*b/x^(1/2)+2/3*(2*a*c+b^2)*x^(3/2)+4/7*b*c*x^(7/2)+2/1
1*c^2*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx = -\frac{2(231a^2+2310abx^2-385b^2x^4-770acx^4-330bcx^6-105c^2x^8)}{1155x^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

output
```
(-2*(231*a^2 + 2310*a*b*x^2 - 385*b^2*x^4 - 770*a*c*x^4 - 330*b*c*x^6 - 10
5*c^2*x^8))/(1155*x^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^2}{x^{7/2}}+\sqrt{x}(2ac+b^2)+\frac{2ab}{x^{3/2}}+2bcx^{5/2}+c^2x^{9/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2}{5x^{5/2}}+\frac{2}{3}x^{3/2}(2ac+b^2)-\frac{4ab}{\sqrt{x}}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

output
```
(-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*(b^2 + 2*a*c)*x^(3/2))/3 + (4*
b*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2c^2x^{\frac{11}{2}}}{11}+\frac{4bc\,x^{\frac{7}{2}}}{7}+\frac{4ac\,x^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{3}{2}}}{3}-\frac{4ab}{\sqrt{x}}-\frac{2a^2}{5x^{\frac{5}{2}}}$	47
default	$\frac{2c^2x^{\frac{11}{2}}}{11}+\frac{4bc\,x^{\frac{7}{2}}}{7}+\frac{4ac\,x^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{3}{2}}}{3}-\frac{4ab}{\sqrt{x}}-\frac{2a^2}{5x^{\frac{5}{2}}}$	47
gosper	$-\frac{2(-105c^2x^8-330bc\,x^6-770ac\,x^4-385b^2x^4+2310ab\,x^2+231a^2)}{1155x^{\frac{5}{2}}}$	49
trager	$-\frac{2(-105c^2x^8-330bc\,x^6-770ac\,x^4-385b^2x^4+2310ab\,x^2+231a^2)}{1155x^{\frac{5}{2}}}$	49
risch	$-\frac{2(-105c^2x^8-330bc\,x^6-770ac\,x^4-385b^2x^4+2310ab\,x^2+231a^2)}{1155x^{\frac{5}{2}}}$	49
orering	$-\frac{2(-105c^2x^8-330bc\,x^6-770ac\,x^4-385b^2x^4+2310ab\,x^2+231a^2)}{1155x^{\frac{5}{2}}}$	49

input
```
int((c*x^4+b*x^2+a)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*c^2*x^(11/2)+4/7*b*c*x^(7/2)+4/3*a*c*x^(3/2)+2/3*b^2*x^(3/2)-4*a*b/x^
(1/2)-2/5*a^2/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2\,(105\,c^2x^8+330\,bcx^6+385\,(b^2+2\,ac)x^4-2310\,abx^2-231\,a^2)}{1155\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="fricas")
```

output
```
2/1155*(105*c^2*x^8 + 330*b*c*x^6 + 385*(b^2 + 2*a*c)*x^4 - 2310*a*b*x^2 -
 231*a^2)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx = -\frac{2a^2}{5x^{\frac{5}{2}}} - \frac{4ab}{\sqrt{x}} + \frac{4acx^{\frac{3}{2}}}{3} + \frac{2b^2x^{\frac{3}{2}}}{3} + \frac{4bcx^{\frac{7}{2}}}{7} + \frac{2c^2x^{\frac{11}{2}}}{11}$$

input
```
integrate((c*x**4+b*x**2+a)**2/x**(7/2),x)
```

output
```
-2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 4*a*c*x**(3/2)/3 + 2*b**2*x**(3/2)/
3 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{7}\,bcx^{\frac{7}{2}} + \frac{2}{3}\,(b^2+2\,ac)x^{\frac{3}{2}} - \frac{2\,(10\,abx^2+a^2)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="maxima")
```

output
```
2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*(b^2 + 2*a*c)*x^(3/2) - 2/5*(10*
a*b*x^2 + a^2)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{7}\,bcx^{\frac{7}{2}} + \frac{2}{3}\,b^2x^{\frac{3}{2}} + \frac{4}{3}\,acx^{\frac{3}{2}} - \frac{2\,(10\,abx^2+a^2)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/x^(7/2),x, algorithm="giac")
```

output
```
2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2) + 4/3*a*c*x^(3/2) -
2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx = x^{3/2}\left(\frac{2\,b^2}{3}+\frac{4\,a\,c}{3}\right)-\frac{\frac{2\,a^2}{5}+4\,b\,a\,x^2}{x^{5/2}}+\frac{2\,c^2\,x^{11/2}}{11}+\frac{4\,b\,c\,x^{7/2}}{7}$$

input
```
int((a + b*x^2 + c*x^4)^2/x^(7/2),x)
```

output
```
x^(3/2)*((4*a*c)/3 + (2*b^2)/3) - ((2*a^2)/5 + 4*a*b*x^2)/x^(5/2) + (2*c^2
*x^(11/2))/11 + (4*b*c*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{\frac{2}{11}c^2x^8+\frac{4}{7}bc\,x^6+\frac{4}{3}ac\,x^4+\frac{2}{3}b^2x^4-4ab\,x^2-\frac{2}{5}a^2}{\sqrt{x}\,x^2}$$

input
```
int((c*x^4+b*x^2+a)^2/x^(7/2),x)
```

output
```
(2*( - 231*a**2 - 2310*a*b*x**2 + 770*a*c*x**4 + 385*b**2*x**4 + 330*b*c*x
**6 + 105*c**2*x**8))/(1155*sqrt(x)*x**2)
```

3.914 $\int x^{5/2}(a+bx^2+cx^4)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 103

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}a\left(b^2+ac\right)x^{15/2}+\frac{2}{19}b\left(b^2+6ac\right)x^{19/2}+\frac{6}{23}c\left(b^2+ac\right)x^{23/2}+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

output
```
2/7*a^3*x^(7/2)+6/11*a^2*b*x^(11/2)+2/5*a*(a*c+b^2)*x^(15/2)+2/19*b*(6*a*c
+b^2)*x^(19/2)+6/23*c*(a*c+b^2)*x^(23/2)+2/9*b*c^2*x^(27/2)+2/31*c^3*x^(31
/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2\left(6705765a^3x^{7/2}+12801915a^2bx^{11/2}+9388071ab^2x^{15/2}+9388071a^2cx^{15/2}+2470545b^3x^{19/2}\right.}{46}$$

input
```
Integrate[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(2*(6705765*a^3*x^(7/2) + 12801915*a^2*b*x^(11/2) + 9388071*a*b^2*x^(15/2)
 + 9388071*a^2*c*x^(15/2) + 2470545*b^3*x^(19/2) + 14823270*a*b*c*x^(19/2)
 + 6122655*b^2*c*x^(23/2) + 6122655*a*c^2*x^(23/2) + 5215595*b*c^2*x^(27/2
) + 1514205*c^3*x^(31/2)))/46940355
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(a^3x^{5/2}+3a^2bx^{9/2}+3cx^{21/2}\left(ac+b^2\right)+bx^{17/2}\left(6ac+b^2\right)+3ax^{13/2}\left(ac+b^2\right)+3bc^2x^{25/2}+c^3x^{29/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

input
```
Int[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*(b^2 + a*c)*x^(15/2))/5 +
 (2*b*(b^2 + 6*a*c)*x^(19/2))/19 + (6*c*(b^2 + a*c)*x^(23/2))/23 + (2*b*c^
2*x^(27/2))/9 + (2*c^3*x^(31/2))/31
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result
gosper	$\frac{2x^{\frac{7}{2}}(1514205c^3x^{12}+5215595b\,c^2x^{10}+6122655a\,c^2x^8+6122655b^2c\,x^8+14823270abc\,x^6+2470545b^3x^6+9388071a^2c\,x^4}{46940355}$
trager	$\frac{2x^{\frac{7}{2}}(1514205c^3x^{12}+5215595b\,c^2x^{10}+6122655a\,c^2x^8+6122655b^2c\,x^8+14823270abc\,x^6+2470545b^3x^6+9388071a^2c\,x^4}{46940355}$
risch	$\frac{2x^{\frac{7}{2}}(1514205c^3x^{12}+5215595b\,c^2x^{10}+6122655a\,c^2x^8+6122655b^2c\,x^8+14823270abc\,x^6+2470545b^3x^6+9388071a^2c\,x^4}{46940355}$
orering	$\frac{2x^{\frac{7}{2}}(1514205c^3x^{12}+5215595b\,c^2x^{10}+6122655a\,c^2x^8+6122655b^2c\,x^8+14823270abc\,x^6+2470545b^3x^6+9388071a^2c\,x^4}{46940355}$
derivativedivides	$\frac{2c^3x^{\frac{31}{2}}}{31}+\frac{2b\,c^2x^{\frac{27}{2}}}{9}+\frac{2(c^2a+2b^2c+c(2ac+b^2))x^{\frac{23}{2}}}{23}+\frac{2(4abc+b(2ac+b^2))x^{\frac{19}{2}}}{19}+\frac{2(a(2ac+b^2)+2b^2a+c\,a^2)x}{15}$
default	$\frac{2c^3x^{\frac{31}{2}}}{31}+\frac{2b\,c^2x^{\frac{27}{2}}}{9}+\frac{2(c^2a+2b^2c+c(2ac+b^2))x^{\frac{23}{2}}}{23}+\frac{2(4abc+b(2ac+b^2))x^{\frac{19}{2}}}{19}+\frac{2(a(2ac+b^2)+2b^2a+c\,a^2)x}{15}$

input
```
int(x^(5/2)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/46940355*x^(7/2)*(1514205*c^3*x^12+5215595*b*c^2*x^10+6122655*a*c^2*x^8+
6122655*b^2*c*x^8+14823270*a*b*c*x^6+2470545*b^3*x^6+9388071*a^2*c*x^4+938
8071*a*b^2*x^4+12801915*a^2*b*x^2+6705765*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2}{46940355}\left(1514205\,c^3x^{15}+5215595\,bc^2x^{13}+6122655\left(b^2c+ac^2\right)x^{11}+2470545\left(b^3+6\,abc\right)x\right.$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
2/46940355*(1514205*c^3*x^15 + 5215595*b*c^2*x^13 + 6122655*(b^2*c + a*c^2
)*x^11 + 2470545*(b^3 + 6*a*b*c)*x^9 + 12801915*a^2*b*x^5 + 9388071*(a*b^2
 + a^2*c)*x^7 + 6705765*a^3*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.25

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{6a^2bx^{\frac{11}{2}}}{11}+\frac{2a^2cx^{\frac{15}{2}}}{5}+\frac{2ab^2x^{\frac{15}{2}}}{5}+\frac{12abcx^{\frac{19}{2}}}{19}+\frac{6ac^2x^{\frac{23}{2}}}{23}+\frac{2b^3x^{\frac{19}{2}}}{19}+\frac{6b^2cx^{\frac{23}{2}}}{23}+\frac{2bc^2x^{\frac{27}{2}}}{9}+\frac{2c^3x^{\frac{31}{2}}}{31}$$

input
```
integrate(x**(5/2)*(c*x**4+b*x**2+a)**3,x)
```

output
```
2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a**2*c*x**(15/2)/5 + 2*a*b**
2*x**(15/2)/5 + 12*a*b*c*x**(19/2)/19 + 6*a*c**2*x**(23/2)/23 + 2*b**3*x**
(19/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x**(27/2)/9 + 2*c**3*x**(31/2
)/31
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,\left(b^2c+ac^2\right)x^{\frac{23}{2}}+\frac{2}{19}\,\left(b^3+6\,abc\right)x^{\frac{19}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{5}\,\left(ab^2+a^2c\right)x^{\frac{15}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*(b^2*c + a*c^2)*x^(23/2) + 2
/19*(b^3 + 6*a*b*c)*x^(19/2) + 6/11*a^2*b*x^(11/2) + 2/5*(a*b^2 + a^2*c)*x
^(15/2) + 2/7*a^3*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,b^2cx^{\frac{23}{2}}+\frac{6}{23}\,ac^2x^{\frac{23}{2}}+\frac{2}{19}\,b^3x^{\frac{19}{2}}+\frac{12}{19}\,abcx^{\frac{19}{2}}+\frac{2}{5}\,ab^2x^{\frac{15}{2}}+\frac{2}{5}\,a^2cx^{\frac{15}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 6/23*a*c^2*
x^(23/2) + 2/19*b^3*x^(19/2) + 12/19*a*b*c*x^(19/2) + 2/5*a*b^2*x^(15/2) +
 2/5*a^2*c*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx=x^{19/2}\left(\frac{2\,b^3}{19}+\frac{12\,a\,c\,b}{19}\right)+\frac{2\,a^3\,x^{7/2}}{7}+\frac{2\,c^3\,x^{31/2}}{31}+\frac{6\,a^2\,b\,x^{11/2}}{11}+\frac{2\,b\,c^2\,x^{27/2}}{9}+\frac{2\,a\,x^{15/2}\,\left(b^2+a\,c\right)}{5}+\frac{6\,c\,x^{23/2}\,\left(b^2+a\,c\right)}{23}$$

input
```
int(x^(5/2)*(a + b*x^2 + c*x^4)^3,x)
```

output
```
x^(19/2)*((2*b^3)/19 + (12*a*b*c)/19) + (2*a^3*x^(7/2))/7 + (2*c^3*x^(31/2
))/31 + (6*a^2*b*x^(11/2))/11 + (2*b*c^2*x^(27/2))/9 + (2*a*x^(15/2)*(a*c
+ b^2))/5 + (6*c*x^(23/2)*(a*c + b^2))/23
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int x^{5/2}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2\sqrt{x}\,x^3(1514205c^3x^{12}+5215595b\,c^2x^{10}+6122655a\,c^2x^8+6122655b^2c\,x^8+14823270abc\,x^6+}{46940355}$$

input
```
int(x^(5/2)*(c*x^4+b*x^2+a)^3,x)
```

output
```
(2*sqrt(x)*x**3*(6705765*a**3 + 12801915*a**2*b*x**2 + 9388071*a**2*c*x**4
 + 9388071*a*b**2*x**4 + 14823270*a*b*c*x**6 + 6122655*a*c**2*x**8 + 24705
45*b**3*x**6 + 6122655*b**2*c*x**8 + 5215595*b*c**2*x**10 + 1514205*c**3*x
**12))/46940355
```

3.915 $\int x^{3/2}(a+bx^2+cx^4)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 103

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}a\left(b^2+ac\right)x^{13/2}+\frac{2}{17}b\left(b^2+6ac\right)x^{17/2}+\frac{2}{7}c\left(b^2+ac\right)x^{21/2}+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

output
```
2/5*a^3*x^(5/2)+2/3*a^2*b*x^(9/2)+6/13*a*(a*c+b^2)*x^(13/2)+2/17*b*(6*a*c+
b^2)*x^(17/2)+2/7*c*(a*c+b^2)*x^(21/2)+6/25*b*c^2*x^(25/2)+2/29*c^3*x^(29/
2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2\left(672945a^3x^{5/2}+1121575a^2bx^{9/2}+776475ab^2x^{13/2}+776475a^2cx^{13/2}+197925b^3x^{17/2}+118\right.}{3364725}$$

input
```
Integrate[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(2*(672945*a^3*x^(5/2) + 1121575*a^2*b*x^(9/2) + 776475*a*b^2*x^(13/2) + 7
76475*a^2*c*x^(13/2) + 197925*b^3*x^(17/2) + 1187550*a*b*c*x^(17/2) + 4806
75*b^2*c*x^(21/2) + 480675*a*c^2*x^(21/2) + 403767*b*c^2*x^(25/2) + 116025
*c^3*x^(29/2)))/3364725
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} \left(a+bx^2+cx^4\right)^3 \, dx$$
$$\Big\downarrow \text{1433}$$
$$\int \left(a^3x^{3/2}+3a^2bx^{7/2}+3cx^{19/2}\left(ac+b^2\right)+bx^{15/2}\left(6ac+b^2\right)+3ax^{11/2}\left(ac+b^2\right)+3bc^2x^{23/2}+c^3x^{27/2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{2}{7}cx^{21/2}\left(ac+b^2\right)+\frac{2}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{6}{13}ax^{13/2}\left(ac+b^2\right)+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

input
```
Int[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*(b^2 + a*c)*x^(13/2))/13 +
(2*b*(b^2 + 6*a*c)*x^(17/2))/17 + (2*c*(b^2 + a*c)*x^(21/2))/7 + (6*b*c^2*
x^(25/2))/25 + (2*c^3*x^(29/2))/29
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result
gosper	$\frac{2x^{\frac{5}{2}}(116025c^3x^{12}+403767b\,c^2x^{10}+480675a\,c^2x^8+480675b^2c\,x^8+1187550abc\,x^6+197925b^3x^6+776475a^2c\,x^4+77647}{3364725}$
trager	$\frac{2x^{\frac{5}{2}}(116025c^3x^{12}+403767b\,c^2x^{10}+480675a\,c^2x^8+480675b^2c\,x^8+1187550abc\,x^6+197925b^3x^6+776475a^2c\,x^4+77647}{3364725}$
risch	$\frac{2x^{\frac{5}{2}}(116025c^3x^{12}+403767b\,c^2x^{10}+480675a\,c^2x^8+480675b^2c\,x^8+1187550abc\,x^6+197925b^3x^6+776475a^2c\,x^4+77647}{3364725}$
orering	$\frac{2x^{\frac{5}{2}}(116025c^3x^{12}+403767b\,c^2x^{10}+480675a\,c^2x^8+480675b^2c\,x^8+1187550abc\,x^6+197925b^3x^6+776475a^2c\,x^4+77647}{3364725}$
derivativedivides	$\frac{2c^3x^{\frac{29}{2}}}{29}+\frac{6b\,c^2x^{\frac{25}{2}}}{25}+\frac{2(c^2a+2b^2c+c(2ac+b^2))x^{\frac{21}{2}}}{21}+\frac{2(4abc+b(2ac+b^2))x^{\frac{17}{2}}}{17}+\frac{2(a(2ac+b^2)+2b^2a+c\,a^2)x}{13}$
default	$\frac{2c^3x^{\frac{29}{2}}}{29}+\frac{6b\,c^2x^{\frac{25}{2}}}{25}+\frac{2(c^2a+2b^2c+c(2ac+b^2))x^{\frac{21}{2}}}{21}+\frac{2(4abc+b(2ac+b^2))x^{\frac{17}{2}}}{17}+\frac{2(a(2ac+b^2)+2b^2a+c\,a^2)x}{13}$

input
```
int(x^(3/2)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3364725*x^(5/2)*(116025*c^3*x^12+403767*b*c^2*x^10+480675*a*c^2*x^8+4806
75*b^2*c*x^8+1187550*a*b*c*x^6+197925*b^3*x^6+776475*a^2*c*x^4+776475*a*b^
2*x^4+1121575*a^2*b*x^2+672945*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int x^{3/2}(a+bx^2+cx^4)^3\,dx=\frac{2}{3364725}\left(116025\,c^3x^{14}+403767\,bc^2x^{12}+480675\,(b^2c+ac^2)x^{10}+197925\,(b^3+6\,abc)x^8+1$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
2/3364725*(116025*c^3*x^14 + 403767*b*c^2*x^12 + 480675*(b^2*c + a*c^2)*x^
10 + 197925*(b^3 + 6*a*b*c)*x^8 + 1121575*a^2*b*x^4 + 776475*(a*b^2 + a^2*
c)*x^6 + 672945*a^3*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.25

$$\int x^{3/2}(a+bx^2+cx^4)^3\,dx=\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2bx^{\frac{9}{2}}}{3}+\frac{6a^2cx^{\frac{13}{2}}}{13}+\frac{6ab^2x^{\frac{13}{2}}}{13}+\frac{12abcx^{\frac{17}{2}}}{17}+\frac{2ac^2x^{\frac{21}{2}}}{7}+\frac{2b^3x^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{21}{2}}}{7}+\frac{6bc^2x^{\frac{25}{2}}}{25}+\frac{2c^3x^{\frac{29}{2}}}{29}$$

input
```
integrate(x**(3/2)*(c*x**4+b*x**2+a)**3,x)
```

output
```
2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a**2*c*x**(13/2)/13 + 6*a*b**2
*x**(13/2)/13 + 12*a*b*c*x**(17/2)/17 + 2*a*c**2*x**(21/2)/7 + 2*b**3*x**(
17/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(25/2)/25 + 2*c**3*x**(29/2)
/29
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,\left(b^2c+ac^2\right)x^{\frac{21}{2}}+\frac{2}{17}\,\left(b^3+6\,abc\right)x^{\frac{17}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{6}{13}\,\left(ab^2+a^2c\right)x^{\frac{13}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*(b^2*c + a*c^2)*x^(21/2) + 2
/17*(b^3 + 6*a*b*c)*x^(17/2) + 2/3*a^2*b*x^(9/2) + 6/13*(a*b^2 + a^2*c)*x^
(13/2) + 2/5*a^3*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,b^2cx^{\frac{21}{2}}+\frac{2}{7}\,ac^2x^{\frac{21}{2}}+\frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{12}{17}\,abcx^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{6}{13}\,a^2cx^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/7*a*c^2*x
^(21/2) + 2/17*b^3*x^(17/2) + 12/17*a*b*c*x^(17/2) + 6/13*a*b^2*x^(13/2) +
 6/13*a^2*c*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx=x^{17/2}\left(\frac{2\,b^3}{17}+\frac{12\,a\,c\,b}{17}\right)+\frac{2\,a^3\,x^{5/2}}{5}+\frac{2\,c^3\,x^{29/2}}{29}+\frac{2\,a^2\,b\,x^{9/2}}{3}+\frac{6\,b\,c^2\,x^{25/2}}{25}+\frac{6\,a\,x^{13/2}\,\left(b^2+a\,c\right)}{13}+\frac{2\,c\,x^{21/2}\,\left(b^2+a\,c\right)}{7}$$

input
```
int(x^(3/2)*(a + b*x^2 + c*x^4)^3,x)
```

output
```
x^(17/2)*((2*b^3)/17 + (12*a*b*c)/17) + (2*a^3*x^(5/2))/5 + (2*c^3*x^(29/2
))/29 + (2*a^2*b*x^(9/2))/3 + (6*b*c^2*x^(25/2))/25 + (6*a*x^(13/2)*(a*c +
 b^2))/13 + (2*c*x^(21/2)*(a*c + b^2))/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2\sqrt{x}\,x^2(116025c^3x^{12}+403767b\,c^2x^{10}+480675a\,c^2x^8+480675b^2c\,x^8+1187550abc\,x^6+1979}{3364725}$$

input
```
int(x^(3/2)*(c*x^4+b*x^2+a)^3,x)
```

output
```
(2*sqrt(x)*x**2*(672945*a**3 + 1121575*a**2*b*x**2 + 776475*a**2*c*x**4 +
776475*a*b**2*x**4 + 1187550*a*b*c*x**6 + 480675*a*c**2*x**8 + 197925*b**3
*x**6 + 480675*b**2*c*x**8 + 403767*b*c**2*x**10 + 116025*c**3*x**12))/336
4725
```

3.916 $\int \sqrt{x}(a+bx^2+cx^4)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 103

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx = \frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}a\left(b^2+ac\right)x^{11/2} + \frac{2}{15}b\left(b^2+6ac\right)x^{15/2}+\frac{6}{19}c\left(b^2+ac\right)x^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

output
```
2/3*a^3*x^(3/2)+6/7*a^2*b*x^(7/2)+6/11*a*(a*c+b^2)*x^(11/2)+2/15*b*(6*a*c+
b^2)*x^(15/2)+6/19*c*(a*c+b^2)*x^(19/2)+6/23*b*c^2*x^(23/2)+2/27*c^3*x^(27
/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx = \frac{2}{3}a^3x^{3/2}+\frac{6}{77}a^2x^{7/2}(11b+7cx^2) + \frac{2ax^{11/2}(285b^2+418bcx^2+165c^2x^4)}{1045} + \frac{2x^{15/2}(3933b^3+9315b^2cx^2+7695bc^2x^4+2185c^3x^6)}{58995}$$

input
```
Integrate[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(2*a^3*x^(3/2))/3 + (6*a^2*x^(7/2)*(11*b + 7*c*x^2))/77 + (2*a*x^(11/2)*(2
85*b^2 + 418*b*c*x^2 + 165*c^2*x^4))/1045 + (2*x^(15/2)*(3933*b^3 + 9315*b
^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x^6))/58995
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^3 \, dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(a^3\sqrt{x}+3a^2bx^{5/2}+3cx^{17/2}\left(ac+b^2\right)+bx^{13/2}\left(6ac+b^2\right)+3ax^{9/2}\left(ac+b^2\right)+3bc^2x^{21/2}+c^3x^{25/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

input
```
Int[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*(b^2 + a*c)*x^(11/2))/11 +
(2*b*(b^2 + 6*a*c)*x^(15/2))/15 + (6*c*(b^2 + a*c)*x^(19/2))/19 + (6*b*c^2
*x^(23/2))/23 + (2*c^3*x^(27/2))/27
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result
gosper	$\frac{2x^{\frac{3}{2}}\left(168245c^{3}x^{12}+592515b\,c^{2}x^{10}+717255a\,c^{2}x^{8}+717255b^{2}c\,x^{8}+1817046abc\,x^{6}+302841b^{3}x^{6}+1238895a^{2}c\,x^{4}+1238\right.}{4542615}$
trager	$\frac{2x^{\frac{3}{2}}\left(168245c^{3}x^{12}+592515b\,c^{2}x^{10}+717255a\,c^{2}x^{8}+717255b^{2}c\,x^{8}+1817046abc\,x^{6}+302841b^{3}x^{6}+1238895a^{2}c\,x^{4}+1238\right.}{4542615}$
risch	$\frac{2x^{\frac{3}{2}}\left(168245c^{3}x^{12}+592515b\,c^{2}x^{10}+717255a\,c^{2}x^{8}+717255b^{2}c\,x^{8}+1817046abc\,x^{6}+302841b^{3}x^{6}+1238895a^{2}c\,x^{4}+1238\right.}{4542615}$
orering	$\frac{2x^{\frac{3}{2}}\left(168245c^{3}x^{12}+592515b\,c^{2}x^{10}+717255a\,c^{2}x^{8}+717255b^{2}c\,x^{8}+1817046abc\,x^{6}+302841b^{3}x^{6}+1238895a^{2}c\,x^{4}+1238\right.}{4542615}$
derivativedivides	$\frac{2c^{3}x^{\frac{27}{2}}}{27}+\frac{6b\,c^{2}x^{\frac{23}{2}}}{23}+\frac{2\left(c^{2}a+2b^{2}c+c\left(2ac+b^{2}\right)\right)x^{\frac{19}{2}}}{19}+\frac{2\left(4abc+b\left(2ac+b^{2}\right)\right)x^{\frac{15}{2}}}{15}+\frac{2\left(a\left(2ac+b^{2}\right)+2b^{2}a+c\,a^{2}\right)x}{11}$
default	$\frac{2c^{3}x^{\frac{27}{2}}}{27}+\frac{6b\,c^{2}x^{\frac{23}{2}}}{23}+\frac{2\left(c^{2}a+2b^{2}c+c\left(2ac+b^{2}\right)\right)x^{\frac{19}{2}}}{19}+\frac{2\left(4abc+b\left(2ac+b^{2}\right)\right)x^{\frac{15}{2}}}{15}+\frac{2\left(a\left(2ac+b^{2}\right)+2b^{2}a+c\,a^{2}\right)x}{11}$

input
```
int(x^(1/2)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/4542615*x^(3/2)*(168245*c^3*x^12+592515*b*c^2*x^10+717255*a*c^2*x^8+7172
55*b^2*c*x^8+1817046*a*b*c*x^6+302841*b^3*x^6+1238895*a^2*c*x^4+1238895*a*
b^2*x^4+1946835*a^2*b*x^2+1514205*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx$$
$$=\frac{2}{4542615}\left(168245\,c^3x^{13}+592515\,bc^2x^{11}+717255\,\left(b^2c+ac^2\right)x^9+302841\,\left(b^3+6\,abc\right)x^7+1946835\,a^2bx\right.$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
2/4542615*(168245*c^3*x^13 + 592515*b*c^2*x^11 + 717255*(b^2*c + a*c^2)*x^
9 + 302841*(b^3 + 6*a*b*c)*x^7 + 1946835*a^2*b*x^3 + 1238895*(a*b^2 + a^2*
c)*x^5 + 1514205*a^3*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx=\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2bx^{\frac{7}{2}}}{7}+\frac{6bc^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{27}{2}}}{27}+\frac{2x^{\frac{19}{2}}\cdot\left(3ac^2+3b^2c\right)}{19}+\frac{2x^{\frac{15}{2}}\cdot\left(6abc+b^3\right)}{15}+\frac{2x^{\frac{11}{2}}\cdot\left(3a^2c+3ab^2\right)}{11}$$

input
```
integrate(x**(1/2)*(c*x**4+b*x**2+a)**3,x)
```

output
```
2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*b*c**2*x**(23/2)/23 + 2*c**3*x
**(27/2)/27 + 2*x**(19/2)*(3*a*c**2 + 3*b**2*c)/19 + 2*x**(15/2)*(6*a*b*c
 + b**3)/15 + 2*x**(11/2)*(3*a**2*c + 3*a*b**2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(a+bx^2+cx^4)^3 \, dx = \frac{2}{27} c^3 x^{\frac{27}{2}} + \frac{6}{23} bc^2 x^{\frac{23}{2}} + \frac{6}{19} (b^2c + ac^2) x^{\frac{19}{2}} + \frac{2}{15} (b^3 + 6\,abc) x^{\frac{15}{2}} + \frac{6}{7} a^2 b x^{\frac{7}{2}} + \frac{6}{11} (ab^2 + a^2c) x^{\frac{11}{2}} + \frac{2}{3} a^3 x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*(b^2*c + a*c^2)*x^(19/2) +
2/15*(b^3 + 6*a*b*c)*x^(15/2) + 6/7*a^2*b*x^(7/2) + 6/11*(a*b^2 + a^2*c)*x
^(11/2) + 2/3*a^3*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \sqrt{x}(a + bx^2 + cx^4)^3 \, dx = \frac{2}{27} c^3 x^{\frac{27}{2}} + \frac{6}{23} bc^2 x^{\frac{23}{2}} + \frac{6}{19} b^2 c x^{\frac{19}{2}} + \frac{6}{19} ac^2 x^{\frac{19}{2}} + \frac{2}{15} b^3 x^{\frac{15}{2}} + \frac{4}{5} abc x^{\frac{15}{2}} + \frac{6}{11} ab^2 x^{\frac{11}{2}} + \frac{6}{11} a^2 c x^{\frac{11}{2}} + \frac{6}{7} a^2 b x^{\frac{7}{2}} + \frac{2}{3} a^3 x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 6/19*a*c^2
*x^(19/2) + 2/15*b^3*x^(15/2) + 4/5*a*b*c*x^(15/2) + 6/11*a*b^2*x^(11/2) +
 6/11*a^2*c*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx=x^{15/2}\left(\frac{2\,b^3}{15}+\frac{4\,a\,c\,b}{5}\right)+\frac{2\,a^3\,x^{3/2}}{3}+\frac{2\,c^3\,x^{27/2}}{27}+\frac{6\,a^2\,b\,x^{7/2}}{7}+\frac{6\,b\,c^2\,x^{23/2}}{23}+\frac{6\,a\,x^{11/2}\,\left(b^2+a\,c\right)}{11}+\frac{6\,c\,x^{19/2}\,\left(b^2+a\,c\right)}{19}$$

input
```
int(x^(1/2)*(a + b*x^2 + c*x^4)^3,x)
```

output
```
x^(15/2)*((2*b^3)/15 + (4*a*b*c)/5) + (2*a^3*x^(3/2))/3 + (2*c^3*x^(27/2))
/27 + (6*a^2*b*x^(7/2))/7 + (6*b*c^2*x^(23/2))/23 + (6*a*x^(11/2)*(a*c + b
^2))/11 + (6*c*x^(19/2)*(a*c + b^2))/19
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx$$
$$=\frac{2\sqrt{x}\,x(168245c^3x^{12}+592515b\,c^2x^{10}+717255a\,c^2x^8+717255b^2c\,x^8+1817046abc\,x^6+302841b^3x^6+12}{4542615}$$

input
```
int(x^(1/2)*(c*x^4+b*x^2+a)^3,x)
```

output
```
(2*sqrt(x)*x*(1514205*a**3 + 1946835*a**2*b*x**2 + 1238895*a**2*c*x**4 + 1
238895*a*b**2*x**4 + 1817046*a*b*c*x**6 + 717255*a*c**2*x**8 + 302841*b**3
*x**6 + 717255*b**2*c*x**8 + 592515*b*c**2*x**10 + 168245*c**3*x**12))/454
2615
```

3.917 $\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}} \, dx = 2a^3\sqrt{x} + \frac{6}{5}a^2bx^{5/2} + \frac{2}{3}a(b^2+ac)\,x^{9/2} + \frac{2}{13}b(b^2+6ac)\,x^{13/2} + \frac{6}{17}c(b^2+ac)\,x^{17/2} + \frac{2}{7}bc^2x^{21/2} + \frac{2}{25}c^3x^{25/2}$$

output
```
2*a^3*x^(1/2)+6/5*a^2*b*x^(5/2)+2/3*a*(a*c+b^2)*x^(9/2)+2/13*b*(6*a*c+b^2)
*x^(13/2)+6/17*c*(a*c+b^2)*x^(17/2)+2/7*b*c^2*x^(21/2)+2/25*c^3*x^(25/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}} \, dx = \frac{2\sqrt{x}(116025a^3 + 7735a^2(9bx^2+5cx^4) + 3x^6(2975b^3 + 6825b^2cx^2 + 5525bc^2x^4 + 1547c^3x^6) + 175a(221}{116025}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(116025*a^3 + 7735*a^2*(9*b*x^2 + 5*c*x^4) + 3*x^6*(2975*b^3 +
6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x^6) + 175*a*(221*b^2*x^4 + 306
*b*c*x^6 + 117*c^2*x^8)))/116025
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^3}{\sqrt{x}}+3a^2bx^{3/2}+3cx^{15/2}(ac+b^2)+bx^{11/2}(6ac+b^2)+3ax^{7/2}(ac+b^2)+3bc^2x^{19/2}+c^3x^{23/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{17}cx^{17/2}(ac+b^2)+\frac{2}{13}bx^{13/2}(6ac+b^2)+\frac{2}{3}ax^{9/2}(ac+b^2)+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

input
```
Int[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]
```

output
```
2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*(b^2 + a*c)*x^(9/2))/3 + (2*b*(
b^2 + 6*a*c)*x^(13/2))/13 + (6*c*(b^2 + a*c)*x^(17/2))/17 + (2*b*c^2*x^(21
/2))/7 + (2*c^3*x^(25/2))/25
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

method	result
trager	$\left(\frac{2}{25}c^3x^{12}+\frac{2}{7}b\,c^2x^{10}+\frac{6}{17}a\,c^2x^8+\frac{6}{17}b^2c\,x^8+\frac{12}{13}abc\,x^6+\frac{2}{13}b^3x^6+\frac{2}{3}a^2c\,x^4+\frac{2}{3}b^2x^4a+\frac{6}{5}\right.$
gosper	$\frac{2\sqrt{x}\,(4641c^3x^{12}+16575b\,c^2x^{10}+20475a\,c^2x^8+20475b^2c\,x^8+53550abc\,x^6+8925b^3x^6+38675a^2c\,x^4+38675b^2x^4a+696}{116025}$
risch	$\frac{2\sqrt{x}\,(4641c^3x^{12}+16575b\,c^2x^{10}+20475a\,c^2x^8+20475b^2c\,x^8+53550abc\,x^6+8925b^3x^6+38675a^2c\,x^4+38675b^2x^4a+696}{116025}$
orering	$\frac{2\sqrt{x}\,(4641c^3x^{12}+16575b\,c^2x^{10}+20475a\,c^2x^8+20475b^2c\,x^8+53550abc\,x^6+8925b^3x^6+38675a^2c\,x^4+38675b^2x^4a+696}{116025}$
derivativedivides	$\frac{2c^3x^{\frac{25}{2}}}{25}+\frac{2b\,c^2x^{\frac{21}{2}}}{7}+\frac{2\left(c^2a+2b^2c+c\left(2ac+b^2\right)\right)x^{\frac{17}{2}}}{17}+\frac{2\left(4abc+b\left(2ac+b^2\right)\right)x^{\frac{13}{2}}}{13}+\frac{2\left(a\left(2ac+b^2\right)+2b^2a+c\,a^2\right)x}{9}$
default	$\frac{2c^3x^{\frac{25}{2}}}{25}+\frac{2b\,c^2x^{\frac{21}{2}}}{7}+\frac{2\left(c^2a+2b^2c+c\left(2ac+b^2\right)\right)x^{\frac{17}{2}}}{17}+\frac{2\left(4abc+b\left(2ac+b^2\right)\right)x^{\frac{13}{2}}}{13}+\frac{2\left(a\left(2ac+b^2\right)+2b^2a+c\,a^2\right)x}{9}$

input
```
int((c*x^4+b*x^2+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/25*c^3*x^12+2/7*b*c^2*x^10+6/17*a*c^2*x^8+6/17*b^2*c*x^8+12/13*a*b*c*x^
6+2/13*b^3*x^6+2/3*a^2*c*x^4+2/3*b^2*x^4*a+6/5*a^2*b*x^2+2*a^3)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}}\,dx$$
$$= \frac{2}{116025}\left(4641\,c^3x^{12} + 16575\,bc^2x^{10} + 20475\,(b^2c+ac^2)x^8 + 8925\,(b^3+6\,abc)x^6 + 69615\,a^2bx^2 + 38675\right.$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/116025*(4641*c^3*x^12 + 16575*b*c^2*x^10 + 20475*(b^2*c + a*c^2)*x^8 + 8
925*(b^3 + 6*a*b*c)*x^6 + 69615*a^2*b*x^2 + 38675*(a*b^2 + a^2*c)*x^4 + 11
6025*a^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}}\,dx = 2a^3\sqrt{x} + \frac{6a^2bx^{\frac{5}{2}}}{5} + \frac{2a^2cx^{\frac{9}{2}}}{3} + \frac{2ab^2x^{\frac{9}{2}}}{3} + \frac{12abcx^{\frac{13}{2}}}{13} + \frac{6ac^2x^{\frac{17}{2}}}{17} + \frac{2b^3x^{\frac{13}{2}}}{13} + \frac{6b^2cx^{\frac{17}{2}}}{17} + \frac{2bc^2x^{\frac{21}{2}}}{7} + \frac{2c^3x^{\frac{25}{2}}}{25}$$

input
```
integrate((c*x**4+b*x**2+a)**3/x**(1/2),x)
```

output
```
2*a**3*sqrt(x) + 6*a**2*b*x**(5/2)/5 + 2*a**2*c*x**(9/2)/3 + 2*a*b**2*x**(
9/2)/3 + 12*a*b*c*x**(13/2)/13 + 6*a*c**2*x**(17/2)/17 + 2*b**3*x**(13/2)/
13 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)/7 + 2*c**3*x**(25/2)/25
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{25}\,c^3x^{\frac{25}{2}} + \frac{2}{7}\,bc^2x^{\frac{21}{2}} + \frac{6}{17}\,b^2cx^{\frac{17}{2}} + \frac{2}{13}\,b^3x^{\frac{13}{2}} + 2\,a^3\sqrt{x} + \frac{2}{15}\left(5\,cx^{\frac{9}{2}} + 9\,bx^{\frac{5}{2}}\right)a^2 + \frac{2}{663}\left(117\,c^2x^{\frac{17}{2}} + 306\,bcx^{\frac{13}{2}} + 221\,b^2x^{\frac{9}{2}}\right)a$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

output
```
2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/13*b^3*x^
(13/2) + 2*a^3*sqrt(x) + 2/15*(5*c*x^(9/2) + 9*b*x^(5/2))*a^2 + 2/663*(117
*c^2*x^(17/2) + 306*b*c*x^(13/2) + 221*b^2*x^(9/2))*a
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{25}\,c^3x^{\frac{25}{2}} + \frac{2}{7}\,bc^2x^{\frac{21}{2}} + \frac{6}{17}\,b^2cx^{\frac{17}{2}} + \frac{6}{17}\,ac^2x^{\frac{17}{2}} + \frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{12}{13}\,abcx^{\frac{13}{2}} + \frac{2}{3}\,ab^2x^{\frac{9}{2}} + \frac{2}{3}\,a^2cx^{\frac{9}{2}} + \frac{6}{5}\,a^2bx^{\frac{5}{2}} + 2\,a^3\sqrt{x}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

output
```
2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 6/17*a*c^2*
x^(17/2) + 2/13*b^3*x^(13/2) + 12/13*a*b*c*x^(13/2) + 2/3*a*b^2*x^(9/2) +
2/3*a^2*c*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}}\,dx = x^{13/2}\left(\frac{2\,b^3}{13}+\frac{12\,a\,c\,b}{13}\right)+2\,a^3\,\sqrt{x}+\frac{2\,c^3\,x^{25/2}}{25}+\frac{6\,a^2\,b\,x^{5/2}}{5}+\frac{2\,b\,c^2\,x^{21/2}}{7}+\frac{2\,a\,x^{9/2}\,(b^2+a\,c)}{3}+\frac{6\,c\,x^{17/2}\,(b^2+a\,c)}{17}$$

input
```
int((a + b*x^2 + c*x^4)^3/x^(1/2),x)
```

output
```
x^(13/2)*((2*b^3)/13 + (12*a*b*c)/13) + 2*a^3*x^(1/2) + (2*c^3*x^(25/2))/2
5 + (6*a^2*b*x^(5/2))/5 + (2*b*c^2*x^(21/2))/7 + (2*a*x^(9/2)*(a*c + b^2))
/3 + (6*c*x^(17/2)*(a*c + b^2))/17
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(4641c^3x^{12}+16575b\,c^2x^{10}+20475a\,c^2x^8+20475b^2c\,x^8+53550abc\,x^6+8925b^3x^6+38675a^2c\,x^4+}{116025}$$

input
```
int((c*x^4+b*x^2+a)^3/x^(1/2),x)
```

output
```
(2*sqrt(x)*(116025*a**3 + 69615*a**2*b*x**2 + 38675*a**2*c*x**4 + 38675*a*
b**2*x**4 + 53550*a*b*c*x**6 + 20475*a*c**2*x**8 + 8925*b**3*x**6 + 20475*
b**2*c*x**8 + 16575*b*c**2*x**10 + 4641*c**3*x**12))/116025
```

3.918 $\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 99

$$\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx = -\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2} + \frac{6}{7}a(b^2+ac)\,x^{7/2}+\frac{2}{11}b(b^2+6ac)\,x^{11/2}+\frac{2}{5}c(b^2+ac)\,x^{15/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

output
```
-2*a^3/x^(1/2)+2*a^2*b*x^(3/2)+6/7*a*(a*c+b^2)*x^(7/2)+2/11*b*(6*a*c+b^2)*
x^(11/2)+2/5*c*(a*c+b^2)*x^(15/2)+6/19*b*c^2*x^(19/2)+2/23*c^3*x^(23/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx = -\frac{2(168245a^3-168245a^2bx^2-72105ab^2x^4-72105a^2cx^4-15295b^3x^6-91770abcx^6-33649b^2cx^8-336}{168245\sqrt{x}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

output
```
(-2*(168245*a^3 - 168245*a^2*b*x^2 - 72105*a*b^2*x^4 - 72105*a^2*c*x^4 - 1
5295*b^3*x^6 - 91770*a*b*c*x^6 - 33649*b^2*c*x^8 - 33649*a*c^2*x^8 - 26565
*b*c^2*x^10 - 7315*c^3*x^12))/(168245*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx$$

$$\Big\downarrow \ 1433$$

$$\int \left(\frac{a^3}{x^{3/2}}+3a^2b\sqrt{x}+3cx^{13/2}(ac+b^2)+bx^{9/2}(6ac+b^2)+3ax^{5/2}(ac+b^2)+3bc^2x^{17/2}+c^3x^{21/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{2}{5}cx^{15/2}(ac+b^2)+\frac{2}{11}bx^{11/2}(6ac+b^2)+\frac{6}{7}ax^{7/2}(ac+b^2)+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

input
```
Int[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

output
```
(-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*(b^2 + a*c)*x^(7/2))/7 + (2*b*(b
^2 + 6*a*c)*x^(11/2))/11 + (2*c*(b^2 + a*c)*x^(15/2))/5 + (6*b*c^2*x^(19/2
))/19 + (2*c^3*x^(23/2))/23
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2c^3 x^{\frac{23}{2}}}{23}+\frac{6b\,c^2 x^{\frac{19}{2}}}{19}+\frac{2a\,c^2 x^{\frac{15}{2}}}{5}+\frac{2b^2 c\,x^{\frac{15}{2}}}{5}+\frac{12abc\,x^{\frac{11}{2}}}{11}+\frac{2b^3 x^{\frac{11}{2}}}{11}+\frac{6a^2 c\,x^{\frac{7}{2}}}{7}+\frac{6a\,b^2 x^{\frac{7}{2}}}{7}+2a^2 b\,x^{\frac{3}{2}}$
default	$\frac{2c^3 x^{\frac{23}{2}}}{23}+\frac{6b\,c^2 x^{\frac{19}{2}}}{19}+\frac{2a\,c^2 x^{\frac{15}{2}}}{5}+\frac{2b^2 c\,x^{\frac{15}{2}}}{5}+\frac{12abc\,x^{\frac{11}{2}}}{11}+\frac{2b^3 x^{\frac{11}{2}}}{11}+\frac{6a^2 c\,x^{\frac{7}{2}}}{7}+\frac{6a\,b^2 x^{\frac{7}{2}}}{7}+2a^2 b\,x^{\frac{3}{2}}$
gosper	$-\frac{2(-7315c^3x^{12}-26565b\,c^2x^{10}-33649a\,c^2x^8-33649b^2c\,x^8-91770abc\,x^6-15295b^3x^6-72105a^2c\,x^4-72105b^2x^4a-16}{168245\sqrt{x}}$
trager	$-\frac{2(-7315c^3x^{12}-26565b\,c^2x^{10}-33649a\,c^2x^8-33649b^2c\,x^8-91770abc\,x^6-15295b^3x^6-72105a^2c\,x^4-72105b^2x^4a-16}{168245\sqrt{x}}$
risch	$-\frac{2(-7315c^3x^{12}-26565b\,c^2x^{10}-33649a\,c^2x^8-33649b^2c\,x^8-91770abc\,x^6-15295b^3x^6-72105a^2c\,x^4-72105b^2x^4a-16}{168245\sqrt{x}}$
orering	$-\frac{2(-7315c^3x^{12}-26565b\,c^2x^{10}-33649a\,c^2x^8-33649b^2c\,x^8-91770abc\,x^6-15295b^3x^6-72105a^2c\,x^4-72105b^2x^4a-16}{168245\sqrt{x}}$

input
```
int((c*x^4+b*x^2+a)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/23*c^3*x^(23/2)+6/19*b*c^2*x^(19/2)+2/5*a*c^2*x^(15/2)+2/5*b^2*c*x^(15/2
)+12/11*a*b*c*x^(11/2)+2/11*b^3*x^(11/2)+6/7*a^2*c*x^(7/2)+6/7*a*b^2*x^(7/
2)+2*a^2*b*x^(3/2)-2*a^3/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2\,(7315\,c^3x^{12}+26565\,bc^2x^{10}+33649\,(b^2c+ac^2)x^8+15295\,(b^3+6\,abc)x^6+1682}{168245\,\sqrt{x}}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="fricas")
```

output
```
2/168245*(7315*c^3*x^12 + 26565*b*c^2*x^10 + 33649*(b^2*c + a*c^2)*x^8 + 1
5295*(b^3 + 6*a*b*c)*x^6 + 168245*a^2*b*x^2 + 72105*(a*b^2 + a^2*c)*x^4 -
168245*a^3)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx = -\frac{2a^3}{\sqrt{x}} + 2a^2bx^{\frac{3}{2}} + \frac{6a^2cx^{\frac{7}{2}}}{7} + \frac{6ab^2x^{\frac{7}{2}}}{7} + \frac{12abcx^{\frac{11}{2}}}{11} + \frac{2ac^2x^{\frac{15}{2}}}{5} + \frac{2b^3x^{\frac{11}{2}}}{11} + \frac{2b^2cx^{\frac{15}{2}}}{5} + \frac{6bc^2x^{\frac{19}{2}}}{19} + \frac{2c^3x^{\frac{23}{2}}}{23}$$

input
```
integrate((c*x**4+b*x**2+a)**3/x**(3/2),x)
```

output
```
-2*a**3/sqrt(x) + 2*a**2*b*x**(3/2) + 6*a**2*c*x**(7/2)/7 + 6*a*b**2*x**(7
/2)/7 + 12*a*b*c*x**(11/2)/11 + 2*a*c**2*x**(15/2)/5 + 2*b**3*x**(11/2)/11
 + 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/19 + 2*c**3*x**(23/2)/23
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{23}\,c^3x^{\frac{23}{2}} + \frac{6}{19}\,bc^2x^{\frac{19}{2}} + \frac{2}{5}\,(b^2c+ac^2)x^{\frac{15}{2}} + \frac{2}{11}\,(b^3+6\,abc)x^{\frac{11}{2}} + 2\,a^2bx^{\frac{3}{2}} + \frac{6}{7}\,(ab^2+a^2c)x^{\frac{7}{2}} - \frac{2\,a^3}{\sqrt{x}}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="maxima")
```

output
```
2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*(b^2*c + a*c^2)*x^(15/2) + 2
/11*(b^3 + 6*a*b*c)*x^(11/2) + 2*a^2*b*x^(3/2) + 6/7*(a*b^2 + a^2*c)*x^(7/
2) - 2*a^3/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{23}\,c^3x^{\frac{23}{2}} + \frac{6}{19}\,bc^2x^{\frac{19}{2}} + \frac{2}{5}\,b^2cx^{\frac{15}{2}} + \frac{2}{5}\,ac^2x^{\frac{15}{2}} + \frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{12}{11}\,abcx^{\frac{11}{2}} + \frac{6}{7}\,ab^2x^{\frac{7}{2}} + \frac{6}{7}\,a^2cx^{\frac{7}{2}} + 2\,a^2bx^{\frac{3}{2}} - \frac{2\,a^3}{\sqrt{x}}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^(3/2),x, algorithm="giac")
```

output
```
2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/5*a*c^2*x
^(15/2) + 2/11*b^3*x^(11/2) + 12/11*a*b*c*x^(11/2) + 6/7*a*b^2*x^(7/2) + 6
/7*a^2*c*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx = x^{11/2}\left(\frac{2\,b^3}{11}+\frac{12\,a\,c\,b}{11}\right)-\frac{2\,a^3}{\sqrt{x}}+\frac{2\,c^3\,x^{23/2}}{23} + 2\,a^2\,b\,x^{3/2}+\frac{6\,b\,c^2\,x^{19/2}}{19}+\frac{6\,a\,x^{7/2}\,(b^2+a\,c)}{7}+\frac{2\,c\,x^{15/2}\,(b^2+a\,c)}{5}$$

input
```
int((a + b*x^2 + c*x^4)^3/x^(3/2),x)
```

output
```
x^(11/2)*((2*b^3)/11 + (12*a*b*c)/11) - (2*a^3)/x^(1/2) + (2*c^3*x^(23/2))
/23 + 2*a^2*b*x^(3/2) + (6*b*c^2*x^(19/2))/19 + (6*a*x^(7/2)*(a*c + b^2))/
7 + (2*c*x^(15/2)*(a*c + b^2))/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{\frac{2}{23}c^3x^{12}+\frac{6}{19}b\,c^2x^{10}+\frac{2}{5}a\,c^2x^8+\frac{2}{5}b^2c\,x^8+\frac{12}{11}abc\,x^6+\frac{2}{11}b^3x^6+\frac{6}{7}a^2c\,x^4+\frac{6}{7}a\,b^2x^4}{\sqrt{x}}$$

input
```
int((c*x^4+b*x^2+a)^3/x^(3/2),x)
```

output
```
(2*( - 168245*a**3 + 168245*a**2*b*x**2 + 72105*a**2*c*x**4 + 72105*a*b**2
*x**4 + 91770*a*b*c*x**6 + 33649*a*c**2*x**8 + 15295*b**3*x**6 + 33649*b**
2*c*x**8 + 26565*b*c**2*x**10 + 7315*c**3*x**12))/(168245*sqrt(x))
```

3.919 $\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx = -\frac{2a^3}{3x^{3/2}} + 6a^2b\sqrt{x} + \frac{6}{5}a\left(b^2+ac\right)x^{5/2} + \frac{2}{9}b\left(b^2+6ac\right)x^{9/2} + \frac{6}{13}c\left(b^2+ac\right)x^{13/2} + \frac{6}{17}bc^2x^{17/2} + \frac{2}{21}c^3x^{21/2}$$

output
```
-2/3*a^3/x^(3/2)+6*a^2*b*x^(1/2)+6/5*a*(a*c+b^2)*x^(5/2)+2/9*b*(6*a*c+b^2)
*x^(9/2)+6/13*c*(a*c+b^2)*x^(13/2)+6/17*b*c^2*x^(17/2)+2/21*c^3*x^(21/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx = -\frac{2(23205a^3 - 208845a^2bx^2 - 41769ab^2x^4 - 41769a^2cx^4 - 7735b^3x^6 - 46410abcx^6 - 16065b^2cx^8 - 16065}{69615x^{3/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

output
```
(-2*(23205*a^3 - 208845*a^2*b*x^2 - 41769*a*b^2*x^4 - 41769*a^2*c*x^4 - 77
35*b^3*x^6 - 46410*a*b*c*x^6 - 16065*b^2*c*x^8 - 16065*a*c^2*x^8 - 12285*b
*c^2*x^10 - 3315*c^3*x^12))/(69615*x^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{\sqrt{x}}+3cx^{11/2}(ac+b^2)+bx^{7/2}(6ac+b^2)+3ax^{3/2}(ac+b^2)+3bc^2x^{15/2}+c^3x^{19/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{13}cx^{13/2}(ac+b^2)+\frac{2}{9}bx^{9/2}(6ac+b^2)+\frac{6}{5}ax^{5/2}(ac+b^2)+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

input
```
Int[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

output
```
(-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*(b^2 + a*c)*x^(5/2))/5 + (2*
b*(b^2 + 6*a*c)*x^(9/2))/9 + (6*c*(b^2 + a*c)*x^(13/2))/13 + (6*b*c^2*x^(1
7/2))/17 + (2*c^3*x^(21/2))/21
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{2c^3x^{\frac{21}{2}}}{21}+\frac{6b\,c^2x^{\frac{17}{2}}}{17}+\frac{6a\,c^2x^{\frac{13}{2}}}{13}+\frac{6b^2c\,x^{\frac{13}{2}}}{13}+\frac{4abc\,x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6a^2c\,x^{\frac{5}{2}}}{5}+\frac{6a\,b^2x^{\frac{5}{2}}}{5}+6a^2b\sqrt{x}-$
default	$\frac{2c^3x^{\frac{21}{2}}}{21}+\frac{6b\,c^2x^{\frac{17}{2}}}{17}+\frac{6a\,c^2x^{\frac{13}{2}}}{13}+\frac{6b^2c\,x^{\frac{13}{2}}}{13}+\frac{4abc\,x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6a^2c\,x^{\frac{5}{2}}}{5}+\frac{6a\,b^2x^{\frac{5}{2}}}{5}+6a^2b\sqrt{x}-$
gosper	$-\frac{2(-3315c^3x^{12}-12285b\,c^2x^{10}-16065a\,c^2x^8-16065b^2c\,x^8-46410abc\,x^6-7735b^3x^6-41769a^2c\,x^4-41769b^2x^4a-208}{69615x^{\frac{3}{2}}}$
trager	$-\frac{2(-3315c^3x^{12}-12285b\,c^2x^{10}-16065a\,c^2x^8-16065b^2c\,x^8-46410abc\,x^6-7735b^3x^6-41769a^2c\,x^4-41769b^2x^4a-208}{69615x^{\frac{3}{2}}}$
risch	$-\frac{2(-3315c^3x^{12}-12285b\,c^2x^{10}-16065a\,c^2x^8-16065b^2c\,x^8-46410abc\,x^6-7735b^3x^6-41769a^2c\,x^4-41769b^2x^4a-208}{69615x^{\frac{3}{2}}}$
orering	$-\frac{2(-3315c^3x^{12}-12285b\,c^2x^{10}-16065a\,c^2x^8-16065b^2c\,x^8-46410abc\,x^6-7735b^3x^6-41769a^2c\,x^4-41769b^2x^4a-208}{69615x^{\frac{3}{2}}}$

input
```
int((c*x^4+b*x^2+a)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*c^3*x^(21/2)+6/17*b*c^2*x^(17/2)+6/13*a*c^2*x^(13/2)+6/13*b^2*c*x^(13
/2)+4/3*a*b*c*x^(9/2)+2/9*b^3*x^(9/2)+6/5*a^2*c*x^(5/2)+6/5*a*b^2*x^(5/2)+
6*a^2*b*x^(1/2)-2/3*a^3/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{(a + bx^2 + cx^4)^3}{x^{5/2}}\,dx = \frac{2\,(3315\,c^3x^{12} + 12285\,bc^2x^{10} + 16065\,(b^2c + ac^2)x^8 + 7735\,(b^3 + 6\,abc)x^6 + 2088}{69615\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="fricas")
```

output
```
2/69615*(3315*c^3*x^12 + 12285*b*c^2*x^10 + 16065*(b^2*c + a*c^2)*x^8 + 77
35*(b^3 + 6*a*b*c)*x^6 + 208845*a^2*b*x^2 + 41769*(a*b^2 + a^2*c)*x^4 - 23
205*a^3)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.27

$$\int \frac{(a + bx^2 + cx^4)^3}{x^{5/2}}\,dx = -\frac{2a^3}{3x^{\frac{3}{2}}} + 6a^2b\sqrt{x} + \frac{6a^2cx^{\frac{5}{2}}}{5} + \frac{6ab^2x^{\frac{5}{2}}}{5} + \frac{4abcx^{\frac{9}{2}}}{3} + \frac{6ac^2x^{\frac{13}{2}}}{13} + \frac{2b^3x^{\frac{9}{2}}}{9} + \frac{6b^2cx^{\frac{13}{2}}}{13} + \frac{6bc^2x^{\frac{17}{2}}}{17} + \frac{2c^3x^{\frac{21}{2}}}{21}$$

input
```
integrate((c*x**4+b*x**2+a)**3/x**(5/2),x)
```

output
```
-2*a**3/(3*x**(3/2)) + 6*a**2*b*sqrt(x) + 6*a**2*c*x**(5/2)/5 + 6*a*b**2*x
**(5/2)/5 + 4*a*b*c*x**(9/2)/3 + 6*a*c**2*x**(13/2)/13 + 2*b**3*x**(9/2)/9
 + 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/17 + 2*c**3*x**(21/2)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{21}\,c^3x^{\frac{21}{2}} + \frac{6}{17}\,bc^2x^{\frac{17}{2}} + \frac{6}{13}\,(b^2c+ac^2)x^{\frac{13}{2}} + \frac{2}{9}\,(b^3+6\,abc)x^{\frac{9}{2}} + 6\,a^2b\sqrt{x} + \frac{6}{5}\,(ab^2+a^2c)x^{\frac{5}{2}} - \frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="maxima")
```

output
```
2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*(b^2*c + a*c^2)*x^(13/2) +
2/9*(b^3 + 6*a*b*c)*x^(9/2) + 6*a^2*b*sqrt(x) + 6/5*(a*b^2 + a^2*c)*x^(5/2
) - 2/3*a^3/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{21}\,c^3x^{\frac{21}{2}} + \frac{6}{17}\,bc^2x^{\frac{17}{2}} + \frac{6}{13}\,b^2cx^{\frac{13}{2}} + \frac{6}{13}\,ac^2x^{\frac{13}{2}} + \frac{2}{9}\,b^3x^{\frac{9}{2}} + \frac{4}{3}\,abcx^{\frac{9}{2}} + \frac{6}{5}\,ab^2x^{\frac{5}{2}} + \frac{6}{5}\,a^2cx^{\frac{5}{2}} + 6\,a^2b\sqrt{x} - \frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^(5/2),x, algorithm="giac")
```

output
```
2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 6/13*a*c^2
*x^(13/2) + 2/9*b^3*x^(9/2) + 4/3*a*b*c*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6/5*
a^2*c*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx = x^{9/2}\left(\frac{2\,b^3}{9}+\frac{4\,a\,c\,b}{3}\right)-\frac{2\,a^3}{3\,x^{3/2}}+\frac{2\,c^3\,x^{21/2}}{21} +6\,a^2\,b\,\sqrt{x}+\frac{6\,b\,c^2\,x^{17/2}}{17}+\frac{6\,a\,x^{5/2}\,(b^2+a\,c)}{5}+\frac{6\,c\,x^{13/2}\,(b^2+a\,c)}{13}$$

input
```
int((a + b*x^2 + c*x^4)^3/x^(5/2),x)
```

output
```
x^(9/2)*((2*b^3)/9 + (4*a*b*c)/3) - (2*a^3)/(3*x^(3/2)) + (2*c^3*x^(21/2))
/21 + 6*a^2*b*x^(1/2) + (6*b*c^2*x^(17/2))/17 + (6*a*x^(5/2)*(a*c + b^2))/
5 + (6*c*x^(13/2)*(a*c + b^2))/13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{\frac{2}{21}c^3x^{12}+\frac{6}{17}b\,c^2x^{10}+\frac{6}{13}a\,c^2x^8+\frac{6}{13}b^2c\,x^8+\frac{4}{3}abc\,x^6+\frac{2}{9}b^3x^6+\frac{6}{5}a^2c\,x^4+\frac{6}{5}a\,b^2x^4}{\sqrt{x}\,x}$$

input
```
int((c*x^4+b*x^2+a)^3/x^(5/2),x)
```

output
```
(2*( - 23205*a**3 + 208845*a**2*b*x**2 + 41769*a**2*c*x**4 + 41769*a*b**2*
x**4 + 46410*a*b*c*x**6 + 16065*a*c**2*x**8 + 7735*b**3*x**6 + 16065*b**2*
c*x**8 + 12285*b*c**2*x**10 + 3315*c**3*x**12))/(69615*sqrt(x)*x)
```

3.920 $\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 99

$$\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}} \, dx = -\frac{2a^3}{5x^{5/2}} - \frac{6a^2b}{\sqrt{x}} + 2a(b^2+ac)\,x^{3/2} + \frac{2}{7}b(b^2+6ac)\,x^{7/2} + \frac{6}{11}c(b^2+ac)\,x^{11/2} + \frac{2}{5}bc^2x^{15/2} + \frac{2}{19}c^3x^{19/2}$$

output
```
-2/5*a^3/x^(5/2)-6*a^2*b/x^(1/2)+2*a*(a*c+b^2)*x^(3/2)+2/7*b*(6*a*c+b^2)*x
^(7/2)+6/11*c*(a*c+b^2)*x^(11/2)+2/5*b*c^2*x^(15/2)+2/19*c^3*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}} \, dx = -\frac{2(1463a^3 + 21945a^2bx^2 - 7315ab^2x^4 - 7315a^2cx^4 - 1045b^3x^6 - 6270abcx^6 - 1995b^2cx^8 - 1995ac^2x^8 -}{7315x^{5/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^3/x^(7/2),x]
```

output
```
(-2*(1463*a^3 + 21945*a^2*b*x^2 - 7315*a*b^2*x^4 - 7315*a^2*c*x^4 - 1045*b
^3*x^6 - 6270*a*b*c*x^6 - 1995*b^2*c*x^8 - 1995*a*c^2*x^8 - 1463*b*c^2*x^1
0 - 385*c^3*x^12))/(7315*x^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(\frac{a^3}{x^{7/2}}+\frac{3a^2b}{x^{3/2}}+3cx^{9/2}(ac+b^2)+bx^{5/2}(6ac+b^2)+3a\sqrt{x}(ac+b^2)+3bc^2x^{13/2}+c^3x^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+\frac{6}{11}cx^{11/2}(ac+b^2)+\frac{2}{7}bx^{7/2}(6ac+b^2)+2ax^{3/2}(ac+b^2)+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

input
```
Int[(a + b*x^2 + c*x^4)^3/x^(7/2),x]
```

output
```
(-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*(b^2 + a*c)*x^(3/2) + (2*b*
(b^2 + 6*a*c)*x^(7/2))/7 + (6*c*(b^2 + a*c)*x^(11/2))/11 + (2*b*c^2*x^(15/
2))/5 + (2*c^3*x^(19/2))/19
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2c^3x^{\frac{19}{2}}}{19}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{6ac^2x^{\frac{11}{2}}}{11}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{12abcx^{\frac{7}{2}}}{7}+\frac{2b^3x^{\frac{7}{2}}}{7}+2a^2cx^{\frac{3}{2}}+2ab^2x^{\frac{3}{2}}-\frac{2a^3}{5x^{\frac{5}{2}}}-$
default	$\frac{2c^3x^{\frac{19}{2}}}{19}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{6ac^2x^{\frac{11}{2}}}{11}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{12abcx^{\frac{7}{2}}}{7}+\frac{2b^3x^{\frac{7}{2}}}{7}+2a^2cx^{\frac{3}{2}}+2ab^2x^{\frac{3}{2}}-\frac{2a^3}{5x^{\frac{5}{2}}}-$
gosper	$-\frac{2(-385c^3x^{12}-1463bc^2x^{10}-1995ac^2x^8-1995b^2cx^8-6270abcx^6-1045b^3x^6-7315a^2cx^4-7315b^2x^4a+21945a^2bx}{7315x^{\frac{5}{2}}}$
trager	$-\frac{2(-385c^3x^{12}-1463bc^2x^{10}-1995ac^2x^8-1995b^2cx^8-6270abcx^6-1045b^3x^6-7315a^2cx^4-7315b^2x^4a+21945a^2bx}{7315x^{\frac{5}{2}}}$
risch	$-\frac{2(-385c^3x^{12}-1463bc^2x^{10}-1995ac^2x^8-1995b^2cx^8-6270abcx^6-1045b^3x^6-7315a^2cx^4-7315b^2x^4a+21945a^2bx}{7315x^{\frac{5}{2}}}$
orering	$-\frac{2(-385c^3x^{12}-1463bc^2x^{10}-1995ac^2x^8-1995b^2cx^8-6270abcx^6-1045b^3x^6-7315a^2cx^4-7315b^2x^4a+21945a^2bx}{7315x^{\frac{5}{2}}}$

input
```
int((c*x^4+b*x^2+a)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/19*c^3*x^(19/2)+2/5*b*c^2*x^(15/2)+6/11*a*c^2*x^(11/2)+6/11*b^2*c*x^(11/
2)+12/7*a*b*c*x^(7/2)+2/7*b^3*x^(7/2)+2*a^2*c*x^(3/2)+2*a*b^2*x^(3/2)-2/5*
a^3/x^(5/2)-6*a^2*b/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2\,(385\,c^3x^{12}+1463\,bc^2x^{10}+1995\,(b^2c+ac^2)x^8+1045\,(b^3+6\,abc)x^6-21945\,a^2}{7315\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="fricas")
```

output
```
2/7315*(385*c^3*x^12 + 1463*b*c^2*x^10 + 1995*(b^2*c + a*c^2)*x^8 + 1045*(
b^3 + 6*a*b*c)*x^6 - 21945*a^2*b*x^2 + 7315*(a*b^2 + a^2*c)*x^4 - 1463*a^3
)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}}\,dx = -\frac{2a^3}{5x^{\frac{5}{2}}}-\frac{6a^2b}{\sqrt{x}}+2a^2cx^{\frac{3}{2}}+2ab^2x^{\frac{3}{2}} +\frac{12abcx^{\frac{7}{2}}}{7}+\frac{6ac^2x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$$

input
```
integrate((c*x**4+b*x**2+a)**3/x**(7/2),x)
```

output
```
-2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a**2*c*x**(3/2) + 2*a*b**2*x**
(3/2) + 12*a*b*c*x**(7/2)/7 + 6*a*c**2*x**(11/2)/11 + 2*b**3*x**(7/2)/7 +
6*b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5 + 2*c**3*x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{19}\,c^3x^{\frac{19}{2}} + \frac{2}{5}\,bc^2x^{\frac{15}{2}} + \frac{6}{11}\,(b^2c+ac^2)x^{\frac{11}{2}} + \frac{2}{7}\,(b^3+6\,abc)x^{\frac{7}{2}} + 2\,(ab^2+a^2c)x^{\frac{3}{2}} - \frac{2\,(15\,a^2bx^2+a^3)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="maxima")
```

output
```
2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*(b^2*c + a*c^2)*x^(11/2) + 2
/7*(b^3 + 6*a*b*c)*x^(7/2) + 2*(a*b^2 + a^2*c)*x^(3/2) - 2/5*(15*a^2*b*x^2
 + a^3)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{19}\,c^3x^{\frac{19}{2}} + \frac{2}{5}\,bc^2x^{\frac{15}{2}} + \frac{6}{11}\,b^2cx^{\frac{11}{2}} + \frac{6}{11}\,ac^2x^{\frac{11}{2}} + \frac{2}{7}\,b^3x^{\frac{7}{2}} + \frac{12}{7}\,abcx^{\frac{7}{2}} + 2\,ab^2x^{\frac{3}{2}} + 2\,a^2cx^{\frac{3}{2}} - \frac{2\,(15\,a^2bx^2+a^3)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^2+a)^3/x^(7/2),x, algorithm="giac")
```

output
```
2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 6/11*a*c^2*
x^(11/2) + 2/7*b^3*x^(7/2) + 12/7*a*b*c*x^(7/2) + 2*a*b^2*x^(3/2) + 2*a^2*
c*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}}\,dx = x^{7/2}\left(\frac{2\,b^3}{7}+\frac{12\,a\,c\,b}{7}\right)-\frac{\frac{2\,a^3}{5}+6\,b\,a^2\,x^2}{x^{5/2}} + \frac{2\,c^3\,x^{19/2}}{19}+\frac{2\,b\,c^2\,x^{15/2}}{5}+2\,a\,x^{3/2}\,\left(b^2+a\,c\right)+\frac{6\,c\,x^{11/2}\,\left(b^2+a\,c\right)}{11}$$

input
```
int((a + b*x^2 + c*x^4)^3/x^(7/2),x)
```

output
```
x^(7/2)*((2*b^3)/7 + (12*a*b*c)/7) - ((2*a^3)/5 + 6*a^2*b*x^2)/x^(5/2) + (
2*c^3*x^(19/2))/19 + (2*b*c^2*x^(15/2))/5 + 2*a*x^(3/2)*(a*c + b^2) + (6*c
*x^(11/2)*(a*c + b^2))/11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{\frac{2}{19}c^3x^{12}+\frac{2}{5}b\,c^2x^{10}+\frac{6}{11}a\,c^2x^8+\frac{6}{11}b^2c\,x^8+\frac{12}{7}abc\,x^6+\frac{2}{7}b^3x^6+2a^2c\,x^4+2a\,b^2x^4+}{\sqrt{x}\,x^2}$$

input
```
int((c*x^4+b*x^2+a)^3/x^(7/2),x)
```

output
```
(2*( - 1463*a**3 - 21945*a**2*b*x**2 + 7315*a**2*c*x**4 + 7315*a*b**2*x**4
 + 6270*a*b*c*x**6 + 1995*a*c**2*x**8 + 1045*b**3*x**6 + 1995*b**2*c*x**8
+ 1463*b*c**2*x**10 + 385*c**3*x**12))/(7315*sqrt(x)*x**2)
```

3.921 $\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 389

$$\begin{aligned}\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx &= \frac{2x^{3/2}}{3c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}}\\&-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}\\&+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}}\\&+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
2/3*x^(3/2)/c-1/2*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/
4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)
^(1/2))^(1/4)-1/2*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/
4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)
^(1/2))^(1/4)+1/2*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1
/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2
)^(1/2))^(1/4)+1/2*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(
1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^
2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.21

$$\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx = \frac{4x^{3/2}-3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{a\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{6c}$$

input
```
Integrate[x^(9/2)/(a + b*x^2 + c*x^4),x]
```

output
```
(4*x^(3/2) - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #1] + b*Lo
g[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(6*c)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1435, 1703, 27, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x^5}{cx^4+bx^2+a}d\sqrt{x}$$

$$\Big\downarrow \text{1703}$$

$$2\left(\frac{x^{3/2}}{3c}-\frac{\int \frac{3x\left(bx^2+a\right)}{cx^4+bx^2+a}d\sqrt{x}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{x^{3/2}}{3c}-\frac{\int \frac{x\left(bx^2+a\right)}{cx^4+bx^2+a}d\sqrt{x}}{c}\right)$$

$$\Big\downarrow \text{1834}$$

$$2\left(\frac{x^{3/2}}{3c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{2x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{c}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{x^{3/2}}{3c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{c}\right)$$

$$\Big\downarrow \text{827}$$

$$2\left(\frac{x^{3/2}}{3c}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)+\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{\sqrt{b^2 \cdots}}}}{2\sqrt{2}\sqrt{\cdots}} \cdots\right.}{c}\right.$$

$$\Big\downarrow \text{218}$$

$$2\left(\frac{x^{3/2}}{3c}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)+\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\right.}{2\,2^{3/4}c^{3/}}\right.}{c}\right.$$

$$\Big\downarrow \; 221$$

$$2\left(\frac{x^{3/2}}{3c}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\right.}{c}\right.$$

input
```
Int[x^(9/2)/(a + b*x^2 + c*x^4),x]
```

output
```
2*(x^(3/2)/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[
b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) +
 (b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])
^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)
]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.17

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6b+\_R^2a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2c}$	65
default	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6b+\_R^2a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2c}$	65
risch	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6b+\_R^2a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2c}$	65

input
```
int(x^(9/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)/c-1/2/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_
R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7019 vs. $2(307) = 614$.

Time = 1.13 (sec) , antiderivative size = 7019, normalized size of antiderivative = 18.04

$$\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^{\frac{9}{2}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
2/3*x^(3/2)/c - integrate((b*x^(5/2) + a*sqrt(x))/(c^2*x^4 + b*c*x^2 + a*c
), x)
```

Giac [F]

$$\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^{\frac{9}{2}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x^(9/2)/(c*x^4 + b*x^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 19.02 (sec) , antiderivative size = 12789, normalized size of antiderivative = 32.88

$$\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^(9/2)/(a + b*x^2 + c*x^4),x)
```

output
```
atan(((((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b^5*c^4 - 512*a^5*b
^3*c^5))/c^3 - (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a
^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-
(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/
2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*
a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(512*a^6*c^8 - 16*a
^3*b^6*c^5 + 160*a^4*b^4*c^6 - 512*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a
*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 28
0*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*
c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(25
6*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))
^(3/4) + (256*x^(1/2)*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11
+ b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*
b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c
+ 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1
/2))/(32*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3
*b^2*c^10)))^(1/4)*1i - (((128*(512*a^6*b*c^6 - 16*a^3*b^7*c^3 + 160*a^4*b
^5*c^4 - 512*a^5*b^3*c^5))/c^3 + (256*x^(1/2)*(-(b^11 + b^6*(-(4*a*c - b^2
)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^
3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(...
```

Reduce [F]

$$\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx=\int \frac{x^{\frac{9}{2}}}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^(9/2)/(c*x^4+b*x^2+a),x)
```

output
```
int(x^(9/2)/(c*x^4+b*x^2+a),x)
```

3.922 $\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 385

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx = \frac{2\sqrt{x}}{c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$
$$+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$
$$+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$
$$+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
2*x^(1/2)/c+1/2*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)
*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(
1/2))^(3/4)+1/2*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)
*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(
1/2))^(3/4)+1/2*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4
)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^
(1/2))^(3/4)+1/2*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/
4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)
^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.21

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx = -\frac{-4\sqrt{x}+\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{a\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{2c}$$

input
```
Integrate[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

output
```
-1/2*(-4*Sqrt[x] + RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #1] +
b*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/c
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1435, 1703, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 1435$$

$$2\int \frac{x^4}{cx^4+bx^2+a}d\sqrt{x}$$

$$\Big\downarrow \; 1703$$

$$2\left(\frac{\sqrt{x}}{c}-\frac{\int \frac{bx^2+a}{cx^4+bx^2+a}d\sqrt{x}}{c}\right)$$

$$\Big\downarrow \; 1752$$

$$2\left(\frac{\sqrt{x}}{c}-\frac{\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d\sqrt{x}+\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d\sqrt{x}}{c}\right)$$

$$\Big\downarrow \; 756$$

$$2\left(\frac{\sqrt{x}}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-\cdots}}}}{\sqrt{\cdots}}\cdots\right.}{c}\cdots\right.$$

$$\Big\downarrow \; 218$$

$$2\left(\frac{\sqrt{x}}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \cdots}{\cdots}\cdots\right.}{c}\cdots\right.$$

$$\Big\downarrow \; 221$$

$$2\left(\frac{\sqrt{x}}{c}-\frac{\frac{1}{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right.}{c}\right.$$

input
```
Int[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

output
```
2*(Sqrt[x]/c - (((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sq
rt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 +
((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4
)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2)/c)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1703
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Simp[d^(2*n)/(c*(m + 2*n*p + 1))   Int[(d
*x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x
^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && N
eQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0
] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.16

method	result	size
risch	$\frac{2\sqrt{x}}{c}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\_R^{4}b+a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2c}$	61
derivativedivides	$\frac{2\sqrt{x}}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}b-a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2c}$	64
default	$\frac{2\sqrt{x}}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}b-a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2c}$	64

input
```
int(x^(7/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/c-1/2/c*sum((_R^4*b+a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootO
f(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4023 vs. $2(307) = 614$.

Time = 0.27 (sec) , antiderivative size = 4023, normalized size of antiderivative = 10.45

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^{\frac{7}{2}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^{\frac{7}{2}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x^(7/2)/(c*x^4 + b*x^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 19.61 (sec) , antiderivative size = 10449, normalized size of antiderivative = 27.14

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^(7/2)/(a + b*x^2 + c*x^4),x)
```

output
```
atan((((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (25
6*x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^
5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c -
3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6 + 16*a^3*b^
5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^
4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1
/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 +
b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (256*
x^(1/2)*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/
(32*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c
^8)))^(1/4)*1i - (((512*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^
2))/c + (256*x^(1/2)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(256*a^5*b*c^6
+ 16*a^3*b^5*c^4 - 128*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*...
```

Reduce [F]

$$\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^{\frac{7}{2}}}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^(7/2)/(c*x^4+b*x^2+a),x)
```

output
```
int(x^(7/2)/(c*x^4+b*x^2+a),x)
```

3.923 $\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 331

$$\begin{aligned}\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx =& -\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}} \\ &+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}} \\ &+\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}} \\ &-\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}\end{aligned}$$

output
```
-1/2*(-b-(-4*a*c+b^2)^(1/2))^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*
a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/2*(-b+(-4*a*c+
b^2)^(1/2))^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(
1/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/2*(-b-(-4*a*c+b^2)^(1/2))^(3/4)
*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^
(3/4)/(-4*a*c+b^2)^(1/2)-1/2*(-b+(-4*a*c+b^2)^(1/2))^(3/4)*arctanh(2^(1/4)
*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.15

$$\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx=\frac{1}{2}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log\left(\sqrt{x}-\#1\right)\#1^3}{b+2c\#1^4}\&\right]$$

input
```
Integrate[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1^3)/(b + 2*c*#1^4) &
]/2
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1435, 1710, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 1435}$$

$$2\int \frac{x^3}{cx^4+bx^2+a}d\sqrt{x}$$

$\downarrow$ 1710

$$2\left(\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{2x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}\right)$$

$\downarrow$ 27

$$2\left(\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}\right)$$

$\downarrow$ 827

$$2\left(\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)+\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\ldots}\right.\right.$$

$\downarrow$ 218

$$2\left(\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)+\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\ldots\right.\right.$$

$\downarrow$ 221

$$2\left(\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\ldots\right.$$

input

```
Int[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

output
```
2*((1 + b/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[
b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) -
ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/
4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (1 - b/Sqrt[b^2 - 4*a*c])*(A
rcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)
*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a
*c])^(1/4))))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.14

method	result	size
derivativedivides	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\_R^{6}\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{2}$	45
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\_R^{6}\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{2}$	45

input
```
int(x^(5/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4449 vs. 2(253) = 506.

Time = 0.14 (sec) , antiderivative size = 4449, normalized size of antiderivative = 13.44

$$\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^{\frac{5}{2}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^{\frac{5}{2}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x^(5/2)/(c*x^4 + b*x^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 20.35 (sec) , antiderivative size = 8093, normalized size of antiderivative = 24.45

$$\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^(5/2)/(a + b*x^2 + c*x^4),x)
```

output
```
- atan(((x^(1/2)*(256*a^3*b^3*c - 768*a^4*b*c^2) + (-(b^7 + b^2*(-(4*a*c -
 b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*c^5 + x^(1/2)*(-(b^7 + b^2*(-(4*a
*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4
*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b
^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072*a^5*c^6 + 8192*a^3*b^4*c^4 - 655
36*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 16384*a^4*b^2*c^4))*(-(b^7 + b^2*(-(4
*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-
(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2
*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i + (x^(1/2)*(256*a^3*b^3*c - 768*a^4
*b*c^2) - (-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 + b^8*
c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(32768*a^5*
c^5 - x^(1/2)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^
2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c^7 +
b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(131072
*a^5*c^6 + 8192*a^3*b^4*c^4 - 65536*a^4*b^2*c^5) + 2048*a^3*b^4*c^3 - 1638
4*a^4*b^2*c^4))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*
a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(256*a^4*c...
```

Reduce [F]

$$\int \frac{x^{5/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^{\frac{5}{2}}}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^(5/2)/(c*x^4+b*x^2+a),x)
```

output
```
int(x^(5/2)/(c*x^4+b*x^2+a),x)
```

3.924 $\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 331

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx = \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\,\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

output
```
1/2*(-b-(-4*a*c+b^2)^(1/2))^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a
*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)-1/2*(-b+(-4*a*c+b
^2)^(1/2))^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1
/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)+1/2*(-b-(-4*a*c+b^2)^(1/2))^(1/4)*
arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(
1/4)/(-4*a*c+b^2)^(1/2)-1/2*(-b+(-4*a*c+b^2)^(1/2))^(1/4)*arctanh(2^(1/4)*
c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.14

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log\left(\sqrt{x}-\#1\right)\#1}{b+2c\#1^4}\&\right]$$

input
```
Integrate[x^(3/2)/(a + b*x^2 + c*x^4),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1)/(b + 2*c*#1^4) & ]/
2
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1435, 1710, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x^2}{cx^4+bx^2+a}d\sqrt{x}$$

$$\Big\downarrow \text{1710}$$

$$2\left(\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d\sqrt{x}+\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d\sqrt{x}\right.$$

$$\Big\downarrow \text{756}$$

$$2\left(\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\right.\right.$$

$$\Big\downarrow \text{218}$$

$$2\left(\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right.\right.$$

$$\Big\downarrow \text{221}$$

$$2\left(\frac{1}{2}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-\right.\right.$$

input
```
Int[x^(3/2)/(a + b*x^2 + c*x^4),x]
```

output
```
2*(((1 + b/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)))
- ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/
4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + ((1 - b/Sqrt[b^2 - 4*a*c]
)*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(
1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 -
4*a*c])^(3/4))))/2)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1710
```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^n/2)*(b/q + 1)   Int[(d*x)^(m
- n)/(b/2 + q/2 + c*x^n), x], x] - Simp[(d^n/2)*(b/q - 1)   Int[(d*x)^(m -
n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.14

method	result	size
derivativedivides	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\_R^{4}\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{2}$	45
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\_R^{4}\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{2}$	45

input
```
int(x^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2157 vs. 2(253) = 506.

Time = 0.10 (sec) , antiderivative size = 2157, normalized size of antiderivative = 6.52

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)))*log((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-
(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) + sqrt(x)) - 1/2*sqrt(sqrt
(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^
4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))
*log(-(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) + sqrt(x)) + 1/2*sqrt(-sqrt(1/2)*sqrt(
-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log((b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(-sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5
))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5) + sqrt(x)) - 1/2*sqrt(-sqrt(1/2)*sqrt(-(b + (b^4*
c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(-(b^4*c - 8*a*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^{\frac{3}{2}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx = \int \frac{x^{\frac{3}{2}}}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x^(3/2)/(c*x^4 + b*x^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 19.13 (sec) , antiderivative size = 8229, normalized size of antiderivative = 24.86

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^(3/2)/(a + b*x^2 + c*x^4),x)
```

output
```
atan(((x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) + (-(b^5 + (-(4*a*c - b^2)^
5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c
^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^
5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c
^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^2*
b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6) - x^(1/2)*(65536*a^4*b*c
^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 +
96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 2048*a^3*b*c^4 - 512*a^2*b^3*c
^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/
4)*1i + (x^(1/2)*(512*a^3*c^4 - 256*a^2*b^2*c^3) - (-(b^5 + (-(4*a*c - b^2
)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6
*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2
)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6
*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(524288*a^5*c^7 - 8192*a^
2*b^6*c^4 + 98304*a^3*b^4*c^5 - 393216*a^4*b^2*c^6) + x^(1/2)*(65536*a^4*b
*c^6 + 4096*a^2*b^5*c^4 - 32768*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^
(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2
+ 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 2048*a^3*b*c^4 - 512*a^2*...
```

Reduce [F]

$$\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx=\int \frac{x^{\frac{3}{2}}}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^(3/2)/(c*x^4+b*x^2+a),x)
```

output
```
int(x^(3/2)/(c*x^4+b*x^2+a),x)
```

3.925 $\int \frac{\sqrt{x}}{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 331

$$\int \frac{\sqrt{x}}{a+bx^2+cx^4} \, dx = -\frac{\sqrt[4]{2}\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{2}\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{2}\sqrt[4]{c}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{2}\sqrt[4]{c}\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
-2^(1/4)*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1
/4))/(-4*a*c+b^2)^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+2^(1/4)*c^(1/4)*arct
an(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/(-4*a*c+b^2)^(1/
2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+2^(1/4)*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x
^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b-(-4*a*c+b^2)^
(1/2))^(1/4)-2^(1/4)*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b
^2)^(1/2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log\left(\sqrt{x}-\#1\right)}{b\#1+2c\#1^5}\&\right]$$

input
```
Integrate[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1 + 2*c*#1^5) & ]/2
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1435, 1711, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x}{cx^4+bx^2+a}d\sqrt{x}$$

$$\Big\downarrow \text{1711}$$

$$2\left(\frac{c\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}}{\sqrt{b^2-4ac}}-\frac{c\int \frac{2x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{2c\int \frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{827}$$

$$2\left(\frac{2c\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{\sqrt{b^2-4ac}-b}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{\ldots}}{2\sqrt{2}\sqrt{c}}\ldots}{\sqrt{b^2-4ac}}\ldots\right.$$

$$\Big\downarrow \text{218}$$

$$2\left(\frac{2c\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4\ldots}}}\ldots}{2\sqrt{\ldots}}\ldots}{\sqrt{b^2-4ac}}\ldots\right.$$

$$\Big\downarrow \text{221}$$

$$2\left(\frac{2c\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\cdots\right.}{\sqrt{b^2-4ac}}\right.$$

input
```
Int[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

output
```
2*((-2*c*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]
/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqr
t[b^2 - 4*a*c])^(1/4))))/Sqrt[b^2 - 4*a*c] + (2*c*(ArcTan[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^
2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/Sqrt[b^2
 - 4*a*c])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1711
```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c
*x^n), x], x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; Free
Q[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.14

method	result	size
derivativedivides	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\_R^{2}\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{2}$	45
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\_R^{2}\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{2}$	45

input
```
int(x^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3209 vs. $2(251) = 502$.

Time = 0.11 (sec) , antiderivative size = 3209, normalized size of antiderivative = 9.69

$$\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{x}}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx = \int \frac{\sqrt{x}}{cx^4+bx^2+a}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/(c*x^4 + b*x^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 6133, normalized size of antiderivative = 18.53

$$\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)/(a + b*x^2 + c*x^4),x)
```

output

```
2*atan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32
*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))
^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3*c^5 - x^(1/2)*(-(
b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 25
6*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(1310
72*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072*a^3*b^2*c^6)*1i)*
1i - 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(1/4) - ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^
2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 -
256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c^6 - 16384*a^2*b^3
*c^5 + x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*
c^3)))^(1/4)*(131072*a^4*c^7 - 4096*a*b^6*c^4 + 40960*a^2*b^4*c^5 - 131072
*a^3*b^2*c^6)*1i)*1i + 256*a*b*c^5*x^(1/2))*(-(b^5 - (-(4*a*c - b^2)^5)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c +
96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4))/(((-(b^5 - (-(4*a*c - b^2)^5)^(
1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(32*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c +
 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(2048*a*b^5*c^4 + 32768*a^3*b*c
^6 - 16384*a^2*b^3*c^5 - x^(1/2)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16...
```

Reduce [F]

$$\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx=\int \frac{\sqrt{x}}{c\,x^4+b\,x^2+a}dx$$

input

```
int(x^(1/2)/(c*x^4+b*x^2+a),x)
```

output

```
int(x^(1/2)/(c*x^4+b*x^2+a),x)
```

3.926 $\int \frac{1}{\sqrt{x}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 331

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)}\,dx = \frac{2^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{2^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{2^{3/4}c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{2^{3/4}c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
2^(3/4)*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/
4))/(-4*a*c+b^2)^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-2^(3/4)*c^(3/4)*arcta
n(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/(-4*a*c+b^2)^(1/2
)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+2^(3/4)*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^
(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b-(-4*a*c+b^2)^(
1/2))^(3/4)-2^(3/4)*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^
2)^(1/2))^(1/4))/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.15

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)}\,dx=\frac{1}{2}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log\left(\sqrt{x}-\#1\right)}{b\#1^3+2c\#1^7}\&\right]$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
```

output
```
RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1^3 + 2*c*#1^7) & ]/
2
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1435, 1685, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)}\,dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{1}{cx^4+bx^2+a}d\sqrt{x}$$

$$\Big\downarrow \text{1685}$$

$$2\left(\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d\sqrt{x}}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d\sqrt{x}}{\sqrt{b^2-4ac}}\right)$$

$$\Big\downarrow \text{756}$$

$$2\left(\frac{c\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{\sqrt{b^2-4ac}-b}}d\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right.}{\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow \text{218}$$

$$2\left(\frac{c\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}}{\sqrt[4]{-\sqrt{b}}}\right.}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2}\right.}\right.}{\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow \text{221}$$

$$2\left(\frac{c\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\text{arc}\right.}{\sqrt{b^2-4ac}}\right.$$

input
```
Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
```

output
```
2*(-((c*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)
]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[
b^2 - 4*a*c])^(3/4))))/Sqrt[b^2 - 4*a*c]) + (c*(-(ArcTan[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 -
 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*
c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/Sqrt[b^2 - 4
*a*c])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1685
```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^n), x], x] - Simp[
c/q   Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.13

method	result	size
derivativedivides	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{2}$	42
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}\right)}{2}$	42

input
```
int(1/x^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3137 vs. $2(251) = 502$.

Time = 0.16 (sec) , antiderivative size = 3137, normalized size of antiderivative = 9.48

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(1/2)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
2*sqrt(x)/a - integrate((c*x^(7/2) + b*x^(3/2))/(a*c*x^4 + a*b*x^2 + a^2),
 x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x)), x)
```

Mupad [B] (verification not implemented)

Time = 19.43 (sec) , antiderivative size = 10401, normalized size of antiderivative = 31.42

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(1/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
- atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3
*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c
^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7
- 512*b^2*c^6 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40
*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 2
56*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(819
2*a*b^7*c^4 - 524288*a^4*b*c^7 - 98304*a^2*b^5*c^5 + 393216*a^3*b^3*c^6) +
 x^(1/2)*(4096*b^7*c^4 - 45056*a*b^5*c^5 - 196608*a^3*b*c^7 + 163840*a^2*b
^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3
*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c
^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4)) + 512*c^7*x
^(1/2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^
4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7
+ b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*
c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*
c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(2048*a*c^7 - 512*b^2*c^6 +
((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 1
1*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^3*b^8 + 256*a^7*c^4 - 16*
a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(8192*a*b^7*c^4 -...
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)}\,dx=\int \frac{1}{\sqrt{x}\,\left(c\,x^4+b\,x^2+a\right)}dx$$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2+a),x)
```

output
```
int(1/x^(1/2)/(c*x^4+b*x^2+a),x)
```

3.927 $\int \frac{1}{x^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 371

$$\begin{aligned}\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)}\,dx &= -\frac{2}{a\sqrt{x}} \\ &\quad -\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} \\ &\quad -\frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}} \\ &\quad +\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} \\ &\quad +\frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-2/a/x^(1/2)-1/2*c^(1/4)*(1-b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x
^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1
/4)-1/2*c^(1/4)*(1+b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-
b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/2*c
^(1/4)*(1-b/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*
c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/2*c^(1/4)*(
1+b/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(
1/2))^(1/4))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)}\,dx = -\frac{\frac{4}{\sqrt{x}}+\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log\left(\sqrt{x}-\#1\right)+c\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{2a}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/2*(4/Sqrt[x] + RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] + c
*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/a
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1435, 1704, 25, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)}\,dx$$

$\Big\downarrow$ 1435

$$2\int \frac{1}{x\left(cx^4+bx^2+a\right)}d\sqrt{x}$$

$\Big\downarrow$ 1704

$$2\left(\frac{\int -\frac{x(cx^2+b)}{cx^4+bx^2+a}d\sqrt{x}}{a}-\frac{1}{a\sqrt{x}}\right)$$

$\Big\downarrow$ 25

$$2\left(-\frac{\int \frac{x(cx^2+b)}{cx^4+bx^2+a}d\sqrt{x}}{a}-\frac{1}{a\sqrt{x}}\right)$$

$\Big\downarrow$ 1834

$$2\left(-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{a}-\frac{1}{a\sqrt{x}}\right)$$

$\Big\downarrow$ 27

$$2\left(-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{a}-\frac{1}{a\sqrt{x}}\right)$$

$\Big\downarrow$ 827

$$2\left(-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{\sqrt{b^2-4ac}}}}{2\sqrt{2}\sqrt{c}}\right.}{a}\right.$$

$\Big\downarrow$ 218

$$2\left(-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\, 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\cdots\right.}{2\, 2^{3/4}c^{3/4}\sqrt[4]{\cdots}}\right.}{a}\right.$$

$\big\downarrow$ 221

$$2\left(-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\, 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\, 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\text{arct}\cdots}{2\,2\cdots}\right.}{a}\right.$$

input
```
Int[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
2*(-(1/(a*Sqrt[x])) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4
)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b
^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4
*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + c*(1 +
 b/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4
*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh
[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3
/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.18

method	result	size
derivativedivides	$-\frac{2}{a\sqrt{x}}-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\_R^{6}c+\_R^{2}b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2a}$	65
default	$-\frac{2}{a\sqrt{x}}-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\_R^{6}c+\_R^{2}b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2a}$	65
risch	$-\frac{2}{a\sqrt{x}}-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\_R^{6}c+\_R^{2}b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2a}$	65

input
```
int(1/x^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/a/x^(1/2)-1/2/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R
=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5778 vs. 2(289) = 578.

Time = 0.97 (sec) , antiderivative size = 5778, normalized size of antiderivative = 15.57

$$\int \frac{1}{x^{3/2}(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(3/2)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^{3/2}(a+bx^2+cx^4)}\,dx = \int \frac{1}{(cx^4+bx^2+a)x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
-2/(a*sqrt(x)) - integrate((c*x^(5/2) + b*sqrt(x))/(a*c*x^4 + a*b*x^2 + a^
2), x)
```

Giac [F]

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \int \frac{1}{(cx^4+bx^2+a)x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)*x^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 20.10 (sec) , antiderivative size = 10573, normalized size of antiderivative = 28.50

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(3/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
2*atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5
*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3
*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6
*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 - x^(1/2)*(
-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(
-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^
7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(131072*a^16*c^8 + 4096*a^12*b^8*c^4
- 49152*a^13*b^6*c^5 + 204800*a^14*b^4*c^6 - 327680*a^15*b^2*c^7)*1i + 204
8*a^11*b^8*c^4 - 22528*a^12*b^6*c^5 + 83968*a^13*b^4*c^6 - 114688*a^14*b^2
*c^7)*1i + 256*a^11*b*c^8*x^(1/2))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) +
 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)
^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8
+ 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) -
 ((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*
c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96
*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(32768*a^15*c^8 + x^(1/2)*(-(b^9 +
 b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^
3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*...
```

Reduce [F]

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)}\,dx=\int \frac{1}{x^{\frac{3}{2}}\left(c\,x^4+b\,x^2+a\right)}dx$$

input
```
int(1/x^(3/2)/(c*x^4+b*x^2+a),x)
```

output
```
int(1/x^(3/2)/(c*x^4+b*x^2+a),x)
```

3.928 $\int \frac{1}{x^{5/2}(a+bx^2+cx^4)} dx$

Optimal result

Integrand size = 20, antiderivative size = 371

$$\int \frac{1}{x^{5/2}(a+bx^2+cx^4)} dx = -\frac{2}{3ax^{3/2}} + \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
-2/3/a/x^(3/2)+1/2*c^(3/4)*(1-b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)
*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^
(3/4)+1/2*c^(3/4)*(1+b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/
(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/2
*c^(3/4)*(1-b/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*
a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/2*c^(3/4)
*(1+b/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)
^(1/2))^(1/4))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^{5/2} (a + bx^2 + cx^4)} dx = -\frac{\frac{4}{x^{3/2}} + 3\text{RootSum}\left[a + b\#1^4 + c\#1^8\&, \frac{b\log(\sqrt{x}-\#1)+c\log(\sqrt{x}-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{6a}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
-1/6*(4/x^(3/2) + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] +
 c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/a
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1435, 1704, 27, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\left(a+bx^2+cx^4\right)}\,dx$$

$\big\downarrow$ 1435

$$2\int \frac{1}{x^2\left(cx^4+bx^2+a\right)}d\sqrt{x}$$

$\big\downarrow$ 1704

$$2\left(\frac{\int -\frac{3\left(cx^2+b\right)}{cx^4+bx^2+a}d\sqrt{x}}{3a}-\frac{1}{3ax^{3/2}}\right)$$

$\big\downarrow$ 27

$$2\left(-\frac{\int \frac{cx^2+b}{cx^4+bx^2+a}d\sqrt{x}}{a}-\frac{1}{3ax^{3/2}}\right)$$

$\big\downarrow$ 1752

$$2\left(-\frac{\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d\sqrt{x}+\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d\sqrt{x}}{a}-\frac{1}{3ax^{3/2}}\right)$$

$\big\downarrow$ 756

$$2\left(-\frac{\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}\cdots}}}{\sqrt{\sqrt{b\cdots}}}\right.}{a}\right.$$

$\big\downarrow$ 218

$$2\left(-\frac{\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x}}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(-\frac{\int \frac{}{\sqrt{\cdots}}\cdots}{\cdots}\right.}{a}\right.$$

$$\Big\downarrow \; 221$$

$$2\left(-\frac{\frac{1}{2}c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\cdots}{a}\right.$$

input
```
Int[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
2*(-1/3*1/(a*x^(3/2)) - ((c*(1 - b/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqr
t[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2
 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + (
c*(1 + b/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt
[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) -
ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)
*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2)/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.16

method	result	size
risch	$-\frac{2}{3a\,x^{\frac{3}{2}}}-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\_R^{4}c+b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2a}$	61
derivativedivides	$-\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}c-b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2a}$	64
default	$-\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{4}c-b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{2a}$	64

input
```
int(1/x^(5/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a/x^(3/2)-1/2/a*sum((_R^4*c+b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=Ro
otOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5046 vs. 2(289) = 578.

Time = 0.72 (sec) , antiderivative size = 5046, normalized size of antiderivative = 13.60

$$\int \frac{1}{x^{5/2}\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**(5/2)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\left(a+bx^2+cx^4\right)}\,dx=\int \frac{1}{(cx^4+bx^2+a)x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
-2/3*(3*b*sqrt(x) + a/x^(3/2))/a^2 + integrate((b*c*x^(7/2) + (b^2 - a*c)*
x^(3/2))/(a^2*c*x^4 + a^2*b*x^2 + a^3), x)
```

Giac [F]

$$\int \frac{1}{x^{5/2}\left(a+bx^2+cx^4\right)}\,dx=\int \frac{1}{(cx^4+bx^2+a)x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)*x^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 22.17 (sec) , antiderivative size = 16557, normalized size of antiderivative = 44.63

$$\int \frac{1}{x^{5/2}\left(a+bx^2+cx^4\right)} dx = \text{Too large to display}$$

input
```
int(1/(x^(5/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
atan(((x^(1/2)*(512*a^10*c^10 - 256*a^9*b^2*c^9) - (-(b^11 + b^6*(-(4*a*c
- b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a
^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b
^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1
/4)*((x^(1/2)*(327680*a^15*b*c^8 + 4096*a^11*b^9*c^4 - 53248*a^12*b^7*c^5
+ 249856*a^13*b^5*c^6 - 491520*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^
2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b
^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 +
 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*
(524288*a^17*c^8 + 8192*a^13*b^8*c^4 - 106496*a^14*b^6*c^5 + 491520*a^15*b
^4*c^6 - 917504*a^16*b^2*c^7))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 11
2*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3
*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^7*b^8 + 256*a^11*c^4 -
16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 4096*a^11*b*c^
9 - 512*a^9*b^5*c^7 + 3072*a^10*b^3*c^8))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^
4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*...
```

Reduce [F]

$$\int \frac{1}{x^{5/2}\left(a+bx^2+cx^4\right)} dx = \int \frac{1}{x^{\frac{5}{2}}\left(c\,x^4+b\,x^2+a\right)}dx$$

input
```
int(1/x^(5/2)/(c*x^4+b*x^2+a),x)
```

output
```
int(1/x^(5/2)/(c*x^4+b*x^2+a),x)
```

3.929 $\int \frac{1}{x^{7/2}(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 412

$$\begin{aligned}\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx &= -\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}\\
&+\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a^2\sqrt[4]{-b-\sqrt{b^2-4ac}}}\\
&+\frac{\sqrt[4]{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}a^2\sqrt[4]{-b+\sqrt{b^2-4ac}}}\\
&-\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}a^2\sqrt[4]{-b-\sqrt{b^2-4ac}}}\\
&-\frac{\sqrt[4]{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}a^2\sqrt[4]{-b+\sqrt{b^2-4ac}}}\end{aligned}$$

output
```
-2/5/a/x^(5/2)+2*b/a^2/x^(1/2)+1/2*c^(1/4)*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1
/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)
/a^2/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/2*c^(1/4)*(b+(-2*a*c+b^2)/(-4*a*c+b^2
)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(
1/4)/a^2/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/2*c^(1/4)*(b-(-2*a*c+b^2)/(-4*a*c
+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
)*2^(1/4)/a^2/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/2*c^(1/4)*(b+(-2*a*c+b^2)/(-
4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^
(1/4))*2^(1/4)/a^2/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx = \frac{-\frac{4(a-5bx^2)}{x^{5/2}}+5\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^2\log\left(\sqrt{x}-\#1\right)-ac\log\left(\sqrt{x}-\#1\right)+bc\log}{b\#1+2c\#1^5}\right.}{10a^2}$$

input
```
Integrate[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
((-4*(a - 5*b*x^2))/x^(5/2) + 5*RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log[S
qrt[x] - #1] - a*c*Log[Sqrt[x] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 +
 2*c*#1^5) & ])/(10*a^2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1435, 1704, 27, 1828, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}dx$$

$$\Big\downarrow \text{ 1435}$$

$$2\int \frac{1}{x^3\left(cx^4+bx^2+a\right)}d\sqrt{x}$$

$$\Big\downarrow \text{ 1704}$$

$$2\left(\frac{\int -\frac{5\left(cx^2+b\right)}{x\left(cx^4+bx^2+a\right)}d\sqrt{x}}{5a}-\frac{1}{5ax^{5/2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2\left(-\frac{\int \frac{cx^2+b}{x\left(cx^4+bx^2+a\right)}d\sqrt{x}}{a}-\frac{1}{5ax^{5/2}}\right)$$

$$\Big\downarrow \text{ 1828}$$

$$2\left(-\frac{-\frac{\int \frac{x\left(b^2+cx^2b-ac\right)}{cx^4+bx^2+a}d\sqrt{x}}{a}-\frac{b}{a\sqrt{x}}}{a}-\frac{1}{5ax^{5/2}}\right)$$

$$\Big\downarrow \text{ 1834}$$

$$2\left(-\frac{-\frac{\frac{1}{2}c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+\frac{1}{2}c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{a}-\frac{b}{a\sqrt{x}}}{a}-\frac{1}{5ax^{5/2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2\left(-\frac{-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{a}-\frac{b}{a\sqrt{x}}}{a}-\frac{1}{5ax^{5/2}}\right)$$

$$\Big\downarrow \text{ 827}$$

$$2\left(-\frac{-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{\sqrt{b^2-4ac}-b}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\ldots\right.}{a}}{a}\right.$$

$$\Big\downarrow \; 218$$

$$2\left(-\frac{-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-\ldots}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4a\ldots}}}\right.}{a}}{a}\right.$$

$$\Big\downarrow \; 221$$

$$2\left(-\frac{-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{\ldots}}{\sqrt[4]{\sqrt{b^2\ldots}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{\ldots}}}\right.}{a}}{a}\right.$$

input
```
Int[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]
```

output
```
2*(-1/5*1/(a*x^(5/2)) - (-(b/(a*Sqrt[x])) - (c*(b - (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4
)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - S
qrt[b^2 - 4*a*c])^(1/4))) + c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(ArcTa
n[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(
3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])
^(1/4))))/a)/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1704
```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Simp[1/(a*d^n*(m + 1))   Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1)
+ c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{
a, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] &&
LtQ[m, -1] && IntegerQ[p]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.20

method	result	size
risch	$-\frac{2(-5b\,x^2+a)}{5a^2x^{\frac{5}{2}}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(bc\_R^6+\left(-ac+b^2\right)\_R^2\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2a^2}$	81
derivativedivides	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-bc\_R^6+\left(ac-b^2\right)\_R^2\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2a^2}-\frac{2}{5a\,x^{\frac{5}{2}}}+\frac{2b}{a^2\sqrt{x}}$	84
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-bc\_R^6+\left(ac-b^2\right)\_R^2\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{2a^2}-\frac{2}{5a\,x^{\frac{5}{2}}}+\frac{2b}{a^2\sqrt{x}}$	84

input
```
int(1/x^(7/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-5*b*x^2+a)/a^2/x^(5/2)+1/2/a^2*sum((b*c*_R^6+(-a*c+b^2)*_R^2)/(2*_R
^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8376 vs. $2(328) = 656$.

Time = 6.70 (sec) , antiderivative size = 8376, normalized size of antiderivative = 20.33

$$\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(7/2)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx = \int \frac{1}{(cx^4+bx^2+a)x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
2/5*(5*b/sqrt(x) - a/x^(5/2))/a^2 + integrate((b*c*x^(5/2) + (b^2 - a*c)*s
qrt(x))/(a^2*c*x^4 + a^2*b*x^2 + a^3), x)
```

Giac [F]

$$\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx = \int \frac{1}{(cx^4+bx^2+a)x^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x^(7/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)*x^(7/2)), x)
```

Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 15149, normalized size of antiderivative = 36.77

$$\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(7/2)*(a + b*x^2 + c*x^4)),x)
```

output
```
atan((((-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^
9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4
*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/
2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5
)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 2
56*a^12*b^2*c^3)))^(3/4)*(x^(1/2)*(-(b^13 + b^8*(-(4*a*c - b^2)^5)^(1/2) +
 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c^3 + 681*a^4*b^5*c^4 - 552
*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2) - 17*a*b^11*c + 15*a^2*b^4
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(-(4*a*c - b^2)^5)^(1/2) -
7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8 + 256*a^13*c^4 - 16*a^10*
b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))^(1/4)*(131072*a^28*c^9 - 409
6*a^23*b^10*c^4 + 57344*a^24*b^8*c^5 - 299008*a^25*b^6*c^6 + 696320*a^26*b
^4*c^7 - 655360*a^27*b^2*c^8) - 131072*a^26*b*c^9 + 2048*a^21*b^11*c^4 - 2
8672*a^22*b^9*c^5 + 151552*a^23*b^7*c^6 - 368640*a^24*b^5*c^7 + 393216*a^2
5*b^3*c^8) + x^(1/2)*(768*a^21*b*c^11 - 256*a^20*b^3*c^10))*(-(b^13 + b^8*
(-(4*a*c - b^2)^5)^(1/2) + 144*a^6*b*c^6 + 115*a^2*b^9*c^2 - 390*a^3*b^7*c
^3 + 681*a^4*b^5*c^4 - 552*a^5*b^3*c^5 + a^4*c^4*(-(4*a*c - b^2)^5)^(1/2)
- 17*a*b^11*c + 15*a^2*b^4*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a^3*b^2*c^3*(
-(4*a*c - b^2)^5)^(1/2) - 7*a*b^6*c*(-(4*a*c - b^2)^5)^(1/2))/(32*(a^9*b^8
 + 256*a^13*c^4 - 16*a^10*b^6*c + 96*a^11*b^4*c^2 - 256*a^12*b^2*c^3)))...
```

Reduce [F]

$$\int \frac{1}{x^{7/2}\left(a+bx^2+cx^4\right)}\,dx = \int \frac{1}{x^{\frac{7}{2}}\left(c\,x^4+b\,x^2+a\right)}dx$$

input
```
int(1/x^(7/2)/(c*x^4+b*x^2+a),x)
```

output
```
int(1/x^(7/2)/(c*x^4+b*x^2+a),x)
```

3.930 $\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 536

$$
\begin{aligned}
\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}\,dx &= -\frac{bx^{3/2}}{2c(b^2-4ac)}+\frac{x^{7/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\
&+\frac{\left(3b^2-14ac+\frac{3b^3-20abc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}c^{7/4}(b^2-4ac)\sqrt[4]{-b-\sqrt{b^2-4ac}}} \\
&+\frac{\left(3b^2-14ac-\frac{3b^3-20abc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}c^{7/4}(b^2-4ac)\sqrt[4]{-b+\sqrt{b^2-4ac}}} \\
&-\frac{\left(3b^2-14ac+\frac{3b^3-20abc}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}c^{7/4}(b^2-4ac)\sqrt[4]{-b-\sqrt{b^2-4ac}}} \\
&-\frac{\left(3b^2-14ac-\frac{3b^3-20abc}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}c^{7/4}(b^2-4ac)\sqrt[4]{-b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

output
```
-1/2*b*x^(3/2)/c/(-4*a*c+b^2)+1/2*x^(7/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+
b*x^2+a)+1/8*(3*b^2-14*a*c+(-20*a*b*c+3*b^3)/(-4*a*c+b^2)^(1/2))*arctan(2^
(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(7/4)/(-4*a
*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/8*(3*b^2-14*a*c-(-20*a*b*c+3*b^3)/
(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))
^(1/4))*2^(1/4)/c^(7/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/8*(3*
b^2-14*a*c+(-20*a*b*c+3*b^3)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x
^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(7/4)/(-4*a*c+b^2)/(-b-(-4
*a*c+b^2)^(1/2))^(1/4)-1/8*(3*b^2-14*a*c-(-20*a*b*c+3*b^3)/(-4*a*c+b^2)^(1
/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4
)/c^(7/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.43

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}\,dx =$$

$$-\frac{4\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log(\sqrt{x}-\#1)-c\log(\sqrt{x}-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]+\frac{\frac{4cx^{3/2}(ab+b^2x^2-2acx^2)}{a+bx^2+cx^4}+\text{RootSum}\left[a+b\#1\right.}{8c^2}$$

input
```
Integrate[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/8*(4*RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] - c*Log[Sqrt[
x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ] + ((4*c*x^(3/2)*(a*b + b^2*x^2 - 2*a*
c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#1^8 & , (-4*b^3*Log[
Sqrt[x] - #1] + 13*a*b*c*Log[Sqrt[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4
- 2*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(b^2 - 4*a*c))/c^
2
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1435, 1701, 1826, 27, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x^7}{(cx^4+bx^2+a)^2}d\sqrt{x}$$

$$\Big\downarrow \text{1701}$$

$$2\left(\frac{x^{7/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{x^3(3bx^2+14a)}{cx^4+bx^2+a}d\sqrt{x}}{4(b^2-4ac)}\right)$$

$$\Big\downarrow \text{1826}$$

$$2\left(\frac{x^{7/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{bx^{3/2}}{c}-\frac{\int \frac{3x\left(\left(3b^2-14ac\right)x^2+3ab\right)}{cx^4+bx^2+a}d\sqrt{x}}{3c}}{4(b^2-4ac)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{x^{7/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{bx^{3/2}}{c}-\frac{\int \frac{x\left(\left(3b^2-14ac\right)x^2+3ab\right)}{cx^4+bx^2+a}d\sqrt{x}}{c}}{4(b^2-4ac)}\right)$$

$$\Big\downarrow \text{1834}$$

$$2\left(\frac{x^{7/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{bx^{3/2}}{c}-\frac{\frac{1}{2}\left(-\frac{3b^3-20abc}{\sqrt{b^2-4ac}}-14ac+3b^2\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+\frac{1}{2}\left(\frac{3b^3-20abc}{\sqrt{b^2-4ac}}-14ac+3b^2\right)\int \cdots}{c}}{4(b^2-4ac)}\right.$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx^{3/2}}{c}-\frac{\left(-\frac{3b^3-20abc}{\sqrt{b^2-4ac}}-14ac+3b^2\right)\int\frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+\left(\frac{3b^3-20abc}{\sqrt{b^2-4ac}}-14ac+3b^2\right)\int\frac{}{2cx^2+b+}}{c}}{4\left(b^2-4ac\right)}\right.$$

$\downarrow$ 827

$$2\left(\frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx^{3/2}}{c}-\frac{\left(\frac{3b^3-20abc}{\sqrt{b^2-4ac}}-14ac+3b^2\right)\left(\frac{\int\frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)}{}}{4\left(b^2-4a\right.}\right.$$

$\downarrow$ 218

$$2\left(\frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{bx^{3/2}}{c}-\frac{\left(\frac{3b^3-20abc}{\sqrt{b^2-4ac}}-14ac+3b^2\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}}{2\sqrt{2}\sqrt{c}}\right.}{}}{4\left(b^2\right.}\right.$$

$\downarrow$ 221

$$2\left(\frac{x^{7/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{bx^{3/2}}{c}-\frac{\left(\frac{3b^3-20abc}{\sqrt{b^2-4ac}}-14ac+3b^2\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{\cdots}}{\sqrt[4]{-\sqrt{b^2\cdots}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2\cdots}}}\cdots\right)}{\cdots}}{\cdots}\right.$$

input
```
Int[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
2*((x^(7/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b*x^(
3/2))/c - ((3*b^2 - 14*a*c + (3*b^3 - 20*a*b*c)/Sqrt[b^2 - 4*a*c])*(ArcTan
[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3
/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-
b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^
(1/4))) + (3*b^2 - 14*a*c - (3*b^3 - 20*a*b*c)/Sqrt[b^2 - 4*a*c])*(ArcTan[
(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/
4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
 + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(
1/4))))/c)/(4*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1701
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2
*p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1
] && GtQ[m, 2*n - 1]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.52 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.28

method	result	siz
derivativedivides	$\frac{-\frac{(2ac-b^2)x^{\frac{7}{2}}}{2c(4ac-b^2)}+\frac{ab\,x^{\frac{3}{2}}}{2c(4ac-b^2)}}{c\,x^4+b\,x^2+a}-\frac{\sum_{\_R=\operatorname{RootOf}(c\_Z^8+\_Z^4b+a)}\frac{\left((-14ac+3b^2)\_R^6+3b\_R^2a\right)\ln(\sqrt{x}-\_R)}{2\_R^7c+\_R^3b}}{8c(4ac-b^2)}$	149
default	$\frac{-\frac{(2ac-b^2)x^{\frac{7}{2}}}{2c(4ac-b^2)}+\frac{ab\,x^{\frac{3}{2}}}{2c(4ac-b^2)}}{c\,x^4+b\,x^2+a}-\frac{\sum_{\_R=\operatorname{RootOf}(c\_Z^8+\_Z^4b+a)}\frac{\left((-14ac+3b^2)\_R^6+3b\_R^2a\right)\ln(\sqrt{x}-\_R)}{2\_R^7c+\_R^3b}}{8c(4ac-b^2)}$	149

input
```
int(x^(13/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/4*(2*a*c-b^2)/c/(4*a*c-b^2)*x^(7/2)+1/4*a*b/c/(4*a*c-b^2)*x^(3/2))/(
c*x^4+b*x^2+a)-1/8/c/(4*a*c-b^2)*sum(((-14*a*c+3*b^2)*_R^6+3*b*_R^2*a)/(2*
_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14601 vs. 2(440) = 880.

Time = 20.29 (sec) , antiderivative size = 14601, normalized size of antiderivative = 27.24

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{13}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((b^2 - 2*a*c)*x^(7/2) + a*b*x^(3/2))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^
2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + integrate(1/4*((3*b^2 - 14*a*
c)*x^(5/2) + 3*a*b*sqrt(x))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2
 + (b^3*c - 4*a*b*c^2)*x^2), x)
```

Giac [F]

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{13}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(13/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 20.29 (sec) , antiderivative size = 28774, normalized size of antiderivative = 53.68

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(13/2)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
- ((x^(7/2)*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x^(3/2))/(2*c*(4*a*c
 - b^2)))/(a + b*x^2 + c*x^4) - atan(((((46036680704*a^12*c^12 - 110592*a^
3*b^18*c^3 + 4423680*a^4*b^16*c^4 - 77783040*a^5*b^14*c^5 + 788037632*a^6*
b^12*c^6 - 5065015296*a^7*b^10*c^7 + 21401960448*a^8*b^8*c^8 - 59401830400
*a^9*b^6*c^9 + 104312340480*a^10*b^4*c^10 - 104991817728*a^11*b^2*c^11)/(1
28*(16384*a^7*c^10 - b^14*c^3 + 28*a*b^12*c^4 - 336*a^2*b^10*c^5 + 2240*a^
3*b^8*c^6 - 8960*a^4*b^6*c^7 + 21504*a^5*b^4*c^8 - 28672*a^6*b^2*c^9)) - (
x^(1/2)*((81*b^8*(-(4*a*c - b^2)^15)^(1/2) - 81*b^23 + 741801984*a^11*b*c^
11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17*c^3 - 10588384*a^4*b^15*c^4 + 6
4704576*a^5*b^13*c^5 - 279571968*a^6*b^11*c^6 + 853174784*a^7*b^9*c^7 - 17
99626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*c^9 - 2038693888*a^10*b^3*c^10 +
 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) + 4023*a*b^21*c + 10746*a^2*b^4*c^
2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^2*c^3*(-(4*a*c - b^2)^15)^(1/2)
- 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(16777216*a^12*c^19 + b^24
*c^7 - 48*a*b^22*c^8 + 1056*a^2*b^20*c^9 - 14080*a^3*b^18*c^10 + 126720*a^
4*b^16*c^11 - 811008*a^5*b^14*c^12 + 3784704*a^6*b^12*c^13 - 12976128*a^7*
b^10*c^14 + 32440320*a^8*b^8*c^15 - 57671680*a^9*b^6*c^16 + 69206016*a^10*
b^4*c^17 - 50331648*a^11*b^2*c^18))))^(1/4)*(6576668672*a^11*c^13 + 36864*a
^3*b^16*c^5 - 1302528*a^4*b^14*c^6 + 20480000*a^5*b^12*c^7 - 185991168*a^6
*b^10*c^8 + 1061683200*a^7*b^8*c^9 - 3886022656*a^8*b^6*c^10 + 88835358...
```

Reduce [F]

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{13}{2}}}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int(x^(13/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(x^(13/2)/(c*x^4+b*x^2+a)^2,x)
```

3.931 $\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 535

$$\begin{aligned}\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}\,dx &= -\frac{\sqrt{x}(ab+(b^2-2ac)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} \\ &+ \frac{\left(10a-\frac{b^2}{c}-\frac{b(b^2-12ac)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\ &+ \frac{\left(10a-\frac{b^2}{c}+\frac{b(b^2-12ac)}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} \\ &+ \frac{\left(10a-\frac{b^2}{c}-\frac{b(b^2-12ac)}{c\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\ &+ \frac{\left(10a-\frac{b^2}{c}+\frac{b(b^2-12ac)}{c\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}\end{aligned}$$

output
```
-1/2*x^(1/2)*(a*b+(-2*a*c+b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/8*(10
*a-b^2/c-b*(-12*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1
/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)/(-b-(-4*a*
c+b^2)^(1/2))^(3/4)+1/8*(10*a-b^2/c+b*(-12*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))*
arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1
/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/8*(10*a-b^2/c-b*(-12*a*c+
b^2)/c/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2
)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)
+1/8*(10*a-b^2/c+b*(-12*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(
1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)/(
-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.44

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}\,dx =$$

$$-\frac{4\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log(\sqrt{x}-\#1)-c\log(\sqrt{x}-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]+\frac{\frac{4c\sqrt{x}(ab+b^2x^2-2acx^2)}{a+bx^2+cx^4}+\text{RootSum}\left[a+b\#1^4\right.}{8c^2}$$

input
```
Integrate[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/8*(4*RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] - c*Log[Sqrt[
x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ] + ((4*c*Sqrt[x]*(a*b + b^2*x^2 - 2*
a*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#1^8 & , (-4*b^3*Lo
g[Sqrt[x] - #1] + 15*a*b*c*Log[Sqrt[x] - #1] + 3*b^2*c*Log[Sqrt[x] - #1]*#
1^4 - 6*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(b^2 - 4*a*
c))/c^2
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1435, 1701, 1826, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x^6}{(cx^4+bx^2+a)^2}d\sqrt{x}$$

$$\Big\downarrow \text{1701}$$

$$2\left(\frac{x^{5/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{x^2(bx^2+10a)}{cx^4+bx^2+a}d\sqrt{x}}{4(b^2-4ac)}\right)$$

$$\Big\downarrow \text{1826}$$

$$2\left(\frac{x^{5/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{b\sqrt{x}}{c}-\frac{\int \frac{(b^2-10ac)x^2+ab}{cx^4+bx^2+a}d\sqrt{x}}{c}}{4(b^2-4ac)}\right)$$

$$\Big\downarrow \text{1752}$$

$$2\left(\frac{x^{5/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{\frac{b\sqrt{x}}{c}-\frac{\frac{1}{2}\left(-\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d\sqrt{x}+\frac{1}{2}\left(\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\int \frac{}{cx}}{c}}{4(b^2-4ac)}\right)$$

$$\Big\downarrow \text{756}$$

$$2\left(\frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{b\sqrt{x}}{c}-\frac{\frac{1}{2}\left(\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\left(-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int\frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}\cdots\right.}{\cdots}}{4\left(b^2-\cdots\right.}\cdots\right.$$

$$\Big\downarrow \text{218}$$

$$2\left(\frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{b\sqrt{x}}{c}-\frac{\frac{1}{2}\left(\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\left(-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}\cdots}}\right.}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3\cdots}}\cdots\right.}{\cdots}}{4\left(\cdots\right.}\cdots\right.$$

$$\Big\downarrow \text{221}$$

$$2\left(\frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\frac{b\sqrt{x}}{c}-\frac{\frac{1}{2}\left(\frac{b\left(b^2-12ac\right)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\cdots}{\sqrt[4]{-\sqrt{b^2\cdots}}}\right.}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-\cdots}\right.}\cdots\right.}{\cdots}}{\cdots}\cdots\right.$$

input
```
Int[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
2*((x^(5/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b*Sqr
t[x])/c - (((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*(-(ArcTa
n[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/
4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-
b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3
/4))))/2 + ((b^2 - 10*a*c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*c])*(-(ArcTa
n[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/
4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-
b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3
/4))))/2)/c)/(4*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1701
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2
*p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1
] && GtQ[m, 2*n - 1]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.27

method	result	size
derivativedivides	$\frac{-\frac{\left(2ac-b^2\right)x^{\frac{5}{2}}}{2c\left(4ac-b^2\right)}+\frac{ab\sqrt{x}}{2c\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\left(10ac-b^2\right)\_R^4-ab\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{8c\left(4ac-b^2\right)}$	146
default	$\frac{-\frac{\left(2ac-b^2\right)x^{\frac{5}{2}}}{2c\left(4ac-b^2\right)}+\frac{ab\sqrt{x}}{2c\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\left(10ac-b^2\right)\_R^4-ab\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{8c\left(4ac-b^2\right)}$	146

input
```
int(x^(11/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/4*(2*a*c-b^2)/c/(4*a*c-b^2)*x^(5/2)+1/4*a*b/c/(4*a*c-b^2)*x^(1/2))/(
c*x^4+b*x^2+a)+1/8/c/(4*a*c-b^2)*sum(((10*a*c-b^2)*_R^4-a*b)/(2*_R^7*c+_R^
3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9180 vs. $2(443) = 886$.

Time = 1.98 (sec) , antiderivative size = 9180, normalized size of antiderivative = 17.16

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{11}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*x^(9/2) + 2*a*x^(5/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*x^2) + integrate(-1/4*(b*x^(7/2) + 10*a*x^(3/2))/((b^2*c -
4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

Giac [F]

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{11}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(11/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 26.05 (sec) , antiderivative size = 31964, normalized size of antiderivative = 59.75

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(11/2)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
2*atan(((((9*a^3*b^9 - 397*a^4*b^7*c + 130000*a^7*b*c^4 + 6549*a^5*b^5*c^2
 - 47800*a^6*b^3*c^3)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*
c^3 - 256*a^3*b^2*c^4)) + ((x^(1/2)*(1006632960*a^10*b*c^11 + 4096*a^3*b^1
5*c^4 + 147456*a^4*b^13*c^5 - 4915200*a^5*b^11*c^6 + 53739520*a^6*b^9*c^7
- 298844160*a^7*b^7*c^8 + 918552576*a^8*b^5*c^9 - 1493172224*a^9*b^3*c^10)
)/(16*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*
b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) - ((-(b^21 + b^6*(-(4*a*c
- b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^
15*c^3 + 404160*a^4*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6
 - 50503680*a^7*b^7*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 -
2500*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(
4*a*c - b^2)^15)^(1/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(1677
7216*a^12*c^17 + b^24*c^5 - 48*a*b^22*c^6 + 1056*a^2*b^20*c^7 - 14080*a^3*
b^18*c^8 + 126720*a^4*b^16*c^9 - 811008*a^5*b^14*c^10 + 3784704*a^6*b^12*c
^11 - 12976128*a^7*b^10*c^12 + 32440320*a^8*b^8*c^13 - 57671680*a^9*b^6*c^
14 + 69206016*a^10*b^4*c^15 - 50331648*a^11*b^2*c^16)))^(1/4)*(167772160*a
^9*c^11 + 40960*a^3*b^12*c^5 - 983040*a^4*b^10*c^6 + 9830400*a^5*b^8*c^7 -
 52428800*a^6*b^6*c^8 + 157286400*a^7*b^4*c^9 - 251658240*a^8*b^2*c^10)*1i
)/(2*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^
4)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) + 73728000*a^10*b*c^10 + 2...
```

Reduce [F]

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{11}{2}}}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int(x^(11/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(x^(11/2)/(c*x^4+b*x^2+a)^2,x)
```

3.932 $\int \frac{x^{9/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 471

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^2}\,dx = \frac{x^{3/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b^2+12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}c^{3/4}(b^2-4ac)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\left(b-\frac{b^2+12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}c^{3/4}(b^2-4ac)\sqrt[4]{-b+\sqrt{b^2-4ac}}} - \frac{\left(b^2+12ac+b\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}c^{3/4}(b^2-4ac)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2+12ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}c^{3/4}(b^2-4ac)\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x^(3/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/8*(b^2+12*a*c+b*(-4
*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1
/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/8*
(b-(12*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4
*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2
))^(1/4)-1/8*(b^2+12*a*c+b*(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(
1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(3/2)/(-b
-(-4*a*c+b^2)^(1/2))^(1/4)-1/8*(b-(12*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctanh
(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-
4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.40

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{8}\left(\frac{4x^{3/2}(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{4\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(\sqrt{x}-\#1)}{b\#1+2c\#1^5}\&\right]}{c} + \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{-4b^2\log(\sqrt{x}-\#1)+10ac\log(\sqrt{x}-\#1)+bc\log(\sqrt{x}-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{c(b^2-4ac)}\right)$$

input
```
Integrate[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((4*x^(3/2)*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (4*RootSu
m[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1 + 2*c*#1^5) & ])/c + Roo
tSum[a + b*#1^4 + c*#1^8 & , (-4*b^2*Log[Sqrt[x] - #1] + 10*a*c*Log[Sqrt[x
] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(c*(b^2 - 4*a*
c))/8
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1435, 1701, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \; 1435$$

$$2\int \frac{x^5}{\left(cx^4+bx^2+a\right)^2}d\sqrt{x}$$

$$\Big\downarrow \; 1701$$

$$2\left(\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{x\left(6a-bx^2\right)}{cx^4+bx^2+a}d\sqrt{x}}{4\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \; 1834$$

$$2\left(\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{1}{2}\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}-\frac{1}{2}\left(\frac{12ac+b^2}{\sqrt{b^2-4ac}}+b\right)\int \frac{2x}{2cx^2+b+\sqrt{b^2-4ac}}d\right.}{4\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\left(\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}\right)-\left(\frac{12ac+b^2}{\sqrt{b^2-4ac}}+b\right)\int \frac{x}{2cx^2+b+\sqrt{b^2-4ac}}d\right.}{4\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 827$$

$$2\left(\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\left(\left(\frac{12ac+b^2}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)\right)-(}{4\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \; 218$$

$$2\left(\frac{x^{3/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\left(\left(\frac{12ac+b^2}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)\right)}{4(b^2-}\right.$$

$$\Big\downarrow \text{221}$$

$$2\left(\frac{x^{3/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\left(\left(\frac{12ac+b^2}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}}}\right.}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}}}\right.\right.}{}\right.$$

input
```
Int[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
2*((x^(3/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (-((b +
 (b^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b -
 Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/
4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2
*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)))) - (b - (b^2 + 12*a*c)/S
qrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*
(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/(4*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1701
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2
*p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1
] && GtQ[m, 2*n - 1]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.26

method	result	size
derivativedivides	$\frac{-\frac{b\,x^{\frac{7}{2}}}{2\left(4ac-b^{2}\right)}-\frac{a\,x^{\frac{3}{2}}}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{6}b+6\_R^{2}a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{32ac-8b^{2}}$	121
default	$\frac{-\frac{b\,x^{\frac{7}{2}}}{2\left(4ac-b^{2}\right)}-\frac{a\,x^{\frac{3}{2}}}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-\_R^{6}b+6\_R^{2}a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{32ac-8b^{2}}$	121

input
```
int(x^(9/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/4*b/(4*a*c-b^2)*x^(7/2)-1/2*a/(4*a*c-b^2)*x^(3/2))/(c*x^4+b*x^2+a)+1
/8/(4*a*c-b^2)*sum((-_R^6*b+6*_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=
RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11817 vs. 2(375) = 750.

Time = 4.69 (sec) , antiderivative size = 11817, normalized size of antiderivative = 25.09

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{9}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*x^(7/2) + 2*a*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*x^2) - integrate(-1/4*(b*x^(5/2) - 6*a*sqrt(x))/((b^2*c - 4
*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

Giac [F]

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{9}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(9/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 19.53 (sec) , antiderivative size = 23808, normalized size of antiderivative = 50.55

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(9/2)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
- ((a*x^(3/2))/(4*a*c - b^2) + (b*x^(7/2))/(2*(4*a*c - b^2)))/(a + b*x^2 +
 c*x^4) - atan(((((5435817984*a^10*b*c^10 - 4096*a^3*b^15*c^3 + 1425408*a^
4*b^13*c^4 - 32833536*a^5*b^11*c^5 + 323747840*a^6*b^9*c^6 - 1714421760*a^
7*b^7*c^7 + 5121245184*a^8*b^5*c^8 - 8170504192*a^9*b^3*c^9)/(128*(b^14 -
16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8960*a^4*b^6*c^4 - 2
1504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) - (x^(1/2)*((b^4*(-(4
*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752
*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*
c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c -
 b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8192
*(16777216*a^12*c^15 + b^24*c^3 - 48*a*b^22*c^4 + 1056*a^2*b^20*c^5 - 1408
0*a^3*b^18*c^6 + 126720*a^4*b^16*c^7 - 811008*a^5*b^14*c^8 + 3784704*a^6*b
^12*c^9 - 12976128*a^7*b^10*c^10 + 32440320*a^8*b^8*c^11 - 57671680*a^9*b^
6*c^12 + 69206016*a^10*b^4*c^13 - 50331648*a^11*b^2*c^14)))^(1/4)*(1207959
552*a^10*c^11 - 204800*a^3*b^14*c^4 + 5210112*a^4*b^12*c^5 - 56229888*a^5*
b^10*c^6 + 332922880*a^6*b^8*c^7 - 1163919360*a^7*b^6*c^8 + 2390753280*a^8
*b^4*c^9 - 2650800128*a^9*b^2*c^10))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^
8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10
*c)))*((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2
*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5...
```

Reduce [F]

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{9}{2}}}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int(x^(9/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(x^(9/2)/(c*x^4+b*x^2+a)^2,x)
```

3.933 $\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 483

$$
\begin{aligned}
\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx &= \frac{\sqrt{x}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} \\
&- \frac{\left(3b^2+4ac+3b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\
&+ \frac{\left(3b^2+4ac-3b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)^{3/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} \\
&- \frac{\left(3b^2+4ac+3b\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\
&+ \frac{\left(3b^2+4ac-3b\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)^{3/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}
\end{aligned}
$$

output
```
1/2*x^(1/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/8*(3*b^2+4*a*c+3*b*
(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))
^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1
/8*(3*b^2+4*a*c-3*b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b
+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c
+b^2)^(1/2))^(3/4)-1/8*(3*b^2+4*a*c+3*b*(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4
)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b
^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/8*(3*b^2+4*a*c-3*b*(-4*a*c+b^2)^
(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3
/4)/c^(1/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.40

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{8}\left(\frac{4\sqrt{x}(2a+bx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{4\mathrm{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(\sqrt{x}-\#1)}{b\#1^3+2c\#1^7}\&\right]}{c} + \frac{\mathrm{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{-4b^2\log(\sqrt{x}-\#1)+14ac\log(\sqrt{x}-\#1)+3bc\log(\sqrt{x}-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{c(b^2-4ac)}\right)$$

input
```
Integrate[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((4*Sqrt[x]*(2*a + b*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (4*RootSu
m[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1^3 + 2*c*#1^7) & ])/c + R
ootSum[a + b*#1^4 + c*#1^8 & , (-4*b^2*Log[Sqrt[x] - #1] + 14*a*c*Log[Sqrt
[x] - #1] + 3*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(c*(b^2
- 4*a*c)))/8
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1435, 1701, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x^4}{(cx^4+bx^2+a)^2}d\sqrt{x}$$

$$\Big\downarrow \text{1701}$$

$$2\left(\frac{\sqrt{x}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{2a-3bx^2}{cx^4+bx^2+a}d\sqrt{x}}{4(b^2-4ac)}\right)$$

$$\Big\downarrow \text{1752}$$

$$2\left(\frac{\sqrt{x}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{1}{2}\left(3b-\frac{4ac+3b^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d\sqrt{x}-\frac{1}{2}\left(\frac{4ac+3b^2}{\sqrt{b^2-4ac}}+3b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2\right.}}}{4(b^2-4ac)}\right.$$

$$\Big\downarrow \text{756}$$

$$2\left(\frac{\sqrt{x}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{-\frac{1}{2}\left(\frac{4ac+3b^2}{\sqrt{b^2-4ac}}+3b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)-}{4(b^2-4a}\right.$$

$$\Big\downarrow \text{218}$$

$$2\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{1}{2}\left(\frac{4ac+3b^2}{\sqrt{b^2-4ac}}+3b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x}}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right.}{4\left(b^2\right.}\right.$$

$\Big\downarrow$ 221

$$2\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-\frac{1}{2}\left(\frac{4ac+3b^2}{\sqrt{b^2-4ac}}+3b\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4a}}}\right.}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right.}\right.}{}\right.$$

input

```
Int[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

output

```
2*((Sqrt[x]*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (-1/2*(
(3*b + (3*b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[
x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(
1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)))) - ((3*b - (3*b^2
+ 4*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt
[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) -
ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)
*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2)/(4*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1701
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2
*p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1
] && GtQ[m, 2*n - 1]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.24

method	result	size
derivativedivides	$\frac{-\frac{b\,x^{\frac{5}{2}}}{2\left(4ac-b^{2}\right)}-\frac{a\sqrt{x}}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-3\_R^{4}b+2a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{32ac-8b^{2}}$	118
default	$\frac{-\frac{b\,x^{\frac{5}{2}}}{2\left(4ac-b^{2}\right)}-\frac{a\sqrt{x}}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-3\_R^{4}b+2a\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{32ac-8b^{2}}$	118

input
```
int(x^(7/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/4*b/(4*a*c-b^2)*x^(5/2)-1/2*a/(4*a*c-b^2)*x^(1/2))/(c*x^4+b*x^2+a)+1
/8/(4*a*c-b^2)*sum((-3*_R^4*b+2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=Roo
tOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7244 vs. 2(383) = 766.

Time = 0.45 (sec) , antiderivative size = 7244, normalized size of antiderivative = 15.00

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{7}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c*x^(9/2) + b*x^(5/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c +
(b^3 - 4*a*b*c)*x^2) - integrate(-1/4*(2*c*x^(7/2) + 5*b*x^(3/2))/((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

Giac [F]

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{7}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(7/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 23.95 (sec) , antiderivative size = 26432, normalized size of antiderivative = 54.72

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(7/2)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
atan((((((x^(1/2)*(603979776*a^9*b*c^11 - 102400*a^2*b^15*c^4 + 2605056*a^
3*b^13*c^5 - 28114944*a^4*b^11*c^6 + 166461440*a^5*b^9*c^7 - 581959680*a^6
*b^7*c^8 + 1195376640*a^7*b^5*c^9 - 1325400064*a^8*b^3*c^10))/(16*(b^12 +
4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 614
4*a^5*b^2*c^5 - 24*a*b^10*c)) - ((-(81*b^17 - 81*b^2*(-(4*a*c - b^2)^15)^(
1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a
^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c
^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(b^24*c + 1677
7216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3 - 14080*a^3*b^18*c^4 +
126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784704*a^6*b^12*c^7 - 1297612
8*a^7*b^10*c^8 + 32440320*a^8*b^8*c^9 - 57671680*a^9*b^6*c^10 + 69206016*a
^10*b^4*c^11 - 50331648*a^11*b^2*c^12)))^(1/4)*(83886080*a^8*b*c^10 + 2048
0*a^2*b^13*c^4 - 491520*a^3*b^11*c^5 + 4915200*a^4*b^9*c^6 - 26214400*a^5*
b^7*c^7 + 78643200*a^6*b^5*c^8 - 125829120*a^7*b^3*c^9))/(2*(b^8 + 256*a^4
*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(81*b^17 - 81*b^
2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*
a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*
c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c + 4*a*c*(-(4*a*c - b^2)^15)^(1/2
))/(8192*(b^24*c + 16777216*a^12*c^13 - 48*a*b^22*c^2 + 1056*a^2*b^20*c^3
- 14080*a^3*b^18*c^4 + 126720*a^4*b^16*c^5 - 811008*a^5*b^14*c^6 + 3784...
```

Reduce [F]

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{7}{2}}}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int(x^(7/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(x^(7/2)/(c*x^4+b*x^2+a)^2,x)
```

3.934 $\int \frac{x^{5/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 450

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^2}\,dx = -\frac{x^{3/2}(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt[4]{c}\left(4b+\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}(b^2-4ac)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}(b^2-4ac)^{3/2}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(4b+\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}(b^2-4ac)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}(b^2-4ac)^{3/2}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
-1/2*x^(3/2)*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*c^(1/4)*(4*b+(-4
*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1
/4))*2^(1/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4)*
(4*b-(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(
1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*
c^(1/4)*(4*b+(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a
*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(
1/4)-1/4*c^(1/4)*(4*b-(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/
(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)
^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.24

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^2}\,dx = \frac{-\frac{4x^{3/2}(b+2cx^2)}{a+bx^2+cx^4} + \text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{3b\log(\sqrt{x}-\#1)-2c\log(\sqrt{x}-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{8(b^2-4ac)}$$

input
```
Integrate[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((-4*x^(3/2)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#
1^8 & , (3*b*Log[Sqrt[x] - #1] - 2*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#
1^5) & ])/(8*(b^2 - 4*a*c))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1435, 1700, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$\Big\downarrow$ 1435

$$2\int \frac{x^3}{\left(cx^4+bx^2+a\right)^2}d\sqrt{x}$$

$\Big\downarrow$ 1700

$$2\left(\frac{\int \frac{x\left(3b-2cx^2\right)}{cx^4+bx^2+a}d\sqrt{x}}{4\left(b^2-4ac\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\Big\downarrow$ 1834

$$2\left(\frac{-c\left(1-\frac{4b}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}-c\left(\frac{4b}{\sqrt{b^2-4ac}}+1\right)\int \frac{2x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{4\left(b^2-4ac\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx\right.}\right.$$

$\Big\downarrow$ 27

$$2\left(\frac{-2c\left(1-\frac{4b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}-2c\left(\frac{4b}{\sqrt{b^2-4ac}}+1\right)\int \frac{x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{4\left(b^2-4ac\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+\right.}\right.$$

$\Big\downarrow$ 827

$$2\left(\frac{-2c\left(\frac{4b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)-2c\left(1-\frac{4b}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{\sqrt{b^2-}}}}{2\sqrt{2}\sqrt{c}}\right.}{4\left(b^2-4ac\right)}\right.$$

$\Big\downarrow$ 218

$$2\left(\frac{-2c\left(\frac{4b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)-2c\left(1-\frac{4b}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\cdots\right.}{2\,2^{3/4}c^{3/4}\cdots}\right.}{4\left(b^2-4ac\right)}\right.$$

$\Big\downarrow$ 221

$$2\left(\frac{-2c\left(\frac{4b}{\sqrt{b^2-4ac}}+1\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)-2c\left(1-\frac{4b}{\sqrt{b^2-4ac}}\right)\left(\cdots\right.}{4\left(b^2-4ac\right)}\right.$$

input
```
Int[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
2*(-1/4*(x^(3/2)*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (-2*
c*(1 + (4*b)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
 - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^
(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) - 2*c*(1 - (4*b)/Sqrt[b^2 -
 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]
/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqr
t[b^2 - 4*a*c])^(1/4))))/(4*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1700
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*((a + b*x^n + c*
x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] - Simp[d^n/(n*(p + 1)*(b^2
- 4*a*c))   Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^
n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ
[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
 1] && LeQ[m, 2*n - 1]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.27

method	result	size
derivativedivides	$\frac{\frac{2c\,x^{\frac{7}{2}}}{8ac-2b^{2}}+\frac{2b\,x^{\frac{3}{2}}}{16ac-4b^{2}}}{c\,x^{4}+b\,x^{2}+a}-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-2\_R^{6}c+3\_R^{2}b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{8\left(4ac-b^{2}\right)}$	121
default	$\frac{\frac{2c\,x^{\frac{7}{2}}}{8ac-2b^{2}}+\frac{2b\,x^{\frac{3}{2}}}{16ac-4b^{2}}}{c\,x^{4}+b\,x^{2}+a}-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-2\_R^{6}c+3\_R^{2}b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{8\left(4ac-b^{2}\right)}$	121

input
```
int(x^(5/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(1/2*c/(4*a*c-b^2)*x^(7/2)+1/4*b/(4*a*c-b^2)*x^(3/2))/(c*x^4+b*x^2+a)-1/
8/(4*a*c-b^2)*sum((-2*_R^6*c+3*_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R
=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10601 vs. 2(350) = 700.

Time = 1.88 (sec) , antiderivative size = 10601, normalized size of antiderivative = 23.56

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{5}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c*x^(7/2) + b*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c +
(b^3 - 4*a*b*c)*x^2) + integrate(-1/4*(2*c*x^(5/2) - 3*b*sqrt(x))/((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2), x)
```

Giac [F]

$$\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{x^{\frac{5}{2}}}{\left(cx^4+bx^2+a\right)^2}\,dx$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(5/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 19.23 (sec) , antiderivative size = 21913, normalized size of antiderivative = 48.70

$$\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
int(x^(5/2)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((b*x^(3/2))/(2*(4*a*c - b^2)) + (c*x^(7/2))/(4*a*c - b^2))/(a + b*x^2 + c
*x^4) - atan(((((110592*a*b^16*c^4 - 134217728*a^9*c^12 - 2433024*a^2*b^14
*c^5 + 21200896*a^3*b^12*c^6 - 87687168*a^4*b^10*c^7 + 133693440*a^5*b^8*c
^8 + 211812352*a^6*b^6*c^9 - 1031798784*a^7*b^4*c^10 + 1107296256*a^8*b^2*
c^11)/(128*(b^14 - 16384*a^7*c^7 + 336*a^2*b^10*c^2 - 2240*a^3*b^8*c^3 + 8
960*a^4*b^6*c^4 - 21504*a^5*b^4*c^5 + 28672*a^6*b^2*c^6 - 28*a*b^12*c)) -
(x^(1/2)*(-(81*b^17 + 81*b^2*(-(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8
+ 960*a^2*b^13*c^2 + 84480*a^3*b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5
*b^7*c^5 - 5259264*a^6*b^5*c^6 + 4587520*a^7*b^3*c^7 - 1184*a*b^15*c - 4*a
*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a*b^24 + 16777216*a^13*c^12 - 48*a^2*
b^22*c + 1056*a^3*b^20*c^2 - 14080*a^4*b^18*c^3 + 126720*a^5*b^16*c^4 - 81
1008*a^6*b^14*c^5 + 3784704*a^7*b^12*c^6 - 12976128*a^8*b^10*c^7 + 3244032
0*a^9*b^8*c^8 - 57671680*a^10*b^6*c^9 + 69206016*a^11*b^4*c^10 - 50331648*
a^12*b^2*c^11)))^(1/4)*(134217728*a^9*c^12 + 36864*a*b^16*c^4 - 909312*a^2
*b^14*c^5 + 9469952*a^3*b^12*c^6 - 53870592*a^4*b^10*c^7 + 180879360*a^5*b
^8*c^8 - 362807296*a^6*b^6*c^9 + 427819008*a^7*b^4*c^10 - 301989888*a^8*b^
2*c^11))/(16*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3
840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(81*b^17 + 81*b^2*(-
(4*a*c - b^2)^15)^(1/2) - 983040*a^8*b*c^8 + 960*a^2*b^13*c^2 + 84480*a^3*
b^11*c^3 - 719360*a^4*b^9*c^4 + 2727936*a^5*b^7*c^5 - 5259264*a^6*b^5*c...
```

Reduce [F]

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{5}{2}}}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int(x^(5/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(x^(5/2)/(c*x^4+b*x^2+a)^2,x)
```

3.935 $\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 442

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2} \, dx = -\frac{\sqrt{x}(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{c^{3/4}\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}(b^2-4ac)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}(b^2-4ac)\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(3+\frac{4b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}(b^2-4ac)\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}(b^2-4ac)\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
-1/2*x^(1/2)*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(3/4)*(3+4*b/(
-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^
(1/4))*2^(3/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(3/4)*(3-4
*b/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/
2))^(1/4))*2^(3/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(3/4)*
(3+4*b/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2
)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(
3/4)*(3-4*b/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*
c+b^2)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.25

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2}\,dx = \frac{-\frac{4\sqrt{x}(b+2cx^2)}{a+bx^2+cx^4}+\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log\left(\sqrt{x}-\#1\right)-6c\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{8\,(b^2-4ac)}$$

input
```
Integrate[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((-4*Sqrt[x]*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1^4 + c*#
1^8 & , (b*Log[Sqrt[x] - #1] - 6*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#
1^7) & ])/(8*(b^2 - 4*a*c))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1435, 1700, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2}\,dx$$

$\Big\downarrow$ 1435

$$2\int \frac{x^2}{(cx^4+bx^2+a)^2}d\sqrt{x}$$

$\Big\downarrow$ 1700

$$2\left(\frac{\int \frac{b-6cx^2}{cx^4+bx^2+a}d\sqrt{x}}{4(b^2-4ac)}-\frac{\sqrt{x}(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$\Big\downarrow$ 1752

$$2\left(\frac{-c\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d\sqrt{x}-c\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d\sqrt{x}}{4(b^2-4ac)}-\frac{\sqrt{x}(b+2cx^2)}{4(b^2-4ac)(a+bx^2}\right.$$

$\Big\downarrow$ 756

$$2\left(\frac{-c\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)-c\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}\cdots}}{\sqrt{\sqrt{b^2-4}\cdots}}\right.}{4(b^2-4ac)}\right.$$

$\Big\downarrow$ 218

$$2\left(\frac{-c\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-c\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-}\cdots}}}{\sqrt{\cdots}}\right.}{4(b^2-4ac)}\right.$$

$\Big\downarrow$ 221

$$2\left(\frac{-c\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-c\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\left(-\right.}{4\left(b^2-4ac\right)}\right.$$

input
```
Int[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
2*(-1/4*(Sqrt[x]*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (-(c
*(3 + (4*b)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - S
qrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)))
 - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1
/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)))) - c*(3 - (4*b)/Sqrt[b^2 - 4*
a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/
(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^
2 - 4*a*c])^(3/4))))/(4*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1700

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*((a + b*x^n + c*
x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] - Simp[d^n/(n*(p + 1)*(b^2
- 4*a*c))   Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^
n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ
[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
 1] && LeQ[m, 2*n - 1]
```

rule 1752

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.27

method	result	size
derivativedivides	$\frac{\frac{2c\,x^{\frac{5}{2}}}{8ac-2b^2}+\frac{2b\sqrt{x}}{16ac-4b^2}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^8+\_Z^4b+a\right)}\frac{\left(6\_R^4c-b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{32ac-8b^2}$	118
default	$\frac{\frac{2c\,x^{\frac{5}{2}}}{8ac-2b^2}+\frac{2b\sqrt{x}}{16ac-4b^2}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^8+\_Z^4b+a\right)}\frac{\left(6\_R^4c-b\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^7c+\_R^3b}}{32ac-8b^2}$	118

input
```
int(x^(3/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(1/2*c/(4*a*c-b^2)*x^(5/2)+1/4*b/(4*a*c-b^2)*x^(1/2))/(c*x^4+b*x^2+a)+1/
8/(4*a*c-b^2)*sum((6*_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(
_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8211 vs. 2(350) = 700.

Time = 0.95 (sec) , antiderivative size = 8211, normalized size of antiderivative = 18.58

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{3}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*x^(9/2) + (b^2 - 2*a*c)*x^(5/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + integrate(-1/4*(b*c*x^(7/2) +
(b^2 + 6*a*c)*x^(3/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a
*b^3 - 4*a^2*b*c)*x^2), x)
```

Giac [F]

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x^{\frac{3}{2}}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(3/2)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 23.64 (sec) , antiderivative size = 28713, normalized size of antiderivative = 64.96

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(x^(3/2)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
atan(((((((b^4*(-(4*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96
*a^2*b^15*c^2 - 2752*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^
5 + 3350528*a^6*b^7*c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 32
4*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b
^2)^15)^(1/2))/(8192*(a^3*b^24 + 16777216*a^15*c^12 - 48*a^4*b^22*c + 1056
*a^5*b^20*c^2 - 14080*a^6*b^18*c^3 + 126720*a^7*b^16*c^4 - 811008*a^8*b^14
*c^5 + 3784704*a^9*b^12*c^6 - 12976128*a^10*b^10*c^7 + 32440320*a^11*b^8*c
^8 - 57671680*a^12*b^6*c^9 + 69206016*a^13*b^4*c^10 - 50331648*a^14*b^2*c^
11)))^(1/4)*(100663296*a^8*c^11 + 4096*a*b^14*c^4 - 73728*a^2*b^12*c^5 + 3
93216*a^3*b^10*c^6 + 655360*a^4*b^8*c^7 - 15728640*a^5*b^6*c^8 + 69206016*
a^6*b^4*c^9 - 134217728*a^7*b^2*c^10))/(2*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*
c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)) - (x^(1/2)*(2048*b^17*c^4 - 30720*a*b
^15*c^5 + 100663296*a^8*b*c^12 + 73728*a^2*b^13*c^6 + 1212416*a^3*b^11*c^7
 - 9830400*a^4*b^9*c^8 + 26738688*a^5*b^7*c^9 - 10485760*a^6*b^5*c^10 - 75
497472*a^7*b^3*c^11))/(8*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3
*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*((b^4*(-(4
*a*c - b^2)^15)^(1/2) - b^19 - 12386304*a^9*b*c^9 + 96*a^2*b^15*c^2 - 2752
*a^3*b^13*c^3 + 55296*a^4*b^11*c^4 - 585216*a^5*b^9*c^5 + 3350528*a^6*b^7*
c^6 - 10665984*a^7*b^5*c^7 + 17891328*a^8*b^3*c^8 + 324*a^2*c^2*(-(4*a*c -
 b^2)^15)^(1/2) + 3*a*b^17*c + 27*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2))/(8...
```

Reduce [F]

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2} \, dx = \int \frac{x^{\frac{3}{2}}}{(c\,x^4+b\,x^2+a)^2} dx$$

input
```
int(x^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(x^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

3.936 $\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 489

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2} \, dx = \frac{x^{3/2}(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}a(b^2-4ac)\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(b+\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}a(b^2-4ac)\sqrt[4]{-b+\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}a(b^2-4ac)\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(b+\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}a(b^2-4ac)\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
1/2*x^(3/2)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/8*c^(1/4)
*(b-(-20*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(
-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-4*a*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(
1/4)+1/8*c^(1/4)*(b+(-20*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/
4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-4*a*c+b^2)/(-b+(-4*a
*c+b^2)^(1/2))^(1/4)-1/8*c^(1/4)*(b-(-20*a*c+b^2)/(-4*a*c+b^2)^(1/2))*arct
anh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-4*a
*c+b^2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/8*c^(1/4)*(b+(-20*a*c+b^2)/(-4*a*c
+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
)*2^(1/4)/a/(-4*a*c+b^2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2}\,dx = -\frac{\frac{4x^{3/2}(b^2-2ac+bcx^2)}{a+bx^2+cx^4}+\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^2\log\left(\sqrt{x}-\#1\right)-10ac\log\left(\sqrt{x}-\#1\right)+bc\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1+2c\#1^5}\&\right]}{8a\left(-b^2+4ac\right)}$$

input
```
Integrate[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

output
```
-1/8*((4*x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a
+ b*#1^4 + c*#1^8 & , (b^2*Log[Sqrt[x] - #1] - 10*a*c*Log[Sqrt[x] - #1] +
b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(a*(-b^2 + 4*a*c))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1435, 1702, 25, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x}{(cx^4+bx^2+a)^2}d\sqrt{x}$$

$$\Big\downarrow \text{1702}$$

$$2\left(\frac{x^{3/2}(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{x(b^2+cx^2b-10ac)}{cx^4+bx^2+a}d\sqrt{x}}{4a(b^2-4ac)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{x(b^2+cx^2b-10ac)}{cx^4+bx^2+a}d\sqrt{x}}{4a(b^2-4ac)}+\frac{x^{3/2}(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{1834}$$

$$2\left(\frac{\frac{1}{2}c\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+\frac{1}{2}c\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{4a(b^2-4ac)}+\frac{x^{3/2}(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{c\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+c\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{4a(b^2-4ac)}+\frac{x^{3/2}(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \text{827}$$

$$2\left(\frac{c\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{\sqrt{b^2-4ac}-b}}\ldots}{2\sqrt{2}\sqrt{c}}\right.}{4a\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \ 218$$

$$2\left(\frac{c\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)+c\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{\ldots}}{\sqrt[4]{\sqrt{b\ldots}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{\sqrt{\ldots}}}\right.}{4a\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \ 221$$

$$2\left(\frac{c\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\left(\frac{\arctan\ldots}{2\ 2^{3/4}\ldots}\right.}{4a\left(b^2-4ac\right)}\right.$$

input
```
Int[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

output
```
2*((x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (c*(b - (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 -
 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c
])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + c*(b + (b^
2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
- ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(
3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/(4*a*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1702
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*
(p + 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && IGtQ[n, 0] && ILtQ[p, -1]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.30

method	result	size
derivativedivides	$\frac{-\frac{b c \,x^{\frac{7}{2}}}{2 a \left(4 a c -b^{2}\right)}+\frac{\left(2 a c -b^{2}\right) x^{\frac{3}{2}}}{2 a \left(4 a c -b^{2}\right)}}{c \,x^{4}+b \,x^{2}+a}+\frac{\sum_{\_R =\operatorname{RootOf}\left(c \,\_Z^{8}+\_Z^{4} b +a \right)}\frac{\left(-b c \,\_R^{6}+\left(10 a c -b^{2}\right) \_R^{2}\right) \ln \left(\sqrt{x}-\_R \right)}{2 \_R^{7} c +\_R^{3} b}}{8 a \left(4 a c -b^{2}\right)}$	149
default	$\frac{-\frac{b c \,x^{\frac{7}{2}}}{2 a \left(4 a c -b^{2}\right)}+\frac{\left(2 a c -b^{2}\right) x^{\frac{3}{2}}}{2 a \left(4 a c -b^{2}\right)}}{c \,x^{4}+b \,x^{2}+a}+\frac{\sum_{\_R =\operatorname{RootOf}\left(c \,\_Z^{8}+\_Z^{4} b +a \right)}\frac{\left(-b c \,\_R^{6}+\left(10 a c -b^{2}\right) \_R^{2}\right) \ln \left(\sqrt{x}-\_R \right)}{2 \_R^{7} c +\_R^{3} b}}{8 a \left(4 a c -b^{2}\right)}$	149

input
```
int(x^(1/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/4*b/a/(4*a*c-b^2)*c*x^(7/2)+1/4*(2*a*c-b^2)/a/(4*a*c-b^2)*x^(3/2))/(
c*x^4+b*x^2+a)+1/8/a/(4*a*c-b^2)*sum((-b*c*_R^6+(10*a*c-b^2)*_R^2)/(2*_R^7
*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13225 vs. 2(397) = 794.

Time = 8.48 (sec) , antiderivative size = 13225, normalized size of antiderivative = 27.04

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{\sqrt{x}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*x^(7/2) + (b^2 - 2*a*c)*x^(3/2))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - integrate(-1/4*(b*c*x^(5/2) +
(b^2 - 10*a*c)*sqrt(x))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (
a*b^3 - 4*a^2*b*c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2}\,dx = \int \frac{\sqrt{x}}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x)/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 20.58 (sec) , antiderivative size = 26373, normalized size of antiderivative = 53.93

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
atan(((((2048*b^19*c^4 - 116736*a*b^17*c^5 - 10905190400*a^9*b*c^13 + 2852
864*a^2*b^15*c^6 - 39247872*a^3*b^13*c^7 + 335708160*a^4*b^11*c^8 - 185742
1312*a^5*b^9*c^9 + 6670516224*a^6*b^7*c^10 - 15042871296*a^7*b^5*c^11 + 19
386073088*a^8*b^3*c^12)/(64*(a^2*b^14 - 16384*a^9*c^7 - 28*a^3*b^12*c + 33
6*a^4*b^10*c^2 - 2240*a^5*b^8*c^3 + 8960*a^6*b^6*c^4 - 21504*a^7*b^4*c^5 +
 28672*a^8*b^2*c^6)) - (x^(1/2)*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1/2) +
73728000*a^10*b*c^10 + 2085*a^2*b^17*c^2 - 36320*a^3*b^15*c^3 + 404160*a^4
*b^13*c^4 - 3001344*a^5*b^11*c^5 + 15064576*a^6*b^9*c^6 - 50503680*a^7*b^7
*c^7 + 108380160*a^8*b^5*c^8 - 134676480*a^9*b^3*c^9 - 2500*a^3*c^3*(-(4*a
*c - b^2)^15)^(1/2) - 69*a*b^19*c + 525*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1
/2) - 39*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2))/(8192*(a^5*b^24 + 16777216*a^1
7*c^12 - 48*a^6*b^22*c + 1056*a^7*b^20*c^2 - 14080*a^8*b^18*c^3 + 126720*a
^9*b^16*c^4 - 811008*a^10*b^14*c^5 + 3784704*a^11*b^12*c^6 - 12976128*a^12
*b^10*c^7 + 32440320*a^13*b^8*c^8 - 57671680*a^14*b^6*c^9 + 69206016*a^15*
b^4*c^10 - 50331648*a^16*b^2*c^11)))^(1/4)*(3355443200*a^10*c^13 - 4096*a*
b^18*c^4 + 196608*a^2*b^16*c^5 - 4005888*a^3*b^14*c^6 + 45580288*a^4*b^12*
c^7 - 320471040*a^5*b^10*c^8 + 1448607744*a^6*b^8*c^9 - 4217372672*a^7*b^6
*c^10 + 7625244672*a^8*b^4*c^11 - 7751073792*a^9*b^2*c^12))/(16*(a^2*b^12
+ 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840
*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^21 + b^6*(-(4*a*c - b^2)^15)^(1...
```

Reduce [F]

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2}\,dx=\int \frac{\sqrt{x}}{(c\,x^4+b\,x^2+a)^2}dx$$

input
```
int(x^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(x^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

3.937 $\int \frac{1}{\sqrt{x}(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 503

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{\sqrt{x}(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} \\
&+ \frac{c^{3/4}\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\
&- \frac{c^{3/4}\left(3b^2-28ac+3b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} \\
&+ \frac{c^{3/4}\left(3b^2-28ac-3b\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\
&- \frac{c^{3/4}\left(3b^2-28ac+3b\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}
\end{aligned}
$$

output
```
1/2*x^(1/2)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/8*c^(3/4)
*(3*b^2-28*a*c-3*b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-
(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(
1/2))^(3/4)-1/8*c^(3/4)*(3*b^2-28*a*c+3*b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/
4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-4*a*c+b^2)^(
3/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/8*c^(3/4)*(3*b^2-28*a*c-3*b*(-4*a*c+b
^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
2^(3/4)/a/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/8*c^(3/4)*(3*
b^2-28*a*c+3*b*(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4
*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-4*a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2
))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^2}\,dx = -\frac{\frac{4\sqrt{x}(b^2-2ac+bcx^2)}{a+bx^2+cx^4}+\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{3b^2\log\left(\sqrt{x}-\#1\right)-14ac\log\left(\sqrt{x}-\#1\right)+3bc\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{8a\,(-b^2+4ac)}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/8*((4*Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a
+ b*#1^4 + c*#1^8 & , (3*b^2*Log[Sqrt[x] - #1] - 14*a*c*Log[Sqrt[x] - #1]
+ 3*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(a*(-b^2 + 4*a*c)
)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1435, 1683, 25, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x} \left(a+b x^2+c x^4\right)^2} \, dx$$

$$\Big\downarrow \text{1435}$$

$$2 \int \frac{1}{\left(c x^4+b x^2+a\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{1683}$$

$$2 \left(\frac{\sqrt{x} \left(-2 a c+b^2+b c x^2\right)}{4 a \left(b^2-4 a c\right) \left(a+b x^2+c x^4\right)}-\frac{\int -\frac{3 b^2+3 c x^2 b-14 a c}{c x^4+b x^2+a}d\sqrt{x}}{4 a \left(b^2-4 a c\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$2 \left(\frac{\int \frac{3 b^2+3 c x^2 b-14 a c}{c x^4+b x^2+a}d\sqrt{x}}{4 a \left(b^2-4 a c\right)}+\frac{\sqrt{x} \left(-2 a c+b^2+b c x^2\right)}{4 a \left(b^2-4 a c\right) \left(a+b x^2+c x^4\right)}\right)$$

$$\Big\downarrow \text{1752}$$

$$2 \left(\frac{\frac{1}{2} c \left(\frac{3 b^2-28 a c}{\sqrt{b^2-4 a c}}+3 b\right) \int \frac{1}{c x^2+\frac{1}{2} \left(b-\sqrt{b^2-4 a c}\right)}d\sqrt{x}-\frac{c \left(-3 b \sqrt{b^2-4 a c}-28 a c+3 b^2\right) \int \frac{1}{c x^2+\frac{1}{2} \left(b+\sqrt{b^2-4 a c}\right)}d\sqrt{x}}{2 \sqrt{b^2-4 a c}}}{4 a \left(b^2-4 a c\right)}+\frac{\sqrt{x} \left(-2 a c+\right.}{4 a \left(b^2-4 a c\right) (a}\right.$$

$$\Big\downarrow \text{756}$$

$$2 \left(\frac{\frac{1}{2} c \left(\frac{3 b^2-28 a c}{\sqrt{b^2-4 a c}}+3 b\right) \left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4 a c}-b-\sqrt{2} \sqrt{c} x}}d\sqrt{x}}{\sqrt{\sqrt{b^2-4 a c}-b}}-\frac{\int \frac{1}{\sqrt{2} \sqrt{c} x+\sqrt{\sqrt{b^2-4 a c}-b}}d\sqrt{x}}{\sqrt{\sqrt{b^2-4 a c}-b}}\right)-\frac{c \left(-3 b \sqrt{b^2-4 a c}-28 a c+3 b^2\right) \left(-\frac{\int \frac{}{\sqrt{-b-\sqrt{b}}}}{\sqrt{-}}\right.}{2}}{4 a \left(b^2-4 a c\right)}\right.$$

$$\Big\downarrow \; 218$$

$$2\left(\frac{\frac{1}{2}c\left(\frac{3b^2-28ac}{\sqrt{b^2-4ac}}+3b\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{2}\sqrt{c}x}}d\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-\frac{c\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\left(-\frac{\int \frac{}{\sqrt{}}}{}\right.}{}}{4a\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \; 221$$

$$2\left(\frac{\frac{1}{2}c\left(\frac{3b^2-28ac}{\sqrt{b^2-4ac}}+3b\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)-\frac{c\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)}{}}{4a\left(b^2-4ac\right)}\right.$$

input
```
Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

output
```
2*((Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (-1/2*(c*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sq
rt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/Sqrt
[b^2 - 4*a*c] + (c*(3*b + (3*b^2 - 28*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2
^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(
-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
))/2)/(4*a*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1683
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-
x)*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c +
 n*(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n
))^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && ILtQ[p, -1]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.29

method	result	size
derivativedivides	$\frac{-\frac{bc\,x^{\frac{5}{2}}}{2a\left(4ac-b^{2}\right)}+\frac{\left(2ac-b^{2}\right)\sqrt{x}}{2a\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-3\_R^{4}bc+14ac-3b^{2}\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{8a\left(4ac-b^{2}\right)}$	144
default	$\frac{-\frac{bc\,x^{\frac{5}{2}}}{2a\left(4ac-b^{2}\right)}+\frac{\left(2ac-b^{2}\right)\sqrt{x}}{2a\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-3\_R^{4}bc+14ac-3b^{2}\right)\ln\left(\sqrt{x}-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{8a\left(4ac-b^{2}\right)}$	144

input
```
int(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/4*b/a/(4*a*c-b^2)*c*x^(5/2)+1/4*(2*a*c-b^2)/a/(4*a*c-b^2)*x^(1/2))/(
c*x^4+b*x^2+a)+1/8/a/(4*a*c-b^2)*sum((-3*_R^4*b*c+14*a*c-3*b^2)/(2*_R^7*c+
_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10274 vs. 2(403) = 806.

Time = 4.17 (sec) , antiderivative size = 10274, normalized size of antiderivative = 20.43

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{1}{\left(cx^4+bx^2+a\right)^2\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((3*b^2*c - 14*a*c^2)*x^(9/2) + (3*b^3 - 13*a*b*c)*x^(5/2) + 4*(a*b^2
- 4*a^2*c)*sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^
2*b^3 - 4*a^3*b*c)*x^2) - integrate(1/4*((3*b^2*c - 14*a*c^2)*x^(7/2) + (3
*b^3 - 17*a*b*c)*x^(3/2))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4
 + (a^2*b^3 - 4*a^3*b*c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^2}\,dx=\int \frac{1}{(cx^4+bx^2+a)^2\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^2*sqrt(x)), x)
```

Mupad [B] (verification not implemented)

Time = 20.23 (sec) , antiderivative size = 35171, normalized size of antiderivative = 69.92

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^2}\,dx=\text{Too large to display}$$

input
```
int(1/(x^(1/2)*(a + b*x^2 + c*x^4)^2),x)
```

output
```
((x^(1/2)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^(5/2))/(2*a*(4*a*c -
 b^2)))/(a + b*x^2 + c*x^4) + atan(((((((81*b^8*(-(4*a*c - b^2)^15)^(1/2)
 - 81*b^23 + 741801984*a^11*b*c^11 - 90126*a^2*b^19*c^2 + 1201623*a^3*b^17
*c^3 - 10588384*a^4*b^15*c^4 + 64704576*a^5*b^13*c^5 - 279571968*a^6*b^11*
c^6 + 853174784*a^7*b^9*c^7 - 1799626752*a^8*b^7*c^8 + 2494119936*a^9*b^5*
c^9 - 2038693888*a^10*b^3*c^10 + 9604*a^4*c^4*(-(4*a*c - b^2)^15)^(1/2) +
4023*a*b^21*c + 10746*a^2*b^4*c^2*(-(4*a*c - b^2)^15)^(1/2) - 26313*a^3*b^
2*c^3*(-(4*a*c - b^2)^15)^(1/2) - 1593*a*b^6*c*(-(4*a*c - b^2)^15)^(1/2))/
(8192*(a^7*b^24 + 16777216*a^19*c^12 - 48*a^8*b^22*c + 1056*a^9*b^20*c^2 -
 14080*a^10*b^18*c^3 + 126720*a^11*b^16*c^4 - 811008*a^12*b^14*c^5 + 37847
04*a^13*b^12*c^6 - 12976128*a^14*b^10*c^7 + 32440320*a^15*b^8*c^8 - 576716
80*a^16*b^6*c^9 + 69206016*a^17*b^4*c^10 - 50331648*a^18*b^2*c^11)))^(1/4)
*(285212672*a^11*b*c^11 - 12288*a^4*b^15*c^4 + 364544*a^5*b^13*c^5 - 46202
88*a^6*b^11*c^6 + 32440320*a^7*b^9*c^7 - 136314880*a^8*b^7*c^8 + 342884352
*a^9*b^5*c^9 - 478150656*a^10*b^3*c^10))/(2*(a^4*b^8 + 256*a^8*c^4 - 16*a^
5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) - (x^(1/2)*(12683575296*a^11*
b*c^13 - 36864*a^2*b^19*c^4 + 1413120*a^3*b^17*c^5 - 23891968*a^4*b^15*c^6
 + 233816064*a^5*b^13*c^7 - 1459421184*a^6*b^11*c^8 + 6023806976*a^7*b^9*c
^9 - 16436428800*a^8*b^7*c^10 + 28575793152*a^9*b^5*c^11 - 28705816576*a^1
0*b^3*c^12))/(16*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^...
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^2}\,dx = \int \frac{1}{\sqrt{x}\,\left(c\,x^4+b\,x^2+a\right)^2}dx$$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

3.938 $\int \frac{1}{x^{3/2}(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 573

$$\int \frac{1}{x^{3/2}(a+bx^2+cx^4)^2}\,dx = -\frac{5b^2-18ac}{2a^2(b^2-4ac)\sqrt{x}} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)\sqrt{x}(a+bx^2+cx^4)} + \frac{\sqrt[4]{c}\left(5b^3-28abc-(5b^2-18ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}a^2(b^2-4ac)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(5b^3-28abc+(5b^2-18ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}a^2(b^2-4ac)^{3/2}\sqrt[4]{-b+\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(5b^3-28abc-(5b^2-18ac)\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}a^2(b^2-4ac)^{3/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(5b^3-28abc+(5b^2-18ac)\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{4\,2^{3/4}a^2(b^2-4ac)^{3/2}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output

```
-1/2*(-18*a*c+5*b^2)/a^2/(-4*a*c+b^2)/x^(1/2)+1/2*(b*c*x^2-2*a*c+b^2)/a/(-
4*a*c+b^2)/x^(1/2)/(c*x^4+b*x^2+a)+1/8*c^(1/4)*(5*b^3-28*a*b*c-(-18*a*c+5*
b^2)*(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(
1/2))^(1/4))*2^(1/4)/a^2/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-
1/8*c^(1/4)*(5*b^3-28*a*b*c+(-18*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*arctan(2^(
1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a^2/(-4*a*c+b^
2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/8*c^(1/4)*(5*b^3-28*a*b*c-(-18*a*
c+5*b^2)*(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b
^2)^(1/2))^(1/4))*2^(1/4)/a^2/(-4*a*c+b^2)^(3/2)/(-b-(-4*a*c+b^2)^(1/2))^(
1/4)+1/8*c^(1/4)*(5*b^3-28*a*b*c+(-18*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*arcta
nh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a^2/(-4*
a*c+b^2)^(3/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.49 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^{3/2} (a + bx^2 + cx^4)^2} \, dx =$$

$$-\frac{-\frac{4(16a^2c-5b^2x^2(b+cx^2)+a(-4b^2+19bcx^2+18c^2x^4))}{(b^2-4ac)\sqrt{x}(a+bx^2+cx^4)} + 4\text{RootSum}\left[a + b\#1^4 + c\#1^8\&, \frac{b\log(\sqrt{x}-\#1)+c\log(\sqrt{x}-\#1)\#}{b\#1+2c\#1^5}\right.}{8a^2}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-1/8*((-4*(16*a^2*c - 5*b^2*x^2*(b + c*x^2) + a*(-4*b^2 + 19*b*c*x^2 + 18*
c^2*x^4)))/((b^2 - 4*a*c)*Sqrt[x]*(a + b*x^2 + c*x^4)) + 4*RootSum[a + b*#
1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1] + c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 +
2*c*#1^5) & ] + RootSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 7
*a*b*c*Log[Sqrt[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 - 2*a*c^2*Log[Sqrt
[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(b^2 - 4*a*c))/a^2
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1435, 1702, 25, 1828, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} (a + bx^2 + cx^4)^2} \, dx$$

$$\Big\downarrow \; 1435$$

$$2\int \frac{1}{x (cx^4 + bx^2 + a)^2} d\sqrt{x}$$

$\big\downarrow$ 1702

$$2\left(\frac{-2ac+b^2+bcx^2}{4a\sqrt{x}\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{5b^2+5cx^2b-18ac}{x\left(cx^4+bx^2+a\right)}d\sqrt{x}}{4a\left(b^2-4ac\right)}\right)$$

$\big\downarrow$ 25

$$2\left(\frac{\int \frac{5b^2+5cx^2b-18ac}{x\left(cx^4+bx^2+a\right)}d\sqrt{x}}{4a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{4a\sqrt{x}\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\big\downarrow$ 1828

$$2\left(\frac{-\frac{\int \frac{x\left(c\left(5b^2-18ac\right)x^2+b\left(5b^2-23ac\right)\right)}{cx^4+bx^2+a}d\sqrt{x}}{a}-\frac{5b^2-18ac}{a\sqrt{x}}}{4a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{4a\sqrt{x}\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\big\downarrow$ 1834

$$2\left(\frac{-\frac{\frac{1}{2}c\left(-\frac{28abc}{\sqrt{b^2-4ac}}+\frac{5b^3}{\sqrt{b^2-4ac}}-18ac+5b^2\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}-\frac{c\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\int \frac{2x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{2\sqrt{b^2-4ac}}}{a}-\frac{5b^2-18}{a\sqrt{x}}}{4a\left(b^2-4ac\right)}\right.$$

$\big\downarrow$ 27

$$2\left(\frac{-\frac{c\left(-\frac{28abc}{\sqrt{b^2-4ac}}+\frac{5b^3}{\sqrt{b^2-4ac}}-18ac+5b^2\right)\int \frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}-\frac{c\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\int \frac{x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{\sqrt{b^2-4ac}}}{a}-\frac{5b^2-18a}{a\sqrt{x}}}{4a\left(b^2-4ac\right)}\right.$$

$\big\downarrow$ 827

$$2\left(\frac{-\frac{c\left(-\frac{28abc}{\sqrt{b^2-4ac}}+\frac{5b^3}{\sqrt{b^2-4ac}}-18ac+5b^2\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{\sqrt{b^2-4ac}-b}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)-\frac{c\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)}{}}{a}}{4a\left(b^2-4ac\right)}\right.$$

$\downarrow$ 218

$$2\left(\frac{-\frac{c\left(-\frac{28abc}{\sqrt{b^2-4ac}}+\frac{5b^3}{\sqrt{b^2-4ac}}-18ac+5b^2\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\int\frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)-\frac{c\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc\right.}{}}{a}}{4a\left(b^2-4ac\right)}\right.$$

$\downarrow$ 221

$$2\left(\frac{-\frac{c\left(-\frac{28abc}{\sqrt{b^2-4ac}}+\frac{5b^3}{\sqrt{b^2-4ac}}-18ac+5b^2\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)-\frac{c\left(-\left(5b^2-18ac\right)\sqrt{b^2-\right.}{}}{a}}{4a\left(b^2-4ac\right)}\right.$$

input
```
Int[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

output
```
2*((b^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*Sqrt[x]*(a + b*x^2 + c*x^4))
 + (-((5*b^2 - 18*a*c)/(a*Sqrt[x])) - (-((c*(5*b^3 - 28*a*b*c - (5*b^2 - 1
8*a*c)*Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2
 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - Arc
Tanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*
c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))))/Sqrt[b^2 - 4*a*c]) + c*(5*b^2 -
18*a*c + (5*b^3)/Sqrt[b^2 - 4*a*c] - (28*a*b*c)/Sqrt[b^2 - 4*a*c])*(ArcTan
[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3
/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-
b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^
(1/4))))/a)/(4*a*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1702
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*
(p + 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && IGtQ[n, 0] && ILtQ[p, -1]
```

rule 1828
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Simp[1/(a*f^n*(m + 1))   Int[(f*x)^(m +
 n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) -
c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x
] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Int
egerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.30

method	result
derivativedivides	$-\frac{2}{a^2\sqrt{x}}-\frac{2\left(\frac{\frac{c(2ac-b^2)x^{\frac{7}{2}}}{16ac-4b^2}+\frac{b(3ac-b^2)x^{\frac{3}{2}}}{16ac-4b^2}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+\_Z^4b+a)}\frac{\left(c(18ac-5b^2)\_R^6+b(23ac-5b^2)\_R^2\right)}{2\_R^7c+\_R^3b}}{64ac-16b^2}\right)}{a^2}$
default	$-\frac{2}{a^2\sqrt{x}}-\frac{2\left(\frac{\frac{c(2ac-b^2)x^{\frac{7}{2}}}{16ac-4b^2}+\frac{b(3ac-b^2)x^{\frac{3}{2}}}{16ac-4b^2}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+\_Z^4b+a)}\frac{\left(c(18ac-5b^2)\_R^6+b(23ac-5b^2)\_R^2\right)}{2\_R^7c+\_R^3b}}{64ac-16b^2}\right)}{a^2}$
risch	$-\frac{2}{a^2\sqrt{x}}-\frac{\frac{\frac{2c(2ac-b^2)x^{\frac{7}{2}}}{16ac-4b^2}+\frac{2b(3ac-b^2)x^{\frac{3}{2}}}{16ac-4b^2}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^8+\_Z^4b+a)}\frac{\left(c(18ac-5b^2)\_R^6+b(23ac-5b^2)\_R^2\right)}{2\_R^7c+\_R^3b}}{32ac-8b^2}}{a^2}$

input
```
int(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^2/x^(1/2)-2/a^2*((1/4*c*(2*a*c-b^2)/(4*a*c-b^2)*x^(7/2)+1/4*b*(3*a*c-
b^2)/(4*a*c-b^2)*x^(3/2))/(c*x^4+b*x^2+a)+1/16/(4*a*c-b^2)*sum((c*(18*a*c-
5*b^2)*_R^6+b*(23*a*c-5*b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=Roo
tOf(_Z^8*c+_Z^4*b+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16004 vs. 2(469) = 938.

Time = 80.48 (sec) , antiderivative size = 16004, normalized size of antiderivative = 27.93

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/x**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{1}{\left(cx^4+bx^2+a\right)^2x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((5*b^2*c - 18*a*c^2)*x^(7/2) + (5*b^3 - 19*a*b*c)*x^(3/2) + 4*(a*b^2
 - 4*a^2*c)/sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a
^2*b^3 - 4*a^3*b*c)*x^2) - integrate(1/4*((5*b^2*c - 18*a*c^2)*x^(5/2) + (
5*b^3 - 23*a*b*c)*sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^
4 + (a^2*b^3 - 4*a^3*b*c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^{3/2} \left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{1}{\left(cx^4+bx^2+a\right)^2 x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^2*x^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 25.43 (sec) , antiderivative size = 31145, normalized size of antiderivative = 54.35

$$\int \frac{1}{x^{3/2} \left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x)
```

output
```
atan(((x^(1/2)*(602332119171072*a^31*b*c^21 - 54080000*a^20*b^23*c^10 + 26
04992000*a^21*b^21*c^11 - 57034444800*a^22*b^19*c^12 + 749118545920*a^23*b
^17*c^13 - 6557747642368*a^24*b^15*c^14 + 40169229778944*a^25*b^13*c^15 -
175670703423488*a^26*b^11*c^16 + 548447002296320*a^27*b^9*c^17 - 119782124
8143360*a^28*b^7*c^18 + 1742819580444672*a^29*b^5*c^19 - 1520311317037056*
a^30*b^3*c^20) + (-(625*b^25 - 625*b^10*(-(4*a*c - b^2)^15)^(1/2) + 310542
3360*a^12*b*c^12 + 638475*a^2*b^21*c^2 - 8264990*a^3*b^19*c^3 + 71483001*a
^4*b^17*c^4 - 434478624*a^5*b^15*c^5 + 1898983360*a^6*b^13*c^6 - 599668992
0*a^7*b^11*c^7 + 13524825600*a^8*b^9*c^8 - 21122310144*a^9*b^7*c^9 + 21483
012096*a^10*b^5*c^10 - 12575047680*a^11*b^3*c^11 + 26244*a^5*c^5*(-(4*a*c
- b^2)^15)^(1/2) - 29625*a*b^23*c - 68475*a^2*b^6*c^2*(-(4*a*c - b^2)^15)^
(1/2) + 181990*a^3*b^4*c^3*(-(4*a*c - b^2)^15)^(1/2) - 171801*a^4*b^2*c^4*
(-(4*a*c - b^2)^15)^(1/2) + 10875*a*b^8*c*(-(4*a*c - b^2)^15)^(1/2))/(8192
*(a^9*b^24 + 16777216*a^21*c^12 - 48*a^10*b^22*c + 1056*a^11*b^20*c^2 - 14
080*a^12*b^18*c^3 + 126720*a^13*b^16*c^4 - 811008*a^14*b^14*c^5 + 3784704*
a^15*b^12*c^6 - 12976128*a^16*b^10*c^7 + 32440320*a^17*b^8*c^8 - 57671680*
a^18*b^6*c^9 + 69206016*a^19*b^4*c^10 - 50331648*a^20*b^2*c^11)))^(3/4)*(3
2768000*a^21*b^34*c^4 - 25649407252758528*a^38*c^21 - 2123366400*a^22*b^32
*c^5 + 64398295040*a^23*b^30*c^6 - 1213399564288*a^24*b^28*c^7 + 158983630
35648*a^25*b^26*c^8 - 153599583715328*a^26*b^24*c^9 + 1132021560639488*...
```

Reduce [F]

$$\int \frac{1}{x^{3/2}\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{1}{x^{\frac{3}{2}}\left(c\,x^4+b\,x^2+a\right)^2}dx$$

input
```
int(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

output
```
int(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

3.939 $\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 617

$$\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx = \frac{x^{9/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{3\sqrt{x}(a(b^2+12ac)+b(b^2+4ac)x^2)}{16c(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3\left(28ab-\frac{b^3}{c}-\frac{b^4-30ab^2c-24a^2c^2}{c\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}(b^2-4ac)^2\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{3\left(b^3-28abc-\frac{b^4-30ab^2c-24a^2c^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}c^{5/4}(b^2-4ac)^2\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{3\left(28ab-\frac{b^3}{c}-\frac{b^4-30ab^2c-24a^2c^2}{c\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}(b^2-4ac)^2\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{3\left(b^3-28abc-\frac{b^4-30ab^2c-24a^2c^2}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}c^{5/4}(b^2-4ac)^2\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output

```
1/4*x^(9/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/16*x^(1/2)*(a*(12
*a*c+b^2)+b*(4*a*c+b^2)*x^2)/c/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/64*(28*a*b
-b^3/c-(-24*a^2*c^2-30*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c
^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)
^2/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3/64*(b^3-28*a*b*c-(-24*a^2*c^2-30*a*b^2*
c+b^4)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)
^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-4*a*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2))^(3/4
)+3/64*(28*a*b-b^3/c-(-24*a^2*c^2-30*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/2))*ar
ctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/
4)/(-4*a*c+b^2)^2/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3/64*(b^3-28*a*b*c-(-24*a^
2*c^2-30*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/
(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(5/4)/(-4*a*c+b^2)^2/(-b+(-4*a*c+
b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.86 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.82

$$\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx = \frac{-\frac{4c^2\sqrt{x}(36a^3c+b^3x^4(3b-cx^2)+abx^2(6b^2+7bcx^2+28c^2x^4)+a^2(3b^2+48bcx^2+68c^2x^4))}{(b^2-4ac)^2(a+bx^2+cx^4)^2}+32c\text{RootSum}}{}$$

input
```
Integrate[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((-4*c^2*Sqrt[x]*(36*a^3*c + b^3*x^4*(3*b - c*x^2) + a*b*x^2*(6*b^2 + 7*b*
c*x^2 + 28*c^2*x^4) + a^2*(3*b^2 + 48*b*c*x^2 + 68*c^2*x^4)))/((b^2 - 4*a*
c)^2*(a + b*x^2 + c*x^4)^2) + 32*c*RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqr
t[x] - #1]/(b*#1^3 + 2*c*#1^7) & ] + (8*RootSum[a + b*#1^4 + c*#1^8 & , (3
*b^4*Log[Sqrt[x] - #1] - 22*a*b^2*c*Log[Sqrt[x] - #1] + 28*a^2*c^2*Log[Sqr
t[x] - #1] + 3*b^3*c*Log[Sqrt[x] - #1]*#1^4 + 6*a*b*c^2*Log[Sqrt[x] - #1]*
#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(a*(b^2 - 4*a*c)) - (3*RootSum[a + b*#1^4 +
 c*#1^8 & , (8*b^6*Log[Sqrt[x] - #1] - 80*a*b^4*c*Log[Sqrt[x] - #1] + 223*
a^2*b^2*c^2*Log[Sqrt[x] - #1] - 140*a^3*c^3*Log[Sqrt[x] - #1] + 8*b^5*c*Lo
g[Sqrt[x] - #1]*#1^4 - 17*a*b^3*c^2*Log[Sqrt[x] - #1]*#1^4 - 36*a^2*b*c^3*
Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(a*(b^2 - 4*a*c)^2))/(64*
c^3)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1435, 1701, 27, 1822, 25, 1826, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x^8}{\left(cx^4+bx^2+a\right)^3}d\sqrt{x}$$

$$\Big\downarrow \text{1701}$$

$$2\left(\frac{x^{9/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\int \frac{3x^4\left(6a-bx^2\right)}{\left(cx^4+bx^2+a\right)^2}d\sqrt{x}}{8\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{x^{9/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\int \frac{x^4\left(6a-bx^2\right)}{\left(cx^4+bx^2+a\right)^2}d\sqrt{x}}{8\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{1822}$$

$$2\left(\frac{x^{9/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(-\frac{\int -\frac{x^2\left(\left(b^2+12ac\right)x^2+40ab\right)}{cx^4+bx^2+a}d\sqrt{x}}{4\left(b^2-4ac\right)}-\frac{x^{5/2}\left(x^2\left(12ac+b^2\right)+8ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)}{8\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{x^{9/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{\int \frac{x^2\left(\left(b^2+12ac\right)x^2+40ab\right)}{cx^4+bx^2+a}d\sqrt{x}}{4\left(b^2-4ac\right)}-\frac{x^{5/2}\left(x^2\left(12ac+b^2\right)+8ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)}{8\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{1826}$$

$$2\left(\frac{x^{9/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{\frac{\sqrt{x}\left(12ac+b^2\right)}{c}-\frac{\int \frac{b\left(b^2-28ac\right)x^2+a\left(b^2+12ac\right)}{cx^4+bx^2+a}d\sqrt{x}}{c}}{4\left(b^2-4ac\right)}-\frac{x^{5/2}\left(x^2\left(12ac+b^2\right)+8ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)}{8\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{1752}$$

$$2\left(\frac{x^{9/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{\frac{\sqrt{x}\left(12ac+b^2\right)}{c}-\frac{\frac{1}{2}\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d\sqrt{x}+\frac{1}{2}\left(\frac{-24a^2c^2-3\cdots}{\sqrt{b^2-\cdots}}\right.\cdots}{c}}{4(b^2-4ac)}\cdots\right.}{8\left(b^2-4ac\right)\cdots}\right.$$

$\downarrow$ 756

$$2\left(\frac{x^{9/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{\frac{\sqrt{x}\left(12ac+b^2\right)}{c}-\frac{\frac{1}{2}\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\left(-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int\frac{\cdots}{\sqrt{2}\sqrt{c}x+\cdots}}{\sqrt{\cdots}}\right.\cdots}{\cdots}}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 218

$$2\left(\frac{x^{9/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{\sqrt{x}\left(12ac+b^2\right)}{c}-\frac{\frac{1}{2}\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\left(-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\cdots\right)}{\sqrt[4]{2}\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$\downarrow$ 221

$$\frac{1}{2}\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\ldots\ \mathrm{arc}\ldots\right.$$

$$2\left(\frac{x^{9/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{\sqrt{x}\left(12ac+b^2\right)}{c}-\frac{\ldots}{\ldots}\right.}{\ldots}\right.$$

input
```
Int[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
2*((x^(9/2)*(2*a + b*x^2))/(8*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*(-
1/4*(x^(5/2)*(8*a*b + (b^2 + 12*a*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (((b^2 + 12*a*c)*Sqrt[x])/c - (((b^3 - 28*a*b*c + (b^4 - 30*a*b^2*c
 - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
- Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4
))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2
^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + ((b^3 - 28*a*b*c - (b
^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4
)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2
 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2)/c)/(4*(
b^2 - 4*a*c))))/(8*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1701
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2
*p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1
] && GtQ[m, 2*n - 1]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1822
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[f^(n - 1)*(f*x)^(m - n + 1)*(a +
b*x^n + c*x^(2*n))^(p + 1)*((b*d - 2*a*e - (b*e - 2*c*d)*x^n)/(n*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[f^n/(n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - n)*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2
*n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
 n - 1] && IntegerQ[p]
```

rule 1826
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Simp[f^n/(c*(
m + n*(2*p + 1) + 1))   Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*
e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Intege
rQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.52 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.45

method	result
derivativedivides	$\frac{-\frac{3a^2(12ac+b^2)\sqrt{x}}{16c(16a^2c^2-8ab^2c+b^4)}-\frac{3ab(8ac+b^2)x^{\frac{5}{2}}}{8c(16a^2c^2-8ab^2c+b^4)}-\frac{(68a^2c^2+7ab^2c+3b^4)x^{\frac{9}{2}}}{16c(16a^2c^2-8ab^2c+b^4)}-\frac{b(28ac-b^2)x^{\frac{13}{2}}}{16(16a^2c^2-8ab^2c+b^4)}}{(cx^4+bx^2+a)^2}+\frac{3\left(\sum_{\_R=\text{RootC}}\right.}{}$
default	$\frac{-\frac{3a^2(12ac+b^2)\sqrt{x}}{16c(16a^2c^2-8ab^2c+b^4)}-\frac{3ab(8ac+b^2)x^{\frac{5}{2}}}{8c(16a^2c^2-8ab^2c+b^4)}-\frac{(68a^2c^2+7ab^2c+3b^4)x^{\frac{9}{2}}}{16c(16a^2c^2-8ab^2c+b^4)}-\frac{b(28ac-b^2)x^{\frac{13}{2}}}{16(16a^2c^2-8ab^2c+b^4)}}{(cx^4+bx^2+a)^2}+\frac{3\left(\sum_{\_R=\text{RootC}}\right.}{}$

input
```
int(x^(15/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-3/32*a^2*(12*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-3/16/c*a*b*
(8*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)-1/32*(68*a^2*c^2+7*a*b^2*c+
3*b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)-1/32*b*(28*a*c-b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+3/64/c/(16*a^2*c^2-8*a*b^2*c+
b^4)*sum((b*(-28*a*c+b^2)*_R^4+12*c*a^2+b^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2
)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14789 vs. 2(521) = 1042.

Time = 8.92 (sec) , antiderivative size = 14789, normalized size of antiderivative = 23.97

$$\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(15/2)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{15}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*(3*(b^2*c + 12*a*c^2)*x^(17/2) + (7*b^3 + 44*a*b*c)*x^(13/2) + 24*a^2
*b*x^(5/2) + (35*a*b^2 + 4*a^2*c)*x^(9/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^
2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*
b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*x^2) - integrate(3/32*((b^2 + 12*a*c)*x^(7/2) + 40*
a*b*x^(3/2))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16
*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Giac [F]

$$\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{15}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(15/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x^(15/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 22.57 (sec) , antiderivative size = 50970, normalized size of antiderivative = 82.61

$$\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(15/2)/(a + b*x^2 + c*x^4)^3,x)
```

output
```
atan(((((3*(3159*a^3*b^14 - 20155392*a^10*c^7 - 367497*a^4*b^12*c + 159002
19*a^5*b^10*c^2 - 299549340*a^6*b^8*c^3 + 1945179360*a^7*b^6*c^4 + 2840323
968*a^8*b^4*c^5 + 164042496*a^9*b^2*c^6))/(65536*(b^18*c - 262144*a^9*c^10
 - 36*a*b^16*c^2 + 576*a^2*b^14*c^3 - 5376*a^3*b^12*c^4 + 32256*a^4*b^10*c
^5 - 129024*a^5*b^8*c^6 + 344064*a^6*b^6*c^7 - 589824*a^7*b^4*c^8 + 589824
*a^8*b^2*c^9)) + ((3*(-(81*(b^33 + b^8*(-(4*a*c - b^2)^25)^(1/2) - 4711042
25280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*b^27*c^3 + 9219696*a^4
*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*b^21*c^6 - 9732052992*
a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 108493078528*a^9*b^15*c^9 + 1315
1174656*a^10*b^13*c^10 + 986354024448*a^11*b^11*c^11 - 3840358219776*a^12*
b^9*c^12 + 7562531438592*a^13*b^7*c^13 - 8212262682624*a^14*b^5*c^14 + 421
3765570560*a^15*b^3*c^15 + 1296*a^4*c^4*(-(4*a*c - b^2)^25)^(1/2) - 157*a*
b^31*c + 4009*a^2*b^4*c^2*(-(4*a*c - b^2)^25)^(1/2) - 54648*a^3*b^2*c^3*(-
(4*a*c - b^2)^25)^(1/2) - 107*a*b^6*c*(-(4*a*c - b^2)^25)^(1/2)))/(3355443
2*(1099511627776*a^20*c^25 + b^40*c^5 - 80*a*b^38*c^6 + 3040*a^2*b^36*c^7
- 72960*a^3*b^34*c^8 + 1240320*a^4*b^32*c^9 - 15876096*a^5*b^30*c^10 + 158
760960*a^6*b^28*c^11 - 1270087680*a^7*b^26*c^12 + 8255569920*a^8*b^24*c^13
 - 44029706240*a^9*b^22*c^14 + 193730707456*a^10*b^20*c^15 - 704475299840*
a^11*b^18*c^16 + 2113425899520*a^12*b^16*c^17 - 5202279137280*a^13*b^14*c^
18 + 10404558274560*a^14*b^12*c^19 - 16647293239296*a^15*b^10*c^20 + 20...
```

Reduce [F]

$$\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx=\int \frac{x^{\frac{15}{2}}}{(c\,x^4+b\,x^2+a)^3}dx$$

input
```
int(x^(15/2)/(c*x^4+b*x^2+a)^3,x)
```

output
```
int(x^(15/2)/(c*x^4+b*x^2+a)^3,x)
```

3.940 $\int \frac{x^{13/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 569

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^3}\,dx = \frac{x^{7/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x^{3/2}(24ab+(5b^2+28ac)x^2)}{16(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\left(5b^3+172abc+\sqrt{b^2-4ac}(5b^2+28ac)\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\,2^{3/4}c^{3/4}(b^2-4ac)^{5/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\left(5b^2+28ac-\frac{5b^3+172abc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\,2^{3/4}c^{3/4}(b^2-4ac)^2\sqrt[4]{-b+\sqrt{b^2-4ac}}} - \frac{\left(5b^3+172abc+\sqrt{b^2-4ac}(5b^2+28ac)\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\,2^{3/4}c^{3/4}(b^2-4ac)^{5/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(5b^2+28ac-\frac{5b^3+172abc}{\sqrt{b^2-4ac}}\right)\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\,2^{3/4}c^{3/4}(b^2-4ac)^2\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output

```
1/4*x^(7/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*x^(3/2)*(24*a*
b+(28*a*c+5*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/64*(5*b^3+172*a*b*c
+(-4*a*c+b^2)^(1/2)*(28*a*c+5*b^2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4
*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2
)^(1/2))^(1/4)+1/64*(5*b^2+28*a*c-(172*a*b*c+5*b^3)/(-4*a*c+b^2)^(1/2))*ar
ctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4
)/(-4*a*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/64*(5*b^3+172*a*b*c+(-4*a
*c+b^2)^(1/2)*(28*a*c+5*b^2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+
b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/
2))^(1/4)-1/64*(5*b^2+28*a*c-(172*a*b*c+5*b^3)/(-4*a*c+b^2)^(1/2))*arctanh
(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/c^(3/4)/(-
4*a*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.76 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.69

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{64}\left(\frac{4x^{3/2}(4a^2(6b-cx^2)+b^2x^4(9b+5cx^2)+a(37b^2x^2+36bcx^4+28c^2x^6))}{(b^2-4ac)^2(a+bx^2+cx^4)^2} + \frac{8\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^3\log(\sqrt{x}-\#1)-13abc\log(\sqrt{x}-\#1)+b^2c\log(\sqrt{x}-\#1)\#1^4+2ac^2\log(\sqrt{x}-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{ac^2(-b^2+4ac)} + \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{8b^5\log(\sqrt{x}-\#1)-136ab^3c\log(\sqrt{x}-\#1)+344a^2bc^2\log(\sqrt{x}-\#1)+8b^4c\log(\sqrt{x}-\#1)\#1^4-}{b\#1+2c\#1^5}\right.}{ac^2(b^2-4ac)^2}\right.$$

input
```
Integrate[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((4*x^(3/2)*(4*a^2*(6*b - c*x^2) + b^2*x^4*(9*b + 5*c*x^2) + a*(37*b^2*x^2
 + 36*b*c*x^4 + 28*c^2*x^6)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) + (8
*RootSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 13*a*b*c*Log[Sqr
t[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 2*a*c^2*Log[Sqrt[x] - #1]*#1^4
)/(b*#1 + 2*c*#1^5) & ])/(a*c^2*(-b^2 + 4*a*c)) + RootSum[a + b*#1^4 + c*#
1^8 & , (8*b^5*Log[Sqrt[x] - #1] - 136*a*b^3*c*Log[Sqrt[x] - #1] + 344*a^2
*b*c^2*Log[Sqrt[x] - #1] + 8*b^4*c*Log[Sqrt[x] - #1]*#1^4 - 11*a*b^2*c^2*L
og[Sqrt[x] - #1]*#1^4 - 36*a^2*c^3*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^
5) & ]/(a*c^2*(b^2 - 4*a*c)^2))/64
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1435, 1701, 1822, 25, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x^7}{(cx^4+bx^2+a)^3}d\sqrt{x}$$

$$\Big\downarrow \text{1701}$$

$$2\left(\frac{x^{7/2}(2a+bx^2)}{8(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int \frac{x^3(14a-5bx^2)}{(cx^4+bx^2+a)^2}d\sqrt{x}}{8(b^2-4ac)}\right)$$

$$\Big\downarrow \text{1822}$$

$$2\left(\frac{x^{7/2}(2a+bx^2)}{8(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{-\frac{\int -\frac{x(72ab-(5b^2+28ac)x^2)}{cx^4+bx^2+a}d\sqrt{x}}{4(b^2-4ac)}-\frac{x^{3/2}(x^2(28ac+5b^2)+24ab)}{4(b^2-4ac)(a+bx^2+cx^4)}}{8(b^2-4ac)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{x^{7/2}(2a+bx^2)}{8(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{\int \frac{x(72ab-(5b^2+28ac)x^2)}{cx^4+bx^2+a}d\sqrt{x}}{4(b^2-4ac)}-\frac{x^{3/2}(x^2(28ac+5b^2)+24ab)}{4(b^2-4ac)(a+bx^2+cx^4)}}{8(b^2-4ac)}\right)$$

$$\Big\downarrow \text{1834}$$

$$2\left(\frac{x^{7/2}(2a+bx^2)}{8(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{-\frac{1}{2}\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}-\frac{1}{2}\left(\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\int \frac{2x}{2cx^2+b+\sqrt{\cdots}}\cdots}{4(b^2-4ac)}}{8(b^2-4ac)}\right.$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{x^{7/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\frac{-\left(\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\int\frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}\right)-\left(\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\int\frac{x}{2cx^2+b+\sqrt{}}}{4(b^2-4ac)}}{8\left(b^2-4ac\right)}\right.$$

$\downarrow$ 827

$$2\left(\frac{x^{7/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{-\left(\left(\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\left(\frac{\int\frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)\right)-}{4(b^2-4ac}\right.$$

$\downarrow$ 218

$$2\left(\frac{x^{7/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{-\left(\left(\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{}}{2\sqrt{2}\sqrt{c}}\right.\right.}{4(b}\right.$$

$\downarrow$ 221

$$2\left(\frac{x^{7/2}(2a+bx^2)}{8(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{-\left(\left(\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right.\right.}{}\right.$$

input
```
Int[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
2*((x^(7/2)*(2*a + b*x^2))/(8*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (-1/4
*(x^(3/2)*(24*a*b + (5*b^2 + 28*a*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (-((5*b^2 + 28*a*c + (5*b^3 + 172*a*b*c)/Sqrt[b^2 - 4*a*c])*(ArcTan
[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3
/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-
b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^
(1/4)))) - (5*b^2 + 28*a*c - (5*b^3 + 172*a*b*c)/Sqrt[b^2 - 4*a*c])*(ArcTa
n[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(
3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])
^(1/4))))/(4*(b^2 - 4*a*c)))/(8*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1701
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2
*p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1
] && GtQ[m, 2*n - 1]
```

rule 1822
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[f^(n - 1)*(f*x)^(m - n + 1)*(a +
b*x^n + c*x^(2*n))^(p + 1)*((b*d - 2*a*e - (b*e - 2*c*d)*x^n)/(n*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[f^n/(n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - n)*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2
*n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
 n - 1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.48 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.43

method	result
derivativedivides	$\frac{\frac{3a^2 b\, x^{\frac{3}{2}}}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(4ac-37b^2\right)x^{\frac{7}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{9b\left(4ac+b^2\right)x^{\frac{11}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{2c\left(28ac+5b^2\right)x^{\frac{15}{2}}}{512a^2c^2-256a\,b^2c+32b^4}}{\left(c\,x^4+b\,x^2+a\right)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8\right.}}{}$
default	$\frac{\frac{3a^2 b\, x^{\frac{3}{2}}}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(4ac-37b^2\right)x^{\frac{7}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{9b\left(4ac+b^2\right)x^{\frac{11}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{2c\left(28ac+5b^2\right)x^{\frac{15}{2}}}{512a^2c^2-256a\,b^2c+32b^4}}{\left(c\,x^4+b\,x^2+a\right)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8\right.}}{}$

input
```
int(x^(13/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(3/4*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)-1/32*a*(4*a*c-37*b^2)/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+9/32*b*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^(11/2)+1/32*c*(28*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(c*
x^4+b*x^2+a)^2-1/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum(((-28*a*c-5*b^2)*_R^6+7
2*b*_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19805 vs. $2(469) = 938$.

Time = 42.02 (sec) , antiderivative size = 19805, normalized size of antiderivative = 34.81

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{13}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*((5*b^2*c + 28*a*c^2)*x^(15/2) + 9*(b^3 + 4*a*b*c)*x^(11/2) + 24*a^2*
b*x^(3/2) + (37*a*b^2 - 4*a^2*c)*x^(7/2))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b
^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*
b^3*c + 16*a^3*b*c^2)*x^2) + integrate(1/32*((5*b^2 + 28*a*c)*x^(5/2) - 72
*a*b*sqrt(x))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 1
6*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Giac [F]

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{13}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(13/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x^(13/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 21.36 (sec) , antiderivative size = 39697, normalized size of antiderivative = 69.77

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(13/2)/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((9*x^(11/2)*(b^3 + 4*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(7/
2)*(37*a*b^2 - 4*a^2*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^(15/2)
*(28*a*c + 5*b^2))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a^2*b*x^(3/2))
/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 +
2*a*b*x^2 + 2*b*c*x^6) - atan(((((386183668047020032*a^16*c^16 + 209715200
0*a^3*b^26*c^3 - 7615312560128*a^4*b^24*c^4 + 295658569334784*a^5*b^22*c^5
 - 5154027327193088*a^6*b^20*c^6 + 52821290217635840*a^7*b^18*c^7 - 350572
668266741760*a^8*b^16*c^8 + 1560295235622273024*a^9*b^14*c^9 - 46282369669
60300032*a^10*b^12*c^10 + 8604139182719238144*a^11*b^10*c^11 - 79240263697
53743360*a^12*b^8*c^12 - 1942353261163970560*a^13*b^6*c^13 + 1182321565924
2749952*a^14*b^4*c^14 - 8419198028392431616*a^15*b^2*c^15)/(268435456*(b^2
8 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 23296*a^3*b^22*c^3 + 256256*
a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*a^6*b^16*c^6 - 56229888*a^7
*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*a^9*b^10*c^9 + 1049624576*a
^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 1526726656*a^12*b^4*c^12 - 93952
4096*a^13*b^2*c^13 - 56*a*b^26*c)) - (x^(1/2)*(-(625*b^31 + 625*b^6*(-(4*a
*c - b^2)^25)^(1/2) - 15192104632320*a^15*b*c^15 - 89000*a^2*b^27*c^2 + 27
186416*a^3*b^25*c^3 - 1342297600*a^4*b^23*c^4 + 25492409600*a^5*b^21*c^5 -
 265188833280*a^6*b^19*c^6 + 1688816578560*a^7*b^17*c^7 - 6664504147968*a^
8*b^15*c^8 + 14462970429440*a^9*b^13*c^9 - 4163326443520*a^10*b^11*c^10...
```

Reduce [F]

$$\int \frac{x^{13/2}}{(a+bx^2+cx^4)^3}\,dx=\int \frac{x^{\frac{13}{2}}}{(c\,x^4+b\,x^2+a)^3}dx$$

input
```
int(x^(13/2)/(c*x^4+b*x^2+a)^3,x)
```

output
```
int(x^(13/2)/(c*x^4+b*x^2+a)^3,x)
```

3.941 $\int \frac{x^{11/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 569

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^3}\,dx = \frac{x^{5/2}(2a+bx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\sqrt{x}(24ab+(7b^2+20ac)\,x^2)}{16\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{3\left(7b^3+36abc+\sqrt{b^2-4ac}(7b^2+20ac)\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{3\left(7b^2+20ac-\frac{b(7b^2+36ac)}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)^2\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} - \frac{3\left(7b^3+36abc+\sqrt{b^2-4ac}(7b^2+20ac)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{3\left(7b^2+20ac-\frac{b(7b^2+36ac)}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\,(b^2-4ac)^2\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output
```
1/4*x^(5/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*x^(1/2)*(24*a*
b+(20*a*c+7*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-3/64*(7*b^3+36*a*b*c+
(-4*a*c+b^2)^(1/2)*(20*a*c+7*b^2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*
a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)
^(1/2))^(3/4)-3/64*(7*b^2+20*a*c-b*(36*a*c+7*b^2)/(-4*a*c+b^2)^(1/2))*arct
an(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/
(-4*a*c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-3/64*(7*b^3+36*a*b*c+(-4*a*c+
b^2)^(1/2)*(20*a*c+7*b^2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2
)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))
^(3/4)-3/64*(7*b^2+20*a*c-b*(36*a*c+7*b^2)/(-4*a*c+b^2)^(1/2))*arctanh(2^(
1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/c^(1/4)/(-4*a*
c+b^2)^2/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.70

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{64}\left(\frac{4\sqrt{x}(12a^2(2b-cx^2)+b^2x^4(11b+7cx^2)+a(39b^2x^2+28bcx^4+20c^2x^6))}{(b^2-4ac)^2(a+bx^2+cx^4)^2}\right.$$
$$+\frac{24\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^3\log(\sqrt{x}-\#1)-5abc\log(\sqrt{x}-\#1)+b^2c\log(\sqrt{x}-\#1)\#1^4+2ac^2\log(\sqrt{x}-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{ac^2(-b^2+4ac)}$$
$$+\frac{3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{8b^5\log(\sqrt{x}-\#1)-72ab^3c\log(\sqrt{x}-\#1)+152a^2bc^2\log(\sqrt{x}-\#1)+8b^4c\log(\sqrt{x}-\#1)\#1^4-}{b\#1^3+2c\#1^7}\right.}{ac^2(b^2-4ac)^2}$$

input
```
Integrate[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((4*Sqrt[x]*(12*a^2*(2*b - c*x^2) + b^2*x^4*(11*b + 7*c*x^2) + a*(39*b^2*x
^2 + 28*b*c*x^4 + 20*c^2*x^6)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) +
(24*RootSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 5*a*b*c*Log[S
qrt[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 2*a*c^2*Log[Sqrt[x] - #1]*#1
^4)/(b*#1^3 + 2*c*#1^7) & ])/(a*c^2*(-b^2 + 4*a*c)) + (3*RootSum[a + b*#1^
4 + c*#1^8 & , (8*b^5*Log[Sqrt[x] - #1] - 72*a*b^3*c*Log[Sqrt[x] - #1] + 1
52*a^2*b*c^2*Log[Sqrt[x] - #1] + 8*b^4*c*Log[Sqrt[x] - #1]*#1^4 - 9*a*b^2*
c^2*Log[Sqrt[x] - #1]*#1^4 - 44*a^2*c^3*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 +
2*c*#1^7) & ])/(a*c^2*(b^2 - 4*a*c)^2))/64
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1435, 1701, 1822, 27, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^3} \, dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x^6}{(cx^4+bx^2+a)^3}d\sqrt{x}$$

$$\Big\downarrow \text{1701}$$

$$2\left(\frac{x^{5/2}(2a+bx^2)}{8(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int \frac{x^2(10a-7bx^2)}{(cx^4+bx^2+a)^2}d\sqrt{x}}{8(b^2-4ac)}\right)$$

$$\Big\downarrow \text{1822}$$

$$2\left(\frac{x^{5/2}(2a+bx^2)}{8(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{-\frac{\int -\frac{3\left(8ab-\left(7b^2+20ac\right)x^2\right)}{cx^4+bx^2+a}d\sqrt{x}}{4(b^2-4ac)}-\frac{\sqrt{x}(x^2(20ac+7b^2)+24ab)}{4(b^2-4ac)(a+bx^2+cx^4)}}{8(b^2-4ac)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{x^{5/2}(2a+bx^2)}{8(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{3\int \frac{8ab-\left(7b^2+20ac\right)x^2}{cx^4+bx^2+a}d\sqrt{x}}{4(b^2-4ac)}-\frac{\sqrt{x}(x^2(20ac+7b^2)+24ab)}{4(b^2-4ac)(a+bx^2+cx^4)}}{8(b^2-4ac)}\right)$$

$$\Big\downarrow \text{1752}$$

$$2\left(\frac{x^{5/2}(2a+bx^2)}{8(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\frac{3\left(-\frac{1}{2}\left(-\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d\sqrt{x}-\frac{1}{2}\left(\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\int \frac{}{cx^2+\frac{}{}} \right.}{4(b^2-4ac)}}{8(b^2-4ac)}\right.$$

$$\Big\downarrow \text{756}$$

$$2\left(\frac{x^{5/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(-\frac{1}{2}\left(\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\left(-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int\frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)\cdots}{4(b^2-\cdots}\cdots\right.$$

$$\Big\downarrow \text{218}$$

$$2\left(\frac{x^{5/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(-\frac{1}{2}\left(\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\left(-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-\cdots}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\cdots\right.\right.}{4\cdots}\cdots\right.$$

$$\Big\downarrow \text{221}$$

$$2\left(\frac{x^{5/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(-\frac{1}{2}\left(\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{\cdots}}{\sqrt[4]{-\sqrt{b^2-\cdots}}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4a\cdots}\right.}\cdots\right.\right.}{\cdots}\cdots\right.$$

input
```
Int[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
2*((x^(5/2)*(2*a + b*x^2))/(8*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (-1/4
*(Sqrt[x]*(24*a*b + (7*b^2 + 20*a*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (3*(-1/2*((7*b^2 + 20*a*c + (7*b^3 + 36*a*b*c)/Sqrt[b^2 - 4*a*c])*(
-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4
)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a
*c])^(3/4))) - ((7*b^2 + 20*a*c - (7*b^3 + 36*a*b*c)/Sqrt[b^2 - 4*a*c])*(
-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4
)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a
*c])^(3/4))))/2))/(4*(b^2 - 4*a*c)))/(8*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1701
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2
*p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1
] && GtQ[m, 2*n - 1]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1822
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[f^(n - 1)*(f*x)^(m - n + 1)*(a +
b*x^n + c*x^(2*n))^(p + 1)*((b*d - 2*a*e - (b*e - 2*c*d)*x^n)/(n*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[f^n/(n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - n)*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[(n - m - 1)*(b*d - 2*a*e) + (2*n*p + 2
*n + m + 1)*(b*e - 2*c*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m,
 n - 1] && IntegerQ[p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.48 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.42

method	result
derivativedivides	$\frac{\frac{3a^2b\sqrt{x}}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{3(4ac-13b^2)a\,x^{\frac{5}{2}}}{16(16a^2c^2-8a\,b^2c+b^4)}+\frac{2b(28ac+11b^2)x^{\frac{9}{2}}}{512a^2c^2-256a\,b^2c+32b^4}+\frac{2c(20ac+7b^2)x^{\frac{13}{2}}}{512a^2c^2-256a\,b^2c+32b^4}}{(c\,x^4+b\,x^2+a)^2}+\frac{3\Bigg(\sum_{\_R=\mathrm{RootOf}(c\_}$
default	$\frac{\frac{3a^2b\sqrt{x}}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{3(4ac-13b^2)a\,x^{\frac{5}{2}}}{16(16a^2c^2-8a\,b^2c+b^4)}+\frac{2b(28ac+11b^2)x^{\frac{9}{2}}}{512a^2c^2-256a\,b^2c+32b^4}+\frac{2c(20ac+7b^2)x^{\frac{13}{2}}}{512a^2c^2-256a\,b^2c+32b^4}}{(c\,x^4+b\,x^2+a)^2}+\frac{3\Bigg(\sum_{\_R=\mathrm{RootOf}(c\_}$

input
```
int(x^(11/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(3/4*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-3/32*(4*a*c-13*b^2)*a/(16*
a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/32*b*(28*a*c+11*b^2)/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^(9/2)+1/32*c*(20*a*c+7*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/
(c*x^4+b*x^2+a)^2+3/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum(((20*a*c+7*b^2)*_R^4
-8*a*b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12814 vs. 2(469) = 938.

Time = 3.09 (sec) , antiderivative size = 12814, normalized size of antiderivative = 22.52

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{11}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/16*(24*b*c^2*x^(17/2) + (41*b^2*c - 20*a*c^2)*x^(13/2) + (13*b^3 + 20*a
*b*c)*x^(9/2) + 3*(3*a*b^2 + 4*a^2*c)*x^(5/2))/((b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*
a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8
*a^2*b^3*c + 16*a^3*b*c^2)*x^2) + integrate(3/32*(8*b*c*x^(7/2) + 5*(3*b^2
 + 4*a*c)*x^(3/2))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Giac [F]

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{11}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(11/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x^(11/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 21.74 (sec) , antiderivative size = 45495, normalized size of antiderivative = 79.96

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int(x^(11/2)/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((x^(9/2)*(11*b^3 + 28*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^
(5/2)*(13*a*b^2 - 4*a^2*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^(13
/2)*(20*a*c + 7*b^2))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a^2*b*x^(1/
2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8
 + 2*a*b*x^2 + 2*b*c*x^6) - atan((((((3*((81*(2401*b^4*(-(4*a*c - b^2)^25)
^(1/2) - 2401*b^29 - 704643072000*a^14*b*c^14 + 1323600*a^2*b^25*c^2 - 282
43200*a^3*b^23*c^3 + 271415040*a^4*b^21*c^4 - 1437284352*a^5*b^19*c^5 + 39
89852160*a^6*b^17*c^6 - 2793799680*a^7*b^15*c^7 - 13327073280*a^8*b^13*c^8
 + 19977994240*a^9*b^11*c^9 + 66059239424*a^10*b^9*c^10 - 143696855040*a^1
1*b^7*c^11 - 230770606080*a^12*b^5*c^12 + 887850270720*a^13*b^3*c^13 + 100
00*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) + 9400*a*b^27*c + 9400*a*b^2*c*(-(4*a
*c - b^2)^25)^(1/2)))/(33554432*(b^40*c + 1099511627776*a^20*c^21 - 80*a*b
^38*c^2 + 3040*a^2*b^36*c^3 - 72960*a^3*b^34*c^4 + 1240320*a^4*b^32*c^5 -
15876096*a^5*b^30*c^6 + 158760960*a^6*b^28*c^7 - 1270087680*a^7*b^26*c^8 +
 8255569920*a^8*b^24*c^9 - 44029706240*a^9*b^22*c^10 + 193730707456*a^10*b
^20*c^11 - 704475299840*a^11*b^18*c^12 + 2113425899520*a^12*b^16*c^13 - 52
02279137280*a^13*b^14*c^14 + 10404558274560*a^14*b^12*c^15 - 1664729323929
6*a^15*b^10*c^16 + 20809116549120*a^16*b^8*c^17 - 19585050869760*a^17*b^6*
c^18 + 13056700579840*a^18*b^4*c^19 - 5497558138880*a^19*b^2*c^20)))^(1/4)
*(351843720888320*a^13*c^15 + 251658240*a^2*b^22*c^4 - 9730785280*a^3*b...
```

Reduce [F]

$$\int \frac{x^{11/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{11}{2}}}{(c\,x^4+b\,x^2+a)^3}dx$$

input
```
int(x^(11/2)/(c*x^4+b*x^2+a)^3,x)
```

output
```
int(x^(11/2)/(c*x^4+b*x^2+a)^3,x)
```

3.942 $\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 533

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3}\,dx = \frac{x^{3/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{3x^{3/2}(5b^2-4ac+8bcx^2)}{16(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{3\sqrt[4]{c}\left(11b^2+20ac+4b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{16\,2^{3/4}(b^2-4ac)^{5/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{3\sqrt[4]{c}\left(11b^2+20ac-4b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{16\,2^{3/4}(b^2-4ac)^{5/2}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{3\sqrt[4]{c}\left(11b^2+20ac+4b\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{16\,2^{3/4}(b^2-4ac)^{5/2}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{3\sqrt[4]{c}\left(11b^2+20ac-4b\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{16\,2^{3/4}(b^2-4ac)^{5/2}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
1/4*x^(3/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/16*x^(3/2)*(8*b*c
*x^2-4*a*c+5*b^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-3/32*c^(1/4)*(11*b^2+20*a
*c+4*b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)
^(1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+3/
32*c^(1/4)*(11*b^2+20*a*c-4*b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x
^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)^(5/2)/(-b+(-4*a
*c+b^2)^(1/2))^(1/4)+3/32*c^(1/4)*(11*b^2+20*a*c+4*b*(-4*a*c+b^2)^(1/2))*a
rctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*
a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-3/32*c^(1/4)*(11*b^2+20*a*c-4
*b*(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1
/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.64

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{64}\left(-\frac{4x^{3/2}(20a^2c+a(7b^2+28bcx^2-12c^2x^4)+bx^2(11b^2+39bcx^2+24c^2x^4))}{(b^2-4ac)^2(a+bx^2+cx^4)^2}\right.$$
$$+\frac{8\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^2\log(\sqrt{x}-\#1)-10ac\log(\sqrt{x}-\#1)+bc\log(\sqrt{x}-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{ab^2c-4a^2c^2}$$
$$-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{8b^4\log(\sqrt{x}-\#1)-133ab^2c\log(\sqrt{x}-\#1)+260a^2c^2\log(\sqrt{x}-\#1)+8b^3c\log(\sqrt{x}-\#1)\#1^4-8}{b\#1+2c\#1^5}\right.}{ac(b^2-4ac)^2}$$

input
```
Integrate[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((-4*x^(3/2)*(20*a^2*c + a*(7*b^2 + 28*b*c*x^2 - 12*c^2*x^4) + b*x^2*(11*b
^2 + 39*b*c*x^2 + 24*c^2*x^4)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) +
(8*RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log[Sqrt[x] - #1] - 10*a*c*Log[Sqr
t[x] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(a*b^2*c -
 4*a^2*c^2) - RootSum[a + b*#1^4 + c*#1^8 & , (8*b^4*Log[Sqrt[x] - #1] - 1
33*a*b^2*c*Log[Sqrt[x] - #1] + 260*a^2*c^2*Log[Sqrt[x] - #1] + 8*b^3*c*Log
[Sqrt[x] - #1]*#1^4 - 8*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5)
& ]/(a*c*(b^2 - 4*a*c)^2))/64
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1435, 1701, 27, 1824, 27, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$$

$$\Big\downarrow \ 1435$$

$$2\int \frac{x^5}{\left(cx^4+bx^2+a\right)^3}d\sqrt{x}$$

$$\Big\downarrow \ 1701$$

$$2\left(\frac{x^{3/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\int \frac{3x\left(2a-3bx^2\right)}{\left(cx^4+bx^2+a\right)^2}d\sqrt{x}}{8\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{x^{3/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\int \frac{x\left(2a-3bx^2\right)}{\left(cx^4+bx^2+a\right)^2}d\sqrt{x}}{8\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \ 1824$$

$$2\left(\frac{x^{3/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{ax\left(7b^2-8cx^2b+20ac\right)}{cx^4+bx^2+a}d\sqrt{x}}{4a(b^2-4ac)}\right)}{8\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{x^{3/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int \frac{x\left(7b^2-8cx^2b+20ac\right)}{cx^4+bx^2+a}d\sqrt{x}}{4(b^2-4ac)}\right)}{8\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \ 1834$$

$$2\left(\frac{x^{3/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{-c\left(4b-\frac{20ac+11b^2}{\sqrt{b^2-4ac}}\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}-c\left(\frac{20ac+11b^2}{\sqrt{b^2-4ac}}+4b\right)}{4(b^2-4ac)}\right.}{8\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{x^{3/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{-2c\left(4b-\frac{20ac+11b^2}{\sqrt{b^2-4ac}}\right)\int\frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}-2c\left(\frac{20ac+11b^2}{\sqrt{b^2-4ac}}+4\right.}{4(b^2-4ac)}\right.}{8\left(b^2-4ac\right)}\right.$$

$\Big\downarrow$ 827

$$2\left(\frac{x^{3/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{-2c\left(\frac{20ac+11b^2}{\sqrt{b^2-4ac}}+4b\right)\left(\frac{\int\frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int\frac{}{\sqrt{-b-\sqrt{}}}}{}\right.}{}\right.}{8\left(\right.}\right.$$

$\Big\downarrow$ 218

$$2\left(\frac{x^{3/2}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\frac{x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-2c\left(\frac{20ac+11b^2}{\sqrt{b^2-4ac}}+4b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 221

$$2\left(\frac{x^{3/2}(2a+bx^2)}{8(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3\left(\frac{x^{3/2}(-4ac+5b^2+8bcx^2)}{4(b^2-4ac)(a+bx^2+cx^4)}-\frac{-2c\left(\frac{20ac+11b^2}{\sqrt{b^2-4ac}}+4b\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{ar}\cdots}{2\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
2*((x^(3/2)*(2*a + b*x^2))/(8*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*((
x^(3/2)*(5*b^2 - 4*a*c + 8*b*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 - (-2*c*(4*b + (11*b^2 + 20*a*c)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt
[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) - 2*c*
(4*b - (11*b^2 + 20*a*c)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[
x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*
a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^
(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/(4*(b^2 - 4*a*
c))))/(8*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1701
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2
*p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1
] && GtQ[m, 2*n - 1]
```

rule 1824
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m
+ 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.48 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.46

method	result
derivativedivides	$\frac{-\frac{a(20ac+7b^2)x^{\frac{3}{2}}}{16(16a^2c^2-8ab^2c+b^4)}-\frac{b(28ac+11b^2)x^{\frac{7}{2}}}{16(16a^2c^2-8ab^2c+b^4)}+\frac{3(4ac-13b^2)cx^{\frac{11}{2}}}{16(16a^2c^2-8ab^2c+b^4)}-\frac{3bc^2x^{\frac{15}{2}}}{2(16a^2c^2-8ab^2c+b^4)}}{(cx^4+bx^2+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(c\dots}\right.}{}$
default	$\frac{-\frac{a(20ac+7b^2)x^{\frac{3}{2}}}{16(16a^2c^2-8ab^2c+b^4)}-\frac{b(28ac+11b^2)x^{\frac{7}{2}}}{16(16a^2c^2-8ab^2c+b^4)}+\frac{3(4ac-13b^2)cx^{\frac{11}{2}}}{16(16a^2c^2-8ab^2c+b^4)}-\frac{3bc^2x^{\frac{15}{2}}}{2(16a^2c^2-8ab^2c+b^4)}}{(cx^4+bx^2+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(c\dots}\right.}{}$

input
```
int(x^(9/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/32*a*(20*a*c+7*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)-1/32*b*(28*a*
c+11*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+3/32*(4*a*c-13*b^2)*c/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^(11/2)-3/4*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2)
)/(c*x^4+b*x^2+a)^2+3/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((-8*b*c*_R^6+(20*a
*c+7*b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18451 vs. $2(429) = 858$.

Time = 24.47 (sec) , antiderivative size = 18451, normalized size of antiderivative = 34.62

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{9}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/16*(24*b*c^2*x^(15/2) + 3*(13*b^2*c - 4*a*c^2)*x^(11/2) + (11*b^3 + 28*
a*b*c)*x^(7/2) + (7*a*b^2 + 20*a^2*c)*x^(3/2))/((b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*
a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8
*a^2*b^3*c + 16*a^3*b*c^2)*x^2) - integrate(3/32*(8*b*c*x^(5/2) - (7*b^2 +
 20*a*c)*sqrt(x))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2
 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Giac [F]

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{9}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x^(9/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 21.36 (sec) , antiderivative size = 37678, normalized size of antiderivative = 70.69

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(9/2)/(a + b*x^2 + c*x^4)^3,x)
```

output
```
- atan(((((27*(5754585088*a*b^27*c^4 + 309622474381721600*a^14*b*c^17 - 16
1128382464*a^2*b^25*c^5 + 1626181992448*a^3*b^23*c^6 - 3983582167040*a^4*b
^21*c^7 - 56328496087040*a^5*b^19*c^8 + 557813172535296*a^6*b^17*c^9 - 196
1803621859328*a^7*b^15*c^10 + 715782069682176*a^8*b^13*c^11 + 158164747655
57760*a^9*b^11*c^12 - 39296545576714240*a^10*b^9*c^13 - 32756650414702592*
a^11*b^7*c^14 + 300756012615335936*a^12*b^5*c^15 - 517069532217475072*a^13
*b^3*c^16))/(268435456*(b^28 + 268435456*a^14*c^14 + 1456*a^2*b^24*c^2 - 2
3296*a^3*b^22*c^3 + 256256*a^4*b^20*c^4 - 2050048*a^5*b^18*c^5 + 12300288*
a^6*b^16*c^6 - 56229888*a^7*b^14*c^7 + 196804608*a^8*b^12*c^8 - 524812288*
a^9*b^10*c^9 + 1049624576*a^10*b^8*c^10 - 1526726656*a^11*b^6*c^11 + 15267
26656*a^12*b^4*c^12 - 939524096*a^13*b^2*c^13 - 56*a*b^26*c)) - (9*x^(1/2)
*((81*(2401*b^4*(-(4*a*c - b^2)^25)^(1/2) - 2401*b^29 - 704643072000*a^14*
b*c^14 + 1323600*a^2*b^25*c^2 - 28243200*a^3*b^23*c^3 + 271415040*a^4*b^21
*c^4 - 1437284352*a^5*b^19*c^5 + 3989852160*a^6*b^17*c^6 - 2793799680*a^7*
b^15*c^7 - 13327073280*a^8*b^13*c^8 + 19977994240*a^9*b^11*c^9 + 660592394
24*a^10*b^9*c^10 - 143696855040*a^11*b^7*c^11 - 230770606080*a^12*b^5*c^12
 + 887850270720*a^13*b^3*c^13 + 10000*a^2*c^2*(-(4*a*c - b^2)^25)^(1/2) +
9400*a*b^27*c + 9400*a*b^2*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a*b^40
 + 1099511627776*a^21*c^20 - 80*a^2*b^38*c + 3040*a^3*b^36*c^2 - 72960*a^4
*b^34*c^3 + 1240320*a^5*b^32*c^4 - 15876096*a^6*b^30*c^5 + 158760960*a^...
```

Reduce [F]

$$\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3}\,dx=\int \frac{x^{\frac{9}{2}}}{(c\,x^4+b\,x^2+a)^3}dx$$

input
```
int(x^(9/2)/(c*x^4+b*x^2+a)^3,x)
```

output
```
int(x^(9/2)/(c*x^4+b*x^2+a)^3,x)
```

3.943 $\int \frac{x^{7/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 533

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^3}\,dx = \frac{\sqrt{x}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{\sqrt{x}(13b^2-4ac+24bcx^2)}{16(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{c^{3/4}\left(41b^2+28ac+36b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{16\sqrt[4]{2}(b^2-4ac)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\left(41b^2+28ac-36b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{16\sqrt[4]{2}(b^2-4ac)^{5/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(41b^2+28ac+36b\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{16\sqrt[4]{2}(b^2-4ac)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\left(41b^2+28ac-36b\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{16\sqrt[4]{2}(b^2-4ac)^{5/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output

```
1/4*x^(1/2)*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/16*x^(1/2)*(24*b*
c*x^2-4*a*c+13*b^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/32*c^(3/4)*(41*b^2+28
*a*c+36*b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b
^2)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)
-1/32*c^(3/4)*(41*b^2+28*a*c-36*b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/
4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)^(5/2)/(-b+(
-4*a*c+b^2)^(1/2))^(3/4)+1/32*c^(3/4)*(41*b^2+28*a*c+36*b*(-4*a*c+b^2)^(1/
2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)
/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/32*c^(3/4)*(41*b^2+28*
a*c-36*b*(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b
^2)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.65

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{64}\left(-\frac{4\sqrt{x}(28a^2c+a(5b^2+36bcx^2-4c^2x^4)+bx^2(9b^2+37bcx^2+24c^2x^4))}{(b^2-4ac)^2(a+bx^2+cx^4)^2}\right.$$
$$+\frac{8\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{3b^2\log(\sqrt{x}-\#1)-14ac\log(\sqrt{x}-\#1)+3bc\log(\sqrt{x}-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{ab^2c-4a^2c^2}$$
$$-\frac{3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{8b^4\log(\sqrt{x}-\#1)-71ab^2c\log(\sqrt{x}-\#1)+140a^2c^2\log(\sqrt{x}-\#1)+8b^3c\log(\sqrt{x}-\#1)\#1^4-}{b\#1^3+2c\#1^7}\right.}{ac(b^2-4ac)^2}$$

input
```
Integrate[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((-4*Sqrt[x]*(28*a^2*c + a*(5*b^2 + 36*b*c*x^2 - 4*c^2*x^4) + b*x^2*(9*b^2
 + 37*b*c*x^2 + 24*c^2*x^4)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2) + (8
*RootSum[a + b*#1^4 + c*#1^8 & , (3*b^2*Log[Sqrt[x] - #1] - 14*a*c*Log[Sqr
t[x] - #1] + 3*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(a*b^2
*c - 4*a^2*c^2) - (3*RootSum[a + b*#1^4 + c*#1^8 & , (8*b^4*Log[Sqrt[x] -
#1] - 71*a*b^2*c*Log[Sqrt[x] - #1] + 140*a^2*c^2*Log[Sqrt[x] - #1] + 8*b^3
*c*Log[Sqrt[x] - #1]*#1^4 - 8*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*
c*#1^7) & ])/(a*c*(b^2 - 4*a*c)^2))/64
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 472, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1435, 1701, 1760, 27, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\left(a+bx^2+cx^4\right)^3}\,dx$$

$\big\downarrow$ 1435

$$2\int \frac{x^4}{\left(cx^4+bx^2+a\right)^3}d\sqrt{x}$$

$\big\downarrow$ 1701

$$2\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\int \frac{2a-11bx^2}{\left(cx^4+bx^2+a\right)^2}d\sqrt{x}}{8\left(b^2-4ac\right)}\right)$$

$\big\downarrow$ 1760

$$2\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\frac{\sqrt{x}\left(-4ac+13b^2+24bcx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{a\left(5b^2-72cx^2b+28ac\right)}{cx^4+bx^2+a}d\sqrt{x}}{4a\left(b^2-4ac\right)}}{8\left(b^2-4ac\right)}\right)$$

$\big\downarrow$ 27

$$2\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\frac{\sqrt{x}\left(-4ac+13b^2+24bcx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{5b^2-72cx^2b+28ac}{cx^4+bx^2+a}d\sqrt{x}}{4\left(b^2-4ac\right)}}{8\left(b^2-4ac\right)}\right)$$

$\big\downarrow$ 1752

$$2\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\frac{\sqrt{x}\left(-4ac+13b^2+24bcx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-c\left(36b-\frac{28ac+41b^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d\sqrt{x}-c\left(\frac{28ac+41b^2}{\sqrt{b^2-4ac}}+36\right.}{4\left(b^2-4ac\right)}}{8\left(b^2-4ac\right)}\right.$$

$\big\downarrow$ 756

$$2\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\frac{\sqrt{x}\left(-4ac+13b^2+24bcx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-c\left(\frac{28ac+41b^2}{\sqrt{b^2-4ac}}+36b\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{}{\sqrt{2}\sqrt{c}x+\sqrt{}}}{\sqrt{-}}\right.}{}}{8\left(b\right.}\right.$$

$\big\downarrow$ 218

$$2\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\frac{\sqrt{x}\left(-4ac+13b^2+24bcx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-c\left(\frac{28ac+41b^2}{\sqrt{b^2-4ac}}+36b\right)\left(-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x}}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{}{\sqrt[4]{}}\right)}{\sqrt[4]{2}\sqrt[4]{}}\right.}{}}{}\right.$$

$\downarrow$ 221

$$2\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\frac{\sqrt{x}\left(-4ac+13b^2+24bcx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{-c\left(\frac{28ac+41b^2}{\sqrt{b^2-4ac}}+36b\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arct}}{}\right.}{}}{}\right.$$

input
```
Int[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
2*((Sqrt[x]*(2*a + b*x^2))/(8*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - ((Sqr
t[x]*(13*b^2 - 4*a*c + 24*b*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
- (-(c*(36*b + (41*b^2 + 28*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(
1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[
b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)))) - c*(36
*b - (41*b^2 + 28*a*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[
x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*
c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(
1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/(4*(b^2 - 4*a*c))
)/(8*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1701
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-d^(2*n - 1))*(d*x)^(m - 2*n + 1)*(2*a + b*x^n)*((a + b*x
^n + c*x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^(2*n)/(n*(p
 + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m - 2*n)*(2*a*(m - 2*n + 1) + b*(m + n*(2
*p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1
] && GtQ[m, 2*n - 1]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1760
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/
(a*n*(p + 1)*(b^2 - 4*a*c))   Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*
d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n
+ c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n
] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.48 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.44

method	result
derivativedivides	$\frac{-\frac{a(28ac+5b^2)\sqrt{x}}{16(16a^2c^2-8ab^2c+b^4)}-\frac{9b(4ac+b^2)x^{\frac{5}{2}}}{16(16a^2c^2-8ab^2c+b^4)}+\frac{2c(4ac-37b^2)x^{\frac{9}{2}}}{512a^2c^2-256ab^2c+32b^4}-\frac{3bc^2x^{\frac{13}{2}}}{2(16a^2c^2-8ab^2c+b^4)}}{(cx^4+bx^2+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z}}{}$
default	$\frac{-\frac{a(28ac+5b^2)\sqrt{x}}{16(16a^2c^2-8ab^2c+b^4)}-\frac{9b(4ac+b^2)x^{\frac{5}{2}}}{16(16a^2c^2-8ab^2c+b^4)}+\frac{2c(4ac-37b^2)x^{\frac{9}{2}}}{512a^2c^2-256ab^2c+32b^4}-\frac{3bc^2x^{\frac{13}{2}}}{2(16a^2c^2-8ab^2c+b^4)}}{(cx^4+bx^2+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z}}{}$

input
```
int(x^(7/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/32*a*(28*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-9/32*b*(4*a*c
+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/32*c*(4*a*c-37*b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^(9/2)-3/4*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*
x^4+b*x^2+a)^2+1/64/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((-72*_R^4*b*c+28*a*c+5*
b^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13878 vs. $2(429) = 858$.

Time = 6.00 (sec) , antiderivative size = 13878, normalized size of antiderivative = 26.04

$$\int \frac{x^{7/2}}{(a + bx^2 + cx^4)^3} dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(a + bx^2 + cx^4)^3} dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{7}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*((5*b^2*c^2 + 28*a*c^3)*x^(17/2) + 2*(5*b^3*c + 16*a*b*c^2)*x^(13/2)
+ (5*b^4 + a*b^2*c + 60*a^2*c^2)*x^(9/2) + (a*b^3 + 20*a^2*b*c)*x^(5/2))/(
(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*
a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*
b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) -
integrate(1/32*((5*b^2*c + 28*a*c^2)*x^(7/2) + 5*(b^3 + 20*a*b*c)*x^(3/2))
/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c
^3)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2), x)
```

Giac [F]

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{7}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x^(7/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 22.48 (sec) , antiderivative size = 47803, normalized size of antiderivative = 89.69

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int(x^(7/2)/(a + b*x^2 + c*x^4)^3,x)
```

output
```
atan(((((171894580*a*b^8*c^7 - 48125*b^10*c^6 - 17210368*a^5*c^11 + 352085
6800*a^2*b^6*c^8 + 3512738432*a^3*b^4*c^9 + 167976704*a^4*b^2*c^10)/(65536
*(b^18 - 262144*a^9*c^9 + 576*a^2*b^14*c^2 - 5376*a^3*b^12*c^3 + 32256*a^4
*b^10*c^4 - 129024*a^5*b^8*c^5 + 344064*a^6*b^6*c^6 - 589824*a^7*b^4*c^7 +
 589824*a^8*b^2*c^8 - 36*a*b^16*c)) + ((((625*b^6*(-(4*a*c - b^2)^25)^(1/2
) - 625*b^31 + 15192104632320*a^15*b*c^15 + 89000*a^2*b^27*c^2 - 27186416*
a^3*b^25*c^3 + 1342297600*a^4*b^23*c^4 - 25492409600*a^5*b^21*c^5 + 265188
833280*a^6*b^19*c^6 - 1688816578560*a^7*b^17*c^7 + 6664504147968*a^8*b^15*
c^8 - 14462970429440*a^9*b^13*c^9 + 4163326443520*a^10*b^11*c^10 + 7045524
2260480*a^11*b^9*c^11 - 206669464207360*a^12*b^7*c^12 + 267459844112384*a^
13*b^5*c^13 - 150009114787840*a^14*b^3*c^14 - 38416*a^3*c^3*(-(4*a*c - b^2
)^25)^(1/2) - 23125*a*b^29*c + 1911000*a^2*b^2*c^2*(-(4*a*c - b^2)^25)^(1/
2) + 54375*a*b^4*c*(-(4*a*c - b^2)^25)^(1/2))/(33554432*(a^3*b^40 + 109951
1627776*a^23*c^20 - 80*a^4*b^38*c + 3040*a^5*b^36*c^2 - 72960*a^6*b^34*c^3
 + 1240320*a^7*b^32*c^4 - 15876096*a^8*b^30*c^5 + 158760960*a^9*b^28*c^6 -
 1270087680*a^10*b^26*c^7 + 8255569920*a^11*b^24*c^8 - 44029706240*a^12*b^
22*c^9 + 193730707456*a^13*b^20*c^10 - 704475299840*a^14*b^18*c^11 + 21134
25899520*a^15*b^16*c^12 - 5202279137280*a^16*b^14*c^13 + 10404558274560*a^
17*b^12*c^14 - 16647293239296*a^18*b^10*c^15 + 20809116549120*a^19*b^8*c^1
6 - 19585050869760*a^20*b^6*c^17 + 13056700579840*a^21*b^4*c^18 - 54975...
```

Reduce [F]

$$\int \frac{x^{7/2}}{(a+bx^2+cx^4)^3} \, dx = \int \frac{x^{\frac{7}{2}}}{(c\,x^4+b\,x^2+a)^3}dx$$

input
```
int(x^(7/2)/(c*x^4+b*x^2+a)^3,x)
```

output

```
int(x^(7/2)/(c*x^4+b*x^2+a)^3,x)
```

3.944 $\int \frac{x^{5/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 594

$$\int \frac{x^{5/2}}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x^{3/2}(b+2cx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3x^{3/2}(b(b^2+4ac)+c(b^2+12ac)\,x^2)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3\sqrt[4]{c}\left(b^2+12ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\ 2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{3\sqrt[4]{c}\left(b^3-68abc+\sqrt{b^2-4ac}(b^2+12ac)\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\ 2^{3/4}a\left(b^2-4ac\right)^{5/2}\sqrt[4]{-b+\sqrt{b^2-4ac}}} - \frac{3\sqrt[4]{c}\left(b^2+12ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\ 2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{3\sqrt[4]{c}\left(b^3-68abc+\sqrt{b^2-4ac}(b^2+12ac)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\ 2^{3/4}a\left(b^2-4ac\right)^{5/2}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
-1/4*x^(3/2)*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/16*x^(3/2)*(b*(4
*a*c+b^2)+c*(12*a*c+b^2)*x^2)/a/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3/64*c^(1/4
)*(b^2+12*a*c-b^3/(-4*a*c+b^2)^(1/2)+68*a*b*c/(-4*a*c+b^2)^(1/2))*arctan(2
^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-4*a*c+b^
2)^2/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+3/64*c^(1/4)*(b^3-68*a*b*c+(-4*a*c+b^2)
^(1/2)*(12*a*c+b^2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2)
)^(1/4))*2^(1/4)/a/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-3/64*c
^(1/4)*(b^2+12*a*c-b^3/(-4*a*c+b^2)^(1/2)+68*a*b*c/(-4*a*c+b^2)^(1/2))*arc
tanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a/(-4*
a*c+b^2)^2/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-3/64*c^(1/4)*(b^3-68*a*b*c+(-4*a*
c+b^2)^(1/2)*(12*a*c+b^2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2
)^(1/2))^(1/4))*2^(1/4)/a/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.36

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^3}\,dx = \frac{\frac{4x^{3/2}\left(3b^2x^2(b+cx^2)^2+4a^2c(7b+17cx^2)+a(-b^3+7b^2cx^2+48bc^2x^4+36c^3x^6)\right)}{(a+bx^2+cx^4)^2}+3\text{RootSum}\left[a+b\#1\right.}{64a\,(}$$

input
```
Integrate[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((4*x^(3/2)*(3*b^2*x^2*(b + c*x^2)^2 + 4*a^2*c*(7*b + 17*c*x^2) + a*(-b^3
+ 7*b^2*c*x^2 + 48*b*c^2*x^4 + 36*c^3*x^6)))/(a + b*x^2 + c*x^4)^2 + 3*Roo
tSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 28*a*b*c*Log[Sqrt[x]
 - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 12*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(
b*#1 + 2*c*#1^5) & ])/(64*a*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1435, 1700, 27, 1824, 25, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x^3}{(cx^4+bx^2+a)^3}d\sqrt{x}$$

$$\Big\downarrow \text{1700}$$

$$2\left(\frac{\int \frac{3x(b-6cx^2)}{(cx^4+bx^2+a)^2}d\sqrt{x}}{8(b^2-4ac)}-\frac{x^{3/2}(b+2cx^2)}{8(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{3\int \frac{x\left(b-6cx^2\right)}{\left(cx^4+bx^2+a\right)^2}d\sqrt{x}}{8\left(b^2-4ac\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \text{1824}$$

$$2\left(\frac{3\left(\frac{x^{3/2}\left(cx^2\left(12ac+b^2\right)+b\left(4ac+b^2\right)\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{x\left(c\left(b^2+12ac\right)x^2+b\left(b^2-28ac\right)\right)}{cx^4+bx^2+a}d\sqrt{x}}{4a\left(b^2-4ac\right)}\right)}{8\left(b^2-4ac\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{3\left(\frac{\int \frac{x\left(c\left(b^2+12ac\right)x^2+b\left(b^2-28ac\right)\right)}{cx^4+bx^2+a}d\sqrt{x}}{4a\left(b^2-4ac\right)}+\frac{x^{3/2}\left(cx^2\left(12ac+b^2\right)+b\left(4ac+b^2\right)\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)}{8\left(b^2-4ac\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \text{1834}$$

$$2\left(\frac{3\left(\frac{\frac{1}{2}c\left(-\frac{68abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+\frac{1}{2}c\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\int \frac{2x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{4a\left(b^2-4ac\right)}+\frac{x^{3/2}}{4}\right.}{8\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{3\left(\frac{c\left(-\frac{68abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\int \frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}+c\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\int \frac{x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{4a\left(b^2-4ac\right)}+\frac{x^{3/2}\left(cx\right.}{4a(b}\right.}{8\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \text{827}$$

$$2\left(\frac{3\left(\frac{c\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)+c\left(-\frac{68abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right.}{4a(b^2-4ac)}}{8\left(b^2-4ac\right)}\right.$$

$\downarrow$ 218

$$2\left(\frac{3\left(\frac{c\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)+c\left(-\frac{68abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+1\right.}{4a(b^2-4ac)}}{8\left(b^2-4ac\right)}\right.$$

$\downarrow$ 221

$$2\left(\frac{3\left(\frac{c\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)+c\left(-\frac{68abc}{\sqrt{b^2-4ac}}+\cdots\right.}{4a(b^2-4ac)}}{8\,(b^2-\cdots}\right.$$

input
```
Int[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
2*(-1/8*(x^(3/2)*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3
*((x^(3/2)*(b*(b^2 + 4*a*c) + c*(b^2 + 12*a*c)*x^2))/(4*a*(b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (c*(b^2 + 12*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)
/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a
*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(
2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4
)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + c*(b^2 + 12*a*c + b^3/Sqrt[b^2 - 4*a*
c] - (68*a*b*c)/Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b +
 Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/
4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2
*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/(4*a*(b^2 - 4*a*c))))/(
8*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1700
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*((a + b*x^n + c*
x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] - Simp[d^n/(n*(p + 1)*(b^2
- 4*a*c))   Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^
n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ
[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
 1] && LeQ[m, 2*n - 1]
```

rule 1824
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m
+ 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.51 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.47

method	result
derivativedivides	$\frac{\frac{2b(28ac-b^2)x^{\frac{3}{2}}}{512a^2c^2-256a\,b^2c+32b^4}+\frac{(68a^2c^2+7a\,b^2c+3b^4)x^{\frac{7}{2}}}{16a(16a^2c^2-8a\,b^2c+b^4)}+\frac{3cb(8ac+b^2)x^{\frac{11}{2}}}{8a(16a^2c^2-8a\,b^2c+b^4)}+\frac{3c^2(12ac+b^2)x^{\frac{15}{2}}}{16a(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}-\frac{3\left(\sum_{\_R=\text{RootO}}\right.}{}$
default	$\frac{\frac{2b(28ac-b^2)x^{\frac{3}{2}}}{512a^2c^2-256a\,b^2c+32b^4}+\frac{(68a^2c^2+7a\,b^2c+3b^4)x^{\frac{7}{2}}}{16a(16a^2c^2-8a\,b^2c+b^4)}+\frac{3cb(8ac+b^2)x^{\frac{11}{2}}}{8a(16a^2c^2-8a\,b^2c+b^4)}+\frac{3c^2(12ac+b^2)x^{\frac{15}{2}}}{16a(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}-\frac{3\left(\sum_{\_R=\text{RootO}}\right.}{}$

input
```
int(x^(5/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/32*b*(28*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)+1/32*(68*a^2*c^2
+7*a*b^2*c+3*b^4)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+3/16*c/a*b*(8*a*c+b
^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(11/2)+3/32*c^2*(12*a*c+b^2)/a/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^(15/2))/(c*x^4+b*x^2+a)^2-3/64/a/(16*a^2*c^2-8*a*b^2*c+
b^4)*sum((c*(-12*a*c-b^2)*_R^6+b*(28*a*c-b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x
^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21124 vs. $2(492) = 984$.

Time = 62.23 (sec) , antiderivative size = 21124, normalized size of antiderivative = 35.56

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{5}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*(3*(b^2*c^2 + 12*a*c^3)*x^(15/2) + 6*(b^3*c + 8*a*b*c^2)*x^(11/2) + (
3*b^4 + 7*a*b^2*c + 68*a^2*c^2)*x^(7/2) - (a*b^3 - 28*a^2*b*c)*x^(3/2))/((
a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a
^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b
^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) + i
ntegrate(3/32*((b^2*c + 12*a*c^2)*x^(5/2) + (b^3 - 28*a*b*c)*sqrt(x))/(a^2
*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x
^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2), x)
```

Giac [F]

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{5}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(5/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x^(5/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 22.13 (sec) , antiderivative size = 42197, normalized size of antiderivative = 71.04

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(5/2)/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((3*x^(11/2)*(b^3*c + 8*a*b*c^2))/(8*(a*b^4 + 16*a^3*c^2 - 8*a^2*b^2*c)) -
 (x^(3/2)*(b^3 - 28*a*b*c))/(16*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^(7/2)
*(3*b^4 + 68*a^2*c^2 + 7*a*b^2*c))/(16*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) +
 (3*c^2*x^(15/2)*(12*a*c + b^2))/(16*(a*b^4 + 16*a^3*c^2 - 8*a^2*b^2*c)))/
(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - atan(((((27*
(3799912185593856*a^15*c^19 + 2097152*b^30*c^4 - 266338304*a*b^28*c^5 + 14
019461120*a^2*b^26*c^6 - 402594463744*a^3*b^24*c^7 + 7074549334016*a^4*b^2
2*c^8 - 81637933056000*a^5*b^20*c^9 + 645335479222272*a^6*b^18*c^10 - 3564
382621532160*a^7*b^16*c^11 + 13728399105196032*a^8*b^14*c^12 - 35694820362
027008*a^9*b^12*c^13 + 65529603635707904*a^10*b^10*c^14 - 3376765135644262
4*a^11*b^8*c^15 - 51215251621806080*a^12*b^6*c^16 + 114542723335192576*a^1
3*b^4*c^17 - 70615034782285824*a^14*b^2*c^18))/(33554432*(a^2*b^28 + 26843
5456*a^16*c^14 - 56*a^3*b^26*c + 1456*a^4*b^24*c^2 - 23296*a^5*b^22*c^3 +
256256*a^6*b^20*c^4 - 2050048*a^7*b^18*c^5 + 12300288*a^8*b^16*c^6 - 56229
888*a^9*b^14*c^7 + 196804608*a^10*b^12*c^8 - 524812288*a^11*b^10*c^9 + 104
9624576*a^12*b^8*c^10 - 1526726656*a^13*b^6*c^11 + 1526726656*a^14*b^4*c^1
2 - 939524096*a^15*b^2*c^13)) - (9*x^(1/2)*(-(81*(b^33 + b^8*(-(4*a*c - b^
2)^25)^(1/2) - 471104225280*a^16*b*c^16 + 10509*a^2*b^29*c^2 - 394248*a^3*
b^27*c^3 + 9219696*a^4*b^25*c^4 - 140233728*a^5*b^23*c^5 + 1424368896*a^6*
b^21*c^6 - 9732052992*a^7*b^19*c^7 + 43376799744*a^8*b^17*c^8 - 1084930...
```

Reduce [F]

$$\int \frac{x^{5/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{5}{2}}}{(c\,x^4+b\,x^2+a)^3}dx$$

input
```
int(x^(5/2)/(c*x^4+b*x^2+a)^3,x)
```

output
```
int(x^(5/2)/(c*x^4+b*x^2+a)^3,x)
```

3.945 $\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 594

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx = -\frac{\sqrt{x}(b+2cx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\sqrt{x}(b(b^2+20ac)+c(b^2+44ac)\,x^2)}{16a\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \frac{3c^{3/4}\left(b^2+44ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}a\,(b^2-4ac)^2\,\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{3c^{3/4}\left(b^3-68abc+\sqrt{b^2-4ac}(b^2+44ac)\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}a\,(b^2-4ac)^{5/2}\,\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} - \frac{3c^{3/4}\left(b^2+44ac-\frac{b^3}{\sqrt{b^2-4ac}}+\frac{68abc}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}a\,(b^2-4ac)^2\,\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{3c^{3/4}\left(b^3-68abc+\sqrt{b^2-4ac}(b^2+44ac)\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}a\,(b^2-4ac)^{5/2}\,\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

output

```
-1/4*x^(1/2)*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*x^(1/2)*(b*(2
0*a*c+b^2)+c*(44*a*c+b^2)*x^2)/a/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-3/64*c^(3/
4)*(b^2+44*a*c-b^3/(-4*a*c+b^2)^(1/2)+68*a*b*c/(-4*a*c+b^2)^(1/2))*arctan(
2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-4*a*c+b
^2)^2/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3/64*c^(3/4)*(b^3-68*a*b*c+(-4*a*c+b^2
)^(1/2)*(44*a*c+b^2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2
))^(1/4))*2^(3/4)/a/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-3/64*
c^(3/4)*(b^2+44*a*c-b^3/(-4*a*c+b^2)^(1/2)+68*a*b*c/(-4*a*c+b^2)^(1/2))*ar
ctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a/(-4
*a*c+b^2)^2/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3/64*c^(3/4)*(b^3-68*a*b*c+(-4*a
*c+b^2)^(1/2)*(44*a*c+b^2))*arctanh(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^
2)^(1/2))^(1/4))*2^(3/4)/a/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.36

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx = \frac{\frac{4\sqrt{x}\left(b^2x^2(b+cx^2)^2+4a^2c(9b+19cx^2)+a(-3b^3+13b^2cx^2+64bc^2x^4+44c^3x^6)\right)}{(a+bx^2+cx^4)^2}+3\text{RootSum}\left[a+b\#\right.}{64a\,(}$$

input
```
Integrate[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((4*Sqrt[x]*(b^2*x^2*(b + c*x^2)^2 + 4*a^2*c*(9*b + 19*c*x^2) + a*(-3*b^3
+ 13*b^2*c*x^2 + 64*b*c^2*x^4 + 44*c^3*x^6)))/(a + b*x^2 + c*x^4)^2 + 3*Ro
otSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] - #1] - 12*a*b*c*Log[Sqrt[x
] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4 + 44*a*c^2*Log[Sqrt[x] - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ])/(64*a*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 516, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1435, 1700, 1760, 27, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x^2}{(cx^4+bx^2+a)^3}\,d\sqrt{x}$$

$$\Big\downarrow \text{1700}$$

$$2\left(\frac{\int \frac{b-22cx^2}{(cx^4+bx^2+a)^2}\,d\sqrt{x}}{8(b^2-4ac)}-\frac{\sqrt{x}(b+2cx^2)}{8(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \; 1760$$

$$2\left(\frac{\frac{\sqrt{x}\left(cx^2\left(44ac+b^2\right)+b\left(20ac+b^2\right)\right)}{4a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{3\left(c\left(b^2+44ac\right)x^2+b\left(b^2-12ac\right)\right)}{cx^4+bx^2+a}d\sqrt{x}}{4a(b^2-4ac)}}{8\left(b^2-4ac\right)}-\frac{\sqrt{x}\left(b+2cx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{3\int \frac{c\left(b^2+44ac\right)x^2+b\left(b^2-12ac\right)}{cx^4+bx^2+a}d\sqrt{x}}{4a(b^2-4ac)}+\frac{\sqrt{x}\left(cx^2\left(44ac+b^2\right)+b\left(20ac+b^2\right)\right)}{4a(b^2-4ac)(a+bx^2+cx^4)}}{8\left(b^2-4ac\right)}-\frac{\sqrt{x}\left(b+2cx^2\right)}{8\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \; 1752$$

$$2\left(\frac{\frac{3\left(\frac{1}{2}c\left(-\frac{68abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d\sqrt{x}+\frac{1}{2}c\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}d\sqrt{x}\right)}{4a(b^2-4ac)}+}{8\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \; 756$$

$$2\left(\frac{\frac{3\left(\frac{1}{2}c\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\left(-\frac{\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{-b-\sqrt{b^2-4ac}}}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)+\frac{1}{2}c\left(-\frac{68abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4ac}}+44\right.}{4a(b^2-4ac)}}{8\left(b^2-4ac\right)}\right.$$

$$\Big\downarrow \; 218$$

$$2\left(\frac{\frac{3\left(\frac{1}{2}c\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\left(-\frac{\int\frac{1}{\sqrt{-b-\sqrt{b^2-4ac}-\sqrt{2}\sqrt{c}x}}d\sqrt{x}}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(-\frac{68abc}{\sqrt{b^2-4ac}}+\frac{b^3}{\sqrt{b^2-4a}}\right.}{4a(b^2-4ac)}}{8\left(b^2-4ac\right)}\right.$$

$$\downarrow 221$$

$$2\left(\frac{\frac{3\left(\frac{1}{2}c\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}\right)+\frac{1}{2}c\left(-\frac{68abc}{\sqrt{b^2-4ac}}\right.}{4a(b^2-4ac)}}{8\left(b^2-\right.}\right.$$

input
```
Int[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

output
```
2*(-1/8*(Sqrt[x]*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((
Sqrt[x]*(b*(b^2 + 20*a*c) + c*(b^2 + 44*a*c)*x^2))/(4*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (3*((c*(b^2 + 44*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*
c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTan
h[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/
4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/2 + (c*(b^2 + 44*a*c + b^3/Sqrt[b^2 -
 4*a*c] - (68*a*b*c)/Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x]
)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c]
)^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/
4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/2))/(4*a*(b^2 - 4*a
*c)))/(8*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1700

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(n - 1)*(d*x)^(m - n + 1)*(b + 2*c*x^n)*((a + b*x^n + c*
x^(2*n))^(p + 1)/(n*(p + 1)*(b^2 - 4*a*c))), x] - Simp[d^n/(n*(p + 1)*(b^2
- 4*a*c))   Int[(d*x)^(m - n)*(b*(m - n + 1) + 2*c*(m + 2*n*(p + 1) + 1)*x^
n)*(a + b*x^n + c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ
[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && ILtQ[p, -1] && GtQ[m, n -
 1] && LeQ[m, 2*n - 1]
```

rule 1752

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1760

```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/
(a*n*(p + 1)*(b^2 - 4*a*c))   Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*
d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n
+ c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n
] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.45

method	result
derivativedivides	$\frac{\frac{3b(12ac-b^2)\sqrt{x}}{16(16a^2c^2-8a\,b^2c+b^4)}+\frac{(76a^2c^2+13a\,b^2c+b^4)x^{\frac{5}{2}}}{16a(16a^2c^2-8a\,b^2c+b^4)}+\frac{cb(32ac+b^2)x^{\frac{9}{2}}}{8a(16a^2c^2-8a\,b^2c+b^4)}+\frac{c^2(44ac+b^2)x^{\frac{13}{2}}}{16a(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}}\right.}{}$
default	$\frac{\frac{3b(12ac-b^2)\sqrt{x}}{16(16a^2c^2-8a\,b^2c+b^4)}+\frac{(76a^2c^2+13a\,b^2c+b^4)x^{\frac{5}{2}}}{16a(16a^2c^2-8a\,b^2c+b^4)}+\frac{cb(32ac+b^2)x^{\frac{9}{2}}}{8a(16a^2c^2-8a\,b^2c+b^4)}+\frac{c^2(44ac+b^2)x^{\frac{13}{2}}}{16a(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^4+b\,x^2+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}}\right.}{}$

input
```
int(x^(3/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(3/32*b*(12*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)+1/32*(76*a^2*c^2
+13*a*b^2*c+b^4)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/16*c/a*b*(32*a*c+b
^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)+1/32*c^2*(44*a*c+b^2)/a/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+3/64/a/(16*a^2*c^2-8*a*b^2*c+b
^4)*sum((c*(44*a*c+b^2)*_R^4-12*a*b*c+b^3)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R
),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15770 vs. $2(492) = 984$.

Time = 15.98 (sec) , antiderivative size = 15770, normalized size of antiderivative = 26.55

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{3}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*(3*(b^3*c^2 - 12*a*b*c^3)*x^(17/2) + (6*b^4*c - 71*a*b^2*c^2 + 44*a^2
*c^3)*x^(13/2) + (3*b^5 - 28*a*b^3*c - 8*a^2*b*c^2)*x^(9/2) + (7*a*b^4 - 5
9*a^2*b^2*c + 76*a^3*c^2)*x^(5/2))/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*
c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c
^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3
*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) + integrate(-3/32*((b^3*c - 12*a*b
*c^2)*x^(7/2) + (b^4 - 13*a*b^2*c - 44*a^2*c^2)*x^(3/2))/(a^3*b^4 - 8*a^4*
b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b
^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), x)
```

Giac [F]

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{3}{2}}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x^(3/2)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 23.63 (sec) , antiderivative size = 54027, normalized size of antiderivative = 90.95

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(3/2)/(a + b*x^2 + c*x^4)^3,x)
```

output
```
atan(((((3*(230850*a*b^11*c^8 - 4455*b^13*c^7 + 24287662080*a^6*b*c^13 - 3
679344*a^2*b^9*c^9 + 8309952*a^3*b^7*c^10 - 548653824*a^4*b^5*c^11 + 97602
27840*a^5*b^3*c^12))/(65536*(a^4*b^18 - 262144*a^13*c^9 - 36*a^5*b^16*c +
576*a^6*b^14*c^2 - 5376*a^7*b^12*c^3 + 32256*a^8*b^10*c^4 - 129024*a^9*b^8
*c^5 + 344064*a^10*b^6*c^6 - 589824*a^11*b^4*c^7 + 589824*a^12*b^2*c^8)) +
 ((3*(-(81*(b^35 + b^10*(-(4*a*c - b^2)^25)^(1/2) + 12505065717760*a^17*b*
c^17 + 3910*a^2*b^31*c^2 - 91335*a^3*b^29*c^3 + 1329320*a^4*b^27*c^4 - 123
56816*a^5*b^25*c^5 + 70316800*a^6*b^23*c^6 - 181190400*a^7*b^21*c^7 - 6687
23200*a^8*b^19*c^8 + 10912870400*a^9*b^17*c^9 - 83490242560*a^10*b^15*c^10
 + 502626713600*a^11*b^13*c^11 - 2379389337600*a^12*b^11*c^12 + 8291284418
560*a^13*b^9*c^13 - 20114959237120*a^14*b^7*c^14 + 31974471237632*a^15*b^5
*c^15 - 29919144837120*a^16*b^3*c^16 - 234256*a^5*c^5*(-(4*a*c - b^2)^25)^
(1/2) - 95*a*b^33*c + 510*a^2*b^6*c^2*(-(4*a*c - b^2)^25)^(1/2) + 2015*a^3
*b^4*c^3*(-(4*a*c - b^2)^25)^(1/2) - 33880*a^4*b^2*c^4*(-(4*a*c - b^2)^25)
^(1/2) - 45*a*b^8*c*(-(4*a*c - b^2)^25)^(1/2)))/(33554432*(a^7*b^40 + 1099
511627776*a^27*c^20 - 80*a^8*b^38*c + 3040*a^9*b^36*c^2 - 72960*a^10*b^34*
c^3 + 1240320*a^11*b^32*c^4 - 15876096*a^12*b^30*c^5 + 158760960*a^13*b^28
*c^6 - 1270087680*a^14*b^26*c^7 + 8255569920*a^15*b^24*c^8 - 44029706240*a
^16*b^22*c^9 + 193730707456*a^17*b^20*c^10 - 704475299840*a^18*b^18*c^11 +
 2113425899520*a^19*b^16*c^12 - 5202279137280*a^20*b^14*c^13 + 10404558...
```

Reduce [F]

$$\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x^{\frac{3}{2}}}{(c\,x^4+b\,x^2+a)^3}dx$$

input
```
int(x^(3/2)/(c*x^4+b*x^2+a)^3,x)
```

output

```
int(x^(3/2)/(c*x^4+b*x^2+a)^3,x)
```

3.946 $\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 658

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}\,dx$$
$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{x^{3/2}(5b^4-45ab^2c+52a^2c^2+bc(5b^2-44ac)\,x^2)}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$
$$-\frac{\sqrt[4]{c}\left(5b^4-54ab^2c+520a^2c^2-b(5b^2-44ac)\,\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\;2^{3/4}a^2\,(b^2-4ac)^{5/2}\,\sqrt[4]{-b-\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt[4]{c}\left(5b^4-54ab^2c+520a^2c^2+b(5b^2-44ac)\,\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\;2^{3/4}a^2\,(b^2-4ac)^{5/2}\,\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$
$$+\frac{\sqrt[4]{c}\left(5b^4-54ab^2c+520a^2c^2-b(5b^2-44ac)\,\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\;2^{3/4}a^2\,(b^2-4ac)^{5/2}\,\sqrt[4]{-b-\sqrt{b^2-4ac}}}$$
$$-\frac{\sqrt[4]{c}\left(5b^4-54ab^2c+520a^2c^2+b(5b^2-44ac)\,\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\;2^{3/4}a^2\,(b^2-4ac)^{5/2}\,\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

output
```
1/4*x^(3/2)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*x^(3
/2)*(5*b^4-45*a*b^2*c+52*c^2*a^2+b*c*(-44*a*c+5*b^2)*x^2)/a^2/(-4*a*c+b^2)
^2/(c*x^4+b*x^2+a)-1/64*c^(1/4)*(5*b^4-54*a*b^2*c+520*c^2*a^2-b*(-44*a*c+5
*b^2)*(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^
(1/2))^(1/4))*2^(1/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)
+1/64*c^(1/4)*(5*b^4-54*a*b^2*c+520*c^2*a^2+b*(-44*a*c+5*b^2)*(-4*a*c+b^2)
^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1
/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/64*c^(1/4)*(5*b
^4-54*a*b^2*c+520*c^2*a^2-b*(-44*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*arctanh(2^
(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a^2/(-4*a*c+b
^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/64*c^(1/4)*(5*b^4-54*a*b^2*c+520
*c^2*a^2+b*(-44*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(
1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b+(-4
*a*c+b^2)^(1/2))^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.47 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{\frac{4x^{3/2}\left(84a^3c^2+5b^3x^2(b+cx^2)^2+a^2c(-69b^2-8bcx^2+52c^2x^4)+ab(9b^3-36b^2cx^2-89bc^2x^4-44c^3x^6)\right)}{(a+bx^2+cx^4)^2}+\text{RootSum}\left[a+b\#1^4+c\#1\right.}{64a^2\left(b^2-\right.}$$

input
```
Integrate[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]
```

output
```
((4*x^(3/2)*(84*a^3*c^2 + 5*b^3*x^2*(b + c*x^2)^2 + a^2*c*(-69*b^2 - 8*b*c
*x^2 + 52*c^2*x^4) + a*b*(9*b^3 - 36*b^2*c*x^2 - 89*b*c^2*x^4 - 44*c^3*x^6
)))/(a + b*x^2 + c*x^4)^2 + RootSum[a + b*#1^4 + c*#1^8 & , (5*b^4*Log[Sqr
t[x] - #1] - 49*a*b^2*c*Log[Sqrt[x] - #1] + 260*a^2*c^2*Log[Sqrt[x] - #1]
+ 5*b^3*c*Log[Sqrt[x] - #1]*#1^4 - 44*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#
1 + 2*c*#1^5) & ])/(64*a^2*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1435, 1702, 25, 1824, 25, 1834, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{x}{(cx^4+bx^2+a)^3}d\sqrt{x}$$

$$\Big\downarrow \text{1702}$$

$$2\left(\frac{x^{3/2}(-2ac+b^2+bcx^2)}{8a(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{\int -\frac{x(5b^2+9cx^2b-26ac)}{(cx^4+bx^2+a)^2}d\sqrt{x}}{8a(b^2-4ac)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{x(5b^2+9cx^2b-26ac)}{(cx^4+bx^2+a)^2}d\sqrt{x}}{8a(b^2-4ac)}+\frac{x^{3/2}(-2ac+b^2+bcx^2)}{8a(b^2-4ac)(a+bx^2+cx^4)^2}\right)$$

$$\Big\downarrow \text{1824}$$

$$2\left(\frac{\frac{x^{3/2}(52a^2c^2+bcx^2(5b^2-44ac)-45ab^2c+5b^4)}{4a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{x\left(5b^4-49acb^2+c\left(5b^2-44ac\right)x^2b+260a^2c^2\right)}{cx^4+bx^2+a}d\sqrt{x}}{4a(b^2-4ac)}}{8a(b^2-4ac)}+\frac{x^{3/2}(-2ac+b^2+bcx^2)}{8a(b^2-4ac)(a+bx^2+cx^4}\right.$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\frac{\int \frac{x\left(5b^4-49acb^2+c\left(5b^2-44ac\right)x^2b+260a^2c^2\right)}{cx^4+bx^2+a}d\sqrt{x}}{4a(b^2-4ac)}+\frac{x^{3/2}(52a^2c^2+bcx^2(5b^2-44ac)-45ab^2c+5b^4)}{4a(b^2-4ac)(a+bx^2+cx^4)}}{8a(b^2-4ac)}+\frac{x^{3/2}(-2ac+b^2+bcx^2)}{8a(b^2-4ac)(a+bx^2+cx^4)^{\cdot}}\right.$$

$$\Big\downarrow \text{1834}$$

$$2\left(\frac{\frac{\frac{c\left(520a^2c^2-54ab^2c+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{2x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}}{2\sqrt{b^2-4ac}}-\frac{c\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{2x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{2\sqrt{b^2-4ac}}}{4a(b^2-4ac)}}{8a\left(b^2-4ac\right)}\right.$$

$\downarrow$ 27

$$2\left(\frac{\frac{\frac{c\left(520a^2c^2-54ab^2c+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{x}{2cx^2+b-\sqrt{b^2-4ac}}d\sqrt{x}}{\sqrt{b^2-4ac}}-\frac{c\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\int \frac{x}{2cx^2+b+\sqrt{b^2-4ac}}d\sqrt{x}}{\sqrt{b^2-4ac}}}{4a(b^2-4ac)}}{8a\left(b^2-4ac\right)}\right.$$

$\downarrow$ 827

$$2\left(\frac{\frac{\frac{c\left(520a^2c^2-54ab^2c+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\left(\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{\sqrt{b^2-4ac}-b}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{2}\sqrt{c}x}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\cdots\right.}{}}{4a(b^2-4ac)}}{8a\left(b^2-4a\cdots\right.}\right.$$

$\downarrow$ 218

$$2\left(\frac{\frac{\frac{c\left(520a^2c^2-54ab^2c+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{2}\sqrt{c}x}}d\sqrt{x}}{2\sqrt{2}\sqrt{c}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(520a^2c^2-54ab^2c-b\left(5b^2-\cdots\right.\right.}{}}{4a(b^2-4ac)}}{8a\left(b^2\cdots\right.}\right.$$

$\downarrow$ 221

$$2\left(\frac{\frac{\frac{c\left(520a^2c^2-54ab^2c+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\left(\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}-\frac{c\left(520a^2c^2-54ab^2\right.}{}}{4a(b^2-4ac)}}{}\right.$$

input
```
Int[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]
```

output
```
2*((x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(8*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)^2) + ((x^(3/2)*(5*b^4 - 45*a*b^2*c + 52*a^2*c^2 + b*c*(5*b^2 - 44*a*c)*x
^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (-((c*(5*b^4 - 54*a*b^2*c +
 520*a^2*c^2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt
[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))))/Sqrt[
b^2 - 4*a*c]) + (c*(5*b^4 - 54*a*b^2*c + 520*a^2*c^2 + b*(5*b^2 - 44*a*c)*
Sqrt[b^2 - 4*a*c])*(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*
c])^(1/4)]/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ArcTanh[(2
^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2*2^(3/4)*c^(3/4)
*(-b + Sqrt[b^2 - 4*a*c])^(1/4))))/Sqrt[b^2 - 4*a*c])/(4*a*(b^2 - 4*a*c)))
/(8*a*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1702
```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b
^2 - 4*a*c))   Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*
(p + 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && IGtQ[n, 0] && ILtQ[p, -1]
```

rule 1824
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^m*
(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m
+ 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + c*(m + n*(2*p + 3) + 1)*(b*d - 2*a*e)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntegerQ[p]
```

rule 1834
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 +
 (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2
 - (2*c*d - b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ
[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n
, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.49

method	result
derivativedivides	$\frac{\frac{3\left(28a^2c^2-23a\,b^2c+3b^4\right)x^{\frac{3}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)a}-\frac{b\left(8a^2c^2+36a\,b^2c-5b^4\right)x^{\frac{7}{2}}}{16a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{c\left(52a^2c^2-89a\,b^2c+10b^4\right)x^{\frac{11}{2}}}{16a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{b\,c^2\left(44ac-5b^2\right)x^{\frac{15}{2}}}{16a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{\_R$
default	$\frac{\frac{3\left(28a^2c^2-23a\,b^2c+3b^4\right)x^{\frac{3}{2}}}{16\left(16a^2c^2-8a\,b^2c+b^4\right)a}-\frac{b\left(8a^2c^2+36a\,b^2c-5b^4\right)x^{\frac{7}{2}}}{16a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{c\left(52a^2c^2-89a\,b^2c+10b^4\right)x^{\frac{11}{2}}}{16a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{b\,c^2\left(44ac-5b^2\right)x^{\frac{15}{2}}}{16a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}+\frac{\_R$

input
```
int(x^(1/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(3/32*(28*a^2*c^2-23*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(3/2)
-1/32*b*(8*a^2*c^2+36*a*b^2*c-5*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2
)+1/32/a^2*c*(52*a^2*c^2-89*a*b^2*c+10*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(
11/2)-1/32*b*c^2*(44*a*c-5*b^2)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(
c*x^4+b*x^2+a)^2+1/64/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((b*c*(-44*a*c+5*b
^2)*_R^6+(260*a^2*c^2-49*a*b^2*c+5*b^4)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)
-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23877 vs. $2(554) = 1108$.

Time = 162.85 (sec) , antiderivative size = 23877, normalized size of antiderivative = 36.29

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{\sqrt{x}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*((5*b^3*c^2 - 44*a*b*c^3)*x^(15/2) + (10*b^4*c - 89*a*b^2*c^2 + 52*a^
2*c^3)*x^(11/2) + (5*b^5 - 36*a*b^3*c - 8*a^2*b*c^2)*x^(7/2) + 3*(3*a*b^4
- 23*a^2*b^2*c + 28*a^3*c^2)*x^(3/2))/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a
^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^
3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) - integrate(-1/32*((5*b^3*c - 4
4*a*b*c^2)*x^(5/2) + (5*b^4 - 49*a*b^2*c + 260*a^2*c^2)*sqrt(x))/(a^3*b^4
- 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4
+ (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{\sqrt{x}}{(cx^4+bx^2+a)^3}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(x)/(c*x^4 + b*x^2 + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 23.59 (sec) , antiderivative size = 46948, normalized size of antiderivative = 71.35

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)/(a + b*x^2 + c*x^4)^3,x)
```

output
```
((x^(11/2)*(10*b^4*c + 52*a^2*c^3 - 89*a*b^2*c^2))/(16*(a^2*b^4 + 16*a^4*c
^2 - 8*a^3*b^2*c)) - (x^(7/2)*(8*a^2*b*c^2 - 5*b^5 + 36*a*b^3*c))/(16*a*(a
*b^4 + 16*a^3*c^2 - 8*a^2*b^2*c)) + (3*x^(3/2)*(3*b^4 + 28*a^2*c^2 - 23*a*
b^2*c))/(16*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^(15/2)*(44*a*c -
5*b^2))/(16*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^4*(2*a*c + b^2) + a^
2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan(((((2097152000*a*b^33*c^4 + 46
6178856428188467200*a^17*b*c^20 - 151833804800*a^2*b^31*c^5 + 534002008064
0*a^3*b^29*c^6 - 120300087803904*a^4*b^27*c^7 + 1933149881761792*a^5*b^25*
c^8 - 23398590986584064*a^6*b^23*c^9 + 219878252263505920*a^7*b^21*c^10 -
1631099300505190400*a^8*b^19*c^11 + 9625014804028588032*a^9*b^17*c^12 - 45
207702606568226816*a^10*b^15*c^13 + 168027072287612076032*a^11*b^13*c^14 -
 487882094458626375680*a^12*b^11*c^15 + 1082673222923122114560*a^13*b^9*c^
16 - 1771946621413479153664*a^14*b^7*c^17 + 2014068018680264916992*a^15*b^
5*c^18 - 1418770116510434197504*a^16*b^3*c^19)/(268435456*(a^6*b^28 + 2684
35456*a^20*c^14 - 56*a^7*b^26*c + 1456*a^8*b^24*c^2 - 23296*a^9*b^22*c^3 +
 256256*a^10*b^20*c^4 - 2050048*a^11*b^18*c^5 + 12300288*a^12*b^16*c^6 - 5
6229888*a^13*b^14*c^7 + 196804608*a^14*b^12*c^8 - 524812288*a^15*b^10*c^9
+ 1049624576*a^16*b^8*c^10 - 1526726656*a^17*b^6*c^11 + 1526726656*a^18*b^
4*c^12 - 939524096*a^19*b^2*c^13)) - (x^(1/2)*(-(625*b^37 - 625*b^12*(-(4*
a*c - b^2)^25)^(1/2) + 1127902032691 2000*a^18*b*c^18 + 2168275*a^2*b^33...
```

Reduce [F]

$$\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{\sqrt{x}}{(c\,x^4+b\,x^2+a)^3}dx$$

input
```
int(x^(1/2)/(c*x^4+b*x^2+a)^3,x)
```

output

```
int(x^(1/2)/(c*x^4+b*x^2+a)^3,x)
```

3.947 $\int \frac{1}{\sqrt{x}(a+bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 658

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^3}\,dx &= \frac{\sqrt{x}(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\sqrt{x}(7b^4-55ab^2c+60a^2c^2+bc(7b^2-52ac)\,x^2)}{16a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} \\
&+ \frac{3c^{3/4}\left(7b^4-66ab^2c+280a^2c^2-b(7b^2-52ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}a^2\,(b^2-4ac)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\
&- \frac{3c^{3/4}\left(7b^4-66ab^2c+280a^2c^2+b(7b^2-52ac)\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}a^2\,(b^2-4ac)^{5/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} \\
&+ \frac{3c^{3/4}\left(7b^4-66ab^2c+280a^2c^2-b(7b^2-52ac)\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}a^2\,(b^2-4ac)^{5/2}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} \\
&- \frac{3c^{3/4}\left(7b^4-66ab^2c+280a^2c^2+b(7b^2-52ac)\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{32\sqrt[4]{2}a^2\,(b^2-4ac)^{5/2}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}
\end{aligned}$$

output
```
1/4*x^(1/2)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/16*x^(1
/2)*(7*b^4-55*a*b^2*c+60*c^2*a^2+b*c*(-52*a*c+7*b^2)*x^2)/a^2/(-4*a*c+b^2)
^2/(c*x^4+b*x^2+a)+3/64*c^(3/4)*(7*b^4-66*a*b^2*c+280*c^2*a^2-b*(-52*a*c+7
*b^2)*(-4*a*c+b^2)^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^
(1/2))^(1/4))*2^(3/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)
-3/64*c^(3/4)*(7*b^4-66*a*b^2*c+280*c^2*a^2+b*(-52*a*c+7*b^2)*(-4*a*c+b^2)
^(1/2))*arctan(2^(1/4)*c^(1/4)*x^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3
/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+3/64*c^(3/4)*(7*b
^4-66*a*b^2*c+280*c^2*a^2-b*(-52*a*c+7*b^2)*(-4*a*c+b^2)^(1/2))*arctanh(2^
(1/4)*c^(1/4)*x^(1/2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a^2/(-4*a*c+b
^2)^(5/2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-3/64*c^(3/4)*(7*b^4-66*a*b^2*c+280
*c^2*a^2+b*(-52*a*c+7*b^2)*(-4*a*c+b^2)^(1/2))*arctanh(2^(1/4)*c^(1/4)*x^(
1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/a^2/(-4*a*c+b^2)^(5/2)/(-b+(-4
*a*c+b^2)^(1/2))^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.40 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^3}\,dx = \frac{\frac{4\sqrt{x}\left(92a^3c^2+7b^3x^2(b+cx^2)^2+a^2c(-79b^2-8bcx^2+60c^2x^4)+ab(11b^3-44b^2cx^2-107bc^2x^4-52c^3x^6)\right)}{(a+bx^2+cx^4)^2}+3\text{RootSum}\left[a+b\#1^4+c\#\right.}{64a^2\,(b^2-}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

output
```
((4*Sqrt[x]*(92*a^3*c^2 + 7*b^3*x^2*(b + c*x^2)^2 + a^2*c*(-79*b^2 - 8*b*c
*x^2 + 60*c^2*x^4) + a*b*(11*b^3 - 44*b^2*c*x^2 - 107*b*c^2*x^4 - 52*c^3*x
^6)))/(a + b*x^2 + c*x^4)^2 + 3*RootSum[a + b*#1^4 + c*#1^8 & , (7*b^4*Log
[Sqrt[x] - #1] - 59*a*b^2*c*Log[Sqrt[x] - #1] + 140*a^2*c^2*Log[Sqrt[x] -
#1] + 7*b^3*c*Log[Sqrt[x] - #1]*#1^4 - 52*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ])/(64*a^2*(b^2 - 4*a*c)^2)
```

Rubi [A] (verified)

Time = 2.39 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1435, 1683, 25, 1760, 27, 1752, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^3}dx$$

$$\Big\downarrow \text{1435}$$

$$2\int \frac{1}{\left(cx^4+bx^2+a\right)^3}d\sqrt{x}$$

$$\Big\downarrow \text{1683}$$

$$2\left(\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{8a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\int -\frac{7b^2+11cx^2b-30ac}{\left(cx^4+bx^2+a\right)^2}d\sqrt{x}}{8a\left(b^2-4ac\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{7b^2+11cx^2b-30ac}{\left(cx^4+bx^2+a\right)^2}d\sqrt{x}}{8a\left(b^2-4ac\right)}+\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{8a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \text{1760}$$

$$2\left(\frac{\frac{\sqrt{x}\left(60a^2c^2+bcx^2\left(7b^2-52ac\right)-55ab^2c+7b^4\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{3\left(7b^4-59acb^2+c\left(7b^2-52ac\right)x^2b+140a^2c^2\right)}{cx^4+bx^2+a}d\sqrt{x}}{4a\left(b^2-4ac\right)}}{8a\left(b^2-4ac\right)}+\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{8a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\frac{3\int \frac{7b^4-59acb^2+c\left(7b^2-52ac\right)x^2b+140a^2c^2}{cx^4+bx^2+a}d\sqrt{x}}{4a\left(b^2-4ac\right)}+\frac{\sqrt{x}\left(60a^2c^2+bcx^2\left(7b^2-52ac\right)-55ab^2c+7b^4\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}}{8a\left(b^2-4ac\right)}+\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{8a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}\right)$$

$$\Big\downarrow \text{1752}$$

$$2\left(\frac{3\left(\frac{c\left(280a^2c^2-66ab^2c+b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}d\sqrt{x}}{2\sqrt{b^2-4ac}}-\frac{c\left(280a^2c^2-66ab^2c-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-\right.}}}{2\sqrt{b^2-4ac}}\right.}{4a(b^2-4ac)}}{8a\left(b^2-4ac\right)}\right.$$

$\downarrow$ 756

$$2\left(\frac{3\left(\frac{c\left(280a^2c^2-66ab^2c+b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\left(-\frac{\int \frac{1}{\sqrt{\sqrt{b^2-4ac}-b}-\sqrt{2}\sqrt{c}x}d\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\int \frac{1}{\sqrt{2}\sqrt{c}x+\sqrt{\sqrt{b^2-4ac}-b}}d\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt{b^2-4ac}}-\frac{c\left(280a^2c^2-66ab^2c-b\left(7b^2-5\right.\right.}{}\right.}{4a(b^2-4ac)}}{8a\left(b^2-\right.}\right.$$

$\downarrow$ 218

$$2\left(\frac{3\left(\frac{c\left(280a^2c^2-66ab^2c+b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\left(-\frac{\int\frac{1}{\sqrt{\sqrt{b^2-4ac}-b-\sqrt{2}\sqrt{c}x}}d\sqrt{x}}{\sqrt{\sqrt{b^2-4ac}-b}}-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{2\sqrt{b^2-4ac}}-\frac{c\left(280a^2c^2-66ab^2c-b\left(7\right.\right.}{}\right)}{4a(b^2-4ac)}}{8a\left(b\right.}\right.$$

$\downarrow$ 221

$$2\left(\frac{3\left(\frac{c\left(280a^2c^2-66ab^2c+b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\left(-\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}\right)}{2\sqrt{b^2-4ac}}-\frac{c\left(280a^2c^2-6\ldots\right.}{\ldots}\right)}{4a\left(b^2-4ac\right)}\ldots}{\ldots}\right.$$

input
```
Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

output
```
2*((Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(8*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)^2) + ((Sqrt[x]*(7*b^4 - 55*a*b^2*c + 60*a^2*c^2 + b*c*(7*b^2 - 52*a*c)*x
^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*(-1/2*(c*(7*b^4 - 66*a*b
^2*c + 280*a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1
/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b
- Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))))/
Sqrt[b^2 - 4*a*c] + (c*(7*b^4 - 66*a*b^2*c + 280*a^2*c^2 + b*(7*b^2 - 52*a
*c)*Sqrt[b^2 - 4*a*c])*(-(ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2
- 4*a*c])^(1/4)]/(2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))) - ArcTa
nh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]/(2^(1/4)*c^(1
/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))))/(2*Sqrt[b^2 - 4*a*c])))/(4*a*(b^2 -
4*a*c)))/(8*a*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1435
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> With[{k = Denominator[m]}, Simp[k/d   Subst[Int[x^(k*(m + 1) - 1)*(a + b
*(x^(2*k)/d^2) + c*(x^(4*k)/d^4))^p, x], x, (d*x)^(1/k)], x]] /; FreeQ[{a,
b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1683
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-
x)*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(a*n*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c +
 n*(p + 1)*(b^2 - 4*a*c) + b*c*(n*(2*p + 3) + 1)*x^n)*(a + b*x^n + c*x^(2*n
))^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && ILtQ[p, -1]
```

rule 1752
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))
   Int[1/(b/2 - q/2 + c*x^n), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   I
nt[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2
 - 4*a*c] ||  !IGtQ[n/2, 0])
```

rule 1760
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p
_), x_Symbol] :> Simp[(-x)*(d*b^2 - a*b*e - 2*a*c*d + (b*d - 2*a*e)*c*x^n)*
((a + b*x^n + c*x^(2*n))^(p + 1)/(a*n*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/
(a*n*(p + 1)*(b^2 - 4*a*c))   Int[Simp[(n*p + n + 1)*d*b^2 - a*b*e - 2*a*c*
d*(2*n*p + 2*n + 1) + (2*n*p + 3*n + 1)*(d*b - 2*a*e)*c*x^n, x]*(a + b*x^n
+ c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n
] && NeQ[b^2 - 4*a*c, 0] && ILtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.48

method	result
derivativedivides	$\frac{\frac{\left(92a^{2}c^{2}-79a\,b^{2}c+11b^{4}\right)\sqrt{x}}{16\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)a}-\frac{b\left(8a^{2}c^{2}+44a\,b^{2}c-7b^{4}\right)x^{\frac{5}{2}}}{16a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c\left(60a^{2}c^{2}-107a\,b^{2}c+14b^{4}\right)x^{\frac{9}{2}}}{16a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{b\,c^{2}\left(52ac-7b^{2}\right)x^{\frac{13}{2}}}{16a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{3\left(\right.}{}$
default	$\frac{\frac{\left(92a^{2}c^{2}-79a\,b^{2}c+11b^{4}\right)\sqrt{x}}{16\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)a}-\frac{b\left(8a^{2}c^{2}+44a\,b^{2}c-7b^{4}\right)x^{\frac{5}{2}}}{16a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c\left(60a^{2}c^{2}-107a\,b^{2}c+14b^{4}\right)x^{\frac{9}{2}}}{16a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{b\,c^{2}\left(52ac-7b^{2}\right)x^{\frac{13}{2}}}{16a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{3\left(\right.}{}$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/32*(92*a^2*c^2-79*a*b^2*c+11*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(1/2
)-1/32*b*(8*a^2*c^2+44*a*b^2*c-7*b^4)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/
2)+1/32/a^2*c*(60*a^2*c^2-107*a*b^2*c+14*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^(9/2)-1/32*b*c^2*(52*a*c-7*b^2)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/
(c*x^4+b*x^2+a)^2+3/64/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*sum((b*c*(-52*a*c+7*
b^2)*_R^4+140*a^2*c^2-59*a*b^2*c+7*b^4)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_
R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17801 vs. 2(554) = 1108.

Time = 44.14 (sec) , antiderivative size = 17801, normalized size of antiderivative = 27.05

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^3}\,dx = \int \frac{1}{(cx^4+bx^2+a)^3\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*(3*(7*b^4*c^2 - 59*a*b^2*c^3 + 140*a^2*c^4)*x^(17/2) + (42*b^5*c - 34
7*a*b^3*c^2 + 788*a^2*b*c^3)*x^(13/2) + (21*b^6 - 121*a*b^4*c - 41*a^2*b^2
*c^2 + 900*a^3*c^3)*x^(9/2) + (49*a*b^5 - 398*a^2*b^3*c + 832*a^3*b*c^2)*x
^(5/2) + 32*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*sqrt(x))/(a^5*b^4 - 8*a^6
*b^2*c + 16*a^7*c^2 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^8 + 2*(
a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 3
2*a^6*c^3)*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^2) - integrate
(3/32*((7*b^4*c - 59*a*b^2*c^2 + 140*a^2*c^3)*x^(7/2) + (7*b^5 - 66*a*b^3*
c + 192*a^2*b*c^2)*x^(3/2))/(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + (a^3*b^4
*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^4 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c
^2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^3}\,dx=\int \frac{1}{\left(cx^4+bx^2+a\right)^3\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^3*sqrt(x)), x)
```

Mupad [B] (verification not implemented)

Time = 23.43 (sec) , antiderivative size = 60099, normalized size of antiderivative = 91.34

$$\int \frac{1}{\sqrt{x}\left(a+bx^2+cx^4\right)^3}\,dx=\text{Too large to display}$$

input
```
int(1/(x^(1/2)*(a + b*x^2 + c*x^4)^3),x)
```

output
```
((x^(9/2)*(14*b^4*c + 60*a^2*c^3 - 107*a*b^2*c^2))/(16*a^2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)) + (x^(1/2)*(11*b^4 + 92*a^2*c^2 - 79*a*b^2*c))/(16*a*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^(5/2)*(8*a^2*b*c^2 - 7*b^5 + 44*a*b^3*c))
/(16*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b*c^2*x^(13/2)*(52*a*c - 7*b^2
))/(16*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2
*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - atan((((((9*x^(1/2)*(154670499702505472 0*a
^19*b*c^19 - 822083584*a^4*b^31*c^4 + 50851741696*a^5*b^29*c^5 - 147367709
9008*a^6*b^27*c^6 + 26523687976960*a^7*b^25*c^7 - 331351626612736*a^8*b^23
*c^8 + 3041476258824192*a^9*b^21*c^9 - 21176692735213568*a^10*b^19*c^10 +
113812892427485184*a^11*b^17*c^11 - 475720885626470400*a^12*b^15*c^12 + 15
45406748670558208*a^13*b^13*c^13 - 3867206695260258304*a^14*b^11*c^14 + 73
15227880965799936*a^15*b^9*c^15 - 10117494892562219008*a^16*b^7*c^16 + 965
0897342106173440*a^17*b^5*c^17 - 5672002255696429056*a^18*b^3*c^18))/(4194
304*(a^8*b^24 + 16777216*a^20*c^12 - 48*a^9*b^22*c + 1056*a^10*b^20*c^2 -
14080*a^11*b^18*c^3 + 126720*a^12*b^16*c^4 - 811008*a^13*b^14*c^5 + 378470
4*a^14*b^12*c^6 - 12976128*a^15*b^10*c^7 + 32440320*a^16*b^8*c^8 - 5767168
0*a^17*b^6*c^9 + 69206016*a^18*b^4*c^10 - 50331648*a^19*b^2*c^11)) - (3*(-
(81*(2401*b^39 - 2401*b^14*(-(4*a*c - b^2)^25)^(1/2) - 2405416566784000*a^
19*b*c^19 + 7445060*a^2*b^35*c^2 - 180851965*a^3*b^33*c^3 + 3112544495*a^4
*b^31*c^4 - 40302663491*a^5*b^29*c^5 + 406936342200*a^6*b^27*c^6 - 3276...
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}\,(a+bx^2+cx^4)^3}\,dx=\int \frac{1}{\sqrt{x}\,\left(c\,x^4+b\,x^2+a\right)^3}dx$$

input
```
int(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x)
```

output
```
int(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x)
```

3.948 $\int x^7\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx = -\frac{b(7b^2-12ac)\,(b+2cx^2)\sqrt{a+bx^2+cx^4}}{256c^4} + \frac{x^4(a+bx^2+cx^4)^{3/2}}{10c} + \frac{(35b^2-32ac-42bcx^2)\,(a+bx^2+cx^4)^{3/2}}{480c^3} + \frac{b(7b^2-12ac)\,(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{9/2}}$$

output
```
-1/256*b*(-12*a*c+7*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^4+1/10*x^4*(c
*x^4+b*x^2+a)^(3/2)/c+1/480*(-42*b*c*x^2-32*a*c+35*b^2)*(c*x^4+b*x^2+a)^(3
/2)/c^3+1/512*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/
2)/(c*x^4+b*x^2+a)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx$$
$$=\frac{\sqrt{a+bx^2+cx^4}(-105b^4+70b^3cx^2+4b^2c(115a-14cx^4)+8bc^2x^2(-29a+6cx^4)+128c^2(-2a^2+acx^4}{3840c^4}$$
$$-\frac{(7b^5-40ab^3c+48a^2bc^2)\log\left(c^4\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)}{512c^{9/2}}$$

input
```
Integrate[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(-105*b^4 + 70*b^3*c*x^2 + 4*b^2*c*(115*a - 14*c*
x^4) + 8*b*c^2*x^2*(-29*a + 6*c*x^4) + 128*c^2*(-2*a^2 + a*c*x^4 + 3*c^2*x
^8)))/(3840*c^4) - ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*Log[c^4*(b + 2*c*x
^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(9/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1434, 1166, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \text{1434}$$
$$\frac{1}{2}\int x^6\sqrt{cx^4+bx^2+a}dx^2$$
$$\Big\downarrow \text{1166}$$
$$\frac{1}{2}\left(\frac{\int -\frac{1}{2}x^2\left(7bx^2+4a\right)\sqrt{cx^4+bx^2+a}dx^2}{5c}+\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{5c}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\left(\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{5c}-\frac{\int x^2\left(7bx^2+4a\right)\sqrt{cx^4+bx^2+a}dx^2}{10c}\right)$$

$$\Big\downarrow 1225$$

$$\frac{1}{2}\left(\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{5c}-\frac{\frac{5b(7b^2-12ac)\int\sqrt{cx^4+bx^2+a}dx^2}{16c^2}-\frac{\left(-32ac+35b^2-42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{24c^2}}{10c}\right)$$

$$\Big\downarrow 1087$$

$$\frac{1}{2}\left(\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{5c}-\frac{\frac{5b(7b^2-12ac)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)}{16c^2}-\frac{\left(-32ac+35b^2-42bcx^2\right)\left(a+bx^2+\right.}{24c^2}}{10c}\right.$$

$$\Big\downarrow 1092$$

$$\frac{1}{2}\left(\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{5c}-\frac{\frac{5b(7b^2-12ac)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)}{16c^2}-\frac{\left(-32ac+35b^2-42bcx^2\right)\left(a+\right.}{24c^2}}{10c}\right.$$

$$\Big\downarrow 219$$

$$\frac{1}{2}\left(\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{5c}-\frac{\frac{5b(7b^2-12ac)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c^2}-\frac{\left(-32ac+35b^2-42bc\right.}{24c}}{10c}\right.$$

input
```
Int[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((x^4*(a + b*x^2 + c*x^4)^(3/2))/(5*c) - (-1/24*((35*b^2 - 32*a*c - 42*b*c
*x^2)*(a + b*x^2 + c*x^4)^(3/2))/c^2 + (5*b*(7*b^2 - 12*a*c)*(((b + 2*c*x^
2)*Sqrt[a + b*x^2 + c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(
2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2))))/(16*c^2))/(10*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{\left(-384c^{4}x^{8}-48b\,c^{3}x^{6}-128a\,c^{3}x^{4}+56b^{2}c^{2}x^{4}+232ab\,c^{2}x^{2}-70x^{2}b^{3}c+256a^{2}c^{2}-460a\,b^{2}c+105b^{4}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{3840c^{4}}+\frac{b(48}{}$
pseudoelliptic	$-\frac{\left(-\frac{45}{32}a^{2}b\,c^{2}+\frac{75}{64}a\,b^{3}c-\frac{105}{512}b^{5}\right)\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)+\left(\left(\frac{7}{32}b^{2}x^{4}+\frac{29}{32}ab\,x^{2}+a^{2}\right)c^{\frac{5}{2}}-\frac{115\left(\frac{7b\,x^{2}}{46}+a\right)b^{2}c^{\frac{3}{2}}}{64}-\frac{(3}{}\right.}{15c^{\frac{9}{2}}}$
default	$\frac{x^{4}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{10c}-\frac{7b\,x^{2}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{80c^{2}}+\frac{7b^{2}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{96c^{3}}-\frac{7b^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}\,x^{2}}{128c^{3}}-\frac{7b^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{256c^{4}}-$
elliptic	$\frac{x^{4}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{10c}-\frac{7b\,x^{2}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{80c^{2}}+\frac{7b^{2}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{96c^{3}}-\frac{7b^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}\,x^{2}}{128c^{3}}-\frac{7b^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{256c^{4}}-$

input
```
int(x^7*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3840*(-384*c^4*x^8-48*b*c^3*x^6-128*a*c^3*x^4+56*b^2*c^2*x^4+232*a*b*c^
2*x^2-70*b^3*c*x^2+256*a^2*c^2-460*a*b^2*c+105*b^4)*(c*x^4+b*x^2+a)^(1/2)/
c^4+1/512*b*(48*a^2*c^2-40*a*b^2*c+7*b^4)/c^(9/2)*ln((1/2*b+c*x^2)/c^(1/2)
+(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.15

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx$$
$$= \left[\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)+\right.$$
$$-\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-2\,(384\,c^5x^8+48\,bc^4x^6-105\,b^4c}{7680\,c^5}$$

input
```
integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/15360*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^4 -
8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c)
 + 4*(384*c^5*x^8 + 48*b*c^4*x^6 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3
 - 8*(7*b^2*c^3 - 16*a*c^4)*x^4 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^2)*sqrt(c
*x^4 + b*x^2 + a))/c^5, -1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sq
rt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4
+ b*c*x^2 + a*c)) - 2*(384*c^5*x^8 + 48*b*c^4*x^6 - 105*b^4*c + 460*a*b^2*
c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^4 + 2*(35*b^3*c^2 - 116*a*b
*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^5]
```

Sympy [F]

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx=\int x^7\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**7*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**7*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.99

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx$$
$$= \frac{1}{3840}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4-\cdots}{\cdots}\right.$$
$$-\frac{(7b^5-40ab^3c+48a^2bc^2)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{512c^{\frac{9}{2}}}$$

input
```
integrate(x^7*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/3840*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 1
6*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*b
^2*c + 256*a^2*c^2)/c^4) - 1/512*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*log(a
bs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.84

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx=\frac{x^4\left(c\,x^4+b\,x^2+a\right)^{3/2}}{10\,c}+\frac{7\,b\left(\frac{a\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2+a}+\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^2\left(c\,x^4+b\,x^2+a\right)^{3/2}}{4\,c}+\frac{5\,b\left(\frac{\left(8\,c\left(c\,x^4+a\right)-3\,b^2+2\,b\,c\,x^2\right)}{24\,c^2}\right.}{}\right.}{20\,c}-\frac{a\left(\frac{\left(8\,c\left(c\,x^4+a\right)-3\,b^2+2\,b\,c\,x^2\right)\sqrt{c\,x^4+b\,x^2+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^4+b\,x^2+a}+\frac{2\,c\,x^2+b}{\sqrt{c}}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{5\,c}$$

input
```
int(x^7*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
(x^4*(a + b*x^2 + c*x^4)^(3/2))/(10*c) + (7*b*((a*((b/(4*c) + x^2/2)*(a +
b*x^2 + c*x^4)^(1/2) + (log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1
/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^2*(a + b*x^2 + c*x^4)^(3/2))/
(4*c) + (5*b*(((8*c*(a + c*x^4) - 3*b^2 + 2*b*c*x^2)*(a + b*x^2 + c*x^4)^(
1/2))/(24*c^2) + (log(2*(a + b*x^2 + c*x^4)^(1/2) + (b + 2*c*x^2)/c^(1/2))
*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(20*c) - (a*(((8*c*(a + c*x^4) -
3*b^2 + 2*b*c*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(24*c^2) + (log(2*(a + b*x^2
 + c*x^4)^(1/2) + (b + 2*c*x^2)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(
5*c)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 4009, normalized size of antiderivative = 23.44

$$\int x^7\sqrt{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

input
```
int(x^7*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(23040*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 +
c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**4*b*c**4 + 38400*sqrt(c)*sq
rt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c
*x**2)/sqrt(4*a*c - b**2))*a**3*b**3*c**3 + 276480*sqrt(c)*sqrt(a + b*x**2
 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4
*a*c - b**2))*a**3*b**2*c**4*x**2 + 276480*sqrt(c)*sqrt(a + b*x**2 + c*x**
4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b
**2))*a**3*b*c**5*x**4 - 37440*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sq
rt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b
**5*c**2 - 115200*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a
+ b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b**4*c**3*x**2
 + 253440*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2
 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b**3*c**4*x**4 + 73728
0*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**
4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b**2*c**5*x**6 + 368640*sqrt(c
)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b +
 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b*c**6*x**8 + 2400*sqrt(c)*sqrt(a + b*
x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sq
rt(4*a*c - b**2))*a*b**7*c - 55680*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((
2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))...
```

3.949 $\int x^5\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 153

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx = \frac{(5b^2-4ac)\,(b+2cx^2)\,\sqrt{a+bx^2+cx^4}}{128c^3} - \frac{5b(a+bx^2+cx^4)^{3/2}}{48c^2} + \frac{x^2(a+bx^2+cx^4)^{3/2}}{8c} - \frac{(b^2-4ac)\,(5b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{7/2}}$$

output
```
1/128*(-4*a*c+5*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^3-5/48*b*(c*x^4+b
*x^2+a)^(3/2)/c^2+1/8*x^2*(c*x^4+b*x^2+a)^(3/2)/c-1/256*(-4*a*c+b^2)*(-4*a
*c+5*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.86

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx = \frac{\sqrt{a+bx^2+cx^4}(15b^3-52abc-10b^2cx^2+24ac^2x^2+8bc^2x^4+48c^3x^6)}{384c^3} + \frac{(5b^4-24ab^2c+16a^2c^2)\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{256c^{7/2}}$$

input
```
Integrate[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(15*b^3 - 52*a*b*c - 10*b^2*c*x^2 + 24*a*c^2*x^2
+ 8*b*c^2*x^4 + 48*c^3*x^6))/(384*c^3) + ((5*b^4 - 24*a*b^2*c + 16*a^2*c^2
)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(256*c^(7/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1434, 1166, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt{a+bx^2+cx^4} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int x^4 \sqrt{cx^4+bx^2+a} dx^2$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{2} \left(\frac{\int -\frac{1}{2} (5bx^2+2a) \sqrt{cx^4+bx^2+a} dx^2}{4c} + \frac{x^2 (a+bx^2+cx^4)^{3/2}}{4c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{x^2 (a+bx^2+cx^4)^{3/2}}{4c} - \frac{\int (5bx^2+2a) \sqrt{cx^4+bx^2+a} dx^2}{8c}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2} \left(\frac{x^2 (a+bx^2+cx^4)^{3/2}}{4c} - \frac{\frac{5b(a+bx^2+cx^4)^{3/2}}{3c} - \frac{(5b^2-4ac) \int \sqrt{cx^4+bx^2+a} dx^2}{2c}}{8c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{3c}-\frac{(5b^2-4ac)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)}{2c}}{8c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{3c}-\frac{(5b^2-4ac)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)}{2c}}{8c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2+cx^4\right)^{3/2}}{4c}-\frac{\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{3c}-\frac{(5b^2-4ac)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{2c}}{8c}\right)$$

input
```
Int[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((x^2*(a + b*x^2 + c*x^4)^(3/2))/(4*c) - ((5*b*(a + b*x^2 + c*x^4)^(3/2))/
(3*c) - ((5*b^2 - 4*a*c)*(((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c) -
((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])
/(8*c^(3/2))))/(2*c))/(8*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{(-48c^3x^6-8bc^2x^4-24ac^2x^2+10b^2cx^2+52abc-15b^3)\sqrt{cx^4+bx^2+a}}{384c^3}-\frac{(16a^2c^2-24ab^2c+5b^4)\ln\left(\frac{\frac{b}{2}+cx^2}{\sqrt{c}}+\sqrt{cx^4+\cdots}\right)}{256c^{\frac{7}{2}}}$
pseudoelliptic	$-\frac{(a^2c^2-\frac{3}{2}ab^2c+\frac{5}{16}b^4)\ln\left(\frac{2cx^2+2\sqrt{cx^4+bx^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)+\left(\frac{13\left(\frac{5bx^2}{26}+a\right)bc^{\frac{3}{2}}}{6}+(-\frac{1}{3}bx^4-ax^2)c^{\frac{5}{2}}-2c^{\frac{7}{2}}x^6-\frac{5\sqrt{c}\,b^3}{8}\right)\sqrt{c\cdots}}{16c^{\frac{7}{2}}}$
default	$\frac{x^2(cx^4+bx^2+a)^{\frac{3}{2}}}{8c}-\frac{5b\left(\frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx^2+b)\sqrt{cx^4+bx^2+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx^2}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}\right)}{16c}-\frac{a\cdots}{\cdots}$
elliptic	$\frac{x^2(cx^4+bx^2+a)^{\frac{3}{2}}}{8c}-\frac{5b\left(\frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx^2+b)\sqrt{cx^4+bx^2+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx^2}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}\right)}{16c}-\frac{a\cdots}{\cdots}$

input
```
int(x^5*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384*(-48*c^3*x^6-8*b*c^2*x^4-24*a*c^2*x^2+10*b^2*c*x^2+52*a*b*c-15*b^3)
*(c*x^4+b*x^2+a)^(1/2)/c^3-1/256*(16*a^2*c^2-24*a*b^2*c+5*b^4)/c^(7/2)*ln(
(1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.98

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx$$

$$=\left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)+4\cdots}{1536\,c^4}\right.$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/1536*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*
c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4
*(48*c^4*x^6 + 8*b*c^3*x^4 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c
^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a
^2*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)
/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(48*c^4*x^6 + 8*b*c^3*x^4 + 15*b^3*c - 52*
a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4]
```

Sympy [F]

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx=\int x^5\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**5*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**5*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.86

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx = \frac{1}{384}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right) + \frac{(5b^4-24ab^2c+16a^2c^2)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{256c^{\frac{7}{2}}}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/384*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a*c^
2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 1/256*(5*b^4 - 24*a*b^2*c + 16*a^
2*c^2)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(
7/2)
```

Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.26

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx = \frac{x^2\left(c\,x^4+b\,x^2+a\right)^{3/2}}{8\,c} - \frac{a\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2+a}+\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{8\,c} - \frac{5\,b\left(\frac{\left(8\,c\left(c\,x^4+a\right)-3\,b^2+2\,b\,c\,x^2\right)\sqrt{c\,x^4+b\,x^2+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^4+b\,x^2+a}+\frac{2\,c\,x^2+b}{\sqrt{c}}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{16\,c}$$

input
```
int(x^5*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
(x^2*(a + b*x^2 + c*x^4)^(3/2))/(8*c) - (a*((b/(4*c) + x^2/2)*(a + b*x^2 +
 c*x^4)^(1/2) + (log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))*(a
*c - b^2/4))/(2*c^(3/2))))/(8*c) - (5*b*(((8*c*(a + c*x^4) - 3*b^2 + 2*b*c
*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(24*c^2) + (log(2*(a + b*x^2 + c*x^4)^(1/
2) + (b + 2*c*x^2)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(16*c)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2818, normalized size of antiderivative = 18.42

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^5*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - 1536*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2
+ c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**3*b*c**3 - 3072*sqrt(c)*s
qrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*
c*x**2)/sqrt(4*a*c - b**2))*a**3*c**4*x**2 + 1920*sqrt(c)*sqrt(a + b*x**2
+ c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*
a*c - b**2))*a**2*b**3*c**2 + 768*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2
*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**
2*b**2*c**3*x**2 - 9216*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*s
qrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b*c**4*x
**4 - 6144*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**
2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*c**5*x**6 + 96*sqrt(c
)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b +
 2*c*x**2)/sqrt(4*a*c - b**2))*a*b**5*c + 4800*sqrt(c)*sqrt(a + b*x**2 + c
*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c
 - b**2))*a*b**4*c**2*x**2 + 13824*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((
2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*
b**3*c**3*x**4 + 9216*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqr
t(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b**2*c**4*x**
6 - 120*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 +
 c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**7 - 1200*sqrt(c)*sqrt(a...
```

3.950 $\int x^3\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx=-\frac{b(b+2cx^2)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{(a+bx^2+cx^4)^{3/2}}{6c}+\frac{b(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}$$

output
```
-1/16*b*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^2+1/6*(c*x^4+b*x^2+a)^(3/2)/c+
1/32*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))
/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx$$
$$=\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}(-3b^2+2bcx^2+8c(a+cx^4))-3(b^3-4abc)\log\left(c^2\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right.\right.}{96c^{5/2}}$$

input
```
Integrate[x^3*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 2*b*c*x^2 + 8*c*(a + c*x^4))
- 3*(b^3 - 4*a*b*c)*Log[c^2*(b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(96*c^(5/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1434, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int x^2\sqrt{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\int \sqrt{cx^4+bx^2+a}dx^2}{2c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)}{2c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)}{2c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(a+bx^2+cx^4)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{2c}\right)$$

input
```
Int[x^3*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((a + b*x^2 + c*x^4)^(3/2)/(3*c) - (b*(((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x
^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(8*c^(3/2))))/(2*c))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

method	result
risch	$\frac{\left(8c^{2}x^{4}+2b c\,x^{2}+8ac-3b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{48c^{2}}-\frac{b\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{5}{2}}}$
default	$\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{6c}-\frac{b\left(\frac{\left(2c\,x^{2}+b\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{8c^{\frac{3}{2}}}\right)}{4c}$
elliptic	$\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{6c}-\frac{b\left(\frac{\left(2c\,x^{2}+b\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{8c^{\frac{3}{2}}}\right)}{4c}$
pseudoelliptic	$\frac{16c^{\frac{5}{2}}x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}+4b\,c^{\frac{3}{2}}x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}+12\ln\left(2\right)abc-3\ln\left(2\right)b^{3}-12\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)abc+3\ln\left(\frac{2c\,x^{2}}{}\right.}{96c^{\frac{5}{2}}}$

input
```
int(x^3*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(8*c^2*x^4+2*b*c*x^2+8*a*c-3*b^2)*(c*x^4+b*x^2+a)^(1/2)/c^2-1/32*b*(4
*a*c-b^2)/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.19

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx$$
$$= \left[-\frac{3\,(b^3-4\,abc)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)-4\,(8\,c^3x^4+2\right.}{192\,c^3}$$
$$-\frac{3\,(b^3-4\,abc)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-2\,(8\,c^3x^4+2\,bc^2x^2-3\,b^2c+8\,ac^2)\sqrt{cx^4+bx^2+}}{96\,c^3}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sq
rt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^4 + 2*b*
c^2*x^2 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^3, -1/96*(3*(b^3 -
 4*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-
c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c + 8*a
*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^3]
```

Sympy [F]

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx=\int x^3\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**3*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx = \frac{1}{48}\,\sqrt{cx^4+bx^2+a}\left(2\left(4\,x^2+\frac{b}{c}\right)x^2-\frac{3\,b^2-8\,ac}{c^2}\right) - \frac{(b^3-4\,abc)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{32\,c^{\frac{5}{2}}}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2) -
 1/32*(b^3 - 4*a*b*c)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sq
rt(c) + b))/c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 19.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx = \frac{\left(8\,c\,\left(c\,x^4+a\right)-3\,b^2+2\,b\,c\,x^2\right)\,\sqrt{c\,x^4+b\,x^2+a}}{48\,c^2} + \frac{\ln\left(2\,\sqrt{c\,x^4+b\,x^2+a}+\frac{2\,c\,x^2+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{32\,c^{5/2}}$$

input
```
int(x^3*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
((8*c*(a + c*x^4) - 3*b^2 + 2*b*c*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(48*c^2)
 + (log(2*(a + b*x^2 + c*x^4)^(1/2) + (b + 2*c*x^2)/c^(1/2))*(b^3 - 4*a*b*
c))/(32*c^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1536, normalized size of antiderivative = 14.22

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^3*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - 96*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 +
c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b*c**2 - 48*sqrt(c)*sqrt(
a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x*
*2)/sqrt(4*a*c - b**2))*a*b**3*c - 384*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*l
og((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2)
)*a*b**2*c**2*x**2 - 384*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*
sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b*c**3*x**
4 + 18*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 +
c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**5 + 96*sqrt(c)*sqrt(a + b*x
**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqr
t(4*a*c - b**2))*b**4*c*x**2 + 96*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2
*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**
3*c**2*x**4 + 192*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a**2*b*c**2 + 384*sqrt
(c)*sqrt(a + b*x**2 + c*x**4)*a**2*c**3*x**2 - 56*sqrt(c)*sqrt(a + b*x**2
+ c*x**4)*a*b**3*c + 192*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*b**2*c**2*x**
2 + 1056*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*b*c**3*x**4 + 896*sqrt(c)*sqr
t(a + b*x**2 + c*x**4)*a*c**4*x**6 - 6*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b
**5 - 104*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b**4*c*x**2 - 200*sqrt(c)*sqrt
(a + b*x**2 + c*x**4)*b**3*c**2*x**4 + 288*sqrt(c)*sqrt(a + b*x**2 + c*x**
4)*b**2*c**3*x**6 + 896*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b*c**4*x**8 +...
```

3.951 $\int x\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int x\sqrt{a+bx^2+cx^4}\,dx = \frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

output
```
1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c-1/16*(-4*a*c+b^2)*arctanh(1/2*(2*c
*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int x\sqrt{a+bx^2+cx^4}\,dx = \frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c} + \frac{(-b^2+4ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{-\sqrt{a}+\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}$$

input
```
Integrate[x*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) + ((-b^2 + 4*a*c)*ArcTanh[(S
qrt[c]*x^2)/(-Sqrt[a] + Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1432, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \sqrt{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)$$

input
```
Int[x*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b
 + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2)))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{(2c\,x^2+b)\sqrt{c\,x^4+b\,x^2+a}}{8c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16c^{\frac{3}{2}}}$	73
risch	$\frac{(2c\,x^2+b)\sqrt{c\,x^4+b\,x^2+a}}{8c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16c^{\frac{3}{2}}}$	73
elliptic	$\frac{(2c\,x^2+b)\sqrt{c\,x^4+b\,x^2+a}}{8c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16c^{\frac{3}{2}}}$	73
pseudoelliptic	$\frac{(2c\,x^2+b)\sqrt{c\,x^4+b\,x^2+a}}{8c}-\frac{\left(ac-\frac{b^2}{4}\right)\left(\ln(2)-\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)\right)}{4c^{\frac{3}{2}}}$	80

input
```
int(x*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c+1/16*(4*a*c-b^2)/c^(3/2)*ln((1/2*b
+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.37

$$\int x\sqrt{a+bx^2+cx^4}\,dx$$
$$=\left[-\frac{(b^2-4\,ac)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^4+bx^2+}}{32\,c^2}\right.$$

input
```
integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*
x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c^2*x^2 + b*c))/c^2, 1/16*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x
^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt
(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c))/c^2]
```

Sympy [F]

$$\int x\sqrt{a+bx^2+cx^4}\,dx=\int x\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate(x*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{a+bx^2+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int x\sqrt{a+bx^2+cx^4}\,dx = \frac{1}{8}\sqrt{cx^4+bx^2+a}\left(2\,x^2+\frac{b}{c}\right) + \frac{(b^2-4\,ac)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{3}{2}}}$$

input
```
integrate(x*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*log(abs(2*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 19.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int x\sqrt{a+bx^2+cx^4}\,dx = \frac{\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2+a}}{2} + \frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\left(a\,c-\frac{b^2}{4}\right)}{4\,c^{3/2}}$$

input
```
int(x*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
((b/(4*c) + x^2/2)*(a + b*x^2 + c*x^4)^(1/2))/2 + (log((a + b*x^2 + c*x^4)
^(1/2) + (b/2 + c*x^2)/c^(1/2))*(a*c - b^2/4))/(4*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 816, normalized size of antiderivative = 9.83

$$\int x\sqrt{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x*(c*x^4+b*x^2+a)^(1/2),x)
```

output

```
(16*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x
**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b*c + 32*sqrt(c)*sqrt(a + b*x**
2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(
4*a*c - b**2))*a*c**2*x**2 - 4*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sq
rt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**3 -
 8*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x*
*4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**2*c*x**2 + 8*sqrt(c)*sqrt(a + b
*x**2 + c*x**4)*a*b*c + 16*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*c**2*x**2 +
 2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b**3 + 20*sqrt(c)*sqrt(a + b*x**2 + c
*x**4)*b**2*c*x**2 + 48*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b*c**2*x**4 + 32
*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*c**3*x**6 + 16*log((2*sqrt(c)*sqrt(a +
b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*c**2 + 32*log((2
*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b
*c**2*x**2 + 32*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/s
qrt(4*a*c - b**2))*a*c**3*x**4 - log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)
+ b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**4 - 8*log((2*sqrt(c)*sqrt(a + b*x**
2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**3*c*x**2 - 8*log((2*sqr
t(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**2*c*
*2*x**4 + 8*a*b**2*c + 32*a*b*c**2*x**2 + 32*a*c**3*x**4 + 8*b**3*c*x**2 +
 40*b**2*c**2*x**4 + 64*b*c**3*x**6 + 32*c**4*x**8)/(16*c*(4*sqrt(a + b...
```

3.952 $\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx=\frac{1}{2}\sqrt{a+bx^2+cx^4}-\frac{1}{2}\sqrt{a}\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+\frac{b\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

output
```
1/2*(c*x^4+b*x^2+a)^(1/2)-1/2*a^(1/2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x
^4+b*x^2+a)^(1/2))+1/4*b*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(
1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx=\frac{1}{2}\sqrt{a+bx^2+cx^4}+\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)-\frac{b\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{4\sqrt{c}}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

output
```
Sqrt[a + b*x^2 + c*x^4]/2 + Sqrt[a]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2
+ c*x^4])/Sqrt[a]] - (b*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4
]])/(4*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1434, 1162, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}dx$$
$$\Big\downarrow \text{1434}$$
$$\frac{1}{2}\int \frac{\sqrt{cx^4+bx^2+a}}{x^2}dx^2$$
$$\Big\downarrow \text{1162}$$
$$\frac{1}{2}\left(\sqrt{a+bx^2+cx^4}-\frac{1}{2}\int -\frac{bx^2+2a}{x^2\sqrt{cx^4+bx^2+a}}dx^2\right)$$
$$\Big\downarrow \text{25}$$
$$\frac{1}{2}\left(\frac{1}{2}\int \frac{bx^2+2a}{x^2\sqrt{cx^4+bx^2+a}}dx^2+\sqrt{a+bx^2+cx^4}\right)$$
$$\Big\downarrow \text{1269}$$
$$\frac{1}{2}\left(\frac{1}{2}\left(b\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2+2a\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2\right)+\sqrt{a+bx^2+cx^4}\right)$$
$$\Big\downarrow \text{1092}$$
$$\frac{1}{2}\left(\frac{1}{2}\left(2b\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}+2a\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2\right)+\sqrt{a+bx^2+cx^4}\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{2}\left(\frac{1}{2}\left(2a\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2+\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}\right)+\sqrt{a+bx^2+cx^4}\right)$$

$$\downarrow 1154$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}-4a\int\frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}\right)+\sqrt{a+bx^2+cx^4}\right)$$

$$\downarrow 219$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}-2\sqrt{a}\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)\right)+\sqrt{a+bx^2+cx^4}\right)$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4] + (-2*Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])] + (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x
^2 + c*x^4])])/Sqrt[c])/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2}+\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4\sqrt{c}}-\frac{\sqrt{a}\,\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2}$	91
elliptic	$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2}+\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4\sqrt{c}}-\frac{\sqrt{a}\,\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2}$	91
pseudoelliptic	$-\frac{\sqrt{a}\,\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)\sqrt{c}-\frac{b\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)}{2}+\frac{b\ln(2)}{2}-\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}}{2\sqrt{c}}$	106

input
```
int((c*x^4+b*x^2+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^4+b*x^2+a)^(1/2)+1/4*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(
1/2))/c^(1/2)-1/2*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x
^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 566, normalized size of antiderivative = 5.19

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx$$
$$= \left[\frac{b\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)+2\,\sqrt{a}c\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2 \cdots}{\cdots}\right)\cdots}{8\,c}\right.$$
$$-\frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-\sqrt{a}c\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)-2\,\sqrt{\cdots}}{4\,c}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/8*(b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 +
a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 +
8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4)
+ 4*sqrt(c*x^4 + b*x^2 + a)*c)/c, -1/4*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(a)*c*
log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2
*a)*sqrt(a) + 8*a^2)/x^4) - 2*sqrt(c*x^4 + b*x^2 + a)*c)/c, 1/8*(4*sqrt(-a
)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a
*b*x^2 + a^2)) + b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4
 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*c
)/c, 1/4*(2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sq
rt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x
^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 +
 b*x^2 + a)*c)/c]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [B] (verification not implemented)

Time = 18.67 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx = \frac{\sqrt{c\,x^4+b\,x^2+a}}{2} - \frac{\sqrt{a}\,\ln\left(\frac{b}{2}+\frac{a}{x^2}+\frac{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2} + \frac{b\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{4\,\sqrt{c}}$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/x,x)
```

output
```
(a + b*x^2 + c*x^4)^(1/2)/2 - (a^(1/2)*log(b/2 + a/x^2 + (a^(1/2)*(a + b*x
^2 + c*x^4)^(1/2))/x^2))/2 + (b*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x
^2)/c^(1/2)))/(4*c^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx = \frac{2\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}+b+2c\,x^2}{\sqrt{4ac-b^2}}\right)b+2\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,b+4\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,c\,x^2}{}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x,x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x*
*4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b + 2*sqrt(c)*sqrt(a + b*x**2 + c*
x**4)*b + 4*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*c*x**2 + 8*sqrt(a + b*x**2 +
 c*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*a*c + 4*
sqrt(c)*int(sqrt(a + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*a*b + 8*s
qrt(c)*int(sqrt(a + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*a*c*x**2 +
 log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**
2))*b**2 + 2*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt
(4*a*c - b**2))*b*c*x**2 + 4*a*c + 4*b*c*x**2 + 4*c**2*x**4)/(4*(2*sqrt(a
 + b*x**2 + c*x**4)*c + sqrt(c)*b + 2*sqrt(c)*c*x**2))
```

3.953 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3} \, dx = -\frac{\sqrt{a+bx^2+cx^4}}{2x^2} - \frac{b\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}} + \frac{1}{2}\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

output
```
-1/2*(c*x^4+b*x^2+a)^(1/2)/x^2-1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^
4+b*x^2+a)^(1/2))/a^(1/2)+1/2*c^(1/2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x
^4+b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3} \, dx = \frac{1}{2}\left(-\frac{\sqrt{a+bx^2+cx^4}}{x^2} + \frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}} - \sqrt{c}\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

output
```
(-(Sqrt[a + b*x^2 + c*x^4]/x^2) + (b*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2
 + c*x^4])/Sqrt[a]])/Sqrt[a] - Sqrt[c]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4]])/2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1434, 1161, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx$$

$$\Big\downarrow \ 1434$$

$$\frac{1}{2}\int \frac{\sqrt{cx^4+bx^2+a}}{x^4}dx^2$$

$$\Big\downarrow \ 1161$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{2cx^2+b}{x^2\sqrt{cx^4+bx^2+a}}dx^2-\frac{\sqrt{a+bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \ 1269$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2c\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2+b\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2\right)-\frac{\sqrt{a+bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \ 1092$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4c\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}+b\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2\right)-\frac{\sqrt{a+bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left(\frac{1}{2}\left(b\int\frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2+2\sqrt{c}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)-\frac{\sqrt{a+bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2\sqrt{c}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-2b\int\frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}\right)-\frac{\sqrt{a+bx^2+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2\sqrt{c}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-\frac{b\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}\right)-\frac{\sqrt{a+bx^2+cx^4}}{x^2}\right)$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

output
```
(-(Sqrt[a + b*x^2 + c*x^4]/x^2) + (-((b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*S
qrt[a + b*x^2 + c*x^4])])/Sqrt[a]) + 2*Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sq
rt[c]*Sqrt[a + b*x^2 + c*x^4])])/2)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2x^{2}}-\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4\sqrt{a}}+\frac{\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2}$
pseudoelliptic	$-\frac{-\sqrt{c}\,x^{2}\sqrt{a}\,\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)+\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)x^{2}}{2}+\sqrt{a}\,\left(\ln(2)\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{a}\,x^{2}}$
default	$-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a}-\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4\sqrt{a}}+\frac{c\sqrt{c\,x^{4}+b\,x^{2}+a}\,x^{2}}{2a}+\frac{\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}\right.}{}$
elliptic	$-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a}-\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4\sqrt{a}}+\frac{c\sqrt{c\,x^{4}+b\,x^{2}+a}\,x^{2}}{2a}+\frac{\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}\right.}{}$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^4+b*x^2+a)^(1/2)/x^2-1/4*b/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^
4+b*x^2+a)^(1/2))/x^2)+1/2*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 601, normalized size of antiderivative = 5.37

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx$$
$$= \left[\frac{2\,a\sqrt{c}x^2\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)+\sqrt{a}bx^2\log\left(-\frac{(b^2+4\,ac)x^4}{\cdots}\right.}{8\,ax^2}\right.$$
$$-\frac{4\,a\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-\sqrt{a}bx^2\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)}{8\,ax^2}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(2*a*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*
x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^2*log(-((b^2 + 4*a*c
)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^
2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2), -1/8*(4*a*sqrt(-c)*x^2*arc
tan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2
+ a*c)) - sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4
 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 + b*x^2 +
 a)*a)/(a*x^2), 1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*
x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + a*sqrt(c)*x^2*log(-8*c^2*
x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) -
4*a*c) - 2*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2), 1/4*(sqrt(-a)*b*x^2*arctan(
1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^
2)) - 2*a*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sq
rt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*sqrt(c*x^4 + b*x^2 + a)*a)/(a*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx = \frac{b\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}} - \frac{1}{2}\sqrt{c}\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right) + \frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/2*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) -
 1/2*sqrt(c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) -
b)) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/((sqrt
(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 18.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx = \frac{\sqrt{c}\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2} - \frac{\sqrt{c\,x^4+b\,x^2+a}}{2\,x^2} - \frac{b\,\ln\left(\frac{b}{2}+\frac{a}{x^2}+\frac{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4\,\sqrt{a}}$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/x^3,x)
```

output
```
(c^(1/2)*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/2 - (a +
b*x^2 + c*x^4)^(1/2)/(2*x^2) - (b*log(b/2 + a/x^2 + (a^(1/2)*(a + b*x^2 +
c*x^4)^(1/2))/x^2))/(4*a^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx = \frac{-\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}+\sqrt{c}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^5+b\,x^3+ax}dx\right)b\,x^2+\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}+b+2c\,x^2}{\sqrt{4ac-b^2}}\right)c\,x^2}{2\sqrt{c}\,x^2}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^3,x)
```

output
```
( - sqrt(c)*sqrt(a + b*x**2 + c*x**4) + sqrt(c)*int(sqrt(a + b*x**2 + c*x*
*4)/(a*x + b*x**3 + c*x**5),x)*b*x**2 + log((2*sqrt(c)*sqrt(a + b*x**2 + c
*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*c*x**2)/(2*sqrt(c)*x**2)
```

3.954 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^5} dx = -\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4} + \frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}$$

output
```
-1/8*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a/x^4+1/16*(-4*a*c+b^2)*arctanh(1/2
*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^5} dx = -\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4} - \frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

output
```
-1/8*((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(a*x^4) - ((b^2 - 4*a*c)*ArcT
anh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1434, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^5} dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{\sqrt{cx^4+bx^2+a}}{x^6}dx^2$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{2}\left(-\frac{(b^2-4ac)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{8a}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{(b^2-4ac)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{4a}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{8a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

output
```
(-1/4*((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(a*x^4) + ((b^2 - 4*a*c)*Arc
Tanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(8*a^(3/2)))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{\left(b\,x^{2}+2a\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{8a\,x^{4}}-\frac{\left(4ac-b^{2}\right)\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{16a^{\frac{3}{2}}}$
pseudoelliptic	$\frac{-4c\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)a\,x^{4}+b^{2}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)x^{4}-2b\sqrt{c\,x^{4}+b\,x^{2}+a}\,x^{2}\sqrt{a}-4\sqrt{c\,x^{4}+b\,x^{2}+}}{16a^{\frac{3}{2}}x^{4}}$
default	$-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{4a\,x^{4}}+\frac{b\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{8a^{2}x^{2}}-\frac{b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{8a^{2}}+\frac{b^{2}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{16a^{\frac{3}{2}}}-\frac{bc\sqrt{c\,x^{4}+b\,x^{2}}}{8a^{2}}$
elliptic	$-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{4a\,x^{4}}+\frac{b\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{8a^{2}x^{2}}-\frac{b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{8a^{2}}+\frac{b^{2}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{16a^{\frac{3}{2}}}-\frac{bc\sqrt{c\,x^{4}+b\,x^{2}}}{8a^{2}}$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a/x^4-1/16*(4*a*c-b^2)/a^(3/2)*ln((
2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx = \left[-\frac{(b^2-4\,ac)\sqrt{a}x^4\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,\sqrt{cx^4+bx^2+a}(abx^2+2\,a^2)}{32\,a^2x^4}, -\frac{(b^2-4\,ac)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+2\,\sqrt{cx^4+bx^2+a}(abx^2+2\,a^2)}{16\,a^2x^4}\right]$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[-1/32*((b^2 - 4*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*
sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4
 + b*x^2 + a)*(a*b*x^2 + 2*a^2))/(a^2*x^4), -1/16*((b^2 - 4*a*c)*sqrt(-a)*
x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a
*b*x^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2))/(a^2*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/x**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^5}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(74) = 148$.

Time = 0.13 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^5}\, dx = -\frac{(b^2 - 4\,ac)\arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a} + \frac{\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^3 b^2 + 4\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^3 ac + 8\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 ab\sqrt{c} + \cdots}{8\left(\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 - a\right)^2 a}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/8*(b^2 - 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a
))/(sqrt(-a)*a) + 1/8*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*b^2 + 4*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))^2*a*b*sqrt(c) + (sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b
^2 + 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*c)/(((sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 + a))^2 - a)^2*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^5}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/x^5,x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx = \frac{-2\sqrt{c\,x^4+b\,x^2+a}\,a-\sqrt{c\,x^4+b\,x^2+a}\,b\,x^2+4\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^5+b\,x^3+ax}dx\right)ac\,x^4-\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^5+b\,x^3+ax}dx\right)b^2x^4}{8a\,x^4}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^5,x)
```

output
```
( - 2*sqrt(a + b*x**2 + c*x**4)*a - sqrt(a + b*x**2 + c*x**4)*b*x**2 + 4*i
nt(sqrt(a + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*a*c*x**4 - int(sqr
t(a + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*b**2*x**4)/(8*a*x**4)
```

3.955 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^7} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^7} \, dx = \frac{b(2a+bx^2)\sqrt{a+bx^2+cx^4}}{16a^2x^4} - \frac{(a+bx^2+cx^4)^{3/2}}{6ax^6} - \frac{b(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}$$

output
```
1/16*b*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4-1/6*(c*x^4+b*x^2+a)^(3/2)
/a/x^6-1/32*b*(-4*a*c+b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)
^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^7} \, dx = \frac{\sqrt{a+bx^2+cx^4}(-8a^2-2abx^2+3b^2x^4-8acx^4)}{48a^2x^6} + \frac{(b^3-4abc)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{5/2}}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(-8*a^2 - 2*a*b*x^2 + 3*b^2*x^4 - 8*a*c*x^4))/(48
*a^2*x^6) + ((b^3 - 4*a*b*c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4
])/Sqrt[a]])/(16*a^(5/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1434, 1157, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{\sqrt{cx^4+bx^2+a}}{x^8}dx^2$$

$$\Big\downarrow \text{1157}$$

$$\frac{1}{2}\left(-\frac{b\int \frac{\sqrt{cx^4+bx^2+a}}{x^6}dx^2}{2a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ax^6}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{2}\left(-\frac{b\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{8a}-\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{2a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ax^6}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{4a}-\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{2a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ax^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{8a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{2a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3ax^6}\right)$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

output
```
(-1/3*(a + b*x^2 + c*x^4)^(3/2)/(a*x^6) - (b*(-1/4*((2*a + b*x^2)*Sqrt[a +
 b*x^2 + c*x^4])/(a*x^4) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]
*Sqrt[a + b*x^2 + c*x^4])])/(8*a^(3/2))))/(2*a))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(8ac\,x^{4}-3b^{2}x^{4}+2ab\,x^{2}+8a^{2}\right)}{48x^{6}a^{2}}+\frac{b\left(4ac-b^{2}\right)\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{32a^{\frac{5}{2}}}$
pseudoelliptic	$\frac{b\,x^{6}\left(ac-\frac{b^{2}}{4}\right)\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)+\frac{\left(-\frac{2x^{2}\left(4c\,x^{2}+b\right)a^{\frac{3}{2}}}{3}+\sqrt{a}\,b^{2}x^{4}-\frac{8a^{\frac{5}{2}}}{3}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{2}}{8a^{\frac{5}{2}}x^{6}}$
default	$-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}+\frac{b\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{8a^{2}x^{4}}-\frac{b^{2}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{16a^{3}x^{2}}+\frac{b^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16a^{3}}-\frac{b^{3}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}}}{x^{2}}\right.}{32a^{\frac{5}{2}}}$
elliptic	$-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}+\frac{b\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{8a^{2}x^{4}}-\frac{b^{2}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{16a^{3}x^{2}}+\frac{b^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16a^{3}}-\frac{b^{3}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}}}{x^{2}}\right.}{32a^{\frac{5}{2}}}$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(c*x^4+b*x^2+a)^(1/2)*(8*a*c*x^4-3*b^2*x^4+2*a*b*x^2+8*a^2)/x^6/a^2+
1/32*b*(4*a*c-b^2)/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/
x^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx$$
$$= \left[-\frac{3\,(b^3-4\,abc)\sqrt{a}x^6\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,(2\,a^2bx^2-(3\,ab^2-8\,a^2c)x}{192\,a^3x^6}\right.$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
[-1/192*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2
 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(2*a^
2*b*x^2 - (3*a*b^2 - 8*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x
^6), 1/96*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 +
a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(2*a^2*b*x^2 - (3
*a*b^2 - 8*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^6)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/x**7,x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/x**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(98) = 196$.

Time = 0.13 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.09

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx = \frac{(b^3-4\,abc)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\sqrt{-a}a^2} - \frac{3\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5 b^3 - 12\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5 abc - 48\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^4 a^2}{}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

output
```
1/16*(b^3 - 4*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(
-a))/(sqrt(-a)*a^2) - 1/48*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*b^
3 - 12*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b*c - 48*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))^4*a^2*c^(3/2) - 8*(sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))^3*a*b^3 - 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^2*b*c -
 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^2*b^2*sqrt(c) - 3*(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*b^3 - 36*(sqrt(c)*x^2 - sqrt(c*x^4 + b
*x^2 + a))*a^3*b*c - 16*a^4*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
 a))^2 - a)^3*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^7}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/x^7,x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/x^7, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx$$
$$=\frac{-8\sqrt{c\,x^4+b\,x^2+a}\,a^2-2\sqrt{c\,x^4+b\,x^2+a}\,ab\,x^2-8\sqrt{c\,x^4+b\,x^2+a}\,ac\,x^4+3\sqrt{c\,x^4+b\,x^2+a}\,b^2x^4-12}{48a^2x^6}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^7,x)
```

output
```
( - 8*sqrt(a + b*x**2 + c*x**4)*a**2 - 2*sqrt(a + b*x**2 + c*x**4)*a*b*x**
2 - 8*sqrt(a + b*x**2 + c*x**4)*a*c*x**4 + 3*sqrt(a + b*x**2 + c*x**4)*b**
2*x**4 - 12*int(sqrt(a + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*a*b*c
*x**6 + 3*int(sqrt(a + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*b**3*x*
*6)/(48*a**2*x**6)
```

3.956 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx = -\frac{(5b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{128a^3x^4} - \frac{(a+bx^2+cx^4)^{3/2}}{8ax^8} + \frac{5b(a+bx^2+cx^4)^{3/2}}{48a^2x^6} + \frac{(b^2-4ac)(5b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{7/2}}$$

output
```
-1/128*(-4*a*c+5*b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a^3/x^4-1/8*(c*x^4
+b*x^2+a)^(3/2)/a/x^8+5/48*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^6+1/256*(-4*a*c+b
^2)*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/
a^(7/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx = \frac{\sqrt{a+bx^2+cx^4}(-48a^3-8a^2bx^2+10ab^2x^4-24a^2cx^4-15b^3x^6+52abcx^6)}{384a^3x^8} + \frac{(-5b^4+24ab^2c-16a^2c^2)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{128a^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(-48*a^3 - 8*a^2*b*x^2 + 10*a*b^2*x^4 - 24*a^2*c*
x^4 - 15*b^3*x^6 + 52*a*b*c*x^6))/(384*a^3*x^8) + ((-5*b^4 + 24*a*b^2*c -
16*a^2*c^2)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(128
*a^(7/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1434, 1167, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{\sqrt{cx^4+bx^2+a}}{x^{10}}dx^2$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{2}\left(-\frac{\int \frac{(2cx^2+5b)\sqrt{cx^4+bx^2+a}}{2x^8}dx^2}{4a}-\frac{(a+bx^2+cx^4)^{3/2}}{4ax^8}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(-\frac{\int \frac{(2cx^2+5b)\sqrt{cx^4+bx^2+a}}{x^8}dx^2}{8a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4ax^8}\right)$$

$$\Big\downarrow \; 1228$$

$$\frac{1}{2}\left(-\frac{-\frac{(5b^2-4ac)\int \frac{\sqrt{cx^4+bx^2+a}}{x^6}dx^2}{2a}-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{3ax^6}}{8a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4ax^8}\right)$$

$$\Big\downarrow \; 1152$$

$$\frac{1}{2}\left(-\frac{-\frac{(5b^2-4ac)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{8a}-\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{2a}-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{3ax^6}}{8a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4ax^8}\right)$$

$$\Big\downarrow \; 1154$$

$$\frac{1}{2}\left(-\frac{-\frac{(5b^2-4ac)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{4a}-\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{2a}-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{3ax^6}}{8a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4ax^8}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(-\frac{-\frac{(5b^2-4ac)\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{8a^{3/2}}-\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{2a}-\frac{5b\left(a+bx^2+cx^4\right)^{3/2}}{3ax^6}}{8a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4ax^8}\right)$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

output
```
(-1/4*(a + b*x^2 + c*x^4)^(3/2)/(a*x^8) - ((-5*b*(a + b*x^2 + c*x^4)^(3/2)
)/(3*a*x^6) - ((5*b^2 - 4*a*c)*(-1/4*((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4
])/(a*x^4) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^
2 + c*x^4])])/(8*a^(3/2))))/(2*a))/(8*a))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{\left(ac-\frac{b^{2}}{4}\right)\left(ac-\frac{5b^{2}}{4}\right)x^{8}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)-\frac{5\left(-\frac{2b\left(\frac{26c\,x^{2}}{5}+b\right)x^{4}a^{\frac{3}{2}}}{3}+\frac{8x^{2}\left(3c\,x^{2}+b\right)a^{\frac{5}{2}}}{15}+\sqrt{a}\,b^{3}x^{6}+\frac{16a^{\frac{7}{2}}}{5}\right)\sqrt{c\,x \ldots}}{8}}{16a^{\frac{7}{2}}x^{8}}$
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(-52abc\,x^{6}+15b^{3}x^{6}+24a^{2}c\,x^{4}-10b^{2}x^{4}a+8a^{2}b\,x^{2}+48a^{3}\right)}{384x^{8}a^{3}}+\frac{\left(16a^{2}c^{2}-24a\,b^{2}c+5b^{4}\right)\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\ldots}{x^{2}}\right)}{256a^{\frac{7}{2}}}$
default	$-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{8a\,x^{8}}+\frac{5b\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{48a^{2}x^{6}}-\frac{5b^{2}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{64a^{3}x^{4}}+\frac{5b^{3}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{128a^{4}x^{2}}-\frac{5b^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{128a^{4}}+\ldots$
elliptic	$-\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{8a\,x^{8}}+\frac{5b\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{48a^{2}x^{6}}-\frac{5b^{2}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{64a^{3}x^{4}}+\frac{5b^{3}\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{128a^{4}x^{2}}-\frac{5b^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{128a^{4}}+\ldots$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/16/a^(7/2)*((a*c-1/4*b^2)*(a*c-5/4*b^2)*x^8*ln((2*a+b*x^2+2*a^(1/2)*(c*x
^4+b*x^2+a)^(1/2))/x^2)-5/8*(-2/3*b*(26/5*c*x^2+b)*x^4*a^(3/2)+8/15*x^2*(3
*c*x^2+b)*a^(5/2)+a^(1/2)*b^3*x^6+16/5*a^(7/2))*(c*x^4+b*x^2+a)^(1/2))/x^8
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx = \left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{a}x^8\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)-4\,((15\,ab^3-52\,a\right.}{1536\,a^4x^8}$$

$$-\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{-a}x^8\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+2\,((15\,ab^3-52\,a^2bc)x^6+8\,a^3bx^2}{768\,a^4x^8}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
[1/1536*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^8*log(-((b^2 + 4*a*
c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a
^2)/x^4) - 4*((15*a*b^3 - 52*a^2*b*c)*x^6 + 8*a^3*b*x^2 - 2*(5*a^2*b^2 - 1
2*a^3*c)*x^4 + 48*a^4)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^8), -1/768*(3*(5*b^
4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^8*arctan(1/2*sqrt(c*x^4 + b*x^2 +
a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^3 - 52*a
^2*b*c)*x^6 + 8*a^3*b*x^2 - 2*(5*a^2*b^2 - 12*a^3*c)*x^4 + 48*a^4)*sqrt(c*
x^4 + b*x^2 + a))/(a^4*x^8)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/x**9,x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/x**9, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(139) = 278$.

Time = 0.13 (sec) , antiderivative size = 617, normalized size of antiderivative = 3.83

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx = -\frac{\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{128\,\sqrt{-a}a^3} + \frac{15\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^7b^4-72\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^7ab^2c+48\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^7}{}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^9,x, algorithm="giac")
```

output
```
-1/128*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/384*(15*(sqrt(c)*x^2 - sqrt(c*
x^4 + b*x^2 + a))^7*b^4 - 72*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a*b
^2*c + 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^2*c^2 - 55*(sqrt(c)*
x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b^4 + 264*(sqrt(c)*x^2 - sqrt(c*x^4 + b
*x^2 + a))^5*a^2*b^2*c + 336*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3
*c^2 + 1152*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^3*b*c^(3/2) + 73*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^2*b^4 + 648*(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))^3*a^3*b^2*c + 336*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
 a))^3*a^4*c^2 + 384*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^3*b^3*sqr
t(c) + 256*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^4*b*c^(3/2) + 15*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^3*b^4 + 312*(sqrt(c)*x^2 - sqrt(c*
x^4 + b*x^2 + a))*a^4*b^2*c + 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a
^5*c^2 + 128*a^5*b*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 -
a)^4*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^9}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/x^9,x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/x^9, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^9}dx$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^9,x)
```

output
```
int((c*x^4+b*x^2+a)^(1/2)/x^9,x)
```

3.957 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx = \frac{b(7b^2-12ac)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}} + \frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8} - \frac{(35b^2-32ac)\left(a+bx^2+cx^4\right)^{3/2}}{480a^3x^6} - \frac{b(7b^2-12ac)\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{9/2}}$$

output
```
1/256*b*(-12*a*c+7*b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a^4/x^4-1/10*(c*
x^4+b*x^2+a)^(3/2)/a/x^10+7/80*b*(c*x^4+b*x^2+a)^(3/2)/a^2/x^8-1/480*(-32*
a*c+35*b^2)*(c*x^4+b*x^2+a)^(3/2)/a^3/x^6-1/512*b*(-12*a*c+7*b^2)*(-4*a*c+
b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx = \frac{\sqrt{a+bx^2+cx^4}(-384a^4-48a^3bx^2+56a^2b^2x^4-128a^3cx^4-70ab^3x^6+232a^2bcx^6+105b^4x^8-460ab^2c}{3840a^4x^{10}} + \frac{(7b^5-40ab^3c+48a^2bc^2)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{256a^{9/2}}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(-384*a^4 - 48*a^3*b*x^2 + 56*a^2*b^2*x^4 - 128*a
^3*c*x^4 - 70*a*b^3*x^6 + 232*a^2*b*c*x^6 + 105*b^4*x^8 - 460*a*b^2*c*x^8
+ 256*a^2*c^2*x^8))/(3840*a^4*x^10) + ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)
*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(256*a^(9/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1434, 1167, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx$$

$$\Big\downarrow \; 1434$$

$$\frac{1}{2}\int \frac{\sqrt{cx^4+bx^2+a}}{x^{12}}dx^2$$

$$\Big\downarrow \; 1167$$

$$\frac{1}{2}\left(-\frac{\int \frac{(4cx^2+7b)\sqrt{cx^4+bx^2+a}}{2x^{10}}dx^2}{5a}-\frac{(a+bx^2+cx^4)^{3/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(-\frac{\int \frac{(4cx^2+7b)\sqrt{cx^4+bx^2+a}}{x^{10}}dx^2}{10a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \ 1237$$

$$\frac{1}{2}\left(-\frac{-\frac{\int \frac{\left(35b^2+14cx^2b-32ac\right)\sqrt{cx^4+bx^2+a}}{2x^8}dx^2}{4a}-\frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{4ax^8}}{10a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(-\frac{-\frac{\int \frac{\left(35b^2+14cx^2b-32ac\right)\sqrt{cx^4+bx^2+a}}{x^8}dx^2}{8a}-\frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{4ax^8}}{10a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \ 1228$$

$$\frac{1}{2}\left(-\frac{-\frac{-\frac{5b\left(7b^2-12ac\right)\int \frac{\sqrt{cx^4+bx^2+a}}{x^6}dx^2}{2a}-\frac{\left(35b^2-32ac\right)\left(a+bx^2+cx^4\right)^{3/2}}{3ax^6}}{8a}-\frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{4ax^8}}{10a}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \ 1152$$

$$\frac{1}{2}\left(-\frac{-\frac{-\frac{5b\left(7b^2-12ac\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{8a}-\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{2a}-\frac{\left(35b^2-32ac\right)\left(a+bx^2+cx^4\right)^{3/2}}{3ax^6}}{8a}-\frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{4ax^8}}{10a}-\right.$$

$$\Big\downarrow \ 1154$$

$$\frac{1}{2}\left(-\frac{-\frac{-\frac{5b\left(7b^2-12ac\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{4a}-\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{2a}-\frac{\left(35b^2-32ac\right)\left(a+bx^2+cx^4\right)^{3/2}}{3ax^6}}{8a}-\frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{4ax^8}}{10a}-\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(-\frac{-\frac{-\frac{5b(7b^2-12ac)\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{8a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{2a}-\frac{(35b^2-32ac)(a+bx^2+cx^4)^{3/2}}{3ax^6}}{8a}-\frac{7b(a+bx^2+cx^4)}{4ax^8}}{10a}\right.$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```

output
```
(-1/5*(a + b*x^2 + c*x^4)^(3/2)/(a*x^10) - ((-7*b*(a + b*x^2 + c*x^4)^(3/2
))/(4*a*x^8) - (-1/3*((35*b^2 - 32*a*c)*(a + b*x^2 + c*x^4)^(3/2))/(a*x^6)
 - (5*b*(7*b^2 - 12*a*c)*(-1/4*((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(a*
x^4) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*
x^4])])/(8*a^(3/2))))/(2*a))/(8*a))/(10*a))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{3\left(\left(ac-\frac{7b^2}{12}\right)\left(ac-\frac{b^2}{4}\right)b\,x^{10}\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)+\frac{16\sqrt{c\,x^4+b\,x^2+a}\left(\left(-\frac{2}{3}c^2x^8-\frac{29}{48}bc\,x^6-\frac{7}{48}b^2x^4\right)a^{\frac{5}{2}}+\frac{35(46}{}\right.}{15}\right.}{32a^{\frac{9}{2}}x^{10}}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\left(-256a^2c^2x^8+460a\,b^2c\,x^8-105b^4x^8-232a^2bc\,x^6+70a\,b^3x^6+128a^3c\,x^4-56a^2b^2x^4+48a^3b\,x^2+384a^4\right)}{3840x^{10}a^4}$
default	$-\frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{10a\,x^{10}}+\frac{7b(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{80a^2x^8}-\frac{7b^2(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{96a^3x^6}+\frac{7b^3(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{128a^4x^4}-\frac{7b^4(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{256a^5x^2}+$
elliptic	$-\frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{10a\,x^{10}}+\frac{7b(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{80a^2x^8}-\frac{7b^2(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{96a^3x^6}+\frac{7b^3(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{128a^4x^4}-\frac{7b^4(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{256a^5x^2}+$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-3/32*((a*c-7/12*b^2)*(a*c-1/4*b^2)*b*x^10*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+
b*x^2+a)^(1/2))/x^2)+16/15*(c*x^4+b*x^2+a)^(1/2)*((-2/3*c^2*x^8-29/48*b*c*
x^6-7/48*b^2*x^4)*a^(5/2)+35/192*(46/7*c*x^2+b)*b^2*x^6*a^(3/2)+1/8*(8/3*c
*x^2+b)*x^2*a^(7/2)-35/128*a^(1/2)*b^4*x^8+a^(9/2)))/a^(9/2)/x^10
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.95

$$\int\frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx$$
$$=\left[\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{a}x^{10}\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\left((105\,ab^4-\right.}{15}\right.$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="fricas")
```

output
```
[1/15360*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^10*log(-((b^2 +
 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a)
+ 8*a^2)/x^4) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^8 - 48*a^4*
b*x^2 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^6 - 384*a^5 + 8*(7*a^3*b^2 - 16*a^4
*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^10), 1/7680*(15*(7*b^5 - 40*a*b^3
*c + 48*a^2*b*c^2)*sqrt(-a)*x^10*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^2*
c + 256*a^3*c^2)*x^8 - 48*a^4*b*x^2 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^6 - 3
84*a^5 + 8*(7*a^3*b^2 - 16*a^4*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^10)
]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/x**11,x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/x**11, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(173) = 346$.

Time = 0.16 (sec) , antiderivative size = 842, normalized size of antiderivative = 4.23

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^11,x, algorithm="giac")
```

output
```
1/256*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/3840*(105*(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))^9*b^5 - 600*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9
*a*b^3*c + 720*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a^2*b*c^2 - 490*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a*b^5 + 2800*(sqrt(c)*x^2 - sqrt(
c*x^4 + b*x^2 + a))^7*a^2*b^3*c - 3360*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
 a))^7*a^3*b*c^2 - 7680*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^4*c^(5
/2) + 896*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^2*b^5 - 5120*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3*b^3*c - 15360*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))^5*a^4*b*c^2 - 24320*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
 a))^4*a^4*b^2*c^(3/2) - 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^
5*c^(5/2) - 790*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^3*b^5 - 9200*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^4*b^3*c - 12000*(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))^3*a^5*b*c^2 - 3840*(sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))^2*a^4*b^4*sqrt(c) - 5120*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))
^2*a^5*b^2*c^(3/2) - 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^6*c^
(5/2) - 105*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^4*b^5 - 3240*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^5*b^3*c - 720*(sqrt(c)*x^2 - sqrt(c*x^4
 + b*x^2 + a))*a^6*b*c^2 - 1280*a^6*b^2*c^(3/2) + 512*a^7*c^(5/2))/(((sqrt
(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^5*a^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^{11}}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/x^11,x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/x^11, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^{11}}dx$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^11,x)
```

output
```
int((c*x^4+b*x^2+a)^(1/2)/x^11,x)
```

3.958 $\int x^4\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 394

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx=\frac{b(8b^2-29ac)\,x\sqrt{a+bx^2+cx^4}}{105c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{x(4b^2+5ac+12bcx^2)\sqrt{a+bx^2+cx^4}}{105c^2}+\frac{x(a+bx^2+cx^4)^{3/2}}{7c}$$
$$-\frac{\sqrt[4]{a}b(8b^2-29ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{105c^{11/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(\frac{b(8b^2-29ac)}{\sqrt{c}}+2\sqrt{a}(2b^2-5ac)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\ldots}\right)\right)}{210c^{9/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/105*b*(-29*a*c+8*b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+c^(1/2)*x
^2)-1/105*x*(12*b*c*x^2+5*a*c+4*b^2)*(c*x^4+b*x^2+a)^(1/2)/c^2+1/7*x*(c*x^
4+b*x^2+a)^(3/2)/c-1/105*a^(1/4)*b*(-29*a*c+8*b^2)*(a^(1/2)+c^(1/2)*x^2)*(
(c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1
/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(11/4)/(c*x^4+b*x^2+a)^
(1/2)+1/210*a^(1/4)*(b*(-29*a*c+8*b^2)/c^(1/2)+2*a^(1/2)*(-5*a*c+2*b^2))*(
a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Inver
seJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^
(9/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.22 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.37

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(10a^2c-4b^3x^2-b^2cx^4+18bc^2x^6+15c^3x^8+a(-4b^2+13bcx^2+25c^2x^4))+ib(8b^2-29}{}$$

input
```
Integrate[x^4*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(10*a^2*c - 4*b^3*x^2 - b^2*c*x^4 +
 18*b*c^2*x^6 + 15*c^3*x^8 + a*(-4*b^2 + 13*b*c*x^2 + 25*c^2*x^4)) + I*b*(
8*b^2 - 29*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^
4 + 37*a*b^2*c - 20*a^2*c^2 + 8*b^3*Sqrt[b^2 - 4*a*c] - 29*a*b*c*Sqrt[b^2
- 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*
Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellipt
icF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(420*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1436, 1602, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \ 1436$$

$$\frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2+cx^4}}{35c}-\frac{\int \frac{x^2\left(2\left(2b^2-5ac\right)x^2+3ab\right)}{\sqrt{cx^4+bx^2+a}}dx}{35c}$$

$$\Big\downarrow \ 1602$$

$$\frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2+cx^4}}{35c}-\frac{\frac{2x\left(2b^2-5ac\right)\sqrt{a+bx^2+cx^4}}{3c}-\frac{\int \frac{b\left(8b^2-29ac\right)x^2+2a\left(2b^2-5ac\right)}{\sqrt{cx^4+bx^2+a}}dx}{3c}}{35c}$$

$$\Big\downarrow \ 1511$$

$$\frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2+cx^4}}{35c}-\frac{\frac{2x\left(2b^2-5ac\right)\sqrt{a+bx^2+cx^4}}{3c}-\frac{\sqrt{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}b\left(8b^2-29ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}}{35c}$$

$$\Big\downarrow \ 27$$

$$\frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2+cx^4}}{35c}-\frac{\frac{2x\left(2b^2-5ac\right)\sqrt{a+bx^2+cx^4}}{3c}-\frac{\sqrt{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{b\left(8b^2-29ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}}{35c}$$

$$\Big\downarrow \ 1416$$

$$\frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2+cx^4}}{35c}-\frac{\frac{2x\left(2b^2-5ac\right)\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{\sqrt[4]{a}\left(\frac{b\left(8b^2-29ac\right)}{\sqrt{c}}+2\sqrt{a}\left(2b^2-5ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-b\left(8b^2\right.}{3c}}{35c}$$

$$\Big\downarrow \ 1509$$

$$\frac{x^3(b+5cx^2)\sqrt{a+bx^2+cx^4}}{35c} - \frac{\frac{2x(2b^2-5ac)\sqrt{a+bx^2+cx^4}}{3c} - \frac{\frac{\sqrt[4]{a}\left(\frac{b(8b^2-29ac)}{\sqrt{c}}+2\sqrt{a}(2b^2-5ac)\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}} - \frac{b(8b^2}{}}{3c}}{35c}$$

input
```
Int[x^4*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(x^3*(b + 5*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c) - ((2*(2*b^2 - 5*a*c)*x
*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (-((b*(8*b^2 - 29*a*c)*(-((x*Sqrt[a + b*
x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 +
 c*x^4])))/Sqrt[c]) + (a^(1/4)*((b*(8*b^2 - 29*a*c))/Sqrt[c] + 2*Sqrt[a]*(
2*b^2 - 5*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]
*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/(3*c))/(35*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1436
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*((2*b*p + c*(m + 4*p - 1)*x^2
)/(c*(m + 4*p + 1)*(m + 4*p - 1))), x] - Simp[2*p*(d^2/(c*(m + 4*p + 1)*(m
+ 4*p - 1)))   Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.21

method	result
default	$\frac{x^{5}\sqrt{c\,x^{4}+b\,x^{2}+a}}{7}+\frac{b\,x^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{35c}+\frac{\left(\frac{2a}{7}-\frac{4b^{2}}{35c}\right)x\sqrt{c\,x^{4}+b\,x^{2}+a}}{3c}-\frac{\left(\frac{2a}{7}-\frac{4b^{2}}{35c}\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\right.}{}}}{12c\sqrt{-}}$
elliptic	$\frac{x^{5}\sqrt{c\,x^{4}+b\,x^{2}+a}}{7}+\frac{b\,x^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{35c}+\frac{\left(\frac{2a}{7}-\frac{4b^{2}}{35c}\right)x\sqrt{c\,x^{4}+b\,x^{2}+a}}{3c}-\frac{\left(\frac{2a}{7}-\frac{4b^{2}}{35c}\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\right.}{}}}{12c\sqrt{-}}$
risch	$\frac{x\left(15c^{2}x^{4}+3bc\,x^{2}+10ac-4b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{105c^{2}}-\frac{b\left(29ac-8b^{2}\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\left(\operatorname{EllipticF}\left(\frac{x}{}\right.\right.}{2\sqrt{-b+\sqrt{}}}$

input
```
int(x^4*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^5*(c*x^4+b*x^2+a)^(1/2)+1/35/c*b*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(2/7*
a-4/35*b^2/c)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(2/7*a-4/35*b^2/c)/c*a*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellipti
cF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/35/c*b*a-2/3*(2/7*a-4/35*b^2/c)/c*b)*a*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(
-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.02

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx$$
$$= \frac{\sqrt{\frac{1}{2}}\left((8\,b^3c-29\,abc^2)x\sqrt{\frac{b^2-4\,ac}{c^2}}-(8\,b^4-29\,ab^2c)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid \frac{bc\sqrt{}}{}}{}$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/210*(sqrt(1/2)*((8*b^3*c - 29*a*b*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (8*b^
4 - 29*a*b^2*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*ellipti
c_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*
sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((8*b^3*c + 10*a
*c^3 - (29*a*b + 4*b^2)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (8*b^4 - 10*a*b*c
^2 - (29*a*b^2 - 4*b^3)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)
/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x)
, 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(15*c^4*x^6 +
 3*b*c^3*x^4 + 8*b^3*c - 29*a*b*c^2 - 2*(2*b^2*c^2 - 5*a*c^3)*x^2)*sqrt(c*
x^4 + b*x^2 + a))/(c^4*x)
```

Sympy [F]

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx = \int x^4\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**4*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}x^4\,dx$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}x^4\,dx$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx=\int x^4\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(x^4*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^4*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{a+bx^2+cx^4}\,dx$$
$$= \frac{10\sqrt{c\,x^4+b\,x^2+a}\,acx-4\sqrt{c\,x^4+b\,x^2+a}\,b^2x+3\sqrt{c\,x^4+b\,x^2+a}\,bc\,x^3+15\sqrt{c\,x^4+b\,x^2+a}\,c^2x^5-1}{1}$$

input
```
int(x^4*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(10*sqrt(a + b*x**2 + c*x**4)*a*c*x - 4*sqrt(a + b*x**2 + c*x**4)*b**2*x +
 3*sqrt(a + b*x**2 + c*x**4)*b*c*x**3 + 15*sqrt(a + b*x**2 + c*x**4)*c**2*
x**5 - 10*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a**2*c +
4*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b**2 - 29*int((
sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a*b*c + 8*int((sq
rt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*b**3)/(105*c**2)
```

3.959 $\int x^2\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 343

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx = -\frac{2(b^2-3ac)\,x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{x(b+3cx^2)\sqrt{a+bx^2+cx^4}}{15c} + \frac{2\sqrt[4]{a}(b^2-3ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}\left(\sqrt{a}b+\frac{2(b^2-3ac)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{5/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-2/15*(-3*a*c+b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^(1/2)+c^(1/2)*x^2)+1
/15*x*(3*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c+2/15*a^(1/4)*(-3*a*c+b^2)*(a^(1/
2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(
sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/
(c*x^4+b*x^2+a)^(1/2)-1/30*a^(1/4)*(a^(1/2)*b+2*(-3*a*c+b^2)/c^(1/2))*(a^(
1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJ
acobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(5/
4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.89 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.40

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx$$
$$= \frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(b+3cx^2)\,(a+bx^2+cx^4)-i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4a}}{b-\sqrt{b^2-}}}}{}$$

input
```
Integrate[x^2*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 3*c*x^2)*(a + b*x^2 + c*x^4) -
 I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3
 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2
 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
- 4*a*c])])/(30*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4
])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1436, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1436}$$

$$\frac{x(b+3cx^2)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\int \frac{2(b^2-3ac)x^2+ab}{\sqrt{cx^4+bx^2+a}}dx}{15c}$$

$$\Big\downarrow \text{1511}$$

$$\frac{x(b+3cx^2)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\sqrt{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}(b^2-3ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{15c}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(b+3cx^2)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\sqrt{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2(b^2-3ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{15c}$$

$$\Big\downarrow \text{1416}$$

$$\frac{x(b+3cx^2)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\frac{\sqrt[4]{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2(b^2-3ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{15c}$$

$$\Big\downarrow \text{1509}$$

$$\frac{x(b+3cx^2)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\frac{\sqrt[4]{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2(b^2-3ac)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+c}{(\sqrt{a}+\sqrt{cx}}}}{\sqrt[4]{}}\right.}{}}{15c}$$

input
```
Int[x^2*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(x*(b + 3*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) - ((-2*(b^2 - 3*a*c)*(-((
x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqr
t[a + b*x^2 + c*x^4])))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*b + (2*(b^2 - 3*a*c))/
Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/(15*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1436
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*((2*b*p + c*(m + 4*p - 1)*x^2
)/(c*(m + 4*p + 1)*(m + 4*p - 1))), x] - Simp[2*p*(d^2/(c*(m + 4*p + 1)*(m
+ 4*p - 1)))   Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.18

method	result
risch	$\frac{x(3c\,x^2+b)\sqrt{c\,x^4+b\,x^2+a}}{15c}-\frac{\left(6ac-2b^2\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+}}$
default	$\frac{x^3\sqrt{c\,x^4+b\,x^2+a}}{5}+\frac{bx\sqrt{c\,x^4+b\,x^2+a}}{15c}-\frac{ba\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+}}{a}}}{2}\right.}{60c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
elliptic	$\frac{x^3\sqrt{c\,x^4+b\,x^2+a}}{5}+\frac{bx\sqrt{c\,x^4+b\,x^2+a}}{15c}-\frac{ba\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+}}{a}}}{2}\right.}{60c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$

input
```
int(x^2*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c-1/15/c*(1/2*(6*a*c-2*b^2)*a*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(
-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2)))+1/4*a*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.03

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx =$$

$$-\frac{2\sqrt{\frac{1}{2}}\left((b^2c-3\,ac^2)x\sqrt{\frac{b^2-4\,ac}{c^2}}-(b^3-3\,abc)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4}{c^2}}}{2}}{}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/30*(2*sqrt(1/2)*((b^2*c - 3*a*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (b^3 - 3
*a*b*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcs
in(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^
2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*b^2*c - (6*a + b)*c^
2)*x*sqrt((b^2 - 4*a*c)/c^2) - (2*b^3 - (6*a*b - b^2)*c)*x)*sqrt(c)*sqrt((
c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt
((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2
*a*c)/(a*c)) - 2*(3*c^3*x^4 + b*c^2*x^2 - 2*b^2*c + 6*a*c^2)*sqrt(c*x^4 +
b*x^2 + a))/(c^3*x)
```

Sympy [F]

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx=\int x^2\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}x^2\,dx$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}x^2\,dx$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx=\int x^2\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int(x^2*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^2*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{a+bx^2+cx^4}\,dx$$
$$=\frac{\sqrt{c\,x^4+b\,x^2+a}\,bx+3\sqrt{c\,x^4+b\,x^2+a}\,c\,x^3-\left(\int\frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)ab+6\left(\int\frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)ac-2\left(\int\right.}{15c}$$

input
```
int(x^2*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2 + c*x**4)*b*x + 3*sqrt(a + b*x**2 + c*x**4)*c*x**3 - int(
sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b + 6*int((sqrt(a + b
*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a*c - 2*int((sqrt(a + b*x**
2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*b**2)/(15*c)
```

3.960 $\int \sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 309

$$\int \sqrt{a+bx^2+cx^4}\,dx = \frac{1}{3}x\sqrt{a+bx^2+cx^4}+\frac{bx\sqrt{a+bx^2+cx^4}}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/3*x*(c*x^4+b*x^2+a)^(1/2)+1/3*b*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)
+c^(1/2)*x^2)-1/3*a^(1/4)*b*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2
)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-
b/a^(1/2)/c^(1/2))^(1/2))/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/6*a^(1/4)*(b+2*a
^(1/2)*c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^
2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c
^(1/2))^(1/2))/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.44

$$\int \sqrt{a+bx^2+cx^4}\,dx$$
$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(a+bx^2+cx^4)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\sqrt{\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) + I*b*(-b + Sqr
t[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*
a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[
I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])])/(12*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a
 + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1404, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \; 1404$$

$$\frac{1}{3}\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx+\frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{3}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)+\frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)+\frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{3}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}d}{\sqrt{c}}\right.$$

$$\frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{3}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2}{}\right.}{}\right.$$

$$\frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(x*Sqrt[a + b*x^2 + c*x^4])/3 + (-((b*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt
[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c])
 + (a^(1/4)*(2*Sqrt[a] + b/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.23

method	result
default	$\frac{x\sqrt{c x^4+b x^2+a}}{3}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c x^4+b x^2+a}}$
risch	$\frac{x\sqrt{c x^4+b x^2+a}}{3}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c x^4+b x^2+a}}$
elliptic	$\frac{x\sqrt{c x^4+b x^2+a}}{3}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c x^4+b x^2+a}}$

input
```
int((c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(c*x^4+b*x^2+a)^(1/2)+1/6*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/6*b*a*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+
(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.97

$$\int \sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{\sqrt{\frac{1}{2}}\left(bcx\sqrt{\frac{b^2-4\,ac}{c^2}}-b^2x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right) \mid \frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac}) - \sqrt{\frac{1}{2}}\left((bc-\right.}{}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(1/2)*(b*c*x*sqrt((b^2 - 4*a*c)/c^2) - b^2*x)*sqrt(c)*sqrt((c*sqr
t((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2
 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)
/(a*c)) - sqrt(1/2)*((b*c - 2*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (b^2 + 2*b*
c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sq
rt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4
*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*(c^2*x^2 + b*
c))/(c^2*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^2+cx^4}\,dx = \frac{\sqrt{c\,x^4+b\,x^2+a}\,x}{3} + \frac{2\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)a}{3} + \frac{\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)b}{3}$$

input

```
int((c*x^4+b*x^2+a)^(1/2),x)
```

output

```
(sqrt(a + b*x**2 + c*x**4)*x + 2*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2
 + c*x**4),x)*a + int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**
4),x)*b)/3
```

3.961 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 303

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2} \, dx = -\frac{\sqrt{a+bx^2+cx^4}}{x}+\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2} - \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}} + \frac{\left(b+2\sqrt{a}\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
-(c*x^4+b*x^2+a)^(1/2)/x+2*c^(1/2)*x*(c*x^4+b*x^2+a)^(1/2)/(a^(1/2)+c^(1/2
)*x^2)-2*a^(1/4)*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c
^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a
^(1/2)/c^(1/2))^(1/2))/(c*x^4+b*x^2+a)^(1/2)+1/2*(b+2*a^(1/2)*c^(1/2))*(a^
(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Inverse
JacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1
/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$
$$= \frac{-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4)+i\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\sqrt{\right.\right.}{}}{}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/x^2,x]
```

output
```
(-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4) + I*(-b + Sqrt[b^2
 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ell
ipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*Sqrt[2]*Sqrt[b^2 - 4*a*c]*x*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2
]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^
2 - 4*a*c])])/(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4]
)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1437, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$
$$\Big\downarrow \text{ 1437}$$

$$\int \frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}}{x}$$

$$\Big\downarrow \text{1511}$$

$$\left(2\sqrt{a}\sqrt{c}+b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-2\sqrt{a}\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}}{x}$$

$$\Big\downarrow \text{27}$$

$$\left(2\sqrt{a}\sqrt{c}+b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-2\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}}{x}$$

$$\Big\downarrow \text{1416}$$

$$-2\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx+\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{a+bx^2+cx^4}}{x}$$

$$\Big\downarrow \text{1509}$$

$$\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-2\sqrt{c}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)-\frac{\sqrt{a+bx^2+cx^4}}{x}$$

input

```
Int[Sqrt[a + b*x^2 + c*x^4]/x^2,x]
```

output
```
-(Sqrt[a + b*x^2 + c*x^4]/x) - 2*Sqrt[c]*(-((x*Sqrt[a + b*x^2 + c*x^4])/(S
qrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])) + ((b
+ 2*Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1437
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Simp[2*(p/(
d^2*(m + 1)))   Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{x}+\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac}\right.}{ac}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{x}+\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac}\right.}{ac}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{x}+\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac}\right.}{ac}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*x^4+b*x^2+a)^(1/2)/x+1/4*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(
4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-c*a*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c
+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
))
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^2}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/x^2,x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2}\,dx = \frac{\sqrt{c\,x^4+b\,x^2+a}+2\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^6+b\,x^4+a\,x^2}dx\right)ax+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)bx}{x}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^2,x)
```

output
```
(sqrt(a + b*x**2 + c*x**4) + 2*int(sqrt(a + b*x**2 + c*x**4)/(a*x**2 + b*x
**4 + c*x**6),x)*a*x + int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4)
,x)*b*x)/x
```

3.962 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 313

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4} \, dx = -\frac{\sqrt{a+bx^2+cx^4}}{3x^3} - \frac{b\sqrt{a+bx^2+cx^4}}{3\sqrt{a}x\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\left(b+2\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/3*(c*x^4+b*x^2+a)^(1/2)/x^3-1/3*b*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)/x/(a^(1
/2)+c^(1/2)*x^2)-1/3*b*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(
1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*
(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/6*(b+2*a^(1/2
)*c^(1/2))*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)
*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2
)/c^(1/2))^(1/2))/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.60 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$
$$= \frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2)\,(a+bx^2+cx^4)+ib\left(-b+\sqrt{b^2-4ac}\right)x^3\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\Big(}{}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/x^4,x]
```

output
```
(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2)*(a + b*x^2 + c*x^4) + I*b*
(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*
Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*
a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(12*a*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1437, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$
$$\Big\downarrow \text{ 1437}$$

$$\frac{1}{3}\int \frac{2cx^2+b}{x^2\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}}{3x^3}$$

$\Big\downarrow$ 1604

$$\frac{1}{3}\left(-\frac{\int -\frac{c\left(bx^2+2a\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{b\sqrt{a+bx^2+cx^4}}{ax}\right)-\frac{\sqrt{a+bx^2+cx^4}}{3x^3}$$

$\Big\downarrow$ 25

$$\frac{1}{3}\left(\frac{\int \frac{c\left(bx^2+2a\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{b\sqrt{a+bx^2+cx^4}}{ax}\right)-\frac{\sqrt{a+bx^2+cx^4}}{3x^3}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{c\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{b\sqrt{a+bx^2+cx^4}}{ax}\right)-\frac{\sqrt{a+bx^2+cx^4}}{3x^3}$$

$\Big\downarrow$ 1511

$$\frac{1}{3}\left(\frac{c\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{b\sqrt{a+bx^2+cx^4}}{ax}\right)-\frac{\sqrt{a+bx^2+cx^4}}{3x^3}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{c\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{b\sqrt{a+bx^2+cx^4}}{ax}\right)-\frac{\sqrt{a+bx^2+cx^4}}{3x^3}$$

$\Big\downarrow$ 1416

$$\frac{1}{3}\left(\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{b\sqrt{a+bx^2}}{ax}\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}}{3x^3}$$

$\Big\downarrow$ 1509

$$\frac{1}{3}\left(\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\text{a}\right.}{\sqrt[4]{c}\sqrt{a+bx^2}}\right.}{}\right.}{a}\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}}{3x^3}$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/x^4,x]
```

output
```
-1/3*Sqrt[a + b*x^2 + c*x^4]/x^3 + (-((b*Sqrt[a + b*x^2 + c*x^4])/(a*x)) +
 (c*(-((b*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/
4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(2*Sqrt[a] + b/S
qrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]
))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/a)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1437
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Simp[2*(p/(
d^2*(m + 1)))   Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(b\,x^2+a)}{3x^3 a}+\frac{c\left(-\frac{b a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\sqrt{-4+\cdots}\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2\cdots}}\cdots\right.}{\cdots}$
default	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{3x^3}-\frac{b\sqrt{c\,x^4+b\,x^2+a}}{3ax}+\frac{c\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\cdots\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}\cdots$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{3x^3}-\frac{b\sqrt{c\,x^4+b\,x^2+a}}{3ax}+\frac{c\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\cdots\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}\cdots$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c*x^4+b*x^2+a)^(1/2)*(b*x^2+a)/x^3/a+1/3*c/a*(-1/2*b*a*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+
2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2
)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1
/2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(abx^3\sqrt{\frac{b^2-4\,ac}{a^2}}-b^2x^3\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}\right)\mid\frac{ab\sqrt{\frac{b^2-4\,ac}{a^2}}+b^2-2\,ac}{2\,ac})+\sqrt{\frac{1}{2}}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(sqrt(1/2)*(a*b*x^3*sqrt((b^2 - 4*a*c)/a^2) - b^2*x^3)*sqrt(a)*sqrt((
a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sq
rt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2
*a*c)/(a*c)) + sqrt(1/2)*((2*a^2 - a*b)*x^3*sqrt((b^2 - 4*a*c)/a^2) + (2*a
*b + b^2)*x^3)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(
arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt
((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*(a*b
*x^2 + a^2))/(a^2*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^4}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/x^4,x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^4}\,dx$$
$$= \frac{-\sqrt{c\,x^4+b\,x^2+a}-\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^8+b\,x^6+a\,x^4}dx\right)a\,x^3+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)c\,x^3}{2x^3}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^4,x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4) - int(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*
x**6 + c*x**8),x)*a*x**3 + int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x
**4),x)*c*x**3)/(2*x**3)
```

3.963 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 363

$$\begin{aligned}\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx &= -\frac{\sqrt{a+bx^2+cx^4}}{5x^5}-\frac{b\sqrt{a+bx^2+cx^4}}{15ax^3}+\frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{15a^{3/2}x\left(\sqrt{a}+\sqrt{c}x^2\right)}\\&\quad+\frac{2\sqrt[4]{c}(b^2-3ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15a^{7/4}\sqrt{a+bx^2+cx^4}}\\&\quad-\frac{\sqrt[4]{c}\left(b\sqrt{c}+\frac{2(b^2-3ac)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30a^{5/4}\sqrt{a+bx^2+cx^4}}\end{aligned}$$

output
```
-1/5*(c*x^4+b*x^2+a)^(1/2)/x^5-1/15*b*(c*x^4+b*x^2+a)^(1/2)/a/x^3+2/15*(-3
*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/a^(3/2)/x/(a^(1/2)+c^(1/2)*x^2)+2/15*c^(1/
4)*(-3*a*c+b^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^
2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^
(1/2))^(1/2))/a^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/30*c^(1/4)*(b*c^(1/2)+2*(-3*
a*c+b^2)/a^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*
x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)
/c^(1/2))^(1/2))/a^(5/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.90 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

$$= \frac{-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(3a^3-2b^2x^6(b+cx^2)+a^2(4bx^2+9cx^4)+a(-b^2x^4+7bcx^6+6c^2x^8))-i(b^2-3ac)\left(-b\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/x^6,x]
```

output
```
(-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(3*a^3 - 2*b^2*x^6*(b + c*x^2) + a^2*(
4*b*x^2 + 9*c*x^4) + a*(-(b^2*x^4) + 7*b*c*x^6 + 6*c^2*x^8)) - I*(b^2 - 3*
a*c)*(-b + Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c
])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b
*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4
*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*
a*c])])/(30*a^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^5*Sqrt[a + b*x^2 + c*x^4
])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1437, 1604, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

$\Big\downarrow$ 1437

$$\frac{1}{5}\int \frac{2cx^2+b}{x^4\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a+bx^2+cx^4}}{5x^5}$$

$\Big\downarrow$ 1604

$$\frac{1}{5}\left(-\frac{\int \frac{bcx^2+2(b^2-3ac)}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax^3}\right)-\frac{\sqrt{a+bx^2+cx^4}}{5x^5}$$

$\Big\downarrow$ 1604

$$\frac{1}{5}\left(-\frac{-\frac{\int -\frac{c\left(2\left(b^2-3ac\right)x^2+ab\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax^3}\right)-\frac{\sqrt{a+bx^2+cx^4}}{5x^5}$$

$\Big\downarrow$ 25

$$\frac{1}{5}\left(-\frac{\frac{\int \frac{c\left(2\left(b^2-3ac\right)x^2+ab\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax^3}\right)-\frac{\sqrt{a+bx^2+cx^4}}{5x^5}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(-\frac{\frac{c\int \frac{2\left(b^2-3ac\right)x^2+ab}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax^3}\right)-\frac{\sqrt{a+bx^2+cx^4}}{5x^5}$$

$\Big\downarrow$ 1511

$$\frac{1}{5}\left(-\frac{\frac{c\left(\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}\left(b^2-3ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax^3}\right)-\frac{\sqrt{a+bx^2+cx^4}}{5x^5}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(-\frac{\frac{c\left(\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\left(b^2-3ac\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax^3}\right)-$$

$$\frac{\sqrt{a+bx^2+cx^4}}{5x^5}$$

$\downarrow$ 1416

$$\frac{1}{5}\left(-\frac{\frac{c\left(\frac{\sqrt[4]{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2\left(b^2-3ac\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2(b^2}{}}{3a}\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}}{5x^5}$$

$\downarrow$ 1509

$$\frac{1}{5}\left(-\frac{\frac{c\left(\frac{\sqrt[4]{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2\left(b^2-3ac\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+}{(\sqrt{a}+\sqrt{c}}}}{\sqrt[4]{}}\right.}{}\right.}{a}}{3a}\right.$$

$$\frac{\sqrt{a+bx^2+cx^4}}{5x^5}$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/x^6,x]
```

output
```
-1/5*Sqrt[a + b*x^2 + c*x^4]/x^5 + (-1/3*(b*Sqrt[a + b*x^2 + c*x^4])/(a*x^
3) - ((-2*(b^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a*x) + (c*((-2*(b^2 - 3*
a*c)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^
(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*b + (2*(b^2 -
 3*a*c))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[
a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/a)/(3*a))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1437
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Simp[2*(p/(
d^2*(m + 1)))   Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(6ac\,x^4-2b^2x^4+ab\,x^2+3a^2)}{15x^5a^2}-\frac{c\left(\frac{\left(6ac-2b^2\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x}{}\right.\right.}{2\sqrt{\frac{-b+}{}}}\right.}{}$
default	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{5x^5}-\frac{b\sqrt{c\,x^4+b\,x^2+a}}{15a\,x^3}-\frac{2(3ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}{15a^2x}-\frac{bc\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{I}}{60a\sqrt{\frac{-b+\sqrt{-4ac}}{a}}}$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{5x^5}-\frac{b\sqrt{c\,x^4+b\,x^2+a}}{15a\,x^3}-\frac{2(3ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}{15a^2x}-\frac{bc\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{I}}{60a\sqrt{\frac{-b+\sqrt{-4ac}}{a}}}$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(c*x^4+b*x^2+a)^(1/2)*(6*a*c*x^4-2*b^2*x^4+a*b*x^2+3*a^2)/x^5/a^2-1/
15*c/a^2*(1/2*(6*a*c-2*b^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4
-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/4*a*b*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$
$$= \frac{2\sqrt{\frac{1}{2}}\left((ab^2-3a^2c)x^5\sqrt{\frac{b^2-4ac}{a^2}}-(b^3-3abc)x^5\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4ac}{a^2}}-b}{a}}E(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4ac}{a^2}}-b}{a}}\right)\mid\frac{ab\sqrt{b}}{}}{}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
1/30*(2*sqrt(1/2)*((a*b^2 - 3*a^2*c)*x^5*sqrt((b^2 - 4*a*c)/a^2) - (b^3 -
3*a*b*c)*x^5)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(a
rcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt(
(b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((a^2*b - 2*a*b^2 + 6
*a^2*c)*x^5*sqrt((b^2 - 4*a*c)/a^2) + (a*b^2 + 2*b^3 - 6*a*b*c)*x^5)*sqrt(
a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*s
qrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2)
+ b^2 - 2*a*c)/(a*c)) - 2*(a^2*b*x^2 - 2*(a*b^2 - 3*a^2*c)*x^4 + 3*a^3)*sq
rt(c*x^4 + b*x^2 + a))/(a^3*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/x**6,x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^6}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/x^6,x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx$$
$$= \frac{-\sqrt{c\,x^4+b\,x^2+a}\,a-2\sqrt{c\,x^4+b\,x^2+a}\,c\,x^4+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^8+b\,x^6+a\,x^4}dx\right)ab\,x^5+2\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)c^2x^5}{5a\,x^5}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/x^6,x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4)*a - 2*sqrt(a + b*x**2 + c*x**4)*c*x**4 + int
(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)*a*b*x**5 + 2*int(
(sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*c**2*x**5)/(5*a*
x**5)
```

3.964 $\int x^7(a+bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 223

$$\int x^7\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{3b(b^2-4ac)\,(3b^2-4ac)\,(b+2cx^2)\,\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b(3b^2-4ac)\,(b+2cx^2)\,(a+bx^2+cx^4)^{3/2}}{256c^4}+\frac{x^4(a+bx^2+cx^4)^{5/2}}{14c}+\frac{(21b^2-16ac-30bcx^2)\,(a+bx^2+cx^4)^{5/2}}{560c^3}-\frac{3b(b^2-4ac)^2\,(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4096c^{11/2}}$$

output
```
3/2048*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^5
-1/256*b*(-4*a*c+3*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c^4+1/14*x^4*(c*
x^4+b*x^2+a)^(5/2)/c+1/560*(-30*b*c*x^2-16*a*c+21*b^2)*(c*x^4+b*x^2+a)^(5/
2)/c^3-3/4096*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.99

$$\int x^7(a+bx^2+cx^4)^{3/2}\,dx = \frac{\sqrt{a+bx^2+cx^4}\left(315b^6-210b^5cx^2+16b^3c^2x^2(91a-9cx^4)+168b^4c(-15a+cx^4)+1024c^3\right.}{}$$
$$+\frac{3b(b^2-4ac)^2\,(3b^2-4ac)\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{4096c^{11/2}}$$

input
```
Integrate[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(315*b^6 - 210*b^5*c*x^2 + 16*b^3*c^2*x^2*(91*a -
 9*c*x^4) + 168*b^4*c*(-15*a + c*x^4) + 1024*c^3*(a + c*x^4)^2*(-2*a + 5*c
*x^4) + 16*b^2*c^2*(343*a^2 - 62*a*c*x^4 + 8*c^2*x^8) + 32*b*c^3*x^2*(-73*
a^2 + 22*a*c*x^4 + 200*c^2*x^8)))/(71680*c^5) + (3*b*(b^2 - 4*a*c)^2*(3*b^
2 - 4*a*c)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(4096*c^(
11/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1434, 1166, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\left(a+bx^2+cx^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int x^6\left(cx^4+bx^2+a\right)^{3/2}dx^2$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{2}\left(\frac{\int -\frac{1}{2}x^2\left(9bx^2+4a\right)\left(cx^4+bx^2+a\right)^{3/2}dx^2}{7c}+\frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{7c}\right)$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(\frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{7c}-\frac{\int x^2\left(9bx^2+4a\right)\left(cx^4+bx^2+a\right)^{3/2}dx^2}{14c}\right)$$

$\big\downarrow$ 1225

$$\frac{1}{2}\left(\frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\int\left(cx^4+bx^2+a\right)^{3/2}dx^2}{8c^2}-\frac{\left(-16ac+21b^2-30bcx^2\right)\left(a+bx^2+cx^4\right)^{5/2}}{20c^2}}{14c}\right)$$

$\big\downarrow$ 1087

$$\frac{1}{2}\left(\frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int\sqrt{cx^4+bx^2+a}dx^2}{16c}\right)}{8c^2}-\frac{\left(-16ac+21b^2-30bcx^2\right)\left(a+bx\right.}{20c^2}}{14c}\right.$$

$\big\downarrow$ 1087

$$\frac{1}{2}\left(\frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right.}{16c}\right.}{8c^2}}{14c}\right.$$

$\big\downarrow$ 1092

$$\frac{1}{2}\left(\frac{x^4(a+bx^2+cx^4)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\left(\frac{(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int\frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2}}}{4c}\right.}{16c}\right.}{8c^2}}{14c}\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{x^4(a+bx^2+cx^4)^{5/2}}{7c}-\frac{\frac{7b(3b^2-4ac)\left(\frac{(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+}{2\sqrt{c}\sqrt{a+}}\right.}{8c^{3/2}}\right.}{16c}\right.}{8c^2}}{14c}\right.$$

input
```
Int[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
((x^4*(a + b*x^2 + c*x^4)^(5/2))/(7*c) - (-1/20*((21*b^2 - 16*a*c - 30*b*c
*x^2)*(a + b*x^2 + c*x^4)^(5/2))/c^2 + (7*b*(3*b^2 - 4*a*c)*(((b + 2*c*x^2
)*(a + b*x^2 + c*x^4)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x^2)*Sqrt
[a + b*x^2 + c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[
c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2))))/(16*c)))/(8*c^2))/(14*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$-\frac{-\frac{105\left(ac-\frac{3b^2}{4}\right)\left(ac-\frac{b^2}{4}\right)^2 b\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)}{64}+\left(\left(\frac{9}{128}b^3x^6+\frac{31}{64}b^2x^4a+\frac{73}{64}a^2b\,x^2+a^3\right)c^{\frac{7}{2}}-\frac{343\left(\frac{3}{98}b^2x^4+\frac{13}{49}\right.}{12}\right.}{}$
risch	$-\frac{\left(-5120c^6x^{12}-6400b\,c^5x^{10}-8192a\,c^5x^8-128b^2c^4x^8-704ab\,c^4x^6+144b^3c^3x^6-1024a^2c^4x^4+992a\,b^2c^3x^4-168b^4c^2x^4+\right.}{71680c^5}$
default	$\frac{13b^3a\,x^2\sqrt{c\,x^4+b\,x^2+a}}{640c^3}-\frac{31b^2a\,x^4\sqrt{c\,x^4+b\,x^2+a}}{2240c^2}+\frac{11ba\,x^6\sqrt{c\,x^4+b\,x^2+a}}{1120c}+\frac{21b^5a\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{1024c^{\frac{9}{2}}}-$
elliptic	$\frac{13b^3a\,x^2\sqrt{c\,x^4+b\,x^2+a}}{640c^3}-\frac{31b^2a\,x^4\sqrt{c\,x^4+b\,x^2+a}}{2240c^2}+\frac{11ba\,x^6\sqrt{c\,x^4+b\,x^2+a}}{1120c}+\frac{21b^5a\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{1024c^{\frac{9}{2}}}-$

input
```
int(x^7*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(-105/64*(a*c-3/4*b^2)*(a*c-1/4*b^2)^2*b*ln((2*c*x^2+2*(c*x^4+b*x^2+
a)^(1/2)*c^(1/2)+b)/c^(1/2))+((9/128*b^3*x^6+31/64*b^2*x^4*a+73/64*a^2*b*x
^2+a^3)*c^(7/2)-343/128*(3/98*b^2*x^4+13/49*a*b*x^2+a^2)*b^2*c^(5/2)+(-1/1
6*b^2*x^8-11/32*a*b*x^6-1/2*a^2*x^4)*c^(9/2)+(-25/8*b*x^10-4*a*x^8)*c^(11/
2)+315/256*b^4*(1/12*b*x^2+a)*c^(3/2)-5/2*c^(13/2)*x^12-315/2048*c^(1/2)*b
^6)*(c*x^4+b*x^2+a)^(1/2)+105/64*ln(2)*(a*c-3/4*b^2)*(a*c-1/4*b^2)^2*b)/c^
(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.40

$$\int x^7 (a + bx^2 + cx^4)^{3/2} \, dx = \left[-\frac{105 \left(3 b^7 - 28 ab^5c + 80 a^2b^3c^2 - 64 a^3bc^3\right)\sqrt{c} \log \left(-8 c^2x^4 - 8 bcx^2 - b^2 - 4 \sqrt{cx^4 + bx^2} \right.}{}\right.$$

input
```
integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/286720*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(
c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
 b)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^12 + 6400*b*c^6*x^10 + 128*(b^2*c^5 +
 64*a*c^6)*x^8 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*
c^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^6 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 12
8*a^2*c^5)*x^4 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^2)*sqr
t(c*x^4 + b*x^2 + a))/c^6, 1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*
c^2 - 64*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
 b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(5120*c^7*x^12 + 6400*b*c^6*x^
10 + 128*(b^2*c^5 + 64*a*c^6)*x^8 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*
b^2*c^3 - 2048*a^3*c^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^6 + 8*(21*b^4*c^3 -
 124*a*b^2*c^4 + 128*a^2*c^5)*x^4 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*
a^2*b*c^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^6]
```

Sympy [F]

$$\int x^7 (a + bx^2 + cx^4)^{3/2} \, dx = \int x^7 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**7*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**7*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^7 \left(a+bx^2+cx^4\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(197) = 394$.

Time = 0.17 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.97

$$\int x^7 \left(a+bx^2+cx^4\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(x^7*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/7680*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2
- 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*
a*b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*log(a
bs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(9/2))*a + 1/
30720*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*x^2 + b/c)*x^2 - (9*b^2*c
^3 - 20*a*c^4)/c^5)*x^2 + (21*b^3*c^2 - 68*a*b*c^3)/c^5)*x^2 - (105*b^4*c
- 448*a*b^2*c^2 + 240*a^2*c^3)/c^5)*x^2 + (315*b^5 - 1680*a*b^3*c + 1808*a
^2*b*c^2)/c^5) + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*
log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(11/2))*
b + 1/430080*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*(12*x^2 + b/c)*x^2
 - (11*b^2*c^4 - 24*a*c^5)/c^6)*x^2 + (99*b^3*c^3 - 316*a*b*c^4)/c^6)*x^2
- (231*b^4*c^2 - 972*a*b^2*c^3 + 512*a^2*c^4)/c^6)*x^2 + (1155*b^5*c - 604
8*a*b^3*c^2 + 6352*a^2*b*c^3)/c^6)*x^2 - (3465*b^6 - 21840*a*b^4*c + 34608
*a^2*b^2*c^2 - 8192*a^3*c^3)/c^6) - 105*(33*b^7 - 252*a*b^5*c + 560*a^2*b^
3*c^2 - 320*a^3*b*c^3)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*s
qrt(c) + b))/c^(13/2))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^7\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int x^7\,\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

input
```
int(x^7*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^7*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 7599, normalized size of antiderivative = 34.08

$$\int x^7\left(a+bx^2+cx^4\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int(x^7*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(860160*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 +
 c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**6*b*c**6 + 3440640*sqrt(c)
*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b +
2*c*x**2)/sqrt(4*a*c - b**2))*a**5*b**3*c**5 + 20643840*sqrt(c)*sqrt(a + b
*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/s
qrt(4*a*c - b**2))*a**5*b**2*c**6*x**2 + 20643840*sqrt(c)*sqrt(a + b*x**2
+ c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*
a*c - b**2))*a**5*b*c**7*x**4 - 3386880*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*
log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2
))*a**4*b**5*c**4 - 1720320*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(
c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**4*b**4
*c**5*x**2 + 67092480*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqr
t(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**4*b**3*c**6*
x**4 + 137625600*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a +
 b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**4*b**2*c**7*x**6
+ 68812800*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**
2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**4*b*c**8*x**8 - 322560*
sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)
 + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**3*b**7*c**3 - 18063360*sqrt(c)*sqr
t(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2...
```

3.965 $\int x^5(a+bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 204

$$\int x^5\left(a+bx^2+cx^4\right)^{3/2}\,dx=-\frac{(b^2-4ac)\,(7b^2-4ac)\,(b+2cx^2)\,\sqrt{a+bx^2+cx^4}}{1024c^4}+\frac{(7b^2-4ac)\,(b+2cx^2)\,(a+bx^2+cx^4)^{3/2}}{384c^3}-\frac{7b(a+bx^2+cx^4)^{5/2}}{120c^2}+\frac{x^2(a+bx^2+cx^4)^{5/2}}{12c}+\frac{(b^2-4ac)^2\,(7b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2048c^{9/2}}$$

output
```
-1/1024*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^4+
1/384*(-4*a*c+7*b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c^3-7/120*b*(c*x^4+
b*x^2+a)^(5/2)/c^2+1/12*x^2*(c*x^4+b*x^2+a)^(5/2)/c+1/2048*(-4*a*c+b^2)^2*
(-4*a*c+7*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(9
/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95

$$\int x^5 (a + bx^2 + cx^4)^{3/2} \, dx = \frac{2\sqrt{c}\sqrt{a + bx^2 + cx^4}(-105b^5 + 70b^4cx^2 + 8b^3c(95a - 7cx^4) + 48b^2c^2x^2(-9a + cx^4) + 160c^3 \cdots}{}$$

input
```
Integrate[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(-105*b^5 + 70*b^4*c*x^2 + 8*b^3*c*(95*
a - 7*c*x^4) + 48*b^2*c^2*x^2*(-9*a + c*x^4) + 160*c^3*x^2*(3*a^2 + 14*a*c
*x^4 + 8*c^2*x^8) + 16*b*c^2*(-81*a^2 + 18*a*c*x^4 + 104*c^2*x^8)) - 15*(b
^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4]])/(30720*c^(9/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1434, 1166, 27, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a + bx^2 + cx^4)^{3/2} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int x^4 (cx^4 + bx^2 + a)^{3/2} \, dx^2$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{2} \left(\frac{\int -\frac{1}{2}(7bx^2 + 2a)(cx^4 + bx^2 + a)^{3/2} \, dx^2}{6c} + \frac{x^2 (a + bx^2 + cx^4)^{5/2}}{6c} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2+cx^4\right)^{5/2}}{6c}-\frac{\int \left(7bx^2+2a\right)\left(cx^4+bx^2+a\right)^{3/2}dx^2}{12c}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2+cx^4\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{5c}-\frac{\left(7b^2-4ac\right)\int \left(cx^4+bx^2+a\right)^{3/2}dx^2}{2c}}{12c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2+cx^4\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{5c}-\frac{\left(7b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^4+bx^2+a}\,dx^2}{16c}\right)}{2c}}{12c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2+cx^4\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{5c}-\frac{\left(7b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4a\right.}{}\right.}{16c}\right.}{2c}}{12c}\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2+cx^4\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{5c}-\frac{\left(7b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4a\right.}{}\right.}{16c}\right.}{2c}}{12c}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2+cx^4\right)^{5/2}}{6c}-\frac{\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{5c}-\frac{(7b^2-4ac)\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4a\right.}{}\right.}{16c}\right.}{2c}}{12c}\right.$$

input
```
Int[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
((x^2*(a + b*x^2 + c*x^4)^(5/2))/(6*c) - ((7*b*(a + b*x^2 + c*x^4)^(5/2))/
(5*c) - ((7*b^2 - 4*a*c)*(((b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(8*c)
- (3*(b^2 - 4*a*c)*(((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c) - ((b^2
- 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^
(3/2))))/(16*c)))/(2*c))/(12*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\left(-1280c^5x^{10}-1664c^4x^8b-2240c^4a\,x^6-48b^2c^3x^6-288ab\,c^3x^4+56c^2x^4b^3-480a^2c^3x^2+432a\,b^2c^2x^2-70x^2c\,b^4+1296a^2\right.}{15360c^4}$
pseudoelliptic	$-\frac{\left(ac-\frac{b^2}{4}\right)^2\left(ac-\frac{7b^2}{4}\right)\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)+\left(\frac{\left(\frac{7}{6}b^3x^4+9a\,b^2x^2+27a^2b\right)c^{\frac{5}{2}}}{10}+\left(-\frac{1}{10}b^2x^6-\frac{3}{5}ab\,x^4-a^2x^2\right)c^{\frac{7}{2}}-\right.}{32c^{\frac{9}{2}}}$
default	$-\frac{15b^4a\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{512c^{\frac{7}{2}}}-\frac{9b^2a\,x^2\sqrt{c\,x^4+b\,x^2+a}}{320c^2}+\frac{3ba\,x^4\sqrt{c\,x^4+b\,x^2+a}}{160c}-\frac{27a^2b\sqrt{c\,x^4+b\,x^2+a}}{320c^2}+$
elliptic	$-\frac{15b^4a\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{512c^{\frac{7}{2}}}-\frac{9b^2a\,x^2\sqrt{c\,x^4+b\,x^2+a}}{320c^2}+\frac{3ba\,x^4\sqrt{c\,x^4+b\,x^2+a}}{160c}-\frac{27a^2b\sqrt{c\,x^4+b\,x^2+a}}{320c^2}+$

input
```
int(x^5*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15360*(-1280*c^5*x^10-1664*b*c^4*x^8-2240*a*c^4*x^6-48*b^2*c^3*x^6-288*
a*b*c^3*x^4+56*b^3*c^2*x^4-480*a^2*c^3*x^2+432*a*b^2*c^2*x^2-70*b^4*c*x^2+
1296*a^2*b*c^2-760*a*b^3*c+105*b^5)*(c*x^4+b*x^2+a)^(1/2)/c^4-1/2048*(64*a
^3*c^3-144*a^2*b^2*c^2+60*a*b^4*c-7*b^6)/c^(9/2)*ln((1/2*b+c*x^2)/c^(1/2)+
(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.21

$$\int x^5(a+bx^2+cx^4)^{3/2}\,dx = \left[-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+\cdots}\right.}{\cdots}\right.$$
$$-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-2\,(1280\,c^6x^{10}+1664\,bc^5\cdots}{\cdots}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/61440*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*
log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)
*sqrt(c) - 4*a*c) - 4*(1280*c^6*x^10 + 1664*b*c^5*x^8 + 16*(3*b^2*c^4 + 14
0*a*c^5)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 - 8*(7*b^3*c^3 -
 36*a*b*c^4)*x^4 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^2)*sqrt(
c*x^4 + b*x^2 + a))/c^5, -1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^
2 - 64*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*
sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(1280*c^6*x^10 + 1664*b*c^5*x^8 +
16*(3*b^2*c^4 + 140*a*c^5)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^
3 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^4 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a
^2*c^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^5]
```

Sympy [F]

$$\int x^5 \left(a + bx^2 + cx^4\right)^{3/2} dx = \int x^5 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**5*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**5*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^5 \left(a + bx^2 + cx^4\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(178) = 356$.

Time = 0.16 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.59

$$\int x^5(a+bx^2+cx^4)^{3/2}\,dx = \frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3(5\right.$$
$$+\frac{1}{7680}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4}{\right.$$
$$+\frac{1}{30720}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(2\left(8\left(10x^2+\frac{b}{c}\right)x^2-\frac{9b^2c^3-20ac^4}{c^5}\right)x^2+\frac{21b^3c^2-68abc^3}{c^5}\right)x^2-\frac{1}{\right.$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a
*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2
*c^2)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(7
/2))*a + 1/7680*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7
*b^2*c^2 - 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b
^4 - 460*a*b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c
^2)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(9/2
))*b + 1/30720*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*x^2 + b/c)*x^2 -
 (9*b^2*c^3 - 20*a*c^4)/c^5)*x^2 + (21*b^3*c^2 - 68*a*b*c^3)/c^5)*x^2 - (1
05*b^4*c - 448*a*b^2*c^2 + 240*a^2*c^3)/c^5)*x^2 + (315*b^5 - 1680*a*b^3*c
 + 1808*a^2*b*c^2)/c^5) + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*
a^3*c^3)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c
^(11/2))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^5(a+bx^2+cx^4)^{3/2}\,dx=\int x^5\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

input
```
int(x^5*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^5*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 5920, normalized size of antiderivative = 29.02

$$\int x^5 (a + bx^2 + cx^4)^{3/2} \, dx = \text{Too large to display}$$

input
```
int(x^5*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
( - 184320*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**
2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**5*b*c**5 - 368640*sqrt(
c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b
+ 2*c*x**2)/sqrt(4*a*c - b**2))*a**5*c**6*x**2 + 261120*sqrt(c)*sqrt(a + b
*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/s
qrt(4*a*c - b**2))*a**4*b**3*c**4 - 460800*sqrt(c)*sqrt(a + b*x**2 + c*x**
4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b
**2))*a**4*b**2*c**5*x**2 - 2949120*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log(
(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a
**4*b*c**6*x**4 - 1966080*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)
*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**4*c**7*x
**6 + 161280*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x
**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**3*b**5*c**3 + 2288640
*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4
) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**3*b**4*c**4*x**2 + 4915200*sqrt(c
)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b +
 2*c*x**2)/sqrt(4*a*c - b**2))*a**3*b**3*c**5*x**4 - 4915200*sqrt(c)*sqrt(
a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x*
*2)/sqrt(4*a*c - b**2))*a**3*b*c**7*x**8 - 1966080*sqrt(c)*sqrt(a + b*x**2
 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqr...
```

3.966 $\int x^3(a+bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\int x^3\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{3b(b^2-4ac)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}-\frac{3b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}$$

output
```
3/256*b*(-4*a*c+b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^3-1/32*b*(2*c*x^2
+b)*(c*x^4+b*x^2+a)^(3/2)/c^2+1/10*(c*x^4+b*x^2+a)^(5/2)/c-3/512*b*(-4*a*c
+b^2)^2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^2+cx^4)^{3/2}\,dx=\frac{\sqrt{a+bx^2+cx^4}\left(15b^4-10b^3cx^2+128c^2\left(a+cx^4\right)^2+4b^2c(-25a+2cx^4)+8bc^2x^2(7a+22c\right.}{1280c^3}+\frac{3b(b^2-4ac)^2\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{512c^{7/2}}$$

input
```
Integrate[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(15*b^4 - 10*b^3*c*x^2 + 128*c^2*(a + c*x^4)^2 +
4*b^2*c*(-25*a + 2*c*x^4) + 8*b*c^2*x^2*(7*a + 22*c*x^4)))/(1280*c^3) + (3
*b*(b^2 - 4*a*c)^2*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(
512*c^(7/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1434, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2} \int x^2 \left(cx^4+bx^2+a\right)^{3/2} dx^2$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{2} \left(\frac{\left(a+bx^2+cx^4\right)^{5/2}}{5c}-\frac{b \int \left(cx^4+bx^2+a\right)^{3/2} dx^2}{2c}\right)$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{2} \left(\frac{\left(a+bx^2+cx^4\right)^{5/2}}{5c}-\frac{b \left(\frac{(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3(b^2-4ac)\int \sqrt{cx^4+bx^2+a}dx^2}{16c}\right)}{2c}\right)$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{5/2}}{5c}-\frac{b\left(\frac{(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)}{16c}\right)}{2c}\right)$$

$\Big\downarrow$ 1092

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{5/2}}{5c}-\frac{b\left(\frac{(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)}{16c}\right)}{2c}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{\left(a+bx^2+cx^4\right)^{5/2}}{5c}-\frac{b\left(\frac{(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)}{2c}\right)$$

input
```
Int[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
((a + b*x^2 + c*x^4)^(5/2)/(5*c) - (b*(((b + 2*c*x^2)*(a + b*x^2 + c*x^4)^
(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(
4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*
x^4])])/(8*c^(3/2))))/(16*c)))/(2*c))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01

method	result
risch	$\frac{\left(128 c^{4} x^{8}+176 b\, c^{3} x^{6}+256 a\, c^{3} x^{4}+8 b^{2} c^{2} x^{4}+56 a b\, c^{2} x^{2}-10 x^{2} b^{3} c+128 a^{2} c^{2}-100 a\, b^{2} c+15 b^{4}\right) \sqrt{c\, x^{4}+b\, x^{2}+a}}{1280 c^{3}}-\frac{3 b \left(16 a^{2} c^{2}-\right.}{}$
pseudoelliptic	$\frac{-\frac{15 \left(a c-\frac{b^{2}}{4}\right)^{2} b \ln \left(\frac{2 c\, x^{2}+2 \sqrt{c\, x^{4}+b\, x^{2}+a}\, \sqrt{c}+b}{\sqrt{c}}\right)}{16}+\left(\left(\frac{1}{16} b^{2} x^{4}+\frac{7}{16} a b\, x^{2}+a^{2}\right) c^{\frac{5}{2}}-\frac{25 \left(\frac{b\, x^{2}}{10}+a\right) b^{2} c^{\frac{3}{2}}}{32}+\left(\frac{11}{8} b\, x^{6}+2 a\, x^{4}\right) c^{\frac{7}{2}}+c^{\frac{9}{2}}\right.}{10 c^{\frac{7}{2}}}$
default	$\frac{3 b^{3} a \ln \left(\frac{\frac{b}{2}+c\, x^{2}}{\sqrt{c}}+\sqrt{c\, x^{4}+b\, x^{2}+a}\right)}{64 c^{\frac{5}{2}}}+\frac{7 b a\, x^{2} \sqrt{c\, x^{4}+b\, x^{2}+a}}{160 c}-\frac{b^{3} x^{2} \sqrt{c\, x^{4}+b\, x^{2}+a}}{128 c^{2}}-\frac{3 b^{5} \ln \left(\frac{\frac{b}{2}+c\, x^{2}}{\sqrt{c}}+\sqrt{c\, x^{4}+b\, x^{2}+a}\right)}{512 c^{\frac{7}{2}}}$
elliptic	$\frac{3 b^{3} a \ln \left(\frac{\frac{b}{2}+c\, x^{2}}{\sqrt{c}}+\sqrt{c\, x^{4}+b\, x^{2}+a}\right)}{64 c^{\frac{5}{2}}}+\frac{7 b a\, x^{2} \sqrt{c\, x^{4}+b\, x^{2}+a}}{160 c}-\frac{b^{3} x^{2} \sqrt{c\, x^{4}+b\, x^{2}+a}}{128 c^{2}}-\frac{3 b^{5} \ln \left(\frac{\frac{b}{2}+c\, x^{2}}{\sqrt{c}}+\sqrt{c\, x^{4}+b\, x^{2}+a}\right)}{512 c^{\frac{7}{2}}}$

input
```
int(x^3*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1280*(128*c^4*x^8+176*b*c^3*x^6+256*a*c^3*x^4+8*b^2*c^2*x^4+56*a*b*c^2*x
^2-10*b^3*c*x^2+128*a^2*c^2-100*a*b^2*c+15*b^4)*(c*x^4+b*x^2+a)^(1/2)/c^3-
3/512*b*(16*a^2*c^2-8*a*b^2*c+b^4)/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4
+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.41

$$\int x^3 \left(a+bx^2+cx^4\right)^{3/2} \, dx = \left[\frac{15 \, (b^5 - 8 \, ab^3c + 16 \, a^2bc^2)\sqrt{c} \log \left(-8 \, c^2x^4 - 8 \, bcx^2 - b^2 + 4 \, \sqrt{cx^4 + bx^2 + a}(2 \, cx^2 + b)\sqrt{c}\right.}{}\right.$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*
c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4
*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8
*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^4 + b
*x^2 + a))/c^4, 1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arcta
n(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 +
a*c)) + 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^
2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(
c*x^4 + b*x^2 + a))/c^4]
```

Sympy [F]

$$\int x^3 \left(a + bx^2 + cx^4\right)^{3/2} \, dx = \int x^3 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**3*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**3*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \left(a + bx^2 + cx^4\right)^{3/2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(128) = 256$.

Time = 0.16 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.72

$$\int x^3(a+bx^2+cx^4)^{3/2}\,dx = \frac{1}{96}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{3(b^3-4abc)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{c}\right.\right.\right.}{c^{\frac{5}{2}}}\right.$$
$$+\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3(5b^4-24ab^2c}{}\right.$$
$$+\frac{1}{7680}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^2-16ac^3}{c^4}\right)x^2+\frac{35b^3c-116abc^2}{c^4}\right)x^2-\frac{105b^4}{}\right.\right.$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2
) - 3*(b^3 - 4*a*b*c)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sq
rt(c) + b))/c^(5/2))*a + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b
/c)*x^2 - (5*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*
b^4 - 24*a*b^2*c + 16*a^2*c^2)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
 + a))*sqrt(c) + b))/c^(7/2))*b + 1/7680*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*
(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 16*a*c^3)/c^4)*x^2 + (35*b^3*c - 116*a
*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*b^2*c + 256*a^2*c^2)/c^4) - 15*(7*b^5
- 40*a*b^3*c + 48*a^2*b*c^2)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
 a))*sqrt(c) + b))/c^(9/2))*c
```

Mupad [B] (verification not implemented)

Time = 17.78 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.49

$$\int x^3(a+bx^2+cx^4)^{3/2}\,dx = \frac{(c\,x^4+b\,x^2+a)^{5/2}}{10\,c}$$
$$-\frac{b\left(\frac{3\,a\left(\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\left(\frac{a}{2\sqrt{c}}-\frac{b^2}{8\,c^{3/2}}\right)+\frac{(2\,c\,x^2+b)\sqrt{c\,x^4+b\,x^2+a}}{4\,c}\right)}{4}+\frac{x^2\,(c\,x^4+b\,x^2+a)^{3/2}}{4}-\frac{3\,b^2\left(\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2}{}\right.\right.}{}\right.}{4\,c}$$

input
```
int(x^3*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
(a + b*x^2 + c*x^4)^(5/2)/(10*c) - (b*((3*a*(log((a + b*x^2 + c*x^4)^(1/2)
 + (b/2 + c*x^2)/c^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^
2)*(a + b*x^2 + c*x^4)^(1/2))/(4*c)))/4 + (x^2*(a + b*x^2 + c*x^4)^(3/2))/
4 - (3*b^2*(log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))*(a/(2*c
^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(4*
c)))/(16*c) + (b*(a + b*x^2 + c*x^4)^(3/2))/(8*c)))/(4*c)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 3844, normalized size of antiderivative = 25.63

$$\int x^3 (a + bx^2 + cx^4)^{3/2} \, dx = \text{Too large to display}$$

input
```
int(x^3*(c*x^4+b*x^2+a)^(3/2),x)
```

output

```
( - 7680*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2
+ c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**4*b*c**4 - 15360*sqrt(c)*
sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2
*c*x**2)/sqrt(4*a*c - b**2))*a**3*b**3*c**3 - 92160*sqrt(c)*sqrt(a + b*x**
2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(
4*a*c - b**2))*a**3*b**2*c**4*x**2 - 92160*sqrt(c)*sqrt(a + b*x**2 + c*x**
4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b
**2))*a**3*b*c**5*x**4 + 6720*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqr
t(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b*
*5*c**2 + 7680*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b
*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b**4*c**3*x**2 -
115200*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 +
c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b**3*c**4*x**4 - 245760*s
qrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)
+ b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b**2*c**5*x**6 - 122880*sqrt(c)*s
qrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*
c*x**2)/sqrt(4*a*c - b**2))*a**2*b*c**6*x**8 + 13440*sqrt(c)*sqrt(a + b*x*
*2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt
(4*a*c - b**2))*a*b**6*c**2*x**2 + 74880*sqrt(c)*sqrt(a + b*x**2 + c*x**4)
*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - ...
```

3.967 $\int x(a+bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 124

$$\int x\left(a+bx^2+cx^4\right)^{3/2}\,dx=-\frac{3(b^2-4ac)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}+\frac{3(b^2-4ac)^2\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}}$$

output
```
-3/128*(-4*a*c+b^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^2+1/16*(2*c*x^2+b)
*(c*x^4+b*x^2+a)^(3/2)/c+3/256*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x^2+b)/c^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int x\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\left(-3b^2+20ac+8bcx^2+8c^2x^4\right)}{128c^2}+\frac{3(-b^2+4ac)^2\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{-\sqrt{a}+\sqrt{a+bx^2+cx^4}}\right)}{128c^{5/2}}$$

input
```
Integrate[x*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 20*a*c + 8*b*c*x^2 + 8*c^
2*x^4))/(128*c^2) + (3*(-b^2 + 4*a*c)^2*ArcTanh[(Sqrt[c]*x^2)/(-Sqrt[a] +
Sqrt[a + b*x^2 + c*x^4])])/(128*c^(5/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1432, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2+cx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \left(cx^4+bx^2+a\right)^{3/2}dx^2$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^4+bx^2+a}dx^2}{16c}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)}{16c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{(b+2cx^2)\,(a+bx^2+cx^4)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int\frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)}{16c}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{(b+2cx^2)\,(a+bx^2+cx^4)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)$$

input
```
Int[x*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(((b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b +
 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c
*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2))))/(16*c))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97

method	result
risch	$\frac{\left(16c^{3}x^{6}+24b\,c^{2}x^{4}+40a\,c^{2}x^{2}+2b^{2}c\,x^{2}+20abc-3b^{3}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{128c^{2}}+\frac{3\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right.}{256c^{\frac{5}{2}}}$
pseudoelliptic	$\frac{\frac{3\left(ac-\frac{b^{2}}{4}\right)^{2}\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)}{16}+\frac{3\left(\frac{5\left(\frac{b\,x^{2}}{10}+a\right)b\,c^{\frac{3}{2}}}{6}+\left(b\,x^{4}+\frac{5}{3}a\,x^{2}\right)c^{\frac{5}{2}}+\frac{2c^{\frac{7}{2}}x^{6}}{3}-\frac{\sqrt{c}\,b^{3}}{8}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{16}-\frac{3\ln(2)\left(\right.}{}}{c^{\frac{5}{2}}}$
default	$-\frac{3b^{2}a\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{3}{2}}}+\frac{5ba\sqrt{c\,x^{4}+b\,x^{2}+a}}{32c}+\frac{b^{2}x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{64c}+\frac{3b^{4}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{256c^{\frac{5}{2}}}$
elliptic	$-\frac{3b^{2}a\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{3}{2}}}+\frac{5ba\sqrt{c\,x^{4}+b\,x^{2}+a}}{32c}+\frac{b^{2}x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{64c}+\frac{3b^{4}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{256c^{\frac{5}{2}}}$

input
```
int(x*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/128*(16*c^3*x^6+24*b*c^2*x^4+40*a*c^2*x^2+2*b^2*c*x^2+20*a*b*c-3*b^3)*(c
*x^4+b*x^2+a)^(1/2)/c^2+3/256*(16*a^2*c^2-8*a*b^2*c+b^4)/c^(5/2)*ln((1/2*b
+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.40

$$\int x\left(a+bx^{2}+cx^{4}\right)^{3/2}\,dx=\left[\frac{3\,(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2})\sqrt{c}\log\left(-8\,c^{2}x^{4}-8\,bcx^{2}-b^{2}-4\,\sqrt{cx^{4}+bx^{2}+a}(2\,cx^{2}+b)\sqrt{c}-\right.}{512\,c^{3}}\right.$$

$$-\frac{3\,(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2})\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}(2\,cx^{2}+b)\sqrt{-c}}{2\,(c^{2}x^{4}+bcx^{2}+ac)}\right)-2\,(16\,c^{4}x^{6}+24\,bc^{3}x^{4}-3\,b^{3}c+20\,abc^{2}+}{256\,c^{3}}$$

input
```
integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^
2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(16
*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^
2)*sqrt(c*x^4 + b*x^2 + a))/c^3, -1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*
sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^
4 + b*c*x^2 + a*c)) - 2*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2
+ 2*(b^2*c^2 + 20*a*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^3]
```

Sympy [F]

$$\int x(a+bx^2+cx^4)^{3/2}\,dx=\int x(a+bx^2+cx^4)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x(a+bx^2+cx^4)^{3/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(106) = 212$.

Time = 0.15 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.51

$$\int x(a+bx^2+cx^4)^{3/2}\,dx = \frac{1}{16}\left(2\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{(b^2-4ac)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}}\right.$$
$$+\frac{1}{96}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{3(b^3-4abc)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right.\right.\right.}{c^{\frac{5}{2}}}\right.$$
$$+\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3(5b^4-24ab^2c-}{}\right.$$

input
```
integrate(x*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + (b^2 - 4*a*c)*log(abs(2*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(3/2))*a + 1/96*(2*s
qrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2) - 3*(b^
3 - 4*a*b*c)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b
))/c^(5/2))*b + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 -
 (5*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*
a*b^2*c + 16*a^2*c^2)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sq
rt(c) + b))/c^(7/2))*c
```

Mupad [B] (verification not implemented)

Time = 17.71 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int x(a+bx^2+cx^4)^{3/2}\,dx = \frac{\left(c\,x^2+\frac{b}{2}\right)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}{8\,c}$$
$$+\frac{\left(3\,a\,c-\frac{3\,b^2}{4}\right)\,\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\,\sqrt{c\,x^4+b\,x^2+a}+\frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{8\,c}$$

input
```
int(x*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
((b/2 + c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(8*c) + ((3*a*c - (3*b^2)/4)*((b
/(4*c) + x^2/2)*(a + b*x^2 + c*x^4)^(1/2) + (log((a + b*x^2 + c*x^4)^(1/2)
 + (b/2 + c*x^2)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(8*c)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2869, normalized size of antiderivative = 23.14

$$\int x\left(a+bx^2+cx^4\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int(x*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(1536*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c
*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**3*b*c**3 + 3072*sqrt(c)*sqrt
(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x
**2)/sqrt(4*a*c - b**2))*a**3*c**4*x**2 - 384*sqrt(c)*sqrt(a + b*x**2 + c*
x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c
- b**2))*a**2*b**3*c**2 + 2304*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sq
rt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b
**2*c**3*x**2 + 9216*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt
(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b*c**4*x**4
 + 6144*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 +
 c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*c**5*x**6 - 96*sqrt(c)*s
qrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*
c*x**2)/sqrt(4*a*c - b**2))*a*b**5*c - 1728*sqrt(c)*sqrt(a + b*x**2 + c*x*
*4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c -
b**2))*a*b**4*c**2*x**2 - 4608*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sq
rt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b**3
*c**3*x**4 - 3072*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a
+ b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b**2*c**4*x**6 +
24*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x*
*4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**7 + 240*sqrt(c)*sqrt(a + b*x...
```

3.968 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 155

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x}\,dx = \frac{(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16c}+\frac{1}{6}(a+bx^2+cx^4)^{3/2} - \frac{1}{2}a^{3/2}\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) - \frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}}$$

output
```
1/16*(2*b*c*x^2+8*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/c+1/6*(c*x^4+b*x^2+a)^(3/
2)-1/2*a^(3/2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))-1/32
*b*(-12*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^
(3/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x}\,dx = \frac{\sqrt{a+bx^2+cx^4}(3b^2+32ac+14bcx^2+8c^2x^4)}{48c} + \frac{(-b^3+12abc)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}} + a^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}} - \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(3*b^2 + 32*a*c + 14*b*c*x^2 + 8*c^2*x^4))/(48*c)
 + ((-b^3 + 12*a*b*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*
x^4])])/(32*c^(3/2)) + a^(3/2)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a] - Sqrt[a + b*
x^2 + c*x^4]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1434, 1162, 25, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^{3/2}}{x^2}dx^2$$

$$\Big\downarrow \text{1162}$$

$$\frac{1}{2}\left(\frac{1}{3}(a+bx^2+cx^4)^{3/2}-\frac{1}{2}\int -\frac{(bx^2+2a)\sqrt{cx^4+bx^2+a}}{x^2}dx^2\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{(bx^2+2a)\sqrt{cx^4+bx^2+a}}{x^2}dx^2+\frac{1}{3}(a+bx^2+cx^4)^{3/2}\right)$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{(8ac+b^2+2bcx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\int -\frac{16a^2c-b(b^2-12ac)x^2}{2x^2\sqrt{cx^4+bx^2+a}}dx^2}{4c}\right)+\frac{1}{3}(a+bx^2+cx^4)^{3/2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{16a^2c-b(b^2-12ac)x^2}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{8c}+\frac{\sqrt{a+bx^2+cx^4}\left(8ac+b^2+2bcx^2\right)}{4c}\right)+\frac{1}{3}\left(a+bx^2+cx^4\right)^{3/2}\right)$$

$\Big\downarrow$ 1269

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{16a^2c\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2-b\left(b^2-12ac\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}+\frac{\sqrt{a+bx^2+cx^4}\left(8ac+b^2+2bcx^2\right)}{4c}\right)+\frac{1}{3}(a$$

$\Big\downarrow$ 1092

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{16a^2c\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2-2b\left(b^2-12ac\right)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{8c}+\frac{\sqrt{a+bx^2+cx^4}\left(8ac+b^2+2bcx^2\right)}{4c}\right)+$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{16a^2c\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^2+cx^4}\left(8ac+b^2+2bcx^2\right)}{4c}\right)+$$

$\Big\downarrow$ 1154

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{-32a^2c\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^2+cx^4}\left(8ac+b^2+2bcx^2\right)}{4c}\right)\right.$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{-16a^{3/2}c\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}}{8c}+\frac{\sqrt{a+bx^2+cx^4}\left(8ac+b^2+2b\right.}{4c}\right.\right.$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```

output
```
((a + b*x^2 + c*x^4)^(3/2)/3 + (((b^2 + 8*a*c + 2*b*c*x^2)*Sqrt[a + b*x^2
+ c*x^4])/(4*c) + (-16*a^(3/2)*c*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
 b*x^2 + c*x^4])] - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqr
t[a + b*x^2 + c*x^4])])/Sqrt[c])/(8*c))/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.24

method	result
default	$\frac{b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16c}-\frac{b^{3}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{3}{2}}}-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2}+\frac{c\,x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{6}+$
elliptic	$\frac{b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16c}-\frac{b^{3}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{3}{2}}}-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2}+\frac{c\,x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{6}+$
pseudoelliptic	$\frac{16c^{\frac{5}{2}}x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}-48\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)a^{\frac{3}{2}}c^{\frac{3}{2}}+28b\,c^{\frac{3}{2}}x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}+64a\,c^{\frac{3}{2}}\sqrt{c\,x^{4}+b\,x^{2}+a}+6b^{2}\sqrt{}}{96c^{\frac{3}{2}}}$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/16*b^2/c*(c*x^4+b*x^2+a)^(1/2)-1/32*b^3/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)
+(c*x^4+b*x^2+a)^(1/2))-1/2*a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a
)^(1/2))/x^2)+1/6*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24*b*x^2*(c*x^4+b*x^2+a)^(
1/2)+3/8*a*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+2/3*a
*(c*x^4+b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 727, normalized size of antiderivative = 4.69

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/192*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4
+ b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 3*(b^3 - 12*a*b*c)*sqrt
(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
+ b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c + 32*a*c^2)*
sqrt(c*x^4 + b*x^2 + a))/c^2, 1/96*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^4
 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^
4) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x
^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 + 14*b*c^2*x^2
+ 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2, 1/192*(96*sqrt(-a)*a*c
^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*
b*x^2 + a^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^
2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^
4 + 14*b*c^2*x^2 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^2, 1/96*
(48*sqrt(-a)*a*c^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-
a)/(a*c*x^4 + a*b*x^2 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqr
t(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2
*(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/
c^2]
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x}\,dx=\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/x,x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x}\,dx=\frac{72\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}+b+2c\,x^2}{\sqrt{4ac-b^2}}\right)abc-6\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,\mathrm{l}}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x,x)
```

output

```
(72*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x
**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b*c - 6*sqrt(c)*sqrt(a + b*x**2
 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4
*a*c - b**2))*b**3 + 64*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*b*c + 128*sqrt
(c)*sqrt(a + b*x**2 + c*x**4)*a*c**2*x**2 + 6*sqrt(c)*sqrt(a + b*x**2 + c*
x**4)*b**3 + 40*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b**2*c*x**2 + 72*sqrt(c)
*sqrt(a + b*x**2 + c*x**4)*b*c**2*x**4 + 32*sqrt(c)*sqrt(a + b*x**2 + c*x*
*4)*c**3*x**6 + 192*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4
)/(a*x + b*x**3 + c*x**5),x)*a**2*c**2 + 96*sqrt(c)*int(sqrt(a + b*x**2 +
c*x**4)/(a*x + b*x**3 + c*x**5),x)*a**2*b*c + 192*sqrt(c)*int(sqrt(a + b*x
**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*a**2*c**2*x**2 + 36*log((2*sqrt(c
)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b**2*c +
 72*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c -
b**2))*a*b*c**2*x**2 - 3*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*
c*x**2)/sqrt(4*a*c - b**2))*b**4 - 6*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x*
*4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**3*c*x**2 + 128*a**2*c**2 + 12*a
*b**2*c + 184*a*b*c**2*x**2 + 160*a*c**3*x**4 + 12*b**3*c*x**2 + 68*b**2*c
**2*x**4 + 88*b*c**3*x**6 + 32*c**4*x**8)/(96*c*(2*sqrt(a + b*x**2 + c*x**
4)*c + sqrt(c)*b + 2*sqrt(c)*c*x**2))
```

3.969 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 150

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3} \, dx = \frac{3}{8}(3b+2cx^2)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+cx^4)^{3/2}}{2x^2} - \frac{3}{4}\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) + \frac{3(b^2+4ac)\arctan\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}}$$

output
```
3/8*(2*c*x^2+3*b)*(c*x^4+b*x^2+a)^(1/2)-1/2*(c*x^4+b*x^2+a)^(3/2)/x^2-3/4*
a^(1/2)*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))+3/16*(4*a
*c+b^2)*arctan(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3} \, dx = \frac{1}{16}\left(\frac{2\sqrt{a+bx^2+cx^4}(-4a+5bx^2+2cx^4)}{x^2} + 24\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right) - \frac{3(b^2+4ac)\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{\sqrt{c}}\right)$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

output
```
((2*Sqrt[a + b*x^2 + c*x^4]*(-4*a + 5*b*x^2 + 2*c*x^4))/x^2 + 24*Sqrt[a]*b
*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] - (3*(b^2 + 4*a*
c)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/Sqrt[c])/16
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1434, 1161, 1231, 25, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{\left(cx^4+bx^2+a\right)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \text{1161}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{\left(2cx^2+b\right)\sqrt{cx^4+bx^2+a}}{x^2}dx^2-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{2}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}-\frac{\int -\frac{c\left(\left(b^2+4ac\right)x^2+4ab\right)}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{4c}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{\int \frac{c\left(\left(b^2+4ac\right)x^2+4ab\right)}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{4c}+\frac{1}{2}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{4}\int \frac{(b^2+4ac)\,x^2+4ab}{x^2\sqrt{cx^4+bx^2+a}}dx^2+\frac{1}{2}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^2}\right)$$

$\big\downarrow$ 1269

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{4}\left(\left(4ac+b^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2+4ab\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2\right)+\frac{1}{2}\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}\right)\right.$$

$\big\downarrow$ 1092

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{4}\left(2\left(4ac+b^2\right)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}+4ab\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2\right)+\frac{1}{2}\left(3b+2cx^2\right)\sqrt{a+bx^2+\right.\right.\right.$$

$\big\downarrow$ 219

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{4}\left(4ab\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2+\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}\right)+\frac{1}{2}\left(3b+2cx^2\right)\sqrt{a+bx^2+c\right.\right.\right.$$

$\big\downarrow$ 1154

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{4}\left(\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}-8ab\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}\right)+\frac{1}{2}\left(3b+2cx^2\right)\sqrt{a+bx^2\right.\right.\right.$$

$\big\downarrow$ 219

$$\frac{1}{2}\left(\frac{3}{2}\left(\frac{1}{4}\left(\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}-4\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)\right)+\frac{1}{2}\left(3b+2cx^2\right)\sqrt{a+\right.\right.\right.$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

output
```
(-((a + b*x^2 + c*x^4)^(3/2)/x^2) + (3*(((3*b + 2*c*x^2)*Sqrt[a + b*x^2 +
c*x^4])/2 + (-4*Sqrt[a]*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2
+ c*x^4])] + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/Sqrt[c])/4))/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.13

method	result
default	$\frac{3b^2\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{16\sqrt{c}}-\frac{a\sqrt{c x^4+b x^2+a}}{2x^2}-\frac{3\sqrt{a}\, b\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)}{4}+\frac{c x^2\sqrt{c x^4+b x^2+a}}{4}$
risch	$\frac{3b^2\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{16\sqrt{c}}-\frac{a\sqrt{c x^4+b x^2+a}}{2x^2}-\frac{3\sqrt{a}\, b\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)}{4}+\frac{c x^2\sqrt{c x^4+b x^2+a}}{4}$
elliptic	$\frac{3b^2\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{16\sqrt{c}}-\frac{a\sqrt{c x^4+b x^2+a}}{2x^2}-\frac{3\sqrt{a}\, b\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)}{4}+\frac{c x^2\sqrt{c x^4+b x^2+a}}{4}$
pseudoelliptic	$\frac{4c^{\frac{3}{2}}\sqrt{c x^4+b x^2+a}\, x^4-12\sqrt{a}\, b\ln\left(\frac{2a+b x^2+2\sqrt{a}\,\sqrt{c x^4+b x^2+a}}{x^2}\right)x^2\sqrt{c}+10b\sqrt{c x^4+b x^2+a}\, x^2\sqrt{c}+12ac x^2\ln\left(\frac{2c x^2+2\sqrt{c x^4-}}{\sqrt{}}\right.}{16x^2\sqrt{c}}$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
3/16*b^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*a/x^2
*(c*x^4+b*x^2+a)^(1/2)-3/4*a^(1/2)*b*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+
a)^(1/2))/x^2)+1/4*c*x^2*(c*x^4+b*x^2+a)^(1/2)+5/8*b*(c*x^4+b*x^2+a)^(1/2)
+3/4*a*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.75

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/32*(12*sqrt(a)*b*c*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x
^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 3*(b^2 + 4*a*c)*sqrt
(c)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*
x^2 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c)*sqrt(c*x^4 +
 b*x^2 + a))/(c*x^2), 1/16*(6*sqrt(a)*b*c*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) -
3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
 b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*x^4 + 5*b*c*x^2 - 4*a*c
)*sqrt(c*x^4 + b*x^2 + a))/(c*x^2), 1/32*(24*sqrt(-a)*b*c*x^2*arctan(1/2*s
qrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) +
 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x
^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^4 + 5*b*c*x^2
- 4*a*c)*sqrt(c*x^4 + b*x^2 + a))/(c*x^2), 1/16*(12*sqrt(-a)*b*c*x^2*arcta
n(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 +
a^2)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2
*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*x^4 + 5*b*c*x^2
 - 4*a*c)*sqrt(c*x^4 + b*x^2 + a))/(c*x^2)]
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x**3,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{3\,ab\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}} + \frac{1}{8}\sqrt{cx^4+bx^2+a}(2\,cx^2+5\,b) - \frac{3\,(b^2+4\,ac)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{16\sqrt{c}} + \frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)ab+2\,a^2\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
3/2*a*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a)
 + 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + 5*b) - 3/16*(b^2 + 4*a*c)*log(ab
s(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/sqrt(c) + 1/2*((
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b + 2*a^2*sqrt(c))/((sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))^2 - a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3}\,dx=\int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^3}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/x^3,x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3}\,dx=\frac{24\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}+b+2c\,x^2}{\sqrt{4ac-b^2}}\right)ac\,x^2+6\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+c}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^3,x)
```

output

```
(24*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x
**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*c*x**2 + 6*sqrt(c)*sqrt(a + b*x
**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqr
t(4*a*c - b**2))*b**2*x**2 - 8*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*b - 16*
sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*c*x**2 + 10*sqrt(c)*sqrt(a + b*x**2 +
c*x**4)*b**2*x**2 + 24*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b*c*x**4 + 8*sqrt
(c)*sqrt(a + b*x**2 + c*x**4)*c**2*x**6 + 48*sqrt(a + b*x**2 + c*x**4)*int
(sqrt(a + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*a*b*c*x**2 + 24*sqrt
(c)*int(sqrt(a + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*a*b**2*x**2 +
 48*sqrt(c)*int(sqrt(a + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*a*b*c
*x**4 + 12*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4
*a*c - b**2))*a*b*c*x**2 + 24*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b
 + 2*c*x**2)/sqrt(4*a*c - b**2))*a*c**2*x**4 + 3*log((2*sqrt(c)*sqrt(a + b
*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**3*x**2 + 6*log((2*s
qrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**2*
c*x**4 - 16*a**2*c + 4*a*b*c*x**2 - 8*a*c**2*x**4 + 20*b**2*c*x**4 + 28*b*
c**2*x**6 + 8*c**3*x**8)/(16*x**2*(2*sqrt(a + b*x**2 + c*x**4)*c + sqrt(c)
*b + 2*sqrt(c)*c*x**2))
```

3.970 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 151

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^5}\,dx = -\frac{3(b-2cx^2)\sqrt{a+bx^2+cx^4}}{8x^2}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{4x^4} - \frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}+\frac{3}{4}b\sqrt{c}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

output
```
-3/8*(-2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/x^2-1/4*(c*x^4+b*x^2+a)^(3/2)/x^4-
3/16*(4*a*c+b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^
(1/2)+3/4*b*c^(1/2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^5}\,dx = \frac{1}{8}\left(\frac{\sqrt{a+bx^2+cx^4}(-2a-5bx^2+4cx^4)}{x^4} + \frac{3(b^2+4ac)\,\mathrm{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}} - 6b\sqrt{c}\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```

output
```
((Sqrt[a + b*x^2 + c*x^4]*(-2*a - 5*b*x^2 + 4*c*x^4))/x^4 + (3*(b^2 + 4*a*
c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/Sqrt[a] - 6*b
*Sqrt[c]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/8
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1434, 1161, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^{3/2}}{x^6}dx^2$$

$$\Big\downarrow \text{1161}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \frac{\left(2cx^2+b\right)\sqrt{cx^4+bx^2+a}}{x^4}dx^2-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^4}\right)$$

$\Big\downarrow$ 1230

$$\frac{1}{2}\left(\frac{3}{4}\left(-\frac{1}{2}\int -\frac{b^2+4cx^2b+4ac}{x^2\sqrt{cx^4+bx^2+a}}dx^2-\frac{\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{x^2}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^4}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{b^2+4cx^2b+4ac}{x^2\sqrt{cx^4+bx^2+a}}dx^2-\frac{\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{x^2}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2x^4}\right)$$

$\Big\downarrow$ 1269

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\left(\left(4ac+b^2\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2+4bc\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2\right)-\frac{\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{x^2}\right)-\right.$$

$\Big\downarrow$ 1092

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\left(\left(4ac+b^2\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2+8bc\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}\right)-\frac{\left(b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{x^2}\right.\right.$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\left(\left(4ac+b^2\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2+4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)-\frac{\left(b-2cx^2\right)\sqrt{a+bx^2}}{x^2}\right.\right.$$

$\Big\downarrow$ 1154

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\left(4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-2\left(4ac+b^2\right)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}\right)-\frac{\left(b-2cx^2\right)\sqrt{a+}}{x^2}\right.\right.$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\left(4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}\right)-\frac{\left(b-2cx^2\right)\sqrt{a+bx}}{x^2}\right.\right.$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```

output
```
(-1/2*(a + b*x^2 + c*x^4)^(3/2)/x^4 + (3*(-(((b - 2*c*x^2)*Sqrt[a + b*x^2
+ c*x^4])/x^2) + (-(((b^2 + 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a
 + b*x^2 + c*x^4])])/Sqrt[a]) + 4*b*Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[
c]*Sqrt[a + b*x^2 + c*x^4])])/2))/4)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$-\frac{3\left(x^{4}\left(ac+\frac{b^{2}}{4}\right)\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)-b\sqrt{c}\,x^{4}\sqrt{a}\,\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)+\frac{\left(a^{\frac{3}{2}}+\left(-2c\,x^{4}+\frac{5}{2}b\,x^{2}\right)\sqrt{c}\right.}{3}\right.}{4\sqrt{a}\,x^{4}}$
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(5b\,x^{2}+2a)}{8x^{4}}-\frac{3\sqrt{a}\,c\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4}-\frac{3b^{2}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{16\sqrt{a}}+c\sqrt{}$
default	$-\frac{a\sqrt{c\,x^{4}+b\,x^{2}+a}}{4x^{4}}-\frac{5b\sqrt{c\,x^{4}+b\,x^{2}+a}}{8x^{2}}-\frac{3b^{2}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{16\sqrt{a}}-\frac{3\sqrt{a}\,c\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+}}{x^{2}}\right.}{4}$
elliptic	$-\frac{a\sqrt{c\,x^{4}+b\,x^{2}+a}}{4x^{4}}-\frac{5b\sqrt{c\,x^{4}+b\,x^{2}+a}}{8x^{2}}-\frac{3b^{2}\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{16\sqrt{a}}-\frac{3\sqrt{a}\,c\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+}}{x^{2}}\right.}{4}$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-3/4/a^(1/2)*(x^4*(a*c+1/4*b^2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1
/2))/x^2)-b*c^(1/2)*x^4*a^(1/2)*ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2
)+b)/c^(1/2))+1/3*(a^(3/2)+(-2*c*x^4+5/2*b*x^2)*a^(1/2))*(c*x^4+b*x^2+a)^(
1/2)+b*c^(1/2)*x^4*a^(1/2)*ln(2))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.72

$$\int \frac{(a + bx^2 + cx^4)^{3/2}}{x^5} \, dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/32*(12*a*b*sqrt(c)*x^4*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 3*(b^2 + 4*a*c)*sqrt(a)*x^4*
log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2
*a)*sqrt(a) + 8*a^2)/x^4) + 4*(4*a*c*x^4 - 5*a*b*x^2 - 2*a^2)*sqrt(c*x^4 +
 b*x^2 + a))/(a*x^4), -1/32*(24*a*b*sqrt(-c)*x^4*arctan(1/2*sqrt(c*x^4 + b
*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 3*(b^2 + 4*a
*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2
 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(4*a*c*x^4 - 5*a*b*x^2 - 2*a
^2)*sqrt(c*x^4 + b*x^2 + a))/(a*x^4), 1/16*(6*a*b*sqrt(c)*x^4*log(-8*c^2*x
^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4
*a*c) + 3*(b^2 + 4*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(4*a*c*x^4 - 5*a*b*x^2
 - 2*a^2)*sqrt(c*x^4 + b*x^2 + a))/(a*x^4), -1/16*(12*a*b*sqrt(-c)*x^4*arc
tan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2
+ a*c)) - 3*(b^2 + 4*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*
(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(4*a*c*x^4 - 5*a*b*x
^2 - 2*a^2)*sqrt(c*x^4 + b*x^2 + a))/(a*x^4)]
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^5}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x**5,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/x**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(125) = 250.

Time = 0.18 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^5}\,dx = -\frac{3}{4}b\sqrt{c}\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right) + \frac{1}{2}\sqrt{cx^4+bx^2+a}c + \frac{3\,(b^2+4\,ac)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\sqrt{-a}} + \frac{5\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3b^2+4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3ac+16\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2ab\sqrt{c}-}{8\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-\right.}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^5,x, algorithm="giac")
```

output
```
-3/4*b*sqrt(c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c)
- b)) + 1/2*sqrt(c*x^4 + b*x^2 + a)*c + 3/8*(b^2 + 4*a*c)*arctan(-(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) + 1/8*(5*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))^3*b^2 + 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a)
)^3*a*c + 16*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a*b*sqrt(c) - 3*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b^2 + 4*(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))*a^2*c - 8*a^2*b*sqrt(c))/((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
 a))^2 - a)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^5}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^5}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/x^5,x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^5}\,dx = \frac{12\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}+b+2c\,x^2}{\sqrt{4ac-b^2}}\right)bc\,x^4-2\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+c}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^5,x)
```

output
```
(12*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x
**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b*c*x**4 - 2*sqrt(c)*sqrt(a + b*x
**2 + c*x**4)*a*b - 4*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*c*x**2 - 5*sqrt(
c)*sqrt(a + b*x**2 + c*x**4)*b**2*x**2 - 6*sqrt(c)*sqrt(a + b*x**2 + c*x**
4)*b*c*x**4 + 8*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*c**2*x**6 + 24*sqrt(a +
b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*
a*c**2*x**4 + 6*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a
*x + b*x**3 + c*x**5),x)*b**2*c*x**4 + 12*sqrt(c)*int(sqrt(a + b*x**2 + c*
x**4)/(a*x + b*x**3 + c*x**5),x)*a*b*c*x**4 + 24*sqrt(c)*int(sqrt(a + b*x*
*2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*a*c**2*x**6 + 3*sqrt(c)*int(sqrt(a
 + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*b**3*x**4 + 6*sqrt(c)*int(s
qrt(a + b*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*b**2*c*x**6 + 6*log((2
*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**
2*c*x**4 + 12*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqr
t(4*a*c - b**2))*b*c**2*x**6 - 4*a**2*c - 14*a*b*c*x**2 + 4*a*c**2*x**4 -
10*b**2*c*x**4 - 2*b*c**2*x**6 + 8*c**3*x**8)/(8*x**4*(2*sqrt(a + b*x**2 +
 c*x**4)*c + sqrt(c)*b + 2*sqrt(c)*c*x**2))
```

3.971 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 20, antiderivative size = 163

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7} dx = -\frac{(2ab+(b^2+8ac)x^2)\sqrt{a+bx^2+cx^4}}{16ax^4} - \frac{(a+bx^2+cx^4)^{3/2}}{6x^6} + \frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{3/2}} + \frac{1}{2}c^{3/2}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

output

```
-1/16*(2*a*b+(8*a*c+b^2)*x^2)*(c*x^4+b*x^2+a)^(1/2)/a/x^4-1/6*(c*x^4+b*x^2
+a)^(3/2)/x^6+1/32*b*(-12*a*c+b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+
b*x^2+a)^(1/2))/a^(3/2)+1/2*c^(3/2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4
+b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{\sqrt{a+bx^2+cx^4}(-8a^2-14abx^2-3b^2x^4-32acx^4)}{48ax^6} + \frac{(b^3-12abc)\operatorname{arctanh}\left(\frac{-\sqrt{c}x^2+\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{3/2}} - \frac{1}{2}c^{3/2}\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(-8*a^2 - 14*a*b*x^2 - 3*b^2*x^4 - 32*a*c*x^4))/(
48*a*x^6) + ((b^3 - 12*a*b*c)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c
*x^4])/Sqrt[a]])/(16*a^(3/2)) - (c^(3/2)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4]])/2
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1434, 1161, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^{3/2}}{x^8}dx^2$$

$$\Big\downarrow \text{1161}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{(2cx^2+b)\sqrt{cx^4+bx^2+a}}{x^6}dx^2 - \frac{(a+bx^2+cx^4)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{1229}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{b(b^2-12ac)-16ac^2x^2}{2x^2\sqrt{cx^4+bx^2+a}}dx^2}{4a}-\frac{\sqrt{a+bx^2+cx^4}\left(x^2\left(8ac+b^2\right)+2ab\right)}{4ax^4}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{b(b^2-12ac)-16ac^2x^2}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{8a}-\frac{\sqrt{a+bx^2+cx^4}\left(x^2\left(8ac+b^2\right)+2ab\right)}{4ax^4}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{b\left(b^2-12ac\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2-16ac^2\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8a}-\frac{\sqrt{a+bx^2+cx^4}\left(x^2\left(8ac+b^2\right)+2ab\right)}{4ax^4}\right)-\right.$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{b\left(b^2-12ac\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2-32ac^2\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{8a}-\frac{\sqrt{a+bx^2+cx^4}\left(x^2\left(8ac+b^2\right)+2ab\right)}{4ax^4}\right)\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{b\left(b^2-12ac\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8a}-\frac{\sqrt{a+bx^2+cx^4}\left(x^2\left(8ac+b^2\right)+\right.}{4ax^4}\right.\right.$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{-2b\left(b^2-12ac\right)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8a}-\frac{\sqrt{a+bx^2+cx^4}\left(x^2\left(8ac+b\right.\right.}{4ax^4}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8a}-\frac{\sqrt{a+bx^2+cx^4}\left(x^2\left(8ac+b^2\right.\right.}{4ax^4}\right.\right.$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

output
```
(-1/3*(a + b*x^2 + c*x^4)^(3/2)/x^6 + (-1/4*((2*a*b + (b^2 + 8*a*c)*x^2)*S
qrt[a + b*x^2 + c*x^4])/(a*x^4) - (-((b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^
2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a]) - 16*a*c^(3/2)*ArcTanh[(
b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*a))/2)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(32ac\,x^4+3b^2x^4+14ab\,x^2+8a^2)}{48x^6a}+\frac{-\frac{b(12ac-b^2)\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}+8c^{\frac{3}{2}}a\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\right.}{16a}$
pseudoelliptic	$-\frac{3\left(b\,x^6\left(ac-\frac{b^2}{12}\right)\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)-\frac{4c^{\frac{3}{2}}a^{\frac{3}{2}}x^6\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)}{3}+\left(\frac{7\left(\frac{16c\,x^2}{7}+b\right)x^2a^{\frac{3}{2}}}{9}+\right.\right.}{8a^{\frac{3}{2}}x^6}$
default	$\frac{c^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2}-\frac{a\sqrt{c\,x^4+b\,x^2+a}}{6x^6}-\frac{7b\sqrt{c\,x^4+b\,x^2+a}}{24x^4}-\frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{16a\,x^2}+\frac{b^3\ln\left(\frac{2a+b\,x^2+2\sqrt{a}}{x^2}\right.}{32a^{\frac{3}{2}}}$
elliptic	$\frac{c^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2}-\frac{a\sqrt{c\,x^4+b\,x^2+a}}{6x^6}-\frac{7b\sqrt{c\,x^4+b\,x^2+a}}{24x^4}-\frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{16a\,x^2}+\frac{b^3\ln\left(\frac{2a+b\,x^2+2\sqrt{a}}{x^2}\right.}{32a^{\frac{3}{2}}}$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(c*x^4+b*x^2+a)^(1/2)*(32*a*c*x^4+3*b^2*x^4+14*a*b*x^2+8*a^2)/x^6/a+
1/16/a*(-1/2*b*(12*a*c-b^2)/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a
)^(1/2))/x^2)+8*c^(3/2)*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 771, normalized size of antiderivative = 4.73

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/192*(48*a^2*c^(3/2)*x^6*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4
 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*
x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4
 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x^6), -1/192*(96*a^2*sqrt(-c)*c*x^
6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c
*x^2 + a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) +
4*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a
))/(a^2*x^6), 1/96*(24*a^2*c^(3/2)*x^6*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 -
4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b
*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)
/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(14*a^2*b*x^2 + (3*a*b^2 + 32*a^2*c)*x^4 +
 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x^6), -1/96*(48*a^2*sqrt(-c)*c*x^6*a
rctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^
2 + a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(14*a^2*b*x^2 +
 (3*a*b^2 + 32*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^2*x^6)]
```

Sympy [F]

$$\int \frac{(a + bx^2 + cx^4)^{3/2}}{x^7}\,dx = \int \frac{(a + bx^2 + cx^4)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x**7,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/x**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(135) = 270$.

Time = 0.20 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.44

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7}\,dx = -\frac{1}{2}\,c^{\frac{3}{2}}\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right) - \frac{(b^3-12\,abc)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\sqrt{-a}a} + \frac{3\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5b^3+60\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5abc+48\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^4ab^2\sqrt{}}{}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-1/2*c^(3/2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) -
b)) - 1/16*(b^3 - 12*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a)
)/sqrt(-a))/(sqrt(-a)*a) + 1/48*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))
^5*b^3 + 60*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b*c + 48*(sqrt(c)*
x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a*b^2*sqrt(c) + 96*(sqrt(c)*x^2 - sqrt(c*
x^4 + b*x^2 + a))^4*a^2*c^(3/2) + 8*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a)
)^3*a*b^3 - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^3*c^(3/2) - 3*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*b^3 + 36*(sqrt(c)*x^2 - sqrt(c*
x^4 + b*x^2 + a))*a^3*b*c + 64*a^4*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 + a))^2 - a)^3*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^7}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/x^7,x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/x^7, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{-8\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,a^2-14\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,ab\,x^2-32\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+}}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^7,x)
```

output
```
( - 8*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a**2 - 14*sqrt(c)*sqrt(a + b*x**2
+ c*x**4)*a*b*x**2 - 32*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*c*x**4 - 3*sqr
t(c)*sqrt(a + b*x**2 + c*x**4)*b**2*x**4 + 36*sqrt(c)*int(sqrt(a + b*x**2
+ c*x**4)/(a*x + b*x**3 + c*x**5),x)*a*b*c*x**6 - 3*sqrt(c)*int(sqrt(a + b
*x**2 + c*x**4)/(a*x + b*x**3 + c*x**5),x)*b**3*x**6 + 24*log((2*sqrt(c)*s
qrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*c**2*x**6)/
(48*sqrt(c)*a*x**6)
```

3.972 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^9} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 133

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^9} \, dx = \frac{3(b^2-4ac)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4} - \frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8} - \frac{3(b^2-4ac)^2 \operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{5/2}}$$

output

```
3/128*(-4*a*c+b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4-1/16*(b*x^2+2
*a)*(c*x^4+b*x^2+a)^(3/2)/a/x^8-3/256*(-4*a*c+b^2)^2*arctanh(1/2*(b*x^2+2*
a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^9} \, dx = \frac{-\frac{\sqrt{a}(2a+bx^2)\sqrt{a+bx^2+cx^4}(8a^2+8abx^2-3b^2x^4+20acx^4)}{x^8} + 3(b^2-4ac)^2 \operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+b}}{\sqrt{a}}\right.}{128a^{5/2}}$$

input

```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

output
```
(-((Sqrt[a]*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4]*(8*a^2 + 8*a*b*x^2 - 3*b
^2*x^4 + 20*a*c*x^4))/x^8) + 3*(b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]*x^2 - Sqrt
[a + b*x^2 + c*x^4])/Sqrt[a]])/(128*a^(5/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1434, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^9}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{\left(cx^4+bx^2+a\right)^{3/2}}{x^{10}}dx^2$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{2}\left(-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^4+bx^2+a}}{x^6}dx^2}{16a}-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{2}\left(-\frac{3\left(b^2-4ac\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{8a}-\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{16a}-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{4a}-\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{16a}-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{8a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{16a}-\frac{(2a+bx^2)\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8}\right)$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

output
```
(-1/8*((2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(a*x^8) - (3*(b^2 - 4*a*c)
*(-1/4*((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(a*x^4) + ((b^2 - 4*a*c)*Ar
cTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(8*a^(3/2))))/(1
6*a))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{3\left(\left(ac-\frac{b^2}{4}\right)^2 x^8 \ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)-\frac{\left(-\frac{2b\,x^4\left(10c\,x^2+b\right)a^{\frac{3}{2}}}{3}-8\left(\frac{5c\,x^2}{3}+b\right)x^2a^{\frac{5}{2}}+\sqrt{a}\,b^3x^6-\frac{16a^{\frac{7}{2}}}{3}\right)\sqrt{c\,x^4+b\,x}}{8}\right.}{16a^{\frac{5}{2}}x^8}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\left(20abc\,x^6-3b^3x^6+40a^2c\,x^4+2b^2x^4a+24a^2b\,x^2+16a^3\right)}{128x^8a^2}-\frac{3\left(16a^2c^2-8a\,b^2c+b^4\right)\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x}}{x^2}\right.}{256a^{\frac{5}{2}}}$
default	$\frac{3b^2c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{32a^{\frac{3}{2}}}-\frac{5bc\sqrt{c\,x^4+b\,x^2+a}}{32a\,x^2}-\frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{64a\,x^4}+\frac{3b^3\sqrt{c\,x^4+b\,x^2+a}}{128a^2x^2}-\frac{3b^4\ln\left(\frac{2a+b}{}\right.}{}$
elliptic	$\frac{3b^2c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{32a^{\frac{3}{2}}}-\frac{5bc\sqrt{c\,x^4+b\,x^2+a}}{32a\,x^2}-\frac{b^2\sqrt{c\,x^4+b\,x^2+a}}{64a\,x^4}+\frac{3b^3\sqrt{c\,x^4+b\,x^2+a}}{128a^2x^2}-\frac{3b^4\ln\left(\frac{2a+b}{}\right.}{}$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-3/16/a^(5/2)*((a*c-1/4*b^2)^2*x^8*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)
^(1/2))/x^2)-1/8*(-2/3*b*x^4*(10*c*x^2+b)*a^(3/2)-8*(5/3*c*x^2+b)*x^2*a^(5
/2)+a^(1/2)*b^3*x^6-16/3*a^(7/2))*(c*x^4+b*x^2+a)^(1/2))/x^8
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.40

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^9}\,dx = \left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{a}x^8\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a}{x^4}\right.}{5}\right.$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
[1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^8*log(-((b^2 + 4*a*c)*x
^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/
x^4) + 4*((3*a*b^3 - 20*a^2*b*c)*x^6 - 24*a^3*b*x^2 - 2*(a^2*b^2 + 20*a^3*
c)*x^4 - 16*a^4)*sqrt(c*x^4 + b*x^2 + a))/(a^3*x^8), 1/256*(3*(b^4 - 8*a*b
^2*c + 16*a^2*c^2)*sqrt(-a)*x^8*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*x^6
 - 24*a^3*b*x^2 - 2*(a^2*b^2 + 20*a^3*c)*x^4 - 16*a^4)*sqrt(c*x^4 + b*x^2
+ a))/(a^3*x^8)]
```

Sympy [F]

$$\int \frac{(a + bx^2 + cx^4)^{3/2}}{x^9} \, dx = \int \frac{(a + bx^2 + cx^4)^{\frac{3}{2}}}{x^9} \, dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x**9,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/x**9, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + bx^2 + cx^4)^{3/2}}{x^9} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. 2(115) = 230.

Time = 0.17 (sec) , antiderivative size = 606, normalized size of antiderivative = 4.56

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^9}\,dx = \frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{128\,\sqrt{-a}a^2}$$
$$-\frac{3\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^7b^4-24\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^7ab^2c-80\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^7a^2c}{}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^9,x, algorithm="giac")
```

output
```
3/128*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b
*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/128*(3*(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))^7*b^4 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a*b^2*c
- 80*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^2*c^2 - 256*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))^6*a^2*b*c^(3/2) - 11*(sqrt(c)*x^2 - sqrt(c*x^4
+ b*x^2 + a))^5*a*b^4 - 168*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^2*
b^2*c - 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3*c^2 - 128*(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^2*b^3*sqrt(c) - 11*(sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 + a))^3*a^2*b^4 - 168*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
a))^3*a^3*b^2*c - 48*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^4*c^2 - 2
56*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^4*b*c^(3/2) + 3*(sqrt(c)*x^
2 - sqrt(c*x^4 + b*x^2 + a))*a^3*b^4 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^
2 + a))*a^4*b^2*c - 80*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^5*c^2)/((
(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^4*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^9}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^9}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/x^9,x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/x^9, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^9}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{x^9}dx$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^9,x)
```

output
```
int((c*x^4+b*x^2+a)^(3/2)/x^9,x)
```

3.973 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{11}} dx$

Optimal result

Integrand size = 20, antiderivative size = 162

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{11}} dx = -\frac{3b(b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{256a^3x^4} + \frac{b(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{32a^2x^8} - \frac{(a+bx^2+cx^4)^{5/2}}{10ax^{10}} + \frac{3b(b^2-4ac)^2 \operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{7/2}}$$

output
```
-3/256*b*(-4*a*c+b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a^3/x^4+1/32*b*(b*
x^2+2*a)*(c*x^4+b*x^2+a)^(3/2)/a^2/x^8-1/10*(c*x^4+b*x^2+a)^(5/2)/a/x^10+3
/512*b*(-4*a*c+b^2)^2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2
))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{11}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^2+cx^4}(128a^4+15b^4x^8-10ab^2x^6(b+10cx^2)+16a^3(11bx^2+16cx^4)+8a^2x^4(b^2+7bcx^2+16c^2x^4))}{x^{10}}}{1280a^{7/2}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]*(128*a^4 + 15*b^4*x^8 - 10*a*b^2*x^6*(
b + 10*c*x^2) + 16*a^3*(11*b*x^2 + 16*c*x^4) + 8*a^2*x^4*(b^2 + 7*b*c*x^2
+ 16*c^2*x^4)))/x^10) - 15*b*(b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a
 + b*x^2 + c*x^4])/Sqrt[a]])/(1280*a^(7/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1434, 1157, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{11}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^{3/2}}{x^{12}}dx^2$$

$$\Big\downarrow \text{1157}$$

$$\frac{1}{2}\left(-\frac{b\int \frac{(cx^4+bx^2+a)^{3/2}}{x^{10}}dx^2}{2a}-\frac{(a+bx^2+cx^4)^{5/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{2}\left(-\frac{b\left(-\frac{3(b^2-4ac)\int\frac{\sqrt{cx^4+bx^2+a}}{x^6}dx^2}{16a}-\frac{(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{8ax^8}\right)}{2a}-\frac{(a+bx^2+cx^4)^{5/2}}{5ax^{10}}\right)$$

$\Big\downarrow$ 1152

$$\frac{1}{2}\left(-\frac{b\left(-\frac{3(b^2-4ac)\left(-\frac{(b^2-4ac)\int\frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{8a}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{16a}-\frac{(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{8ax^8}\right)}{2a}-\frac{(a+bx^2+cx^4)}{5ax^{10}}\right.$$

$\Big\downarrow$ 1154

$$\frac{1}{2}\left(-\frac{b\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\int\frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{4a}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{16a}-\frac{(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{8ax^8}\right)}{2a}-\frac{(a+bx^2+cx^4)}{5ax^{10}}\right.$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(-\frac{b\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{8a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{16a}-\frac{(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{8ax^8}\right)}{2a}-\frac{(a+bx^2}{5a}\right.$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]
```

output
```
(-1/5*(a + b*x^2 + c*x^4)^(5/2)/(a*x^10) - (b*(-1/8*((2*a + b*x^2)*(a + b*
x^2 + c*x^4)^(3/2))/(a*x^8) - (3*(b^2 - 4*a*c)*(-1/4*((2*a + b*x^2)*Sqrt[a
 + b*x^2 + c*x^4])/(a*x^4) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[
a]*Sqrt[a + b*x^2 + c*x^4])])/(8*a^(3/2))))/(16*a)))/(2*a))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{\frac{3\left(ac-\frac{b^2}{4}\right)^2 b\,x^{10}\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{32}-\frac{\sqrt{c\,x^4+b\,x^2+a}\left(\frac{x^4\left(16c^2x^4+7bc\,x^2+b^2\right)a^{\frac{5}{2}}}{16}-\frac{5b^2x^6\left(10c\,x^2+b\right)a^{\frac{3}{2}}}{64}+\left(2c\,x^4+\right.\right.}{10}}{a^{\frac{7}{2}}x^{10}}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\left(128a^2c^2x^8-100a\,b^2c\,x^8+15b^4x^8+56a^2bc\,x^6-10a\,b^3x^6+256a^3c\,x^4+8a^2b^2x^4+176a^3b\,x^2+128a^4\right)}{1280x^{10}a^3}+\frac{3}{}$
default	$-\frac{3b^3c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{64a^{\frac{5}{2}}}+\frac{5b^2c\sqrt{c\,x^4+b\,x^2+a}}{64a^2x^2}-\frac{7bc\sqrt{c\,x^4+b\,x^2+a}}{160a\,x^4}+\frac{3b\,c^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b}}{x^2}\right.}{32a^{\frac{3}{2}}}$
elliptic	$-\frac{3b^3c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{64a^{\frac{5}{2}}}+\frac{5b^2c\sqrt{c\,x^4+b\,x^2+a}}{64a^2x^2}-\frac{7bc\sqrt{c\,x^4+b\,x^2+a}}{160a\,x^4}+\frac{3b\,c^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b}}{x^2}\right.}{32a^{\frac{3}{2}}}$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
3/32/a^(7/2)*((a*c-1/4*b^2)^2*b*x^10*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+
a)^(1/2))/x^2)-16/15*(c*x^4+b*x^2+a)^(1/2)*(1/16*x^4*(16*c^2*x^4+7*b*c*x^2
+b^2)*a^(5/2)-5/64*b^2*x^6*(10*c*x^2+b)*a^(3/2)+(2*c*x^4+11/8*b*x^2)*a^(7/
2)+15/128*a^(1/2)*b^4*x^8+a^(9/2)))/x^10
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.36

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx = \left[\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\sqrt{a}x^{10}\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}}{x^4}\right.}{}\right.$$

$$-\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\sqrt{-a}x^{10}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)+2\left(\left(15\,ab^4-100\,a^2b^2c+128\,a^3c^2\right)x\right.}{2560\,a^4x^{10}}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="fricas")
```

output
```
[1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^10*log(-((b^2 + 4*a
*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*
a^2)/x^4) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^8 + 176*a^4*b*x^
2 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^6 + 128*a^5 + 8*(a^3*b^2 + 32*a^4*c)*x^4)
*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^10), -1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^
2*b*c^2)*sqrt(-a)*x^10*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sq
rt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3
*c^2)*x^8 + 176*a^4*b*x^2 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^6 + 128*a^5 + 8*(
a^3*b^2 + 32*a^4*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^10)]
```

Sympy [F]

$$\int \frac{(a + bx^2 + cx^4)^{3/2}}{x^{11}}\, dx = \int \frac{(a + bx^2 + cx^4)^{\frac{3}{2}}}{x^{11}}\, dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x**11,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/x**11, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + bx^2 + cx^4)^{3/2}}{x^{11}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(140) = 280$.

Time = 0.20 (sec) , antiderivative size = 832, normalized size of antiderivative = 5.14

$$\int \frac{(a + bx^2 + cx^4)^{3/2}}{x^{11}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^11,x, algorithm="giac")
```

output
```
-3/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4
+ b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/1280*(15*(sqrt(c)*x^2 - sqrt(c*
x^4 + b*x^2 + a))^9*b^5 - 120*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a*
b^3*c + 240*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a^2*b*c^2 + 1280*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^8*a^3*c^(5/2) - 70*(sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 + a))^7*a*b^5 + 560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a)
)^7*a^2*b^3*c + 2720*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^3*b*c^2 +
 5120*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^3*b^2*c^(3/2) + 128*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^2*b^5 + 2560*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))^5*a^3*b^3*c + 3840*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
a))^5*a^4*b*c^2 + 1280*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^3*b^4*s
qrt(c) + 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^4*b^2*c^(3/2) +
2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^5*c^(5/2) + 70*(sqrt(c)*x
^2 - sqrt(c*x^4 + b*x^2 + a))^3*a^3*b^5 + 2000*(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))^3*a^4*b^3*c + 2400*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*
a^5*b*c^2 + 2560*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^5*b^2*c^(3/2)
 - 15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^4*b^5 + 120*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))*a^5*b^3*c + 1040*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))*a^6*b*c^2 + 256*a^7*c^(5/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^2 - a)^5*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{11}}\,dx=\int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^{11}}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/x^11,x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/x^11, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{11}}\,dx=\int \frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{x^{11}}dx$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^11,x)
```

output
```
int((c*x^4+b*x^2+a)^(3/2)/x^11,x)
```

3.974 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}} dx$

Optimal result

Integrand size = 20, antiderivative size = 216

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}} dx = \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{1024a^4x^4} - \frac{(7b^2-4ac)(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{384a^3x^8} - \frac{(a+bx^2+cx^4)^{5/2}}{12ax^{12}} + \frac{7b(a+bx^2+cx^4)^{5/2}}{120a^2x^{10}} - \frac{(b^2-4ac)^2(7b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2048a^{9/2}}$$

output
```
1/1024*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(1/2)/a^4/x
^4-1/384*(-4*a*c+7*b^2)*(b*x^2+2*a)*(c*x^4+b*x^2+a)^(3/2)/a^3/x^8-1/12*(c*
x^4+b*x^2+a)^(5/2)/a/x^12+7/120*b*(c*x^4+b*x^2+a)^(5/2)/a^2/x^10-1/2048*(-
4*a*c+b^2)^2*(-4*a*c+7*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a
)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^2+cx^4}(1280a^5-105b^5x^{10}+10ab^3x^8(7b+76cx^2)+64a^4(26bx^2+35cx^4)+48a^3x^4(b^2+6bcx^2+10c}{x^{12}}}{15}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]*(1280*a^5 - 105*b^5*x^10 + 10*a*b^3*x^
8*(7*b + 76*c*x^2) + 64*a^4*(26*b*x^2 + 35*c*x^4) + 48*a^3*x^4*(b^2 + 6*b*
c*x^2 + 10*c^2*x^4) - 8*a^2*b*x^6*(7*b^2 + 54*b*c*x^2 + 162*c^2*x^4)))/x^1
2) + 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*
x^2 + c*x^4])/Sqrt[a]])/(15360*a^(9/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1434, 1167, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^{3/2}}{x^{14}}dx^2$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{2}\left(-\frac{\int \frac{(2cx^2+7b)(cx^4+bx^2+a)^{3/2}}{2x^{12}}dx^2}{6a}-\frac{(a+bx^2+cx^4)^{5/2}}{6ax^{12}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{(2cx^2+7b)\left(cx^4+bx^2+a\right)^{3/2}}{x^{12}}dx^2}{12a}-\frac{\left(a+bx^2+cx^4\right)^{5/2}}{6ax^{12}}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2}\left(-\frac{-\frac{(7b^2-4ac)\int \frac{\left(cx^4+bx^2+a\right)^{3/2}}{x^{10}}dx^2}{2a}-\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{5ax^{10}}}{12a}-\frac{\left(a+bx^2+cx^4\right)^{5/2}}{6ax^{12}}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{2}\left(-\frac{-\frac{(7b^2-4ac)\left(-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^4+bx^2+a}}{x^6}dx^2}{16a}-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8}\right)}{2a}-\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{5ax^{10}}}{12a}-\frac{\left(a+bx^2+cx^4\right)^{5/2}}{6ax^{12}}\right)$$

$$\Big\downarrow \text{1152}$$

$$\frac{1}{2}\left(-\frac{-\frac{(7b^2-4ac)\left(-\frac{3\left(b^2-4ac\right)\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{8a}-\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{16a}-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8}\right)}{2a}-\frac{7b\left(a+bx^2+cx^4\right.}{5ax^{10}}}{12a}\right.$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(-\frac{-\frac{(7b^2-4ac)\left(-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{4a}-\frac{\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{16a}-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8ax^8}\right)}{2a}-\frac{7b\left(a+bx^2+cx^{\cdot}\right.}{5ax^{10}}}{12a}\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(-\frac{-\frac{(7b^2-4ac)\left(-\frac{3(b^2-4ac)\left(\frac{(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{8a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{4ax^4}\right)}{16a}-\frac{(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{8ax^8}\right)}{2a}-\frac{7b(a+b}{5}}{12a}\right.$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```

output
```
(-1/6*(a + b*x^2 + c*x^4)^(5/2)/(a*x^12) - ((-7*b*(a + b*x^2 + c*x^4)^(5/2
))/(5*a*x^10) - ((7*b^2 - 4*a*c)*(-1/8*((2*a + b*x^2)*(a + b*x^2 + c*x^4)^
(3/2))/(a*x^8) - (3*(b^2 - 4*a*c)*(-1/4*((2*a + b*x^2)*Sqrt[a + b*x^2 + c*
x^4])/(a*x^4) + ((b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b
*x^2 + c*x^4])])/(8*a^(3/2))))/(16*a)))/(2*a))/(12*a))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\left(ac-\frac{b^2}{4}\right)^2\left(ac-\frac{7b^2}{4}\right)x^{12}\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)-\frac{52\sqrt{c\,x^4+b\,x^2+a}\left(-\frac{7\left(\frac{162}{7}c^2x^4+\frac{54}{7}bc\,x^2+b^2\right)b\,x^6a^{\frac{5}{2}}}{208}+\frac{3x^4\left(10c^2x\right.}{}\right.}{}}{32a^{\frac{9}{2}}x^{12}}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\left(-1296a^2b\,c^2x^{10}+760a\,b^3c\,x^{10}-105x^{10}b^5+480a^3c^2x^8-432a^2b^2c\,x^8+70a\,x^8b^4+288a^3bc\,x^6-56a^2x^6b^3-\right.}{15360x^{12}a^4}$
default	$\frac{15b^4c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{512a^{\frac{7}{2}}}-\frac{19b^3c\sqrt{c\,x^4+b\,x^2+a}}{384a^3x^2}+\frac{9b^2c\sqrt{c\,x^4+b\,x^2+a}}{320a^2x^4}-\frac{9b^2c^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+}}{x^2}\right.}{128a^{\frac{5}{2}}}$
elliptic	$\frac{15b^4c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{512a^{\frac{7}{2}}}-\frac{19b^3c\sqrt{c\,x^4+b\,x^2+a}}{384a^3x^2}+\frac{9b^2c\sqrt{c\,x^4+b\,x^2+a}}{320a^2x^4}-\frac{9b^2c^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+}}{x^2}\right.}{128a^{\frac{5}{2}}}$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
1/32/a^(9/2)*((a*c-1/4*b^2)^2*(a*c-7/4*b^2)*x^12*ln((2*a+b*x^2+2*a^(1/2)*(
c*x^4+b*x^2+a)^(1/2))/x^2)-52/15*(c*x^4+b*x^2+a)^(1/2)*(-7/208*(162/7*c^2*
x^4+54/7*b*c*x^2+b^2)*b*x^6*a^(5/2)+3/104*x^4*(10*c^2*x^4+6*b*c*x^2+b^2)*a
^(7/2)+35/832*(76/7*c*x^2+b)*b^3*x^8*a^(3/2)+x^2*(35/26*c*x^2+b)*a^(9/2)-1
05/1664*a^(1/2)*b^5*x^10+10/13*a^(11/2)))/x^12
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.19

$$\int\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx=\left[-\frac{15\left(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3\right)\sqrt{a}x^{12}\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx}}{a}\right.}{}\right.$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="fricas")
```

output
```
[-1/61440*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*
x^12*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^
2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*
b*c^2)*x^10 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^8 - 1664*a^5*
b*x^2 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^6 - 1280*a^6 - 16*(3*a^4*b^2 + 140*a^
5*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^12), 1/30720*(15*(7*b^6 - 60*a*b
^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((105*a*
b^5 - 760*a^2*b^3*c + 1296*a^3*b*c^2)*x^10 - 2*(35*a^2*b^4 - 216*a^3*b^2*c
 + 240*a^4*c^2)*x^8 - 1664*a^5*b*x^2 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^6 - 12
80*a^6 - 16*(3*a^4*b^2 + 140*a^5*c)*x^4)*sqrt(c*x^4 + b*x^2 + a))/(a^5*x^1
2)]
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x^{13}}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x**13,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/x**13, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. $2(190) = 380$.

Time = 0.20 (sec) , antiderivative size = 1235, normalized size of antiderivative = 5.72

$$\int \frac{(a + bx^2 + cx^4)^{3/2}}{x^{13}} dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^13,x, algorithm="giac")
```

output
```
1/1024*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*arctan(-(sqrt(c
)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/15360*(105*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^11*b^6 - 900*(sqrt(c)*x^2 - sqrt(c*
x^4 + b*x^2 + a))^11*a*b^4*c + 2160*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a)
)^11*a^2*b^2*c^2 - 960*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^11*a^3*c^3
- 595*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^9*a*b^6 + 5100*(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))^9*a^2*b^4*c - 12240*(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))^9*a^3*b^2*c^2 - 15040*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))
^9*a^4*c^3 - 76800*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^8*a^4*b*c^(5/2)
 + 1386*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^2*b^6 - 11880*(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))^7*a^3*b^4*c - 97440*(sqrt(c)*x^2 - sqrt(c*
x^4 + b*x^2 + a))^7*a^4*b^2*c^2 - 24960*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^7*a^5*c^3 - 112640*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^4*b^3
*c^(3/2) - 61440*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^6*a^5*b*c^(5/2) -
 1686*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^3*b^6 - 42600*(sqrt(c)*x
^2 - sqrt(c*x^4 + b*x^2 + a))^5*a^4*b^4*c - 128160*(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))^5*a^5*b^2*c^2 - 24960*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
 a))^5*a^6*c^3 - 15360*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^4*b^5*s
qrt(c) - 61440*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^5*b^3*c^(3/2) -
 92160*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^4*a^6*b*c^(5/2) - 595*(s...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}}\,dx=\int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^{13}}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/x^13,x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/x^13, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{13}}\,dx=\frac{-960\sqrt{c\,x^4+b\,x^2+a}\,a^3bc+400\sqrt{c\,x^4+b\,x^2+a}\,a^2b^3-1248\sqrt{c\,x^4+b\,x^2+a}\,a}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^13,x)
```

output
```
( - 960*sqrt(a + b*x**2 + c*x**4)*a**3*b*c + 400*sqrt(a + b*x**2 + c*x**4)
*a**2*b**3 - 1248*sqrt(a + b*x**2 + c*x**4)*a**2*b**2*c*x**2 - 1680*sqrt(a
 + b*x**2 + c*x**4)*a**2*b*c**2*x**4 - 160*sqrt(a + b*x**2 + c*x**4)*a**2*
c**3*x**6 + 520*sqrt(a + b*x**2 + c*x**4)*a*b**4*x**2 + 664*sqrt(a + b*x**
2 + c*x**4)*a*b**3*c*x**4 + 144*sqrt(a + b*x**2 + c*x**4)*a*b**2*c**2*x**6
 - 160*sqrt(a + b*x**2 + c*x**4)*a*b*c**3*x**8 + 320*sqrt(a + b*x**2 + c*x
**4)*a*c**4*x**10 + 15*sqrt(a + b*x**2 + c*x**4)*b**5*x**4 - 18*sqrt(a + b
*x**2 + c*x**4)*b**4*c*x**6 + 24*sqrt(a + b*x**2 + c*x**4)*b**3*c**2*x**8
- 48*sqrt(a + b*x**2 + c*x**4)*b**2*c**3*x**10 - 11520*int(sqrt(a + b*x**2
 + c*x**4)/(12*a**2*c*x**7 - 5*a*b**2*x**7 + 12*a*b*c*x**9 + 12*a*c**2*x**
11 - 5*b**3*x**9 - 5*b**2*c*x**11),x)*a**4*c**4*x**12 + 30720*int(sqrt(a +
 b*x**2 + c*x**4)/(12*a**2*c*x**7 - 5*a*b**2*x**7 + 12*a*b*c*x**9 + 12*a*c
**2*x**11 - 5*b**3*x**9 - 5*b**2*c*x**11),x)*a**3*b**2*c**3*x**12 - 21600*
int(sqrt(a + b*x**2 + c*x**4)/(12*a**2*c*x**7 - 5*a*b**2*x**7 + 12*a*b*c*x
**9 + 12*a*c**2*x**11 - 5*b**3*x**9 - 5*b**2*c*x**11),x)*a**2*b**4*c**2*x*
*12 + 5760*int(sqrt(a + b*x**2 + c*x**4)/(12*a**2*c*x**7 - 5*a*b**2*x**7 +
 12*a*b*c*x**9 + 12*a*c**2*x**11 - 5*b**3*x**9 - 5*b**2*c*x**11),x)*a*b**6
*c*x**12 - 525*int(sqrt(a + b*x**2 + c*x**4)/(12*a**2*c*x**7 - 5*a*b**2*x*
*7 + 12*a*b*c*x**9 + 12*a*c**2*x**11 - 5*b**3*x**9 - 5*b**2*c*x**11),x)*b*
*8*x**12)/(960*a*b*x**12*(12*a*c - 5*b**2))
```

3.975 $\int x^4(a+bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 493

$$\int x^4\left(a+bx^2+cx^4\right)^{3/2}\,dx=-\frac{8b(2b^2-9ac)\,(b^2-3ac)\,x\sqrt{a+bx^2+cx^4}}{1155c^{7/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{x(8b^4-21ab^2c-30a^2c^2+3bc(8b^2-31ac)\,x^2)\sqrt{a+bx^2+cx^4}}{1155c^3}$$
$$-\frac{x(3(2b^2+ac)+14bcx^2)\,(a+bx^2+cx^4)^{3/2}}{231c^2}+\frac{x(a+bx^2+cx^4)^{5/2}}{11c}$$
$$+\frac{8\sqrt[4]{a}b(2b^2-9ac)\,(b^2-3ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{1155c^{15/4}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{a}\left(\frac{8b(2b^2-9ac)\,(b^2-3ac)}{\sqrt{c}}+\sqrt{a}(8b^4-51ab^2c+60a^2c^2)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\right.\right.}{2310c^{13/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-8/1155*b*(-9*a*c+2*b^2)*(-3*a*c+b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(7/2)/(a^(
1/2)+c^(1/2)*x^2)+1/1155*x*(8*b^4-21*a*b^2*c-30*c^2*a^2+3*b*c*(-31*a*c+8*b
^2)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^3-1/231*x*(14*b*c*x^2+3*a*c+6*b^2)*(c*x^4
+b*x^2+a)^(3/2)/c^2+1/11*x*(c*x^4+b*x^2+a)^(5/2)/c+8/1155*a^(1/4)*b*(-9*a*
c+2*b^2)*(-3*a*c+b^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1
/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1
/2)/c^(1/2))^(1/2))/c^(15/4)/(c*x^4+b*x^2+a)^(1/2)-1/2310*a^(1/4)*(8*b*(-9
*a*c+2*b^2)*(-3*a*c+b^2)/c^(1/2)+a^(1/2)*(60*a^2*c^2-51*a*b^2*c+8*b^4))*(a
^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Invers
eJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(
13/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.44 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.33

$$\int x^4(a+bx^2+cx^4)^{3/2}\,dx=\frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(60a^3c^2+a^2c(-51b^2+92bcx^2+255c^2x^4)+a(8b^4-57b^3cx^2-14b^2c^2x^4+3\cdots}{\cdots}$$

input
```
Integrate[x^4*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(60*a^3*c^2 + a^2*c*(-51*b^2 + 92*b
*c*x^2 + 255*c^2*x^4) + a*(8*b^4 - 57*b^3*c*x^2 - 14*b^2*c^2*x^4 + 367*b*c
^3*x^6 + 300*c^4*x^8) + x^2*(8*b^5 + 2*b^4*c*x^2 - b^3*c^2*x^4 + 145*b^2*c
^3*x^6 + 245*b*c^4*x^8 + 105*c^5*x^10)) - (4*I)*b*(2*b^4 - 15*a*b^2*c + 27
*a^2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - S
qrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*
c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-8*b^6 + 68
*a*b^4*c - 159*a^2*b^2*c^2 + 60*a^3*c^3 + 8*b^5*Sqrt[b^2 - 4*a*c] - 60*a*b
^3*c*Sqrt[b^2 - 4*a*c] + 108*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4
*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*
a*c])])/(2310*c^4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1436, 27, 1596, 25, 1602, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1436}$$

$$\frac{x^3 (b+3cx^2) \left(a+bx^2+cx^4\right)^{3/2}}{33c}-\frac{\int 3x^2 \left(2 \left(b^2-3ac\right) x^2+ab\right) \sqrt{cx^4+bx^2+a}dx}{33c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^3 (b+3cx^2) \left(a+bx^2+cx^4\right)^{3/2}}{33c}-\frac{\int x^2 \left(2 \left(b^2-3ac\right) x^2+ab\right) \sqrt{cx^4+bx^2+a}dx}{11c}$$

$$\Big\downarrow \text{ 1596}$$

$$\frac{x^3\left(b+3cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{33c}-\frac{\frac{\int -\frac{x^2\left(\left(8b^4-51acb^2+60a^2c^2\right)x^2+2ab\left(3b^2-16ac\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{35c}+\frac{x^3\left(10cx^2\left(b^2-3ac\right)+b\left(ac+2b^2\right)\right)\sqrt{a+bx^2+cx^4}}{35c}}{11c}$$

$\downarrow$ 25

$$\frac{x^3\left(b+3cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{33c}-\frac{\frac{x^3\left(10cx^2\left(b^2-3ac\right)+b\left(ac+2b^2\right)\right)\sqrt{a+bx^2+cx^4}}{35c}-\frac{\int \frac{x^2\left(\left(8b^4-51acb^2+60a^2c^2\right)x^2+2ab\left(3b^2-16ac\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{35c}}{11c}$$

$\downarrow$ 1602

$$\frac{x^3\left(b+3cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{33c}-\frac{\frac{x^3\left(10cx^2\left(b^2-3ac\right)+b\left(ac+2b^2\right)\right)\sqrt{a+bx^2+cx^4}}{35c}-\frac{\frac{x\left(60a^2c^2-51ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{3c}-\frac{\int \frac{8b\left(2b^2-9ac\right)\left(b^2-3ac\right)x^2+a\left(8b^4-51acb^2+60a^2c^2\right)}{\sqrt{cx^4+bx^2+a}}dx}{3c}}{35c}}{11c}$$

$\downarrow$ 1511

$$\frac{x^3\left(b+3cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{33c}-\frac{\frac{x^3\left(10cx^2\left(b^2-3ac\right)+b\left(ac+2b^2\right)\right)\sqrt{a+bx^2+cx^4}}{35c}-\frac{\frac{x\left(60a^2c^2-51ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{3c}-\frac{\sqrt{a}\left(\sqrt{a}\left(60a^2c^2-51ab^2c+8b^4\right)+\frac{8b\left(2b^2-9ac\right)\left(b^2-3ac\right)}{\sqrt{c}}\right)\int}{}}{35c}}{11c}$$

$\downarrow$ 27

$$\frac{x^3\left(b+3cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{33c}-\frac{\frac{x^3\left(10cx^2\left(b^2-3ac\right)+b\left(ac+2b^2\right)\right)\sqrt{a+bx^2+cx^4}}{35c}-\frac{\frac{x\left(60a^2c^2-51ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{3c}-\frac{\sqrt{a}\left(\sqrt{a}\left(60a^2c^2-51ab^2c+8b^4\right)+\frac{8b\left(2b^2-9ac\right)\left(b^2-3ac\right)}{\sqrt{c}}\right)\int}{3}}{35c}}{11c}$$

$\downarrow$ 1416

$$\frac{x^3\left(b+3cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{33c}-\frac{\frac{x^3\left(10cx^2\left(b^2-3ac\right)+b\left(ac+2b^2\right)\right)\sqrt{a+bx^2+cx^4}}{35c}-\frac{x\left(60a^2c^2-51ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}\left(60a^2c^2-51ab^2c+8b^4\right)+\frac{8b\left(2b^2-9ac\right)\left(b^2-3ac\right)}{\sqrt{c}}\right)}{2\sqrt[4]{}}}{}}{11c}$$

$$\Big\downarrow \text{1509}$$

$$\frac{x^3\left(b+3cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{33c}-$$

$$\frac{\frac{x^3\left(10cx^2\left(b^2-3ac\right)+b\left(ac+2b^2\right)\right)\sqrt{a+bx^2+cx^4}}{35c}-\frac{\frac{x\left(60a^2c^2-51ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{3c}-\frac{\sqrt[4]{a}\left(\sqrt{a}\left(60a^2c^2-51ab^2c+8b^4\right)+\frac{8b\left(2b^2-9ac\right)\left(b^2-3ac\right)}{\sqrt{c}}\right)}{2\ldots}}{}}{}$$

input
```
Int[x^4*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^3*(b + 3*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(33*c) - ((x^3*(b*(2*b^2 + a
*c) + 10*c*(b^2 - 3*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c) - (((8*b^4 -
 51*a*b^2*c + 60*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - ((-8*b*(2*b^2
 - 9*a*c)*(b^2 - 3*a*c)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*
x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a
]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c] + (a^(1/4)*((8
*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c))/Sqrt[c] + Sqrt[a]*(8*b^4 - 51*a*b^2*c +
60*a^2*c^2))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/(3*c))/(35*c))/(11*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1436
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*((2*b*p + c*(m + 4*p - 1)*x^2
)/(c*(m + 4*p + 1)*(m + 4*p - 1))), x] - Simp[2*p*(d^2/(c*(m + 4*p + 1)*(m
+ 4*p - 1)))   Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1596
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Simp[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a +
 b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c
*e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &
& NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[
p] || IntegerQ[m])
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.37

method	result
default	$\frac{c\,x^{9}\sqrt{c\,x^{4}+b\,x^{2}+a}}{11}+\frac{4b\,x^{7}\sqrt{c\,x^{4}+b\,x^{2}+a}}{33}+\frac{\left(\frac{13ac}{11}+\frac{b^{2}}{33}\right)x^{5}\sqrt{c\,x^{4}+b\,x^{2}+a}}{7c}+\frac{\left(\frac{38ab}{33}-\frac{6\left(\frac{13ac}{11}+\frac{b^{2}}{33}\right)b}{7c}\right)x^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{5c}+$
elliptic	$\frac{c\,x^{9}\sqrt{c\,x^{4}+b\,x^{2}+a}}{11}+\frac{4b\,x^{7}\sqrt{c\,x^{4}+b\,x^{2}+a}}{33}+\frac{\left(\frac{13ac}{11}+\frac{b^{2}}{33}\right)x^{5}\sqrt{c\,x^{4}+b\,x^{2}+a}}{7c}+\frac{\left(\frac{38ab}{33}-\frac{6\left(\frac{13ac}{11}+\frac{b^{2}}{33}\right)b}{7c}\right)x^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{5c}+$
risch	$\frac{x\left(105c^{4}x^{8}+140b\,c^{3}x^{6}+195a\,c^{3}x^{4}+5b^{2}c^{2}x^{4}+32ab\,c^{2}x^{2}-6x^{2}b^{3}c+60a^{2}c^{2}-51a\,b^{2}c+8b^{4}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{1155c^{3}}-\frac{4b\left(27a^{2}c^{2}-15a\,b^{2}c+2b\right.}{}$

input
```
int(x^4*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/11*c*x^9*(c*x^4+b*x^2+a)^(1/2)+4/33*b*x^7*(c*x^4+b*x^2+a)^(1/2)+1/7*(13/
11*a*c+1/33*b^2)/c*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(38/33*a*b-6/7*(13/11*a*c
+1/33*b^2)/c*b)/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(a^2-5/7*(13/11*a*c+1/33*b
^2)/c*a-4/5*(38/33*a*b-6/7*(13/11*a*c+1/33*b^2)/c*b)/c*b)/c*x*(c*x^4+b*x^2
+a)^(1/2)-1/12*(a^2-5/7*(13/11*a*c+1/33*b^2)/c*a-4/5*(38/33*a*b-6/7*(13/11
*a*c+1/33*b^2)/c*b)/c*b)/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-
2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5*(38/3
3*a*b-6/7*(13/11*a*c+1/33*b^2)/c*b)/c*a-2/3*(a^2-5/7*(13/11*a*c+1/33*b^2)/
c*a-4/5*(38/33*a*b-6/7*(13/11*a*c+1/33*b^2)/c*b)/c*b)/c*b)*a*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+
2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2
)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.07

$$\int x^4\left(a+bx^2+cx^4\right)^{3/2}\,dx =$$

$$-\frac{8\sqrt{\frac{1}{2}}\left(\left(2\,b^5c-15\,ab^3c^2+27\,a^2bc^3\right)x\sqrt{\frac{b^2-4\,ac}{c^2}}-\left(2\,b^6-15\,ab^4c+27\,a^2b^2c^2\right)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin}{}$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2310*(8*sqrt(1/2)*((2*b^5*c - 15*a*b^3*c^2 + 27*a^2*b*c^3)*x*sqrt((b^2
- 4*a*c)/c^2) - (2*b^6 - 15*a*b^4*c + 27*a^2*b^2*c^2)*x)*sqrt(c)*sqrt((c*s
qrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b
^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*
c)/(a*c)) - sqrt(1/2)*((16*b^5*c - 60*a^2*c^4 + 3*(72*a^2*b + 17*a*b^2)*c^
3 - 8*(15*a*b^3 + b^4)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (16*b^6 + 60*a^2*b
*c^3 + 3*(72*a^2*b^2 - 17*a*b^3)*c^2 - 8*(15*a*b^4 - b^5)*c)*x)*sqrt(c)*sq
rt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*
sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2
 - 2*a*c)/(a*c)) - 2*(105*c^6*x^10 + 140*b*c^5*x^8 + 5*(b^2*c^4 + 39*a*c^5
)*x^6 - 16*b^5*c + 120*a*b^3*c^2 - 216*a^2*b*c^3 - 2*(3*b^3*c^3 - 16*a*b*c
^4)*x^4 + (8*b^4*c^2 - 51*a*b^2*c^3 + 60*a^2*c^4)*x^2)*sqrt(c*x^4 + b*x^2
+ a))/(c^5*x)
```

Sympy [F]

$$\int x^4 (a + bx^2 + cx^4)^{3/2} \, dx = \int x^4 (a + bx^2 + cx^4)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**4*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**4*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int x^4 (a + bx^2 + cx^4)^{3/2} \, dx = \int (cx^4 + bx^2 + a)^{\frac{3}{2}} x^4 \, dx$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4, x)
```

Giac [F]

$$\int x^4 (a+bx^2+cx^4)^{3/2}\,dx=\int (cx^4+bx^2+a)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a+bx^2+cx^4)^{3/2}\,dx=\int x^4\,\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

input
```
int(x^4*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^4*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int x^4 (a+bx^2+cx^4)^{3/2}\,dx=\frac{60\sqrt{c\,x^4+b\,x^2+a}\,a^2c^2x-51\sqrt{c\,x^4+b\,x^2+a}\,a\,b^2cx+32\sqrt{c\,x^4+b\,x^2+a}\,ab\,c^2x^3+195\sqrt{c}}{}$$

input
```
int(x^4*(c*x^4+b*x^2+a)^(3/2),x)
```

output

```
(60*sqrt(a + b*x**2 + c*x**4)*a**2*c**2*x - 51*sqrt(a + b*x**2 + c*x**4)*a
*b**2*c*x + 32*sqrt(a + b*x**2 + c*x**4)*a*b*c**2*x**3 + 195*sqrt(a + b*x*
*2 + c*x**4)*a*c**3*x**5 + 8*sqrt(a + b*x**2 + c*x**4)*b**4*x - 6*sqrt(a +
 b*x**2 + c*x**4)*b**3*c*x**3 + 5*sqrt(a + b*x**2 + c*x**4)*b**2*c**2*x**5
 + 140*sqrt(a + b*x**2 + c*x**4)*b*c**3*x**7 + 105*sqrt(a + b*x**2 + c*x**
4)*c**4*x**9 - 60*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a
**3*c**2 + 51*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a**2*
b**2*c - 8*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b**4 -
 216*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a**2*b*
c**2 + 120*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a
*b**3*c - 16*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)
*b**5)/(1155*c**3)
```

3.976 $\int x^2(a+bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 445

$$\int x^2(a+bx^2+cx^4)^{3/2}\,dx=\frac{(8b^4-57ab^2c+84a^2c^2)\,x\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{x(b(4b^2-9ac)+6c(2b^2-7ac)\,x^2)\sqrt{a+bx^2+cx^4}}{315c^2}+\frac{x(3b+7cx^2)\,(a+bx^2+cx^4)^{3/2}}{63c}$$
$$-\frac{\sqrt[4]{a}(8b^4-57ab^2c+84a^2c^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(4\sqrt{a}b(b^2-6ac)+\frac{8b^4-57ab^2c+84a^2c^2}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{630c^{9/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/315*(84*a^2*c^2-57*a*b^2*c+8*b^4)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a^(1/
2)+c^(1/2)*x^2)-1/315*x*(b*(-9*a*c+4*b^2)+6*c*(-7*a*c+2*b^2)*x^2)*(c*x^4+b
*x^2+a)^(1/2)/c^2+1/63*x*(7*c*x^2+3*b)*(c*x^4+b*x^2+a)^(3/2)/c-1/315*a^(1/
4)*(84*a^2*c^2-57*a*b^2*c+8*b^4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a
^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/
2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)+1/630*a^(1/4
)*(4*a^(1/2)*b*(-6*a*c+b^2)+(84*a^2*c^2-57*a*b^2*c+8*b^4)/c^(1/2))*(a^(1/2
)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(9/4)/
(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.97 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.35

$$\int x^2(a+bx^2+cx^4)^{3/2}\,dx = \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(-4b^4x^2-b^3cx^4+53b^2c^2x^6+85bc^3x^8+35c^4x^{10}+a^2c(24b+77cx^2)+a(-$$

input
```
Integrate[x^2*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b^4*x^2 - b^3*c*x^4 + 53*b^2*c^
2*x^6 + 85*b*c^3*x^8 + 35*c^4*x^10 + a^2*c*(24*b + 77*c*x^2) + a*(-4*b^3 +
 27*b^2*c*x^2 + 151*b*c^2*x^4 + 112*c^3*x^6)) + I*(8*b^4 - 57*a*b^2*c + 84
*a^2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - S
qrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*
c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^5 + 65
*a*b^3*c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57*a*b^2*c*Sqrt[b^2 -
 4*a*c] + 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(1260*c^3*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1436, 1490, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2+cx^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1436}$$

$$\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c}-\frac{\int\left(2\left(2b^2-7ac\right)x^2+ab\right)\sqrt{cx^4+bx^2+a}dx}{21c}$$

$$\Big\downarrow \text{1490}$$

$$\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c}-\frac{\frac{\int-\frac{\left(8b^4-57acb^2+84a^2c^2\right)x^2+4ab\left(b^2-6ac\right)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\frac{x\sqrt{a+bx^2+cx^4}\left(6cx^2\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)}{15c}}{21c}$$

$$\Big\downarrow \text{25}$$

$$\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c}-\frac{\frac{x\left(6cx^2\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\int\frac{\left(8b^4-57acb^2+84a^2c^2\right)x^2+4ab\left(b^2-6ac\right)}{\sqrt{cx^4+bx^2+a}}dx}{15c}}{21c}$$

$$\Big\downarrow \text{1511}$$

$$\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c}-\frac{\frac{x\left(6cx^2\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\frac{\sqrt{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(84a^2c^2-57ab^2c+8b^4\right)\int\frac{\sqrt{\ldots}}{\sqrt{a}\sqrt{\ldots}}}{\sqrt{c}}}{15c}}{21c}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c}-\frac{\frac{x\left(6cx^2\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\frac{\sqrt{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(84a^2c^2-57ab^2c+8b^4\right)\int\frac{\sqrt{a}-\sqrt{\ldots}}{\sqrt{cx^4+b\ldots}}}{\sqrt{c}}}{15c}}{21c}$$

$$\Big\downarrow \text{1416}$$

$$\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c}-\frac{\frac{x\left(6cx^2\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\ldots}{\sqrt[4]{\ldots}}\right.\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}\ldots}{15c}}{21c}$$

$$\Big\downarrow \text{ 1509}$$

$$\frac{x(3b+7cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{63c}-$$

$$\frac{\frac{x\left(6cx^2\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}}{\sqrt[4]{}}\right.\right.}{2c^{3/4}\sqrt{a+bx^2+cx^4}}}{21c}$$

input
```
Int[x^2*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(3*b + 7*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(63*c) - ((x*(b*(4*b^2 - 9*a
*c) + 6*c*(2*b^2 - 7*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) - (-(((8*b^
4 - 57*a*b^2*c + 84*a^2*c^2)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqr
t[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/
4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[a]*b*Sqrt[c]*(b^2 - 6*a*c))*(
Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/(15*c))/(21*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1436
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*((2*b*p + c*(m + 4*p - 1)*x^2
)/(c*(m + 4*p + 1)*(m + 4*p - 1))), x] - Simp[2*p*(d^2/(c*(m + 4*p + 1)*(m
+ 4*p - 1)))   Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 4.05 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.22

method	result
default	$\frac{c\,x^{7}\sqrt{c\,x^{4}+b\,x^{2}+a}}{9}+\frac{10b\,x^{5}\sqrt{c\,x^{4}+b\,x^{2}+a}}{63}+\frac{\left(\frac{11ac}{9}+\frac{b^{2}}{21}\right)x^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{5c}+\frac{\left(\frac{76ab}{63}-\frac{4\left(\frac{11ac}{9}+\frac{b^{2}}{21}\right)b}{5c}\right)x\sqrt{c\,x^{4}+b\,x^{2}+a}}{3c}-$
elliptic	$\frac{c\,x^{7}\sqrt{c\,x^{4}+b\,x^{2}+a}}{9}+\frac{10b\,x^{5}\sqrt{c\,x^{4}+b\,x^{2}+a}}{63}+\frac{\left(\frac{11ac}{9}+\frac{b^{2}}{21}\right)x^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{5c}+\frac{\left(\frac{76ab}{63}-\frac{4\left(\frac{11ac}{9}+\frac{b^{2}}{21}\right)b}{5c}\right)x\sqrt{c\,x^{4}+b\,x^{2}+a}}{3c}-$
risch	$\frac{x\left(35c^{3}x^{6}+50b\,c^{2}x^{4}+77a\,c^{2}x^{2}+3b^{2}c\,x^{2}+24abc-4b^{3}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{315c^{2}}-\frac{\left(84a^{2}c^{2}-57a\,b^{2}c+8b^{4}\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\cdots}}{\cdots}$

input

```
int(x^2*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/9*c*x^7*(c*x^4+b*x^2+a)^(1/2)+10/63*b*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(11/
9*a*c+1/21*b^2)/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(76/63*a*b-4/5*(11/9*a*c+1
/21*b^2)/c*b)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(76/63*a*b-4/5*(11/9*a*c+1/21
*b^2)/c*b)/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b
*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(a^2-3/5*(11/9*a*c+1/21*b
^2)/c*a-2/3*(76/63*a*b-4/5*(11/9*a*c+1/21*b^2)/c*b)/c*b)*a*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*
(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.06

$$\int x^2(a+bx^2+cx^4)^{3/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left((8\,b^4c-57\,ab^2c^2+84\,a^2c^3)x\sqrt{\frac{b^2-4\,ac}{c^2}}-(8\,b^5-57\,ab^3c+84\,a^2bc^2)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/630*(sqrt(1/2)*((8*b^4*c - 57*a*b^2*c^2 + 84*a^2*c^3)*x*sqrt((b^2 - 4*a*
c)/c^2) - (8*b^5 - 57*a*b^3*c + 84*a^2*b*c^2)*x)*sqrt(c)*sqrt((c*sqrt((b^2
 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a
*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)
) - sqrt(1/2)*((8*b^4*c + 12*(7*a^2 + 2*a*b)*c^3 - (57*a*b^2 + 4*b^3)*c^2)
*x*sqrt((b^2 - 4*a*c)/c^2) - (8*b^5 + 12*(7*a^2*b - 2*a*b^2)*c^2 - (57*a*b
^3 - 4*b^4)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic
_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*s
qrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(35*c^5*x^8 + 50*b*c^4*x^
6 + 8*b^4*c - 57*a*b^2*c^2 + 84*a^2*c^3 + (3*b^2*c^3 + 77*a*c^4)*x^4 - 4*(
b^3*c^2 - 6*a*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^4*x)
```

Sympy [F]

$$\int x^2(a+bx^2+cx^4)^{3/2}\,dx = \int x^2(a+bx^2+cx^4)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**2*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int x^2 (a+bx^2+cx^4)^{3/2}\,dx = \int (cx^4+bx^2+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2 (a+bx^2+cx^4)^{3/2}\,dx = \int (cx^4+bx^2+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a+bx^2+cx^4)^{3/2}\,dx = \int x^2\,(c\,x^4+b\,x^2+a)^{3/2}\,dx$$

input
```
int(x^2*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^2*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int x^2(a+bx^2+cx^4)^{3/2}\,dx = \frac{24\sqrt{c\,x^4+b\,x^2+a}\,abcx+77\sqrt{c\,x^4+b\,x^2+a}\,a\,c^2x^3-4\sqrt{c\,x^4+b\,x^2+a}\,b^3x+3\sqrt{c\,x^4+b\,x^2+a}}{}$$

input
```
int(x^2*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(24*sqrt(a + b*x**2 + c*x**4)*a*b*c*x + 77*sqrt(a + b*x**2 + c*x**4)*a*c**
2*x**3 - 4*sqrt(a + b*x**2 + c*x**4)*b**3*x + 3*sqrt(a + b*x**2 + c*x**4)*
b**2*c*x**3 + 50*sqrt(a + b*x**2 + c*x**4)*b*c**2*x**5 + 35*sqrt(a + b*x**
2 + c*x**4)*c**3*x**7 - 24*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x
**4),x)*a**2*b*c + 4*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x
)*a*b**3 + 84*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x
)*a**2*c**2 - 57*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4
),x)*a*b**2*c + 8*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**
4),x)*b**4)/(315*c**2)
```

3.977 $\int (a+bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 381

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx=-\frac{2b(b^2-8ac)\,x\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{x(b^2+10ac+3bcx^2)\sqrt{a+bx^2+cx^4}}{35c}+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}+\frac{2\sqrt[4]{a}b(b^2-8ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b(b^2-8ac)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70c^{5/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-2/35*b*(-8*a*c+b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^(1/2)+c^(1/2)*x^2)
+1/35*x*(3*b*c*x^2+10*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/c+1/7*x*(c*x^4+b*x^2+
a)^(3/2)+2/35*a^(1/4)*b*(-8*a*c+b^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a
)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/70*a^(1
/4)*(a^(1/2)*(-20*a*c+b^2)+2*b*(-8*a*c+b^2)/c^(1/2))*(a^(1/2)+c^(1/2)*x^2)
*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(
c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(5/4)/(c*x^4+b*x^2+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.70 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.40

$$\int (a+bx^2+cx^4)^{3/2}\, dx = \frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(15a^2c+a(b^2+23bcx^2+20c^2x^4)+x^2(b^3+9b^2cx^2+13bc^2x^4+5c^3x^6))-ib}{}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(15*a^2*c + a*(b^2 + 23*b*c*x^2 + 2
0*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*x^4 + 5*c^3*x^6)) - I*b*(b^
2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b -
Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 + 9*a
*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSi
nh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])])/(70*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*
x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1404, 1490, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx$$

$\Big\downarrow$ 1404

$$\frac{3}{7}\int \left(bx^2+2a\right)\sqrt{cx^4+bx^2+a}dx+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

$\Big\downarrow$ 1490

$$\frac{3}{7}\left(\frac{\int -\frac{2b(b^2-8ac)x^2+a(b^2-20ac)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\frac{x\sqrt{a+bx^2+cx^4}\left(10ac+b^2+3bcx^2\right)}{15c}\right)+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

$\Big\downarrow$ 25

$$\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\int \frac{2b(b^2-8ac)x^2+a(b^2-20ac)}{\sqrt{cx^4+bx^2+a}}dx}{15c}\right)+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

$\Big\downarrow$ 1511

$$\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\sqrt{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b(b^2-8ac)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}b(b^2-8ac)\int -}{\sqrt{}}}{15c}\right.$$
$$\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\sqrt{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b(b^2-8ac)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2b(b^2-8ac)\int \frac{\sqrt{c}}{\sqrt{cx}}}{\sqrt{c}}}{15c}\right.$$
$$\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

$\Big\downarrow$ 1416

$$\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\right.\right.}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}}{15c}\right.$$

$$\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

$\downarrow$ 1509

$$\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\right.\right.}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.$$

$$\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(a + b*x^2 + c*x^4)^(3/2))/7 + (3*((x*(b^2 + 10*a*c + 3*b*c*x^2)*Sqrt[a
 + b*x^2 + c*x^4])/(15*c) - ((-2*b*(b^2 - 8*a*c)*(-((x*Sqrt[a + b*x^2 + c*
x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*(b^2 - 20*a*c) + (2*b*(b^2 - 8*a*c))/Sqrt[c
])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/(15*c)))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.24

method	result
default	$\frac{c x^5\sqrt{c x^4+b x^2+a}}{7}+\frac{8b x^3\sqrt{c x^4+b x^2+a}}{35}+\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)x\sqrt{c x^4+b x^2+a}}{3c}+\frac{\left(a^2-\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)a}{3c}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x}{a}}}{}$
elliptic	$\frac{c x^5\sqrt{c x^4+b x^2+a}}{7}+\frac{8b x^3\sqrt{c x^4+b x^2+a}}{35}+\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)x\sqrt{c x^4+b x^2+a}}{3c}+\frac{\left(a^2-\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)a}{3c}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x}{a}}}{}$
risch	$\frac{x(5c^2x^4+8bc x^2+15ac+b^2)\sqrt{c x^4+b x^2+a}}{35c}+\frac{-\frac{b\left(8ac-b^2\right)a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(x\sqrt{2}\sqrt{}\right.\right.}{}}{\sqrt{\frac{-b+\sqrt{-4a}}{a}}}$

input
```
int((c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*c*x^5*(c*x^4+b*x^2+a)^(1/2)+8/35*b*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(9/7*
a*c+3/35*b^2)/c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(a^2-1/3*(9/7*a*c+3/35*b^2)/c*
a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2
)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(46/35*a*b-2/3*(9/7*a*c+3/35*b^2)/c*b
)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.04

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx =$$

$$-\frac{2\sqrt{\frac{1}{2}}\left((b^3c-8\,abc^2)x\sqrt{\frac{b^2-4\,ac}{c^2}}-(b^4-8\,ab^2c)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,a}{c^2}}}{2\,}\right.}{}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/70*(2*sqrt(1/2)*((b^3*c - 8*a*b*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (b^4 -
 8*a*b^2*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(
arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt
((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*b^3*c + 20*a*c^3
 - (16*a*b + b^2)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (2*b^4 - 20*a*b*c^2 - (
16*a*b^2 - b^3)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elli
ptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b
*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*(5*c^4*x^6 + 8*b*c^3*
x^4 - 2*b^3*c + 16*a*b*c^2 + (b^2*c^2 + 15*a*c^3)*x^2)*sqrt(c*x^4 + b*x^2
+ a))/(c^3*x)
```

Sympy [F]

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{15\sqrt{c\,x^4+b\,x^2+a}\,acx+\sqrt{c\,x^4+b\,x^2+a}\,b^2x+8\sqrt{c\,x^4+b\,x^2+a}\,bc\,x^3+5\sqrt{c\,x^4+b\,x^2+\cdots}}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(15*sqrt(a + b*x**2 + c*x**4)*a*c*x + sqrt(a + b*x**2 + c*x**4)*b**2*x + 8
*sqrt(a + b*x**2 + c*x**4)*b*c*x**3 + 5*sqrt(a + b*x**2 + c*x**4)*c**2*x**
5 + 20*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a**2*c - int
(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b**2 + 16*int((sqrt(
a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a*b*c - 2*int((sqrt(a
+ b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*b**3)/(35*c)
```

3.978 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 361

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{(b^2+12ac)\,x\sqrt{a+bx^2+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{5}x(7b+6cx^2)\sqrt{a+bx^2+cx^4} - \frac{(a+bx^2+cx^4)^{3/2}}{x} - \frac{\sqrt[4]{a}(b^2+12ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{5c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}(b^2+8\sqrt{a}b\sqrt{c}+12ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/5*(12*a*c+b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)+1/5
*x*(6*c*x^2+7*b)*(c*x^4+b*x^2+a)^(1/2)-(c*x^4+b*x^2+a)^(3/2)/x-1/5*a^(1/4)
*(12*a*c+b^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)
^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1
/2))^(1/2))/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/10*a^(1/4)*(b^2+8*a^(1/2)*b*c^
(1/2)+12*a*c)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)
^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(
1/2))^(1/2))/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.83 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(-5a^2-3abx^2+2b^2x^4-4acx^4+3bcx^6+c^2x^8)+i(b^2+12ac)\,(}{}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^2,x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-5*a^2 - 3*a*b*x^2 + 2*b^2*x^4 - 4*a
*c*x^4 + 3*b*c*x^6 + c^2*x^8) + I*(b^2 + 12*a*c)*(-b + Sqrt[b^2 - 4*a*c])*
x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*
b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/
(b - Sqrt[b^2 - 4*a*c])] - I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 12*
a*c*Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(20*c*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1437, 1490, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{1437}$$

$$3\int \left(2cx^2+b\right)\sqrt{cx^4+bx^2+a}dx-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}$$

$\big\downarrow$ 1490

$$3\left(\frac{\int \frac{c\left(\left(b^2+12ac\right)x^2+8ab\right)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\frac{1}{15}x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}$$

$\big\downarrow$ 27

$$3\left(\frac{1}{15}\int \frac{\left(b^2+12ac\right)x^2+8ab}{\sqrt{cx^4+bx^2+a}}dx+\frac{1}{15}x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}$$

$\big\downarrow$ 1511

$$3\left(\frac{1}{15}\left(\frac{\sqrt{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(12ac+b^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)+\frac{1}{15}x\left(7b+6cx^2\right)\sqrt{a\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}$$

$\big\downarrow$ 27

$$3\left(\frac{1}{15}\left(\frac{\sqrt{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(12ac+b^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)+\frac{1}{15}x\left(7b+6cx^2\right)\sqrt{a+bx\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}$$

$\big\downarrow$ 1416

$$3\left(\frac{1}{15}\left(\frac{\sqrt[4]{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\right.\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}$$

$\big\downarrow$ 1509

$$3\left(\frac{1}{15}\left(\frac{\sqrt[4]{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\right.\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/x^2,x]
```

output
```
-((a + b*x^2 + c*x^4)^(3/2)/x) + 3*((x*(7*b + 6*c*x^2)*Sqrt[a + b*x^2 + c*
x^4])/15 + (-(((b^2 + 12*a*c)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sq
rt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1
/4)*(b^2 + 8*Sqrt[a]*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]
))/15)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1437
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Simp[2*(p/(
d^2*(m + 1)))   Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\sqrt{c x^4+b x^2+a}\,(-c x^4-2b x^2+5a)}{5x}-\frac{(12ac+b^2)a\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\sqrt{4+\frac{2(b+\sqrt{-4ac+b^2})x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{-b}}{}\right.\right.}{10\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}$
default	$-\frac{a\sqrt{c x^4+b x^2+a}}{x}+\frac{c x^3\sqrt{c x^4+b x^2+a}}{5}+\frac{2bx\sqrt{c x^4+b x^2+a}}{5}+\frac{2ab\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\sqrt{4+\frac{2(b+\sqrt{-4ac+b^2})x^2}{a}}\,\text{El}}{5\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}$
elliptic	$-\frac{a\sqrt{c x^4+b x^2+a}}{x}+\frac{c x^3\sqrt{c x^4+b x^2+a}}{5}+\frac{2bx\sqrt{c x^4+b x^2+a}}{5}+\frac{2ab\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\sqrt{4+\frac{2(b+\sqrt{-4ac+b^2})x^2}{a}}\,\text{El}}{5\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(c*x^4+b*x^2+a)^(1/2)*(-c*x^4-2*b*x^2+5*a)/x-1/10*(12*a*c+b^2)*a*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^
(1/2)))+2/5*a*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+
b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x**2,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx=\int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx=\int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^2}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/x^2,x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx=\frac{7\sqrt{c\,x^4+b\,x^2+a}\,ac+\sqrt{c\,x^4+b\,x^2+a}\,b^2+2\sqrt{c\,x^4+b\,x^2+a}\,bc\,x^2+\sqrt{c\,x^4+}}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^2,x)
```

output
```
(7*sqrt(a + b*x**2 + c*x**4)*a*c + sqrt(a + b*x**2 + c*x**4)*b**2 + 2*sqrt
(a + b*x**2 + c*x**4)*b*c*x**2 + sqrt(a + b*x**2 + c*x**4)*c**2*x**4 + 12*
int(sqrt(a + b*x**2 + c*x**4)/(a*x**2 + b*x**4 + c*x**6),x)*a**2*c*x + int
(sqrt(a + b*x**2 + c*x**4)/(a*x**2 + b*x**4 + c*x**6),x)*a*b**2*x + 8*int(
sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b*c*x)/(5*c*x)
```

3.979 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 353

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4} dx = \frac{8b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3(\sqrt{a}+\sqrt{c}x^2)} - \frac{(3b-2cx^2)\sqrt{a+bx^2+cx^4}}{3x} - \frac{(a+bx^2+cx^4)^{3/2}}{3x^3} - \frac{8\sqrt[4]{a}b\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3\sqrt{a+bx^2+cx^4}} + \frac{(3b^2+8\sqrt{a}b\sqrt{c}+4ac)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
8*b*c^(1/2)*x*(c*x^4+b*x^2+a)^(1/2)/(3*a^(1/2)+3*c^(1/2)*x^2)-1/3*(-2*c*x^
2+3*b)*(c*x^4+b*x^2+a)^(1/2)/x-1/3*(c*x^4+b*x^2+a)^(3/2)/x^3-8/3*a^(1/4)*b
*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(
1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^
(1/2))/(c*x^4+b*x^2+a)^(1/2)+1/6*(3*b^2+8*a^(1/2)*b*c^(1/2)+4*a*c)*(a^(1/2
)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/
c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.61 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4}\,dx = \frac{2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(-a^2-5abx^2-4b^2x^4-3bcx^6+c^2x^8)+4ib\left(-b+\sqrt{b^2-4ac}\right)x^3}{}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^4,x]
```

output
```
(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-a^2 - 5*a*b*x^2 - 4*b^2*x^4 - 3*b*c*x
^6 + c^2*x^8) + (4*I)*b*(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c]
 + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
 + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
] - I*(-b^2 + 4*a*c + 4*b*Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(
6*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1437, 1594, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4}\,dx$$

$$\Big\downarrow \text{1437}$$

$$\int \frac{\left(2cx^2+b\right)\sqrt{cx^4+bx^2+a}}{x^2}dx-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{1594}$$

$$-\frac{1}{3}\int -\frac{3b^2+8cx^2b+4ac}{\sqrt{cx^4+bx^2+a}}dx-\frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{3x}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{3b^2+8cx^2b+4ac}{\sqrt{cx^4+bx^2+a}}dx-\frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{3x}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{3}\left(\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-8\sqrt{a}b\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx\right)-\frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{3x}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-8b\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx\right)-\frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{3x}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{3}\left(\frac{\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-8b\sqrt{c}\int \right.$$
$$\frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{3x}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{3}\left(\frac{\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-8b\sqrt{c}\left(\right.\right.$$
$$\frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{3x}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3}$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/x^4,x]
```

output
```
-1/3*((3*b - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/x - (a + b*x^2 + c*x^4)^(3/
2)/(3*x^3) + (-8*b*Sqrt[c]*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[
c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt
[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqr
t[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])) + ((3*b^2 + 8*Sqrt[a
]*b*Sqrt[c] + 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1437
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Simp[2*(p/(
d^2*(m + 1)))   Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.21

method	result
default	$-\frac{a\sqrt{c x^4+b x^2+a}}{3x^3}-\frac{4b\sqrt{c x^4+b x^2+a}}{3x}+\frac{cx\sqrt{c x^4+b x^2+a}}{3}+\frac{(\frac{4ac}{3}+b^2)\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\sqrt{4+\frac{2(b+\sqrt{-4ac+b^2})x^2}{a}}}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}$
elliptic	$-\frac{a\sqrt{c x^4+b x^2+a}}{3x^3}-\frac{4b\sqrt{c x^4+b x^2+a}}{3x}+\frac{cx\sqrt{c x^4+b x^2+a}}{3}+\frac{(\frac{4ac}{3}+b^2)\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\sqrt{4+\frac{2(b+\sqrt{-4ac+b^2})x^2}{a}}}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}$
risch	$-\frac{\sqrt{c x^4+b x^2+a}\,(-c x^4+4b x^2+a)}{3x^3}+\frac{b^2\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\sqrt{4+\frac{2(b+\sqrt{-4ac+b^2})x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4+b x^2+a}}$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3*(c*x^4+b*x^2+a)^(1/2)-4/3*b*(c*x^4+b*x^2+a)^(1/2)/x+1/3*c*x*(c*
x^4+b*x^2+a)^(1/2)+1/4*(4/3*a*c+b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-4/3*b*c
*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4}\,dx=\int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x**4,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4}\,dx=\int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^4}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/x^4,x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4}\,dx=\frac{-5\sqrt{c\,x^4+b\,x^2+a}\,a+4\sqrt{c\,x^4+b\,x^2+a}\,b\,x^2+\sqrt{c\,x^4+b\,x^2+a}\,c\,x^4-12\left(\int \right.}{3x^3}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^4,x)
```

output
```
( - 5*sqrt(a + b*x**2 + c*x**4)*a + 4*sqrt(a + b*x**2 + c*x**4)*b*x**2 + s
qrt(a + b*x**2 + c*x**4)*c*x**4 - 12*int(sqrt(a + b*x**2 + c*x**4)/(a*x**4
 + b*x**6 + c*x**8),x)*a**2*x**3 + 3*int(sqrt(a + b*x**2 + c*x**4)/(a + b*
x**2 + c*x**4),x)*b**2*x**3)/(3*x**3)
```

3.980 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 365

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^6}\,dx = -\frac{\left(b^2+12ac\right)\sqrt{a+bx^2+cx^4}}{5\sqrt{a}x\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{5x^3} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5} - \frac{\sqrt[4]{c}\left(b^2+12ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{5a^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{c}\left(b^2+8\sqrt{a}b\sqrt{c}+12ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10a^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/5*(12*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)/x/(a^(1/2)+c^(1/2)*x^2)-1/
5*(-6*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/x^3-1/5*(c*x^4+b*x^2+a)^(3/2)/x^5-1/5
*c^(1/4)*(12*a*c+b^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1
/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1
/2)/c^(1/2))^(1/2))/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/10*c^(1/4)*(b^2+8*a^(1
/2)*b*c^(1/2)+12*a*c)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1
/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(
1/2)/c^(1/2))^(1/2))/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.88 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^6}\,dx = \frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a^3+b^2x^6(b+cx^2)+a^2(3bx^2+8cx^4)+a(3b^2x^4+9bcx^6+7c^2x$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^6,x]
```

output
```
(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a^3 + b^2*x^6*(b + c*x^2) + a^2*(3*b*
x^2 + 8*c*x^4) + a*(3*b^2*x^4 + 9*b*c*x^6 + 7*c^2*x^8)) + I*(b^2 + 12*a*c)
*(-b + Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*
x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^3 + 4*a*b*c +
 b^2*Sqrt[b^2 - 4*a*c] + 12*a*c*Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*
c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c
])])/(20*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^5*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1437, 1594, 25, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^6}\,dx$$

$$\Big\downarrow \text{ 1437}$$

$$\frac{3}{5}\int \frac{\left(2cx^2+b\right)\sqrt{cx^4+bx^2+a}}{x^4}dx-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{1594}$$

$$\frac{3}{5}\left(-\frac{1}{3}\int -\frac{b^2+8cx^2b+12ac}{x^2\sqrt{cx^4+bx^2+a}}dx-\frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{3x^3}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{5}\left(\frac{1}{3}\int \frac{b^2+8cx^2b+12ac}{x^2\sqrt{cx^4+bx^2+a}}dx-\frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{3x^3}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{1604}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(-\frac{\int -\frac{c\left(\left(b^2+12ac\right)x^2+8ab\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\left(12ac+b^2\right)\sqrt{a+bx^2+cx^4}}{ax}\right)-\frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{3x^3}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{\int \frac{c\left(\left(b^2+12ac\right)x^2+8ab\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\left(12ac+b^2\right)\sqrt{a+bx^2+cx^4}}{ax}\right)-\frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{3x^3}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{c\int \frac{\left(b^2+12ac\right)x^2+8ab}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\left(12ac+b^2\right)\sqrt{a+bx^2+cx^4}}{ax}\right)-\frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{3x^3}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{1511}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{c\left(\frac{\sqrt{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\left(12ac+b^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\left(12ac+b^2\right)\sqrt{a+bx^2+cx^4}}{ax}\right)-$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}$$

$\downarrow$ 27

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{c\left(\frac{\sqrt{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(12ac+b^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\left(12ac+b^2\right)\sqrt{a+bx^2+cx^4}}{ax}\right)-\frac{(b-$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}$$

$\downarrow$ 1416

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{c\left(\frac{\sqrt[4]{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(12ac+b^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}d}{\sqrt{c}}\right.}{a}\right.\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5}$$

$\downarrow$ 1509

$$\frac{3}{5}\left(\frac{1}{3}\left(\frac{c\left(\frac{\sqrt[4]{a}(8\sqrt{a}b\sqrt{c}+12ac+b^2)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{(12ac+b^2)\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x\right.}{}}{}\right.}{a}\right.\right.$$

$$\frac{(a+bx^2+cx^4)^{3/2}}{5x^5}$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/x^6,x]
```

output
```
-1/5*(a + b*x^2 + c*x^4)^(3/2)/x^5 + (3*(-1/3*((b - 6*c*x^2)*Sqrt[a + b*x^
2 + c*x^4])/x^3 + (-(((b^2 + 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a*x)) + (c*
(-(((b^2 + 12*a*c)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2))
 + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqr
t[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(b^2 +
8*Sqrt[a]*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])))/a)/3))/
5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1437
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Simp[2*(p/(
d^2*(m + 1)))   Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(7ac\,x^4+b^2x^4+2ab\,x^2+a^2)}{5x^5a}+\frac{c\left(-\frac{(12ac+b^2)a\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\sqrt{4+\frac{2(b+\sqrt{-4ac+b^2})x^2}{a}}\left(\text{EllipticF}\left(\ldots\right.\right.}{2\sqrt{\frac{-b+\ldots}{}}}\right.}{\ldots}$
default	$-\frac{a\sqrt{c\,x^4+b\,x^2+a}}{5x^5}-\frac{2b\sqrt{c\,x^4+b\,x^2+a}}{5x^3}-\frac{(7ac+b^2)\sqrt{c\,x^4+b\,x^2+a}}{5ax}+\frac{2bc\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\sqrt{4+\frac{2(b+\sqrt{-4ac+b^2})x^2}{a}}\ldots}{5\sqrt{\frac{-b+\sqrt{-4ac\ldots}}{a}}\ldots}$
elliptic	$-\frac{a\sqrt{c\,x^4+b\,x^2+a}}{5x^5}-\frac{2b\sqrt{c\,x^4+b\,x^2+a}}{5x^3}-\frac{(7ac+b^2)\sqrt{c\,x^4+b\,x^2+a}}{5ax}+\frac{2bc\sqrt{2}\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\sqrt{4+\frac{2(b+\sqrt{-4ac+b^2})x^2}{a}}\ldots}{5\sqrt{\frac{-b+\sqrt{-4ac\ldots}}{a}}\ldots}$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(c*x^4+b*x^2+a)^(1/2)*(7*a*c*x^4+b^2*x^4+2*a*b*x^2+a^2)/x^5/a+1/5*c/a
*(-1/2*(12*a*c+b^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+2*a*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^6}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^6}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x**6,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^6}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^6}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^6}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^6}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/x^6,x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^6}\,dx = \frac{-\sqrt{c\,x^4+b\,x^2+a}\,a^2-10\sqrt{c\,x^4+b\,x^2+a}\,ab\,x^2-7\sqrt{c\,x^4+b\,x^2+a}\,ac\,x^4+15}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^6,x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4)*a**2 - 10*sqrt(a + b*x**2 + c*x**4)*a*b*x**2
 - 7*sqrt(a + b*x**2 + c*x**4)*a*c*x**4 + 15*sqrt(a + b*x**2 + c*x**4)*b**
2*x**4 - 24*int(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)*a*
*2*b*x**5 + 12*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),
x)*a*c**2*x**5 - 15*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x
**4),x)*b**2*c*x**5)/(5*a*x**5)
```

3.981 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 411

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^8}\,dx = -\frac{(b^2-20ac)\sqrt{a+bx^2+cx^4}}{35ax^3} + \frac{2b(b^2-8ac)\sqrt{a+bx^2+cx^4}}{35a^{3/2}x\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{3(b+10cx^2)\sqrt{a+bx^2+cx^4}}{35x^5} - \frac{(a+bx^2+cx^4)^{3/2}}{7x^7} + \frac{2b\sqrt[4]{c}(b^2-8ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35a^{7/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{c}\left(\sqrt{c}(b^2-20ac)+\frac{2b(b^2-8ac)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70a^{5/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/35*(-20*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/a/x^3+2/35*b*(-8*a*c+b^2)*(c*x^4
+b*x^2+a)^(1/2)/a^(3/2)/x/(a^(1/2)+c^(1/2)*x^2)-3/35*(10*c*x^2+b)*(c*x^4+b
*x^2+a)^(1/2)/x^5-1/7*(c*x^4+b*x^2+a)^(3/2)/x^7+2/35*b*c^(1/4)*(-8*a*c+b^2
)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*El
lipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/
a^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/70*c^(1/4)*(c^(1/2)*(-20*a*c+b^2)+2*b*(-8*
a*c+b^2)/a^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*
x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)
/c^(1/2))^(1/2))/a^(5/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.07 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^8}\,dx = \frac{-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(5a^4-2b^3x^8(b+cx^2)+a^3(13bx^2+20cx^4)+abx^6(-b^2+17bcx$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^8,x]
```

output
```
(-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(5*a^4 - 2*b^3*x^8*(b + c*x^2) + a^3*(
13*b*x^2 + 20*c*x^4) + a*b*x^6*(-b^2 + 17*b*c*x^2 + 16*c^2*x^4) + 3*a^2*(3
*b^2*x^4 + 13*b*c*x^6 + 5*c^2*x^8)) - I*b*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4
*a*c])*x^7*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellip
ticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 + 9*a*b^2*c - 20*a^2*c^2 + b^3
*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*x^7*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c]
+ 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
)/(70*a^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^7*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1437, 1594, 1604, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^8}\,dx$$

$$\Big\downarrow \; 1437$$

$$\frac{3}{7}\int \frac{(2cx^2+b)\sqrt{cx^4+bx^2+a}}{x^6}dx-\frac{(a+bx^2+cx^4)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{1594}$$

$$\frac{3}{7}\left(\frac{1}{5}\int \frac{b^2-8cx^2b-20ac}{x^4\sqrt{cx^4+bx^2+a}}dx-\frac{(b+10cx^2)\sqrt{a+bx^2+cx^4}}{5x^5}\right)-\frac{(a+bx^2+cx^4)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{1604}$$

$$\frac{3}{7}\left(\frac{1}{5}\left(-\frac{\int \frac{c(b^2-20ac)x^2+2b(b^2-8ac)}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a}-\frac{(b^2-20ac)\sqrt{a+bx^2+cx^4}}{3ax^3}\right)-\frac{(b+10cx^2)\sqrt{a+bx^2+cx^4}}{5x^5}\right)-\frac{(a+bx^2+cx^4)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{1604}$$

$$\frac{3}{7}\left(\frac{1}{5}\left(-\frac{-\frac{\int -\frac{c\left(2b\left(b^2-8ac\right)x^2+a\left(b^2-20ac\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{2b(b^2-8ac)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{(b^2-20ac)\sqrt{a+bx^2+cx^4}}{3ax^3}\right)-\frac{(b+10cx^2)\sqrt{a+bx^2+cx^4}}{5x^5}\right)-\frac{(a+bx^2+cx^4)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{7}\left(\frac{1}{5}\left(-\frac{\frac{\int \frac{c\left(2b\left(b^2-8ac\right)x^2+a\left(b^2-20ac\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{2b(b^2-8ac)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{(b^2-20ac)\sqrt{a+bx^2+cx^4}}{3ax^3}\right)-\frac{(b+10cx^2)\sqrt{a+bx^2+cx^4}}{5x^5}\right)-\frac{(a+bx^2+cx^4)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{7}\left(\frac{1}{5}\left(-\frac{\frac{c\int \frac{2b\left(b^2-8ac\right)x^2+a\left(b^2-20ac\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{2b(b^2-8ac)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{(b^2-20ac)\sqrt{a+bx^2+cx^4}}{3ax^3}\right)-\frac{(b+10cx^2)\sqrt{a+bx^2+cx^4}}{5x^5}\right)-\frac{(a+bx^2+cx^4)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{1511}$$

$$\frac{3}{7}\left(\frac{1}{5}\left(-\frac{\frac{c\left(\sqrt{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}b\left(b^2-8ac\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2b(b^2-8ac)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{(b^2}{}\right.\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7}$$

$\downarrow$ 27

$$\frac{3}{7}\left(\frac{1}{5}\left(-\frac{\frac{c\left(\sqrt{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2b\left(b^2-8ac\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2b(b^2-8ac)\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{(b^2-2}{}\right.\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7}$$

$\downarrow$ 1416

$$\frac{3}{7}\left(\frac{1}{5}\left(-\frac{\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2b\left(b^2-8ac\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2}}}{\sqrt{c}}\right.}{a}}{3a}\right.\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7}$$

$\downarrow$ 1509

$$\frac{3}{7}\left(\frac{1}{5}\left(-\frac{\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2b\left(b^2-8ac\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\cdots\right.}{\cdots}\right.}{\cdots}\right.}{a}}{3a}\right.\right.$$

$$\frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7}$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/x^8,x]
```

output
```
-1/7*(a + b*x^2 + c*x^4)^(3/2)/x^7 + (3*(-1/5*((b + 10*c*x^2)*Sqrt[a + b*x
^2 + c*x^4])/x^5 + (-1/3*((b^2 - 20*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a*x^3)
- ((-2*b*(b^2 - 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a*x) + (c*((-2*b*(b^2 - 8
*a*c)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(
Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c
^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*(b^2 - 20*a*
c) + (2*b*(b^2 - 8*a*c))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/a)/
(3*a))/5))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1437
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^p/(d*(m + 1))), x] - Simp[2*(p/(
d^2*(m + 1)))   Int[(d*x)^(m + 2)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p - 1)
, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && L
tQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1594
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[2*(p/(f^2
*(m + 1)*(m + 4*p + 3)))   Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Si
mp[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && G
tQ[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
 || IntegerQ[m])
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.20

method	result
default	$-\frac{a\sqrt{c\,x^{4}+b\,x^{2}+a}}{7x^{7}}-\frac{8b\sqrt{c\,x^{4}+b\,x^{2}+a}}{35x^{5}}-\frac{\left(15ac+b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{35a\,x^{3}}-\frac{2b\left(8ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{35a^{2}x}+\frac{\left(c^{2}-\frac{c\left(15ac+b^{2}\right)}{35a}\right)\sqrt{2}\,\sqrt{\cdots}}{\cdots}$
elliptic	$-\frac{a\sqrt{c\,x^{4}+b\,x^{2}+a}}{7x^{7}}-\frac{8b\sqrt{c\,x^{4}+b\,x^{2}+a}}{35x^{5}}-\frac{\left(15ac+b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{35a\,x^{3}}-\frac{2b\left(8ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{35a^{2}x}+\frac{\left(c^{2}-\frac{c\left(15ac+b^{2}\right)}{35a}\right)\sqrt{2}\,\sqrt{\cdots}}{\cdots}$
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(16abc\,x^{6}-2b^{3}x^{6}+15a^{2}c\,x^{4}+b^{2}x^{4}a+8a^{2}b\,x^{2}+5a^{3}\right)}{35x^{7}a^{2}}+\frac{c\left(-\frac{b\left(8ac-b^{2}\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{\cdots}\right)\cdots}{\cdots}}}{\cdots}\cdots\right.}{\cdots}$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a*(c*x^4+b*x^2+a)^(1/2)/x^7-8/35*b*(c*x^4+b*x^2+a)^(1/2)/x^5-1/35*(15
*a*c+b^2)/a*(c*x^4+b*x^2+a)^(1/2)/x^3-2/35*b*(8*a*c-b^2)/a^2*(c*x^4+b*x^2+
a)^(1/2)/x+1/4*(c^2-1/35*c*(15*a*c+b^2)/a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1
/35*b*c*(8*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^8}\,dx = \frac{2\sqrt{\frac{1}{2}}\left((ab^3-8\,a^2bc)x^7\sqrt{\frac{b^2-4\,ac}{a^2}}-(b^4-8\,ab^2c)x^7\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E(\arcsin\Bigg(}{}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
1/70*(2*sqrt(1/2)*((a*b^3 - 8*a^2*b*c)*x^7*sqrt((b^2 - 4*a*c)/a^2) - (b^4
- 8*a*b^2*c)*x^7)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic
_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*s
qrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((a^2*b^2 - 2*a*b
^3 - 4*(5*a^3 - 4*a^2*b)*c)*x^7*sqrt((b^2 - 4*a*c)/a^2) + (a*b^3 + 2*b^4 -
 4*(5*a^2*b + 4*a*b^2)*c)*x^7)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b
)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)
), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + 2*(2*(a*b^3 -
8*a^2*b*c)*x^6 - 8*a^3*b*x^2 - (a^2*b^2 + 15*a^3*c)*x^4 - 5*a^4)*sqrt(c*x^
4 + b*x^2 + a))/(a^3*x^7)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/x**8,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/x**8, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/x^8,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{x^8}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/x^8,x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/x^8, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^8}\,dx = \frac{-\sqrt{c\,x^4+b\,x^2+a}\,a-7\sqrt{c\,x^4+b\,x^2+a}\,c\,x^4+8\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^{10}+b\,x^8+a\,x^6}dx\right)ab\,x^7-12}{7x^7}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/x^8,x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4)*a - 7*sqrt(a + b*x**2 + c*x**4)*c*x**4 + 8*i
nt(sqrt(a + b*x**2 + c*x**4)/(a*x**6 + b*x**8 + c*x**10),x)*a*b*x**7 - 12*
int(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)*a*c*x**7 + 7*i
nt(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)*b**2*x**7)/(7*x
**7)
```

3.982 $\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 121

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{x^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{(15b^2-16ac-10bcx^2)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{b(5b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}$$

output
```
1/6*x^4*(c*x^4+b*x^2+a)^(1/2)/c+1/48*(-10*b*c*x^2-16*a*c+15*b^2)*(c*x^4+b*
x^2+a)^(1/2)/c^3-1/32*b*(-12*a*c+5*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c
*x^4+b*x^2+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a+bx^2+cx^4}(15b^2-16ac-10bcx^2+8c^2x^4)}{48c^3} + \frac{(5b^3-12abc)\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{32c^{7/2}}$$

input
```
Integrate[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(15*b^2 - 16*a*c - 10*b*c*x^2 + 8*c^2*x^4))/(48*c
^3) + ((5*b^3 - 12*a*b*c)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x
^4]])/(32*c^(7/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1434, 1166, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^6}{\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{2}\left(\frac{\int -\frac{x^2(5bx^2+4a)}{2\sqrt{cx^4+bx^2+a}}dx^2}{3c}+\frac{x^4\sqrt{a+bx^2+cx^4}}{3c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{x^4\sqrt{a+bx^2+cx^4}}{3c}-\frac{\int \frac{x^2(5bx^2+4a)}{\sqrt{cx^4+bx^2+a}}dx^2}{6c}\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{2}\left(\frac{x^4\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{3b(5b^2-12ac)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c^2}-\frac{(-16ac+15b^2-10bcx^2)\sqrt{a+bx^2+cx^4}}{4c^2}}{6c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{x^4\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{3b(5b^2-12ac)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c^2}-\frac{(-16ac+15b^2-10bcx^2)\sqrt{a+bx^2+cx^4}}{4c^2}}{6c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^4\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{3b(5b^2-12ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{(-16ac+15b^2-10bcx^2)\sqrt{a+bx^2+cx^4}}{4c^2}}{6c}\right)$$

input
```
Int[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((x^4*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (-1/4*((15*b^2 - 16*a*c - 10*b*c*x^
2)*Sqrt[a + b*x^2 + c*x^4])/c^2 + (3*b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(5/2)))/(6*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{\left(-8c^{2}x^{4}+10bc\,x^{2}+16ac-15b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{48c^{3}}+\frac{b\left(12ac-5b^{2}\right)\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{7}{2}}}$
default	$\frac{x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{6c}-\frac{5b\,x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{24c^{2}}+\frac{5b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16c^{3}}-\frac{5b^{3}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{7}{2}}}+\frac{3ba\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}\right.}{}$
elliptic	$\frac{x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}}{6c}-\frac{5b\,x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{24c^{2}}+\frac{5b^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}{16c^{3}}-\frac{5b^{3}\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{32c^{\frac{7}{2}}}+\frac{3ba\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}\right.}{}$
pseudoelliptic	$\frac{16c^{\frac{5}{2}}x^{4}\sqrt{c\,x^{4}+b\,x^{2}+a}-20b\,c^{\frac{3}{2}}x^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}-36\ln\left(2\right)abc+15\ln\left(2\right)b^{3}+36\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)abc-15\ln\left(\right.}{96c^{\frac{7}{2}}}$

input
```
int(x^7/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-8*c^2*x^4+10*b*c*x^2+16*a*c-15*b^2)*(c*x^4+b*x^2+a)^(1/2)/c^3+1/32
*b*(12*a*c-5*b^2)/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.99

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[-\frac{3\,(5\,b^3-12\,abc)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)-4\,(8\,c^3x^4-}{192\,c^4}\right.$$

input
```
integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4
*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^4 - 1
0*b*c^2*x^2 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/96*(3*(
5*b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*c^3*x^4 - 10*b*c^2*x^2 + 15*b
^2*c - 16*a*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^4]
```

Sympy [F]

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**7/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{48}\sqrt{cx^4+bx^2+a}\left(2\,x^2\left(\frac{4\,x^2}{c}-\frac{5\,b}{c^2}\right)+\frac{15\,b^2-16\,ac}{c^3}\right) + \frac{(5\,b^3-12\,abc)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{32\,c^{\frac{7}{2}}}$$

input
```
integrate(x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*sqrt(c*x^4 + b*x^2 + a)*(2*x^2*(4*x^2/c - 5*b/c^2) + (15*b^2 - 16*a*c
)/c^3) + 1/32*(5*b^3 - 12*a*b*c)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x
^2 + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^7}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(x^7/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^7/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1548, normalized size of antiderivative = 12.79

$$\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(288*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) +
b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b*c**3 + 96*sqrt(a + b*x**2 + c*x**
4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b
**2))*a*b**3*c**2 + 1152*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a +
 b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b**2*c**3*x**2 + 1
152*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b
 + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b*c**4*x**4 - 90*sqrt(a + b*x**2 + c*x*
*4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c -
b**2))*b**5*c - 480*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x*
*2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**4*c**2*x**2 - 480*sqrt
(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x
**2)/sqrt(4*a*c - b**2))*b**3*c**3*x**4 - 384*sqrt(a + b*x**2 + c*x**4)*a*
*2*b*c**3 - 768*sqrt(a + b*x**2 + c*x**4)*a**2*c**4*x**2 + 328*sqrt(a + b*
x**2 + c*x**4)*a*b**3*c**2 - 96*sqrt(a + b*x**2 + c*x**4)*a*b**2*c**3*x**2
 - 1824*sqrt(a + b*x**2 + c*x**4)*a*b*c**4*x**4 - 640*sqrt(a + b*x**2 + c*
x**4)*a*c**5*x**6 + 30*sqrt(a + b*x**2 + c*x**4)*b**5*c + 520*sqrt(a + b*x
**2 + c*x**4)*b**4*c**2*x**2 + 1096*sqrt(a + b*x**2 + c*x**4)*b**3*c**3*x*
*4 + 288*sqrt(a + b*x**2 + c*x**4)*b**2*c**4*x**6 + 128*sqrt(a + b*x**2 +
c*x**4)*b*c**5*x**8 + 512*sqrt(a + b*x**2 + c*x**4)*c**6*x**10 + 432*sqrt(
c)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c ...
```

3.983 $\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{a+bx^2+cx^4}}{4c}+\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}$$

output
```
-3/8*b*(c*x^4+b*x^2+a)^(1/2)/c^2+1/4*x^2*(c*x^4+b*x^2+a)^(1/2)/c+1/16*(-4*
a*c+3*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{(-3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{(-3b^2+4ac)\log\left(bc^2+2c^3x^2-2c^{5/2}\sqrt{a+bx^2+cx^4}\right)}{16c^{5/2}}$$

input
```
Integrate[x^5/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((-3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + ((-3*b^2 + 4*a*c)*Log
[b*c^2 + 2*c^3*x^2 - 2*c^(5/2)*Sqrt[a + b*x^2 + c*x^4]])/(16*c^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1434, 1166, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{2}\left(\frac{\int -\frac{3bx^2+2a}{2\sqrt{cx^4+bx^2+a}}dx^2}{2c}+\frac{x^2\sqrt{a+bx^2+cx^4}}{2c}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{a+bx^2+cx^4}}{2c}-\frac{\int \frac{3bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx^2}{4c}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{a+bx^2+cx^4}}{2c}-\frac{\frac{3b\sqrt{a+bx^2+cx^4}}{c}-\frac{(3b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{2c}}{4c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{a+bx^2+cx^4}}{2c}-\frac{\frac{3b\sqrt{a+bx^2+cx^4}}{c}-\frac{(3b^2-4ac)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{c}}{4c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{a+bx^2+cx^4}}{2c}-\frac{\frac{3b\sqrt{a+bx^2+cx^4}}{c}-\frac{(3b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}}{4c}\right)$$

input
```
Int[x^5/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((x^2*Sqrt[a + b*x^2 + c*x^4])/(2*c) - ((3*b*Sqrt[a + b*x^2 + c*x^4])/c -
((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])
])/(2*c^(3/2)))/(4*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\left(-2 c \,x^{2}+3 b \right) \sqrt{c \,x^{4}+b \,x^{2}+a}}{8 c^{2}}-\frac{\left(4 a c -3 b^{2}\right) \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right)}{16 c^{\frac{5}{2}}}$
default	$\frac{x^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}}{4 c}-\frac{3 b \sqrt{c \,x^{4}+b \,x^{2}+a}}{8 c^{2}}+\frac{3 b^{2} \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right)}{16 c^{\frac{5}{2}}}-\frac{a \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right)}{4 c^{\frac{3}{2}}}$
elliptic	$\frac{x^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}}{4 c}-\frac{3 b \sqrt{c \,x^{4}+b \,x^{2}+a}}{8 c^{2}}+\frac{3 b^{2} \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right)}{16 c^{\frac{5}{2}}}-\frac{a \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right)}{4 c^{\frac{3}{2}}}$
pseudoelliptic	$\frac{4 c^{\frac{3}{2}} x^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}+4 a c \ln \left(2\right)-3 b^{2} \ln \left(2\right)-4 a c \ln \left(\frac{2 c \,x^{2}+2 \sqrt{c \,x^{4}+b \,x^{2}+a}\, \sqrt{c}+b}{\sqrt{c}}\right)+3 \ln \left(\frac{2 c \,x^{2}+2 \sqrt{c \,x^{4}+b \,x^{2}+a}\, \sqrt{c}+b}{\sqrt{c}}\right) b^{2}-6 b}{16 c^{\frac{5}{2}}}$

input
```
int(x^5/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*c*x^2+3*b)*(c*x^4+b*x^2+a)^(1/2)/c^2-1/16*(4*a*c-3*b^2)/c^(5/2)*l
n((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.95

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx = \left[-\frac{(3\,b^2-4\,ac)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^4+bx^2+a}(2\,c^2x^2-3\,bc)}{32\,c^3}, -\frac{(3\,b^2-4\,ac)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-2\,\sqrt{cx^4+bx^2+a}(2\,c^2x^2-3\,bc)}{16\,c^3}\right]$$

input
```
integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(
c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 +
 a)*(2*c^2*x^2 - 3*b*c))/c^3, -1/16*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*s
qrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) -
 2*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 - 3*b*c))/c^3]
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**5/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{8}\sqrt{cx^4+bx^2+a}\left(\frac{2\,x^2}{c}-\frac{3\,b}{c^2}\right) - \frac{(3\,b^2-4\,ac)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{5}{2}}}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2/c - 3*b/c^2) - 1/16*(3*b^2 - 4*a*c)*log
(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{x^5}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(x^5/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^5/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 862, normalized size of antiderivative = 8.29

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx=\text{Too large to display}$$

input
```
int(x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

output

```
( - 16*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)
+ b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b*c**2 - 32*sqrt(a + b*x**2 + c*x**4
)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b*
*2))*a*c**3*x**2 + 12*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*
x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**3*c + 24*sqrt(a + b*
x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sq
rt(4*a*c - b**2))*b**2*c**2*x**2 - 24*sqrt(a + b*x**2 + c*x**4)*a*b*c**2 +
 16*sqrt(a + b*x**2 + c*x**4)*a*c**3*x**2 - 6*sqrt(a + b*x**2 + c*x**4)*b*
*3*c - 44*sqrt(a + b*x**2 + c*x**4)*b**2*c**2*x**2 - 16*sqrt(a + b*x**2 +
c*x**4)*b*c**3*x**4 + 32*sqrt(a + b*x**2 + c*x**4)*c**4*x**6 - 16*sqrt(c)*
log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2
))*a**2*c**2 + 8*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*
c*x**2)/sqrt(4*a*c - b**2))*a*b**2*c - 32*sqrt(c)*log((2*sqrt(c)*sqrt(a +
b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b*c**2*x**2 - 32*sq
rt(c)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c
- b**2))*a*c**3*x**4 + 3*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)
+ b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**4 + 24*sqrt(c)*log((2*sqrt(c)*sqrt(
a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**3*c*x**2 + 24*
sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*
c - b**2))*b**2*c**2*x**4 - 24*sqrt(c)*a*b**2*c - 32*sqrt(c)*a*b*c**2*x...
```

3.984 $\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

output
```
1/2*(c*x^4+b*x^2+a)^(1/2)/c-1/4*b*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b
*x^2+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a+bx^2+cx^4}}{2c} - \frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

input
```
Integrate[x^3/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
Sqrt[a + b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
 + b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1434, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{c}-\frac{b\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{2c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{c}-\frac{b\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2+cx^4}}{c}-\frac{b\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}\right)$$

input
```
Int[x^3/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]/c - (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(2*c^(3/2)))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}$	56
risch	$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}$	56
elliptic	$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}$	56
pseudoelliptic	$\frac{b\ln\left(2\right)-b\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}}{4c^{\frac{3}{2}}}$	65

input
```
int(x^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^4+b*x^2+a)^(1/2)/c-1/4*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+
b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.37

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[\frac{b\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)+4\,\sqrt{cx^4+bx^2+a}c}{8\,c^2}, \frac{b\sqrt{-c}\,a}{}\right.$$

input
```
integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(b*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 +
a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*c)/c^2, 1/4*
(b*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2
*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*c)/c^2]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**3/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{b\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{4\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^4+bx^2+a}}{2\,c}$$

input
```
integrate(x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*b*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/c^(3
/2) + 1/2*sqrt(c*x^4 + b*x^2 + a)/c
```

Mupad [B] (verification not implemented)

Time = 17.78 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2+a}}{2\,c} - \frac{b\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{4\,c^{3/2}}$$

input
```
int(x^3/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
(a + b*x^2 + c*x^4)^(1/2)/(2*c) - (b*log((a + b*x^2 + c*x^4)^(1/2) + (b/2
+ c*x^2)/c^(1/2)))/(4*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.66

$$\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{-2\sqrt{c\,x^4+b\,x^2+a}\,\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}+b+2c\,x^2}{\sqrt{4ac-b^2}}\right)bc+2\sqrt{c\,x^4+b\,x^2+a}\,bc+4\sqrt{c\,x^4+b\,x^2+a}\,c^2x^2-\sqrt{c}}{4c^2\left(2\sqrt{c}\,\sqrt{c\,x^4+b}\right.}$$

input
```
int(x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) +
 b + 2*c*x**2)/sqrt(4*a*c - b**2))*b*c + 2*sqrt(a + b*x**2 + c*x**4)*b*c +
 4*sqrt(a + b*x**2 + c*x**4)*c**2*x**2 - sqrt(c)*log((2*sqrt(c)*sqrt(a + b
*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**2 - 2*sqrt(c)*log((
2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b*
c*x**2 + 4*sqrt(c)*a*c + 4*sqrt(c)*b*c*x**2 + 4*sqrt(c)*c**2*x**4)/(4*c**2
*(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2))
```

3.985 $\int \frac{x}{\sqrt{a+bx^2+cx^4}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}} \, dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

output
```
1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}} \, dx = -\frac{\log \left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{2\sqrt{c}}$$

input
```
Integrate[x/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
-1/2*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]/Sqrt[c]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1432, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1092}$$

$$\int \frac{1}{4c-x^4}d\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

input
```
Int[x/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}$	35
elliptic	$\frac{\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}$	35
pseudoelliptic	$\frac{-\ln(2)+\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{\sqrt{c}}\right)}{2\sqrt{c}}$	43

input
```
int(x/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.74

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx = \left[\frac{\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)}{4\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)}{2\,c}\right]$$

input
```
integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
 + b)*sqrt(c) - 4*a*c)/sqrt(c), -1/2*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^
2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c))/c]
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{2\sqrt{c}}$$

input
```
integrate(x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) + b))/sqrt(
c)
```

Mupad [B] (verification not implemented)

Time = 18.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

input
```
int(x/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))/(2*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}+b+2c\,x^2}{\sqrt{4ac-b^2}}\right)}{2c}$$

input
```
int(x/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a
*c - b**2)))/(2*c)
```

3.986 $\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

output
```
-1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1434, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1154}$$

$$-\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{a}}$	39
elliptic	$-\frac{\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{a}}$	39
pseudoelliptic	$-\frac{\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{a}}$	39

input
```
int(1/x/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.82

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \left[\frac{\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)}{2\,a}\right]$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^
2 + 2*a)*sqrt(a) + 8*a^2)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^4
 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))/a]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 17.69 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\ln\left(\frac{1}{x^2}\right)}{2\sqrt{a}} - \frac{\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\,\sqrt{a}}$$

input
```
int(1/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
- log(1/x^2)/(2*a^(1/2)) - log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) +
 b*x^2)/(2*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.98

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a}\,\sqrt{4ac-b^2}\,\sqrt{-4ac+b^2}\,atan\left(\frac{4\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,ac-\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,b^2+4a\,c^2x^2-b^2c\,x^2}{\sqrt{c}\,\sqrt{a}\,\sqrt{4ac-b^2}\,\sqrt{-4ac+b^2}}\right)}{a\,(4ac-b^2)}$$

input
```
int(1/x/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(a)*sqrt(4*a*c - b**2)*sqrt( - 4*a*c + b**2)*atan((4*sqrt(c)*sqrt(
a + b*x**2 + c*x**4)*a*c - sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b**2 + 4*a*c*
*2*x**2 - b**2*c*x**2)/(sqrt(c)*sqrt(a)*sqrt(4*a*c - b**2)*sqrt( - 4*a*c +
 b**2))))/(a*(4*a*c - b**2))
```

3.987 $\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{b\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

output
```
-1/2*(c*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*
x^4+b*x^2+a)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*Sqrt[a + b*x^2 + c*x^4]/(a*x^2) - (b*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a +
b*x^2 + c*x^4])/Sqrt[a]])/(2*a^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1434, 1157, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1157}$$

$$\frac{1}{2}\left(-\frac{b\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2+cx^4}}{ax^2}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{a}-\frac{\sqrt{a+bx^2+cx^4}}{ax^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{ax^2}\right)$$

input
```
Int[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-(Sqrt[a + b*x^2 + c*x^4]/(a*x^2)) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}$	63
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}$	63
elliptic	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}$	63
pseudoelliptic	$\frac{b\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)x^{2}-2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{4a^{\frac{3}{2}}\,x^{2}}$	67

input
```
int(1/x^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*b/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*
x^4+b*x^2+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.49

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \left[\frac{\sqrt{a}bx^2\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}a}{8\,a^2x^2},\ -\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+2\,\sqrt{cx^4+bx^2+a}a}{4\,a^2x^2}\right]$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b
*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*
a)/(a^2*x^2), -1/4*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x
^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*
a)/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx = -\frac{b\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}a} + \frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)a}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)
*a) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a)
```

Mupad [B] (verification not implemented)

Time = 17.75 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\frac{b\,x^2}{2}+a}{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}\right)}{4\,a^{3/2}} - \frac{\sqrt{c\,x^4+b\,x^2+a}}{2\,a\,x^2}$$

input
```
int(1/(x^3*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
(b*atanh((a + (b*x^2)/2)/(a^(1/2)*(a + b*x^2 + c*x^4)^(1/2))))/(4*a^(3/2))
 - (a + b*x^2 + c*x^4)^(1/2)/(2*a*x^2)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+b\,x^2+a}\,x^3}dx$$

input
```
int(1/x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(1/(sqrt(a + b*x**2 + c*x**4)*x**3),x)
```

3.988 $\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}$$

output
```
-1/4*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^2-1/16*
(-4*a*c+3*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5
/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \frac{(-2a+3bx^2)\sqrt{a+bx^2+cx^4}}{8a^2x^4} + \frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{5/2}}$$

input
```
Integrate[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-2*a + 3*b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((3*b^2 - 4*a*c)*
ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(5/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1434, 1167, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{2}\left(-\frac{\int \frac{2cx^2+3b}{2x^4\sqrt{cx^4+bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{2cx^2+3b}{x^4\sqrt{cx^4+bx^2+a}}dx^2}{4a}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^4}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2}\left(-\frac{-\frac{(3b^2-4ac)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{2a}-\frac{3b\sqrt{a+bx^2+cx^4}}{ax^2}}{4a}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^4}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(-\frac{\frac{(3b^2-4ac)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{a}-\frac{3b\sqrt{a+bx^2+cx^4}}{ax^2}}{4a}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{\frac{(3b^2-4ac)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}-\frac{3b\sqrt{a+bx^2+cx^4}}{ax^2}}{4a}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^4}\right)$$

input
```
Int[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-1/2*Sqrt[a + b*x^2 + c*x^4]/(a*x^4) - ((-3*b*Sqrt[a + b*x^2 + c*x^4])/(a
*x^2) + ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
 c*x^4])])/(2*a^(3/2)))/(4*a))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,\left(-3b\,x^2+2a\right)}{8a^2x^4}+\frac{\left(4ac-3b^2\right)\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{16a^{\frac{5}{2}}}$
default	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{4a\,x^4}+\frac{3b\sqrt{c\,x^4+b\,x^2+a}}{8a^2x^2}-\frac{3b^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{16a^{\frac{5}{2}}}+\frac{c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4a^{\frac{3}{2}}}$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{4a\,x^4}+\frac{3b\sqrt{c\,x^4+b\,x^2+a}}{8a^2x^2}-\frac{3b^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{16a^{\frac{5}{2}}}+\frac{c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4a^{\frac{3}{2}}}$
pseudoelliptic	$\frac{4c\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)a\,x^4-3b^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)x^4+6b\sqrt{c\,x^4+b\,x^2+a}\,x^2\sqrt{a}-4\sqrt{c\,x^4+b\,x^2+}}{16a^{\frac{5}{2}}x^4}$

input
```
int(1/x^5/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(c*x^4+b*x^2+a)^(1/2)*(-3*b*x^2+2*a)/a^2/x^4+1/16/a^(5/2)*(4*a*c-3*b^
2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \left[-\frac{(3\,b^2-4\,ac)\sqrt{a}x^4\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}(3\,abx^2-2}{32\,a^3x^4}\right.$$

input
```
integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((3*b^2 - 4*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 +
4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x
^4 + b*x^2 + a)*(3*a*b*x^2 - 2*a^2))/(a^3*x^4), 1/16*((3*b^2 - 4*a*c)*sqrt
(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^
4 + a*b*x^2 + a^2)) + 2*sqrt(c*x^4 + b*x^2 + a)*(3*a*b*x^2 - 2*a^2))/(a^3*
x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(90) = 180$.

Time = 0.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \frac{(3\,b^2-4\,ac)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^2} - \frac{3\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3b^2-4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3ac-5\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)ab^2 -}{8\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^2a^2}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*(3*b^2 - 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-
a))/(sqrt(-a)*a^2) - 1/8*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*b^2
- 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a*c - 5*(sqrt(c)*x^2 - sqrt(
c*x^4 + b*x^2 + a))*a*b^2 - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*
c - 8*a^2*b*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^
2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{1}{x^5\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/(x^5*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^5*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4+b\,x^2+a}\,x^5}dx$$

input
```
int(1/x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(1/(sqrt(a + b*x**2 + c*x**4)*x**5),x)
```

3.989 $\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{6ax^6}+\frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{48a^3x^2} + \frac{b(5b^2-12ac)\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}$$

output
```
-1/6*(c*x^4+b*x^2+a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4-1/48
*(-16*a*c+15*b^2)*(c*x^4+b*x^2+a)^(1/2)/a^3/x^2+1/32*b*(-12*a*c+5*b^2)*arc
tanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{a+bx^2+cx^4}(-8a^2+10abx^2-15b^2x^4+16acx^4)}{48a^3x^6} + \frac{(-5b^3+12abc)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{7/2}}$$

input
```
Integrate[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[a + b*x^2 + c*x^4]*(-8*a^2 + 10*a*b*x^2 - 15*b^2*x^4 + 16*a*c*x^4))/
(48*a^3*x^6) + ((-5*b^3 + 12*a*b*c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2
+ c*x^4])/Sqrt[a]])/(16*a^(7/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1434, 1167, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^8\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{2}\left(-\frac{\int \frac{4cx^2+5b}{2x^6\sqrt{cx^4+bx^2+a}}dx^2}{3a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{4cx^2+5b}{x^6\sqrt{cx^4+bx^2+a}}dx^2}{6a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{2}\left(-\frac{-\frac{\int \frac{15b^2+10cx^2b-16ac}{2x^4\sqrt{cx^4+bx^2+a}}dx^2}{2a}-\frac{5b\sqrt{a+bx^2+cx^4}}{2ax^4}}{6a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{-\frac{\int \frac{15b^2+10cx^2b-16ac}{x^4\sqrt{cx^4+bx^2+a}}dx^2}{4a}-\frac{5b\sqrt{a+bx^2+cx^4}}{2ax^4}}{6a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2}\left(-\frac{-\frac{-\frac{3b(5b^2-12ac)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{2a}-\frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{ax^2}}{4a}-\frac{5b\sqrt{a+bx^2+cx^4}}{2ax^4}}{6a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(-\frac{-\frac{\frac{3b(5b^2-12ac)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{a}-\frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{ax^2}}{4a}-\frac{5b\sqrt{a+bx^2+cx^4}}{2ax^4}}{6a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{-\frac{\frac{3b(5b^2-12ac)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}-\frac{(15b^2-16ac)\sqrt{a+bx^2+cx^4}}{ax^2}}{4a}-\frac{5b\sqrt{a+bx^2+cx^4}}{2ax^4}}{6a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^6}\right)$$

input
```
Int[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-1/3*Sqrt[a + b*x^2 + c*x^4]/(a*x^6) - ((-5*b*Sqrt[a + b*x^2 + c*x^4])/(2
*a*x^4) - (-(((15*b^2 - 16*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a*x^2)) + (3*b*(
5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]
)/(2*a^(3/2)))/(4*a))/(6*a))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(-16 a c\,x^{4}+15 b^{2} x^{4}-10 a b\,x^{2}+8 a^{2}\right)}{48 a^{3} x^{6}}-\frac{b \left(12 a c-5 b^{2}\right) \ln\left(\frac{2 a+b\,x^{2}+2 \sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{32 a^{\frac{7}{2}}}$
pseudoelliptic	$-\frac{3 \left(b\,x^{6} \left(a c-\frac{5 b^{2}}{12}\right) \ln\left(\frac{2 a+b\,x^{2}+2 \sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)+\frac{5 \left(-\frac{2 \left(\frac{8 c\,x^{2}}{5}+b\right) x^{2} a^{\frac{3}{2}}}{3}+\sqrt{a}\,b^{2} x^{4}+\frac{8 a^{\frac{5}{2}}}{15}\right) \sqrt{c\,x^{4}+b\,x^{2}+a}}{6}\right)}{8 a^{\frac{7}{2}} x^{6}}$
default	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{6 a\,x^{6}}+\frac{5 b \sqrt{c\,x^{4}+b\,x^{2}+a}}{24 a^{2} x^{4}}-\frac{5 b^{2} \sqrt{c\,x^{4}+b\,x^{2}+a}}{16 a^{3} x^{2}}+\frac{5 b^{3} \ln\left(\frac{2 a+b\,x^{2}+2 \sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{32 a^{\frac{7}{2}}}-\frac{3 b c \ln\left(\frac{2 a+b\,x^{2}}{}\right.}{}$
elliptic	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{6 a\,x^{6}}+\frac{5 b \sqrt{c\,x^{4}+b\,x^{2}+a}}{24 a^{2} x^{4}}-\frac{5 b^{2} \sqrt{c\,x^{4}+b\,x^{2}+a}}{16 a^{3} x^{2}}+\frac{5 b^{3} \ln\left(\frac{2 a+b\,x^{2}+2 \sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{32 a^{\frac{7}{2}}}-\frac{3 b c \ln\left(\frac{2 a+b\,x^{2}}{}\right.}{}$

input
```
int(1/x^7/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(c*x^4+b*x^2+a)^(1/2)*(-16*a*c*x^4+15*b^2*x^4-10*a*b*x^2+8*a^2)/a^3/
x^6-1/32*b*(12*a*c-5*b^2)/a^(7/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^
(1/2))/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.83

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[-\frac{3\,(5\,b^3-12\,abc)\sqrt{a}x^6\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)-4\,(10\,a^2bx^2-(15\,ab^2-16}{192\,a^4x^6}\right.$$
$$-\frac{3\,(5\,b^3-12\,abc)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)-2\,(10\,a^2bx^2-(15\,ab^2-16\,a^2c)x^4-8\,a^3)\sqrt{}}{96\,a^4x^6}$$

input
```
integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(5*b^3 - 12*a*b*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*
x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(1
0*a^2*b*x^2 - (15*a*b^2 - 16*a^2*c)*x^4 - 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/
(a^4*x^6), -1/96*(3*(5*b^3 - 12*a*b*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4
+ b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(10*a^2
*b*x^2 - (15*a*b^2 - 16*a^2*c)*x^4 - 8*a^3)*sqrt(c*x^4 + b*x^2 + a))/(a^4*
x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**7*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(123) = 246$.

Time = 0.13 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.31

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx = -\frac{(5\,b^3-12\,abc)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}a^3}$$
$$+\frac{15\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5b^3-36\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^5abc-40\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3a}{}$$

input
```
integrate(1/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/16*(5*b^3 - 12*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/s
qrt(-a))/(sqrt(-a)*a^3) + 1/48*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))
^5*b^3 - 36*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*a*b*c - 40*(sqrt(c)*
x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a*b^3 + 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))^3*a^2*b*c + 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*a^3*c^(
3/2) + 33*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*b^3 + 36*(sqrt(c)*x^
2 - sqrt(c*x^4 + b*x^2 + a))*a^3*b*c + 48*a^3*b^2*sqrt(c) - 32*a^4*c^(3/2)
)/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^3*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x^7\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/(x^7*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^7*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \text{too large to display}$$

input
```
int(1/x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - 128*sqrt(a + b*x**2 + c*x**4)*a**5*c**3 - 96*sqrt(a + b*x**2 + c*x**4)
*a**4*b**2*c**2 - 128*sqrt(a + b*x**2 + c*x**4)*a**4*b*c**3*x**2 + 832*sqr
t(a + b*x**2 + c*x**4)*a**4*c**4*x**4 + 80*sqrt(a + b*x**2 + c*x**4)*a**3*
b**4*c + 336*sqrt(a + b*x**2 + c*x**4)*a**3*b**3*c**2*x**2 - 48*sqrt(a + b
*x**2 + c*x**4)*a**3*b**2*c**3*x**4 - 20*sqrt(a + b*x**2 + c*x**4)*a**2*b*
*5*c*x**2 - 824*sqrt(a + b*x**2 + c*x**4)*a**2*b**4*c**2*x**4 - 50*sqrt(a
+ b*x**2 + c*x**4)*a*b**7*x**2 + 130*sqrt(a + b*x**2 + c*x**4)*a*b**6*c*x*
*4 + 75*sqrt(a + b*x**2 + c*x**4)*b**8*x**4 + 288*sqrt(a)*log(sqrt(a + b*x
**2 + c*x**4) - sqrt(a))*a**3*b*c**4*x**6 + 96*sqrt(a)*log(sqrt(a + b*x**2
 + c*x**4) - sqrt(a))*a**2*b**3*c**3*x**6 - 270*sqrt(a)*log(sqrt(a + b*x**
2 + c*x**4) - sqrt(a))*a*b**5*c**2*x**6 + 75*sqrt(a)*log(sqrt(a + b*x**2 +
 c*x**4) - sqrt(a))*b**7*c*x**6 - 288*sqrt(a)*log(sqrt(a + b*x**2 + c*x**4
) + sqrt(a))*a**3*b*c**4*x**6 - 96*sqrt(a)*log(sqrt(a + b*x**2 + c*x**4) +
 sqrt(a))*a**2*b**3*c**3*x**6 + 270*sqrt(a)*log(sqrt(a + b*x**2 + c*x**4)
+ sqrt(a))*a*b**5*c**2*x**6 - 75*sqrt(a)*log(sqrt(a + b*x**2 + c*x**4) + s
qrt(a))*b**7*c*x**6 - 9216*int(sqrt(a + b*x**2 + c*x**4)/(8*a**3*b*c**2*x*
*5 + 8*a**3*c**3*x**7 + 6*a**2*b**3*c*x**5 + 14*a**2*b**2*c**2*x**7 + 16*a
**2*b*c**3*x**9 + 8*a**2*c**4*x**11 - 5*a*b**5*x**5 + a*b**4*c*x**7 + 12*a
*b**3*c**2*x**9 + 6*a*b**2*c**3*x**11 - 5*b**6*x**7 - 10*b**5*c*x**9 - 5*b
**4*c**2*x**11),x)*a**7*b**2*c**5*x**6 + 13504*int(sqrt(a + b*x**2 + c*...
```

3.990 $\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 313

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{x\sqrt{a+bx^2+cx^4}}{3c}-\frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{2\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(2b+\sqrt{a}\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/3*x*(c*x^4+b*x^2+a)^(1/2)/c-2/3*b*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^(1/
2)+c^(1/2)*x^2)+2/3*a^(1/4)*b*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1
/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(
2-b/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/6*a^(1/4)*(2*b
+a^(1/2)*c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*
x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)
/c^(1/2))^(1/2))/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.61 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.42

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(a+bx^2+cx^4)-ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\operatorname{arcsinh}\left(\sqrt{\right.\right.}{}$$

input
```
Integrate[x^4/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) - I*b*(-b + Sqr
t[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*
c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(
2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c]
)/(b - Sqrt[b^2 - 4*a*c])])/(6*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a
+ b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1442, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 1442$$

$$\frac{x\sqrt{a+bx^2+cx^4}}{3c}-\frac{\int \frac{2bx^2+a}{\sqrt{cx^4+bx^2+a}}dx}{3c}$$

$\Big\downarrow$ 1511

$$\frac{x\sqrt{a+bx^2+cx^4}}{3c}-\frac{\sqrt{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}$$

$\Big\downarrow$ 27

$$\frac{x\sqrt{a+bx^2+cx^4}}{3c}-\frac{\sqrt{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}$$

$\Big\downarrow$ 1416

$$\frac{x\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}$$

$\Big\downarrow$ 1509

$$\frac{x\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.}{\sqrt{c}}}{3c}$$

input
```
Int[x^4/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - ((-2*b*(-((x*Sqrt[a + b*x^2 + c*x^4])/
(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqr
t[c] + (a^(1/4)*(Sqrt[a] + (2*b)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4
]))/(3*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.24

method	result
default	$\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+\right.}}{ac}}}{2}\right.}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
risch	$\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}}{}$
elliptic	$\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+\right.}}{ac}}}{2}\right.}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$

input
```
int(x^4/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(c*x^4+b*x^2+a)^(1/2)/c-1/12/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3/c
*b*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx =$$

$$-\frac{2\sqrt{\frac{1}{2}}\left(bcx\sqrt{\frac{b^2-4\,ac}{c^2}}-b^2x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac})-\sqrt{\frac{1}{2}}\left((2\right.}{}$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(1/2)*(b*c*x*sqrt((b^2 - 4*a*c)/c^2) - b^2*x)*sqrt(c)*sqrt((c*
sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((
b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a
*c)/(a*c)) - sqrt(1/2)*((2*b*c - c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (2*b^2 +
 b*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin
(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2
- 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*sqrt(c*x^4 + b*x^2 + a)*(c^2*x^2 -
 2*b*c))/(c^3*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**4/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(x^4/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^4/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2+a}\,x-\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)a-2\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)b}{3c}$$

input
```
int(x^4/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2 + c*x**4)*x - int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 +
 c*x**4),x)*a - 2*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**
4),x)*b)/(3*c)
```

3.991 $\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 267

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)-a^(1/4)*(a^(1/2)+c^(
1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*
arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(3/4)/(c*x^4
+b*x^2+a)^(1/2)+1/2*a^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2
)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2
-b/a^(1/2)/c^(1/2))^(1/2))/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{i\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\text{Ell}\right.}{2\sqrt{2}c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[x^2/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((I/2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b +
Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))
/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1459}$$

$$\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}}{\sqrt{c}}$$

input
```
Int[x^2/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
-((-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/
4)*Sqrt[a + b*x^2 + c*x^4]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2
+ c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.81

method	result
default	$-\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)-\text{Elliptic}\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(b+\sqrt{-4ac+b^{2}}\right)}$
elliptic	$-\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)-\text{Elliptic}\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(b+\sqrt{-4ac+b^{2}}\right)}$

input
```
int(x^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(
1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(cx\sqrt{\frac{b^2-4\,ac}{c^2}}-bx\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac})-\sqrt{\frac{1}{2}}\left(cx\sqrt{\frac{b^2-\cdots}{c\cdots}}\right.}{2\,c^2x}$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(1/2)*(c*x*sqrt((b^2 - 4*a*c)/c^2) - b*x)*sqrt(c)*sqrt((c*sqrt((b
^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4
*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*
c)) - sqrt(1/2)*(c*x*sqrt((b^2 - 4*a*c)/c^2) - b*x)*sqrt(c)*sqrt((c*sqrt((
b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 -
4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a
*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*c)/(c^2*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(x^2/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(x^2/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx$$

input
```
int(x^2/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)
```

3.992 $\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 114

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
1/2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*
InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2
))/a^(1/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.63

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[1/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c
/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \; 1416$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

input
```
Int[1/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$	144
elliptic	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$	144

input
```
int(1/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{\frac{1}{2}}\left(a\sqrt{\frac{b^2-4\,ac}{a^2}}+b\right)\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}F(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}\right)\mid\frac{ab\sqrt{\frac{b^2-4\,ac}{a^2}}+b^2-2\,ac}{2\,ac})}{2\,\sqrt{ac}}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*(a*sqrt((b^2 - 4*a*c)/a^2) + b)*sqrt((a*sqrt((b^2 - 4*a*c)/
a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2)
 - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c))/(sqrt(a)
*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(1/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx$$

input
```
int(1/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)
```

3.993 $\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 294

$$\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{ax}+\frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-(c*x^4+b*x^2+a)^(1/2)/a/x+c^(1/2)*x*(c*x^4+b*x^2+a)^(1/2)/a/(a^(1/2)+c^(1
/2)*x^2)-c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x
^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c
^(1/2))^(1/2))/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/2*c^(1/4)*(a^(1/2)+c^(1/2)*
x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arc
tan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(c*x^4+b*x
^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.33 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \frac{-\frac{4(a+bx^2+cx^4)}{x}+\frac{i\sqrt{2}\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\text{Elliptic}\right.}{\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}}}{4a\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
((-4*(a + b*x^2 + c*x^4))/x + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - Ellipt
icF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(4*a*S
qrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1443, 27, 1459, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1443}$$

$$\frac{\int \frac{cx^2}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}}{ax}$$

$$\Big\downarrow \ 27$$

$$\frac{c\int \frac{x^2}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2+cx^4}}{ax}$$

$$\Big\downarrow \ 1459$$

$$\frac{c\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{a+bx^2+cx^4}}{ax}$$

$$\Big\downarrow \ 27$$

$$\frac{c\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{a+bx^2+cx^4}}{ax}$$

$$\Big\downarrow \ 1416$$

$$\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{a+bx^2+cx^4}}{ax}$$

$$\Big\downarrow \ 1509$$

$$\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}}{\sqrt{c}}\right.}{a}$$

$$\frac{\sqrt{a+bx^2+cx^4}}{ax}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-(Sqrt[a + b*x^2 + c*x^4]/(a*x)) + (c*(-((-((x*Sqrt[a + b*x^2 + c*x^4])/(S
qrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/Sqrt[c
]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.81

method	result
default	$-\frac{\sqrt{c x^4+b x^2+a}}{a x}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4\ldots}\right)}{ac}}}{2}\right.\right.\ldots}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c x^4+b x^2+a}\,\left(b+\sqrt{-4\ldots}\right)}$
risch	$-\frac{\sqrt{c x^4+b x^2+a}}{a x}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4\ldots}\right)}{ac}}}{2}\right.\right.\ldots}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c x^4+b x^2+a}\,\left(b+\sqrt{-4\ldots}\right)}$
elliptic	$-\frac{\sqrt{c x^4+b x^2+a}}{a x}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4\ldots}\right)}{ac}}}{2}\right.\right.\ldots}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c x^4+b x^2+a}\,\left(b+\sqrt{-4\ldots}\right)}$

input
```
int(1/x^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^4+b*x^2+a)^(1/2)/a/x-1/2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{\frac{1}{2}}\left(ax\sqrt{\frac{b^2-4\,ac}{a^2}}-bx\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}\right) \mid \frac{ab\sqrt{\frac{b^2-4\,ac}{a^2}}+b^2-2\,ac}{2\,ac}) - \sqrt{\frac{1}{2}}\left(ax\right.}{2}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(a*x*sqrt((b^2 - 4*a*c)/a^2) - b*x)*sqrt(a)*sqrt((a*sqrt((
b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2
- 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a
*c)) - sqrt(1/2)*(a*x*sqrt((b^2 - 4*a*c)/a^2) - b*x)*sqrt(a)*sqrt((a*sqrt(
(b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2
 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(
a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*a)/(a^2*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}\,x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}\,x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x^2\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/(x^2*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^6+b\,x^4+a\,x^2}dx$$

input
```
int(1/x^2/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a*x**2 + b*x**4 + c*x**6),x)
```

3.994 $\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 317

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2+cx^4}}{3a^{3/2}x\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{2b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\sqrt{a+bx^2+cx^4}} - \frac{\left(2b+\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/3*(c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(c*x^4+b*x^2+a)^(1/2)/a^(3/2)/x/(a^
(1/2)+c^(1/2)*x^2)+2/3*b*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a
^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/
2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/6*(2*b+a^(1
/2)*c^(1/2))*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/
2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1
/2)/c^(1/2))^(1/2))/a^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.59 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a-2bx^2)(a+bx^2+cx^4)-ib\left(-b+\sqrt{b^2-4ac}\right)x^3\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E}{}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a - 2*b*x^2)*(a + b*x^2 + c*x^4) - I*
b*(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + a*c + b*
Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*
a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(6*a^2*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1443, 25, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$
$$\Big\downarrow \text{1443}$$

$$\frac{\int -\frac{cx^2+2b}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\Big\downarrow$ 25

$$-\frac{\int \frac{cx^2+2b}{x^2\sqrt{cx^4+bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\Big\downarrow$ 1604

$$-\frac{-\frac{\int -\frac{c(2bx^2+a)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{2b\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int \frac{c(2bx^2+a)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{2b\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\Big\downarrow$ 27

$$-\frac{\frac{c\int \frac{2bx^2+a}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{2b\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\Big\downarrow$ 1511

$$-\frac{\frac{c\left(\sqrt{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2b\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\Big\downarrow$ 27

$$-\frac{\frac{c\left(\sqrt{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2b\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$\Big\downarrow$ 1416

$$-\frac{\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2b\sqrt{a+bx^2+cx^4}}{ax}}{3a}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}$$

$$\Big\downarrow \text{1509}$$

$$-\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\frac{2b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.}{\sqrt{c}}\right.}{a}}{3a} \frac{\sqrt{a+bx^2+cx^4}}{3ax^3}$$

input
```
Int[1/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/3*Sqrt[a + b*x^2 + c*x^4]/(a*x^3) - ((-2*b*Sqrt[a + b*x^2 + c*x^4])/(a*
x) + (c*((-2*b*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (
a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]
))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c] + (a^(1/4)*(Sqrt[a] + (
2*b)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/a)/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(-2b\,x^{2}+a\right)}{3a^{2}x^{3}}-\frac{c\left(\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\sqrt{-4+\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\right.}{}$
default	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{c\,x^{4}+b\,x^{2}+a}}{3a^{2}x}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2}\right.}{12a\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$
elliptic	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{c\,x^{4}+b\,x^{2}+a}}{3a^{2}x}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2}\right.}{12a\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$

input
```
int(1/x^4/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c*x^4+b*x^2+a)^(1/2)*(-2*b*x^2+a)/a^2/x^3-1/3*c/a^2*(1/4*a*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2))-b*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{\frac{1}{2}}\left(abx^3\sqrt{\frac{b^2-4ac}{a^2}}-b^2x^3\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4ac}{a^2}}-b}{a}}E\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4ac}{a^2}}-b}{a}}\right)\mid\frac{ab\sqrt{\frac{b^2-4ac}{a^2}}+b^2-2ac}{2ac}\right)+\sqrt{\frac{1}{2}}(}{}$$

input
```
integrate(1/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(1/2)*(a*b*x^3*sqrt((b^2 - 4*a*c)/a^2) - b^2*x^3)*sqrt(a)*sqrt(
(a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*s
qrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 -
2*a*c)/(a*c)) + sqrt(1/2)*((a^2 - 2*a*b)*x^3*sqrt((b^2 - 4*a*c)/a^2) + (a*
b + 2*b^2)*x^3)*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f
(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqr
t((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*(2*
a*b*x^2 - a^2))/(a^3*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}x^4}\,dx$$

input
```
integrate(1/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}x^4}\,dx$$

input
```
integrate(1/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{x^4\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^8+b\,x^6+a\,x^4}dx$$

input
```
int(1/x^4/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a*x**4 + b*x**6 + c*x**8),x)
```

3.995 $\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{(15b^2+16ac+10bcx^2)\sqrt{a+bx^2-cx^4}}{48c^3} - \frac{b(5b^2+12ac)\arctan\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{32c^{7/2}}$$

output
```
-1/6*x^4*(-c*x^4+b*x^2+a)^(1/2)/c-1/48*(10*b*c*x^2+16*a*c+15*b^2)*(-c*x^4+
b*x^2+a)^(1/2)/c^3-1/32*b*(12*a*c+5*b^2)*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(
-c*x^4+b*x^2+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\sqrt{a+bx^2-cx^4}(-15b^2-16ac-10bcx^2-8c^2x^4)}{48c^3} + \frac{(5b^3+12abc)\arctan\left(\frac{\sqrt{c}x^2}{-\sqrt{a}+\sqrt{a+bx^2-cx^4}}\right)}{16c^{7/2}}$$

input
```
Integrate[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
(Sqrt[a + b*x^2 - c*x^4]*(-15*b^2 - 16*a*c - 10*b*c*x^2 - 8*c^2*x^4))/(48*
c^3) + ((5*b^3 + 12*a*b*c)*ArcTan[(Sqrt[c]*x^2)/(-Sqrt[a] + Sqrt[a + b*x^2
 - c*x^4])])/(16*c^(7/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1434, 1166, 27, 1225, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{x^6}{\sqrt{-cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 1166}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{x^2(5bx^2+4a)}{2\sqrt{-cx^4+bx^2+a}}dx^2}{3c}-\frac{x^4\sqrt{a+bx^2-cx^4}}{3c}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{\int \frac{x^2(5bx^2+4a)}{\sqrt{-cx^4+bx^2+a}}dx^2}{6c}-\frac{x^4\sqrt{a+bx^2-cx^4}}{3c}\right)$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{1}{2}\left(\frac{\frac{3b(12ac+5b^2)\int \frac{1}{\sqrt{-cx^4+bx^2+a}}dx^2}{8c^2}-\frac{(16ac+15b^2+10bcx^2)\sqrt{a+bx^2-cx^4}}{4c^2}}{6c}-\frac{x^4\sqrt{a+bx^2-cx^4}}{3c}\right)$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{2}\left(\frac{\frac{3b(12ac+5b^2)\int \frac{1}{-x^4-4c}d\frac{b-2cx^2}{\sqrt{-cx^4+bx^2+a}}}{4c^2}-\frac{(16ac+15b^2+10bcx^2)\sqrt{a+bx^2-cx^4}}{4c^2}}{6c}-\frac{x^4\sqrt{a+bx^2-cx^4}}{3c}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{-\frac{3b(12ac+5b^2)\arctan\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{8c^{5/2}}-\frac{\sqrt{a+bx^2-cx^4}(16ac+15b^2+10bcx^2)}{4c^2}}{6c}-\frac{x^4\sqrt{a+bx^2-cx^4}}{3c}\right)$$

input
```
Int[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
(-1/3*(x^4*Sqrt[a + b*x^2 - c*x^4])/c + (-1/4*((15*b^2 + 16*a*c + 10*b*c*x
^2)*Sqrt[a + b*x^2 - c*x^4])/c^2 - (3*b*(5*b^2 + 12*a*c)*ArcTan[(b - 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(8*c^(5/2)))/(6*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{\left(8c^{2}x^{4}+10bc\,x^{2}+16ac+15b^{2}\right)\sqrt{-c\,x^{4}+b\,x^{2}+a}}{48c^{3}}+\frac{b\left(12ac+5b^{2}\right)\arctan\left(\frac{\sqrt{c}\left(x^{2}-\frac{b}{2c}\right)}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{32c^{\frac{7}{2}}}$
pseudoelliptic	$\frac{-16x^{4}\sqrt{-c\,x^{4}+b\,x^{2}+a}\,c^{\frac{5}{2}}-20b\,x^{2}\sqrt{-c\,x^{4}+b\,x^{2}+a}\,c^{\frac{3}{2}}-36ba\arctan\left(\frac{-2c\,x^{2}+b}{2\sqrt{c}\,\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)c-15b^{3}\arctan\left(\frac{-2c\,x^{2}+b}{2\sqrt{c}\,\sqrt{-c\,x^{4}+b}\right.}{96c^{\frac{7}{2}}}$
default	$-\frac{x^{4}\sqrt{-c\,x^{4}+b\,x^{2}+a}}{6c}-\frac{5b\,x^{2}\sqrt{-c\,x^{4}+b\,x^{2}+a}}{24c^{2}}-\frac{5b^{2}\sqrt{-c\,x^{4}+b\,x^{2}+a}}{16c^{3}}+\frac{5b^{3}\arctan\left(\frac{\sqrt{c}\left(x^{2}-\frac{b}{2c}\right)}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{32c^{\frac{7}{2}}}+\frac{3ba\arctan}{}$
elliptic	$-\frac{x^{4}\sqrt{-c\,x^{4}+b\,x^{2}+a}}{6c}-\frac{5b\,x^{2}\sqrt{-c\,x^{4}+b\,x^{2}+a}}{24c^{2}}-\frac{5b^{2}\sqrt{-c\,x^{4}+b\,x^{2}+a}}{16c^{3}}+\frac{5b^{3}\arctan\left(\frac{\sqrt{c}\left(x^{2}-\frac{b}{2c}\right)}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{32c^{\frac{7}{2}}}+\frac{3ba\arctan}{}$

input
```
int(x^7/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(8*c^2*x^4+10*b*c*x^2+16*a*c+15*b^2)/c^3*(-c*x^4+b*x^2+a)^(1/2)+1/32
*b*(12*a*c+5*b^2)/c^(7/2)*arctan(c^(1/2)*(x^2-1/2/c*b)/(-c*x^4+b*x^2+a)^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.01

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx$$
$$= \left[-\frac{3\,(5\,b^3+12\,abc)\sqrt{-c}\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}(2\,cx^2-b)\sqrt{-c}-4\,ac\right)+4\,(8\,c^3x\right.}{192\,c^4}$$
$$-\frac{3\,(5\,b^3+12\,abc)\sqrt{c}\arctan\left(\frac{\sqrt{-cx^4+bx^2+a}(2\,cx^2-b)\sqrt{c}}{2\,(c^2x^4-bcx^2-ac)}\right)+2\,(8\,c^3x^4+10\,bc^2x^2+15\,b^2c+16\,ac^2)\sqrt{-cx^4+}}{96\,c^4}$$

input
```
integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(5*b^3 + 12*a*b*c)*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4
*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*(8*c^3*x^4 +
 10*b*c^2*x^2 + 15*b^2*c + 16*a*c^2)*sqrt(-c*x^4 + b*x^2 + a))/c^4, -1/96*
(3*(5*b^3 + 12*a*b*c)*sqrt(c)*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2
 - b)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c)) + 2*(8*c^3*x^4 + 10*b*c^2*x^2 + 1
5*b^2*c + 16*a*c^2)*sqrt(-c*x^4 + b*x^2 + a))/c^4]
```

Sympy [F]

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate(x**7/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**7/sqrt(a + b*x**2 - c*x**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.23

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{-cx^4+bx^2+a}x^4}{6\,c} - \frac{5\,\sqrt{-cx^4+bx^2+a}bx^2}{24\,c^2} - \frac{5\,b^3\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{32\,c^{\frac{7}{2}}} - \frac{3\,ab\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{8\,c^{\frac{5}{2}}} - \frac{5\,\sqrt{-cx^4+bx^2+a}b^2}{16\,c^3} - \frac{\sqrt{-cx^4+bx^2+a}a}{3\,c^2}$$

input

```
integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output

```
-1/6*sqrt(-c*x^4 + b*x^2 + a)*x^4/c - 5/24*sqrt(-c*x^4 + b*x^2 + a)*b*x^2/
c^2 - 5/32*b^3*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(7/2) - 3/8*a*b*
arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(5/2) - 5/16*sqrt(-c*x^4 + b*x^
2 + a)*b^2/c^3 - 1/3*sqrt(-c*x^4 + b*x^2 + a)*a/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{1}{48}\,\sqrt{-cx^4+bx^2+a}\left(2\,x^2\left(\frac{4\,x^2}{c}+\frac{5\,b}{c^2}\right)+\frac{15\,b^2+16\,ac}{c^3}\right) - \frac{(5\,b^3+12\,abc)\log\left(\left|2\left(\sqrt{-c}x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{32\,\sqrt{-c}c^3}$$

input

```
integrate(x^7/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output

```
-1/48*sqrt(-c*x^4 + b*x^2 + a)*(2*x^2*(4*x^2/c + 5*b/c^2) + (15*b^2 + 16*a
*c)/c^3) - 1/32*(5*b^3 + 12*a*b*c)*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 +
 b*x^2 + a))*sqrt(-c) + b))/(sqrt(-c)*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{x^7}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(x^7/(a + b*x^2 - c*x^4)^(1/2),x)
```

output
```
int(x^7/(a + b*x^2 - c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.44

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{-36\sqrt{c}\,asin\left(\frac{-2c\,x^2+b}{\sqrt{4ac+b^2}}\right)abc-15\sqrt{c}\,asin\left(\frac{-2c\,x^2+b}{\sqrt{4ac+b^2}}\right)b^3+16\sqrt{c}\,\sqrt{4ac+b^2}\,ac+22\sqrt{c}\,\sqrt{4ac+b^2}\,b^2-32\sqrt{-}}{96}$$

input
```
int(x^7/(-c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - 36*sqrt(c)*asin((b - 2*c*x**2)/sqrt(4*a*c + b**2))*a*b*c - 15*sqrt(c)*
asin((b - 2*c*x**2)/sqrt(4*a*c + b**2))*b**3 + 16*sqrt(c)*sqrt(4*a*c + b**
2)*a*c + 22*sqrt(c)*sqrt(4*a*c + b**2)*b**2 - 32*sqrt(a + b*x**2 - c*x**4)
*a*c**2 - 30*sqrt(a + b*x**2 - c*x**4)*b**2*c - 20*sqrt(a + b*x**2 - c*x**
4)*b*c**2*x**2 - 16*sqrt(a + b*x**2 - c*x**4)*c**3*x**4)/(96*c**4)
```

3.996 $\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2} - \frac{x^2\sqrt{a+bx^2-cx^4}}{4c} - \frac{(3b^2+4ac)\arctan\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}$$

output
```
-3/8*b*(-c*x^4+b*x^2+a)^(1/2)/c^2-1/4*x^2*(-c*x^4+b*x^2+a)^(1/2)/c-1/16*(4
*a*c+3*b^2)*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2))/c^(5/2
)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{(-3b-2cx^2)\sqrt{a+bx^2-cx^4}}{8c^2} + \frac{(3b^2+4ac)\arctan\left(\frac{\sqrt{c}x^2}{-\sqrt{a}+\sqrt{a+bx^2-cx^4}}\right)}{8c^{5/2}}$$

input
```
Integrate[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
((-3*b - 2*c*x^2)*Sqrt[a + b*x^2 - c*x^4])/(8*c^2) + ((3*b^2 + 4*a*c)*ArcT
an[(Sqrt[c]*x^2)/(-Sqrt[a] + Sqrt[a + b*x^2 - c*x^4])])/(8*c^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1434, 1166, 27, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{-cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 1166}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{3bx^2+2a}{2\sqrt{-cx^4+bx^2+a}}dx^2}{2c}-\frac{x^2\sqrt{a+bx^2-cx^4}}{2c}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{\int \frac{3bx^2+2a}{\sqrt{-cx^4+bx^2+a}}dx^2}{4c}-\frac{x^2\sqrt{a+bx^2-cx^4}}{2c}\right)$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{2}\left(\frac{\frac{(4ac+3b^2)\int \frac{1}{\sqrt{-cx^4+bx^2+a}}dx^2}{2c}-\frac{3b\sqrt{a+bx^2-cx^4}}{c}}{4c}-\frac{x^2\sqrt{a+bx^2-cx^4}}{2c}\right)$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{1}{2}\left(\frac{\frac{(4ac+3b^2)\int \frac{1}{-x^4-4c}d\frac{b-2cx^2}{\sqrt{-cx^4+bx^2+a}}}{c}-\frac{3b\sqrt{a+bx^2-cx^4}}{c}}{4c}-\frac{x^2\sqrt{a+bx^2-cx^4}}{2c}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{-\frac{(4ac+3b^2)\arctan\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2c^{3/2}}-\frac{3b\sqrt{a+bx^2-cx^4}}{c}}{4c}-\frac{x^2\sqrt{a+bx^2-cx^4}}{2c}\right)$$

input
```
Int[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
(-1/2*(x^2*Sqrt[a + b*x^2 - c*x^4])/c + ((-3*b*Sqrt[a + b*x^2 - c*x^4])/c
- ((3*b^2 + 4*a*c)*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]
)])/(2*c^(3/2)))/(4*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{(2cx^2+3b)\sqrt{-cx^4+bx^2+a}}{8c^2}+\frac{(4ac+3b^2)\arctan\left(\frac{\sqrt{c}\left(x^2-\frac{b}{2c}\right)}{\sqrt{-cx^4+bx^2+a}}\right)}{16c^{\frac{5}{2}}}$
pseudoelliptic	$\frac{-4x^2\sqrt{-cx^4+bx^2+a}\,c^{\frac{3}{2}}-4a\arctan\left(\frac{-2cx^2+b}{2\sqrt{c}\,\sqrt{-cx^4+bx^2+a}}\right)c-3b^2\arctan\left(\frac{-2cx^2+b}{2\sqrt{c}\,\sqrt{-cx^4+bx^2+a}}\right)-6b\sqrt{-cx^4+bx^2+a}\,\sqrt{c}}{16c^{\frac{5}{2}}}$
default	$-\frac{x^2\sqrt{-cx^4+bx^2+a}}{4c}-\frac{3b\sqrt{-cx^4+bx^2+a}}{8c^2}+\frac{3b^2\arctan\left(\frac{\sqrt{c}\left(x^2-\frac{b}{2c}\right)}{\sqrt{-cx^4+bx^2+a}}\right)}{16c^{\frac{5}{2}}}+\frac{a\arctan\left(\frac{\sqrt{c}\left(x^2-\frac{b}{2c}\right)}{\sqrt{-cx^4+bx^2+a}}\right)}{4c^{\frac{3}{2}}}$
elliptic	$-\frac{x^2\sqrt{-cx^4+bx^2+a}}{4c}-\frac{3b\sqrt{-cx^4+bx^2+a}}{8c^2}+\frac{3b^2\arctan\left(\frac{\sqrt{c}\left(x^2-\frac{b}{2c}\right)}{\sqrt{-cx^4+bx^2+a}}\right)}{16c^{\frac{5}{2}}}+\frac{a\arctan\left(\frac{\sqrt{c}\left(x^2-\frac{b}{2c}\right)}{\sqrt{-cx^4+bx^2+a}}\right)}{4c^{\frac{3}{2}}}$

input
```
int(x^5/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(2*c*x^2+3*b)/c^2*(-c*x^4+b*x^2+a)^(1/2)+1/16*(4*a*c+3*b^2)/c^(5/2)*a
rctan(c^(1/2)*(x^2-1/2/c*b)/(-c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.97

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx$$
$$= \left[-\frac{(3\,b^2+4\,ac)\sqrt{-c}\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}(2\,cx^2-b)\sqrt{-c}-4\,ac\right)+4\,\sqrt{-cx^4 -}}{32\,c^3}\right.$$
$$\left.-\frac{(3\,b^2+4\,ac)\sqrt{c}\arctan\left(\frac{\sqrt{-cx^4+bx^2+a}(2\,cx^2-b)\sqrt{c}}{2\,(c^2x^4-bcx^2-ac)}\right)+2\,\sqrt{-cx^4+bx^2+a}(2\,c^2x^2+3\,bc)}{16\,c^3}\right]$$

input
```
integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((3*b^2 + 4*a*c)*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(
-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*sqrt(-c*x^4 + b*x^
2 + a)*(2*c^2*x^2 + 3*b*c))/c^3, -1/16*((3*b^2 + 4*a*c)*sqrt(c)*arctan(1/2
*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c))
 + 2*sqrt(-c*x^4 + b*x^2 + a)*(2*c^2*x^2 + 3*b*c))/c^3]
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate(x**5/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**5/sqrt(a + b*x**2 - c*x**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{-cx^4+bx^2+a}x^2}{4\,c} - \frac{3\,b^2 \arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{16\,c^{\frac{5}{2}}} - \frac{a \arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{4\,c^{\frac{3}{2}}} - \frac{3\,\sqrt{-cx^4+bx^2+a}b}{8\,c^2}$$

input
```
integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(-c*x^4 + b*x^2 + a)*x^2/c - 3/16*b^2*arcsin(-(2*c*x^2 - b)/sqrt(
b^2 + 4*a*c))/c^(5/2) - 1/4*a*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(
3/2) - 3/8*sqrt(-c*x^4 + b*x^2 + a)*b/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{1}{8}\,\sqrt{-cx^4+bx^2+a}\left(\frac{2\,x^2}{c}+\frac{3\,b}{c^2}\right) - \frac{\left(3\,b^2+4\,ac\right)\log\left(\left|2\,\left(\sqrt{-c}x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{16\,\sqrt{-c}c^2}$$

input
```
integrate(x^5/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(-c*x^4 + b*x^2 + a)*(2*x^2/c + 3*b/c^2) - 1/16*(3*b^2 + 4*a*c)*l
og(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/(sqrt(-c
)*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{x^5}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(x^5/(a + b*x^2 - c*x^4)^(1/2),x)
```

output
```
int(x^5/(a + b*x^2 - c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{-4\sqrt{c}\,asin\left(\frac{-2c\,x^2+b}{\sqrt{4ac+b^2}}\right)ac-3\sqrt{c}\,asin\left(\frac{-2c\,x^2+b}{\sqrt{4ac+b^2}}\right)b^2-6\sqrt{-c\,x^4+b\,x^2+a}\,bc-4\sqrt{-c\,x^4+b\,x^2+a}\,c^2x^2}{16c^3}$$

input
```
int(x^5/(-c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - 4*sqrt(c)*asin((b - 2*c*x**2)/sqrt(4*a*c + b**2))*a*c - 3*sqrt(c)*asin
((b - 2*c*x**2)/sqrt(4*a*c + b**2))*b**2 - 6*sqrt(a + b*x**2 - c*x**4)*b*c
 - 4*sqrt(a + b*x**2 - c*x**4)*c**2*x**2)/(16*c**3)
```

3.997 $\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx=-\frac{\sqrt{a+bx^2-cx^4}}{2c}-\frac{b\arctan\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}$$

output
```
-1/2*(-c*x^4+b*x^2+a)^(1/2)/c-1/4*b*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^
4+b*x^2+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx=-\frac{\sqrt{a+bx^2-cx^4}}{2c}+\frac{b\arctan\left(\frac{\sqrt{c}x^2}{-\sqrt{a}+\sqrt{a+bx^2-cx^4}}\right)}{2c^{3/2}}$$

input
```
Integrate[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
-1/2*Sqrt[a + b*x^2 - c*x^4]/c + (b*ArcTan[(Sqrt[c]*x^2)/(-Sqrt[a] + Sqrt[
a + b*x^2 - c*x^4])])/(2*c^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1434, 1160, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{-cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{\sqrt{-cx^4+bx^2+a}}dx^2}{2c}-\frac{\sqrt{a+bx^2-cx^4}}{c}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{-x^4-4c}d\frac{b-2cx^2}{\sqrt{-cx^4+bx^2+a}}}{c}-\frac{\sqrt{a+bx^2-cx^4}}{c}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{b\arctan\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2c^{3/2}}-\frac{\sqrt{a+bx^2-cx^4}}{c}\right)$$

input
```
Int[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
(-(Sqrt[a + b*x^2 - c*x^4]/c) - (b*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 - c*x^4])])/(2*c^(3/2)))/2
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{\sqrt{-c\,x^{4}+b\,x^{2}+a}}{2c}-\frac{b\arctan\left(\frac{-2c\,x^{2}+b}{2\sqrt{c}\,\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{4c^{\frac{3}{2}}}$	57
default	$-\frac{\sqrt{-c\,x^{4}+b\,x^{2}+a}}{2c}+\frac{b\arctan\left(\frac{\sqrt{c}\,\left(x^{2}-\frac{b}{2c}\right)}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{4c^{\frac{3}{2}}}$	58
risch	$-\frac{\sqrt{-c\,x^{4}+b\,x^{2}+a}}{2c}+\frac{b\arctan\left(\frac{\sqrt{c}\,\left(x^{2}-\frac{b}{2c}\right)}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{4c^{\frac{3}{2}}}$	58
elliptic	$-\frac{\sqrt{-c\,x^{4}+b\,x^{2}+a}}{2c}+\frac{b\arctan\left(\frac{\sqrt{c}\,\left(x^{2}-\frac{b}{2c}\right)}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{4c^{\frac{3}{2}}}$	58

input
```
int(x^3/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-c*x^4+b*x^2+a)^(1/2)/c-1/4*b*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^
4+b*x^2+a)^(1/2))/c^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.41

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx = \left[-\frac{b\sqrt{-c}\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}(2\,cx^2-b)\sqrt{-c}-4\,ac\right)+4\,\sqrt{-cx^4+bx^2+a}c}{8\,c^2}, -\frac{b\sqrt{c}\arctan\left(\frac{\sqrt{-cx^4+bx^2+a}(2\,cx^2-b)\sqrt{c}}{2\,(c^2x^4-bcx^2-ac)}\right)+2\,\sqrt{-cx^4+bx^2+a}c}{4\,c^2}\right]$$

input
```
integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(b*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-c*x^4 + b*x^2
+ a)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c) + 4*sqrt(-c*x^4 + b*x^2 + a)*c)/c^2,
-1/4*(b*sqrt(c)*arctan(1/2*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 - b)*sqrt(c)/
(c^2*x^4 - b*c*x^2 - a*c)) + 2*sqrt(-c*x^4 + b*x^2 + a)*c)/c^2]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate(x**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**3/sqrt(a + b*x**2 - c*x**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{b\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{4\,c^{\frac{3}{2}}} - \frac{\sqrt{-cx^4+bx^2+a}}{2\,c}$$

input
```
integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*b*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/c^(3/2) - 1/2*sqrt(-c*x^4
+ b*x^2 + a)/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{b\log\left(\left|2\left(\sqrt{-c}x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{4\sqrt{-c}c} - \frac{\sqrt{-cx^4+bx^2+a}}{2\,c}$$

input
```
integrate(x^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/4*b*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/
(sqrt(-c)*c) - 1/2*sqrt(-c*x^4 + b*x^2 + a)/c
```

Mupad [B] (verification not implemented)

Time = 18.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{-c\,x^4+b\,x^2+a}}{2\,c} - \frac{b\,\ln\left(\frac{\frac{b}{2}-c\,x^2}{\sqrt{-c}}+\sqrt{-c\,x^4+b\,x^2+a}\right)}{4\,{\left(-c\right)}^{3/2}}$$

input
```
int(x^3/(a + b*x^2 - c*x^4)^(1/2),x)
```

output
```
- (a + b*x^2 - c*x^4)^(1/2)/(2*c) - (b*log((b/2 - c*x^2)/(-c)^(1/2) + (a +
 b*x^2 - c*x^4)^(1/2)))/(4*(-c)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{-\sqrt{c}\,asin\left(\frac{-2c\,x^2+b}{\sqrt{4ac+b^2}}\right)b-2\sqrt{-c\,x^4+b\,x^2+a}\,c}{4c^2}$$

input
```
int(x^3/(-c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(c)*asin((b - 2*c*x**2)/sqrt(4*a*c + b**2))*b - 2*sqrt(a + b*x**2
- c*x**4)*c)/(4*c**2)
```

3.998 $\int \frac{x}{\sqrt{a+bx^2-cx^4}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}} \, dx = -\frac{\arctan\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

output
```
-1/2*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}} \, dx = \frac{\arctan\left(\frac{\sqrt{c}x^2}{-\sqrt{a}+\sqrt{a+bx^2-cx^4}}\right)}{\sqrt{c}}$$

input
```
Integrate[x/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
ArcTan[(Sqrt[c]*x^2)/(-Sqrt[a] + Sqrt[a + b*x^2 - c*x^4])]/Sqrt[c]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1432, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}}dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{-cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1092}$$

$$\int \frac{1}{-4c-x^4}d\frac{b-2cx^2}{\sqrt{a+bx^2-cx^4}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

input
```
Int[x/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
-1/2*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])]/Sqrt[c]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$-\frac{\arctan\left(\frac{-2 c\,x^{2}+b}{2 \sqrt{c}\,\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{2 \sqrt{c}}$	35
default	$\frac{\arctan\left(\frac{\sqrt{c}\,\left(x^{2}-\frac{b}{2 c}\right)}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{2 \sqrt{c}}$	36
elliptic	$\frac{\arctan\left(\frac{\sqrt{c}\,\left(x^{2}-\frac{b}{2 c}\right)}{\sqrt{-c\,x^{4}+b\,x^{2}+a}}\right)}{2 \sqrt{c}}$	36

input
```
int(x/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(1/2*(-2*c*x^2+b)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.82

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx = \left[-\frac{\sqrt{-c}\log\left(8\,c^2x^4-8\,bcx^2+b^2-4\,\sqrt{-cx^4+bx^2+a}(2\,cx^2-b)\sqrt{-c}-4\,ac\right)}{4\,c}, -\frac{\arctan\left(\frac{\sqrt{-cx^4+bx^2+a}(2\,cx^2-b)\sqrt{c}}{2\,(c^2x^4-bcx^2-ac)}\right)}{2\,\sqrt{c}}\right]$$

input
```
integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-c)*log(8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*sqrt(-c*x^4 + b*x^2 + a
)*(2*c*x^2 - b)*sqrt(-c) - 4*a*c)/c, -1/2*arctan(1/2*sqrt(-c*x^4 + b*x^2 +
 a)*(2*c*x^2 - b)*sqrt(c)/(c^2*x^4 - b*c*x^2 - a*c))/sqrt(c)]
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate(x/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*x**2 - c*x**4), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx=-\frac{\arcsin\left(-\frac{2\,cx^2-b}{\sqrt{b^2+4\,ac}}\right)}{2\,\sqrt{c}}$$

input
```
integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arcsin(-(2*c*x^2 - b)/sqrt(b^2 + 4*a*c))/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\log\left(\left|2\left(\sqrt{-c}x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{2\sqrt{-c}}$$

input
```
integrate(x/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/sq
rt(-c)
```

Mupad [B] (verification not implemented)

Time = 19.44 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\ln\left(\frac{\frac{b}{2}-c\,x^2}{\sqrt{-c}}+\sqrt{-c\,x^4+b\,x^2+a}\right)}{2\sqrt{-c}}$$

input
```
int(x/(a + b*x^2 - c*x^4)^(1/2),x)
```

output
```
log((b/2 - c*x^2)/(-c)^(1/2) + (a + b*x^2 - c*x^4)^(1/2))/(2*(-c)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{c}\,asin\left(\frac{-2c\,x^2+b}{\sqrt{4ac+b^2}}\right)}{2c}$$

input
```
int(x/(-c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(c)*asin((b - 2*c*x**2)/sqrt(4*a*c + b**2)))/(2*c)
```

3.999 $\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{\arctan\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

output
```
-1/2*arctan(1/2*(-b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2-a)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{\arctan\left(\frac{\sqrt{c}x^2-\sqrt{-a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output
```
-(ArcTan[(Sqrt[c]*x^2 - Sqrt[-a + b*x^2 + c*x^4])/Sqrt[a]]/Sqrt[a])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1434, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{cx^4+bx^2-a}}dx^2$$

$$\Big\downarrow \text{1154}$$

$$-\int \frac{1}{-x^4-4a}d\left(-\frac{2a-bx^2}{\sqrt{cx^4+bx^2-a}}\right)$$

$$\Big\downarrow \text{217}$$

$$-\frac{\arctan\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])]/Sqrt[a]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{2\sqrt{-a}}$	45
elliptic	$-\frac{\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{2\sqrt{-a}}$	45
pseudoelliptic	$-\frac{\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{2\sqrt{-a}}$	45

input
```
int(1/x/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.74

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx = \left[-\frac{\sqrt{-a}\log\left(\frac{(b^2-4\,ac)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}(bx^2-2\,a)\sqrt{-a}+8\,a^2}{x^4}\right)}{4\,a},\ \frac{\arctan\left(\frac{\sqrt{cx^4+bx^2-a}(bx^2-2\,a)\sqrt{a}}{2\,(acx^4+abx^2-a^2)}\right)}{2\,\sqrt{a}}\right]$$

input
```
integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a)*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 -
 a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4)/a, 1/2*arctan(1/2*sqrt(c*x^4 + b*
x^2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2))/sqrt(a)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x/(c*x**4+b*x**2-a)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-a + b*x**2 + c*x**4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{\arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}x^2}\right)}{2\,\sqrt{a}}$$

input
```
integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqrt(a))/sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 17.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{\ln\left(\frac{1}{x^2}\right)}{2\sqrt{-a}} - \frac{\ln\left(2\sqrt{-a}\sqrt{c\,x^4+b\,x^2-a}-2\,a+b\,x^2\right)}{2\sqrt{-a}}$$

input
```
int(1/(x*(b*x^2 - a + c*x^4)^(1/2)),x)
```

output
```
- log(1/x^2)/(2*(-a)^(1/2)) - log(2*(-a)^(1/2)*(b*x^2 - a + c*x^4)^(1/2) -
 2*a + b*x^2)/(2*(-a)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\sqrt{a}\,atan\left(\frac{\sqrt{c}\sqrt{c\,x^4+b\,x^2-a}+c\,x^2}{\sqrt{c}\sqrt{a}}\right)}{a}$$

input
```
int(1/x/(c*x^4+b*x^2-a)^(1/2),x)
```

output
```
(sqrt(a)*atan((sqrt(c)*sqrt( - a + b*x**2 + c*x**4) + c*x**2)/(sqrt(c)*sqr
t(a))))/a
```

3.1000 $\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2} - \frac{b\arctan\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

output
```
1/2*(c*x^4+b*x^2-a)^(1/2)/a/x^2-1/4*b*arctan(1/2*(-b*x^2+2*a)/a^(1/2)/(c*x
^4+b*x^2-a)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\sqrt{-a+bx^2+cx^4}}{2ax^2} - \frac{b\arctan\left(\frac{\sqrt{c}x^2-\sqrt{-a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

input
```
Integrate[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output
```
Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) - (b*ArcTan[(Sqrt[c]*x^2 - Sqrt[-a + b*
x^2 + c*x^4])/Sqrt[a]])/(2*a^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1434, 1157, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{cx^4+bx^2-a}}dx^2$$

$$\Big\downarrow \text{1157}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{x^2\sqrt{cx^4+bx^2-a}}dx^2}{2a}+\frac{\sqrt{-a+bx^2+cx^4}}{ax^2}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{\sqrt{-a+bx^2+cx^4}}{ax^2}-\frac{b\int \frac{1}{-x^4-4a}d\left(-\frac{2a-bx^2}{\sqrt{cx^4+bx^2-a}}\right)}{a}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\sqrt{-a+bx^2+cx^4}}{ax^2}-\frac{b\arctan\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2a^{3/2}}\right)$$

input
```
Int[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[-a + b*x^2 + c*x^4]/(a*x^2) - (b*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqr
t[-a + b*x^2 + c*x^4]])])/(2*a^(3/2)))/2
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\sqrt{c\,x^{4}+b\,x^{2}-a}}{2a\,x^{2}}-\frac{b\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{4a\sqrt{-a}}$	74
elliptic	$\frac{\sqrt{c\,x^{4}+b\,x^{2}-a}}{2a\,x^{2}}-\frac{b\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{4a\sqrt{-a}}$	74
pseudoelliptic	$\frac{b\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)x^{2}-2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{4x^{2}\left(-a\right)^{\frac{3}{2}}}$	77
risch	$-\frac{-c\,x^{4}-b\,x^{2}+a}{2a\,x^{2}\sqrt{c\,x^{4}+b\,x^{2}-a}}-\frac{b\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{4a\sqrt{-a}}$	88

input
```
int(1/x^3/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^4+b*x^2-a)^(1/2)/a/x^2-1/4*b/a/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(
1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx = \left[-\frac{\sqrt{-a}bx^2\log\left(\frac{(b^2-4\,ac)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}(bx^2-2\,a)\sqrt{-a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2-a}a}{8\,a^2x^2}, \frac{\sqrt{a}bx^2\arctan\left(\frac{\sqrt{cx}}{}\right.}{}\right.$$

input
```
integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(-a)*b*x^2*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 - 4*sqrt(c*x^4 +
b*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 - a
)*a)/(a^2*x^2), 1/4*(sqrt(a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 - a)*(b*x
^2 - 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2)) + 2*sqrt(c*x^4 + b*x^2 - a)*a
)/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x**3/(c*x**4+b*x**2-a)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(-a + b*x**2 + c*x**4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{b\arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}x^2}\right)}{4\,a^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2-a}}{2\,ax^2}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*b*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(3/2)
+ 1/2*sqrt(c*x^4 + b*x^2 - a)/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx = \frac{b\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}} - \frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}\right)b-2\,a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}\right)^2+a\right)a}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
1/2*b*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqrt(a))/a^(3/2) - 1
/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))*b - 2*a*sqrt(c))/(((sqrt(c)*x^
2 - sqrt(c*x^4 + b*x^2 - a))^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 17.80 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx=\frac{\sqrt{c\,x^4+b\,x^2-a}}{2\,a\,x^2}-\frac{b\,\mathrm{atanh}\left(\frac{a-\frac{b\,x^2}{2}}{\sqrt{-a}\,\sqrt{c\,x^4+b\,x^2-a}}\right)}{4\,{\left(-a\right)}^{3/2}}$$

input
```
int(1/(x^3*(b*x^2 - a + c*x^4)^(1/2)),x)
```

output
```
(b*x^2 - a + c*x^4)^(1/2)/(2*a*x^2) - (b*atanh((a - (b*x^2)/2)/((-a)^(1/2)
*(b*x^2 - a + c*x^4)^(1/2))))/(4*(-a)^(3/2))
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4+b\,x^2-a}\,x^3}dx$$

input
```
int(1/x^3/(c*x^4+b*x^2-a)^(1/2),x)
```

output
```
int(1/(sqrt( - a + b*x**2 + c*x**4)*x**3),x)
```

3.1001 $\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx=\frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}-\frac{(3b^2+4ac)\arctan\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}$$

output
```
1/4*(c*x^4+b*x^2-a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x^2-1/16*(
4*a*c+3*b^2)*arctan(1/2*(-b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2-a)^(1/2))/a^(5/2
)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx=\frac{(2a+3bx^2)\sqrt{-a+bx^2+cx^4}}{8a^2x^4}+\frac{(-3b^2-4ac)\arctan\left(\frac{\sqrt{c}x^2-\sqrt{-a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{5/2}}$$

input
```
Integrate[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output
```
((2*a + 3*b*x^2)*Sqrt[-a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((-3*b^2 - 4*a*c)
*ArcTan[(Sqrt[c]*x^2 - Sqrt[-a + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(5/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1434, 1167, 27, 1228, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt{cx^4+bx^2-a}}dx^2$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{2}\left(\frac{\int \frac{2cx^2+3b}{2x^4\sqrt{cx^4+bx^2-a}}dx^2}{2a}+\frac{\sqrt{-a+bx^2+cx^4}}{2ax^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{2cx^2+3b}{x^4\sqrt{cx^4+bx^2-a}}dx^2}{4a}+\frac{\sqrt{-a+bx^2+cx^4}}{2ax^4}\right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2}\left(\frac{\frac{(4ac+3b^2)\int \frac{1}{x^2\sqrt{cx^4+bx^2-a}}dx^2}{2a}+\frac{3b\sqrt{-a+bx^2+cx^4}}{ax^2}}{4a}+\frac{\sqrt{-a+bx^2+cx^4}}{2ax^4}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{\frac{3b\sqrt{-a+bx^2+cx^4}}{ax^2}-\frac{(4ac+3b^2)\int \frac{1}{-x^4-4a}d\left(-\frac{2a-bx^2}{\sqrt{cx^4+bx^2-a}}\right)}{a}}{4a}+\frac{\sqrt{-a+bx^2+cx^4}}{2ax^4}\right)$$

$$\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\frac{3b\sqrt{-a+bx^2+cx^4}}{ax^2}-\frac{(4ac+3b^2)\arctan\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2a^{3/2}}}{4a}+\frac{\sqrt{-a+bx^2+cx^4}}{2ax^4}\right)$$

input
```
Int[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^4) + ((3*b*Sqrt[-a + b*x^2 + c*x^4])/(a*x
^2) - ((3*b^2 + 4*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c
*x^4])])/(2*a^(3/2)))/(4*a))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{\left(-c\,x^{4}-b\,x^{2}+a\right)\left(3b\,x^{2}+2a\right)}{8a^{2}x^{4}\sqrt{c\,x^{4}+b\,x^{2}-a}}-\frac{\left(4ac+3b^{2}\right)\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{16a^{2}\sqrt{-a}}$
default	$\frac{\sqrt{c\,x^{4}+b\,x^{2}-a}}{4a\,x^{4}}+\frac{3b\sqrt{c\,x^{4}+b\,x^{2}-a}}{8a^{2}x^{2}}-\frac{3b^{2}\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{16a^{2}\sqrt{-a}}-\frac{c\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{4a\sqrt{-a}}$
elliptic	$\frac{\sqrt{c\,x^{4}+b\,x^{2}-a}}{4a\,x^{4}}+\frac{3b\sqrt{c\,x^{4}+b\,x^{2}-a}}{8a^{2}x^{2}}-\frac{3b^{2}\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{16a^{2}\sqrt{-a}}-\frac{c\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)}{4a\sqrt{-a}}$
pseudoelliptic	$\frac{-4c\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)a\,x^{4}-3b^{2}\ln\left(\frac{-2a+b\,x^{2}+2\sqrt{-a}\,\sqrt{c\,x^{4}+b\,x^{2}-a}}{x^{2}}\right)x^{4}+6b\sqrt{c\,x^{4}+b\,x^{2}-a}\,x^{2}\sqrt{-a}-4(-a}{16x^{4}(-a)^{\frac{5}{2}}}$

input
```
int(1/x^5/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-c*x^4-b*x^2+a)*(3*b*x^2+2*a)/a^2/x^4/(c*x^4+b*x^2-a)^(1/2)-1/16/a^2
*(4*a*c+3*b^2)/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2
))/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx$$
$$= \left[-\frac{(3\,b^2+4\,ac)\sqrt{-a}x^4\log\left(\frac{(b^2-4\,ac)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}(bx^2-2\,a)\sqrt{-a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2-a}(3\,abx^2+2}{32\,a^3x^4}\right.$$

input
```
integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((3*b^2 + 4*a*c)*sqrt(-a)*x^4*log(((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 -
4*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*sqrt(c*
x^4 + b*x^2 - a)*(3*a*b*x^2 + 2*a^2))/(a^3*x^4), 1/16*((3*b^2 + 4*a*c)*sqr
t(a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4
 + a*b*x^2 - a^2)) + 2*sqrt(c*x^4 + b*x^2 - a)*(3*a*b*x^2 + 2*a^2))/(a^3*x
^4)]
```

Sympy [F]

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x**5/(c*x**4+b*x**2-a)**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt(-a + b*x**2 + c*x**4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{3\,b^2\arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}x^2}\right)}{16\,a^{\frac{5}{2}}} - \frac{c\arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}x^2}\right)}{4\,a^{\frac{3}{2}}} + \frac{3\sqrt{cx^4+bx^2-a}b}{8\,a^2x^2} + \frac{\sqrt{cx^4+bx^2-a}}{4\,ax^4}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
-3/16*b^2*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(5/
2) - 1/4*c*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(3
/2) + 3/8*sqrt(c*x^4 + b*x^2 - a)*b/(a^2*x^2) + 1/4*sqrt(c*x^4 + b*x^2 - a
)/(a*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(96) = 192.

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.95

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx = \frac{(3\,b^2+4\,ac)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}} - \frac{3\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}\right)^3b^2+4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}\right)^3ac+5\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}\right)ab^2-}{8\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}\right)^2+a\right)^2a^2}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
1/8*(3*b^2 + 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sqrt(a
))/a^(5/2) - 1/8*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^3*b^2 + 4*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^3*a*c + 5*(sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 - a))*a*b^2 - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))*a^2*c - 8*a^
2*b*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^2 + a)^2*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{1}{x^5\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

input
```
int(1/(x^5*(b*x^2 - a + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^5*(b*x^2 - a + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4+b\,x^2-a}\,x^5}dx$$

input
```
int(1/x^5/(c*x^4+b*x^2-a)^(1/2),x)
```

output
```
int(1/(sqrt( - a + b*x**2 + c*x**4)*x**5),x)
```

3.1002 $\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx=\frac{\sqrt{-a+bx^2+cx^4}}{6ax^6}+\frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4}+\frac{(15b^2+16ac)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}-\frac{b(5b^2+12ac)\arctan\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}}$$

output
```
1/6*(c*x^4+b*x^2-a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x^4+1/48*
(16*a*c+15*b^2)*(c*x^4+b*x^2-a)^(1/2)/a^3/x^2-1/32*b*(12*a*c+5*b^2)*arctan
(1/2*(-b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2-a)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx=\frac{\sqrt{-a+bx^2+cx^4}(8a^2+10abx^2+15b^2x^4+16acx^4)}{48a^3x^6}+\frac{(-5b^3-12abc)\arctan\left(\frac{\sqrt{c}x^2-\sqrt{-a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{7/2}}$$

input
```
Integrate[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[-a + b*x^2 + c*x^4]*(8*a^2 + 10*a*b*x^2 + 15*b^2*x^4 + 16*a*c*x^4))/
(48*a^3*x^6) + ((-5*b^3 - 12*a*b*c)*ArcTan[(Sqrt[c]*x^2 - Sqrt[-a + b*x^2
+ c*x^4])/Sqrt[a]])/(16*a^(7/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1434, 1167, 27, 1237, 27, 1228, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{1}{x^8\sqrt{cx^4+bx^2-a}}dx^2$$

$$\Big\downarrow \text{1167}$$

$$\frac{1}{2}\left(\frac{\int \frac{4cx^2+5b}{2x^6\sqrt{cx^4+bx^2-a}}dx^2}{3a}+\frac{\sqrt{-a+bx^2+cx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{4cx^2+5b}{x^6\sqrt{cx^4+bx^2-a}}dx^2}{6a}+\frac{\sqrt{-a+bx^2+cx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{15b^2+10cx^2b+16ac}{2x^4\sqrt{cx^4+bx^2-a}}dx^2}{2a}+\frac{5b\sqrt{-a+bx^2+cx^4}}{2ax^4}}{6a}+\frac{\sqrt{-a+bx^2+cx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{15b^2+10cx^2b+16ac}{x^4\sqrt{cx^4+bx^2-a}}dx^2}{4a}+\frac{5b\sqrt{-a+bx^2+cx^4}}{2ax^4}}{6a}+\frac{\sqrt{-a+bx^2+cx^4}}{3ax^6}\right)$$

$$\Big\downarrow \textbf{1228}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{3b(12ac+5b^2)\int \frac{1}{x^2\sqrt{cx^4+bx^2-a}}dx^2}{2a}+\frac{(16ac+15b^2)\sqrt{-a+bx^2+cx^4}}{ax^2}}{4a}+\frac{5b\sqrt{-a+bx^2+cx^4}}{2ax^4}}{6a}+\frac{\sqrt{-a+bx^2+cx^4}}{3ax^6}\right)$$

$$\Big\downarrow \textbf{1154}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{(16ac+15b^2)\sqrt{-a+bx^2+cx^4}}{ax^2}-\frac{3b(12ac+5b^2)\int \frac{1}{-x^4-4a}d\left(-\frac{2a-bx^2}{\sqrt{cx^4+bx^2-a}}\right)}{a}}{4a}+\frac{5b\sqrt{-a+bx^2+cx^4}}{2ax^4}}{6a}+\frac{\sqrt{-a+bx^2+cx^4}}{3ax^6}\right)$$

$$\Big\downarrow \textbf{217}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{(16ac+15b^2)\sqrt{-a+bx^2+cx^4}}{ax^2}-\frac{3b(12ac+5b^2)\arctan\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2a^{3/2}}}{4a}+\frac{5b\sqrt{-a+bx^2+cx^4}}{2ax^4}}{6a}+\frac{\sqrt{-a+bx^2+cx^4}}{3ax^6}\right)$$

input

```
Int[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```

output

```
(Sqrt[-a + b*x^2 + c*x^4]/(3*a*x^6) + ((5*b*Sqrt[-a + b*x^2 + c*x^4])/(2*a
*x^4) + (((15*b^2 + 16*a*c)*Sqrt[-a + b*x^2 + c*x^4])/(a*x^2) - (3*b*(5*b^
2 + 12*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])])/(2
*a^(3/2)))/(4*a))/(6*a))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{\frac{3 b\, x^{6} \left(a c +\frac{5 b^{2}}{12}\right) \ln \left(\frac{-2 a +b \,x^{2}+2 \sqrt{-a}\, \sqrt{c \,x^{4}+b \,x^{2}-a}}{x^{2}}\right)}{8}-\frac{5 \left(-\frac{2 \left(\frac{8 c \,x^{2}}{5}+b \right) x^{2} \left(-a \right)^{\frac{3}{2}}}{3}+b^{2} x^{4} \sqrt{-a}+\frac{8 \left(-a \right)^{\frac{5}{2}}}{15}\right) \sqrt{c \,x^{4}+b \,x^{2}-a}}{16}}{x^{6} \left(-a \right)^{\frac{7}{2}}}$
risch	$-\frac{\left(-c \,x^{4}-b \,x^{2}+a \right) \left(16 a c \,x^{4}+15 b^{2} x^{4}+10 a b \,x^{2}+8 a^{2}\right)}{48 a^{3} x^{6} \sqrt{c \,x^{4}+b \,x^{2}-a}}-\frac{b \left(12 a c +5 b^{2}\right) \ln \left(\frac{-2 a +b \,x^{2}+2 \sqrt{-a}\, \sqrt{c \,x^{4}+b \,x^{2}-a}}{x^{2}}\right)}{32 a^{3} \sqrt{-a}}$
default	$\frac{\sqrt{c \,x^{4}+b \,x^{2}-a}}{6 a \,x^{6}}+\frac{5 b \sqrt{c \,x^{4}+b \,x^{2}-a}}{24 a^{2} x^{4}}+\frac{5 b^{2} \sqrt{c \,x^{4}+b \,x^{2}-a}}{16 a^{3} x^{2}}-\frac{5 b^{3} \ln \left(\frac{-2 a +b \,x^{2}+2 \sqrt{-a}\, \sqrt{c \,x^{4}+b \,x^{2}-a}}{x^{2}}\right)}{32 a^{3} \sqrt{-a}}-\frac{3 b c \ln \left(\frac{-2 a +b}{}\right.}{}$
elliptic	$\frac{\sqrt{c \,x^{4}+b \,x^{2}-a}}{6 a \,x^{6}}+\frac{5 b \sqrt{c \,x^{4}+b \,x^{2}-a}}{24 a^{2} x^{4}}+\frac{5 b^{2} \sqrt{c \,x^{4}+b \,x^{2}-a}}{16 a^{3} x^{2}}-\frac{5 b^{3} \ln \left(\frac{-2 a +b \,x^{2}+2 \sqrt{-a}\, \sqrt{c \,x^{4}+b \,x^{2}-a}}{x^{2}}\right)}{32 a^{3} \sqrt{-a}}-\frac{3 b c \ln \left(\frac{-2 a +b}{}\right.}{}$

input
```
int(1/x^7/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/8/(-a)^(7/2)*(b*x^6*(a*c+5/12*b^2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*
x^2-a)^(1/2))/x^2)-5/6*(-2/3*(8/5*c*x^2+b)*x^2*(-a)^(3/2)+b^2*x^4*(-a)^(1/
2)+8/15*(-a)^(5/2))*(c*x^4+b*x^2-a)^(1/2))/x^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.77

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx$$
$$= \left[-\frac{3\,(5\,b^3+12\,abc)\sqrt{-a}x^6\log\left(\frac{(b^2-4\,ac)x^4-8\,abx^2-4\,\sqrt{cx^4+bx^2-a}(bx^2-2\,a)\sqrt{-a}+8\,a^2}{x^4}\right)-4\,(10\,a^2bx^2+(15\,ab^2+\right.}{192\,a^4x^6}$$

input
```
integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(5*b^3 + 12*a*b*c)*sqrt(-a)*x^6*log(((b^2 - 4*a*c)*x^4 - 8*a*b*
x^2 - 4*sqrt(c*x^4 + b*x^2 - a)*(b*x^2 - 2*a)*sqrt(-a) + 8*a^2)/x^4) - 4*(
10*a^2*b*x^2 + (15*a*b^2 + 16*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 - a))
/(a^4*x^6), 1/96*(3*(5*b^3 + 12*a*b*c)*sqrt(a)*x^6*arctan(1/2*sqrt(c*x^4 +
 b*x^2 - a)*(b*x^2 - 2*a)*sqrt(a)/(a*c*x^4 + a*b*x^2 - a^2)) + 2*(10*a^2*b
*x^2 + (15*a*b^2 + 16*a^2*c)*x^4 + 8*a^3)*sqrt(c*x^4 + b*x^2 - a))/(a^4*x^
6)]
```

Sympy [F]

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/x**7/(c*x**4+b*x**2-a)**(1/2),x)
```

output
```
Integral(1/(x**7*sqrt(-a + b*x**2 + c*x**4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx = -\frac{5\,b^3\arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}x^2}\right)}{32\,a^{\frac{7}{2}}} - \frac{3\,bc\arcsin\left(-\frac{b}{\sqrt{b^2+4\,ac}}+\frac{2\,a}{\sqrt{b^2+4\,ac}x^2}\right)}{8\,a^{\frac{5}{2}}} + \frac{5\,\sqrt{cx^4+bx^2-a}b^2}{16\,a^3x^2} + \frac{\sqrt{cx^4+bx^2-a}c}{3\,a^2x^2} + \frac{5\,\sqrt{cx^4+bx^2-a}b}{24\,a^2x^4} + \frac{\sqrt{cx^4+bx^2-a}}{6\,ax^6}$$

input
```
integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
-5/32*b^3*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^(7/
2) - 3/8*b*c*arcsin(-b/sqrt(b^2 + 4*a*c) + 2*a/(sqrt(b^2 + 4*a*c)*x^2))/a^
(5/2) + 5/16*sqrt(c*x^4 + b*x^2 - a)*b^2/(a^3*x^2) + 1/3*sqrt(c*x^4 + b*x^
2 - a)*c/(a^2*x^2) + 5/24*sqrt(c*x^4 + b*x^2 - a)*b/(a^2*x^4) + 1/6*sqrt(c
*x^4 + b*x^2 - a)/(a*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(131) = 262.

Time = 0.14 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.23

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx = \frac{(5\,b^3+12\,abc)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}}{\sqrt{a}}\right)}{16\,a^{\frac{7}{2}}} - \frac{15\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}\right)^5 b^3 + 36\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}\right)^5 abc + 40\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2-a}\right)^3 c}{}$$

input
```
integrate(1/x^7/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
1/16*(5*b^3 + 12*a*b*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))/sq
rt(a))/a^(7/2) - 1/48*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^5*b^3 +
36*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^5*a*b*c + 40*(sqrt(c)*x^2 - sqr
t(c*x^4 + b*x^2 - a))^3*a*b^3 + 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))
^3*a^2*b*c - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^2*a^3*c^(3/2) + 33
*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))*a^2*b^3 - 36*(sqrt(c)*x^2 - sqrt(
c*x^4 + b*x^2 - a))*a^3*b*c - 48*a^3*b^2*sqrt(c) - 32*a^4*c^(3/2))/(((sqrt
(c)*x^2 - sqrt(c*x^4 + b*x^2 - a))^2 + a)^3*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{1}{x^7\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

input
```
int(1/(x^7*(b*x^2 - a + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^7*(b*x^2 - a + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+b\,x^2-a}\,x^7}dx$$

input
```
int(1/x^7/(c*x^4+b*x^2-a)^(1/2),x)
```

output
```
int(1/(sqrt( - a + b*x**2 + c*x**4)*x**7),x)
```

3.1003 $\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 409

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{x\sqrt{a+bx^2-cx^4}}{3c} - \frac{b\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}\left(b^2+ac-b\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}$$

output
```
-1/3*x*(-c*x^4+b*x^2+a)^(1/2)/c-1/6*b*(b-(4*a*c+b^2)^(1/2))*(b+(4*a*c+b^2)
^(1/2))^(1/2)*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*x^2/(b+(4*a*c
+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*x/(b+(4*a*c+b^2)^(1/2))^(1/2
),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(5/2)/(-c
*x^4+b*x^2+a)^(1/2)+1/6*(b+(4*a*c+b^2)^(1/2))^(1/2)*(b^2+a*c-b*(4*a*c+b^2)
^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^
(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*x/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+
(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(5/2)/(-c*x^4+b
*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.53 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.12

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx$$
$$= \frac{2c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x(-a-bx^2+cx^4)-i\sqrt{2}b\left(-b+\sqrt{b^2+4ac}\right)\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{-b+\sqrt{b^2+4ac}+2cx^2}{-b+\sqrt{b^2+4ac}}}E\Big(i\mathrm{arcs}}{}$$

input
```
Integrate[x^4/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
(2*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x*(-a - b*x^2 + c*x^4) - I*Sqrt[2]
*b*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sq
rt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 +
 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x
], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-b^2 - a*
c + b*Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*
a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x],
(b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*c^2*Sqrt[-(c/(b + Sqr
t[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1442, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx$$
$$\Big\downarrow \text{ 1442}$$

$$\frac{\int \frac{2bx^2+a}{\sqrt{-cx^4+bx^2+a}}dx}{3c}-\frac{x\sqrt{a+bx^2-cx^4}}{3c}$$

$\Big\downarrow$ 1514

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{2bx^2+a}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{3c\sqrt{a+bx^2-cx^4}}-\frac{x\sqrt{a+bx^2-cx^4}}{3c}$$

$\Big\downarrow$ 399

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{c}-\frac{b\left(b-\sqrt{4ac+b^2}\right)\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4a}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4a}}}}}{c}\right.}{3c\sqrt{a+bx^2-cx^4}}$$

$$\frac{x\sqrt{a+bx^2-cx^4}}{3c}$$

$\Big\downarrow$ 321

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}c^{3/2}}-\frac{b\left(b-\sqrt{4\right.}}{}\right.}{3c\sqrt{a+bx^2-cx^4}}$$

$$\frac{x\sqrt{a+bx^2-cx^4}}{3c}$$

$\Big\downarrow$ 327

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}c^{3/2}}-\frac{b\left(b-\sqrt{4\right.}}{}\right.}{3c\sqrt{a+bx^2-cx^4}}$$

$$\frac{x\sqrt{a+bx^2-cx^4}}{3c}$$

input
```
Int[x^4/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
-1/3*(x*Sqrt[a + b*x^2 - c*x^4])/c + (Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(-((b*(b - Sqrt[b^2 +
4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c
])])/(Sqrt[2]*c^(3/2))) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^2 + a*c - b*Sqrt
[b^2 + 4*a*c])*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*
a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2)
)))/(3*c*Sqrt[a + b*x^2 - c*x^4])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1442
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
x] - Simp[d^4/(c*(m + 4*p + 1))   Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.96

method	result
default	$-\frac{x\sqrt{-c\,x^{4}+b\,x^{2}+a}}{3c}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^{2}}\right)}{ac}}}{2}\right.}{12c\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}\,\sqrt{-c\,x^{4}+b\,x^{2}+a}}$
risch	$-\frac{x\sqrt{-c\,x^{4}+b\,x^{2}+a}}{3c}+\frac{\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}\,\sqrt{-c\,x^{4}+b\,x^{2}+a}}}{}$
elliptic	$-\frac{x\sqrt{-c\,x^{4}+b\,x^{2}+a}}{3c}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^{2}}\right)}{ac}}}{2}\right.}{12c\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}\,\sqrt{-c\,x^{4}+b\,x^{2}+a}}$

input
```
int(x^4/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(-c*x^4+b*x^2+a)^(1/2)/c+1/12/c*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3/c*b
*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/
(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2
)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.77

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{2\sqrt{\frac{1}{2}}\left(b\sqrt{-c}cx\sqrt{\frac{b^2+4ac}{c^2}}+b^2\sqrt{-c}x\right)\sqrt{\frac{c\sqrt{\frac{b^2+4ac}{c^2}}+b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2+4ac}{c^2}}+b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2+4ac}{c^2}}-b^2-2ac}{2ac})-\cdots}{\cdots}$$

input
```
integrate(x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(1/2)*(b*sqrt(-c)*c*x*sqrt((b^2 + 4*a*c)/c^2) + b^2*sqrt(-c)*x
)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt
((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)/x), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) -
 b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*b*c + c^2)*sqrt(-c)*x*sqrt((b^2 + 4*a
*c)/c^2) + (2*b^2 - b*c)*sqrt(-c)*x)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/
c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)/x),
 1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) - b^2 - 2*a*c)/(a*c)) + 2*sqrt(-c*x^4 +
b*x^2 + a)*(c^2*x^2 + 2*b*c))/(c^3*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate(x**4/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**4/sqrt(a + b*x**2 - c*x**4), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{x^4}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(-c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{x^4}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{x^4}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(x^4/(a + b*x^2 - c*x^4)^(1/2),x)
```

output
```
int(x^4/(a + b*x^2 - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{-\sqrt{-c\,x^4+b\,x^2+a}\,x+\left(\int \frac{\sqrt{-c\,x^4+b\,x^2+a}}{-c\,x^4+b\,x^2+a}dx\right)a+2\left(\int \frac{\sqrt{-c\,x^4+b\,x^2+a}\,x^2}{-c\,x^4+b\,x^2+a}dx\right)b}{3c}$$

input
```
int(x^4/(-c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**2 - c*x**4)*x + int(sqrt(a + b*x**2 - c*x**4)/(a + b*x**
2 - c*x**4),x)*a + 2*int((sqrt(a + b*x**2 - c*x**4)*x**2)/(a + b*x**2 - c*
x**4),x)*b)/(3*c)
```

3.1004 $\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 377

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}} + \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\cdots}{b-\cdots}\right.}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

output
```
-1/4*(b-(4*a*c+b^2)^(1/2))*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b-(4*a*
c+b^2)^(1/2)))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(
1/2)*c^(1/2)*x/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*
c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(3/2)/(-c*x^4+b*x^2+a)^(1/2)+1/4*(b-(4*a*c
+b^2)^(1/2))*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))
^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*x
/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))
^(1/2))*2^(1/2)/c^(3/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.09 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{i\left(-b+\sqrt{b^2+4ac}\right)\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|-\frac{b+\sqrt{b^2+4ac}}{-b+\sqrt{b^2+4ac}}\right.\right.}{2\sqrt{2}c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{a+bx^2-cx^4}}$$

input
```
Integrate[x^2/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
((-1/2*I)*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a
*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b
+ Sqrt[b^2 + 4*a*c]))] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^
2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))]))/(
Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1460, 389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx$$

$$\Big\downarrow \text{1460}$$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{\sqrt{a+bx^2-cx^4}}$$

$$\Big\downarrow \ 389$$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(b-\sqrt{4ac+b^2}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{2c}-\frac{\left(b-\sqrt{4ac+b^2}\right)\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{2c}\right)}{\sqrt{a+bx^2-cx^4}}$$

$$\Big\downarrow \ 321$$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}}-\frac{\left(b-\sqrt{4ac+b^2}\right)\ldots}{}\right.}{\sqrt{a+bx^2-cx^4}}$$

$$\Big\downarrow \ 327$$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}}-\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\ldots}}{}\right.}{\sqrt{a+bx^2-cx^4}}$$

input
```
Int[x^2/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
(Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[
b^2 + 4*a*c])]*(-1/2*((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*
EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sq
rt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2)) + ((b - Sqrt[
b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 +
 4*a*c])])/(2*Sqrt[2]*c^(3/2))))/Sqrt[a + b*x^2 - c*x^4]
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 1460
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(
b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[x^2/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sq
rt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.58

method	result
default	$-\frac{a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\left(\operatorname{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)-\operatorname{EllipticE}\left(x\sqrt{\cdots}\right.\right.}{2\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\sqrt{-cx^4+bx^2+a}\left(b+\sqrt{4ac+b^2}\right)}$
elliptic	$-\frac{a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\left(\operatorname{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right)-\operatorname{EllipticE}\left(x\sqrt{\cdots}\right.\right.}{2\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\sqrt{-cx^4+bx^2+a}\left(b+\sqrt{4ac+b^2}\right)}$

input
```
int(x^2/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1
/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^
(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a
/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(\sqrt{-c}cx\sqrt{\frac{b^2+4\,ac}{c^2}}+b\sqrt{-c}x\right)\sqrt{\frac{c\sqrt{\frac{b^2+4\,ac}{c^2}}+b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2+4\,ac}{c^2}}+b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2+4\,ac}{c^2}}-b^2-2\,ac}{2\,ac})-\sqrt{\frac{1}{2}}(}{}$$

input
```
integrate(x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(sqrt(-c)*c*x*sqrt((b^2 + 4*a*c)/c^2) + b*sqrt(-c)*x)*sqrt
((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sq
rt((b^2 + 4*a*c)/c^2) + b)/c)/x), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c^2) - b^2 -
 2*a*c)/(a*c)) - sqrt(1/2)*(sqrt(-c)*c*x*sqrt((b^2 + 4*a*c)/c^2) + b*sqrt(
-c)*x)*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)*elliptic_f(arcsin(sqrt(1/2)
*sqrt((c*sqrt((b^2 + 4*a*c)/c^2) + b)/c)/x), 1/2*(b*c*sqrt((b^2 + 4*a*c)/c
^2) - b^2 - 2*a*c)/(a*c)) + 2*sqrt(-c*x^4 + b*x^2 + a)*c)/(c^2*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate(x**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + b*x**2 - c*x**4), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{x^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{x^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate(x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{x^2}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(x^2/(a + b*x^2 - c*x^4)^(1/2),x)
```

output
```
int(x^2/(a + b*x^2 - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{\sqrt{-c\,x^4+b\,x^2+a}\,x^2}{-c\,x^4+b\,x^2+a}dx$$

input
```
int(x^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(a + b*x**2 - c*x**4)*x**2)/(a + b*x**2 - c*x**4),x)
```

3.1005 $\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 169

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

output
```
1/2*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(1
-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c^(1/2)*x/(b+(4*a*
c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*2
^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right),-\frac{b+\sqrt{b^2+4ac}}{-b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{a+bx^2-cx^4}}$$

input
```
Integrate[1/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
 Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 +
4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt[
2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1417, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}dx$$

$$\Big\downarrow \text{1417}$$

$$\frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{\sqrt{a+bx^2-cx^4}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

input
```
Int[1/Sqrt[a + b*x^2 - c*x^4],x]
```

output
```
(Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*S
qrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2
+ 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}\,\sqrt{-c\,x^{4}+b\,x^{2}+a}}$	145
elliptic	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}\,\sqrt{-c\,x^{4}+b\,x^{2}+a}}$	145

input
```
int(1/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(
4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(a^{\frac{3}{2}}\sqrt{\frac{b^2+4\,ac}{a^2}}+\sqrt{a}b\right)\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}-b}{a}}F\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}-b}{a}}\right)\mid -\frac{ab\sqrt{\frac{b^2+4\,ac}{a^2}}+b^2+2\,ac}{2\,ac}\right)}{2\,ac}$$

input
```
integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/2)*(a^(3/2)*sqrt((b^2 + 4*a*c)/a^2) + sqrt(a)*b)*sqrt((a*sqrt((
b^2 + 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2
+ 4*a*c)/a^2) - b)/a)), -1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) + b^2 + 2*a*c)/(
a*c))/(a*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate(1/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*x**2 - c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}}\,dx$$

input
```
integrate(1/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/(a + b*x^2 - c*x^4)^(1/2),x)
```

output
```
int(1/(a + b*x^2 - c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{\sqrt{-c\,x^4+b\,x^2+a}}{-c\,x^4+b\,x^2+a}dx$$

input
```
int(1/(-c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 - c*x**4)/(a + b*x**2 - c*x**4),x)
```

3.1006 $\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 408

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{a+bx^2-cx^4}}{ax} + \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}a\sqrt{c}\sqrt{a+bx^2-cx^4}} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}a\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

output
```
-(-c*x^4+b*x^2+a)^(1/2)/a/x+1/4*(b-(4*a*c+b^2)^(1/2))*(b+(4*a*c+b^2)^(1/2)
)^(1/2)*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^
(1/2)))^(1/2)*EllipticE(2^(1/2)*c^(1/2)*x/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+
(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a/c^(1/2)/(-c*x^4
+b*x^2+a)^(1/2)-1/4*(b-(4*a*c+b^2)^(1/2))*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2
*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2
)*EllipticF(2^(1/2)*c^(1/2)*x/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^
(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/a/c^(1/2)/(-c*x^4+b*x^2+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx$$
$$= \frac{-\frac{4a}{x}-4bx+4cx^3+\frac{i\left(-b+\sqrt{b^2+4ac}\right)\sqrt{2+\frac{4cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\left(E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)-\text{Ellip}\right.}{\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}}}{4a\sqrt{a+bx^2-cx^4}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

output
```
((-4*a)/x - 4*b*x + 4*c*x^3 + (I*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[2 + (4*c*x^
2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(
EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sq
rt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sq
rt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2
 + 4*a*c])]))/Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))])/(4*a*Sqrt[a + b*x^2 - c*
x^4])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1443, 25, 27, 1460, 389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}dx$$
$$\Big\downarrow \text{ 1443}$$
$$\frac{\int -\frac{cx^2}{\sqrt{-cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2-cx^4}}{ax}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{cx^2}{\sqrt{-cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2-cx^4}}{ax}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\int \frac{x^2}{\sqrt{-cx^4+bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2-cx^4}}{ax}$$

$$\Big\downarrow \text{1460}$$

$$-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{x^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{a\sqrt{a+bx^2-cx^4}}-\frac{\sqrt{a+bx^2-cx^4}}{ax}$$

$$\Big\downarrow \text{389}$$

$$-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(b-\sqrt{4ac+b^2}\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{2c}-\frac{\left(b-\sqrt{4ac+b^2}\right)\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{2c}\right)}{a\sqrt{a+bx^2-cx^4}}$$

$$\frac{\sqrt{a+bx^2-cx^4}}{ax}$$

$$\Big\downarrow \text{321}$$

$$-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}}-\frac{\left(b-\sqrt{4ac+b\right.}}{}\right)}{a\sqrt{a+bx^2-cx^4}}$$

$$\frac{\sqrt{a+bx^2-cx^4}}{ax}$$

$$\Big\downarrow \text{327}$$

$$-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}}-\frac{\left(b-\sqrt{4ac+b\right.}}{}\right.}{a\sqrt{a+bx^2-cx^4}}$$

$$\frac{\sqrt{a+bx^2-cx^4}}{ax}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

output
```
-(Sqrt[a + b*x^2 - c*x^4]/(a*x)) - (c*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(-1/2*((b - Sqrt[b^2 +
 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*
c])])/(Sqrt[2]*c^(3/2)) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a
*c]]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b
 + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2))))/(a*S
qrt[a + b*x^2 - c*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1460
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(
b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[x^2/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sq
rt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && NegQ[c/a]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.59

method	result
default	$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{a x}+\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right.\right.}{2\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}\,\left(b+\sqrt{4ac+\right.}}$
risch	$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{a x}+\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right.\right.}{2\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}\,\left(b+\sqrt{4ac+\right.}}$
elliptic	$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{a x}+\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\frac{\sqrt{-4-\frac{2b\left(b+\sqrt{4ac+b^2}\right)}{ac}}}{2}\right.\right.}{2\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}\,\left(b+\sqrt{4ac+\right.}}$

input
```
int(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-c*x^4+b*x^2+a)^(1/2)/a/x+1/2*c*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)
*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1
/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c
)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-
2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(a^{\frac{3}{2}}x\sqrt{\frac{b^2+4\,ac}{a^2}}-\sqrt{a}bx\right)\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}-b}{a}}E\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}-b}{a}}\right)\mid-\frac{ab\sqrt{\frac{b^2+4\,ac}{a^2}}+b^2+2\,ac}{2\,ac}\right)-\sqrt{\frac{1}{2}}}{}$$

input
```
integrate(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(a^(3/2)*x*sqrt((b^2 + 4*a*c)/a^2) - sqrt(a)*b*x)*sqrt((a*
sqrt((b^2 + 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt
((b^2 + 4*a*c)/a^2) - b)/a)), -1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) + b^2 + 2*
a*c)/(a*c)) - sqrt(1/2)*(a^(3/2)*x*sqrt((b^2 + 4*a*c)/a^2) - sqrt(a)*b*x)*
sqrt((a*sqrt((b^2 + 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt
((a*sqrt((b^2 + 4*a*c)/a^2) - b)/a)), -1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) +
b^2 + 2*a*c)/(a*c)) + 2*sqrt(-c*x^4 + b*x^2 + a)*a)/(a^2*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate(1/x**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**2 - c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}\,x^2}\,dx$$

input
```
integrate(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}\,x^2}\,dx$$

input
```
integrate(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{x^2\,\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/(x^2*(a + b*x^2 - c*x^4)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2 - c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{\sqrt{-c\,x^4+b\,x^2+a}}{-c\,x^6+b\,x^4+a\,x^2}dx$$

input
```
int(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 - c*x**4)/(a*x**2 + b*x**4 - c*x**6),x)
```

3.1007 $\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 445

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx = -\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}$$
$$-\frac{b\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}$$
$$+\frac{\sqrt{b+\sqrt{b^2+4ac}}\left(b^2+ac-b\sqrt{b^2+4ac}\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4a}}}\right.\right.}{3\sqrt{2}a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

output
```
-1/3*(-c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(-c*x^4+b*x^2+a)^(1/2)/a^2/x-1/6*b
*(b-(4*a*c+b^2)^(1/2))*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b-(4*a*c+b^
2)^(1/2)))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)*EllipticE(2^(1/2)
*c^(1/2)*x/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^
2)^(1/2)))^(1/2))*2^(1/2)/a^2/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/6*(b+(4*a*c
+b^2)^(1/2))^(1/2)*(b^2+a*c-b*(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)
^(1/2)))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(2^(1/2)*c
^(1/2)*x/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)
^(1/2)))^(1/2))*2^(1/2)/a^2/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.50 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx = \frac{-2\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}(a-2bx^2)\left(a+bx^2-cx^4\right)-i\sqrt{2}b\left(-b+\sqrt{b^2+4ac}\right)x^3\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{-b+\sqrt{b^2+4ac}+2}{-b+\sqrt{b^2+4ac}}}}{}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^2 - c*x^4]),x]
```

output
```
(-2*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*(a - 2*b*x^2)*(a + b*x^2 - c*x^4) -
 I*Sqrt[2]*b*(-b + Sqrt[b^2 + 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*
c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-
b + Sqrt[b^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2
 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt
[2]*(-b^2 - a*c + b*Sqrt[b^2 + 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2
*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(
-b + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^
2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*a^2
*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x^3*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1443, 25, 1604, 27, 1514, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx$$

$$\Big\downarrow \text{ 1443}$$

$$\frac{\int -\frac{2b-cx^2}{x^2\sqrt{-cx^4+bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}$$

$\big\downarrow$ 25

$$-\frac{\int \frac{2b-cx^2}{x^2\sqrt{-cx^4+bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}$$

$\big\downarrow$ 1604

$$-\frac{-\frac{\int \frac{c\left(2bx^2+a\right)}{\sqrt{-cx^4+bx^2+a}}dx}{a}-\frac{2b\sqrt{a+bx^2-cx^4}}{ax}}{3a}-\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}$$

$\big\downarrow$ 27

$$-\frac{-\frac{c\int \frac{2bx^2+a}{\sqrt{-cx^4+bx^2+a}}dx}{a}-\frac{2b\sqrt{a+bx^2-cx^4}}{ax}}{3a}-\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}$$

$\big\downarrow$ 1514

$$-\frac{-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\int \frac{2bx^2+a}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{a\sqrt{a+bx^2-cx^4}}-\frac{2b\sqrt{a+bx^2-cx^4}}{ax}}{3a}-\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}$$

$\big\downarrow$ 399

$$-\frac{-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{c}-\frac{b\left(b-\sqrt{4ac+b^2}\right)\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}dx}{c}\right)}{a\sqrt{a+bx^2-cx^4}}-}{3a}$$

$$\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}$$

$\big\downarrow$ 321

$$-\frac{-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}c^{3/2}}-\frac{b\left(b-\sqrt{4ac+b^2}\right)\int\frac{\sqrt{1}}{\sqrt{1}}}{c}\right.}{a\sqrt{a+bx^2-cx^4}}}{3a}$$

$$\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}$$

$$\Big\downarrow \text{327}$$

$$-\frac{-\frac{c\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right),\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}c^{3/2}}-\frac{b\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4a}}}{}\right.}{a\sqrt{a+bx^2-cx^4}}}{3a}$$

$$\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}$$

input
```
Int[1/(x^4*Sqrt[a + b*x^2 - c*x^4]),x]
```

output
```
-1/3*Sqrt[a + b*x^2 - c*x^4]/(a*x^3) - ((-2*b*Sqrt[a + b*x^2 - c*x^4])/(a*
x) - (c*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b
+ Sqrt[b^2 + 4*a*c])]*(-((b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*
a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (
b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2))) + (Sqr
t[b + Sqrt[b^2 + 4*a*c]]*(b^2 + a*c - b*Sqrt[b^2 + 4*a*c])*EllipticF[ArcSi
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c]
)/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2))))/(a*Sqrt[a + b*x^2 - c*x^4]
))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1443
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Sim
p[1/(a*d^2*(m + 1))   Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{\sqrt{-c\,x^4+b\,x^2+a}\,(-2b\,x^2+a)}{3a^2x^3}+\frac{c\left(\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},\sqrt{-4-\frac{2b(}{}}\right.}{4\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}}\right.}{}$
default	$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{-c\,x^4+b\,x^2+a}}{3a^2x}+\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2}\right.}{12a\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}}$
elliptic	$-\frac{\sqrt{-c\,x^4+b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{-c\,x^4+b\,x^2+a}}{3a^2x}+\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2}\right.}{12a\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\,\sqrt{-c\,x^4+b\,x^2+a}}$

input
```
int(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-c*x^4+b*x^2+a)^(1/2)*(-2*b*x^2+a)/a^2/x^3+1/3*c/a^2*(1/4*a*2^(1/2)/
((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*
(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1
/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(
1/2))/a/c)^(1/2))-b*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*
x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ell
ipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*
c+b^2)^(1/2))/a/c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx = \frac{2\sqrt{\frac{1}{2}}\left(a^{\frac{3}{2}}bx^3\sqrt{\frac{b^2+4\,ac}{a^2}}-\sqrt{a}b^2x^3\right)\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}-b}{a}}E(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2+4\,ac}{a^2}}-b}{a}}\right)\,|-\frac{ab\sqrt{\frac{b^2+4\,ac}{a^2}}+b^2+2\,ac}{2\,ac})+\sqrt{}}{}$$

input
```
integrate(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(1/2)*(a^(3/2)*b*x^3*sqrt((b^2 + 4*a*c)/a^2) - sqrt(a)*b^2*x^3)
*sqrt((a*sqrt((b^2 + 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqr
t((a*sqrt((b^2 + 4*a*c)/a^2) - b)/a)), -1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) +
 b^2 + 2*a*c)/(a*c)) + sqrt(1/2)*((a^2 - 2*a*b)*sqrt(a)*x^3*sqrt((b^2 + 4*
a*c)/a^2) + (a*b + 2*b^2)*sqrt(a)*x^3)*sqrt((a*sqrt((b^2 + 4*a*c)/a^2) - b
)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 + 4*a*c)/a^2) - b)/a)
), -1/2*(a*b*sqrt((b^2 + 4*a*c)/a^2) + b^2 + 2*a*c)/(a*c)) + 2*sqrt(-c*x^4
 + b*x^2 + a)*(2*a*b*x^2 - a^2))/(a^3*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx$$

input
```
integrate(1/x**4/(-c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a + b*x**2 - c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}x^4}\,dx$$

input
```
integrate(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx = \int \frac{1}{\sqrt{-cx^4+bx^2+a}x^4}\,dx$$

input
```
integrate(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{1}{x^4\,\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2 - c*x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2 - c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx=\int \frac{\sqrt{-c\,x^4+b\,x^2+a}}{-c\,x^8+b\,x^6+a\,x^4}dx$$

input
```
int(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 - c*x**4)/(a*x**4 + b*x**6 - c*x**8),x)
```

3.1008 $\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 190

$$\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x^6(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{bx^4\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5b^2-12ac)x^2)\sqrt{a+bx^2+cx^4}}{8c^3(b^2-4ac)} + \frac{3(5b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{7/2}}$$

output
```
x^6*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*x^4*(c*x^4+b*x^2+a)^(
1/2)/c/(-4*a*c+b^2)-1/8*(b*(-52*a*c+15*b^2)-2*c*(-12*a*c+5*b^2)*x^2)*(c*x^
4+b*x^2+a)^(1/2)/c^3/(-4*a*c+b^2)+3/16*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^2
+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{4a^2c(-13b+6cx^2)+b^2x^2(15b^2+5bcx^2-2c^2x^4)+a(15b^3-62b^2cx^2-20bc^2x}{8c^3(-b^2+4ac)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{3(-5b^2+4ac)\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{16c^{7/2}}$$

input
```
Integrate[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(4*a^2*c*(-13*b + 6*c*x^2) + b^2*x^2*(15*b^2 + 5*b*c*x^2 - 2*c^2*x^4) + a*
(15*b^3 - 62*b^2*c*x^2 - 20*b*c^2*x^4 + 8*c^3*x^6))/(8*c^3*(-b^2 + 4*a*c)*
Sqrt[a + b*x^2 + c*x^4]) + (3*(-5*b^2 + 4*a*c)*Log[b + 2*c*x^2 - 2*Sqrt[c]
*Sqrt[a + b*x^2 + c*x^4]])/(16*c^(7/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1434, 1164, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^8}{(cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{1164}$$

$$\frac{1}{2}\left(\frac{2x^6(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{2\int \frac{3x^4(bx^2+2a)}{\sqrt{cx^4+bx^2+a}}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{2x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{6\int\frac{x^4\left(bx^2+2a\right)}{\sqrt{cx^4+bx^2+a}}dx^2}{b^2-4ac}\right)$$

$\downarrow$ 1236

$$\frac{1}{2}\left(\frac{2x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{6\left(\frac{\int-\frac{x^2\left(\left(5b^2-12ac\right)x^2+4ab\right)}{2\sqrt{cx^4+bx^2+a}}dx^2}{3c}+\frac{bx^4\sqrt{a+bx^2+cx^4}}{3c}\right)}{b^2-4ac}\right)$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{2x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{6\left(\frac{bx^4\sqrt{a+bx^2+cx^4}}{3c}-\frac{\int\frac{x^2\left(\left(5b^2-12ac\right)x^2+4ab\right)}{\sqrt{cx^4+bx^2+a}}dx^2}{6c}\right)}{b^2-4ac}\right)$$

$\downarrow$ 1225

$$\frac{1}{2}\left(\frac{2x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{6\left(\frac{bx^4\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c^2}-\frac{\left(b\left(15b^2-52ac\right)-2cx^2\left(5b^2-12ac\right)\right)\sqrt{}}{4c^2}}{6c}\right.}{b^2-4ac}\right.$$

$\downarrow$ 1092

$$\frac{1}{2}\left(\frac{2x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{6\left(\frac{bx^4\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int\frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c^2}-\frac{\left(b\left(15b^2-52ac\right)-2cx^2\left(5b^2-12a\right.\right.}{4c^2}}{6c}\right.}{b^2-4ac}\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{2x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{6\left(\frac{bx^4\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{\left(b\left(15b^2-52ac\right)-2cx^2\left(5\cdots\right.\right.}{4\cdots}}{6c}\right.}{b^2-4ac}\right.$$

input
```
Int[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
((2*x^6*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (6*((b*x^
4*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (-1/4*((b*(15*b^2 - 52*a*c) - 2*c*(5*b^
2 - 12*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/c^2 + (3*(b^2 - 4*a*c)*(5*b^2 -
4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(5
/2)))/(6*c)))/(b^2 - 4*a*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{48\left(\left(-\frac{5}{24}b^3x^4+\frac{31}{12}a\,b^2x^2+\frac{13}{6}a^2b\right)c^{\frac{3}{2}}+\left(\frac{1}{12}b^2x^6+\frac{5}{6}ab\,x^4-a^2x^2\right)c^{\frac{5}{2}}-\frac{a\,c^{\frac{7}{2}}x^6}{3}-\frac{5b^3\sqrt{c}\,\left(b\,x^2+a\right)}{8}+\left(-\ln(2)+\ln\left(\frac{2c\,x^2+2\sqrt{}}{}\right.\right.\right.}{c^{\frac{7}{2}}\sqrt{c\,x^4+b\,x^2+a}\,(64ac-16b^2)}$
risch	$-\frac{(-2c\,x^2+7b)\sqrt{c\,x^4+b\,x^2+a}}{8c^3}-\frac{3c(4ac-5b^2)\left(-\frac{x^2}{2c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(2c\,x^2+b\right)}{c\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}\right)}{4c}+\frac{\ln(}{}\right.}{8c^3}$
default	$\frac{x^6}{4c\sqrt{c\,x^4+b\,x^2+a}}-\frac{5b\,x^4}{8c^2\sqrt{c\,x^4+b\,x^2+a}}-\frac{15b^2x^2}{16c^3\sqrt{c\,x^4+b\,x^2+a}}+\frac{15b^3}{32c^4\sqrt{c\,x^4+b\,x^2+a}}+\frac{15b^4x^2}{16c^3(4ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}$
elliptic	$\frac{x^6}{4c\sqrt{c\,x^4+b\,x^2+a}}-\frac{5b\,x^4}{8c^2\sqrt{c\,x^4+b\,x^2+a}}-\frac{15b^2x^2}{16c^3\sqrt{c\,x^4+b\,x^2+a}}+\frac{15b^3}{32c^4\sqrt{c\,x^4+b\,x^2+a}}+\frac{15b^4x^2}{16c^3(4ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}$

input
```
int(x^9/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-48/c^(7/2)/(c*x^4+b*x^2+a)^(1/2)*((-5/24*b^3*x^4+31/12*a*b^2*x^2+13/6*a^2
*b)*c^(3/2)+(1/12*b^2*x^6+5/6*a*b*x^4-a^2*x^2)*c^(5/2)-1/3*a*c^(7/2)*x^6-5
/8*b^3*c^(1/2)*(b*x^2+a)+(-ln(2)+ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/
2)+b)/c^(1/2)))*(a*c-1/4*b^2)*(c*x^4+b*x^2+a)^(1/2)*(a*c-5/4*b^2))/(64*a*c
-16*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 591, normalized size of antiderivative = 3.11

$$\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx = \left[-\frac{3\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2+(5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^4+(5\,b^5-24\,ab\right.}{}\right.$$

$$-\frac{3\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2+(5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^4+(5\,b^5-24\,ab^3c+16\,a^2bc^2)x^2\right)\sqrt{-c}\arctan}{16\left(ab^2c^4-\right.}$$

input
```
integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/32*(3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^4*c - 24*a*b^2*c^2 +
 16*a^2*c^3)*x^4 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2)*sqrt(c)*log(-8
*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(
c) - 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - 5
*(b^3*c^2 - 4*a*b*c^3)*x^4 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^2)*s
qrt(c*x^4 + b*x^2 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^4 +
 (b^3*c^4 - 4*a*b*c^5)*x^2), -1/16*(3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2
 + (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (5*b^5 - 24*a*b^3*c + 16*a^
2*b*c^2)*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sq
rt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^6 - 15*a*b^
3*c + 52*a^2*b*c^2 - 5*(b^3*c^2 - 4*a*b*c^3)*x^4 - (15*b^4*c - 62*a*b^2*c^
2 + 24*a^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^
2*c^5 - 4*a*c^6)*x^4 + (b^3*c^4 - 4*a*b*c^5)*x^2)]
```

Sympy [F]

$$\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^9}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**9/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**9/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.12

$$\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,(b^2c^2-4\,ac^3)x^2}{b^2c^3-4\,ac^4}-\frac{5\,(b^3c-4\,abc^2)}{b^2c^3-4\,ac^4}\right)x^2-\frac{15\,b^4-62\,ab^2c+24\,a^2c^2}{b^2c^3-4\,ac^4}\right)x^2-\frac{15\,ab^3-52\,a^2bc}{b^2c^3-4\,ac^4}}{8\sqrt{cx^4+bx^2+a}} - \frac{3\,(5\,b^2-4\,ac)\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{7}{2}}}$$

input
```
integrate(x^9/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/8*(((2*(b^2*c^2 - 4*a*c^3)*x^2/(b^2*c^3 - 4*a*c^4) - 5*(b^3*c - 4*a*b*c^
2)/(b^2*c^3 - 4*a*c^4))*x^2 - (15*b^4 - 62*a*b^2*c + 24*a^2*c^2)/(b^2*c^3
- 4*a*c^4))*x^2 - (15*a*b^3 - 52*a^2*b*c)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^4
+ b*x^2 + a) - 3/16*(5*b^2 - 4*a*c)*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^9}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int(x^9/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^9/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 5905, normalized size of antiderivative = 31.08

$$\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^9/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
( - 1536*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4
) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**5*c**5 - 1536*sqrt(a + b*x**2 + c
*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c
 - b**2))*a**4*b**2*c**4 - 19968*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*
sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**4*b*c**5*
x**2 - 19968*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*
x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**4*c**6*x**4 + 4800*sqrt(a + b
*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/s
qrt(4*a*c - b**2))*a**3*b**4*c**3 + 18432*sqrt(a + b*x**2 + c*x**4)*log((2
*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**
3*b**3*c**4*x**2 - 24576*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a +
 b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**3*b**2*c**5*x**4
- 86016*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)
 + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**3*b*c**6*x**6 - 43008*sqrt(a + b*x
**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqr
t(4*a*c - b**2))*a**3*c**7*x**8 - 480*sqrt(a + b*x**2 + c*x**4)*log((2*sqr
t(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b*
*6*c**2 + 10560*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 +
 c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b**5*c**3*x**2 + 67392*s
qrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b +...
```

3.1009 $\int \frac{x^7}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 134

$$\int \frac{x^7}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{(3b^2-8ac-2bcx^2)\sqrt{a+bx^2+cx^4}}{2c^2(b^2-4ac)} - \frac{3b\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}}$$

output
```
x^4*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*(-2*b*c*x^2-8*a*c+3
*b^2)*(c*x^4+b*x^2+a)^(1/2)/c^2/(-4*a*c+b^2)-3/4*b*arctanh(1/2*(2*c*x^2+b)
/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int \frac{x^7}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-3ab^2+8a^2c-3b^3x^2+10abcx^2-b^2cx^4+4ac^2x^4}{2c^2(-b^2+4ac)\sqrt{a+bx^2+cx^4}} + \frac{3b\log\left(bc^2+2c^3x^2-2c^{5/2}\sqrt{a+bx^2+cx^4}\right)}{4c^{5/2}}$$

input
```
Integrate[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-3*a*b^2 + 8*a^2*c - 3*b^3*x^2 + 10*a*b*c*x^2 - b^2*c*x^4 + 4*a*c^2*x^4)/
(2*c^2*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (3*b*Log[b*c^2 + 2*c^3*x^
2 - 2*c^(5/2)*Sqrt[a + b*x^2 + c*x^4]])/(4*c^(5/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1434, 1164, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^6}{(cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{1164}$$

$$\frac{1}{2}\left(\frac{2x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{2\int \frac{2x^2(bx^2+2a)}{\sqrt{cx^4+bx^2+a}}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{2x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{4\int \frac{x^2(bx^2+2a)}{\sqrt{cx^4+bx^2+a}}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{2}\left(\frac{2x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{4\left(\frac{3b(b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c^2}-\frac{(-8ac+3b^2-2bcx^2)\sqrt{a+bx^2+cx^4}}{4c^2}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{2x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{4\left(\frac{3b(b^2-4ac)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c^2}-\frac{(-8ac+3b^2-2bcx^2)\sqrt{a+bx^2+cx^4}}{4c^2}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{4\left(\frac{3b(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{(-8ac+3b^2-2bcx^2)\sqrt{a+bx^2+cx^4}}{4c^2}\right)}{b^2-4ac}\right)$$

input
```
Int[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
((2*x^4*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (4*(-1/4*
((3*b^2 - 8*a*c - 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/c^2 + (3*b*(b^2 - 4*
a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(5/2
))))/(b^2 - 4*a*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$\frac{\frac{x^4}{2\sqrt{c\,x^4+b\,x^2+a}}-\frac{3b\left(-\frac{x^2}{c\sqrt{c\,x^4+b\,x^2+a}}+\frac{b\left(b\,x^2+2a\right)}{c\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}+\frac{-\ln(2)+\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)}{4}+\frac{2a\left(b\,x^2+\right.}{\left(4ac-b^2\right)\sqrt{c\,x}}}{c}$
default	$\frac{x^4}{2c\sqrt{c\,x^4+b\,x^2+a}}-\frac{3b\left(-\frac{x^2}{c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(2c\,x^2+b\right)}{c\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{c^{\frac{3}{2}}}\right.}{4c}$
elliptic	$\frac{x^4}{2c\sqrt{c\,x^4+b\,x^2+a}}-\frac{3b\left(-\frac{x^2}{c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(2c\,x^2+b\right)}{c\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{c^{\frac{3}{2}}}\right.}{4c}$
risch	$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2c^2}-\frac{b^3x^2}{4c^2(4ac-b^2)\sqrt{c\,x^4+b\,x^2+a}}+\frac{2a^2}{c\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}+\frac{b^2a}{2c^2\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}+\frac{3}{4c^2\sqrt{c\,x}}$

input
```
int(x^7/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*x^4/(c*x^4+b*x^2+a)^(1/2)-3/4*b*(-x^2/c/(c*x^4+b*x^2+a)^(1/2)+b/c
*(b*x^2+2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+1/c^(3/2)*(-ln(2)+ln((2*c*x
^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))))+2*a*(b*x^2+2*a)/(4*a*c-b^
2)/(c*x^4+b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.43

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\left[\frac{3\left((b^3c-4\,abc^2)x^4+ab^3-4\,a^2bc+(b^4-4\,ab^2c)x^2\right)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2\right.}{8\,(ab^2}\right.$$

input
```
integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*((b^3*c - 4*a*b*c^2)*x^4 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x
^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(
2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^4 + 3*a*b^2*c - 8
*a^2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^3
 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2), 1/4*(
3*((b^3*c - 4*a*b*c^2)*x^4 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^2)*sq
rt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4
+ b*c*x^2 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^4 + 3*a*b^2*c - 8*a^2*c^2 + (
3*b^3*c - 10*a*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*b^2*c^3 - 4*a^2*c^4
 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2)]
```

Sympy [F]

$$\int \frac{x^7}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**7/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**7/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{x^7}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\left(\frac{(b^2c-4\,ac^2)x^2}{b^2c^2-4\,ac^3}+\frac{3\,b^3-10\,abc}{b^2c^2-4\,ac^3}\right)x^2+\frac{3\,ab^2-8\,a^2c}{b^2c^2-4\,ac^3}}{2\,\sqrt{cx^4+bx^2+a}} + \frac{3\,b\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{4\,c^{\frac{5}{2}}}$$

input
```
integrate(x^7/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(((b^2*c - 4*a*c^2)*x^2/(b^2*c^2 - 4*a*c^3) + (3*b^3 - 10*a*b*c)/(b^2*
c^2 - 4*a*c^3))*x^2 + (3*a*b^2 - 8*a^2*c)/(b^2*c^2 - 4*a*c^3))/sqrt(c*x^4
+ b*x^2 + a) + 3/4*b*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqr
t(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7}{\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int(x^7/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^7/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 3725, normalized size of antiderivative = 27.80

$$\int \frac{x^7}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^7/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
( - 384*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)
 + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**3*b**2*c**3 - 768*sqrt(a + b*x**2
+ c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*
a*c - b**2))*a**3*b*c**4*x**2 - 1152*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt
(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b**
3*c**3*x**2 - 3456*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**
2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b**2*c**4*x**4 - 2304
*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b +
2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b*c**5*x**6 + 24*sqrt(a + b*x**2 + c*x*
*4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c -
b**2))*a*b**6*c + 240*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*
x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b**5*c**2*x**2 - 192*
sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2
*c*x**2)/sqrt(4*a*c - b**2))*a*b**4*c**3*x**4 - 2688*sqrt(a + b*x**2 + c*x
**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c -
 b**2))*a*b**3*c**4*x**6 - 3840*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*s
qrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b**2*c**5*x
**8 - 1536*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x*
*4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b*c**6*x**10 + 24*sqrt(a + b*x**
2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sq...
```

3.1010 $\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}} \, dx = \frac{x^2(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{a+bx^2+cx^4}}{c(b^2-4ac)} + \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

output
```
x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*(c*x^4+b*x^2+a)^(1/2)
/c/(-4*a*c+b^2)+1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))
/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}} \, dx = \frac{b^2x^2+a(b-2cx^2)}{c(-b^2+4ac)\sqrt{a+bx^2+cx^4}} + \frac{\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

input
```
Integrate[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(b^2*x^2 + a*(b - 2*c*x^2))/(c*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*c^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1434, 1164, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{x^4}{(cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{1164}$$

$$\frac{1}{2}\left(\frac{2x^2(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{2\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx^2}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{2x^2(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{2\left(\frac{b\sqrt{a+bx^2+cx^4}}{c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{2c}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left(\frac{2x^2(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{2\left(\frac{b\sqrt{a+bx^2+cx^4}}{c}-\frac{(b^2-4ac)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{c}\right)}{b^2-4ac}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{2x^2(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{2\left(\frac{b\sqrt{a+bx^2+cx^4}}{c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}\right)}{b^2-4ac}\right)$$

input
```
Int[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
((2*x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (2*((b*Sq
rt[a + b*x^2 + c*x^4])/c - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]
*Sqrt[a + b*x^2 + c*x^4])])/(2*c^(3/2))))/(b^2 - 4*a*c))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.92

method	result	size
pseudoelliptic	$-\frac{x^2}{2c\sqrt{c\,x^4+b\,x^2+a}}+\frac{b\left(b\,x^2+2a\right)}{8\sqrt{c\,x^4+b\,x^2+a}\,c\left(ac-\frac{b^2}{4}\right)}-\frac{\ln(2)-\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}}$	106
default	$-\frac{x^2}{2c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(2c\,x^2+b\right)}{c\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}\right)}{4c}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2c^{\frac{3}{2}}}$	124
elliptic	$-\frac{x^2}{2c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^4+b\,x^2+a}}-\frac{b\left(2c\,x^2+b\right)}{c\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}\right)}{4c}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2c^{\frac{3}{2}}}$	124

input
```
int(x^5/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2/c/(c*x^4+b*x^2+a)^(1/2)+1/8*b*(b*x^2+2*a)/(c*x^4+b*x^2+a)^(1/2)/c
/(a*c-1/4*b^2)-1/2*(ln(2)-ln((2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c
^(1/2)))/c^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.37

$$\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2\right.}{4\,(ab^2c^2-4\,a^2c^3+(b^2c^3\right.$$

$$-\frac{((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)+2\sqrt{cx^4+bx^2+a}}{2\,(ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^4+(b^3c^2-4\,abc^3)x^2)}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt
(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
+ b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a)*(a*b*c + (b^2*c - 2*a*c^
2)*x^2))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a
*b*c^3)*x^2), -1/2*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-
c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*(a*b*c + (b^2*c
- 2*a*c^2)*x^2))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c
^2 - 4*a*b*c^3)*x^2)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^5}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**5/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{\frac{(b^2-2\,ac)x^2}{b^2c-4\,ac^2}+\frac{ab}{b^2c-4\,ac^2}}{\sqrt{cx^4+bx^2+a}} - \frac{\log\left(\left|2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{3}{2}}}$$

input
```
integrate(x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-((b^2 - 2*a*c)*x^2/(b^2*c - 4*a*c^2) + a*b/(b^2*c - 4*a*c^2))/sqrt(c*x^4
+ b*x^2 + a) - 1/2*log(abs(2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(
c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 17.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,c^{3/2}} + \frac{\frac{a\,b}{2}-x^2\left(a\,c-\frac{b^2}{2}\right)}{2\,c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^4+b\,x^2+a}}$$

input
```
int(x^5/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2))/(2*c^(3/2)) + ((a*b
)/2 - x^2*(a*c - b^2/2))/(2*c*(a*c - b^2/4)*(a + b*x^2 + c*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1887, normalized size of antiderivative = 16.41

$$\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^5/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(16*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b
 + 2*c*x**2)/sqrt(4*a*c - b**2))*a**2*b*c**2 + 32*sqrt(a + b*x**2 + c*x**4
)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b*
*2))*a**2*c**3*x**2 - 4*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a +
b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*b**3*c + 8*sqrt(a +
 b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)
/sqrt(4*a*c - b**2))*a*b**2*c**2*x**2 + 48*sqrt(a + b*x**2 + c*x**4)*log((
2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*
b*c**3*x**4 + 32*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2
+ c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*a*c**4*x**6 - 4*sqrt(a + b*x
**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqr
t(4*a*c - b**2))*b**4*c*x**2 - 12*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)
*sqrt(a + b*x**2 + c*x**4) + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**3*c**2*x
**4 - 8*sqrt(a + b*x**2 + c*x**4)*log((2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)
 + b + 2*c*x**2)/sqrt(4*a*c - b**2))*b**2*c**3*x**6 - 8*sqrt(a + b*x**2 +
c*x**4)*a**2*b*c**2 - 48*sqrt(a + b*x**2 + c*x**4)*a**2*c**3*x**2 + 10*sqr
t(a + b*x**2 + c*x**4)*a*b**3*c + 20*sqrt(a + b*x**2 + c*x**4)*a*b**2*c**2
*x**2 - 64*sqrt(a + b*x**2 + c*x**4)*a*b*c**3*x**4 - 64*sqrt(a + b*x**2 +
c*x**4)*a*c**4*x**6 + 10*sqrt(a + b*x**2 + c*x**4)*b**4*c*x**2 + 40*sqrt(a
 + b*x**2 + c*x**4)*b**3*c**2*x**4 + 32*sqrt(a + b*x**2 + c*x**4)*b**2*...
```

3.1011 $\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 36

$$\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{2a+bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

output
```
(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{2a+bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[x^3/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1434, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1434}$$

$$\frac{1}{2}\int \frac{x^2}{(cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 1158}$$

$$\frac{2a+bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[x^3/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

method	result	size
gosper	$-\frac{b\,x^{2}+2a}{\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}$	38
default	$-\frac{b\,x^{2}+2a}{\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}$	38
trager	$-\frac{b\,x^{2}+2a}{\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}$	38
elliptic	$-\frac{b\,x^{2}+2a}{\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}$	38
pseudoelliptic	$\frac{-b\,x^{2}-2a}{\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}$	38
orering	$-\frac{b\,x^{2}+2a}{\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}$	38

input
```
int(x^3/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.86

$$\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{cx^4+bx^2+a}(bx^2+2\,a)}{(b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2}$$

input
```
integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a
^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**3/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\frac{bx^2}{b^2-4\,ac} + \frac{2\,a}{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}$$

input
```
integrate(x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(b*x^2/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 17.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{b\,x^2+2\,a}{(4\,a\,c-b^2)\,\sqrt{c\,x^4+b\,x^2+a}}$$

input
```
int(x^3/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
-(2*a + b*x^2)/((4*a*c - b^2)*(a + b*x^2 + c*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 605, normalized size of antiderivative = 16.81

$$\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-8\sqrt{c\,x^4+b\,x^2+a}\,a^2c^2-6\sqrt{c\,x^4+b\,x^2}}{c\left(16\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,a^2bc+32\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,a^2c^2x^2-4\sqrt{c}\,\sqrt{c\,x^4+b}\right.}$$

input
```
int(x^3/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
( - 8*sqrt(a + b*x**2 + c*x**4)*a**2*c**2 - 6*sqrt(a + b*x**2 + c*x**4)*a*
b**2*c - 28*sqrt(a + b*x**2 + c*x**4)*a*b*c**2*x**2 - 16*sqrt(a + b*x**2 +
 c*x**4)*a*c**3*x**4 - 5*sqrt(a + b*x**2 + c*x**4)*b**3*c*x**2 - 20*sqrt(a
 + b*x**2 + c*x**4)*b**2*c**2*x**4 - 16*sqrt(a + b*x**2 + c*x**4)*b*c**3*x
**6 - 12*sqrt(c)*a**2*b*c - 16*sqrt(c)*a**2*c**2*x**2 - sqrt(c)*a*b**3 - 2
4*sqrt(c)*a*b**2*c*x**2 - 44*sqrt(c)*a*b*c**2*x**4 - 16*sqrt(c)*a*c**3*x**
6 - sqrt(c)*b**4*x**2 - 13*sqrt(c)*b**3*c*x**4 - 28*sqrt(c)*b**2*c**2*x**6
 - 16*sqrt(c)*b*c**3*x**8)/(c*(16*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a**2*b
*c + 32*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a**2*c**2*x**2 - 4*sqrt(c)*sqrt(
a + b*x**2 + c*x**4)*a*b**3 + 8*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*b**2*c
*x**2 + 48*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*b*c**2*x**4 + 32*sqrt(c)*sq
rt(a + b*x**2 + c*x**4)*a*c**3*x**6 - 4*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*
b**4*x**2 - 12*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b**3*c*x**4 - 8*sqrt(c)*s
qrt(a + b*x**2 + c*x**4)*b**2*c**2*x**6 + 16*a**3*c**2 + 48*a**2*b*c**2*x*
*2 + 48*a**2*c**3*x**4 - a*b**4 - 8*a*b**3*c*x**2 + 24*a*b**2*c**2*x**4 +
64*a*b*c**3*x**6 + 32*a*c**4*x**8 - b**5*x**2 - 9*b**4*c*x**4 - 16*b**3*c*
*2*x**6 - 8*b**2*c**3*x**8))
```

3.1012 $\int \frac{x}{(a+bx^2+cx^4)^{3/2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 36

$$\int \frac{x}{(a+bx^2+cx^4)^{3/2}} dx = -\frac{b+2cx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

output
```
-(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x}{(a+bx^2+cx^4)^{3/2}} dx = \frac{-b-2cx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[x/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-b - 2*c*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1432, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2+cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{(cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{1088}$$

$$-\frac{b+2cx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[x/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
-((b + 2*c*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]))
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{2c\,x^{2}+b}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$	36
default	$\frac{2c\,x^{2}+b}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$	36
trager	$\frac{2c\,x^{2}+b}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$	36
elliptic	$\frac{2c\,x^{2}+b}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$	36
pseudoelliptic	$\frac{2c\,x^{2}+b}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$	36
orering	$\frac{2c\,x^{2}+b}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$	36

input
```
int(x/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*c*x^2+b)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.86

$$\int \frac{x}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{\sqrt{cx^4+bx^2+a}(2\,cx^2+b)}{(b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2}$$

input
```
integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F]

$$\int \frac{x}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{x}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{\frac{2\,cx^2}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}$$

input
```
integrate(x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-(2*c*x^2/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 17.94 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{x}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{2\,c\,x^2+b}{\left(4\,a\,c-b^2\right)\,\sqrt{c\,x^4+b\,x^2+a}}$$

input
```
int(x/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
(b + 2*c*x^2)/((4*a*c - b^2)*(a + b*x^2 + c*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 216, normalized size of antiderivative = 6.00

$$\int \frac{x}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{4\sqrt{c\,x^4+b\,x^2+a}\,bc+8\sqrt{c\,x^4+b\,x^2+}}{4\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,abc+8\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,a\,c^2x^2-\sqrt{c}\,\sqrt{c\,x^4+b\,x^2+a}\,b}$$

input
```
int(x/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(4*sqrt(a + b*x**2 + c*x**4)*b*c + 8*sqrt(a + b*x**2 + c*x**4)*c**2*x**2 +
 4*sqrt(c)*a*c + sqrt(c)*b**2 + 8*sqrt(c)*b*c*x**2 + 8*sqrt(c)*c**2*x**4)/
(4*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*b*c + 8*sqrt(c)*sqrt(a + b*x**2 + c
*x**4)*a*c**2*x**2 - sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b**3 - 2*sqrt(c)*sq
rt(a + b*x**2 + c*x**4)*b**2*c*x**2 + 8*a**2*c**2 - 2*a*b**2*c + 8*a*b*c**
2*x**2 + 8*a*c**3*x**4 - 2*b**3*c*x**2 - 2*b**2*c**2*x**4)
```

3.1013 $\int \frac{1}{x(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\operatorname{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

output
```
(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*arctanh(1/2*(
b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{-b^2+2ac-bcx^2}{a\left(-b^2+4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-b^2 + 2*a*c - b*c*x^2)/(a*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ArcT
anh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]/a^(3/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1434, 1165, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^2+cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int \frac{1}{x^2 (cx^4+bx^2+a)^{3/2}} dx^2$$

$$\Big\downarrow \text{1165}$$

$$\frac{1}{2} \left(\frac{2(-2ac+b^2+bcx^2)}{a (b^2-4ac) \sqrt{a+bx^2+cx^4}}-\frac{2 \int -\frac{b^2-4ac}{2x^2\sqrt{cx^4+bx^2+a}}dx^2}{a (b^2-4ac)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{a}+\frac{2(-2ac+b^2+bcx^2)}{a (b^2-4ac) \sqrt{a+bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2} \left(\frac{2(-2ac+b^2+bcx^2)}{a (b^2-4ac) \sqrt{a+bx^2+cx^4}}-\frac{2 \int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2} \left(\frac{2(-2ac+b^2+bcx^2)}{a (b^2-4ac) \sqrt{a+bx^2+cx^4}}-\frac{\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{3/2}}\right)$$

input
```
Int[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - A
rcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/a^(3/2))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

method	result	size
pseudoelliptic	$\frac{1}{2a\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{b\left(2c\,x^{2}+b\right)}{8\sqrt{c\,x^{4}+b\,x^{2}+a}\,\left(ac-\frac{b^{2}}{4}\right)a}-\frac{\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}$	98
default	$\frac{1}{2a\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{b\left(2c\,x^{2}+b\right)}{2a\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}$	99
elliptic	$\frac{1}{2a\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{b\left(2c\,x^{2}+b\right)}{2a\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}$	99

input
```
int(1/x/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(c*x^4+b*x^2+a)^(1/2)-1/8*b*(2*c*x^2+b)/(c*x^4+b*x^2+a)^(1/2)/(a*c-1
/4*b^2)/a-1/2/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(77) = 154.

Time = 0.11 (sec) , antiderivative size = 389, normalized size of antiderivative = 4.37

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \left[\frac{\left((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2\right)\sqrt{a}\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4}{}\right.}{4\left(a^3b^2-4\,a^4c+(a^2b^2c-4\,a^3c^2\right.}\right.$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt
(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(c*x^4
 + b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3
 - 4*a^3*b*c)*x^2), 1/2*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*s
qrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(
c*x^4 + b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^
2*b^3 - 4*a^3*b*c)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(1/(x*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{\frac{abcx^2}{a^2b^2-4\,a^3c}+\frac{ab^2-2\,a^2c}{a^2b^2-4\,a^3c}}{\sqrt{cx^4+bx^2+a}}+\frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}$$

input
```
integrate(1/x/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(a*b*c*x^2/(a^2*b^2 - 4*a^3*c) + (a*b^2 - 2*a^2*c)/(a^2*b^2 - 4*a^3*c))/sq
rt(c*x^4 + b*x^2 + a) + arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sq
rt(-a))/(sqrt(-a)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{x\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 887, normalized size of antiderivative = 9.97

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*sqrt(a + b*x**2 + c*x**4)*sqrt(4*a*c - b**2)*sqrt( - 4
*a*c + b**2)*atan((4*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*c - sqrt(c)*sqrt(
a + b*x**2 + c*x**4)*b**2 + 4*a*c**2*x**2 - b**2*c*x**2)/(sqrt(c)*sqrt(a)*
sqrt(4*a*c - b**2)*sqrt( - 4*a*c + b**2)))*b - 2*sqrt(c)*sqrt(a)*sqrt(a +
b*x**2 + c*x**4)*sqrt(4*a*c - b**2)*sqrt( - 4*a*c + b**2)*atan((4*sqrt(c)*
sqrt(a + b*x**2 + c*x**4)*a*c - sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b**2 + 4
*a*c**2*x**2 - b**2*c*x**2)/(sqrt(c)*sqrt(a)*sqrt(4*a*c - b**2)*sqrt( - 4*
a*c + b**2)))*c*x**2 - 2*sqrt(a)*sqrt(4*a*c - b**2)*sqrt( - 4*a*c + b**2)*
atan((4*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*c - sqrt(c)*sqrt(a + b*x**2 +
c*x**4)*b**2 + 4*a*c**2*x**2 - b**2*c*x**2)/(sqrt(c)*sqrt(a)*sqrt(4*a*c -
b**2)*sqrt( - 4*a*c + b**2)))*a*c - 2*sqrt(a)*sqrt(4*a*c - b**2)*sqrt( - 4
*a*c + b**2)*atan((4*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*c - sqrt(c)*sqrt(
a + b*x**2 + c*x**4)*b**2 + 4*a*c**2*x**2 - b**2*c*x**2)/(sqrt(c)*sqrt(a)*
sqrt(4*a*c - b**2)*sqrt( - 4*a*c + b**2)))*b*c*x**2 - 2*sqrt(a)*sqrt(4*a*c
 - b**2)*sqrt( - 4*a*c + b**2)*atan((4*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a
*c - sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b**2 + 4*a*c**2*x**2 - b**2*c*x**2)
/(sqrt(c)*sqrt(a)*sqrt(4*a*c - b**2)*sqrt( - 4*a*c + b**2)))*c**2*x**4 + 4
*sqrt(a + b*x**2 + c*x**4)*a**2*c**2 - 3*sqrt(a + b*x**2 + c*x**4)*a*b**2*
c - 4*sqrt(a + b*x**2 + c*x**4)*a*b*c**2*x**2 + 4*sqrt(c)*a**2*c**2*x**2 -
 sqrt(c)*a*b**3 - 5*sqrt(c)*a*b**2*c*x**2 - 4*sqrt(c)*a*b*c**2*x**4)/(a...
```

3.1014 $\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 139

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{2a^2\left(b^2-4ac\right)x^2} + \frac{3b\,\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}}$$

output
```
(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2+a)^(1/2)-1/2*(-8*a*c+3
*b^2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)/x^2+3/4*b*arctanh(1/2*(b*x^2+
2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{-4a^2c+3b^2x^2(b+cx^2)+a(b^2-10bcx^2-8c^2x^4)}{2a^2\left(-b^2+4ac\right)x^2\sqrt{a+bx^2+cx^4}} - \frac{3b\,\text{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{5/2}}$$

input
```
Integrate[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2*x^4))/(2*a
^2*(-b^2 + 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]) - (3*b*ArcTanh[(Sqrt[c]*x^2
 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1434, 1165, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2+cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2} \int \frac{1}{x^4 (cx^4+bx^2+a)^{3/2}} dx^2$$

$$\Big\downarrow \text{1165}$$

$$\frac{1}{2} \left(\frac{2(-2ac+b^2+bcx^2)}{ax^2 (b^2-4ac) \sqrt{a+bx^2+cx^4}} - \frac{2 \int -\frac{3b^2+2cx^2b-8ac}{2x^4\sqrt{cx^4+bx^2+a}} dx^2}{a (b^2-4ac)} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{\int \frac{3b^2+2cx^2b-8ac}{x^4\sqrt{cx^4+bx^2+a}} dx^2}{a (b^2-4ac)} + \frac{2(-2ac+b^2+bcx^2)}{ax^2 (b^2-4ac) \sqrt{a+bx^2+cx^4}} \right)$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{2} \left(\frac{-\frac{3b(b^2-4ac) \int \frac{1}{x^2\sqrt{cx^4+bx^2+a}} dx^2}{2a} - \frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{ax^2}}{a (b^2-4ac)} + \frac{2(-2ac+b^2+bcx^2)}{ax^2 (b^2-4ac) \sqrt{a+bx^2+cx^4}} \right)$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2}\left(\frac{\frac{3b(b^2-4ac)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{a}-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{ax^2}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^2\right)}{ax^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\frac{3b(b^2-4ac)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{ax^2}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^2\right)}{ax^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}\right)$$

input
```
Int[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4])
 + (-(((3*b^2 - 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a*x^2)) + (3*b*(b^2 - 4*a
*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)
))/(a*(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{-\frac{3\sqrt{c x^4+b x^2+a}\left(a c-\frac{b^2}{4}\right) a b x^2 \ln\left(\frac{2a+b x^2+2\sqrt{a}\sqrt{c x^4+b x^2+a}}{x^2}\right)}{2}+a^{\frac{3}{2}}\left(c a^2+\left(2c^2x^4+\frac{5}{2}bcx^2-\frac{1}{4}b^2\right)a-\frac{3b^2x^2\left(cx^2+b\right)}{4}\right)}{2\sqrt{c x^4+b x^2+a}\left(ac-\frac{b^2}{4}\right)a^{\frac{7}{2}}x^2}$
default	$-\frac{1}{2a x^2\sqrt{cx^4+bx^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{cx^4+bx^2+a}}-\frac{b(2cx^2+b)}{a(4ac-b^2)\sqrt{cx^4+bx^2+a}}-\frac{\ln\left(\frac{2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}}{x^2}\right)}{a^{\frac{3}{2}}}\right)}{4a}-\frac{2\cdots}{a(4ac-\cdots}$
elliptic	$-\frac{1}{2a x^2\sqrt{cx^4+bx^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{cx^4+bx^2+a}}-\frac{b(2cx^2+b)}{a(4ac-b^2)\sqrt{cx^4+bx^2+a}}-\frac{\ln\left(\frac{2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}}{x^2}\right)}{a^{\frac{3}{2}}}\right)}{4a}-\frac{2\cdots}{a(4ac-\cdots}$
risch	$-\frac{\sqrt{cx^4+bx^2+a}}{2a^2x^2}+\frac{b^2x^2c}{a^2(4ac-b^2)\sqrt{cx^4+bx^2+a}}+\frac{b^3}{4a^2(4ac-b^2)\sqrt{cx^4+bx^2+a}}-\frac{2c^2x^2}{a(4ac-b^2)\sqrt{cx^4+bx^2+a}}-\frac{\cdots}{4a^2\sqrt{c\cdots}}$

input
```
int(1/x^3/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-3/2*(c*x^4+b*x^2+a)^(1/2)*(a*c-1/4*b^2)*a*b*x^2*ln((2*a+b*x^2+2*a^(
1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+a^(3/2)*(c*a^2+(2*c^2*x^4+5/2*b*c*x^2-1/4
*b^2)*a-3/4*b^2*x^2*(c*x^2+b)))/(c*x^4+b*x^2+a)^(1/2)/(a*c-1/4*b^2)/a^(7/2
)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 485, normalized size of antiderivative = 3.49

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\left[\frac{3\left(\left(b^3c-4\,abc^2\right)x^6+\left(b^4-4\,ab^2c\right)x^4+\left(ab^3-4\,a^2bc\right)x^2\right)\sqrt{a}\log\left(-\frac{(b^2+4\,ac)x\cdots}{\cdots}\right)\cdots}{8\left(\left(a^3b^2c-\cdots\right.\right.}\right.$$

$$-\frac{3\left(\left(b^3c-4\,abc^2\right)x^6+\left(b^4-4\,ab^2c\right)x^4+\left(ab^3-4\,a^2bc\right)x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+2\left(\left(3\,ab^2\cdots\right.\right.}{4\left(\left(a^3b^2c-4\,a^4c^2\right)x^6+\left(a^3b^3-4\,a^4bc\right)x^4+\left(a^4b^2\cdots\right.\right.}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*((b^3*c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*
b*c)*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x
^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((3*a*b^2*c - 8*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(c*x^4 + b*x^2 +
a))/((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 -
4*a^5*c)*x^2), -1/4*(3*((b^3*c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 +
(a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^
2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)*
x^4 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(c*x^4 + b*x^2 +
 a))/((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 -
 4*a^5*c)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)^{3/2}}\, dx = \int \frac{1}{x^3 (a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/x**3/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)^{3/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^3 \left(a+bx^2+cx^4\right)^{3/2}}\,dx = -\frac{\frac{\left(a^2b^2c-2\,a^3c^2\right)x^2}{a^4b^2-4\,a^5c}+\frac{a^2b^3-3\,a^3bc}{a^4b^2-4\,a^5c}}{\sqrt{cx^4+bx^2+a}} - \frac{3\,b\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a^2} + \frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)a^2}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-((a^2*b^2*c - 2*a^3*c^2)*x^2/(a^4*b^2 - 4*a^5*c) + (a^2*b^3 - 3*a^3*b*c)/
(a^4*b^2 - 4*a^5*c))/sqrt(c*x^4 + b*x^2 + a) - 3/2*b*arctan(-(sqrt(c)*x^2
- sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/2*((sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*
x^2 + a))^2 - a)*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{x^3\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,dx$$

input
```
int(1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x^3*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{1}{\sqrt{c\,x^4 + b\,x^2 + a}\,a\,x^3 + \sqrt{c\,x^4 + b\,x^2 + a}\,b\,x^5 + \sqrt{c\,x^4 + b\,x^2 + a}\,c\,x^7} dx$$

input
```
int(1/x^3/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(1/(sqrt(a + b*x**2 + c*x**4)*a*x**3 + sqrt(a + b*x**2 + c*x**4)*b*x**5
 + sqrt(a + b*x**2 + c*x**4)*c*x**7),x)
```

3.1015 $\int \frac{1}{x^5(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 195

$$\int \frac{1}{x^5\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^4\sqrt{a+bx^2+cx^4}} - \frac{(5b^2-12ac)\sqrt{a+bx^2+cx^4}}{4a^2\left(b^2-4ac\right)x^4} + \frac{b(15b^2-52ac)\sqrt{a+bx^2+cx^4}}{8a^3\left(b^2-4ac\right)x^2} - \frac{3(5b^2-4ac)\,\mathrm{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{7/2}}$$

output
```
(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^4/(c*x^4+b*x^2+a)^(1/2)-1/4*(-12*a*c+
5*b^2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)/x^4+1/8*b*(-52*a*c+15*b^2)*(
c*x^4+b*x^2+a)^(1/2)/a^3/(-4*a*c+b^2)/x^2-3/16*(-4*a*c+5*b^2)*arctanh(1/2*
(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5 (a+bx^2+cx^4)^{3/2}}\,dx = \frac{-8a^3c-15b^3x^4(b+cx^2)+2a^2(b^2+10bcx^2-12c^2x^4)+abx^2(-5b^2+62bcx}{8a^3(-b^2+4ac)\,x^4\sqrt{a+bx^2+cx^4}} + \frac{3(5b^2-4ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{7/2}}$$

input

```
Integrate[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output

```
(-8*a^3*c - 15*b^3*x^4*(b + c*x^2) + 2*a^2*(b^2 + 10*b*c*x^2 - 12*c^2*x^4)
 + a*b*x^2*(-5*b^2 + 62*b*c*x^2 + 52*c^2*x^4))/(8*a^3*(-b^2 + 4*a*c)*x^4*S
qrt[a + b*x^2 + c*x^4]) + (3*(5*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a
 + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(7/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1434, 1165, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 1434$$

$$\frac{1}{2}\int \frac{1}{x^6 (cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \; 1165$$

$$\frac{1}{2}\left(\frac{2(-2ac+b^2+bcx^2)}{ax^4(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{2\int -\frac{5b^2+4cx^2b-12ac}{2x^6\sqrt{cx^4+bx^2+a}}dx^2}{a(b^2-4ac)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\int \frac{5b^2+4cx^2b-12ac}{x^6\sqrt{cx^4+bx^2+a}}dx^2}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^2\right)}{ax^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}\right)$$

$$\Big\downarrow \ 1237$$

$$\frac{1}{2}\left(\frac{-\frac{\int \frac{2c\left(5b^2-12ac\right)x^2+b\left(15b^2-52ac\right)}{2x^4\sqrt{cx^4+bx^2+a}}dx^2}{2a}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{2ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^2\right)}{ax^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(\frac{-\frac{\int \frac{2c\left(5b^2-12ac\right)x^2+b\left(15b^2-52ac\right)}{x^4\sqrt{cx^4+bx^2+a}}dx^2}{4a}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{2ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx^2\right)}{ax^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}\right)$$

$$\Big\downarrow \ 1228$$

$$\frac{1}{2}\left(\frac{-\frac{-\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int \frac{1}{x^2\sqrt{cx^4+bx^2+a}}dx^2}{2a}-\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^2+cx^4}}{ax^2}}{4a}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{2ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right.}{ax^4\left(b^2-4ac\right)\sqrt{a+bx^2}}\right.$$

$$\Big\downarrow \ 1154$$

$$\frac{1}{2}\left(\frac{-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int \frac{1}{4a-x^4}d\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}}{a}-\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^2+cx^4}}{ax^2}}{4a}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{2ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right.}{ax^4\left(b^2-4ac\right)\sqrt{a+bx^2}}\right.$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left(\frac{-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\text{arctanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}-\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^2+cx^4}}{ax^2}}{4a}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{2ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2\right.}{ax^4\left(b^2-4ac\right)\sqrt{a}}\right.$$

input

```
Int[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
((2*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*x^4*Sqrt[a + b*x^2 + c*x^4])
 + (-1/2*((5*b^2 - 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a*x^4) - (-((b*(15*b^
2 - 52*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a*x^2)) + (3*(b^2 - 4*a*c)*(5*b^2 -
4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3
/2)))/(4*a))/(a*(b^2 - 4*a*c)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{-\frac{5\left(-\frac{52}{5}c^2x^4-\frac{62}{5}bcx^2+b^2\right)bx^2a^{\frac{3}{2}}}{32}+\frac{3\left(-c^2x^4+\frac{5}{6}bcx^2+\frac{1}{12}b^2\right)a^{\frac{5}{2}}}{4}-\frac{a^{\frac{7}{2}}c}{4}+\frac{3x^4\left(-10b^3\left(cx^2+b\right)\sqrt{a}+\ln\left(\frac{2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2}}{x^2}\right.\right.}{64}}{a^{\frac{7}{2}}\sqrt{cx^4+bx^2+a}\left(ac-\frac{b^2}{4}\right)x^4}$
default	$-\frac{1}{4ax^4\sqrt{cx^4+bx^2+a}}-\frac{5b\left(-\frac{1}{ax^2\sqrt{cx^4+bx^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{cx^4+bx^2+a}}-\frac{b\left(2cx^2+b\right)}{a\left(4ac-b^2\right)\sqrt{cx^4+bx^2+a}}-\frac{\ln\left(\frac{2a+bx^2+2\sqrt{a}\sqrt{cx^4}}{x^2}\right.}{a^{\frac{3}{2}}}\right.}{2a}\right.}{8a}$
elliptic	$-\frac{1}{4ax^4\sqrt{cx^4+bx^2+a}}-\frac{5b\left(-\frac{1}{ax^2\sqrt{cx^4+bx^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{cx^4+bx^2+a}}-\frac{b\left(2cx^2+b\right)}{a\left(4ac-b^2\right)\sqrt{cx^4+bx^2+a}}-\frac{\ln\left(\frac{2a+bx^2+2\sqrt{a}\sqrt{cx^4}}{x^2}\right.}{a^{\frac{3}{2}}}\right.}{2a}\right.}{8a}$
risch	$-\frac{\sqrt{cx^4+bx^2+a}\left(-7bx^2+2a\right)}{8a^3x^4}-\frac{-\frac{7b^3\left(2cx^2+b\right)}{\left(4ac-b^2\right)\sqrt{cx^4+bx^2+a}}-\frac{4c^2a\left(bx^2+2a\right)}{\sqrt{cx^4+bx^2+a}\left(4ac-b^2\right)}+\frac{7b^2c\left(bx^2+2a\right)}{\sqrt{cx^4+bx^2+a}\left(4ac-b^2\right)}+3a(4ac-}{}$

input
```
int(1/x^5/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/4/a^(7/2)*(-5/24*(-52/5*c^2*x^4-62/5*b*c*x^2+b^2)*b*x^2*a^(3/2)+(-c^2*x^
4+5/6*b*c*x^2+1/12*b^2)*a^(5/2)-1/3*a^(7/2)*c+1/16*x^4*(-10*b^3*(c*x^2+b)*
a^(1/2)+ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*(c*x^4+b*x^2+a
)^(1/2)*(16*a^2*c^2-24*a*b^2*c+5*b^4)))/(c*x^4+b*x^2+a)^(1/2)/(a*c-1/4*b^2
)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 615, normalized size of antiderivative = 3.15

$$\int \frac{1}{x^5\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\left[-\frac{3\left(\left(5b^4c-24ab^2c^2+16a^2c^3\right)x^8+\left(5b^5-24ab^3c+16a^2bc^2\right)x^6+\left(5ab^4\right.\right.}{}\right.$$

input
```
integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/32*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^8 + (5*b^5 - 24*a*b^3*c
 + 16*a^2*b*c^2)*x^6 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^4)*sqrt(a)*
log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2
*a)*sqrt(a) + 8*a^2)/x^4) - 4*((15*a*b^3*c - 52*a^2*b*c^2)*x^6 - 2*a^3*b^2
 + 8*a^4*c + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^4 + 5*(a^2*b^3 - 4*a
^3*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^4*b^2*c - 4*a^5*c^2)*x^8 + (a^4*
b^3 - 4*a^5*b*c)*x^6 + (a^5*b^2 - 4*a^6*c)*x^4), 1/16*(3*((5*b^4*c - 24*a*
b^2*c^2 + 16*a^2*c^3)*x^8 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^6 + (5*a
*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^4)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*((15*a*b^3*
c - 52*a^2*b*c^2)*x^6 - 2*a^3*b^2 + 8*a^4*c + (15*a*b^4 - 62*a^2*b^2*c + 2
4*a^3*c^2)*x^4 + 5*(a^2*b^3 - 4*a^3*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a
^4*b^2*c - 4*a^5*c^2)*x^8 + (a^4*b^3 - 4*a^5*b*c)*x^6 + (a^5*b^2 - 4*a^6*c
)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5 (a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{1}{x^5 (a + bx^2 + cx^4)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 (a + bx^2 + cx^4)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.79

$$\int \frac{1}{x^5 (a+bx^2+cx^4)^{3/2}}\,dx = \frac{\frac{(a^3b^3c-3\,a^4bc^2)x^2}{a^6b^2-4\,a^7c}+\frac{a^3b^4-4\,a^4b^2c+2\,a^5c^2}{a^6b^2-4\,a^7c}}{\sqrt{cx^4+bx^2+a}} + \frac{3\,(5\,b^2-4\,ac)\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^3} - \frac{7\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3b^2-4\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^3ac+8\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2ab\sqrt{c}-}{8\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)^2-\right.}$$

input
```
integrate(1/x^5/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((a^3*b^3*c - 3*a^4*b*c^2)*x^2/(a^6*b^2 - 4*a^7*c) + (a^3*b^4 - 4*a^4*b^2*
c + 2*a^5*c^2)/(a^6*b^2 - 4*a^7*c))/sqrt(c*x^4 + b*x^2 + a) + 3/8*(5*b^2 -
 4*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a
)*a^3) - 1/8*(7*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*b^2 - 4*(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^
2 + a))^2*a*b*sqrt(c) - 9*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a*b^2 -
4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*a^2*c - 16*a^2*b*sqrt(c))/(((sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{x^5\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,dx$$

input
```
int(1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x^5*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 \left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\sqrt{c\,x^4+b\,x^2+a}\,a\,x^5+\sqrt{c\,x^4+b\,x^2+a}\,b\,x^7+\sqrt{c\,x^4+b\,x^2+a}\,c\,x^9}dx$$

input
```
int(1/x^5/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(1/(sqrt(a + b*x**2 + c*x**4)*a*x**5 + sqrt(a + b*x**2 + c*x**4)*b*x**7
 + sqrt(a + b*x**2 + c*x**4)*c*x**9),x)
```

3.1016 $\int \frac{x^6}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 384

$$\int \frac{x^6}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{x(ab+(b^2-2ac)\,x^2)}{c\,(b^2-4ac)\,\sqrt{a+bx^2+cx^4}}+\frac{2(b^2-3ac)\,x\sqrt{a+bx^2+cx^4}}{c^{3/2}\,(b^2-4ac)\,(\sqrt{a}+\sqrt{c}x^2)}-\frac{2\sqrt[4]{a}(b^2-3ac)\,(\sqrt{a}+\sqrt{c}x^2)\,\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{7/4}\,(b^2-4ac)\,\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}(2b^2+\sqrt{a}b\sqrt{c}-6ac)\,(\sqrt{a}+\sqrt{c}x^2)\,\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{7/4}\,(b^2-4ac)\,\sqrt{a+bx^2+cx^4}}$$

output

```
-x*(a*b+(-2*a*c+b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+2*(-3*a*c+b
^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)-2*a
^(1/4)*(-3*a*c+b^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2
)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2
)/c^(1/2))^(1/2))/c^(7/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*a^(1/4)*(
2*b^2+a^(1/2)*b*c^(1/2)-6*a*c)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(
1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.93 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.27

$$\int \frac{x^6}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(b^2x^2+a(b-2cx^2)) - i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4}}{b+\sqrt{b^2}}}}{}$$

input
```
Integrate[x^6/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2*x^2 + a*(b - 2*c*x^2)) - I*(b^
2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b -
Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a
*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a
*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c
/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/(2*c^2*(-b^2 + 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2
 + c*x^4])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1440, 27, 1602, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \ 1440$$

$$\frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\int \frac{3x^2\left(bx^2+2a\right)}{\sqrt{cx^4+bx^2+a}}dx}{b^2-4ac}$$

$\Big\downarrow$ 27

$$\frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{3\int \frac{x^2\left(bx^2+2a\right)}{\sqrt{cx^4+bx^2+a}}dx}{b^2-4ac}$$

$\Big\downarrow$ 1602

$$\frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{3\left(\frac{bx\sqrt{a+bx^2+cx^4}}{3c}-\frac{\int \frac{2\left(b^2-3ac\right)x^2+ab}{\sqrt{cx^4+bx^2+a}}dx}{3c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 1511

$$\frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{3\left(\frac{bx\sqrt{a+bx^2+cx^4}}{3c}-\frac{\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}\left(b^2-3ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 27

$$\frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{3\left(\frac{bx\sqrt{a+bx^2+cx^4}}{3c}-\frac{\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\left(b^2-3ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{3c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 1416

$$\frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{3\left(\frac{bx\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{\sqrt[4]{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2\left(b^2-3ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+}}}{\sqrt{c}}}{3c}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{1509}$$

$$\frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-$$

$$\frac{3\left(\frac{bx\sqrt{a+bx^2+cx^4}}{3c}-\frac{\frac{\sqrt[4]{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2\left(b^2-3ac\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{\ldots}\right.}{\ldots}\right.}{\ldots}}{3c}\right.}{b^2-4ac}$$

input
```
Int[x^6/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x^3*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (3*((b*x*Sqr
t[a + b*x^2 + c*x^4])/(3*c) - ((-2*(b^2 - 3*a*c)*(-((x*Sqrt[a + b*x^2 + c*
x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*b + (2*(b^2 - 3*a*c))/Sqrt[c])*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sq
rt[a + b*x^2 + c*x^4]))/(3*c)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.26

method	result
default	$-\frac{2c\left(\frac{(2ac-b^2)x^3}{2c^2(4ac-b^2)}-\frac{abx}{2c^2(4ac-b^2)}\right)}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}-\frac{ab\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\right.}{4c(4ac-b^2)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}$
elliptic	$-\frac{2c\left(\frac{(2ac-b^2)x^3}{2c^2(4ac-b^2)}-\frac{abx}{2c^2(4ac-b^2)}\right)}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}-\frac{ab\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\right.}{4c(4ac-b^2)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}$

input
```
int(x^6/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/2/c^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3-1/2*a*b/c^2/(4*a*c-b^2)*x)/((x^4
+1/c*b*x^2+1/c*a)*c)^(1/2)-1/4*a*b/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))-1/2*(1/c+(2*a*c-b^2)/c/(4*a*c-b^2))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Elliptic
F(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.60

$$\int \frac{x^6}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{\frac{1}{2}}\left((b^3c-3\,abc^2)x^5+(b^4-3\,ab^2c)x^3+(ab^3-3\,a^2bc)x-((b^2c^2-3\,ac^3)x^5+(b^3c-3\,abc^2)x^3+(ab^2\right.}{}$$

input
```
integrate(x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(1/2)*((b^3*c - 3*a*b*c^2)*x^5 + (b^4 - 3*a*b^2*c)*x^3 + (a*b^
3 - 3*a^2*b*c)*x - ((b^2*c^2 - 3*a*c^3)*x^5 + (b^3*c - 3*a*b*c^2)*x^3 + (a
*b^2*c - 3*a^2*c^2)*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2
- 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*
c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c))
 - sqrt(1/2)*((2*b^3*c - (6*a*b - b^2)*c^2)*x^5 + (2*b^4 - (6*a*b^2 - b^3)
*c)*x^3 + (2*a*b^3 - (6*a^2*b - a*b^2)*c)*x - ((2*b^2*c^2 - (6*a + b)*c^3)
*x^5 + (2*b^3*c - (6*a*b + b^2)*c^2)*x^3 + (2*a*b^2*c - (6*a^2 + a*b)*c^2)
*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/
c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x),
 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*((b^2*c^2 - 4*
a*c^3)*x^4 + 2*a*b^2*c - 6*a^2*c^2 + (2*b^3*c - 7*a*b*c^2)*x^2)*sqrt(c*x^4
 + b*x^2 + a))/((b^2*c^4 - 4*a*c^5)*x^5 + (b^3*c^3 - 4*a*b*c^4)*x^3 + (a*b
^2*c^3 - 4*a^2*c^4)*x)
```

Sympy [F]

$$\int \frac{x^6}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^6}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**6/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^6}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{x^6}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{x^6}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(x^6/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^6/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2+cx^4)^{3/2}}\,dx=\frac{2\sqrt{c\,x^4+b\,x^2+a}\,bx+\sqrt{c\,x^4+b\,x^2+a}\,c\,x^3-2\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+}\right.}{}$$

input
```
int(x^6/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**2 + c*x**4)*b*x + sqrt(a + b*x**2 + c*x**4)*c*x**3 - 2*in
t(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 +
2*b*c*x**6 + c**2*x**8),x)*a**2*b - 2*int(sqrt(a + b*x**2 + c*x**4)/(a**2
+ 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b**2*
x**2 - 2*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b
**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b*c*x**4 - 3*int((sqrt(a + b*x**2
+ c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 +
 c**2*x**8),x)*a**2*c - 3*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a
*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b*c*x**2 -
 3*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 +
b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*c**2*x**4)/(c**2*(a + b*x**2 + c*
x**4))
```

3.1017 $\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 342

$$\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}(b^2-4ac)(\sqrt{a}+\sqrt{c}x^2)} + \frac{\sqrt[4]{a}b(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2(b-2\sqrt{a}\sqrt{c})c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*x*(c*x^4+b*x^2+a)^(1/2)
/c^(1/2)/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)+a^(1/4)*b*(a^(1/2)+c^(1/2)*x^2
)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c
^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(3/4)/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)^(1/2)-1/2*a^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a
^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/(b-2*a^(1/2)*c^(1/2))/c^(3/4)/(c*x^4+b*x^2
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.58 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.32

$$\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(2a+bx^2)-ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4}}}}{}$$

input
```
Integrate[x^4/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(2*a + b*x^2) - I*b*(-b + Sqrt[b^2
- 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*
Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellipt
icE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
- 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
 - Sqrt[b^2 - 4*a*c])])/(4*c*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1440, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \ 1440$$

$$\frac{x(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\int\frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx}{b^2-4ac}$$

$\downarrow$ 1511

$$\frac{x(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}b\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{b^2-4ac}$$

$\downarrow$ 27

$$\frac{x(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{b\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{b^2-4ac}$$

$\downarrow$ 1416

$$\frac{x(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{b^2-4ac}$$

$\downarrow$ 1509

$$\frac{x(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.}{\sqrt{c}}}{b^2-4ac}$$

input
```
Int[x^4/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (-((b*(-((x*Sq
rt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a
+ b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(2*Sqrt[a] + b/Sqrt[c])*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*
Sqrt[a + b*x^2 + c*x^4]))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1440
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*
(p + 1)*(b^2 - 4*a*c))), x] + Simp[d^4/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.32

method	result
default	$-\frac{2c\left(\frac{b\,x^{3}}{2\left(4ac-b^{2}\right)c}+\frac{ax}{c\left(4ac-b^{2}\right)}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\sqrt{-\cdots}\right)}{2\left(4ac-b^{2}\right)\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$
elliptic	$-\frac{2c\left(\frac{b\,x^{3}}{2\left(4ac-b^{2}\right)c}+\frac{ax}{c\left(4ac-b^{2}\right)}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\sqrt{-\cdots}\right)}{2\left(4ac-b^{2}\right)\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$

input
```
int(x^4/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/2*b/(4*a*c-b^2)/c*x^3+a/c/(4*a*c-b^2)*x)/((x^4+1/c*b*x^2+1/c*a)*c)
^(1/2)+1/2*a/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*a
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/
(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(
1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/
a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.31

$$\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(b^2cx^4+b^3x^2+ab^2-(abcx^4+ab^2x^2+a^2b)\sqrt{\frac{b^2-4\,ac}{a^2}}\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-}{a}}\right.}{}$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(b^2*c*x^4 + b^3*x^2 + a*b^2 - (a*b*c*x^4 + a*b^2*x^2 + a^
2*b)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)
/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a))
, 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*a
*b + b^2)*c*x^4 + 2*a^2*b + a*b^2 + (2*a*b^2 + b^3)*x^2 + ((2*a^2 - a*b)*c
*x^4 + 2*a^3 - a^2*b + (2*a^2*b - a*b^2)*x^2)*sqrt((b^2 - 4*a*c)/a^2))*sqr
t(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x
*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2
) + b^2 - 2*a*c)/(a*c)) - 2*(a*b*c*x^3 + 2*a^2*c*x)*sqrt(c*x^4 + b*x^2 + a
))/(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2
*b*c^2)*x^2)
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^4}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**4/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^4}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^4}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^4}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(x^4/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^4/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-\sqrt{c\,x^4+b\,x^2+a}\,x+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx\right)a^2+\left(\int \frac{\sqrt{}}{c^2x^8+2bc\,x^6+}\right.}{c\left(c\,x^4+b\,x\right.}$$

input
```
int(x^4/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2 + c*x**4)*x + int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*
a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2 + int(
sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*
b*c*x**6 + c**2*x**8),x)*a*b*x**2 + int(sqrt(a + b*x**2 + c*x**4)/(a**2 +
2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*c*x**4)
/(c*(a + b*x**2 + c*x**4))
```

3.1018 $\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 341

$$\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{x(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output

```
-x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+2*c^(1/2)*x*(c*x^4+b*x^2
+a)^(1/2)/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)-2*a^(1/4)*c^(1/4)*(a^(1/2)+c^
(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/(-4*a*c+b^2)/
(c*x^4+b*x^2+a)^(1/2)+1/2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+
c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b
/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/(b-2*a^(1/2)*c^(1/2))/c^(1/4)/(c*x^4+b*x^
2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.56 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(b+2cx^2)+i\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4a}}}}{}$$

input
```
Integrate[x^2/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 2*c*x^2) + I*(-b + Sqrt[b^2 - 4
*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])] - I*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c
/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/(2*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*
x^4])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1439, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 1439}$$

$$\frac{\int \frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}dx}{b^2-4ac}-\frac{x(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1511

$$\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-2\sqrt{a}\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{b^2-4ac}-\frac{x(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 27

$$\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-2\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{b^2-4ac}-\frac{x(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1416

$$\frac{\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-2\sqrt{c}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{b^2-4ac}-\frac{x(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1509

$$\frac{\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-2\sqrt{c}\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.}{b^2-4ac}$$

$$\frac{x(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[x^2/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
-((x*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) + (-2*Sqrt[c]
*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4
)*Sqrt[a + b*x^2 + c*x^4])) + ((b + 2*Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*S
qrt[a + b*x^2 + c*x^4]))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1439
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
1)*(b^2 - 4*a*c))), x] - Simp[d^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.31

method	result
default	$-\frac{2c\left(-\frac{x^3}{4ac-b^2}-\frac{bx}{2\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}-\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\sqrt{-4\cdots}\right.}{4\left(4ac-b^2\right)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
elliptic	$-\frac{2c\left(-\frac{x^3}{4ac-b^2}-\frac{bx}{2\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}-\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\sqrt{-4\cdots}\right.}{4\left(4ac-b^2\right)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$

input
```
int(x^2/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/(4*a*c-b^2)*x^3-1/2*b/(4*a*c-b^2)/c*x)/((x^4+1/c*b*x^2+1/c*a)*c)^
(1/2)-1/4*b/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b
+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+c/(4*a*c-b^2)*a*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{2\sqrt{\frac{1}{2}}\left(bc^2x^4+b^2cx^2+abc-(ac^2x^4+abcx^2+a^2c)\sqrt{\frac{b^2-4\,ac}{a^2}}\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}}{}$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(1/2)*(b*c^2*x^4 + b^2*c*x^2 + a*b*c - (a*c^2*x^4 + a*b*c*x^2 +
 a^2*c)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) -
 b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/
a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((
b^2*c + 2*b*c^2)*x^4 + a*b^2 + 2*a*b*c + (b^3 + 2*b^2*c)*x^2 + ((a*b*c - 2
*a*c^2)*x^4 + a^2*b - 2*a^2*c + (a*b^2 - 2*a*b*c)*x^2)*sqrt((b^2 - 4*a*c)/
a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sq
rt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4
*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - 2*(2*a*c^2*x^3 + a*b*c*x)*sqrt(c*x^4 +
b*x^2 + a))/(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*
c - 4*a^2*b*c^2)*x^2)
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(x**2/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(x^2/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(x^2/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(x^2/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**
2*x**4 + 2*b*c*x**6 + c**2*x**8),x)
```

3.1019 $\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 353

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x(b^2-2ac+bcx^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)(\sqrt{a}+\sqrt{c}x^2)} + \frac{b\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}(b-2\sqrt{a}\sqrt{c})\sqrt{a+bx^2+cx^4}}$$

output
```
x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*c^(1/2)*x*(c*
x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)+b*c^(1/4)*(a^(1/2)
+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(-
4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4
+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*
x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/
(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(b^2-2ac+bcx^2)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\Big(i\text{arcsinh}}{}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-3/2),x]
```

output
```
-1/4*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2) + I*b*(
-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
 + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[
b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 1405$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\int \frac{c\left(bx^2+2a\right)}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1511$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1416$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1509$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.}{\sqrt{}}\right.}{a\left(b^2-4ac\right)}$$

input
```
Int[(a + b*x^2 + c*x^4)^(-3/2),x]
```

output
```
(x*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (c
*(-((b*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(2*Sqrt[a] + b/Sqrt
[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/
4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.36

method	result
default	$-\frac{2c\left(\frac{b\,x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)x}{2a\left(4ac-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^{2}}{a\left(4ac-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{}}{}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$
elliptic	$-\frac{2c\left(\frac{b\,x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)x}{2a\left(4ac-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^{2}}{a\left(4ac-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{}}{}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$

input
```
int(1/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/2*b/a/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+1/c
*b*x^2+1/c*a)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*
(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(b^2cx^4+b^3x^2+ab^2-(abcx^4+ab^2x^2+a^2b)\sqrt{\frac{b^2-4ac}{a^2}}\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4ac}{a^2}}-b}{a}}E(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4ac}{a^2}}-}{a}}\right.}{}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(b^2*c*x^4 + b^3*x^2 + a*b^2 - (a*b*c*x^4 + a*b^2*x^2 + a^
2*b)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)
/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a))
, 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*a
*b + b^2)*c*x^4 + 2*a^2*b + a*b^2 + (2*a*b^2 + b^3)*x^2 + ((2*a^2 - a*b)*c
*x^4 + 2*a^3 - a^2*b + (2*a^2*b - a*b^2)*x^2)*sqrt((b^2 - 4*a*c)/a^2))*sqr
t(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x
*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2
) + b^2 - 2*a*c)/(a*c)) - 2*(a*b*c*x^3 + (a*b^2 - 2*a^2*c)*x)*sqrt(c*x^4 +
 b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 -
 4*a^3*b*c)*x^2)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(1/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(1/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(1/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4
+ 2*b*c*x**6 + c**2*x**8),x)
```

3.1020 $\int \frac{1}{x^2(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 385

$$\int \frac{1}{x^2 (a+bx^2+cx^4)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a(b^2-4ac)\,x\sqrt{a+bx^2+cx^4}} - \frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{a^{3/2}(b^2-4ac)\,x\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{2\sqrt[4]{c}(b^2-3ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{7/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{\left(2b-3\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}$$

output
```
(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)^(1/2)-2*(-3*a*c+b^2)*
(c*x^4+b*x^2+a)^(1/2)/a^(3/2)/(-4*a*c+b^2)/x/(a^(1/2)+c^(1/2)*x^2)-2*c^(1/
4)*(-3*a*c+b^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^
2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^
(1/2))^(1/2))/a^(7/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*(2*b-3*a^(1/2
)*c^(1/2))*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)
*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2
)/c^(1/2))^(1/2))/a^(7/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.89 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^2 (a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(-4a^2c+2b^2x^2(b+cx^2)+a(b^2-7bcx^2-6c^2x^4))-i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{\cdots}}{b-\cdots}}\cdots}{\cdots}$$

input
```
Integrate[1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-1/2*(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-4*a^2*c + 2*b^2*x^2*(b + c*x^2)
+ a*(b^2 - 7*b*c*x^2 - 6*c^2*x^4)) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*
c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[
I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] -
 3*a*c*Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2
 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x]
, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a^2*(b^2 - 4*a*c)*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1441, 25, 1604, 25, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1441}$$

$$\frac{-2ac+b^2+bcx^2}{ax\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\int -\frac{bcx^2+2\left(b^2-3ac\right)}{x^2\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bcx^2+2\left(b^2-3ac\right)}{x^2\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{1604}$$

$$\frac{-\frac{\int -\frac{c\left(2\left(b^2-3ac\right)x^2+ab\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{ax}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{c\left(2\left(b^2-3ac\right)x^2+ab\right)}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{ax}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{c\int \frac{2\left(b^2-3ac\right)x^2+ab}{\sqrt{cx^4+bx^2+a}}dx}{a}-\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{ax}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{\frac{c\left(\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}\left(b^2-3ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{ax}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{c\left(\sqrt{a}\left(\frac{2\left(b^2-3ac\right)}{\sqrt{c}}+\sqrt{a}b\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\left(b^2-3ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{ax}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\frac{c\left(\frac{\sqrt[4]{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2(b^2-3ac)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a}-\frac{2(b^2-3ac)\cdots}{\cdots}}{a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx^2}{ax\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{\frac{c\left(\frac{\sqrt[4]{a}\left(\frac{2(b^2-3ac)}{\sqrt{c}}+\sqrt{a}b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2(b^2-3ac)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\cdots}{\sqrt[4]{c}\sqrt{a+\cdots}}\cdots\right)}{\cdots}\right)}{a}\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{-2ac+b^2+bcx^2}{ax\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x*Sqrt[a + b*x^2 + c*x^4]) + ((-2
*(b^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a*x) + (c*((-2*(b^2 - 3*a*c)*(-((
x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqr
t[a + b*x^2 + c*x^4])))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*b + (2*(b^2 - 3*a*c))/
Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/a)/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1604
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Simp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + b*x^2
 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Maple [A] (verified)

Time = 6.16 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{a^2 x}-\frac{2c\left(\frac{(2ac-b^2)x^3}{2(4ac-b^2)a^2}+\frac{b(3ac-b^2)x}{2a^2(4ac-b^2)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(-\frac{b}{a^2}+\frac{b(3ac-b^2)}{a^2(4ac-b^2)}\right)\sqrt{2}\,\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\,\sqrt{4+\frac{2(b+\sqrt{-4ac}}{a}}}{4\sqrt{\frac{-b+\sqrt{-4ac+b}}{a}}}$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{a^2 x}-\frac{2c\left(\frac{(2ac-b^2)x^3}{2(4ac-b^2)a^2}+\frac{b(3ac-b^2)x}{2a^2(4ac-b^2)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(-\frac{b}{a^2}+\frac{b(3ac-b^2)}{a^2(4ac-b^2)}\right)\sqrt{2}\,\sqrt{4-\frac{2(-b+\sqrt{-4ac+b^2})x^2}{a}}\,\sqrt{4+\frac{2(b+\sqrt{-4ac}}{a}}}{4\sqrt{\frac{-b+\sqrt{-4ac+b}}{a}}}$
risch	Expression too large to display

input
```
int(1/x^2/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(c*x^4+b*x^2+a)^(1/2)/x-2*c*(1/2*(2*a*c-b^2)/(4*a*c-b^2)/a^2*x^3+1/
2*b*(3*a*c-b^2)/a^2/(4*a*c-b^2)/c*x)/((x^4+1/c*b*x^2+1/c*a)*c)^(1/2)+1/4*(
-b/a^2+b*(3*a*c-b^2)/a^2/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(c
/a^2+c*(2*a*c-b^2)/(4*a*c-b^2)/a^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(
1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{2\sqrt{\frac{1}{2}}\Big((b^3c-3\,abc^2)x^5+(b^4-3\,ab^2c)x^3+(ab^3-3\,a^2bc)x-((ab^2c-3\,a^2c^2}{}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(1/2)*((b^3*c - 3*a*b*c^2)*x^5 + (b^4 - 3*a*b^2*c)*x^3 + (a*b^3
 - 3*a^2*b*c)*x - ((a*b^2*c - 3*a^2*c^2)*x^5 + (a*b^3 - 3*a^2*b*c)*x^3 + (
a^2*b^2 - 3*a^3*c)*x)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 -
 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a
*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c))
+ sqrt(1/2)*((6*a*b*c^2 - (a*b^2 + 2*b^3)*c)*x^5 - (a*b^3 + 2*b^4 - 6*a*b^
2*c)*x^3 - (a^2*b^2 + 2*a*b^3 - 6*a^2*b*c)*x - ((6*a^2*c^2 + (a^2*b - 2*a*
b^2)*c)*x^5 + (a^2*b^2 - 2*a*b^3 + 6*a^2*b*c)*x^3 + (a^3*b - 2*a^2*b^2 + 6
*a^3*c)*x)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2
) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) -
b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - 2*(2*(a*b
^2*c - 3*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (2*a*b^3 - 7*a^2*b*c)*x^2)*sqr
t(c*x^4 + b*x^2 + a))/((a^3*b^2*c - 4*a^4*c^2)*x^5 + (a^3*b^3 - 4*a^4*b*c)
*x^3 + (a^4*b^2 - 4*a^5*c)*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{1}{x^2 (c\,x^4 + b\,x^2 + a)^{3/2}} \, dx$$

input
```
int(1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^{3/2}} \, dx = \int \frac{\sqrt{c\,x^4 + b\,x^2 + a}}{c^2 x^{10} + 2bc\,x^8 + 2ac\,x^6 + b^2 x^6 + 2ab\,x^4 + a^2 x^2} dx$$

input
```
int(1/x^2/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a**2*x**2 + 2*a*b*x**4 + 2*a*c*x**6 + b**2*
x**6 + 2*b*c*x**8 + c**2*x**10),x)
```

3.1021 $\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 50

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}+\frac{x\sqrt{bx^2+cx^4}}{3c}$$

output
```
-2/3*b*(c*x^4+b*x^2)^(1/2)/c^2/x+1/3*x*(c*x^4+b*x^2)^(1/2)/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{(-2b+cx^2)\sqrt{x^2(b+cx^2)}}{3c^2x}$$

input
```
Integrate[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

output
```
((-2*b + c*x^2)*Sqrt[x^2*(b + c*x^2)])/(3*c^2*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3, 1421, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{2a-2(a+1)+bx^2+cx^4+2}}dx$$
$$\Big\downarrow \; 3$$
$$\int \frac{x^4}{\sqrt{bx^2+cx^4}}dx$$
$$\Big\downarrow \; 1421$$
$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx}{3c}$$
$$\Big\downarrow \; 1420$$
$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

input
```
Int[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

output
```
(-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3*c)
```

Defintions of rubi rules used

rule 3
```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

rule 1421
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))), x] - Simp[b
*d^2*((m + 2*p - 1)/(c*(m + 4*p + 1)))   Int[(d*x)^(m - 2)*(b*x^2 + c*x^4)^
p, x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && IGtQ[Simplify[(m
 + 2*p - 1)/2], 0] && NeQ[m + 4*p + 1, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

method	result	size
trager	$-\frac{\left(-c\,x^{2}+2 b \right) \sqrt{c\,x^{4}+b\,x^{2}}}{3 c^{2} x}$	32
gosper	$-\frac{\left(c\,x^{2}+b \right) \left(-c\,x^{2}+2 b \right) x}{3 c^{2} \sqrt{c\,x^{4}+b\,x^{2}}}$	37
default	$-\frac{\left(c\,x^{2}+b \right) \left(-c\,x^{2}+2 b \right) x}{3 c^{2} \sqrt{c\,x^{4}+b\,x^{2}}}$	37
risch	$-\frac{x \left(c\,x^{2}+b \right) \left(-c\,x^{2}+2 b \right)}{3 \sqrt{x^{2} \left(c\,x^{2}+b \right)}\, c^{2}}$	37
orering	$-\frac{\left(c\,x^{2}+b \right) \left(-c\,x^{2}+2 b \right) x}{3 c^{2} \sqrt{c\,x^{4}+b\,x^{2}}}$	37

input
```
int(x^4/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-c*x^2+2*b)/c^2/x*(c*x^4+b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.60

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\sqrt{cx^4+bx^2}(cx^2-2\,b)}{3\,c^2x}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b)/(c^2*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(x**4/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**4/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{c^2x^4-bcx^2-2\,b^2}{3\,\sqrt{cx^2+b}c^2}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)/(sqrt(c*x^2 + b)*c^2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{2\,b^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,c^2} + \frac{(cx^2+b)^{\frac{3}{2}}}{3\,c^2\mathrm{sgn}\,(x)} - \frac{\sqrt{cx^2+b}b}{c^2\mathrm{sgn}\,(x)}$$

input
```
integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*b^(3/2)*sgn(x)/c^2 + 1/3*(c*x^2 + b)^(3/2)/(c^2*sgn(x)) - sqrt(c*x^2 +
 b)*b/(c^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.88 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{2\,b}{3\,c^2}-\frac{x^2}{3\,c}\right)}{x}$$

input
```
int(x^4/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
-((b*x^2 + c*x^4)^(1/2)*((2*b)/(3*c^2) - x^2/(3*c)))/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.44

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^2+b}\,(c\,x^2-2b)}{3c^2}$$

input
```
int(x^4/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(b + c*x**2)*( - 2*b + c*x**2))/(3*c**2)
```

3.1022 $\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 58

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

output
```
1/2*(c*x^4+b*x^2)^(1/2)/c-1/2*b*arctanh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/c
^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.43

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$$
$$=\frac{x\left(\sqrt{c}x(b+cx^2)+2b\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}-\sqrt{b+cx^2}}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

output
```
(x*(Sqrt[c]*x*(b + c*x^2) + 2*b*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/(Sqrt[
b] - Sqrt[b + c*x^2])]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3, 1424, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{2a-2(a+1)+bx^2+cx^4+2}}dx$$

$$\Big\downarrow \text{ 3}$$

$$\int \frac{x^3}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1424}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{2}\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2}{2c}\right)$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{1}{2}\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+bx^2}}}{c}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{\sqrt{bx^2+cx^4}}{c}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}\right)$$

input
```
Int[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

output
```
(Sqrt[b*x^2 + c*x^4]/c - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/c^
(3/2))/2
```

Defintions of rubi rules used

rule 3
```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

method	result	size
pseudoelliptic	$\frac{b\ln(2)-\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b+2\sqrt{x^2(c\,x^2+b)}\,\sqrt{c}}{4c^{\frac{3}{2}}}$	63
default	$\frac{x\sqrt{c\,x^2+b}\,\left(x\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}-b\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)c\right)}{2\sqrt{c\,x^4+b\,x^2}\,c^{\frac{5}{2}}}$	64
risch	$\frac{x^2\left(c\,x^2+b\right)}{2c\sqrt{x^2\left(c\,x^2+b\right)}}-\frac{b\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)x\sqrt{c\,x^2+b}}{2c^{\frac{3}{2}}\sqrt{x^2\left(c\,x^2+b\right)}}$	75

input
```
int(x^3/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/c^(3/2)*(b*ln(2)-ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1/2
))*b+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.97

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \left[\frac{b\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}c}{4\,c^2}, \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}c}{2\,c^2}\right.$$

input
```
integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(b*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) + 2*sqrt
(c*x^4 + b*x^2)*c)/c^2, 1/2*(b*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c
)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*c)/c^2]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{x^3}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{4\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^4+bx^2}}{2\,c}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) + 1/2*sqrt
(c*x^4 + b*x^2)/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{b\log\left(|b|\right)\mathrm{sgn}(x)}{4\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^2+b}x}{2\,c\mathrm{sgn}\,(x)} + \frac{b\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{2\,c^{\frac{3}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*b*log(abs(b))*sgn(x)/c^(3/2) + 1/2*sqrt(c*x^2 + b)*x/(c*sgn(x)) + 1/2
*b*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.64 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}}{2\,c} - \frac{b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,c^{3/2}}$$

input
```
int(x^3/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
(b*x^2 + c*x^4)^(1/2)/(2*c) - (b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^
4)^(1/2)))/(4*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^2+b}\,cx-\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)b}{2c^2}$$

input
```
int(x^3/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(b + c*x**2)*c*x - sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b)
)*b)/(2*c**2)
```

3.1023 $\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 22

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\frac{\sqrt{bx^2+cx^4}}{cx}$$

output
```
(c*x^4+b*x^2)^(1/2)/c/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\frac{\sqrt{x^2\left(b+cx^2\right)}}{cx}$$

input
```
Integrate[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

output
```
Sqrt[x^2*(b + c*x^2)]/(c*x)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3, 1420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{2a-2(a+1)+bx^2+cx^4+2}}dx$$
$$\Big\downarrow \text{ 3}$$
$$\int \frac{x^2}{\sqrt{bx^2+cx^4}}dx$$
$$\Big\downarrow \text{ 1420}$$
$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

input
```
Int[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

output
```
Sqrt[b*x^2 + c*x^4]/(c*x)
```

Defintions of rubi rules used

rule 3
```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

rule 1420
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d^3*(d*x)^(m - 3)*((b*x^2 + c*x^4)^(p + 1)/(2*c*(p + 1))), x] /; FreeQ[{b,
 c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 2*p - 1, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
trager	$\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{cx}$	21
gosper	$\frac{\left(c\,x^{2}+b\right)x}{c\sqrt{c\,x^{4}+b\,x^{2}}}$	26
default	$\frac{\left(c\,x^{2}+b\right)x}{c\sqrt{c\,x^{4}+b\,x^{2}}}$	26
risch	$\frac{x\left(c\,x^{2}+b\right)}{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,c}$	26
orering	$\frac{\left(c\,x^{2}+b\right)x}{c\sqrt{c\,x^{4}+b\,x^{2}}}$	26

input
```
int(x^2/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x^4+b*x^2)^(1/2)/c/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\frac{\sqrt{cx^4+bx^2}}{cx}$$

input
```
integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^4 + b*x^2)/(c*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{x^2}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(x**2/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\sqrt{cx^2+b}}{c}$$

input
```
integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(c*x^2 + b)/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{\sqrt{b}\mathrm{sgn}(x)}{c} + \frac{\sqrt{cx^2+b}}{c\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(b)*sgn(x)/c + sqrt(c*x^2 + b)/(c*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 18.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}}{c\,x}$$

input
```
int(x^2/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
(b*x^2 + c*x^4)^(1/2)/(c*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^2+b}}{c}$$

input
```
int(x^2/(c*x^4+b*x^2)^(1/2),x)
```

output
```
sqrt(b + c*x**2)/c
```

3.1024 $\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 31

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

output
```
arctanh(c^(1/2)*x^2/(c*x^4+b*x^2)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\frac{x\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)}{\sqrt{c}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

output
```
(x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]])/(Sqrt[c]*Sqrt[x^2
*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3, 1424, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{2a-2(a+1)+bx^2+cx^4+2}}dx$$

$$\Big\downarrow \text{ 3}$$

$$\int \frac{x}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \text{ 1424}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{cx^4+bx^2}}dx^2$$

$$\Big\downarrow \text{ 1091}$$

$$\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{bx^2+cx^4}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

input
```
Int[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

output
```
ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Defintions of rubi rules used

rule 3
```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1424
```
Int[(x_)^(m_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
Subst[Int[x^((m - 1)/2)*(b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{b, c, m,
 p}, x] &&  !IntegerQ[p] && IntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

method	result	size
pseudoelliptic	$\frac{-\ln(2)+\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)}{2\sqrt{c}}$	42
default	$\frac{x\sqrt{c\,x^{2}+b}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)}{\sqrt{c\,x^{4}+b\,x^{2}}\,\sqrt{c}}$	44

input
```
int(x/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-ln(2)+ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1/2)))/c^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.39

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \left[\frac{\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{2\,\sqrt{c}},\; -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)}{c}\right]$$

input
```
integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c), -sqrt(-c)*
arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b))/c]
```

Sympy [F]

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{x}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(x/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(x/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{2\,\sqrt{c}}$$

input
```
integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\log(|b|)\operatorname{sgn}(x)}{2\sqrt{c}} - \frac{\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{\sqrt{c}\operatorname{sgn}(x)}$$

input
```
integrate(x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*log(abs(b))*sgn(x)/sqrt(c) - log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(s
qrt(c)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 18.44 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,\sqrt{c}}$$

input
```
int(x/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2))/(2*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{c}\,x}{\sqrt{b}}\right)}{c}$$

input
```
int(x/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(c)*log((sqrt(b + c*x**2) + sqrt(c)*x)/sqrt(b)))/c
```

3.1025 $\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 30

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

output
```
-arctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{x\sqrt{b+cx^2}\,\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{x^2\left(b+cx^2\right)}}$$

input
```
Integrate[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

output
```
-((x*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[x^2*(
b + c*x^2)]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2a-2(a+1)+bx^2+cx^4+2}}dx$$

$$\Big\downarrow \; 3$$

$$\int \frac{1}{\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \; 1400$$

$$-\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

output
```
-(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Defintions of rubi rules used

rule 3
```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(24) = 48.

Time = 0.51 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

method	result	size
default	$-\frac{x\sqrt{c\,x^{2}+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)}{\sqrt{c\,x^{4}+b\,x^{2}}\,\sqrt{b}}$	50

input
```
int(1/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(
1/2)+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \left[\frac{\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{bx}\right)}{b}\right]$$

input
```
integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3)/sqrt(b), sq
rt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(b*x))/b]
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/sqrt(b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+bx^2}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}+\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\operatorname{sgn}(x)}$$

input
```
integrate(1/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + arctan(sqrt(c*x^2 + b)/sqrt(-b
))/(sqrt(-b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(1/(b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(1/(b*x^2 + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\sqrt{b}\left(\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)-\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)\right)}{b}$$

input
```
int(1/(c*x^4+b*x^2)^(1/2),x)
```

output
```
(sqrt(b)*(log((sqrt(b + c*x**2) - sqrt(b) + sqrt(c)*x)/sqrt(b)) - log((sqr
t(b + c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))))/b
```

3.1026 $\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 23

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

output
```
-(c*x^4+b*x^2)^(1/2)/b/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{\sqrt{x^2(b+cx^2)}}{bx^2}$$

input
```
Integrate[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

output
```
-(Sqrt[x^2*(b + c*x^2)]/(b*x^2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{2a-2(a+1)+bx^2+cx^4+2}}dx$$

$$\Big\downarrow \; 3$$

$$\int \frac{1}{x\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \; 1422$$

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

input
```
Int[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

output
```
-(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Defintions of rubi rules used

rule 3
```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
trager	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}}}{b\,x^{2}}$	22
gosper	$-\frac{c\,x^{2}+b}{b\sqrt{c\,x^{4}+b\,x^{2}}}$	26
default	$-\frac{c\,x^{2}+b}{b\sqrt{c\,x^{4}+b\,x^{2}}}$	26
risch	$-\frac{c\,x^{2}+b}{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,b}$	26
pseudoelliptic	$-\frac{c\,x^{2}+b}{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,b}$	26
orering	$-\frac{c\,x^{2}+b}{b\sqrt{c\,x^{4}+b\,x^{2}}}$	26

input
```
int(1/x/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^4+b*x^2)^(1/2)/b/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(c*x^4 + b*x^2)/(b*x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{x\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(1/x/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(c*x^4 + b*x^2)/(b*x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{2\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)\operatorname{sgn}(x)}$$

input
```
integrate(1/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}}{b\,x^2}$$

input
```
int(1/(x*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
-(b*x^2 + c*x^4)^(1/2)/(b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{-\sqrt{c\,x^2+b}-\sqrt{c}\,x}{bx}$$

input
```
int(1/x/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - (sqrt(b + c*x**2) + sqrt(c)*x))/(b*x)
```

3.1027 $\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 59

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=-\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}$$

output
```
-1/2*(c*x^4+b*x^2)^(1/2)/b/x^3+1/2*c*arctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2)
)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\frac{-\sqrt{b}(b+cx^2)+cx^2\sqrt{b+cx^2}\,\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{2b^{3/2}x\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

output
```
(-(Sqrt[b]*(b + c*x^2)) + c*x^2*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sq
rt[b]])/(2*b^(3/2)*x*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{2a-2(a+1)+bx^2+cx^4+2}}dx$$

$$\Big\downarrow \ 3$$

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \ 1430$$

$$-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

$$\Big\downarrow \ 1400$$

$$\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

$$\Big\downarrow \ 219$$

$$\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

input
```
Int[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

output
```
-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]])/(2*b^(3/2))
```

Defintions of rubi rules used

rule 3
```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\sqrt{c\,x^{2}+b}\,\left(-c\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)b\,x^{2}+\sqrt{c\,x^{2}+b}\,b^{\frac{3}{2}}\right)}{2x\sqrt{c\,x^{4}+b\,x^{2}}\,b^{\frac{5}{2}}}$	73
risch	$-\frac{c\,x^{2}+b}{2bx\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{c\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)x\sqrt{c\,x^{2}+b}}{2b^{\frac{3}{2}}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	82

input
```
int(1/x^2/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x*(c*x^2+b)^(1/2)*(-c*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*x^2+(c*x^
2+b)^(1/2)*b^(3/2))/(c*x^4+b*x^2)^(1/2)/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \left[\frac{\sqrt{b}cx^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}b}{4\,b^2x^3},\ -\frac{\sqrt{-b}cx^3\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{bx}\right)+\sqrt{cx^4+bx^2}b}{2\,b^2x^3}\right]$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x
^3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3), -1/2*(sqrt(-b)*c*x^3*arctan(sqrt
(c*x^4 + b*x^2)*sqrt(-b)/(b*x)) + sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{x^2\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2}x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{c\left(\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{\sqrt{cx^2+b}}{bcx^2}\right)}{2\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*c*(arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(c*x^2 + b)/(b
*c*x^2))/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 17.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{\left(\frac{\sqrt{c}\,x^2\,\sqrt{c+\frac{b}{x^2}}}{2\,b}+\frac{c^{3/2}\,x^3\,\mathrm{asin}\left(\frac{\sqrt{b}\,1{}\mathrm{i}}{\sqrt{c}\,x}\right)\,1{}\mathrm{i}}{2\,b^{3/2}}\right)\,\sqrt{\frac{b}{c\,x^2}+1}}{x\,\sqrt{c\,x^4+b\,x^2}}$$

input
```
int(1/(x^2*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
-(((c^(1/2)*x^2*(c + b/x^2)^(1/2))/(2*b) + (c^(3/2)*x^3*asin((b^(1/2)*1i)/
(c^(1/2)*x))*1i)/(2*b^(3/2)))*(b/(c*x^2) + 1)^(1/2))/(x*(b*x^2 + c*x^4)^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{-\sqrt{c\,x^2+b}\,b-\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c\,x^2+\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c\,x^2}{2b^2x^2}$$

input
```
int(1/x^2/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - sqrt(b + c*x**2)*b - sqrt(b)*log((sqrt(b + c*x**2) - sqrt(b) + sqrt(c)
*x)/sqrt(b))*c*x**2 + sqrt(b)*log((sqrt(b + c*x**2) + sqrt(b) + sqrt(c)*x)
/sqrt(b))*c*x**2)/(2*b**2*x**2)
```

3.1028 $\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 52

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=-\frac{\sqrt{bx^2+cx^4}}{3bx^4}+\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}$$

output
```
-1/3*(c*x^4+b*x^2)^(1/2)/b/x^4+2/3*c*(c*x^4+b*x^2)^(1/2)/b^2/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\frac{\sqrt{x^2(b+cx^2)}(-b+2cx^2)}{3b^2x^4}$$

input
```
Integrate[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[x^2*(b + c*x^2)]*(-b + 2*c*x^2))/(3*b^2*x^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3, 1423, 1422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{2a-2(a+1)+bx^2+cx^4+2}}dx$$
$$\Big\downarrow \text{3}$$
$$\int \frac{1}{x^3\sqrt{bx^2+cx^4}}dx$$
$$\Big\downarrow \text{1423}$$
$$-\frac{2c\int \frac{1}{x\sqrt{cx^4+bx^2}}dx}{3b}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$
$$\Big\downarrow \text{1422}$$
$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

input
```
Int[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

output
```
-1/3*Sqrt[b*x^2 + c*x^4]/(b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^2)
```

Defintions of rubi rules used

rule 3
```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

rule 1422
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[(-d)*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(2*b*(p + 1))), x] /; FreeQ[{b
, c, d, m, p}, x] &&  !IntegerQ[p] && EqQ[m + 4*p + 3, 0]
```

rule 1423
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && ILtQ[Simplify[(m +
 4*p + 3)/2], 0] && NeQ[m + 2*p + 1, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

method	result	size
trager	$-\frac{\left(-2c\,x^{2}+b\right)\sqrt{c\,x^{4}+b\,x^{2}}}{3b^{2}x^{4}}$	30
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-2c\,x^{2}+b\right)}{3x^{2}b^{2}\sqrt{c\,x^{4}+b\,x^{2}}}$	37
default	$-\frac{\left(c\,x^{2}+b\right)\left(-2c\,x^{2}+b\right)}{3x^{2}b^{2}\sqrt{c\,x^{4}+b\,x^{2}}}$	37
risch	$-\frac{\left(c\,x^{2}+b\right)\left(-2c\,x^{2}+b\right)}{3x^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,b^{2}}$	37
pseudoelliptic	$-\frac{\left(c\,x^{2}+b\right)\left(-2c\,x^{2}+b\right)}{3x^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,b^{2}}$	37
orering	$-\frac{\left(c\,x^{2}+b\right)\left(-2c\,x^{2}+b\right)}{3x^{2}b^{2}\sqrt{c\,x^{4}+b\,x^{2}}}$	37

input
```
int(1/x^3/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-2*c*x^2+b)/b^2/x^4*(c*x^4+b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\frac{\sqrt{cx^4+bx^2}(2\,cx^2-b)}{3\,b^2x^4}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b)/(b^2*x^4)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\int \frac{1}{x^3\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

input
```
integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\frac{2\sqrt{cx^4+bx^2}c}{3\,b^2x^2}-\frac{\sqrt{cx^4+bx^2}}{3\,bx^4}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - 1/3*sqrt(c*x^4 + b*x^2)/(b*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx=\frac{4\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)c^{\frac{3}{2}}}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^3\operatorname{sgn}\left(x\right)}$$

input
```
integrate(1/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
4/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*c^(3/2)/(((sqrt(c)*x - sqrt(c*
x^2 + b))^2 - b)^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 17.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{(b-2\,c\,x^2)\,\sqrt{c\,x^4+b\,x^2}}{3\,b^2\,x^4}$$

input
```
int(1/(x^3*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
-((b - 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*b^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{-\sqrt{c\,x^2+b}\,b+2\sqrt{c\,x^2+b}\,c\,x^2-2\sqrt{c}\,c\,x^3}{3b^2x^3}$$

input
```
int(1/x^3/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - sqrt(b + c*x**2)*b + 2*sqrt(b + c*x**2)*c*x**2 - 2*sqrt(c)*c*x**3)/(3*
b**2*x**3)
```

3.1029 $\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 87

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3} - \frac{3c^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}$$

output
```
-1/4*(c*x^4+b*x^2)^(1/2)/b/x^5+3/8*c*(c*x^4+b*x^2)^(1/2)/b^2/x^3-3/8*c^2*a
rctanh(b^(1/2)*x/(c*x^4+b*x^2)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\sqrt{b}(-2b^2+bcx^2+3c^2x^4)-3c^2x^4\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{8b^{5/2}x^3\sqrt{x^2(b+cx^2)}}$$

input
```
Integrate[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

output
```
(Sqrt[b]*(-2*b^2 + b*c*x^2 + 3*c^2*x^4) - 3*c^2*x^4*Sqrt[b + c*x^2]*ArcTan
h[Sqrt[b + c*x^2]/Sqrt[b]])/(8*b^(5/2)*x^3*Sqrt[x^2*(b + c*x^2)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3, 1430, 1430, 1400, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{2a-2(a+1)+bx^2+cx^4+2}}dx$$

$$\Big\downarrow \ 3$$

$$\int \frac{1}{x^4\sqrt{bx^2+cx^4}}dx$$

$$\Big\downarrow \ 1430$$

$$-\frac{3c\int \frac{1}{x^2\sqrt{cx^4+bx^2}}dx}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

$$\Big\downarrow \ 1430$$

$$-\frac{3c\left(-\frac{c\int \frac{1}{\sqrt{cx^4+bx^2}}dx}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

$$\Big\downarrow \ 1400$$

$$-\frac{3c\left(\frac{c\int \frac{1}{1-\frac{bx^2}{cx^4+bx^2}}d\frac{x}{\sqrt{cx^4+bx^2}}}{2b}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

$$\Big\downarrow \ 219$$

$$-\frac{3c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}\right)}{4b}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

input
```
Int[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

output
```
-1/4*Sqrt[b*x^2 + c*x^4]/(b*x^5) - (3*c*(-1/2*Sqrt[b*x^2 + c*x^4]/(b*x^3)
+ (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(3/2))))/(4*b)
```

Defintions of rubi rules used

rule 3
```
Int[(u_.)*((a_) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[u*(b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n]
 && EqQ[a, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1400
```
Int[1/Sqrt[(b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> -Subst[Int[1/(1 - b*x
^2), x], x, x/Sqrt[b*x^2 + c*x^4]] /; FreeQ[{b, c}, x]
```

rule 1430
```
Int[((d_.)*(x_))^(m_)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp
[d*(d*x)^(m - 1)*((b*x^2 + c*x^4)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[c*(
(m + 4*p + 3)/(b*d^2*(m + 2*p + 1)))   Int[(d*x)^(m + 2)*(b*x^2 + c*x^4)^p,
 x], x] /; FreeQ[{b, c, d, m, p}, x] &&  !IntegerQ[p] && LtQ[m + 2*p + 1, 0
]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{\sqrt{c\,x^{2}+b}\,\left(3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)b\,c^{2}x^{4}-3b^{\frac{3}{2}}\sqrt{c\,x^{2}+b}\,c\,x^{2}+2\sqrt{c\,x^{2}+b}\,b^{\frac{5}{2}}\right)}{8x^{3}\sqrt{c\,x^{4}+b\,x^{2}}\,b^{\frac{7}{2}}}$	94
risch	$-\frac{\left(c\,x^{2}+b\right)\left(-3c\,x^{2}+2b\right)}{8b^{2}x^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{3c^{2}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)x\sqrt{c\,x^{2}+b}}{8b^{\frac{5}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	94

input
```
int(1/x^4/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/x^3*(c*x^2+b)^(1/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*c^2*x^4-
3*b^(3/2)*(c*x^2+b)^(1/2)*c*x^2+2*(c*x^2+b)^(1/2)*b^(5/2))/(c*x^4+b*x^2)^(
1/2)/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \left[\frac{3\sqrt{b}c^2x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}(3\,bcx^2-2\,b^2)}{16\,b^3x^5}, \frac{3\sqrt{-b}c^2x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{bx}\right)}{8\,b^3x^5}\right.$$

input
```
integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(b)*c^2*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(
b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5), 1/8*(3*sq
rt(-b)*c^2*x^5*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(b*x)) + sqrt(c*x^4 + b
*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5)]
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{x^4\sqrt{x^2\,(b+cx^2)}}\,dx$$

input
```
integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2}x^4}\,dx$$

input
```
integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \frac{\frac{3\,c^3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2} + \frac{3\,(cx^2+b)^{\frac{3}{2}}c^3-5\,\sqrt{cx^2+b}bc^3}{b^2c^2x^4}}{8\,c\text{sgn}\left(x\right)}$$

input
```
integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*(3*c^3*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*(c*x^2 + b
)^(3/2)*c^3 - 5*sqrt(c*x^2 + b)*b*c^3)/(b^2*c^2*x^4))/(c*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx = \int \frac{1}{x^4\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

input
```
int(1/(x^4*(b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$$
$$= \frac{-2\sqrt{c\,x^2+b}\,b^2+3\sqrt{c\,x^2+b}\,bc\,x^2+3\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}-\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c^2x^4-3\sqrt{b}\log\left(\frac{\sqrt{c\,x^2+b}+\sqrt{b}+\sqrt{c}\,x}{\sqrt{b}}\right)c^2x^4}{8b^3x^4}$$

input
```
int(1/x^4/(c*x^4+b*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(b + c*x**2)*b**2 + 3*sqrt(b + c*x**2)*b*c*x**2 + 3*sqrt(b)*log(
(sqrt(b + c*x**2) - sqrt(b) + sqrt(c)*x)/sqrt(b))*c**2*x**4 - 3*sqrt(b)*lo
g((sqrt(b + c*x**2) + sqrt(b) + sqrt(c)*x)/sqrt(b))*c**2*x**4)/(8*b**3*x**
4)
```

3.1030 $\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 108

$$\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}} \, dx = \frac{x\sqrt{a+cx^4}}{3c} - \frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$

output
```
1/3*x*(c*x^4+a)^(1/2)/c-1/6*a^(3/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1
/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*
2^(1/2))/c^(5/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.57

$$\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}} \, dx = \frac{x\left(a+cx^4-a\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)\right)}{3c\sqrt{a+cx^4}}$$

input
```
Integrate[x^4/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

output
```
(x*(a + c*x^4 - a*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((
c*x^4)/a)]))/(3*c*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+(2b-2(b+1)+2)x^2+cx^4}}dx$$
$$\Big\downarrow \text{ 4}$$
$$\int \frac{x^4}{\sqrt{a+cx^4}}dx$$
$$\Big\downarrow \text{ 843}$$
$$\frac{x\sqrt{a+cx^4}}{3c}-\frac{a\int \frac{1}{\sqrt{cx^4+a}}dx}{3c}$$
$$\Big\downarrow \text{ 761}$$
$$\frac{x\sqrt{a+cx^4}}{3c}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$

input
```
Int[x^4/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

output
```
(x*Sqrt[a + c*x^4])/(3*c) - (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/
2])/(6*c^(5/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 4
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{x\sqrt{c\,x^{4}+a}}{3c}-\frac{a\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	91
risch	$\frac{x\sqrt{c\,x^{4}+a}}{3c}-\frac{a\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	91
elliptic	$\frac{x\sqrt{c\,x^{4}+a}}{3c}-\frac{a\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	91

input
```
int(x^4/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(c*x^4+a)^(1/2)/c-1/3/c*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = -\frac{\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1) - \sqrt{cx^4+a}\,x}{3\,c}$$

input
```
integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(c)*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) - sqrt(c
*x^4 + a)*x)/c
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.34

$$\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(c*x**4+a)**(1/2),x)
```

output
```
x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqr
t(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate(x^4/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{x^4}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int(x^4/(a + c*x^4)^(1/2),x)
```

output
```
int(x^4/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{\sqrt{c\,x^4+a}\,x-\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)a}{3c}$$

input
```
int(x^4/(c*x^4+a)^(1/2),x)
```

output
```
(sqrt(a + c*x**4)*x - int(sqrt(a + c*x**4)/(a + c*x**4),x)*a)/(3*c)
```

3.1031 $\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 18

$$\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\frac{\sqrt{a+cx^4}}{2c}$$

output
```
1/2*(c*x^4+a)^(1/2)/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\frac{\sqrt{a+cx^4}}{2c}$$

input
```
Integrate[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

output
```
Sqrt[a + c*x^4]/(2*c)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+(2b-2(b+1)+2)x^2+cx^4}}dx$$

$$\Big\downarrow \text{ 4}$$

$$\int \frac{x^3}{\sqrt{a+cx^4}}dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\sqrt{a+cx^4}}{2c}$$

input
```
Int[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

output
```
Sqrt[a + c*x^4]/(2*c)
```

Defintions of rubi rules used

rule 4
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15
derivativedivides	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15
default	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15
trager	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15
risch	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15
elliptic	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15
pseudoelliptic	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15
orering	$\frac{\sqrt{c\,x^{4}+a}}{2c}$	15

input
```
int(x^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^4+a)^(1/2)/c
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\frac{\sqrt{cx^4+a}}{2\,c}$$

input
```
integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(c*x^4 + a)/c
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \begin{cases} \frac{\sqrt{a+cx^4}}{2c} & \text{for } c \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(c*x**4+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + c*x**4)/(2*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{\sqrt{cx^4+a}}{2\,c}$$

input
```
integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^4 + a)/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{\sqrt{cx^4+a}}{2\,c}$$

input
```
integrate(x^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^4 + a)/c
```

Mupad [B] (verification not implemented)

Time = 18.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\frac{\sqrt{c\,x^4+a}}{2\,c}$$

input
```
int(x^3/(a + c*x^4)^(1/2),x)
```

output
```
(a + c*x^4)^(1/2)/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\frac{\sqrt{c}\,\sqrt{c\,x^4+a}\,x^2+a+c\,x^4}{2c\left(\sqrt{c\,x^4+a}+\sqrt{c}\,x^2\right)}$$

input
```
int(x^3/(c*x^4+a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a + c*x**4)*x**2 + a + c*x**4)/(2*c*(sqrt(a + c*x**4) + sqrt
(c)*x**2))
```

3.1032 $\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}$$

output
```
x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)-a^(1/4)*(a^(1/2)+c^(1/2)*x
^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/
4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*(a^(1/2)+c
^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.24

$$\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{x^3\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)}{3\sqrt{a+cx^4}}$$

input
```
Integrate[x^2/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

output
```
(x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(
3*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+(2b-2(b+1)+2)x^2+cx^4}}\,dx$$

$$\Big\downarrow \text{ 4}$$

$$\int \frac{x^2}{\sqrt{a+cx^4}}dx$$

$$\Big\downarrow \text{ 834}$$

$$\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}}{\sqrt{c}}$$

input
```
Int[x^2/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

output
```
-((-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4]))/Sqrt[c]) + (a^(1/4)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 4
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.46

method	result	size
default	$\frac{i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$	97
elliptic	$\frac{i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$	97

input
```
int(x^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*
c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+\sqrt{cx^4+a}}{cx}$$

input
```
integrate(x^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) - sqrt(c)*x
*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + sqrt(c*x^4 + a))/(c
*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.18

$$\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(c*x**4+a)**(1/2),x)
```

output
```
x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqr
t(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a + (2 + 2b - 2(1 + b))x^2 + cx^4}} \, dx = \int \frac{x^2}{\sqrt{cx^4 + a}} \, dx$$

input
```
integrate(x^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a + (2 + 2b - 2(1 + b))x^2 + cx^4}} \, dx = \int \frac{x^2}{\sqrt{cx^4 + a}} \, dx$$

input
```
integrate(x^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a + (2 + 2b - 2(1 + b))x^2 + cx^4}} \, dx = \int \frac{x^2}{\sqrt{c\,x^4 + a}} \, dx$$

input
```
int(x^2/(a + c*x^4)^(1/2),x)
```

output
```
int(x^2/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx$$

input
```
int(x^2/(c*x^4+a)^(1/2),x)
```

output
```
int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)
```

3.1033 $\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 30

$$\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

output
```
1/2*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\frac{\log\left(\sqrt{c}x^2+\sqrt{a+cx^4}\right)}{2\sqrt{c}}$$

input
```
Integrate[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

output
```
Log[Sqrt[c]*x^2 + Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+(2b-2(b+1)+2)x^2+cx^4}}dx$$

$$\Big\downarrow \; 4$$

$$\int \frac{x}{\sqrt{a+cx^4}}dx$$

$$\Big\downarrow \; 807$$

$$\frac{1}{2}\int \frac{1}{\sqrt{cx^4+a}}dx^2$$

$$\Big\downarrow \; 224$$

$$\frac{1}{2}\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}$$

$$\Big\downarrow \; 219$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

input
```
Int[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

output
```
ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

Defintions of rubi rules used

rule 4
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{4}+a}}{x^{2}\sqrt{c}}\right)}{2\sqrt{c}}$	23
default	$\frac{\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{2\sqrt{c}}$	24
elliptic	$\frac{\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{2\sqrt{c}}$	24

input
```
int(x/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c^(1/2)*arctanh((c*x^4+a)^(1/2)/x^2/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.20

$$\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \left[\frac{\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)}{4\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-c}}{cx^2}\right)}{2\,c}\right]$$

input
```
integrate(x/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a)/sqrt(c), -1/2*sqrt(
-c)*arctan(sqrt(c*x^4 + a)*sqrt(-c)/(c*x^2))/c]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

input
```
integrate(x/(c*x**4+a)**(1/2),x)
```

output
```
asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{a + (2 + 2b - 2(1 + b))x^2 + cx^4}}\, dx = -\frac{\log\left(-\frac{\sqrt{c} - \frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c} + \frac{\sqrt{cx^4+a}}{x^2}}\right)}{4\sqrt{c}}$$

input
```
integrate(x/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/x^2))
/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{a + (2 + 2b - 2(1 + b))x^2 + cx^4}}\, dx = -\frac{\log\left(\left|-\sqrt{c}x^2 + \sqrt{cx^4 + a}\right|\right)}{2\sqrt{c}}$$

input
```
integrate(x/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-sqrt(c)*x^2 + sqrt(c*x^4 + a)))/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a + (2 + 2b - 2(1 + b))x^2 + cx^4}}\, dx = \int \frac{x}{\sqrt{c\,x^4 + a}}\, dx$$

input
```
int(x/(a + c*x^4)^(1/2),x)
```

output
```
int(x/(a + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

input
```
int(x/(c*x^4+a)^(1/2),x)
```

output
```
log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))/(2*sqrt(c))
```

3.1034 $\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Invers
eJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(1/4)/(c*x^4+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = -\frac{i\sqrt{1+\frac{cx^4}{a}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

input
```
Integrate[1/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

output
```
((-I)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x]
, -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+(2b-2(b+1)+2)x^2+cx^4}}dx$$

$$\Big\downarrow \; 4$$

$$\int \frac{1}{\sqrt{a+cx^4}}dx$$

$$\Big\downarrow \; 761$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

input
```
Int[1/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

output
```
((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^
4])
```

Defintions of rubi rules used

rule 4
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	70
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	70

input
```
int(1/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = -\frac{\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1\right)}{c}$$

input
```
integrate(1/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1)/c
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x**4+a)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a
)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate(1/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate(1/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 18.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\frac{x\,\sqrt{\frac{c\,x^4}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\sqrt{c\,x^4+a}}$$

input
```
int(1/(a + c*x^4)^(1/2),x)
```

output
```
(x*((c*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(c*x^4)/a))/(a + c*x^
4)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx$$

input
```
int(1/(c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a + c*x**4),x)
```

3.1035 $\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 27

$$\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=-\frac{\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
-1/2*arctanh((c*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=-\frac{\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

output
```
-1/2*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+(2b-2(b+1)+2)x^2+cx^4}}dx$$

$$\Big\downarrow \ 4$$

$$\int \frac{1}{x\sqrt{a+cx^4}}dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4}\int \frac{1}{x^4\sqrt{cx^4+a}}dx^4$$

$$\Big\downarrow \ 73$$

$$\frac{\int \frac{1}{\frac{x^8}{c}-\frac{a}{c}}d\sqrt{cx^4+a}}{2c}$$

$$\Big\downarrow \ 221$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

output
```
-1/2*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/Sqrt[a]
```

Defintions of rubi rules used

rule 4
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{4}+a}}{\sqrt{a}}\right)}{2\sqrt{a}}$	20
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{2\sqrt{a}}$	29
elliptic	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{2\sqrt{a}}$	29

input
```
int(1/x/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh((c*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \left[\frac{\log\left(\frac{cx^4-2\sqrt{cx^4+a}\sqrt{a}+2a}{x^4}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^4+a}}\right)}{2a}\right]$$

input
```
integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4)/sqrt(a), 1/2*sqrt(
-a)*arctan(sqrt(-a)/sqrt(c*x^4 + a))/a]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}x^2}\right)}{2\sqrt{a}}$$

input
```
integrate(1/x/(c*x**4+a)**(1/2),x)
```

output
```
-asinh(sqrt(a)/(sqrt(c)*x**2))/(2*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

input
```
integrate(1/x/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 18.85 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
int(1/(x*(a + c*x^4)^(1/2)),x)
```

output
```
-atanh((a + c*x^4)^(1/2)/a^(1/2))/(2*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$$
$$=\frac{\sqrt{a}\left(\log\left(\frac{\sqrt{c\,x^4+a}-\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)-\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)\right)}{2a}$$

input
```
int(1/x/(c*x^4+a)^(1/2),x)
```

output
```
(sqrt(a)*(log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a)) - log((
sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a))))/(2*a)
```

3.1036 $\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 232

$$\begin{aligned}
&\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx \\
&= -\frac{\sqrt{a+cx^4}}{ax}+\frac{\sqrt{c}x\sqrt{a+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+cx^4}} \\
&\quad+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+cx^4}}
\end{aligned}$$

output
```
-(c*x^4+a)^(1/2)/a/x+c^(1/2)*x*(c*x^4+a)^(1/2)/a/(a^(1/2)+c^(1/2)*x^2)-c^(
1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Ellip
ticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(c*x^4+a)^(1/2)
+1/2*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/
2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(3/4)/(c*x^4
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = -\frac{\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{cx^4}{a}\right)}{x\sqrt{a+cx^4}}$$

input
```
Integrate[1/(x^2*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

output
```
-((Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((c*x^4)/a)])/(x
*Sqrt[a + c*x^4]))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+(2b-2(b+1)+2)x^2+cx^4}}\,dx$$

$$\Big\downarrow \text{4}$$

$$\int \frac{1}{x^2\sqrt{a+cx^4}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{c\int \frac{x^2}{\sqrt{cx^4+a}}dx}{a}-\frac{\sqrt{a+cx^4}}{ax}$$

$$\Big\downarrow \text{834}$$

$$\frac{c\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{a+cx^4}}{ax}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{c\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{a+cx^4}}{ax}$$

$$\Big\downarrow \textbf{761}$$

$$\frac{c\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{a+cx^4}}{ax}$$

$$\Big\downarrow \textbf{1510}$$

$$\frac{c\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}}{\sqrt{c}}\right)}{a}$$

$$\frac{\sqrt{a+cx^4}}{ax}$$

input
```
Int[1/(x^2*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

output
```
-(Sqrt[a + c*x^4]/(a*x)) + (c*(-((-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]
*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a
 + c*x^4]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
2*c^(3/4)*Sqrt[a + c*x^4])))/a
```

Defintions of rubi rules used

rule 4
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{\sqrt{c\,x^4+a}}{ax}+\frac{i\sqrt{c}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	115
risch	$-\frac{\sqrt{c\,x^4+a}}{ax}+\frac{i\sqrt{c}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	115
elliptic	$-\frac{\sqrt{c\,x^4+a}}{ax}+\frac{i\sqrt{c}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	115

input
```
int(1/x^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^4+a)^(1/2)/a/x+I*c^(1/2)/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1
/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(El
lipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/
2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = -\frac{\sqrt{a}x\left(-\frac{c}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid-1)-\sqrt{a}x\left(-\frac{c}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid-1)+\sqrt{cx^4+a}}{ax}$$

input
```
integrate(1/x^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(a)*x*(-c/a)^(3/4)*elliptic_e(arcsin(x*(-c/a)^(1/4)), -1) - sqrt(a)*
x*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1) + sqrt(c*x^4 + a))/(
a*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(c*x**4+a)**(1/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a
)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+a}x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\int \frac{1}{\sqrt{cx^4+ax^2}}\,dx$$

input
```
integrate(1/x^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 18.90 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=-\frac{\sqrt{\frac{a}{c\,x^4}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{a}{c\,x^4}\right)}{3\,x\,\sqrt{c\,x^4+a}}$$

input
```
int(1/(x^2*(a + c*x^4)^(1/2)),x)
```

output
```
-((a/(c*x^4) + 1)^(1/2)*hypergeom([1/2, 3/4], 7/4, -a/(c*x^4)))/(3*x*(a +
c*x^4)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=\int \frac{\sqrt{c\,x^4+a}}{c\,x^6+a\,x^2}dx$$

input
```
int(1/x^2/(c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a*x**2 + c*x**6),x)
```

3.1037 $\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 21

$$\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=-\frac{\sqrt{a+cx^4}}{2ax^2}$$

output
```
-1/2*(c*x^4+a)^(1/2)/a/x^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=-\frac{\sqrt{a+cx^4}}{2ax^2}$$

input
```
Integrate[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

output
```
-1/2*Sqrt[a + c*x^4]/(a*x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+(2b-2(b+1)+2)x^2+cx^4}}dx$$
$$\Big\downarrow \text{4}$$
$$\int \frac{1}{x^3\sqrt{a+cx^4}}dx$$
$$\Big\downarrow \text{796}$$
$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

input
```
Int[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

output
```
-1/2*Sqrt[a + c*x^4]/(a*x^2)
```

Defintions of rubi rules used

rule 4
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{\sqrt{c\,x^{4}+a}}{2a\,x^{2}}$	18
default	$-\frac{\sqrt{c\,x^{4}+a}}{2a\,x^{2}}$	18
trager	$-\frac{\sqrt{c\,x^{4}+a}}{2a\,x^{2}}$	18
risch	$-\frac{\sqrt{c\,x^{4}+a}}{2a\,x^{2}}$	18
elliptic	$-\frac{\sqrt{c\,x^{4}+a}}{2a\,x^{2}}$	18
pseudoelliptic	$-\frac{\sqrt{c\,x^{4}+a}}{2a\,x^{2}}$	18
orering	$-\frac{\sqrt{c\,x^{4}+a}}{2a\,x^{2}}$	18

input
```
int(1/x^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^4+a)^(1/2)/a/x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx=-\frac{\sqrt{cx^4+a}}{2\,ax^2}$$

input
```
integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(c*x^4 + a)/(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = -\frac{\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{2a}$$

input
```
integrate(1/x**3/(c*x**4+a)**(1/2),x)
```

output
```
-sqrt(c)*sqrt(a/(c*x**4) + 1)/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = -\frac{\sqrt{cx^4+a}}{2\,ax^2}$$

input
```
integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(c*x^4 + a)/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{\sqrt{c}}{\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2-a}$$

input
```
integrate(1/x^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
sqrt(c)/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 18.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = -\frac{\sqrt{c\,x^4+a}}{2\,a\,x^2}$$

input
```
int(1/(x^3*(a + c*x^4)^(1/2)),x)
```

output
```
-(a + c*x^4)^(1/2)/(2*a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.38

$$\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{-2\sqrt{c}\,\sqrt{c\,x^4+a}\,x^2-a-2c\,x^4}{2a\,x^2\left(\sqrt{c\,x^4+a}+\sqrt{c}\,x^2\right)}$$

input
```
int(1/x^3/(c*x^4+a)^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(a + c*x**4)*x**2 - a - 2*c*x**4)/(2*a*x**2*(sqrt(a + c*
x**4) + sqrt(c)*x**2))
```

3.1038 $\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 110

$$\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = -\frac{\sqrt{a+cx^4}}{3ax^3}-\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}$$

output
```
-1/3*(c*x^4+a)^(1/2)/a/x^3-1/6*c^(3/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a
^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1
/2*2^(1/2))/a^(5/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = -\frac{\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{cx^4}{a}\right)}{3x^3\sqrt{a+cx^4}}$$

input
```
Integrate[1/(x^4*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

output
```
-1/3*(Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((c*x^4)/a)])
/(x^3*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+(2b-2(b+1)+2)x^2+cx^4}}dx$$

$$\Big\downarrow \text{ 4}$$

$$\int \frac{1}{x^4\sqrt{a+cx^4}}dx$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{c\int \frac{1}{\sqrt{cx^4+a}}dx}{3a}-\frac{\sqrt{a+cx^4}}{3ax^3}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3ax^3}$$

input
```
Int[1/(x^4*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

output
```
-1/3*Sqrt[a + c*x^4]/(a*x^3) - (c^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 1/2])/(6*a^(5/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 4
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] &&
 EqQ[b, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\sqrt{c\,x^4+a}}{3a\,x^3}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	93
risch	$-\frac{\sqrt{c\,x^4+a}}{3a\,x^3}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	93
elliptic	$-\frac{\sqrt{c\,x^4+a}}{3a\,x^3}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	93

input
```
int(1/x^4/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c*x^4+a)^(1/2)/a/x^3-1/3*c/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*
c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipti
cF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{\sqrt{a}x^3\left(-\frac{c}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right) \mid -1\right)-\sqrt{cx^4+a}}{3\,ax^3}$$

input
```
integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(a)*x^3*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1) - sqr
t(c*x^4 + a))/(a*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4},\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**4/(c*x**4+a)**(1/2),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a
)*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}x^4}\,dx$$

input
```
integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}x^4}\,dx$$

input
```
integrate(1/x^4/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{1}{x^4\sqrt{c\,x^4+a}}\,dx$$

input
```
int(1/(x^4*(a + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(a + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx = \int \frac{\sqrt{c\,x^4+a}}{c\,x^8+a\,x^4}dx$$

input
```
int(1/x^4/(c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a*x**4 + c*x**8),x)
```

3.1039 $\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
-3/8*a*x*(b*x^2+a)^(1/2)/b^2+1/4*x^3*(b*x^2+a)^(1/2)/b+3/8*a^2*arctanh(b^(
1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{a+bx^2}(-3ax+2bx^3)}{8b^2}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4b^{5/2}}$$

input
```
Integrate[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
(Sqrt[a + b*x^2]*(-3*a*x + 2*b*x^3))/(8*b^2) + (3*a^2*ArcTanh[(Sqrt[b]*x)/
(-Sqrt[a] + Sqrt[a + b*x^2])])/(4*b^(5/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {5, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^2+(2c-2(c+1)+2)x^4}}dx$$

$$\Big\downarrow \; 5$$

$$\int \frac{x^4}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \; 262$$

$$\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}$$

$$\Big\downarrow \; 262$$

$$\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}$$

$$\Big\downarrow \; 224$$

$$\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}$$

$$\Big\downarrow \; 219$$

$$\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}$$

input
```
Int[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
(x^3*Sqrt[a + b*x^2])/(4*b) - (3*a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b)
```

Defintions of rubi rules used

rule 5
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{x(-2b\,x^2+3a)\sqrt{b\,x^2+a}}{8b^2}+\frac{3a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}$	51
pseudoelliptic	$\frac{2b^{\frac{3}{2}}x^3\sqrt{b\,x^2+a}-3ax\sqrt{b\,x^2+a}\,\sqrt{b}+3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)a^2}{8b^{\frac{5}{2}}}$	62
default	$\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}$	63

input
```
int(x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(-2*b*x^2+3*a)*(b*x^2+a)^(1/2)/b^2+3/8*a^2/b^(5/2)*ln(b^(1/2)*x+(b*
x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.70

$$\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$$
$$=\left[\frac{3\,a^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,b^2x^3-3\,abx)\sqrt{bx^2+a}}{16\,b^3},\right.$$
$$\left.-\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,b^2x^3-3\,abx)\sqrt{bx^2+a}}{8\,b^3}\right]$$

input
```
integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
2*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3, -1/8*(3*a^2*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.30

$$\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{3a^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{\sqrt{a}x^3}{8b\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}} + \frac{x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**4/(b*x**2+a)**(1/2),x)
```

output
```
-3*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*x**3/(8*b*sqrt(1 + b*x
**2/a)) + 3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + x**5/(4*sqrt(a)*s
qrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{bx^2+a}x^3}{4\,b} - \frac{3\,\sqrt{bx^2+a}ax}{8\,b^2} + \frac{3\,a^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*x^3/b - 3/8*sqrt(b*x^2 + a)*a*x/b^2 + 3/8*a^2*arcsinh(
b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{1}{8}\sqrt{bx^2+a}x\left(\frac{2\,x^2}{b}-\frac{3\,a}{b^2}\right) - \frac{3\,a^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*x*(2*x^2/b - 3*a/b^2) - 3/8*a^2*log(abs(-sqrt(b)*x + s
qrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \int \frac{x^4}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(1/2),x)
```

output
```
int(x^4/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{-3\sqrt{b\,x^2+a}\,abx+2\sqrt{b\,x^2+a}\,b^2x^3+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2}{8b^3}$$

input
```
int(x^4/(b*x^2+a)^(1/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a*b*x + 2*sqrt(a + b*x**2)*b**2*x**3 + 3*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2)/(8*b**3)
```

3.1040 $\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 36

$$\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{a\sqrt{a+bx^2}}{b^2}+\frac{(a+bx^2)^{3/2}}{3b^2}$$

output
```
-a*(b*x^2+a)^(1/2)/b^2+1/3*(b*x^2+a)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{(-2a+bx^2)\sqrt{a+bx^2}}{3b^2}$$

input
```
Integrate[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {5, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^2+(2c-2(c+1)+2)x^4}}dx$$

$$\Big\downarrow \text{ 5}$$

$$\int \frac{x^3}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{\sqrt{bx^2+a}}{b}-\frac{a}{b\sqrt{bx^2+a}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx^2}}{b^2}\right)$$

input
```
Int[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
((-2*a*Sqrt[a + b*x^2])/b^2 + (2*(a + b*x^2)^(3/2))/(3*b^2))/2
```

Defintions of rubi rules used

rule 5
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{(b\,x^{2}-2a)\sqrt{b\,x^{2}+a}}{3b^{2}}$	24
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,(-b\,x^{2}+2a)}{3b^{2}}$	25
trager	$-\frac{\sqrt{b\,x^{2}+a}\,(-b\,x^{2}+2a)}{3b^{2}}$	25
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(-b\,x^{2}+2a)}{3b^{2}}$	25
orering	$-\frac{\sqrt{b\,x^{2}+a}\,(-b\,x^{2}+2a)}{3b^{2}}$	25
default	$\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}$	34

input
```
int(x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^2-2*a)*(b*x^2+a)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{bx^2+a}(bx^2-2\,a)}{3\,b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*x^2 + a)*(b*x^2 - 2*a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), N
e(b, 0)), (x**4/(4*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{bx^2+a}\,x^2}{3\,b} - \frac{2\,\sqrt{bx^2+a}\,a}{3\,b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*x^2/b - 2/3*sqrt(b*x^2 + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}}{3\,b^2} - \frac{\sqrt{bx^2+a}\,a}{b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/3*(b*x^2 + a)^(3/2)/b^2 - sqrt(b*x^2 + a)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 17.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

input
```
int(x^3/(a + b*x^2)^(1/2),x)
```

output
```
-((a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{b\,x^2+a}\,(b\,x^2-2a)}{3b^2}$$

input
```
int(x^3/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 2*a + b*x**2))/(3*b**2)
```

3.1041 $\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 49

$$\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
1/2*x*(b*x^2+a)^(1/2)/b-1/2*a*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

input
```
Integrate[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
(x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*
x^2])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2+(2c-2(c+1)+2)x^4}}dx$$
$$\Big\downarrow \text{ 5}$$
$$\int \frac{x^2}{\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \text{ 262}$$
$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}$$
$$\Big\downarrow \text{ 224}$$
$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

input
```
Int[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
(x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^
(3/2))
```

Defintions of rubi rules used

rule 5
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}$	39
risch	$\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}$	39
pseudoelliptic	$-\frac{-\sqrt{b\,x^{2}+a}\,x\sqrt{b}+\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a}{2b^{\frac{3}{2}}}$	41

input
```
int(x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/b*(b*x^2+a)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$$
$$=\left[\frac{2\sqrt{bx^2+a}bx+a\sqrt{b}\log\left(-2\,bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4\,b^2},\frac{\sqrt{bx^2+a}bx+a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

input
```
integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*x + a*sqrt(-b)*arctan(sqrt(-b)*
x/sqrt(b*x^2 + a)))/b^2]
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=\frac{\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

input
```
integrate(x**2/(b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{bx^2+a}x}{2\,b} - \frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*x/b - 1/2*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{bx^2+a}x}{2\,b} + \frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*x/b + 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
3/2)
```

Mupad [B] (verification not implemented)

Time = 17.94 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \begin{cases} \frac{x^3}{3\sqrt{a}} & \text{if } b=0 \\ \frac{x\sqrt{b\,x^2+a}}{2\,b} - \frac{a\,\ln\left(2\sqrt{b}\,x+2\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} & \text{if } b\neq 0 \end{cases}$$

input
```
int(x^2/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, x^3/(3*a^(1/2)), b ~= 0, (x*(a + b*x^2)^(1/2))/(2*b) - (
a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{b\,x^2+a}\,bx-\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a}{2b^2}$$

input
```
int(x^2/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*b*x - sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a)/(2*b**2)
```

3.1042 $\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 15

$$\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{a+bx^2}}{b}$$

output
```
(b*x^2+a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{a+bx^2}}{b}$$

input
```
Integrate[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
Sqrt[a + b*x^2]/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {5, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^2+(2c-2(c+1)+2)x^4}}dx$$
$$\Big\downarrow \ 5$$
$$\int \frac{x}{\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \ 241$$
$$\frac{\sqrt{a+bx^2}}{b}$$

input
```
Int[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
Sqrt[a + b*x^2]/b
```

Defintions of rubi rules used

rule 5
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
derivativedivides	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
default	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
trager	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
risch	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
pseudoelliptic	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
orering	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14

input
```
int(x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=\frac{\sqrt{bx^2+a}}{b}$$

input
```
integrate(x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(b*x^2 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{bx^2+a}}{b}$$

input
```
integrate(x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{bx^2+a}}{b}$$

input
```
integrate(x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)/b
```

Mupad [B] (verification not implemented)

Time = 17.92 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=\frac{\sqrt{b\,x^2+a}}{b}$$

input
```
int(x/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=\frac{\sqrt{b\,x^2+a}}{b}$$

input
```
int(x/(b*x^2+a)^(1/2),x)
```

output
```
sqrt(a + b*x**2)/b
```

3.1043 $\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

output
```
arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2+(2c-2(c+1)+2)x^4}}dx$$

$$\Big\downarrow \; 5$$

$$\int \frac{1}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \; 224$$

$$\int \frac{1}{1-\frac{bx^2}{a+bx^2}}d\frac{x}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Defintions of rubi rules used

rule 5
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$	21
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{\sqrt{b}}$	22

input
```
int(1/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\,\sqrt{b}},\ -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x**2+a)**(1/2),x)
```

output
```
asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{1}{2}\sqrt{bx^2+a}x - \frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(
b)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

input
```
int(1/(a + b*x^2)^(1/2),x)
```

output
```
log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)}{b}$$

input
```
int(1/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)))/b
```

3.1044 $\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 25

$$\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
-(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^2+(2c-2(c+1)+2)x^4}}dx$$

$$\Big\downarrow \text{5}$$

$$\int \frac{1}{x\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
-(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Defintions of rubi rules used

rule 5
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	20
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}$	29

input
```
int(1/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \left[\frac{\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)}{2\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)}{a}\right]$$

input
```
integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)/sqrt(a), sqrt(-a)
*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a)/a]
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x**2+a)**(1/2),x)
```

output
```
-asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 18.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(a + b*x^2)^(1/2)),x)
```

output
```
-atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{a}\left(\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)-\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{a}$$

input

```
int(1/x/(b*x^2+a)^(1/2),x)
```

output

```
(sqrt(a)*(log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a)) - log((sqr
t(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))))/a
```

3.1045 $\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 19

$$\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=-\frac{\sqrt{a+bx^2}}{ax}$$

output
```
-(b*x^2+a)^(1/2)/a/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=-\frac{\sqrt{a+bx^2}}{ax}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
-(Sqrt[a + b*x^2]/(a*x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {5, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^2+(2c-2(c+1)+2)x^4}}dx$$

$$\Big\downarrow \text{ 5}$$

$$\int \frac{1}{x^2\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{\sqrt{a+bx^2}}{ax}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
-(Sqrt[a + b*x^2]/(a*x))
```

Defintions of rubi rules used

rule 5
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
default	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
trager	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
risch	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
pseudoelliptic	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
orering	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18

input
```
int(1/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)^(1/2)/a/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=-\frac{\sqrt{bx^2+a}}{ax}$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(b*x^2 + a)/(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{\sqrt{bx^2+a}}{ax}$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(b*x^2 + a)/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{\sqrt{b\,x^2+a}}{a\,x}$$

input
```
int(1/(x^2*(a + b*x^2)^(1/2)),x)
```

output
```
-(a + b*x^2)^(1/2)/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{-\sqrt{b\,x^2+a}-\sqrt{b}\,x}{ax}$$

input
```
int(1/x^2/(b*x^2+a)^(1/2),x)
```

output
```
( - (sqrt(a + b*x**2) + sqrt(b)*x))/(a*x)
```

3.1046 $\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 50

$$\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=-\frac{\sqrt{a+bx^2}}{2ax^2}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=-\frac{\sqrt{a+bx^2}}{2ax^2}+\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
-1/2*Sqrt[a + b*x^2]/(a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(
3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5, 243, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^2+(2c-2(c+1)+2)x^4}}dx$$

$$\Big\downarrow \text{5}$$

$$\int \frac{1}{x^3\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)$$

input
```
Int[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
(-(Sqrt[a + b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
)/2
```

Defintions of rubi rules used

rule 5
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right) b\,x^{2}-\sqrt{a}\,\sqrt{b\,x^{2}+a}}{2 a^{\frac{3}{2}} x^{2}}$	43
default	$-\frac{\sqrt{b\,x^{2}+a}}{2 a\,x^{2}}+\frac{b \ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}$	48
risch	$-\frac{\sqrt{b\,x^{2}+a}}{2 a\,x^{2}}+\frac{b \ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}$	48

input
```
int(1/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctanh((b*x^2+a)^(1/2)/a^(1/2))*b*x^2-a^(1/2)*(b*x^2+a)^(1/2))/a^(3/
2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.16

$$\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \left[\frac{\sqrt{a}bx^2\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\sqrt{bx^2+a}a}{4\,a^2x^2}, -\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+\sqrt{bx^2+a}a}{2\,a^2x^2}\right]$$

input
```
integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) -
2*sqrt(b*x^2 + a)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(sqrt(b*x^2 + a
)*sqrt(-a)/a) + sqrt(b*x^2 + a)*a)/(a^2*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate(1/x**3/(b*x**2+a)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a*
*(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^2+a}}{2\,ax^2}$$

input
```
integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/2*sqrt(b*x^2 + a)/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{2}\,b\left(\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx^2+a}}{abx^2}\right)$$

input
```
integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*(arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^2 + a)/(a
*b*x^2))
```

Mupad [B] (verification not implemented)

Time = 17.64 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{\sqrt{b\,x^2+a}}{2\,a\,x^2}$$

input
```
int(1/(x^3*(a + b*x^2)^(1/2)),x)
```

output
```
(b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (a + b*x^2)^(1/2)/(2*a*
x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{-\sqrt{b\,x^2+a}\,a-\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b\,x^2+\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b\,x^2}{2a^2x^2}$$

input
```
int(1/x^3/(b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**2)*a - sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)
*x)/sqrt(a))*b*x**2 + sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)
/sqrt(a))*b*x**2)/(2*a**2*x**2)
```

3.1047 $\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 44

$$\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=-\frac{\sqrt{a+bx^2}}{3ax^3}+\frac{2b\sqrt{a+bx^2}}{3a^2x}$$

output
```
-1/3*(b*x^2+a)^(1/2)/a/x^3+2/3*b*(b*x^2+a)^(1/2)/a^2/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=\frac{\sqrt{a+bx^2}(-a+2bx^2)}{3a^2x^3}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
(Sqrt[a + b*x^2]*(-a + 2*b*x^2))/(3*a^2*x^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^2+(2c-2(c+1)+2)x^4}}dx$$
$$\Big\downarrow \text{ 5}$$
$$\int \frac{1}{x^4\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \text{ 245}$$
$$-\frac{2b\int \frac{1}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2}}{3ax^3}$$
$$\Big\downarrow \text{ 242}$$
$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

input
```
Int[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
-1/3*Sqrt[a + b*x^2]/(a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Defintions of rubi rules used

rule 5
```
Int[(u_.)*((a_.) + (c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
 Int[u*(a + b*x^n)^p, x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[j, 2*n] && EqQ
[c, 0]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2 b\,x^{2}+a \right)}{3 a^{2} x^{3}}$	26
trager	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2 b\,x^{2}+a \right)}{3 a^{2} x^{3}}$	26
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2 b\,x^{2}+a \right)}{3 a^{2} x^{3}}$	26
pseudoelliptic	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2 b\,x^{2}+a \right)}{3 a^{2} x^{3}}$	26
orering	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2 b\,x^{2}+a \right)}{3 a^{2} x^{3}}$	26
default	$-\frac{\sqrt{b\,x^{2}+a}}{3 a\,x^{3}}+\frac{2 b \sqrt{b\,x^{2}+a}}{3 a^{2} x}$	37

input
```
int(1/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(-2*b*x^2+a)/a^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=\frac{(2\,bx^2-a)\sqrt{bx^2+a}}{3\,a^2x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*b*x^2 - a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(3*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{2\sqrt{bx^2+a}b}{3\,a^2x}-\frac{\sqrt{bx^2+a}}{3\,ax^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(b*x^2 + a)*b/(a^2*x) - 1/3*sqrt(b*x^2 + a)/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx = \frac{4\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
4/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*b^(3/2)/((sqrt(b)*x - sqrt(b*x
^2 + a))^2 - a)^3
```

Mupad [B] (verification not implemented)

Time = 17.98 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=-\frac{\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3}$$

input
```
int(1/(x^4*(a + b*x^2)^(1/2)),x)
```

output
```
-((a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx=\frac{-\sqrt{b\,x^2+a}\,a+2\sqrt{b\,x^2+a}\,b\,x^2-2\sqrt{b}\,b\,x^3}{3a^2x^3}$$

input
```
int(1/x^4/(b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**2)*a + 2*sqrt(a + b*x**2)*b*x**2 - 2*sqrt(b)*b*x**3)/(3*
a**2*x**3)
```

3.1048 $\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx=\frac{\sqrt{cx^4}}{2c}$$

output
```
1/2*(c*x^4)^(1/2)/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx=\frac{x^4}{2\sqrt{cx^4}}$$

input
```
Integrate[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

output
```
x^4/(2*Sqrt[c*x^4])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1, 21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{2a-2(a+1)+cx^4+2}}dx$$

$$\Big\downarrow \ 1$$

$$\int \frac{x^3}{\sqrt{cx^4}}dx$$

$$\Big\downarrow \ 21$$

$$\frac{1}{4}\int \frac{1}{\sqrt{cx^4}}dx^4$$

$$\Big\downarrow \ 17$$

$$\frac{\sqrt{cx^4}}{2c}$$

input
```
Int[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

output
```
Sqrt[c*x^4]/(2*c)
```

Defintions of rubi rules used

rule 1
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^{4}}{2\sqrt{c\,x^{4}}}$	13
derivativedivides	$\frac{\sqrt{c\,x^{4}}}{2c}$	13
default	$\frac{x^{4}}{2\sqrt{c\,x^{4}}}$	13
risch	$\frac{x^{4}}{2\sqrt{c\,x^{4}}}$	13
pseudoelliptic	$\frac{\sqrt{c\,x^{4}}}{2c}$	13
orering	$\frac{x^{4}}{2\sqrt{c\,x^{4}}}$	13
trager	$\frac{(x-1)(1+x)\sqrt{c\,x^{4}}}{2c\,x^{2}}$	22

input
```
int(x^3/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^4/(c*x^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx=\frac{\sqrt{cx^4}}{2\,c}$$

input
```
integrate(x^3/(c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(c*x^4)/c
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{x^4}{2\sqrt{cx^4}}$$

input
```
integrate(x**3/(c*x**4)**(1/2),x)
```

output
```
x**4/(2*sqrt(c*x**4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{\sqrt{cx^4}}{2\,c}$$

input
```
integrate(x^3/(c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^4)/c
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{x^2}{2\,\sqrt{c}}$$

input
```
integrate(x^3/(c*x^4)^(1/2),x, algorithm="giac")
```

output
```
1/2*x^2/sqrt(c)
```

Mupad [B] (verification not implemented)

Time = 17.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{\sqrt{x^4}}{2\sqrt{c}}$$

input
```
int(x^3/(c*x^4)^(1/2),x)
```

output
```
(x^4)^(1/2)/(2*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{\sqrt{c}\,x^2}{2c}$$

input
```
int(x^3/(c*x^4)^(1/2),x)
```

output
```
(sqrt(c)*x**2)/(2*c)
```

3.1049 $\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 13

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{1}{2\sqrt{cx^4}}$$

output
```
-1/2/(c*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{1}{2\sqrt{cx^4}}$$

input
```
Integrate[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

output
```
-1/2*1/Sqrt[c*x^4]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1, 21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{2a-2(a+1)+cx^4+2}}dx$$
$$\Big\downarrow \text{ 1}$$
$$\int \frac{1}{x\sqrt{cx^4}}dx$$
$$\Big\downarrow \text{ 21}$$
$$\frac{1}{4}c\int \frac{1}{(cx^4)^{3/2}}dx^4$$
$$\Big\downarrow \text{ 17}$$
$$-\frac{1}{2\sqrt{cx^4}}$$

input
```
Int[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

output
```
-1/2*1/Sqrt[c*x^4]
```

Defintions of rubi rules used

rule 1
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{1}{2\sqrt{c\,x^{4}}}$	10
derivativedivides	$-\frac{1}{2\sqrt{c\,x^{4}}}$	10
default	$-\frac{1}{2\sqrt{c\,x^{4}}}$	10
risch	$-\frac{1}{2\sqrt{c\,x^{4}}}$	10
pseudoelliptic	$-\frac{1}{2\sqrt{c\,x^{4}}}$	10
orering	$-\frac{1}{2\sqrt{c\,x^{4}}}$	10
trager	$\frac{(x-1)(1+x)\sqrt{c\,x^{4}}}{2c\,x^{4}}$	22

input
```
int(1/x/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(c*x^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{\sqrt{cx^4}}{2\,cx^4}$$

input
```
integrate(1/x/(c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(c*x^4)/(c*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{2\sqrt{cx^4}}$$

input
```
integrate(1/x/(c*x**4)**(1/2),x)
```

output
```
-1/(2*sqrt(c*x**4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{2\sqrt{cx^4}}$$

input
```
integrate(1/x/(c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/2/sqrt(c*x^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{2\sqrt{c}x^2}$$

input
```
integrate(1/x/(c*x^4)^(1/2),x, algorithm="giac")
```

output
```
-1/2/(sqrt(c)*x^2)
```

Mupad [B] (verification not implemented)

Time = 17.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{2\sqrt{c}\,\sqrt{x^4}}$$

input
```
int(1/(x*(c*x^4)^(1/2)),x)
```

output
```
-1/(2*c^(1/2)*(x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{\sqrt{c}}{2c\,x^2}$$

input
```
int(1/x/(c*x^4)^(1/2),x)
```

output
```
( - sqrt(c))/(2*c*x**2)
```

3.1050 $\int \frac{1}{x^5\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 14

$$\int \frac{1}{x^5\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{c}{6\left(cx^4\right)^{3/2}}$$

output
```
-1/6*c/(c*x^4)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{1}{6x^4\sqrt{cx^4}}$$

input
```
Integrate[1/(x^5*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

output
```
-1/6*1/(x^4*Sqrt[c*x^4])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{2a-2(a+1)+cx^4+2}}dx$$

$$\Big\downarrow \text{ 1}$$

$$\int \frac{1}{x^5\sqrt{cx^4}}dx$$

$$\Big\downarrow \text{ 21}$$

$$\frac{1}{4}c^2\int \frac{1}{\left(cx^4\right)^{5/2}}dx^4$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{c}{6\left(cx^4\right)^{3/2}}$$

input
```
Int[1/(x^5*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

output
```
-1/6*c/(c*x^4)^(3/2)
```

Defintions of rubi rules used

rule 1
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{6x^4\sqrt{c\,x^4}}$	13
default	$-\frac{1}{6x^4\sqrt{c\,x^4}}$	13
risch	$-\frac{1}{6x^4\sqrt{c\,x^4}}$	13
pseudoelliptic	$-\frac{1}{6x^4\sqrt{c\,x^4}}$	13
orering	$-\frac{1}{6x^4\sqrt{c\,x^4}}$	13
trager	$\frac{(x-1)(x^5+x^4+x^3+x^2+x+1)\sqrt{c\,x^4}}{6c\,x^8}$	34

input
```
int(1/x^5/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^4/(c*x^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^5\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{\sqrt{cx^4}}{6\,cx^8}$$

input
```
integrate(1/x^5/(c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(c*x^4)/(c*x^8)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^5\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{6x^4\sqrt{cx^4}}$$

input
```
integrate(1/x**5/(c*x**4)**(1/2),x)
```

output
```
-1/(6*x**4*sqrt(c*x**4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{6\sqrt{cx^4}x^4}$$

input
```
integrate(1/x^5/(c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/6/(sqrt(c*x^4)*x^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^5\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{6\sqrt{c}x^6}$$

input
```
integrate(1/x^5/(c*x^4)^(1/2),x, algorithm="giac")
```

output
```
-1/6/(sqrt(c)*x^6)
```

Mupad [B] (verification not implemented)

Time = 17.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^5\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{6\sqrt{c}\,(x^4)^{3/2}}$$

input
```
int(1/(x^5*(c*x^4)^(1/2)),x)
```

output
```
-1/(6*c^(1/2)*(x^4)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^5\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{\sqrt{c}}{6c\,x^6}$$

input
```
int(1/x^5/(c*x^4)^(1/2),x)
```

output
```
( - sqrt(c))/(6*c*x**6)
```

3.1051 $\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 17

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{x\sqrt{cx^4}}{3c}$$

output
```
1/3*x*(c*x^4)^(1/2)/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{x^5}{3\sqrt{cx^4}}$$

input
```
Integrate[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

output
```
x^5/(3*Sqrt[c*x^4])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1, 22, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{2a-2(a+1)+cx^4+2}}dx$$

$$\Big\downarrow \; 1$$

$$\int \frac{x^4}{\sqrt{cx^4}}dx$$

$$\Big\downarrow \; 22$$

$$\frac{\int \sqrt{cx^4}dx}{c}$$

$$\Big\downarrow \; 20$$

$$\frac{\sqrt{cx^4}\int x^2dx}{cx^2}$$

$$\Big\downarrow \; 15$$

$$\frac{x\sqrt{cx^4}}{3c}$$

input
```
Int[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

output
```
(x*Sqrt[c*x^4])/(3*c)
```

Defintions of rubi rules used

rule 1
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 22
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[1/a^(m/n)   Int[(
a*x^n)^(p + m/n), x], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[m/n] && LtQ[
p*(m/n), 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{x^{5}}{3\sqrt{c\,x^{4}}}$	13
default	$\frac{x^{5}}{3\sqrt{c\,x^{4}}}$	13
risch	$\frac{x^{5}}{3\sqrt{c\,x^{4}}}$	13
orering	$\frac{x^{5}}{3\sqrt{c\,x^{4}}}$	13
trager	$\frac{\left(x^{2}+x+1\right)\left(x-1\right)\sqrt{c\,x^{4}}}{3c\,x^{2}}$	25

input
```
int(x^4/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^5/(c*x^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{\sqrt{cx^4}x}{3\,c}$$

input
```
integrate(x^4/(c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(c*x^4)*x/c
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{x^5}{3\sqrt{cx^4}}$$

input
```
integrate(x**4/(c*x**4)**(1/2),x)
```

output
```
x**5/(3*sqrt(c*x**4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{x^5}{3\,\sqrt{cx^4}}$$

input
```
integrate(x^4/(c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
1/3*x^5/sqrt(c*x^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{x^3}{3\sqrt{c}}$$

input
```
integrate(x^4/(c*x^4)^(1/2),x, algorithm="giac")
```

output
```
1/3*x^3/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x^4}{\sqrt{c\,x^4}}\,dx$$

input
```
int(x^4/(c*x^4)^(1/2),x)
```

output
```
int(x^4/(c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{\sqrt{c}\,x^3}{3c}$$

input
```
int(x^4/(c*x^4)^(1/2),x)
```

output
```
(sqrt(c)*x**3)/(3*c)
```

3.1052 $\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 13

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}} \, dx = \frac{x^3}{\sqrt{cx^4}}$$

output
```
x^3/(c*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}} \, dx = \frac{x^3}{\sqrt{cx^4}}$$

input
```
Integrate[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

output
```
x^3/Sqrt[c*x^4]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 23, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{2a-2(a+1)+cx^4+2}}dx$$
$$\Big\downarrow \ 1$$
$$\int \frac{x^2}{\sqrt{cx^4}}dx$$
$$\Big\downarrow \ 23$$
$$\frac{x^2\int 1dx}{\sqrt{cx^4}}$$
$$\Big\downarrow \ 24$$
$$\frac{x^3}{\sqrt{cx^4}}$$

input
```
Int[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

output
```
x^3/Sqrt[c*x^4]
```

Defintions of rubi rules used

rule 1
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{x^{3}}{\sqrt{c\,x^{4}}}$	12
risch	$\frac{x^{3}}{\sqrt{c\,x^{4}}}$	12
orering	$\frac{x^{3}}{\sqrt{c\,x^{4}}}$	12
trager	$\frac{(x-1)\sqrt{c\,x^{4}}}{c\,x^{2}}$	18

input
```
int(x^2/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^3/(c*x^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{\sqrt{cx^4}}{cx}$$

input
```
integrate(x^2/(c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^4)/(c*x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{x^3}{\sqrt{cx^4}}$$

input
```
integrate(x**2/(c*x**4)**(1/2),x)
```

output
```
x**3/sqrt(c*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{x^3}{\sqrt{cx^4}}$$

input
```
integrate(x^2/(c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
x^3/sqrt(c*x^4)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.38

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{x}{\sqrt{c}}$$

input
```
integrate(x^2/(c*x^4)^(1/2),x, algorithm="giac")
```

output
```
x/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x^2}{\sqrt{c\,x^4}}\,dx$$

input
```
int(x^2/(c*x^4)^(1/2),x)
```

output
```
int(x^2/(c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{\sqrt{c}\,x}{c}$$

input
```
int(x^2/(c*x^4)^(1/2),x)
```

output
```
(sqrt(c)*x)/c
```

3.1053 $\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx=\frac{x^2\log(x)}{\sqrt{cx^4}}$$

output
```
x^2*ln(x)/(c*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx=\frac{x^2\log(x)}{\sqrt{cx^4}}$$

input
```
Integrate[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

output
```
(x^2*Log[x])/Sqrt[c*x^4]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1, 23, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{2a-2(a+1)+cx^4+2}}dx$$
$$\Big\downarrow \text{1}$$
$$\int \frac{x}{\sqrt{cx^4}}dx$$
$$\Big\downarrow \text{23}$$
$$\frac{x^2\int \frac{1}{x}dx}{\sqrt{cx^4}}$$
$$\Big\downarrow \text{14}$$
$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

input
```
Int[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

output
```
(x^2*Log[x])/Sqrt[c*x^4]
```

Defintions of rubi rules used

rule 1
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$\frac{\sqrt{c\,x^{2}}}{c}$	12
default	$\frac{x^{2} \ln\left(x\right)}{\sqrt{c\,x^{4}}}$	14
risch	$\frac{x^{2} \ln\left(x\right)}{\sqrt{c\,x^{4}}}$	14

input
```
int(x/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2)^(1/2)/c
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{\sqrt{cx^4}\log(x)}{cx^2}$$

input
```
integrate(x/(c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(c*x^4)*log(x)/(c*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{x^2\log(x)}{\sqrt{cx^4}}$$

input
```
integrate(x/(c*x**4)**(1/2),x)
```

output
```
x**2*log(x)/sqrt(c*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{x^2\log(x)}{\sqrt{cx^4}}$$

input
```
integrate(x/(c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
x^2*log(x)/sqrt(c*x^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{\log\left(x^2\sqrt{|c|}\right)}{2\sqrt{c}}$$

input
```
integrate(x/(c*x^4)^(1/2),x, algorithm="giac")
```

output
```
1/2*log(x^2*sqrt(abs(c)))/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \int \frac{x}{\sqrt{c\,x^4}}\,dx$$

input
```
int(x/(c*x^4)^(1/2),x)
```

output
```
int(x/(c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = \frac{\sqrt{c}\log(x)}{c}$$

input
```
int(x/(c*x^4)^(1/2),x)
```

output
```
(sqrt(c)*log(x))/c
```

3.1054 $\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 12

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{x}{\sqrt{cx^4}}$$

output
```
-x/(c*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{x}{\sqrt{cx^4}}$$

input
```
Integrate[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

output
```
-(x/Sqrt[c*x^4])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2a-2(a+1)+cx^4+2}}dx$$

$$\Big\downarrow \; 1$$

$$\int \frac{1}{\sqrt{cx^4}}dx$$

$$\Big\downarrow \; 20$$

$$\frac{x^2\int \frac{1}{x^2}dx}{\sqrt{cx^4}}$$

$$\Big\downarrow \; 15$$

$$-\frac{x}{\sqrt{cx^4}}$$

input
```
Int[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

output
```
-(x/Sqrt[c*x^4])
```

Defintions of rubi rules used

rule 1
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{x}{\sqrt{c\,x^{4}}}$	11
default	$-\frac{x}{\sqrt{c\,x^{4}}}$	11
risch	$-\frac{x}{\sqrt{c\,x^{4}}}$	11
orering	$-\frac{x}{\sqrt{c\,x^{4}}}$	11
trager	$\frac{(x-1)\sqrt{c\,x^{4}}}{c\,x^{3}}$	18

input
```
int(1/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x/(c*x^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{\sqrt{cx^4}}{cx^3}$$

input
```
integrate(1/(c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(c*x^4)/(c*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{x}{\sqrt{cx^4}}$$

input
```
integrate(1/(c*x**4)**(1/2),x)
```

output
```
-x/sqrt(c*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{x}{\sqrt{cx^4}}$$

input
```
integrate(1/(c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
-x/sqrt(c*x^4)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{\sqrt{c}x}$$

input
```
integrate(1/(c*x^4)^(1/2),x, algorithm="giac")
```

output
```
-1/(sqrt(c)*x)
```

Mupad [B] (verification not implemented)

Time = 17.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{\sqrt{x^4}}{\sqrt{c}\,x^3}$$

input
```
int(1/(c*x^4)^(1/2),x)
```

output
```
-(x^4)^(1/2)/(c^(1/2)*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{\sqrt{c}}{cx}$$

input
```
int(1/(c*x^4)^(1/2),x)
```

output
```
( - sqrt(c))/(c*x)
```

3.1055 $\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{1}{3x\sqrt{cx^4}}$$

output
```
-1/3/x/(c*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{1}{3x\sqrt{cx^4}}$$

input
```
Integrate[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

output
```
-1/3*1/(x*Sqrt[c*x^4])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{2a-2(a+1)+cx^4+2}}dx$$

$$\Big\downarrow \; 1$$

$$\int \frac{1}{x^2\sqrt{cx^4}}dx$$

$$\Big\downarrow \; 23$$

$$\frac{x^2\int \frac{1}{x^4}dx}{\sqrt{cx^4}}$$

$$\Big\downarrow \; 15$$

$$-\frac{1}{3x\sqrt{cx^4}}$$

input
```
Int[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

output
```
-1/3*1/(x*Sqrt[c*x^4])
```

Defintions of rubi rules used

rule 1
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{1}{3x\sqrt{c\,x^4}}$	13
default	$-\frac{1}{3x\sqrt{c\,x^4}}$	13
risch	$-\frac{1}{3x\sqrt{c\,x^4}}$	13
orering	$-\frac{1}{3x\sqrt{c\,x^4}}$	13
trager	$\frac{(x-1)(x^2+x+1)\sqrt{c\,x^4}}{3c\,x^5}$	25

input
```
int(1/x^2/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x/(c*x^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{\sqrt{cx^4}}{3\,cx^5}$$

input
```
integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(c*x^4)/(c*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{3x\sqrt{cx^4}}$$

input
```
integrate(1/x**2/(c*x**4)**(1/2),x)
```

output
```
-1/(3*x*sqrt(c*x**4))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{3\sqrt{cx^4}x}$$

input
```
integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/3/(sqrt(c*x^4)*x)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{3\sqrt{c}x^3}$$

input
```
integrate(1/x^2/(c*x^4)^(1/2),x, algorithm="giac")
```

output
```
-1/3/(sqrt(c)*x^3)
```

Mupad [B] (verification not implemented)

Time = 17.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{3\sqrt{c}\,x\,\sqrt{x^4}}$$

input
```
int(1/(x^2*(c*x^4)^(1/2)),x)
```

output
```
-1/(3*c^(1/2)*x*(x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{\sqrt{c}}{3c\,x^3}$$

input
```
int(1/x^2/(c*x^4)^(1/2),x)
```

output
```
( - sqrt(c))/(3*c*x**3)
```

3.1056 $\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{1}{4x^2\sqrt{cx^4}}$$

output
```
-1/4/x^2/(c*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{cx^2}{4\,(cx^4)^{3/2}}$$

input
```
Integrate[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

output
```
-1/4*(c*x^2)/(c*x^4)^(3/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{2a-2(a+1)+cx^4+2}}dx$$
$$\Big\downarrow \text{ 1}$$
$$\int \frac{1}{x^3\sqrt{cx^4}}dx$$
$$\Big\downarrow \text{ 23}$$
$$\frac{x^2\int \frac{1}{x^5}dx}{\sqrt{cx^4}}$$
$$\Big\downarrow \text{ 15}$$
$$-\frac{1}{4x^2\sqrt{cx^4}}$$

input
```
Int[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

output
```
-1/4*1/(x^2*Sqrt[c*x^4])
```

Defintions of rubi rules used

rule 1
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{1}{4x^{2}\sqrt{c\,x^{4}}}$	13
default	$-\frac{1}{4x^{2}\sqrt{c\,x^{4}}}$	13
risch	$-\frac{1}{4x^{2}\sqrt{c\,x^{4}}}$	13
pseudoelliptic	$-\frac{1}{3x^{2}\sqrt{c\,x^{2}}}$	13
orering	$-\frac{1}{4x^{2}\sqrt{c\,x^{4}}}$	13
trager	$\frac{(x-1)\left(x^{3}+x^{2}+x+1\right)\sqrt{c\,x^{4}}}{4c\,x^{6}}$	28

input
```
int(1/x^3/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^2/(c*x^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{\sqrt{cx^4}}{4\,cx^6}$$

input
```
integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(c*x^4)/(c*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{4x^2\sqrt{cx^4}}$$

input
```
integrate(1/x**3/(c*x**4)**(1/2),x)
```

output
```
-1/(4*x**2*sqrt(c*x**4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{4\sqrt{cx^4}x^2}$$

input
```
integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/4/(sqrt(c*x^4)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{4\sqrt{c}x^4}$$

input
```
integrate(1/x^3/(c*x^4)^(1/2),x, algorithm="giac")
```

output
```
-1/4/(sqrt(c)*x^4)
```

Mupad [B] (verification not implemented)

Time = 17.64 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{1}{4\sqrt{c}\,x^2\sqrt{x^4}}$$

input
```
int(1/(x^3*(c*x^4)^(1/2)),x)
```

output
```
-1/(4*c^(1/2)*x^2*(x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{\sqrt{c}}{4c\,x^4}$$

input
```
int(1/x^3/(c*x^4)^(1/2),x)
```

output
```
( - sqrt(c))/(4*c*x**4)
```

3.1057 $\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{1}{5x^3\sqrt{cx^4}}$$

output
```
-1/5/x^3/(c*x^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{cx}{5\,(cx^4)^{3/2}}$$

input
```
Integrate[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

output
```
-1/5*(c*x)/(c*x^4)^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1, 23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{2a-2(a+1)+cx^4+2}}dx$$
$$\Big\downarrow \text{ 1}$$
$$\int \frac{1}{x^4\sqrt{cx^4}}dx$$
$$\Big\downarrow \text{ 23}$$
$$\frac{x^2\int \frac{1}{x^6}dx}{\sqrt{cx^4}}$$
$$\Big\downarrow \text{ 15}$$
$$-\frac{1}{5x^3\sqrt{cx^4}}$$

input
```
Int[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

output
```
-1/5*1/(x^3*Sqrt[c*x^4])
```

Defintions of rubi rules used

rule 1
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*(b*x^n)^p, x]
 /; FreeQ[{a, b, n, p}, x] && EqQ[a, 0]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{1}{5x^{3}\sqrt{c\,x^{4}}}$	13
default	$-\frac{1}{5x^{3}\sqrt{c\,x^{4}}}$	13
risch	$-\frac{1}{5x^{3}\sqrt{c\,x^{4}}}$	13
orering	$-\frac{1}{5x^{3}\sqrt{c\,x^{4}}}$	13
trager	$\frac{(x-1)(x^{4}+x^{3}+x^{2}+x+1)\sqrt{c\,x^{4}}}{5c\,x^{7}}$	31

input
```
int(1/x^4/(c*x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^3/(c*x^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx=-\frac{\sqrt{cx^4}}{5\,cx^7}$$

input
```
integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*sqrt(c*x^4)/(c*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{5x^3\sqrt{cx^4}}$$

input
```
integrate(1/x**4/(c*x**4)**(1/2),x)
```

output
```
-1/(5*x**3*sqrt(c*x**4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{5\sqrt{cx^4}x^3}$$

input
```
integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/5/(sqrt(c*x^4)*x^3)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{5\sqrt{c}x^5}$$

input
```
integrate(1/x^4/(c*x^4)^(1/2),x, algorithm="giac")
```

output
```
-1/5/(sqrt(c)*x^5)
```

Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{1}{5\sqrt{c}\,x^3\,\sqrt{x^4}}$$

input
```
int(1/(x^4*(c*x^4)^(1/2)),x)
```

output
```
-1/(5*c^(1/2)*x^3*(x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx = -\frac{\sqrt{c}}{5c\,x^5}$$

input
```
int(1/x^4/(c*x^4)^(1/2),x)
```

output
```
( - sqrt(c))/(5*c*x**5)
```

3.1058 $\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 12

$$\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^5}{5\sqrt{a}}$$

output
```
1/5*x^5/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^5}{5\sqrt{a}}$$

input
```
Integrate[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
x^5/(5*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+(2c-2(c+1)+2)x^4}}dx$$
$$\Big\downarrow \ 2$$
$$\int \frac{x^4}{\sqrt{a}}dx$$
$$\Big\downarrow \ 15$$
$$\frac{x^5}{5\sqrt{a}}$$

input
```
Int[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
x^5/(5*Sqrt[a])
```

Defintions of rubi rules used

rule 2
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{x^5}{5\sqrt{a}}$	9
default	$\frac{x^5}{5\sqrt{a}}$	9
norman	$\frac{x^5}{5\sqrt{a}}$	9
parallelrisch	$\frac{x^5}{5\sqrt{a}}$	9
orering	$\frac{x^5}{5\sqrt{a}}$	9

input
```
int(x^4/a^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^5}{5\sqrt{a}}$$

input
```
integrate(x^4/a^(1/2),x, algorithm="fricas")
```

output
```
1/5*x^5/sqrt(a)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^5}{5\sqrt{a}}$$

input
```
integrate(x**4/a**(1/2),x)
```

output
```
x**5/(5*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^5}{5\sqrt{a}}$$

input
```
integrate(x^4/a^(1/2),x, algorithm="maxima")
```

output
```
1/5*x^5/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^5}{5\sqrt{a}}$$

input
```
integrate(x^4/a^(1/2),x, algorithm="giac")
```

output
```
1/5*x^5/sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^5}{5\sqrt{a}}$$

input
```
int(x^4/a^(1/2),x)
```

output
```
x^5/(5*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{a}\,x^5}{5a}$$

input
```
int(x^4/a^(1/2),x)
```

output
```
(sqrt(a)*x**5)/(5*a)
```

3.1059 $\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 12

$$\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^4}{4\sqrt{a}}$$

output
```
1/4*x^4/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^4}{4\sqrt{a}}$$

input
```
Integrate[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
x^4/(4*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+(2c-2(c+1)+2)x^4}}dx$$

$$\Big\downarrow \; 2$$

$$\int \frac{x^3}{\sqrt{a}}dx$$

$$\Big\downarrow \; 15$$

$$\frac{x^4}{4\sqrt{a}}$$

input
```
Int[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
x^4/(4*Sqrt[a])
```

Defintions of rubi rules used

rule 2
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{x^4}{4\sqrt{a}}$	9
default	$\frac{x^4}{4\sqrt{a}}$	9
norman	$\frac{x^4}{4\sqrt{a}}$	9
parallelrisch	$\frac{x^4}{4\sqrt{a}}$	9
orering	$\frac{x^4}{4\sqrt{a}}$	9

input
```
int(x^3/a^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^4}{4\sqrt{a}}$$

input
```
integrate(x^3/a^(1/2),x, algorithm="fricas")
```

output
```
1/4*x^4/sqrt(a)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^4}{4\sqrt{a}}$$

input
```
integrate(x**3/a**(1/2),x)
```

output
```
x**4/(4*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^4}{4\,\sqrt{a}}$$

input
```
integrate(x^3/a^(1/2),x, algorithm="maxima")
```

output
```
1/4*x^4/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^4}{4\,\sqrt{a}}$$

input
```
integrate(x^3/a^(1/2),x, algorithm="giac")
```

output
```
1/4*x^4/sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^4}{4\sqrt{a}}$$

input
```
int(x^3/a^(1/2),x)
```

output
```
x^4/(4*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{\sqrt{a}\,x^4}{4a}$$

input
```
int(x^3/a^(1/2),x)
```

output
```
(sqrt(a)*x**4)/(4*a)
```

3.1060 $\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 12

$$\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^3}{3\sqrt{a}}$$

output
```
1/3*x^3/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^3}{3\sqrt{a}}$$

input
```
Integrate[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
x^3/(3*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+(2c-2(c+1)+2)x^4}}dx$$

$$\Big\downarrow \; 2$$

$$\int \frac{x^2}{\sqrt{a}}dx$$

$$\Big\downarrow \; 15$$

$$\frac{x^3}{3\sqrt{a}}$$

input
```
Int[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
x^3/(3*Sqrt[a])
```

Defintions of rubi rules used

rule 2
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{x^3}{3\sqrt{a}}$	9
default	$\frac{x^3}{3\sqrt{a}}$	9
norman	$\frac{x^3}{3\sqrt{a}}$	9
parallelrisch	$\frac{x^3}{3\sqrt{a}}$	9
orering	$\frac{x^3}{3\sqrt{a}}$	9

input
```
int(x^2/a^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^3}{3\sqrt{a}}$$

input
```
integrate(x^2/a^(1/2),x, algorithm="fricas")
```

output
```
1/3*x^3/sqrt(a)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^3}{3\sqrt{a}}$$

input
```
integrate(x**2/a**(1/2),x)
```

output
```
x**3/(3*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^3}{3\sqrt{a}}$$

input
```
integrate(x^2/a^(1/2),x, algorithm="maxima")
```

output
```
1/3*x^3/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^3}{3\sqrt{a}}$$

input
```
integrate(x^2/a^(1/2),x, algorithm="giac")
```

output
```
1/3*x^3/sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^3}{3\sqrt{a}}$$

input
```
int(x^2/a^(1/2),x)
```

output
```
x^3/(3*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{\sqrt{a}\,x^3}{3a}$$

input
```
int(x^2/a^(1/2),x)
```

output
```
(sqrt(a)*x**3)/(3*a)
```

3.1061 $\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 12

$$\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^2}{2\sqrt{a}}$$

output
```
1/2*x^2/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x^2}{2\sqrt{a}}$$

input
```
Integrate[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
x^2/(2*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+(2c-2(c+1)+2)x^4}}dx$$

$$\Big\downarrow \; 2$$

$$\int \frac{x}{\sqrt{a}}dx$$

$$\Big\downarrow \; 15$$

$$\frac{x^2}{2\sqrt{a}}$$

input
```
Int[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
x^2/(2*Sqrt[a])
```

Defintions of rubi rules used

rule 2
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{x^2}{2\sqrt{a}}$	9
default	$\frac{x^2}{2\sqrt{a}}$	9
norman	$\frac{x^2}{2\sqrt{a}}$	9
parallelrisch	$\frac{x^2}{2\sqrt{a}}$	9
orering	$\frac{x^2}{2\sqrt{a}}$	9

input
```
int(x/a^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^2}{2\sqrt{a}}$$

input
```
integrate(x/a^(1/2),x, algorithm="fricas")
```

output
```
1/2*x^2/sqrt(a)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^2}{2\sqrt{a}}$$

input
```
integrate(x/a**(1/2),x)
```

output
```
x**2/(2*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^2}{2\,\sqrt{a}}$$

input
```
integrate(x/a^(1/2),x, algorithm="maxima")
```

output
```
1/2*x^2/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^2}{2\,\sqrt{a}}$$

input
```
integrate(x/a^(1/2),x, algorithm="giac")
```

output
```
1/2*x^2/sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x^2}{2\sqrt{a}}$$

input
```
int(x/a^(1/2),x)
```

output
```
x^2/(2*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{\sqrt{a}\,x^2}{2a}$$

input
```
int(x/a^(1/2),x)
```

output
```
(sqrt(a)*x**2)/(2*a)
```

3.1062 $\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 7

$$\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x}{\sqrt{a}}$$

output
```
x/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x}{\sqrt{a}}$$

input
```
Integrate[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
x/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+(2c-2(c+1)+2)x^4}}dx$$
$$\Big\downarrow \; 2$$
$$\int \frac{1}{\sqrt{a}}dx$$
$$\Big\downarrow \; 24$$
$$\frac{x}{\sqrt{a}}$$

input
```
Int[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

output
```
x/Sqrt[a]
```

Defintions of rubi rules used

rule 2
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x}{\sqrt{a}}$	6
norman	$\frac{x}{\sqrt{a}}$	6
parallelrisch	$\frac{x}{\sqrt{a}}$	6
orering	$\frac{x}{\sqrt{a}}$	6

input
```
int(1/a^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x}{\sqrt{a}}$$

input
```
integrate(1/a^(1/2),x, algorithm="fricas")
```

output
```
x/sqrt(a)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{x}{\sqrt{a}}$$

input
```
integrate(1/a**(1/2),x)
```

output
```
x/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x}{\sqrt{a}}$$

input
```
integrate(1/a^(1/2),x, algorithm="maxima")
```

output
```
x/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x}{\sqrt{a}}$$

input
```
integrate(1/a^(1/2),x, algorithm="giac")
```

output
```
x/sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{x}{\sqrt{a}}$$

input
```
int(1/a^(1/2),x)
```

output
```
x/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{\sqrt{a}\,x}{a}$$

input
```
int(1/a^(1/2),x)
```

output
```
(sqrt(a)*x)/a
```

3.1063 $\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 8

$$\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{\log(x)}{\sqrt{a}}$$

output
```
ln(x)/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{\log(x)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
Log[x]/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+(2c-2(c+1)+2)x^4}}dx$$
$$\Big\downarrow \text{ 2}$$
$$\int \frac{1}{\sqrt{a}x}dx$$
$$\Big\downarrow \text{ 14}$$
$$\frac{\log(x)}{\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
Log[x]/Sqrt[a]
```

Defintions of rubi rules used

rule 2
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\ln(x)}{\sqrt{a}}$	7
norman	$\frac{\ln(x)}{\sqrt{a}}$	7
parallelrisch	$\frac{\ln(x)}{\sqrt{a}}$	7

input
```
int(1/x/a^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{\log{(x)}}{\sqrt{a}}$$

input
```
integrate(1/x/a^(1/2),x, algorithm="fricas")
```

output
```
log(x)/sqrt(a)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{\log{(x)}}{\sqrt{a}}$$

input
```
integrate(1/x/a**(1/2),x)
```

output
```
log(x)/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{\log(x)}{\sqrt{a}}$$

input
```
integrate(1/x/a^(1/2),x, algorithm="maxima")
```

output
```
log(x)/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{\log(|x|)}{\sqrt{a}}$$

input
```
integrate(1/x/a^(1/2),x, algorithm="giac")
```

output
```
log(abs(x))/sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 17.54 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = \frac{\ln(x)}{\sqrt{a}}$$

input
```
int(1/(a^(1/2)*x),x)
```

output
```
log(x)/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=\frac{\sqrt{a}\log(x)}{a}$$

input
```
int(1/x/a^(1/2),x)
```

output
```
(sqrt(a)*log(x))/a
```

3.1064 $\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 10

$$\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=-\frac{1}{\sqrt{a}x}$$

output
```
-1/x/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=-\frac{1}{\sqrt{a}x}$$

input
```
Integrate[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
-(1/(Sqrt[a]*x))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+(2c-2(c+1)+2)x^4}}\,dx$$
$$\Big\downarrow \ 2$$
$$\int \frac{1}{\sqrt{a}x^2}dx$$
$$\Big\downarrow \ 15$$
$$-\frac{1}{\sqrt{a}x}$$

input
```
Int[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
-(1/(Sqrt[a]*x))
```

Defintions of rubi rules used

rule 2
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{1}{x\sqrt{a}}$	9
default	$-\frac{1}{x\sqrt{a}}$	9
norman	$-\frac{1}{x\sqrt{a}}$	9
parallelrisch	$-\frac{1}{x\sqrt{a}}$	9
orering	$-\frac{1}{x\sqrt{a}}$	9

input
```
int(1/x^2/a^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/x/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{\sqrt{a}x}$$

input
```
integrate(1/x^2/a^(1/2),x, algorithm="fricas")
```

output
```
-1/(sqrt(a)*x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{\sqrt{a}x}$$

input
```
integrate(1/x**2/a**(1/2),x)
```

output
```
-1/(sqrt(a)*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{\sqrt{a}x}$$

input
```
integrate(1/x^2/a^(1/2),x, algorithm="maxima")
```

output
```
-1/(sqrt(a)*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{\sqrt{a}x}$$

input
```
integrate(1/x^2/a^(1/2),x, algorithm="giac")
```

output
```
-1/(sqrt(a)*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=-\frac{1}{\sqrt{a}\,x}$$

input
```
int(1/(a^(1/2)*x^2),x)
```

output
```
-1/(a^(1/2)*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=-\frac{\sqrt{a}}{ax}$$

input
```
int(1/x^2/a^(1/2),x)
```

output
```
( - sqrt(a))/(a*x)
```

3.1065 $\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 12

$$\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=-\frac{1}{2\sqrt{a}x^2}$$

output
```
-1/2/x^2/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=-\frac{1}{2\sqrt{a}x^2}$$

input
```
Integrate[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
-1/2*1/(Sqrt[a]*x^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+(2c-2(c+1)+2)x^4}}dx$$
$$\Big\downarrow \; 2$$
$$\int \frac{1}{\sqrt{a}x^3}dx$$
$$\Big\downarrow \; 15$$
$$-\frac{1}{2\sqrt{a}x^2}$$

input
```
Int[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
-1/2*1/(Sqrt[a]*x^2)
```

Defintions of rubi rules used

rule 2
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{1}{2x^2\sqrt{a}}$	9
default	$-\frac{1}{2x^2\sqrt{a}}$	9
norman	$-\frac{1}{2x^2\sqrt{a}}$	9
parallelrisch	$-\frac{1}{2x^2\sqrt{a}}$	9
orering	$-\frac{1}{2x^2\sqrt{a}}$	9

input
```
int(1/x^3/a^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{2\sqrt{a}x^2}$$

input
```
integrate(1/x^3/a^(1/2),x, algorithm="fricas")
```

output
```
-1/2/(sqrt(a)*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{2\sqrt{a}x^2}$$

input
```
integrate(1/x**3/a**(1/2),x)
```

output
```
-1/(2*sqrt(a)*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{2\,\sqrt{a}x^2}$$

input
```
integrate(1/x^3/a^(1/2),x, algorithm="maxima")
```

output
```
-1/2/(sqrt(a)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{2\,\sqrt{a}x^2}$$

input
```
integrate(1/x^3/a^(1/2),x, algorithm="giac")
```

output
```
-1/2/(sqrt(a)*x^2)
```

Mupad [B] (verification not implemented)

Time = 17.72 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=-\frac{1}{2\sqrt{a}\,x^2}$$

input
```
int(1/(a^(1/2)*x^3),x)
```

output
```
-1/(2*a^(1/2)*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=-\frac{\sqrt{a}}{2a\,x^2}$$

input
```
int(1/x^3/a^(1/2),x)
```

output
```
( - sqrt(a))/(2*a*x**2)
```

3.1066 $\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 12

$$\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=-\frac{1}{3\sqrt{a}x^3}$$

output
```
-1/3/x^3/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx=-\frac{1}{3\sqrt{a}x^3}$$

input
```
Integrate[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
-1/3*1/(Sqrt[a]*x^3)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+(2c-2(c+1)+2)x^4}}dx$$
$$\Big\downarrow \; 2$$
$$\int \frac{1}{\sqrt{a}x^4}dx$$
$$\Big\downarrow \; 15$$
$$-\frac{1}{3\sqrt{a}x^3}$$

input
```
Int[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

output
```
-1/3*1/(Sqrt[a]*x^3)
```

Defintions of rubi rules used

rule 2
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[u*a^p, x] /; F
reeQ[{a, b, n, p}, x] && EqQ[b, 0]
```

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{1}{3x^{3}\sqrt{a}}$	9
default	$-\frac{1}{3x^{3}\sqrt{a}}$	9
norman	$-\frac{1}{3x^{3}\sqrt{a}}$	9
parallelrisch	$-\frac{1}{3x^{3}\sqrt{a}}$	9
orering	$-\frac{1}{3x^{3}\sqrt{a}}$	9

input
```
int(1/x^4/a^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{3\sqrt{a}x^3}$$

input
```
integrate(1/x^4/a^(1/2),x, algorithm="fricas")
```

output
```
-1/3/(sqrt(a)*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{3\sqrt{a}x^3}$$

input
```
integrate(1/x**4/a**(1/2),x)
```

output
```
-1/(3*sqrt(a)*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{3\sqrt{a}x^3}$$

input
```
integrate(1/x^4/a^(1/2),x, algorithm="maxima")
```

output
```
-1/3/(sqrt(a)*x^3)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{3\sqrt{a}x^3}$$

input
```
integrate(1/x^4/a^(1/2),x, algorithm="giac")
```

output
```
-1/3/(sqrt(a)*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{1}{3\sqrt{a}\,x^3}$$

input
```
int(1/(a^(1/2)*x^4),x)
```

output
```
-1/(3*a^(1/2)*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx = -\frac{\sqrt{a}}{3a\,x^3}$$

input
```
int(1/x^4/a^(1/2),x)
```

output
```
( - sqrt(a))/(3*a*x**3)
```

3.1067 $\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 147

$$\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2/5*(d*x)^(5/2)*(c*x^4+b*x^2+a)^(1/2)*AppellF1(5/4,-1/2,-1/2,9/4,-2*c*x^2/
(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(147) = 294.

Time = 11.39 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.48

$$\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\frac{2d\sqrt{dx}\left(5(2b+5cx^2)(a+bx^2+cx^4)-10ab\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2c}{b+\sqrt{b^2-4ac}}}\right.}{}$$

input
```
Integrate[(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*d*Sqrt[d*x]*(5*(2*b + 5*c*x^2)*(a + b*x^2 + c*x^4) - 10*a*b*Sqrt[(b - S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 2*
(-3*b^2 + 10*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])]))/(225*c*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{a+bx^2+cx^4}\int (dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int (dx)^{\frac{3}{2}} \sqrt{c\,x^{4}+b\,x^{2}+a}dx$$

input
```
int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(dx)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)*d*x, x)
```

Sympy [F]

$$\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int (dx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((d*x)**(3/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}(dx)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2), x)
```

Giac [F]

$$\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}(dx)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\int (d\,x)^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx=\frac{2\sqrt{d}\,d\left(4\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,a+3\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,b\,x^2-10\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4}}{c\,x^4+}\right)\right)}{27}$$

input
```
int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(d)*d*(4*sqrt(x)*sqrt(a + b*x**2 + c*x**4)*a + 3*sqrt(x)*sqrt(a + b
*x**2 + c*x**4)*b*x**2 - 10*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4)*x**3)/(
a + b*x**2 + c*x**4),x)*a*c + 3*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4)*x**
3)/(a + b*x**2 + c*x**4),x)*b**2 - 2*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4
))/(a*x + b*x**3 + c*x**5),x)*a**2))/(27*b)
```

3.1068 $\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 147

$$\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx = \frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2/3*(d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2)*AppellF1(3/4,-1/2,-1/2,7/4,-2*c*x^2/
(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(147) = 294.

Time = 11.31 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.33

$$\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx = \frac{2x\sqrt{dx}\left(21(a+bx^2+cx^4)+28a\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2}{-b+}\right.\right.}{147\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*x*Sqrt[d*x]*(21*(a + b*x^2 + c*x^4) + 28*a*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 6*b*x^2*Sqrt[(b - S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(
147*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{a+bx^2+cx^4}\int \sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}dx}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \sqrt{dx}\,\sqrt{c\,x^{4}+b\,x^{2}+a}dx$$

input
```
int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}\sqrt{dx}\,dx$$

input
```
integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Sympy [F]

$$\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((d*x)**(1/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(d*x)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}\sqrt{dx}\,dx$$

input
```
integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Giac [F]

$$\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\sqrt{dx}\,dx$$

input
```
integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{d\,x}\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx = \frac{2\sqrt{d}\left(\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,x+\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)b+2\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)a\right)}{7}$$

input
```
int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(d)*(sqrt(x)*sqrt(a + b*x**2 + c*x**4)*x + int((sqrt(x)*sqrt(a + b*
x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*b + 2*int((sqrt(x)*sqrt(a +
b*x**2 + c*x**4))/(a + b*x**2 + c*x**4),x)*a))/7
```

3.1069 $\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2*(d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)*AppellF1(1/4,-1/2,-1/2,5/4,-2*c*x^2/(b
-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(145) = 290.

Time = 11.29 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.36

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx = \frac{2x\left(5(a+bx^2+cx^4)+20a\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4a}}\right.\right.}{25\sqrt{dx}\sqrt{a+bx^2+}}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/Sqrt[d*x],x]
```

output
```
(2*x*(5*(a + b*x^2 + c*x^4) + 20*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 2*b*x^2*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(25*Sqrt[d*x]
*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{a+bx^2+cx^4}\int \frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}{\sqrt{dx}}dx}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2\sqrt{dx}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/Sqrt[d*x],x]
```

output
```
(2*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/4, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{\sqrt{d x}}dx$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x)
```

output
```
int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx=\int \frac{\sqrt{cx^4+bx^2+a}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(d*x), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx=\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/(d*x)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/sqrt(d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx=\int \frac{\sqrt{cx^4+bx^2+a}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx=\int \frac{\sqrt{cx^4+bx^2+a}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{\sqrt{d\,x}}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/(d*x)^(1/2),x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx =\frac{2\sqrt{d}\left(\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}-\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,x^3}{c\,x^4+b\,x^2+a}dx\right)c+\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}}{c\,x^5+b\,x^3+ax}dx\right)a\right)}{3d}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x)
```

output
```
(2*sqrt(d)*(sqrt(x)*sqrt(a + b*x**2 + c*x**4) - int((sqrt(x)*sqrt(a + b*x**
*2 + c*x**4)*x**3)/(a + b*x**2 + c*x**4),x)*c + int((sqrt(x)*sqrt(a + b*x*
*2 + c*x**4))/(a*x + b*x**3 + c*x**5),x)*a))/(3*d)
```

3.1070 $\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx = -\frac{2\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(-\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output

```
-2*(c*x^4+b*x^2+a)^(1/2)*AppellF1(-1/4,-1/2,-1/2,3/4,-2*c*x^2/(b-(-4*a*c+b
^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(d*x)^(1/2)/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 345 vs. 2(145) = 290.

Time = 11.29 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx = \frac{x\left(-42(a+bx^2+cx^4)+28bx^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/(d*x)^(3/2),x]
```

output
```
(x*(-42*(a + b*x^2 + c*x^4) + 28*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 24*c*x^4*Sqrt[(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*(d*
x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{a+bx^2+cx^4}\int \frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}{(dx)^{3/2}}dx}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{394}$$

$$-\frac{2\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(-\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/(d*x)^(3/2),x]
```

output
```
(-2*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{(dx)^{\frac{3}{2}}}dx$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x)
```

output
```
int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(d^2*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/(d*x)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/(d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/(d*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{(d\,x)^{3/2}}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/(d*x)^(3/2),x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/(d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx = \frac{2\sqrt{d}\left(\sqrt{c\,x^4+b\,x^2+a}-\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)c+\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}}{c\,x^6+b\,x^4+a\,x^2}dx\right.}{\sqrt{x}\,d^2}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x)
```

output
```
(2*sqrt(d)*(sqrt(a + b*x**2 + c*x**4) - sqrt(x)*int((sqrt(x)*sqrt(a + b*x*
*2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*c + sqrt(x)*int((sqrt(x)*sqrt(
a + b*x**2 + c*x**4))/(a*x**2 + b*x**4 + c*x**6),x)*a))/(sqrt(x)*d**2)
```

3.1071 $\int (dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int (dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2} \, dx=\frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2/5*a*(d*x)^(5/2)*(c*x^4+b*x^2+a)^(1/2)*AppellF1(5/4,-3/2,-3/2,9/4,-2*c*x^
2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 459 vs. 2(148) = 296.

Time = 11.62 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.10

$$\int (dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2} \, dx=\frac{2d\sqrt{dx}\left(5(-28b^4x^2-8b^3cx^4+305b^2c^2x^6+480bc^3x^8+195c^4x^{10}+a^2c(176b+455cx^2)+a(\right.}{}$$

input
```
Integrate[(d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*d*Sqrt[d*x]*(5*(-28*b^4*x^2 - 8*b^3*c*x^4 + 305*b^2*c^2*x^6 + 480*b*c^3
*x^8 + 195*c^4*x^10 + a^2*c*(176*b + 455*c*x^2) + a*(-28*b^3 + 196*b^2*c*x
^2 + 916*b*c^2*x^4 + 650*c^3*x^6)) + 20*a*b*(7*b^2 - 44*a*c)*Sqrt[(b - Sqr
t[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 4*(2
1*b^4 - 157*a*b^2*c + 260*a^2*c^2)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(16575*c^2*Sqrt[a + b*x^2
+ c*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1461, 394\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2} \, dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{a\sqrt{a+bx^2+cx^4} \int (dx)^{3/2} \left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{3/2} \left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2} dx}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4} \operatorname{AppellF1}\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*a*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*
d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int (dx)^{\frac{3}{2}} \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}d x$$

input
```
int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int (dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}(dx)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*d*x^5 + b*d*x^3 + a*d*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Sympy [F]

$$\int (dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int (dx)^{\frac{3}{2}}\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x)**(3/2)*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((d*x)**(3/2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int (dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}(dx)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2), x)
```

Giac [F]

$$\int (dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}} \, dx$$

input
```
integrate((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int (d\,x)^{3/2} \left(c\,x^4+b\,x^2+a\right)^{3/2} \, dx$$

input
```
int((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int (dx)^{3/2} \left(a+bx^2 +cx^4\right)^{3/2} \, dx = \frac{2\sqrt{d}\,d\Big(208\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,a^2c-20\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,a\,b^2+273\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}}{}$$

input
```
int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(d)*d*(208*sqrt(x)*sqrt(a + b*x**2 + c*x**4)*a**2*c - 20*sqrt(x)*sq
rt(a + b*x**2 + c*x**4)*a*b**2 + 273*sqrt(x)*sqrt(a + b*x**2 + c*x**4)*a*b
*c*x**2 + 12*sqrt(x)*sqrt(a + b*x**2 + c*x**4)*b**3*x**2 + 171*sqrt(x)*sqr
t(a + b*x**2 + c*x**4)*b**2*c*x**4 + 117*sqrt(x)*sqrt(a + b*x**2 + c*x**4)
*b*c**2*x**6 - 520*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4)*x**3)/(a + b*x**
2 + c*x**4),x)*a**2*c**2 + 314*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4)*x**3
)/(a + b*x**2 + c*x**4),x)*a*b**2*c - 42*int((sqrt(x)*sqrt(a + b*x**2 + c*
x**4)*x**3)/(a + b*x**2 + c*x**4),x)*b**4 - 104*int((sqrt(x)*sqrt(a + b*x*
*2 + c*x**4))/(a*x + b*x**3 + c*x**5),x)*a**3*c + 10*int((sqrt(x)*sqrt(a +
 b*x**2 + c*x**4))/(a*x + b*x**3 + c*x**5),x)*a**2*b**2))/(1989*b*c)
```

3.1072 $\int \sqrt{dx}(a+bx^2+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}\,\mathrm{AppellF1}\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2/3*a*(d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2)*AppellF1(3/4,-3/2,-3/2,7/4,-2*c*x^
2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 417 vs. 2(148) = 296.

Time = 11.53 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.82

$$\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{2x\sqrt{dx}\Big(7(12ab^2+209a^2c+12b^3x^2+328abcx^2+131b^2cx^4+286ac^2x^4+196bc^2x^6+77c^3x}{}$$

input
```
Integrate[Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*x*Sqrt[d*x]*(7*(12*a*b^2 + 209*a^2*c + 12*b^3*x^2 + 328*a*b*c*x^2 + 131
*b^2*c*x^4 + 286*a*c^2*x^4 + 196*b*c^2*x^6 + 77*c^3*x^8) - 28*a*(3*b^2 - 4
4*a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4,
 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])] + 12*b*(-5*b^2 + 36*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(8085*c*Sqrt[a + b*x^
2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{a\sqrt{a+bx^2+cx^4}\int \sqrt{dx}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*a*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*
d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \sqrt{dx}\,\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}dx$$

input
```
int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{dx}\,dx$$

input
```
integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

Sympy [F]

$$\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x)**(1/2)*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(sqrt(d*x)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{dx}\,dx$$

input
```
integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

Giac [F]

$$\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{dx}\,dx$$

input
```
integrate((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \sqrt{d\,x}\,\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

input
```
int((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{dx}(a+bx^2 +cx^4)^{3/2}\,dx=\frac{2\sqrt{d}\left(209\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,acx+12\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,b^2x+119\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,b\right.}{}$$

input
```
int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(d)*(209*sqrt(x)*sqrt(a + b*x**2 + c*x**4)*a*c*x + 12*sqrt(x)*sqrt(
a + b*x**2 + c*x**4)*b**2*x + 119*sqrt(x)*sqrt(a + b*x**2 + c*x**4)*b*c*x*
*3 + 77*sqrt(x)*sqrt(a + b*x**2 + c*x**4)*c**2*x**5 + 216*int((sqrt(x)*sqr
t(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a*b*c - 30*int((sqrt
(x)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*b**3 + 264*in
t((sqrt(x)*sqrt(a + b*x**2 + c*x**4))/(a + b*x**2 + c*x**4),x)*a**2*c - 18
*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4))/(a + b*x**2 + c*x**4),x)*a*b**2))
/(1155*c)
```

3.1073 $\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 146

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{dx}}\,dx = \frac{2a\sqrt{dx}\sqrt{a+bx^2+cx^4}\,\operatorname{AppellF1}\left(\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
2*a*(d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)*AppellF1(1/4,-3/2,-3/2,5/4,-2*c*x^2/
(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 415 vs. 2(146) = 292.

Time = 11.51 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{dx}}\,dx = \frac{2x\left(5(4ab^2+51a^2c+4b^3x^2+76abcx^2+29b^2cx^4+66ac^2x^4+40bc^2x^6+15c^3x^{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/Sqrt[d*x],x]
```

output
```
(2*x*(5*(4*a*b^2 + 51*a^2*c + 4*b^3*x^2 + 76*a*b*c*x^2 + 29*b^2*c*x^4 + 66
*a*c^2*x^4 + 40*b*c^2*x^6 + 15*c^3*x^8) - 20*a*(b^2 - 36*a*c)*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 4*b
*(3*b^2 - 28*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])]))/(975*c*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{a\sqrt{a+bx^2+cx^4}\int \frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}{\sqrt{dx}}dx}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2a\sqrt{dx}\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/Sqrt[d*x],x]
```

output
```
(2*a*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2, 5/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{(c\,x^{4}+b\,x^{2}+a)^{\frac{3}{2}}}{\sqrt{dx}}dx$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x)
```

output
```
int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{\sqrt{dx}}\,dx=\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)/(d*x), x)
```

Sympy [F]

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{\sqrt{dx}}\,dx=\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/(d*x)**(1/2),x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/sqrt(d*x), x)
```

Maxima [F]

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{\sqrt{dx}}\,dx=\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x), x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{dx}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{dx}}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{\sqrt{d\,x}}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/(d*x)^(1/2),x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{dx}}\,dx = \frac{2\sqrt{d}\left(53\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,a+15\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,b\,x^2+9\sqrt{x}\,\sqrt{c\,x^4+b\,x\right.}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x)
```

output
```
(2*sqrt(d)*(53*sqrt(x)*sqrt(a + b*x**2 + c*x**4)*a + 15*sqrt(x)*sqrt(a + b
*x**2 + c*x**4)*b*x**2 + 9*sqrt(x)*sqrt(a + b*x**2 + c*x**4)*c*x**4 - 56*i
nt((sqrt(x)*sqrt(a + b*x**2 + c*x**4)*x**3)/(a + b*x**2 + c*x**4),x)*a*c +
 6*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4)*x**3)/(a + b*x**2 + c*x**4),x)*b
**2 + 32*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4))/(a*x + b*x**3 + c*x**5),x
)*a**2))/(117*d)
```

3.1074 $\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 146

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx = -\frac{2a\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(-\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
-2*a*(c*x^4+b*x^2+a)^(1/2)*AppellF1(-1/4,-3/2,-3/2,3/4,-2*c*x^2/(b-(-4*a*c
+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(d*x)^(1/2)/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 384 vs. 2(146) = 292.

Time = 11.52 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.63

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx = \frac{x\left(14(-77a^2-64abx^2+13b^2x^4-70acx^4+20bcx^6+7c^2x^8)+896abx^2\sqrt{\frac{b-\sqrt{b^2}}{b-}}\right.}{}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x]
```

output
```
(x*(14*(-77*a^2 - 64*a*b*x^2 + 13*b^2*x^4 - 70*a*c*x^4 + 20*b*c*x^6 + 7*c^
2*x^8) + 896*a*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2
- 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2
)/(-b + Sqrt[b^2 - 4*a*c])] + 24*(b^2 + 28*a*c)*x^4*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(539*(d*x)^(
3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{(dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{a\sqrt{a+bx^2+cx^4}\int \frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}{(dx)^{3/2}}dx}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{394}$$

$$-\frac{2a\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(-\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x]
```

output
```
(-2*a*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}dx$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x)
```

output
```
int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)/(d^2*x^2), x)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/(d*x)**(3/2),x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/(d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/(d*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx=\int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx=\int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{(d\,x)^{3/2}}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(dx)^{3/2}}\,dx=\frac{2\sqrt{d}\left(115\sqrt{c\,x^4+b\,x^2+a}\,a+13\sqrt{c\,x^4+b\,x^2+a}\,b\,x^2+7\sqrt{c\,x^4+b\,x^2+a}\,c\,x^4\right.}{}$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x)
```

output
```
(2*sqrt(d)*(115*sqrt(a + b*x**2 + c*x**4)*a + 13*sqrt(a + b*x**2 + c*x**4)
*b*x**2 + 7*sqrt(a + b*x**2 + c*x**4)*c*x**4 - 120*sqrt(x)*int((sqrt(x)*sq
rt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a*c + 6*sqrt(x)*int
((sqrt(x)*sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*b**2 +
96*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4))/(a*x**2 + b*x**4 + c*x*
*6),x)*a**2))/(77*sqrt(x)*d**2)
```

3.1075 $\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 147

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2(dx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-\cdots}}\right.}{5d\sqrt{a+bx^2+cx^4}}$$

output
```
2/5*(d*x)^(5/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)*AppellF1(5/4,1/2,1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 11.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.18

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2x(dx)^{3/2}\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2c\cdots}{-b+\sqrt{\cdots}}\right.}{5\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[(d*x)^(3/2)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*x*(d*x)^(3/2)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*App
ellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])])/(5*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\int \frac{(dx)^{3/2}}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(d*x)^(3/2)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*d*Sqrt[a + b*x
^2 + c*x^4])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{(dx)^{\frac{3}{2}}}{\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)*d*x/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((d*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d*x)**(3/2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(d\,x)^{3/2}}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d*x)^(3/2)/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d*x)^(3/2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx=\sqrt{d}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,x}{c\,x^4+b\,x^2+a}dx\right)d$$

input
```
int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
sqrt(d)*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4)*x)/(a + b*x**2 + c*x**4),x)
*d
```

3.1076 $\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 147

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{a+bx^2+cx^4}}$$

output
```
2/3*(d*x)^(3/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 11.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{2x\sqrt{dx}\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[Sqrt[d*x]/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*x*Sqrt[d*x]*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\int \frac{\sqrt{dx}}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{a+bx^2+cx^4}}$$

input
```
Int[Sqrt[d*x]/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*d*Sqrt[a + b*x
^2 + c*x^4])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{\sqrt{dx}}{\sqrt{c\,x^{4}+b\,x^{2}+a}}dx$$

input
```
int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{\sqrt{dx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((d*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(d*x)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{\sqrt{dx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{\sqrt{dx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{\sqrt{d\,x}}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d*x)^(1/2)/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d*x)^(1/2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx=\sqrt{d}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)$$

input
```
int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
sqrt(d)*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4))/(a + b*x**2 + c*x**4),x)
```

3.1077 $\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx = \frac{2\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{a+bx^2+cx^4}}$$

output
```
2*(d*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a
*c+b^2)^(1/2)))^(1/2)*AppellF1(1/4,1/2,1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 11.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx = \frac{2x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[1/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(2*x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1
/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
 - 4*a*c])])/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{a+bx^2+cx^4}}$$

input
```
Int[1/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[a + b*x^2 +
 c*x^4])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{dx}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}dx$$

input
```
int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c*d*x^5 + b*d*x^3 + a*d*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(d*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{d\,x}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}}{c\,x^5+b\,x^3+ax}dx\right)}{d}$$

input
```
int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4))/(a*x + b*x**3 + c*x**5),x
))/d
```

3.1078 $\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = -\frac{2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(-\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

output
```
-2*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2)*AppellF1(-1/4,1/2,1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 348 vs. 2(145) = 290.

Time = 11.37 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.40

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \frac{x\left(-42(a+bx^2+cx^4)+14bx^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\right.}{}$$

input
```
Integrate[1/((d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(x*(-42*(a + b*x^2 + c*x^4) + 14*b*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 18*c*x^4*Sqrt[(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*a*(
d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \ 1461$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \ 394$$

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

input
```
Int[1/((d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*Sqrt[d*x]*Sqrt[a + b*x^2
 + c*x^4])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{1}{(dx)^{\frac{3}{2}} \sqrt{c\,x^{4}+b\,x^{2}+a}}dx$$

input
```
int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}\,(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c*d^2*x^6 + b*d^2*x^4 + a*d^2*
x^2), x)
```

Sympy [F]

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/(d*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/((d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}\,(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}\,(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{{\left(d\,x\right)}^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{d}\,\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}}{c\,x^6+b\,x^4+a\,x^2}dx\right)}{d^2}$$

input
```
int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4))/(a*x**2 + b*x**4 + c*x**6
),x))/d**2
```

3.1079 $\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 150

$$\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{2(dx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5ad\sqrt{a+bx^2+cx^4}}$$

output
```
2/5*(d*x)^(5/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)*AppellF1(5/4,3/2,3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/a/d/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 348 vs. 2(150) = 300.

Time = 11.32 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.32

$$\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{d\sqrt{dx}\left(-5(b+2cx^2)+5b\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\mathrm{AppellF1}\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4}\right.\right.}{}$$

input
```
Integrate[(d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(d*Sqrt[d*x]*(-5*(b + 2*c*x^2) + 5*b*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 2*c*x^2*Sqrt[(b - Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*(b^2 - 4
*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\int \frac{(dx)^{3/2}}{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5ad\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*d*Sqrt[a + b
*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{(dx)^{\frac{3}{2}}}{(c\,x^{4}+b\,x^{2}+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)*d*x/(c^2*x^8 + 2*b*c*x^6 + (b^2
 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((d*x)**(3/2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{{\left(d\,x\right)}^{3/2}}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int((d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx=\sqrt{d}\,\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\,x}{c^{2}x^{8}+2bc\,x^{6}+2ac\,x^{4}+b^{2}x^{4}+2ab\,x^{2}+a^{2}}d x \right) d$$

input
```
int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
sqrt(d)*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4)*x)/(a**2 + 2*a*b*x**2 + 2*a
*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*d
```

3.1080 $\int \frac{\sqrt{dx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 150

$$\int \frac{\sqrt{dx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{2(dx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{\ldots}}\right)}{3ad\sqrt{a+bx^2+cx^4}}$$

output
```
2/3*(d*x)^(3/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)*AppellF1(3/4,3/2,3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/a/d/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 367 vs. $2(150) = 300$.

Time = 11.42 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.45

$$\int \frac{\sqrt{dx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{x\sqrt{dx}\left(-21(b^2-2ac+bcx^2)+7(b^2+2ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{A}\ldots\right.}{\ldots}$$

input
```
Integrate[Sqrt[d*x]/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*Sqrt[d*x]*(-21*(b^2 - 2*a*c + b*c*x^2) + 7*(b^2 + 2*a*c)*Sqrt[(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*b*c
*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1
/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])]))/(21*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\int \frac{\sqrt{dx}}{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3ad\sqrt{a+bx^2+cx^4}}$$

input
```
Int[Sqrt[d*x]/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*d*Sqrt[a + b
*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{\sqrt{dx}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{dx}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{dx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2
*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{dx}}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(sqrt(d*x)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{dx}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{dx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{dx}}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{dx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{\sqrt{d\,x}}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((d*x)^(1/2)/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d*x)^(1/2)/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{dx}}{(a+bx^2+cx^4)^{3/2}}\,dx=\sqrt{d}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx\right)$$

input
```
int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
sqrt(d)*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4))/(a**2 + 2*a*b*x**2 + 2*a*c
*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)
```

3.1081 $\int \frac{1}{\sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int \frac{1}{\sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{2\sqrt{dx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{ad\sqrt{a+bx^2+cx^4}}$$

output
```
2*(d*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a
*c+b^2)^(1/2)))^(1/2)*AppellF1(1/4,3/2,3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/a/d/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 366 vs. 2(148) = 296.

Time = 11.41 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.47

$$\int \frac{1}{\sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{x\left(-5(b^2-2ac+bcx^2)-5(b^2-6ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{Ap}\right.}{}$$

input
```
Integrate[1/(Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(x*(-5*(b^2 - 2*a*c + b*c*x^2) - 5*(b^2 - 6*a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + b*c*x^2*Sqrt[(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4,
 (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))
/(5*a*(-b^2 + 4*a*c)*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{\sqrt{dx}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{394}$$

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{a+bx^2+cx^4}}$$

input
```
Int[1/(Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*Sqrt[a + b*x^2
 + c*x^4])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{1}{\sqrt{dx}\,\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c^2*d*x^9 + 2*b*c*d*x^7 + (b^2
 + 2*a*c)*d*x^5 + 2*a*b*d*x^3 + a^2*d*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{\sqrt{dx}\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(1/(sqrt(d*x)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^2+a)^{\frac{3}{2}}\sqrt{dx}}\,dx$$

input
```
integrate(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{\sqrt{d\,x}\,(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int(1/((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/((d*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{dx}\,(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{\sqrt{x}\,a^2+2\sqrt{x}\,ab\,x^2+2\sqrt{x}\,ac\,x^4+\sqrt{x}\,b^2x^4+2\sqrt{x}\,bc\,x^6+\sqrt{x}\,c^2x^8}dx\right)}{d}$$

input
```
int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(a + b*x**2 + c*x**4)/(sqrt(x)*a**2 + 2*sqrt(x)*a*b*x**2
+ 2*sqrt(x)*a*c*x**4 + sqrt(x)*b**2*x**4 + 2*sqrt(x)*b*c*x**6 + sqrt(x)*c*
*2*x**8),x))/d
```

3.1082 $\int \frac{1}{(dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}} \, dx = -\frac{2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

output
```
-2*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2)*AppellF1(-1/4,3/2,3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^2/(b+(-4*a*c+b^2)^(1/2)))/a/d/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 409 vs. 2(148) = 296.

Time = 11.54 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.76

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}} \, dx =$$

$$-\frac{x\left(-7(8a^2c-3b^2x^2(b+cx^2)+a(-2b^2+11bcx^2+10c^2x^4))-7b(b^2-3ac)\,x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4}}{b+\sqrt{b^2}}}\right.}{}$$

input
```
Integrate[1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-1/7*(x*(-7*(8*a^2*c - 3*b^2*x^2*(b + c*x^2) + a*(-2*b^2 + 11*b*c*x^2 + 10
*c^2*x^4)) - 7*b*(b^2 - 3*a*c)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 3*c*(-3*b^2 + 10*a*c)*x^4*Sqrt
[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2,
11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])]))/(a^2*(b^2 - 4*a*c)*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\int \frac{1}{(dx)^{3/2}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{394}$$

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

input
```
Int[1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*Sqrt[d*x]*Sqrt[a + b*x
^2 + c*x^4])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{1}{(dx)^{\frac{3}{2}} (c\,x^4+b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)/(c^2*d^2*x^10 + 2*b*c*d^2*x^8 +
 (b^2 + 2*a*c)*d^2*x^6 + 2*a*b*d^2*x^4 + a^2*d^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{(dx)^{\frac{3}{2}} \left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(1/((d*x)**(3/2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{(d\,x)^{3/2}\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int(1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(dx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{\sqrt{d}\left(-2\sqrt{c\,x^4+b\,x^2+a}-5\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx\right)ac$$

input
```
int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

output

```
(sqrt(d)*( - 2*sqrt(a + b*x**2 + c*x**4) - 5*sqrt(x)*int((sqrt(x)*sqrt(a +
 b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*
c*x**6 + c**2*x**8),x)*a*c - 5*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2 + c*x*
*4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*
x**8),x)*b*c*x**2 - 5*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4)*x**2)
/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*
c**2*x**4 - 3*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4))/(a**2 + 2*a*
b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b - 3*sqrt(
x)*int((sqrt(x)*sqrt(a + b*x**2 + c*x**4))/(a**2 + 2*a*b*x**2 + 2*a*c*x**4
 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*b**2*x**2 - 3*sqrt(x)*int((sqrt(
x)*sqrt(a + b*x**2 + c*x**4))/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4
+ 2*b*c*x**6 + c**2*x**8),x)*b*c*x**4))/(sqrt(x)*a*d**2*(a + b*x**2 + c*x*
*4))
```

3.1083 $\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx = \frac{a^3(dx)^{1+m}}{d(1+m)} + \frac{3a^2b(dx)^{3+m}}{d^3(3+m)} + \frac{3a(b^2+ac)\,(dx)^{5+m}}{d^5(5+m)} + \frac{b(b^2+6ac)\,(dx)^{7+m}}{d^7(7+m)} + \frac{3c(b^2+ac)\,(dx)^{9+m}}{d^9(9+m)} + \frac{3bc^2(dx)^{11+m}}{d^{11}(11+m)} + \frac{c^3(dx)^{13+m}}{d^{13}(13+m)}$$

output
```
a^3*(d*x)^(1+m)/d/(1+m)+3*a^2*b*(d*x)^(3+m)/d^3/(3+m)+3*a*(a*c+b^2)*(d*x)^
(5+m)/d^5/(5+m)+b*(6*a*c+b^2)*(d*x)^(7+m)/d^7/(7+m)+3*c*(a*c+b^2)*(d*x)^(9
+m)/d^9/(9+m)+3*b*c^2*(d*x)^(11+m)/d^11/(11+m)+c^3*(d*x)^(13+m)/d^13/(13+m
)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.71

$$\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx = x(dx)^m \left(\frac{a^3}{1+m}+\frac{3a^2bx^2}{3+m}+\frac{3a(b^2+ac)\,x^4}{5+m} +\frac{b(b^2+6ac)\,x^6}{7+m}+\frac{3c(b^2+ac)\,x^8}{9+m}+\frac{3bc^2x^{10}}{11+m}+\frac{c^3x^{12}}{13+m}\right)$$

input
```
Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```

output
```
x*(d*x)^m*(a^3/(1 + m) + (3*a^2*b*x^2)/(3 + m) + (3*a*(b^2 + a*c)*x^4)/(5
+ m) + (b*(b^2 + 6*a*c)*x^6)/(7 + m) + (3*c*(b^2 + a*c)*x^8)/(9 + m) + (3*
b*c^2*x^10)/(11 + m) + (c^3*x^12)/(13 + m))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx$$

$$\Big\downarrow \text{1433}$$

$$\int \left(a^3(dx)^m+\frac{3a^2b(dx)^{m+2}}{d^2}+\frac{3c\left(ac+b^2\right)(dx)^{m+8}}{d^8}+\frac{b\left(6ac+b^2\right)(dx)^{m+6}}{d^6}+\frac{3a\left(ac+b^2\right)(dx)^{m+4}}{d^4}+\frac{3bc^2(dx}{d^{10}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2b(dx)^{m+3}}{d^3(m+3)}+\frac{3c\left(ac+b^2\right)(dx)^{m+9}}{d^9(m+9)}+\frac{b\left(6ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{3a\left(ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{3bc^2(dx)^{m+11}}{d^{11}(m+11)}+\frac{c^3(dx)^{m+13}}{d^{13}(m+13)}$$

input
```
Int[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```

output
```
(a^3*(d*x)^(1 + m))/(d*(1 + m)) + (3*a^2*b*(d*x)^(3 + m))/(d^3*(3 + m)) +
(3*a*(b^2 + a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (b*(b^2 + 6*a*c)*(d*x)^(7
+ m))/(d^7*(7 + m)) + (3*c*(b^2 + a*c)*(d*x)^(9 + m))/(d^9*(9 + m)) + (3*b
*c^2*(d*x)^(11 + m))/(d^11*(11 + m)) + (c^3*(d*x)^(13 + m))/(d^13*(13 + m)
)
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 781 vs. 2(156) = 312.

Time = 0.30 (sec) , antiderivative size = 782, normalized size of antiderivative = 5.01

method	result
gosper	$\frac{x(c^3m^6x^{12}+36c^3m^5x^{12}+3b\,c^2m^6x^{10}+505c^3m^4x^{12}+114b\,c^2m^5x^{10}+3480c^3m^3x^{12}+3a\,c^2m^6x^8+3b^2c\,m^6x^8+1665b\,c^2m^4x^1}{}$
risch	$\frac{x(c^3m^6x^{12}+36c^3m^5x^{12}+3b\,c^2m^6x^{10}+505c^3m^4x^{12}+114b\,c^2m^5x^{10}+3480c^3m^3x^{12}+3a\,c^2m^6x^8+3b^2c\,m^6x^8+1665b\,c^2m^4x^1}{}$
orering	$\frac{x(c^3m^6x^{12}+36c^3m^5x^{12}+3b\,c^2m^6x^{10}+505c^3m^4x^{12}+114b\,c^2m^5x^{10}+3480c^3m^3x^{12}+3a\,c^2m^6x^8+3b^2c\,m^6x^8+1665b\,c^2m^4x^1}{}$
parallelrisch	Expression too large to display

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(c^3*m^6*x^12+36*c^3*m^5*x^12+3*b*c^2*m^6*x^10+505*c^3*m^4*x^12+114*b*c^
2*m^5*x^10+3480*c^3*m^3*x^12+3*a*c^2*m^6*x^8+3*b^2*c*m^6*x^8+1665*b*c^2*m^
4*x^10+12139*c^3*m^2*x^12+120*a*c^2*m^5*x^8+120*b^2*c*m^5*x^8+11820*b*c^2*
m^3*x^10+19524*c^3*m*x^12+6*a*b*c*m^6*x^6+1839*a*c^2*m^4*x^8+b^3*m^6*x^6+1
839*b^2*c*m^4*x^8+42117*b*c^2*m^2*x^10+10395*c^3*x^12+252*a*b*c*m^5*x^6+13
584*a*c^2*m^3*x^8+42*b^3*m^5*x^6+13584*b^2*c*m^3*x^8+68706*b*c^2*m*x^10+3*
a^2*c*m^6*x^4+3*a*b^2*m^6*x^4+4074*a*b*c*m^4*x^6+49881*a*c^2*m^2*x^8+679*b
^3*m^4*x^6+49881*b^2*c*m^2*x^8+36855*b*c^2*x^10+132*a^2*c*m^5*x^4+132*a*b^
2*m^5*x^4+31752*a*b*c*m^3*x^6+83064*a*c^2*m*x^8+5292*b^3*m^3*x^6+83064*b^2
*c*m*x^8+3*a^2*b*m^6*x^2+2259*a^2*c*m^4*x^4+2259*a*b^2*m^4*x^4+122010*a*b*
c*m^2*x^6+45045*a*c^2*x^8+20335*b^3*m^2*x^6+45045*b^2*c*x^8+138*a^2*b*m^5*
x^2+18840*a^2*c*m^3*x^4+18840*a*b^2*m^3*x^4+209916*a*b*c*m*x^6+34986*b^3*m
*x^6+a^3*m^6+2505*a^2*b*m^4*x^2+77937*a^2*c*m^2*x^4+77937*a*b^2*m^2*x^4+11
5830*a*b*c*x^6+19305*b^3*x^6+48*a^3*m^5+22620*a^2*b*m^3*x^2+142308*a^2*c*m
*x^4+142308*a*b^2*m*x^4+925*a^3*m^4+104277*a^2*b*m^2*x^2+81081*a^2*c*x^4+8
1081*a*b^2*x^4+9120*a^3*m^3+219162*a^2*b*m*x^2+48259*a^3*m^2+135135*a^2*b*
x^2+129072*a^3*m+135135*a^3)*(d*x)^m/(13+m)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)
/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(156) = 312$.

Time = 0.08 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.81

$$\int (dx)^m \left(a + bx^2 + cx^4\right)^3 \, dx$$
$$= \frac{((c^3m^6 + 36\,c^3m^5 + 505\,c^3m^4 + 3480\,c^3m^3 + 12139\,c^3m^2 + 19524\,c^3m + 10395\,c^3)x^{13} + 3\,(bc^2m^6 + 38\,bc}{}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

output
```
((c^3*m^6 + 36*c^3*m^5 + 505*c^3*m^4 + 3480*c^3*m^3 + 12139*c^3*m^2 + 1952
4*c^3*m + 10395*c^3)*x^13 + 3*(b*c^2*m^6 + 38*b*c^2*m^5 + 555*b*c^2*m^4 +
3940*b*c^2*m^3 + 14039*b*c^2*m^2 + 22902*b*c^2*m + 12285*b*c^2)*x^11 + 3*(
(b^2*c + a*c^2)*m^6 + 40*(b^2*c + a*c^2)*m^5 + 613*(b^2*c + a*c^2)*m^4 + 4
528*(b^2*c + a*c^2)*m^3 + 15015*b^2*c + 15015*a*c^2 + 16627*(b^2*c + a*c^2
)*m^2 + 27688*(b^2*c + a*c^2)*m)*x^9 + ((b^3 + 6*a*b*c)*m^6 + 42*(b^3 + 6*
a*b*c)*m^5 + 679*(b^3 + 6*a*b*c)*m^4 + 5292*(b^3 + 6*a*b*c)*m^3 + 19305*b^
3 + 115830*a*b*c + 20335*(b^3 + 6*a*b*c)*m^2 + 34986*(b^3 + 6*a*b*c)*m)*x^
7 + 3*((a*b^2 + a^2*c)*m^6 + 44*(a*b^2 + a^2*c)*m^5 + 753*(a*b^2 + a^2*c)*
m^4 + 6280*(a*b^2 + a^2*c)*m^3 + 27027*a*b^2 + 27027*a^2*c + 25979*(a*b^2
+ a^2*c)*m^2 + 47436*(a*b^2 + a^2*c)*m)*x^5 + 3*(a^2*b*m^6 + 46*a^2*b*m^5
+ 835*a^2*b*m^4 + 7540*a^2*b*m^3 + 34759*a^2*b*m^2 + 73054*a^2*b*m + 45045
*a^2*b)*x^3 + (a^3*m^6 + 48*a^3*m^5 + 925*a^3*m^4 + 9120*a^3*m^3 + 48259*a
^3*m^2 + 129072*a^3*m + 135135*a^3)*x)*(d*x)^m/(m^7 + 49*m^6 + 973*m^5 + 1
0045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4332 vs. $2(143) = 286$.

Time = 0.89 (sec) , antiderivative size = 4332, normalized size of antiderivative = 27.77

$$\int (dx)^m \left(a + bx^2 + cx^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(c*x**4+b*x**2+a)**3,x)
```

output
```
Piecewise(((-a**3/(12*x**12) - 3*a**2*b/(10*x**10) - 3*a**2*c/(8*x**8) - 3
*a*b**2/(8*x**8) - a*b*c/x**6 - 3*a*c**2/(4*x**4) - b**3/(6*x**6) - 3*b**2
*c/(4*x**4) - 3*b*c**2/(2*x**2) + c**3*log(x))/d**13, Eq(m, -13)), ((-a**3
/(10*x**10) - 3*a**2*b/(8*x**8) - a**2*c/(2*x**6) - a*b**2/(2*x**6) - 3*a*
b*c/(2*x**4) - 3*a*c**2/(2*x**2) - b**3/(4*x**4) - 3*b**2*c/(2*x**2) + 3*b
*c**2*log(x) + c**3*x**2/2)/d**11, Eq(m, -11)), ((-a**3/(8*x**8) - a**2*b/
(2*x**6) - 3*a**2*c/(4*x**4) - 3*a*b**2/(4*x**4) - 3*a*b*c/x**2 + 3*a*c**2
*log(x) - b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*x**4/4)
/d**9, Eq(m, -9)), ((-a**3/(6*x**6) - 3*a**2*b/(4*x**4) - 3*a**2*c/(2*x**2
) - 3*a*b**2/(2*x**2) + 6*a*b*c*log(x) + 3*a*c**2*x**2/2 + b**3*log(x) + 3
*b**2*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6)/d**7, Eq(m, -7)), ((-a**3/
(4*x**4) - 3*a**2*b/(2*x**2) + 3*a**2*c*log(x) + 3*a*b**2*log(x) + 3*a*b*c
*x**2 + 3*a*c**2*x**4/4 + b**3*x**2/2 + 3*b**2*c*x**4/4 + b*c**2*x**6/2 +
c**3*x**8/8)/d**5, Eq(m, -5)), ((-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a**2
*c*x**2/2 + 3*a*b**2*x**2/2 + 3*a*b*c*x**4/2 + a*c**2*x**6/2 + b**3*x**4/4
 + b**2*c*x**6/2 + 3*b*c**2*x**8/8 + c**3*x**10/10)/d**3, Eq(m, -3)), ((a*
*3*log(x) + 3*a**2*b*x**2/2 + 3*a**2*c*x**4/4 + 3*a*b**2*x**4/4 + a*b*c*x*
*6 + 3*a*c**2*x**8/8 + b**3*x**6/6 + 3*b**2*c*x**8/8 + 3*b*c**2*x**10/10 +
 c**3*x**12/12)/d, Eq(m, -1)), (a**3*m**6*x*(d*x)**m/(m**7 + 49*m**6 + 973
*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.25

$$\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx = \frac{c^3 d^m x^{13} x^m}{m+13} + \frac{3\,bc^2 d^m x^{11} x^m}{m+11} + \frac{3\,b^2 c d^m x^9 x^m}{m+9} + \frac{3\,ac^2 d^m x^9 x^m}{m+9} + \frac{b^3 d^m x^7 x^m}{m+7} + \frac{6\,abcd^m x^7 x^m}{m+7} + \frac{3\,ab^2 d^m x^5 x^m}{m+5} + \frac{3\,a^2 c d^m x^5 x^m}{m+5} + \frac{3\,a^2 b d^m x^3 x^m}{m+3} + \frac{(dx)^{m+1} a^3}{d(m+1)}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

output
```
c^3*d^m*x^13*x^m/(m + 13) + 3*b*c^2*d^m*x^11*x^m/(m + 11) + 3*b^2*c*d^m*x^
9*x^m/(m + 9) + 3*a*c^2*d^m*x^9*x^m/(m + 9) + b^3*d^m*x^7*x^m/(m + 7) + 6*
a*b*c*d^m*x^7*x^m/(m + 7) + 3*a*b^2*d^m*x^5*x^m/(m + 5) + 3*a^2*c*d^m*x^5*
x^m/(m + 5) + 3*a^2*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a^3/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. $2(156) = 312$.

Time = 0.14 (sec) , antiderivative size = 1132, normalized size of antiderivative = 7.26

$$\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

output
```
((d*x)^m*c^3*m^6*x^13 + 36*(d*x)^m*c^3*m^5*x^13 + 3*(d*x)^m*b*c^2*m^6*x^11
 + 505*(d*x)^m*c^3*m^4*x^13 + 114*(d*x)^m*b*c^2*m^5*x^11 + 3480*(d*x)^m*c^
3*m^3*x^13 + 3*(d*x)^m*b^2*c*m^6*x^9 + 3*(d*x)^m*a*c^2*m^6*x^9 + 1665*(d*x
)^m*b*c^2*m^4*x^11 + 12139*(d*x)^m*c^3*m^2*x^13 + 120*(d*x)^m*b^2*c*m^5*x^
9 + 120*(d*x)^m*a*c^2*m^5*x^9 + 11820*(d*x)^m*b*c^2*m^3*x^11 + 19524*(d*x)
^m*c^3*m*x^13 + (d*x)^m*b^3*m^6*x^7 + 6*(d*x)^m*a*b*c*m^6*x^7 + 1839*(d*x)
^m*b^2*c*m^4*x^9 + 1839*(d*x)^m*a*c^2*m^4*x^9 + 42117*(d*x)^m*b*c^2*m^2*x^
11 + 10395*(d*x)^m*c^3*x^13 + 42*(d*x)^m*b^3*m^5*x^7 + 252*(d*x)^m*a*b*c*m
^5*x^7 + 13584*(d*x)^m*b^2*c*m^3*x^9 + 13584*(d*x)^m*a*c^2*m^3*x^9 + 68706
*(d*x)^m*b*c^2*m*x^11 + 3*(d*x)^m*a*b^2*m^6*x^5 + 3*(d*x)^m*a^2*c*m^6*x^5
+ 679*(d*x)^m*b^3*m^4*x^7 + 4074*(d*x)^m*a*b*c*m^4*x^7 + 49881*(d*x)^m*b^2
*c*m^2*x^9 + 49881*(d*x)^m*a*c^2*m^2*x^9 + 36855*(d*x)^m*b*c^2*x^11 + 132*
(d*x)^m*a*b^2*m^5*x^5 + 132*(d*x)^m*a^2*c*m^5*x^5 + 5292*(d*x)^m*b^3*m^3*x
^7 + 31752*(d*x)^m*a*b*c*m^3*x^7 + 83064*(d*x)^m*b^2*c*m*x^9 + 83064*(d*x)
^m*a*c^2*m*x^9 + 3*(d*x)^m*a^2*b*m^6*x^3 + 2259*(d*x)^m*a*b^2*m^4*x^5 + 22
59*(d*x)^m*a^2*c*m^4*x^5 + 20335*(d*x)^m*b^3*m^2*x^7 + 122010*(d*x)^m*a*b*
c*m^2*x^7 + 45045*(d*x)^m*b^2*c*x^9 + 45045*(d*x)^m*a*c^2*x^9 + 138*(d*x)^
m*a^2*b*m^5*x^3 + 18840*(d*x)^m*a*b^2*m^3*x^5 + 18840*(d*x)^m*a^2*c*m^3*x^
5 + 34986*(d*x)^m*b^3*m*x^7 + 209916*(d*x)^m*a*b*c*m*x^7 + (d*x)^m*a^3*m^6
*x + 2505*(d*x)^m*a^2*b*m^4*x^3 + 77937*(d*x)^m*a*b^2*m^2*x^5 + 77937*(...
```

Mupad [B] (verification not implemented)

Time = 17.94 (sec) , antiderivative size = 546, normalized size of antiderivative = 3.50

$$\int (dx)^m \left(a+bx^2+cx^4\right)^3 \,dx$$
$$=\frac{a^3\,x\,{\left(d\,x\right)}^m\,\left(m^6+48\,m^5+925\,m^4+9120\,m^3+48259\,m^2+129072\,m+135135\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{c^3\,x^{13}\,{\left(d\,x\right)}^m\,\left(m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{3\,a^2\,b\,x^3\,{\left(d\,x\right)}^m\,\left(m^6+46\,m^5+835\,m^4+7540\,m^3+34759\,m^2+73054\,m+45045\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{3\,b\,c^2\,x^{11}\,{\left(d\,x\right)}^m\,\left(m^6+38\,m^5+555\,m^4+3940\,m^3+14039\,m^2+22902\,m+12285\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{3\,a\,x^5\,{\left(d\,x\right)}^m\,\left(b^2+a\,c\right)\,\left(m^6+44\,m^5+753\,m^4+6280\,m^3+25979\,m^2+47436\,m+27027\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{b\,x^7\,{\left(d\,x\right)}^m\,\left(b^2+6\,a\,c\right)\,\left(m^6+42\,m^5+679\,m^4+5292\,m^3+20335\,m^2+34986\,m+19305\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}+\frac{3\,c\,x^9\,{\left(d\,x\right)}^m\,\left(b^2+a\,c\right)\,\left(m^6+40\,m^5+613\,m^4+4528\,m^3+16627\,m^2+27688\,m+15015\right)}{m^7+49\,m^6+973\,m^5+10045\,m^4+57379\,m^3+177331\,m^2+264207\,m+135135}$$

input
```
int((d*x)^m*(a + b*x^2 + c*x^4)^3,x)
```

output
```
(a^3*x*(d*x)^m*(129072*m + 48259*m^2 + 9120*m^3 + 925*m^4 + 48*m^5 + m^6 +
 135135))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^
6 + m^7 + 135135) + (c^3*x^13*(d*x)^m*(19524*m + 12139*m^2 + 3480*m^3 + 50
5*m^4 + 36*m^5 + m^6 + 10395))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*
m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (3*a^2*b*x^3*(d*x)^m*(73054*m + 3
4759*m^2 + 7540*m^3 + 835*m^4 + 46*m^5 + m^6 + 45045))/(264207*m + 177331*
m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (3*b*c^2*
x^11*(d*x)^m*(22902*m + 14039*m^2 + 3940*m^3 + 555*m^4 + 38*m^5 + m^6 + 12
285))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 +
m^7 + 135135) + (3*a*x^5*(d*x)^m*(a*c + b^2)*(47436*m + 25979*m^2 + 6280*m
^3 + 753*m^4 + 44*m^5 + m^6 + 27027))/(264207*m + 177331*m^2 + 57379*m^3 +
 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135) + (b*x^7*(d*x)^m*(6*a*c + b^
2)*(34986*m + 20335*m^2 + 5292*m^3 + 679*m^4 + 42*m^5 + m^6 + 19305))/(264
207*m + 177331*m^2 + 57379*m^3 + 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 1351
35) + (3*c*x^9*(d*x)^m*(a*c + b^2)*(27688*m + 16627*m^2 + 4528*m^3 + 613*m
^4 + 40*m^5 + m^6 + 15015))/(264207*m + 177331*m^2 + 57379*m^3 + 10045*m^4
 + 973*m^5 + 49*m^6 + m^7 + 135135)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 782, normalized size of antiderivative = 5.01

$$\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx$$
$$= \frac{x^m d^m x(c^3m^6x^{12} + 36c^3m^5x^{12} + 3b\,c^2m^6x^{10} + 505c^3m^4x^{12} + 114b\,c^2m^5x^{10} + 3480c^3m^3x^{12} + 3a\,c^2m^6x^8}{}$$

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^3,x)
```

output
```
(x**m*d**m*x*(a**3*m**6 + 48*a**3*m**5 + 925*a**3*m**4 + 9120*a**3*m**3 +
48259*a**3*m**2 + 129072*a**3*m + 135135*a**3 + 3*a**2*b*m**6*x**2 + 138*a
**2*b*m**5*x**2 + 2505*a**2*b*m**4*x**2 + 22620*a**2*b*m**3*x**2 + 104277*
a**2*b*m**2*x**2 + 219162*a**2*b*m*x**2 + 135135*a**2*b*x**2 + 3*a**2*c*m*
*6*x**4 + 132*a**2*c*m**5*x**4 + 2259*a**2*c*m**4*x**4 + 18840*a**2*c*m**3
*x**4 + 77937*a**2*c*m**2*x**4 + 142308*a**2*c*m*x**4 + 81081*a**2*c*x**4
+ 3*a*b**2*m**6*x**4 + 132*a*b**2*m**5*x**4 + 2259*a*b**2*m**4*x**4 + 1884
0*a*b**2*m**3*x**4 + 77937*a*b**2*m**2*x**4 + 142308*a*b**2*m*x**4 + 81081
*a*b**2*x**4 + 6*a*b*c*m**6*x**6 + 252*a*b*c*m**5*x**6 + 4074*a*b*c*m**4*x
**6 + 31752*a*b*c*m**3*x**6 + 122010*a*b*c*m**2*x**6 + 209916*a*b*c*m*x**6
 + 115830*a*b*c*x**6 + 3*a*c**2*m**6*x**8 + 120*a*c**2*m**5*x**8 + 1839*a*
c**2*m**4*x**8 + 13584*a*c**2*m**3*x**8 + 49881*a*c**2*m**2*x**8 + 83064*a
*c**2*m*x**8 + 45045*a*c**2*x**8 + b**3*m**6*x**6 + 42*b**3*m**5*x**6 + 67
9*b**3*m**4*x**6 + 5292*b**3*m**3*x**6 + 20335*b**3*m**2*x**6 + 34986*b**3
*m*x**6 + 19305*b**3*x**6 + 3*b**2*c*m**6*x**8 + 120*b**2*c*m**5*x**8 + 18
39*b**2*c*m**4*x**8 + 13584*b**2*c*m**3*x**8 + 49881*b**2*c*m**2*x**8 + 83
064*b**2*c*m*x**8 + 45045*b**2*c*x**8 + 3*b*c**2*m**6*x**10 + 114*b*c**2*m
**5*x**10 + 1665*b*c**2*m**4*x**10 + 11820*b*c**2*m**3*x**10 + 42117*b*c**
2*m**2*x**10 + 68706*b*c**2*m*x**10 + 36855*b*c**2*x**10 + c**3*m**6*x**12
 + 36*c**3*m**5*x**12 + 505*c**3*m**4*x**12 + 3480*c**3*m**3*x**12 + 12...
```

3.1084 $\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx = \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{3+m}}{d^3(3+m)}+\frac{(b^2+2ac)\,(dx)^{5+m}}{d^5(5+m)} +\frac{2bc(dx)^{7+m}}{d^7(7+m)}+\frac{c^2(dx)^{9+m}}{d^9(9+m)}$$

output
```
a^2*(d*x)^(1+m)/d/(1+m)+2*a*b*(d*x)^(3+m)/d^3/(3+m)+(2*a*c+b^2)*(d*x)^(5+m
)/d^5/(5+m)+2*b*c*(d*x)^(7+m)/d^7/(7+m)+c^2*(d*x)^(9+m)/d^9/(9+m)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx = x(dx)^m \left(\frac{a^2}{1+m}+\frac{2abx^2}{3+m}+\frac{(b^2+2ac)\,x^4}{5+m}+\frac{2bcx^6}{7+m} +\frac{c^2x^8}{9+m}\right)$$

input
```
Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

output
```
x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + ((b^2 + 2*a*c)*x^4)/(5 + m)
 + (2*b*c*x^6)/(7 + m) + (c^2*x^8)/(9 + m))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^2 dx$$

$$\Big\downarrow \text{ 1433}$$

$$\int \left(a^2(dx)^m+\frac{\left(2ac+b^2\right)(dx)^{m+4}}{d^4}+\frac{2ab(dx)^{m+2}}{d^2}+\frac{2bc(dx)^{m+6}}{d^6}+\frac{c^2(dx)^{m+8}}{d^8}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+7}}{d^7(m+7)}+\frac{c^2(dx)^{m+9}}{d^9(m+9)}$$

input
```
Int[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 + m)) + ((
b^2 + 2*a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (2*b*c*(d*x)^(7 + m))/(d^7*(7
+ m)) + (c^2*(d*x)^(9 + m))/(d^9*(9 + m))
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(101) = 202$.

Time = 0.14 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.98

method	result
gosper	$x(c^2m^4x^8+16c^2m^3x^8+2bc\,m^4x^6+86c^2m^2x^8+36bc\,m^3x^6+176m\,x^8c^2+2ac\,m^4x^4+b^2m^4x^4+208bc\,m^2x^6+105c^2x^8+40ac\,m$
risch	$x(c^2m^4x^8+16c^2m^3x^8+2bc\,m^4x^6+86c^2m^2x^8+36bc\,m^3x^6+176m\,x^8c^2+2ac\,m^4x^4+b^2m^4x^4+208bc\,m^2x^6+105c^2x^8+40ac\,m$
orering	$x(c^2m^4x^8+16c^2m^3x^8+2bc\,m^4x^6+86c^2m^2x^8+36bc\,m^3x^6+176m\,x^8c^2+2ac\,m^4x^4+b^2m^4x^4+208bc\,m^2x^6+105c^2x^8+40ac\,m$
parallelrisch	$x^9(dx)^m c^2m^4+16x^9(dx)^m c^2m^3+86x^9(dx)^m c^2m^2+176x^9(dx)^m c^2m+x^5(dx)^m b^2m^4+20x^5(dx)^m b^2m^3+270x^7(dx)^m bc+13$

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(c^2*m^4*x^8+16*c^2*m^3*x^8+2*b*c*m^4*x^6+86*c^2*m^2*x^8+36*b*c*m^3*x^6+
176*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*x^4+208*b*c*m^2*x^6+105*c^2*x^8+40*a*c
*m^3*x^4+20*b^2*m^3*x^4+444*b*c*m*x^6+2*a*b*m^4*x^2+260*a*c*m^2*x^4+130*b^
2*m^2*x^4+270*b*c*x^6+44*a*b*m^3*x^2+600*a*c*m*x^4+300*b^2*m*x^4+a^2*m^4+3
28*a*b*m^2*x^2+378*a*c*x^4+189*b^2*x^4+24*a^2*m^3+916*a*b*m*x^2+206*a^2*m^
2+630*a*b*x^2+744*a^2*m+945*a^2)*(d*x)^m/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(101) = 202.

Time = 0.07 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.39

$$\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx = \frac{((c^2m^4+16\,c^2m^3+86\,c^2m^2+176\,c^2m+105\,c^2)x^9+2\,(bcm^4+18\,bcm^3+104\,bcm^2+222\,bcm+135\,bc}{}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
((c^2*m^4 + 16*c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^9 + 2*(b*c*m^
4 + 18*b*c*m^3 + 104*b*c*m^2 + 222*b*c*m + 135*b*c)*x^7 + ((b^2 + 2*a*c)*m
^4 + 20*(b^2 + 2*a*c)*m^3 + 130*(b^2 + 2*a*c)*m^2 + 189*b^2 + 378*a*c + 30
0*(b^2 + 2*a*c)*m)*x^5 + 2*(a*b*m^4 + 22*a*b*m^3 + 164*a*b*m^2 + 458*a*b*m
 + 315*a*b)*x^3 + (a^2*m^4 + 24*a^2*m^3 + 206*a^2*m^2 + 744*a^2*m + 945*a^
2)*x)*(d*x)^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. 2(90) = 180.

Time = 0.51 (sec) , antiderivative size = 1435, normalized size of antiderivative = 14.21

$$\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(c*x**4+b*x**2+a)**2,x)
```

output
```
Piecewise(((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(4*x**4) -
 b*c/x**2 + c**2*log(x))/d**9, Eq(m, -9)), ((-a**2/(6*x**6) - a*b/(2*x**4)
 - a*c/x**2 - b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2)/d**7, Eq(m, -7))
, ((-a**2/(4*x**4) - a*b/x**2 + 2*a*c*log(x) + b**2*log(x) + b*c*x**2 + c*
*2*x**4/4)/d**5, Eq(m, -5)), ((-a**2/(2*x**2) + 2*a*b*log(x) + a*c*x**2 +
b**2*x**2/2 + b*c*x**4/2 + c**2*x**6/6)/d**3, Eq(m, -3)), ((a**2*log(x) +
a*b*x**2 + a*c*x**4/2 + b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8)/d, Eq(m, -
1)), (a**2*m**4*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
+ 945) + 24*a**2*m**3*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 206*a**2*m**2*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m
**2 + 1689*m + 945) + 744*a**2*m*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 9
50*m**2 + 1689*m + 945) + 945*a**2*x*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 +
 950*m**2 + 1689*m + 945) + 2*a*b*m**4*x**3*(d*x)**m/(m**5 + 25*m**4 + 230
*m**3 + 950*m**2 + 1689*m + 945) + 44*a*b*m**3*x**3*(d*x)**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 328*a*b*m**2*x**3*(d*x)**m/(m**
5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 916*a*b*m*x**3*(d*x)**
m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 630*a*b*x**3*(d*
x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 2*a*c*m**4*x
**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 40*a*
c*m**3*x**5*(d*x)**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx = \frac{c^2d^mx^9x^m}{m+9} + \frac{2\,bcd^mx^7x^m}{m+7} + \frac{b^2d^mx^5x^m}{m+5} + \frac{2\,acd^mx^5x^m}{m+5} + \frac{2\,abd^mx^3x^m}{m+3} + \frac{(dx)^{m+1}\,a^2}{d(m+1)}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
c^2*d^m*x^9*x^m/(m + 9) + 2*b*c*d^m*x^7*x^m/(m + 7) + b^2*d^m*x^5*x^m/(m +
 5) + 2*a*c*d^m*x^5*x^m/(m + 5) + 2*a*b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1
)*a^2/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(101) = 202$.

Time = 0.11 (sec) , antiderivative size = 449, normalized size of antiderivative = 4.45

$$\int (dx)^m \left(a+bx^2+cx^4\right)^2 dx$$
$$= \frac{(dx)^m c^2m^4x^9 + 16 (dx)^m c^2m^3x^9 + 2 (dx)^m bcm^4x^7 + 86 (dx)^m c^2m^2x^9 + 36 (dx)^m bcm^3x^7 + 176 (dx)^m}{}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
((d*x)^m*c^2*m^4*x^9 + 16*(d*x)^m*c^2*m^3*x^9 + 2*(d*x)^m*b*c*m^4*x^7 + 86
*(d*x)^m*c^2*m^2*x^9 + 36*(d*x)^m*b*c*m^3*x^7 + 176*(d*x)^m*c^2*m*x^9 + (d
*x)^m*b^2*m^4*x^5 + 2*(d*x)^m*a*c*m^4*x^5 + 208*(d*x)^m*b*c*m^2*x^7 + 105*
(d*x)^m*c^2*x^9 + 20*(d*x)^m*b^2*m^3*x^5 + 40*(d*x)^m*a*c*m^3*x^5 + 444*(d
*x)^m*b*c*m*x^7 + 2*(d*x)^m*a*b*m^4*x^3 + 130*(d*x)^m*b^2*m^2*x^5 + 260*(d
*x)^m*a*c*m^2*x^5 + 270*(d*x)^m*b*c*x^7 + 44*(d*x)^m*a*b*m^3*x^3 + 300*(d*
x)^m*b^2*m*x^5 + 600*(d*x)^m*a*c*m*x^5 + (d*x)^m*a^2*m^4*x + 328*(d*x)^m*a
*b*m^2*x^3 + 189*(d*x)^m*b^2*x^5 + 378*(d*x)^m*a*c*x^5 + 24*(d*x)^m*a^2*m^
3*x + 916*(d*x)^m*a*b*m*x^3 + 206*(d*x)^m*a^2*m^2*x + 630*(d*x)^m*a*b*x^3
+ 744*(d*x)^m*a^2*m*x + 945*(d*x)^m*a^2*x)/(m^5 + 25*m^4 + 230*m^3 + 950*m
^2 + 1689*m + 945)
```

Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.57

$$\int (dx)^m \left(a+bx^2+cx^4\right)^2 dx = (d\,x)^m \left(\frac{c^2\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{x^5\,\left(b^2+2\,a\,c\right)\,\left(m^4+20\,m^3+130\,m^2+300\,m+189\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{a^2\,x\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{2\,a\,b\,x^3\,\left(m^4+22\,m^3+164\,m^2+458\,m+315\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{2\,b\,c\,x^7\,\left(m^4+18\,m^3+104\,m^2+222\,m+135\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}\right)$$

input
```
int((d*x)^m*(a + b*x^2 + c*x^4)^2,x)
```

output
```
(d*x)^m*((c^2*x^9*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2
 + 230*m^3 + 25*m^4 + m^5 + 945) + (x^5*(2*a*c + b^2)*(300*m + 130*m^2 + 2
0*m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (a
^2*x*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(1689*m + 950*m^2 + 230*m^3 +
 25*m^4 + m^5 + 945) + (2*a*b*x^3*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/
(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (2*b*c*x^7*(222*m + 10
4*m^2 + 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 +
945))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.98

$$\int (dx)^m \left(a + bx^2 + cx^4\right)^2 \, dx$$
$$= \frac{x^m d^m x(c^2m^4x^8 + 16c^2m^3x^8 + 2bc\,m^4x^6 + 86c^2m^2x^8 + 36bc\,m^3x^6 + 176c^2m\,x^8 + 2ac\,m^4x^4 + b^2m^4x^4 +}{}$$

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^2,x)
```

output
```
(x**m*d**m*x*(a**2*m**4 + 24*a**2*m**3 + 206*a**2*m**2 + 744*a**2*m + 945*
a**2 + 2*a*b*m**4*x**2 + 44*a*b*m**3*x**2 + 328*a*b*m**2*x**2 + 916*a*b*m*
x**2 + 630*a*b*x**2 + 2*a*c*m**4*x**4 + 40*a*c*m**3*x**4 + 260*a*c*m**2*x*
*4 + 600*a*c*m*x**4 + 378*a*c*x**4 + b**2*m**4*x**4 + 20*b**2*m**3*x**4 +
130*b**2*m**2*x**4 + 300*b**2*m*x**4 + 189*b**2*x**4 + 2*b*c*m**4*x**6 + 3
6*b*c*m**3*x**6 + 208*b*c*m**2*x**6 + 444*b*c*m*x**6 + 270*b*c*x**6 + c**2
*m**4*x**8 + 16*c**2*m**3*x**8 + 86*c**2*m**2*x**8 + 176*c**2*m*x**8 + 105
*c**2*x**8))/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
```

3.1085 $\int (dx)^m (a+bx^2+cx^4) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int (dx)^m (a+bx^2+cx^4) \, dx = \frac{a(dx)^{1+m}}{d(1+m)} + \frac{b(dx)^{3+m}}{d^3(3+m)} + \frac{c(dx)^{5+m}}{d^5(5+m)}$$

output
```
a*(d*x)^(1+m)/d/(1+m)+b*(d*x)^(3+m)/d^3/(3+m)+c*(d*x)^(5+m)/d^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int (dx)^m (a+bx^2+cx^4) \, dx = x(dx)^m \left(\frac{a}{1+m} + \frac{bx^2}{3+m} + \frac{cx^4}{5+m}\right)$$

input
```
Integrate[(d*x)^m*(a + b*x^2 + c*x^4),x]
```

output
```
x*(d*x)^m*(a/(1 + m) + (b*x^2)/(3 + m) + (c*x^4)/(5 + m))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^2+cx^4\right) \, dx$$
$$\Big\downarrow \text{ 1433}$$
$$\int \left(a(dx)^m+\frac{b(dx)^{m+2}}{d^2}+\frac{c(dx)^{m+4}}{d^4}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+3}}{d^3(m+3)}+\frac{c(dx)^{m+5}}{d^5(m+5)}$$

input
```
Int[(d*x)^m*(a + b*x^2 + c*x^4),x]
```

output
```
(a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(3 + m))/(d^3*(3 + m)) + (c*(d*x)
^(5 + m))/(d^5*(5 + m))
```

Defintions of rubi rules used

rule 1433
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] ||  !IntegerQ[(m + 1)/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

method	result
norman	$\frac{ax\,\mathrm{e}^{m\ln(dx)}}{1+m}+\frac{b\,x^{3}\mathrm{e}^{m\ln(dx)}}{3+m}+\frac{c\,x^{5}\mathrm{e}^{m\ln(dx)}}{5+m}$
gosper	$\frac{x\left(c\,m^{2}x^{4}+4cm\,x^{4}+b\,m^{2}x^{2}+3c\,x^{4}+6bm\,x^{2}+a\,m^{2}+5b\,x^{2}+8am+15a\right)(dx)^{m}}{(5+m)(3+m)(1+m)}$
risch	$\frac{x\left(c\,m^{2}x^{4}+4cm\,x^{4}+b\,m^{2}x^{2}+3c\,x^{4}+6bm\,x^{2}+a\,m^{2}+5b\,x^{2}+8am+15a\right)(dx)^{m}}{(5+m)(3+m)(1+m)}$
orering	$\frac{x\left(c\,m^{2}x^{4}+4cm\,x^{4}+b\,m^{2}x^{2}+3c\,x^{4}+6bm\,x^{2}+a\,m^{2}+5b\,x^{2}+8am+15a\right)(dx)^{m}}{(5+m)(3+m)(1+m)}$
parallelrisch	$\frac{x^{5}(dx)^{m}c\,m^{2}+4x^{5}(dx)^{m}cm+3x^{5}(dx)^{m}c+x^{3}(dx)^{m}b\,m^{2}+6x^{3}(dx)^{m}bm+5x^{3}(dx)^{m}b+x(dx)^{m}a\,m^{2}+8x(dx)^{m}am+15x(dx)^{m}}{(5+m)(3+m)(1+m)}$

input
```
int((d*x)^m*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
a/(1+m)*x*exp(m*ln(d*x))+b/(3+m)*x^3*exp(m*ln(d*x))+c/(5+m)*x^5*exp(m*ln(d
*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int (dx)^m\left(a+bx^2+cx^4\right)\,dx = \frac{((cm^2+4\,cm+3\,c)x^5+(bm^2+6\,bm+5\,b)x^3+(am^2+8\,am+15\,a)x)(dx)^m}{m^3+9\,m^2+23\,m+15}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
((c*m^2 + 4*c*m + 3*c)*x^5 + (b*m^2 + 6*b*m + 5*b)*x^3 + (a*m^2 + 8*a*m +
15*a)*x)*(d*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(42) = 84$.

Time = 0.28 (sec) , antiderivative size = 299, normalized size of antiderivative = 5.75

$$\int (dx)^m \left(a + bx^2 + cx^4\right) \, dx$$
$$= \begin{cases} \frac{-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)}{d^5} \\ \frac{-\frac{a}{2x^2} + b\log(x) + \frac{cx^2}{2}}{d^3} \\ \frac{a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}}{d} \\ \frac{am^2x(dx)^m}{m^3+9m^2+23m+15} + \frac{8amx(dx)^m}{m^3+9m^2+23m+15} + \frac{15ax(dx)^m}{m^3+9m^2+23m+15} + \frac{bm^2x^3(dx)^m}{m^3+9m^2+23m+15} + \frac{6bmx^3(dx)^m}{m^3+9m^2+23m+15} + \frac{5bx^3(dx)^m}{m^3+9m^2+23m+} \end{cases}$$

input
```
integrate((d*x)**m*(c*x**4+b*x**2+a),x)
```

output
```
Piecewise(((-a/(4*x**4) - b/(2*x**2) + c*log(x))/d**5, Eq(m, -5)), ((-a/(2
*x**2) + b*log(x) + c*x**2/2)/d**3, Eq(m, -3)), ((a*log(x) + b*x**2/2 + c*
x**4/4)/d, Eq(m, -1)), (a*m**2*x*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 8*
a*m*x*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a*x*(d*x)**m/(m**3 + 9*m**
2 + 23*m + 15) + b*m**2*x**3*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 6*b*m*
x**3*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 5*b*x**3*(d*x)**m/(m**3 + 9*m*
*2 + 23*m + 15) + c*m**2*x**5*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 4*c*m
*x**5*(d*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 3*c*x**5*(d*x)**m/(m**3 + 9*m
**2 + 23*m + 15), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int (dx)^m \left(a + bx^2 + cx^4\right) \, dx = \frac{cd^m x^5 x^m}{m+5} + \frac{bd^m x^3 x^m}{m+3} + \frac{(dx)^{m+1} a}{d(m+1)}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
c*d^m*x^5*x^m/(m + 5) + b*d^m*x^3*x^m/(m + 3) + (d*x)^(m + 1)*a/(d*(m + 1)
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(52) = 104$.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.29

$$\int (dx)^m \left(a + bx^2 + cx^4\right) dx = \frac{(dx)^m cm^2x^5 + 4 (dx)^m cmx^5 + (dx)^m bm^2x^3 + 3 (dx)^m cx^5 + 6 (dx)^m bmx^3 + (dx)^m am^2x + 5 (dx)^m bx^3}{m^3 + 9 m^2 + 23 m + 15}$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
((d*x)^m*c*m^2*x^5 + 4*(d*x)^m*c*m*x^5 + (d*x)^m*b*m^2*x^3 + 3*(d*x)^m*c*x
^5 + 6*(d*x)^m*b*m*x^3 + (d*x)^m*a*m^2*x + 5*(d*x)^m*b*x^3 + 8*(d*x)^m*a*m
*x + 15*(d*x)^m*a*x)/(m^3 + 9*m^2 + 23*m + 15)
```

Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.71

$$\int (dx)^m \left(a + bx^2 + cx^4\right) dx = (d\,x)^m \left(\frac{b\,x^3\,(m^2 + 6\,m + 5)}{m^3 + 9\,m^2 + 23\,m + 15} + \frac{c\,x^5\,(m^2 + 4\,m + 3)}{m^3 + 9\,m^2 + 23\,m + 15} + \frac{a\,x\,(m^2 + 8\,m + 15)}{m^3 + 9\,m^2 + 23\,m + 15}\right)$$

input
```
int((d*x)^m*(a + b*x^2 + c*x^4),x)
```

output
```
(d*x)^m*((b*x^3*(6*m + m^2 + 5))/(23*m + 9*m^2 + m^3 + 15) + (c*x^5*(4*m +
 m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (a*x*(8*m + m^2 + 15))/(23*m + 9*m^
2 + m^3 + 15))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int (dx)^m \left(a+bx^2+cx^4\right) \, dx = \frac{x^m d^m x(c\,m^2x^4+4cm\,x^4+b\,m^2x^2+3c\,x^4+6bm\,x^2+a\,m^2+5b\,x^2+8am+15a)}{m^3+9m^2+23m+15}$$

input
```
int((d*x)^m*(c*x^4+b*x^2+a),x)
```

output
```
(x**m*d**m*x*(a*m**2 + 8*a*m + 15*a + b*m**2*x**2 + 6*b*m*x**2 + 5*b*x**2
+ c*m**2*x**4 + 4*c*m*x**4 + 3*c*x**4))/(m**3 + 9*m**2 + 23*m + 15)
```

3.1086 $\int \frac{(dx)^m}{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 173

$$\int \frac{(dx)^m}{a+bx^2+cx^4} \, dx = \frac{2c(dx)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

output
```
2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b
^2)^(1/2)))/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))/d/(1+m)-2*c*(d*x)^(1
+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/
(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/d/(1+m)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.47

$$\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx = \frac{(dx)^m\text{RootSum}\left[a+b\#1^2+c\#1^4\&,\frac{\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1+2c\#1^3}\&\right]}{2m}$$

input
```
Integrate[(d*x)^m/(a + b*x^2 + c*x^4),x]
```

output
```
((d*x)^m*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric2F1[-m, -m, 1 - m,
-(#1/(x - #1))]/((x/(x - #1))^m*(b*#1 + 2*c*#1^3)) & ])/(2*m)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1451, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \; 1451$$

$$\frac{c\int \frac{2(dx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{2(dx)^m}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 27$$

$$\frac{2c\int \frac{(dx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{(dx)^m}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 278$$

$$\frac{2c(dx)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

input
```
Int[(d*x)^m/(a + b*x^2 + c*x^4),x]
```

output
```
(2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 +
 m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])
*d*(1 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1451
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[c/q   Int[(d*x)^m/(b/2 - q/2 + c*x^2), x]
, x] - Simp[c/q   Int[(d*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b,
c, d, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{(dx)^m}{c\,x^4+b\,x^2+a}dx$$

input
```
int((d*x)^m/(c*x^4+b*x^2+a),x)
```

output
```
int((d*x)^m/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx=\int \frac{(dx)^m}{cx^4+bx^2+a}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx=\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx$$

input
```
integrate((d*x)**m/(c*x**4+b*x**2+a),x)
```

output
```
Integral((d*x)**m/(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx=\int \frac{(dx)^m}{cx^4+bx^2+a}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx=\int \frac{(dx)^m}{cx^4+bx^2+a}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx=\int \frac{(d\,x)^m}{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((d*x)^m/(a + b*x^2 + c*x^4),x)
```

output
```
int((d*x)^m/(a + b*x^2 + c*x^4), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx = d^m\left(\int \frac{x^m}{c\,x^4+b\,x^2+a}dx\right)$$

input
```
int((d*x)^m/(c*x^4+b*x^2+a),x)
```

output
```
d**m*int(x**m/(a + b*x**2 + c*x**4),x)
```

3.1087 $\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 315

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx = \frac{(dx)^{1+m}(b^2-2ac+bcx^2)}{2a(b^2-4ac)\,d\,(a+bx^2+cx^4)} + \frac{c\left(b^2(1-m)+b\sqrt{b^2-4ac}(1-m)-4ac(3-m)\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2a(b^2-4ac)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{c\left(b^2(1-m)-b\sqrt{b^2-4ac}(1-m)-4ac(3-m)\right)(dx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2a(b^2-4ac)^{3/2}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

output
```
1/2*(d*x)^(1+m)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^4+b*x^2+a)+1/2*c
*(b^2*(1-m)+b*(-4*a*c+b^2)^(1/2)*(1-m)-4*a*c*(3-m))*(d*x)^(1+m)*hypergeom(
[1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)
^(3/2)/(b-(-4*a*c+b^2)^(1/2))/d/(1+m)-1/2*c*(b^2*(1-m)-b*(-4*a*c+b^2)^(1/2
)*(1-m)-4*a*c*(3-m))*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c
*x^2/(b+(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))/d
/(1+m)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.25

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx = \frac{x(dx)^m \operatorname{AppellF1}\left(\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{a^2(1+m)}$$

input
```
Integrate[(d*x)^m/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*(d*x)^m*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(a^2*(1 + m))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1441, 25, 1608, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 1441}$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^2\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{(dx)^m\left((1-m)b^2+c(1-m)x^2b-2ac(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(dx)^m\left((1-m)b^2+c(1-m)x^2b-2ac(3-m)\right)}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^2\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 1608}$$

$$\frac{\frac{c\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)\int \frac{2(dx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}-\frac{c\left(-b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)\int \frac{2(dx)^m}{2cx^2+b+\sqrt{b^2-4ac}}dx}{2\sqrt{b^2-4ac}}}{2a\left(b^2-4ac\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^2\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{c\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)\int \frac{(dx)^m}{2cx^2+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{c\left(-b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)\int \frac{(dx)^m}{2cx^2+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}}{2a\left(b^2-4ac\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^2\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \ 278$$

$$\frac{\frac{c(dx)^{m+1}\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(1-m)\sqrt{b^2-4ac}-4ac\right.}{}}{2a\left(b^2-4ac\right)}$$

$$\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^2\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(d*x)^m/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 +
c*x^4)) + ((c*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m))*
(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - S
qrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m))
- (c*(b^2*(1 - m) - b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m))*(d*x)^(1
+ m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m)))/(2*a*(b^
2 - 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 1441
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))
  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

rule 1608
```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d -
b*e)/(2*q))   Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d
- b*e)/(2*q))   Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \frac{(dx)^m}{(c\,x^4 + b\,x^2 + a)^2}dx$$

input
```
int((d*x)^m/(c*x^4+b*x^2+a)^2,x)
```

output
```
int((d*x)^m/(c*x^4+b*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(dx)^m}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^
2), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx$$

input
```
integrate((d*x)**m/(c*x**4+b*x**2+a)**2,x)
```

output
```
Integral((d*x)**m/(a + b*x**2 + c*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(dx)^m}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx=\int \frac{(dx)^m}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx=\int \frac{(d\,x)^m}{(c\,x^4+b\,x^2+a)^2}\,dx$$

input
```
int((d*x)^m/(a + b*x^2 + c*x^4)^2,x)
```

output
```
int((d*x)^m/(a + b*x^2 + c*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx=d^m\left(\int \frac{x^m}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx\right)$$

input
```
int((d*x)^m/(c*x^4+b*x^2+a)^2,x)
```

output
```
d**m*int(x**m/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c
**2*x**8),x)
```

3.1088 $\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 158

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} \, dx = \frac{a(dx)^{1+m}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{3}{2},-\frac{3}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
a*(d*x)^(1+m)*(c*x^4+b*x^2+a)^(1/2)*AppellF1(1/2+1/2*m,-3/2,-3/2,3/2+1/2*m
,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/
(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 357 vs. 2(158) = 316.

Time = 1.74 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.26

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} \, dx = \frac{x(dx)^m\sqrt{a+bx^2+cx^4}\Big(a(15+8m+m^2)\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx}{-b+\sqrt{b^2}}\right.}{}$$

input
```
Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(d*x)^m*Sqrt[a + b*x^2 + c*x^4]*(a*(15 + 8*m + m^2)*AppellF1[(1 + m)/2,
 -1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])] + (1 + m)*x^2*(b*(5 + m)*AppellF1[(3 + m)/2, -1/2, -
1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])] + c*(3 + m)*x^2*AppellF1[(5 + m)/2, -1/2, -1/2, (7 + m)/2, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))/
((1 + m)*(3 + m)*(5 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{a\sqrt{a+bx^2+cx^4}\int (dx)^m \left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}dx}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{394}$$

$$\frac{a(dx)^{m+1}\sqrt{a+bx^2+cx^4}\operatorname{AppellF1}\left(\frac{m+1}{2},-\frac{3}{2},-\frac{3}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(a*(d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2, -3/2, (
3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int (dx)^m \left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}dx$$

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} (dx)^m dx$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} dx = \int (dx)^m \left(a+bx^2+cx^4\right)^{\frac{3}{2}} dx$$

input
```
integrate((d*x)**m*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((d*x)**m*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} (dx)^m dx$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} (dx)^m dx$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} dx = \int (d\,x)^m \left(c\,x^4+b\,x^2+a\right)^{3/2} dx$$

input
```
int((d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d*x)^m*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} dx = \text{too large to display}$$

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(d**m*(x**m*sqrt(a + b*x**2 + c*x**4)*a*c*m**2*x + 14*x**m*sqrt(a + b*x**2
 + c*x**4)*a*c*m*x + 45*x**m*sqrt(a + b*x**2 + c*x**4)*a*c*x + 3*x**m*sqrt
(a + b*x**2 + c*x**4)*b**2*x + x**m*sqrt(a + b*x**2 + c*x**4)*b*c*m**2*x**
3 + 11*x**m*sqrt(a + b*x**2 + c*x**4)*b*c*m*x**3 + 24*x**m*sqrt(a + b*x**2
 + c*x**4)*b*c*x**3 + x**m*sqrt(a + b*x**2 + c*x**4)*c**2*m**2*x**5 + 8*x*
*m*sqrt(a + b*x**2 + c*x**4)*c**2*m*x**5 + 15*x**m*sqrt(a + b*x**2 + c*x**
4)*c**2*x**5 + 12*int((x**m*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*m**3 + 15*a
*m**2 + 71*a*m + 105*a + b*m**3*x**2 + 15*b*m**2*x**2 + 71*b*m*x**2 + 105*
b*x**2 + c*m**3*x**4 + 15*c*m**2*x**4 + 71*c*m*x**4 + 105*c*x**4),x)*a*b*c
*m**4 + 228*int((x**m*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*m**3 + 15*a*m**2
+ 71*a*m + 105*a + b*m**3*x**2 + 15*b*m**2*x**2 + 71*b*m*x**2 + 105*b*x**2
 + c*m**3*x**4 + 15*c*m**2*x**4 + 71*c*m*x**4 + 105*c*x**4),x)*a*b*c*m**3
+ 1572*int((x**m*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*m**3 + 15*a*m**2 + 71*
a*m + 105*a + b*m**3*x**2 + 15*b*m**2*x**2 + 71*b*m*x**2 + 105*b*x**2 + c*
m**3*x**4 + 15*c*m**2*x**4 + 71*c*m*x**4 + 105*c*x**4),x)*a*b*c*m**2 + 466
8*int((x**m*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*m**3 + 15*a*m**2 + 71*a*m +
 105*a + b*m**3*x**2 + 15*b*m**2*x**2 + 71*b*m*x**2 + 105*b*x**2 + c*m**3*
x**4 + 15*c*m**2*x**4 + 71*c*m*x**4 + 105*c*x**4),x)*a*b*c*m + 5040*int((x
**m*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*m**3 + 15*a*m**2 + 71*a*m + 105*a +
 b*m**3*x**2 + 15*b*m**2*x**2 + 71*b*m*x**2 + 105*b*x**2 + c*m**3*x**4 ...
```

3.1089 $\int (dx)^m\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int (dx)^m\sqrt{a+bx^2+cx^4}\,dx = \frac{(dx)^{1+m}\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

output
```
(d*x)^(1+m)*(c*x^4+b*x^2+a)^(1/2)*AppellF1(1/2+1/2*m,-1/2,-1/2,3/2+1/2*m,-
2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/(1
+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^
(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.56 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int (dx)^m\sqrt{a+bx^2+cx^4}\,dx = \frac{x(dx)^m\sqrt{a+bx^2+cx^4}\,\text{AppellF1}\left(\frac{1+m}{2},-\frac{1}{2},-\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}}$$

input
```
Integrate[(d*x)^m*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(x*(d*x)^m*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3 + m)
/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])
])/((1 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \sqrt{a+bx^2+cx^4} \, dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{a+bx^2+cx^4} \int (dx)^m \sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1} \sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1} dx}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1} \sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

$$\Big\downarrow \text{394}$$

$$\frac{(dx)^{m+1} \sqrt{a+bx^2+cx^4} \operatorname{AppellF1}\left(\frac{m+1}{2},-\frac{1}{2},-\frac{1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1) \sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1} \sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

input
```
Int[(d*x)^m*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3
+ m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int (dx)^m \sqrt{c\,x^4+b\,x^2+a}dx$$

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (dx)^m\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m\sqrt{a+bx^2+cx^4}\,dx=\int (dx)^m\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((d*x)**m*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d*x)**m*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int (dx)^m\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}(dx)^m\,dx$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \sqrt{a+bx^2+cx^4}\, dx = \int (d\,x)^m\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((d*x)^m*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d*x)^m*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int (dx)^m \sqrt{a+bx^2+cx^4}\, dx$$
$$= \frac{d^m \left(x^m \sqrt{c\,x^4+b\,x^2+a}\,x + \left(\int \frac{x^m \sqrt{c\,x^4+b\,x^2+a}\,x^2}{cm\,x^4+3c\,x^4+bm\,x^2+3b\,x^2+am+3a}dx\right) bm + 3\left(\int \frac{x^m \sqrt{c\,x^4+b\,x^2+a}\,x^2}{cm\,x^4+3c\,x^4+bm\,x^2+3b\,x^2+am+3a}dx\right.\right.}{m+3}$$

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(d**m*(x**m*sqrt(a + b*x**2 + c*x**4)*x + int((x**m*sqrt(a + b*x**2 + c*x*
*4)*x**2)/(a*m + 3*a + b*m*x**2 + 3*b*x**2 + c*m*x**4 + 3*c*x**4),x)*b*m +
 3*int((x**m*sqrt(a + b*x**2 + c*x**4)*x**2)/(a*m + 3*a + b*m*x**2 + 3*b*x
**2 + c*m*x**4 + 3*c*x**4),x)*b + 2*int((x**m*sqrt(a + b*x**2 + c*x**4))/(
a*m + 3*a + b*m*x**2 + 3*b*x**2 + c*m*x**4 + 3*c*x**4),x)*a*m + 6*int((x**
m*sqrt(a + b*x**2 + c*x**4))/(a*m + 3*a + b*m*x**2 + 3*b*x**2 + c*m*x**4 +
 3*c*x**4),x)*a))/(m + 3)
```

3.1090 $\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1+m}{2},\frac{1}{2},\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^2+cx^4}}$$

output
```
(d*x)^(1+m)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c
+b^2)^(1/2)))^(1/2)*AppellF1(1/2+1/2*m,1/2,1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/(c*x^4+b*x^2+a)^(1
/2)
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{x(dx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\operatorname{AppellF1}\left(\frac{1+m}{2},\frac{1}{2},\frac{1}{2},\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[(d*x)^m/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[
(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^2)/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\int \frac{(dx)^m}{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1}}dx}{\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{394}$$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,\text{AppellF1}\left(\frac{m+1}{2},\frac{1}{2},\frac{1}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(d*x)^m/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1
 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{(dx)^m}{\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(dx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((d*x)**m/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d*x)**m/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(dx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(d\,x)^m}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d*x)^m/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d*x)^m/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx=d^m\left(\int \frac{x^m\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)$$

input
```
int((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
d**m*int((x**m*sqrt(a + b*x**2 + c*x**4))/(a + b*x**2 + c*x**4),x)
```

3.1091 $\int \frac{(dx)^m}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{(dx)^m}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{(dx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\text{AppellF1}\left(\frac{1+m}{2},\frac{3}{2},\frac{3}{2},\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\right.}{ad(1+m)\sqrt{a+bx^2+cx^4}}$$

output
```
(d*x)^(1+m)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c
+b^2)^(1/2)))^(1/2)*AppellF1(1/2+1/2*m,3/2,3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/a/d/(1+m)/(c*x^4+b*x^2+a)^
(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 10.87 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{x(dx)^m\left(-b+\sqrt{b^2-4ac}-2cx^2\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}\text{AppellF}}{\left(-b+\sqrt{b^2-4ac}\right)(1+m)\left(a+bx^2+cx\right.}$$

input
```
Integrate[(d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(d*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]))^(3/2)*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((-b +
 Sqrt[b^2 - 4*a*c])*(1 + m)*(a + b*x^2 + c*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1461}$$

$$\frac{\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\int \frac{(dx)^m}{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{3/2}\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^{3/2}}dx}{a\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{394}$$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\operatorname{AppellF1}\left(\frac{m+1}{2},\frac{3}{2},\frac{3}{2},\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*
(1 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{(dx)^{m}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^{3/2}}\,dx=\int \frac{(dx)^{m}}{(cx^{4}+bx^{2}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a
*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(dx)^m}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**m/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((d*x)**m/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(dx)^m}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(d\,x)^m}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d*x)^m/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x)
```

output

```
(d**m*(x**m*sqrt(a + b*x**2 + c*x**4)*b*m*x + 2*x**m*sqrt(a + b*x**2 + c*x
**4)*b*x - x**m*sqrt(a + b*x**2 + c*x**4)*c*m*x**3 + x**m*sqrt(a + b*x**2
+ c*x**4)*c*x**3 + int((x**m*sqrt(a + b*x**2 + c*x**4)*x**6)/(a**2*m**2 +
3*a**2*m + 2*a**2 + 2*a*b*m**2*x**2 + 6*a*b*m*x**2 + 4*a*b*x**2 + 2*a*c*m*
*2*x**4 + 6*a*c*m*x**4 + 4*a*c*x**4 + b**2*m**2*x**4 + 3*b**2*m*x**4 + 2*b
**2*x**4 + 2*b*c*m**2*x**6 + 6*b*c*m*x**6 + 4*b*c*x**6 + c**2*m**2*x**8 +
3*c**2*m*x**8 + 2*c**2*x**8),x)*a*c**2*m**4 + 3*int((x**m*sqrt(a + b*x**2
+ c*x**4)*x**6)/(a**2*m**2 + 3*a**2*m + 2*a**2 + 2*a*b*m**2*x**2 + 6*a*b*m
*x**2 + 4*a*b*x**2 + 2*a*c*m**2*x**4 + 6*a*c*m*x**4 + 4*a*c*x**4 + b**2*m*
*2*x**4 + 3*b**2*m*x**4 + 2*b**2*x**4 + 2*b*c*m**2*x**6 + 6*b*c*m*x**6 + 4
*b*c*x**6 + c**2*m**2*x**8 + 3*c**2*m*x**8 + 2*c**2*x**8),x)*a*c**2*m**3 +
 int((x**m*sqrt(a + b*x**2 + c*x**4)*x**6)/(a**2*m**2 + 3*a**2*m + 2*a**2
+ 2*a*b*m**2*x**2 + 6*a*b*m*x**2 + 4*a*b*x**2 + 2*a*c*m**2*x**4 + 6*a*c*m*
x**4 + 4*a*c*x**4 + b**2*m**2*x**4 + 3*b**2*m*x**4 + 2*b**2*x**4 + 2*b*c*m
**2*x**6 + 6*b*c*m*x**6 + 4*b*c*x**6 + c**2*m**2*x**8 + 3*c**2*m*x**8 + 2*
c**2*x**8),x)*a*c**2*m**2 - 3*int((x**m*sqrt(a + b*x**2 + c*x**4)*x**6)/(a
**2*m**2 + 3*a**2*m + 2*a**2 + 2*a*b*m**2*x**2 + 6*a*b*m*x**2 + 4*a*b*x**2
 + 2*a*c*m**2*x**4 + 6*a*c*m*x**4 + 4*a*c*x**4 + b**2*m**2*x**4 + 3*b**2*m
*x**4 + 2*b**2*x**4 + 2*b*c*m**2*x**6 + 6*b*c*m*x**6 + 4*b*c*x**6 + c**2*m
**2*x**8 + 3*c**2*m*x**8 + 2*c**2*x**8),x)*a*c**2*m - 2*int((x**m*sqrt(...
```

3.1092 $\int x^7(a+bx^2+cx^4)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 257

$$\int x^7\left(a+bx^2+cx^4\right)^p\,dx=\frac{x^4(a+bx^2+cx^4)^{1+p}}{4c(2+p)}+\frac{(b^2(2+p)(3+p)-2ac(3+2p)-2bc(1+p)(3+p)x^2)\left(a+bx^2+cx^4\right)^{1+p}}{8c^3(1+p)(2+p)(3+2p)}-\frac{2^{-2+p}b(6ac-b^2(3+p))\left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^2+cx^4\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2\right.}{c^3\sqrt{b^2-4ac}(1+p)(3+2p)}$$

output
```
1/4*x^4*(c*x^4+b*x^2+a)^(p+1)/c/(2+p)+1/8*(b^2*(2+p)*(3+p)-2*a*c*(3+2*p)-2
*b*c*(p+1)*(3+p)*x^2)*(c*x^4+b*x^2+a)^(p+1)/c^3/(p+1)/(2+p)/(3+2*p)-2^(-2+
p)*b*(6*a*c-b^2*(3+p))*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^2)/(-4*a*c+b^2)^(1/2)
)^(-1-p)*(c*x^4+b*x^2+a)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+(-4*a*c+b^
2)^(1/2)+2*c*x^2)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)/(p+1)/(3+2*p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.47 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.63

$$\int x^7(a+bx^2+cx^4)^p\,dx$$
$$=\frac{1}{8}x^8\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\,\text{AppellF1}\left(4,-p,-p,5,-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^7*(a + b*x^2 + c*x^4)^p,x]
```

output
```
(x^8*(a + b*x^2 + c*x^4)^p*AppellF1[4, -p, -p, 5, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(8*((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1434, 1166, 25, 1225, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7(a+bx^2+cx^4)^p\,dx$$
$$\Big\downarrow \text{1434}$$
$$\frac{1}{2}\int x^6(cx^4+bx^2+a)^p\,dx^2$$
$$\Big\downarrow \text{1166}$$

$$\frac{1}{2}\left(\frac{\int -x^2\left(b(p+3)x^2+2a\right)\left(cx^4+bx^2+a\right)^p dx^2}{2c(p+2)}+\frac{x^4\left(a+bx^2+cx^4\right)^{p+1}}{2c(p+2)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{x^4\left(a+bx^2+cx^4\right)^{p+1}}{2c(p+2)}-\frac{\int x^2\left(b(p+3)x^2+2a\right)\left(cx^4+bx^2+a\right)^p dx^2}{2c(p+2)}\right)$$

$$\Big\downarrow \; 1225$$

$$\frac{1}{2}\left(\frac{x^4\left(a+bx^2+cx^4\right)^{p+1}}{2c(p+2)}-\frac{-\frac{b(p+2)\left(6ac-b^2(p+3)\right)\int \left(cx^4+bx^2+a\right)^p dx^2}{2c^2(2p+3)}-\frac{\left(-2ac(2p+3)+b^2(p+2)(p+3)-2bc(p+1)(p+3)x^2\right)\left(a+bx\right.}{2c^2(p+1)(2p+3)}}{2c(p+2)}\right.$$

$$\Big\downarrow \; 1096$$

$$\frac{1}{2}\left(\frac{x^4\left(a+bx^2+cx^4\right)^{p+1}}{2c(p+2)}-\frac{\frac{b2^p(p+2)\left(6ac-b^2(p+3)\right)\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^2+cx^4\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+\right.}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}}}{2c(p+2)}\right.$$

input
```
Int[x^7*(a + b*x^2 + c*x^4)^p,x]
```

output
```
((x^4*(a + b*x^2 + c*x^4)^(1 + p))/(2*c*(2 + p)) - (-1/2*((b^2*(2 + p)*(3
+ p) - 2*a*c*(3 + 2*p) - 2*b*c*(1 + p)*(3 + p)*x^2)*(a + b*x^2 + c*x^4)^(1
 + p))/(c^2*(1 + p)*(3 + 2*p)) + (2^p*b*(2 + p)*(6*a*c - b^2*(3 + p))*(-((
b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 +
 c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*
p)))/(2*c*(2 + p)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int x^{7}\left(c\,x^{4}+b\,x^{2}+a\right)^{p}dx$$

input
```
int(x^7*(c*x^4+b*x^2+a)^p,x)
```

output
```
int(x^7*(c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^7 \left(a+bx^2+cx^4\right)^p \, dx = \int \left(cx^4+bx^2+a\right)^p x^7 \, dx$$

input
```
integrate(x^7*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p*x^7, x)
```

Sympy [F]

$$\int x^7 \left(a+bx^2+cx^4\right)^p \, dx = \int x^7 \left(a+bx^2+cx^4\right)^p \, dx$$

input
```
integrate(x**7*(c*x**4+b*x**2+a)**p,x)
```

output
```
Integral(x**7*(a + b*x**2 + c*x**4)**p, x)
```

Maxima [F]

$$\int x^7\left(a+bx^2+cx^4\right)^p\,dx=\int\left(cx^4+bx^2+a\right)^px^7\,dx$$

input
```
integrate(x^7*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*x^7, x)
```

Giac [F]

$$\int x^7\left(a+bx^2+cx^4\right)^p\,dx=\int\left(cx^4+bx^2+a\right)^px^7\,dx$$

input
```
integrate(x^7*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^7\left(a+bx^2+cx^4\right)^p\,dx=\int x^7\left(c\,x^4+b\,x^2+a\right)^p\,dx$$

input
```
int(x^7*(a + b*x^2 + c*x^4)^p,x)
```

output
```
int(x^7*(a + b*x^2 + c*x^4)^p, x)
```

Reduce [F]

$$\int x^7(a+bx^2+cx^4)^p\,dx = \text{too large to display}$$

input
```
int(x^7*(c*x^4+b*x^2+a)^p,x)
```

output
```
(4*(a + b*x**2 + c*x**4)**p*a**2*c*p**2 + 20*(a + b*x**2 + c*x**4)**p*a**2
*c*p + 18*(a + b*x**2 + c*x**4)**p*a**2*c - (a + b*x**2 + c*x**4)**p*a*b**
2*p**2 - 5*(a + b*x**2 + c*x**4)**p*a*b**2*p - 6*(a + b*x**2 + c*x**4)**p*
a*b**2 - 4*(a + b*x**2 + c*x**4)**p*a*b*c*p**3*x**2 - 20*(a + b*x**2 + c*x
**4)**p*a*b*c*p**2*x**2 - 18*(a + b*x**2 + c*x**4)**p*a*b*c*p*x**2 + 8*(a
+ b*x**2 + c*x**4)**p*a*c**2*p**3*x**4 + 16*(a + b*x**2 + c*x**4)**p*a*c**
2*p**2*x**4 + 6*(a + b*x**2 + c*x**4)**p*a*c**2*p*x**4 + (a + b*x**2 + c*x
**4)**p*b**3*p**3*x**2 + 5*(a + b*x**2 + c*x**4)**p*b**3*p**2*x**2 + 6*(a
+ b*x**2 + c*x**4)**p*b**3*p*x**2 - 2*(a + b*x**2 + c*x**4)**p*b**2*c*p**3
*x**4 - 7*(a + b*x**2 + c*x**4)**p*b**2*c*p**2*x**4 - 3*(a + b*x**2 + c*x*
*4)**p*b**2*c*p*x**4 + 4*(a + b*x**2 + c*x**4)**p*b*c**2*p**3*x**6 + 6*(a
+ b*x**2 + c*x**4)**p*b*c**2*p**2*x**6 + 2*(a + b*x**2 + c*x**4)**p*b*c**2
*p*x**6 + 8*(a + b*x**2 + c*x**4)**p*c**3*p**3*x**8 + 24*(a + b*x**2 + c*x
**4)**p*c**3*p**2*x**8 + 22*(a + b*x**2 + c*x**4)**p*c**3*p*x**8 + 6*(a +
b*x**2 + c*x**4)**p*c**3*x**8 - 192*int(((a + b*x**2 + c*x**4)**p*x**3)/(4
*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2 + 4*c*p**2*x
**4 + 8*c*p*x**4 + 3*c*x**4),x)*a**2*c**2*p**5 - 960*int(((a + b*x**2 + c*
x**4)**p*x**3)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*
x**2 + 4*c*p**2*x**4 + 8*c*p*x**4 + 3*c*x**4),x)*a**2*c**2*p**4 - 1680*int
(((a + b*x**2 + c*x**4)**p*x**3)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**...
```

3.1093 $\int x^5(a+bx^2+cx^4)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 223

$$\int x^5\left(a+bx^2+cx^4\right)^p\,dx=-\frac{b(2+p)\left(a+bx^2+cx^4\right)^{1+p}}{4c^2(1+p)(3+2p)}+\frac{x^2\left(a+bx^2+cx^4\right)^{1+p}}{2c(3+2p)}+\frac{2^{-1+p}(2ac-b^2(2+p))\left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^2+cx^4\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+\right.}{c^2\sqrt{b^2-4ac}(1+p)(3+2p)}$$

output

```
-1/4*b*(2+p)*(c*x^4+b*x^2+a)^(p+1)/c^2/(p+1)/(3+2*p)+1/2*x^2*(c*x^4+b*x^2+
a)^(p+1)/c/(3+2*p)+2^(-1+p)*(2*a*c-b^2*(2+p))*(-(b-(-4*a*c+b^2)^(1/2)+2*c*
x^2)/(-4*a*c+b^2)^(1/2))^(-1-p)*(c*x^4+b*x^2+a)^(p+1)*hypergeom([-p, p+1],
[2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^2)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b
^2)^(1/2)/(p+1)/(3+2*p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.43 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.73

$$\int x^5\left(a+bx^2+cx^4\right)^p\,dx = \frac{1}{6}x^6\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p \operatorname{AppellF1}\left(3,-p,-p,4,-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^5*(a + b*x^2 + c*x^4)^p,x]
```

output
```
(x^6*(a + b*x^2 + c*x^4)^p*AppellF1[3, -p, -p, 4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(6*((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1434, 1166, 25, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+bx^2+cx^4\right)^p\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int x^4\left(cx^4+bx^2+a\right)^p\,dx^2$$

$$\Big\downarrow \text{1166}$$

$$\frac{1}{2}\left(\frac{\int -\left(\left(b(p+2)x^2+a\right)\left(cx^4+bx^2+a\right)^p\right)dx^2}{c(2p+3)}+\frac{x^2\left(a+bx^2+cx^4\right)^{p+1}}{c(2p+3)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2+cx^4\right)^{p+1}}{c(2p+3)}-\frac{\int \left(b(p+2)x^2+a\right)\left(cx^4+bx^2+a\right)^p dx^2}{c(2p+3)}\right)$$

$$\Big\downarrow \textbf{1160}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2+cx^4\right)^{p+1}}{c(2p+3)}-\frac{\frac{\left(2ac-b^2(p+2)\right)\int \left(cx^4+bx^2+a\right)^p dx^2}{2c}+\frac{b(p+2)\left(a+bx^2+cx^4\right)^{p+1}}{2c(p+1)}}{c(2p+3)}\right)$$

$$\Big\downarrow \textbf{1096}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2+cx^4\right)^{p+1}}{c(2p+3)}-\frac{\frac{b(p+2)\left(a+bx^2+cx^4\right)^{p+1}}{2c(p+1)}-\frac{2^p\left(2ac-b^2(p+2)\right)\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^2+cx^4\right)^{p+1}\text{Hypergeom}}{c(p+1)\sqrt{b^2-4ac}}}{c(2p+3)}\right.$$

input
```
Int[x^5*(a + b*x^2 + c*x^4)^p,x]
```

output
```
((x^2*(a + b*x^2 + c*x^4)^(1 + p))/(c*(3 + 2*p)) - ((b*(2 + p)*(a + b*x^2
+ c*x^4)^(1 + p))/(2*c*(1 + p)) - (2^p*(2*a*c - b^2*(2 + p))*(-((b - Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c*x^4)^(
1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(2*Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p)))/(c*(3 + 2*p)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int x^{5} \left(c \,x^{4}+b \,x^{2}+a \right)^{p}d x$$

input
```
int(x^5*(c*x^4+b*x^2+a)^p,x)
```

output
```
int(x^5*(c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^5 \left(a+bx^2+cx^4\right)^p \, dx = \int { {\left(c x^{4} + b x^{2} + a\right)}^{p} x^{5} \,d x }$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p*x^5, x)
```

Sympy [F]

$$\int x^5 \left(a+bx^2+cx^4\right)^p \, dx = \int x^5 \left(a+bx^2+cx^4\right)^p \, dx$$

input
```
integrate(x**5*(c*x**4+b*x**2+a)**p,x)
```

output
```
Integral(x**5*(a + b*x**2 + c*x**4)**p, x)
```

Maxima [F]

$$\int x^5 \left(a+bx^2+cx^4\right)^p \, dx = \int \left(cx^4+bx^2+a\right)^p x^5 \, dx$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*x^5, x)
```

Giac [F]

$$\int x^5 \left(a+bx^2+cx^4\right)^p \, dx = \int \left(cx^4+bx^2+a\right)^p x^5 \, dx$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 (a + bx^2 + cx^4)^p \, dx = \int x^5 \left(c\,x^4 + b\,x^2 + a\right)^p \, dx$$

input
```
int(x^5*(a + b*x^2 + c*x^4)^p,x)
```

output
```
int(x^5*(a + b*x^2 + c*x^4)^p, x)
```

Reduce [F]

$$\int x^5 (a + bx^2 + cx^4)^p \, dx = \text{Too large to display}$$

input
```
int(x^5*(c*x^4+b*x^2+a)^p,x)
```

output
```
( - 4*(a + b*x**2 + c*x**4)**p*a**2*c*p - 4*(a + b*x**2 + c*x**4)**p*a**2*
c + (a + b*x**2 + c*x**4)**p*a*b**2*p + 2*(a + b*x**2 + c*x**4)**p*a*b**2
+ 4*(a + b*x**2 + c*x**4)**p*a*b*c*p**2*x**2 + 4*(a + b*x**2 + c*x**4)**p*
a*b*c*p*x**2 - (a + b*x**2 + c*x**4)**p*b**3*p**2*x**2 - 2*(a + b*x**2 + c
*x**4)**p*b**3*p*x**2 + 2*(a + b*x**2 + c*x**4)**p*b**2*c*p**2*x**4 + (a +
 b*x**2 + c*x**4)**p*b**2*c*p*x**4 + 4*(a + b*x**2 + c*x**4)**p*b*c**2*p**
2*x**6 + 6*(a + b*x**2 + c*x**4)**p*b*c**2*p*x**6 + 2*(a + b*x**2 + c*x**4
)**p*b*c**2*x**6 + 64*int(((a + b*x**2 + c*x**4)**p*x**3)/(4*a*p**2 + 8*a*
p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2 + 4*c*p**2*x**4 + 8*c*p*x*
*4 + 3*c*x**4),x)*a**2*c**2*p**4 + 192*int(((a + b*x**2 + c*x**4)**p*x**3)
/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2 + 4*c*p**
2*x**4 + 8*c*p*x**4 + 3*c*x**4),x)*a**2*c**2*p**3 + 176*int(((a + b*x**2 +
 c*x**4)**p*x**3)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3
*b*x**2 + 4*c*p**2*x**4 + 8*c*p*x**4 + 3*c*x**4),x)*a**2*c**2*p**2 + 48*in
t(((a + b*x**2 + c*x**4)**p*x**3)/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2
+ 8*b*p*x**2 + 3*b*x**2 + 4*c*p**2*x**4 + 8*c*p*x**4 + 3*c*x**4),x)*a**2*c
**2*p - 32*int(((a + b*x**2 + c*x**4)**p*x**3)/(4*a*p**2 + 8*a*p + 3*a + 4
*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2 + 4*c*p**2*x**4 + 8*c*p*x**4 + 3*c*x*
*4),x)*a*b**2*c*p**5 - 176*int(((a + b*x**2 + c*x**4)**p*x**3)/(4*a*p**2 +
 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2 + 4*c*p**2*x**4 + ...
```

3.1094 $\int x^3(a+bx^2+cx^4)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 160

$$\int x^3(a+bx^2+cx^4)^p \, dx = \frac{(a+bx^2+cx^4)^{1+p}}{4c(1+p)} + \frac{2^{-1+p} b \left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p} (a+bx^2+cx^4)^{1+p} \text{Hypergeometric2F1}\left(-p, 1+p, 2+p, \frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right.}{c\sqrt{b^2-4ac}(1+p)}$$

output
```
1/4*(c*x^4+b*x^2+a)^(p+1)/c/(p+1)+2^(-1+p)*b*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x
^2)/(-4*a*c+b^2)^(1/2))^(-1-p)*(c*x^4+b*x^2+a)^(p+1)*hypergeom([-p, p+1],[
2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^2)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)
^(1/2)/(p+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01

$$\int x^3(a+bx^2+cx^4)^p\,dx = \frac{1}{4}x^4\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\,\text{AppellF1}\left(2,-p,-p,3,-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^3*(a + b*x^2 + c*x^4)^p,x]
```

output
```
(x^4*(a + b*x^2 + c*x^4)^p*AppellF1[2, -p, -p, 3, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1434, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx^2+cx^4)^p\,dx$$

$$\Big\downarrow \; 1434$$

$$\frac{1}{2}\int x^2(cx^4+bx^2+a)^p\,dx^2$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{2}\left(\frac{(a+bx^2+cx^4)^{p+1}}{2c(p+1)}-\frac{b\int(cx^4+bx^2+a)^p\,dx^2}{2c}\right)$$

$$\Big\downarrow \; 1096$$

$$\frac{1}{2}\left(\frac{b2^p\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}+\right.$$

input
```
Int[x^3*(a + b*x^2 + c*x^4)^p,x]
```

output
```
((a + b*x^2 + c*x^4)^(1 + p)/(2*c*(1 + p)) + (2^p*b*(-((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c*x^4)^(1 + p)*Hy
pergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(2*Sqr
t[b^2 - 4*a*c])])/(c*Sqrt[b^2 - 4*a*c]*(1 + p)))/2
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int x^3 \left(c\,x^4 + b\,x^2 + a\right)^p dx$$

input
```
int(x^3*(c*x^4+b*x^2+a)^p,x)
```

output
```
int(x^3*(c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^3 \left(a + bx^2 + cx^4\right)^p \, dx = \int \left(cx^4 + bx^2 + a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p*x^3, x)
```

Sympy [F]

$$\int x^3 \left(a + bx^2 + cx^4\right)^p \, dx = \int x^3 \left(a + bx^2 + cx^4\right)^p \, dx$$

input
```
integrate(x**3*(c*x**4+b*x**2+a)**p,x)
```

output
```
Integral(x**3*(a + b*x**2 + c*x**4)**p, x)
```

Maxima [F]

$$\int x^3 (a + bx^2 + cx^4)^p \, dx = \int (cx^4 + bx^2 + a)^p x^3 \, dx$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*x^3, x)
```

Giac [F]

$$\int x^3 (a + bx^2 + cx^4)^p \, dx = \int (cx^4 + bx^2 + a)^p x^3 \, dx$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (a + bx^2 + cx^4)^p \, dx = \int x^3 \left(c\,x^4 + b\,x^2 + a\right)^p \, dx$$

input
```
int(x^3*(a + b*x^2 + c*x^4)^p,x)
```

output
```
int(x^3*(a + b*x^2 + c*x^4)^p, x)
```

Reduce [F]

$$\int x^3 (a + bx^2 + cx^4)^p \, dx$$
$$= \frac{-(c\,x^4 + b\,x^2 + a)^p\,a + (c\,x^4 + b\,x^2 + a)^p\,bp\,x^2 + 2(c\,x^4 + b\,x^2 + a)^p\,cp\,x^4 + (c\,x^4 + b\,x^2 + a)^p\,c\,x^4 + 16\Big(}{}$$

input
```
int(x^3*(c*x^4+b*x^2+a)^p,x)
```

output
```
( - (a + b*x**2 + c*x**4)**p*a + (a + b*x**2 + c*x**4)**p*b*p*x**2 + 2*(a
+ b*x**2 + c*x**4)**p*c*p*x**4 + (a + b*x**2 + c*x**4)**p*c*x**4 + 16*int(
((a + b*x**2 + c*x**4)**p*x**3)/(2*a*p + a + 2*b*p*x**2 + b*x**2 + 2*c*p*x
**4 + c*x**4),x)*a*c*p**3 + 24*int(((a + b*x**2 + c*x**4)**p*x**3)/(2*a*p
+ a + 2*b*p*x**2 + b*x**2 + 2*c*p*x**4 + c*x**4),x)*a*c*p**2 + 8*int(((a +
 b*x**2 + c*x**4)**p*x**3)/(2*a*p + a + 2*b*p*x**2 + b*x**2 + 2*c*p*x**4 +
 c*x**4),x)*a*c*p - 4*int(((a + b*x**2 + c*x**4)**p*x**3)/(2*a*p + a + 2*b
*p*x**2 + b*x**2 + 2*c*p*x**4 + c*x**4),x)*b**2*p**3 - 6*int(((a + b*x**2
+ c*x**4)**p*x**3)/(2*a*p + a + 2*b*p*x**2 + b*x**2 + 2*c*p*x**4 + c*x**4)
,x)*b**2*p**2 - 2*int(((a + b*x**2 + c*x**4)**p*x**3)/(2*a*p + a + 2*b*p*x
**2 + b*x**2 + 2*c*p*x**4 + c*x**4),x)*b**2*p)/(4*c*(2*p**2 + 3*p + 1))
```

3.1095 $\int x(a+bx^2+cx^4)^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int x(a+bx^2+cx^4)^p\,dx = -\frac{2^p\left(-\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^2+cx^4)^{1+p}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{b+\sqrt{b^2-4ac}+2cx^2}{2\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}(1+p)}$$

output
```
-2^p*(-(b-(-4*a*c+b^2)^(1/2)+2*c*x^2)/(-4*a*c+b^2)^(1/2))^(-1-p)*(c*x^4+b*
x^2+a)^(p+1)*hypergeom([-p, p+1],[2+p],1/2*(b+(-4*a*c+b^2)^(1/2)+2*c*x^2)/
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/(p+1)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07

$$\int x(a+bx^2+cx^4)^p\,dx = \frac{2^{-2+p}\left(b-\sqrt{b^2-4ac}+2cx^2\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\,\text{Hypergeometric2F1}\left(-p,1+p,2+p\right.}{c(1+p)}$$

input
```
Integrate[x*(a + b*x^2 + c*x^4)^p,x]
```

output
```
(2^(-2 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(a + b*x^2 + c*x^4)^p*Hyperg
eometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)/(2*Sqrt[b
^2 - 4*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*
a*c])^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{1432, 1096\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a + bx^2 + cx^4)^p \, dx$$
$$\Big\downarrow \text{ 1432}$$
$$\frac{1}{2}\int (cx^4 + bx^2 + a)^p \, dx^2$$
$$\Big\downarrow \text{ 1096}$$
$$-\frac{2^p\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}(a+bx^2+cx^4)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

input
```
Int[x*(a + b*x^2 + c*x^4)^p,x]
```

output
```
-((2^p*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(
a + b*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1 + p))
)
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Maple [F]

$$\int x\left(c\,x^{4}+b\,x^{2}+a\right)^{p}dx$$

input
```
int(x*(c*x^4+b*x^2+a)^p,x)
```

output
```
int(x*(c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int x\left(a+bx^2+cx^4\right)^p\,dx=\int \left(cx^4+bx^2+a\right)^p x\,dx$$

input
```
integrate(x*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p*x, x)
```

Sympy [F]

$$\int x\left(a+bx^2+cx^4\right)^p\,dx=\int x\left(a+bx^2+cx^4\right)^p\,dx$$

input
```
integrate(x*(c*x**4+b*x**2+a)**p,x)
```

output
```
Integral(x*(a + b*x**2 + c*x**4)**p, x)
```

Maxima [F]

$$\int x\left(a+bx^2+cx^4\right)^p\,dx=\int \left(cx^4+bx^2+a\right)^p x\,dx$$

input
```
integrate(x*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+bx^2+cx^4\right)^p\,dx=\int \left(cx^4+bx^2+a\right)^p x\,dx$$

input
```
integrate(x*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+bx^2+cx^4\right)^p\,dx=\int x\,\left(c\,x^4+b\,x^2+a\right)^p\,dx$$

input
```
int(x*(a + b*x^2 + c*x^4)^p,x)
```

output
```
int(x*(a + b*x^2 + c*x^4)^p, x)
```

Reduce [F]

$$\int x\left(a+bx^2+cx^4\right)^p\,dx = \frac{2(c\,x^4+b\,x^2+a)^p\,a+(c\,x^4+b\,x^2+a)^p\,b\,x^2-16\left(\int\frac{(c\,x^4+b\,x^2+a)^p\,x^3}{2cp\,x^4+c\,x^4+2bp\,x^2+b\,x^2+2ap+a}dx\right)ac\,p^2-8\left(\int\frac{(c\,x}{2cp\,x^4+c\,x}\right.}{2b\,(2}$$

input
```
int(x*(c*x^4+b*x^2+a)^p,x)
```

output
```
(2*(a + b*x**2 + c*x**4)**p*a + (a + b*x**2 + c*x**4)**p*b*x**2 - 16*int((
(a + b*x**2 + c*x**4)**p*x**3)/(2*a*p + a + 2*b*p*x**2 + b*x**2 + 2*c*p*x*
*4 + c*x**4),x)*a*c*p**2 - 8*int(((a + b*x**2 + c*x**4)**p*x**3)/(2*a*p +
a + 2*b*p*x**2 + b*x**2 + 2*c*p*x**4 + c*x**4),x)*a*c*p + 4*int(((a + b*x*
*2 + c*x**4)**p*x**3)/(2*a*p + a + 2*b*p*x**2 + b*x**2 + 2*c*p*x**4 + c*x*
*4),x)*b**2*p**2 + 2*int(((a + b*x**2 + c*x**4)**p*x**3)/(2*a*p + a + 2*b*
p*x**2 + b*x**2 + 2*c*p*x**4 + c*x**4),x)*b**2*p)/(2*b*(2*p + 1))
```

3.1096 $\int \frac{(a+bx^2+cx^4)^p}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 152

$$\int \frac{(a+bx^2+cx^4)^p}{x}\,dx = \frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4}}{2cx^2}\right.}{p}$$

output
```
4^(-1+p)*(c*x^4+b*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,-1/2*(b-(-4*a*c+b^2)^
(1/2))/c/x^2,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^2)/p/(((b-(-4*a*c+b^2)^(1/2)+
2*c*x^2)/c/x^2)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^2)/c/x^2)^p)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^p}{x}\,dx = \frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b+\sqrt{b^2-4}}{2cx^2}\right.}{p}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^p/x,x]
```

output
```
(4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b
+ Sqrt[b^2 - 4*a*c])/(c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/(p*((b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(c*x^2))^p)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1434, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^p}{x}\,dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^p}{x^2}dx^2$$

$$\Big\downarrow \text{1178}$$

$$-2^{2p-1}\left(\frac{1}{x^2}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p\int\left(\frac{b-\sqrt{b^2-4ac}}{2cx^2}\right.$$

$$\Big\downarrow \text{150}$$

$$\frac{2^{2p-2}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p\,\text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\right.}{p}$$

input
```
Int[(a + b*x^2 + c*x^4)^p/x,x]
```

output
```
(2^(-2 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(
b - Sqrt[b^2 - 4*a*c])/(c*x^2), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^2)])/(p*
((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(c*x^2))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \frac{(c\,x^4 + b\,x^2 + a)^p}{x}dx$$

input
```
int((c*x^4+b*x^2+a)^p/x,x)
```

output
```
int((c*x^4+b*x^2+a)^p/x,x)
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x}\,dx=\int \frac{(cx^4+bx^2+a)^p}{x}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p/x, x)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x}\,dx=\int \frac{(a+bx^2+cx^4)^p}{x}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**p/x,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**p/x, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x}\,dx=\int \frac{(cx^4+bx^2+a)^p}{x}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x}\,dx = \int \frac{(cx^4+bx^2+a)^p}{x}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^p}{x}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^p}{x}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^p/x,x)
```

output
```
int((a + b*x^2 + c*x^4)^p/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x}\,dx = \frac{(c\,x^4+b\,x^2+a)^p+2\left(\int \frac{(c\,x^4+b\,x^2+a)^p}{c\,x^5+b\,x^3+ax}dx\right)ap-2\left(\int \frac{(c\,x^4+b\,x^2+a)^p x^3}{c\,x^4+b\,x^2+a}dx\right)cp}{2p}$$

input
```
int((c*x^4+b*x^2+a)^p/x,x)
```

output
```
((a + b*x**2 + c*x**4)**p + 2*int((a + b*x**2 + c*x**4)**p/(a*x + b*x**3 +
 c*x**5),x)*a*p - 2*int(((a + b*x**2 + c*x**4)**p*x**3)/(a + b*x**2 + c*x*
*4),x)*c*p)/(2*p)
```

3.1097 $\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 166

$$\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx = -\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2(1-p),-\right.}{(1-2p)x^2}$$

output
```
-2^(-1+2*p)*(c*x^4+b*x^2+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,-1/2*(b-(-4*a*c+b
^2)^(1/2))/c/x^2,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^2)/(1-2*p)/x^2/(((b-(-4*a
*c+b^2)^(1/2)+2*c*x^2)/c/x^2)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^2)/c/x^2)^p
)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx = \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{b+\sqrt{b}}{2c}\right.}{(-1+2p)x^2}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^p/x^3,x]
```

output
```
(2^(-1 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/
2*(b + Sqrt[b^2 - 4*a*c])/(c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((
-1 + 2*p)*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(c*x^2))^p)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1434, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx$$

$$\Big\downarrow \ 1434$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^p}{x^4}dx^2$$

$$\Big\downarrow \ 1178$$

$$-2^{2p-1}\left(\frac{1}{x^2}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p\int\left(\frac{b-\sqrt{b^2-4ac}}{2cx^2}\right.$$

$$\Big\downarrow \ 150$$

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p\operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{b-\sqrt{b^2-4ac}}{2cx^2}\right.}{(1-2p)x^2}$$

input
```
Int[(a + b*x^2 + c*x^4)^p/x^3,x]
```

output
```
-((2^(-1 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -
1/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^2), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^2)]
)/((1 - 2*p)*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \frac{(c\,x^4 + b\,x^2 + a)^p}{x^3} dx$$

input
```
int((c*x^4+b*x^2+a)^p/x^3,x)
```

output
```
int((c*x^4+b*x^2+a)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx=\int \frac{(cx^4+bx^2+a)^p}{x^3}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p/x^3, x)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx=\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**p/x**3,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**p/x**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx=\int \frac{(cx^4+bx^2+a)^p}{x^3}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx=\int \frac{(cx^4+bx^2+a)^p}{x^3}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx=\int \frac{(c\,x^4+b\,x^2+a)^p}{x^3}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^p/x^3,x)
```

output
```
int((a + b*x^2 + c*x^4)^p/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx$$
$$=\frac{-(c\,x^4+b\,x^2+a)^p\,bp+(c\,x^4+b\,x^2+a)^p\,b-(c\,x^4+b\,x^2+a)^p\,c\,x^2+2\left(\int \frac{(c\,x^4+b\,x^2+a)^p}{cp\,x^5-c\,x^5+bp\,x^3-b\,x^3+apx-ax}dx\right)$$

input
```
int((c*x^4+b*x^2+a)^p/x^3,x)
```

output
```
( - (a + b*x**2 + c*x**4)**p*b*p + (a + b*x**2 + c*x**4)**p*b - (a + b*x**
2 + c*x**4)**p*c*x**2 + 2*int((a + b*x**2 + c*x**4)**p/(a*p*x - a*x + b*p*
x**3 - b*x**3 + c*p*x**5 - c*x**5),x)*b**2*p**3*x**2 - 4*int((a + b*x**2 +
 c*x**4)**p/(a*p*x - a*x + b*p*x**3 - b*x**3 + c*p*x**5 - c*x**5),x)*b**2*
p**2*x**2 + 2*int((a + b*x**2 + c*x**4)**p/(a*p*x - a*x + b*p*x**3 - b*x**
3 + c*p*x**5 - c*x**5),x)*b**2*p*x**2 + 4*int(((a + b*x**2 + c*x**4)**p*x*
*3)/(a*p - a + b*p*x**2 - b*x**2 + c*p*x**4 - c*x**4),x)*c**2*p**2*x**2 -
4*int(((a + b*x**2 + c*x**4)**p*x**3)/(a*p - a + b*p*x**2 - b*x**2 + c*p*x
**4 - c*x**4),x)*c**2*p*x**2 + 4*int(((a + b*x**2 + c*x**4)**p*x)/(a*p - a
 + b*p*x**2 - b*x**2 + c*p*x**4 - c*x**4),x)*b*c*p**3*x**2 - 6*int(((a + b
*x**2 + c*x**4)**p*x)/(a*p - a + b*p*x**2 - b*x**2 + c*p*x**4 - c*x**4),x)
*b*c*p**2*x**2 + 2*int(((a + b*x**2 + c*x**4)**p*x)/(a*p - a + b*p*x**2 -
b*x**2 + c*p*x**4 - c*x**4),x)*b*c*p*x**2)/(2*b*x**2*(p - 1))
```

3.1098 $\int \frac{(a+bx^2+cx^4)^p}{x^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 164

$$\int \frac{(a+bx^2+cx^4)^p}{x^5}\,dx = -\frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p\operatorname{AppellF1}\left(2(1-p),-p,-p,3-2p,-\frac{b}{}\right.}{(1-p)x^4}$$

output
```
-4^(-1+p)*(c*x^4+b*x^2+a)^p*AppellF1(2-2*p,-p,-p,3-2*p,-1/2*(b-(-4*a*c+b^2
)^(1/2))/c/x^2,-1/2*(b+(-4*a*c+b^2)^(1/2))/c/x^2)/(1-p)/x^4/(((b-(-4*a*c+b
^2)^(1/2)+2*c*x^2)/c/x^2)^p)/(((b+(-4*a*c+b^2)^(1/2)+2*c*x^2)/c/x^2)^p)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2+cx^4)^p}{x^5}\,dx = \frac{4^{-1+p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p\operatorname{AppellF1}\left(2-2p,-p,-p,3-2p,-\frac{b+\sqrt{b^2}}{2cx}\right.}{(-1+p)x^4}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^p/x^5,x]
```

output
```
(4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -1/2*
(b + Sqrt[b^2 - 4*a*c])/(c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((-1
 + p)*x^4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(c*x^2))^p)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1434, 1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^p}{x^5} \, dx$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{(cx^4+bx^2+a)^p}{x^6}dx^2$$

$$\Big\downarrow \text{1178}$$

$$-2^{2p-1}\left(\frac{1}{x^2}\right)^{2p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p\int \left(\frac{b-\sqrt{b^2-4ac}}{2cx^2}\right.$$

$$\Big\downarrow \text{150}$$

$$-\frac{2^{2p-2}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(2-2p,-p,-p,3-2p,-\frac{b-\sqrt{b^2-4a}}{2cx^2}\right.}{(1-p)x^4}$$

input
```
Int[(a + b*x^2 + c*x^4)^p/x^5,x]
```

output
```
-((2^(-2 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -
1/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^2), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^2)]
)/((1 - p)*x^4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + Sqrt[b^
2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \frac{(c\,x^4 + b\,x^2 + a)^p}{x^5} dx$$

input
```
int((c*x^4+b*x^2+a)^p/x^5,x)
```

output
```
int((c*x^4+b*x^2+a)^p/x^5,x)
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^5}\,dx = \int \frac{(cx^4+bx^2+a)^p}{x^5}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x^5,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p/x^5, x)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^5}\,dx = \int \frac{(a+bx^2+cx^4)^p}{x^5}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**p/x**5,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**p/x**5, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^5}\,dx = \int \frac{(cx^4+bx^2+a)^p}{x^5}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x^5,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p/x^5, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^5}\,dx=\int \frac{(cx^4+bx^2+a)^p}{x^5}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x^5,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^p}{x^5}\,dx=\int \frac{(c\,x^4+b\,x^2+a)^p}{x^5}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^p/x^5,x)
```

output
```
int((a + b*x^2 + c*x^4)^p/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^5}\,dx$$
$$=\frac{-(c\,x^4+b\,x^2+a)^p+2\left(\int \frac{(c\,x^4+b\,x^2+a)^p}{cp\,x^7-2c\,x^7+bp\,x^5-2b\,x^5+ap\,x^3-2a\,x^3}dx\right)b\,p^2x^4-4\left(\int \frac{(c\,x^4+b\,x^2+a)^p}{cp\,x^7-2c\,x^7+bp\,x^5-2b\,x^5+ap\,x^3-2a\,x^3}\right.}{4x^4}$$

input
```
int((c*x^4+b*x^2+a)^p/x^5,x)
```

output
```
( - (a + b*x**2 + c*x**4)**p + 2*int((a + b*x**2 + c*x**4)**p/(a*p*x**3 -
2*a*x**3 + b*p*x**5 - 2*b*x**5 + c*p*x**7 - 2*c*x**7),x)*b*p**2*x**4 - 4*i
nt((a + b*x**2 + c*x**4)**p/(a*p*x**3 - 2*a*x**3 + b*p*x**5 - 2*b*x**5 + c
*p*x**7 - 2*c*x**7),x)*b*p*x**4 + 4*int((a + b*x**2 + c*x**4)**p/(a*p*x -
2*a*x + b*p*x**3 - 2*b*x**3 + c*p*x**5 - 2*c*x**5),x)*c*p**2*x**4 - 8*int(
(a + b*x**2 + c*x**4)**p/(a*p*x - 2*a*x + b*p*x**3 - 2*b*x**3 + c*p*x**5 -
 2*c*x**5),x)*c*p*x**4)/(4*x**4)
```

3.1099 $\int x^4(a+bx^2+cx^4)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int x^4(a+bx^2+cx^4)^p\,dx=\frac{1}{5}x^5\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/5*x^5*(c*x^4+b*x^2+a)^p*AppellF1(5/2,-p,-p,7/2,-2*c*x^2/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int x^4(a+bx^2+cx^4)^p\,dx=\frac{1}{5}x^5\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p\operatorname{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^4*(a + b*x^2 + c*x^4)^p,x]
```

output
```
(x^5*(a + b*x^2 + c*x^4)^p*AppellF1[5/2, -p, -p, 7/2, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(5*((b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2+cx^4\right)^p \, dx$$

$$\Big\downarrow \text{1461}$$

$$\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p\int x^4\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right.$$

$$\Big\downarrow \text{394}$$

$$\frac{1}{5}x^5\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p \text{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2cx^2}{b-\sqrt{b^2-4a}}\right.$$

input
```
Int[x^4*(a + b*x^2 + c*x^4)^p,x]
```

output
```
(x^5*(a + b*x^2 + c*x^4)^p*AppellF1[5/2, -p, -p, 7/2, (-2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int x^{4}\left(c\,x^{4}+b\,x^{2}+a\right)^{p}dx$$

input
```
int(x^4*(c*x^4+b*x^2+a)^p,x)
```

output
```
int(x^4*(c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^4\left(a+bx^2+cx^4\right)^p\,dx=\int\left(cx^4+bx^2+a\right)^px^4\,dx$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p*x^4, x)
```

Sympy [F]

$$\int x^4 (a + bx^2 + cx^4)^p \, dx = \int x^4 (a + bx^2 + cx^4)^p \, dx$$

input
```
integrate(x**4*(c*x**4+b*x**2+a)**p,x)
```

output
```
Integral(x**4*(a + b*x**2 + c*x**4)**p, x)
```

Maxima [F]

$$\int x^4 (a + bx^2 + cx^4)^p \, dx = \int (cx^4 + bx^2 + a)^p x^4 \, dx$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*x^4, x)
```

Giac [F]

$$\int x^4 (a + bx^2 + cx^4)^p \, dx = \int (cx^4 + bx^2 + a)^p x^4 \, dx$$

input
```
integrate(x^4*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a + bx^2 + cx^4)^p \, dx = \int x^4 \left(c x^4 + b x^2 + a\right)^p dx$$

input
```
int(x^4*(a + b*x^2 + c*x^4)^p,x)
```

output
```
int(x^4*(a + b*x^2 + c*x^4)^p, x)
```

Reduce [F]

$$\int x^4 (a + bx^2 + cx^4)^p \, dx = \text{too large to display}$$

input
```
int(x^4*(c*x^4+b*x^2+a)^p,x)
```

output
```
(16*(a + b*x**2 + c*x**4)**p*a*c*p**2*x + 12*(a + b*x**2 + c*x**4)**p*a*c*
p*x - 4*(a + b*x**2 + c*x**4)**p*b**2*p**2*x - 6*(a + b*x**2 + c*x**4)**p*
b**2*p*x + 8*(a + b*x**2 + c*x**4)**p*b*c*p**2*x**3 + 2*(a + b*x**2 + c*x*
*4)**p*b*c*p*x**3 + 16*(a + b*x**2 + c*x**4)**p*c**2*p**2*x**5 + 16*(a + b
*x**2 + c*x**4)**p*c**2*p*x**5 + 3*(a + b*x**2 + c*x**4)**p*c**2*x**5 - 10
24*int((a + b*x**2 + c*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a +
64*b*p**3*x**2 + 144*b*p**2*x**2 + 92*b*p*x**2 + 15*b*x**2 + 64*c*p**3*x**
4 + 144*c*p**2*x**4 + 92*c*p*x**4 + 15*c*x**4),x)*a**2*c*p**5 - 3072*int((
a + b*x**2 + c*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**
3*x**2 + 144*b*p**2*x**2 + 92*b*p*x**2 + 15*b*x**2 + 64*c*p**3*x**4 + 144*
c*p**2*x**4 + 92*c*p*x**4 + 15*c*x**4),x)*a**2*c*p**4 - 3200*int((a + b*x*
*2 + c*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**2 +
 144*b*p**2*x**2 + 92*b*p*x**2 + 15*b*x**2 + 64*c*p**3*x**4 + 144*c*p**2*x
**4 + 92*c*p*x**4 + 15*c*x**4),x)*a**2*c*p**3 - 1344*int((a + b*x**2 + c*x
**4)**p/(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**2 + 144*b*p
**2*x**2 + 92*b*p*x**2 + 15*b*x**2 + 64*c*p**3*x**4 + 144*c*p**2*x**4 + 92
*c*p*x**4 + 15*c*x**4),x)*a**2*c*p**2 - 180*int((a + b*x**2 + c*x**4)**p/(
64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**2 + 144*b*p**2*x**2
+ 92*b*p*x**2 + 15*b*x**2 + 64*c*p**3*x**4 + 144*c*p**2*x**4 + 92*c*p*x**4
 + 15*c*x**4),x)*a**2*c*p + 256*int((a + b*x**2 + c*x**4)**p/(64*a*p**3...
```

3.1100 $\int x^2(a+bx^2+cx^4)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int x^2\left(a+bx^2+cx^4\right)^p\,dx=\frac{1}{3}x^3\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

output
```
1/3*x^3*(c*x^4+b*x^2+a)^p*AppellF1(3/2,-p,-p,5/2,-2*c*x^2/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int x^2\left(a+bx^2+cx^4\right)^p\,dx=\frac{1}{3}x^3\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[x^2*(a + b*x^2 + c*x^4)^p,x]
```

output
```
(x^3*(a + b*x^2 + c*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(3*((b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx^2+cx^4)^p \, dx$$

$\Big\downarrow$ 1461

$$\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p\int x^2\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right.$$

$\Big\downarrow$ 394

$$\frac{1}{3}x^3\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p\operatorname{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2cx^2}{b-\sqrt{b^2-4a}}\right.$$

input
```
Int[x^2*(a + b*x^2 + c*x^4)^p,x]
```

output
```
(x^3*(a + b*x^2 + c*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*(1 + (2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int x^{2} \left(c\,x^{4}+b\,x^{2}+a\right)^{p}dx$$

input
```
int(x^2*(c*x^4+b*x^2+a)^p,x)
```

output
```
int(x^2*(c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^2 \left(a+bx^2+cx^4\right)^p \, dx = \int \left(cx^4+bx^2+a\right)^p x^2 \, dx$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p*x^2, x)
```

Sympy [F]

$$\int x^2(a+bx^2+cx^4)^p\,dx=\int x^2(a+bx^2+cx^4)^p\,dx$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)**p,x)
```

output
```
Integral(x**2*(a + b*x**2 + c*x**4)**p, x)
```

Maxima [F]

$$\int x^2(a+bx^2+cx^4)^p\,dx=\int (cx^4+bx^2+a)^p x^2\,dx$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(a+bx^2+cx^4)^p\,dx=\int (cx^4+bx^2+a)^p x^2\,dx$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+bx^2+cx^4\right)^p\,dx=\int x^2\,\left(c\,x^4+b\,x^2+a\right)^p\,dx$$

input
```
int(x^2*(a + b*x^2 + c*x^4)^p,x)
```

output
```
int(x^2*(a + b*x^2 + c*x^4)^p, x)
```

Reduce [F]

$$\int x^2\left(a+bx^2+cx^4\right)^p\,dx=\text{Too large to display}$$

input
```
int(x^2*(c*x^4+b*x^2+a)^p,x)
```

output
```
(2*(a + b*x**2 + c*x**4)**p*b*p*x + 4*(a + b*x**2 + c*x**4)**p*c*p*x**3 +
(a + b*x**2 + c*x**4)**p*c*x**3 - 32*int((a + b*x**2 + c*x**4)**p/(16*a*p*
*2 + 16*a*p + 3*a + 16*b*p**2*x**2 + 16*b*p*x**2 + 3*b*x**2 + 16*c*p**2*x*
*4 + 16*c*p*x**4 + 3*c*x**4),x)*a*b*p**3 - 32*int((a + b*x**2 + c*x**4)**p
/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**2 + 16*b*p*x**2 + 3*b*x**2 + 16*
c*p**2*x**4 + 16*c*p*x**4 + 3*c*x**4),x)*a*b*p**2 - 6*int((a + b*x**2 + c*
x**4)**p/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**2 + 16*b*p*x**2 + 3*b*x*
*2 + 16*c*p**2*x**4 + 16*c*p*x**4 + 3*c*x**4),x)*a*b*p + 256*int(((a + b*x
**2 + c*x**4)**p*x**2)/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**2 + 16*b*p
*x**2 + 3*b*x**2 + 16*c*p**2*x**4 + 16*c*p*x**4 + 3*c*x**4),x)*a*c*p**4 +
320*int(((a + b*x**2 + c*x**4)**p*x**2)/(16*a*p**2 + 16*a*p + 3*a + 16*b*p
**2*x**2 + 16*b*p*x**2 + 3*b*x**2 + 16*c*p**2*x**4 + 16*c*p*x**4 + 3*c*x**
4),x)*a*c*p**3 + 112*int(((a + b*x**2 + c*x**4)**p*x**2)/(16*a*p**2 + 16*a
*p + 3*a + 16*b*p**2*x**2 + 16*b*p*x**2 + 3*b*x**2 + 16*c*p**2*x**4 + 16*c
*p*x**4 + 3*c*x**4),x)*a*c*p**2 + 12*int(((a + b*x**2 + c*x**4)**p*x**2)/(
16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**2 + 16*b*p*x**2 + 3*b*x**2 + 16*c*
p**2*x**4 + 16*c*p*x**4 + 3*c*x**4),x)*a*c*p - 64*int(((a + b*x**2 + c*x**
4)**p*x**2)/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**2 + 16*b*p*x**2 + 3*b
*x**2 + 16*c*p**2*x**4 + 16*c*p*x**4 + 3*c*x**4),x)*b**2*p**4 - 96*int(((a
 + b*x**2 + c*x**4)**p*x**2)/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**2...
```

3.1101 $\int (a+bx^2+cx^4)^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 133

$$\int (a+bx^2+cx^4)^p \, dx = x\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

output
```
x*(c*x^4+b*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^p)/(
(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.21

$$\int (a+bx^2+cx^4)^p \, dx = x\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)$$

input
```
Integrate[(a + b*x^2 + c*x^4)^p,x]
```

output
```
(x*(a + b*x^2 + c*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1418, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^p \, dx$$

$$\Big\downarrow \text{1418}$$

$$\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p\int \left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+\right.$$

$$\Big\downarrow \text{333}$$

$$x\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\right.$$

input
```
Int[(a + b*x^2 + c*x^4)^p,x]
```

output
```
(x*(a + b*x^2 + c*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1418
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p]))   Int[(1
 + 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b,
c, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Maple [F]

$$\int \left(c\,x^{4}+b\,x^{2}+a\right)^{p}dx$$

input
```
int((c*x^4+b*x^2+a)^p,x)
```

output
```
int((c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx^2+cx^4\right)^p\,dx=\int \left(cx^{4}+bx^{2}+a\right)^{p}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p, x)
```

Sympy [F]

$$\int \left(a+bx^2+cx^4\right)^p\,dx=\int \left(a+bx^2+cx^4\right)^p\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**p,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**p, x)
```

Maxima [F]

$$\int \left(a+bx^2+cx^4\right)^p\,dx=\int \left(cx^4+bx^2+a\right)^p\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^2+cx^4\right)^p\,dx=\int \left(cx^4+bx^2+a\right)^p\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2+cx^4\right)^p\,dx=\int \left(c\,x^4+b\,x^2+a\right)^p\,dx$$

input
```
int((a + b*x^2 + c*x^4)^p,x)
```

output
```
int((a + b*x^2 + c*x^4)^p, x)
```

Reduce [F]

$$\int \left(a+bx^2+cx^4\right)^p\,dx$$
$$=\frac{\left(c\,x^4+b\,x^2+a\right)^p x+16\left(\int \frac{\left(c\,x^4+b\,x^2+a\right)^p}{4cp\,x^4+c\,x^4+4bp\,x^2+b\,x^2+4ap+a}dx\right)a\,p^2+4\left(\int \frac{\left(c\,x^4+b\,x^2+a\right)^p}{4cp\,x^4+c\,x^4+4bp\,x^2+b\,x^2+4ap+a}dx\right)ap+}{4p+1}$$

input
```
int((c*x^4+b*x^2+a)^p,x)
```

output
```
((a + b*x**2 + c*x**4)**p*x + 16*int((a + b*x**2 + c*x**4)**p/(4*a*p + a +
 4*b*p*x**2 + b*x**2 + 4*c*p*x**4 + c*x**4),x)*a*p**2 + 4*int((a + b*x**2
+ c*x**4)**p/(4*a*p + a + 4*b*p*x**2 + b*x**2 + 4*c*p*x**4 + c*x**4),x)*a*
p + 8*int(((a + b*x**2 + c*x**4)**p*x**2)/(4*a*p + a + 4*b*p*x**2 + b*x**2
 + 4*c*p*x**4 + c*x**4),x)*b*p**2 + 2*int(((a + b*x**2 + c*x**4)**p*x**2)/
(4*a*p + a + 4*b*p*x**2 + b*x**2 + 4*c*p*x**4 + c*x**4),x)*b*p)/(4*p + 1)
```

3.1102 $\int \frac{(a+bx^2+cx^4)^p}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 136

$$\int \frac{(a+bx^2+cx^4)^p}{x^2} dx =$$

$$-\frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-\cdots}}\right.}{x}$$

output
```
-(c*x^4+b*x^2+a)^p*AppellF1(-1/2,-p,-p,1/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))
,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/x/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^p)
/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2+cx^4)^p}{x^2} dx =$$

$$-\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2\cdots}}\right.}{x}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^p/x^2,x]
```

output
```
-(((a + b*x^2 + c*x^4)^p*AppellF1[-1/2, -p, -p, 1/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(x*((b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{1461, 394\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^p}{x^2}dx$$

$$\Big\downarrow \text{1461}$$

$$\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p\int \frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^p}{x^2}dx$$

$$\Big\downarrow \text{394}$$

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p \text{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{x}$$

input
```
Int[(a + b*x^2 + c*x^4)^p/x^2,x]
```

output
```
-(((a + b*x^2 + c*x^4)^p*AppellF1[-1/2, -p, -p, 1/2, (-2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{(c\,x^4+b\,x^2+a)^p}{x^2}dx$$

input
```
int((c*x^4+b*x^2+a)^p/x^2,x)
```

output
```
int((c*x^4+b*x^2+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^2}\,dx = \int \frac{(cx^4+bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^2}\,dx=\int \frac{(a+bx^2+cx^4)^p}{x^2}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**p/x**2,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**p/x**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^2}\,dx=\int \frac{(cx^4+bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^2}\,dx=\int \frac{(cx^4+bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^p}{x^2}\,dx=\int \frac{(c\,x^4+b\,x^2+a)^p}{x^2}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^p/x^2,x)
```

output
```
int((a + b*x^2 + c*x^4)^p/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^2}\,dx = \frac{(c\,x^4+b\,x^2+a)^p+4\left(\int \frac{(c\,x^4+b\,x^2+a)^p}{2cp\,x^6-c\,x^6+2bp\,x^4-b\,x^4+2ap\,x^2-a\,x^2}dx\right)a\,p^2x-2\left(\int \frac{(c\,x^4+b\,x^2+a)^p}{2cp\,x^6-c\,x^6+2bp\,x^4-b\,x^4+2ap\,x^2-a\,x^2}dx\right.}{x\,(2p-1)}$$

input
```
int((c*x^4+b*x^2+a)^p/x^2,x)
```

output
```
((a + b*x**2 + c*x**4)**p + 4*int((a + b*x**2 + c*x**4)**p/(2*a*p*x**2 - a
*x**2 + 2*b*p*x**4 - b*x**4 + 2*c*p*x**6 - c*x**6),x)*a*p**2*x - 2*int((a
+ b*x**2 + c*x**4)**p/(2*a*p*x**2 - a*x**2 + 2*b*p*x**4 - b*x**4 + 2*c*p*x
**6 - c*x**6),x)*a*p*x - 4*int(((a + b*x**2 + c*x**4)**p*x**2)/(2*a*p - a
+ 2*b*p*x**2 - b*x**2 + 2*c*p*x**4 - c*x**4),x)*c*p**2*x + 2*int(((a + b*x
**2 + c*x**4)**p*x**2)/(2*a*p - a + 2*b*p*x**2 - b*x**2 + 2*c*p*x**4 - c*x
**4),x)*c*p*x)/(x*(2*p - 1))
```

3.1103 $\int \frac{(a+bx^2+cx^4)^p}{x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{(a+bx^2+cx^4)^p}{x^4} \, dx = -\frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(-\frac{3}{2},-p,-p,-\frac{1}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2}{b+\sqrt{}}\right.}{3x^3}$$

output
```
-1/3*(c*x^4+b*x^2+a)^p*AppellF1(-3/2,-p,-p,-1/2,-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/x^3/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)))^p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2+cx^4)^p}{x^4} \, dx = -\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(-\frac{3}{2},-p,-p,-\frac{1}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2}{-b+\sqrt{}}\right.}{3x^3}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^p/x^4,x]
```

output
```
-1/3*((a + b*x^2 + c*x^4)^p*AppellF1[-3/2, -p, -p, -1/2, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(x^3*((b - Sqrt[b^
2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^p}{x^4}\,dx$$

$$\Big\downarrow \ 1461$$

$$\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p\int \frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^2}{b+\sqrt{b^2-4ac}}+1\right)^p}{x^4}dx$$

$$\Big\downarrow \ 394$$

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p\operatorname{AppellF1}\left(-\frac{3}{2},-p,-p,-\frac{1}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3x^3}$$

input
```
Int[(a + b*x^2 + c*x^4)^p/x^4,x]
```

output
```
-1/3*((a + b*x^2 + c*x^4)^p*AppellF1[-3/2, -p, -p, -1/2, (-2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(x^3*(1 + (2*c*x^2
)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int \frac{(c\,x^4+b\,x^2+a)^p}{x^4}dx$$

input
```
int((c*x^4+b*x^2+a)^p/x^4,x)
```

output
```
int((c*x^4+b*x^2+a)^p/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^4}\,dx=\int \frac{(cx^4+bx^2+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x^4,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^p}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)**p/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^4}\,dx = \int \frac{(cx^4+bx^2+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x^4,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^4}\,dx = \int \frac{(cx^4+bx^2+a)^p}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^p/x^4,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^p}{x^4}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^p}{x^4}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^p/x^4,x)
```

output
```
int((a + b*x^2 + c*x^4)^p/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^p}{x^4}\,dx = \frac{-(c\,x^4+b\,x^2+a)^p+4\left(\int \frac{(c\,x^4+b\,x^2+a)^p}{2cp\,x^6-3c\,x^6+2bp\,x^4-3b\,x^4+2ap\,x^2-3a\,x^2}dx\right)b\,p^2x^3-6\left(\int \frac{(c\,x^4+b\,x^2+a)^p}{2cp\,x^6-3c\,x^6+2bp\,x^4-3b\,x^4+2ap\,x^2 \cdots}\right.}{3 \cdots}$$

input
```
int((c*x^4+b*x^2+a)^p/x^4,x)
```

output
```
( - (a + b*x**2 + c*x**4)**p + 4*int((a + b*x**2 + c*x**4)**p/(2*a*p*x**2
- 3*a*x**2 + 2*b*p*x**4 - 3*b*x**4 + 2*c*p*x**6 - 3*c*x**6),x)*b*p**2*x**3
 - 6*int((a + b*x**2 + c*x**4)**p/(2*a*p*x**2 - 3*a*x**2 + 2*b*p*x**4 - 3*
b*x**4 + 2*c*p*x**6 - 3*c*x**6),x)*b*p*x**3 + 8*int((a + b*x**2 + c*x**4)*
*p/(2*a*p - 3*a + 2*b*p*x**2 - 3*b*x**2 + 2*c*p*x**4 - 3*c*x**4),x)*c*p**2
*x**3 - 12*int((a + b*x**2 + c*x**4)**p/(2*a*p - 3*a + 2*b*p*x**2 - 3*b*x*
*2 + 2*c*p*x**4 - 3*c*x**4),x)*c*p*x**3)/(3*x**3)
```

3.1104 $\int (dx)^m (a+bx^2+cx^4)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 155

$$\int (dx)^m (a+bx^2+cx^4)^p \, dx$$
$$= \frac{(dx)^{1+m}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(\frac{1+m}{2},-p,-p,\frac{3+m}{2},-\frac{2cx^2}{b-\sqrt{b^2-4a}}\right)}{d(1+m)}$$

output
```
(d*x)^(1+m)*(c*x^4+b*x^2+a)^p*AppellF1(1/2+1/2*m,-p,-p,3/2+1/2*m,-2*c*x^2/
(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*x^
2/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15

$$\int (dx)^m (a+bx^2+cx^4)^p \, dx$$
$$= \frac{x(dx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{AppellF1}\left(\frac{1+m}{2},-p,-p,\frac{3+m}{2},-\frac{2cx^2}{b+\sqrt{b^2-4a}}\right)}{1+m}$$

input
```
Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^p,x]
```

output
```
(x*(d*x)^m*(a + b*x^2 + c*x^4)^p*AppellF1[(1 + m)/2, -p, -p, (3 + m)/2, (-
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((1
 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \, dx$$

$$\Big\downarrow \text{1461}$$

$$\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p\int (dx)^m\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2cx^2}{b+\sqrt{b^2-}\right.$$

$$\Big\downarrow \text{394}$$

$$\frac{(dx)^{m+1}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p \operatorname{AppellF1}\left(\frac{m+1}{2},-p,-p,\frac{m+3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{}{b}\right.}{d(m+1)}$$

input
```
Int[(d*x)^m*(a + b*x^2 + c*x^4)^p,x]
```

output
```
((d*x)^(1 + m)*(a + b*x^2 + c*x^4)^p*AppellF1[(1 + m)/2, -p, -p, (3 + m)/2
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
/(d*(1 + m)*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int (dx)^m \left(c\,x^4+b\,x^2+a\right)^p dx$$

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^p,x)
```

output
```
int((d*x)^m*(c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \,dx=\int \left(cx^4+bx^2+a\right)^p (dx)^m \,dx$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(c*x**4+b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \, dx = \int \left(cx^4+bx^2+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Giac [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \, dx = \int \left(cx^4+bx^2+a\right)^p (dx)^m \, dx$$

input
```
integrate((d*x)^m*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \, dx = \int (d\,x)^m \left(c\,x^4+b\,x^2+a\right)^p dx$$

input
```
int((d*x)^m*(a + b*x^2 + c*x^4)^p,x)
```

output
```
int((d*x)^m*(a + b*x^2 + c*x^4)^p, x)
```

Reduce [F]

$$\int (dx)^m \left(a+bx^2+cx^4\right)^p \, dx$$
$$= \frac{d^m \left(x^m (c\,x^4+b\,x^2+a)^p\,x + 2\left(\int \frac{x^m (c\,x^4+b\,x^2+a)^p x^2}{cm\,x^4+4cp\,x^4+c\,x^4+bm\,x^2+4bp\,x^2+b\,x^2+am+4ap+a}dx\right) bmp + 8\left(\int \frac{x}{cm\,x^4+4cp\,x^4+c\,x} \cdots\right.\right.}{\cdots}$$

input
```
int((d*x)^m*(c*x^4+b*x^2+a)^p,x)
```

output
```
(d**m*(x**m*(a + b*x**2 + c*x**4)**p*x + 2*int((x**m*(a + b*x**2 + c*x**4)
**p*x**2)/(a*m + 4*a*p + a + b*m*x**2 + 4*b*p*x**2 + b*x**2 + c*m*x**4 + 4
*c*p*x**4 + c*x**4),x)*b*m*p + 8*int((x**m*(a + b*x**2 + c*x**4)**p*x**2)/
(a*m + 4*a*p + a + b*m*x**2 + 4*b*p*x**2 + b*x**2 + c*m*x**4 + 4*c*p*x**4
+ c*x**4),x)*b*p**2 + 2*int((x**m*(a + b*x**2 + c*x**4)**p*x**2)/(a*m + 4*
a*p + a + b*m*x**2 + 4*b*p*x**2 + b*x**2 + c*m*x**4 + 4*c*p*x**4 + c*x**4)
,x)*b*p + 4*int((x**m*(a + b*x**2 + c*x**4)**p)/(a*m + 4*a*p + a + b*m*x**
2 + 4*b*p*x**2 + b*x**2 + c*m*x**4 + 4*c*p*x**4 + c*x**4),x)*a*m*p + 16*in
t((x**m*(a + b*x**2 + c*x**4)**p)/(a*m + 4*a*p + a + b*m*x**2 + 4*b*p*x**2
 + b*x**2 + c*m*x**4 + 4*c*p*x**4 + c*x**4),x)*a*p**2 + 4*int((x**m*(a + b
*x**2 + c*x**4)**p)/(a*m + 4*a*p + a + b*m*x**2 + 4*b*p*x**2 + b*x**2 + c*
m*x**4 + 4*c*p*x**4 + c*x**4),x)*a*p))/(m + 4*p + 1)
```

3.1105 $\int (ex)^{-7-4p} \left(a+bx^2+cx^4\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 267

$$\int (ex)^{-7-4p} \left(a+bx^2+cx^4\right)^p \, dx = \frac{(ex)^{-4p} \left(-2a(1+p)+b(2+p)x^2\right) \left(a+bx^2+cx^4\right)^{1+p}}{4a^2e^7 \left(3+5p+2p^2\right) x^6} - \frac{\left(-2ac+b^2(2+p)\right) (ex)^{-4p} \left(b+\sqrt{b^2-4ac}+2cx^2\right) \left(\frac{2a+\left(b+\sqrt{b^2-4ac}\right)x^2}{2a+\left(b-\sqrt{b^2-4ac}\right)x^2}\right)^{-p} \left(a+bx^2+cx^4\right)^p \text{Hypergeor}}{4a^2 \left(b+\sqrt{b^2-4ac}\right) e^7 \left(3+8p+4p^2\right) x^2}$$

output
```
1/4*(-2*a*(p+1)+b*(2+p)*x^2)*(c*x^4+b*x^2+a)^(p+1)/a^2/e^7/(2*p^2+5*p+3)/x
^6/((e*x)^(4*p))-1/4*(-2*a*c+b^2*(2+p))*(b+(-4*a*c+b^2)^(1/2)+2*c*x^2)*(c*
x^4+b*x^2+a)^p*hypergeom([-p, -1-2*p],[-2*p],-(-4*a*c+b^2)^(1/2)*x^2/(a+2*
a*c*x^2/(b+(-4*a*c+b^2)^(1/2))))/a^2/(b+(-4*a*c+b^2)^(1/2))/e^7/(4*p^2+8*p
+3)/x^2/((e*x)^(4*p))/(((2*a+(b+(-4*a*c+b^2)^(1/2))*x^2)/(2*a+(b-(-4*a*c+b
^2)^(1/2))*x^2))^p)
```

Mathematica [F]

$$\int (ex)^{-7-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int (ex)^{-7-4p} \left(a+bx^2+cx^4\right)^p \, dx$$

input
```
Integrate[(e*x)^(-7 - 4*p)*(a + b*x^2 + c*x^4)^p,x]
```

output
```
Integrate[(e*x)^(-7 - 4*p)*(a + b*x^2 + c*x^4)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-4p-7} \left(a+bx^2+cx^4\right)^p \, dx$$

$$\Big\downarrow \text{ 1461}$$

$$\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^2+cx^4\right)^p \int (ex)^{-4p-7} \left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^p \left(\frac{2c}{b+\sqrt{b}}\right.$$

$$\Big\downarrow \text{ 1014}$$

$$x^{4p+7}(ex)^{-4p-7} \left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^2+cx^4\right)^p \int x^{-4p-7}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+\right.$$

$$\Big\downarrow \text{ 7299}$$

$$x^{4p+7}(ex)^{-4p-7} \left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^2+cx^4\right)^p \int x^{-4p-7}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+\right.$$

input
```
Int[(e*x)^(-7 - 4*p)*(a + b*x^2 + c*x^4)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1014
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q
_.), x_Symbol] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a +
b*x^n)^p*(c + d*x^n)^q, x], x, v], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && LinearPairQ[u, v, x]
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-7-4p} \left(c\,x^{4}+b\,x^{2}+a\right)^{p} dx$$

input
```
int((e*x)^(-7-4*p)*(c*x^4+b*x^2+a)^p,x)
```

output
```
int((e*x)^(-7-4*p)*(c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int (ex)^{-7-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int \left(cx^4+bx^2+a\right)^p (ex)^{-4\,p-7} \, dx$$

input
```
integrate((e*x)^(-7-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p - 7), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-7-4p} \left(a+bx^2+cx^4\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(-7-4*p)*(c*x**4+b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-7-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int (cx^4+bx^2+a)^p (ex)^{-4p-7} \, dx$$

input
```
integrate((e*x)^(-7-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p - 7), x)
```

Giac [F]

$$\int (ex)^{-7-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int (cx^4+bx^2+a)^p (ex)^{-4p-7} \, dx$$

input
```
integrate((e*x)^(-7-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p - 7), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-7-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int \frac{(c\,x^4+b\,x^2+a)^p}{(e\,x)^{4\,p+7}} \,dx$$

input
```
int((a + b*x^2 + c*x^4)^p/(e*x)^(4*p + 7),x)
```

output
```
int((a + b*x^2 + c*x^4)^p/(e*x)^(4*p + 7), x)
```

Reduce [F]

$$\int (ex)^{-7-4p} \left(a+bx^2+cx^4\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x)^(-7-4*p)*(c*x^4+b*x^2+a)^p,x)
```

output
```
( - 8*(a + b*x**2 + c*x**4)**p*a**3*c*p**2 - 12*(a + b*x**2 + c*x**4)**p*a
**3*c*p - 4*(a + b*x**2 + c*x**4)**p*a**3*c + 4*(a + b*x**2 + c*x**4)**p*a
**2*b**2*p**3 + 10*(a + b*x**2 + c*x**4)**p*a**2*b**2*p**2 + 8*(a + b*x**2
 + c*x**4)**p*a**2*b**2*p + 2*(a + b*x**2 + c*x**4)**p*a**2*b**2 - 4*(a +
b*x**2 + c*x**4)**p*a**2*b*c*p**2*x**2 - 2*(a + b*x**2 + c*x**4)**p*a**2*b
*c*p*x**2 - 8*(a + b*x**2 + c*x**4)**p*a**2*c**2*p**2*x**4 - 8*(a + b*x**2
 + c*x**4)**p*a**2*c**2*p*x**4 + 2*(a + b*x**2 + c*x**4)**p*a*b**3*p**3*x*
*2 + 3*(a + b*x**2 + c*x**4)**p*a*b**3*p**2*x**2 + (a + b*x**2 + c*x**4)**
p*a*b**3*p*x**2 + 4*(a + b*x**2 + c*x**4)**p*a*b**2*c*p**3*x**4 + 10*(a +
b*x**2 + c*x**4)**p*a*b**2*c*p**2*x**4 + 8*(a + b*x**2 + c*x**4)**p*a*b**2
*c*p*x**4 + 4*(a + b*x**2 + c*x**4)**p*a*b*c**2*p*x**6 + 2*(a + b*x**2 + c
*x**4)**p*a*b*c**2*x**6 - (a + b*x**2 + c*x**4)**p*b**4*p**3*x**4 - 3*(a +
 b*x**2 + c*x**4)**p*b**4*p**2*x**4 - 2*(a + b*x**2 + c*x**4)**p*b**4*p*x*
*4 - 128*x**(4*p)*int((a + b*x**2 + c*x**4)**p/(8*x**(4*p)*a**2*c*p**2*x +
 16*x**(4*p)*a**2*c*p*x + 6*x**(4*p)*a**2*c*x - 4*x**(4*p)*a*b**2*p**3*x -
 12*x**(4*p)*a*b**2*p**2*x - 11*x**(4*p)*a*b**2*p*x - 3*x**(4*p)*a*b**2*x
+ 8*x**(4*p)*a*b*c*p**2*x**3 + 16*x**(4*p)*a*b*c*p*x**3 + 6*x**(4*p)*a*b*c
*x**3 + 8*x**(4*p)*a*c**2*p**2*x**5 + 16*x**(4*p)*a*c**2*p*x**5 + 6*x**(4*
p)*a*c**2*x**5 - 4*x**(4*p)*b**3*p**3*x**3 - 12*x**(4*p)*b**3*p**2*x**3 -
11*x**(4*p)*b**3*p*x**3 - 3*x**(4*p)*b**3*x**3 - 4*x**(4*p)*b**2*c*p**3...
```

3.1106 $\int (ex)^{-5-4p} \left(a+bx^2+cx^4\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 269

$$\int (ex)^{-5-4p} \left(a+bx^2+cx^4\right)^p \, dx = -\frac{(ex)^{-4(1+p)} \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right) \left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right) \left(a+bx^2+cx^4\right)^p}{4e(1+p)} + \frac{b(ex)^{-2-4p} \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right) \left(\frac{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\right)^{-p} \left(a+bx^2+cx^4\right)^p \text{Hypergeometric2F1}\left(-1-2p,-p,-2\right.}{4ae^3(1+2p)}$$

output
```
-1/4*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))
*(c*x^4+b*x^2+a)^p/e/(p+1)/((e*x)^(4*p+4))+1/4*b*(e*x)^(-2-4*p)*(1+2*c*x^2
/(b-(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^p*hypergeom([-p, -1-2*p],[-2*p],(
-4*a*c+b^2)^(1/2)*x^2/a/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))))/a/e^3/(1+2*p)/
(((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))))^p
)
```

Mathematica [F]

$$\int (ex)^{-5-4p}\left(a+bx^2+cx^4\right)^p\,dx=\int (ex)^{-5-4p}\left(a+bx^2+cx^4\right)^p\,dx$$

input
```
Integrate[(e*x)^(-5 - 4*p)*(a + b*x^2 + c*x^4)^p,x]
```

output
```
Integrate[(e*x)^(-5 - 4*p)*(a + b*x^2 + c*x^4)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-4p-5}\left(a+bx^2+cx^4\right)^p\,dx$$

$$\Big\downarrow \text{1461}$$

$$\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p\int (ex)^{-4p-5}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^p\left(\frac{2c}{b+\sqrt{b}}\right.$$

$$\Big\downarrow \text{1014}$$

$$x^{4p+5}(ex)^{-4p-5}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p\int x^{-4p-5}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+\right.$$

$$\Big\downarrow \text{7299}$$

$$x^{4p+5}(ex)^{-4p-5}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p\int x^{-4p-5}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+\right.$$

input
```
Int[(e*x)^(-5 - 4*p)*(a + b*x^2 + c*x^4)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1014
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q
_.), x_Symbol] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a +
b*x^n)^p*(c + d*x^n)^q, x], x, v], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && LinearPairQ[u, v, x]
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (ex)^{-5-4p} \left(c\,x^4+b\,x^2+a\right)^p dx$$

input
```
int((e*x)^(-5-4*p)*(c*x^4+b*x^2+a)^p,x)
```

output
```
int((e*x)^(-5-4*p)*(c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int (ex)^{-5-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int \left(cx^4+bx^2+a\right)^p (ex)^{-4\,p-5} \, dx$$

input
```
integrate((e*x)^(-5-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p - 5), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-5-4p} \left(a+bx^2+cx^4\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(-5-4*p)*(c*x**4+b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-5-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int (cx^4+bx^2+a)^p (ex)^{-4p-5} \, dx$$

input
```
integrate((e*x)^(-5-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p - 5), x)
```

Giac [F]

$$\int (ex)^{-5-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int (cx^4+bx^2+a)^p (ex)^{-4p-5} \, dx$$

input
```
integrate((e*x)^(-5-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-5-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int \frac{\left(c\,x^4+b\,x^2+a\right)^p}{\left(e\,x\right)^{4\,p+5}} \, dx$$

input
```
int((a + b*x^2 + c*x^4)^p/(e*x)^(4*p + 5),x)
```

output
```
int((a + b*x^2 + c*x^4)^p/(e*x)^(4*p + 5), x)
```

Reduce [F]

$$\int (ex)^{-5-4p} \left(a+bx^2+cx^4\right)^p \, dx$$
$$= \frac{-2(c\,x^4+b\,x^2+a)^p\,ap-(c\,x^4+b\,x^2+a)^p\,a-(c\,x^4+b\,x^2+a)^p\,bp\,x^2+(c\,x^4+b\,x^2+a)^p\,c\,x^4+16x^{4p}}{}$$

input
```
int((e*x)^(-5-4*p)*(c*x^4+b*x^2+a)^p,x)
```

output
```
( - 2*(a + b*x**2 + c*x**4)**p*a*p - (a + b*x**2 + c*x**4)**p*a - (a + b*x
**2 + c*x**4)**p*b*p*x**2 + (a + b*x**2 + c*x**4)**p*c*x**4 + 16*x**(4*p)*
int((a + b*x**2 + c*x**4)**p/(2*x**(4*p)*a*p*x + x**(4*p)*a*x + 2*x**(4*p)
*b*p*x**3 + x**(4*p)*b*x**3 + 2*x**(4*p)*c*p*x**5 + x**(4*p)*c*x**5),x)*a*
c*p**3*x**4 + 24*x**(4*p)*int((a + b*x**2 + c*x**4)**p/(2*x**(4*p)*a*p*x +
 x**(4*p)*a*x + 2*x**(4*p)*b*p*x**3 + x**(4*p)*b*x**3 + 2*x**(4*p)*c*p*x**
5 + x**(4*p)*c*x**5),x)*a*c*p**2*x**4 + 8*x**(4*p)*int((a + b*x**2 + c*x**
4)**p/(2*x**(4*p)*a*p*x + x**(4*p)*a*x + 2*x**(4*p)*b*p*x**3 + x**(4*p)*b*
x**3 + 2*x**(4*p)*c*p*x**5 + x**(4*p)*c*x**5),x)*a*c*p*x**4 - 4*x**(4*p)*i
nt((a + b*x**2 + c*x**4)**p/(2*x**(4*p)*a*p*x + x**(4*p)*a*x + 2*x**(4*p)*
b*p*x**3 + x**(4*p)*b*x**3 + 2*x**(4*p)*c*p*x**5 + x**(4*p)*c*x**5),x)*b**
2*p**3*x**4 - 6*x**(4*p)*int((a + b*x**2 + c*x**4)**p/(2*x**(4*p)*a*p*x +
x**(4*p)*a*x + 2*x**(4*p)*b*p*x**3 + x**(4*p)*b*x**3 + 2*x**(4*p)*c*p*x**5
 + x**(4*p)*c*x**5),x)*b**2*p**2*x**4 - 2*x**(4*p)*int((a + b*x**2 + c*x**
4)**p/(2*x**(4*p)*a*p*x + x**(4*p)*a*x + 2*x**(4*p)*b*p*x**3 + x**(4*p)*b*
x**3 + 2*x**(4*p)*c*p*x**5 + x**(4*p)*c*x**5),x)*b**2*p*x**4)/(4*x**(4*p)*
e**(4*p)*a*e**5*x**4*(2*p**2 + 3*p + 1))
```

3.1107 $\int (ex)^{-3-4p} \left(a+bx^2+cx^4\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 179

$$\int (ex)^{-3-4p} \left(a+bx^2+cx^4\right)^p \, dx = -\frac{(ex)^{-2-4p} \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right) \left(\frac{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\right)^{-p} \left(a+bx^2+cx^4\right)^p \text{Hypergeometric2F1}\left(-1-2p,-p,-2p\right.}{2e(1+2p)}$$

output

```
-1/2*(e*x)^(-2-4*p)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^p*h
ypergeom([-p, -1-2*p],[-2*p],(-4*a*c+b^2)^(1/2)*x^2/a/(1+2*c*x^2/(b-(-4*a*
c+b^2)^(1/2))))/e/(1+2*p)/(((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/(1+2*c*x^2/
(b-(-4*a*c+b^2)^(1/2))))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int (ex)^{-3-4p} \left(a+bx^2+cx^4\right)^p \, dx = \frac{(ex)^{-4p} \left(b-\sqrt{b^2-4ac}+2cx^2\right) \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^p \left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p} \left(a+bx^2+cx^4\right)^p \text{Hypergeometric2F}}{2\left(-b+\sqrt{b^2-4ac}\right) e^3 (1+2p) x^2}$$

input
```
Integrate[(e*x)^(-3 - 4*p)*(a + b*x^2 + c*x^4)^p,x]
```

output
```
((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))
^p*(a + b*x^2 + c*x^4)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[b^2
 - 4*a*c]*x^2)/(2*a + (b + Sqrt[b^2 - 4*a*c])*x^2)])/(2*(-b + Sqrt[b^2 - 4
*a*c])*e^3*(1 + 2*p)*x^2*(e*x)^(4*p)*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
))^p)
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1461, 394\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-4p-3} \left(a+bx^2+cx^4\right)^p \, dx$$

$$\Big\downarrow \text{1461}$$

$$\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^2+cx^4\right)^p \int (ex)^{-4p-3} \left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^p \left(\frac{2c}{b+\sqrt{b}}\right.$$

$$\Big\downarrow \text{394}$$

$$-\frac{(ex)^{-4p-2} \left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{p+1} \left(a+bx^2+cx^4\right)^p \text{Hypergeometric2F1}\left(-2p-1,-p,-2p,-\right.}{2e(2p+1)}$$

input
```
Int[(e*x)^(-3 - 4*p)*(a + b*x^2 + c*x^4)^p,x]
```

output
```
-1/2*((e*x)^(-2 - 4*p)*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^(1 + p)*(a
+ b*x^2 + c*x^4)^p*Hypergeometric2F1[-1 - 2*p, -p, -2*p, (-2*((c*x^2)/(b -
 Sqrt[b^2 - 4*a*c]) - (c*x^2)/(b + Sqrt[b^2 - 4*a*c])))/(1 + (2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]))])/(e*(1 + 2*p)*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]
))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int (ex)^{-3-4p} \left(c\,x^{4}+b\,x^{2}+a\right)^{p} dx$$

input
```
int((e*x)^(-3-4*p)*(c*x^4+b*x^2+a)^p,x)
```

output
```
int((e*x)^(-3-4*p)*(c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int (ex)^{-3-4p} \left(a + bx^2 + cx^4\right)^p \, dx = \int (cx^4 + bx^2 + a)^p (ex)^{-4p-3} \, dx$$

input
```
integrate((e*x)^(-3-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p - 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{-3-4p} \left(a + bx^2 + cx^4\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(-3-4*p)*(c*x**4+b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{-3-4p} \left(a + bx^2 + cx^4\right)^p \, dx = \int (cx^4 + bx^2 + a)^p (ex)^{-4p-3} \, dx$$

input
```
integrate((e*x)^(-3-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p - 3), x)
```

Giac [F]

$$\int (ex)^{-3-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int (cx^4+bx^2+a)^p (ex)^{-4p-3} \, dx$$

input
```
integrate((e*x)^(-3-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-3-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int \frac{(c\,x^4+b\,x^2+a)^p}{(e\,x)^{4\,p+3}} \, dx$$

input
```
int((a + b*x^2 + c*x^4)^p/(e*x)^(4*p + 3),x)
```

output
```
int((a + b*x^2 + c*x^4)^p/(e*x)^(4*p + 3), x)
```

Reduce [F]

$$\int (ex)^{-3-4p} \left(a+bx^2+cx^4\right)^p \, dx = \frac{-(c\,x^4+b\,x^2+a)^p\,b-2(c\,x^4+b\,x^2+a)^p\,c\,x^2-16x^{4p}\left(\int \frac{(c\,x^4+b\,x^2+a)^p}{2x^{4p}apx+x^{4p}ax+2x^{4p}bp\,x^3+x^{4p}b\,x^3+2x^{4p}cp\,x^5+x^{4p}c\,x^5}dx\right.}{}$$

input
```
int((e*x)^(-3-4*p)*(c*x^4+b*x^2+a)^p,x)
```

output
```
( - (a + b*x**2 + c*x**4)**p*b - 2*(a + b*x**2 + c*x**4)**p*c*x**2 - 16*x*
*(4*p)*int((a + b*x**2 + c*x**4)**p/(2*x**(4*p)*a*p*x + x**(4*p)*a*x + 2*x
**(4*p)*b*p*x**3 + x**(4*p)*b*x**3 + 2*x**(4*p)*c*p*x**5 + x**(4*p)*c*x**5
),x)*a*c*p**2*x**2 - 8*x**(4*p)*int((a + b*x**2 + c*x**4)**p/(2*x**(4*p)*a
*p*x + x**(4*p)*a*x + 2*x**(4*p)*b*p*x**3 + x**(4*p)*b*x**3 + 2*x**(4*p)*c
*p*x**5 + x**(4*p)*c*x**5),x)*a*c*p*x**2 + 4*x**(4*p)*int((a + b*x**2 + c*
x**4)**p/(2*x**(4*p)*a*p*x + x**(4*p)*a*x + 2*x**(4*p)*b*p*x**3 + x**(4*p)
*b*x**3 + 2*x**(4*p)*c*p*x**5 + x**(4*p)*c*x**5),x)*b**2*p**2*x**2 + 2*x**
(4*p)*int((a + b*x**2 + c*x**4)**p/(2*x**(4*p)*a*p*x + x**(4*p)*a*x + 2*x*
*(4*p)*b*p*x**3 + x**(4*p)*b*x**3 + 2*x**(4*p)*c*p*x**5 + x**(4*p)*c*x**5)
,x)*b**2*p*x**2)/(2*x**(4*p)*e**(4*p)*b*e**3*x**2*(2*p + 1))
```

3.1108 $\int (ex)^{-1-4p} \left(a+bx^2+cx^4\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 150

$$\int (ex)^{-1-4p} \left(a+bx^2+cx^4\right)^p \, dx = -\frac{(ex)^{-4p} \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p} \left(a+bx^2+cx^4\right)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{2c}{b-\sqrt{b}}\right.}{4ep}$$

output
```
-1/4*(c*x^4+b*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,-2*c*x^2/(b-(-4*a*c+b^2)^
(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/e/p/((e*x)^(4*p))/((1+2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19

$$\int (ex)^{-1-4p} \left(a+bx^2+cx^4\right)^p \, dx = -\frac{(ex)^{-4p} \left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p} \left(a+bx^2+cx^4\right)^p \operatorname{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{}{b+}\right.}{4ep}$$

input
```
Integrate[(e*x)^(-1 - 4*p)*(a + b*x^2 + c*x^4)^p,x]
```

output
```
-1/4*((a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(e*p*(e*x)^(4*p
)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{-4p-1} \left(a+bx^2+cx^4\right)^p \, dx$$

$$\Big\downarrow \text{1461}$$

$$\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^2+cx^4\right)^p \int (ex)^{-4p-1} \left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^p \left(\frac{2c}{b+\sqrt{b}}\right.$$

$$\Big\downarrow \text{394}$$

$$-\frac{(ex)^{-4p} \left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^2+cx^4\right)^p \text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\right.}{4ep}$$

input
```
Int[(e*x)^(-1 - 4*p)*(a + b*x^2 + c*x^4)^p,x]
```

output
```
-1/4*((a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (-2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(e*p*(e*x)^(4*p
)*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int (ex)^{-1-4p} \left(c\,x^{4}+b\,x^{2}+a\right)^{p} dx$$

input
```
int((e*x)^(-1-4*p)*(c*x^4+b*x^2+a)^p,x)
```

output
```
int((e*x)^(-1-4*p)*(c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int (ex)^{-1-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int \left(cx^4+bx^2+a\right)^p (ex)^{-4\,p-1} \, dx$$

input
```
integrate((e*x)^(-1-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p - 1), x)
```

Sympy [F]

$$\int (ex)^{-1-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int (ex)^{-4p-1} \left(a+bx^2+cx^4\right)^p \, dx$$

input
```
integrate((e*x)**(-1-4*p)*(c*x**4+b*x**2+a)**p,x)
```

output
```
Integral((e*x)**(-4*p - 1)*(a + b*x**2 + c*x**4)**p, x)
```

Maxima [F]

$$\int (ex)^{-1-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int (cx^4+bx^2+a)^p (ex)^{-4p-1} \, dx$$

input
```
integrate((e*x)^(-1-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p - 1), x)
```

Giac [F]

$$\int (ex)^{-1-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int (cx^4+bx^2+a)^p (ex)^{-4p-1} \, dx$$

input
```
integrate((e*x)^(-1-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int \frac{\left(c\,x^4+b\,x^2+a\right)^p}{\left(e\,x\right)^{4\,p+1}} \, dx$$

input
```
int((a + b*x^2 + c*x^4)^p/(e*x)^(4*p + 1),x)
```

output
```
int((a + b*x^2 + c*x^4)^p/(e*x)^(4*p + 1), x)
```

Reduce [F]

$$\int (ex)^{-1-4p} \left(a+bx^2+cx^4\right)^p \, dx = \frac{\int \frac{(c\,x^4+b\,x^2+a)^p}{x^{4p}x}dx}{e^{4p}e}$$

input
```
int((e*x)^(-1-4*p)*(c*x^4+b*x^2+a)^p,x)
```

output
```
int((a + b*x**2 + c*x**4)**p/(x**(4*p)*x),x)/(e**(4*p)*e)
```

3.1109 $\int (ex)^{1-4p} \left(a+bx^2+cx^4\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\int (ex)^{1-4p} \left(a+bx^2+cx^4\right)^p \, dx$$
$$= \frac{(ex)^{2-4p}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{}{b-}\right.}{2e(1-2p)}$$

output
```
1/2*(e*x)^(2-4*p)*(c*x^4+b*x^2+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,-2*c*x^2/(b
-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/e/(1-2*p)/((1+2*c*x^
2/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.14

$$\int (ex)^{1-4p} \left(a+bx^2+cx^4\right)^p \, dx$$
$$= \frac{ex^2(ex)^{-4p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p \operatorname{AppellF1}\left(1-2p,-p,-p,2-2p,-\right.}{2-4p}$$

input
```
Integrate[(e*x)^(1 - 4*p)*(a + b*x^2 + c*x^4)^p,x]
```

output
```
(e*x^2*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((2 - 4*p)*
(e*x)^(4*p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*
((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1461, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{1-4p} \left(a+bx^2+cx^4\right)^p \, dx$$

$$\Big\downarrow \text{1461}$$

$$\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^2+cx^4\right)^p \int (ex)^{1-4p} \left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^p \left(\frac{2cx^2}{b+\sqrt{b^2}}\right.$$

$$\Big\downarrow \text{394}$$

$$\frac{(ex)^{2-4p} \left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^2+cx^4\right)^p \text{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{2cx^2}{b-\sqrt{b^2-4}}\right.}{2e(1-2p)}$$

input
```
Int[(e*x)^(1 - 4*p)*(a + b*x^2 + c*x^4)^p,x]
```

output
```
((e*x)^(2 - 4*p)*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p)
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
/(2*e*(1 - 2*p)*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c]))^p)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1461
```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b + R
t[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^F
racPart[p]))   Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Maple [F]

$$\int (ex)^{1-4p} \left(c\,x^{4}+b\,x^{2}+a\right)^{p} dx$$

input
```
int((e*x)^(1-4*p)*(c*x^4+b*x^2+a)^p,x)
```

output
```
int((e*x)^(1-4*p)*(c*x^4+b*x^2+a)^p,x)
```

Fricas [F]

$$\int (ex)^{1-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int \left(cx^4+bx^2+a\right)^p (ex)^{-4\,p+1} \, dx$$

input
```
integrate((e*x)^(1-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{1-4p} \left(a+bx^2+cx^4\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(1-4*p)*(c*x**4+b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{1-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int \left(cx^4+bx^2+a\right)^p (ex)^{-4p+1} \, dx$$

input
```
integrate((e*x)^(1-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p + 1), x)
```

Giac [F]

$$\int (ex)^{1-4p} \left(a+bx^2+cx^4\right)^p \, dx = \int \left(cx^4+bx^2+a\right)^p (ex)^{-4p+1} \, dx$$

input
```
integrate((e*x)^(1-4*p)*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^p*(e*x)^(-4*p + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{1-4p}\left(a+bx^2+cx^4\right)^p\,dx=\int (e\,x)^{1-4\,p}\,\left(c\,x^4+b\,x^2+a\right)^p\,dx$$

input
```
int((e*x)^(1 - 4*p)*(a + b*x^2 + c*x^4)^p,x)
```

output
```
int((e*x)^(1 - 4*p)*(a + b*x^2 + c*x^4)^p, x)
```

Reduce [F]

$$\int (ex)^{1-4p}\left(a+bx^2+cx^4\right)^p\,dx$$
$$=\frac{e\left(-2\left(c\,x^4+b\,x^2+a\right)^p a+\left(c\,x^4+b\,x^2+a\right)^p b\,x^2-8x^{4p}\left(\int \frac{\left(c\,x^4+b\,x^2+a\right)^p}{x^{4p}ax+x^{4p}b\,x^3+x^{4p}c\,x^5}dx\right)a^2p+2x^{4p}\left(\int \cdots\right.\right.}{2x^{4p}e^{4p}b}$$

input
```
int((e*x)^(1-4*p)*(c*x^4+b*x^2+a)^p,x)
```

output
```
(e*( - 2*(a + b*x**2 + c*x**4)**p*a + (a + b*x**2 + c*x**4)**p*b*x**2 - 8*
x**(4*p)*int((a + b*x**2 + c*x**4)**p/(x**(4*p)*a*x + x**(4*p)*b*x**3 + x*
*(4*p)*c*x**5),x)*a**2*p + 2*x**(4*p)*int(((a + b*x**2 + c*x**4)**p*x**3)/
(x**(4*p)*a + x**(4*p)*b*x**2 + x**(4*p)*c*x**4),x)*b**2*p))/(2*x**(4*p)*e
**(4*p)*b)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file